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di Fisica Nucleare

MW determination at hadron colliders: QCD uncertainties

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Theory Challenges in the Precision Era of the Large Hadron Collider
Galileo Galilei Institute, August 28th 2023

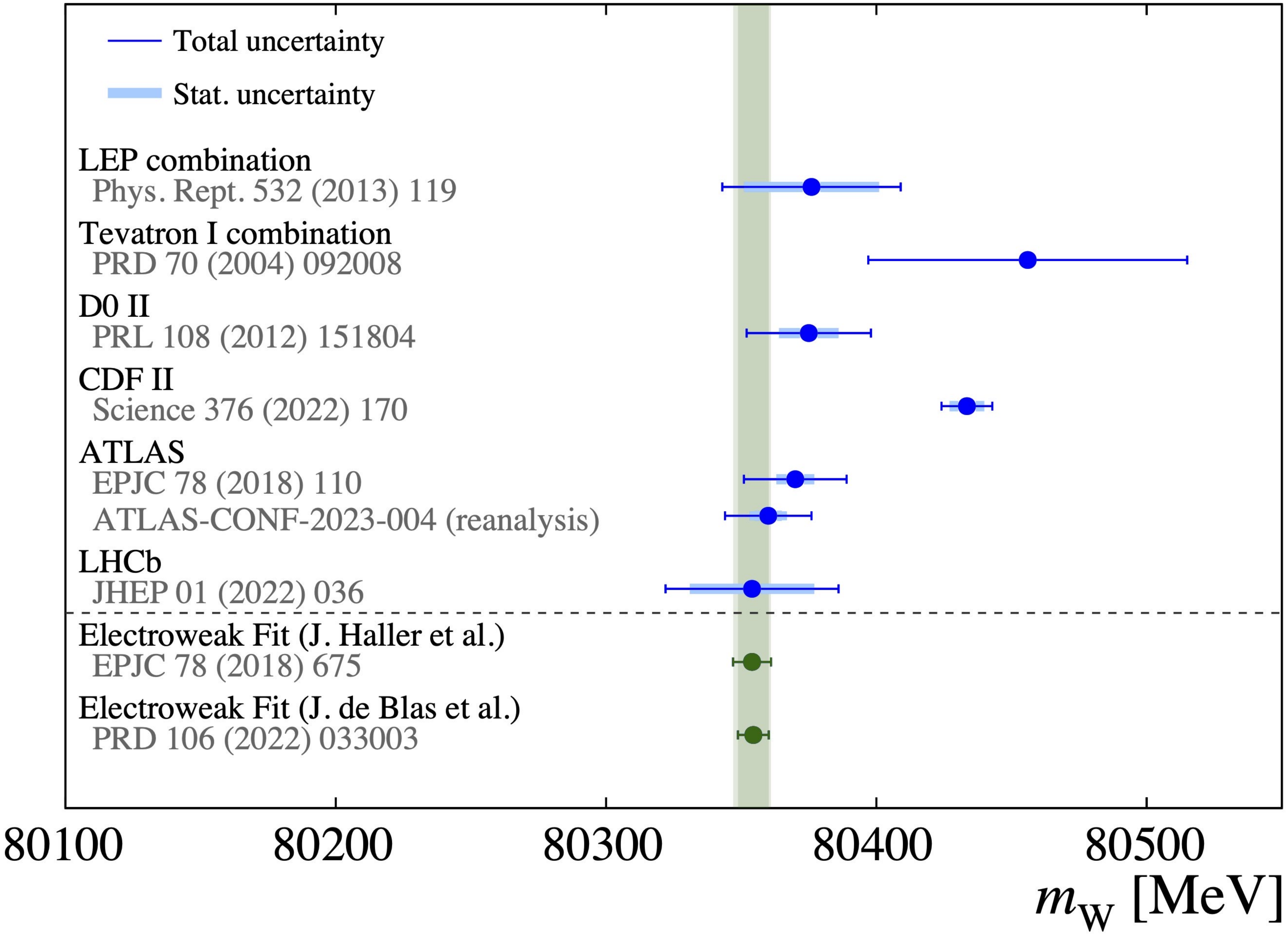
references: L.Rottoli, P.Torrielli, AV, arXiv:2301.04059
LHC-TeV MW combination WG, arXiv:2308.09417

Outline of the talk

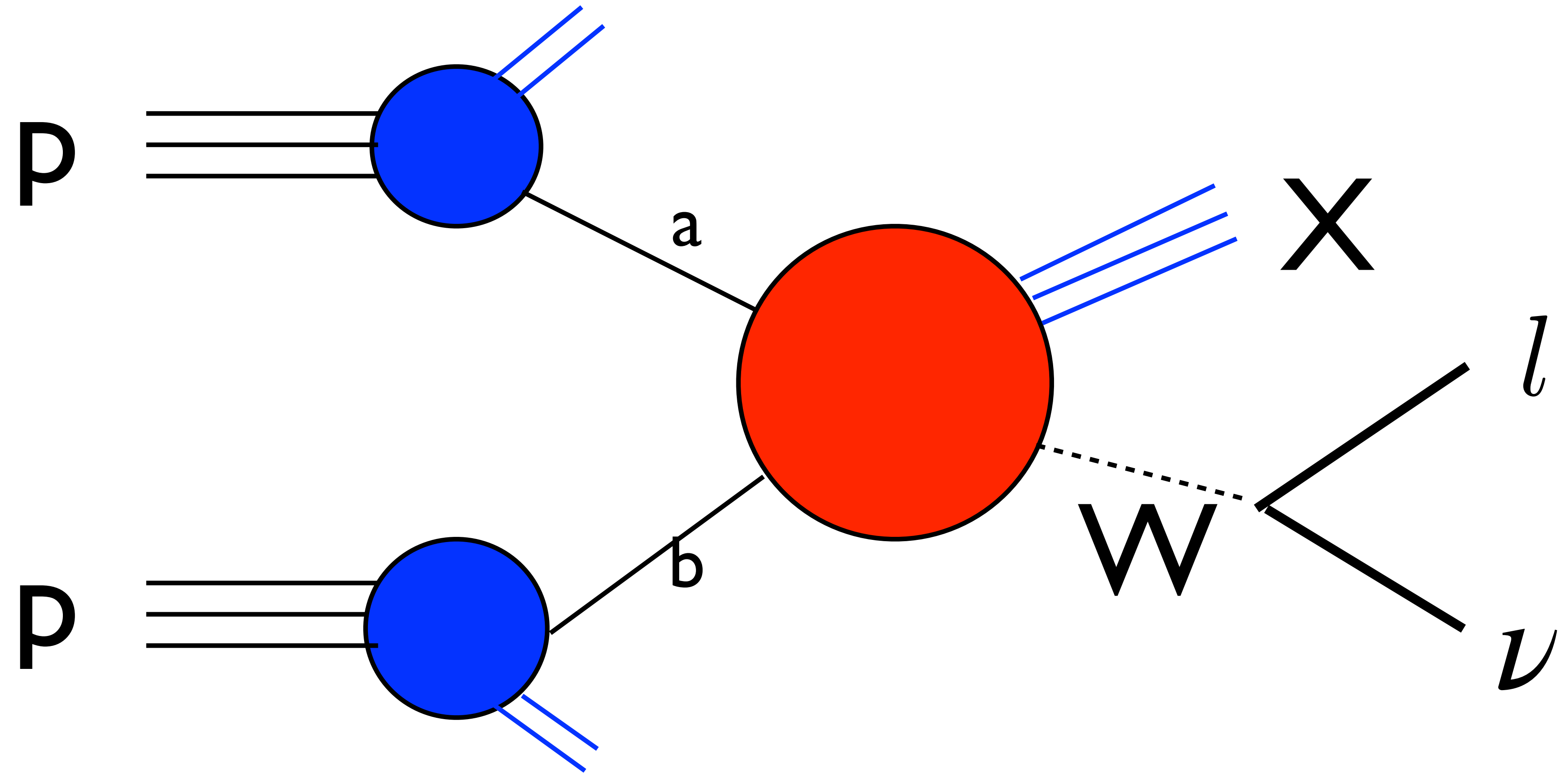
- The modelling of the QCD effects and the difficult estimate of the associated uncertainties

- Proposal of a new observable, suitable for a transparent discussion of the uncertainties on m_W

- Issues in the combination of different experimental results for m_W



m_W determination at hadron colliders



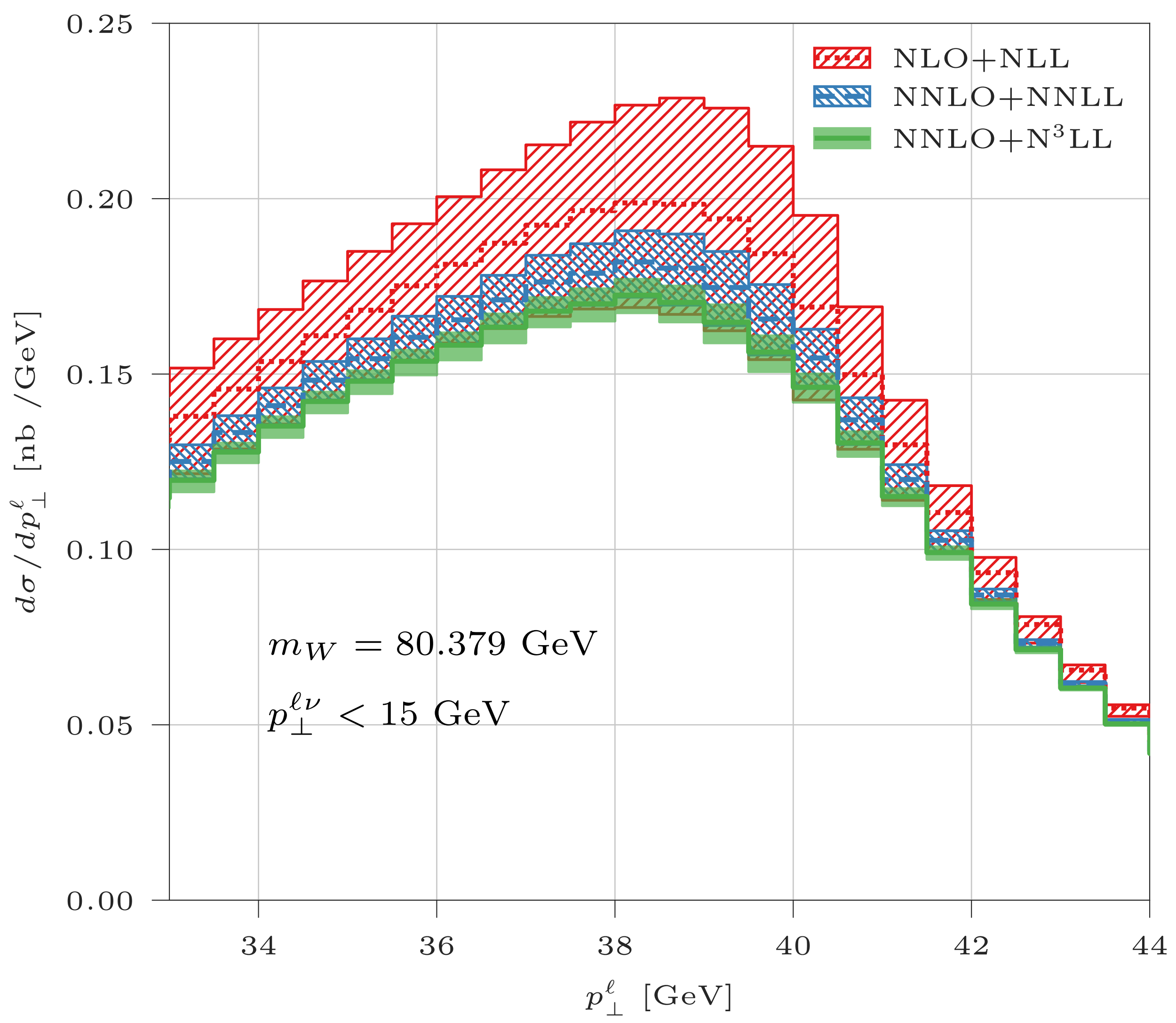
- In charged-current DY,
  it is NOT possible to reconstruct the lepton-neutrino invariant mass
  Full reconstruction is possible (but not easy) only in the transverse plane

- A generic observable has a linear response to an m_W variation
  With a goal for the relative error of 10^{-4}, the problem seems to be unsolvable

- m_W extracted from the study of the shape of the $p_\perp^l, M_\perp$ and $E_\perp^{miss}$ distributions in CC-DY
  thanks to the jacobian peak that enhances the sensitivity to m_W

$$\frac{d}{dp_\perp^2} \to \frac{2}{s}\frac{1}{\sqrt{1-4p_\perp^2/s}}\frac{d}{d\cos\theta} \sim \frac{d}{dp_\perp^2} \to \frac{2}{s}\frac{1}{\sqrt{1-4p_\perp^2/m_W^2}}\frac{d}{d\cos\theta}$$

→ enhanced sensitivity at the 10^{-3} level ($p_\perp^l$ distribution)
or even at the 10^{-2} level ($M_\perp$ distribution)



m_W determination at hadron colliders: template fitting

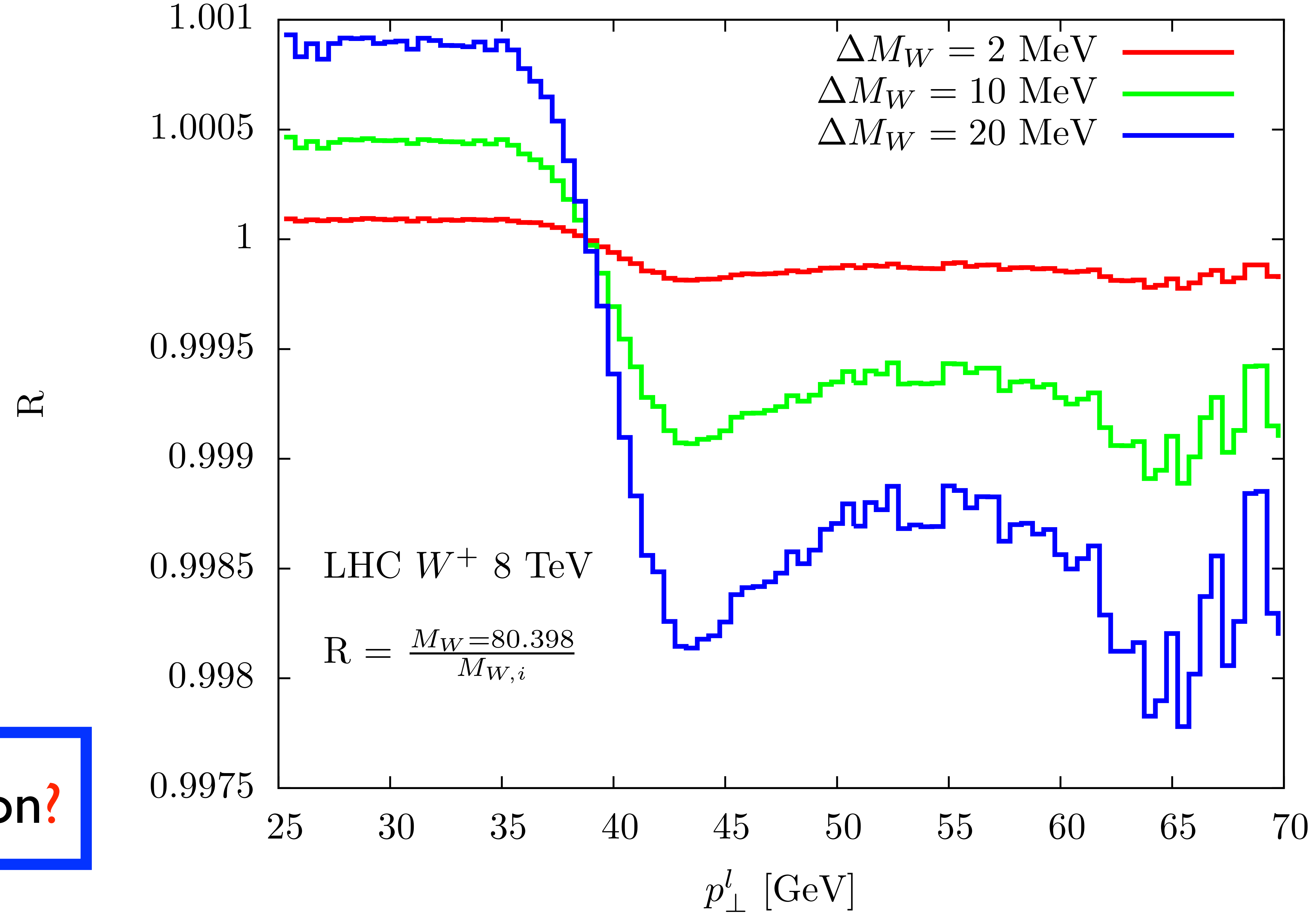
Given one experimental kinematical distribution

- we compute the corresponding theoretical distribution for several hypotheses of one Lagrangian input parameters (e.g. m_W)
- we compute, for each $m_W^{(k)}$ hypothesis, a χ_k^2 defined in a certain interval around the jacobian peak (fitting window)
- we look for the minimum of the χ^2 distribution

The m_W value associated to the position of the minimum of the χ^2 distribution is the experimental result

A determination at the 10^{-4} level requires
a control over the shape of the distributions at the per mille level

The theoretical uncertainties of the templates
contribute to the theoretical systematic error on m_W

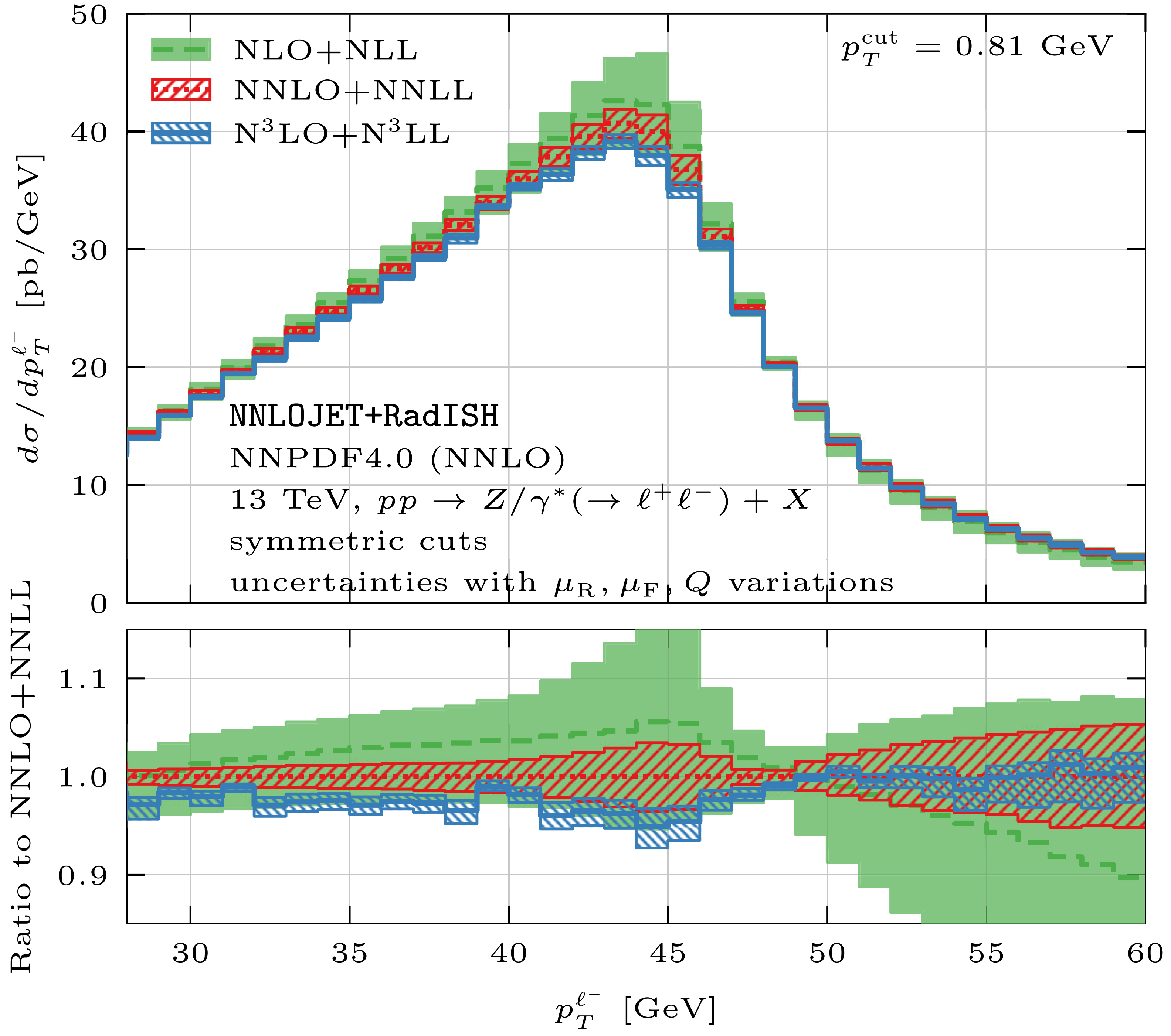


How are QCD scale variations handled, in the template preparation?

Template fitting: description of the single lepton transverse momentum distribution

The template fitting procedure is acceptable if the data are described by the theoretical distribution with high quality

X.Chen, T.Gehrmann, N.Glover, A.Huss, P.Monni, E.Re, L.Rottoli, P.Torrielli, arXiv:2203.01565



Scale variation of the N3LO+N3LL prediction for ptlep provides a set of equally good templates
but the width of the uncertainty band is at the few percent level
a factor 10 larger than the naive estimate would require !

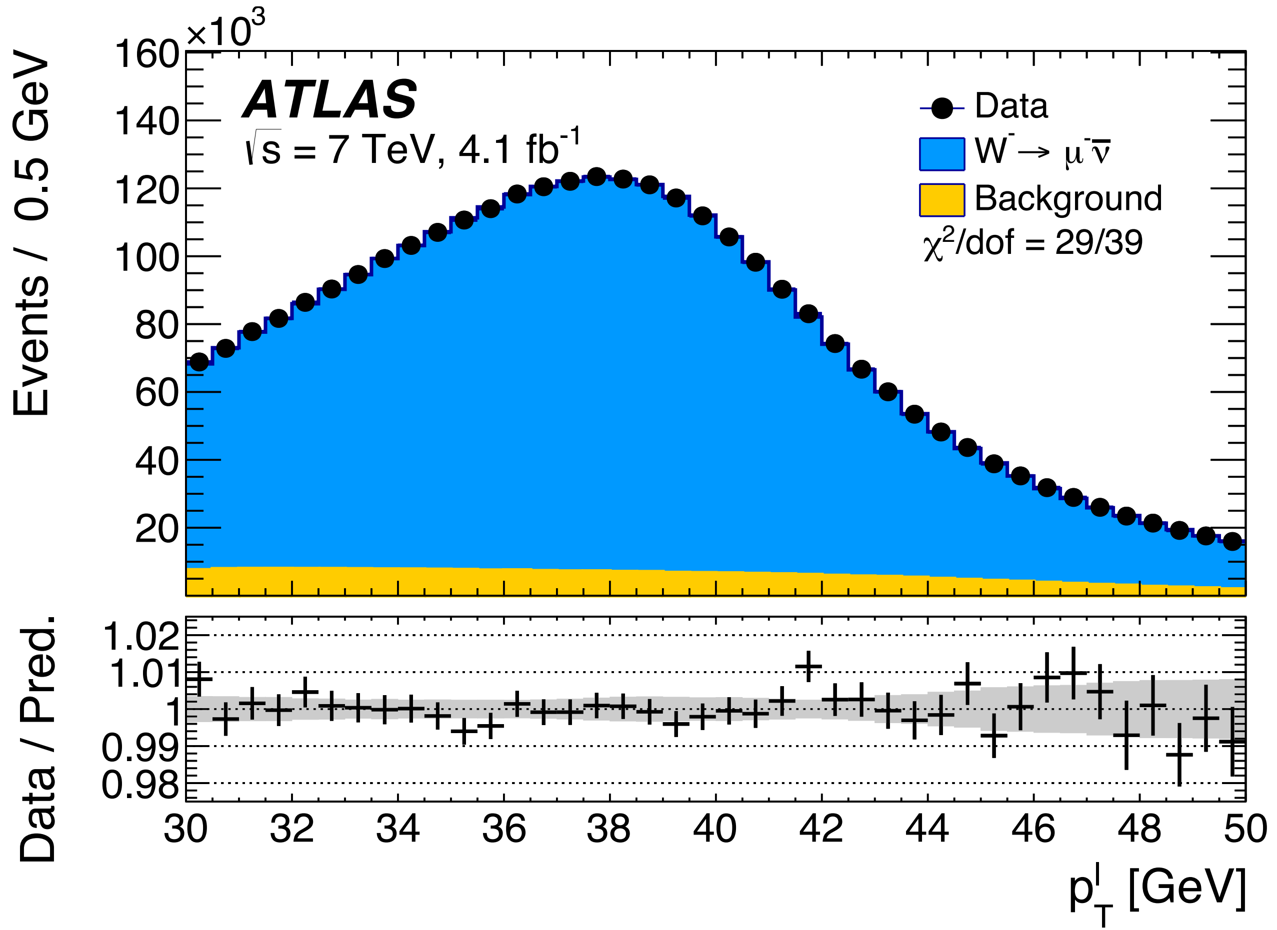
→ data driven approach

a Monte Carlo event generator is tuned to the data in NCDY ($p_\perp^Z$)
for one QCD scale choice

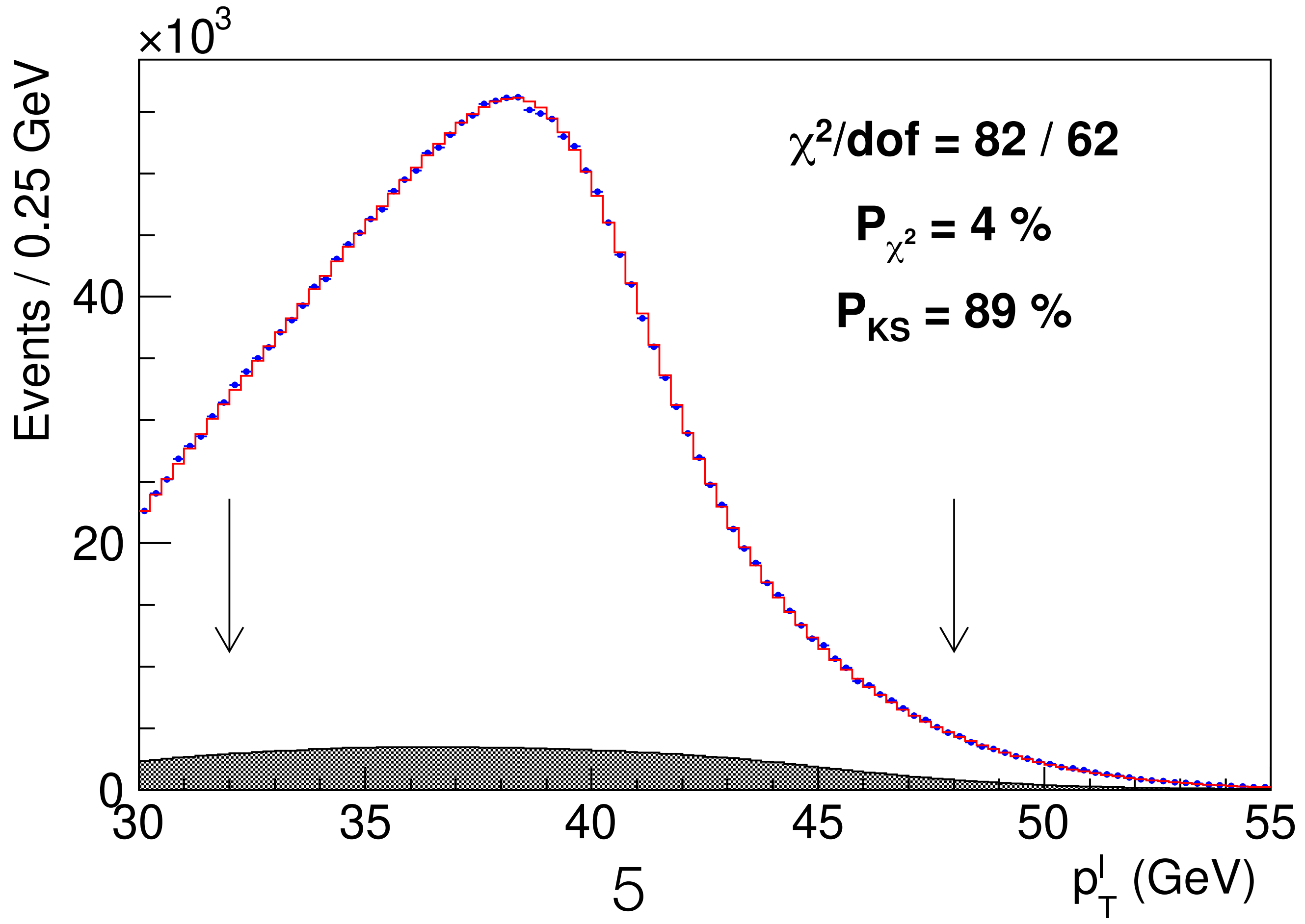
↓

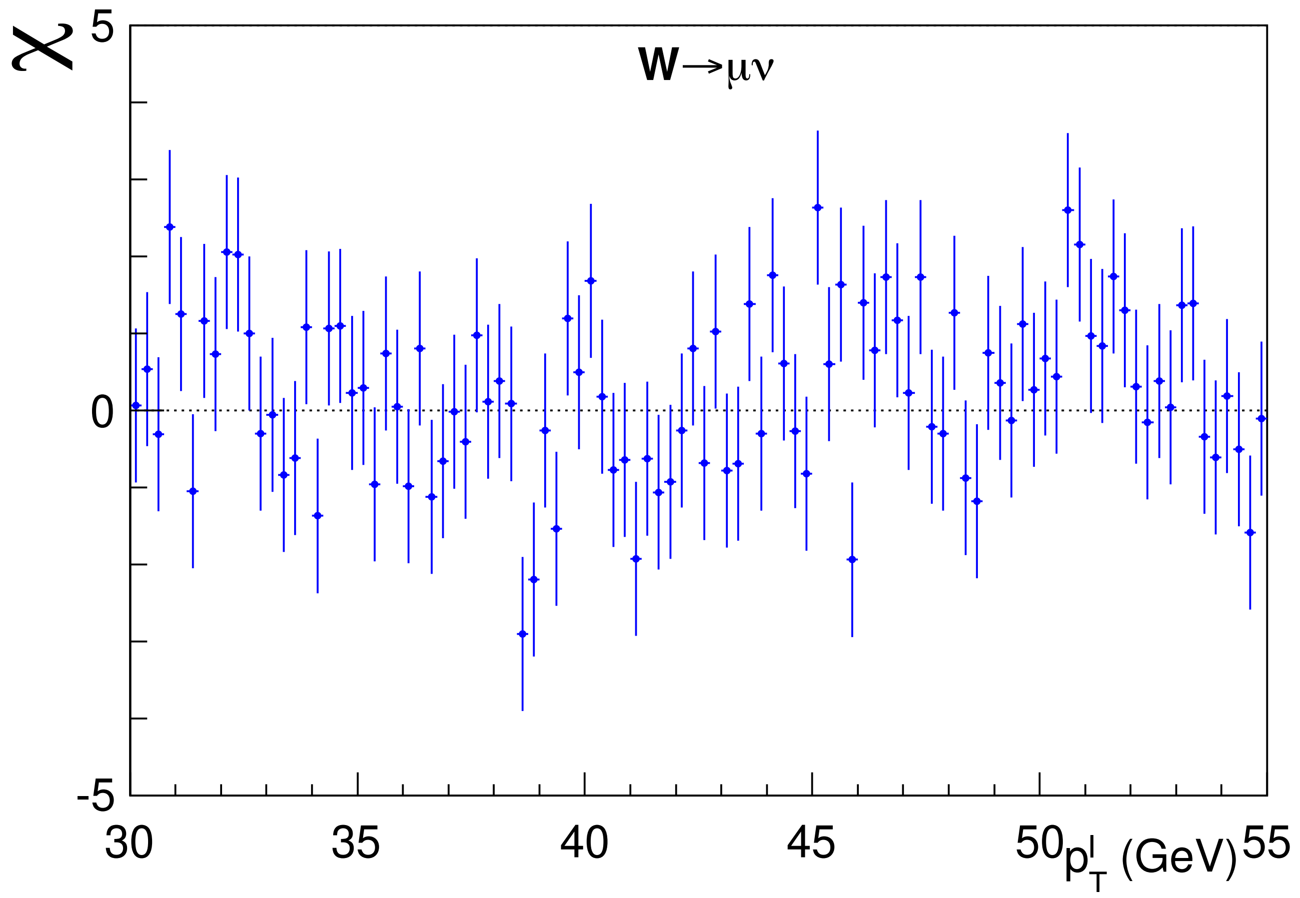
the same parameters are then used to prepare the CCDY templates

Eur.Phys.J.C 78 (2018) 2, 110, *Eur.Phys.J.C* 78 (2018) 11, 898 (erratum)



CDF collaboration, Scince 376, 170-176 (2022)





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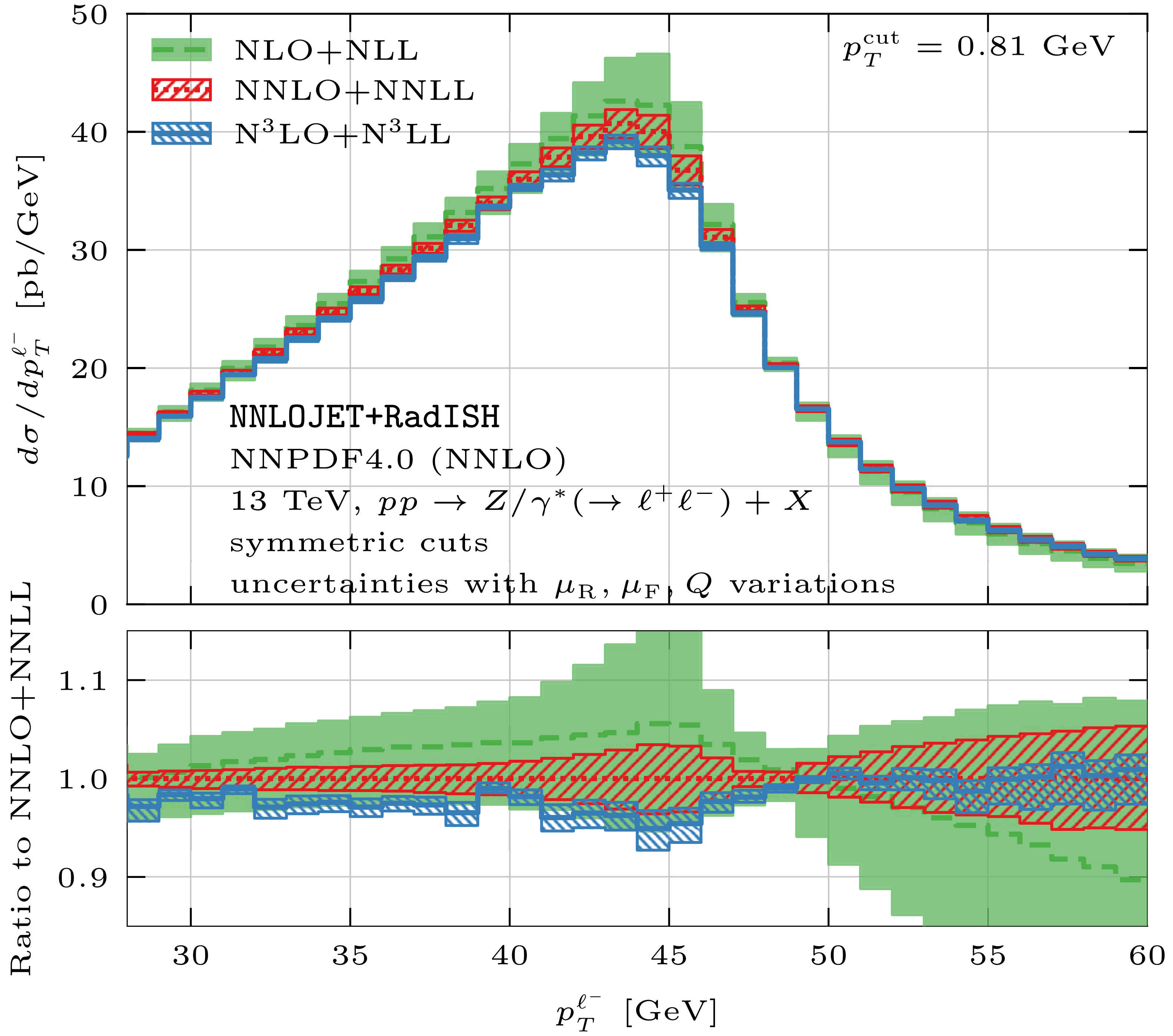

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a factor 10 larger than the naive estimate would require !

→ data driven approach

a Monte Carlo event generator is tuned to the data in NCDY ($p_\perp^Z$)
for one QCD scale choice

↓

the same parameters are then used to prepare the CCDY templates

What are the limitations of the transfer of information from NCDY to CCDY ?

Comments on the data driven approach

- The Monte Carlo event generators typically have NLO+(N)LL QCD perturbative accuracy
  → to match the data they might require a reweighing factor larger than a code N3LO+N3LL

- The tuning to the data should be done in association to QCD scale variations
  → starting from different pQCD scale choices, we can achieve by construction the same description of NCDY with different reweighing functions
  but
  we should check how the different alternatives behave when propagated to CCDY

- The tuning assumes that the reweighing factor derived from $p_\perp^Z$
  applies equally well to the $p_\perp^W$ and to the lepton transverse momentum in CCDY

- The tuning assumes that the missing factor taken from the data is universal, i.e. identical for NCDY and CCDY
  but
  several elements of difference:
  - masses and phase-space factors, acceptances
  - different electric charges (QED corrections)
  - different initial states (→ PDFs, heavy quarks effects)

- It is possible that BSM physics is reabsorbed in the tuning

- The interpretation of the fitted value is not necessarily the SM lagrangian parameter

Comments on the χ^2 minimisation in the template fit

$$\chi^2 = (\vec{d} - \vec{t})^T \cdot C^{-1} \cdot (\vec{d} - \vec{t})$$

$$C = \Sigma_{stat} + \Sigma_{syst,exp} + \Sigma_{MC} + \Sigma_{PDF} + \Sigma_{syst,th}$$

The $\Sigma_{syst,th}$ contribution to the covariance matrix is never included, because of the non-statistical nature of theory uncertainties

The χ^2 minimisation leads to sensible and stable results only when the deviation of the data from the templates is comparable to the size of the eigenvalues of the covariance matrix

but

the lepton transverse momentum distribution has large O(1%) scale uncertainties in pQCD, much larger than 0.1% ;

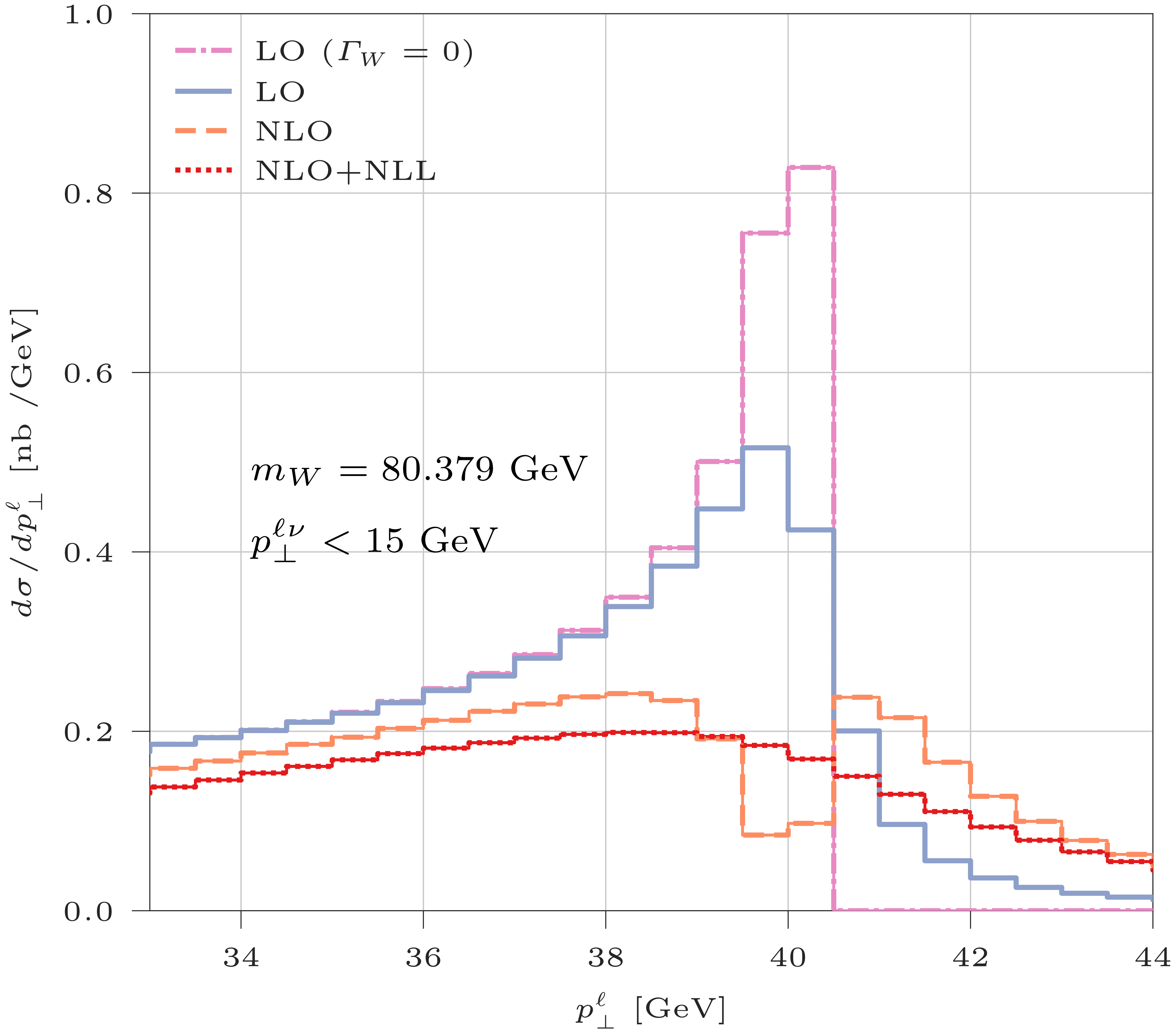
the absence of $\Sigma_{syst,th}$ makes the usage of the χ^2 minimisation procedure extremely unstable

→ the data driven approach remains the only way to pursue a template fit approach
at the price of losing the possibility to study the theoretical uncertainties (pQCD scale variations) on the modelling

MW from a jacobian asymmetry

L.Rottoli, P.Torrielli, AV, arXiv:2301.04059

The lepton transverse momentum distribution in charged-current Drell-Yan



The lepton transverse momentum distribution has a jacobian peak induced by the factor $1/\sqrt{1-\frac{s}{4p_\perp^2}}$.

When studying the W resonance region, the peak appears at $p_\perp \sim \frac{m_W}{2}$

Kinematical end point at $\frac{m_W}{2}$ at LO

The decay width allows to populate the upper tail of the distribution

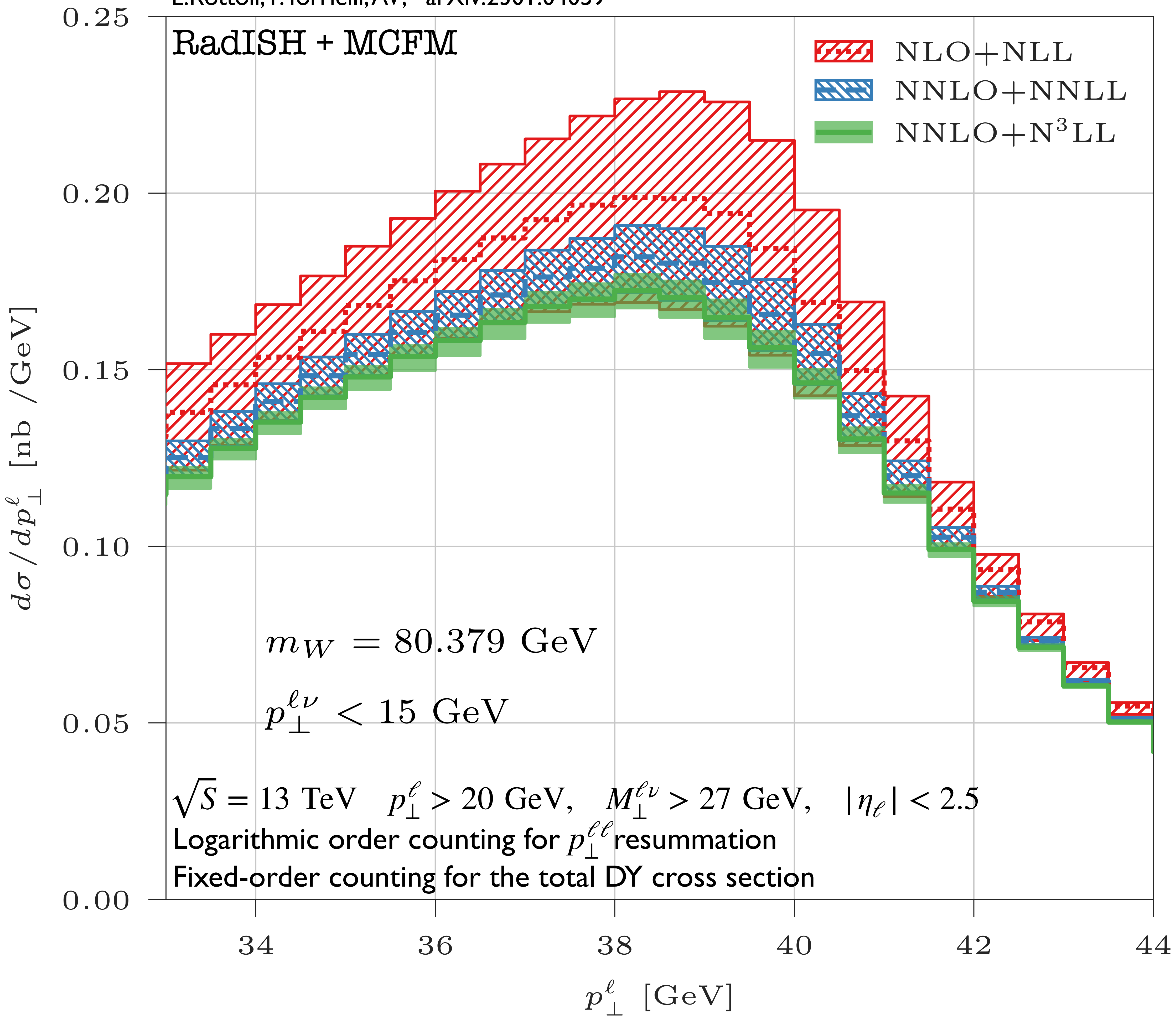
Sensitivity to soft radiation → double peak at NLO-QCD

The QCD-ISR next-to-leading-log resummation broadens the distribution and cures the sensitivity to soft radiation at the jacobian peak.

In the $p_\perp^\ell$ spectrum the sensitivity to m_W and important QCD features are closely intertwined

The lepton transverse momentum distribution in charged-current Drell-Yan

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059



Impressive progress in QCD calculations

X.Chen, T.Gehrmann, N.Glover, A.Huss, P.Monni, E.Re, L.Rottoli, P.Torrielli, arXiv:2203.01565
X.Chen, T.Gehrmann, N.Glover, A.Huss, T.yang, H.Zhu, arXiv: 2205.11426
J.Campbell, T.Neumann, arXiv:2207.07056
S.Camarda, L.Cieri, G.Ferrera, arXiv:2303.12781

Uncertainty band based on canonical scale variations

$$\mu_{R,F} = \xi_{R,F}\sqrt{(M^{\ell\nu})^2 + (p_\perp^{\ell\nu})^2}\,, \quad \mu_Q = \xi_Q M^{\ell\nu}$$

$\xi_{R,F} \in (1/2,1,2)$ excluding ratios=4 (7 variations)

$(\xi_R, \xi_F) = (1,1)$ and $\xi_Q = (1/4,1)$ (2 variations)

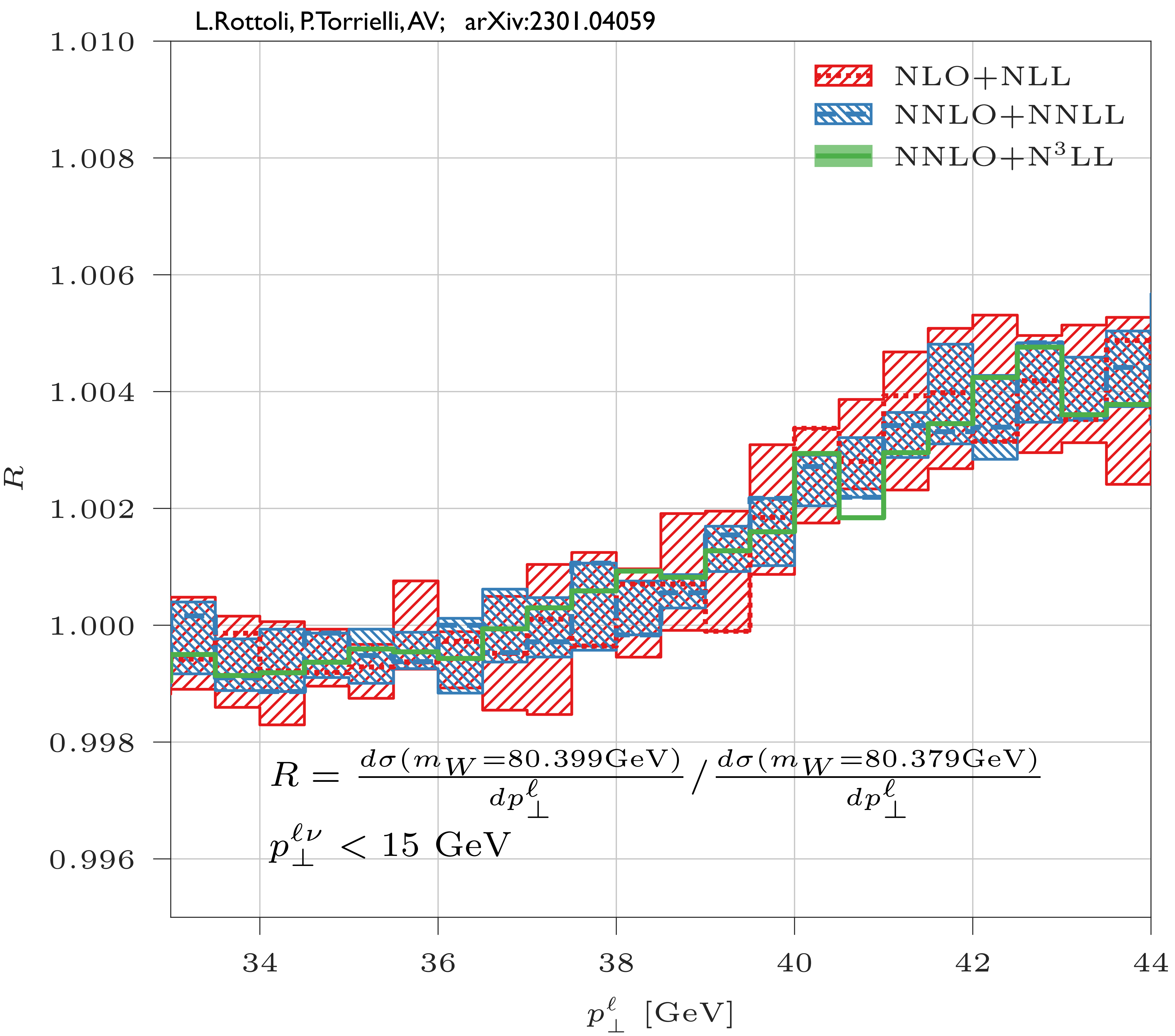
At NNLO+N3LL, residual ±2% uncertainty

The peak of the distribution is located at $p_\perp \sim 38.5$ GeV

The point of maximal sensitivity to m_W is shifted by :

- $\Gamma_W/2$ compared to the nominal value $m_W/2$
- the effect of resummed QCD radiation

Sensitivity to the W boson mass: independence from QCD approximation



The determination of m_W requires the possibility to appreciate the distortion of the distribution induced by 2 different mass hypotheses

A shift by $\Delta m_W = 20$ MeV distorts the distribution at few per mille level

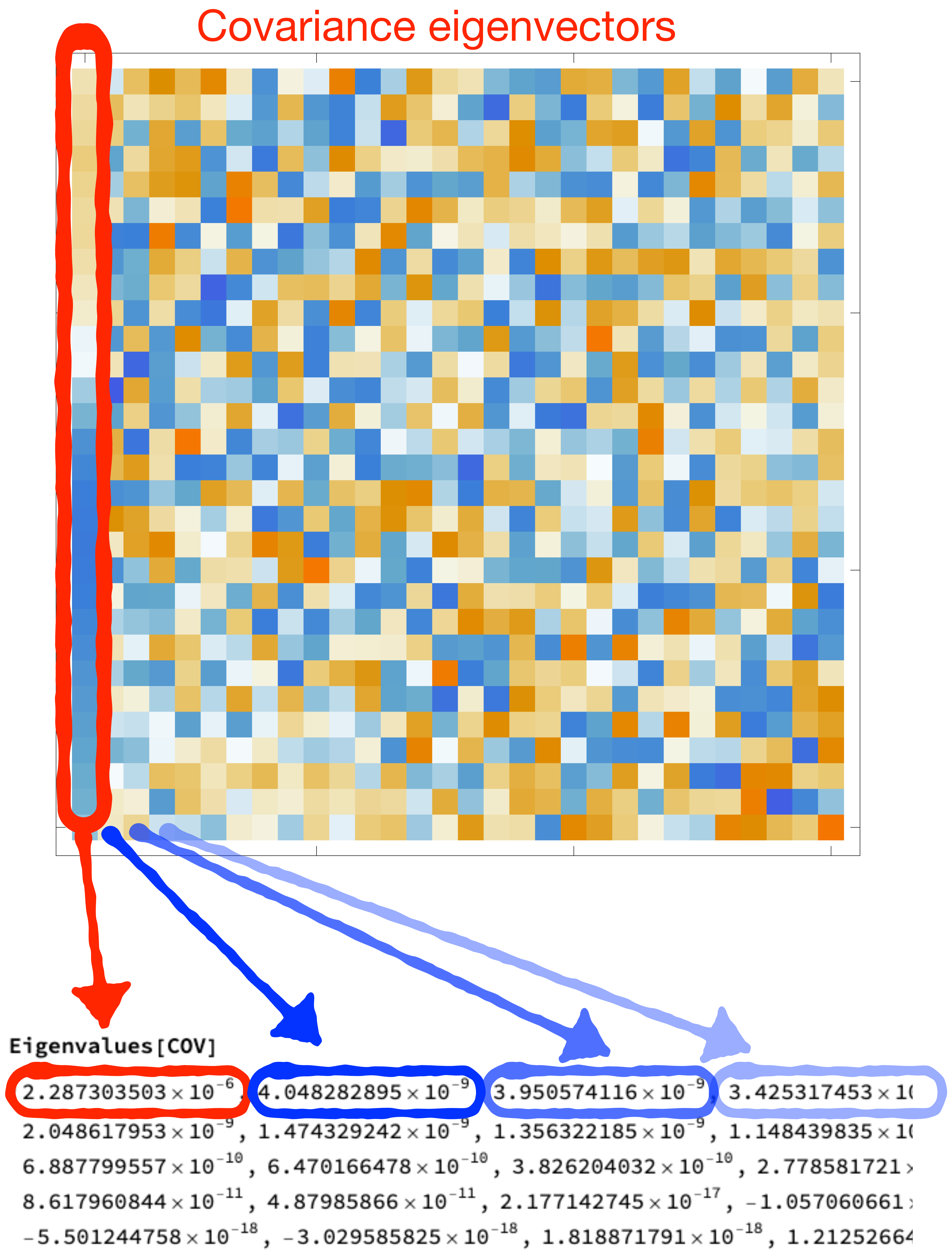
In pure QCD,
the distortion is independent of the QCD approximation or scale choice

The process can be factorized in production (with QCD effects) times propagation and decay of the W boson.
The sensitivity to m_W stems from the propagation and decay part

The sensitivity to m_W is independent of the QCD approximation
The central value and the uncertainty on m_W instead do depend on the QCD approximation

Where is the sensitivity to m_W ? Which bins are the most relevant?
The study of the covariance matrix for m_W variations shows that one specific combination of bins carries the bulk of the sensitivity to m_W → following this indication, we design a new observable

Sensitivity to the W boson mass: covariance with respect to m_W variations



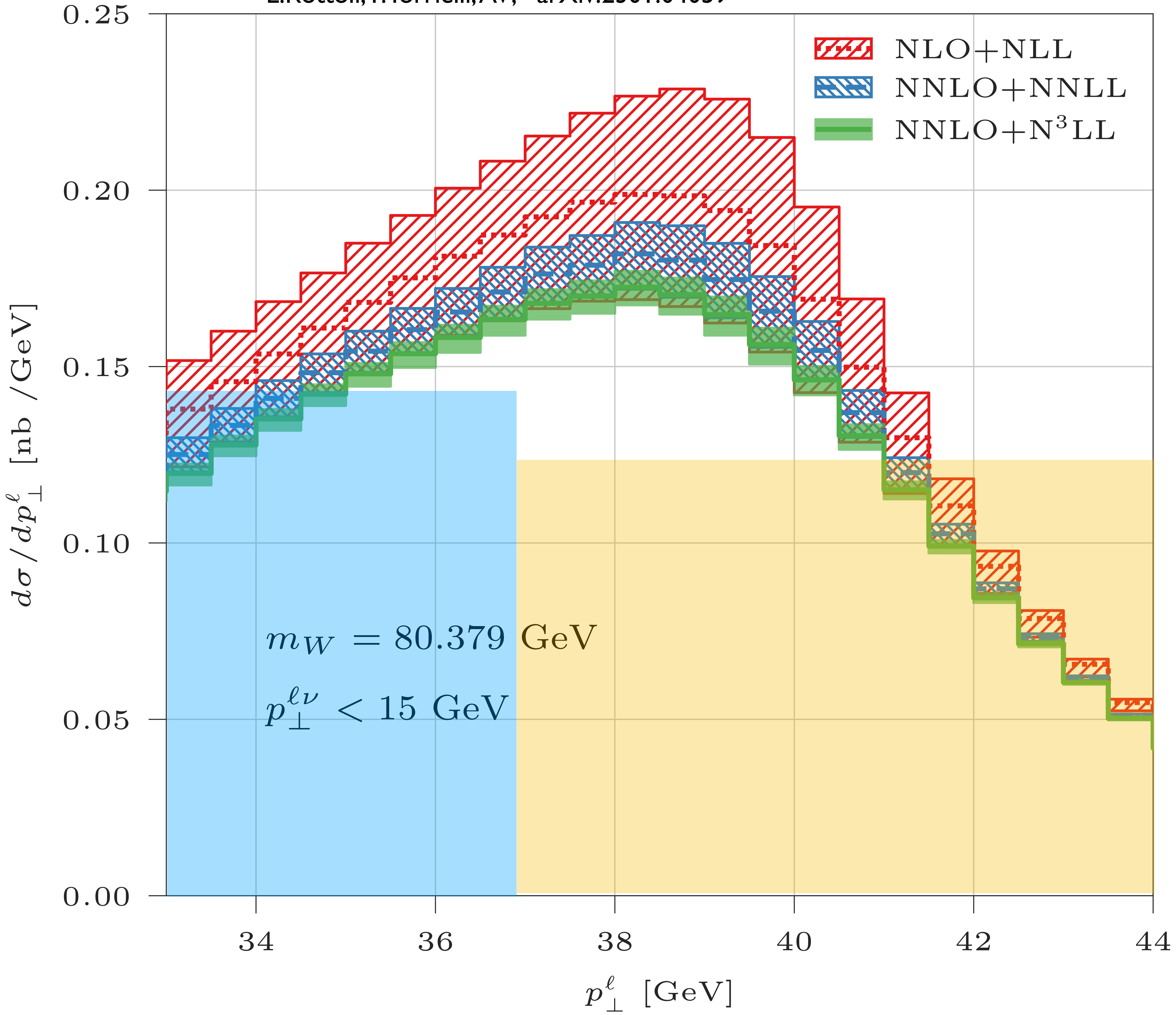
Eigenvalues[COV]

$2.287303503 \times 10^{-6}$, $4.048282895 \times 10^{-9}$, $3.950574116 \times 10^{-9}$, 3.425317453×1
$2.048617953 \times 10^{-9}$, $1.474329242 \times 10^{-9}$, $1.356322185 \times 10^{-9}$, 1.148439835×1
$6.887799557 \times 10^{-10}$, $6.470166478 \times 10^{-10}$, $3.826204032 \times 10^{-10}$, 2.778581721
$8.617960844 \times 10^{-11}$, $4.87985866 \times 10^{-11}$, $2.177142745 \times 10^{-17}$, -1.057060661
$-5.501244758 \times 10^{-18}$, $-3.029585825 \times 10^{-18}$, $1.818871791 \times 10^{-18}$, 1.2125266

- The $p_\perp^\ell$ spectrum includes N bins.
- After the rotation which diagonalises the m_W covariance, we have N linear combinations of the primary bins.
- The combination associated to the (by far) largest eigenvalue exhibits a very clear and simple pattern
- The point where the coefficients change sign is very stable at different orders in QCD and with different bin ranges and it is found at $p_\perp^\ell \sim 37$ GeV

The jacobian asymmetry $\mathscr{A}_{p_\perp^\ell}$

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059



$$L_{p_\perp^\ell} \equiv \int_{p_\perp^{\ell,\mathrm{min}}}^{p_\perp^{\ell,\mathrm{mid}}} dp_\perp^\ell \frac{d\sigma}{dp_\perp^\ell}, \qquad U_{p_\perp^\ell} \equiv \int_{p_\perp^{\ell,\mathrm{mid}}}^{p_\perp^{\ell,\mathrm{max}}} dp_\perp^\ell \frac{d\sigma}{dp_\perp^\ell}$$

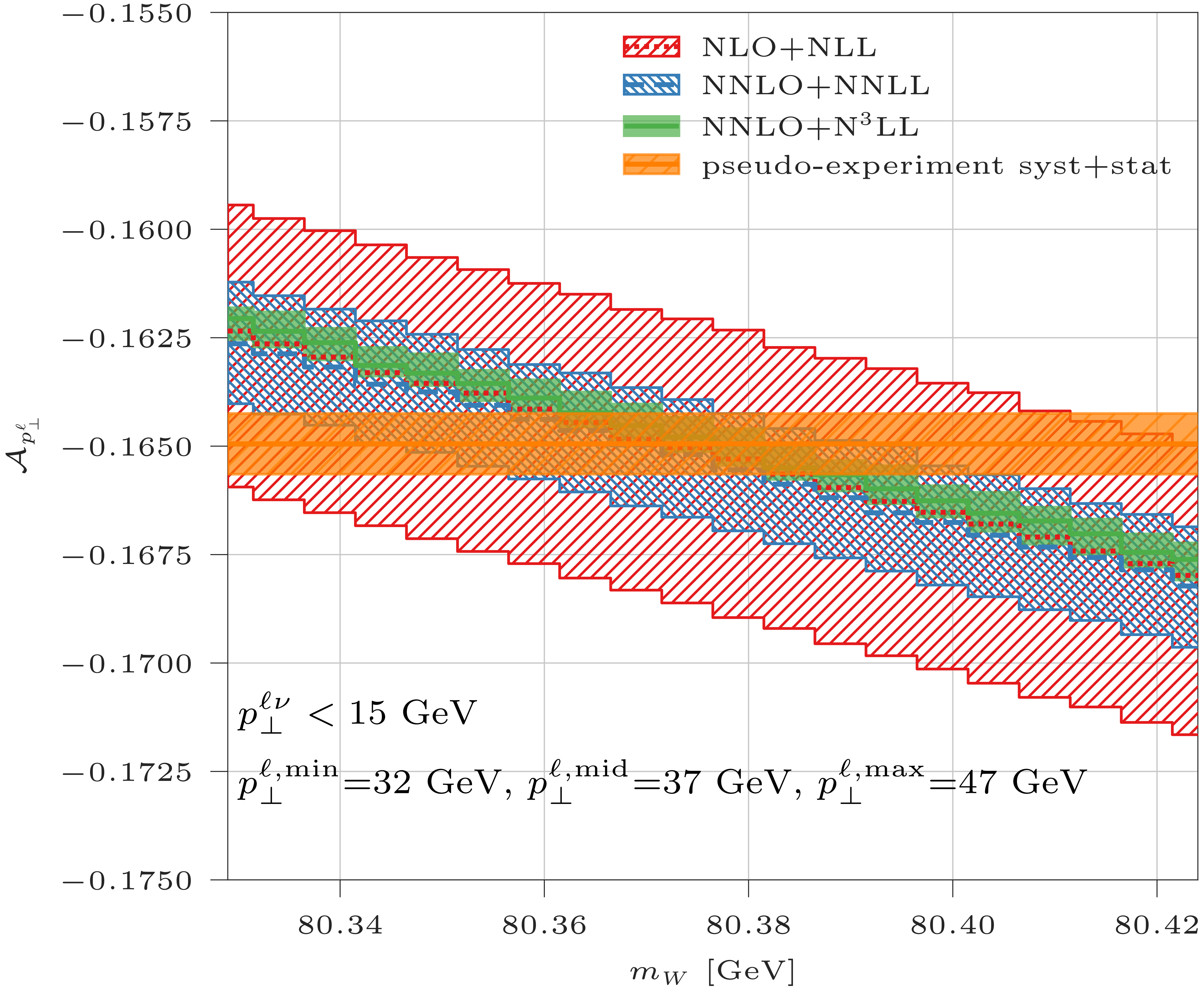
$$\mathcal{A}_{p_\perp^\ell}(p_\perp^{\ell,\mathrm{min}}, p_\perp^{\ell,\mathrm{mid}}, p_\perp^{\ell,\mathrm{max}}) \equiv \frac{L_{p_\perp^\ell} - U_{p_\perp^\ell}}{L_{p_\perp^\ell} + U_{p_\perp^\ell}}$$

The asymmetry is an observable (i.e. it is measurable via counting): its value is one single scalar number
It depends only on the edges of the two defining bins

Increasing m_W shifts the position of the peak to the right → Events migrate from the blue to the orange bin
→ The asymmetry decreases

The jacobian asymmetry $\mathscr{A}_{p_\perp^\ell}$ as a function of m_W

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059



The asymmetry $\mathscr{A}_{p_\perp}$ has a linear dependence on m_W,
stemming from the linear dependence on the end-point position

The slope of the asymmetry expresses the sensitivity to m_W,
in a given setup $(p_\perp^{\ell,min}, p_\perp^{\ell,mid}, p_\perp^{\ell,max})$

The slope is the same with every QCD approximation
(factorization of QCD effects, perturbative and non-perturbative)

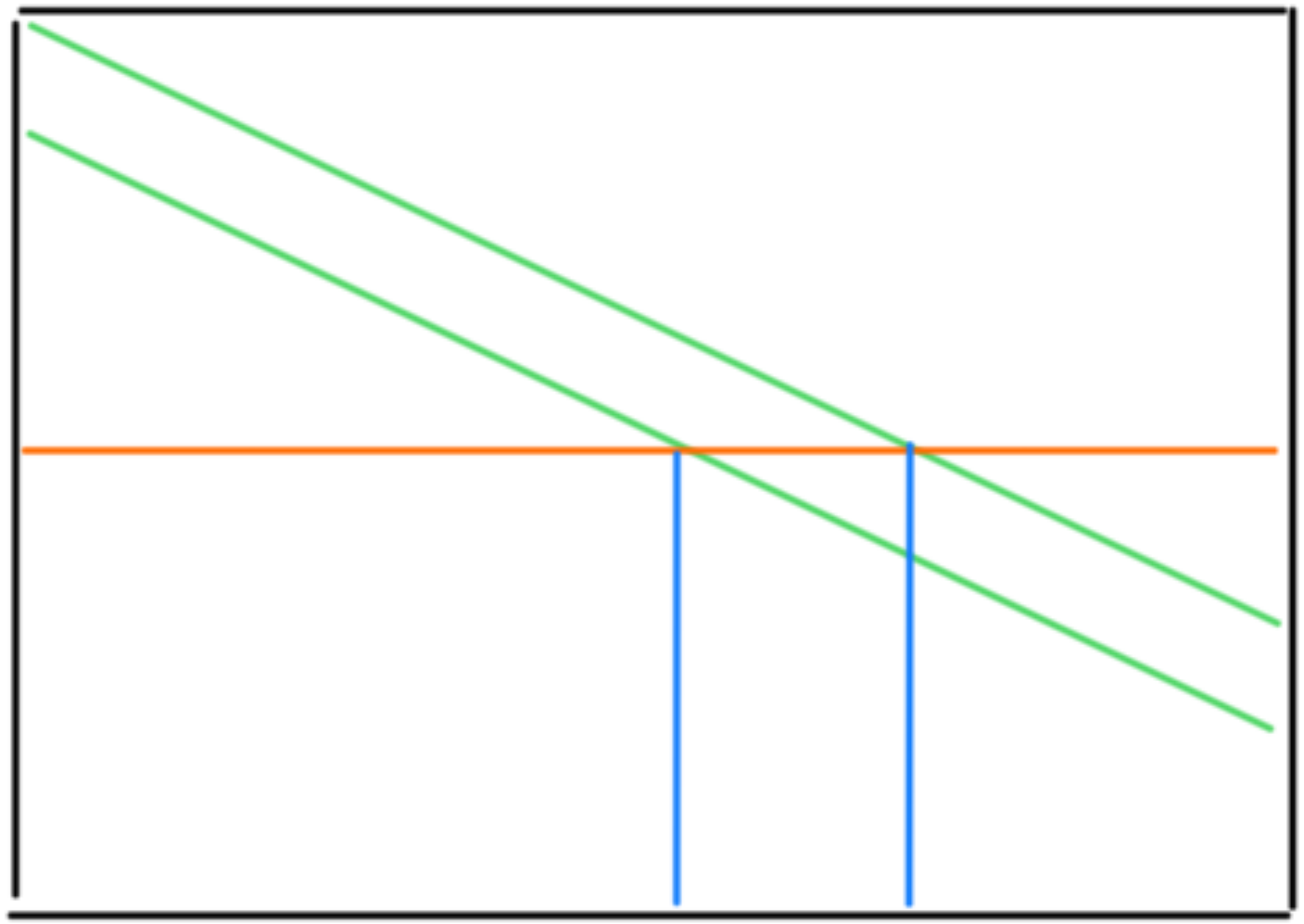
The "large" size of the two bins $\mathscr{O}(5-10)$ GeV leads to

- small statistical errors
- excellent stability of the QCD results (inclusive quantity)
- ease to unfold the data to particle level (m_W combination)

The experimental value and the theoretical predictions can be directly compared (m_W from the intersection of two lines)

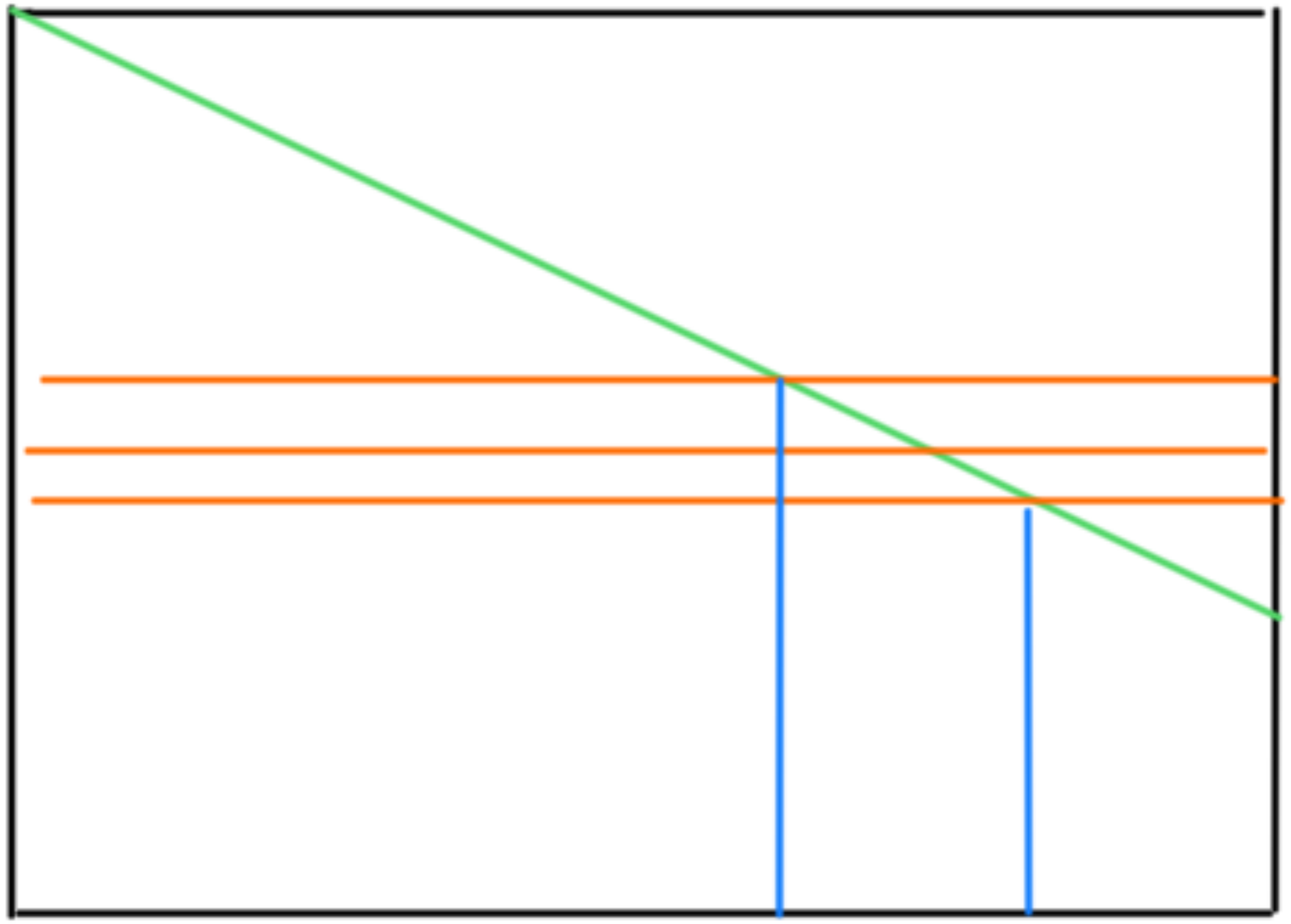
The main systematics on the two fiducial cross sections is related to the lepton momentum scale resolution

Reading the uncertainties on m_W



Δm_W^{th}

m_W^{exp}

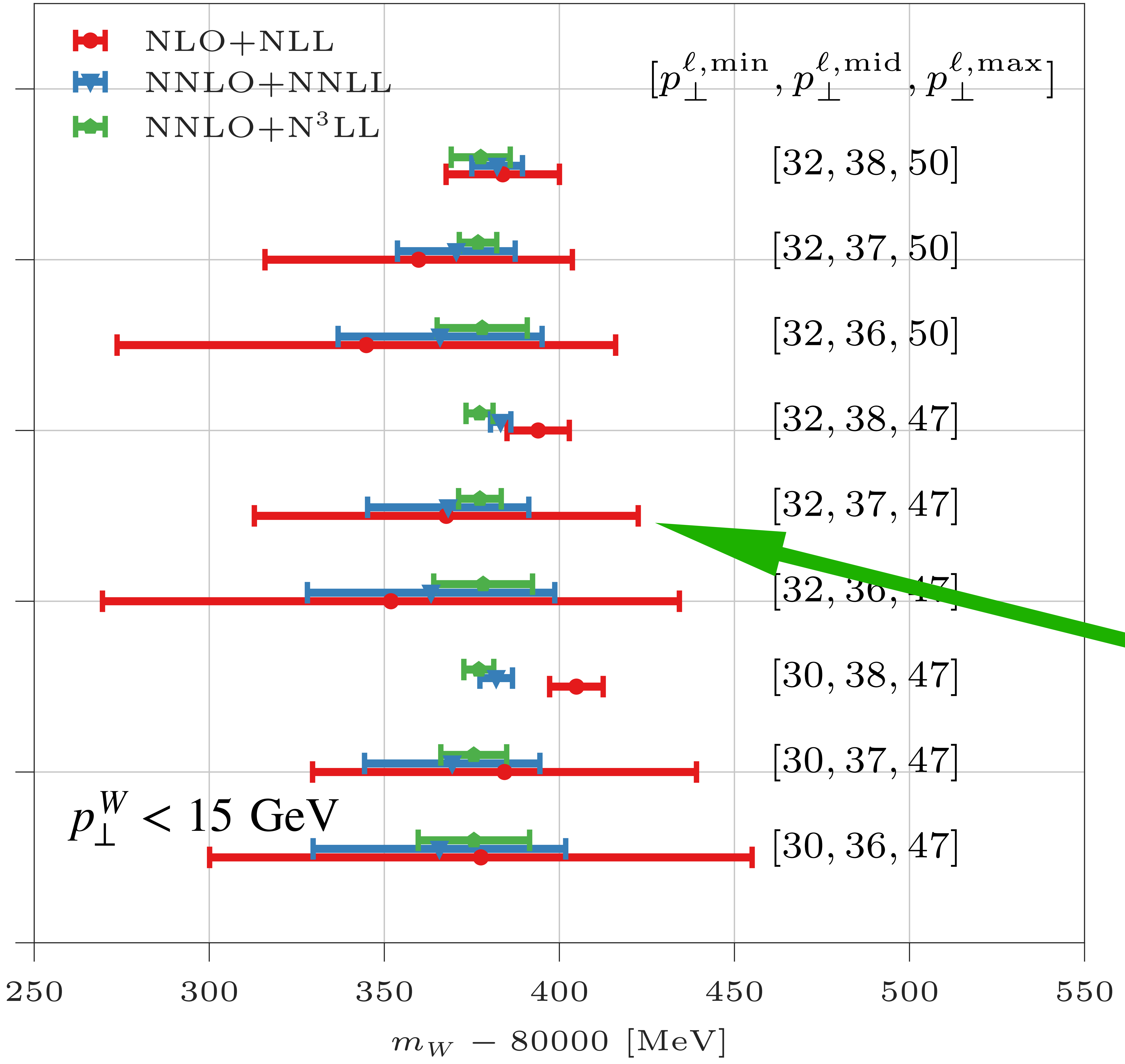


Δm_W^{exp}

m_W determination at the LHC as a function of the $\mathscr{A}_{p_\perp^\ell}$ parameters (low pile-up setup)

as pseudo-experimental value we choose the NNLO+N3LL result with $m_W = 80.379$

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059



Important role of the N3LL corrections

We first check the convergence order-by-order.
If we observe it, then we take the size of the m_W interval as estimator of the residual pQCD uncertainty

We do not trust the scale variations alone
→ cfr the choice with $p_\perp^{\ell,mid} = 38$ GeV

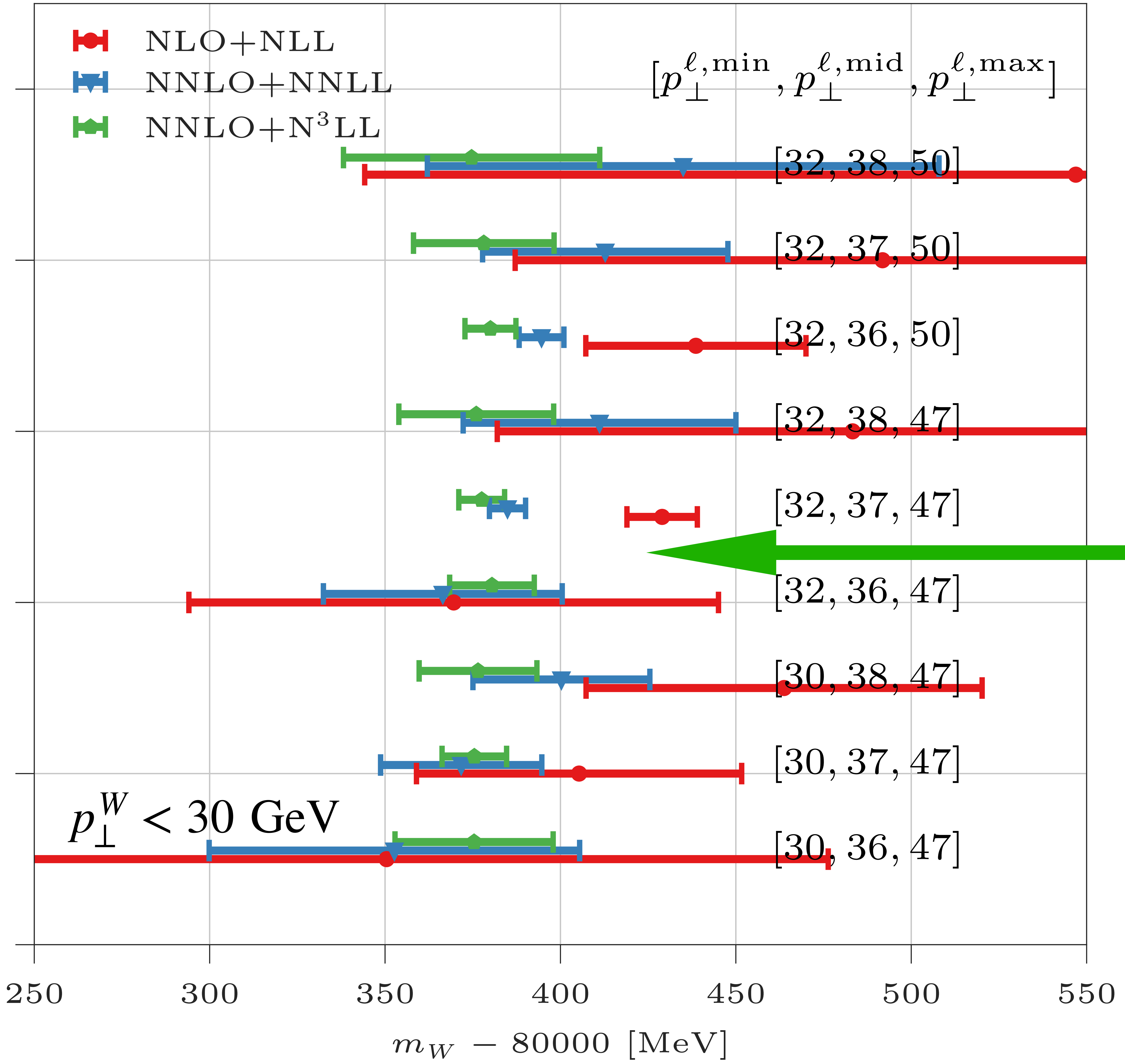
A pQCD uncertainty at the ± 5 MeV level is achievable based on CCDY data alone

The choice of the midpoint is important to identify two regions with excellent QCD convergence

m_W determination at the LHC as a function of the $\mathscr{A}_{p_\perp^\ell}$ parameters (high pile-up setup)

as pseudo-experimental value we choose the NNLO+N3LL result with $m_W = 80.379$

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059



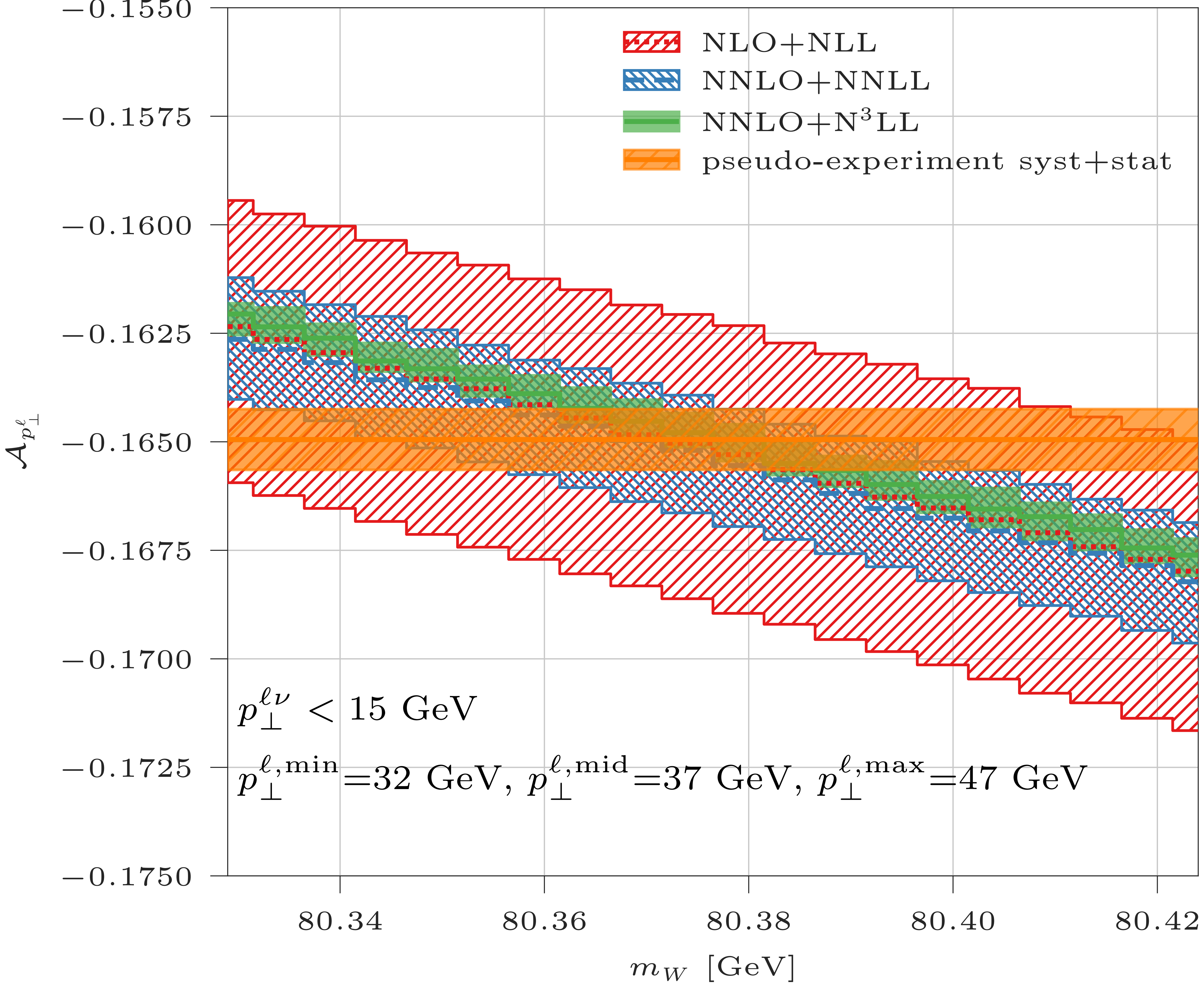
Clear impact of the acceptance cut on $p_\perp^W$

Important role of the N3LL corrections

A pQCD uncertainty below ± 10 MeV level is achievable based on CCDY data alone

The choice of the midpoint is important to identify two regions with excellent QCD convergence

What's missing?



L.Rottoli, P.Torrielli, AV; arXiv:2301.04059

The excellent convergence in pQCD of the asymmetry $\mathscr{A}_{p_\perp}$ is the best possible starting point to discuss

- the impact on the central m_W value of
  - missing perturbative corrections (QED, QCDxEW)
  - non-perturbative effects

→ each effect yields a vertical offset of $\mathscr{A}_{p_\perp^\ell}$ → m_W shift
QED corrections might also change the slope
(preliminary studies show mild QED effects)

→ the non-perturbative effects are a refinement of the study
- impact on top of NNLO+N3LL is expected moderate
- not a crucial element (as in the template fit case)

- the propagation of the uncertainties

→ the linearity of the dependence on m_W allows
an easy propagation of each uncertainty source

The asymmetry in pure pQCD is just one component of the $p_\perp^\ell$ spectrum
→ additional measurements are needed, to achieve an accurate description of the data

Compatibility and combination
of
world W-boson mass measurements

LHC-TeV MW working group,   arXiv:2308.09417

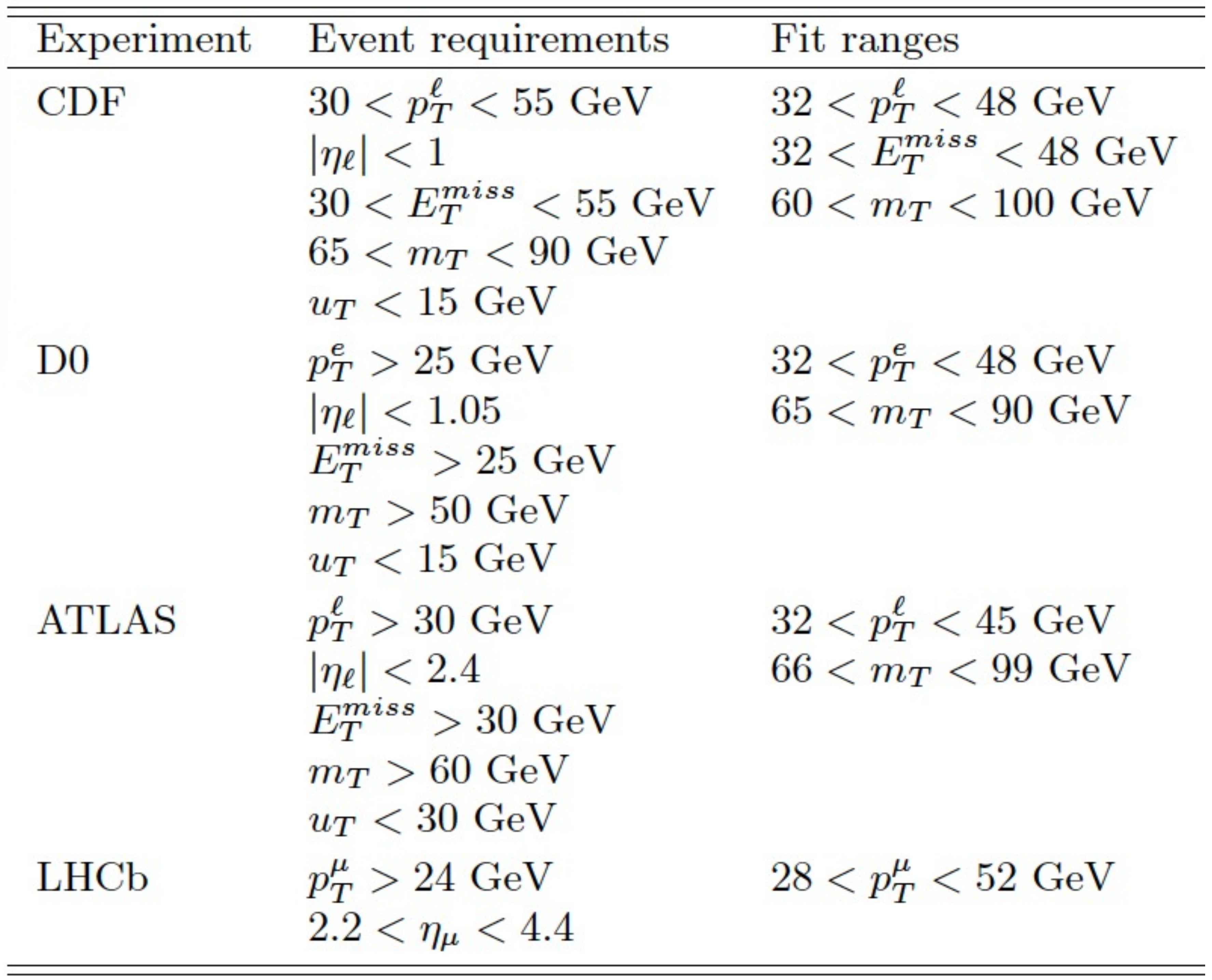
slides prepared by W.Barter in collaboration with the LHC-TeV MW WG

CDF, Science 376 (2022) 170; D0, PRL 103 (2009) 141801 and PRD 89 (2014) 012005;
ATLAS, EPJC 78 (2018) 110; LHCb, JHEP 01 (2022) 036; LEP, Phys Rept 532 (2013) 119

Input Measurements for combination

- CDF – $p\bar{p}$ collisions @ $\sqrt{s}$ = 1.96 TeV; fit variables are p_T^l , p_T^ν and m_T.
- D0 – two separate measurements using $p\bar{p}$ collisions @ $\sqrt{s}$ = 1.96 TeV; fit variables are p_T^e, m_T and p_T^ν.
- ATLAS – pp collisions @ $\sqrt{s}$ = 7 TeV; central region at LHC; fit variables are p_T^l and m_T.
  [Original analysis used following agreement to use published results]
- LHCb – pp collisions @ $\sqrt{s}$ = 13 TeV; forward region at LHC; fit variable is q/p_T^μ.

- LEP – legacy combination from LEP experiments.



Experiment	Event requirements	Fit ranges
CDF	$30 < p_T^\ell < 55$ GeV $\lvert\eta_\ell\rvert < 1$ $30 < E_T^{miss} < 55$ GeV $65 < m_T < 90$ GeV $u_T < 15$ GeV	$32 < p_T^\ell < 48$ GeV $32 < E_T^{miss} < 48$ GeV $60 < m_T < 100$ GeV
D0	$p_T^e > 25$ GeV $\lvert\eta_\ell\rvert < 1.05$ $E_T^{miss} > 25$ GeV $m_T > 50$ GeV $u_T < 15$ GeV	$32 < p_T^e < 48$ GeV $65 < m_T < 90$ GeV
ATLAS	$p_T^\ell > 30$ GeV $\lvert\eta_\ell\rvert < 2.4$ $E_T^{miss} > 30$ GeV $m_T > 60$ GeV $u_T < 30$ GeV	$32 < p_T^\ell < 45$ GeV $66 < m_T < 99$ GeV
LHCb	$p_T^\mu > 24$ GeV $2.2 < \eta_\mu < 4.4$	$28 < p_T^\mu < 52$ GeV

QCD challenges

The measurements span two decades → remarkable theoretical progress

The analyses are based on different PDF sets and event generators, with different theoretical content

- D0: RESBOS CP (N2LO, N2LL) with CTEQ66 PDFs (NLO)
- CDF: RESBOS C (NLO, N2LL) with CTEQ6M PDFs (NLO) [CDF publication applied a correction to reproduce Resbos2 + NNPDF3.1]
- ATLAS: POWHEG + Pythia8 (NLO+PS) with DYTurbo for Angular Distribution (N2LO) with CT10 PDFs (NNLO)
- LHCb: POWHEG + Pythia8 (NLO+PS) with DYTurbo for Angular Distribution (N2LO) with averaged result from MSHT20, NNPDF31 and CT18 PDFs (NLO)

The combination study seeks to "update" the measurements to a common QCD framework before their compatibility is assessed and, eventually, the results are combined

$$m_W^{update} = \underset{\text{Published value}}{m_W^{ref}} + \overset{\text{Update to common PDF}}{\delta m_W^{PDF}} + \underset{\text{Common W polarisation}}{\delta m_W^{pol}} + \overset{\text{Additional (small) updates}}{\delta m_W^{other}}$$

The LHCb measurement has been "repeated", using the same code framework but different PDF sets
Effect of updates on other measurements estimated with two simulated samples from two models

Fitting pseudodata

The impact on m_W is estimated by fitting reference and updated distribution using the same fitting model

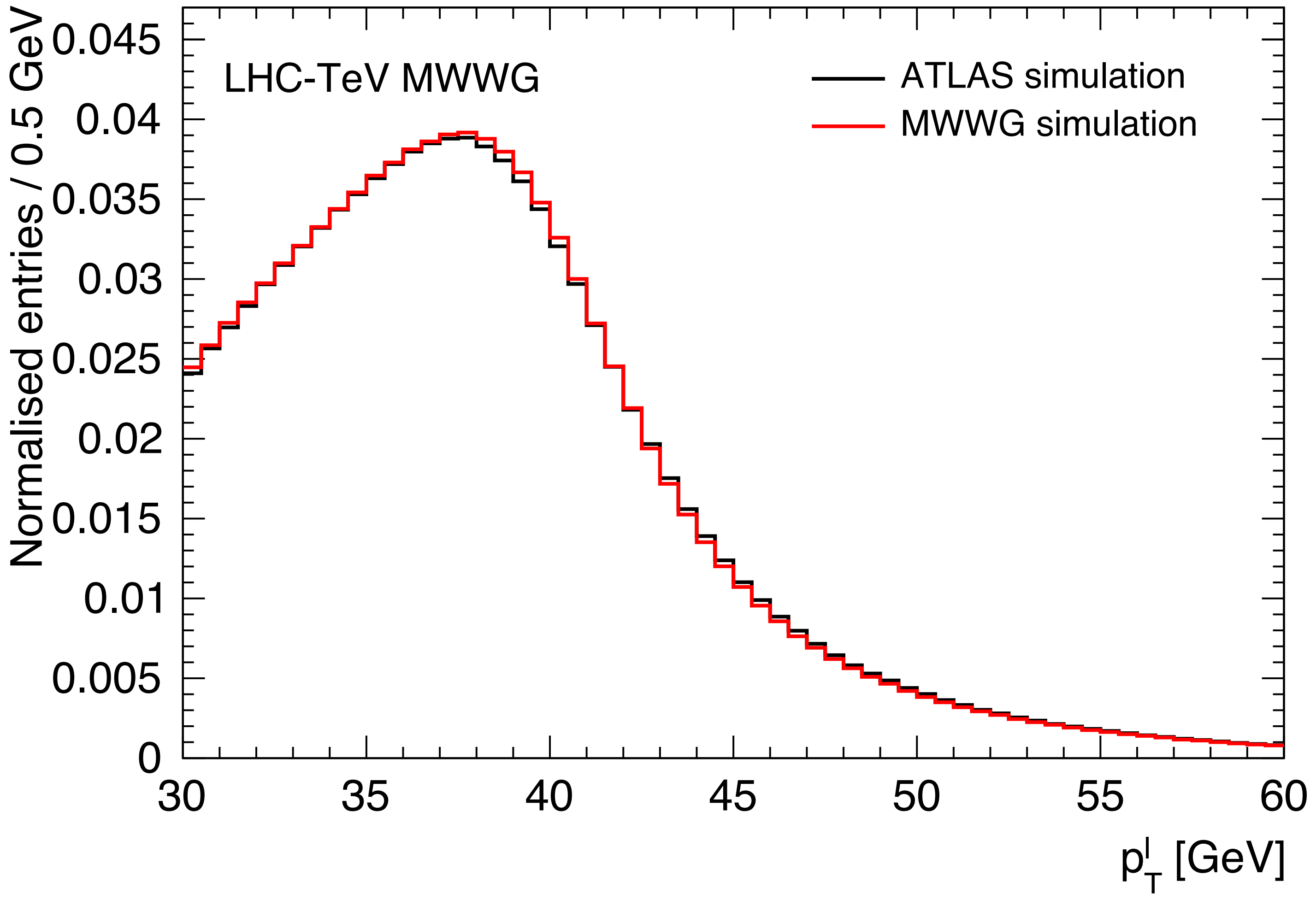
The comparison of PDF effects has been performed using the Wj-MINNLO event generator

The reference generators for the study of pQCD corrections are ResBos (CDF,D0) and DYTurbo (ATLAS, LHCb)

Detector emulation

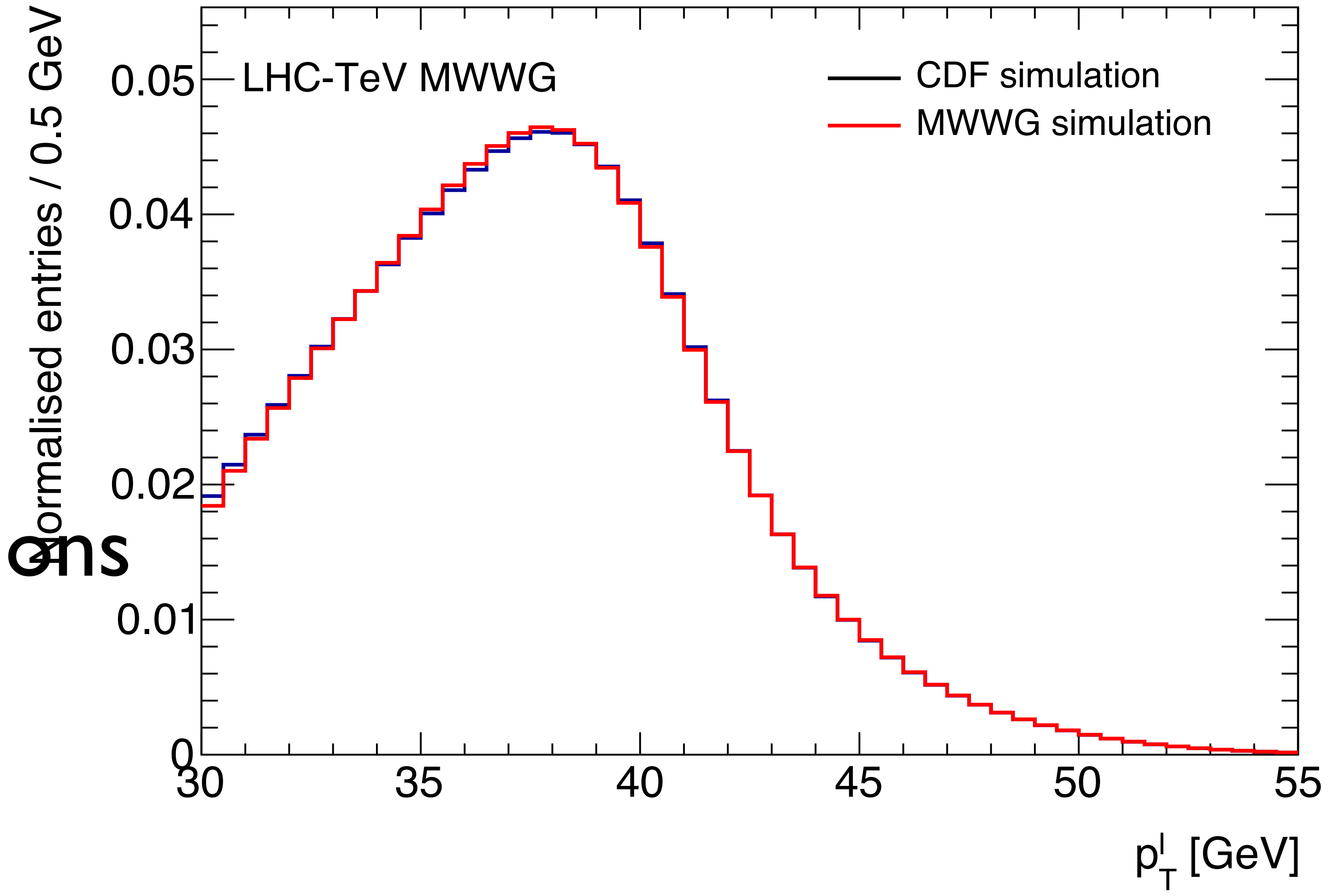
The ATLAS, CDF and D0 detectors have been emulated

- η- and $p_\perp$-dependent smearing of leptons
- Recoil modelling includes lepton removal and event activity effects
- Agreement typically at the percent level between the full simulation and the LHC-TeV MWWG emulation
- Small imperfections in the emulation lead to MeV-level uncertainties on δm_W



The $p_\perp^Z$ ($p_\perp^W$) constraint

After all the updates, the distributions are reweighed to reproduce the exp. $p_\perp^Z$ distributions
The constraints by $p_\perp^W$ are also included, when available.



PDF effects from the study of the $p_\perp^\ell$ or $p_\perp^\nu$ distributions

δm_W^{PDF}

PDF set	D0 p_T^ℓ	D0 p_T^ν	CDF p_T^ℓ	CDF p_T^ν	ATLAS W^+	ATLAS W^-	LHCb
CTEQ6	−17.0	−17.7	0.0	0.0	–	–	–
CTEQ6.6	0.0	0.0	15.0	17.0	–	–	–
CT10	0.4	−1.3	16.0	16.3	0.0	0.0	–
CT14	−9.7	−10.6	5.8	6.8	-1.2	−5.8	1.1
CT18	−8.2	−9.3	7.2	7.7	12.1	−2.3	−6.0
ABMP16	−19.6	−21.5	−1.4	−2.4	−22.5	−3.1	7.7
MMHT2014	−10.4	−12.7	6.1	5.5	−2.6	9.9	−10.8
MSHT20	−13.7	−15.4	3.6	4.1	−20.9	4.5	−2.0
NNPDF3.1	−1.0	−1.2	14.0	15.1	−14.1	-1.8	6.0
NNPDF4.0	6.7	8.1	20.8	24.1	−22.4	6.9	8.3

$\sigma_{PDF}(m_W)$

PDF set	D0	CDF	ATLAS	LHCb
CTEQ6	–	14.1	–	–
CTEQ6.6	15.1	–	–	–
CT10	–	–	9.2	–
CT14	13.8	12.4	11.4	10.8
CT18	14.9	13.4	10.0	12.2
ABMP16	4.5	3.9	4.0	3.0
MMHT2014	8.8	7.7	8.8	8.0
MSHT20	9.4	8.5	7.8	6.8
NNPDF3.1	7.7	6.6	7.4	7.0
NNPDF4.0	8.6	7.7	5.3	4.1

The Tevatron combination did not consider

$$\delta m_W^{PDF}(\text{CTEQ6,CTEQ6.6}) \sim 17\ \text{MeV}$$

Uncertainties here in some cases larger than in original publications
e.g.for CDF the NNPDF3.1 uncertainty from 3.9 to 6.6 MeV

Compatibility of PDF sets with Drell-Yan data

Measurement	NNPDF3.1	NNPDF4.0	MMHT14	MSHT20	CT14	CT18	ABMP16
CDF y_Z	24 / 28	28 / 28	30 / 28	32 / 28	29 / 28	27 / 28	31 / 28
CDF A_W	11 / 13	14 / 13	12 / 13	28 / 13	12 / 13	11 / 13	21 / 13
D0 y_Z	22 / 28	23 / 28	23 / 28	24 / 28	22 / 28	22 / 28	22 / 28
D0 $W \to e\nu$ A_ℓ	22 / 13	23 / 13	52 / 13	42 / 13	21 / 13	19 / 13	26 / 13
D0 $W \to \mu\nu$ A_ℓ	12 / 10	12 / 10	11 / 10	11 / 10	11 / 10	12 / 10	11 / 10
ATLAS peak CC y_Z	13 / 12	13 / 12	58 / 12	17 / 12	12 / 12	11 / 12	18 / 12
ATLAS W^- y_ℓ	12 / 11	12 / 11	33 / 11	16 / 11	13 / 11	10 / 11	14 / 11
ATLAS W^+ y_ℓ	9 / 11	9 / 11	15 / 11	12 / 11	9 / 11	9 / 11	10 / 11
Correlated χ^2	75	62	210	88	81	41	83
Total χ^2 / d.o.f.	200 / 126	196 / 126	444 / 126	270 / 126	210 / 126	162 / 126	236 / 126
p(χ^2, n)	0.003%	0.007%	$< 10^{-10}$	$< 10^{-10}$	0.0004%	1.5%	10^{-8}

No PDF set provides a good description of the full Tevatron+LHC dataset

Best description given by CT18 (which has larger uncertainties)

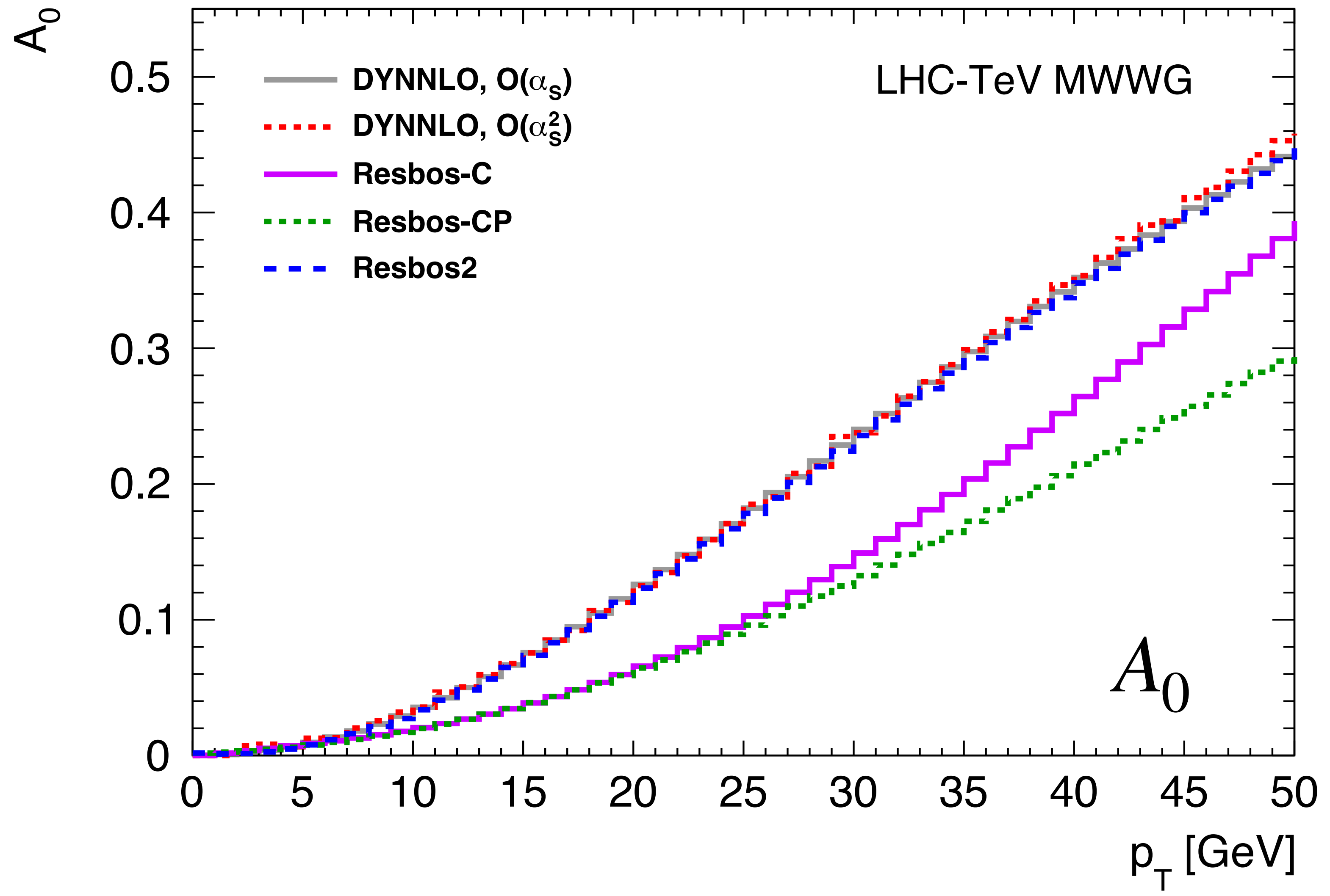
CT18 therefore taken as the default PDF set

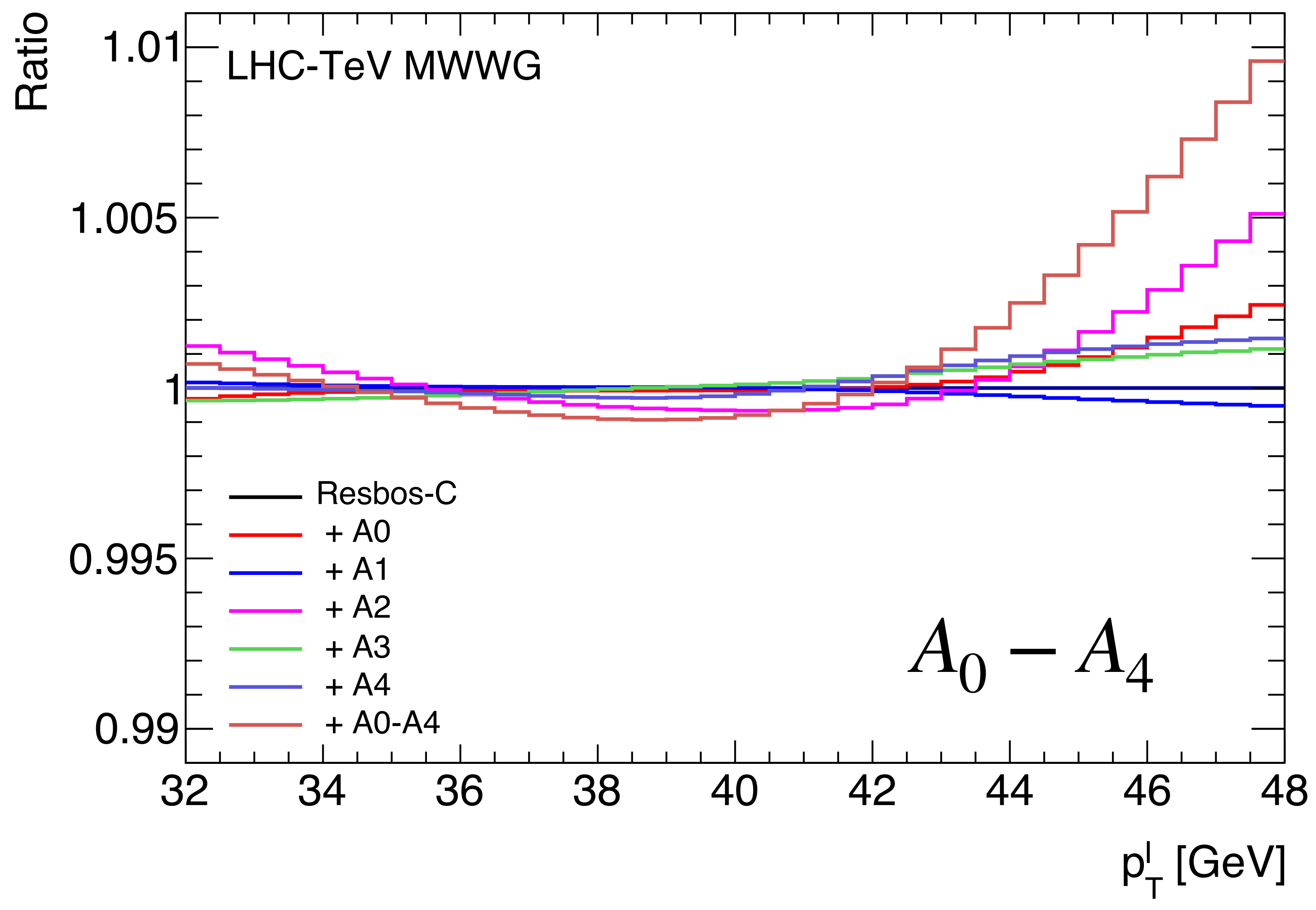
Leptonic angular distributions and QCD corrections

$$\frac{d\sigma}{dp_\perp^W dy_W dm_W d\Omega} = \frac{d\sigma}{dp_\perp^W dy_W dm_W}\left\{1+\cos^2\theta + \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\sin 2\theta\cos\phi + \frac{1}{2}A_2\sin^2\theta\cos 2\phi + A_3\sin\theta\cos\phi + A_4\cos\theta + A_5\sin^2\theta\sin 2\phi + A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi\right\}$$

ATLAS and LHCb use DYTurbo and quote an uncertainty on the A_i → no additional corrections

Fits to data using ResBos-C (CDF) or ResBos-CP (D0) ported so that the $A_0 - A_4$ combinations matches the ResBos2 prediction at $\mathcal{O}(\alpha_s)$





CDF δm_W^{pol}

Coefficient	m_T	p_T^ℓ	p_T^ν
A_0	-6.3	-2.6	-9.1
A_1	1.1	1.3	0.3
A_2	-0.7	0.4	-3.2
A_3	-2.1	-4.2	1.0
A_4	-1.4	-3.3	-1.6
$A_0 - A_4$	-9.5	-8.4	-12.5
ResBos2	-10.2±1.1	-7.6±1.2	-11.8±1.4
Difference	-0.7±1.1	0.8±1.2	0.7±1.4

D0 δm_W^{pol}

Coefficient	m_T	p_T^ℓ	p_T^ν
A_0	-9.8	-7.3	-15.6
A_1	1.9	2.4	1.8
A_2	3.0	3.3	-2.7
A_3	-1.6	-2.9	0.4
A_4	0.2	-2.3	0.5
$A_0 - A_4$	-6.4	-6.9	-15.8
ResBos2	-7.8±1.0	-6.6±1.1	-16.5±1.2
Difference	-1.4±1.0	0.3±1.1	-0.7±1.2

Combination of the different m_W determinations

Results combined using BLUE

Validation by reproducing internal experimental combinations

The CDF measurement contains an *a posteriori* shift $\delta m_W \sim 3\ \mathrm{MeV}$

accounting for (CTEQ6M→NNPDF3.1, mass modelling, polarisation effects) removed before the combination

PDF correlations in the combination

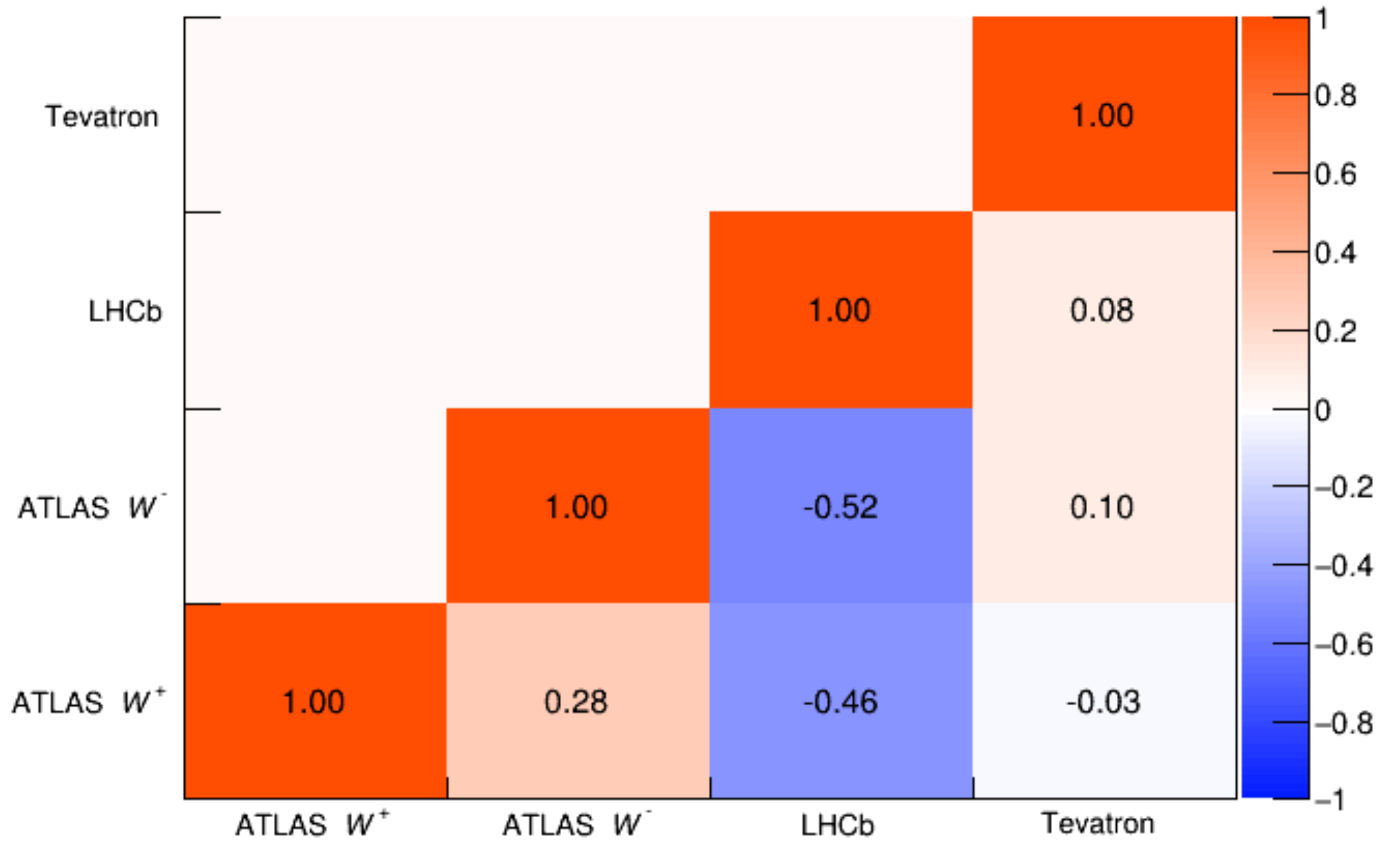
Correlations needed in the combination

Significantly different correlations between the various PDF sets

PDF anti-correlations between experiments leads to more stable results and reduced PDF dependence

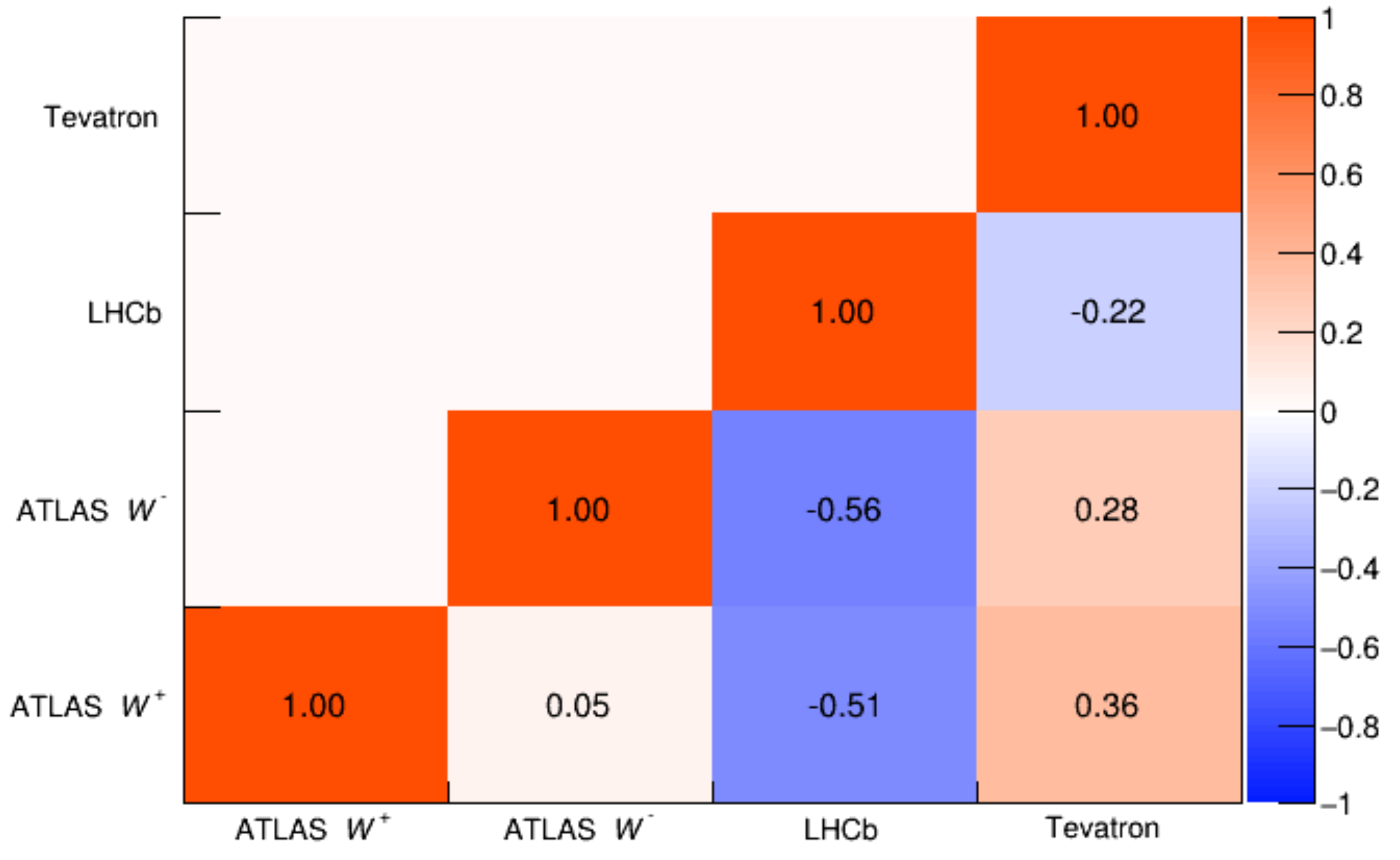
cfr. G.Bozzi, L.Citelli, AV, M.Vesterinen, arXiv:1501.05587, arXiv:1508.06954

CT18



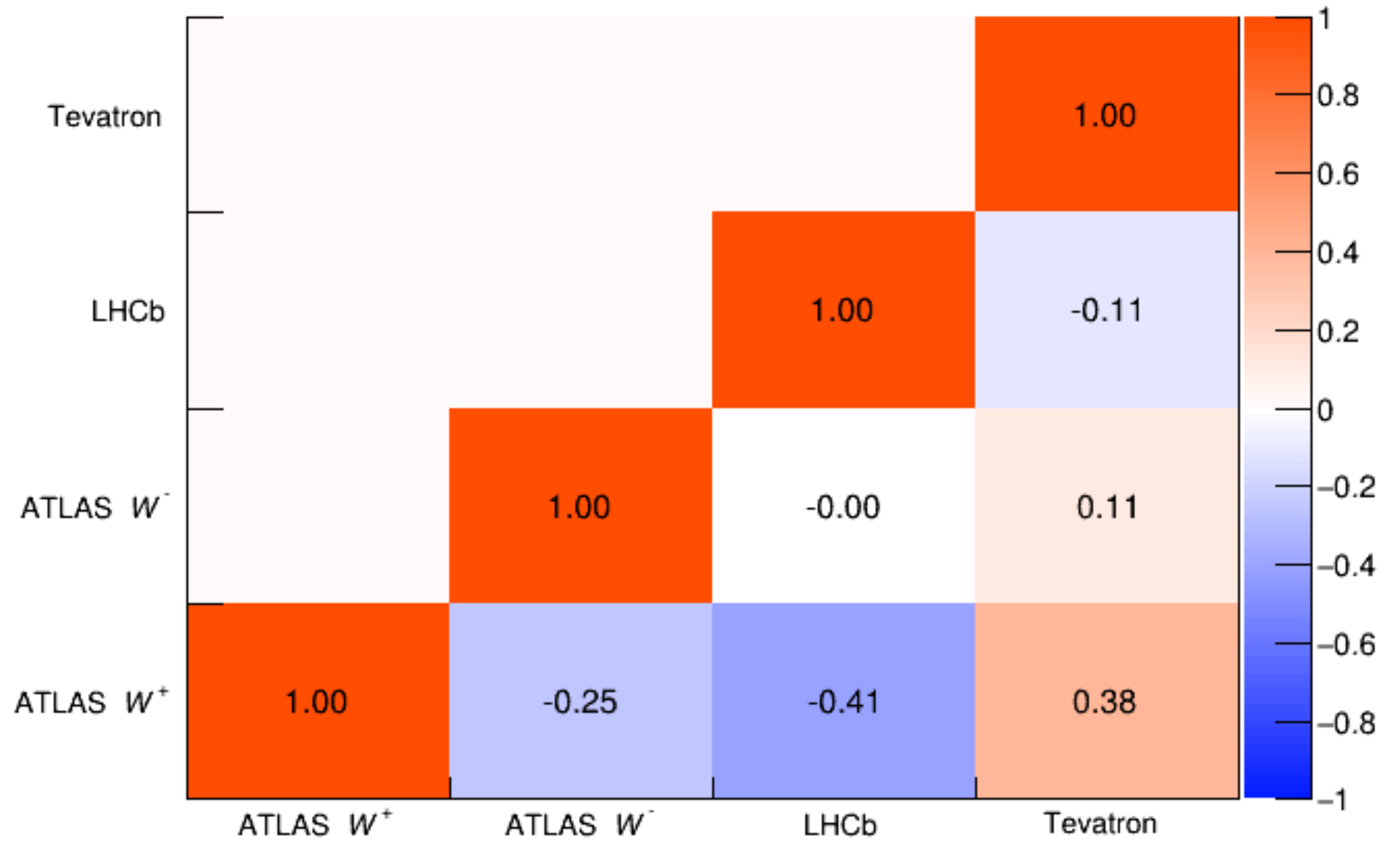
	ATLAS W^+	ATLAS W^-	LHCb	Tevatron
Tevatron				1.00
LHCb			1.00	0.08
ATLAS W^-		1.00	-0.52	0.10
ATLAS W^+	1.00	0.28	-0.46	-0.03

MSHT20



	ATLAS W^+	ATLAS W^-	LHCb	Tevatron
Tevatron				1.00
LHCb			1.00	-0.22
ATLAS W^-		1.00	-0.56	0.28
ATLAS W^+	1.00	0.05	-0.51	0.36

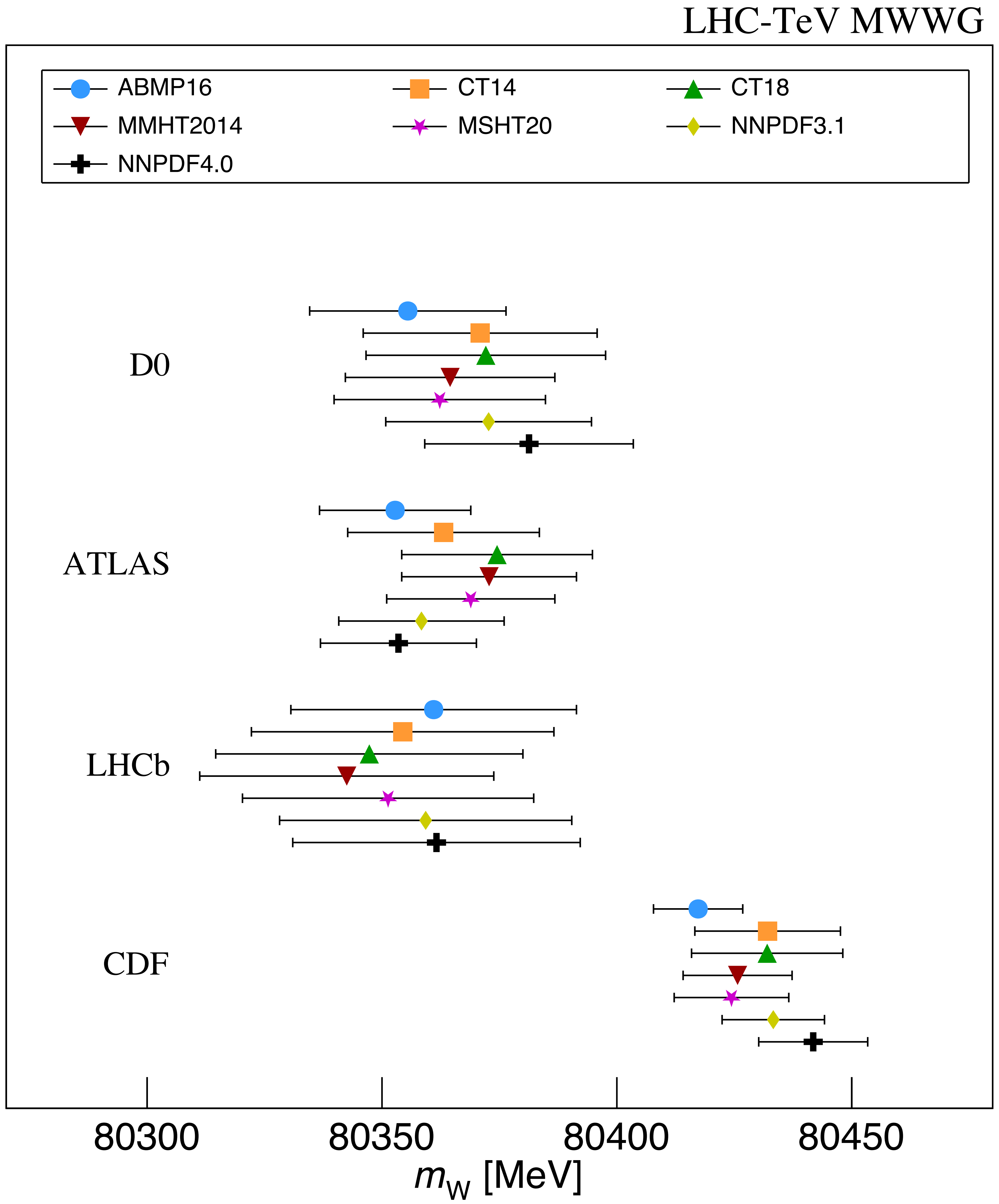
NNPDF4.0



	ATLAS W^+	ATLAS W^-	LHCb	Tevatron
Tevatron				1.00
LHCb			1.00	-0.11
ATLAS W^-		1.00	-0.00	0.11
ATLAS W^+	1.00	-0.25	-0.41	0.38

Combination

Input measurements with updates applied



All experiments (4 d.o.f.)				
PDF set	m_W	σ_{PDF}	χ^2	$p(\chi^2, n)$
ABMP16	80392.7 ± 7.5	3.2	29	0.0008%
CT14	80393.0 ± 10.9	7.1	16	0.3%
CT18	80394.6 ± 11.5	7.7	15	0.5%
MMHT2014	80398.0 ± 9.2	5.8	17	0.2%
MSHT20	80395.1 ± 9.3	5.8	16	0.3%
NNPDF3.1	80403.0 ± 8.7	5.3	23	0.1%
NNPDF4.0	80403.1 ± 8.9	5.3	28	0.001%

No combination of all measurements provides a good χ^2 probability

the full combination is disfavoured

Sub-combinations

Combinations with CDF excluded have good compatibility: $m_W = 80369.2 \pm 13.3$ MeV (CT18)

the χ^2 probability is 91%

relative weights: 42% (ATLAS), 23% (D0), 18% (LHCb), 16% (LEP)

The difference between "All-CDF" and the updated CDF value here is $3.6\ \sigma$ with CT18

Conclusions about the m_W combination effort

Extensive effort to provide a common treatment of PDF and pQCD modelling for the m_W determination at hadron colliders

The updated treatment is unable to solve the tension between the existing measurements

The full combination $m_W = 80394.6 \pm 11.5 \text{ MeV}$ (CT18) is disfavoured due to low χ^2 probability (0.5%)

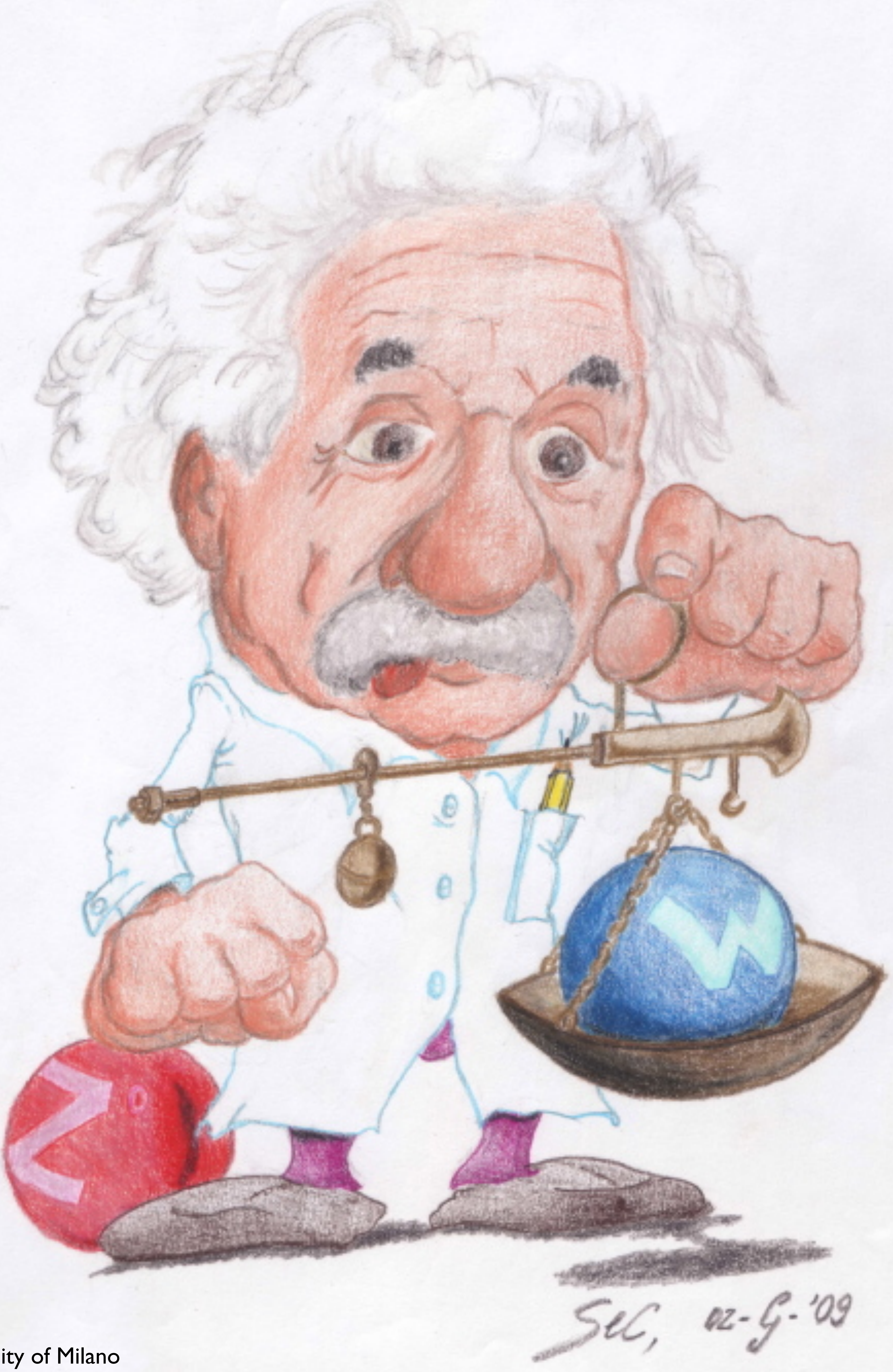
The combination with CDF excluded $m_W = 80369.2 \pm 13.3 \text{ MeV}$ (CT18) has good χ^2 probability (91%)

Conclusions on the m_W determination from the jacobian asymmetry

- The shape of the CC-DY kinematical distributions depends on a non-trivial combination of QCD effects and the m_W value
  → disentangling QCD from m_W is the problem under discussion

- The templates used to fit the data are prepared relying on specific choices in pQCD (i.e. perturbative order and μ_R, μ_F, μ_Q)
  → scale variations in the preparation of the templates are a necessary step
  to properly estimate the pQCD uncertainty

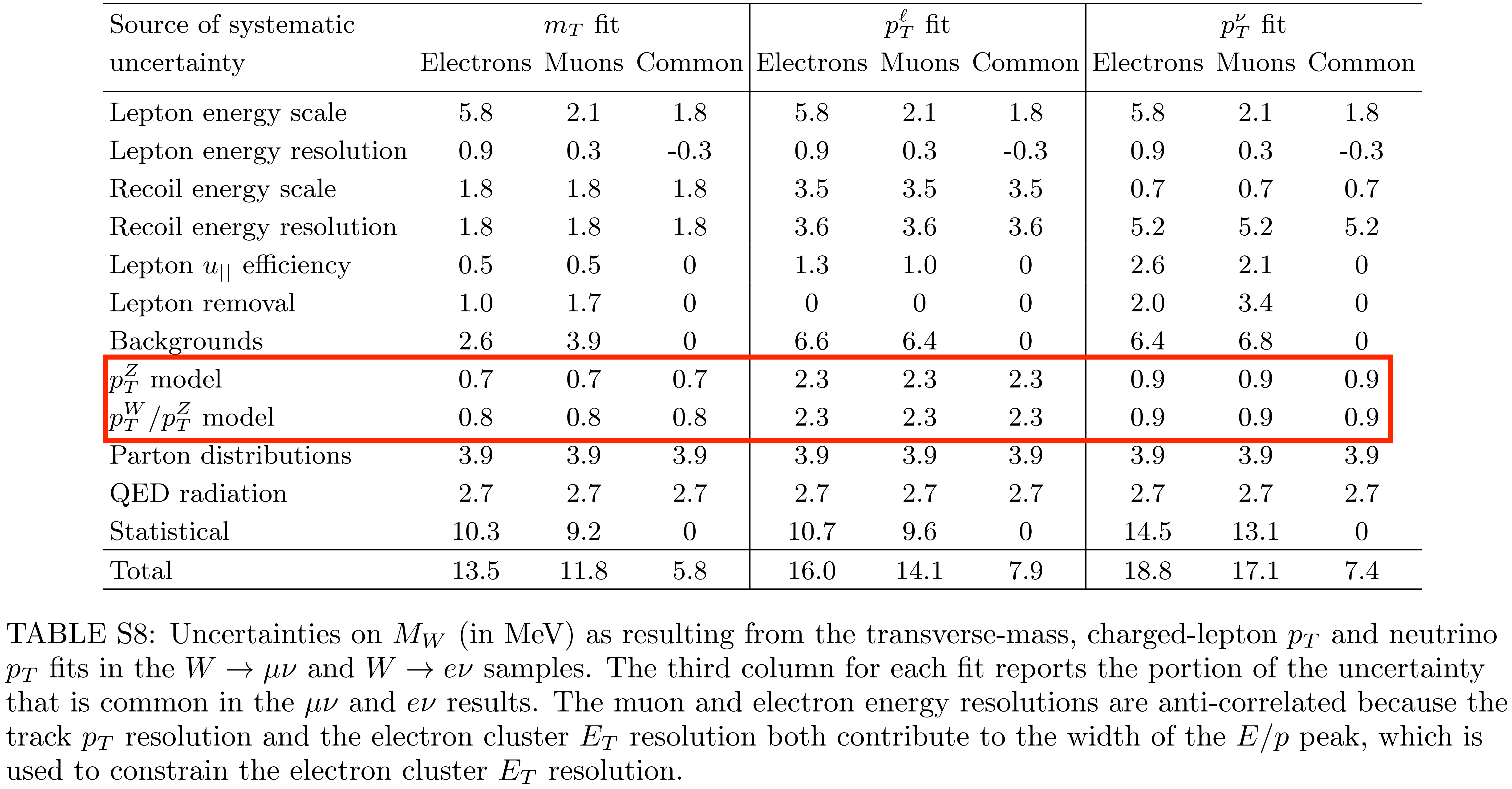
- The study of the pQCD uncertainties is problematic within a template fit procedure (very precise data vs large pQCD unc.)
  → the usage of data improves the accuracy of the data description, it does not improve the precision of the model
  → the asymmetries $\mathscr{A}_{p_\perp^\ell}$, $\mathscr{A}_{M_\perp^{\ell\nu}}$ might help the discussion, with a simpler procedure of assessment
  of the pQCD uncertainty and of all higher-order effects
  → with such observables it is easy to profit of the impressive progress in pQCD calculations



Thank you

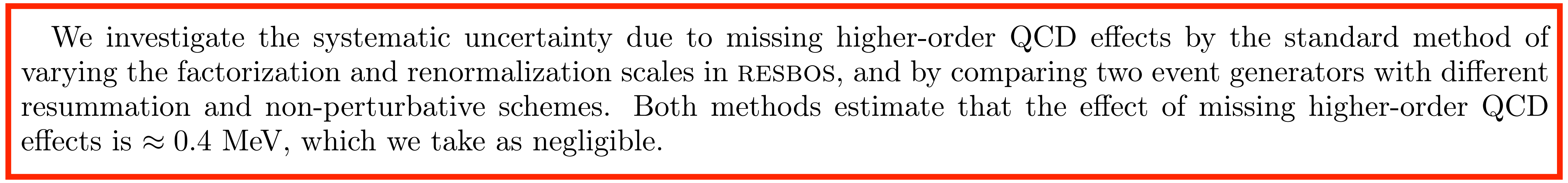
Backup

Uncertainty estimates by the CDF collaboration, Science 376, 170-176 (2022)



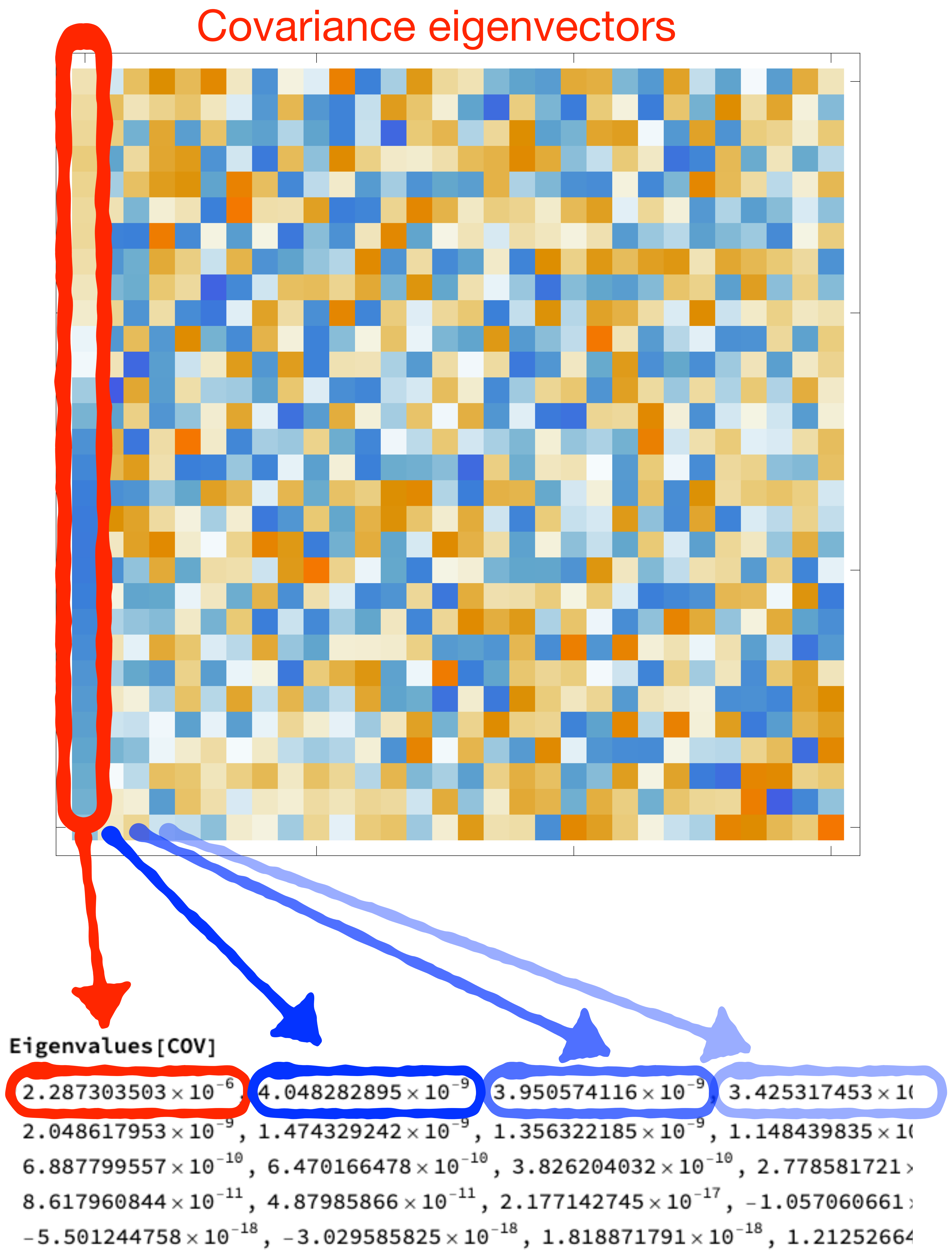
Source of systematic	m_T fit			p_T^ℓ fit			p_T^ν fit		
uncertainty	Electrons	Muons	Common	Electrons	Muons	Common	Electrons	Muons	Common
Lepton energy scale	5.8	2.1	1.8	5.8	2.1	1.8	5.8	2.1	1.8
Lepton energy resolution	0.9	0.3	-0.3	0.9	0.3	-0.3	0.9	0.3	-0.3
Recoil energy scale	1.8	1.8	1.8	3.5	3.5	3.5	0.7	0.7	0.7
Recoil energy resolution	1.8	1.8	1.8	3.6	3.6	3.6	5.2	5.2	5.2
Lepton $u_{\|\|}$ efficiency	0.5	0.5	0	1.3	1.0	0	2.6	2.1	0
Lepton removal	1.0	1.7	0	0	0	0	2.0	3.4	0
Backgrounds	2.6	3.9	0	6.6	6.4	0	6.4	6.8	0
p_T^Z model	0.7	0.7	0.7	2.3	2.3	2.3	0.9	0.9	0.9
p_T^W/p_T^Z model	0.8	0.8	0.8	2.3	2.3	2.3	0.9	0.9	0.9
Parton distributions	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9
QED radiation	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7
Statistical	10.3	9.2	0	10.7	9.6	0	14.5	13.1	0
Total	13.5	11.8	5.8	16.0	14.1	7.9	18.8	17.1	7.4

TABLE S8: Uncertainties on M_W (in MeV) as resulting from the transverse-mass, charged-lepton p_T and neutrino p_T fits in the $W \to \mu\nu$ and $W \to e\nu$ samples. The third column for each fit reports the portion of the uncertainty that is common in the $\mu\nu$ and $e\nu$ results. The muon and electron energy resolutions are anti-correlated because the track p_T resolution and the electron cluster E_T resolution both contribute to the width of the E/p peak, which is used to constrain the electron cluster E_T resolution.



We investigate the systematic uncertainty due to missing higher-order QCD effects by the standard method of varying the factorization and renormalization scales in RESBOS, and by comparing two event generators with different resummation and non-perturbative schemes. Both methods estimate that the effect of missing higher-order QCD effects is ≈ 0.4 MeV, which we take as negligible.

Loss of information ?



Covariance eigenvectors

```
Eigenvalues[COV]
2.287303503×10^-6, 4.048282895×10^-9, 3.950574116×10^-9, 3.425317453×1
2.048617953×10^-9, 1.474329242×10^-9, 1.356322185×10^-9, 1.148439835×1
6.887799557×10^-10, 6.470166478×10^-10, 3.826204032×10^-10, 2.778581721
8.617960844×10^-11, 4.87985866×10^-11, 2.177142745×10^-17, -1.057060661
-5.501244758×10^-18, -3.029585825×10^-18, 1.818871791×10^-18, 1.21252664
```

- The $p_\perp^\ell$ spectrum includes N bins.
- After the rotation which diagonalises the m_W covariance, we have N linear combinations of the primary bins.
- We keep only one combination, the asymmetry, out of N.
  Are we losing information ?

- The amount of information available depends:
  -on the sensitivity of each observable to m_W
  -on the uncertainties affecting the observable

- the jacobian asymmetry has
  the largest sensitivity to m_W among the N combinations
  a very low pQCD uncertainty

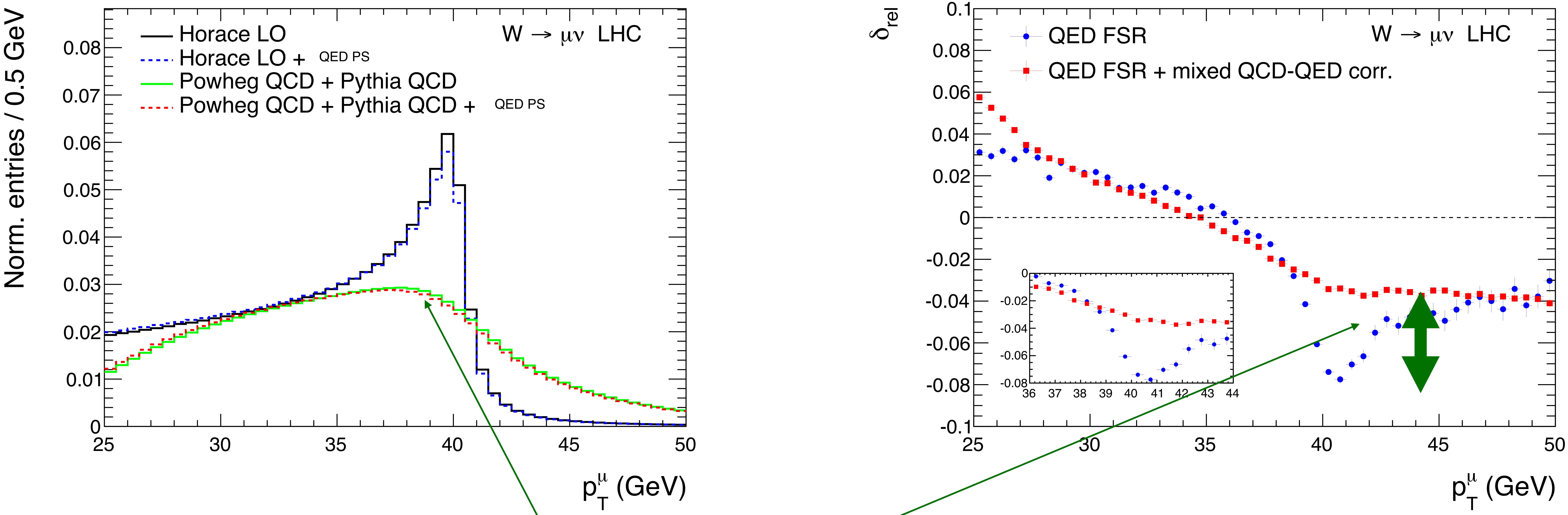
- the remaining N-1 combinations have
  quite low sensitivity to m_W (cfr. the eigenvalues)
  possibly large QCD uncertainties (in progress)

If the amount of information is related to "signal/noise",
the asymmetry has very low pQCD noise.

The remaining N-1 combinations describe the QCD features
of the $p_\perp^\ell$ spectrum → disentangling m_W from pQCD
→possible increase of the total QCD uncertainty

Interplay of QCD and QED corrections

C.Carloni Calame, M.Chiesa, H.Martinez, G.Montagna, O.Nicrosini, F.Piccinini, AV, arXiv:1612.02841



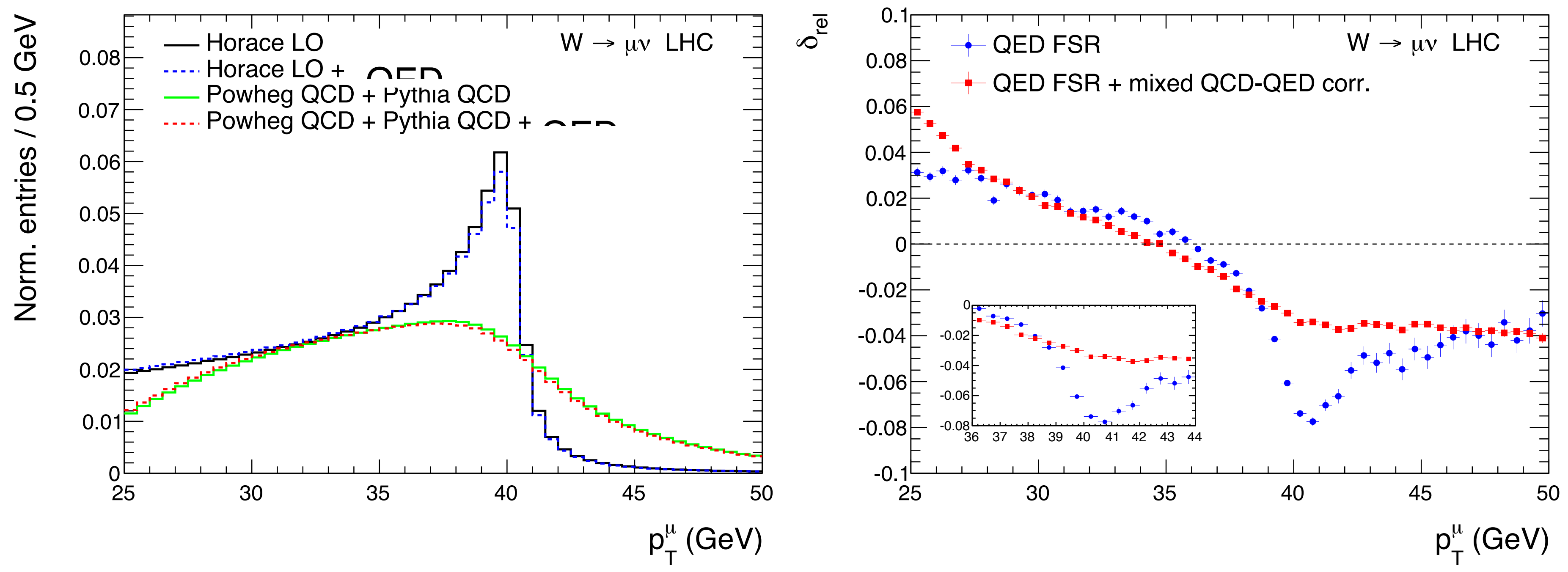
- very large impact of initial-state QCD radiation on the ptlep distribution
- large radiative corrections due to QED final state radiation at the jacobian peak
- very large interplay of QCD and QED corrections redefining the precise shape of the jacobian peak

NLO-QCD + QCDPS + QEDPS is the lowest order meaningful approximation of this observable

the precise size of the mixed QCDxQED corrections (and uncertainties) depends on the choice for the QCD modelling

Impact of EW and mixed QCDxEW corrections on MW

C.Carloni Calame, M.Chiesa, H.Martinez, G.Montagna, O.Nicrosini, F.Piccinini, AV, arXiv:1612.02841



	$pp \to W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: LO Pseudo–data accuracy	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 $W^+ \to \mu^+\nu$ p_T^ℓ	 $W^+ \to e^+\nu$ M_T	 $W^+ \to e^+\nu$ p_T^ℓ
1	HORACE only FSR-LL at $\mathcal{O}(\alpha)$	-94±1	-104±1	-204±1	-230±2
2	HORACE FSR-LL	-89±1	-97±1	-179±1	-195±1
3	HORACE NLO-EW with QED shower	-90±1	-94±1	-177±1	-190±2
4	HORACE FSR-LL + Pairs	-94±1	-102±1	-182±2	-199±1
5	PHOTOS FSR-LL	-92±1	-100±2	-182±1	-199±2

- QED FSR plays the major role
- subleading QED and weak induce further O(4 MeV) shifts

the impact on MW of the mixed QCD QED-FSR corrections strongly depends on the underlying QCD shape/model

	$pp \to W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: NLO-QCD+QCD$_{\rm PS}$ Pseudodata accuracy	 QED FSR	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 $W^+ \to \mu^+\nu$ p_T^ℓ	 $W^+ \to e^+\nu$(dres) M_T	 $W^+ \to e^+\nu$(dres) p_T^ℓ
1	NLO-QCD+(QCD+QED)$_{\rm PS}$	PYTHIA	-95.2±0.6	-400±3	-38.0±0.6	-149±2
2	NLO-QCD+(QCD+QED)$_{\rm PS}$	PHOTOS	-88.0±0.6	-368±2	-38.4±0.6	-150±3
3	NLO-(QCD+EW)+(QCD+QED)$_{\rm PS}$`two-rad`	PYTHIA	-89.0±0.6	-371±3	-38.8±0.6	-157±3
4	NLO-(QCD+EW)+(QCD+QED)$_{\rm PS}$`two-rad`	PHOTOS	-88.6±0.6	-370±3	-39.2±0.6	-159±2

the bulk of the corrections is included in the analyses

- what is the associated uncertainty ?
- what happens if
  we change the underlying QCD model ?

can we constrain the formulation, for the $\alpha\alpha_s$ contribution ?

very stable behaviour of the $M_\perp$ distribution in contrast to the $p_\perp^l$ case

Sensitivity to the W boson mass: covariance w.r.t. MW variations

The sensitivity to m_W can be quantified by means of a matrix of covariance w.r.t. m_W variations

$$\mathscr{C}_{ij} \equiv \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle \quad \text{with} \quad \langle \sigma \rangle \equiv \frac{1}{N_W} \sum_{k=1}^{N_W} \sigma(m_W = m_W^{(k)})$$

and σ_i represents the i-th bin of the $p_\perp^\ell$ distribution

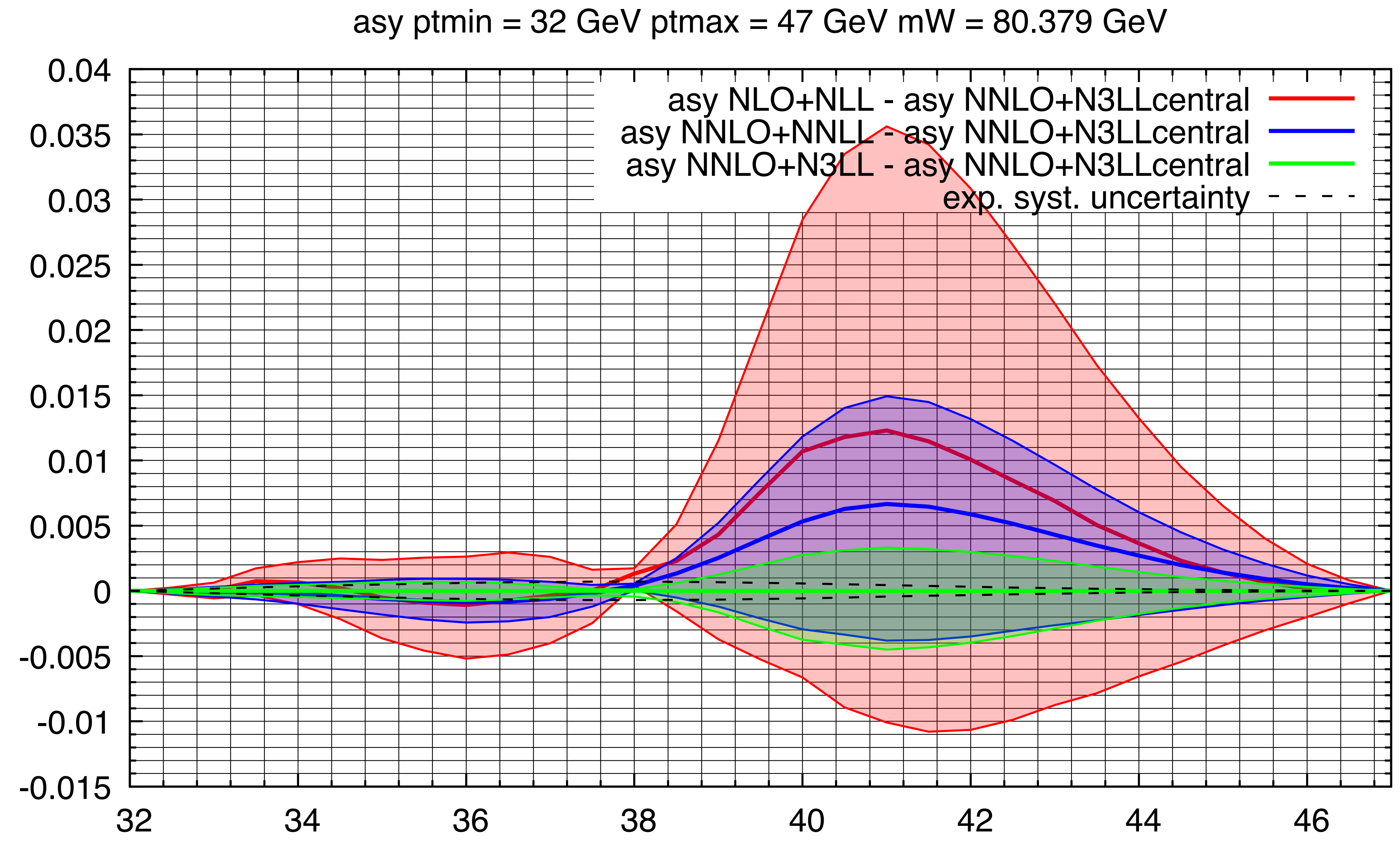
The diagonalization of the covariance matrix yields N_{bins} linear combinations of the σ_i transforming independently of each other under m_W variations

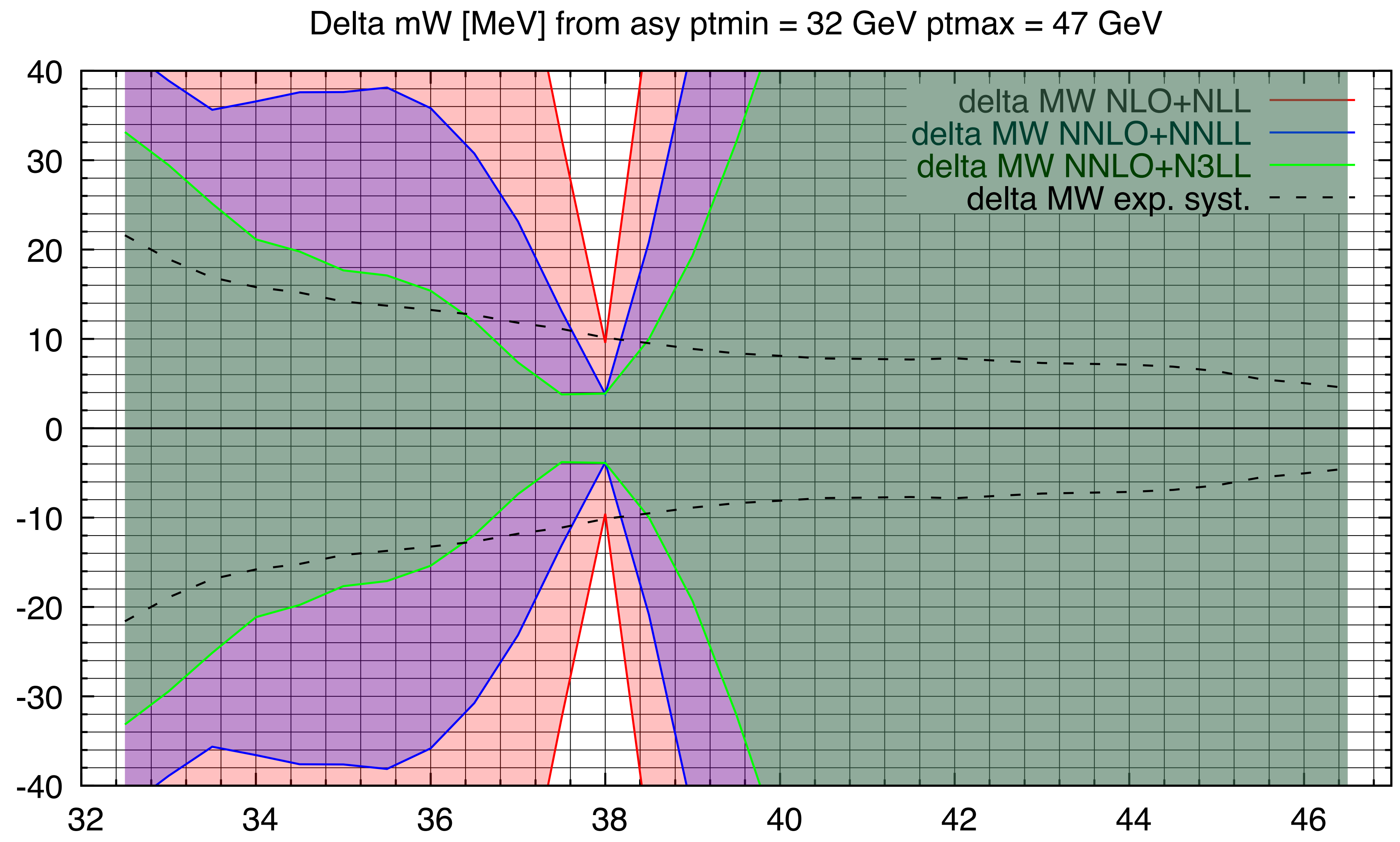
The eigenvalues express the sensitivity for a given Δm_W shift, and help classifying the different combinations

The first eigenvalue is 560 times the second one (in size)
The associated linear combination has a peculiar structure:
all coefficients are positive (negative) for $p_\perp^\ell < 37 \quad (p_\perp^\ell > 37)$ GeV
Explicit check that the value $p_\perp^\ell \sim 37$ is very stable changing QCD approximation or bin range

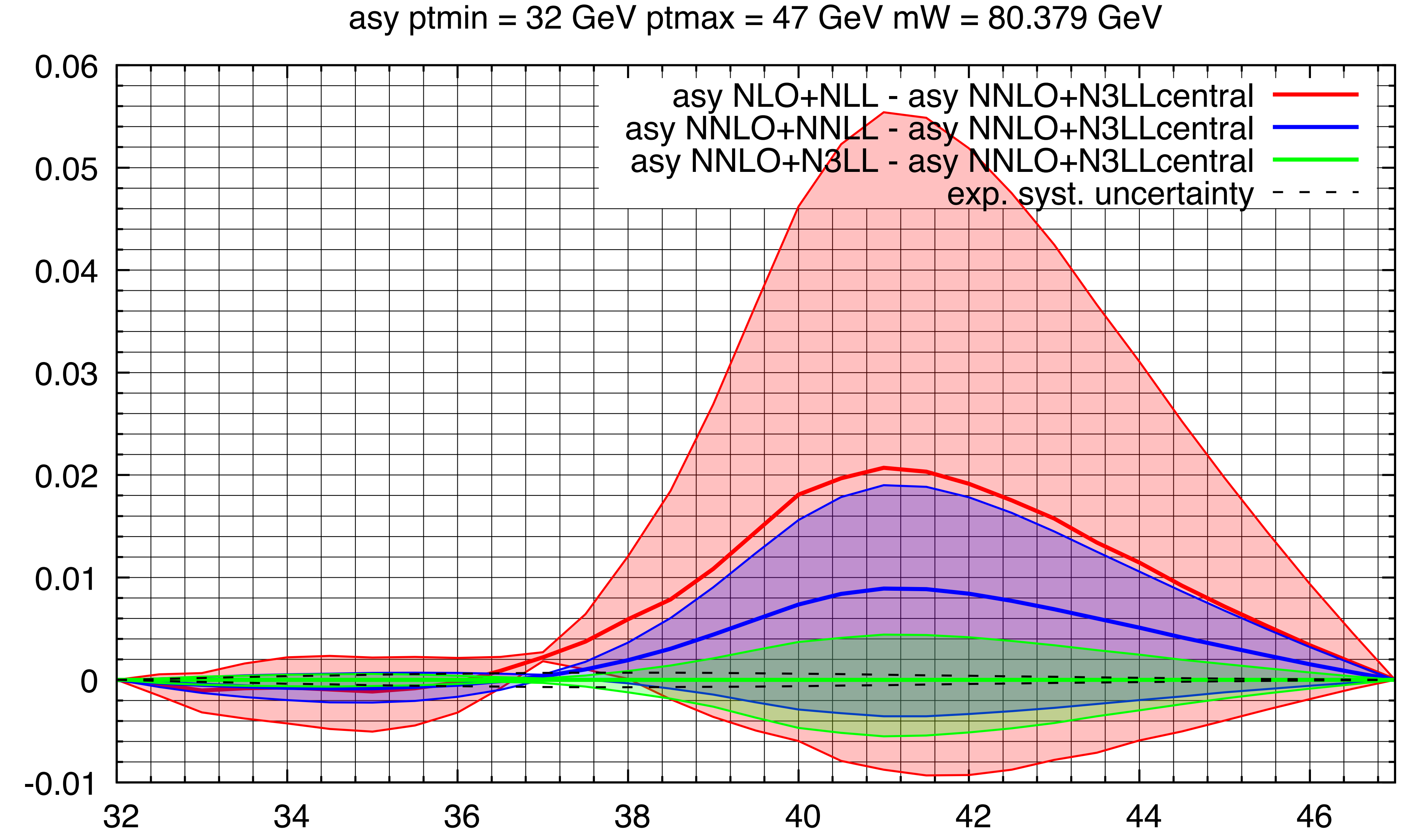
This value can be appreciated also in the plot of the ratio → indication for the definition of a new observable

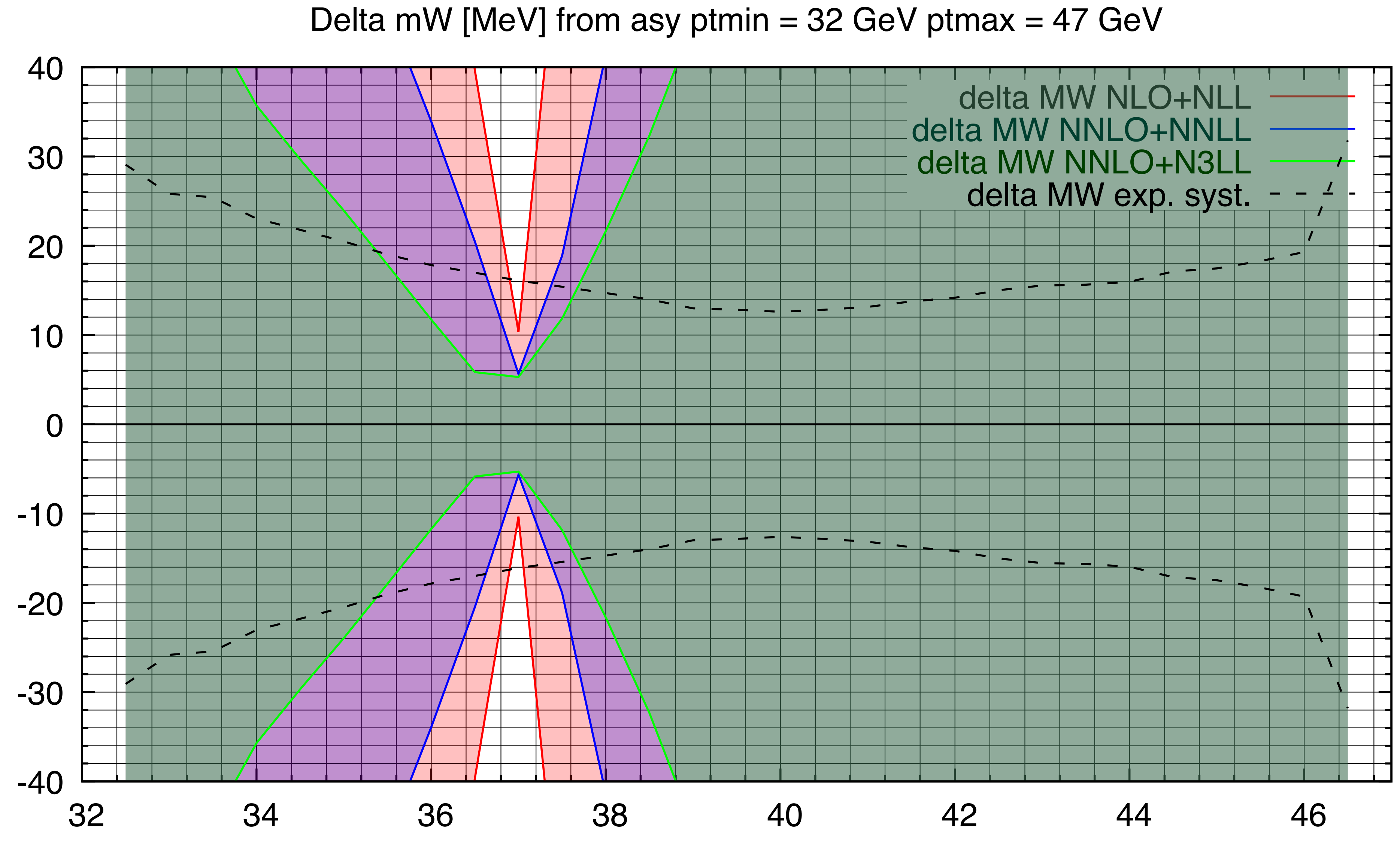
The lepton transverse momentum spectrum as a function of $p_\perp^{\ell,\,mid}$





$p_\perp^W < 15$ GeV

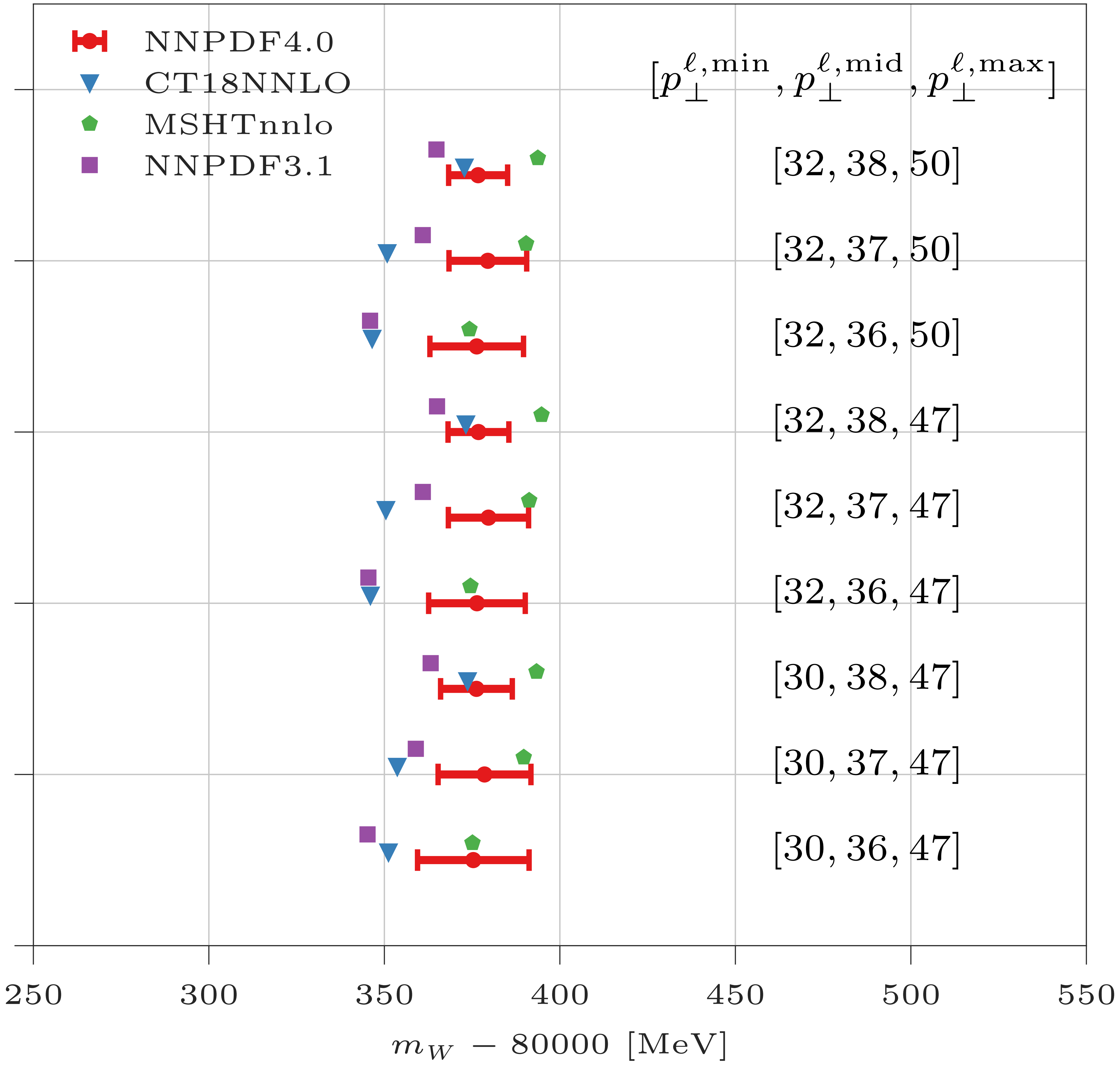




$p_\perp^W < 30$ GeV

for $p_\perp^{\ell,\,mid}$ we observe a good pQCD convergence (comparison of central values) for $p_\perp^{\ell,mid} < 37$ GeV

PDF uncertainties



L.Rottoli, P.Torrielli, AV; arXiv:2301.04059

- the PDF uncertainties on m_W are evaluated in a conservative way using the 100 replicae of the NNPDF4.0 - NLO set
  → $\delta m_W^{PDF} = \pm\, 11$ MeV

- the spread of the central values of CT18NNLO, MSHTnnlo, NNPDF4.0 if of ~ 30 MeV

- this size of the uncertainty is expected:
  $\mathscr{A}_{p_\perp^\ell}$ is one single observable, particularly sensitive to PDF variations

→ more information is needed to mitigate this problem

1) in situ profiling
(e.g. use additional bins of the $p_\perp^\ell$ distribution)

2) combination of results in different rapidity acceptance regions
(e.g. LHCb combined with ATLAS/CMS)

3) combination of results for W^+ and W^-

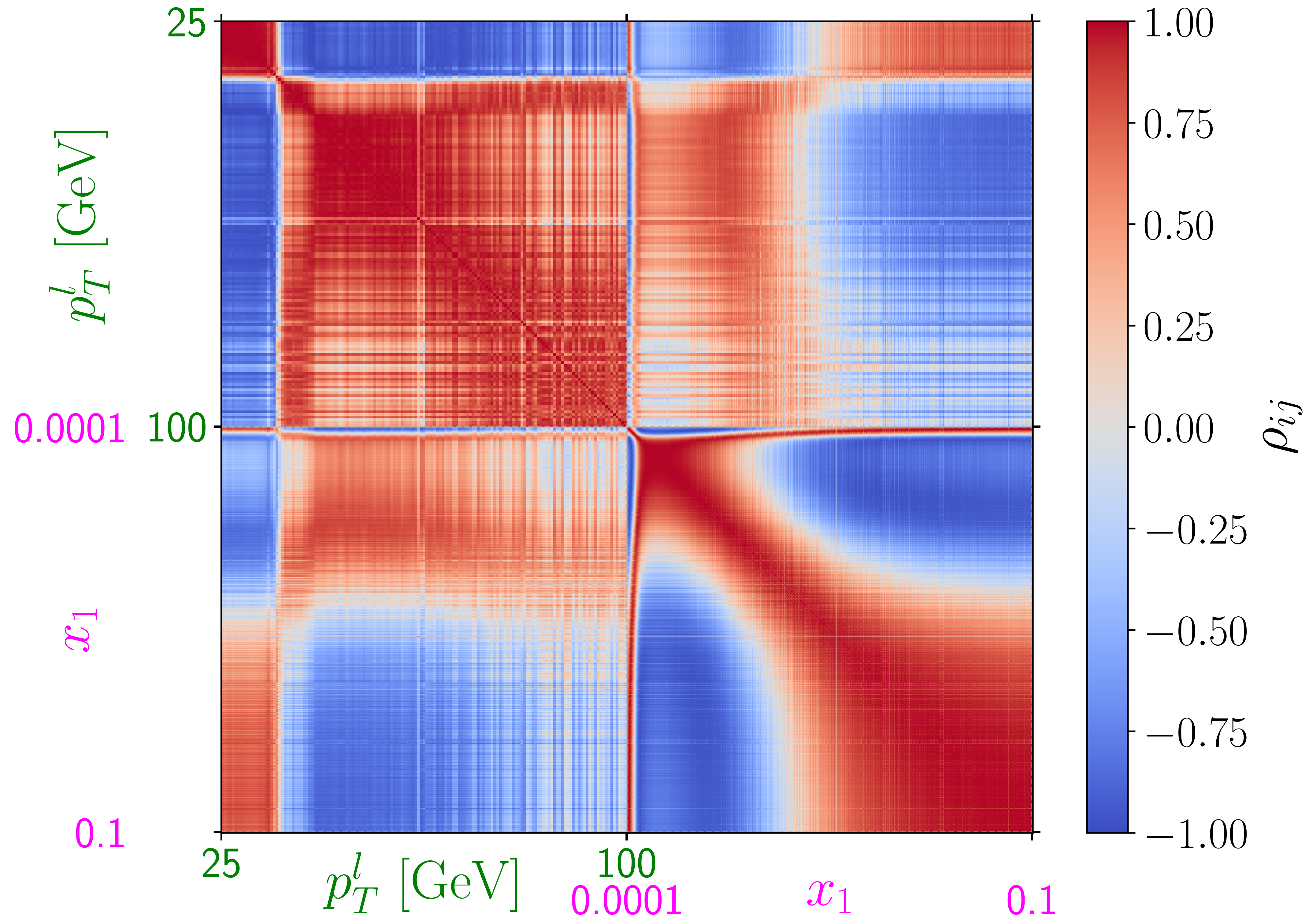
PDF uncertainty on MW: exploiting the theoretical constraints

E.Bagnaschi, AV, Phys.Rev.Lett.126 (2021) 4, 041801

all PDF replicas are correlated because the parton densities are developed in the same QCD framework
1) obey sum rules, 2) satisfy DGLAP equations, 3) are based on the same data set

the "unitarity constraint" of each parton density affects the parton-parton luminosities, which, convoluted with the partonic xsec, in turn affect the hadron-level xsec

$$\rho_{ij} = \frac{\langle \left(\mathcal{O}_i - \langle \mathcal{O}_i \rangle_{PDF}\right) \left(\mathcal{O}_j - \langle \mathcal{O}_j \rangle_{PDF}\right) \rangle_{PDF}}{\sigma_i \sigma_j}$$



$$\chi^2_{k,min} = \sum_{r,s \in bins} \left(\mathcal{T}_{0,k} - \mathcal{D}^{exp}\right)_r C^{-1}_{rs} \left(\mathcal{T}_{0,k} - \mathcal{D}^{exp}\right)_s$$

$$C = \Sigma_{PDF} + \Sigma_{stat} + \Sigma_{MC} + \Sigma_{exp\,syst}$$ total covariance

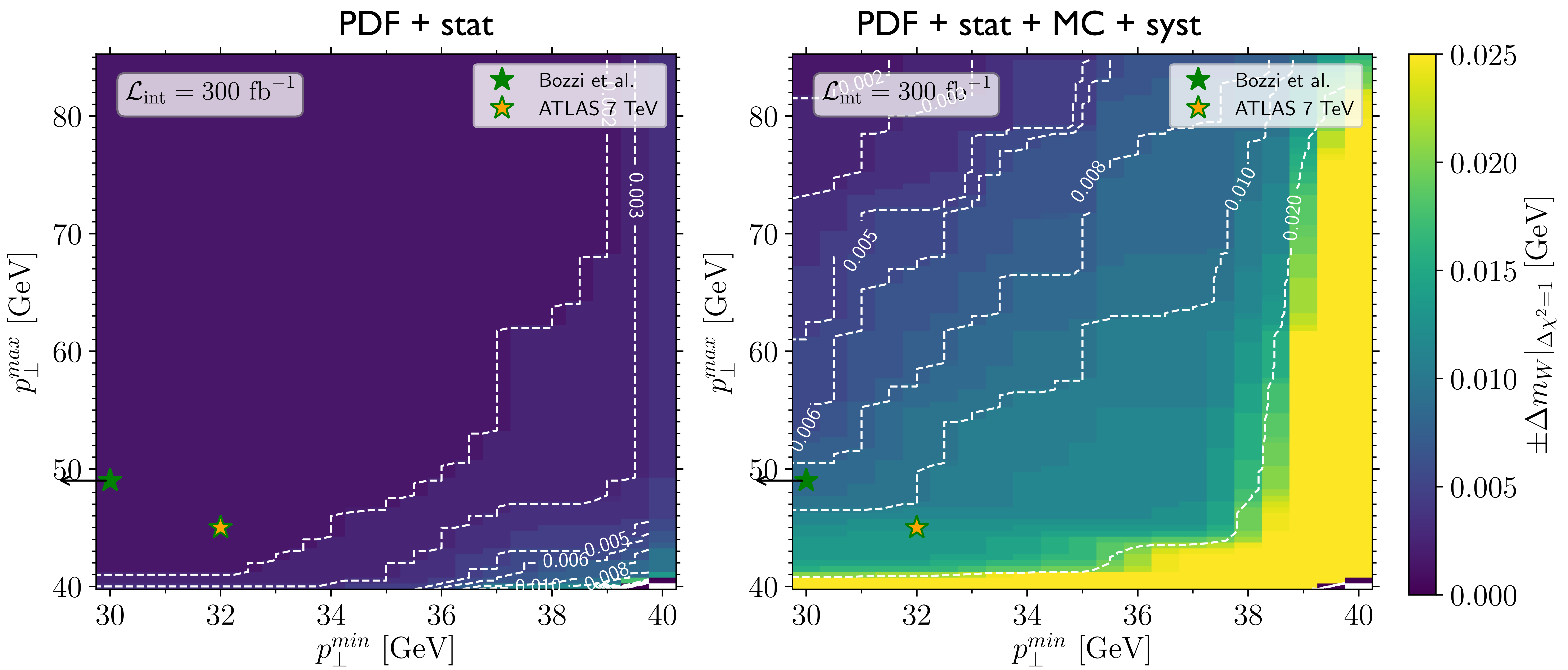
Inserting the information about PDFs in the covariance matrix leads to a profiling action "in situ", given by the data themselves

the PDF uncertainty can be reduced to the few MeV level thanks to the strong anti correlated behaviour of the two tails of $p_\perp^\ell$

PDF uncertainty on MW: exploiting the theoretical constraints

E.Bagnaschi, AV, Phys.Rev.Lett.126 (2021) 4, 041801

scan over fitting windows for normalised distributions



PDF + stat

PDF + stat + MC + syst

$$\chi^2_{k,\,min} = \sum_{r,s\in bins} \left(\mathcal{T}_{0,k} - \mathcal{D}^{exp}\right)_r C^{-1}_{rs} \left(\mathcal{T}_{0,k} - \mathcal{D}^{exp}\right)_s$$

total uncertainty determined with $\Delta\chi^2 = 1$ rule

$$C = \Sigma_{PDF} + \Sigma_{stat} + \Sigma_{MC} + \Sigma_{exp\,syst}$$

total covariance

- The PDF uncertainty is not a limiting factor for MW with high luminosity and a "perfect" detector
- The MC statistics needed is of at least O(100B) of simulated events (several weeks on 1000 cores cluster)

m_W determination and the usage of NC-DY data

- Assuming the validity of the scale uncertainty bands as estimator of the pQCD on m_W,
  we see that - the predictions of $\mathscr{A}_{p_\perp^\ell}$ from CC-DY alone, including N3LL contributions, are promising
  - the procedure to estimate the pQCD uncertainty is robust

- is the estimate of the m_W central value from $\mathscr{A}_{p_\perp^\ell}$ reliable in pure pQCD ?
  are the CC-DY data well described ?

- can we improve the analysis by means of the inclusion of NC-DY data, notably the $p_\perp^Z$ distribution ?

The inclusion of the information from the $p_\perp^Z$ distribution
improves the accuracy of the data description
does not improve the precision of the model (i.e. it does not reduce the QCD uncertainty)

We discuss this statement using $\mathscr{A}_{p_\perp^\ell}$ as a tool to inspect the NC *vs* CC interplay

Information transfer from NCDY to CCDY : a validation exercise

- NNLO+N3LL with central scales $\mu_R = \mu_F = \mu_Q = 1$ is our MC truth = pseudodata both for NCDY and CCDY
- we take NNLO+NNLL as theory model

- for different scale choices we compute the reweighing functions from NNLO+NNLL to the $p_\perp^Z$ pseudodata

$$\mathcal{R}(\mu_R, \mu_F, \mu_Q; p_\perp^Z) = \left(\frac{d\sigma^{NNLO+N3LL}(1,1,1)}{dp_\perp^Z} \right) \left(\frac{d\sigma^{NNLO+NNLL}(\mu_R, \mu_F, \mu_Q)}{dp_\perp^Z} \right)^{-1}$$

NC-DY

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NC-DY

- we then use the appropriate reweighing function in CCDY at NNLO+NNLL for each different scale choice

$$\frac{d\sigma^{NNLO+NNLL-rwg}(\mu_R, \mu_F, \mu_Q)}{dp_\perp^W} = \mathscr{R}(\mu_R, \mu_F, \mu_Q; p_\perp^W) \frac{d\sigma^{NNLO+NNLL}(\mu_R, \mu_F, \mu_Q)}{dp_\perp^W}$$

CC-DY

- we compare the reweighed results and the CCDY pseudodata and study the residual scale dependence

$$\frac{d\sigma^{NNLO+NNLL-rwg}(\mu_R, \mu_F, \mu_Q)}{dp_\perp^W} \leftrightarrow \frac{d\sigma^{NNLO+N3LL}(1,1,1)}{dp_\perp^W}$$

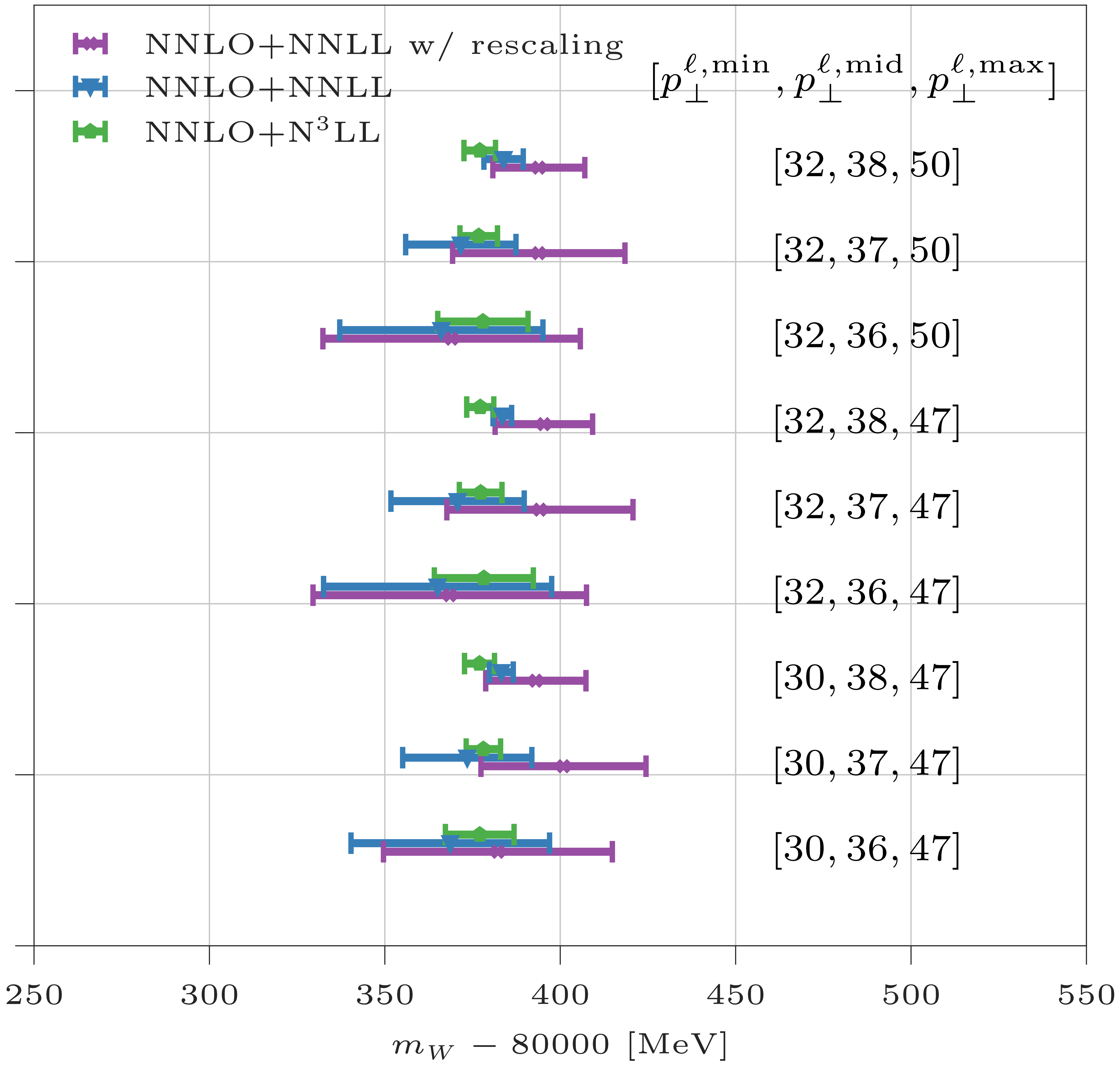
CC-DY

- naive expectation: since by construction all the scale choices match the $p_\perp^Z$ pseudodata,
then also in CC-DY we should find the same (i.e. no scale dependence) for the $p_\perp^W$ distribution

- which is the impact of the reweighing on the CC-DY $p_\perp^\ell$ distribution ? is it the same as in the $p_\perp^W$ case?

Information transfer from NCDY to CCDY : a validation exercise

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059



- we determine m_W using the three sets of distributions:
  - plain NNLO+NNLL
  - reweighed NNLO+NNLL
  - NNLO+N3LL

- the pQCD uncertainty on m_W estimated with or without reweighing is of similar size (in our case the NNLO+NNLL QCD uncertainty)

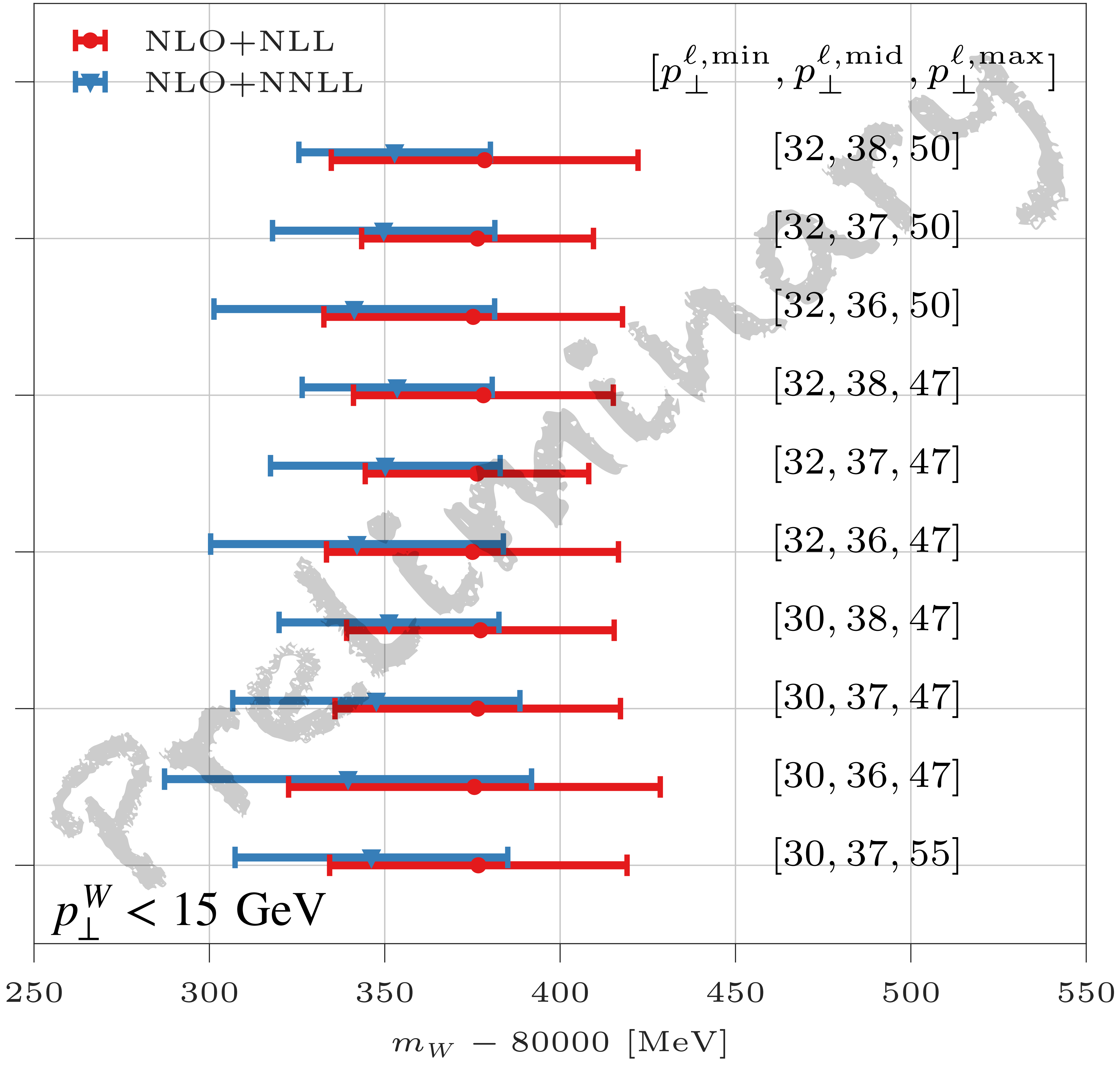
→ the usage of the $p_\perp^Z$ information
improves the accuracy of the data description
crucial for the central value estimate
does not improve the precision of the templates
(beyond that of the theoretical fitting model)

→ usage of the highest available perturbative order is recommended to minimize the pQCD systematics in the transfer from Z to W

m_W determination at the Tevatron as a function of the $\mathscr{A}_{p_\perp^\ell}$ parameters (no $p_\perp^Z$ reweighing)

as pseudo-experimental value we choose the NNLO+N3LL result with $m_W = 80.379$

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059



- we compute $\mathscr{A}_{p_\perp^\ell}$ at the Tevatron, from CC-DY, as a function of m_W
  we vary the QCD scales in the canonical ranges
- in the most optimistic configuration, at NLO+NNLL,
  a range of values $\Delta m_W \sim \pm 30$ MeV is found
- NLO+NNLL is the same perturbative accuracy available in ResBos
- it is difficult to expect a very significant uncertainty reduction thanks to the $p_\perp^Z$ data information only (cfr. previous slides)

→ usage of the highest available perturbative order is recommended to minimize the pQCD systematics in the transfer from Z to W

m_W determination at the Tevatron as a function of the $\mathscr{A}_{M_\perp^{\ell\nu}}$ parameters (no $p_\perp^Z$ reweighing)

as pseudo-experimental value we choose the NNLO+N3LL result with $m_W = 80.379$

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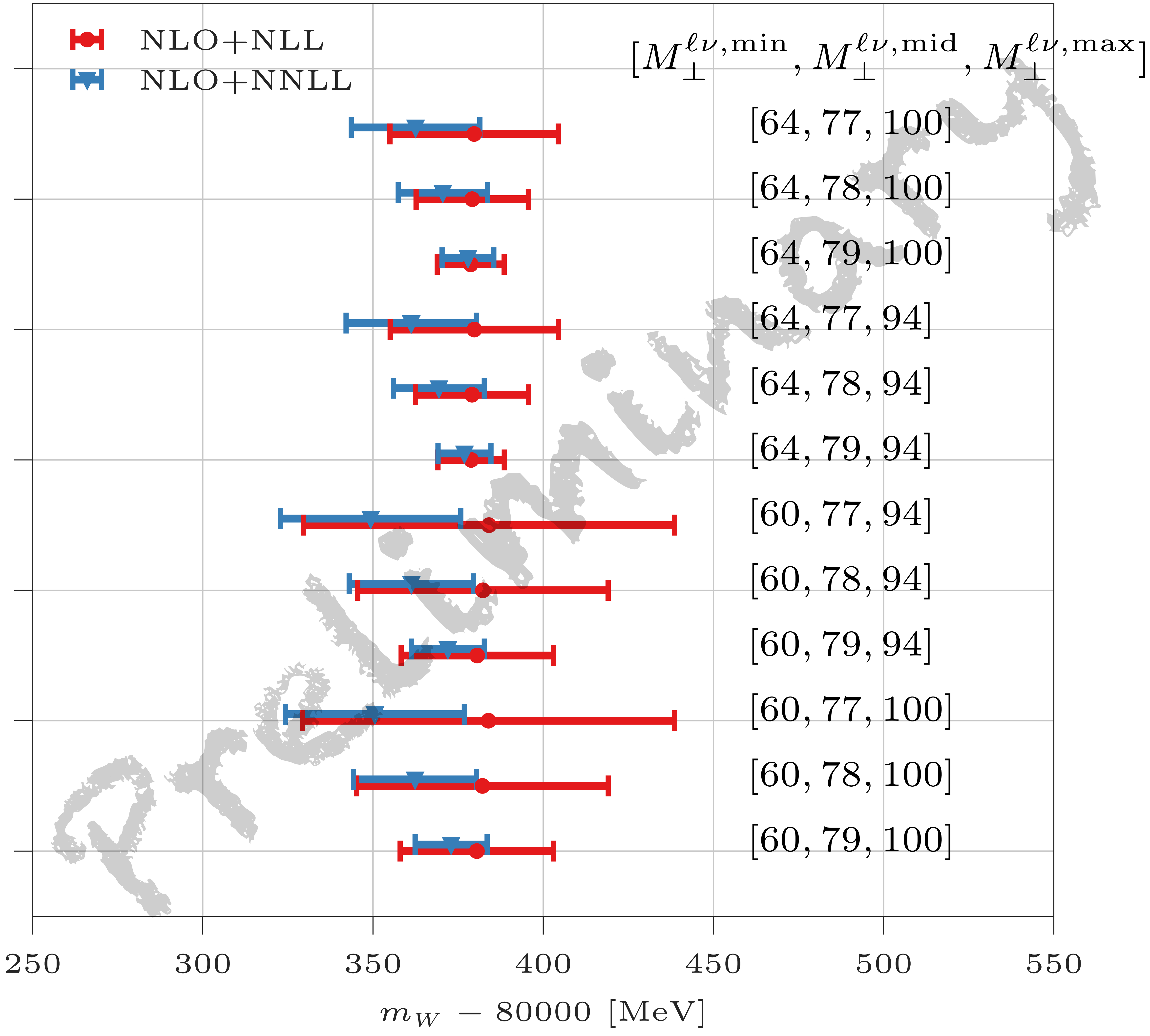

- we compute $\mathscr{A}_{M_\perp^{\ell\nu}}$ at the Tevatron, from CC-DY, as a function of m_W we vary the QCD scales in the canonical ranges
- NLO+NNLL is the same perturbative accuracy available in ResBos
- we neglect important detector simulation effects → optimistic estimates for the uncertainty
- in the most optimistic configuration, at NLO+NNLL, a range of values $\Delta m_W \sim \pm 10$ MeV is found