FACTORIZATION AND RESUMMATION AT NEXT-TO-LEADING POWER

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"Theory Challenges in the Precision Era
of the Large Hadron Collider", GGI, 29/08/2023

OUTLINE

- **Soft-collinear radiation at NLP**
- **Endpoint divergences at NLP**
- **NLP LLs in Thrust in the two-jet limit**
- **NLP NNLO in Drell-Yan near threshold**

In collaboration with
M. Beneke, A. Broggio, M. Garny, S. Jaskiewicz, R. Szafron, J. Strohm J. Wang,

Based on
JHEP 20 (2020), 078, [arXiv:1912.01585 [hep-ph]],

JHEP 10 (2020), 196, [arXiv:2008.04943 [hep-ph]],

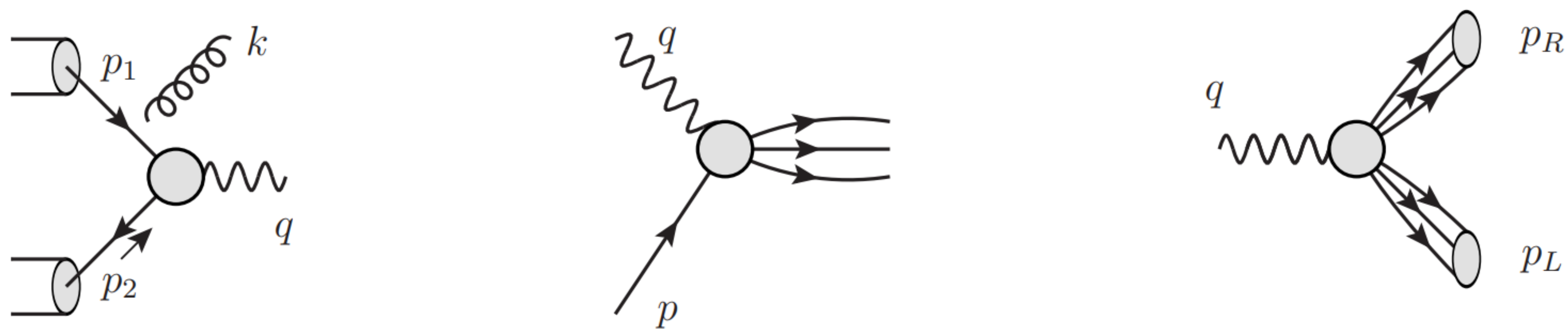
JHEP 10 (2021), 061, [arXiv:2107.07353 [hep-ph]],

JHEP 07 (2022), 144, [arXiv:2205.04479 [hep-ph]],

[arXiv:2306.06037 [hep-ph]].

PARTICLE SCATTERING NEAR KINEMATIC LIMITS

- Consider Drell-Yan, DIS near partonic threshold and Thrust in the back-to-back jet limit:



$(p_1 + p_2)^2 \equiv \hat{s}, \quad z \equiv \frac{q^2}{\hat{s}} \to 1$

$Q^2 = -q^2, \quad x \equiv \frac{Q^2}{2p\cdot q} \to 1$

$q^2 \gg p_L^2, p_R^2,$

$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|} \to 1$

- The partonic cross section has singular expansion

$$\Delta_{ab}(\xi) \sim \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \left[c_n \delta(1-\xi) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-\xi)}{1-\xi}\right]_+ + d_{nm} \ln^m(1-\xi) \right) + \dots \right],$$

LP **NLP**

with $\xi = z$ for DY, $\xi = x$ for DIS, and $\xi = T$ for Thrust.

- Resummation of large logarithms relevant for precision phenomenology.
  - → well understood at LP (up to N3LL),
  - → progress toward resummation at NLP, yet no systematic approach so far.

PARTICLE SCATTERING NEAR KINEMATIC LIMITS

- Subject of intense work in the past few years!
- Within SCET:

  Beneke, Campanario, Mannel, Pecjak, 2004; Larkoski, Neill, Stewart, 2014; Kolodrubetz, Moult, Stewart, 2016; Feige, Kolodrubetz, Moult, Stewart, 2017; Beneke, Garny, Szafron, Wang, 2017-2019; Moult, Rothen, Stewart, Tackmann, Zhu, 2016/17; Boughezal, Liu, Petriello, 2016/17; Moult, Stewart, Vita, Zhu, 2018; Moult, Stewart, Vita, 2019; Beneke, Broggio, Garny, Jaskiewicz, Szafron, LV, Wang, 2018; Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2019; Beneke, Broggio, Jaskiewicz, LV, 2019; Broggio, Jaskiewicz, LV, 2021/22; Beneke, Bobeth, Szafron, 2017; Alte, König, Neubert, 2018; Moult et al., 2019; Liu, Neubert, 2019; Wang, 2019; Liu, Mecaj, Neubert, Wang, 2020; Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020; Liu, Neubert, Schnubel, Wang, 2021; Ebert, Moult, Stewart, Tackmann, Vita, Zhu, 2018; Moult, Vita, Yan, 2019; Beneke, Hager, Szafron, 2021; Beneke, Garny, Jaskiewicz, Szafron, LV, Wang 2020; Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang 2022 + …

- And “diagrammatic” methods:

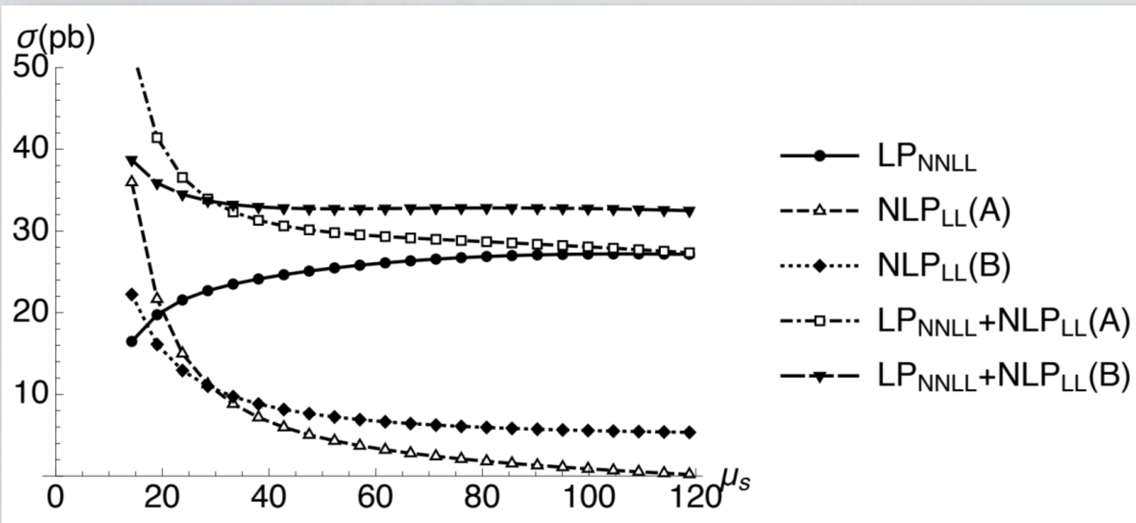
  Del Duca, 1990; Laenen, Magnea, Stavenga, 2008, Laenen, Stavenga, White, 2008; Laenen, Magnea, Stavenga, White, 2010; Bonocore, Laenen, Magnea, LV, White, 2014, 2015, 2016; Bahjat-Abbas, Sinninghe Damsté, LV, White, 2018; Bahjat-Abbas, Bonocore, Sinninghe Damsté, Laenen, Magnea, LV, White, 2019; Liu, Penin, 2017/18; Anastasiou, Penin, 2020; Cieri, Oleari, Rocco, 2019; Oleari, Rocco 2020; van Beekveld, Beenakker, Laenen, White, 2019; van Beekveld, Laenen, Sinninghe Damsté, LV, 2021; + …

- Several topics considered:

  LBKD theorem, operator bases, renormalization, N-jettiness subtraction, thrust distribution, Drell-Yan and Higgs production near threshold, DIS for x → 1, QED effects in B decays, New Physics decay, Higgs decay through bottom loops, TMD factorization, energy-energy correlation in N = 4 SYM, gravitation, …

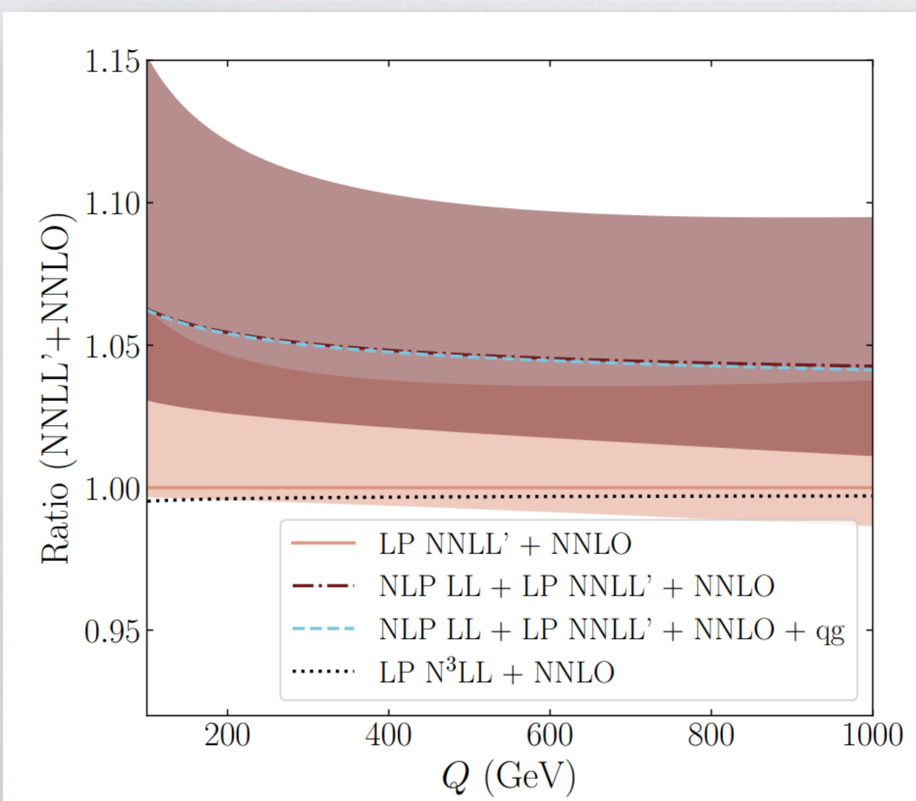
PARTICLE SCATTERING NEAR THRESHOLD

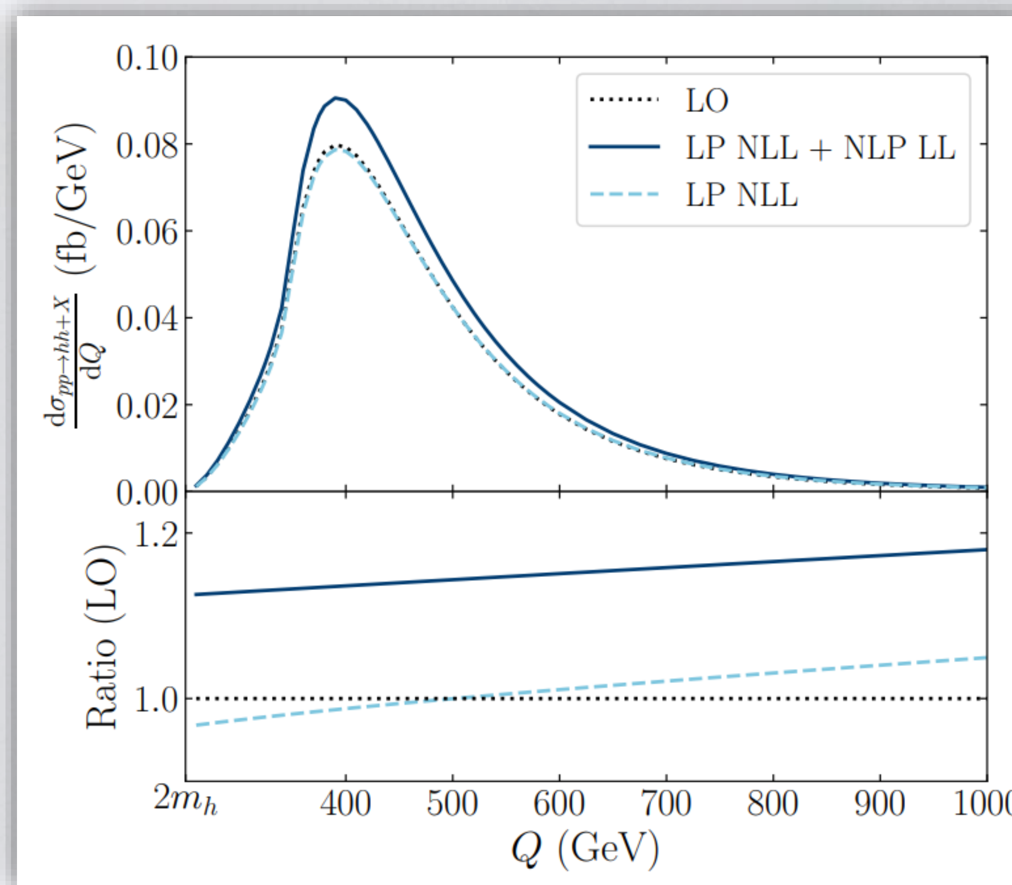
- Phenomenological analyses have shown LLs at NLP to be competitive with NNLLs at LP: relevant for precision physics.



Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2019

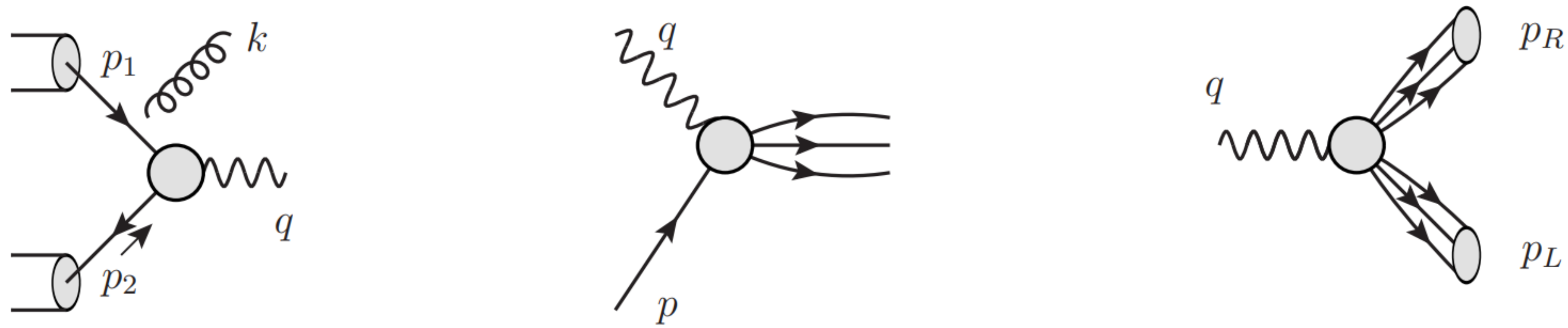
van Beekveld, Laenen, Sinninghe Damsté, LV, 2021.





PARTICLE SCATTERING NEAR KINEMATIC LIMITS

- Consider Drell-Yan, DIS near partonic threshold and Thrust in the back-to-back jet limit:



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- These limits involve a dynamical enhancement of soft and collinear radiation.
- Factorize soft and collinear radiation from the hard interaction:

  → by means of Soft-Collinear Effective Field Theory (SCET);
  → by means of a diagrammatic approach in QCD.

- Here we discuss the first method.

SOFT-COLLINEAR EFFECTIVE FIELD THEORY

- Effective Lagrangian and operators made of collinear and soft fields.

$$\mathcal{L}_{\rm SCET} = \sum_i \mathcal{L}_{c_i} + \mathcal{L}_s,$$

Bauer, Fleming, Pirjol, Stewart, 2000,2001; Beneke, Chapovsky, Diehl, Feldmann, 2002; Hill, Neubert 2002.

$$\mathcal{O}_n = \int dt_1 \ldots dt_n \, \mathcal{C}(t_1,\ldots,t_n)\, \phi_1(t_1 n_{1+}) \ldots \phi_n(t_n n_{n+}).$$

- Constructed to reproduce a scattering process as obtained with the method of regions.
- The cross section factorizes into a hard scattering kernel, and matrix elements of soft and collinear fields.

$$\sigma \sim \mathcal{H} \otimes \mathcal{J}_1 \otimes \ldots \otimes \mathcal{J}_n \otimes \mathcal{S}.$$

Hard matching coefficient

Jet functions – matrix elements of collinear fields

Soft function – matrix element of soft fields

- Renormalize UV divergences of EFT operators and obtain renormalization group equations.
- Each function depends on a single scale: solving the RGE resums large logarithms.

See e.g. Becher, Neubert 2006

FACTORIZATION IN SCET: LP

- Factorization theorem at LP are "simple" due to soft-collinear decoupling:
- There is a single eikonal soft-collinear interaction at LP:

Beneke, Chapovsky, Diehl, Feldmann, 2002

$$\mathcal{L}_c^{(0)} = \bar{\xi}\left(in_- D_c + g_s n_- A_s(x_-) + i\not{D}_{\perp c}\frac{1}{in_+ D_c} i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\xi + \mathcal{L}_{c,\mathrm{YM}}^{(0)},$$

$$\text{where} \qquad iD_c = i\partial + g_s A_c, \qquad x_-^\mu = n_+ \cdot x\,\frac{n_-^\mu}{2}.$$

- This can be removed by means of a field redefinition:

Bauer, Pirjol, Stewart, 2001

$$\xi(x) \to Y(x_-)\xi(x), \qquad A_c^\mu(x) \to Y(x_-)A_c^\mu(x)Y^\dagger(x_-), \qquad Y^\dagger in_- D_s Y = in_-\partial,$$

$$\text{with} \qquad Y(x) = \mathcal{P}\exp\left(ig_s\int_{-\infty}^0 ds\, n_- A_s(x+sn_-)\right).$$

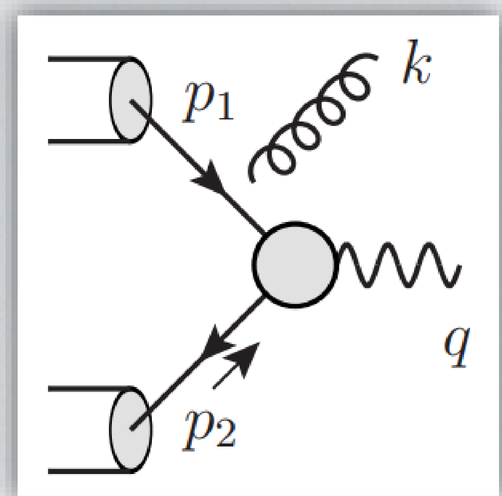
- No soft-collinear interactions are left at LP:

$$\mathcal{L}_c^{(0)} = \bar{\xi}\left(in_- D_c + \cancel{g_s n_- A_s(x_-)} + i\not{D}_{\perp c}\frac{1}{in_+ D_c} i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\xi + \mathcal{L}_{c,\mathrm{YM}}^{(0)}.$$

FACTORIZATION IN SCET: LP

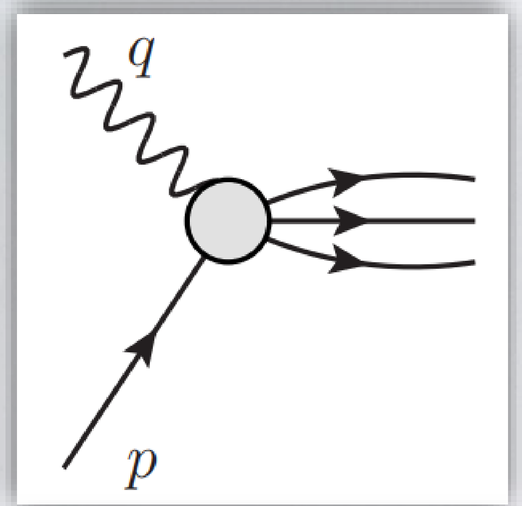
- One obtains "classical" factorization theorem at LP:

Sterman, 1987;
Catani,Trentadue, 1989;
Catani, Turnock, Webber, Trentadue, 1991;
Catani, Trentadue, Turnock, Webber, 1993



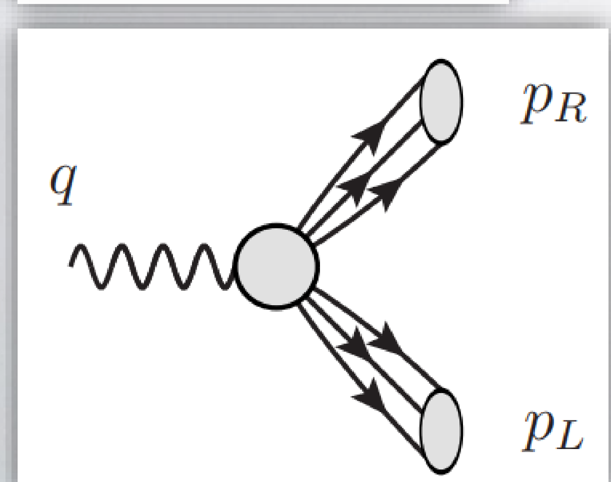
$$\frac{d\sigma}{dQ^2} = |C^{\mathrm{A}0}|^2 \times f_{a/A} \otimes f_{b/B} \otimes S_{\mathrm{DY}}\big(Q(1-z)\big),$$

Becher, Neubert, Xu 2007;



$$F_2 = |C^{\mathrm{A}0}|^2 \times Q^2 \times f_{a/A} \otimes J^{(q)}_{\overline{hc}},$$

Becher, Neubert, Pecjak 2006;



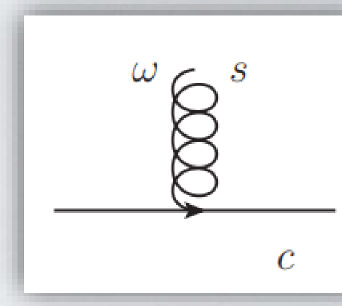
$$\frac{d\sigma}{d\tau} = |C^{\mathrm{A}0}|^2 \times J^{(q)}_{c} \otimes J^{(\bar{q})}_{\bar{c}} \otimes S_{\mathrm{LP}}.$$

Becher, Schwartz, 2008

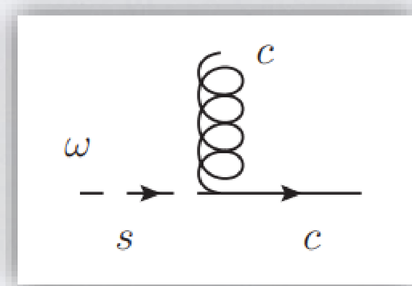
FACTORIZATION IN SCET: NLP

- Soft-collinear interactions are still present at subleading power, and one needs to take into account several effecs: *Beneke, Chapovsky, Diehl, Feldmann, 2002; Beneke, Feldmann, 2002*
- Subleading Lagrangian: $\mathcal{L}_{c_i} = \mathcal{L}_{c_i}^{(0)} + \mathcal{L}_{c_i}^{(1)} + \mathcal{L}_{c_i}^{(2)} + \ldots$ where e.g.

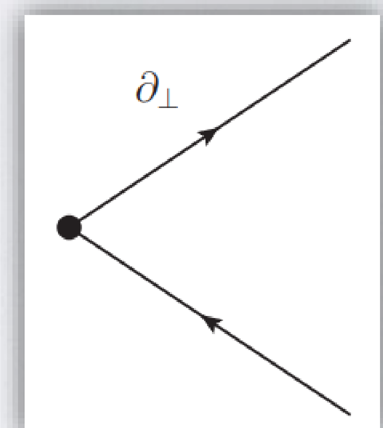
$$\mathcal{L}_c^{(1)\mathrm{gluon}}(x) = \bar{\xi}\left[x_\perp^\mu n_-^\nu W_c g_s F^s_{\mu\nu}(x_-) W_c^\dagger\right]\frac{\not{n}_+}{2}\xi,$$



$$\mathcal{L}_c^{(1)\mathrm{quark}}(x) = \bar{q}(x_-) W_c^\dagger i\not{D}_{\perp c}\xi.$$

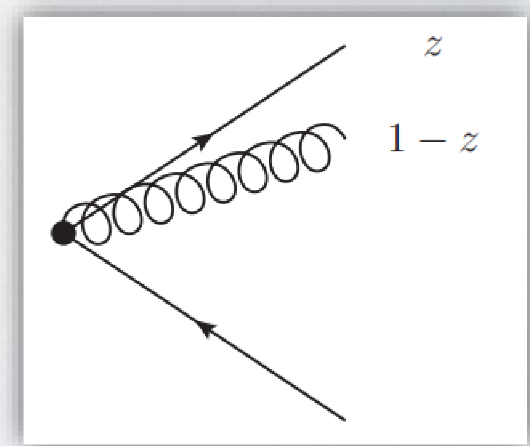


- Power-suppressed operators, e.g.



$$J_\rho^{A0,A1}(t,\bar{t}) = \bar{\chi}_{\bar{c}}(\bar{t}n_-)\, n_{+\rho}\, i\partial_\perp \chi_c(tn_+),$$

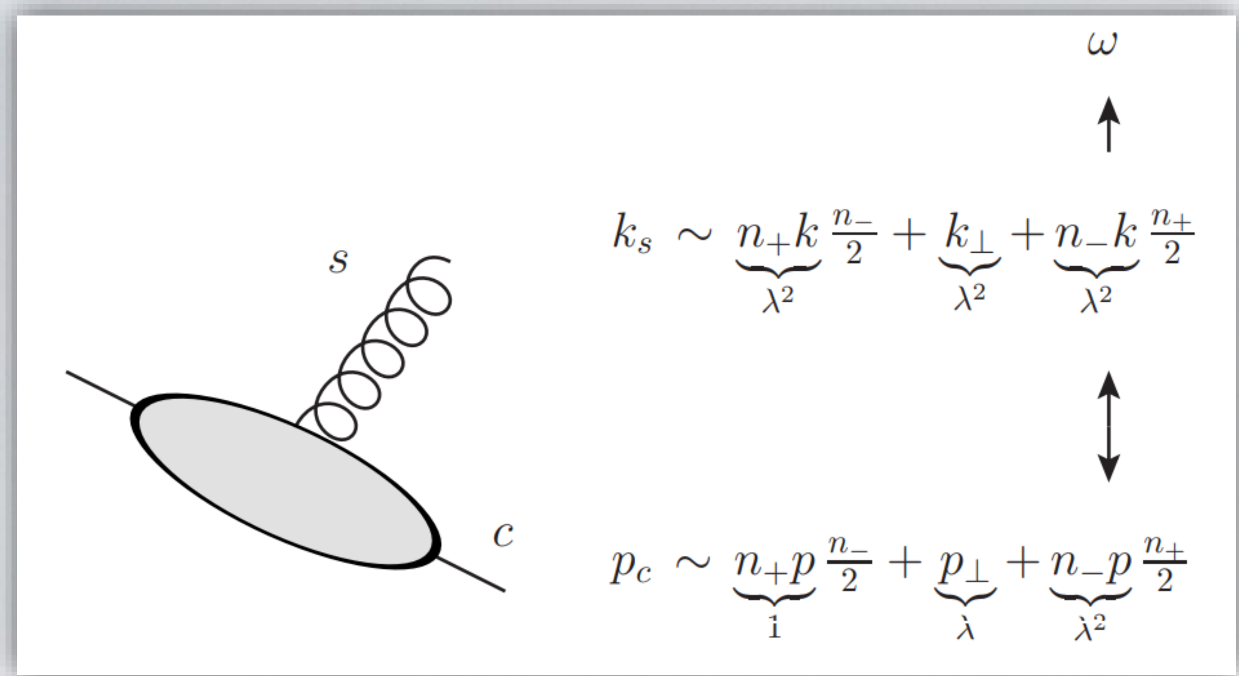
$$J_\rho^{A0,B1}(t_1,t_2,\bar{t}) = \bar{\chi}_{\bar{c}}(\bar{t}n_-)\, n_{\pm\rho}\, \not{\mathcal{A}}_{\perp c}(t_2 n_+)\chi_c(t_1 n_+).$$



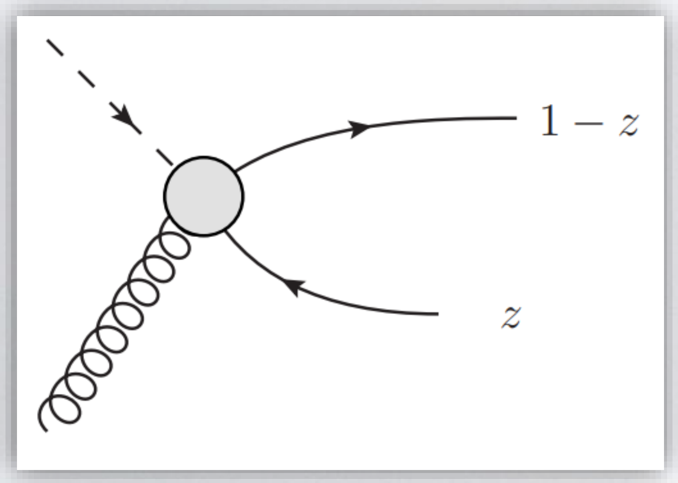
Beneke at Al, 2017-19; see also Stewart at Al., 2014-19

FACTORIZATION IN SCET: NLP

- Matrix elements involve convolutions over momentum fractions/soft momenta components:



$$\longrightarrow \int d\omega\, J(\omega)\, S(\omega),$$



$$\longrightarrow \int_0^1 dz \left(\frac{\mu^2}{s_{qg} z \bar{z}}\right)^{\epsilon} \mathcal{P}_{qg}(s_{qg}, z)\Big|_{s_{qg}=Q^2\frac{1-x}{x}}\,.$$

FACTORIZATION IN SCET: NLP

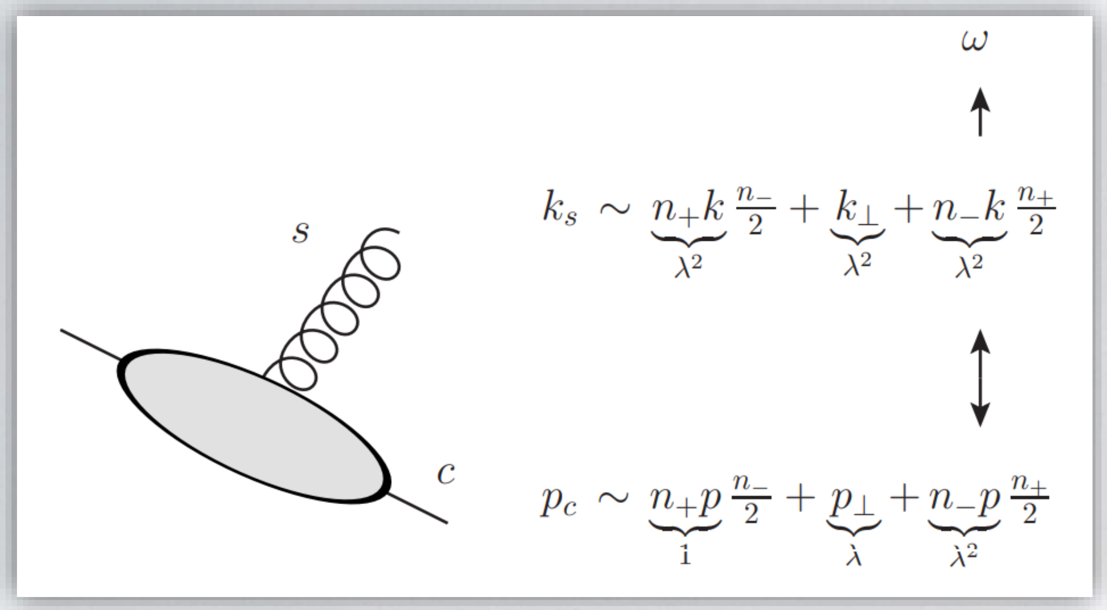
- Convolutions are divergent in $d = 4$!

First observed in Beneke, LV 2008;

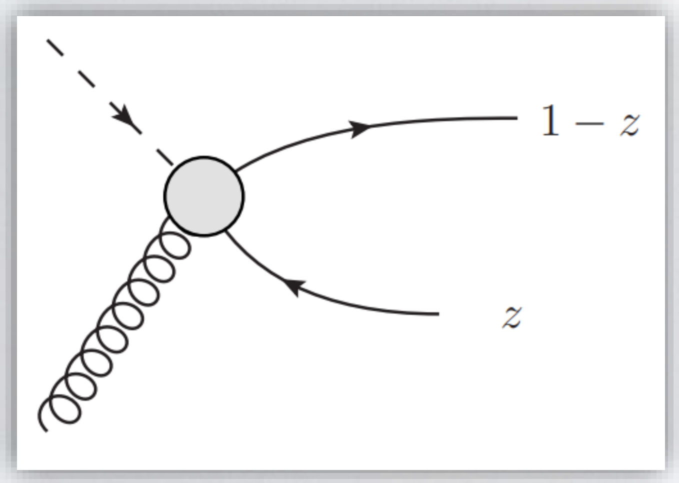
Liu, Mecaj, Neubert, Wang, 2019-2020;

Beneke, Broggio, Garny, Jaskiewicz, Szafron, LV, Wang, 2018

Beneke, Broggio, Jaskiewicz, LV, 2019



$$\longrightarrow \int_0^\Omega d\omega \, \underbrace{\left(n_+p\,\omega\right)^{-\epsilon}}_{\text{collinear piece}} \, \underbrace{\frac{1}{\omega^{1+\epsilon}} \frac{1}{(\Omega-\omega)^\epsilon}}_{\text{soft piece}},$$



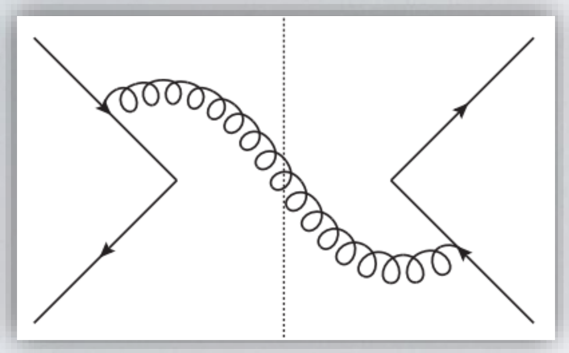
$$\longrightarrow \int_0^1 dz \left(\frac{\mu^2}{s_{qg} z\bar{z}}\right)^\epsilon \frac{\alpha_s C_F}{2\pi} \frac{(1-z)^2}{z}\Big|_{s_{qg}=Q^2\frac{1-x}{x}}.$$

- Cannot apply the standard RGE methods directly to the collinear and soft functions.

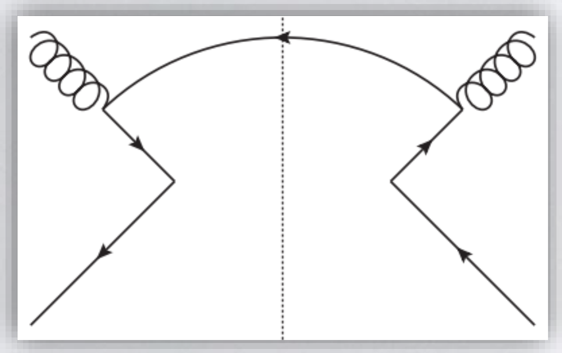
INTERLUDE: DIAGONAL VS OFF-DIAGONAL

- At LP only "diagonal" $q\bar{q}$ channel contributes, endpoint divergences relevant at NLL.
- The "off-diagonal" $g\bar{q},\ qg$ channels start at NLP:

Beneke, Broggio, Jaskiewicz, LV, 2019



$$\Delta_{q\bar{q}}(z) = \Delta_{q\bar{q}}(z)|_{\rm LP} + \underbrace{\Delta_{q\bar{q}}(z)|_{\rm NLP}}_{\underbrace{\Delta^{kin}_{q\bar{q}}(z)|_{\rm NLP}}_{\text{starts at NLL}} + \underbrace{\Delta^{dyn}_{q\bar{q}}(z)|_{\rm NLP}}_{\underbrace{\sum_i \int \{d\omega\} J_i(\{\omega\}) S_i(\{\omega\})}_{\sim \frac{\alpha_s}{4\pi} \sum_i \int\{d\omega\} \left[\underbrace{S_i^{(n)}(\{\omega\})}_{\text{starts at LL}} + \underbrace{\sum_{m=1}^{n-1} J_i^{(m)}(\{\omega\}) S_i^{(n-m)}(\{\omega\})}_{\text{starts at NLL}}\right]}}}$$

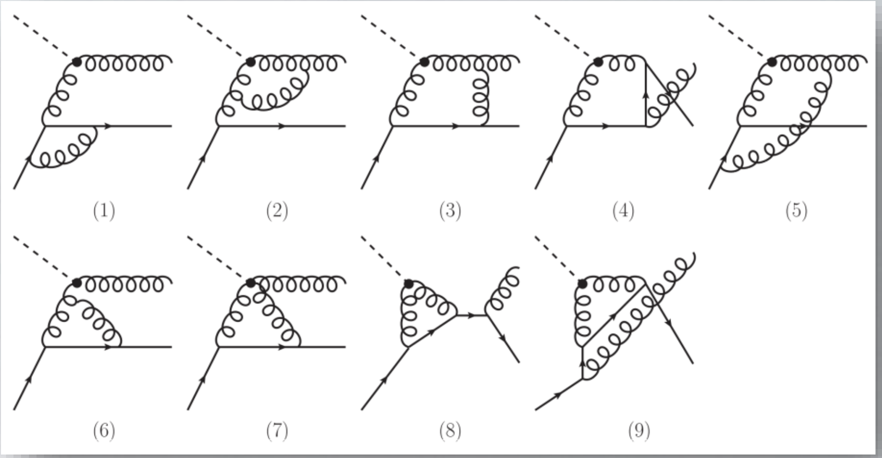


$$\Delta_{g\bar{q}}(z) = \underbrace{\Delta_{g\bar{q}}(z)|_{\rm NLP}}_{\underbrace{\Delta^{dyn}_{g\bar{q}}(z)|_{\rm NLP}}_{\underbrace{\int \{d\omega\} J(\{\omega\}) S(\{\omega\})}_{\text{starts at LL}}}}$$

- Off-diagonal channels have a simpler structure, but endpoint divergences relevant at LL.

ENDPOINT DIVERGENCES

- Let's investigate the endpoint divergence in off-diagonal gluon DIS in some more detail:



$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\text{loop}} = \mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}} \frac{\alpha_s}{\pi}\frac{1}{\epsilon^2} \times \left(\mathbf{T}_1\cdot\mathbf{T}_0 \left(\frac{\mu^2}{zQ^2}\right)^\epsilon + \mathbf{T}_2\cdot\mathbf{T}_0 \left(\frac{\mu^2}{\bar{z}Q^2}\right)^\epsilon + \mathbf{T}_1\cdot\mathbf{T}_2 \left[\left(\frac{\mu^2}{Q^2}\right)^\epsilon - \left(\frac{\mu^2}{zQ^2}\right)^\epsilon + \left(\frac{\mu^2}{zs_{qg}}\right)^\epsilon \right] \right) + \mathcal{O}(\epsilon^{-1}).$$

- The **T1.T2** term contains a single pole, but: promoted to leading pole after integration!
- Compare exact integration:

Beneke, Garny, Jaskiewicz, Szafron, LV, Wang 2020

$$\frac{1}{\epsilon^2}\int_0^1 dz\, \frac{1}{z^{1+\epsilon}}\left(1 - z^{-\epsilon}\right) = -\frac{1}{2\epsilon^3},$$

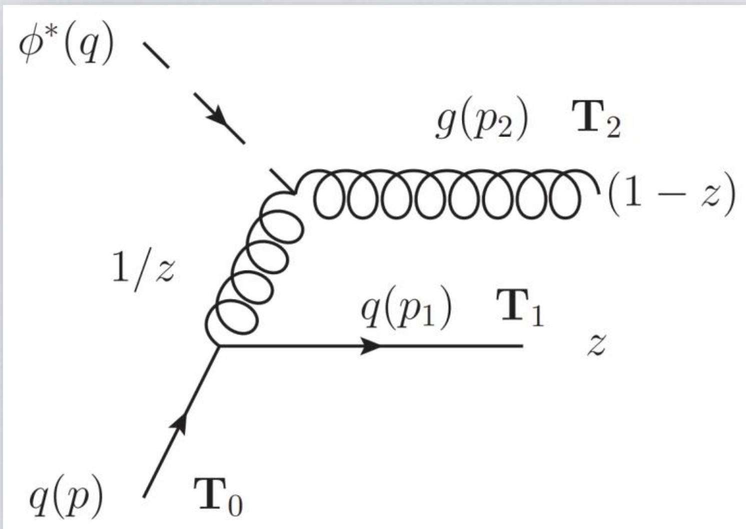
vs integration after expansion:

$$\frac{1}{\epsilon^2}\int_0^1 dz\, \frac{1}{z^{1+\epsilon}}\left(\epsilon \ln z - \frac{\epsilon^2}{2!}\ln^2 z + \frac{\epsilon^2}{3!}\ln^3 z + \cdots\right) = -\frac{1}{\epsilon^3} + \frac{1}{\epsilon^3} - \frac{1}{\epsilon^3} + \cdots .$$

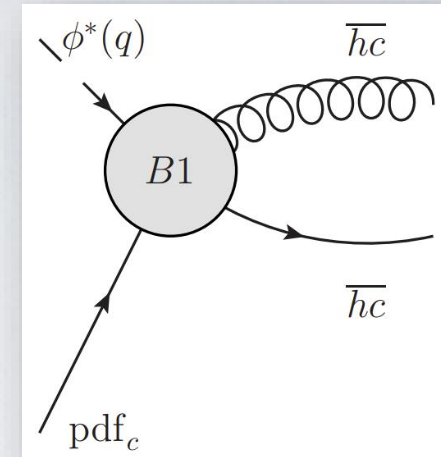
- Expansion in ε not possible before integration! The pole associated to **T1.T2** does not originate from the standard cups anomalous dimension.

BREACKDOWN OF FACTORIZATION NEAR THE ENDPOINT

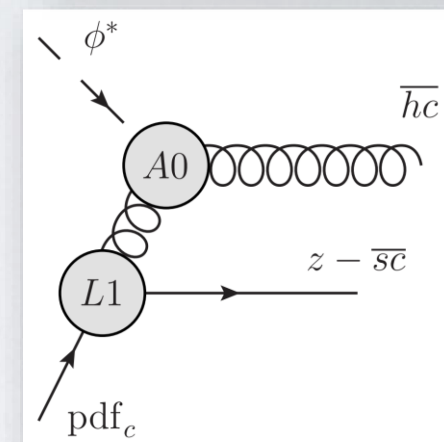
- What happens for $z \to 0$?



For z ~ 1 intermediate propagator is hard



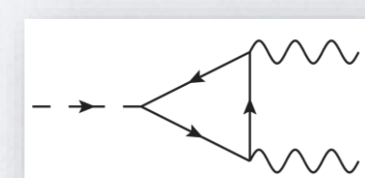
For z « 1 intermediate propagator cannot be integrated out



- Dynamic scale: zQ^2.
- In the endpoint region new counting parameter, $\lambda^2 \ll z \ll 1$.
- New modes contribute: z-softcollinear.
- Need re-factorization:

$$\underbrace{C^{B1}(Q,z)}_{\text{multi-scale function}} J^{B1}(z) \xrightarrow{z\to 0} C^{A0}(Q^2)\int d^4x\, \mathbf{T}\Big[J^{A0}, \mathcal{L}_{\xi q_{z-\overline{sc}}}(x)\Big] = \underbrace{C^{A0}(Q^2) D^{B1}(zQ^2,\mu^2)}_{\text{single-scale functions}} J^{B1}_{z-\overline{sc}}.$$

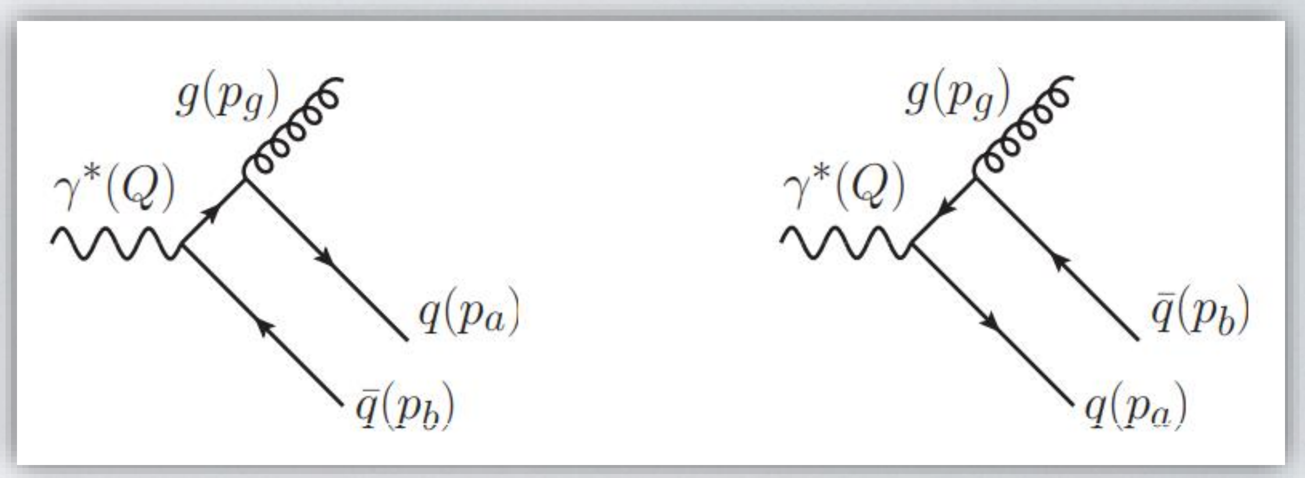
- Similar re-factorization proven in Liu, Mecaj, Neubert, Wang 2020.



OFF-DIAGONAL "GLUON" THRUST

- Consider the power-suppressed contribution to Thrust in the two-jet region:

$$e^+e^- \to \gamma^* \to [g]_c + [q\bar{q}]_{\bar{c}}.$$



- Within SCET one has two contributions:
  "Direct" term (B-type) and time-ordered product soft-quark term (A-type):

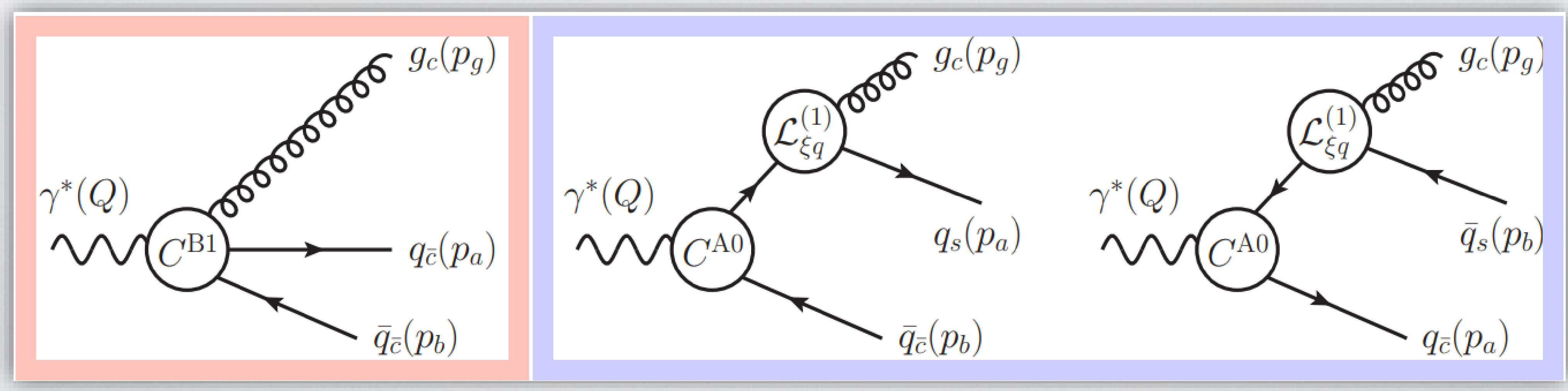
$$\bar{\psi}\gamma_\perp^\mu\psi(0) = \int dt d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t})\, \bar{\chi}_c(tn_+)\gamma_\perp^\mu\chi_{\bar{c}}(\bar{t}n_-) + (c \leftrightarrow \bar{c})$$

$$+ \sum_{i=1,2} \int dt d\bar{t}_1 d\bar{t}_2\, \widetilde{C}_i^{B1}(t,\bar{t}_1,\bar{t}_2)\, \bar{\chi}_{\bar{c}}(\bar{t}_1 n_-)\Gamma_i^{\mu\nu}\mathcal{A}_{c\perp\nu}(tn_+)\chi_{\bar{c}}(\bar{t}_2 n_-) + \ldots$$

"Soft quark Sudakov" in Moult, Stewart, Vita, Zhu, 2019

$$\mathcal{L}_{\xi q}(x) = \bar{q}_s(x_-)\not{\mathcal{A}}_{c\perp}(x)\chi_c(x) + \text{h.c.}\,.$$

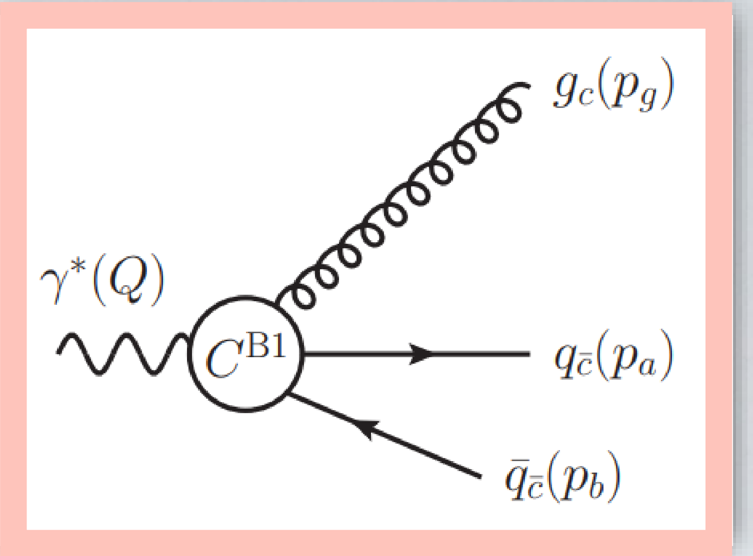
Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang 2022



OFF-DIAGONAL "GLUON" THRUST

- "Direct" B-type term expressed in hard, (anti-)collinear and soft function:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\Big|_{\rm B} \sim \int_0^1 dr\, dr'\, C^{B1}(r)\, C^{B1}(r') \times \mathcal{J}_{\bar{c}}^{(q\bar{q})}(r,r') \otimes \mathcal{J}_c^{(g)} \otimes S^{(g)}.$$

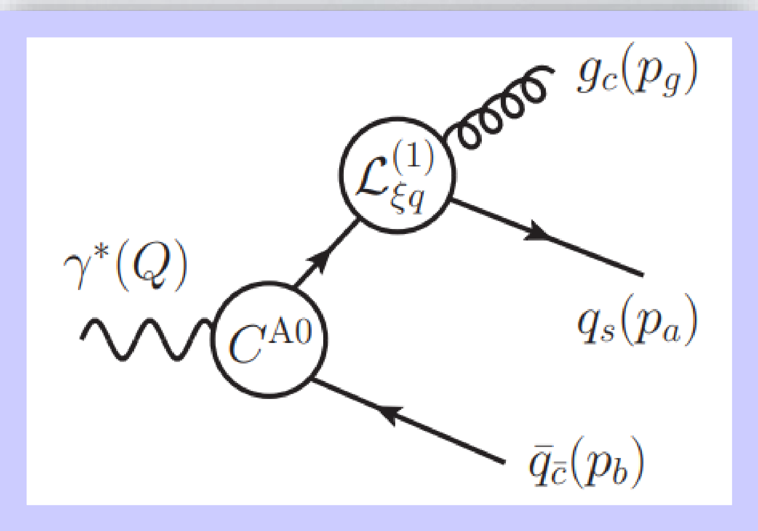


- It develops endpoint divergences when the quark *(r→0)* or anti-quark *(r→1)* become soft:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\Big|_{\rm B} \propto \int_0^1 dr\left[\frac{1}{r^{1+\epsilon}} + \frac{1}{(1-r)^{1+\epsilon}}\right].$$

- Time-ordered product A-type term expressed in hard, (anti-)collinear and soft function:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\Big|_{\rm A} \sim \int_0^\infty d\omega\, d\omega'\, |C^{A0}|^2 \times \mathcal{J}_{\bar{c}}^{(\bar{q})} \otimes \mathcal{J}_c(\omega,\omega') \otimes S_{\rm NLP}(\omega,\omega').$$

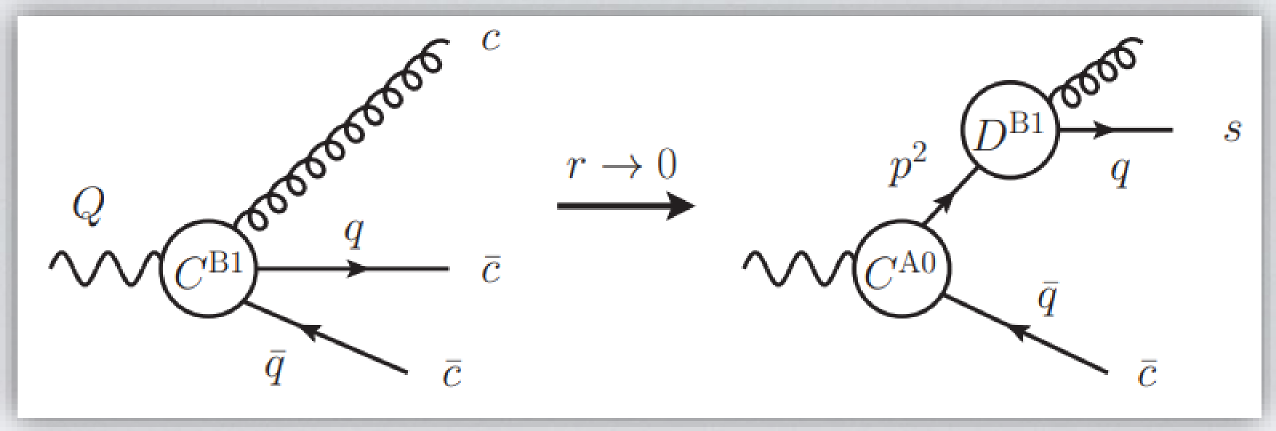


- It develops endpoint divergences when the soft quark or anti-quark become energetic *(ω→∞)*:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\Big|_{\rm A} \propto 2\int_{M_R^2/Q}^\infty d\omega\, \frac{1}{\omega^{1+\epsilon}}.$$

OFF-DIAGONAL "GLUON" THRUST

- As for DIS, in the *r→0* (or *r→1*) limit, the B1 coefficient is a two-scale object, which refactorizes, since the intermediate state develops an on-shell pole:

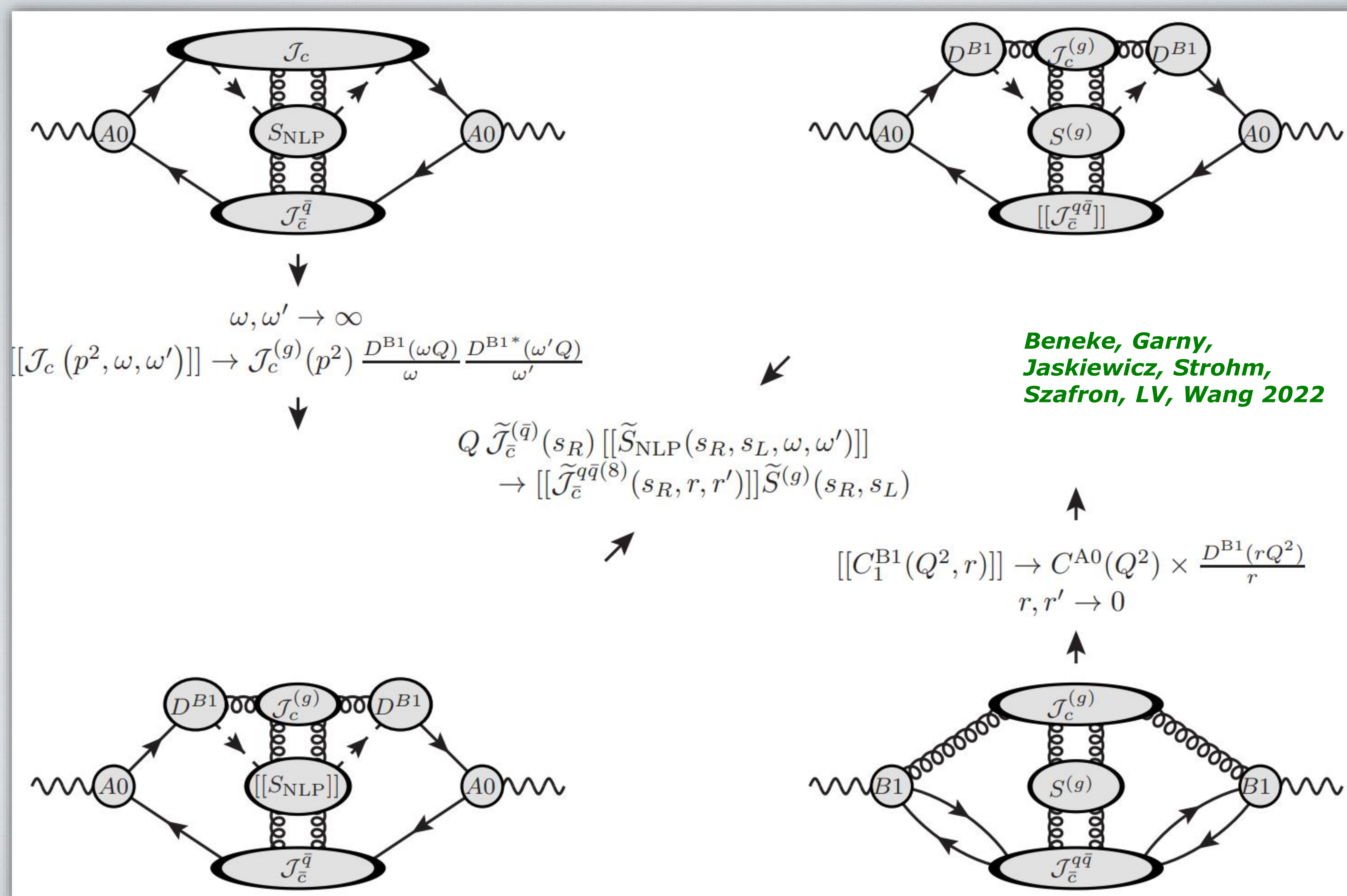


$$C_1^{\mathrm{B1}}(Q^2, r) = C^{\mathrm{A0}}(Q^2) \times \frac{D^{\mathrm{B1}}(rQ^2)}{r} + \mathcal{O}(r^0)\,.$$

- In *d* dimensions the *1/ε* poles from the divergent convolution integrals cancels. The integrands of A and B match in the asymptotic limits *ω,ω′→∞* (A-type) and r,r′→0(1) (B-type).

- This allows a rearrangement between the terms that makes them separately finite, provided two additional refactorization conditions hold for the soft and jet functions:

OFF-DIAGONAL “GLUON” THRUST



Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang 2022

OFF-DIAGONAL "GLUON" THRUST

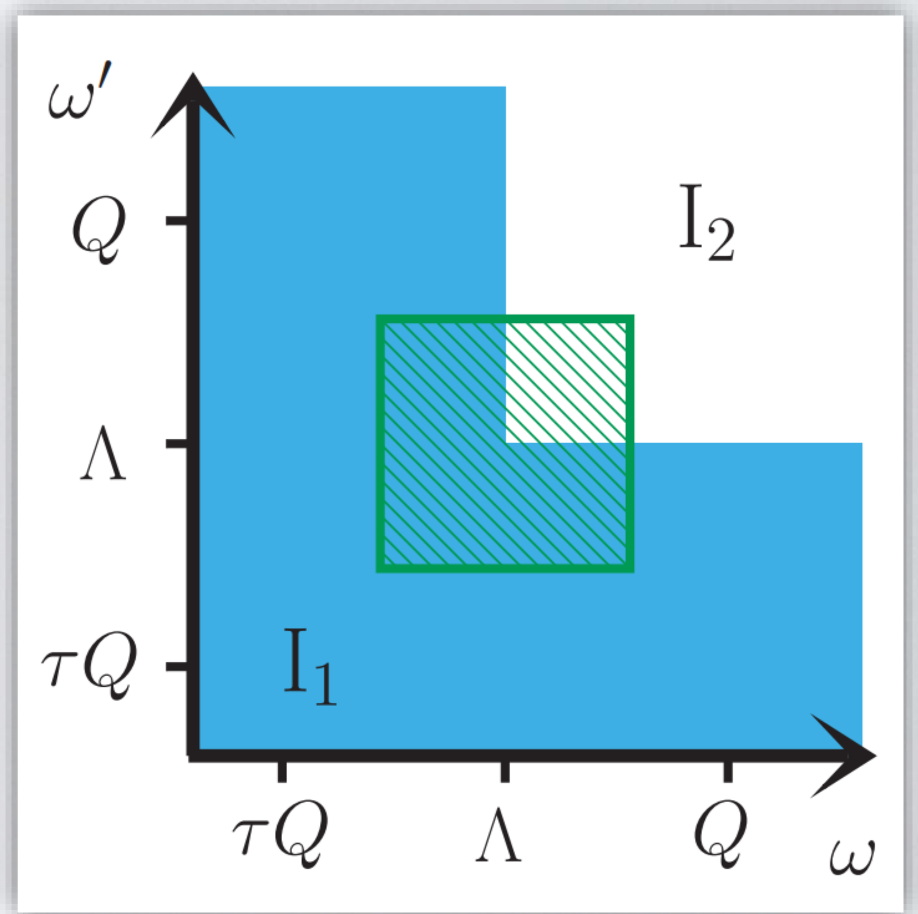
- Refactorization can be achieved as follows: define the asymptotic (scaleless) integral

$$0 = \frac{2C_F}{Q}\, f(\epsilon)\, |C^{\mathrm{A0}}(Q^2)|^2 \widetilde{\mathcal{J}}_{\bar{c}}^{(\bar{q})}(s_R) \widetilde{\mathcal{J}}_{c}^{(g)}(s_L)$$
$$\times \int_0^\infty d\omega d\omega' \, \frac{D^{\mathrm{B1}}(\omega Q)}{\omega} \frac{D^{\mathrm{B1}*}(\omega' Q)}{\omega'} \left[\!\left[\widetilde{S}_{\mathrm{NLP}}(s_R, s_L, \omega, \omega') \right]\!\right].$$

Then split the integral over the two regions

$$0 = I_1 + I_2.$$

by introducing a factorization parameter Λ, and subtract I_1 from the B-type term and I_2 from the A-type term, which makes both endpoint-finite as $d \to 4$.



- One can then proceed with standard resummation methods.

OFF-DIAGONAL "GLUON" THRUST

- Refactorized factorization formula:

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$$\frac{1}{\sigma_0}\frac{\widetilde{d\sigma}}{ds_R ds_L}|_{\mathrm{A-type}} = \frac{2C_F}{Q} f(\epsilon)\,|C^{\mathrm{A0}}(Q^2)|^2\,\widetilde{\mathcal{J}}_{\bar{c}}^{(\bar{q})}(s_R)\int_0^\infty d\omega d\omega'$$
$$\times\Big\{\widetilde{\mathcal{J}}_c(s_L,\omega,\omega')\,\widetilde{S}_{\mathrm{NLP}}(s_R,s_L,\omega,\omega')$$
$$-\,\theta(\omega-\Lambda)\theta(\omega'-\Lambda)\,[\![\widetilde{\mathcal{J}}_c(s_L,\omega,\omega')]\!]\,[\![\widetilde{S}_{\mathrm{NLP}}(s_R,s_L,\omega,\omega')]\!]$$
$$+\,\widetilde{\widetilde{\mathcal{J}}}_c(s_L,\omega,\omega')\,\widetilde{\widetilde{S}}_{\mathrm{NLP}}(s_R,s_L,\omega,\omega')\Big\},$$

$$\frac{1}{\sigma_0}\frac{\widetilde{d\sigma}}{ds_R ds_L}|_{\substack{\mathrm{B-type}\\ \mathrm{i=i'=1}}} = \frac{2C_F}{Q^2} f(\epsilon)\,\widetilde{\mathcal{J}}_c^{(g)}(s_L)\,\widetilde{S}^{(g)}(s_R,s_L)\int_0^\infty dr dr'$$
$$\times\Big[\theta(1-r)\theta(1-r')\,C_1^{\mathrm{B1}*}(Q^2,r')C_1^{\mathrm{B1}}(Q^2,r)\,\widetilde{\mathcal{J}}_{\bar{c}}^{q\bar{q}(8)}(s_R,r,r')$$
$$-\,[1-\theta(r-\Lambda/Q)\theta(r'-\Lambda/Q)]$$
$$\times\,[\![C_1^{\mathrm{B1}*}(Q^2,r')]\!]_0\,[\![C_1^{\mathrm{B1}}(Q^2,r)]\!]_0\,[\![\widetilde{\mathcal{J}}_{\bar{c}}^{q\bar{q}(8)}(s_R,r,r')]\!]_0\Big].$$

- *Λ* dependence cancels between the two terms. Each separately independent of *dim reg μ.*
- In principle valid to any log accuracy. At LL only the subtraction terms contribute do to the extra log from large ω/small r.

OFF-DIAGONAL "GLUON" THRUST

- D^{B1} appears as a universal coefficient that renormalizes soft quark emission. Its double logarithms are proportional to the change of colour charge of the collinear particles:

$$\langle g_c^a(p_c) q_{\overline{sc}}(p_{\overline{sc}})| \int d^4x \, T\{\bar{\chi}_c(0), \mathcal{L}_{\xi q}(x)\} |0\rangle = g_s \bar{u}(p_{\overline{sc}}) t^a \not{\epsilon}_{c\perp}(p_c) \frac{in_+ p_c}{p^2} \frac{\not{n}_-}{2} D^{B1}(p^2) \,.$$

- It appear in Thrust, DIS, DY, as well as H → gg. Up to one loop:

$$D^{B1}(p^2) = 1 + \frac{\alpha_s}{4\pi}(C_F - C_A)\left(\frac{2}{\epsilon^2} - 1 - \frac{\pi^2}{6}\right)\left(\frac{\mu^2}{-p^2 - i\varepsilon}\right)^{\epsilon} + \mathcal{O}(\alpha_s^2).$$

- D^{B1} has (non-local) anomalous dimension

$$\frac{d}{d\ln\mu} D^{B1}(p^2) = \int_0^\infty d\hat{p}^2 \, \gamma_D(\hat{p}^2, p^2) D^{B1}(\hat{p}^2) \,,$$

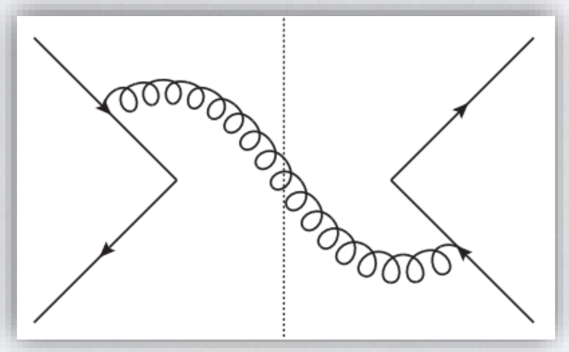
with

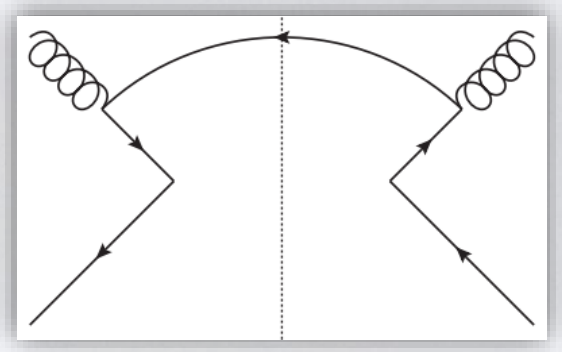
$$\gamma_D(\hat{p}^2, p^2) = \frac{\alpha_s(C_F - C_A)}{\pi} \delta(\hat{p}^2 - p^2) \ln\left(\frac{\mu^2}{-p^2 - i\varepsilon}\right) + \frac{\alpha_s}{\pi}\left(\frac{C_A}{2} - C_F\right) p^2 \left[\frac{\theta(\hat{p}^2 - p^2)}{\hat{p}^2(\hat{p}^2 - p^2)} + \frac{\theta(p^2 - \hat{p}^2)}{p^2(p^2 - \hat{p}^2)}\right]_+ \,.$$

Liu, Mecaj, Neubert, Wang 2019-22; Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020; Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang 2022.

DRELL YAN AT NLP

- D^{B1} is an example of new universal functions appearing at NLP.
- In general, these functions are more involved to compute compared to their LP counterparts, as they depend on more variables.
- On the other hands, refactorization conditions imposes additional constraints, as we have seen in case of thrust.
- Important to collect data on these functions. In this respect, in the past few years we have completed the calculation of all terms contributing to Drell-Yan at NLP, up to NNLO: this includes jet functions at NLO, and soft functions at NNLO.





$$\Delta^{dyn}_{q\bar{q},g\bar{q}}(z)|_{\rm NLP} \sim \sum_i \int \{d\omega\}\, J_i(\{\omega\})\, S_i(\{\omega\}).$$

qqb: J at one loop in
Beneke, Broggio, Jaskiewicz, LV, 2019

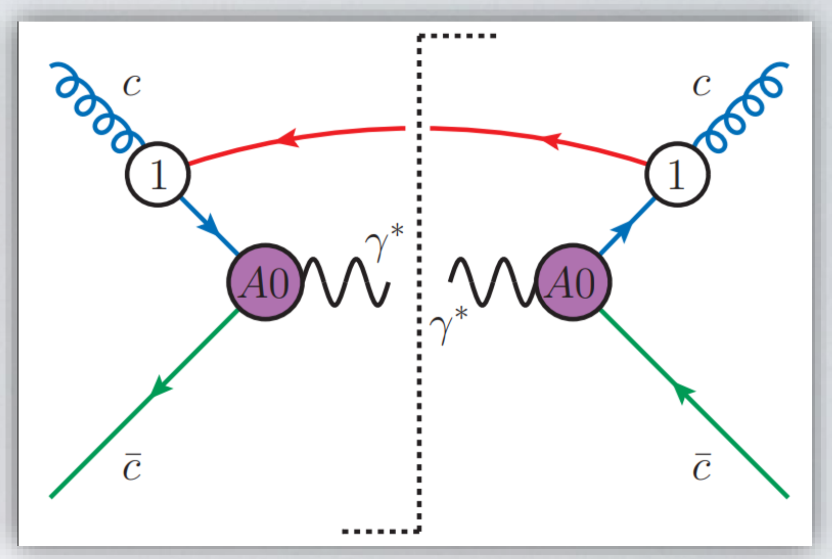
qg: J at one loop in
Broggio, Jaskiewicz, LV, 2023

qqb: S at two loops in
Broggio, Jaskiewicz, LV, 2021

qg: S at otwo loops in
Broggio, Jaskiewicz, LV, 2023

DRELL YAN AT NLP: QG CHANNEL

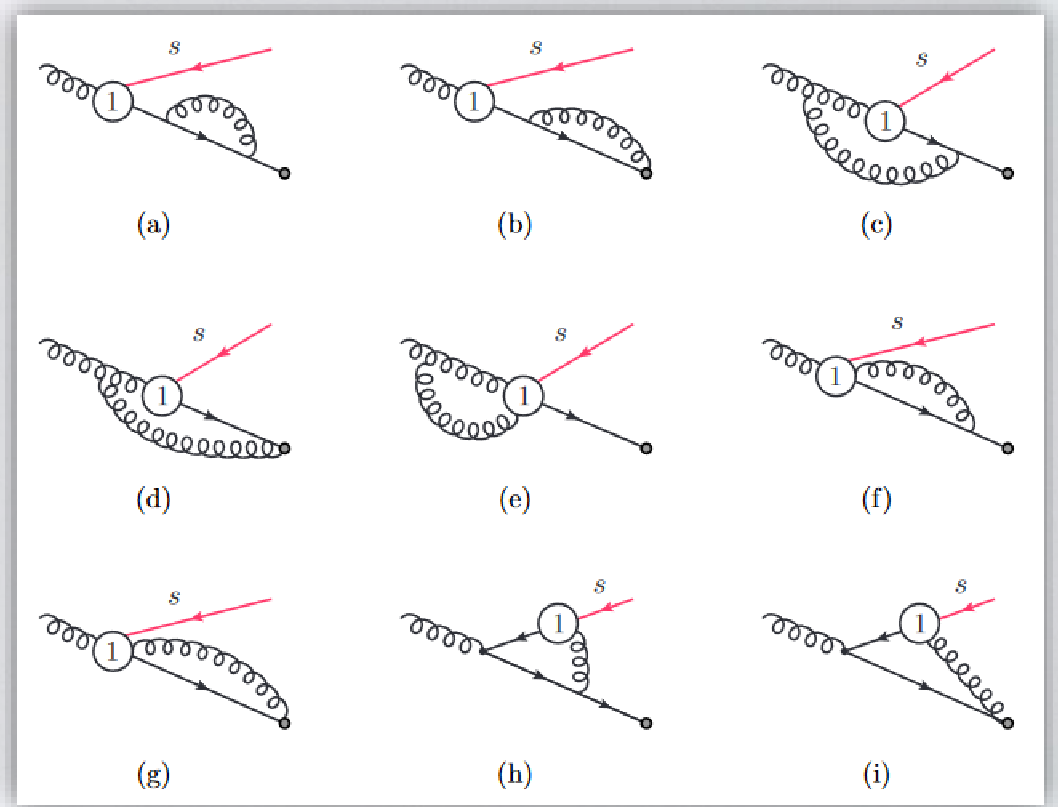
- Focus on the gqb channel: only a single term contribute, involving the emission of a soft quark:



$$\Delta_{g\bar{q}}|_{\rm NLP}(z) = 8H(Q^2) \int d\omega\, d\omega'\, G^*_{\xi q}(x_a n_+ p_A; \omega')\, G_{\xi q}(x_a n_+ p_A; \omega)\, S(\Omega, \omega, \omega').$$

- We have calculated the collinear function at one loop: it corresponds to the coefficient D^{B1}, and it is thus known to two loops.

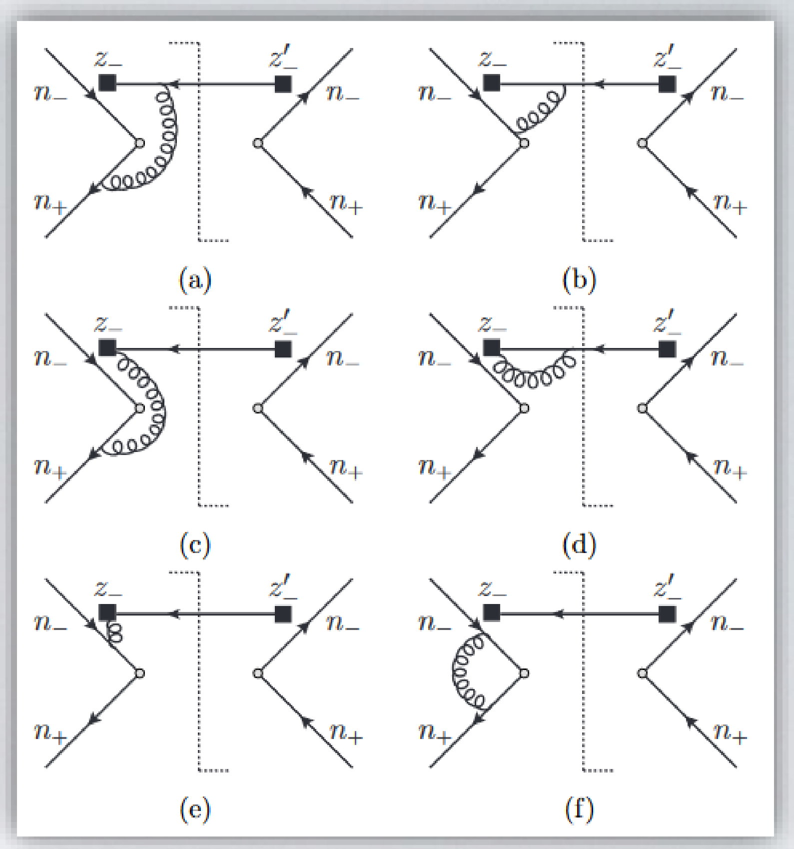
Broggio, Jaskiewicz, LV, 2023



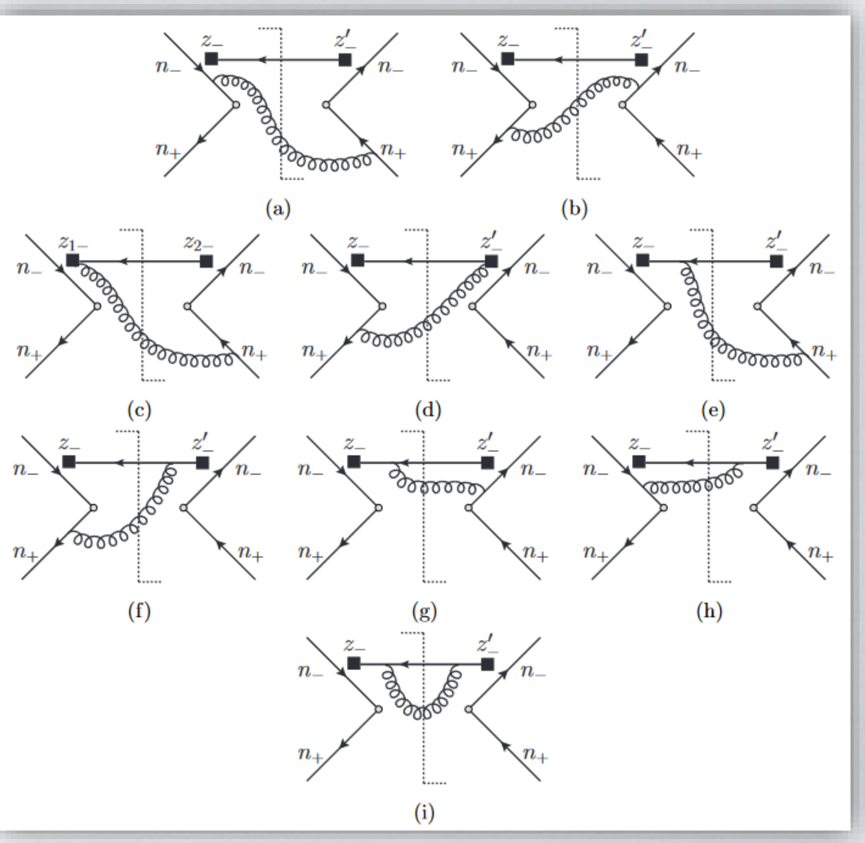
DRELL YAN AT NLP: QG CHANNEL

- The soft function at two loops is more involved: there is a real-virtual contribution and a real-real term, that we calculate by means of standard Feynman parametrization and by means of differential equations:

$$
\begin{aligned}
S_{g\bar{q}}^{(2)1r1v}(\Omega,\omega,\omega') &= \frac{\alpha_s^2\, T_F}{(4\pi)^2}(2C_F - C_A)\frac{e^{2\epsilon\gamma_E}\,\Gamma[1+\epsilon]}{\epsilon\,\Gamma[1-\epsilon]} \\
&\times \mathrm{Re}\bigg\{\frac{1}{(-\omega)\omega'}\bigg[\frac{\omega+\omega'}{\omega'}\,{}_2F_1\Big(1,1+\epsilon,1-\epsilon,\frac{\omega}{\omega'}\Big)-1\bigg]\bigg(\frac{\mu^4}{(-\omega)\omega'(\Omega-\omega')^2}\bigg)^{\epsilon}\theta(-\omega) \\
&+ \frac{2(\omega+\omega')}{\omega\omega'(\omega'-\omega)}\bigg(\frac{\mu^4}{(\omega'-\omega)^2(\Omega-\omega')^2}\bigg)^{\epsilon}\frac{\Gamma[1-\epsilon]^2}{\Gamma[1-2\epsilon]}\theta(\omega'-\omega)\bigg\}\theta(\omega')\theta(\Omega-\omega'),
\end{aligned}
$$



$$
\begin{aligned}
S_{g\bar{q}}^{(2)2r0v}(\Omega,\omega,\omega') &= \frac{\alpha_s^2 T_F}{(4\pi)^2}\bigg\{C_F\frac{e^{2\epsilon\gamma_E}\Gamma[1-\epsilon]}{\epsilon^2}\frac{1}{\omega}\bigg[\frac{4}{\Gamma[1-3\epsilon]}\bigg(\frac{\mu^4}{\omega(\Omega-\omega)^3}\bigg)^{\epsilon} \\
&\quad + \frac{(4-\epsilon)\Gamma[2-\epsilon]}{(1-2\epsilon)\Gamma[1-2\epsilon]^2}\bigg(\frac{\mu^4}{\omega^2(\Omega-\omega)^2}\bigg)^{\epsilon}\bigg]\delta(\omega-\omega')\theta(\Omega-\omega)\theta(\omega) \\
&+ (C_A-2C_F)\frac{2e^{2\epsilon\gamma_E}}{\epsilon\Gamma[1-2\epsilon]}\frac{\omega+\omega'}{\omega\omega'(\omega'-\omega)}\bigg(\frac{\mu^4}{\omega(\omega'-\omega)(\Omega-\omega')^2}\bigg)^{\epsilon} \\
&\quad \times \bigg[{}_2F_1\Big(1,-\epsilon,1-\epsilon,\frac{\omega}{\omega-\omega'}\Big)-1\bigg]\theta(\omega)\theta(\omega')\theta(\omega'-\omega)\theta(\Omega-\omega')\bigg\}.
\end{aligned}
$$



Broggio, Jaskiewicz, LV, 2023

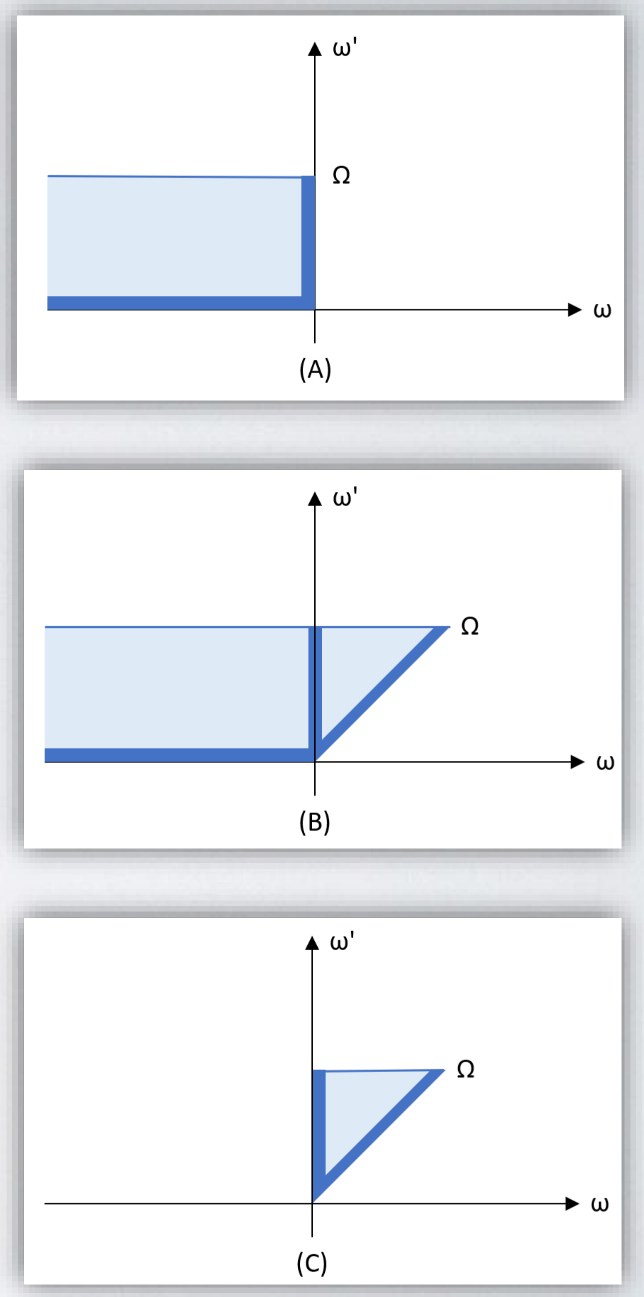
DRELL YAN AT NLP: QG CHANNEL

- Endpoint divergences: at NLO these arise for $\omega \to 0$:

$$\Delta^{(1)}_{g\bar{q}}(z)|_{\rm NLP} = 2\int d\omega\, d\omega'\, S^{(1)}(\Omega,\omega,\omega'),$$

with

$$S^{(1)}_{g\bar{q}}(\Omega,\omega,\omega') = \frac{\alpha_s T_F}{4\pi}\frac{e^{\epsilon\gamma_E}}{\Gamma[1-\epsilon]}\frac{1}{\omega}\left(\frac{\mu^2}{\omega\,(\Omega-\omega)}\right)^{\epsilon}\delta(\omega-\omega')\,\theta(\Omega-\omega)\theta(\omega).$$



- The endpoint divergence needs to be removed by a corresponding divergent term in the PDF, which needs to be factorized near $x \to 1$.
- At NNLO the structure of endpoint divergences appear more involved:

$$\begin{aligned} S^{(2)}(\omega,\omega') &= \hat{S}^{(2)}(\omega)\,\delta(\omega-\omega')\,\theta(\Omega-\omega)\theta(\omega) \\ &+ S^{(2A)}(\omega,\omega')\,\theta(-\omega)\theta(\omega')\theta(\Omega-\omega') \\ &+ S^{(2B)}(\omega,\omega')\,\theta(\omega'-\omega)\theta(\omega')\theta(\Omega-\omega') \\ &+ S^{(2C)}(\omega,\omega')\,\theta(\omega)\theta(\omega')\theta(\omega'-\omega)\theta(\Omega-\omega'). \end{aligned}$$

- A formally finite soft function can still be defined, but we need consistency conditions with PDF factorization for validation.

CONCLUSION

- In the past few years, a lot of work has been devoted to understand the structure of large logarithms at next-to-leading power.

- We have now frameworks (SCET and diagrammatic approach) which allows us derive factorization theorems at general subleading power, at arbitrary loop accuracy, and to resum LLs at NLP, for color singlet processes.

- The next task is to understand how these methods can be applied beyond NLP LLs, and to more involved processes.

EXTRA: DIAGONAL VS OFF-DIAGONAL

- In general:

$$\Delta_{ab} \sim H \sum_i \int \{d\omega\}\, J_{ab,i}(\{\omega\})\, S_{ab,i}(\{\omega\}),$$

Beneke, Broggio, Jaskiewicz, LV, 2019,

Broggio, Jaskiewicz, LV, 2023

- One has e.g.

$$J^{K\,(1)}_{q\bar{q};1,1;\gamma\beta,fe}(n_+q, n_+p;\, \omega) = -\frac{\alpha_s}{4\pi}\delta_{\gamma\beta}\mathbf{T}^K_{fe}\,\frac{1}{(n_+p)}\left(\frac{n_+p\,\omega}{\mu^2}\right)^{-\epsilon}\frac{e^{\epsilon\,\gamma_E}\,\Gamma[1+\epsilon]\Gamma[1-\epsilon]^2}{(1-\epsilon)(1+\epsilon)\Gamma[2-2\epsilon]}$$
$$\times\left(C_F\left(-\frac{4}{\epsilon}+3+8\epsilon+\epsilon^2\right)-C_A\left(-5+8\epsilon+\epsilon^2\right)\right)\delta(n_+q-n_+p)\,,$$

$$S^{(1)}_{q\bar{q};1}(\Omega,\omega) = \frac{\alpha_s C_F}{2\pi}\,\frac{\mu^{2\epsilon}e^{\epsilon\gamma_E}}{\Gamma[1-\epsilon]}\,\frac{1}{\omega^{1+\epsilon}}\,\frac{1}{(\Omega-\omega)^\epsilon}\,\theta(\omega)\theta(\Omega-\omega)\,,$$

for the diagonal channel, and

$$G^{\eta,B\,(1)}_{g\bar{q};\gamma\alpha,fb}(n_+q, n_+p;\omega) = -\frac{\alpha_s}{4\pi}\,\mathbf{T}^B_{fb}(C_F-C_A)\left(\frac{n_+p\,\omega}{\mu^2}\right)^{-\epsilon}\frac{2-4\epsilon-\epsilon^2}{\epsilon^2}$$
$$\times\frac{e^{\epsilon\gamma_E}\Gamma[1+\epsilon]\Gamma[1-\epsilon]^2}{\Gamma[2-2\epsilon]}\left(\frac{\not{n}_-}{2}\gamma^\eta_\perp\right)_{\gamma\alpha}\delta(n_+q-n_+p)\,,$$

$$S^{(1)}_{g\bar{q}}(\Omega,\omega,\omega') = \frac{\alpha_s\, T_F}{4\pi}\,\frac{e^{\epsilon\gamma_E}}{\Gamma[1-\epsilon]}\,\frac{1}{\omega}\left(\frac{\mu^2}{\omega\,(\Omega-\omega)}\right)^{\epsilon}\delta(\omega-\omega')\,\theta(\Omega-\omega)\theta(\omega)\,,$$

for the off-diagonal one.