The Strong CP Problem and Its Implications

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Axion as Solution of Strong CP Problem

Axion-like particles arise in various contexts; today focus on axion as solution of strong CP Problem. This allows us to narrow somewhat the possible theoretical issues, phenomenologies and cosmologies.

Outline

1. The Strong CP Problem (briefly)
2. Three solutions of the Strong CP Problem: An Assessment
3. Theoretical issues with the Peccei-Quinn symmetry.
4. Possible values of the Peccei-Quinn scale and some corresponding theoretical issues.

Strong CP Problem

As all of us here are well aware, we can add to the QCD lagrangian

$$\mathcal{L}_\theta = \frac{\theta}{16\pi^2} F\tilde{F}. \quad (1)$$

This term violates *parity*, and thus CP.

While $F\tilde{F}$ is a total derivative,

$$F\tilde{F} = \partial_\mu K^\mu; \quad (2)$$

K^μ is not gauge invariant. If there are important configurations, say, in the path integral, for which the A^3 term falls off as $1/r^3$, then we can't drop the surface term. Instantons provide examples of such contributions.

If the absence of a ninth light Goldstone is accounted for by the anomaly, then $\theta F\tilde{F}$ has real physical effects. Indeed, from $SU(3) \times SU(3)$ current algebra, i.e. current algebra absent the ninth boson, one can compute the electric dipole moment of the neutron.

The neutron EDM calculation constraints θ to be less than 10^{-9}, and this raises a puzzle: Why such a small dimensionless number?

$\theta \to 0$: strong interactions preserve CP. If not for the fact that the rest of the SM violates CP, would be *natural*.

Among naturalness problems, the strong CP problem is special in that it is of almost no consequence. If the cosmological constant was a few orders of magnitude larger than observed, the universe would be dramatically different. The same is true for the value of the weak scale and of the light quark and lepton masses. But if θ were, say, 10^{-3}, nuclear physics would hardly be different than we observe, since effects of θ are shielded by small quark masses. So it is hard to imagine an anthropic solution.

Possible Resolutions

1. $m_u = 0$ If true, $u \to e^{-i\frac{\theta}{2}\gamma_5} u$ eliminates θ from the lagrangian. An *effective* m_u might be generated from non-perturbative effects in the theory (Georgi, McArthur; Kaplan, Manohar) Could result as an accident of discrete flavor symmetries (Banks, Nir, Seiberg), or a result of “anomalous" discrete symmetries as in string theory (M.D.)
2. CP exact microscopically, $\theta = 0$; spontaneous breaking gives the CKM phase but leads, under suitable conditions, to small effective θ (Nelson, Barr). In critical string theories, CP is an exact (gauge) symmetry, spontaneously broken at generic points in typical moduli spaces. A plausible framework.
3. A new, light particle called the axion dynamically cancels off θ.

Problems with each of these solutions:

1. $m_u = 0$. Lattice computations seem to rule out (the required non-perturbative effects do not seem to be large enough). Large N arguments also rule out (M.D., Ben Lehman)
2. Spontaneous CP: generically, a large θ is generated once CP is spontaneously broken. Model building acrobatics required to avoid. What would single out such theories?
3. Axions: we will shortly turn to promise and limitations.

Summary of lattice results for light quark masses

Current results from lattice simulations (summarized by the FLAG working group) are inconsistent with $m_u = 0$.

$$m_u = 2.16\ (9)(7)\mathrm{MeV} \quad m_d = 4.68\ (14)(7)\mathrm{MeV} \tag{3}$$

$$m_s = 93.5(2.5)\mathrm{MeV}$$

Numbers are in $\overline{MS}$ scheme at 2 GeV.

So m_u is many standard deviations from zero. Large N also rules out Probably end of story, but some proposals for dedicated tests (Kitano), calibrations (Dine, Draper, Festuccia).

Simple realization of the NB structure and some challenges

Complex scalars η_i with complex (CP-violating) vev's.
Additional vectorlike quark with charge $1/3$.

$$\mathcal{L} = \mu \bar{q} q + \lambda_{if} \eta_i \bar{d}_f q + y_{fg} Q_f \bar{d}_g \phi \qquad (4)$$

where ϕ is Higgs; y, λ, μ real.

$$M = \begin{pmatrix} \mu & B \\ 0 & m_d \end{pmatrix} \qquad (5)$$

$B_f = \lambda_{if} \eta_i$ is complex. M has real determinant.

Requirements for a successful NB Solution

1. Symmetries: It is important that η_i not couple to $\bar{q}q$, for example. So, e.g., η's complex, subject to a Z_N symmetry.
2. Coincidences of scale: if only one field η, CKM angle vanishes (can make *d* quark mass matrix real by an overall phase redefinition). Need at least two, and their vev's (times suitable couplings) have to be quite close:

$$\delta_{CKM} \propto \frac{B_{small}}{B_{large}} \tag{6}$$

3. Similarly, μ (which might represent vev of another field) can not be much larger than η_i, and if much smaller the Yukawa's and *B*'s have to have special features.

All of this is to say that the NB solution is not particularly generic and requires model building acrobatics.

The Peccei-Quinn Symmetry

In a somewhat streamlined language, the Peccei-Quinn proposal was to replace θ by a dynamical field: $\theta \rightarrow \frac{a(x)}{f_a}$

It is assumed that $a \rightarrow a + \omega f_a$ is a good symmetry of the theory, *violated only by effects of QCD*. Without QCD, θ can take any value.

In QCD *by itself*, the energy is necessarily stationary when

$$\theta_{eff} = \langle \frac{a}{f_a} \rangle = 0. \tag{7}$$

This is simply because *CP* is a good symmetry of QCD if $\theta = 0$, so the vacuum energy (potential) must be an odd function of θ.

One can do better, calculating, again using what we know about chiral symmetry in QCD, the axion potential:

$$V(a) = m_\pi^2 f_\pi^2 \frac{\sqrt{m_u m_d}}{m_u + m_d} \frac{a^2}{2 f_a^2} \tag{8}$$

This gives, for the axion mass:

$$m_a = 0.6 \text{ meV} \left(\frac{10^{10} \text{ GeV}}{f_a} \right). \tag{9}$$

Peccei and Quinn actually constructed a model for this phenomenon, which was a modest extension of the Standard Model with an extra Higgs doublet. They didn't phrase the problem in quite the way I did above, and didn't appreciate that their model had a light, pseudoscalar particle, a. This was recognized by Weinberg and Wilczek, who calculated its mass and the properties of its interactions. It quickly become clear that the original axion idea was not experimentally viable.

But allowing for a larger decay constant – and a lighter axion – evaded accelerator constraints. Avoiding astrophysical problems placed a lower limit on the decay constant of order 10^{10} GeV. Early examples: KSVZ, DFSS, but phenomenon is generic.

Axion *Quality*

A theoretical question: Why are there axions at all? More precisely, why should there be a Peccei-Quinn symmetry, and how good a symmetry does this have to be?

General belief (supported by studies of string theory): *a theory of quantum gravity does not possess (exact) global symmetries.*

Then hopeless? No: symmetry might be an accidental consequence of other symmetries.

Example: discrete symmetries.

$$\phi \rightarrow \phi e^{\frac{2\pi i}{N}}. \tag{10}$$

So leading symmetry breaking terms in potential might take the form:

$$\mathcal{L}_{symm-breaking} = \frac{\phi^N}{M_p^{N-4}} \tag{11}$$

If N is large, these terms would seem very small. But they have to be *extremely* small to insure the smallness of θ. One needs, e.g., the linear term in the a potential

$$V = \frac{1}{2} m_a^2 \, a^2 + \Gamma a + \ldots \tag{12}$$

to be such that

$$\frac{\Gamma}{m_a^2} < 10^{-10} f_a \tag{13}$$

This translates into a requirement that $N > 12$, if $f_a = 10^{11}$; even larger for larger f_a

Why should this be?

Axions in String Theory

So axions, from the perspective of effective field theory, are surprising. It has long been known, however that axions are common in string theory, indeed axion-like objects seem ubiquitous. The corresponding PQ symmetries are typically good to all orders of perturbation theory, so if the couplings are weak, symmetry violating effects should be exponentially small.

Examples:

1. Heterotic string contains an axion (always) which couples universally to all of the gauge groups.
2. In string theories, antisymmetric tensor fields in higher dimensions become pseudoscalars in four dimensions with axion type couplings.
3. All of these fields exhibit approximate, continuous shift symmetries, $a \to a + \omega f_a$. They typically exhibit *exact* discrete shift symmetries, $a \to a + 2\pi f_a$. The breaking of the continuous symmetries is suppressed, at weak coupling, by $e^{-2\pi/\alpha}$.

Might expect large axion decay constants ($f_a \sim 10^{16}$ GeV?). But we have already said that such limits might well not hold.

Critiques of the Standard Computation of the Axion Dark Matter Density

In recent years, a number of questions have been raised about the reliability of the standard computation of the axion dark matter density. I don't have time to review these thoroughly here, but they will be subjects of discussion at this workshop. Here I list a few and express my own views.

- If the PQ transition occurs after inflation, axion cosmic strings have an infrared divergent tension and carry a great deal of energy. It has been argued that this leads to an enhanced axion dark matter density, altering the prediction of f_a with consequences for experiments like ADMX.

  My Claim: The infrared divergence implies that the strings cannot be considered in isolation but one needs both strings and low momentum axions in the effective theory. The extra energy is dumped in high momentum axions, which don't contribute to the dark matter density.

- If the PQ transition occurs after inflation, there are domain walls. It has been argued that these lead to an enhanced axion dark matter density; there is a large parameter associated with this i.r. divergence..

  My Claim: Provided that there are symmetry violating effects which eliminate the domain walls before they dominate the energy density of the universe, domains of higher energy collapse. Most of their energy is converted to kinetic energy of the domain walls, which is in turn converted, at the final stages of collapse, to extremely relativistic axions, which again don't contribute significantly to the dark energy budget

Unconventional cosmologies

It has long been recognized that the underlying cosmological assumptions of the standard calculation may not hold, and that there is good theoretical motivation to consider lighter axions with larger decay constants. If there is an underlying supersymmetry, for example, the *saxion*, the scalar partner of the axion, leads to severe problems with nucleosynthesis, unless there is significant entropy production at a relatively late stage of evolution. In such a picture, the universe was likely never much hotter than 10 MeV.

String theory as a setting for the PQ Solution

So string theory might be a setting for the axion. If there is an exponentially small parameter (e.g. accounting for hierarchies) this parameter might explain why the Peccei-Quinn symmetry is of the necessary quality.

Suggests high f_a, modified cosmology.

String scale axions

So axions are quite natural in string theory, and large decay constants might be consistent with cosmology. Detection clearly challenging. Recently pursued by P.Graham, S. Rajendran, and others.

Outlook

Axions are a well-motivated dark matter candidate.

The most straightforward approach suggests they should be detectable in cavity experiments.

Theoretical arguments, however, suggest that in a more complete cosmology, axions might be lighter.

In either case, their detection and study would be an extraordinary development.