Collinear Contributions to QCD Resummations for Hard-Scattering Observables

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based on

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in collaboration with Stefano Catani

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Outline

- Introduction & Motivation

    Fixed Order Contribution Vs Resummed Contribution

    Transverse Momentum Resummation

- Computation of Collinear Contributions

- Perturbative Results up to NNLO

- Summary

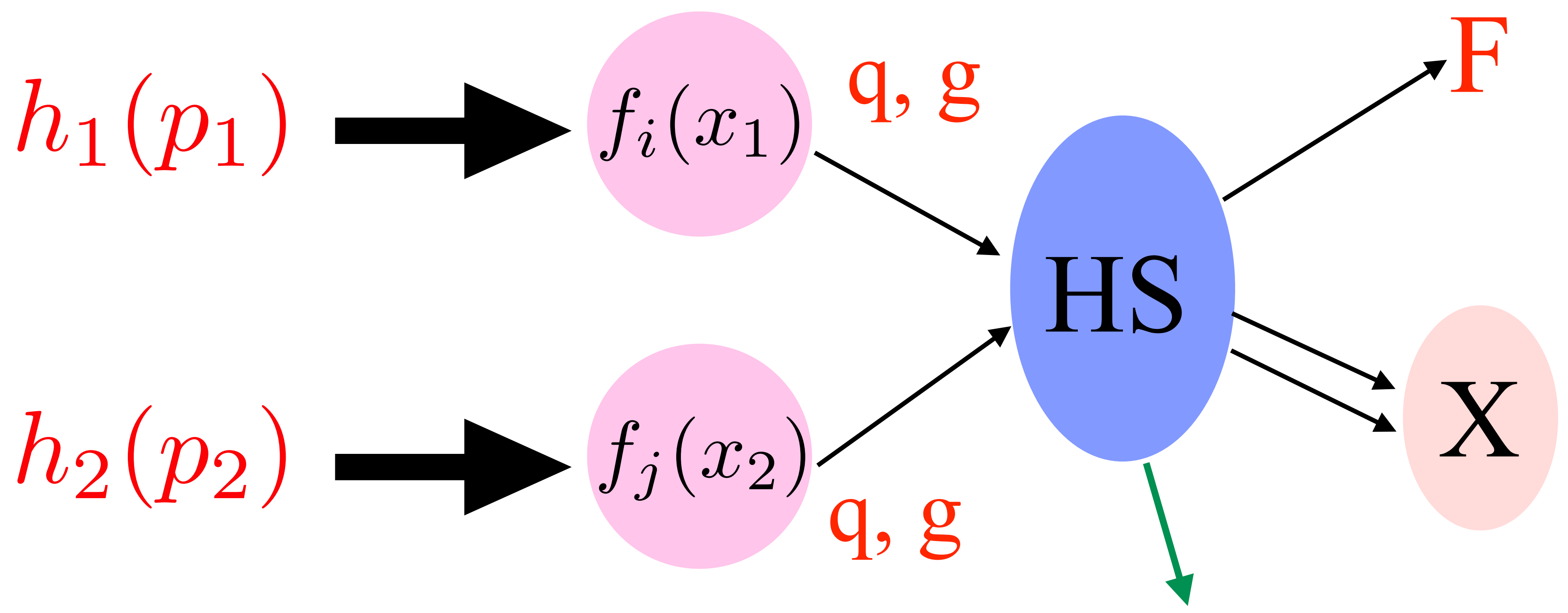
Parton Model: Short Range Factorizes from the Long Range

* For a generic hadro production process

Additional radiation

$$h_1(p_1) + h_2(p_2) \to F(\{q_i\}) + X$$

$$c_i(x_1p_1) + c_j(x_2p_2) \to F(q \equiv \sum_i q_i) \Big| \; q_\mu q^\mu = Q^2 = M^2$$



x_i : momentum fraction

Partonic cross section: $\hat{\sigma}_{ij} = \hat{\sigma}_{ij}^{(0)} + \alpha_S \hat{\sigma}_{ij}^{(1)} + \ldots$

non-perturbative

Long distance

$$\boxed{\sigma_{h_1,h_2} = \sum_{i,j} \int dx_1 dx_2 f_i(x_1) f_j(x_2) \hat{\sigma}_{ij} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)}$$

Short distance

PDFs

perturbative

Differential Cross Section: Analytic Structure

- Theoretical separation

$$\frac{d\sigma_F}{dq_T} = \frac{d\sigma_F^{\text{sing.}}}{dq_T}(\equiv [d\sigma_F]) + \frac{d\sigma_F^{\text{reg.}}}{dq_T}$$

Spoils the reliability of FO predictions

$$\left\{ \left[\frac{1}{q_T^2} \ln^n \left(\frac{M^2}{q_T^2} \right) \right]_+ , \delta(q_T^2) \right\}$$

Process independent w.r.t. the system F

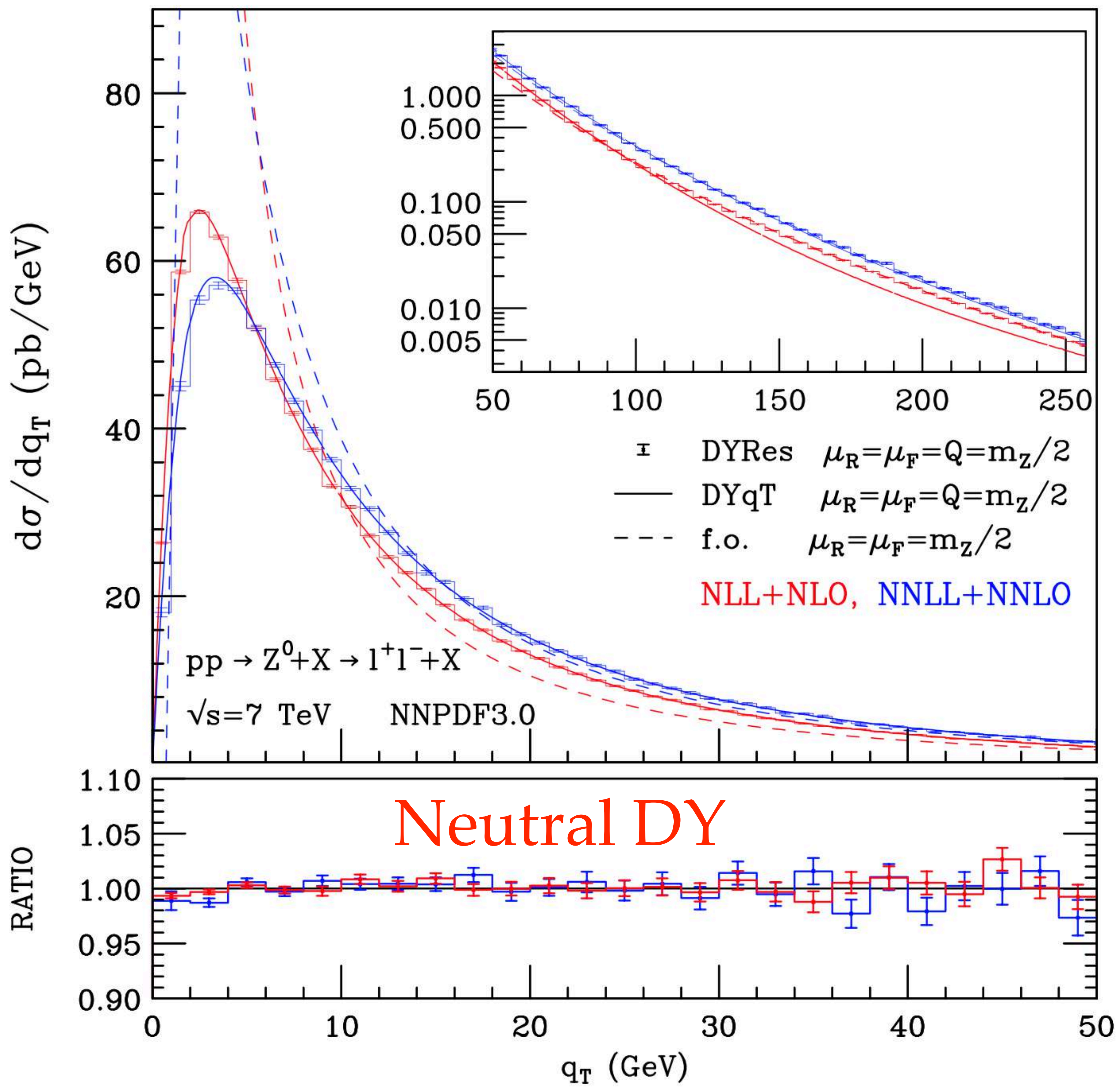
$$\lim_{Q_0 \to 0} \int_0^{Q_0} dq_T \frac{d\sigma_F^{\text{reg.}}}{dq_T} = 0$$

Process dependent

- Good news: singular contributions are of infrared (soft and collinear) origin. Hence, they have some degree of universality that leads to Resummation of these terms.

$$[g(\mathbf{q_T})]_+ = \lim_{q_0 \to 0} \left[\theta(q_T - q_0) g(\mathbf{q_T}) - \delta^{(2)}(\mathbf{q_T}) \int d^2\mathbf{k_T} g(\mathbf{k_T}) \theta(\mu_0 - k_T) \theta(k_T - q_0) \right]$$

Why Resummation? Fixed Order Vs Resummed Predictions



Catani, de Florian, Ferrera, Grazzini (1507.06937)

- For a relative order 'n' compared to Born

$$\alpha_s^n \boxed{\frac{1}{q_T^2}\ln^{2n-1}\left(\frac{M^2}{q_T^2}\right), \frac{1}{q_T^2}\ln^{2n-2}\left(\frac{M^2}{q_T^2}\right), \ldots, \frac{1}{q_T^2}\ln\left(\frac{M^2}{q_T^2}\right), \frac{1}{q_T^2}}$$

Impact parameter space conjugate to q_T

$$\boxed{\ln^{2n}\left(\frac{M^2b^2}{b_0^2}\right)}$$

LL: $\boxed{\alpha_s^n \ln^{n+1}\left(\frac{M^2b^2}{b_0^2}\right)}$

NLL: $\boxed{\alpha_s^n \ln^{n}\left(\frac{M^2b^2}{b_0^2}\right)}$

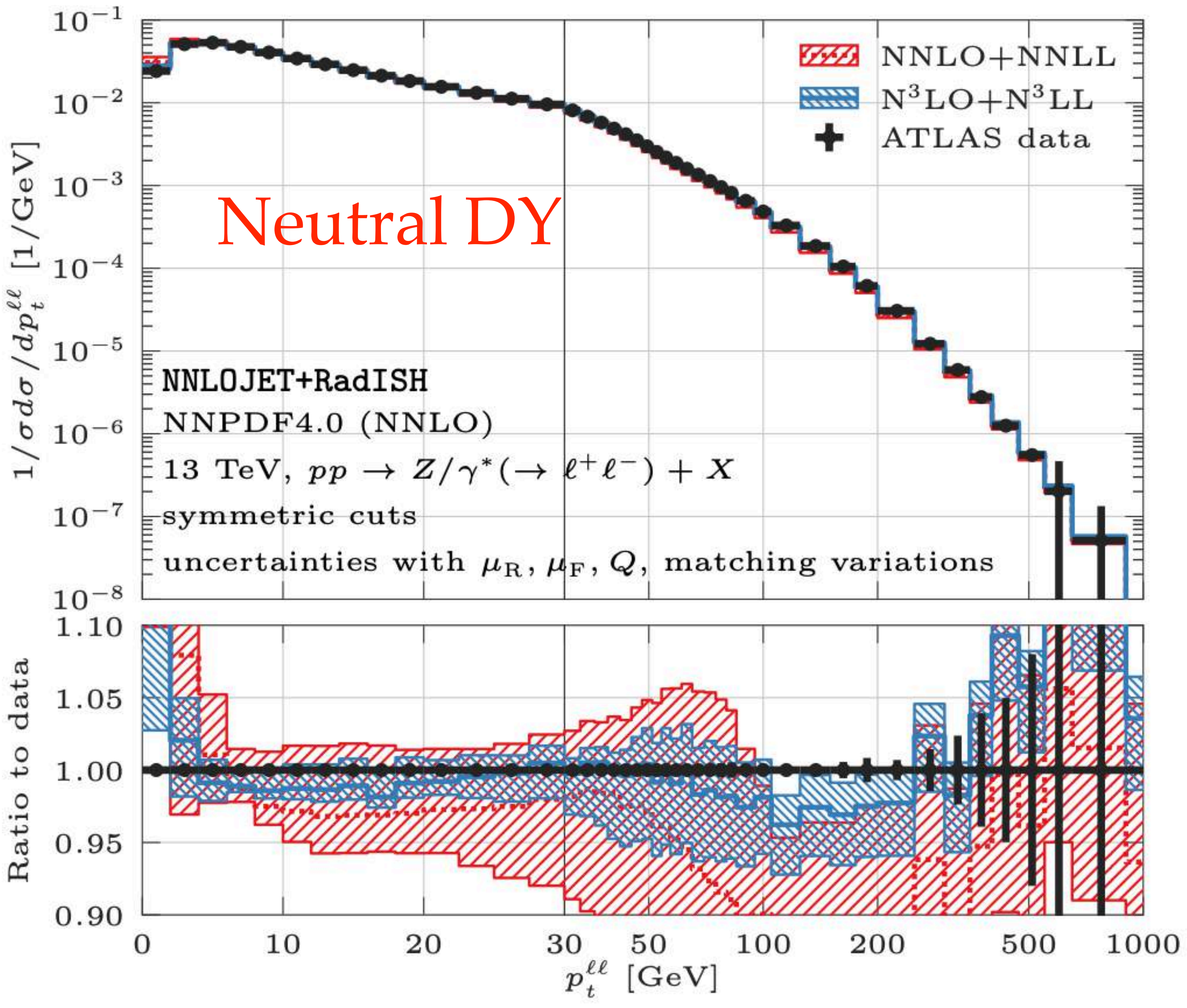
$b_0 = 2e^{-\gamma_E}$

NNLL: $\boxed{\alpha_s^n \ln^{n-1}\left(\frac{M^2b^2}{b_0^2}\right)}$

Next-to-next-to-leading logarithmic accuracy

- Low transverse momentum region: while the FO results are not reliable, the resummed results are smooth.
- High transverse momentum region: significant contributions are from the regular parts, making resummation ineffective.

Resummed Prediction Vs Data



Chen, Gehrmann, Glover, Huss, Monni, Rottoli, Re, Torrielli (2203.01565); Camarda, Cieri, Ferrera (2103.04974)

- Bulk of the events are produced in the low transverse momentum region.
- Theory is consistent with the data within the uncertainties.
- From NNLO+NNLL to N3LO+N3LL: reduction in uncertainties and improvement in data agreement.
- Improved theory predictions helps in precise determination of the mass of the W-boson and other crucial SM parameters.

$\boxed{\text{Alessandro Vicini's talk}}$

How are these large logarithms resummed ?

Transverse Momentum Resummation

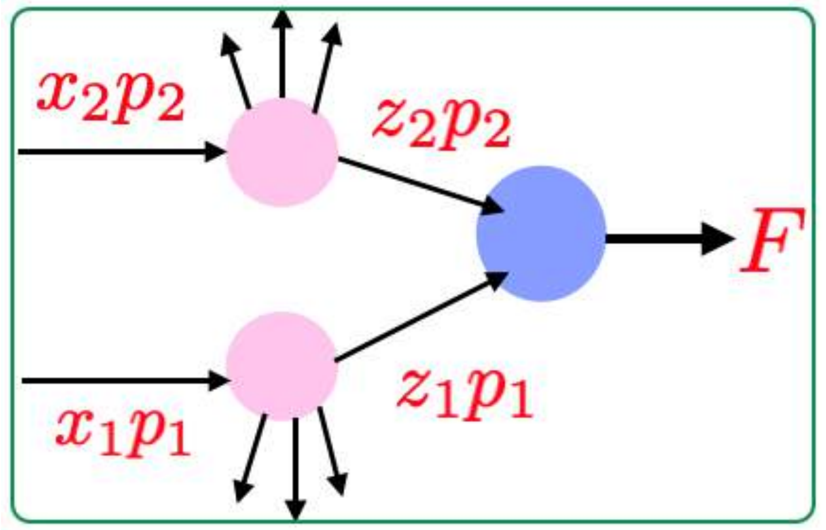
* Using the formalism of qT-resummation [1], singular part of the differential cross section in b-space has the following structure

Born contribution

Sudakov soft contribution

$$[d\sigma_F] = \frac{M^2}{s} \sum_{c=q,\bar{q},g} \left[d\sigma^{(0)}_{c\bar{c},F}\right] \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}.\mathbf{q_T}} S_c(M,b)$$
$$\times \sum_{a_1,a_2} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} \boxed{\left[H^F C_1 C_2\right]_{c\bar{c};a_1 a_2}} f_{a_1/h_1}\left(x_1/z_1, b_0^2/b^2\right) f_{a_2/h_2}\left(x_2/z_2, b_0^2/b^2\right)$$

Hard collinear factor → This Talk!



* Other equivalent formulations do exist in the literature that are based either on TMD factorisation or on SCET methods [2]

[1] Collins, Soper, Sterman (1985); Catani, Cieri, de Florian, Ferrera, Grazzini (1311.1654)

[2] J Collins, Foundations of perturbative QCD; Becher, Neubert (1007.4005); Echevarria, Idilbi, Scimemi (1111.4996); Chiu, Jain, Neill, Rothstein (1202.0814)

Hard Collinear Factor: Structure

- For the gluon gluon fusion channel

Hard functions

Specify the system F

$$\left[H^F C_1 C_2\right]_{gg;a_1 a_2} = H^F_{\mu_1\nu_1,\mu_2\nu_2}\left(x_1 p_1, x_2 p_2; \Omega; \alpha_s(M^2)\right)$$
$$\times C^{\mu_1\nu_1}_{ga_1}\left(z_1; p_1, p_2, \mathbf{b}; \alpha_s(b_0^2/b^2)\right) C^{\mu_2\nu_2}_{ga_2}\left(z_2; p_1, p_2, \mathbf{b}; \alpha_s(b_0^2/b^2)\right)$$

Collinear functions

where [1]

Azimuthal dependent part

$$C^{\mu\nu}_{ga}\left(z; p_1, p_2, \mathbf{b}; \alpha_s\right) = d^{\mu\nu}(p_1, p_2) C_{ga}(z, \alpha_s) + D^{\mu\nu}(p_1, p_2; \mathbf{b}) G_{ga}(z; \alpha_s)$$

$$d^{\mu\nu}(p_1, p_2) = -g^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1 . p_2}$$

Azimuthal independent part

$$D^{\mu\nu}(p_1, p_2; \mathbf{b}) = d^{\mu\nu}(p_1, p_2) - 2\frac{b^\mu b^\nu}{\mathbf{b}^2}$$

$$b^\mu = (0, \mathbf{b}, 0)$$

- For the quark anti-quark annihilation channel, it has a relatively simple structure.

No tensor structure

[1] Catani, Grazzini (1011.3918)

Collinear Functions: Review from Literature

- Azimuthally independent collinear functions are recently known to N3LO in QCD coupling

$$C_{ab}(z;\alpha_S) = \delta_{ab}\delta(1-z) + \frac{\alpha_S}{\pi} C_{ab}^{(1)}(z) + \left(\frac{\alpha_S}{\pi}\right)^2 C_{ab}^{(2)}(z) + \left(\frac{\alpha_S}{\pi}\right)^3 C_{ab}^{(3)}(z) + \sum_{n=4}^{+\infty}\left(\frac{\alpha_S}{\pi}\right)^n C_{ab}^{(n)}(z)$$

[1] [2] [3]

- Azimuthally dependent collinear functions are recently known to NNLO in perturbation series

$$G_{ga}(z;\alpha_s) = \frac{\alpha_s}{\pi} G_{ga}^{(1)}(z) + \left(\frac{\alpha_s}{\pi}\right)^2 G_{ga}^{(2)}(z) + \sum_{n=3}^{+\infty}\left(\frac{\alpha_s}{\pi}\right) G_{ga}^{(n)}(z)$$

[4] [5]

- Similar functions do exist for the processes related by crossing such as SIDIS and production of hadrons from a pair of leptons and they are called Time-Like collinear functions or Fragmenting Jet functions.

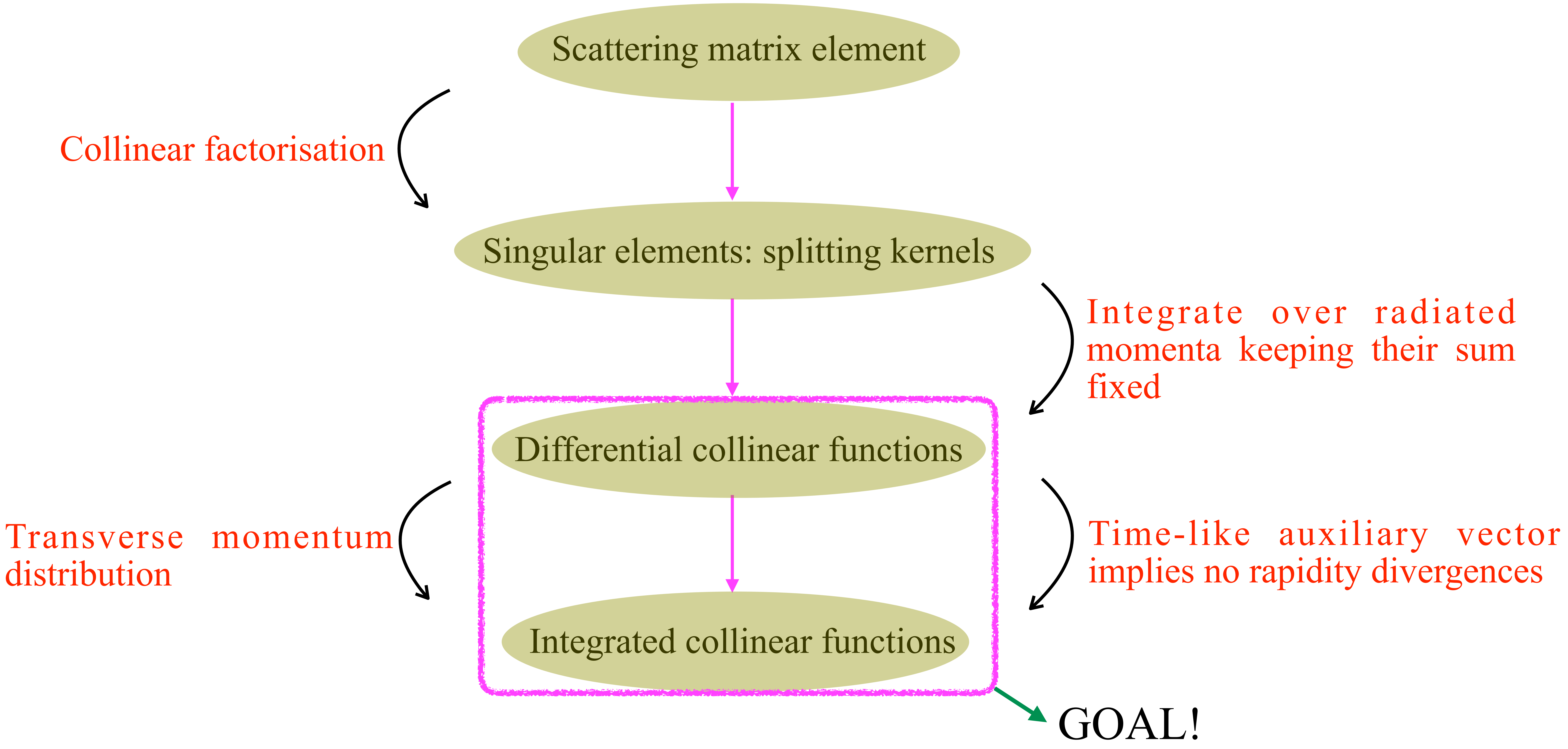
[1] de Florian, Grazzini (0108273); [4] Catani, Grazzini (1011.3918)

[2] Catani, Grazzini (1106.4652); Catani, Cieri, de Florian, Ferrera, Grazzini (1209.0158); Gehrmann, Lubbert, Yang (1209.0682, 1403.6451); Echevarria, Scimemi, Vladimirov (1604.07869); Luo, Wang, Xu, Yang, Yang, Zhu (1908.03831); Luo, Yang, Zhu, Zhu (1909.13820)

[3] Luo, Yang, Zhu, Zhu (1912.05778); Ebert, Mistlberger, Vita (2006.05329); Luo, Yang, Zhu, Zhu (2012.03256)

[5] Luo, Yang, Zhu, Zhu (1909.13820); Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov (1907.03780)

Our Novel Computational Method



Stefano Catani + PKD (2208.05840)

QCD Factorization in the Collinear Limit

* The collinear factorisation of hard scattering matrix element having N collinear partons in its most general form is given by

Auxiliary vector

Reduced ME

$$|\mathcal{M}(\{q_i\}; k_1, \ldots, k_N)|^2 = \langle \mathcal{M}(\{q_i\}; \tilde{k}) | \boxed{\boldsymbol{\mathcal{P}}(\{q_i\}; k_1, \ldots, k_N; n)} | \mathcal{M}(\{q_i\}; \tilde{k}) \rangle + \ldots$$

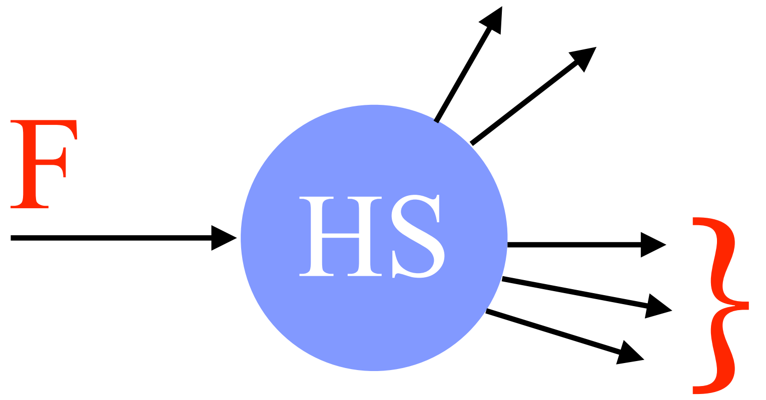
Dependence on non-collinear partons

Collinear limit of $\sum_{i=1}^{N} k_i$

Collinear splitting kernel

Non-singular terms

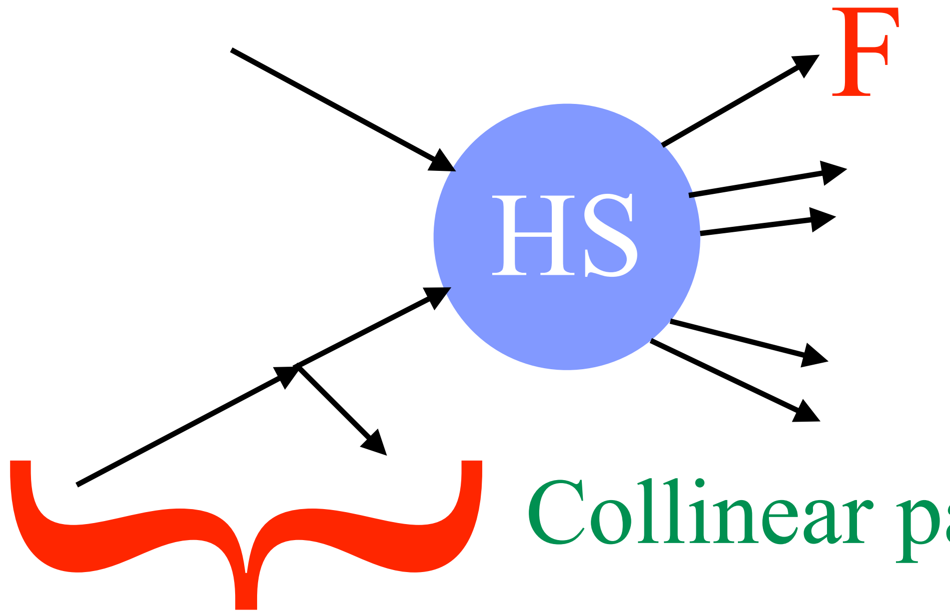
* The TL collinear region is defined by $\boxed{\{k_i^0\} > 0}$



Collinear partons

* The splitting kernel is process independent and this property of factorisation is called strict collinear factorisation.

* The SL collinear region is defined by $\boxed{k_i^0 < 0}$



Collinear partons

* Strict collinear factorisation is instead violated in SL collinear region [1].

[1] Catani, de Florian, Rodrigo (1112.4405)

Comments on the Use of Auxiliary Vector

- In addition to its dependence on spin and colour indices, the splitting kernel depends on scalar functions of collinear momenta of the form [1]

$$\boxed{s_{ij} = 2k_i k_j}$$

$$\boxed{\frac{x_i}{x_j} = \frac{nk_i}{nk_j}}$$

- In the literature, the splitting kernels are usually calculated using a light-like auxiliary vector. Indeed this choice is very convenient for direct computations of the splitting kernels. However, we emphasise that one can also set

$$\boxed{n^2 \neq 0}$$

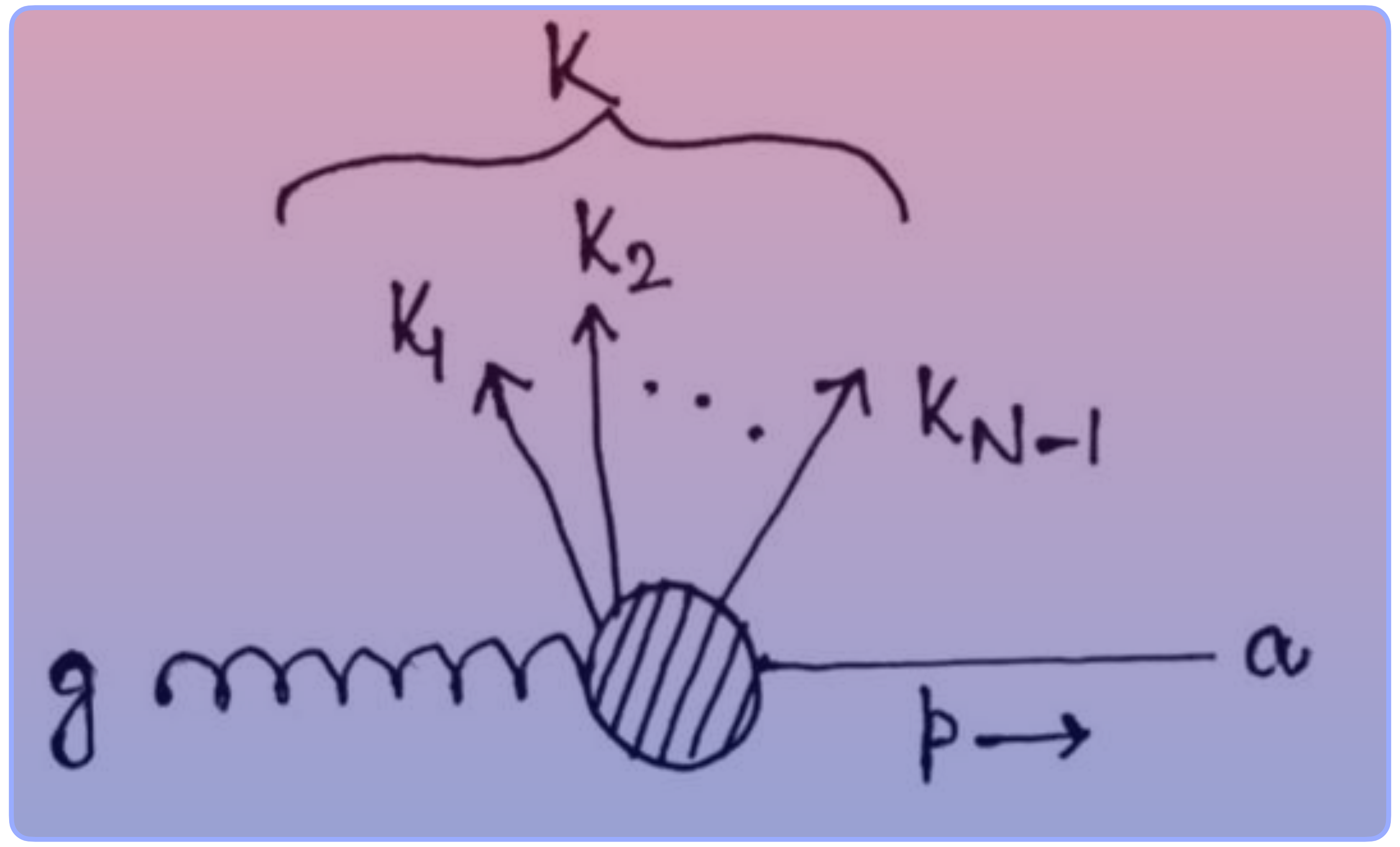
This work —Time-like $(n^2 > 0)$

- The change only affects the non-singular/power-suppressed terms in the collinear limit, and hence neglected.

[1] Catani, Grazzini (9908523)

Differential Collinear Functions

* We define the differential collinear functions for the gluon channel in the TL region as follows [1]



$$\mathcal{F}_{ga}^{\mathrm{TL}\,\mu\nu}(p,k;n) = \sum_{N=2}^{+\infty} \left[\prod_{m=1}^{N-1} \int \frac{d^d k_m}{(2\pi)^{d-1}} \delta_+ \left(k_m^2\right) \right] \delta^{(d)}\left(k - \sum_{i=1}^{N-1} k_i\right) \times \sum_{a_1,\dots,a_{N-1}} \left. \frac{\widetilde{\mathcal{P}}^{\mu\nu}_{g\to a_1\dots a_N}(k_1,\dots,k_N;n)}{\mathrm{SF}\left(a_1,\dots,a_{N-1}\right)} \right|_{\substack{k_N=p\\ a_N=a}}$$

Bose symmetry factor

* TL splitting kernels for various splitting processes are fully known to second [2] and third order [3] in QCD.

* SL Splitting kernels for various splitting processes are fully known to second order and partially known to third order [4] in the QCD strong coupling.

[1] Stefano Catani + PKD; [4] Catani, de Florian, Rodrigo, Forshaw, Seymour, Siodmok, Dixon, Herrmann, Yan, Zhu

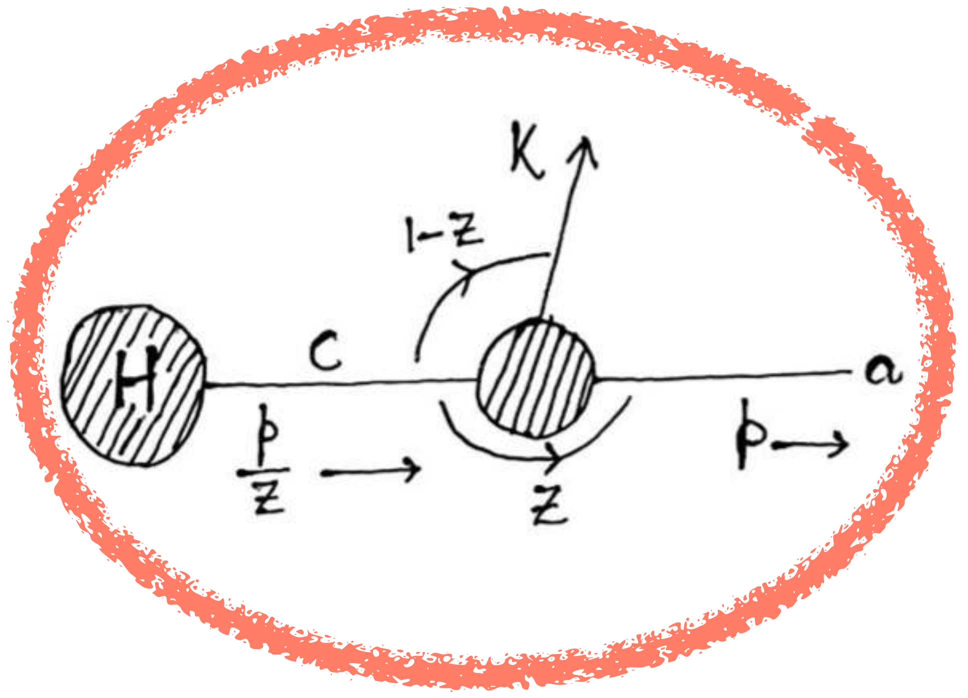
[2] Bern, Del Duca, Kilgore, Schmidt, Catani, Grazzini, Campbell, Glover, Kosower, Uwer, Sborlini, de Florian, Rodrigo

[3] Catani, de Florian, Rodrigo, Del Duca, Frizzo, Maltoni, Birthwright, Glover, Khoze, Marquard, Duhr, Haindl, Lazopoulos, Michel, Sborlini, Rodrigo, Badger, Buciuni, Peraro, Bern, Dixon, Kosower, Gehrmann, Jaquier, Czakon, Sapeta

Integrated Collinear Functions: Time-Like Region

* We define the transverse momentum dependent collinear functions for the gluon case as follows

$$F_{ga}^{\mathrm{TL}\,\mu\nu}(z;p/z,\mathbf{q_T};n) = \delta(1-z)\,\delta^{(d-2)}(\mathbf{q_T})\;\delta_{ga}\;d^{\mu\nu}(p;n) + \int d^dk\;\delta^{(d-2)}(\mathbf{k_T}+\mathbf{q_T})\;\delta\!\left(\frac{k^+}{p^+}-\frac{1-z}{z}\right)\;\mathcal{F}_{ga}^{\mathrm{TL}\,\mu\nu}(p,k;n)$$



$d^{d-2}\mathbf{k_T}\,dk^+\,dk^-$

Phase space in the collinear limit

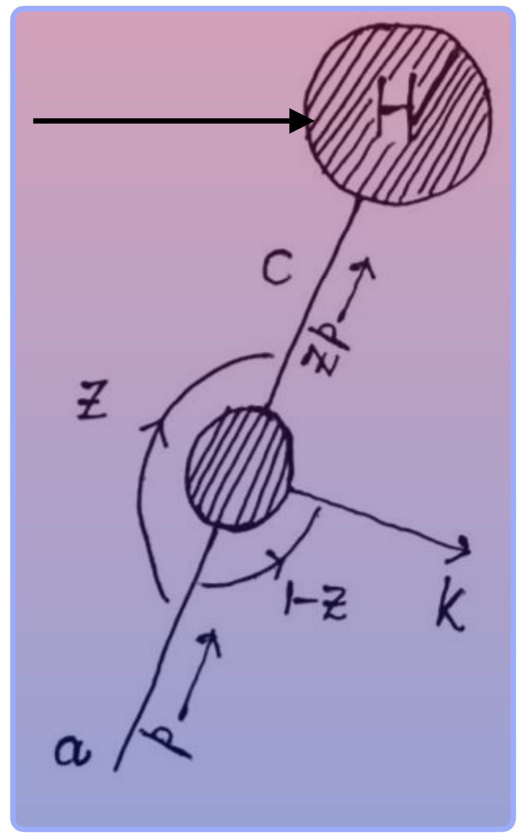
$$F_{ga}^{\mathrm{TL}\,\mu\nu}(z;p/z,\mathbf{q_T};n) = d^{\mu\nu}(p;n)\;F_{ga,\,\mathrm{az.in.}}^{\mathrm{TL}}\!\left(z;\mathbf{q_T}^2,\frac{n^2\mathbf{q_T}^2}{(2np/z)^2}\right) + D^{\mu\nu}(p,n;\mathbf{q_T},\epsilon)\;F_{ga,\,\mathrm{corr.}}^{\mathrm{TL}}\!\left(z;\mathbf{q_T}^2,\frac{n^2\mathbf{q_T}^2}{(2np/z)^2}\right)$$

* For the quark channel, we have only the azimuthal-independent type contributions.

Stefano Catani + PKD (2208.05840)

Integrated Collinear Functions: Space-Like Region

* We define the transverse momentum dependent collinear functions for both quark and gluon splitting as follows



$$\mathbf{F}_{ca}(\{q_i\}; z; zp, \mathbf{q_T}; n) = \mathbf{1}\, \delta(1-z)\, \delta^{(d-2)}(\mathbf{q_T})\, \delta_{ca} + z \int d^dk\, \delta^{(d-2)}(\mathbf{k_T} + \mathbf{q_T})\, \delta\left(\frac{k^+}{p^+} - 1 + z\right) \boldsymbol{\mathcal{F}}_{ca}(\{q_i\}; p, k; n)$$

$$d^{d-2}\mathbf{k_T}\, dk^+\, dk^-$$

Phase space in the collinear limit

* Note that until now our framework is quite general i.e. without restricting to any perturbative order.

* Up to NNLO, SL collinear functions are also process independent. Hence, like TL region the functions we will be dealing with are as follows

$$F^{\mu\nu}_{ga}(z; zp, \mathbf{q_T}; n)\ , \quad F_{ga,\,\text{az.in.}}\left(z; \mathbf{q_T}^2, \frac{n^2 \mathbf{q_T}^2}{(2zpn)^2}\right)\ , \quad F_{ga,\,\text{corr.}}\left(z; \mathbf{q_T}^2, \frac{n^2 \mathbf{q_T}^2}{(2zpn)^2}\right), \quad F_{ca}\left(z; \mathbf{q_T}^2, \frac{n^2 \mathbf{q_T}^2}{(2zpn)^2}\right)$$

Stefano Catani + PKD (2208.05840)

Other Applications: Zero Jettiness Beam Functions

* Our method of computing collinear functions can be extended and applied to other observables of interest.

* For example, we can define the zero-jettiness partonic beam functions for both quark and gluon splitting in the SL region as follows

$$\boxed{\begin{aligned}\boldsymbol{\mathcal{B}}_{ca}(\{q_i\};z;zp,t;n) &= \mathbf{1}\,\delta(1-z)\,\delta(t)\;\delta_{ca} \\ &\quad + z\int d^dk\,\delta(t-2zpk)\;\delta\!\left(\frac{k^+}{p^+}-1+z\right)\,\boldsymbol{\mathcal{F}}_{ca}(\{q_i\};p,k;n)\end{aligned}}$$

Transverse virtuality ~ Zero-jettiness

Stewart, Tackmann, Waalewijn (1002.2213); Berger, Marcantonini, Stewart, Tackmann, Waalewijn (1012.4480); Ritzmann, Waalewijn (1407.3272); Gaunt, Stahlhofen, Tackmann (1401.5478); Gaunt, Stahlhofen, Tackmann (1405.1044); Ebert, Mistlberger, Vita (2006.03056); Baranowski, Behring, Melnikov, Tancredi, Wever (2211.05722)

Comments on Collinear Functions in SCET

* There are related definitions of TMD collinear functions from SCET. These functions are defined in a process independent way using auxiliary Wilson line operators along light-like directions.

* For TL case at the partonic level, SCET functions are equivalent to our collinear functions by using a light-like auxiliary vector [1].

* Same equivalence holds true for the SL region is only up to second order in strong coupling. This is due to the fact that ours results are in general process dependent and this dependency goes away if we are within NNLO in perturbation theory.

[1] Ritzmann, Waalewijn (1407.3272)

TMD Functions & Rapidity Divergences

- Order-by-order perturbative computations of qT differential distributions at any values of qT can be carried out in exact form without encountering rapidity divergences and small-qT behaviour can be obtained by simply neglecting sub-dominant terms.

- Rapidity divergences are artefact of a priori approximations that are introduced in matrix element and qT dependent phase space to evaluate only the dominant terms of the cross section in the small qT-region.

Phase space in the collinear limit

ME in the collinear limit

$$\int d^d k \, \delta^{(d-2)}(\mathbf{k_T} + \mathbf{q_T}) \; \delta\left(\frac{k^+}{p^+} - \frac{1-z}{z}\right) \; \mathcal{F}_{ga}^{\mathrm{TL}\,\mu\nu}(p,k;n)$$

$d^{d-2}\mathbf{k_T} dk^+ dk^-$

Validity of collinear factorisation formula $k^- \lesssim \mathcal{O}(k^+)$ $\xrightarrow[\text{Simplification}]{\text{Phase space}}$ $0 < k^- < +\infty$ Only if the differential collinear function is well behaved in the limit $k^- \to \infty$

- For the computation of individual components such as soft functions, collinear functions etc. there are many rapidity regulators that exist in the literature and they are introduced at the integrated level.

- We avoid rapidity divergences in our computation by introducing a time-like auxiliary vector at the matrix element level.

No Rapidity Divergent Terms!

* Consider the singular term

$$\frac{1}{1-z_n} = \frac{np}{nk} = \frac{p^+}{k^+ + \frac{n^2}{2np}\frac{k^-}{n^-}p^+} = \frac{1}{1-z+\frac{n^2 \mathbf{q_T}^2}{(1-z)(2np)^2}} \xrightarrow{n^2=0} \boxed{\frac{1}{1-z}}$$

Divergent for $z \to 1$

* Using a time-like auxiliary vector

$$\frac{1}{1-z+\frac{\lambda}{1-z}} = \left(\frac{1-z}{(1-z)^2+\lambda}\right)_+ + \delta(1-z)\int_0^1 dz' \frac{1-z'}{(1-z')^2+\lambda}$$
$$= \frac{1}{2}\ln\left(\frac{1}{\lambda}\right)\delta(1-z) + \left(\frac{1}{1-z}\right)_+ + \mathcal{O}(\sqrt{\lambda})$$

$\lambda = \frac{n^2 \mathbf{q_T}^2}{(2np)^2}$

Sub-dominant in the limit $\mathbf{q_T} \to 0$

Stefano Catani + PKD (2208.05840)

Differential Collinear Functions

* SL differential collinear functions up to NNLO have the following perturbative expansion

$$\boxed{\mathcal{F}(p,k;n) = \mathcal{F}^{(1R)}(p,k;n) + \left[\, \mathcal{F}^{(2R)}(p,k;n) + \mathcal{F}^{(1R1V)}(p,k;n) \,\right] + \mathcal{O}(\alpha_S^3)}$$

Single real radiation

Double real radiation

One-loop real-virtual

* Azimuthally independent contributions @ $\mathcal{O}\left(\alpha_S\right)$

$$S_\epsilon = (4\pi\, e^{-\gamma_E})^\epsilon \qquad z_n = \frac{n(p-k)}{np}$$

$$\mathcal{F}^{(1R)}_{ca,\,\text{az.in.}}(p,k;n) = \frac{\alpha_S^u\,\mu_0^{2\epsilon}\,S_\epsilon}{\pi}\,\frac{e^{\epsilon\gamma_E}}{\pi^{1-\epsilon}}\,\frac{\delta_+(k^2)}{pk}\,\frac{1}{z_n}\,\widehat{P}_{ca}(z_n;\epsilon)$$

Real contributions to LO AP splitting functions

* Azimuthally dependent contributions @ $\mathcal{O}\left(\alpha_S\right)$

$$\boxed{\begin{aligned}
\mathcal{F}^{(1R)}_{gg,\,\text{corr.}}(p,k;n) &= -\,\frac{\alpha_S^u\,\mu_0^{2\epsilon}\,S_\epsilon}{\pi}\,\frac{e^{\epsilon\gamma_E}}{\pi^{1-\epsilon}}\,\frac{\delta_+(k^2)\,C_A}{pk}\,\frac{1-z_n}{z_n^2}\,, \\
\mathcal{F}^{(1R)}_{ga,\,\text{corr.}}(p,k;n) &= -\,\frac{\alpha_S^u\,\mu_0^{2\epsilon}\,S_\epsilon}{\pi}\,\frac{e^{\epsilon\gamma_E}}{\pi^{1-\epsilon}}\,\frac{\delta_+(k^2)\,C_F}{pk}\,\frac{1-z_n}{z_n^2}\,, \qquad a = q,\bar{q}
\end{aligned}}$$

Collinear Functions in Momentum Space

* At the bare level, azimuthally independent collinear functions at NLO are obtained as follows

$$F^{(1R)}_{ca,\,\mathrm{az.in.}}\left(z;\mathbf{q_T}^2,\frac{n^2\mathbf{q_T}^2}{(2zpn)^2}\right)=\frac{\alpha_S^u\,\mu_0^{2\epsilon}\,S_\epsilon}{\pi}\,\frac{e^{\epsilon\gamma_E}}{\pi^{1-\epsilon}\,\mathbf{q_T}^2}\left\{\widehat{P}^{\,\mathrm{reg}}_{ca}(z;\epsilon)+\delta_{ca}\,A_c^{(1)}\left[\left(\frac{1}{1-z}\right)_+-\frac{1}{2}\ln\left(\frac{n^2\mathbf{q_T}^2}{(2zpn)^2}\right)\delta(1-z)\right]\right\}$$

$$\widehat{P}_{ca}(x;\epsilon)=\widehat{P}^{\,\mathrm{sing}}_{ca}(x)+\widehat{P}^{\,\mathrm{reg}}_{ca}(x;\epsilon)\quad\Big|\quad\widehat{P}^{\,\mathrm{sing}}_{ca}(x)=\delta_{ca}\,\frac{A_c^{(1)}}{1-x}$$

* Azimuthally correlated collinear functions at $\mathcal{O}(\alpha_S)$ are obtained as follows

$$F^{(1R)}_{ga,\,\mathrm{corr.}}\left(z;\mathbf{q_T}^2,\frac{n^2\mathbf{q_T}^2}{(2zpn)^2}\right)=-\frac{\alpha_S^u\,\mu_0^{2\epsilon}\,S_\epsilon}{\pi}\,\frac{e^{\epsilon\gamma_E}}{\pi^{1-\epsilon}\,\mathbf{q_T}^2}\,C_a\,\frac{1-z}{z}$$

* Formulae for the Fourier transformation from momentum space to conjugate impact parameter space

$$\int d^{d-2}\mathbf{q_T}\,e^{-i\mathbf{b}.\mathbf{q_T}}\,\frac{\ln^m(\mathbf{q_T}^2)}{(\mathbf{q_T}^2)^{1+\delta}}\xrightarrow{d=4-2\epsilon}\frac{d^m}{d\rho^m}\bigg|_{\rho=0}\pi^{1-\epsilon}\left(\frac{\mathbf{b}^2}{4}\right)^{\epsilon+\delta-\rho}\frac{\Gamma(\rho-\epsilon-\delta)}{\Gamma(1+\delta-\rho)}\,,$$

$$\int d^{d-2}\mathbf{q_T}\,e^{-i\mathbf{b}.\mathbf{q_T}}\,\frac{\ln^m(\mathbf{q_T}^2)}{(\mathbf{q_T}^2)^{1+\delta}}D^{\mu\nu}(p,n;\mathbf{q_T})\xrightarrow{d=4-2\epsilon}-\frac{d^m}{d\rho^m}\bigg|_{\rho=0}\pi^{1-\epsilon}\left(\frac{\mathbf{b}^2}{4}\right)^{\epsilon+\delta-\rho}\frac{\Gamma(1+\rho-\epsilon-\delta)}{\Gamma(2+\delta-\rho)}D^{\mu\nu}(p,n;\mathbf{b})\,.$$

Factorisation of IR Divergences

- Infrared factorisation in b-space

$$\widetilde{F}_{ca,\mathrm{az.in.}}\left(z;\frac{\mathbf{b}^2}{b_0^2},\frac{n^2b_0^2}{(2zpn)^2\,\mathbf{b}^2}\right)=Z_c\left(\alpha_S(b_0^2/\mathbf{b}^2),\frac{n^2b_0^2}{(2zpn)^2\,\mathbf{b}^2}\right)\sum_b\int_z^1\frac{dx}{x}\widetilde{C}_{cb}\left(x;\alpha_S(b_0^2/\mathbf{b}^2),\epsilon,\frac{n^2b_0^2}{(2zpn)^2\,\mathbf{b}^2}\right)\widetilde{\Gamma}_{ba}(z/x;b_0^2/\mathbf{b}^2)$$

$$\widetilde{F}_{ga,\mathrm{corr.}}\left(z;\frac{\mathbf{b}^2}{b_0^2},\frac{n^2b_0^2}{(2zpn)^2\,\mathbf{b}^2}\right)=Z_g\left(\alpha_S(b_0^2/\mathbf{b}^2),\frac{n^2b_0^2}{(2zpn)^2\,\mathbf{b}^2}\right)\sum_b\int_z^1\frac{dx}{x}\widetilde{G}_{gb}\left(x;\alpha_S(b_0^2/\mathbf{b}^2),\epsilon,\frac{n^2b_0^2}{(2zpn)^2\,\mathbf{b}^2}\right)\widetilde{\Gamma}_{ba}(z/x;b_0^2/\mathbf{b}^2)$$

$$\rightarrow\ \widetilde{C}_{cb}\left(z;\alpha_S,\epsilon=0,\frac{n^2b_0^2}{(2zpn)^2\mathbf{b}^2}\right)=C_{cb}(z;\alpha_S)\,,\ \widetilde{G}_{gb}\left(z;\alpha_S,\epsilon=0,\frac{n^2b_0^2}{(2zpn)^2\mathbf{b}^2}\right)=G_{gb}(z;\alpha_S)$$

$$\rightarrow\ \widetilde{\Gamma}_{ba}(z;\mu_F^2)=\delta_{ba}\,\delta(1-z)-\frac{\alpha_S(\mu_F^2)}{\pi}\,\frac{P_{ba}^{(1)}(z)}{\epsilon}+\mathcal{O}(\alpha_S^2)$$

- Infrared factorisation factors for both gluon and quark are as follows

$$Z_g=1+\frac{\alpha_S}{\pi}\left[\frac{C_A}{2}\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\left(\frac{n^2b_0^2}{(2znp)^2b^2}\right)\right)+\frac{\beta_0}{\epsilon}-C_A\frac{\pi^2}{24}\right]+\mathcal{O}(\alpha_S^2)$$

$$Z_q=1+\frac{\alpha_S}{\pi}\left[\frac{C_F}{2}\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\left(\frac{n^2b_0^2}{(2znp)^2b^2}\right)\right)+\frac{3}{4}\frac{C_F}{\epsilon}-C_F\frac{\pi^2}{24}\right]+\mathcal{O}(\alpha_S^2)$$

Perturbative Results at NLO

* Collinear functions in the Space-Like Region [1]

$$C_{gq}^{(1)}(z) = \frac{1}{2} C_F z, \qquad G_{gq}^{(1)}(z) = C_F \frac{1-z}{z}$$
$$C_{qg}^{(1)}(z) = T_R z(1-z)\,, \qquad G_{gg}^{(1)}(z) = C_A \frac{1-z}{z}$$
$$C_{qq}^{(1)}(z) = \frac{1}{2} C_F (1-z)\,,$$

* Collinear functions in the Time-Like Region [2]

$$C_{ca}^{\mathrm{TL}\,(1)}(z) = -\widehat{P}'_{ac}(z;\epsilon=0) + 2P_{ac}^{(1)}(z)\,\ln z$$

$$G_{gg}^{\mathrm{TL}\,(1)}(z) = C_A\, z(1-z)\,,\ G_{gq}^{\mathrm{TL}\,(1)}(z) = -T_R\, z(1-z)$$

* Our results are in full agreement with those in the literature.

[1] de Florian, Grazzini (0108273); Catani, Grazzini (1106.4652)

[2] Luo et al. (1908.03831,1909.13820); Echevarria et al. (1604. 07869); Nadolsky et al. (9906280)

Perturbative Results at NNLO: Space-Like Region

* Azimuthally correlated collinear functions are obtained as follows

$$G_{gg}^{(2)}(z) = C_A^2\left\{-\frac{37}{36z}+\frac{31}{18}-\frac{13z}{12}+\frac{11z^2}{36}-\ln(z)\left[\frac{1}{z}+\frac{19}{12}\right]+\frac{\ln^2(z)}{2}+\frac{1-z}{z}\left[\mathrm{Li}_2(z)-\frac{\pi^2}{6}\right]\right\}$$
$$+C_F N_f\left\{\frac{(1-z)^3}{2z}-\frac{1}{4}\ln^2(z)\right\}+C_A N_f\left\{-\frac{17}{36z}+\frac{4}{9}+\frac{z}{12}+\frac{z^2}{36}-\frac{1}{6}\ln(z)\right\}$$

$$G_{gq}^{(2)}(z) = C_F^2\left\{-\frac{1-z}{2}+\frac{5}{4}\ln(z)-\frac{1}{4}\ln^2(z)-\frac{1-z}{2z}\left[\ln(1-z)+\ln^2(1-z)\right]\right\}$$
$$+C_F N_f\left\{-\frac{1-z}{3z}\left[\frac{2}{3}+\ln(1-z)\right]\right\}+C_A C_F\left\{-\frac{11}{18z}+\frac{10}{9}-\frac{z}{2}-\ln(z)\left[\frac{1}{z}+\frac{5}{2}\right]\right.$$
$$\left.+\frac{1}{2}\ln^2(z)+\frac{1-z}{z}\left[\frac{5}{6}\ln(1-z)+\frac{1}{2}\ln^2(1-z)+\mathrm{Li}_2(z)-\frac{\pi^2}{6}\right]\right\}$$

* Our results are in full agreement with those in the literature [1].

[1] Luo, Yang, Zhu, Zhu (1909.13820); Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov (1907.03780)

Perturbative Results at NNLO: Time-Like Region

* Azimuthally correlated collinear functions are obtained as follows

$$G_{gg}^{\mathrm{TL}(2)}(z)=C_F N_f\left\{\frac{1}{18z}+\frac{1}{2}+z-\frac{14z^2}{9}+\ln(z)\left[-\frac{1}{3z}+1+\frac{3z}{2}\right]+\frac{3z}{4}\ln^2(z)\right\}$$
$$+C_A N_f\left\{-\frac{1}{36z}-\frac{1}{12}-\frac{4z}{9}+\frac{17z^2}{36}-\frac{z}{6}\ln(z)\right\}+C_A^2\left\{-\frac{1}{36z}-\frac{5}{12}-\frac{20z}{9}+\frac{11z^2}{4}\right.$$
$$\left.+\ln(z)\left[\frac{1}{3z}-1-\frac{67z}{12}+z^2\right]+z(1-z)\left[\ln(z)\ln(1-z)-\mathrm{Li}_2(1-z)\right]-\ln^2(z)\left[3z-\frac{z^2}{2}\right]\right\}$$

$$G_{gq}^{\mathrm{TL}(2)}(z)=C_F\left\{-\frac{1}{8}+\frac{3z}{4}-\frac{5z^2}{8}-\ln(z)\left[\frac{1}{4}+\frac{3z}{8}-\frac{z^2}{4}\right]+\ln^2(z)\left[\frac{3z}{8}-\frac{3z^2}{4}\right]\right.$$
$$\left.+z(1-z)\left[\frac{1}{4}\ln(1-z)-\frac{3}{2}\ln(z)\ln(1-z)+\frac{1}{4}\ln^2(1-z)-\mathrm{Li}_2(z)-\frac{\pi^2}{12}\right]\right\}$$
$$+N_f\left\{z(1-z)\left[\frac{1}{9}-\frac{1}{6}\ln(z)+\frac{1}{6}\ln(1-z)\right]\right\}+C_A\left\{\ln(z)\left[\frac{1}{4}+\frac{13z}{6}-\frac{17z^2}{12}\right]\right.$$
$$\left.+\ln^2(z)\left[\frac{3z}{4}+\frac{z^2}{2}\right]+z(1-z)\left[-\frac{25}{36}-\frac{5}{12}\ln(1-z)-\frac{1}{2}\mathrm{Li}_2(1-z)-\frac{1}{4}\ln^2(1-z)+\frac{\pi^2}{4}\right]\right\}$$

* Our results are in full agreement with those in the literature [1].

[1] Luo, Yang, Zhu, Zhu (1909.13820)

Summary

* Fixed order predictions can be plagued with large logarithmic corrections stemming from soft and collinear regions of the phase space.

* For better prediction, one needs to resum these large logarithms systematically to all orders in the perturbation theory.

* Large logarithmic contributions to an observable can be obtained from the singular elements associated with the QCD factorisation of scattering matrix elements in soft and collinear limits.

* Singular terms associated with the QCD factorisation and the corresponding integrated functions are important elements for various algorithms which aim to compute fully differential cross sections for various observables.

Summary

- I presented our method to compute both SL and TL collinear functions for QCD resummations using respective splitting kernels for the scattering matrix element.

- To compute these functions, we defined a differential version at the intermediate level and integrated them using proper observable definition to obtain specific collinear functions for transverse momentum resummation.

- For the azimuthally independent collinear functions, we have presented results up to NLO and for the azimuthally correlated case, we have results up to NNLO in perturbation theory.

- In our computation, we have stressed on the point that SL collinear functions, in general, can be process dependent and this dependency comes from splitting kernels to collinear functions.

- Instead of using a regulator to cure rapidity divergences those are present in the transverse momentum case, we use a time-like auxiliary vector to avoid them at the matrix element level.

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Thank You !