Towards NNLO+PS for processes with colored partons and jets

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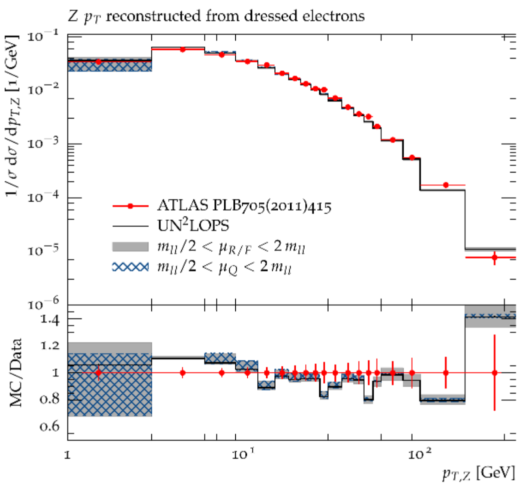
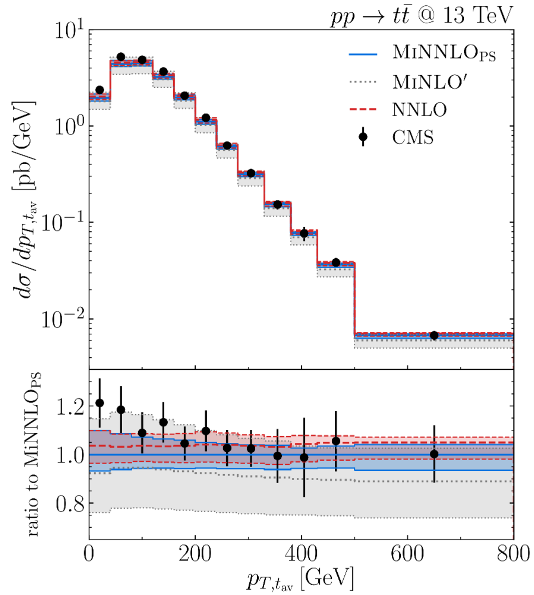
Motivation

- The increasing experimental precision of LHC measurements challenges existing generators, pushing the request for higher accuracy
- The state-of-the-art is the inclusion of NNLO corrections into parton-shower Monte Carlo
- Three main approach to the problem:

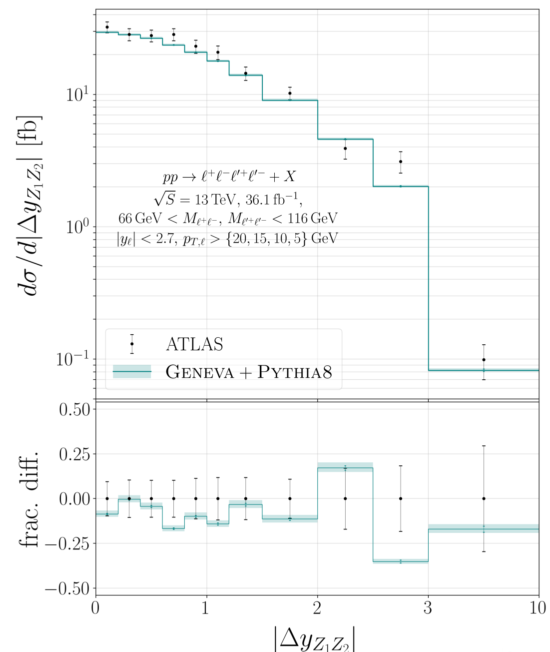
UNNLOPS



MiNNLOPS



GENEVA

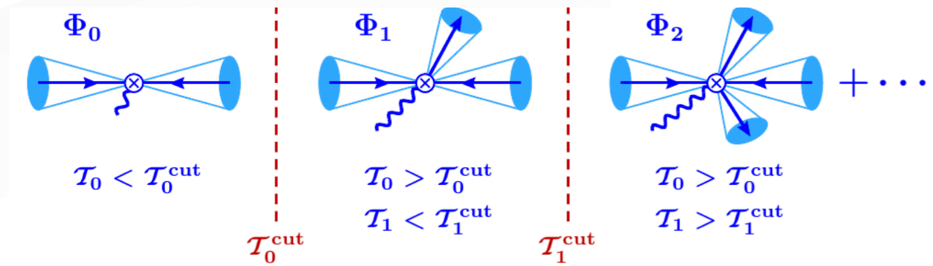


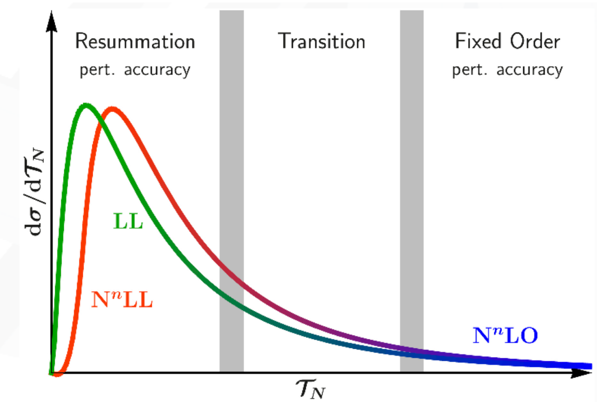
Also NNLO+PS with sector showers available for e^+e^-

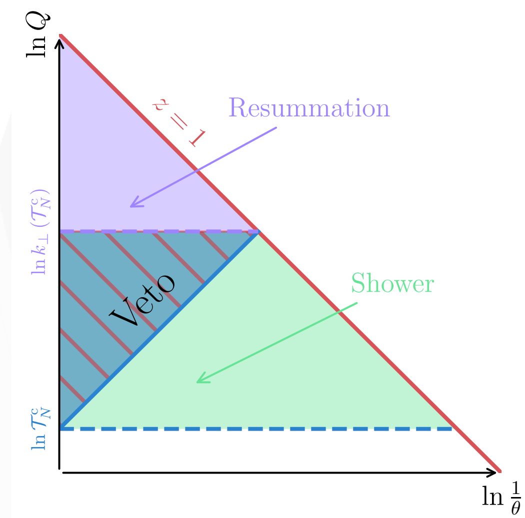
[Campbell et al. 2108.07133]

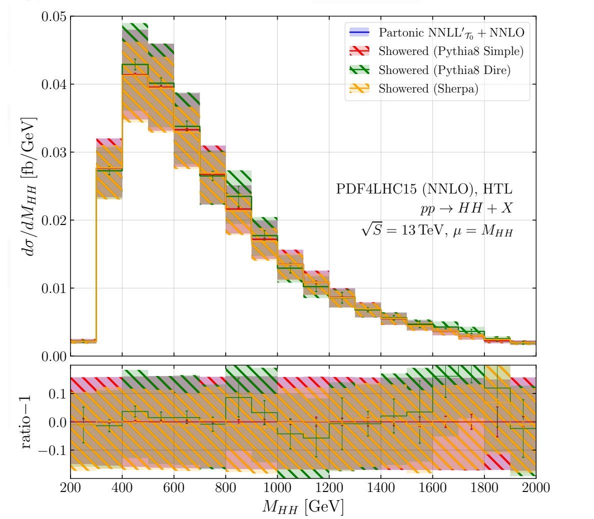
The Geneva method

- Monte Carlo fully-differential event generation at higher-orders (NNLO)
- Resummation plays a key role in the defining the events in a physically sensible way
- Results at partonic level can be further evolved by different shower matching and hadronization models









Resolution parameters for N extra emissions

- The key idea is the introduction of a resolution variable r_N that measure the hardness of the $N+1$-th emission in the Φ_N phase space.

- For color singlet production one can have $r_0 = q_T, p_T^j, k_T$-ness,….

- N-jettiness is a valid resolution variable: given an M-particle phase space point with $M \geq N$

$$\mathcal{T}_N(\Phi_M) = \sum_k \min\{\hat{q}_a \cdot p_k, \hat{q}_b \cdot p_k, \hat{q}_1 \cdot p_k, \ldots, \hat{q}_N \cdot p_k\}$$

- The limit $\tau_N \to 0$ describes a N-jet event where the unresolved emissions are collinear to the final state jets/initial state beams or soft

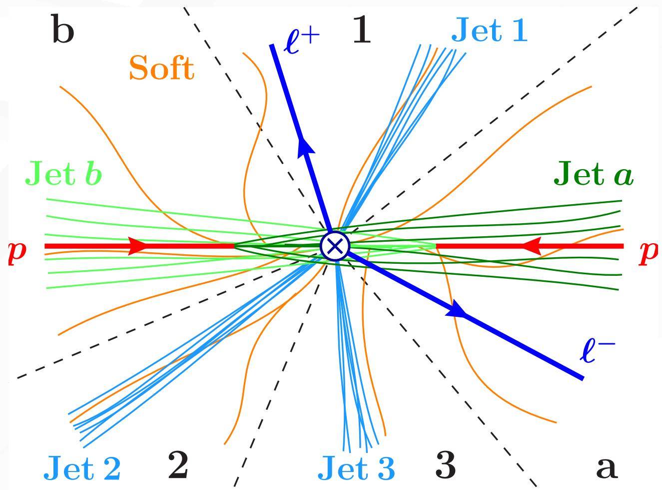
- For color-singlet final states, it reduces to 0-jettiness

$$\mathcal{T}_0 = \sum_k |\vec{p}_{kT}|\, e^{-|\eta_k - Y|}$$

[Stewart, Tackmann,Waalewijn `09,`10]

- When an extra jet is present 1-jettiness used for r_1

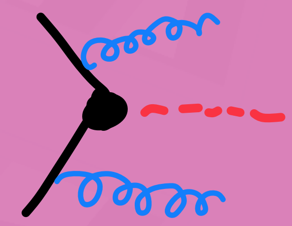
$$\mathcal{T}_1 = \sum_k \min\{\frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_J \cdot p_k}{Q_J}\}$$



Partitioning phase space with resolution cuts

NNLO example : start with two widely separated emission.
Can be described well with LO_2 matrix elements.
What happens when emissions start growing closer and closer ?

$$\frac{d\sigma}{d\Phi_2}(r_0 > r_0^{cut}, r_1 > r_1^{cut}) =$$



$r_0 > r_0^{cut}$

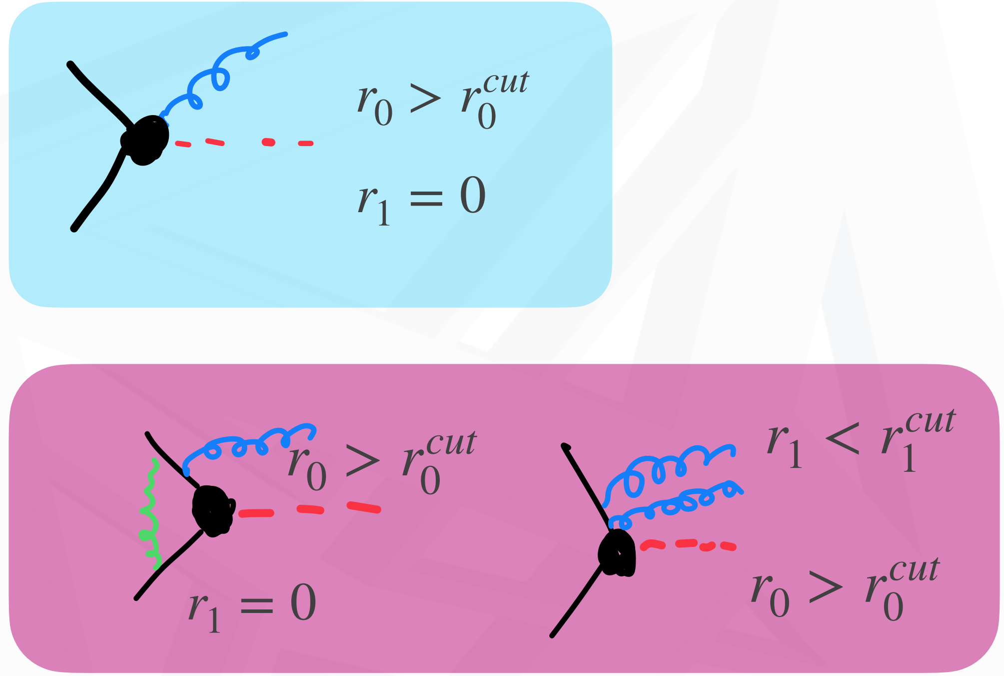
$r_1 > r_1^{cut}$

The logarithms of the resolution parameters grow larger and larger. They need to be resummed to give a physically-sensible description. This takes care of their IR divergencies.
Generated events must have integrated cross section LO_2 accurate and the full N+2-body kinematics must be retained.

Partitioning phase space with resolution cuts

Next: one hard and one unresolved

$$\frac{d\sigma}{d\Phi_1}(r_1^{cut}) = \left\{ \right.$$



$$d\Phi_1 = d\Phi_0 dr_0 dz d\varphi$$

When one emission becomes unresolved r_1^{cut} must be resummed.

Integrated quantities require NLO$_1$ accuracy via local subtraction $\frac{d\Phi_2}{d\Phi_1}\theta(r_1 < r_1^{cut})$.

Φ_2 differential information below r_1^{cut} is lost during projection to Φ_1.

No difference for preserved quantities, in general can be made a power correction in r_1^{cut}.

Mapping that preserves r_0 singular behavior is required for correct event definition.

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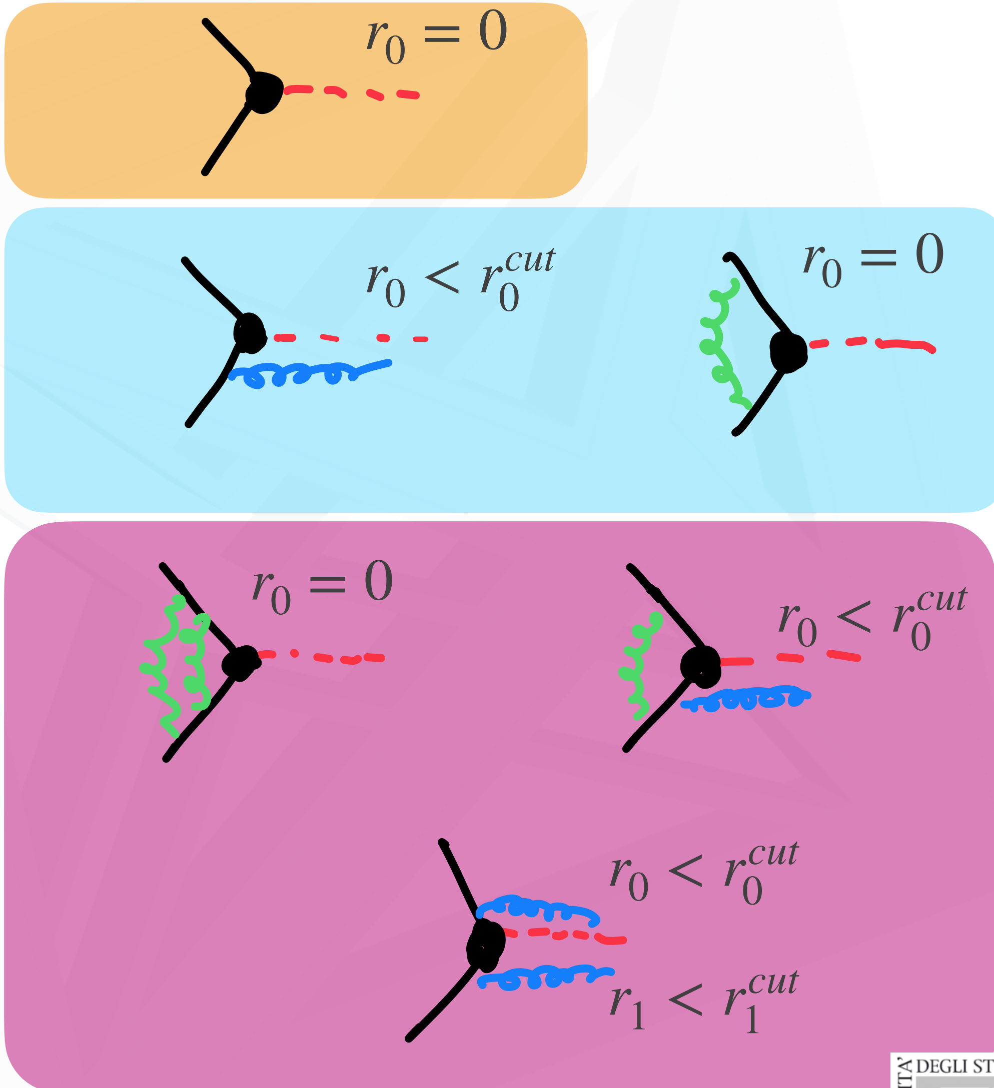
Partitioning phase space with resolution cuts

Last: two unresolved

$$\frac{d\sigma}{d\Phi_0}(r_0^{cut}) = \Bigg\{$$



Zero jet bin must have NNLO_0 integrated accuracy. N-jettiness subtraction used.

The resummation of both r_0^{cut} and r_1^{cut} ensures physically sensible xsec and IR-finite events.

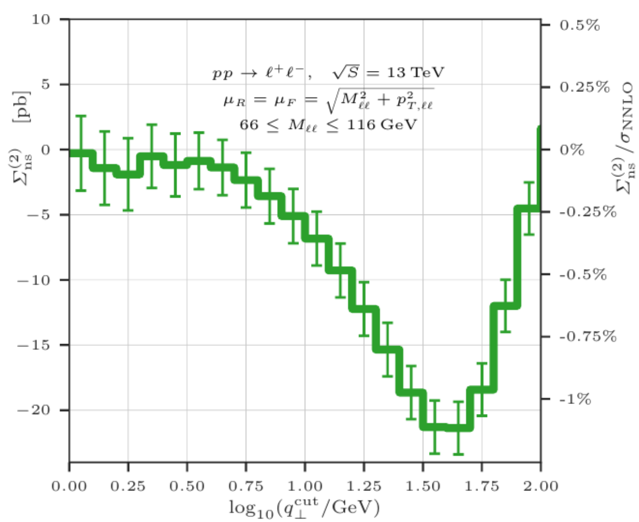
Resummation of resolution parameters

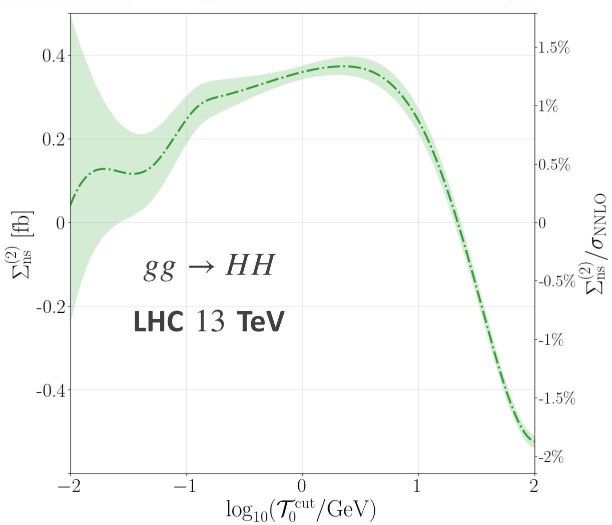
Resumming resolutions parameters not really a new idea, SMCs do it since the '80s with Sudakov factors

The key difference is that using the proper resummation at higher orders has several benefits: systematically improvable (NLL,NNLL,N3LL,…), lowering theoretical uncertainty at each step. Including primed accuracy captures the exact singular behaviour at $\delta(r_N)$.

The higher the accuracy the lower the cuts can be pushed without risking missing higher logarithms being numerically relevant. The lower the cuts the smaller the nonsingular power corrections due to phase-space projections will affect the results differentially.

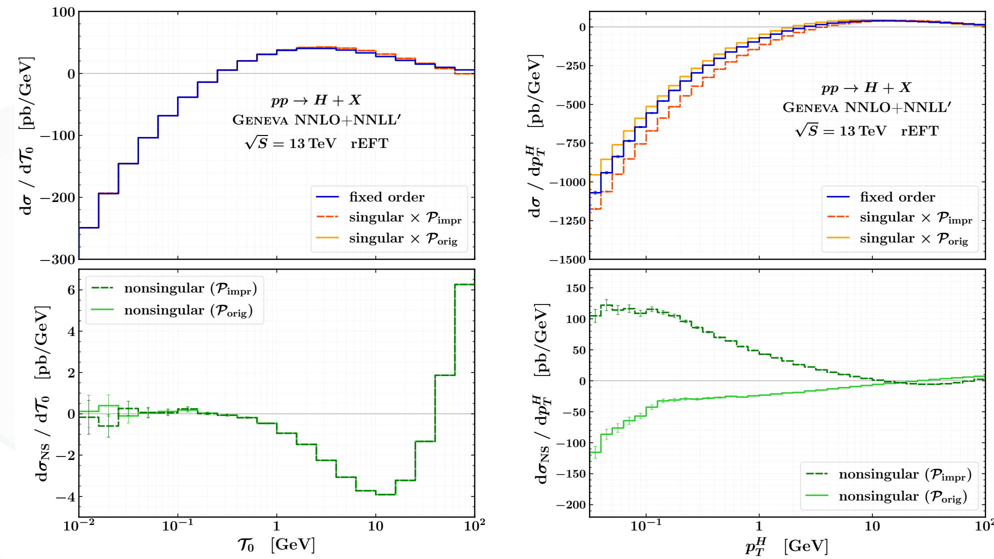
For NNLO event generation one needs at least NNLL'$_{r_0}$ + NNLO accuracy to control the full α_s^2 singular contributions.





From resummation to event generation

Resummed formulae need to be made more differential via splitting functions, capturing the singular behaviour of different resolution variables as best as they can.



Final GENEVA partonic formulae combine resummation and matching to fixed-order

$$\frac{d\sigma^{\mathsf{MC}_0}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = \frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) + \frac{d\sigma_0^{\mathrm{nons}}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}})$$

$$\frac{d\sigma_0^{\mathrm{nons}}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = \frac{d\sigma_0^{\mathrm{NNLO}_0}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) - \left[\frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}})\right]_{\mathrm{NNLO}_0}$$

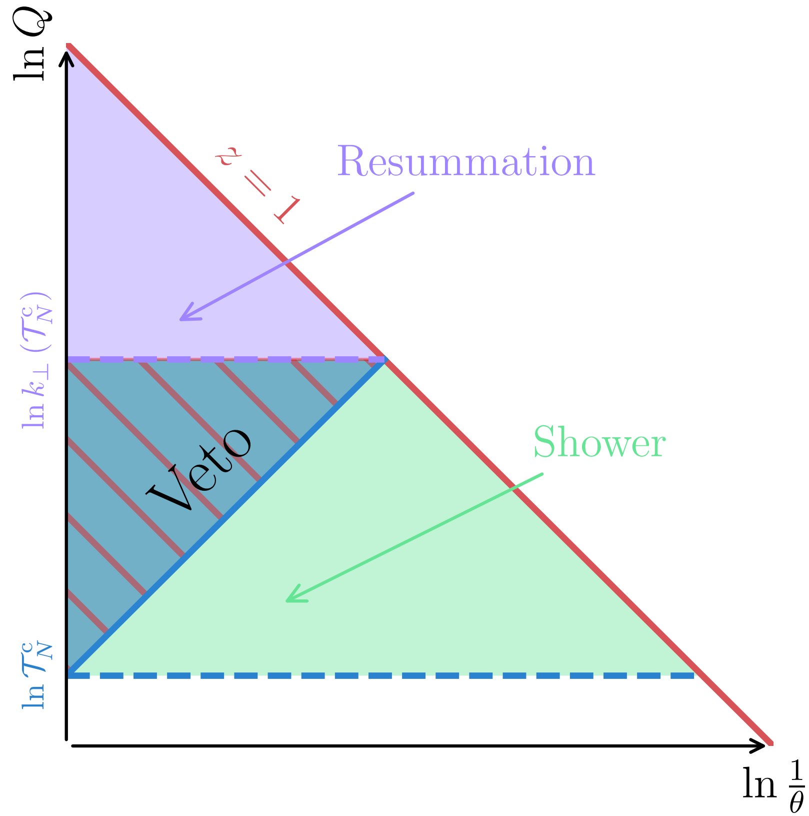
$$\frac{d\sigma^{\mathsf{MC}_1}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}; \mathcal{T}_1^{\mathrm{cut}}) = \frac{d\sigma_{\geq 1}^{C}}{d\Phi_1} U_1(\Phi_1, \mathcal{T}_1^{\mathrm{cut}})\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) + \frac{d\sigma_1^{\mathrm{match}}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}; \mathcal{T}_1^{\mathrm{cut}})$$

$$\frac{d\sigma^{\mathsf{MC}_{\geq 2}}}{d\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}, \mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}}) = \frac{d\sigma_{\geq 1}^{C}}{d\Phi_1} U_1'(\Phi_1, \mathcal{T}_1)\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}})\Big|_{\Phi_1 = \Phi_1^{\mathcal{T}}(\Phi_2)} \times \mathcal{P}(\Phi_2)\, \theta(\mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}}) + \frac{d\sigma_{\geq 2}^{\mathrm{match}}}{d\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}, \mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}})$$

Interface with the parton shower

$\mathcal{T}_N(\Phi_{N+1})$ measures the hardness of the N+1-th emission

- If shower ordered in k_T, start from largest value allowed by N-jettiness
- Let the shower evolve unconstrained.
- At the end veto an event if after $M \geq 1$ shower emissions $\mathcal{T}_N(\Phi_{N+M}) > \mathcal{T}_N(\Phi_N + 1)$ and **retry** the whole shower.



$$\mathcal{T}_{N+M-1}(\Phi_{N+M}) \leq \mathcal{T}_{N+M-2}(\Phi_{N+M}) \leq \ldots \leq \mathcal{T}_N(\Phi_{N+M})$$

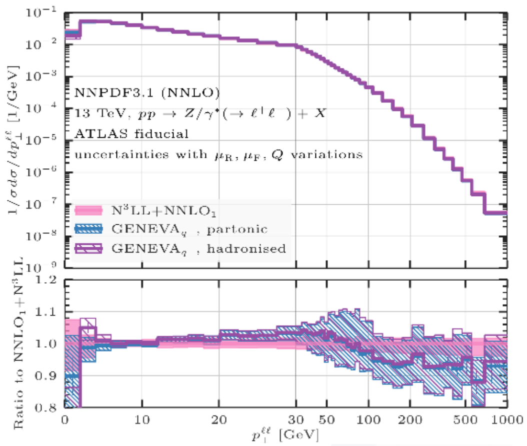
Ensures the relevant phase space is correctly covered to avoid spoiling the resummation accuracy for $\mathcal{T}$ and the shower accuracy for other observables.

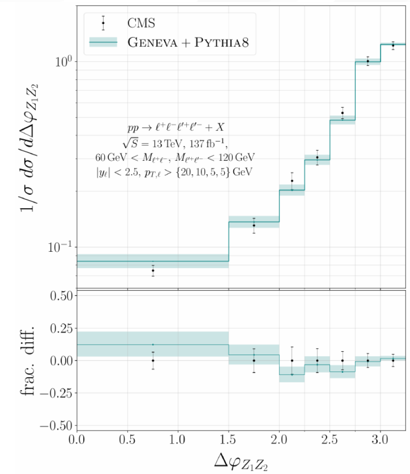
0-jet and 1-jet bins are treated differently: starting scale is resolution cutoff.

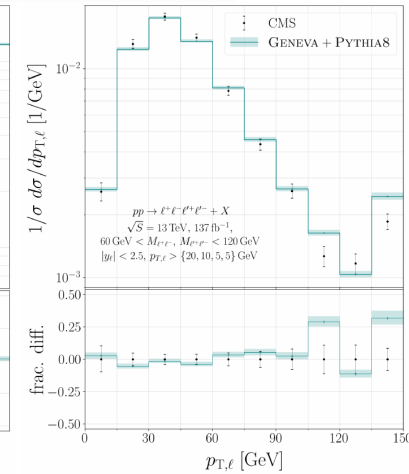
Method rather independent from shower used: PYTHIA8, DIRE & SHERPA.

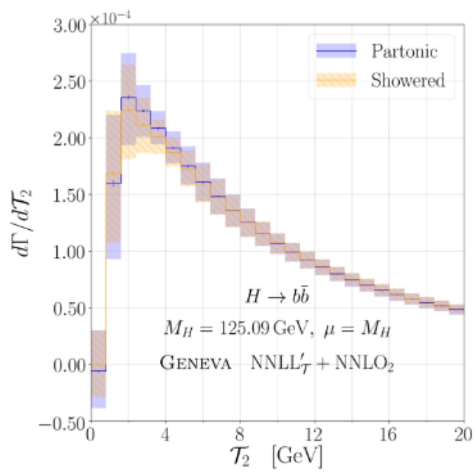
Implemented processes

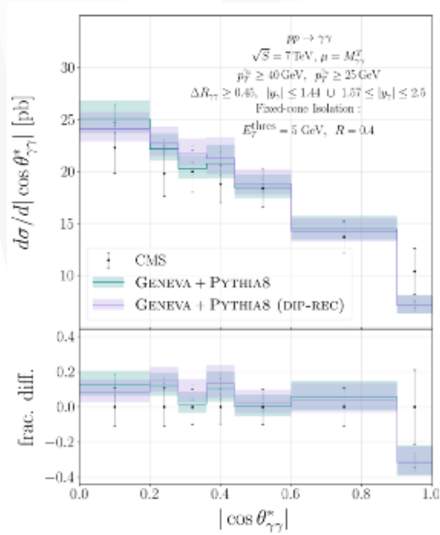
Method has been tested and validated with several color singlet production processes:
DY, ZZ, $W\gamma$, VH, $\gamma\gamma$, ggH, ggHH, Higgs decays using both zero-jettiness and q_T

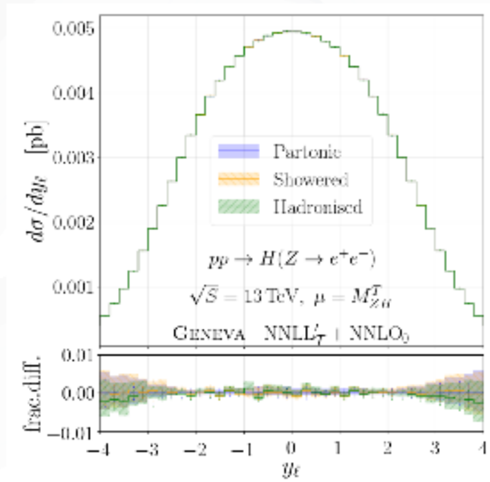












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Using the jet pT as resolution variable

GENEVA recently extended to jet veto resummation in [Gavardi et al. 2308.11577].

Factorization most easily derived for cumulant of the cross-section. SCET II problem. Numerical derivative to get the spectrum. For hardest-jet we have

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_0}(p_T^{\mathrm{cut}},\mu,\nu)=\sum_{a,b} H_{ab}(\Phi_0,\mu)B_a(Q,p_T^{\mathrm{cut}},R,x_a,\mu,\nu)B_b(Q,p_T^{\mathrm{cut}},R,x_b,\mu,\nu)S_{ab}(p_T^{\mathrm{cut}},R,\mu,\nu)$$

Two loop Beam and Soft functions recently computed in [Abreu et al. 2207.07037, 2204.02987]

Focus on $W^+W^- \to \mu^+\nu_\mu e^-\bar{\nu}_e$ with jet veto, in 4-flavor scheme to avoid top contaminations. Massless two-loop hard function taken from `qqVVamp` [Gehrmann et al. 1503.04812]

Interface to SCETlib [Tackmann et al.] allows to perform also resummation also for pT of the second jet at the cumulant level. Refactorization of soft sector into global soft, soft-coll and nonglobal contributions [Cal et al.]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_1}(p_T^{\mathrm{cut}},\mu,\nu)=\sum_{\kappa} H_\kappa(\Phi_1,\mu)B_a(Q,p_T^{\mathrm{cut}},R,x_a,\mu,\nu)\,B_b(Q,p_T^{\mathrm{cut}},R,x_b,\mu,\nu)S_\kappa(p_T^{\mathrm{cut}},y_J,\mu,\nu)$$
$$\times\,\mathcal{S}_j^R(p_T^{\mathrm{cut}}R,\mu)\,J_j(p_T^J R,\mu)\mathcal{S}_j^{\mathrm{NG}}\left(\frac{p_T^{\mathrm{cut}}}{p_T^J}\right).$$

[Banfi et al. hep-ph/0206076]

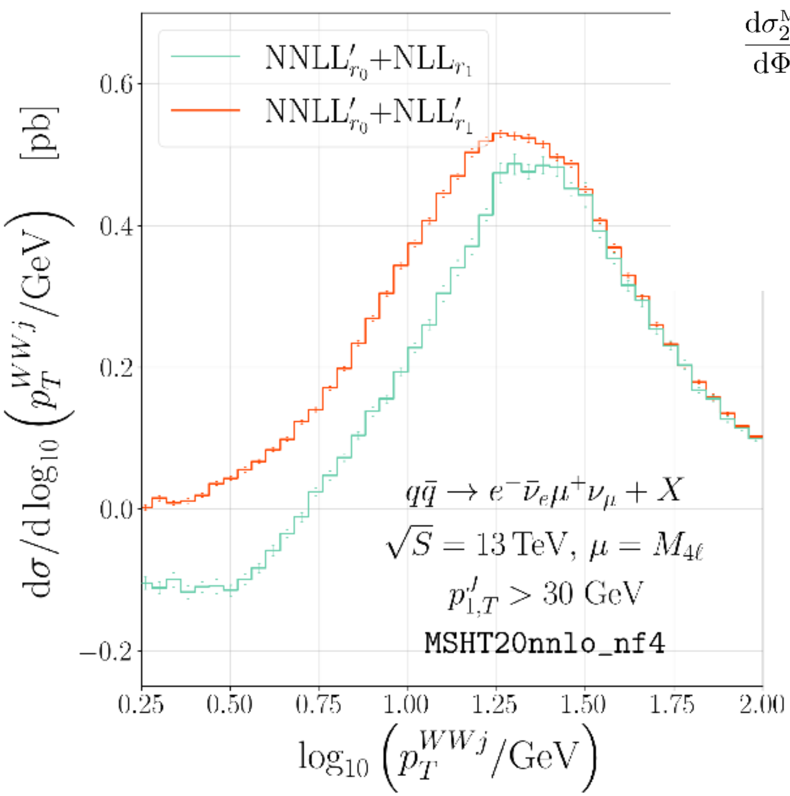
Resumming second jet resolution at NLL’ in GENEVA

Extension of the GENEVA approach to include resummation of r_1^{cut} to NLL’ accuracy

Now truly capturing the correct nonsingular behaviour when approaching the single-jet limit

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_1^{\mathrm{MC}}}{\mathrm{d}\Phi_1}(r_1^{\mathrm{cut}}) = & \Bigg\{ \Bigg[\frac{\mathrm{d}\sigma^{\mathrm{NNLL}'_{r_0}}}{\mathrm{d}\Phi_0\,\mathrm{d}r_0} - \frac{\mathrm{d}\sigma^{\mathrm{NNLL}'_{r_0}}}{\mathrm{d}\Phi_0\,\mathrm{d}r_0}\bigg|_{\mathrm{NLO}_1} \Bigg] \mathcal{P}_{0\to1}(\Phi_1)\, U_1(\Phi_1, r_1^{\mathrm{cut}}) \\
& + \frac{\mathrm{d}\sigma^{\mathrm{NLO}_1}}{\mathrm{d}\Phi_1}(r_1^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma^{\mathrm{NLL}'_{r_1}}}{\mathrm{d}\Phi_1}(r_1^{\mathrm{cut}}) - \frac{\mathrm{d}\sigma^{\mathrm{NLL}'_{r_1}}}{\mathrm{d}\Phi_1}(r_1^{\mathrm{cut}})\bigg|_{\mathrm{NLO}_1} \Bigg\}\, \theta(r_0 > r_0^{\mathrm{cut}}) \\
& + \frac{\mathrm{d}\sigma^{\mathrm{LO}_1}_{\mathrm{nonproj}}}{\mathrm{d}\Phi_1}\, \theta(r_0 < r_0^{\mathrm{cut}})
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_2^{\mathrm{MC}}}{\mathrm{d}\Phi_2} = & \Bigg\{ \Bigg[\frac{\mathrm{d}\sigma^{\mathrm{NNLL}'_{r_0}}}{\mathrm{d}\Phi_0\,\mathrm{d}r_0} - \frac{\mathrm{d}\sigma^{\mathrm{NNLL}'_{r_0}}}{\mathrm{d}\Phi_0\,\mathrm{d}r_0}\bigg|_{\mathrm{NLO}_1} \Bigg] \mathcal{P}_{0\to1}(\Phi_1)\, U_1'(\Phi_1, r_1)\, \mathcal{P}_{1\to2}(\Phi_2) \\
& + \frac{\mathrm{d}\sigma^{\mathrm{LO}_2}}{\mathrm{d}\Phi_2} + \Bigg[\frac{\mathrm{d}\sigma^{\mathrm{NLL}'_{r_1}}}{\mathrm{d}\Phi_1\,\mathrm{d}r_1} - \frac{\mathrm{d}\sigma^{\mathrm{NLL}'_{r_1}}}{\mathrm{d}\Phi_1\,\mathrm{d}r_1}\bigg|_{\mathrm{LO}_2} \Bigg] \mathcal{P}_{1\to2}(\Phi_2) \Bigg\}\, \theta(r_1 > r_1^{\mathrm{cut}})\, \theta(r_0 > r_0^{\mathrm{cut}}) \\
& + \frac{\mathrm{d}\sigma^{\mathrm{LO}_2}_{\mathrm{nonproj}}}{\mathrm{d}\Phi_2}\, \theta(r_1 < r_1^{\mathrm{cut}})\, \theta(r_0 > r_0^{\mathrm{cut}})\,.
\end{aligned}
$$



NLL’ accuracy of the second jet only maintained in presence of an hard first jet.

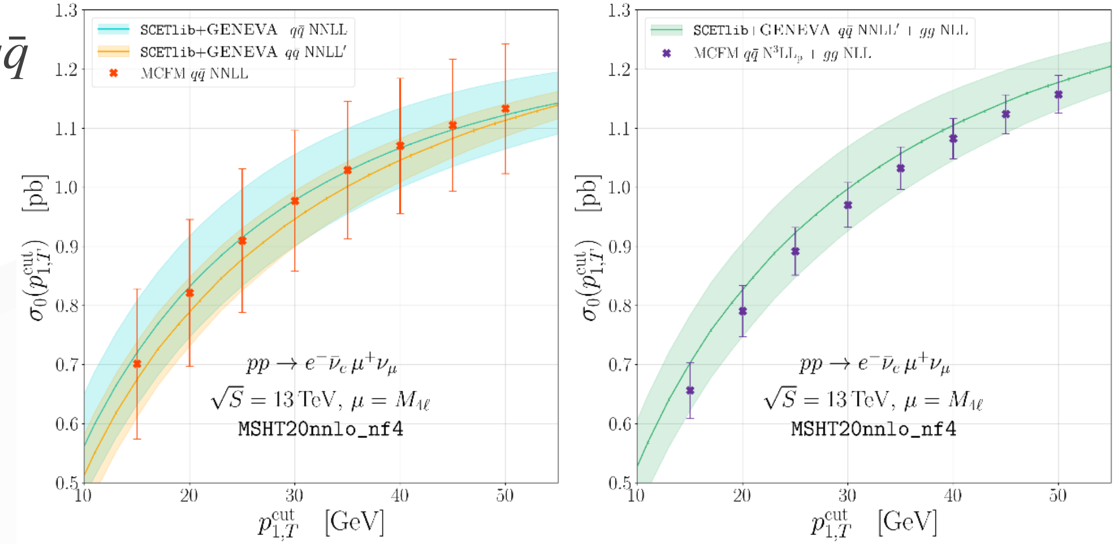
Resummation formula not able to handle the $r_0 \sim r_1 \ll \mu_H$ hierarchy, double resummation required there.

Validation of WW production

We include the resummation of the $q\bar{q}$ channel at NNLL' and the gg channel at NLL

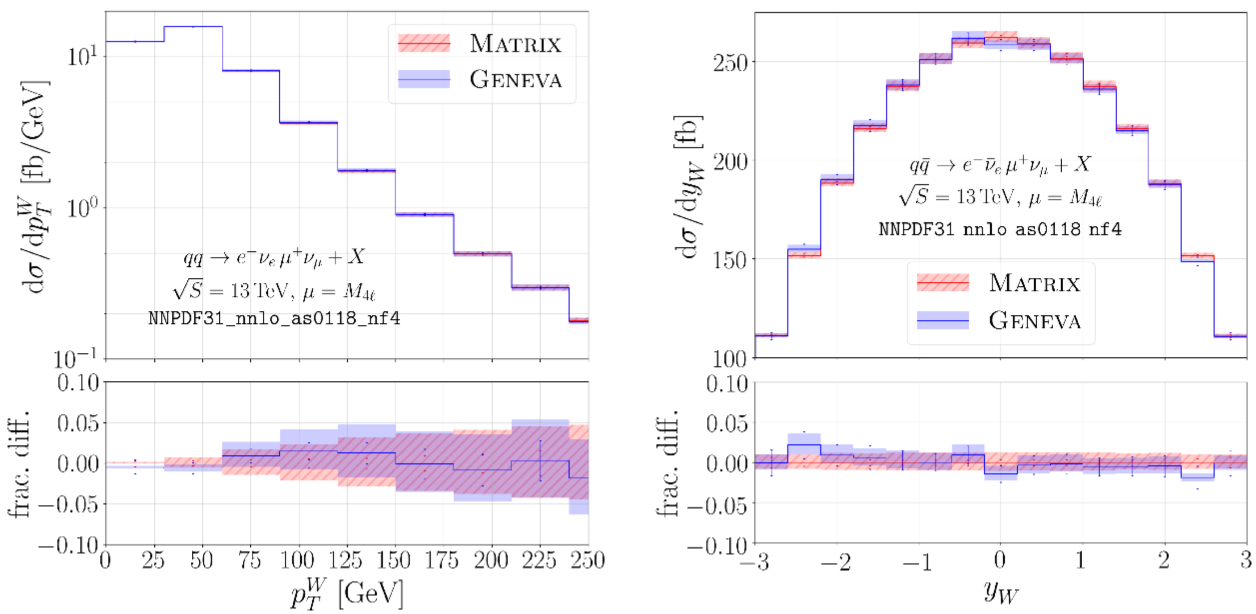
Jet veto resummation available in MCFM up to partial N3LL accuracy. Different treatment of uncertainties.

[Campbell et al. 2301.11768]

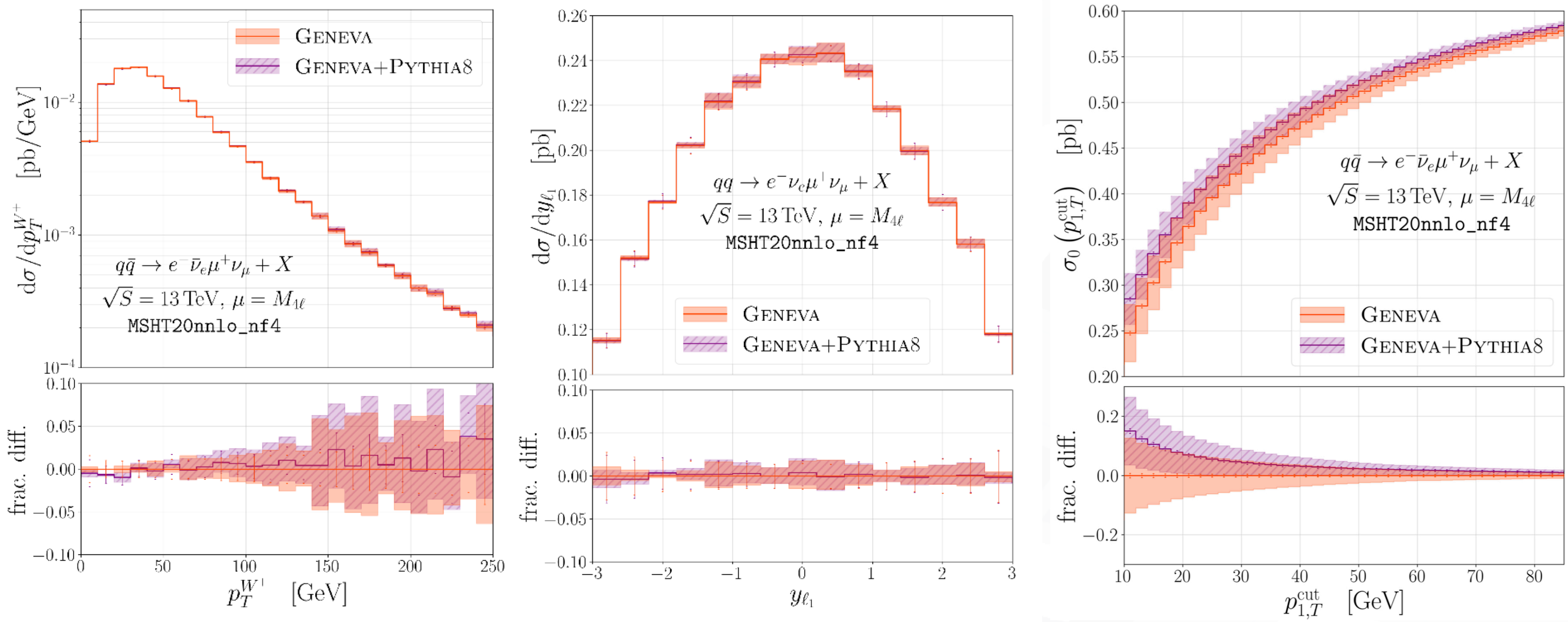


NNLO validation against MATRIX

[Grazzini et al. 1711.06631]



Showering



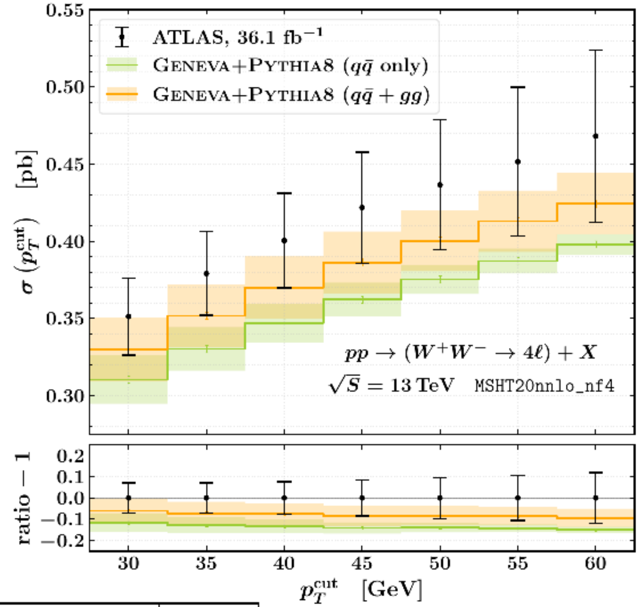
Inclusive quantities well-preserved by the shower, pT of the hardest jet is extremely sensitive to shower effects and gets mildly shifted. Few percent effect at 30 GeV.

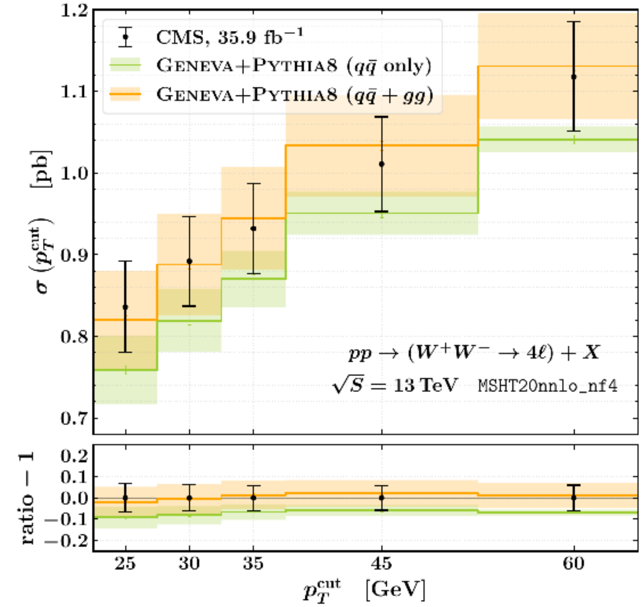
This is entirely due to FSR emissions, choice of resolution variable preserves partonic accuracy when only ISR emissions are allowed.

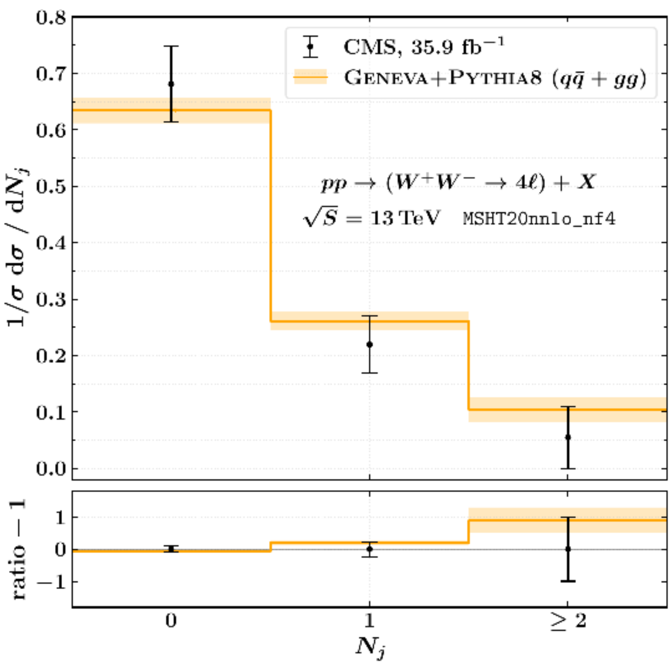
Data comparison

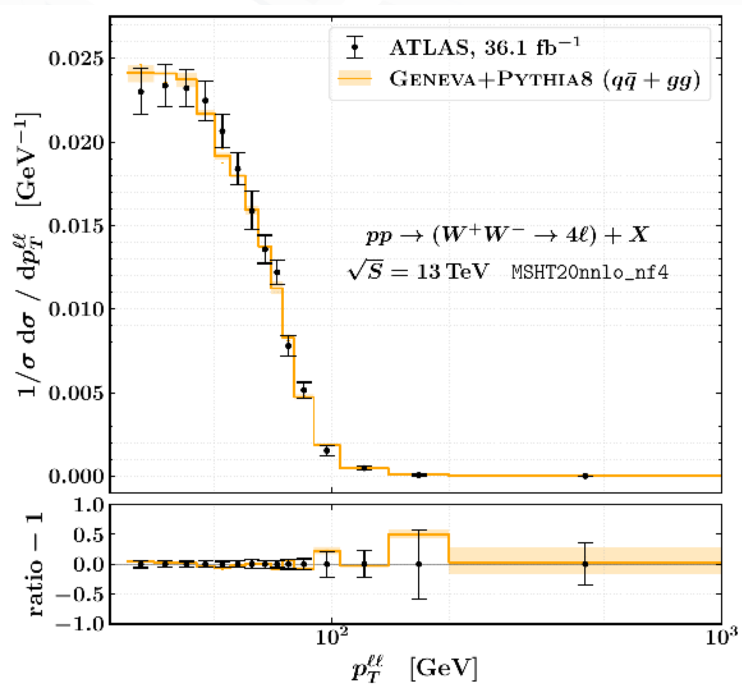
Inclusion of gg channel necessary for agreement with data.

Extension of gg channel to NLO+NLL' ongoing









Zero-jettiness factorization for top-quark pairs

Factorization formula derived using SCET+HQET in the region where $M_{t\bar{t}} \sim m_t \sim \sqrt{\hat{s}}$ are all hard scales. [SA et al. 2111.03632]

In case of boosted regime $M_{t\bar{t}} \gg m_t$ one would instead need a modified two-jettiness [Fleming, Hoang,Mantry,Stewart `07][Bachu,Hoang,Mateu,Pathak,Stewart `21]

$$\frac{d\sigma}{d\Phi_0 d\tau_B} = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \int dt_a\, dt_b\, B_i(t_a, z_a, \mu)\, B_j(t_b, z_b, \mu)\, \mathrm{Tr}\left[\mathbf{H}_{ij}(\Phi_0, \mu)\, \mathbf{S}_{ij}\left(M\tau_B - \frac{t_a + t_b}{M}, \Phi_0, \mu\right)\right]$$

Beam functions [Stewart, Tackmann, Waalewijn, [1002.2213], known up to N^{3}LO

Hard functions (color matrices)

Soft functions (color matrices)

It is convenient to transform the soft and beam functions in Laplace space to solve the RG equations, the factorization formula is turn into a product of (matrix) functions

$$\mathscr{L}\left[\frac{d\sigma}{d\Phi_0 d\tau_B}\right] = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \tilde{B}_i\left(\ln\frac{M\kappa}{\mu^2}, z_a\right) \tilde{B}_j\left(\ln\frac{M\kappa}{\mu^2}, z_b\right) \mathrm{Tr}\left[\mathbf{H}_{ij}\left(\ln\frac{M^2}{\mu^2}, \Phi_0\right) \tilde{\mathbf{S}}_{ij}\left(\ln\frac{\mu^2}{\kappa^2}, \Phi_0\right)\right]$$

Zero-jettiness resummation for top pairs

Resummed formula valid up to NNLL' accuracy

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_0 \mathrm{d}\tau_B} = U(\mu_h, \mu_B, \mu_s, L_h, L_s)$$
$$\times \,\mathrm{Tr}\Big\{\mathbf{u}(\beta_t, \theta, \mu_h, \mu_s)\,\mathbf{H}(M, \beta_t, \theta, \mu_h)\,\mathbf{u}^\dagger(\beta_t, \theta, \mu_h, \mu_s)\,\tilde{\mathbf{S}}_B(\partial_{\eta_s} + L_s, \beta_t, \theta, \mu_s)\Big\}$$
$$\times \,\tilde{B}_a(\partial_{\eta_B} + L_B, z_a, \mu_B)\tilde{B}_b(\partial_{\eta'_B} + L_B, z_b, \mu_B)\frac{1}{\tau_B^{1-\eta_{\mathrm{tot}}}}\frac{e^{-\gamma_E \eta_{\mathrm{tot}}}}{\Gamma(\eta_{\mathrm{tot}})}\,.$$

where

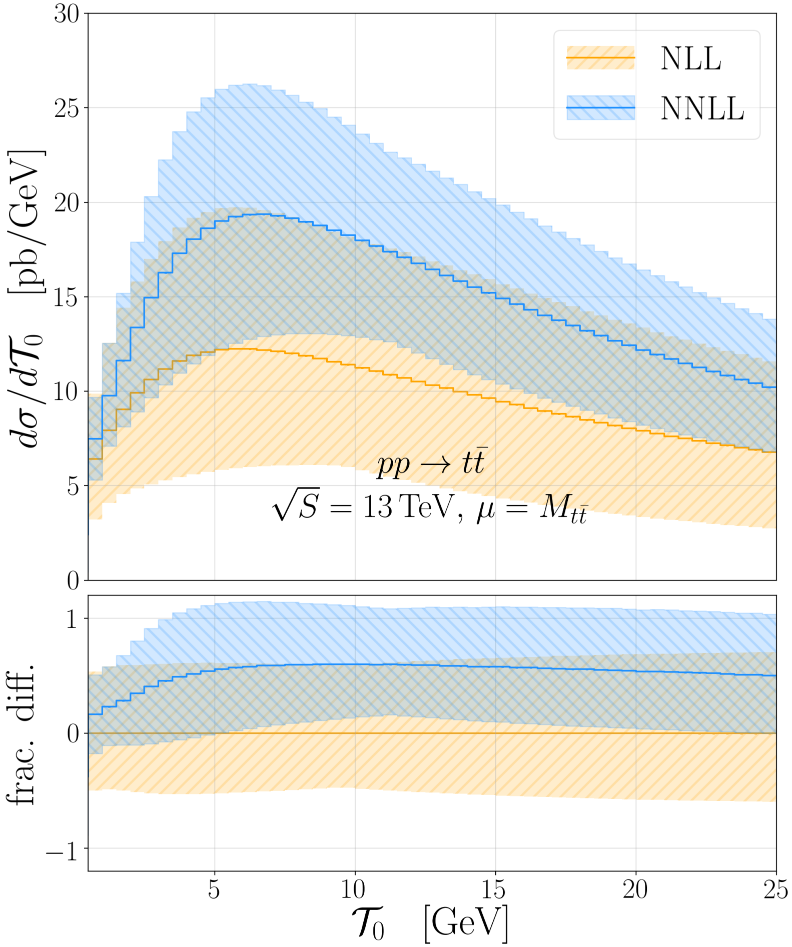
$$U(\mu_h, \mu_B, \mu_s, L_h, L_s) =$$
$$\exp\left[4S(\mu_h, \mu_B) + 4S(\mu_s, \mu_B) + 2a_{\gamma^B}(\mu_s, \mu_B) - 2a_\Gamma(\mu_h, \mu_B)\,L_h - 2a_\Gamma(\mu_s, \mu_B)\,L_s\right]$$

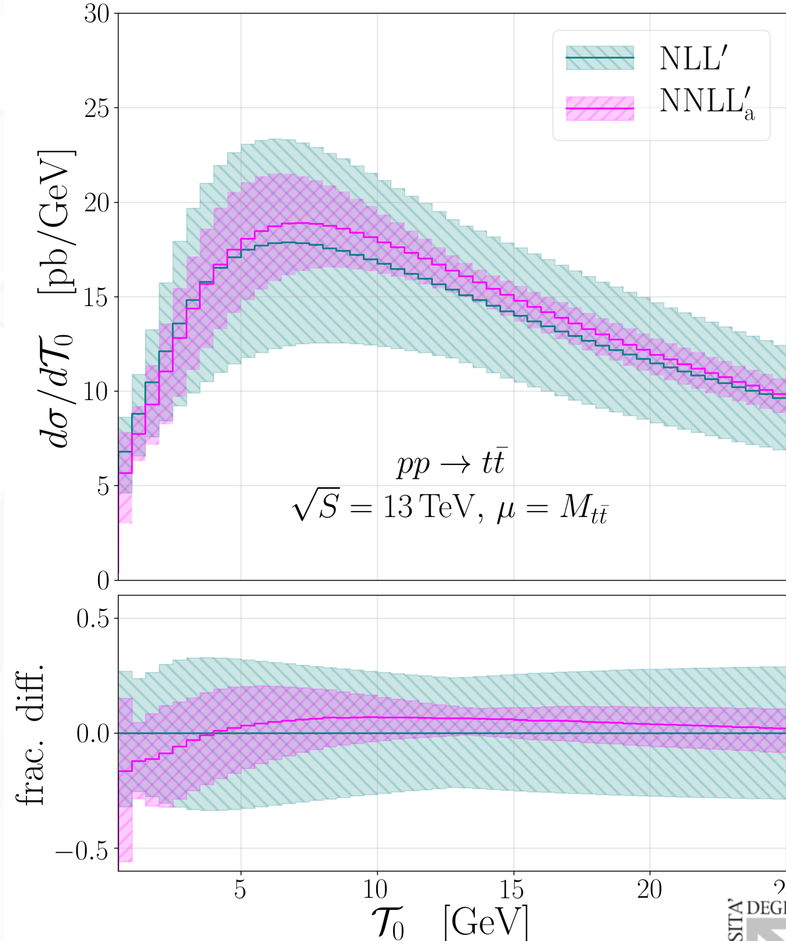
and $L_s = \ln(M^2/\mu_s^2)$, $L_h = \ln(M^2/\mu_h^2)$, $L_B = \ln(M^2/\mu_B^2)$ and $\eta_{\mathrm{tot}} = 2\eta_S + \eta_B + \eta_{B'}$

The final accuracy depends on the availability of the perturbative ingredients

Resummed results

$\mathrm{NNLL}'_{\mathrm{a}}$ is our best prediction, it includes NNLO beam functions, all mixed NLO x NLO terms, NNLL evolution matrices, all NNLO soft logarithmic terms. Resummation is switched off via profile scales

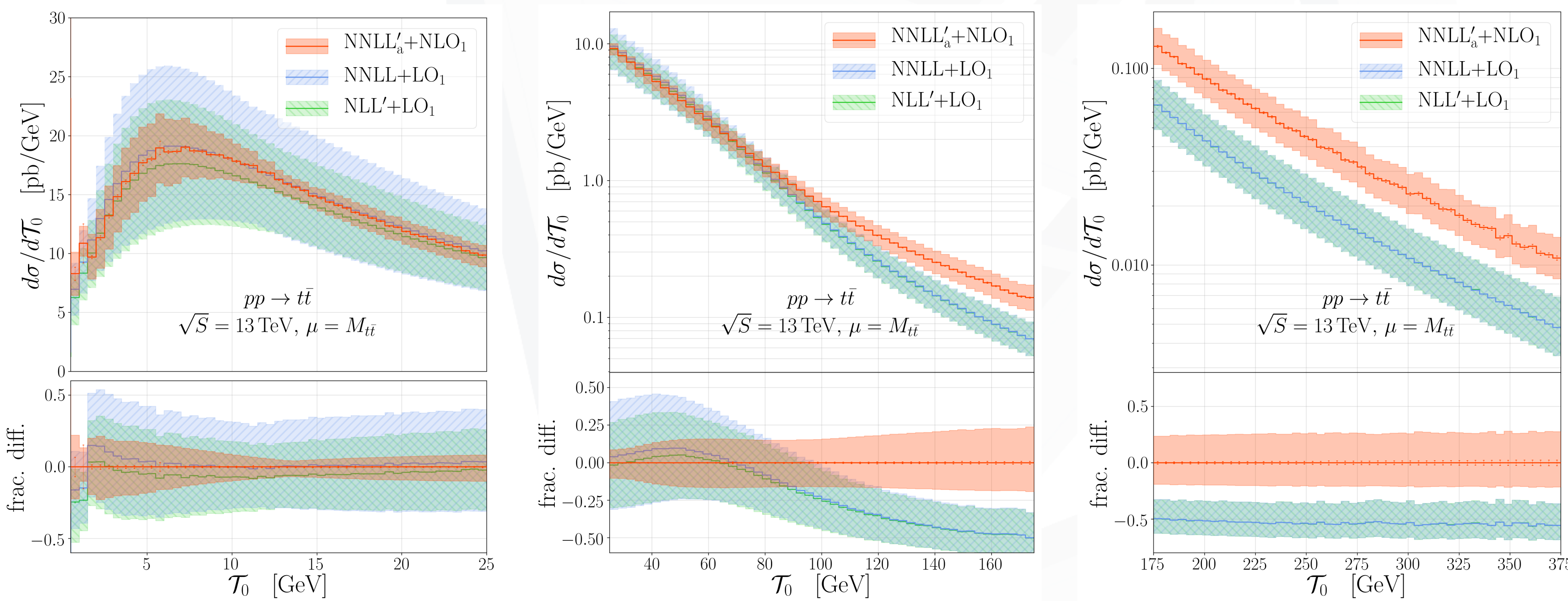




Matched results

Matching to $t\bar{t} + j$ @NLO improves the perturbative accuracy across the whole spectrum

$$\frac{\mathrm{d}\sigma^{\mathrm{match}}}{\mathrm{d}\mathcal{T}_0} = \frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0} + \frac{\mathrm{d}\sigma^{\mathrm{FO}}}{\mathrm{d}\mathcal{T}_0} - \left[\frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0}\right]_{\mathrm{FO}}$$



Extension to full NNLL' and to event generation is in progress.

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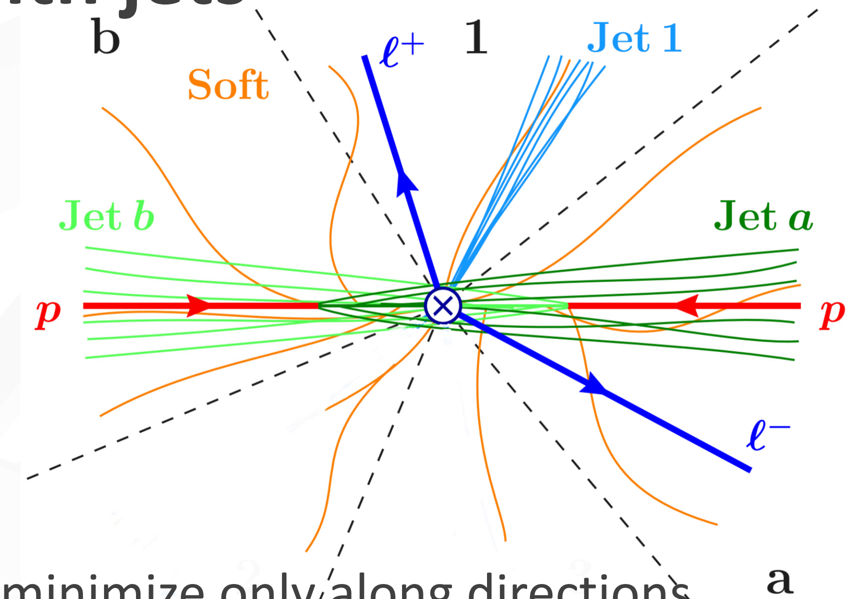
Extension to processes with jets



- Focus of color-singlet plus jet production

$$\mathcal{T}_1 = \sum_k \min\{\frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_J \cdot p_k}{Q_J}\}$$

- To remove energy-dependence and minimize only along directions $Q_i = 2E_i$'s must be frame-dependent

$$\hat{\mathcal{T}}_1 = \sum_k \min\{\frac{\hat{n}_a \cdot \hat{p}_k}{\rho_a}, \frac{\hat{n}_b \cdot \hat{p}_k}{\rho_b}, \frac{\hat{n}_J \cdot \hat{p}_k}{\rho_J}\}$$

- The choice of the ρ_i's determines the frame in which the one-jettiness resummation is performed. We focus on 3 choices: LAB , UB -frame $Y_{Vj} = 0$ and CS-frame $Y_V = 0$

$$\rho_a = e^{\hat{Y}_V}\,,$$
$$\rho_b = e^{-\hat{Y}_V}\,,$$
$$\rho_J = \frac{e^{-\hat{Y}_V}(\hat{p}_J)_+ + e^{\hat{Y}_V}(\hat{p}_J)_-}{2\hat{E}_J}$$

Resummation of one-jettiness for Z+jet

Factorization formula in the region $\mathcal{T}_1 \ll Q$ hard scale $\sqrt{s}, M_{\ell^+\ell^-}, M_{T,\ell^+\ell^-}, \mathcal{T}_0$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_1 \mathrm{d}\mathcal{T}_1} = \sum_{\kappa=\{q\bar{q}g, qgq, ggg\}} H_\kappa(\Phi_1) \int \mathrm{d}t_a \mathrm{d}t_b \mathrm{d}s_J \, B_{\kappa_a}(t_a) B_{\kappa_b}(t_b) J_{\kappa_J}(s_J) \times S_\kappa\left(n_{a,b}\cdot n_J, \mathcal{T}_1 - \frac{t_a}{Q_a} - \frac{t_b}{Q_b} - \frac{s_J}{Q_J}\right)$$

We left the choice of the frame free, keeping in mind the issues for GENEVA.

It is convenient to transform the soft, beam and jet functions in Laplace space to solve the RG equations, the factorization formula is turn into a product.
The color factorizes trivially in soft and hard functions for 3 colored partons.

$$\mathcal{L}\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_1 \mathrm{d}\mathcal{T}_1}\right] = \sum_\kappa H_\kappa(\Phi_1)\, \tilde{S}_\kappa\left(\ln\frac{\lambda_E^2}{\mu^2}\right) \tilde{B}_{\kappa_a}\left(\ln\frac{Q_a\,\lambda_E}{\mu^2}\right) \tilde{B}_{\kappa_b}\left(\ln\frac{Q_b\,\lambda_E}{\mu^2}\right) \tilde{J}_{\kappa_J}\left(\ln\frac{Q_J\,\lambda_E}{\mu^2}\right)$$

Hard, soft, beam and jet functions

Hard functions known analytically up to 2-loops. [Gehrmann, Tancredi et al. `12, `22]

From NNLL' accuracy include the loop-squared $gg \to Zg$

Beam and jet boundary conditions known up to 3-loop [Mistlberger et al. `20]

[Becher, Bell `10] [Gaunt et al. `14]

We compute the one-loop soft boundary terms as on-the-fly integrals using results in [Jouttenus et al. `11]

$$S^{\kappa\,(1)}_{\mathcal{T}_1,-1} = 2c^\kappa_s\Big[L^2_{ab} - \frac{\pi^2}{6} + 2(I_{ab,c} + I_{ba,c})\Big] + 2c^\kappa_t\Big[L^2_{ac} - \frac{\pi^2}{6} + 2(I_{ac,b} + I_{ca,b})\Big]$$
$$+ 2c^\kappa_u\Big[L^2_{bc} - \frac{\pi^2}{6} + 2(I_{bc,a} + I_{cb,a})\Big]$$

$$I_{ij,m} \equiv I_0\Big(\frac{\hat{s}_{jm}}{\hat{s}_{ij}}, \frac{\hat{s}_{im}}{\hat{s}_{ij}}\Big) \ln\frac{\hat{s}_{jm}}{\hat{s}_{ij}} + I_1\Big(\frac{\hat{s}_{jm}}{\hat{s}_{ij}}, \frac{\hat{s}_{im}}{\hat{s}_{ij}}\Big)$$

Also studied for different jet measures in [Bertolini et al. `17]

The 2-loop contribution $S^{\kappa\,(2)}_{\mathcal{T}_1\,-1}$ is provided by SoftSERVE collaboration (thanks to Bahman Dehnadi), in the form of an interpolation grid [Bell, Rahn, Talbert `18]

We validated the approach comparing to the interpolation used in MCFM.

[Campbell, Ellis, Mondini, Williams `18]

Resummed formula

We can combine the solutions for the hard, soft, jet and beam functions to obtain

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_1 \mathrm{d}\mathcal{T}_1} = \sum_\kappa \exp\Bigg\{ & 4(C_{\kappa_a} + C_{\kappa_b}) K_{\Gamma_\mathrm{cusp}}(\mu_B, \mu_H) + 4 C_{\kappa_J} K_{\Gamma_\mathrm{cusp}}(\mu_J, \mu_H) \\
& - 2(C_{\kappa_a} + C_{\kappa_b} + C_{\kappa_J}) K_{\Gamma_\mathrm{cusp}}(\mu_S, \mu_H) - 2 C_{\kappa_J} L_J \, \eta_{\Gamma_\mathrm{cusp}}(\mu_J, \mu_H) \\
& - 2(C_{\kappa_a} L_B + C_{\kappa_b} L'_B) \eta_{\Gamma_\mathrm{cusp}}(\mu_B, \mu_H) + \Bigg[C_{\kappa_a} \ln\left(\frac{Q_a^2 u}{st}\right) + C_{\kappa_b} \ln\left(\frac{Q_b^2 t}{su}\right) \\
& \quad + C_{\kappa_j} \ln\left(\frac{Q_J^2 s}{tu}\right) + (C_{\kappa_a} + C_{\kappa_b} + C_{\kappa_J}) L_S \Bigg] \eta_{\Gamma_\mathrm{cusp}}(\mu_S, \mu_H) + K_{\gamma_\mathrm{tot}} \Bigg\} \\
& \times \tilde{B}_{\kappa_a}(\partial_{\eta_B} + L_B, x_a, \mu_B) \tilde{B}_{\kappa_b}(\partial_{\eta'_B} + L'_B, x_b, \mu_B) \, \tilde{J}_{\kappa_J}(\partial_{\eta_J} + L_J, \mu_J) \\
& \times H_\kappa(\Phi_1, \mu_H) \tilde{S}^{\kappa}_{\mathcal{T}_1}\left(\partial_{\eta_S} + L_S, \mu_S\right) \frac{Q^{-\eta_\mathrm{tot}}}{\mathcal{T}_1^{1-\eta_\mathrm{tot}}} \frac{\eta_\mathrm{tot} \, e^{-\gamma_E \eta_\mathrm{tot}}}{\Gamma(1+\eta_\mathrm{tot})} + \mathcal{O}\left(\frac{\mathcal{T}_1}{Q}\right)
\end{aligned}
$$

where we have defined

$$
L_H = \ln\left(\frac{Q^2}{\mu_H^2}\right), \qquad L_B = \ln\left(\frac{Q_a Q}{\mu_B^2}\right), \qquad L'_B = \ln\left(\frac{Q_b Q}{\mu_B^2}\right)
$$

$$
L_J = \ln\left(\frac{Q_J Q}{\mu_J^2}\right) \quad \text{and} \quad L_S = \ln\left(\frac{Q^2}{\mu_S^2}\right).
$$

$$
\eta_\mathrm{tot} = -2(C_{\kappa_a} + C_{\kappa_b}) \eta_{\Gamma_\mathrm{cusp}}(\mu_B, \mu_J) + 2(C_{\kappa_a} + C_{\kappa_b} + C_{\kappa_J}) \eta_{\Gamma_\mathrm{cusp}}(\mu_S, \mu_J).
$$

$$
\begin{aligned}
K_{\gamma_\mathrm{tot}} = & -2 n_g K_{\gamma_C^g}(\mu_S, \mu_H) + 2(n_g - 3) K_{\gamma_C^q}(\mu_S, \mu_H) \\
& - (n_g - n_g^{\kappa_J}) K_{\gamma_J^g}(\mu_J, \mu_B) - n_g K_{\gamma_J^g}(\mu_S, \mu_J) \\
& + (n_g - 2 - n_g^{\kappa_J}) K_{\gamma_J^q}(\mu_J, \mu_B) + (n_g - 3) K_{\gamma_J^q}(\mu_S, \mu_J)
\end{aligned}
$$

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Resummation formula up to NNLL' accuracy

$$
\begin{aligned}
\frac{\mathrm{d}\sigma^{\mathrm{NNLL}'}}{\mathrm{d}\Phi_1 \mathrm{d}\mathcal{T}_1} = \sum_\kappa \exp\Big\{ & 4(C_{\kappa_a}+C_{\kappa_b}) K^{\mathrm{NNLL}}_{\Gamma_{\mathrm{cusp}}}(\mu_B,\mu_H) + 4C_{\kappa_J} K^{\mathrm{NNLL}}_{\Gamma_{\mathrm{cusp}}}(\mu_J,\mu_H) \\
& - 2(C_{\kappa_a}+C_{\kappa_b}+C_{\kappa_J}) K^{\mathrm{NNLL}}_{\Gamma_{\mathrm{cusp}}}(\mu_S,\mu_H) - 2C_{\kappa_J} L_J\, \eta^{\mathrm{NNLL}}_{\Gamma_{\mathrm{cusp}}}(\mu_J,\mu_H) \\
& - 2(C_{\kappa_a} L_B + C_{\kappa_b} L'_B)\eta^{\mathrm{NNLL}}_{\Gamma_{\mathrm{cusp}}}(\mu_B,\mu_H) + \Big[C_{\kappa_a} \ln\Big(\frac{Q_a^2 u}{st}\Big) + C_{\kappa_b} \ln\Big(\frac{Q_b^2 t}{su}\Big) \\
& \quad + C_{\kappa_j} \ln\Big(\frac{Q_J^2 s}{tu}\Big) + (C_{\kappa_a}+C_{\kappa_b}+C_{\kappa_J}) L_S \Big] \eta^{\mathrm{NNLL}}_{\Gamma_{\mathrm{cusp}}}(\mu_S,\mu_H) + K^{\mathrm{NNLL}}_{\gamma_{\mathrm{tot}}} \Big\} \\
\times \Big\{ & H^{(0)}_\kappa(\Phi_1,\mu_H) \Big[f_{\kappa_a}(x_a,\mu_B) f_{\kappa_b}(x_b,\mu_B) \Big(1 + \tilde{S}^{\kappa\,(1)}_{\mathcal{T}_1}(\partial_{\eta_S}+L_S,\mu_S) + \tilde{J}^{(1)}_{\kappa_J}(\partial_{\eta_J}+L_J,\mu_J) \\
& + \tilde{S}^{\kappa\,(1)}_{\mathcal{T}_1}(\partial_{\eta_S}+L_S,\mu_S)\, \tilde{J}^{(1)}_{\kappa_J}(\partial_{\eta_J}+L_J,\mu_J) + \tilde{S}^{\kappa\,(2)}_{\mathcal{T}_1}(\partial_{\eta_S}+L_S,\mu_S) + \tilde{J}^{(2)}_{\kappa_J}(\partial_{\eta_J}+L_J,\mu_J) \Big) \\
& + \Big(\tilde{B}^{(1)}_{\kappa_a}(\partial_{\eta_B}+L_B,x_a,\mu_B) \Big(1 + \tilde{S}^{\kappa\,(1)}_{\mathcal{T}_1}(\partial_{\eta_S}+L_S,\mu_S) + \tilde{J}^{(1)}_{\kappa_J}(\partial_{\eta_J}+L_J,\mu_J) \Big) \\
& + \tilde{B}^{(2)}_{\kappa_a}(\partial_{\eta_B}+L_B,x_a,\mu_B) \Big) f_{\kappa_b}(x_b,\mu_B) + f_{\kappa_a}(x_a,\mu_B) \Big(\tilde{B}^{(2)}_{\kappa_b}(\partial_{\eta'_B}+L'_B,x_b,\mu_B) \\
& + \tilde{B}^{(1)}_{\kappa_b}(\partial_{\eta'_B}+L'_B,x_b,\mu_B) \Big(1 + \tilde{S}^{\kappa\,(1)}_{\mathcal{T}_1}(\partial_{\eta_S}+L_S,\mu_S) + \tilde{J}^{(1)}_{\kappa_J}(\partial_{\eta_J}+L_J,\mu_J) \Big) \Big) \Big] \\
& + H^{(1)}_\kappa(\Phi_1,\mu_H) \Big[f_{\kappa_a}(x_a,\mu_B) f_{\kappa_b}(x_b,\mu_B) \Big(1 + \tilde{S}^{\kappa\,(1)}_{\mathcal{T}_1}(\partial_{\eta_S}+L_S,\mu_S) + \tilde{J}^{(1)}_{\kappa_J}(\partial_{\eta_J}+L_J,\mu_J) \Big) \\
& + \Big(\tilde{B}^{(1)}_{\kappa_a}(\partial_{\eta_B}+L_B,x_a,\mu_B) f_{\kappa_b}(x_b,\mu_B) + f_{\kappa_a}(x_a,\mu_B) \tilde{B}^{(1)}_{\kappa_b}(\partial_{\eta'_B}+L'_B,x_b,\mu_B) \Big) \Big] \\
& + H^{(2)}_\kappa(\Phi_1,\mu_H) f_{\kappa_a}(x_a,\mu_B) f_{\kappa_b}(x_b,\mu_B) \Big\} \\
\times & \frac{Q^{-\eta^{\mathrm{NNLL}}_{\mathrm{tot}}}}{\mathcal{T}_1^{1-\eta^{\mathrm{NNLL}}_{\mathrm{tot}}}} \frac{\eta^{\mathrm{NNLL}}_{\mathrm{tot}}\, e^{-\gamma_E \eta^{\mathrm{NNLL}}_{\mathrm{tot}}}}{\Gamma(1+\eta^{\mathrm{NNLL}}_{\mathrm{tot}})} .
\end{aligned}
$$

Providing

3-loop cusp an. dim

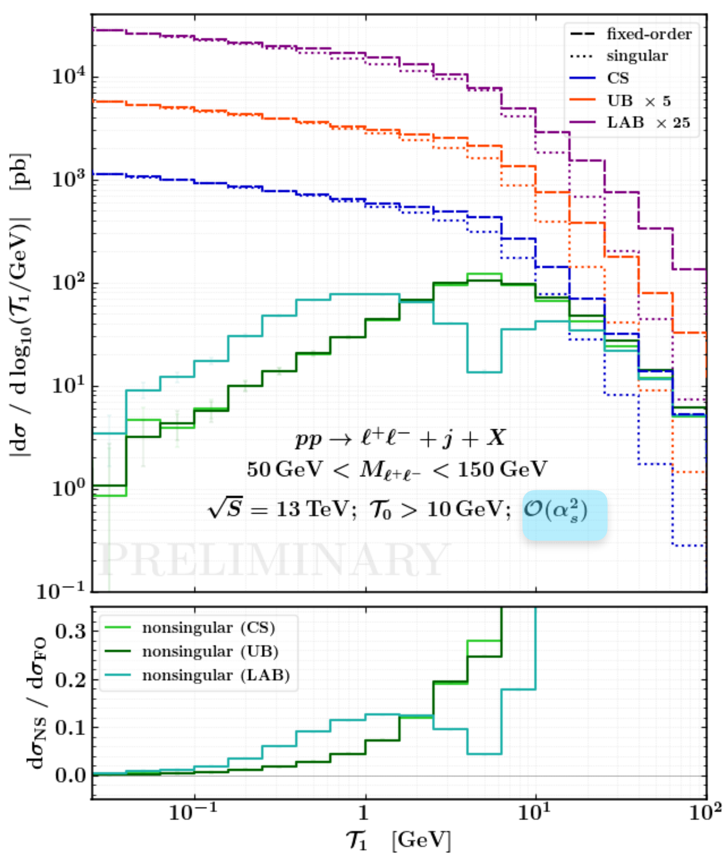
2-loop non cusp

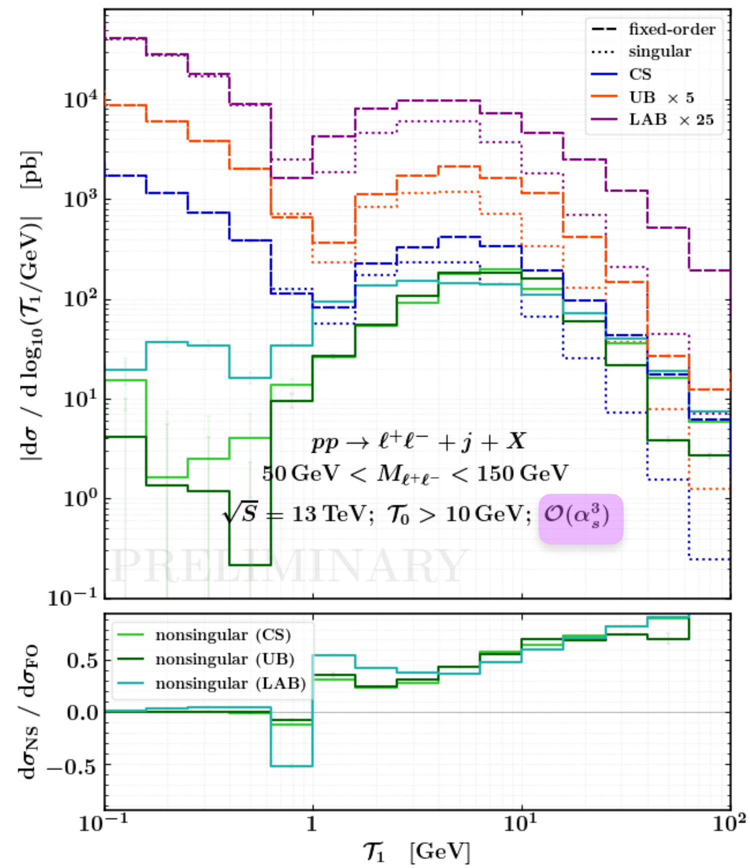
2-loop boundary terms (Hard, Soft, Beam, Jet)

we can reach NNLL' accuracy

Nonsingular behavior

- Different $\mathscr{T}_1$ choices have different subleading PC
- Investigated for one-jettiness subtraction at LL NLP [Boughezal, Isgro', Petriello `20]





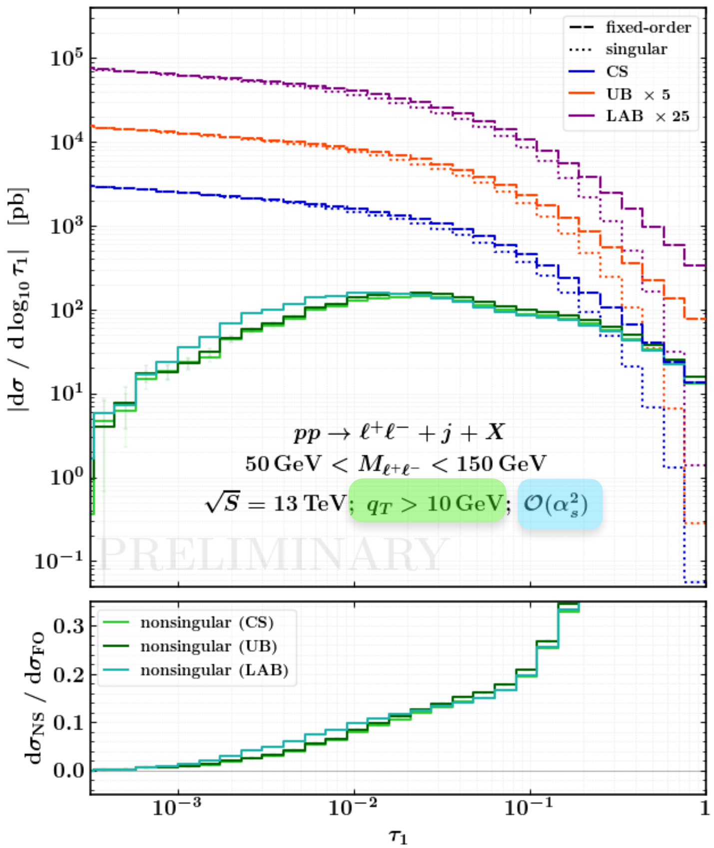
- CS frame as good as UB across different cuts. LAB consistently worse

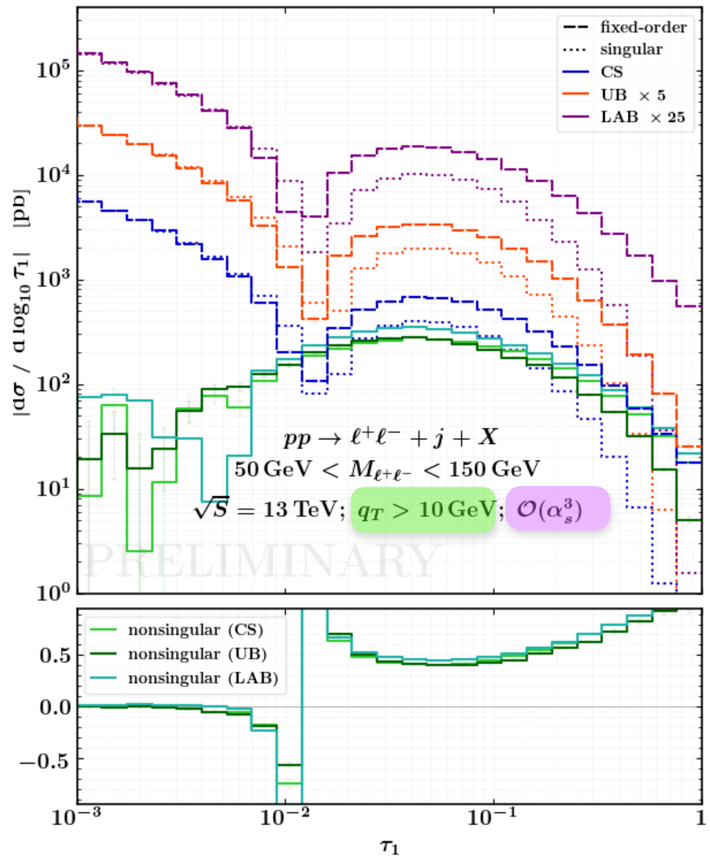
Nonsingular behavior

Dimensionless definition

$$\tau_1 = 2\mathcal{T}_1/\sqrt{M^2_{\ell^+\ell^-} + q_T^2}$$

- Reduced differences when cutting on Z boson trans. momentum q_T





N^3LL resummation for 3 colored partons

channel ($q\bar{q}g$, qgq, ggg,...), hard anomalous dimension has the form [T. Becher and M. Neubert 1908.11379]

$$\Gamma(\{\underline{s}\},\mu) = \frac{\gamma_{\rm cusp}(\alpha_s)}{2}\left[(C_{R_3}-C_{R_1}-C_{R_2})\ln\frac{\mu^2}{(-s_{12})} + \text{cyclic permutations}\right]$$

4-loop

$$+\gamma^1(\alpha_s)+\gamma^2(\alpha_s)+\gamma^3(\alpha_s)+\frac{C_A^2}{8}f(\alpha_s)\left(C_{R_1}+C_{R_2}+C_{R_3}\right)$$

3-loop

$$+\sum_{(i,j)}\left[-f(\alpha_s)\,\mathcal{T}_{iijj}+\sum_R g^R(\alpha_s)\left(3\mathcal{D}^R_{iijj}+4\mathcal{D}^R_{iiij}\right)\ln\frac{\mu^2}{-s_{ij}}\right]$$

we explicitly evaluated these contributions as functions of N_c using colour space formalism

$$\mathcal{D}^R_{ijkl} = d_R^{abcd}\mathbf{T}_i^a\mathbf{T}_j^b\mathbf{T}_k^c\mathbf{T}_l^d \qquad \mathcal{T}_{ijkl} = f^{ade}f^{bce}\left(\mathbf{T}_i^a\mathbf{T}_j^b\mathbf{T}_k^c\mathbf{T}_l^d\right)_+$$

$$d_R^{a_1\ldots a_n} = \mathrm{Tr}_R\left(\boldsymbol{T}^{a_1}\ldots\boldsymbol{T}^{a_n}\right)_+ \equiv \frac{1}{n!}\sum_\pi \mathrm{Tr}\left(\boldsymbol{T}_R^{a_{\pi(1)}}\ldots\boldsymbol{T}_R^{a_{\pi(n)}}\right)$$

We found the following relations $\quad \Gamma_{\mathcal{G}}^{ij} = -\sum_{R=F,A} g^R(\alpha_s)\frac{3\langle\mathcal{D}^R_{iijj}\rangle+4\langle\mathcal{D}^R_{iiij}\rangle}{\langle\mathcal{M}|\mathcal{M}\rangle}$

symmetrized in a,b →

$$\Gamma_{\mathcal{G}}^{\{ab\}} = \sum_{R=F,A} g^R(\alpha_s)\left[C_4(R_a,R)+C_4(R_b,R)-C_4(R_c,R)\right]$$

$$\Gamma_{\mathcal{G}}^{\{ac\}} = \sum_{R=F,A} g^R(\alpha_s)\left[C_4(R_a,R)+C_4(R_c,R)-C_4(R_b,R)\right]$$

$$\Gamma_{\mathcal{G}}^{\{bc\}} = \sum_{R=F,A} g^R(\alpha_s)\left[C_4(R_b,R)+C_4(R_c,R)-C_4(R_a,R)\right]$$

Quartic Casimirs

$$C_4(R_i,R) = \frac{d_{R_i}^{abcd}d_R^{abcd}}{N_{R_i}}$$

N^3LL resummation for 3 colored partons

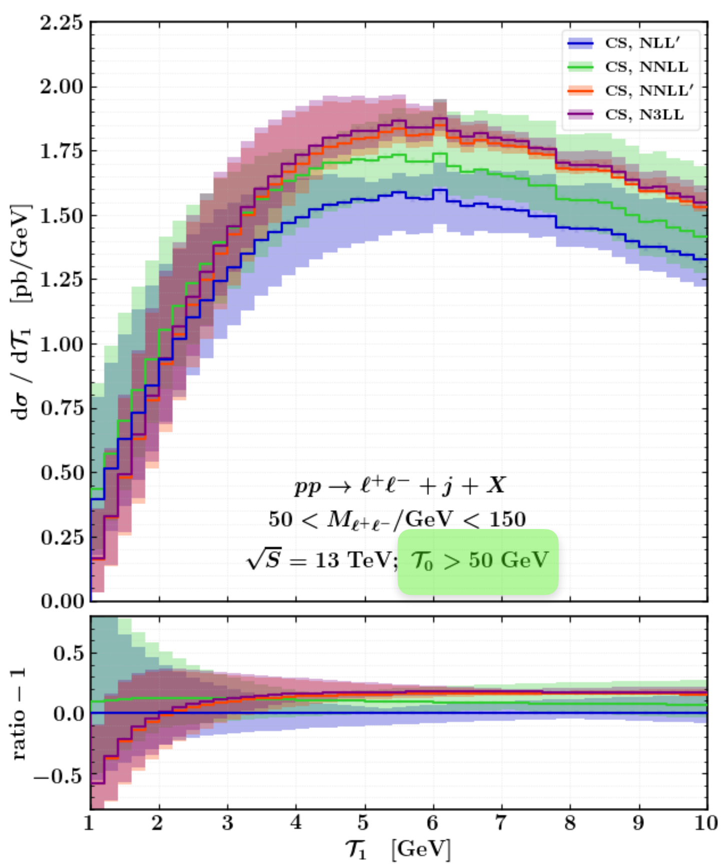
- Final resummation formula at N3LL

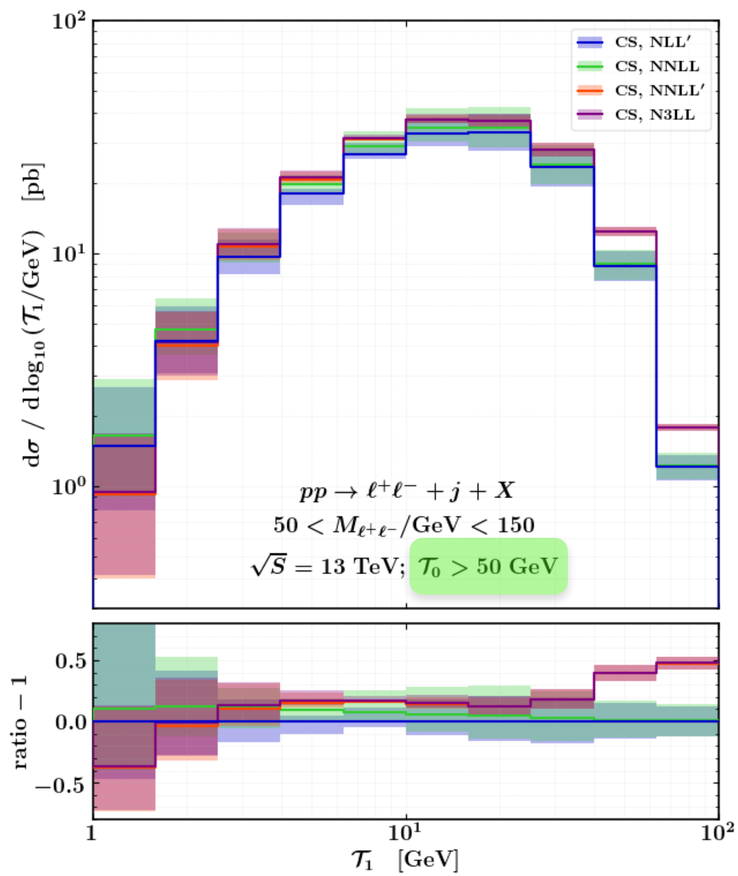
$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_1 \mathrm{d}\mathcal{T}_1} = \sum_\kappa \exp\Bigg\{ & 4(C_{\kappa_a}+C_{\kappa_b})K_{\Gamma_{\rm cusp}}(\mu_B,\mu_H) + 4C_{\kappa_J}K_{\Gamma_{\rm cusp}}(\mu_J,\mu_H) \\
& - 2(C_{\kappa_a}+C_{\kappa_b}+C_{\kappa_J})K_{\Gamma_{\rm cusp}}(\mu_S,\mu_H) - 2C_{\kappa_J}L_J\,\eta_{\Gamma_{\rm cusp}}(\mu_J,\mu_H) \\
& - 2(C_{\kappa_a}L_B + C_{\kappa_b}L'_B)\eta_{\Gamma_{\rm cusp}}(\mu_B,\mu_H) + \Bigg[C_{\kappa_a}\ln\left(\frac{Q_a^2 u}{st}\right) + C_{\kappa_b}\ln\left(\frac{Q_b^2 t}{su}\right) \\
& \quad + C_{\kappa_j}\ln\left(\frac{Q_J^2 s}{tu}\right) + (C_{\kappa_a}+C_{\kappa_b}+C_{\kappa_J})L_S\Bigg]\eta_{\Gamma_{\rm cusp}}(\mu_S,\mu_H) + K_{\gamma_{\rm tot}} \\
& + \sum_{R=F,A}\Bigg[8\left(C_4(R_a,R)+C_4(R_b,R)\right)K_{g^R}(\mu_B,\mu_H) + 8\,C_4(R_c,R)K_{g^R}(\mu_J,\mu_H) \\
& - 4\left(C_4(R_a,R)+C_4(R_b,R)+C_4(R_c,R)\right)K_{g^R}(\mu_S,\mu_H) \\
& - 4\,\eta_{g^R}(\mu_B,\mu_H)\left(C_4(R_a,R)L_B + C_4(R_b,R)L'_B\right) - 4\eta_{g^R}(\mu_J,\mu_H)\,C_4(R_c,R)L_J \\
& + 2\Bigg(C_4(R_a,R)\ln\left(\frac{Q_a^2 u}{st}\right) + C_4(R_b,R)\ln\left(\frac{Q_b^2 t}{su}\right) + C_4(R_c,R)\ln\left(\frac{Q_j^2 s}{tu}\right) \\
& + \left(C_4(R_a,R)+C_4(R_b,R)+C_4(R_c,R)\right)L_S\Bigg)\eta_{g^R}(\mu_S,\mu_H)\Bigg]\Bigg\} \\
& \times \tilde{B}_{\kappa_a}(\partial_{\eta_B}+L_B,x_a,\mu_B)\tilde{B}_{\kappa_b}(\partial_{\eta'_B}+L'_B,x_b,\mu_B)\,\tilde{J}_{\kappa_J}(\partial_{\eta_J}+L_J,\mu_J) \\
& \times H_\kappa(\Phi_1,\mu_H)\tilde{S}^\kappa(\partial_{\eta_S}+L_S,\mu_S)\frac{Q^{-\eta_{\rm tot}}}{\mathcal{T}_1^{1-\eta_{\rm tot}}}\frac{\eta_{\rm tot}\,e^{-\gamma_E\eta_{\rm tot}}}{\Gamma(1+\eta_{\rm tot})} + \mathcal{O}\left(\frac{\mathcal{T}_1}{Q}\right),
\end{aligned}
$$

$$
\begin{aligned}
\eta_{\rm tot} = {} & 2(C_{\kappa_a}+C_{\kappa_b})\eta_{\Gamma_{\rm cusp}}(\mu_S,\mu_B) + 2C_{\kappa_c}\eta_{\Gamma_{\rm cusp}}(\mu_S,\mu_J) \\
& + \sum_{R=F,A}\Big[4\big(C_4(R_a,R)+C_4(R_b,R)\big)\eta_{g^R}(\mu_S,\mu_B) + 4C_4(R_c,R)\eta_{g^R}(\mu_S,\mu_J)\Big]
\end{aligned}
$$

Resummed results

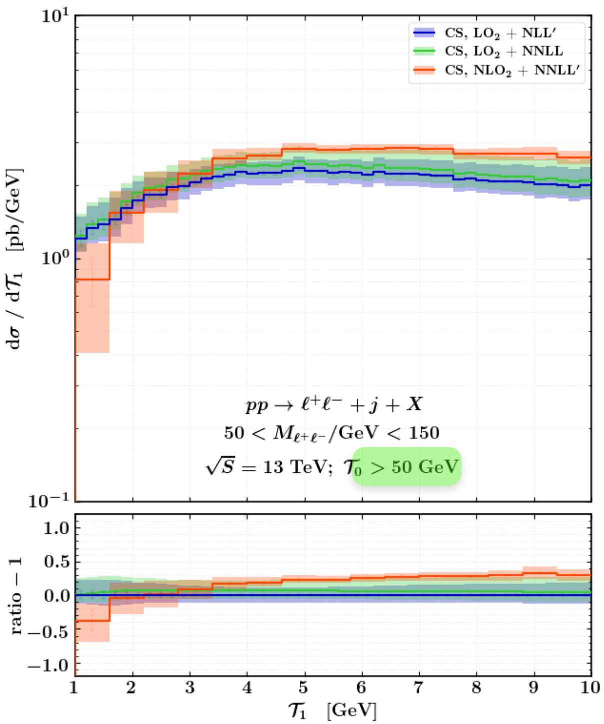
- Using profile scales to switch off resummation at $\mu_H = \sqrt{M^2_{\ell^+\ell^-} + q_T^2}$
- Summing in quadrature profile scales variations and fixed-order ones
- Nice convergence and reduction of theoretical uncertainties

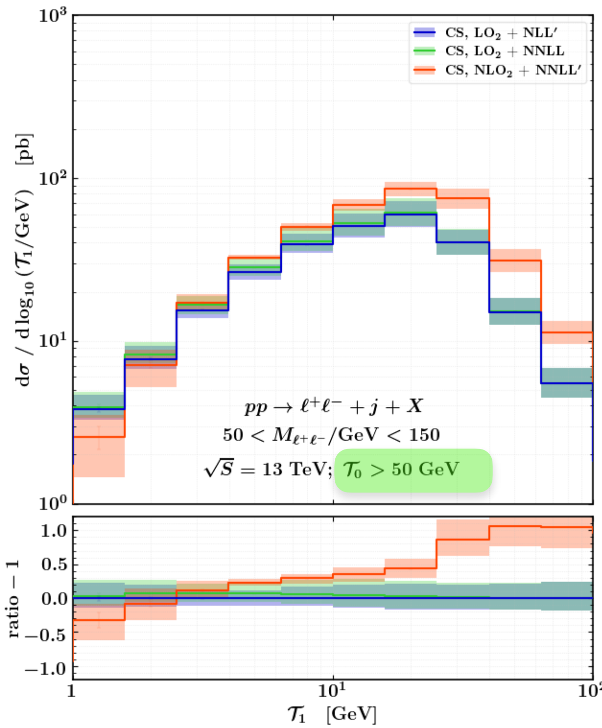


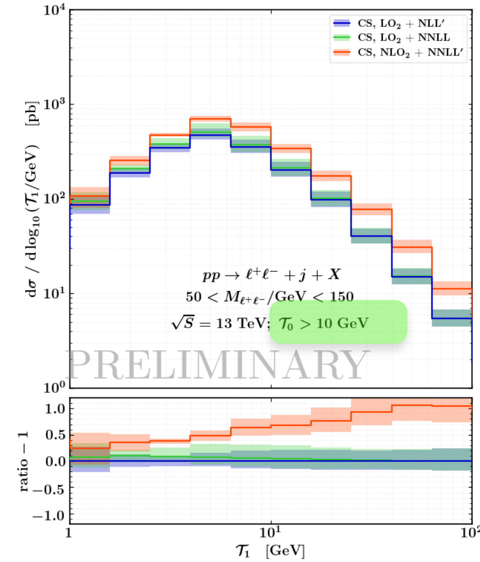


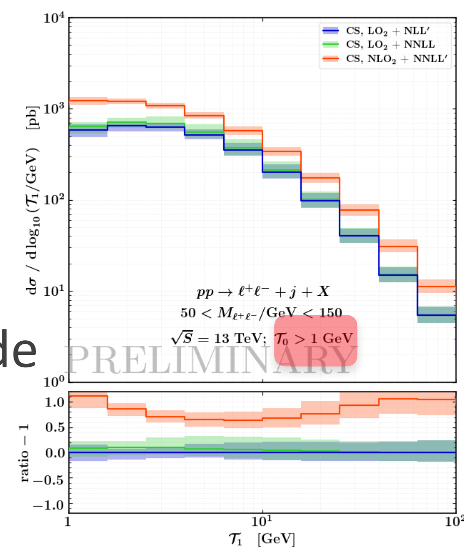
Matched results

$$\frac{d\sigma^{\text{match.}}}{d\Phi_1 d\mathcal{T}_1} = \frac{d\sigma^{\text{res.}}}{d\Phi_1 d\mathcal{T}_1} + \frac{d\sigma^{\text{f.o.}}}{d\Phi_1 d\mathcal{T}_1} - \frac{d\sigma^{\text{res.exp.}}}{d\Phi_1 d\mathcal{T}_1}$$







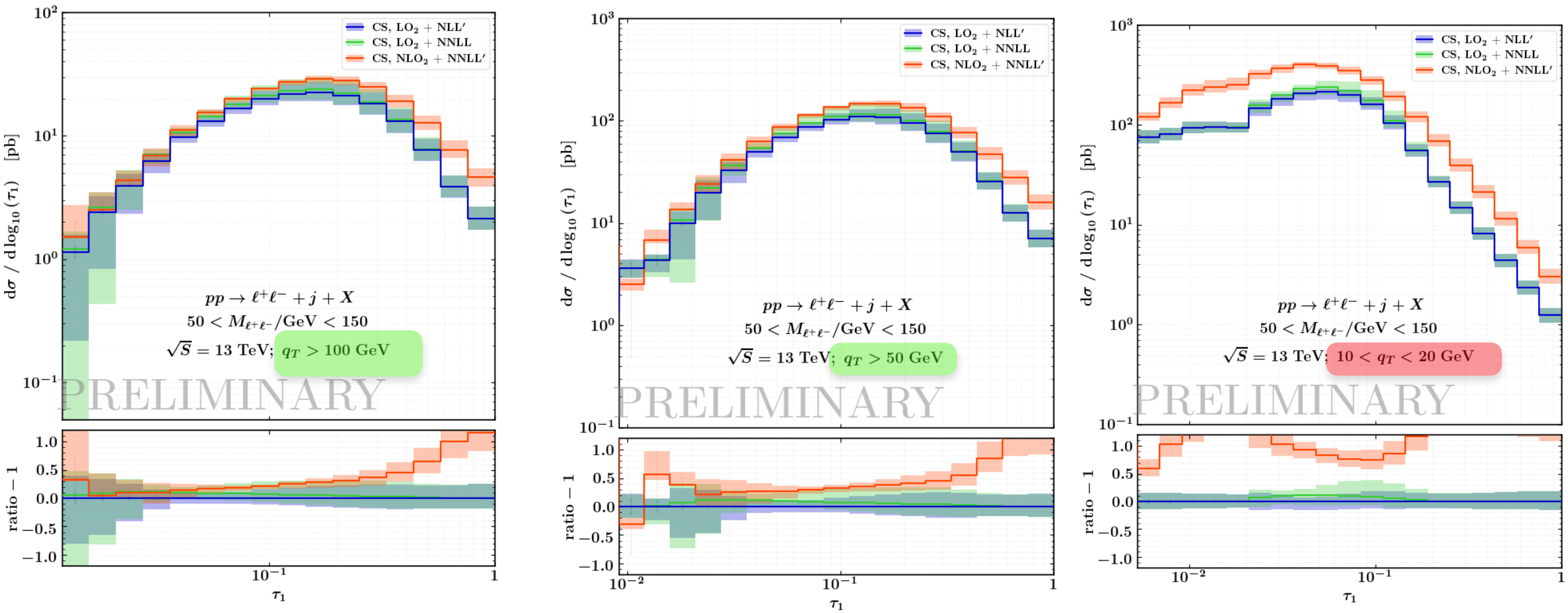


- $\mathcal{O}(\alpha_s^3)$ gives sizable contribution, important to include it for small values of $\mathcal{T}_0$
- Nonsingular divergent for $\mathcal{T}_0 \to 0$. Joint $(\mathcal{T}_0, \mathcal{T}_1)$ resummation required to handle both divergencies

Matched results

Dimensionless definition

$$\tau_1 = 2\mathcal{T}_1/\sqrt{M^2_{\ell^+\ell^-} + q_T^2}$$



- Similar issues with nonsingular behaviour when $q_T \to 0$

Conclusion and outlook

- The inclusion of state-of-the-art theoretical predictions in SMC generators is mandatory to match the experimental precision and fully exploit the discovery potential of LHC measurements
- GENEVA method allows for interfacing higher-order resummation of resolution variables in event generation with NNLO accuracy and parton showers.
- Several color-singlet processes implements, using different resolution variables: N-jettiness, qT, jet veto…
- First steps in extending the method to massive colored particles and jets presented.
- Implemented one-jettiness resummation, prerequisite for Vj@NNLO+PS in GENEVA. Studied different $\mathcal{T}_1$ definitions, performed resummation up to N3LL and matched to corresponding fixed-order. Observed nice convergence and reduction of theory unc. in presence of an hard jet.
- EW and QED corrections will also play an increasingly important role.

Thank you for your attention.

BACKUP

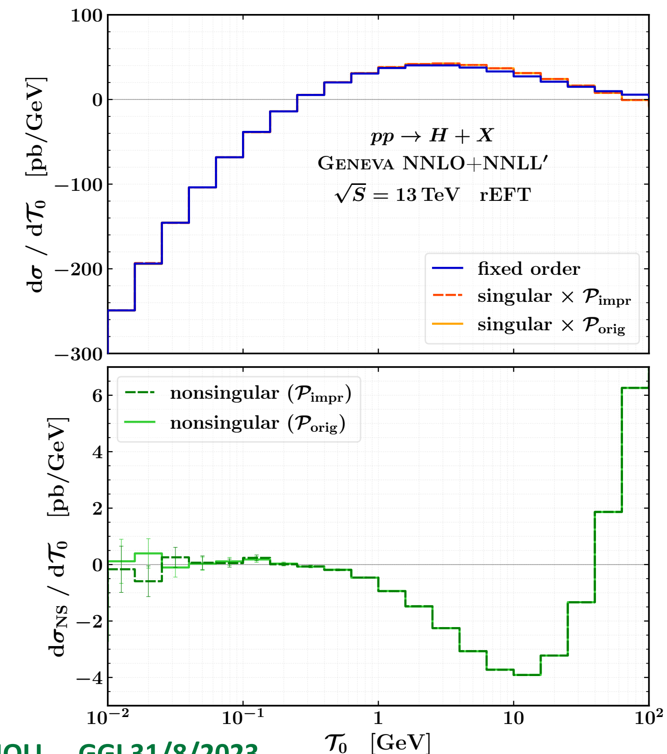
Spreading out the resummation to other variables

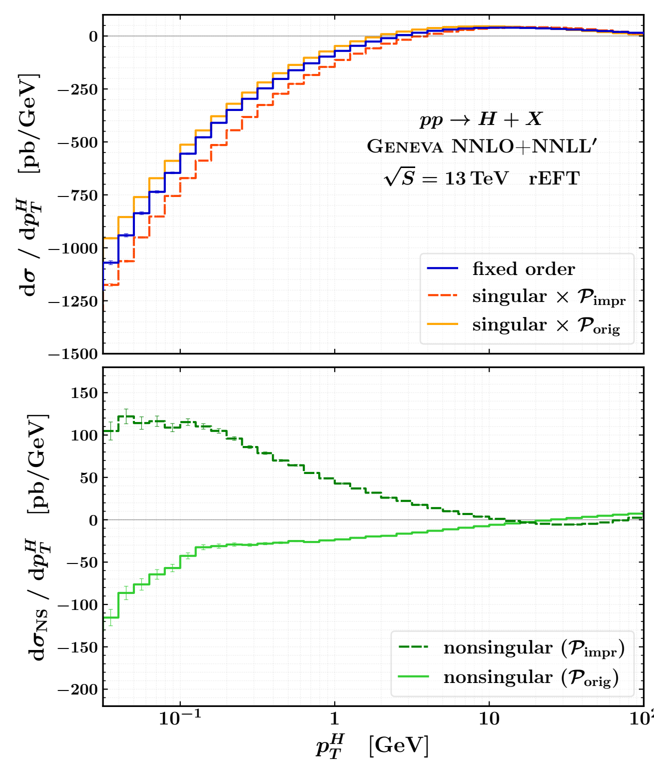
Splitting functions are required to make resummed spectrum fully-differential.

$$\frac{d\sigma}{d\Phi_N dr_N} P_{N\to N+1} \to \frac{d\sigma}{d\Phi_{N+1}}$$

New on-the-fly evaluation and better functional forms captures better the singular behavior of matrix elements also for different resolution variables.

$$P_{N\to N+1}(\Phi_{N+1}) = \frac{f_{kj}(\Phi_N, \mathcal{T}_N, z)}{\sum_{k'=1}^{N+2} \int_{z_{\min}^{k'}(\Phi_N,\mathcal{T}_N)}^{z_{\max}^{k'}(\Phi_N,\mathcal{T}_N)} dz'\, J_{k'}(\Phi_N, \mathcal{T}_N, z')\, I_{\phi}^{k'}(\Phi_N, \mathcal{T}_N, z') \sum_{j'=1}^{n_{\text{split}}^{k'}} f_{k'j'}(\Phi_N, \mathcal{T}_N, z')}$$





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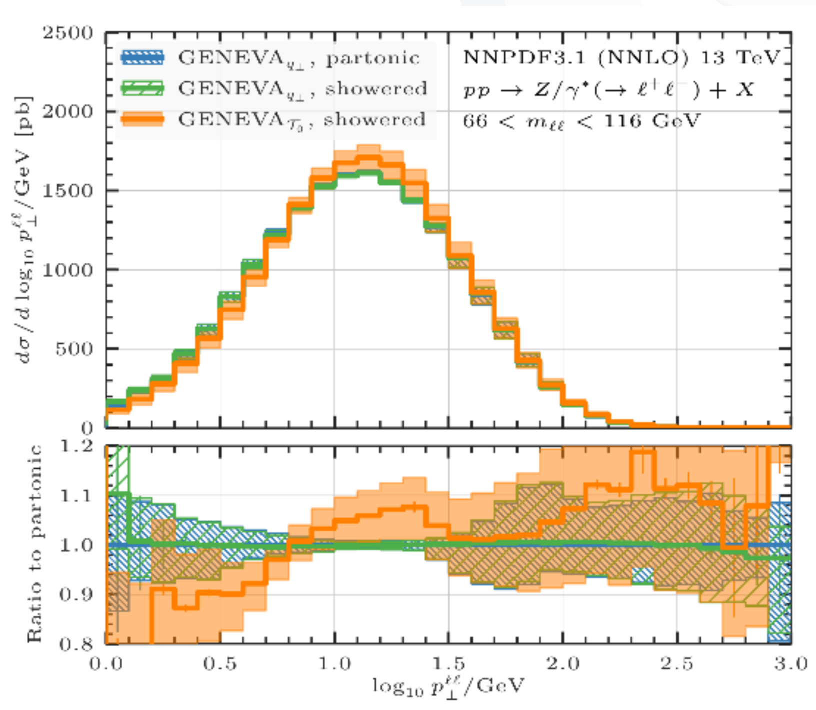
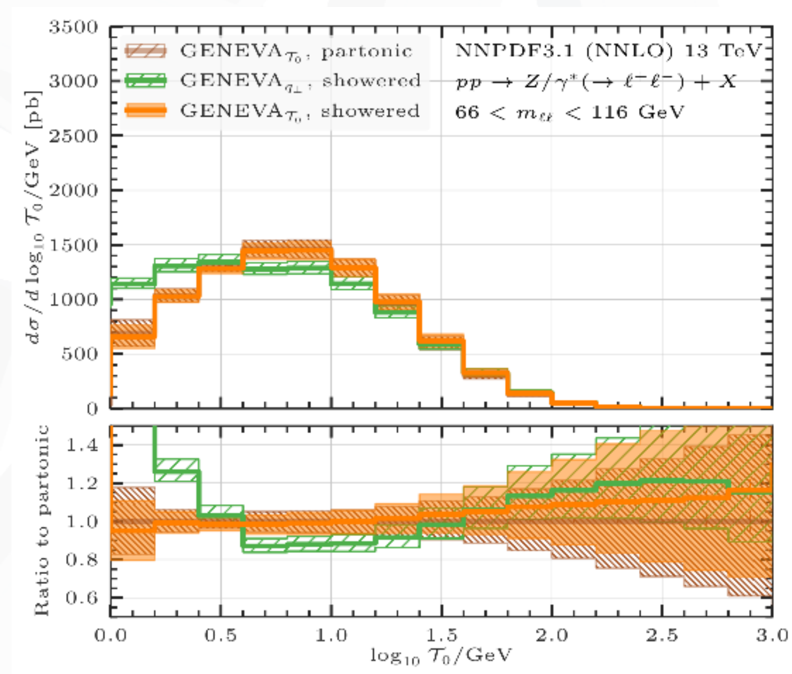
Interface with the parton shower

Effect of shower on resolution variables different from what is resummed more marked, albeit shower accuracy is maintained.
GENEVA framework allows this comparison for DY when resumming q_T or $\mathscr{T}_0$

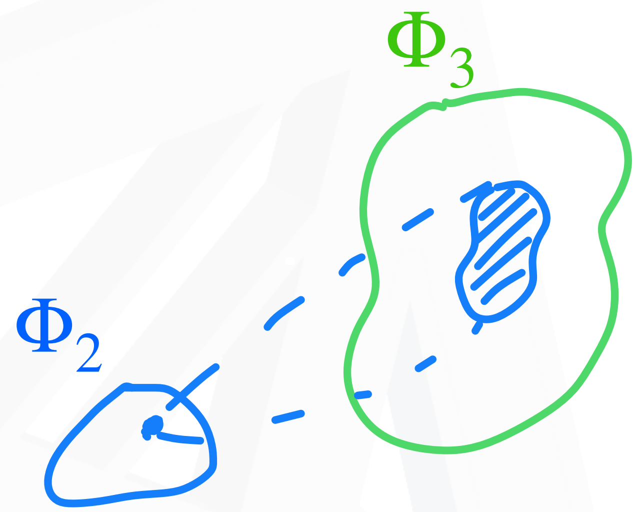
Best approach here would be joint $(\mathscr{T}_0, \vec{q}_T)$ resummation, avoids need of splitting func.





One-jettiness in GENEVA

- For the correct IR definition of an NNLO event weight $\frac{d\sigma^{\mathrm{MC}}}{d\Phi_1 d\mathcal{T}_1 dz d\varphi}$ one needs to preserve the resolution parameter when performing the $\Phi_2 \to \Phi_3$ splitting in the NLO_2 calculation, $\mathcal{T}_1$- preserving map required $\mathcal{T}_1(\Phi_2) = \mathcal{T}_1(\Phi_3)$



- Using a jet-algorithm to find the directions or using the exact $\mathcal{T}_1$ definition makes it impractical to find this map. Alternatively, use similar variable that has the same log structure and different $\alpha_s^2 \delta(\mathcal{T}_1)$
- We introduce a fully-recursive version of one-jettiness $\mathcal{T}_1^{\mathrm{FR}}$ which we use for the fixed-order calculation. The idea is that at each step one finds the closest particles in the one-jettiness metric, merge them and continue. N-jettiness as a clustering procedure.