Precise predictions for the associated production of Higgs and W bosons with a $t\bar{t}$ pair at the LHC

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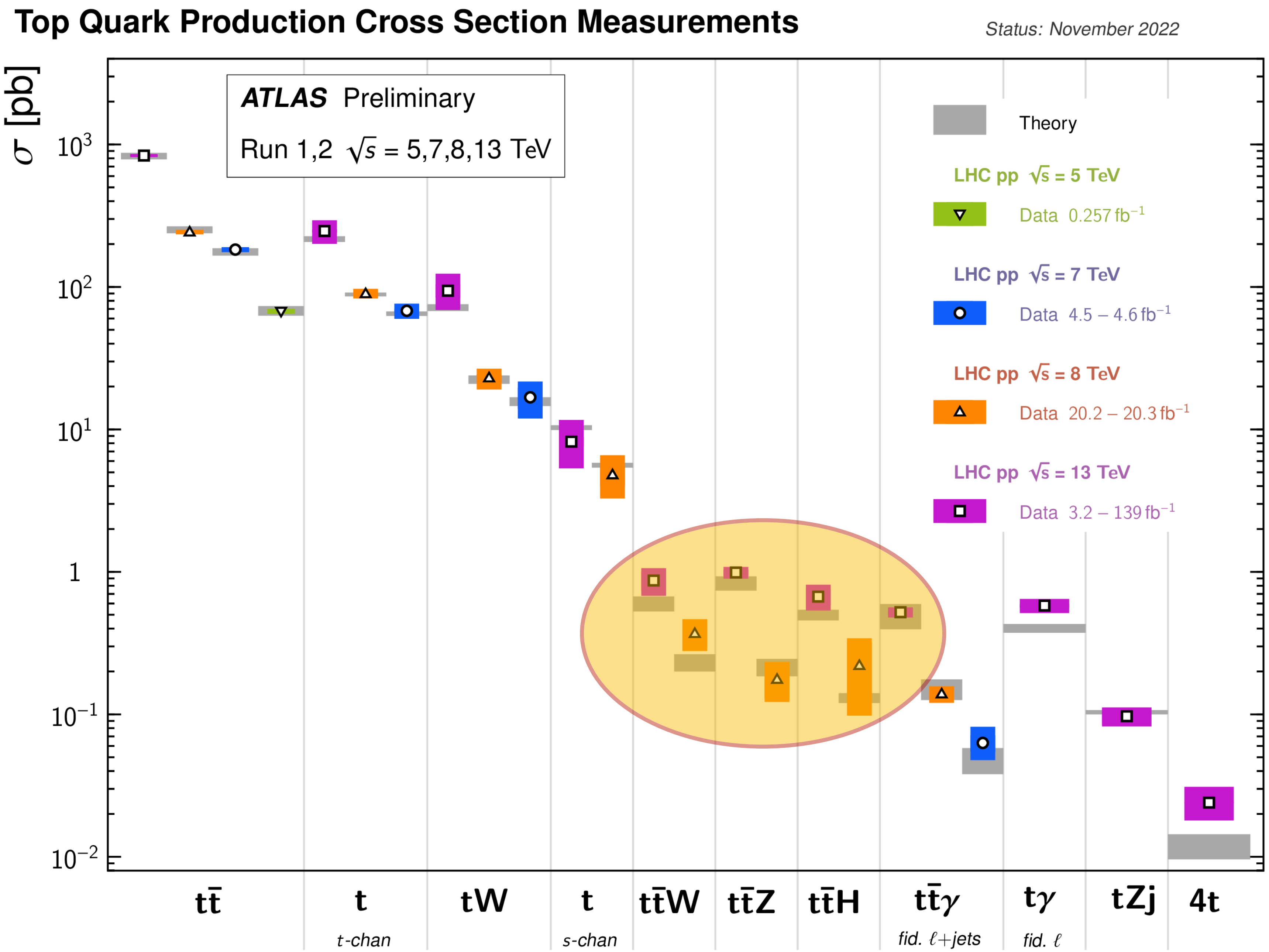
QQF

The production of a top-quark pair together with a vector or Higgs boson is among the most massive SM signatures at hadron colliders

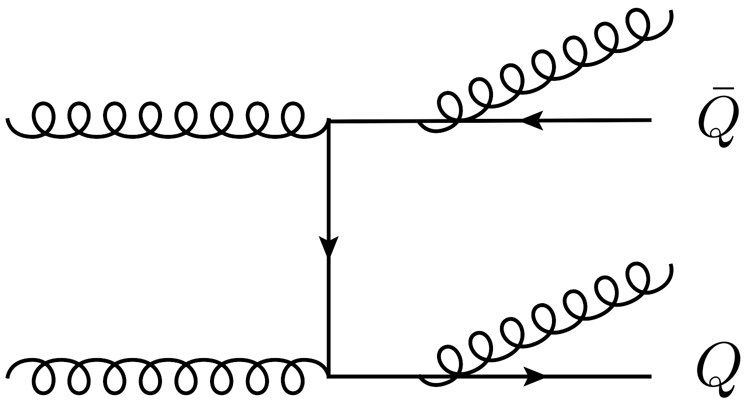


The cross sections are much smaller than tt but already measured

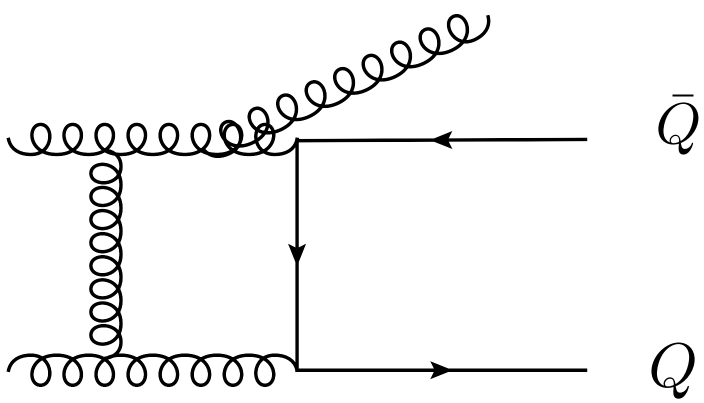
A deep understanding of these processes is crucial to characterise the top-quark interactions

NNLO QCD corrections needed

NNLO: building blocks

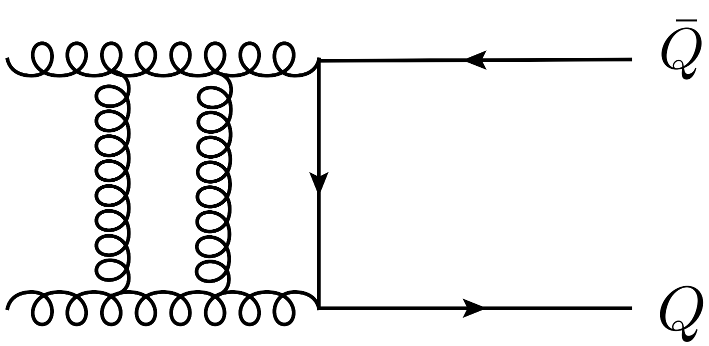


Tree-level amplitudes with two additional partons



One-loop amplitudes with one additional parton (to be evaluated in unresolved regions where instabilities may arise)

All the three contributions separately divergent !



Two-loop amplitudes → currently the major bottleneck (new class of functions, charting new territory…) + one loop squared

Crucial to keep the calculation fully differential: corrections for fiducial and inclusive rates may be significantly different (H in VBF, WW…)

Our method

Catani, MG (2007)

Consider the hard-scattering process $pp \to F + X$

F colourless system (vector, Higgs Boson(s)...)

Use a dimensionless resolution variable $r > r_{\rm cut}$ (e.g. $r = q_T/Q$)

Real contribution with one additional resolved jet, divergent as $r_{\rm cut} \to 0$

Subtraction counterterm that cancels the $r_{\rm cut} \to 0$ singularity

$$d\sigma^{F+X}_{NNLO} = \mathscr{H}^F_{NNLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+\text{jets}}_{NLO} - d\sigma^{CT,F}_{NNLO} \right] + \mathcal{O}(r^p_{\rm cut})$$

Virtual contribution after subtraction of IR singularities + collinear and large-angle soft radiation (beam, jet and soft function)

Power suppressed contribution whose size determines the efficiency of the computation

Structure of $\mathscr{H}^F$ and $d\sigma^{CT,F}$ can be obtained from all-order resummation: now known even at N³LO

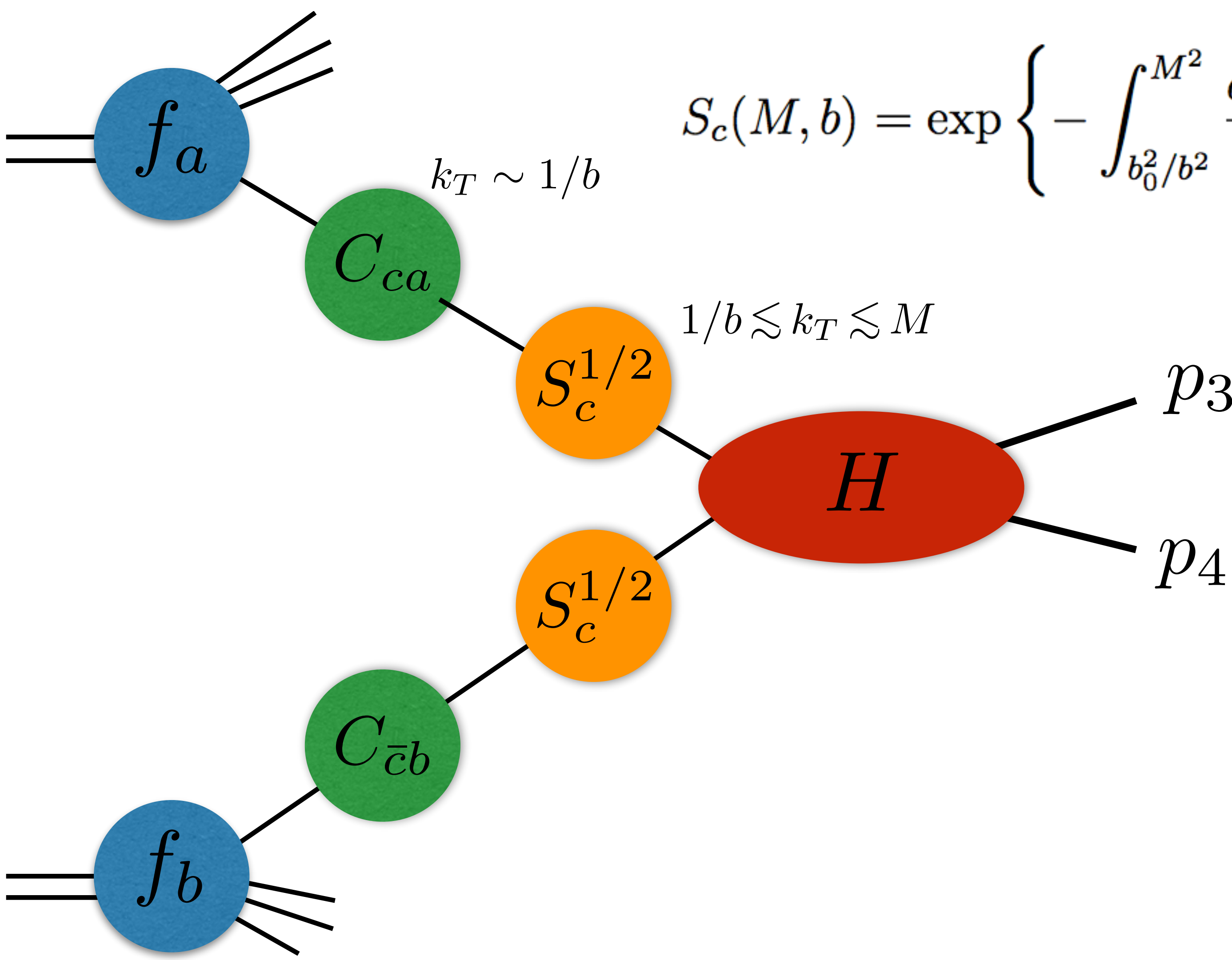
The resummation formula

J.Collins, D.Soper, G.Sterman (1984)
S.Catani, D. de Florian, MG (2000); S.Catani, MG (2010)

$$\frac{d\sigma_F^{(\text{sing})}(p_1,p_2;\mathbf{q_T},M,y,\mathbf{\Omega})}{d^2\mathbf{q_T}\;dM^2\;dy\;d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma^{(0)}_{c\bar{c},F}\right]\int \frac{d^2\mathbf{b}}{(2\pi)^2}\;e^{i\mathbf{b}\cdot\mathbf{q_T}}\;S_c(M,b)$$

$$\times\sum_{a_1,a_2}\int_{x_1}^{1}\frac{dz_1}{z_1}\int_{x_2}^{1}\frac{dz_2}{z_2}\;\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2}\;f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\;f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)$$

$$S_c(M,b)=\exp\left\{-\int_{b_0^2/b^2}^{M^2}\frac{dq^2}{q^2}\left[A_c(\alpha_S(q^2))\;\ln\frac{M^2}{q^2}+B_c(\alpha_S(q^2))\right]\right\}$$



C coefficients embody collinear radiation at scale 1/b

S_c embodies soft and flavour conserving collinear radiation in the region $1/b < k_T < M$

H^F includes hard radiation at scales $k_T \sim M$

Extension to heavy-quark production

Catani, Torre, MG (2014)

$$\frac{d\sigma^{(\mathrm{sing})}(P_1, P_2; \mathbf{q_T}, M, y, \mathbf{\Omega})}{d^2\mathbf{q_T}\; dM^2\; dy\; d\mathbf{\Omega}} = \frac{M^2}{2P_1 \cdot P_2} \sum_{c=q,\bar{q},g} \left[d\sigma^{(0)}_{c\bar{c}}\right] \int \frac{d^2\mathbf{b}}{(2\pi)^2}\; e^{i\mathbf{b}\cdot\mathbf{q_T}}\; S_c(M, b)$$

$$\times \sum_{a_1,a_2} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2}\; \left[(\mathbf{H}\,\mathbf{\Delta})\, C_1 C_2\right]_{c\bar{c};a_1 a_2}\; f_{a_1/h_1}(x_1/z_1, b_0^2/b^2)\; f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

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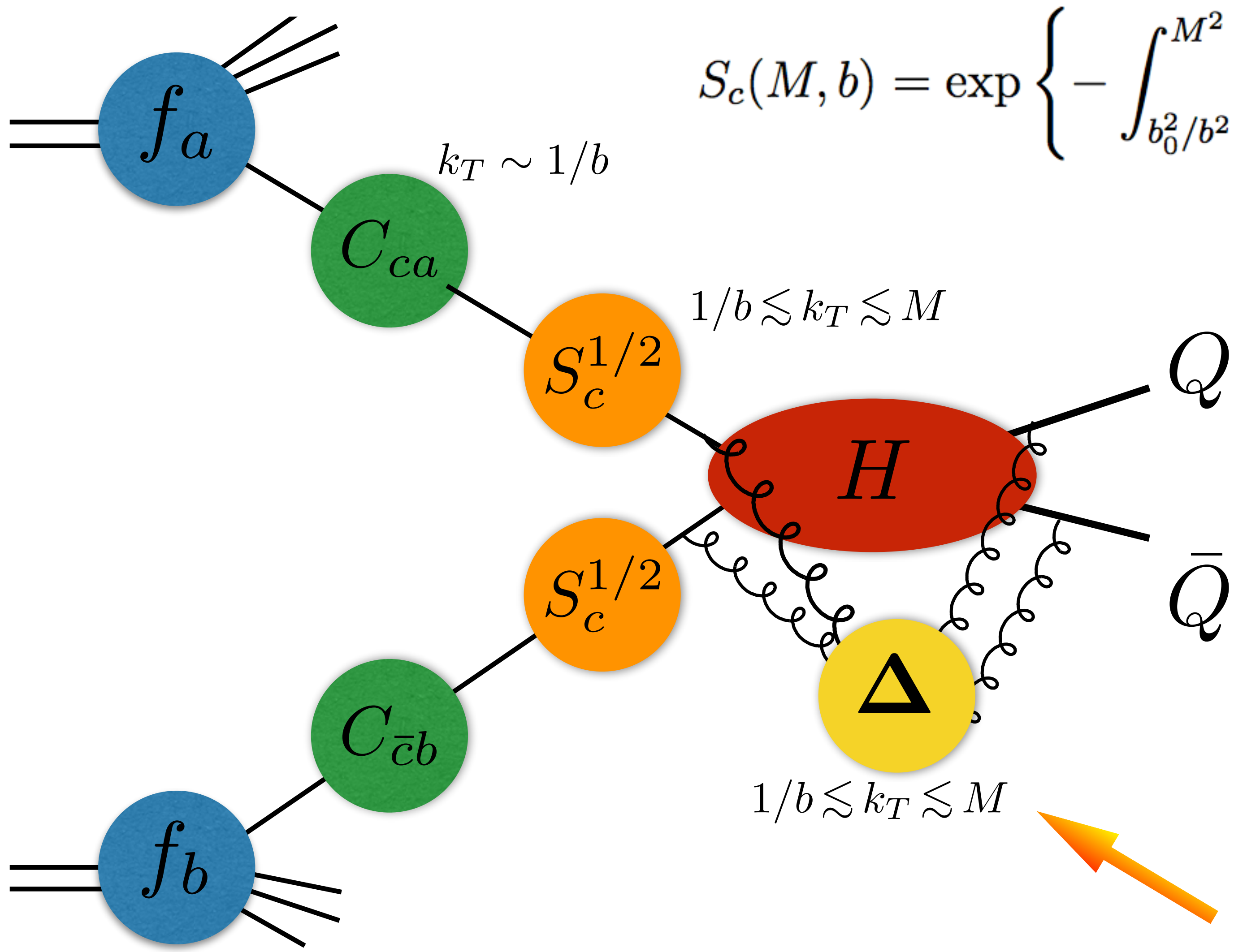

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H^F includes hard radiation at scales $k_T \sim M$

Additional radiative factor of purely soft origin (starts to contribute at NLL)

Extension to heavy-quark production

Catani, Devoto, Kallweit,Mazzitelli, Sargsyan, MG (2019)

We obtain a similar structure for the subtraction formula with some differences

$$d\sigma^{t\bar{t}}_{(N)NLO} = \mathcal{H}^{t\bar{t}}_{(N)NLO} \otimes d\sigma^{t\bar{t}}_{LO} + \left[d\sigma^{t\bar{t}+\mathrm{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right]$$

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✅ Modified subtraction counterterm fully known

Additional perturbative ingredient: soft anomalous dimension Γ_t known at NNLO

Mitov, Sterman, Sung (2009)
Neubert et al (2009)

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Additional perturbative ingredient: soft anomalous dimension Γ_t known at NNLO

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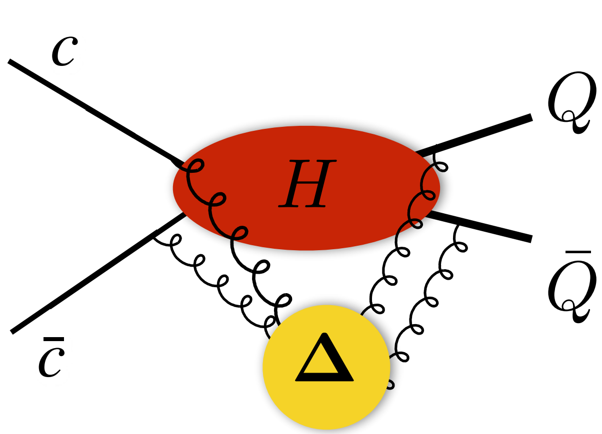
✅ Additional soft contributions needed to evaluate $\mathcal{H}^{t\bar{t}}_{NNLO}$

Catani, Devoto, Mazzitelli, MG (2023)



They can be computed by integrating a suitably subtracted soft current

Soft contributions at NLO

Catani, Torre, MG (2014)

Standard soft current contain the correct soft behaviour but also additional initial state collinear singularities

$$-\mathbf{J}(k)^2 = \sum_{i,j=1}^{4} \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)} \mathbf{T}_i \cdot \mathbf{T}_j$$

These singular contributions are already accounted for in the calculation of colour-singlets

We define a suitably subtracted soft current

$$-\mathbf{J}(k)^2|_{\rm sub} = \sum_{J=3,4} \left[\frac{p_J^2}{(p_J \cdot k)^2} \mathbf{T}_J^2 + \sum_{i=1,2} \left(\frac{p_i \cdot p_J}{p_J \cdot k} - \frac{p_1 \cdot p_2}{(p_1 + p_2) \cdot k} \right) \frac{2\, \mathbf{T}_i \cdot \mathbf{T}_J}{p_i \cdot k} \right] + \frac{2 p_3 \cdot p_4}{(p_3 \cdot k)(p_4 \cdot k)} \mathbf{T}_3 \cdot \mathbf{T}_4$$

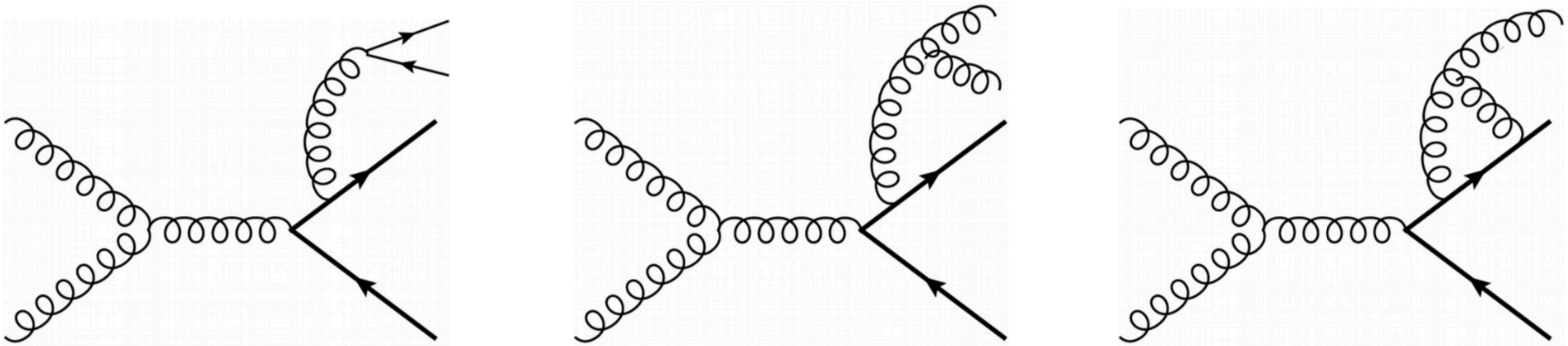
final state (heavy-quark) emitters

Initial state (massless) emitters

Note: subtraction of colourless contributions ensures that no rapidity divergences are present in our soft function, contrary to what happens in SCET

Soft contributions at NNLO

Catani, Devoto, Mazzitelli, MG (2023)



Three classes of contributions: singular structure fully known

- Emission of a soft quark-antiquark pair — Catani, MG (2000)
- Emission of two soft gluons — Catani, MG (2000); Czakon (2011)
- Soft-gluon emission at one loop — Catani, MG (2000); Bierenbaum, Czakon, Mitov (2011); Czakon, Mitov (2018)

Construct suitably subtracted soft current for each of these contribution

Intermediate results contain $1/\varepsilon^3$ poles $\Rightarrow$ add up to $1/\varepsilon^2$ in the end

MATRIX v2.1.0

Kallweit, Wiesemann, MG (June 2017)
+ Buonocore, Devoto, Mazzitelli, Rottoli.......

```
------------------------------------------------------------------------------------------------------
process_id     ||   process                          ||   description
------------------------------------------------------------------------------------------------------
pph21          >>   p p --> H                        >>   on-shell Higgs production (NNLO)
ppz01          >>   p p --> Z                        >>   on-shell Z production (NNLO,NLO EW)
ppw01          >>   p p --> W^-                      >>   on-shell W- production with CKM (NNLO)
ppwx01         >>   p p --> W^+                      >>   on-shell W+ production with CKM (NNLO)
ppeex02        >>   p p --> e^- e^+                  >>   Z production with decay (NNLO,NLO EW)
ppnenex02      >>   p p --> v_e^- v_e^+              >>   Z production with decay (NNLO,NLO EW)
ppenex02       >>   p p --> e^- v_e^+                >>   W- production with decay and CKM (NNLO,NLO EW)
ppexne02       >>   p p --> e^+ v_e^-                >>   W+ production with
ppaa02         >>   p p --> gamma gamma              >>   gamma gamma product
ppeexa03       >>   p p --> e^- e^+ gamma            >>   Z gamma production
ppnenexa03     >>   p p --> v_e^- v_e^+ gamma        >>   Z gamma production
ppenexa03      >>   p p --> e^- v_e^+ gamma          >>   W- gamma productio
ppexnea03      >>   p p --> e^+ v_e^- gamma          >>   W+ gamma production with decay (NNLO)
ppzz02         >>   p p --> Z Z                      >>   on-shell ZZ production (NNLO)
ppwxw02        >>   p p --> W^+ W^-                  >>   on-shell WW production (NNLO)
ppemexmx04     >>   p p --> e^- mu^- e^+ mu^+        >>   ZZ production with deca
ppeeexex04     >>   p p --> e^- e^- e^+ e^+          >>   ZZ production with deca
ppeexnmnmx04   >>   p p --> e^- e^+ v_mu^- v_mu^+    >>   ZZ production with deca
ppemxnmnex04   >>   p p --> e^- mu^+ v_mu^- v_e^+    >>   WW production with deca
ppeexnenex04   >>   p p --> e^- e^+ v_e^- v_e^+      >>   ZZ/WW production with d
ppemexnmx04    >>   p p --> e^- mu^- e^+ v_mu^+      >>   W-Z production with decay (NNLO,NLO EW)
ppeeexnex04    >>   p p --> e^- e^- e^+ v_e^+        >>   W-Z production with decay (NNLO,NLO EW)
ppeexmxnm04    >>   p p --> e^- e^+ mu^+ v_mu^-      >>   W+Z production with decay (NNLO,NLO EW)
ppeexexne04    >>   p p --> e^- e^+ e^+ v_e^-        >>   W+Z production with decay (NNLO,NLO EW)
ppttx20        >>   p p --> top anti-top             >>   on-shell top-pair production (NNLO)
ppaaa03        >>   p p --> gamma gamma gamma        >>   gamma gamma gamma production (NNLO)
|============>>
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NNLO QCD + NLO EW for all the single and massive diboson processes

NLO QCD for loop induced gg contribution for WW and ZZ

MATRIX v2.1.0

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ppeexnmnmx04   >>   p p --> e^- e^+ v_mu^- v_mu^+        >>   ZZ production with deca
ppemxnmnex04   >>   p p --> e^- mu^+ v_mu^- v_e^+        >>   WW production with deca
ppeexnenex04   >>   p p --> e^- e^+ v_e^- v_e^+          >>   ZZ/WW production with d
ppemexnmx0                              v_mu^+           >>   W-Z production with decay
ppeeexnex0                              v_e^+            >>   W-Z production with decay (NNLO,NLO EW)
                                        v_mu^-           >>   W+Z production with decay (NNLO,NLO EW)
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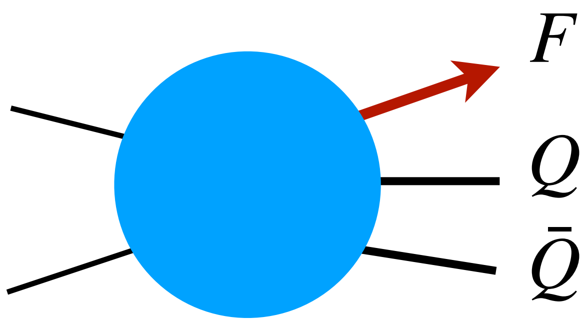
NLO QCD for loop induced gg contribution for WW and ZZ

$t\bar{t}$ and $\gamma\gamma\gamma$ now available

QQF



When the heavy quark pair is accompanied by a colourless system the resummation and subtraction formalisms can be applied in an analogous way with just two additional complications

Catani, Fabre, Kallweit, MG (2020)

- The colourless system takes away momentum and the computation of the soft function has to be extended accordingly

  Devoto, Mazzitelli, (to appear)

- For some important processes ($t\bar{t}Z$, $WWb\bar{b}$....) three-parton correlators are non vanishing and also contribute to the soft integrals (this is not the case for $t\bar{t}$ and $t\bar{t}H$ and $t\bar{t}W$)

How about two loop amplitudes ?

Two loop $2 \to 3$ amplitudes are at the frontier: several massless computations now completed ($\gamma\gamma\gamma$, $\gamma\gamma+$jet, $\gamma+$2jets, 3jets, $W+4$ partons..) typically first using the leading colour approximation but not with masses around !

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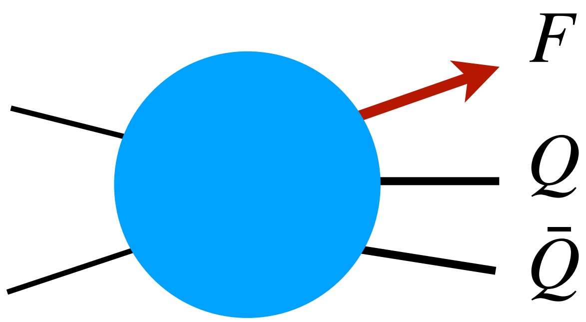

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➡ **Use approximated form of two-loop contribution to obtain first NNLO results for $t\bar{t}H$, $Wb\bar{b}$ and $t\bar{t}W$**

Catani, Devoto, Kallweit,Mazzitelli, Savoini, MG (2022)

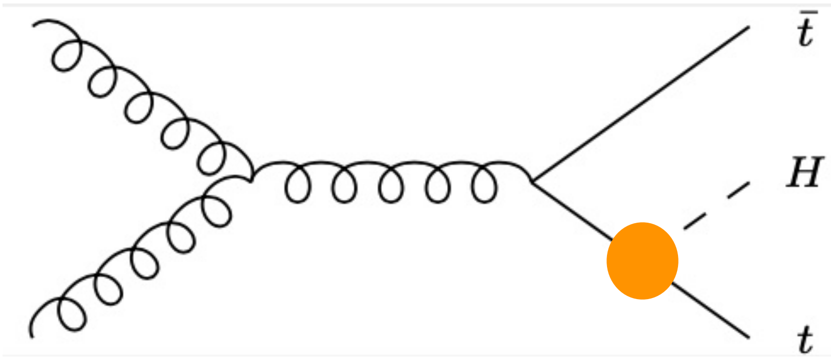
Buonocore, Devoto, Kallweit,Mazzitelli, Rottoli, Savoini (2022)

Buonocore, Devoto, Kallweit,Mazzitelli, Rottoli, Savoini, MG (2023)

ttH

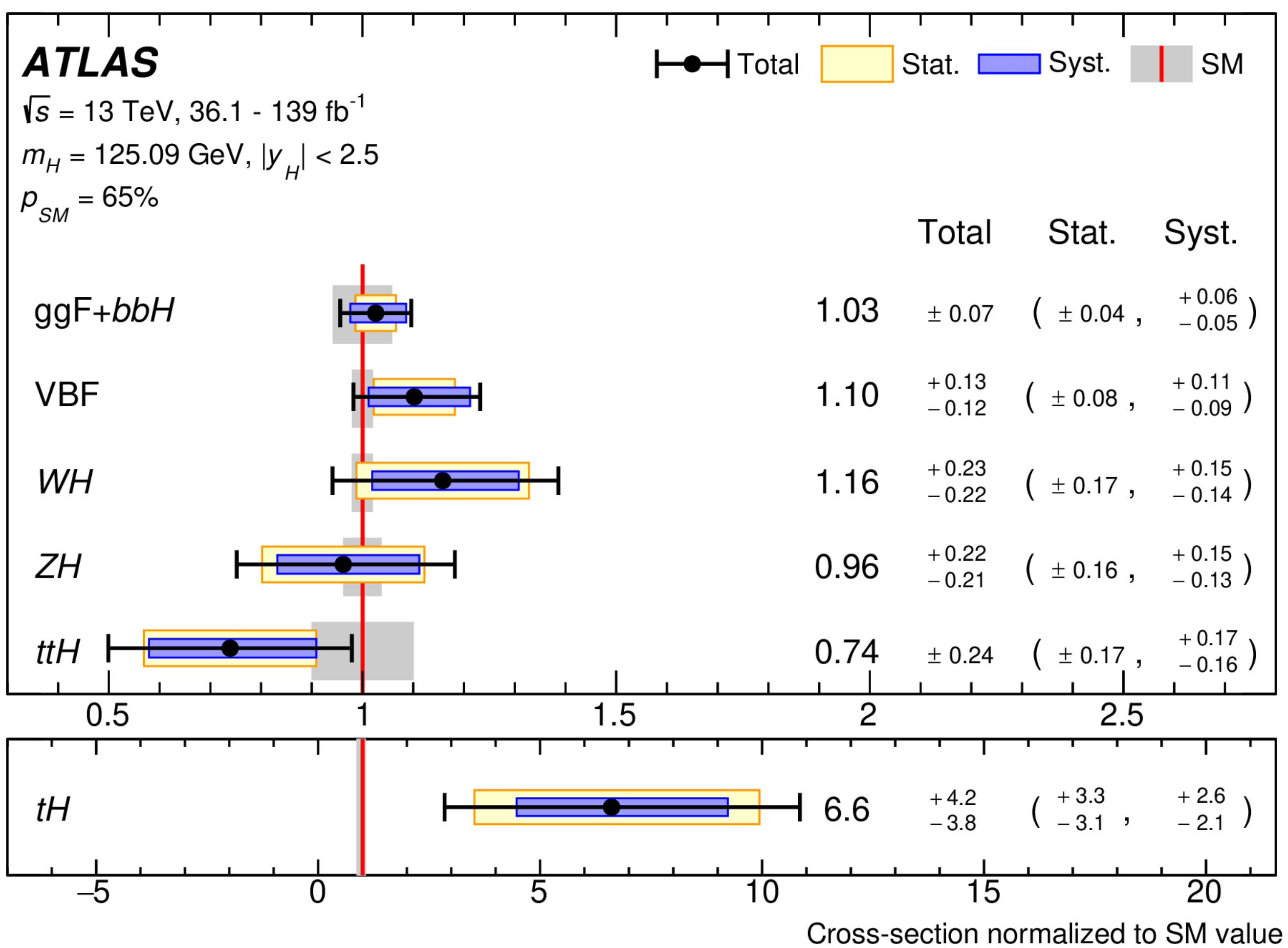
Catani, Devoto, Kallweit, Mazzitelli, Savoini, MG (2022)

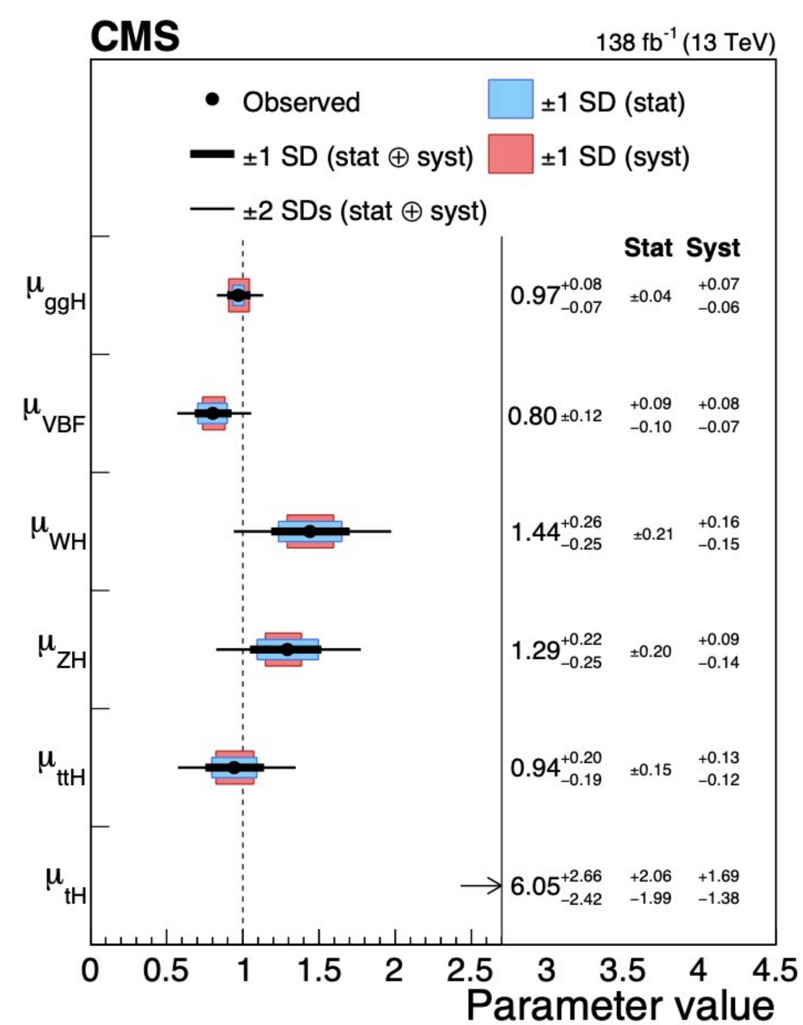
The associated production of the Higgs boson with a top-quark pair is a crucial process at the LHC



It allows a direct extraction of the top Yukawa

Experimental uncertainties are now at the $\mathcal{O}(20\%)$ level





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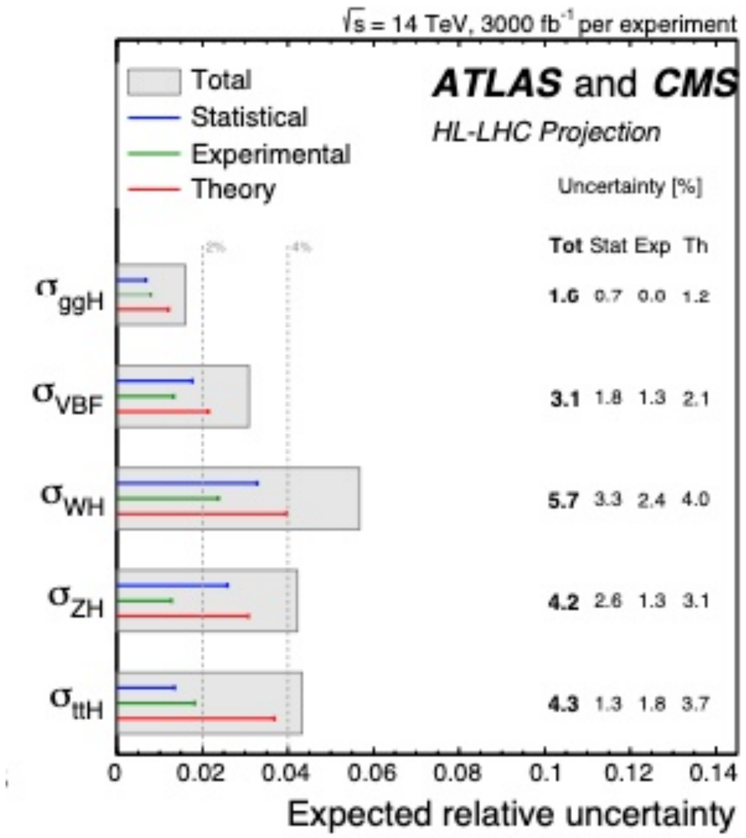


Experimental precision expected to get to the $\mathcal{O}(2\%)$ level at the end of HL-LHC

Current predictions based on NLO QCD+EW (+ resummations) and affected by $\mathcal{O}(10\%)$ uncertainty

NNLO QCD needed to bring theory uncertainty down to the $\mathcal{O}(2\%)$ level expected

First step completed by evaluation of the contribution of the off-diagonal partonic channels

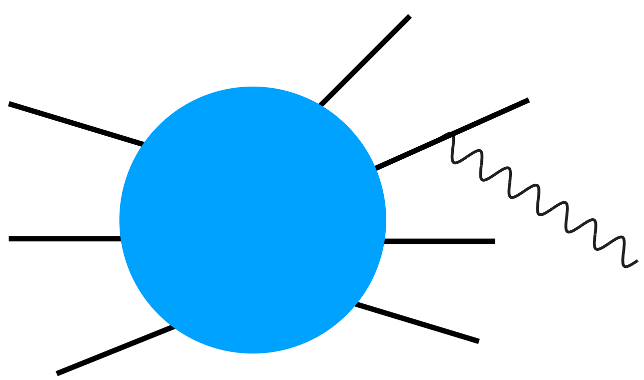
Catani, Fabre,Kallweit, MG(2020)

Missing ingredients are the two-loop $gg \to t\bar{t}H$ and $q\bar{q} \to t\bar{t}H$ amplitudes

The idea:
use a soft-Higgs approximation only for the missing two-loop amplitude

Soft-Higgs radiation

When a soft photon (or gluon) is emitted in a high-energy process the corresponding amplitudes obey well known factorisation formulae



$$\mathcal{M}(\{p_i\}, k) \simeq J^\mu(k)\epsilon_\mu(k)\mathcal{M}(\{p_i\}) \qquad J^\mu(k) = \sum_i e_i \frac{p_i^\mu}{p_i \cdot k}$$

Soft photon: large wavelength

➡ Does not "see" the details of the hard process but only external charges

An analogous formula holds for the emission of a soft scalar off heavy quarks

$$\mathcal{M}(\{p_i\}, k) \simeq J(k)\mathcal{M}(\{p_i\})$$

At tree level it is straightforward to show that

heavy-quark mass

$$J(k) = \sum_i \frac{m}{v}\frac{m}{p_i \cdot k}$$

heavy-quark momenta

Soft-Higgs radiation

This formula can be extended to all orders in the QCD coupling α_S

$$\boxed{\mathscr{M}(\{p_i\}, k) \simeq F(\alpha_S(\mu_R); m/\mu_R)\, J(k) \mathscr{M}(\{p_i\})}$$

Soft-Higgs radiation

This formula can be extended to all orders in the QCD coupling α_S

$$\mathcal{M}(\{p_i\},k) \simeq F(\alpha_S(\mu_R);m/\mu_R)\,J(k)\mathcal{M}(\{p_i\})$$

Physical picture: Higgs soft current essentially "abelian": no corrections beyond LO except for over all normalisation

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Physical picture: Higgs soft current essentially "abelian": no corrections beyond LO except for over all normalisation

The perturbative function $F(\alpha_S(\mu_R); m/\mu_R)$ can be extracted from the soft limit of the scalar form factor of the heavy quark

Bernreuther et al (2005)
Blümlein et al (2017)

$$F(\alpha_S(\mu_R); m/\mu_R) = 1 + \frac{\alpha_S(\mu_R)}{2\pi}(-3C_F)$$
$$+ \left(\frac{\alpha_S(\mu_R)}{2\pi}\right)^2 \left(\frac{33}{4}C_F^2 - \frac{185}{12}C_F C_A + \frac{13}{6}C_F(n_L+1) - 6C_F\beta_0 \ln\frac{\mu_R^2}{m^2}\right) + \mathcal{O}(\alpha_S^3)$$

Alternatively, it can be derived by using Higgs low-energy theorems

See e.g. Kniehl and Spira (1995)

Soft-Higgs radiation

We have done several checks of our factorisation formula by assuming a very light and soft Higgs boson

$$\mathcal{M}(\{p_i\},k) \simeq F(\alpha_S(\mu_R); m/\mu_R)\, J(k) \mathcal{M}(\{p_i\})$$

- We have tested it numerically with Openloops up to one-loop order in the case of $t\bar{t}H$ production
- We have tested it numerically with Recola up to one-loop order in the case of $t\bar{t}t\bar{t}H$ production

The formula can be useful to cross check future exact calculations of QCD amplitudes with heavy quarks and a Higgs boson

Can it be used to complete the NNLO calculation for $t\bar{t}H$ production ?

Remarkably, yes !

The computation

The starting point is again the q_T subtraction formula

$$d\sigma = \mathcal{H} \otimes d\sigma_{\mathrm{LO}} + \left[d\sigma_{\mathrm{R}} - d\sigma_{\mathrm{CT}}\right]$$

All the ingredients in this formula for $t\bar{t}H$ are now available and implemented in MATRIX except the two-loop virtual amplitudes entering $\mathcal{H}$

We define

$$\mathcal{H} = H\delta(1 - z_1)\delta(1 - z_2) + \delta\mathcal{H}$$

$$H^{(n)} = \frac{2\mathrm{Re}\left(\mathcal{M}_{\mathrm{fin}}^{(n)}\mathcal{M}^{(0)*}\right)}{|\mathcal{M}^{(0)}|^2}$$

with

$$H = 1 + \frac{\alpha_S(\mu_R)}{2\pi}H^{(1)} + \left(\frac{\alpha_S(\mu_R)}{2\pi}\right)^2 H^{(2)} + \ldots.$$

$$|\mathcal{M}_{\mathrm{fin}}(\mu_{IR})\rangle = \mathbf{Z}^{-1}(\mu_{IR})|\mathcal{M}\rangle$$

IR subtraction

For $n = 2$ this definition allows us to single out the only missing ingredient in the NNLO calculation, that is, the coefficient $H^{(2)}$

Note that all the remaining terms are computed exactly (including $|\mathcal{M}_{\mathrm{fin}}^{(1)}|^2$)

We have used our factorisation formula to construct approximations of the $H^{(1)}$ and $H^{(2)}$ coefficients

Since the Higgs is not at all soft, in order to use the factorisation formula we have to introduce a mapping that from a $t\bar{t}H$ event defines a $t\bar{t}$ event with no Higgs boson

To this purpose we use the q_T recoil prescription

Catani, Ferrera, de Florian, MG (2016)

With this prescription the momentum of the Higgs boson is equally reabsorbed by the initial state partons, leaving the top and antitop momenta unchanged

The required tree-level and one-loop amplitudes are obtained using Openloops

The $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$ two-loop amplitudes needed to apply our approximation are those provided by Czakon et al.

Bärnreuther, Czakon, Fiedler (2013)

Setup: NNPDF31 NNLO partons with 3-loop α_S
$m_H = 125\,\mathrm{GeV}$ and $m_t = 173.3\,\mathrm{GeV}$

Central values for factorisation and renormalisation scales
$\mu_F = \mu_R = (2m_t + m_H)/2$

Our first check is on the LO cross sections: we find that the soft approximation overestimates it by

- gg channel: a factor of 2.3 at $\sqrt{s} = 13\,\mathrm{TeV}$ and a factor of 2 at $\sqrt{s} = 100\,\mathrm{TeV}$
- $q\bar{q}$ channel: a factor of 1.11 at $\sqrt{s} = 13\,\mathrm{TeV}$ and a factor of 1.06 at $\sqrt{s} = 100\,\mathrm{TeV}$

These are absolute LO predictions: in our calculation we will actually need to approximate $H^{(1)}$ and $H^{(2)}$ that are normalised to LO matrix elements

$$H^{(n)} = \frac{2\mathrm{Re}\left(\mathscr{M}^{(n)}_{\mathrm{fin}}\mathscr{M}^{0)*}\right)}{|\mathscr{M}^{(0)}|^2}$$

We expect this approximation to work better than simply computing $2\mathrm{Re}\left(\mathscr{M}^{(n)}_{\mathrm{fin}}\mathscr{M}^{(0)*}\right)$: effective reweighing of LO cross section

When computing virtual amplitudes we will set the infrared subtraction scale μ_{IR} to the invariant mass of the final state system

	$\sqrt{s} = 13\,\mathrm{TeV}$		$\sqrt{s} = 100\,\mathrm{TeV}$	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
σ_{LO}	261.58	129.47	23055	2323.7
$\Delta\sigma_{\mathrm{NLO,H}}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\mathrm{NLO,H}}\vert_{\mathrm{soft}}$	61.98	7.413	5612	206.0

We now move to NLO and compare the exact contribution from $H^{(1)}$ to the one computed in the soft approximation

The hard contribution computed in the soft approximation is underestimated by just 30 % in the gg channel and by 5 % in the $q\bar{q}$

The mismatch that we observe at NLO can be used to estimate the uncertainty of our approximation at NNLO

The quality of our final result will depend on the size of the contribution we approximate

	$\sqrt{s} = 13$ TeV		$\sqrt{s} = 100$ TeV	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
σ_{LO}	261.58	129.47	23055	2323.7
$\Delta\sigma_{\mathrm{NLO,H}}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\mathrm{NLO,H}}\vert_{\mathrm{soft}}$	61.98	7.413	5612	206.0
$\Delta\sigma_{\mathrm{NNLO,H}}\vert_{\mathrm{soft}}$	$-2.980(3)$	2.622(0)	$-239.4(4)$	65.45(1)

At NNLO the hard contribution is about 1% of the LO cross section in the gg channel and 2% in the $q\bar{q}$ channel

We can therefore anticipate that at NNLO the uncertainties due to the soft approximation will be rather small.

But how can we estimate these uncertainties ?

We have carefully studied the stability of our results under variations of the approximation procedure

- We have varied the recoil procedure: reabsorbing the Higgs momentum in just one of the initial state partons leads to negligible differences

- We have repeated our computation by using different subtraction scales at which the finite part of the two-loop virtual amplitude in $H^{(2)}$ is defined

When varying μ_{IR} from $M/2$ to $2M$ and adding the exact evolution terms from these scales back to M

- In the gg channel we find ${}^{+164\%}_{-25\%}$ at 13 TeV and ${}^{+142\%}_{-20\%}$ at 100 TeV

- In the $q\bar{q}$ channel we find ${}^{+4\%}_{-0\%}$ at 13 TeV and ${}^{+3\%}_{-0\%}$ at 100 TeV

To define our uncertainties we start from the NLO result: the hard contribution computed in the soft approximation is underestimated by just 30% in the gg channel and by 5% in the $q\bar{q}$ therefore the NNLO uncertainty cannot be smaller than these values

We multiply these uncertainties by a tolerance factor of 3

We finally combine the gg and $q\bar{q}$ uncertainties linearly ➡ $\pm 0.6\,\%$ on σ_{NNLO}

Results

σ [pb]	$\sqrt{s} = 13\,\mathrm{TeV}$	$\sqrt{s} = 100\,\mathrm{TeV}$
σ_{LO}	$0.3910\,^{+31.3\%}_{-22.2\%}$	$25.38\,^{+21.1\%}_{-16.0\%}$
σ_{NLO}	$0.4875\,^{+5.6\%}_{-9.1\%}$	$36.43\,^{+9.4\%}_{-8.7\%}$
σ_{NNLO}	$0.5070\,(31)^{+0.9\%}_{-3.0\%}$	$37.20(25)\,^{+0.1\%}_{-2.2\%}$

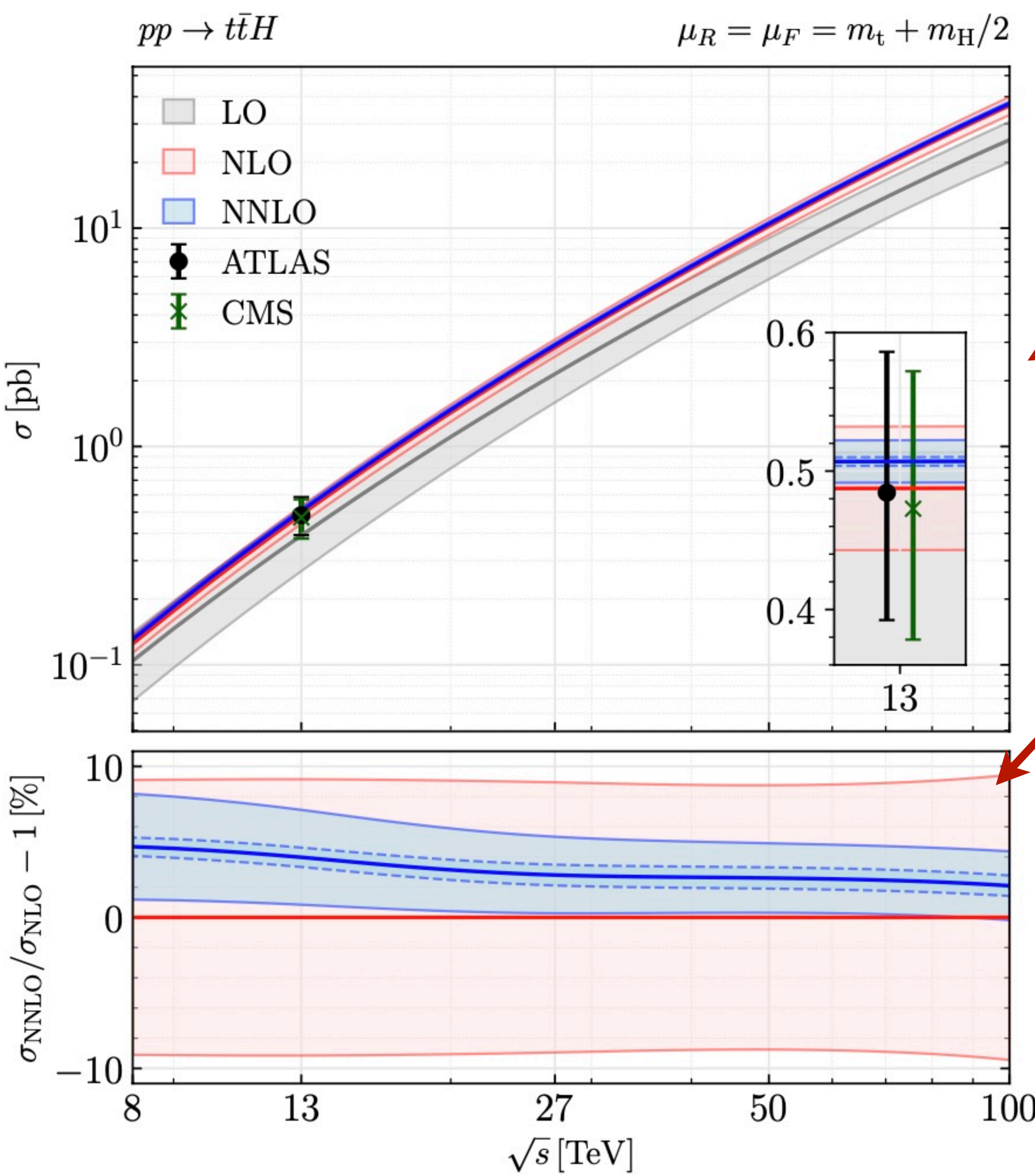
NLO effect is about +25 % at 13 TeV and +44 % at 100 TeV

NNLO effect is about +4 % at 13 TeV and +2 % at 100 TeV

Significant reduction of perturbative uncertainties

Errors in bracket obtained combining uncertainty from the soft approximation and the q_T subtraction systematics (same procedure used in MATRIX)

Results



ATLAS and CMS results from Nature 2022 papers

Perturbative uncertainties estimated by symmetrising the standard 7-point scale variation

Dashed band: residual error from soft approx+systematics

Note that: sensible comparison with data should eventually be done including NLO EW corrections ($+1.7\,\%$ at $\sqrt{s} = 13\,\mathrm{TeV}$)

ttW

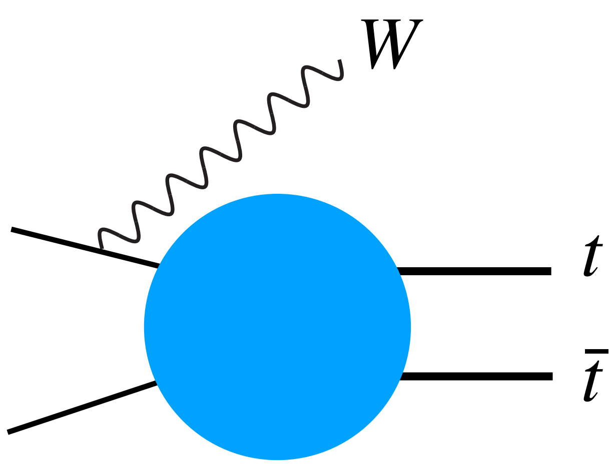
Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, Savoini, MG (2023)

Among the ttV signatures, ttW is special because it involves both EW and top sectors

It is at the same time a signal and a background to ttH and tttt and new physics searches

Since the top quark quickly decays into a W and a b jet, the signature is characterised by 3 W bosons

→ It provides an irreducible source of same-sign dilepton pairs relevant for many BSM searches



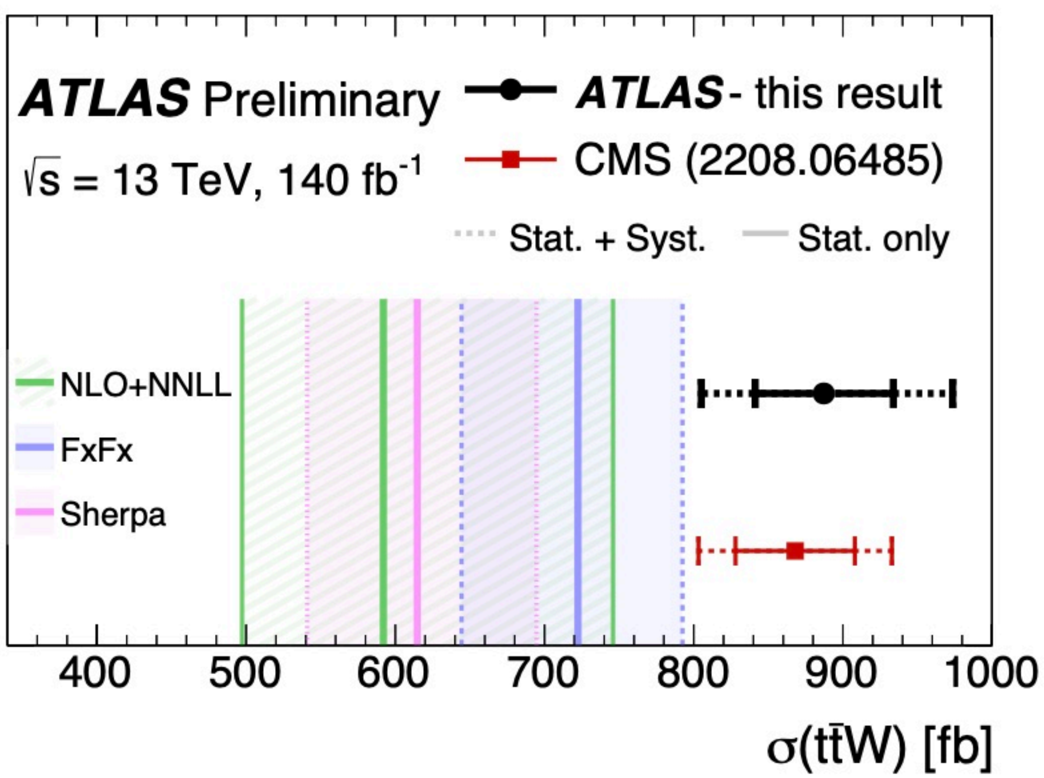
It is special compared to other ttF $(F = H, Z, \gamma)$ signatures because the W can only be emitted by the initial-state light quarks (no gg channel at LO)

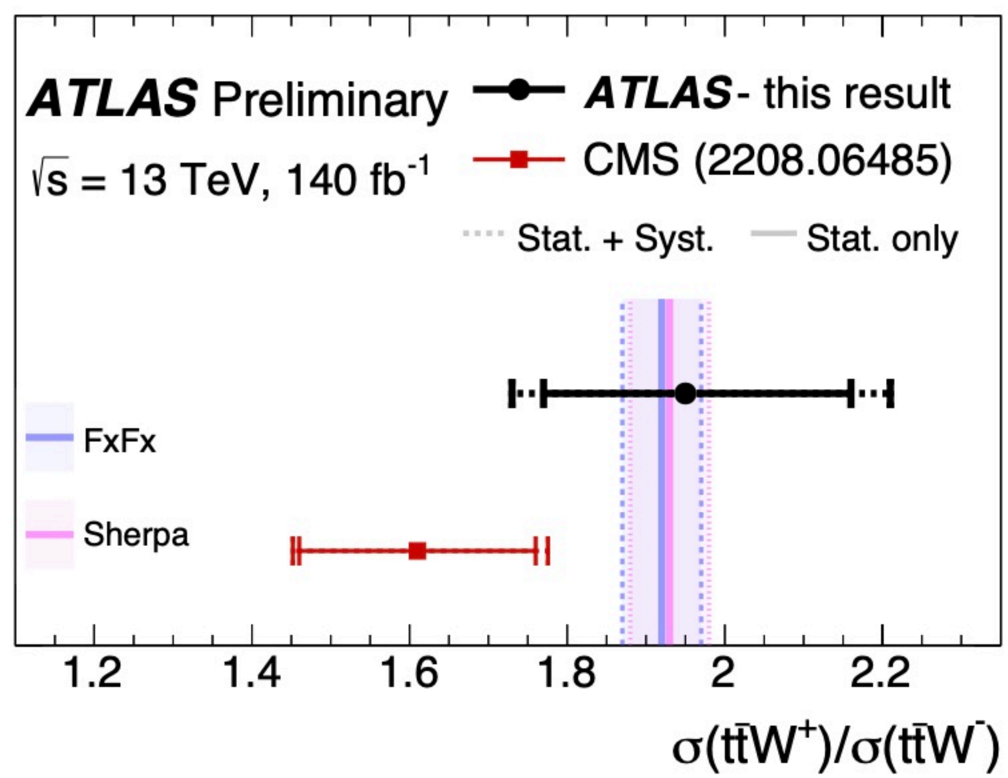
$t\bar{t}W$

Measurements by ATLAS and CMS at $\sqrt{s} = 8\,\text{TeV}$ and $\sqrt{s} = 13\,\text{TeV}$ showed that the $t\bar{t}W$ rate is consistently higher than the SM prediction

This discrepancy is also confirmed by indirect measurements of $t\bar{t}W$ in the context of $t\bar{t}H$ and 4top analyses

The most recent measurements confirm this picture with a slight excess at the $1\sigma - 2\sigma$ level





ttW

Theory predictions still essentially based on NLO QCD and EW predictions

Badger, Campbell, Ellis (2010); Campbell, Ellis (2012);
Dror, Farina, Salvioni, Serra (2015);
Frixione, Hirschi, Pagani, Shao, Zaro (2015);
Bevilacqua et al. (2020); Denner, Pelliccioli (2020)

+ soft-gluon resummation

Broggio et al (2016); Kulesza et al (2019)

+ multijet merging (FxFx)

Frixione, Frederix (2010); Frederix, Tsinikos (2021)

Current theory reference

NNLO computation could be carried out analogously to ttH if the two-loop Wtt amplitude were available

Can we obtain an estimate of the missing two-loop contribution ? Yes !

We constructed and tested two different approximations of the two-loop amplitude

1) Use soft approximation for W emission with momentum k and polarisation $\varepsilon(k)$ to express ttW amplitude in terms of the $q\bar{q} \to t\bar{t}$ amplitude

$$\mathcal{M}(\{p_i\}, k, \mu_R; \epsilon) \simeq \frac{g}{\sqrt{2}} \left(\frac{p_2 \cdot \varepsilon^*(k)}{p_2 \cdot k} - \frac{p_1 \cdot \varepsilon^*(k)}{p_1 \cdot k} \right) \mathcal{M}_L(\{p_i\}, \mu_R; \epsilon)$$

$q_L \bar{q}_R \to t\bar{t}$ virtual amplitude

Bärnreuther et al. (2013)
Mastrolia et al (2022)

2) Start from massless W+4 parton amplitudes

Abreu et al. (2021)

Use a "massification" procedure to obtain the leading terms in a $m_Q/Q \ll 1$ expansion

Penin (2006)
Moch, Mitov (2007)
Becher, Melnikov (2007)

$$\mathcal{M}(\{p_i\}, k; \mu_R; \epsilon) \simeq Z_{[q]}^{(m_Q|0)}(\alpha_S(\mu), m_Q/\mu, \epsilon) \mathcal{M}^{(m_Q=0)}(\{p_i\}, k; \mu_R; \epsilon)$$

Universal perturbatively computable factor

Successfully applied to the NNLO computation of Wbb

Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, Savoini (2023)

As done for $t\bar{t}H$ we have used our factorisation formulas to construct approximations of the $H^{(1)}$ and $H^{(2)}$ coefficients

To properly define our approximations we need momentum mappings

- For the soft-W approximation we absorb the W momentum into the top quarks, thus preserving the invariant mass of the event

- For the massification we map the momenta of the massive top quarks into massless momenta by preserving the four-momentum of the pair

Required tree-level and one-loop amplitudes obtained using Openloops and Recola

- The $q\bar{q} \to t\bar{t}$ two-loop amplitudes needed to apply our soft approximation are those provided by Czakon et al.

Bärnreuther, Czakon, Fiedler (2013); Mastrolia et al (2022)

- The W+4 parton massless two-loop amplitudes needed to use massification are those from Abreu et al (leading colour approximation)

Abreu et al (2021)

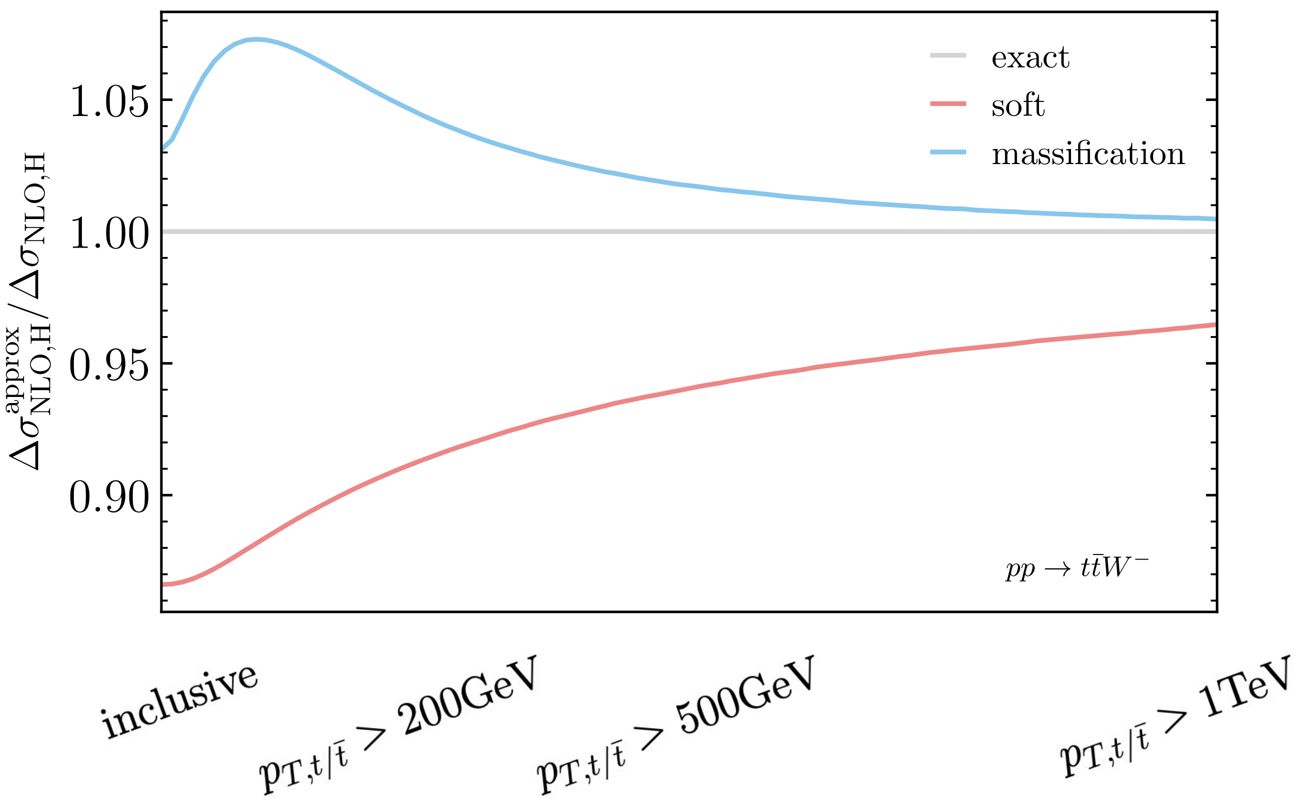
Setup: NNPDF31_nnlo_as_0118_luxqed partons with 3-loop α_S

$\sqrt{s} = 13$ TeV

Central values for factorisation and renormalisation scales $\mu_F = \mu_R = (2m_t + m_W)/2 \equiv = M/2$

ttW

Buonocore, Devoto, Kallweit,
Mazzitelli,Rottoli, Savoini, MG (2023)



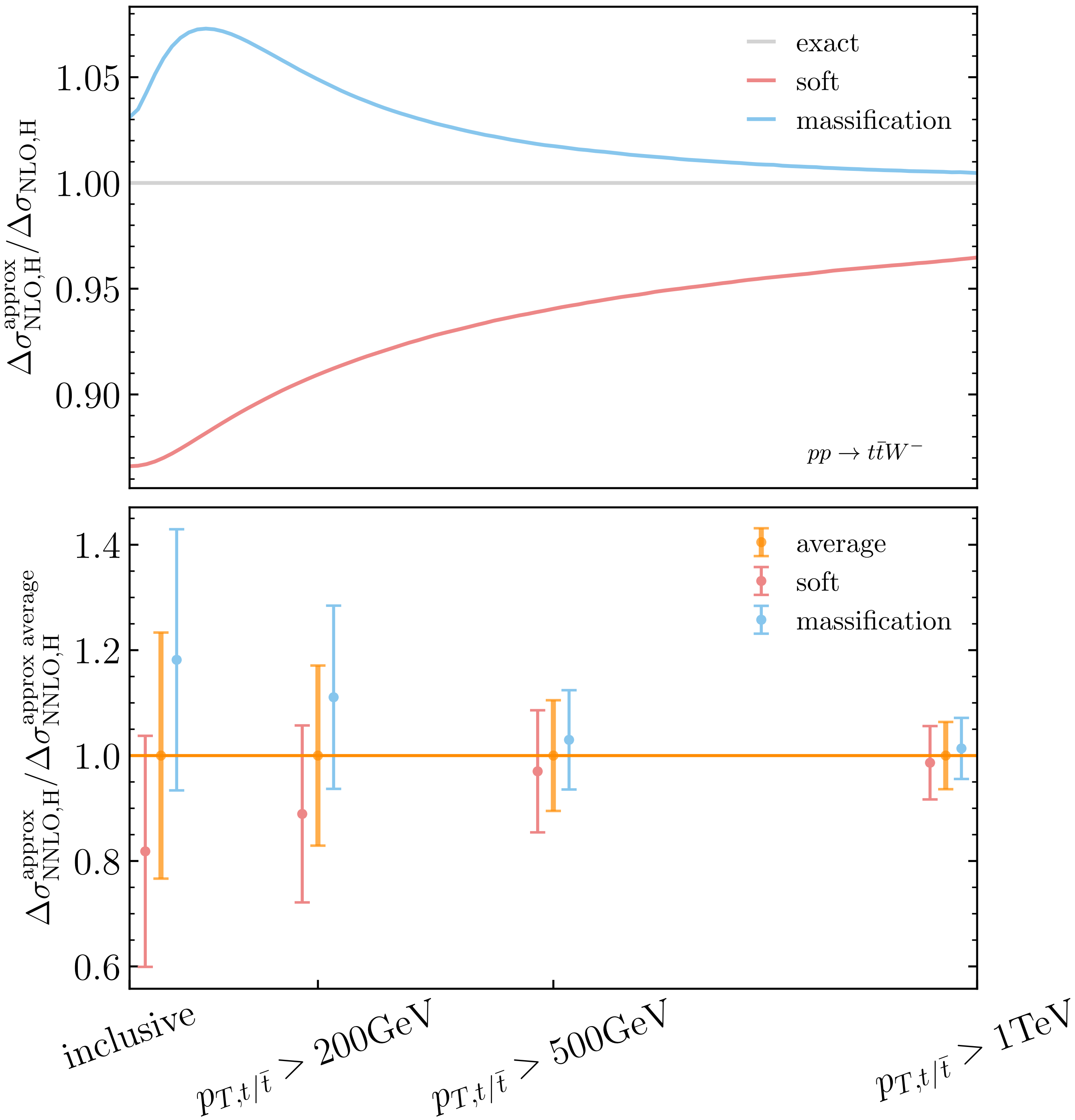
Both approximations provide a good estimate of the exact one-loop contribution

Soft approximation undershoots the exact results while massification tends to overshoot it

Clear asymptotic behaviour towards exact result for high p_T of the top quarks where both approximations are expected to work

ttW

Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, Savoini, MG (2023)



The pattern is preserved at NNLO: massified result systematically higher than soft approximation

We define the uncertainty of each approximation as the maximum between what we obtain varying the subtraction scale $1/2 \leq \mu_{\text{IR}}/Q \leq 2$ and twice the NLO deviation

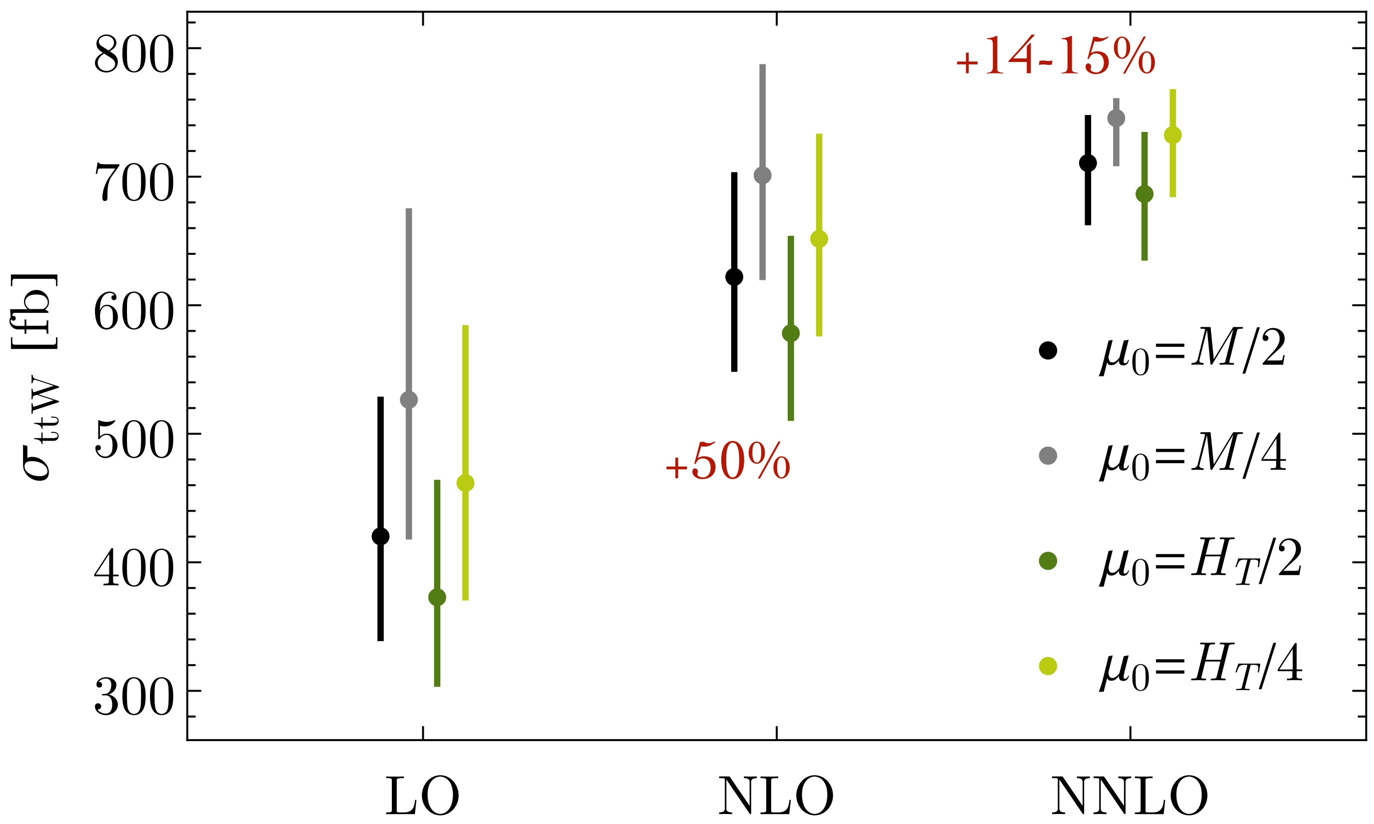
➡ Our best prediction obtained as average of the two with linear combination of uncertainties

Final uncertainty on two-loop contribution about 25% and similar to what obtained in recent $2 \to 3$ calculations in leading color approximation

Abreu et al (2023)

Impact of two-loop virtual contribution: 6-7% of NNLO cross section

Perturbative uncertainties



Our predictions are obtained by using $\mu_0 = M/2$ as central scale and performing standard 7-point scale variations

We have repeated our calculation using $H_T/2$, $H_T/4$ and $M/4$ as central scales

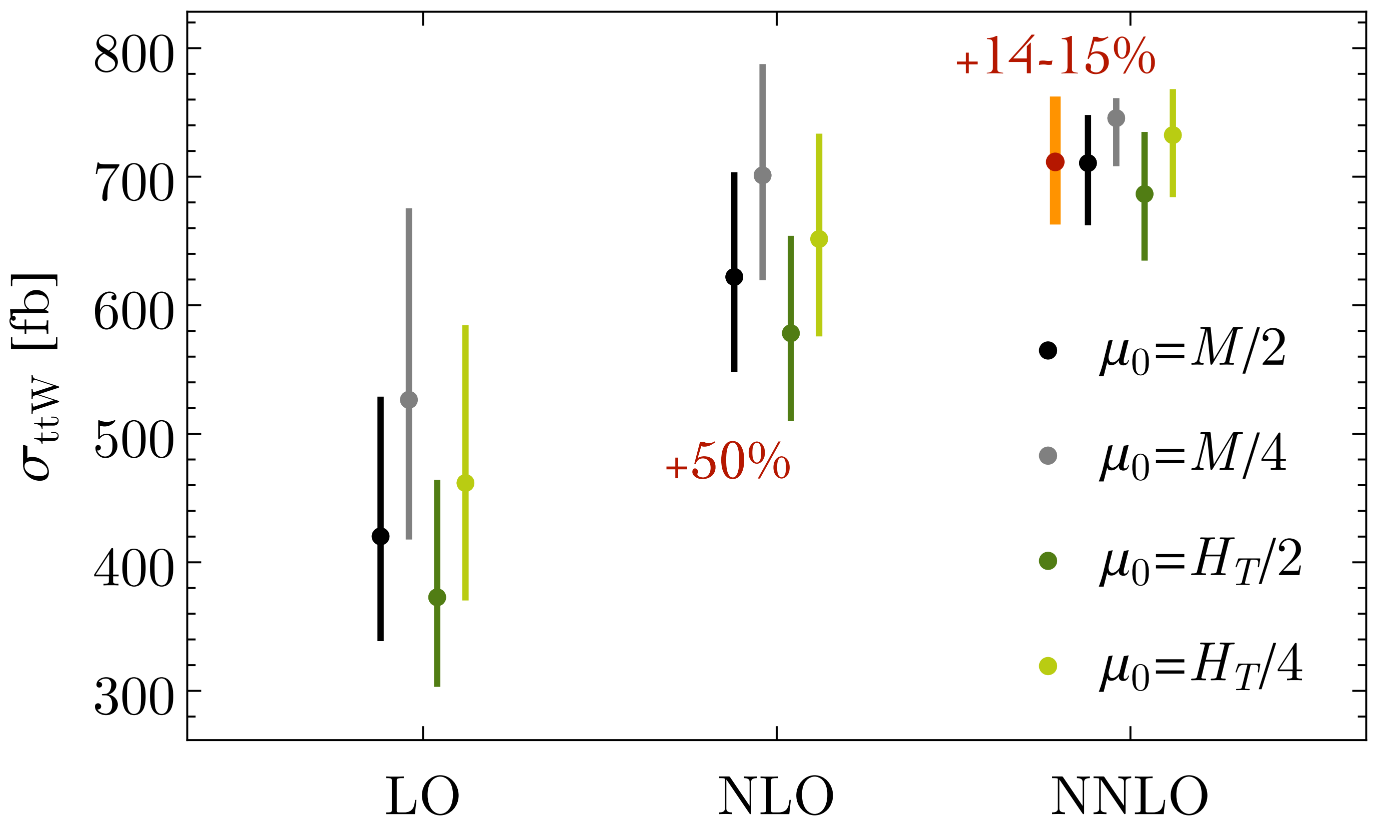
The four predictions are fully consistent within their uncertainties

Symmetrising the $M/2$ scale uncertainty we obtain an upper bound that is almost identical to that of $\mu_0 = M/4$ and $\mu_0 = H_T/4$

We find that the NNLO correction is dominated by virtual and real corrections in the qg channel: no new large contribution from channels opening up at NNLO (as gg)

➡ We take the $\mu_0 = M/2$ as reference and use symmetrised scale variations as estimate of our uncertainties

Perturbative uncertainties



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ttW

	$\sigma_{t\bar{t}W^+}$ [fb]	$\sigma_{t\bar{t}W^-}$ [fb]	$\sigma_{t\bar{t}W}$ [fb]	$\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$
$\mathrm{LO_{QCD}}$	$283.4^{+25.3\%}_{-18.8\%}$	$136.8^{+25.2\%}_{-18.8\%}$	$420.2^{+25.3\%}_{-18.8\%}$	$2.071^{+3.2\%}_{-3.2\%}$
$\mathrm{NLO_{QCD}}$	$416.9^{+12.5\%}_{-11.4\%}$	$205.1^{+13.2\%}_{-11.7\%}$	$622.0^{+12.7\%}_{-11.5\%}$	$2.033^{+3.0\%}_{-3.4\%}$
$\mathrm{NNLO_{QCD}}$	$475.2^{+4.8\%}_{-6.4\%} \pm 1.9\%$	$235.5^{+5.1\%}_{-6.6\%} \pm 1.9\%$	$710.7^{+4.9\%}_{-6.5\%} \pm 1.9\%$	$2.018^{+1.6\%}_{-1.2\%}$
$\mathrm{NNLO_{QCD}+NLO_{EW}}$	$497.5^{+6.6\%}_{-6.6\%} \pm 1.8\%$	$247.9^{+7.0\%}_{-7.0\%} \pm 1.8\%$	$745.3^{+6.7\%}_{-6.7\%} \pm 1.8\%$	$2.007^{+2.1\%}_{-2.1\%}$
ATLAS [11]	$585^{+6.0\%+8.0\%}_{-5.8\%-7.5\%}$	$301^{+9.3\%+11.6\%}_{-9.0\%-10.3\%}$	$890^{+5.6\%+7.9\%}_{-5.6\%-7.9\%}$	$1.95^{+10.8\%+8.2\%}_{-9.2\%\ -6.7\%}$
CMS [10]	$553^{+5.4\%+5.4\%}_{-5.4\%-5.4\%}$	$343^{+7.6\%+7.3\%}_{-7.6\%-7.3\%}$	$868^{+4.6\%+5.9\%}_{-4.6\%-5.9\%}$	$1.61^{+9.3\%+4.3\%}_{-9.3\%-3.1\%}$

Conservative estimate of uncertainty from missing exact two-loop amplitudes

Large NLO QCD corrections (+50%)

Moderate NNLO corrections (+14-15%)

All subdominant LO and NLO contributions at $\mathcal{O}(\alpha^3)$, $\mathcal{O}(\alpha_S^2\alpha^2)$, $\mathcal{O}(\alpha_S\alpha^3)$, $\mathcal{O}(\alpha^4)$ consistently included and denoted as NLO EW: effect is +5%

$\sigma(t\bar{t}W^+)/\sigma(t\bar{t}W^-)$ only slightly decreases increasing the perturbative order

ttW

	$\sigma_{t\bar{t}W^+}$ [fb]	$\sigma_{t\bar{t}W^-}$ [fb]	$\sigma_{t\bar{t}W}$ [fb]	$\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$
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Large NLO QCD corrections (+50%)

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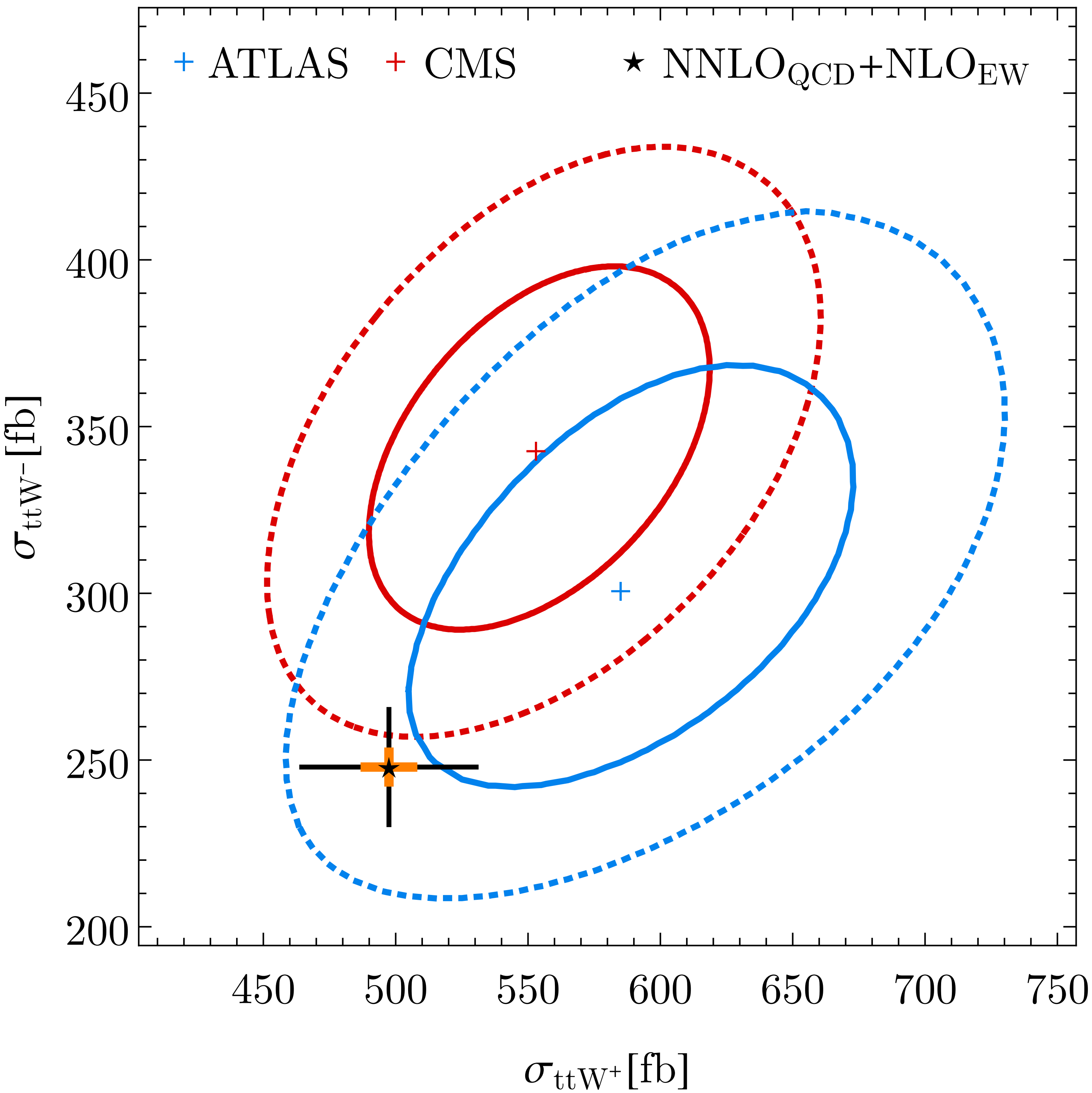
All subdominant LO and NLO contributions at $\mathcal{O}(\alpha^3)$, $\mathcal{O}(\alpha_S^2\alpha^2)$, $\mathcal{O}(\alpha_S\alpha^3)$, $\mathcal{O}(\alpha^4)$ consistently included and denoted as NLO EW: effect is +5%

$\sigma(t\bar{t}W^+)/\sigma(t\bar{t}W^-)$ only slightly decreases increasing the perturbative order

ttW



The comparison with the ATLAS and CMS results shows that discrepancy remains at the 1-2σ level

Inclusion of NNLO corrections significantly reduces perturbative uncertainties

Our result is fully consistent with FxFx prediction but with smaller uncertainties

$$\sigma_{t\bar{t}W}^{\mathrm{FxFx}} = 722.4^{+9.7\%}_{-10.8\%}\ \mathrm{fb}$$

Summary

- Processes in which a $t\bar{t}$ pair is produced together with a vector or Higgs boson are crucial to characterise the top quark interactions but theoretical prediction have still relatively large uncertainties

  → NNLO QCD predictions needed

- For the hadronic production of heavy quarks the q_T subtraction method has proven to be extremely efficient

- The recently completed evaluation of the soft-parton contributions at low transverse momentum allows us to compute NNLO corrections for heavy-quark production plus a colourless system, provided the relevant amplitudes are available

- We have now applied our framework to evaluate NNLO corrections to $t\bar{t}H$ and $t\bar{t}W$ production, by using suitable approximations of the two-loop contributions

Summary

- For $t\bar{t}H$ the approximation is based on a soft-Higgs factorisation formula that has been presented, for the first time, to NNLO accuracy

- In the case of $t\bar{t}W$ we have used both a soft approximation and a massification procedure and they give consistent results within their uncertainties

- Together with $b\bar{b}W$ these are the first computations for $2 \to 3$ processes with massive coloured particles at this perturbative order

- NNLO corrections are moderate and lead to a significant reduction of perturbative uncertainties

- In the case of $t\bar{t}W$ the tension with ATLAS and CMS data remains at the $1\sigma - 2\sigma$ level

Backup

Stability of the subtraction procedure

$$d\sigma^{F}_{(N)NLO} = \mathcal{H}^{F}_{(N)NLO} \otimes d\sigma^{F}_{LO} + \left[d\sigma^{F+\mathrm{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right]$$

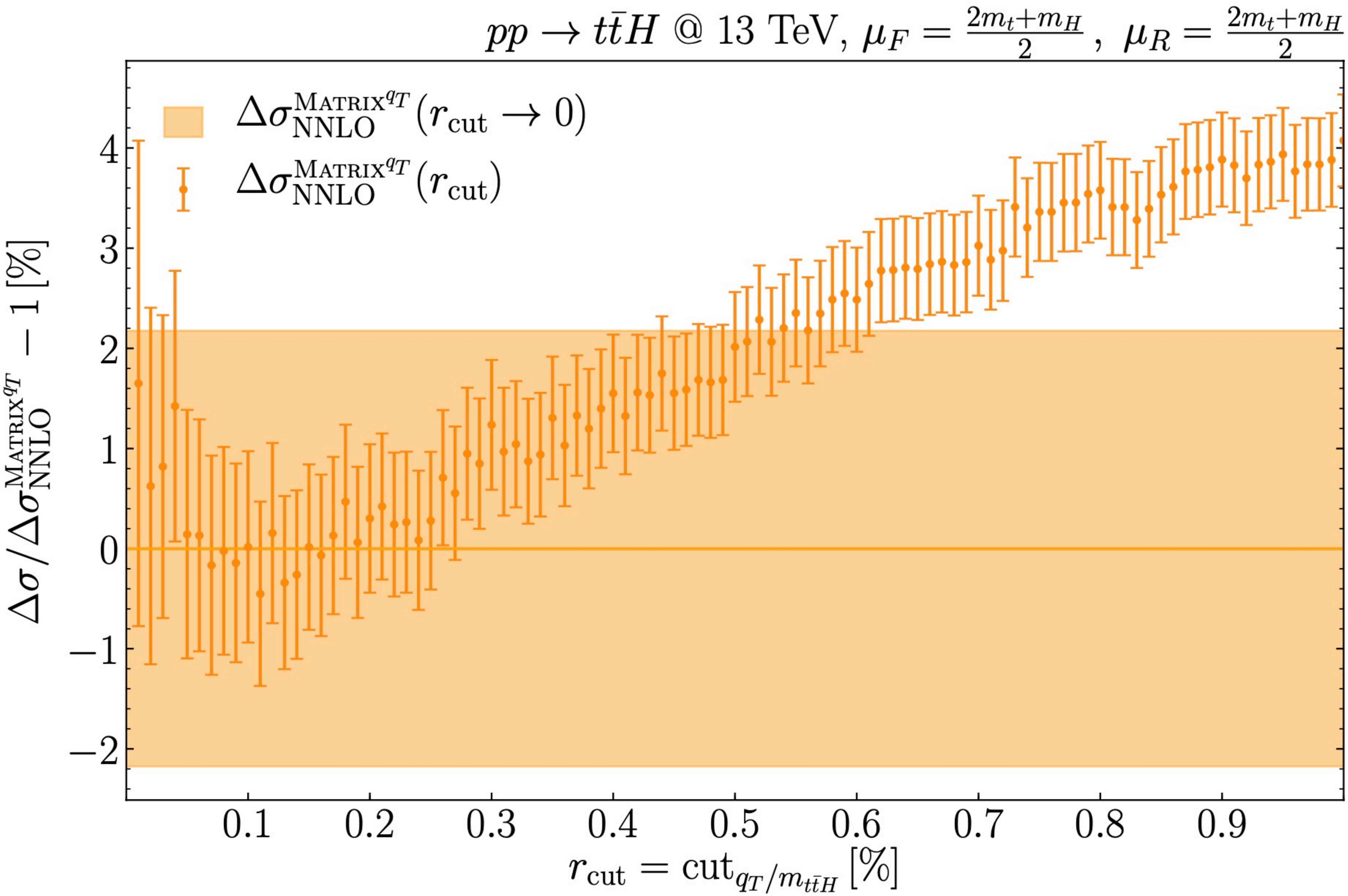
The q_T subtraction counterterm is non-local → the difference in the square bracket is evaluated with a cut-off r_{cut} on the ratio $r = q_T/Q$

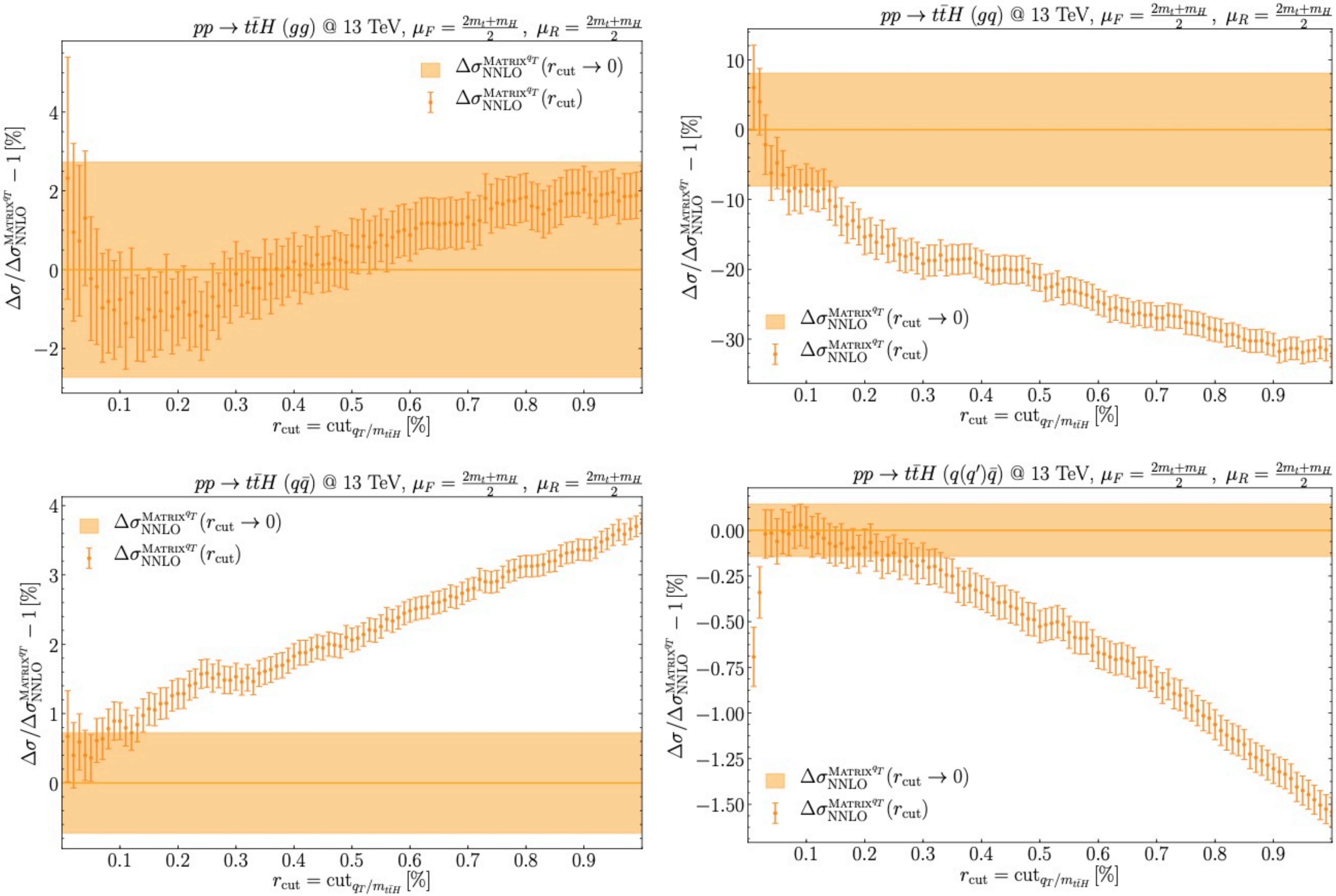
In MATRIX q_T subtraction indeed works as a slicing method

It is important to monitor the dependence of our results on r_{cut}

MATRIX allows for a simultaneous evaluation of the NNLO cross section for different values of r_{cut}

The dependence on r_{cut} is used by the code to provide an estimate of the systematic uncertainty in any NNLO run


$pp \to t\bar{t}H$ @ 13 TeV, $\mu_F = \frac{2m_t+m_H}{2}$, $\mu_R = \frac{2m_t+m_H}{2}$
$\Delta\sigma^{\mathrm{MATRIX}^{q_T}}_{\mathrm{NNLO}}(r_{\mathrm{cut}} \to 0)$
$\Delta\sigma^{\mathrm{MATRIX}^{q_T}}_{\mathrm{NNLO}}(r_{\mathrm{cut}})$
$\Delta\sigma/\Delta\sigma^{\mathrm{MATRIX}^{q_T}}_{\mathrm{NNLO}} - 1\,[\%]$
$r_{\mathrm{cut}} = \mathrm{cut}_{q_T/m_{t\bar{t}H}}\,[\%]$
4
3
2
1
0
−1
−2
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9



$pp \to t\bar{t}H\ (gg)$ @ 13 TeV, $\mu_F = \frac{2m_t+m_H}{2}$, $\mu_R = \frac{2m_t+m_H}{2}$
$pp \to t\bar{t}H\ (gq)$ @ 13 TeV, $\mu_F = \frac{2m_t+m_H}{2}$, $\mu_R = \frac{2m_t+m_H}{2}$
$pp \to t\bar{t}H\ (q\bar{q})$ @ 13 TeV, $\mu_F = \frac{2m_t+m_H}{2}$, $\mu_R = \frac{2m_t+m_H}{2}$
$pp \to t\bar{t}H\ (q(q')\bar{q})$ @ 13 TeV, $\mu_F = \frac{2m_t+m_H}{2}$, $\mu_R = \frac{2m_t+m_H}{2}$
$\Delta\sigma_{\rm NNLO}^{\rm MATRIX^{q_T}}(r_{\rm cut} \to 0)$
$\Delta\sigma_{\rm NNLO}^{\rm MATRIX^{q_T}}(r_{\rm cut})$
$\Delta\sigma/\Delta\sigma_{\rm NNLO}^{\rm MATRIX^{q_T}} - 1$ [%]
$r_{\rm cut} = {\rm cut}_{q_T/m_{t\bar{t}H}}$ [%]

NLO results

$$\mathbf{F}_t^{(1)}(y_{34}) = (\mathbf{T}_3^2 + \mathbf{T}_4^2)\ \ln\left(\frac{m_T^2}{m^2}\right) + \ (\mathbf{T}_3 + \mathbf{T}_4)^2\ \mathrm{Li}_2\left(-\frac{\mathbf{p}_\mathbf{T}^2}{m^2}\right) + \mathbf{T}_3 \cdot \mathbf{T}_4\ \frac{1}{v}\, L_{34}$$

$$\begin{aligned}\mathbf{\Gamma}_t^{(1)}(y_{34}) = -\frac{1}{4}\Bigg\{&(\mathbf{T}_3^2 + \mathbf{T}_4^2)\ (1 - i\pi) + \sum_{\substack{i=1,2\\ j=3,4}} \mathbf{T}_i \cdot \mathbf{T}_j\ \ln\frac{(2p_i \cdot p_j)^2}{M^2 m^2}\\ &+ 2\ \mathbf{T}_3 \cdot \mathbf{T}_4\left[\frac{1}{2v}\ln\left(\frac{1+v}{1-v}\right) - i\pi\left(\frac{1}{v} + 1\right)\right]\Bigg\}\ .\end{aligned}$$

$$v = \sqrt{1 - \frac{m^4}{(p_3 \cdot p_4)^2}}$$

Relative velocity

$$\begin{aligned}L_{34} = &\ln\left(\frac{1+v}{1-v}\right)\ \ln\left(\frac{m_T^2}{m^2}\right) - 2\,\mathrm{Li}_2\left(\frac{2v}{1+v}\right) - \frac{1}{4}\ln^2\left(\frac{1+v}{1-v}\right)\\ &+ 2\left[\mathrm{Li}_2\left(1 - \sqrt{\frac{1-v}{1+v}}\, e^{y_{34}}\right) + \mathrm{Li}_2\left(1 - \sqrt{\frac{1-v}{1+v}}\, e^{-y_{34}}\right) + \frac{1}{2}\, y_{34}^2\right]\end{aligned}$$

$$y_{34} = y_3 - y_4$$

Extension to heavy-quark production

S.Catani, A.Torre, MG (2014)

$$(\mathbf{H}\,\mathbf{\Delta})_{c\bar{c}} = \frac{\langle\, \widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}} \,|\, \mathbf{\Delta} \,|\, \widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}} \,\rangle}{\alpha_S^2(M^2)\; |\mathcal{M}^{(0)}_{c\bar{c}\to Q\bar{Q}}(p_1, p_2; p_3, p_4)|^2}$$

$|\, \widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}} \,\rangle$ subtracted virtual amplitude

$$\mathbf{\Delta}(\mathbf{b}, M; y_{34}, \phi_3) = \mathbf{V}^\dagger(b, M; y_{34})\;\; \mathbf{D}(\alpha_S(b_0^2/b^2); \phi_{3b}, y_{34})\; \mathbf{V}(b, M; y_{34})$$

$$\mathbf{V}(b, M; y_{34}) = \overline{P}_q\; \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\, \mathbf{\Gamma}_t(\alpha_S(q^2); y_{34}) \right\}$$

$\alpha_S^n L^m$ terms $n \geq m$

soft anomalous dimension

$\mathbf{\Gamma}_t^{(1)}$ and $\mathbf{\Gamma}_t^{(2)}$ directly related to singular structure of $|\mathcal{M}_{c\bar{c}\to Q\bar{Q}}\rangle$

Mitov, Sterman, Sung (2009)
Neubert et al (2009)

$\mathbf{D}(\alpha_S; \phi_{3b}, y_{34})$ embodies azimuthal correlations at scale 1/b

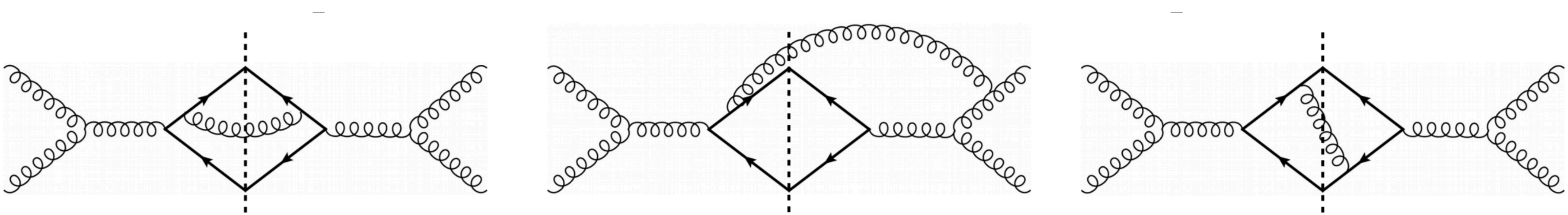
$$\langle\, \mathbf{D}(\alpha_S; \phi_{3b}, y_{34})\rangle_{\mathrm{av.}} = 1$$

Soft contributions at NLO

Catani, Torre, MG (2014)

We need to compute the integral of the subtracted soft current over the phase space of the unresolved gluon

$$\int d^d k \, \delta_+(k^2) \, e^{i\mathbf{b}\cdot\mathbf{k}_T} \, \mathbf{J}^2(k)|_{\rm sub}$$

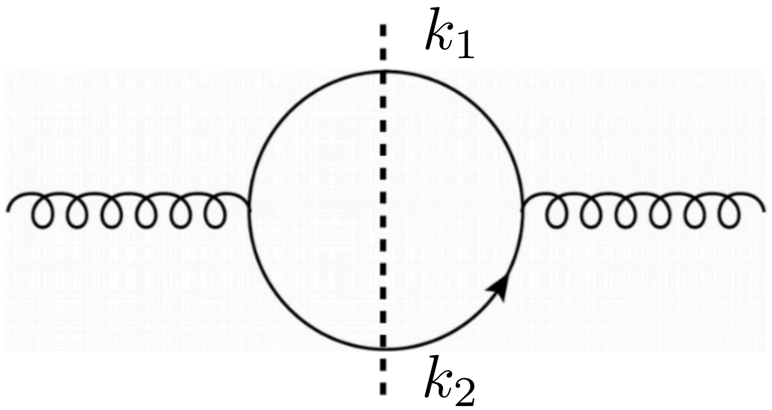


$$\widetilde{\mathbf{I}}^{(1)}_{c\bar{c}\to Q\bar{Q}}\left(\epsilon, \frac{M^2}{\mu_R^2}\right) = -\frac{1}{2}\left(\frac{M^2}{\mu_R^2}\right)^{-\epsilon}\left\{\left(\frac{1}{\epsilon^2} + i\pi\frac{1}{\epsilon} - \frac{\pi^2}{12}\right)(\mathbf{T}_1^2 + \mathbf{T}_2^2) + \frac{2}{\epsilon}\,\gamma_c - \frac{4}{\epsilon}\,\mathbf{\Gamma}_t^{(1)}(y_{34}) + \mathbf{F}_t^{(1)}(y_{34})\right\}$$

Singular structure from initial state radiation

Additional soft contribution obtained from integration of the subtracted soft current

1) Soft quark-antiquark pair



$$\mathbf{J}(k)\big|^2 \longrightarrow \mathbf{J}_\mu(k_1+k_2)\,\Pi^{\mu\nu}(k_1,k_2)\,\mathbf{J}_\nu(k_1+k_2)$$

$$\Pi^{\mu\nu}(k_1,k_2) = \frac{T_R}{(k_1\cdot k_2)^2}\left(-g^{\mu\nu}k_1\cdot k_2 + k_1^\mu k_2^\nu + k_2^\mu k_1^\nu\right)$$

We integrate over k_1 and k_2 at fixed $k_1 + k_2$

$\Rightarrow$ $\delta_+(k^2)$ replaced by $\dfrac{F(\epsilon)}{(k^2)^{1+\epsilon}}$

In practice we need to compute

$$\langle \int d^dk\,(k^2)^{-1-\epsilon}\,e^{i\mathbf{b}\cdot\mathbf{k}_T}\,\mathbf{J}(k)^2|_{\mathrm{sub}}\rangle_{\hat{b}}$$

2) Two soft gluons

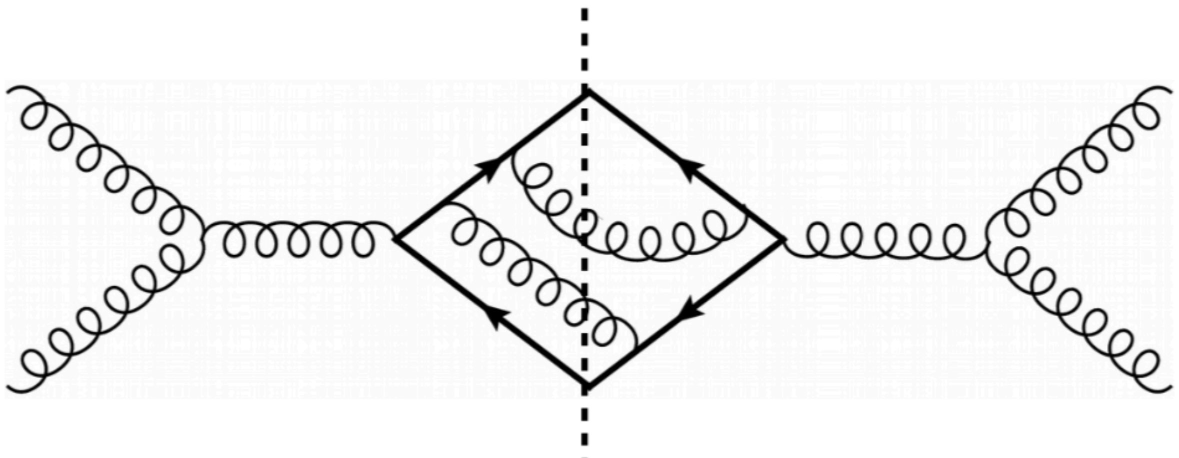
The square of the double soft current can be written as

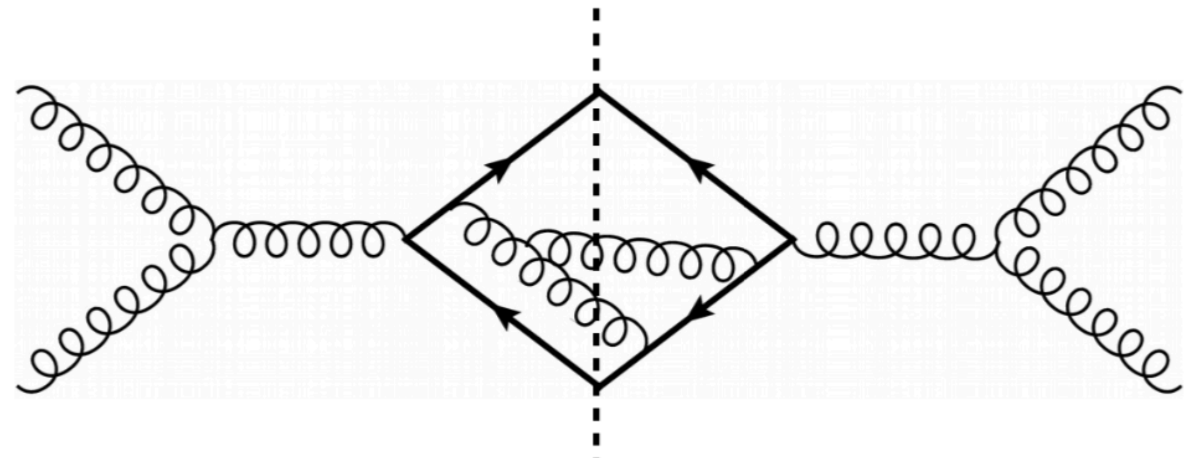
$$\frac{1}{2}\left\{\mathbf{J}^2(k_1), \mathbf{J}^2(k_2)\right\} - C_A \sum_{i,j=1}^{n} \mathbf{T}_i \cdot \mathbf{T}_j\, \mathcal{S}_{ij}(k_1, k_2)$$

Catani, MG (2000)
Czakon (2011)

Iteration of NLO contribution

Purely non-abelian part





Some of the terms in $S_{ij}(k_1, k_2)$ can be treated like the quark-antiquark terms

$$\mathcal{S}_{ij}(k_1, k_2) \to \tilde{\mathcal{S}}_{ij}(k_1, k_2)$$

remaining terms

massive emitters

$$\tilde{\mathcal{S}}_{ij}(k_1, k_2) = \tilde{\mathcal{S}}_{ij}^{m=0}(k_1, k_2) + \left(m_i^2\, \tilde{\mathcal{S}}_{ij}^{m\neq 0}(k_1, k_2) + m_j^2\, \tilde{\mathcal{S}}_{ji}^{m\neq 0}(k_1, k_2)\right)$$

2) Two soft gluons

Strategy: first integrate over k_1 and k_2 at fixed $k_1 + k_2$ in the CM frame of k

$$\int_{(12)} \tilde{\mathcal{S}}_{ij}^{m=0} = \frac{(k^2)^{-1-\epsilon}(p_i \cdot p_j)}{(p_i \cdot k)(p_j \cdot k)} \left[(1 + \vec{n}_i \cdot \vec{n}_j)\, \mathcal{A}_{11}^{+} - 2\,(1 - \vec{n}_i \cdot \vec{n}_j)\, \mathcal{A}_{11}^{-} + \mathcal{A}_{10} + \mathcal{A}_{01} \right]$$

$$\int_{(12)} \tilde{\mathcal{S}}_{ij}^{m\neq 0} = \frac{(k^2)^{-1-\epsilon}}{(p_j \cdot k)^2} \left[(1 - \vec{n}_i \cdot \vec{n}_j)\, \mathcal{A}_{11}^{-} - (1 + \vec{n}_i \cdot \vec{n}_j)\, \mathcal{A}_{11}^{+} - \frac{1}{2}\mathcal{A}_{1,-1}^{+} + 3\mathcal{A}_{10} - \frac{1}{2}\mathcal{A}_{00} \right]$$

Angular integrals defined as

$$\mathcal{A}_{i,j}^{\pm} = \int_0^{\pi} d\theta \int_0^{\pi} d\phi\, \frac{\sin^{n-3}\theta \sin^{n-4}\phi}{(1 - a_i \cos\theta)^i (1 \pm a_j \cos\chi \cos\theta \pm a_j \sin\chi \sin\theta \cos\phi)^j}$$

$$a_i = |\vec{n}_i| \qquad \cos\chi = \hat{n}_i \cdot \hat{n}_j \qquad \vec{n}_i = \vec{p}_i / E_i$$ in CM frame of k

$$\vec{n}_i^2 = 1 - \frac{k^2 m_i^2}{(p_i \cdot k)^2}\,, \qquad \vec{n}_j^2 = 1 - \frac{k^2 m_j^2}{(p_j \cdot k)^2}\,, \qquad \vec{n}_i \cdot \vec{n}_j = 1 - \frac{k^2 (p_i \cdot p_j)}{(p_i \cdot k)(p_j \cdot k)}$$

2) Two soft gluons

Then split the result in singular and regular part as $k^2 \to 0$

$$\int_{(12)} \tilde{\mathcal{S}}_{ij}^{m=0} = \frac{(k^2)^{-1-\epsilon}(p_i \cdot p_j)}{(p_i \cdot k)(p_j \cdot k)} \left\{ \frac{1}{\epsilon} \left[s(\vec{n}_i, \vec{n}_j) + \epsilon\, r(\vec{n}_i, \vec{n}_j) \right] \right\}$$

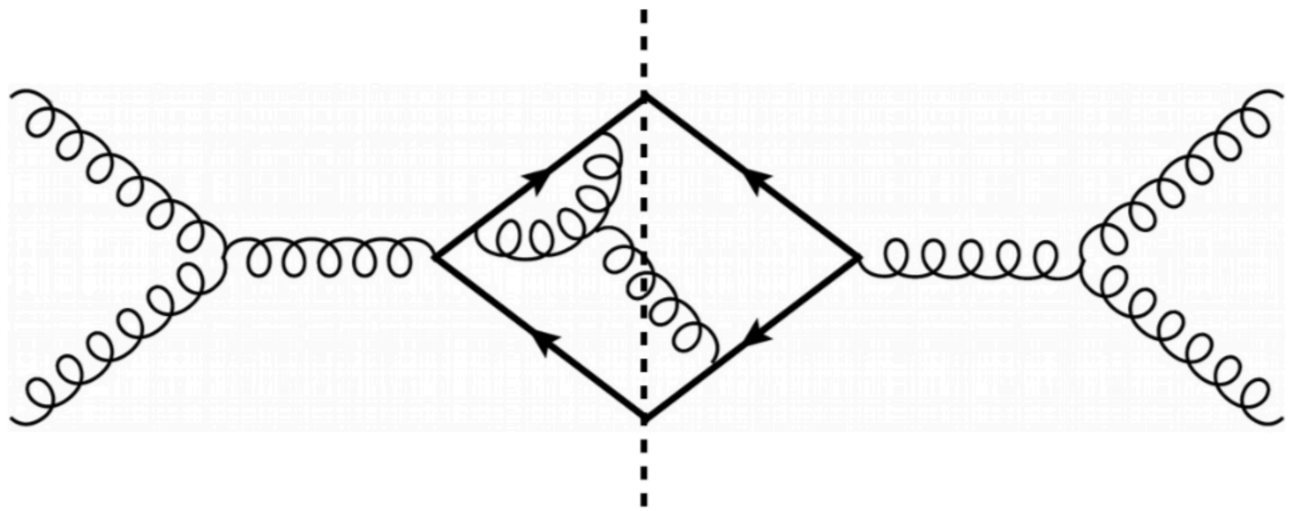
Vanishing when $k^2 \to 0$

We managed to integrate analytically all singular and all regular parts except for S_{34}

Results written in terms of MPL up to weight 4

Remaining contributions integrated numerically

3) Soft-gluon emission at one loop



- massless case

Catani, MG (2000)

- extension to massive emitters

Bierenbaum, Czakon, Mitov (2011)
Czakon, Mitov (2018)

- simplified version

Bierenbaum, Czakon, Mitov (2011)

$$J_\mu^{(1)} J^{(0),\mu} + \text{c.c.} \sim \sum_{i,j} \mathbf{T}_i \cdot \mathbf{T}_j \sum_{n=-2}^{1} \epsilon^n R_{ij}^{(n)}$$

Explicit double poles in ε: we need up to O(ε)

All integrals computed analytically except for a subset of contributions to R_{34}

Wbb

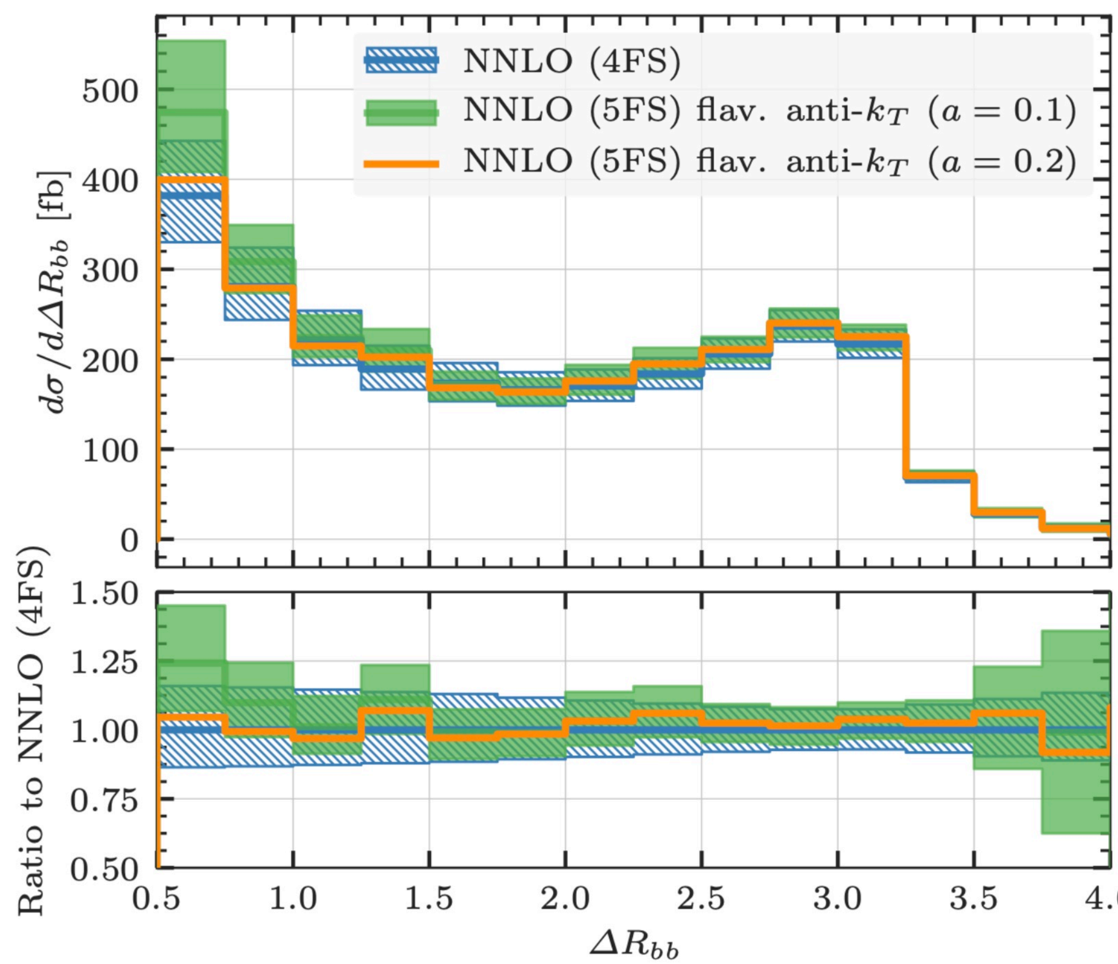
Buonocore, Devoto, Kallweit, Mazzitelli,
Rottoli, Savoini (2023)

Massification procedure successfully applied to carry out first NNLO computation of Wbb with massive bottom quarks

order	σ^{4FS} [fb]	$\sigma^{5FS}_{a=0.05}$ [fb]	$\sigma^{5FS}_{a=0.1}$ [fb]	$\sigma^{5FS}_{a=0.2}$ [fb]
LO	$210.42(2)^{+21.4\%}_{-16.2\%}$	$262.52(10)^{+21.4\%}_{-16.1\%}$	$262.47(10)^{+21.4\%}_{-16.1\%}$	$261.71(10)^{+21.4\%}_{-16.1\%}$
NLO	$468.01(5)^{+17.8\%}_{-13.8\%}$	$500.9(8)^{+16.1\%}_{-12.8\%}$	$497.8(8)^{+16.0\%}_{-12.7\%}$	$486.3(8)^{+15.5\%}_{-12.5\%}$
NNLO	$649.9(1.6)^{+12.6\%}_{-11.0\%}$	$690(7)^{+10.9\%}_{-9.7\%}$	$677(7)^{+10.4\%}_{-9.4\%}$	$647(7)^{+9.5\%}_{-9.4\%}$

Using massive bottom quarks in the 4FS avoids ambiguities related to the use of flavoured jet algorithms

Comparison against the massless computation (using flavoured anti-k_T algorithm) shows overall good agreement within uncertainties



Soft-Higgs radiation

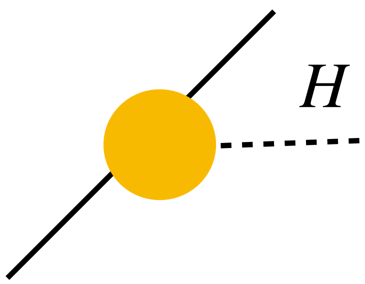
The basic observation is that at the bare amplitude level we have

$$\lim_{k\to 0} \mathcal{M}^{\text{bare}}(\{p_i\},k) = \frac{m_0}{v}\sum_i \frac{m_0}{p_i\cdot k}\mathcal{M}^{\text{bare}}(\{p_i\})$$

The renormalisation of the heavy-quark mass and wave-function induce a modification of the Higgs coupling to the heavy quark

The bare amplitude for the soft-scalar emission is

$$\lim_{k\to 0} \mathcal{M}^{\text{bare}}_{t\to tH}(p,k) = \frac{1}{v} m_0 \frac{\partial}{\partial m_0}\mathcal{M}^{\text{bare}}_{t\to t}(p)\Bigg|_{p^2=m^2}$$



where

$$\mathcal{M}^{\text{bare}}_{t\to t}(p,k) = \bar{t}_0(p)\big(-m_{t,0} - \Sigma(p)\big)\, t_0(p)$$

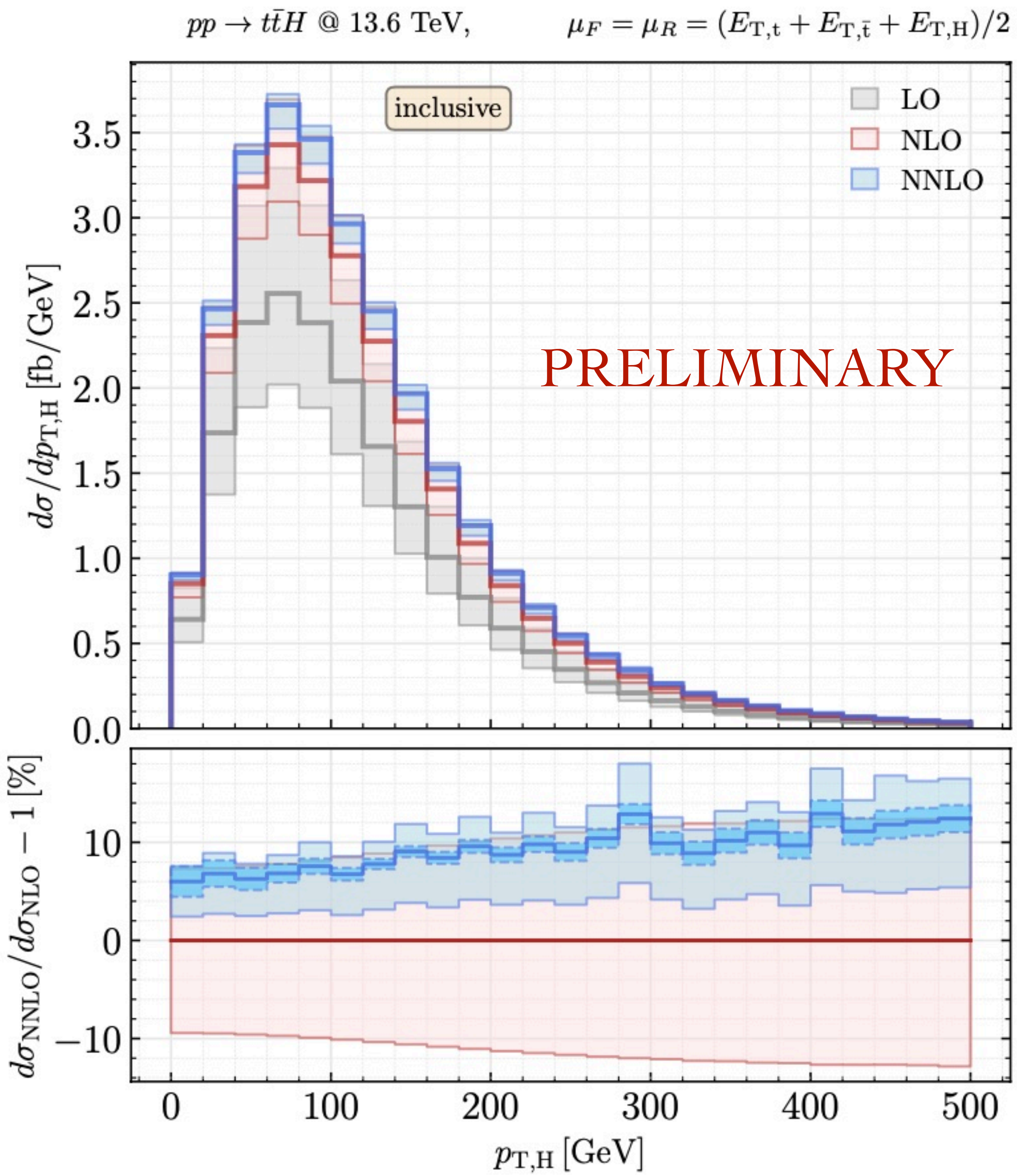
Broadhurst, Gray, Schilcher (1991)
Gray, Broadhurst, Grafe, Schilcher (1990)

By using the results of the $\mathcal{O}(\alpha_S^2)$ contribution to the heavy-quark self energy $\Sigma(p)$ and carrying out the wave function and mass renormalisation we recover the function $F(\alpha_S(\mu_R); m/\mu_R)$ discussed before

Check at $\mathcal{O}(\alpha_S^3)$ in progress

Fael, Lange, Schönwald, Steinhauser (2022,2023)
Chetyrkin, Kniehl, Steinhauser (1997)
Melnikov, Ritbergen (2000)

Higgs p_T spectrum



Uncertainties from soft-approximation over the Higgs p_T spectrum remain of the same order (a similar uncertainty is obtained by using μ_{IR} variations)

At first sight this is counterintuitive since at large $p_{T,H}$ the soft approximation is expected to become worse !

However at large $p_{T,H}$ the role of the gg channel is reduced and the $q\bar{q}$ channel, which is under better control, plays the major role

Differences with other approaches

The idea of a treating the Higgs as a parton radiating off the top quark was used already in the past

Effective Higgs approximation in early NLO calculations: introduce a function expressing the probability to extract the Higgs boson from the top quark

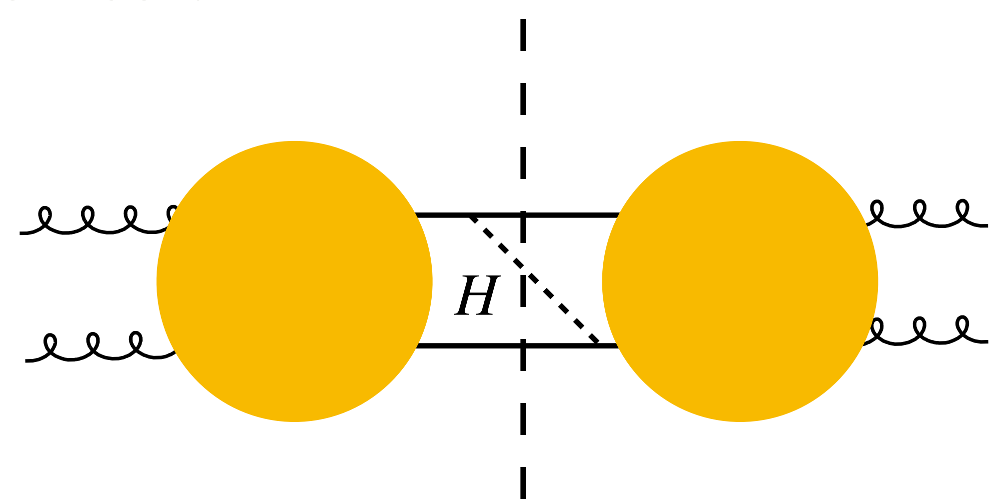
Dawson and Reina (1997)

Fragmentation functions $D_{t \to H}$ and $D_{g \to H}$ evaluated at NLO

Brancaccio, Czakon, Gerenet, Krämer (2021)

These approaches are based on a collinear approximation

Our approximation is purely soft (collinear non-soft emissions are neglected but soft quantum interferences are included)



Moreover, we apply it only to the finite part of the two-loop contribution