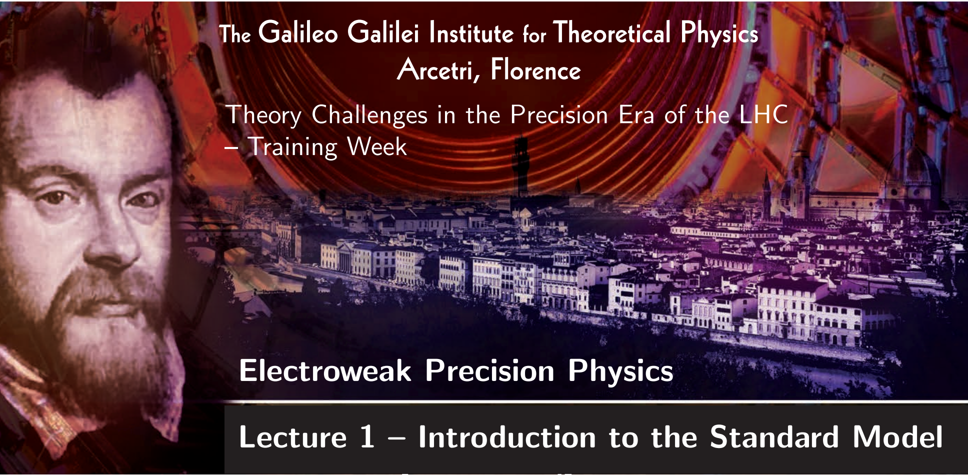

The Galileo Galilei Institute for Theoretical Physics
Arcetri, Florence

Theory Challenges in the Precision Era of the LHC
– Training Week

Electroweak Precision Physics

Lecture 1 – Introduction to the Standard Model

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Electroweak (EW) phenomenology before the GSW model

Some phenomenological facts:

- discovery of the weak interaction via radioactive β-decay of nuclei:
  $\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}, \qquad \mathrm{p} \to \mathrm{n} + \mathrm{e}^+ + \nu_\mathrm{e}$ (not possible for free protons)

- terminology "weak":
  interaction at low energy has very short range
  $\hookrightarrow$ long life time of weakly decaying particles:

strong int.:	$\rho \to 2\pi,$	$\tau \sim 10^{-22}$s
elmg. int.:	$\pi \to 2\gamma,$	$\tau \sim 10^{-16}$s
weak int.:	$\pi^- \to \mu^- + \bar{\nu}_\mu$	$\tau \sim 10^{-8}$s
	$\mu^- \to \mathrm{e}^- + \bar{\nu}_\mathrm{e} + \nu_\mu,$	$\tau \sim 10^{-6}$s

- lepton-number conservation: $\mu^- \not\to \mathrm{e}^- + \gamma \quad (\mathrm{BR} \lesssim 4 \cdot 10^{-13})$
  $\Rightarrow$ $L_\mathrm{e}, L_\mu, L_\tau$ individually conserved:
  $L_\mathrm{e} = +1$ for $\mathrm{e}^-, \nu_\mathrm{e}, \qquad L_\mathrm{e} = -1$ for $\mathrm{e}^+, \bar{\nu}_\mathrm{e},$ etc.

  (For massive ν's with different Dirac masses, only $L_\mathrm{e}+L_\mu+L_\tau$ is conserved.)

- parity violation (Wu et al. 1957):
  e.g.: $K^+ \to \underbrace{2\pi, 3\pi}_{\text{final states of different parity}}$

  $^{60}\mathrm{Co} \to {}^{60}\mathrm{Ni}^* + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$
  $\hookrightarrow$ polarization inversion does not yield inversion of spectra

The Fermi model

(Fermi 1933, further developed by Feynman, Gell-Mann and others after 1958)

Lagrangian for "current–current interaction" of four fermions:

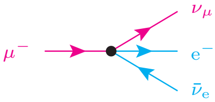
$$\mathcal{L}_{\mathrm{Fermi}}(x) = -2\sqrt{2}G_\mu J^\dagger_\rho(x) J^\rho(x), \qquad G_\mu = 1.16639 \times 10^{-5}\,\mathrm{GeV}^{-2}$$

$$\text{with} \quad J_\rho(x) = J^{\mathrm{lep}}_\rho(x) + J^{\mathrm{had}}_\rho(x) = \text{charged weak current}$$

- Leptonic part J^{lep}_ρ of J_ρ:

  $$J^{\mathrm{lep}}_\rho = \overline{\psi_{\nu_\mathrm{e}}}\gamma_\rho\omega_-\psi_\mathrm{e} + \overline{\psi_{\nu_\mu}}\gamma_\rho\omega_-\psi_\mu \qquad \omega_\pm = \tfrac{1}{2}(1\pm\gamma_5) = \text{chirality projectors}$$

  - only left-handed fermions ($\omega_-\psi$), right-handed anti-fermions ($\overline{\psi}\omega_+$) feel (charged-current) weak interactions $\Rightarrow$ maximal P-violation
  - doublet structure: $\begin{pmatrix}\nu_\mathrm{e}\\ \mathrm{e}^-\end{pmatrix}$, $\begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}$, later completed by $\begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}$
  - $(J^{\mathrm{lep},\rho})^\dagger J^{\mathrm{lep}}_\rho$ induces muon decay:



- Hadronic part $J_\rho^{\rm had}$ of J_ρ:

Relevant quarks for energies $\lesssim 1\,\mathrm{GeV}$: $\quad \mathrm{u,d,s,c}$
$\hookrightarrow$ meson ($q\bar{q}$) and baryon (qqq) spectra

Question: doublet structure $\begin{pmatrix}\mathrm{u}\\ \mathrm{d}\end{pmatrix}$, $\begin{pmatrix}\mathrm{c}\\ \mathrm{s}\end{pmatrix}$?

Problem: e.g. annihilation of $\mathrm{u\bar{s}}$ pair would not be allowed,
but is observed: $\underbrace{K^+}_{\mathrm{u\bar{s}}\text{ pair in quark model}} \to \mu^+\nu_\mu$

Solution (Cabibbo 1963):

u-c-mixing and d-s-mixing in weak interaction

$\hookrightarrow$ doublets $\begin{pmatrix}\mathrm{u}\\ \mathrm{d}'\end{pmatrix}$, $\begin{pmatrix}\mathrm{c}\\ \mathrm{s}'\end{pmatrix}$ with $\begin{pmatrix}\mathrm{d}'\\ \mathrm{s}'\end{pmatrix} = U_\mathrm{C}\begin{pmatrix}\mathrm{d}\\ \mathrm{s}\end{pmatrix}$,

orthogonal Cabbibo matrix $\quad U_\mathrm{C} = \begin{pmatrix}\cos\theta_\mathrm{C} & \sin\theta_\mathrm{C}\\ -\sin\theta_\mathrm{C} & \cos\theta_\mathrm{C}\end{pmatrix}$,

empirical result: $\quad \theta_\mathrm{C} \approx 13°$

$$J_\rho^{\rm had} = \overline{\psi_\mathrm{u}}\gamma_\rho\omega_-\psi_{\mathrm{d}'} + \overline{\psi_\mathrm{c}}\gamma_\rho\omega_-\psi_{\mathrm{s}'}$$

Remarks on the Fermi model:

- universal coupling G_μ for all transitions
  ($U_\mathrm{C}^\dagger U_\mathrm{C} = \mathbf{1}$ is part of universality)
- no (pseudo-)scalar or tensor couplings, such as $(\overline{\psi}\psi)(\overline{\psi}\psi)$, $(\overline{\psi}\psi)(\overline{\psi}\gamma_5\psi)$, etc., necessary to describe low-energy experiments ($E \lesssim 1\,\mathrm{GeV}$)
- Problems:
  - cross sections for $\nu_\mu \mathrm{e} \to \nu_\mathrm{e}\mu$, etc., grow for energy $E \to \infty$ as E^2
    $\hookrightarrow$ unitarity violation !
  - no consistent evaluation of higher perturbative orders possible
    (no cancellation of UV divergences)
    $\hookrightarrow$ non-renormalizability !

"Intermediate-vector-boson (IVB) model"

Idea: "resolution" of four-fermion interaction by vector-boson exchange

Lagrangian:

$$\begin{aligned}
\mathcal{L}_{\mathrm{IVB}} &= \mathcal{L}_{0,\mathrm{ferm}} + \mathcal{L}_{0,\mathrm{W}} + \mathcal{L}_{\mathrm{int}}, \\
\mathcal{L}_{0,\mathrm{ferm}} &= \overline{\psi_f}(\mathrm{i}\partial\!\!\!/ - m_f)\psi_f, \qquad \text{(summation over } f \text{ assumed)} \\
\mathcal{L}_{0,\mathrm{W}} &= -\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) + M_\mathrm{W}^2 W_\mu^+ W^{-,\mu},
\end{aligned}$$

$$\text{with } W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp \mathrm{i} W_\mu^2), \qquad W_\mu^i \text{ real}$$

$\mathrm{W}^\pm$ are vector bosons with electric charge $\pm e$ and mass M_W.

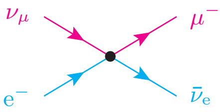
Propagator: $G_{\mu\nu}^{\mathrm{WW}}(k) = \dfrac{-\mathrm{i}}{k^2 - M_\mathrm{W}^2}\left(g_{\mu\nu} - \dfrac{k_\mu k_\nu}{M_\mathrm{W}^2}\right), \quad k = \text{momentum}$

Interaction Lagrangian: $\mathcal{L}_{\mathrm{int}} = \dfrac{g_\mathrm{W}}{\sqrt{2}}\left(J^\rho W_\rho^+ + J^{\rho\dagger} W_\rho^-\right),$

$J^\rho =$ charged weak current as in Fermi model

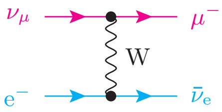
Four-fermion interaction in process $\nu_\mu \mathrm{e}^- \to \mu^- \nu_\mathrm{e}$

Fermi model:



$$-\mathrm{i}2\sqrt{2}G_\mu\, g_{\rho\sigma}$$
$$\times \left[\bar{u}_{\mu^-}\gamma^\rho\omega_- u_{\nu_\mu}\right]\left[\bar{u}_{\nu_\mathrm{e}}\gamma^\sigma\omega_- u_{\mathrm{e}^-}\right]$$

IVB model:



$$\frac{\mathrm{i}}{2}g_\mathrm{W}^2\frac{1}{k^2-M_\mathrm{W}^2}\left(g_{\rho\sigma}-\frac{k_\rho k_\sigma}{M_\mathrm{W}^2}\right)$$
$$\times \left[\bar{u}_{\mu^-}\gamma^\rho\omega_- u_{\nu_\mu}\right]\left[\bar{u}_{\nu_\mathrm{e}}\gamma^\sigma\omega_- u_{\mathrm{e}^-}\right]$$

$\Rightarrow$ identification for $|k| \ll M_\mathrm{W}$: $\quad 2\sqrt{2}G_\mu = \dfrac{g_\mathrm{W}^2}{2M_\mathrm{W}^2}$

Consequences for the high-energy behaviour:

- k^ρ terms: $\quad \bar{u}_{\nu_\mathrm{e}}\not{k}\omega_- u_{\mathrm{e}^-} = \bar{u}_{\nu_\mathrm{e}}(\not{p}_\mathrm{e}-\not{p}_{\nu_\mathrm{e}})\omega_- u_{\mathrm{e}^-} = m_\mathrm{e}\bar{u}_{\nu_\mathrm{e}}\omega_- u_{\mathrm{e}^-}$
  $\hookrightarrow$ no extra factors of scattering energy E
- propagator $1/(k^2-M_\mathrm{W}^2) \sim 1/E^2$ for $|k| \sim E \gg M_\mathrm{W}$
  $\hookrightarrow$ damping of amplitude in high-energy limit by factor $1/E^2$

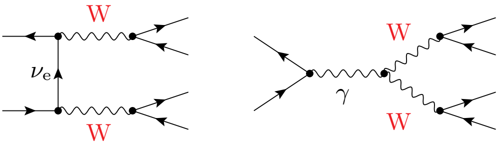
$\Rightarrow$ cross section $\underset{E\to\infty}{\sim}$ const$/E^2$, $\qquad \Rightarrow$ No unitarity violation !

Comments on the IVB model:

- Formal similiarity with QED interaction: $\quad J^\rho W^+_\rho + \text{h.c.} \;\longleftrightarrow\; j^\rho_{\mathrm{elmg}} A_\rho$
- Intermediate vector bosons can be produced, e.g.

$$\underbrace{\mathrm{u\bar{d}}}_{\text{in pp collision}} \quad\longrightarrow\quad \underbrace{\mathrm{W}^+ \to f\bar{f}'}_{\mathrm{W}^\pm \text{ unstable}} \qquad \text{(discovery 1983 at CERN)}$$

- Problems:
  - unitarity violations in cross sections with longitudinal W bosons, e.g.



  - non-renormalizability
    (no consistent treatment of higher perturbative orders)

$\hookrightarrow$ Solution by spontaneously broken gauge theories !

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The principle of local gauge invariance

QED as U(1) gauge theory:

Lagrangian $\mathcal{L}_{0,\mathrm{ferm}} = \overline{\psi_f}(\mathrm{i}\not{\partial} - m_f)\psi_f$ has global phase symmetry:

$$\psi_f \to \psi'_f = \exp\{-\mathrm{i}Q_f e\theta\}\psi_f, \quad \overline{\psi_f} \to \overline{\psi'_f} = \overline{\psi_f}\exp\{+\mathrm{i}Q_f e\theta\}$$

with space-time-independent group parameter θ

"Gauging the symmetry": demand local symmetry, $\theta \to \theta(x)$

To maintain local symmetry, extend theory by "minimal substitution":

$$\partial^\mu \to D^\mu = \partial^\mu + \mathrm{i}Q_f e A^\mu(x) = \text{"covariant derivative"},$$

$$A^\mu(x) = \text{spin-1 gauge field (photon)}.$$

Transformation property of photon $A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu\theta(x)$ ensures

- $D_\mu\psi_f \to (D_\mu\psi_f)' = D'_\mu\psi'_f = \exp\{-\mathrm{i}Q_f e\theta\}(D_\mu\psi_f)$
- gauge invariance of field-strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Gauge-invariant Lagrangian of QED:

$$\mathcal{L}_{\mathrm{QED}} = \overline{\psi_f}(\mathrm{i}\not{\partial} - Q_f e\not{A} - m_f)\psi_f - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Non-Abelian gauge theory (Yang–Mills theory):

Starting point:

Lagrangian $\mathcal{L}_\Phi(\Phi, \partial_\mu\Phi)$ of free or self-interacting fields
with "internal symmetry":

- $\Phi = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix}$ = multiplet of a compact Lie group G:

  $$\Phi \to \Phi' = U(\theta)\Phi, \quad U(\theta) = \exp\{-\mathrm{i}gT^a\theta^a\} = \text{unitary},$$
  $$T^a = \text{group generators}, \quad [T^a, T^b] = \mathrm{i}C^{abc}T^c, \quad \mathrm{Tr}(T^aT^b) = \tfrac{1}{2}\delta^{ab}$$

- $\mathcal{L}_\Phi$ is invariant under G: $\quad \mathcal{L}_\Phi(\Phi, \partial_\mu\Phi) = \mathcal{L}_\Phi(\Phi', \partial_\mu\Phi')$

Example: self-interacting (complex) boson multiplet

$$\mathcal{L}_\Phi = (\partial_\mu\Phi)^\dagger(\partial^\mu\Phi) - m^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 \quad (m = \text{common boson mass}, \ \lambda = \text{coupling strength})$$

Gauging the symmetry by minimal substitution:

$$\mathcal{L}_\Phi(\Phi, \partial_\mu\Phi) \quad \to \quad \mathcal{L}_\Phi(\Phi, D_\mu\Phi) \quad \text{with} \ D_\mu = \partial_\mu + \mathrm{i}gT^aA^a_\mu(x),$$

g = gauge coupling,
T^a = generator of G in Φ representation,
$A^a_\mu(x)$ = gauge fields

Transformation property of gauge fields:

- $\mathcal{L}_\Phi(\Phi, D_\mu\Phi)$ local invariant if $\quad D_\mu\Phi \;\to\; (D_\mu\Phi)' = D'_\mu\Phi' = U(\theta)(D_\mu\Phi)$

  $\Rightarrow \; T^a A'^a_\mu \;=\; U T^a A^a_\mu U^\dagger - \frac{\mathrm{i}}{g} U(\partial_\mu U^\dagger), \qquad A^a_\mu A^{a,\mu} = \text{not gauge invariant}$

  infinitesimal form: $\quad \delta A^a_\mu = g C^{abc}\delta\theta^b A^c_\mu + \partial_\mu \delta\theta^a$

- covariant definition of field strength: $\quad [D_\mu, D_\nu] = \mathrm{i} g T^a F^a_{\mu\nu}$

  $\Rightarrow \; T^a F^a_{\mu\nu} \;\to\; T^a F'^a_{\mu\nu} = U T^a F^a_{\mu\nu} U^\dagger, \qquad F^a_{\mu\nu}F^{a,\mu\nu} = \text{gauge invariant}$

  explicit form: $\quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g C^{abc} A^b_\mu A^c_\nu$

Yang–Mills Lagrangian for gauge and matter fields:

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} F^a_{\mu\nu}F^{a,\mu\nu} + \mathcal{L}_\Phi(\Phi, D_\mu\Phi)$$

- Lagrangian contains terms of order $(\partial A)A^2$, A^4 in F^2 part

  $\hookrightarrow$ cubic and quartic gauge-boson self-interactions

- gauge coupling determines gauge-boson–matter and gauge-boson self-interaction $\;\to\;$ unification of interactions

- mass term $M^2(A^a_\mu A^{a,\mu})$ for gauge bosons forbidden by gauge invariance

  $\hookrightarrow$ gauge bosons of unbroken Yang–Mills theory are massless

Quantum chromodynamics — gauge theory of strong interactions

- **Gauge group:** $\mathrm{SU}(3)_c$, dim. $= 8$

  structure constants f^{abc}, gauge coupling g_s, $\alpha_\mathrm{s} = \dfrac{g_\mathrm{s}^2}{4\pi}$

- **Gauge bosons:** 8 massless gluons g with fields $A^a_\mu(x)$, $a = 1, \dots, 8$

- **Matter fermions:** quarks q (spin-$\frac{1}{2}$) with flavours $q = \mathrm{d,u,s,c,b,t}$

  in fundamental representation:

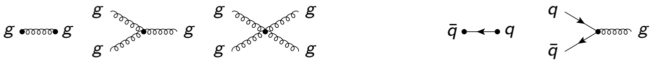
$$\psi_q(x) \equiv q(x) = \begin{pmatrix} q_r(x) \\ q_g(x) \\ q_b(x) \end{pmatrix} = \text{colour triplet}$$

$$T^a = \frac{\lambda^a}{2}, \quad \text{Gell-Mann matrices } \lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \text{ etc.}$$

- **Lagrangian:**

$$\mathcal{L}_\mathrm{QCD} = -\tfrac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \sum_q \overline{\psi_q}(\mathrm{i}\not{D} - m_q)\psi_q$$

$$= -\tfrac{1}{4} \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_\mathrm{s} f^{abc} A^b_\mu A^c_\nu\right)^2 + \sum_q \overline{\psi_q} \left(\mathrm{i}\not{\partial} - g_\mathrm{s} \frac{\lambda^a}{2} \not{A}^a - m_q\right) \psi_q$$



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The Standard Model (SM) of electroweak interaction
(Glashow–Salam–Weinberg model)

The gauge group for EW interaction

Why unification of weak and elmg. interaction ?

- similiarity: spin-1 fields couple to matter currents formed by spin-$\frac{1}{2}$ fields
- elmg. coupling of charged $\mathrm{W}^{\pm}$ bosons

$\gamma, \mathrm{W}^{+}, \mathrm{W}^{-}$ as gauge bosons of group SU(2) ? – No!

Reasons:

- charge operator Q cannot be SU(2) generator, since $\mathrm{Tr}\, Q \neq 0$

  for fermion doublets: $Q = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$ for $\begin{pmatrix} \nu_\mathrm{e} \\ \mathrm{e}^- \end{pmatrix}$, etc.

  Possible way out: additional heavy fermions like E^+ as partner to e^- ?
  $\hookrightarrow$ no experimental confirmation !
- $\mathrm{W}^{\pm}$ couplings parity violating, but γ coupling parity invariant

Minimal solution: $\mathrm{SU}(2)_{\mathrm{I}} \times \mathrm{U}(1)_{\mathrm{Y}}$

- $\mathrm{SU}(2)_{\mathrm{I}}$ $\rightarrow$ weak isospin group with gauge fields W^+, W^-, W^0
- $\mathrm{U}(1)_{\mathrm{Y}}$ $\rightarrow$ weak hypercharge with gauge field B

W^0 and B carry identical quantum numbers

$\hookrightarrow$ two neutral gauge bosons γ, Z as mixed states

Experiment: 1973 discovery of neutral weak currents at CERN

$\hookrightarrow$ indirect confirmation of Z exchange

1983 discovery of $\mathrm{W}^\pm$ and Z bosons at CERN

Fermion sector and minimal substitution

Multiplet structure:

Distinguish between left-/right-handed parts of fermions: $\psi^{\mathrm{L}} = \omega_- \psi$, $\psi^{\mathrm{R}} = \omega_+ \psi$

- ψ^{L} couple to $\mathrm{W}^\pm$ $\rightarrow$ group ψ^{L} into $\mathrm{SU(2)_I}$ doublets, weak isospin $T_{\mathrm{I}}^a = \frac{\sigma^a}{2}$
- ψ^{R} do not couple to $\mathrm{W}^\pm$ $\rightarrow$ ψ^{R} are $\mathrm{SU(2)_I}$ singlets, weak isospin $T_{\mathrm{I}}^a = 0$
- $\psi^{\mathrm{L/R}}$ couple to γ in the same way

  $\hookrightarrow$ adjust coupling to $\mathrm{U(1)_Y}$ (i.e. fix weak hypercharges $Y^{\mathrm{L/R}}$ for $\psi^{\mathrm{L/R}}$) such that elmg. coupling results: $\mathcal{L}_{\mathrm{int,QED}} = -Q_f e \overline{\psi_f} \not{A} \psi_f$

Fermion content of the SM:

(ignoring possible right-handed neutrinos)

					T_{I}^3	Q
leptons:	$\Psi_L^{\mathrm{L}} =$	$\begin{pmatrix} \nu_{\mathrm{e}}^{\mathrm{L}} \\ \mathrm{e}^{\mathrm{L}} \end{pmatrix},$	$\begin{pmatrix} \nu_{\mu}^{\mathrm{L}} \\ \mu^{\mathrm{L}} \end{pmatrix},$	$\begin{pmatrix} \nu_{\tau}^{\mathrm{L}} \\ \tau^{\mathrm{L}} \end{pmatrix},$	$+\frac{1}{2}$ $-\frac{1}{2}$	0 -1
	$\psi_\ell^{\mathrm{R}} =$	$\mathrm{e}^{\mathrm{R}},$	$\mu^{\mathrm{R}},$	$\tau^{\mathrm{R}},$	0	-1
quarks: (Each quark exists in 3 colours!)	$\Psi_Q^{\mathrm{L}} =$	$\begin{pmatrix} \mathrm{u}^{\mathrm{L}} \\ \mathrm{d}^{\mathrm{L}} \end{pmatrix},$	$\begin{pmatrix} \mathrm{c}^{\mathrm{L}} \\ \mathrm{s}^{\mathrm{L}} \end{pmatrix},$	$\begin{pmatrix} \mathrm{t}^{\mathrm{L}} \\ \mathrm{b}^{\mathrm{L}} \end{pmatrix},$	$+\frac{1}{2}$ $-\frac{1}{2}$	$+\frac{2}{3}$ $-\frac{1}{3}$
	$\psi_u^{\mathrm{R}} =$	$\mathrm{u}^{\mathrm{R}},$	$\mathrm{c}^{\mathrm{R}},$	$\mathrm{t}^{\mathrm{R}},$	0	$+\frac{2}{3}$
	$\psi_d^{\mathrm{R}} =$	$\mathrm{d}^{\mathrm{R}},$	$\mathrm{s}^{\mathrm{R}},$	$\mathrm{b}^{\mathrm{R}},$	0	$-\frac{1}{3}$

Free Lagrangian of (still massless) fermions:

$$\mathcal{L}_{0,\mathrm{ferm}} = \mathrm{i}\overline{\psi_f}\partial\!\!\!/\psi_f = \mathrm{i}\overline{\Psi^{\mathrm{L}}_L}\partial\!\!\!/\Psi^{\mathrm{L}}_L + \mathrm{i}\overline{\Psi^{\mathrm{L}}_Q}\partial\!\!\!/\Psi^{\mathrm{L}}_Q + \mathrm{i}\overline{\psi^{\mathrm{R}}_\ell}\partial\!\!\!/\psi^{\mathrm{R}}_\ell + \mathrm{i}\overline{\psi^{\mathrm{R}}_u}\partial\!\!\!/\psi^{\mathrm{R}}_u + \mathrm{i}\overline{\psi^{\mathrm{R}}_d}\partial\!\!\!/\psi^{\mathrm{R}}_d$$

Minimal substitution:

$$\partial_\mu \to D_\mu = \partial_\mu - \mathrm{i}g_2 T^a_{\mathrm{I}} W^a_\mu + \mathrm{i}g_1 \tfrac{1}{2} Y B_\mu = D^{\mathrm{L}}_\mu \omega_- + D^{\mathrm{R}}_\mu \omega_+,$$

$$D^{\mathrm{L}}_\mu = \partial_\mu - \frac{\mathrm{i}g_2}{\sqrt{2}}\begin{pmatrix} 0 & W^+_\mu \\ W^-_\mu & 0 \end{pmatrix} - \frac{\mathrm{i}}{2}\begin{pmatrix} g_2 W^3_\mu - g_1 Y^{\mathrm{L}} B_\mu & 0 \\ 0 & -g_2 W^3_\mu - g_1 Y^{\mathrm{L}} B_\mu \end{pmatrix},$$

$$D^{\mathrm{R}}_\mu = \partial_\mu + \mathrm{i}g_1 \tfrac{1}{2} Y^{\mathrm{R}} B_\mu$$

Photon identification:

"Weinberg rotation": $\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix}\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}$, $c_{\mathrm{W}} = \cos\theta_{\mathrm{W}}$, $s_{\mathrm{W}} = \sin\theta_{\mathrm{W}}$, θ_{W} = weak mixing angle

$$D^{\mathrm{L}}_\mu\Big|_{A_\mu} = -\frac{\mathrm{i}}{2}A_\mu\begin{pmatrix} -g_2 s_{\mathrm{W}} - g_1 c_{\mathrm{W}} Y^{\mathrm{L}} & 0 \\ 0 & g_2 s_{\mathrm{W}} - g_1 c_{\mathrm{W}} Y^{\mathrm{L}} \end{pmatrix} \overset{!}{=} \mathrm{i}eA_\mu\begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}$$

- charged difference in doublet $Q_1 - Q_2 = 1 \quad \to \quad g_2 = \dfrac{e}{s_{\mathrm{W}}}$
- normalize $Y^{\mathrm{L/R}}$ such that $g_1 = \dfrac{e}{c_{\mathrm{W}}}$

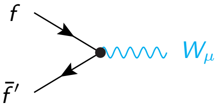
  $\hookrightarrow$ Y fixed by "Gell-Mann–Nishijima relation": $Q = T^3_{\mathrm{I}} + \dfrac{Y}{2}$

Fermion–gauge-boson interaction:

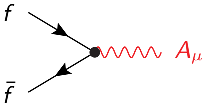
$$\mathcal{L}_{\mathrm{ferm,YM}} = \frac{e}{\sqrt{2}s_{\mathrm{W}}}\overline{\Psi_F^{\mathrm{L}}}\begin{pmatrix} 0 & \not{W}^+ \\ \not{W}^- & 0 \end{pmatrix}\Psi_F^{\mathrm{L}} + \frac{e}{2c_{\mathrm{W}}s_{\mathrm{W}}}\overline{\Psi_F^{\mathrm{L}}}\sigma^3 \not{Z}\Psi_F^{\mathrm{L}}$$

$$- e\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f\overline{\psi_f}\not{Z}\psi_f - eQ_f\overline{\psi_f}\not{A}\psi_f \qquad (f = \text{all fermions, } F = \text{all doublets})$$

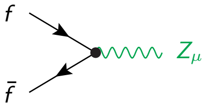
Feynman rules:



$$\frac{\mathrm{i}e}{\sqrt{2}s_{\mathrm{W}}}\gamma_\mu\omega_-$$



$$-\mathrm{i}Q_f e\gamma_\mu$$



$$\mathrm{i}e\gamma_\mu(g_f^+\omega_+ + g_f^-\omega_-) = \mathrm{i}e\gamma_\mu(v_f - a_f\gamma_5)$$

$$\text{with} \quad g_f^+ = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f, \quad g_f^- = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f + \frac{T^3_{\mathrm{I},f}}{c_{\mathrm{W}}s_{\mathrm{W}}},$$

$$v_f = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f + \frac{T^3_{\mathrm{I},f}}{2c_{\mathrm{W}}s_{\mathrm{W}}}, \quad a_f = \frac{T^3_{\mathrm{I},f}}{2c_{\mathrm{W}}s_{\mathrm{W}}}$$

Gauge-boson sector

Yang–Mills Lagrangian for gauge fields:

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

Field-strength tensors:

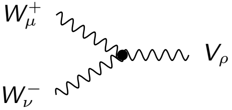
$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Lagrangian in terms of "physical" fields:

$$\begin{aligned}
\mathcal{L}_{\mathrm{YM}} = & -\frac{1}{2}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) \\
& -\frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \\
& + \text{(trilinear interaction terms involving } AW^+W^-,\ ZW^+W^-) \\
& + \text{(quadrilinear interaction terms involving} \\
& \qquad AAW^+W^-,\ AZW^+W^-,\ ZZW^+W^-,\ W^+W^-W^+W^-)
\end{aligned}$$

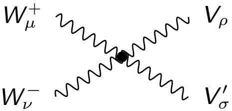
Feynman rules for gauge-boson self-interactions:

(fields and momenta incoming)



$$\mathrm{i}eC_{WWV}\big[\; g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu\big]$$

$$\text{with} \quad C_{WW\gamma} = 1, \quad C_{WWZ} = -\frac{c_\mathrm{W}}{s_\mathrm{W}}$$



$$\mathrm{i}e^2 C_{WWVV'}\left[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\nu\rho}\right]$$

$$\text{with} \quad C_{WW\gamma\gamma} = -1, \qquad C_{WW\gamma Z} = \frac{c_\mathrm{W}}{s_\mathrm{W}},$$
$$C_{WWZZ} = -\frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}, \quad C_{WWWW} = \frac{1}{s_\mathrm{W}^2}$$

Higgs sector and spontaneous symmetry breaking

Idea: spontaneous breakdown of $\mathrm{SU}(2)_\mathrm{I}\times\mathrm{U}(1)_\mathrm{Y}$ symmetry $\to$ $\mathrm{U}(1)_\mathrm{elmg}$ symmetry

$\hookrightarrow$ masses for $\mathrm{W}^\pm$ and Z bosons, but γ remains massless

Note: choice of scalar extension of massless model involves freedom

GSW model:

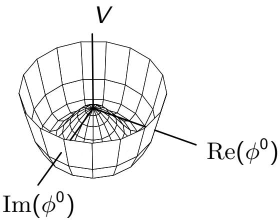
Minimal scalar sector with complex scalar doublet $\Phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}$, $Y_\Phi = 1$

Scalar self-interaction via Higgs potential:

$$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2, \quad \mu^2,\lambda>0,$$

$$= \mathrm{SU}(2)_\mathrm{I}\times\mathrm{U}(1)_\mathrm{Y} \text{ symmetric}$$

$$V(\Phi) = \text{minimal for} \quad |\Phi| = \sqrt{\frac{2\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}} > 0$$



Ground state Φ_0 (=vacuum expectation value of Φ) not unique,

specific choice $\Phi_0 = \begin{pmatrix}0\\ \frac{v}{\sqrt{2}}\end{pmatrix}$ not gauge invariant $\Rightarrow$ spontaneous symmetry breaking!

Elmg. gauge invariance unbroken, since $\quad Q\Phi_0 = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\Phi_0 = 0$

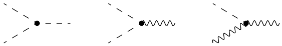
Field excitations in Φ:

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}\left(v + H(x) + \mathrm{i}\chi(x)\right) \end{pmatrix}$$

Gauge-invariant Lagrangian of Higgs sector: $\quad (\phi^- = (\phi^+)^\dagger)$

$$\begin{aligned}
\mathcal{L}_\mathrm{H} &= (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \qquad \text{with } D_\mu = \partial_\mu - \mathrm{i} g_2 \frac{\sigma^a}{2} W_\mu^a + \mathrm{i}\frac{g_1}{2} B_\mu \\
&= (\partial_\mu \phi^+)(\partial^\mu \phi^-) - \frac{\mathrm{i}ev}{2s_\mathrm{W}}(W_\mu^+ \partial^\mu \phi^- - W_\mu^- \partial^\mu \phi^+) + \frac{e^2 v^2}{4 s_\mathrm{W}^2} W_\mu^+ W^{-,\mu} \\
&\quad + \frac{1}{2}(\partial\chi)^2 + \frac{ev}{2c_\mathrm{W} s_\mathrm{W}} Z_\mu \partial^\mu \chi + \frac{e^2 v^2}{4 c_\mathrm{W}^2 s_\mathrm{W}^2} Z^2 + \frac{1}{2}(\partial H)^2 - \mu^2 H^2
\end{aligned}$$

$+$ (trilinear SSS, SSV, SVV interactions)



$+$ (quadrilinear $SSSS$, $SSVV$ interactions)

Implications:

- gauge-boson masses: $\quad M_\mathrm{W} = \frac{ev}{2s_\mathrm{W}}, \quad M_\mathrm{Z} = \frac{ev}{2c_\mathrm{W} s_\mathrm{W}} = \frac{M_\mathrm{W}}{c_\mathrm{W}}, \quad M_\gamma = 0$
- physical Higgs boson H: $\quad M_\mathrm{H} = \sqrt{2\mu^2}$ = free parameter
- would-be Goldstone bosons $\phi^\pm$, χ: $\quad$ unphysical degrees of freedom

Fermion masses and Yukawa couplings

Ordinary Dirac mass terms $m_f\overline{\psi_f}\psi_f = m_f(\overline{\psi_f^{\mathrm{L}}}\psi_f^{\mathrm{R}} + \overline{\psi_f^{\mathrm{R}}}\psi_f^{\mathrm{L}})$ not gauge invariant

$\hookrightarrow$ introduce fermion masses by (gauge-invariant) Yukawa interaction

Lagrangian for Yukawa couplings:

$$\mathcal{L}_{\mathrm{Yuk}} \;=\; -\overline{\Psi_L^{\mathrm{L}}}G_\ell\psi_\ell^{\mathrm{R}}\Phi - \overline{\Psi_Q^{\mathrm{L}}}G_u\psi_u^{\mathrm{R}}\tilde{\Phi} - \overline{\Psi_Q^{\mathrm{L}}}G_d\psi_d^{\mathrm{R}}\Phi + \text{h.c.}$$

- $G_\ell, G_u, G_d = 3\times 3$ matrices in 3-dim. space of generations (ν masses ignored)
- $\tilde{\Phi} = i\sigma^2\Phi^* = \begin{pmatrix}\phi^{0*}\\ -\phi^-\end{pmatrix} =$ charge conjugate Higgs doublet, $Y_{\tilde{\Phi}} = -1$

Fermion mass terms:

mass terms $=$ bilinear terms in $\mathcal{L}_{\mathrm{Yuk}}$, obtained by setting $\Phi \to \Phi_0$:

$$\mathcal{L}_{m_f} \;=\; -\frac{v}{\sqrt{2}}\overline{\psi_\ell^{\mathrm{L}}}G_\ell\psi_\ell^{\mathrm{R}} - \frac{v}{\sqrt{2}}\overline{\psi_u^{\mathrm{L}}}G_u\psi_u^{\mathrm{R}} - \frac{v}{\sqrt{2}}\overline{\psi_d^{\mathrm{L}}}G_d\psi_d^{\mathrm{R}} + \text{h.c.}$$

$\hookrightarrow$ diagonalization by unitary field transformations $\quad (f = l, u, d)$

$\hat{\psi}_f^{\mathrm{L/R}} \equiv U_f^{\mathrm{L/R}}\psi_f^{\mathrm{L/R}}$ such that $\frac{v}{\sqrt{2}}U_f^{\mathrm{L}}G_f(U_f^{\mathrm{R}})^\dagger = \mathrm{diag}(m_f)$

$\Rightarrow$ standard form: $\mathcal{L}_{m_f} \;=\; -m_f\overline{\hat{\psi}_f^{\mathrm{L}}}\hat{\psi}_f^{\mathrm{R}} + \text{h.c.} \;=\; -m_f\overline{\hat{\psi}_f}\hat{\psi}_f$

Quark mixing:

- ψ_f correspond to eigenstates of the gauge interaction
- $\hat{\psi}_f$ correspond to mass eigenstates,
  for massless neutrinos define $\quad \hat{\psi}^{\mathrm{L}}_\nu \equiv U^{\mathrm{L}}_\ell \psi^{\mathrm{L}}_\nu \quad \to$ no lepton-flavour changing

Yukawa and gauge interactions in terms of mass eigenstates:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Yuk}} = & -\frac{\sqrt{2}m_\ell}{v}\left(\phi^+\overline{\hat{\psi}^{\mathrm{L}}_{\nu_\ell}}\hat{\psi}^{\mathrm{R}}_\ell + \phi^-\overline{\hat{\psi}^{\mathrm{R}}_\ell}\hat{\psi}^{\mathrm{L}}_{\nu_\ell}\right) + \frac{\sqrt{2}m_u}{v}\left(\phi^+\overline{\hat{\psi}^{\mathrm{R}}_u} V \hat{\psi}^{\mathrm{L}}_d + \phi^-\overline{\hat{\psi}^{\mathrm{L}}_d} V^\dagger \hat{\psi}^{\mathrm{R}}_u\right) \\
& -\frac{\sqrt{2}m_d}{v}\left(\phi^+\overline{\hat{\psi}^{\mathrm{L}}_u} V \hat{\psi}^{\mathrm{R}}_d + \phi^-\overline{\hat{\psi}^{\mathrm{R}}_d} V^\dagger \hat{\psi}^{\mathrm{L}}_u\right) - \frac{m_f}{v}\mathrm{i}\,\mathrm{sgn}(T^3_{\mathrm{I},f})\chi\,\overline{\hat{\psi}_f}\gamma_5\hat{\psi}_f \\
& -\frac{m_f}{v}(v+H)\,\overline{\hat{\psi}_f}\hat{\psi}_f,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{ferm,YM}} = & \frac{e}{\sqrt{2}s_{\mathrm{W}}}\overline{\hat{\Psi}^{\mathrm{L}}_L}\begin{pmatrix} 0 & \not{W}^+ \\ \not{W}^- & 0 \end{pmatrix}\hat{\psi}^{\mathrm{L}}_L + \frac{e}{\sqrt{2}s_{\mathrm{W}}}\overline{\hat{\Psi}^{\mathrm{L}}_Q}\begin{pmatrix} 0 & V\not{W}^+ \\ V^\dagger\not{W}^- & 0 \end{pmatrix}\hat{\psi}^{\mathrm{L}}_Q \\
& + \frac{e}{2c_{\mathrm{W}}s_{\mathrm{W}}}\overline{\hat{\Psi}^{\mathrm{L}}_F}\sigma^3\not{Z}\hat{\Psi}^{\mathrm{L}}_F - e\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f\overline{\hat{\psi}_f}\not{Z}\hat{\psi}_f - eQ_f\overline{\hat{\psi}_f}\not{A}\hat{\psi}_f
\end{aligned}
$$

- only charged-current coupling of quarks modified by $V = U^{\mathrm{L}}_u (U^{\mathrm{L}}_d)^\dagger =$ unitary
  (Cabibbo–Kobayashi–Maskawa (CKM) matrix)
- Higgs–fermion coupling strength $= \dfrac{m_f}{v}$

Features of the CKM mixing:

- V = 3-dim. generalization of Cabibbo matrix U_{C}
- V is parametrized by 4 free parameters: 3 real angles, 1 complex phase

  $\hookrightarrow$ complex phase is the only source of CP violation in SM

  Counting of physical parameters:

  $$\begin{pmatrix}\#\text{real} \\ \text{d.o.f. in } V\end{pmatrix} - \begin{pmatrix}\#\text{unitarity} \\ \text{relations}\end{pmatrix} - \begin{pmatrix}\#\text{phase diffs. of} \\ u\text{-type quarks}\end{pmatrix} - \begin{pmatrix}\#\text{phase diffs. of} \\ d\text{-type quarks}\end{pmatrix}$$
  $$- \begin{pmatrix}\#\text{phase diff. between} \\ u\text{- and } d\text{-type quarks}\end{pmatrix}$$
  $$= 18 - 9 - 2 - 2 - 1 = 4$$

- no flavour-changing neutral currents in lowest order,
  flavour-changing suppressed by factors $G_\mu(m_{q_1}^2 - m_{q_2}^2)$ in higher orders
  ("Glashow–Iliopoulos–Maiani mechanism")

Quantization of the EW SM

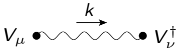
- describes particle creation and annihilation
- requires gauge-fixing for perturbation theory (existence of propagators)

Common approach: Faddeev–Popov method with R_ξ gauge-fixing

- R_ξ gauge-fixing Lagrangian: ($\xi_V^{(\prime)}$ = arbitrary gauge parameters)

$$\mathcal{L}_{\rm fix} = -\tfrac{1}{2\xi_A}(C^A)^2 - \tfrac{1}{2\xi_Z}(C^Z)^2 - \tfrac{1}{\xi_W}C^+C^-$$
$$C^\pm = \partial^\mu W_\mu^\pm \mp {\rm i} M_{\rm W}\xi'_W\phi^\pm, \quad C^Z = \partial^\mu Z_\mu - M_{\rm Z}\xi'_Z\chi, \quad C^A = \partial^\mu A_\mu$$

$\hookrightarrow$ elimination of ϕW and χZ mixing for $\xi'_V = \xi_V$ (propagator decoupling), simple vector propagators in 't Hooft–Feynman gauge ($\xi_V^{(\prime)} = 1$):



$$G_{\mu\nu}^{VV^\dagger}(k) = \frac{-{\rm i}g_{\mu\nu}}{k^2 - M_V^2}, \quad V = W, A, Z$$

- Faddeev–Popov Lagrangian with unphysical ghost fields $u^a, \bar{u}^a$ ($a = \pm, A, Z$)

$$\mathcal{L}_{\rm FP} = -\int {\rm d}^4y\, \bar{u}^a(x)\frac{\delta C^a(x)}{\delta\theta^b(y)}u^b(y), \qquad (\theta^a = \text{gauge group parameters})$$

- Green functions obey Slavnov–Taylor identities (from BRS symmetry), involving ghost contributions

Perturbative evaluation of the EW SM

- Input parameters: $\alpha = e^2/(4\pi), M_\mathrm{W}, M_\mathrm{Z}, M_\mathrm{H}, m_f, V$

  $\hookrightarrow$ non-trivial issue to find

  - appropriate field-theoretical definitions ("renormalization scheme")
  - appropriate phenomenological input ("input parameter scheme")

- Renormalizability:
  - UV finiteness guaranteed
  - perturbative approximation controllable (all orders defined)

- Complications:
  - almost all particles unstable
  - many mass scales in amplitudes and loop integrals
  - IR (soft and/or collinear) singularities

- EW corrections:

  generic size $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_\mathrm{s}^2)$ suggests NLO EW $\sim$ NNLO QCD,
  but systematic enhancements possible, e.g.

  - by photon emission
    $\hookrightarrow$ kinematical effects, mass-singular log's $\propto \alpha \ln(m_\ell/Q)$
  - at high energies
    $\hookrightarrow$ EW Sudakov log's $\propto (\alpha/s_\mathrm{W}^2) \ln^2(M_\mathrm{W}/Q)$ and subleading log's

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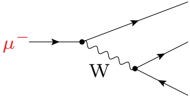
Electroweak precision physics before the LHC era

Key experiments for EW precision physics

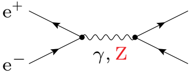
- **Muon decay:** $\mu^- \to \nu_\mu \mathrm{e}^- \bar{\nu}_\mathrm{e}$



determination of the Fermi constant

$$G_\mu = \frac{\pi \alpha M_\mathrm{Z}^2}{\sqrt{2} M_\mathrm{W}^2 (M_\mathrm{Z}^2 - M_\mathrm{W}^2)} + \dots$$

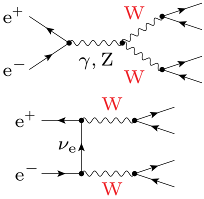
- **Z production** (LEP1/SLC): $\mathrm{e}^+\mathrm{e}^- \to \mathrm{Z} \to f\bar{f}$



various precision measurements at the Z resonance: $M_\mathrm{Z}, \Gamma_\mathrm{Z}, \sigma_\mathrm{had}, A_\mathrm{FB}, A_\mathrm{LR}$, etc.

$\Rightarrow$ good knowledge of the $\mathrm{Z}f\bar{f}$ sector

- **W-pair production** (LEP2): $\mathrm{e}^+\mathrm{e}^- \to \mathrm{WW} \to 4f(+\gamma)$



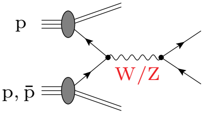
– measurement of M_W

– $\gamma\mathrm{WW}/\mathrm{ZWW}$ couplings

– quartic couplings: $\gamma\gamma\mathrm{WW}$, $\gamma\mathrm{ZWW}$

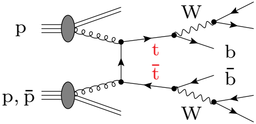
Key experiments for EW precision physics (continued)

- W/Z production (Tevatron/LHC): $\mathrm{pp}, \mathrm{p\bar{p}} \to \mathrm{W} \to \ell\nu_\ell(+\gamma)$
  $\mathrm{pp}, \mathrm{p\bar{p}} \to \mathrm{Z} \to \ell^+\ell^-$



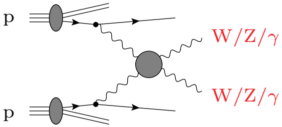
– measurement of M_W
– bounds on $\gamma\mathrm{WW}$ coupling
– measurement of $\sin^2\theta_\mathrm{eff}^\mathrm{lept}$

- top-quark production (Tevatron/LHC): $\mathrm{pp}, \mathrm{p\bar{p}} \to \mathrm{t\bar{t}} \to 6f$



– measurement of m_t

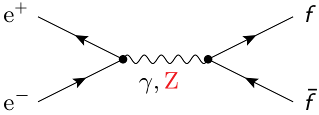
- gauge-boson scattering (LHC)

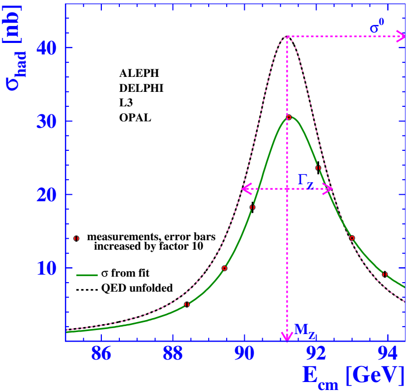


– measurement of couplings $\mathrm{WWWW}, \mathrm{ZZWW},$ etc.
– sensitivity to EW symmetry breaking

+ much more @ LHC ! (Higgs physics, WWW production, etc.)

Precision study of the Z line shape





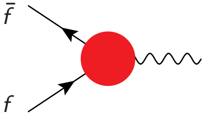
Unfolded resonance:

$$\sigma_{\rm res}(s) = \sigma^0 \, \frac{s\,\Gamma_{\rm Z}^2}{\left| s - M_{\rm Z}^2 + {\rm i} M_{\rm Z} \Gamma_{\rm Z} \frac{s}{M_{\rm Z}^2} \right|^2}$$

Resonance observables:

- Z massand width: $M_{\rm Z}, \Gamma_{\rm Z}$
- peak cross section: $\sigma^0_{\rm had}$
- various asymmetries: $A_{\rm FB}, A_{\rm LR}$, etc.
- ratios of decay widths: $R_\ell = \frac{\Gamma_{\rm had}}{\Gamma_\ell}$, etc.

Effective Z-boson–fermion couplings

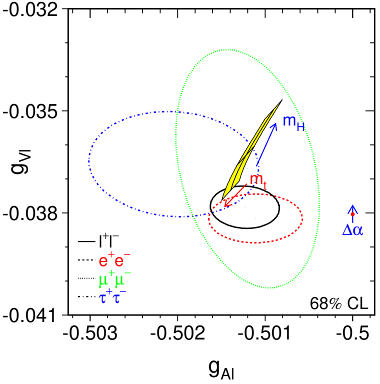


$$Z_\mu \quad = \quad ie\gamma_\mu(g_{Vf} - g_{Af}\gamma_5)$$

Leptonic couplings from
LEP1 asymmetry measurements:

e.g.
$$A_{\mathrm{FB}}^{0,f} = \frac{3}{4}\mathcal{A}_{\mathrm{e}}\mathcal{A}_f$$
$$\mathcal{A}_f = \frac{2g_{Vf}g_{Af}}{g_{Vf}^2 + g_{Af}^2}$$

LEPEWWG '04



Good agreement with SM

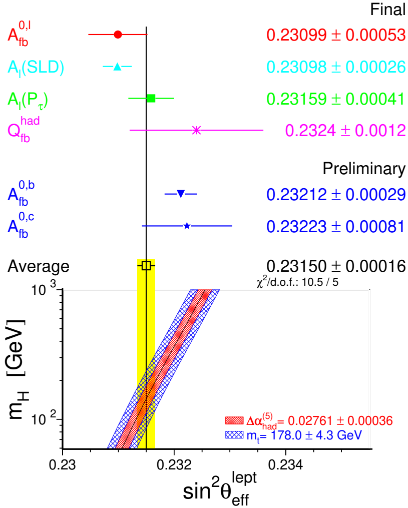
- lepton universality confirmed
- constraints on m_{t} and M_{H}

Translation of effective couplings into effective weak mixing angle

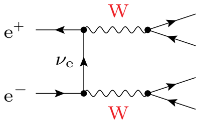
$$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} = \frac{1}{4}\left(1 - \mathrm{Re}\left\{\frac{g_{Vl}}{g_{Al}}\right\}\right)$$

Important features:

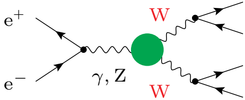
- combination of very different observables
- $\sim 3\sigma$ difference between $A_{\mathrm{FB}}^{0,b}$(LEP) and A_ℓ(SLD)
- high sensitivity to M_{H}



W-pair production $e^+e^- \to WW \to 4f(+\gamma)$



dominates near W-pair threshold



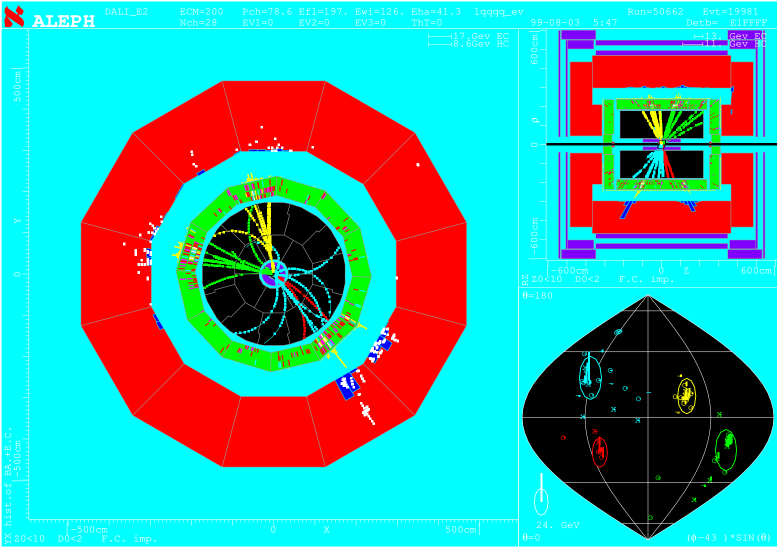
contains $\gamma WW/ZWW$ couplings

Physics goals:

- non-abelian gauge-boson self-interactions
  $\hookrightarrow$ constrain non-standard $\gamma WW/ZWW$ couplings
- W-pair cross section σ_{WW}
- precision measurement of W mass M_W

$\Rightarrow$ Theoretical requirement:
understand $2 \to 4$ process with 0.5% precision

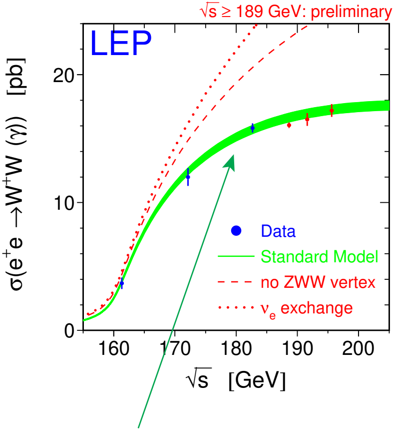
A typical 4-jet event observed at ALEPH



Made on 3-Aug-1999 14:42:48 by lancon with DALI_E2.
Filename: DC050662_019981_990803_1442.PS

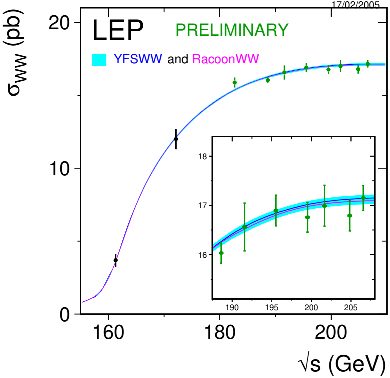
Total WW cross section at LEP2

Status of 1999: (LEPEWWG ’99)



GENTLE (Bardin et al.)
only universal EW corrections
↪ theoretical uncertainty $\sim \pm 2\%$

Final result: (LEPEWWG ’05)

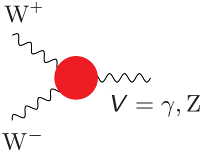


YFSWW (Jadach et al.) / RacoonWW (Denner et al.)
non-universal corrections included
↪ th. uncertainty $\sim \pm 0.5\%$ for $\sqrt{s} > 170\,\mathrm{GeV}$

(Non-)standard TGCs

Gaemers, Gounaris '79; Hagiwara, Hikasa, Peccei, Zeppenfeld '87; Bilenky, Kneur, Renard, Schildknecht '93; etc.

General parametrization (C- and P-conserving):



$$\begin{aligned}\mathcal{L}_{VWW} &= -\mathrm{i}eg_{VWW}\Big\{ g_1^V \big(W^+_{\mu\nu}W^{-,\mu}V^\nu - W^{-,\mu\nu}W^+_\mu V_\nu\big) \\ &\qquad + \kappa_V W^+_\mu W^-_\nu V^{\mu\nu} + \frac{\lambda_V}{M_\mathrm{W}^2} W^+_{\rho\mu} W^{-,\mu}_{\nu} V^{\nu\rho}\Big\}\end{aligned}$$

Meaning for static W^+ bosons:

$$\begin{aligned} Q_\mathrm{W} &= eg_1^\gamma &&= \text{electric charge } (= e \text{ by charge conservation}) \\ \mu_\mathrm{W} &= \frac{e}{2M_\mathrm{W}}\left(g_1^\gamma + \kappa_\gamma + \lambda_\gamma\right) &&= \text{magnetic dipole moment} \\ q_\mathrm{W} &= -\frac{e}{M_\mathrm{W}^2}\left(\kappa_\gamma - \lambda_\gamma\right) &&= \text{electric quadrupole moment}\end{aligned}$$

SM values:

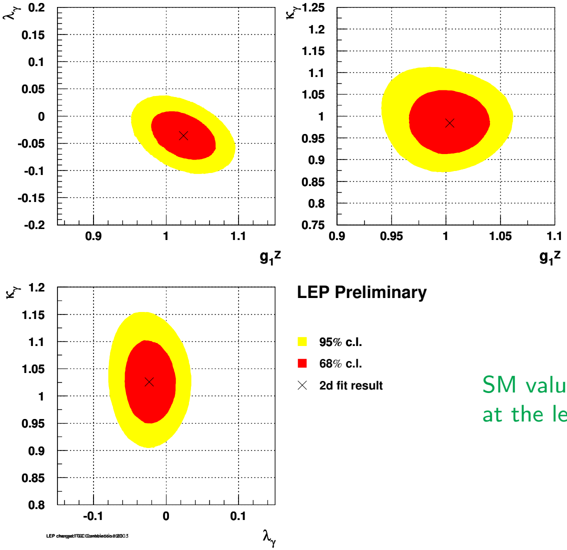
$$g_1^V = \kappa_V = 1, \qquad \lambda_V = 0$$

Restriction to SU(2)×U(1)-symmetric dim-6 operators:

$$\kappa_\mathrm{Z} = g_1^\mathrm{Z} - (\kappa_\gamma - 1)\tan^2\theta_\mathrm{w}, \qquad \lambda_\mathrm{Z} = \lambda_\gamma$$

LEP2 constraints on charged TGCs

LEPEWWG '04

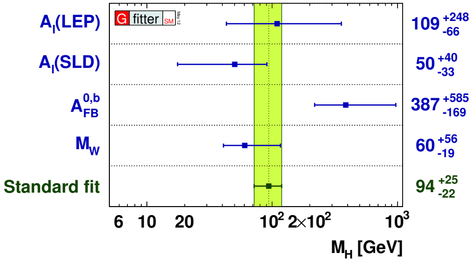


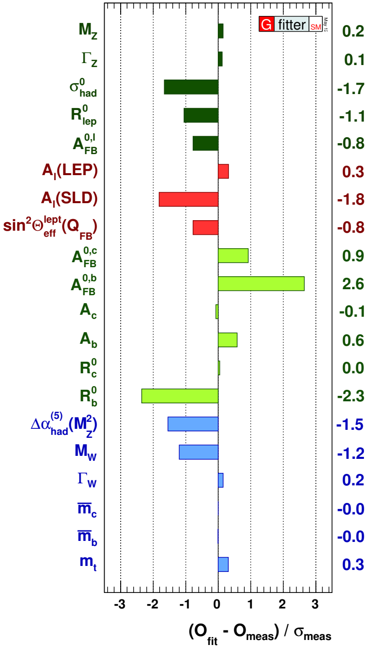
$$\Delta g_1^Z = -0.009^{+0.022}_{-0.021}$$
$$\Delta\kappa_\gamma = -0.016^{+0.042}_{-0.047}$$
$$\lambda_\gamma = -0.016^{+0.021}_{-0.023}$$

SM values verified
at the level of 2–4%

Global status of the SM before Summer 2012

- SM fit yields very good agreement (all "pulls" $\lesssim 2\sigma$)
- Tension between $A_\ell(\mathrm{SLD}) = A^\ell_{\mathrm{LR}}(\mathrm{SLD})$ and $A^{0,\mathrm{b}}_{\mathrm{FB}}(\mathrm{LEP})$
  $\hookrightarrow$ $A^\ell_{\mathrm{FB}}(\mathrm{LHC})$ will be interesting!
- SM fit predicts a light Higgs boson with $M_{\mathrm{H}} \sim 100\,\mathrm{GeV}$

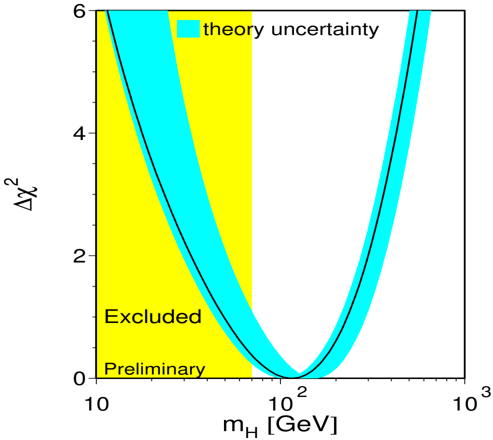




Bounds on M_{H} (95% C.L.) – a brief history

LEPEWWG '97–'12

Status Summer 1997

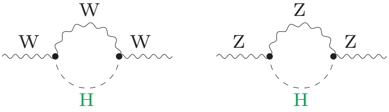


$M_{\mathrm{H}} > 114.4\,\mathrm{GeV}$ (LEPHIGGS '02)

$\mathrm{e}^+\mathrm{e}^- \not\to \mathrm{ZH}$ at LEP2

SM fit favours

perturbative regime for M_{H}



Highest sensitivity via

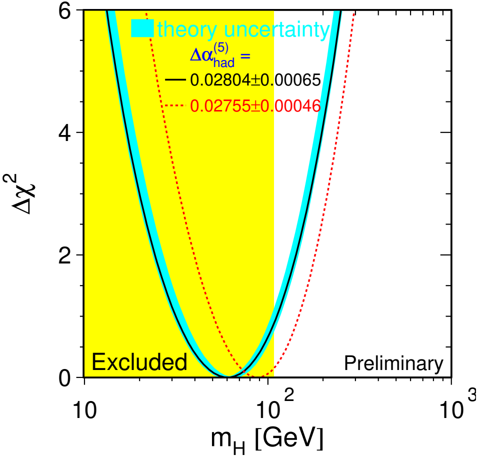
"high-precision observables":

m_{t}, M_{W}, $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$, etc.

Bounds on M_{H} (95% C.L.) – a brief history

LEPEWWG '97–'12

Status Summer 2000

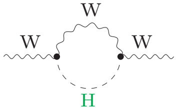


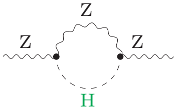
$M_{\mathrm{H}} > 114.4\,\mathrm{GeV}$ (LEPHIGGS '02)

$\mathrm{e}^+\mathrm{e}^- \not\to \mathrm{ZH}$ at LEP2

SM fit favours

$M_{\mathrm{H}} \lesssim 200\,\mathrm{GeV}$

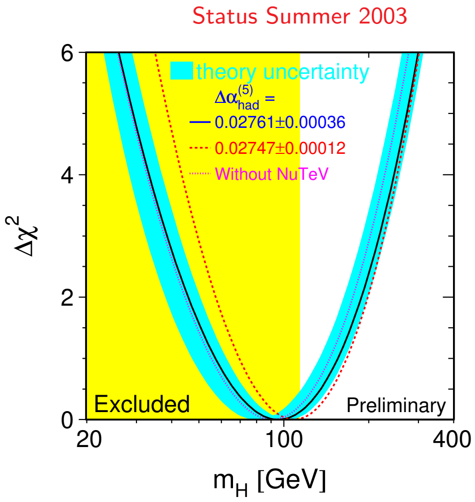




Highest sensitivity via
"high-precision observables":
m_{t}, M_{W}, $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$, etc.

Bounds on M_H (95% C.L.) – a brief history

LEPEWWG '97–'12

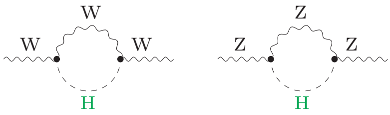


$M_\mathrm{H} > 114.4\,\mathrm{GeV}$ (LEPHIGGS '02)

$\mathrm{e^+e^-} \not\to \mathrm{ZH}$ at LEP2

SM fit favours

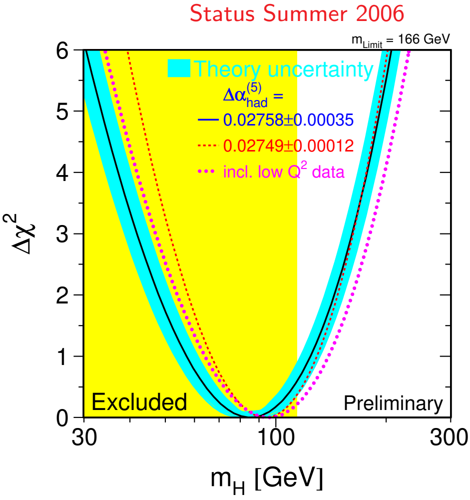
$M_\mathrm{H} \lesssim 200\,\mathrm{GeV}$



Highest sensitivity via "high-precision observables": m_t, M_W, $\sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}$, etc.

Bounds on M_H (95% C.L.) – a brief history

LEPEWWG '97–'12

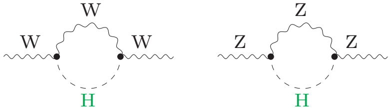


$M_\mathrm{H} > 114.4\,\mathrm{GeV}$ (LEPHIGGS '02)

$e^+e^- \not\to ZH$ at LEP2

SM fit favours

$M_\mathrm{H} < 166\,\mathrm{GeV}$



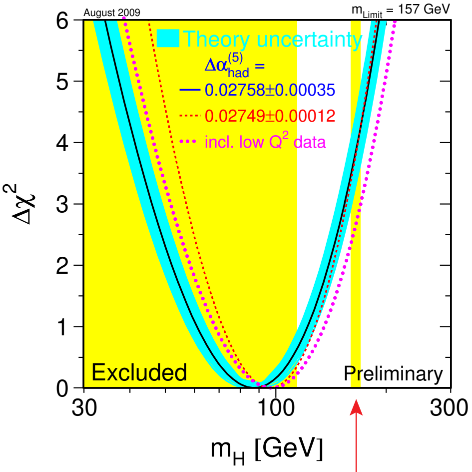
Highest sensitivity via "high-precision observables":

m_t, M_W, $\sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}$, etc.

Bounds on M_{H} (95% C.L.) – a brief history

LEPEWWG '97–'12

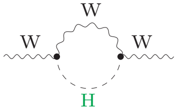
Status Summer 2009

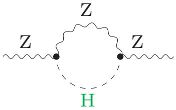


Tevatron excludes
M_{H} window around $2M_{\mathrm{W}}$

$M_{\mathrm{H}} > 114.4\,\mathrm{GeV}$ (LEPHIGGS '02)
$\mathrm{e^+e^-} \not\to \mathrm{ZH}$ at LEP2

SM fit favours
$M_{\mathrm{H}} < 157\,\mathrm{GeV}$



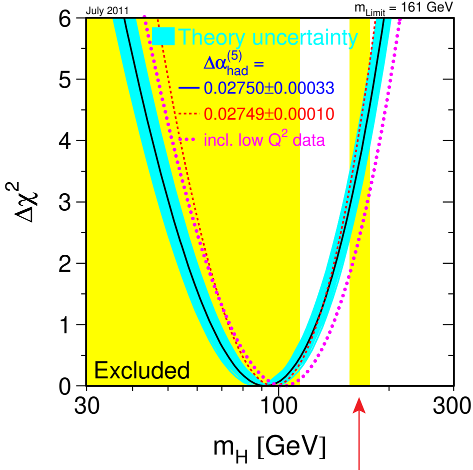


Highest sensitivity via
"high-precision observables":
m_{t}, M_{W}, $\sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}$, etc.

Bounds on M_H (95% C.L.) – a brief history

LEPEWWG '97–'12

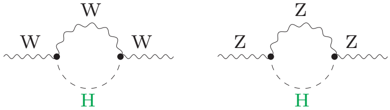
Status Summer 2011



Tevatron excludes
M_H window around $2M_\mathrm{W}$

$M_\mathrm{H} > 114.4\,\mathrm{GeV}$ (LEPHIGGS '02)
$\mathrm{e^+e^-} \not\to \mathrm{ZH}$ at LEP2

SM fit favours
$M_\mathrm{H} < 161\,\mathrm{GeV}$

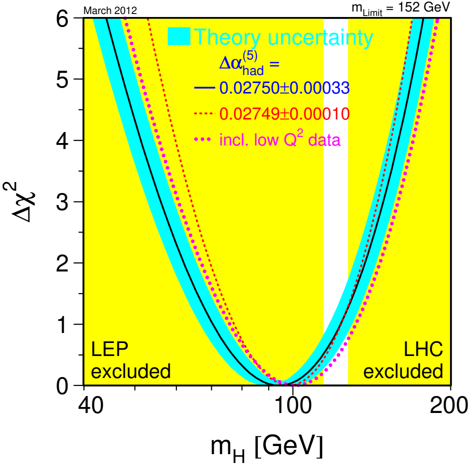


Highest sensitivity via
"high-precision observables":
m_t, M_W, $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$, etc.

Bounds on M_H (95% C.L.) – a brief history

LEPEWWG '97–'12

Status before Summer 2012



Open window not excluded by the LHC:

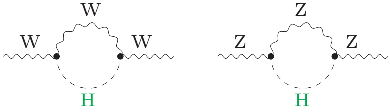
$122\,\mathrm{GeV} < M_\mathrm{H} < 127\,\mathrm{GeV}$

$M_\mathrm{H} > 114.4\,\mathrm{GeV}$ (LEPHIGGS '02)

$\mathrm{e}^+\mathrm{e}^- \not\to \mathrm{ZH}$ at LEP2

SM fit favours

$M_\mathrm{H} < 152\,\mathrm{GeV}$



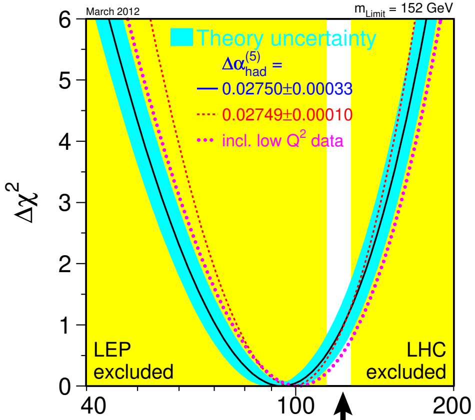
Highest sensitivity via "high-precision observables":

m_t, M_W, $\sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}$, etc.

Bounds on M_{H} (95% C.L.) – a brief history

LEPEWWG '97–'12

Status before Summer 2012



Higgs-boson discovery
in July 2012!

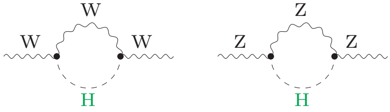
Open w … C:

$122\,\mathrm{GeV} < M_{\mathrm{H}} < 127\,\mathrm{GeV}$

$M_{\mathrm{H}} > 114.4\,\mathrm{GeV}$ (LEPHIGGS '02)

$\mathrm{e}^+\mathrm{e}^- \not\to \mathrm{ZH}$ at LEP2

SM fit favours
$M_{\mathrm{H}} < 152\,\mathrm{GeV}$



Highest sensitivity via
"high-precision observables":
m_{t}, M_{W}, $\sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}$, etc.

Status of the SM after 2012

- SM particle content experimentally completely established
- Collider data in very good agreement with SM predictions (radiative corrections essential)

  $\Rightarrow$ SM confirmed as Quantum Field Theory!
- Successful constraints of respective mass ranges before top-quark and Higgs-boson discoveries

  $\Rightarrow$ Major triumph of EW precision physics!

Upcoming lectures:

- theoretical background of EW higher-order calculations
- salient features of EW corrections
- EW phenomenology and EW precision physics at the LHC

Literature

Textbooks:

- M. Böhm, A. Denner, H. Joos, "Gauge Theories of the Strong and Electroweak Interaction"
- M.E. Peskin, D.V. Schroeder, "An Introduction to Quantum Field Theory"
- M.D. Schwartz, "Quantum Field Theory and the Standard Model"
- G. Sterman, "Quantum Field Theory"
- S. Weinberg, "The Quantum theory of fields. Vol. 1: Foundations"
- S. Weinberg, "The Quantum Theory of Fields, Vol. 2: Modern Applications"

Reviews on electroweak corrections: (for original papers see references therein)

- A. Denner, "Techniques for calculation of electroweak radiative corrections at the one loop level and results for W physics at LEP-200," Fortsch. Phys. **41** (1993), 307-420 [arXiv:0709.1075 [hep-ph]].
- A. Denner, S. Dittmaier, "Electroweak Radiative Corrections for Collider Physics", Phys.Rept. 864 (2020) 1-163, arXiv:1912.06823 [hep-ph].