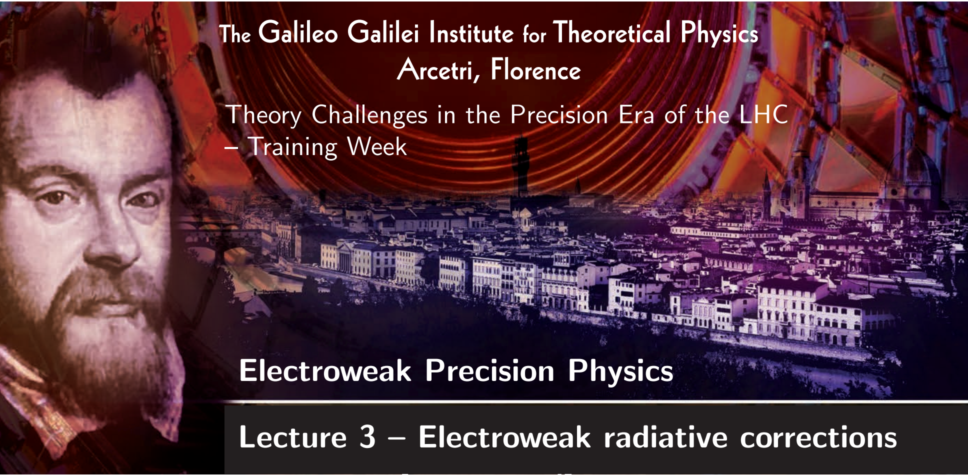

The Galileo Galilei Institute for Theoretical Physics
Arcetri, Florence

Theory Challenges in the Precision Era of the LHC
– Training Week

Electroweak Precision Physics

Lecture 3 – Electroweak radiative corrections

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Relevance of electroweak (EW) corrections at the LHC

Precision measurements at the LHC

- cross-section uncertainties for single-W/Z production:
  Δ(luminosity) $\sim 4\%$, Δ(PDF) $\sim 2{-}3\%$
- often 1% precision on shapes of distributions or ratios of cross sections
- high-precision measurements of M_W, $\sin^2\theta_\mathrm{eff}^\mathrm{lept}$:
  $\Delta M_\mathrm{W}/M_\mathrm{W} \lesssim 2\cdot 10^{-4}$, $\Delta\sin^2\theta_\mathrm{eff}^\mathrm{lept}/\sin^2\theta_\mathrm{eff}^\mathrm{lept} \lesssim 4\cdot 10^{-4}$
- energy reach deep into the TeV range with several-% precision

Size of EW corrections

generic size $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_\mathrm{s}^2) \sim 1\%$ suggests NLO EW $\sim$ NNLO QCD

but systematic enhancements possible, e.g.

- by photon emission
  $\hookrightarrow$ kinematical effects, mass-singular logs $\propto \alpha\ln(m_\mu/Q)$ for muons, etc.,
  often several-10% effects near shoulders of distributions
- at high energies
  $\hookrightarrow$ EW Sudakov logs $\propto (\alpha/s_\mathrm{W}^2)\ln^2(M_\mathrm{W}/Q)$ and subleading logs,
  typically several-10% effects in the TeV range

Peculiarities of EW corrections $\to$ subjects of this lecture

Large universal corrections

- induced by photonic vacuum polarization
  and corrections to the ρ-parameter
- can often be absorbed into leading-order predictions
  by appropriate choice of EW input parameter scheme

Instability of W and Z bosons

- realistic observables have to be defined via decay products (leptons, γs, jets)
- off-shell effects $\sim \mathcal{O}(\Gamma/M) \sim \mathcal{O}(\alpha)$ are part of the NLO EW corrections

Photon–jet separation

- non-trivial due to $q \to q + \gamma$ splitting

  $\hookrightarrow$ separation, e.g., by quark-to-photon "fragmentation function"
- complication by photon-induced jets via $\gamma^* \to q\bar{q}$

  $\hookrightarrow$ description by "fragmentation" or "conversion function"

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Electroweak input parameter schemes

SM input parameters: (natural choice)

$\alpha_{\rm s}$, α, $M_{\rm W}$, $M_{\rm Z}$, $M_{\rm H}$, m_f, $V_{\rm CKM}$

Issues:

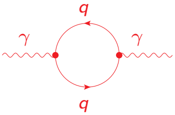
- Setting of α: process-specific choice to
  - avoid sensitivity to non-perturbative light-quark masses
  - minimize universal EW corrections

  Schemes: fix $M_{\rm W}$, $M_{\rm Z}$, and α
  - $\alpha(0)$-scheme: $\alpha = \alpha(0) = 1/137.0\dots$
  - $\alpha(M_{\rm Z})$-scheme: $\alpha = \alpha(M_{\rm Z}^2) \approx 1/129$
  - G_μ-scheme: $\alpha = \alpha_{G_\mu} = \sqrt{2}G_\mu M_{\rm W}^2(1 - M_{\rm W}^2/M_{\rm Z}^2)/\pi \approx 1/132$

  $\hookrightarrow$ Some arbitrariness of $\sim 3{-}6\%$ per factor of α in LO prediction

- Warnings / pitfalls:
  - α must not be set diagram by diagram, but global factors like $\alpha(0)^m\alpha_{G_\mu}^n$ in gauge-invariant contributions mandatory !
  - weak mixing angle: $s_{\rm W} \neq$ free parameter if $M_{\rm W}$ and $M_{\rm Z}$ are fixed !
  - Yukawa couplings are uniquely fixed by fermion masses !

Running electromagnetic coupling $\alpha(s)$:



becomes sensitive to unphysical quark masses m_q
for $|s|$ in GeV range and below (non-perturbative regime)
$\hookrightarrow$ δZ_e and δZ_{AA} involve $\ln m_f$ with $f = q, \ell$

Solution: fit hadronic part of $\Delta\alpha(s) \; = \; -\mathrm{Re}\{\Sigma^{AA}_{\mathrm{T,R}}(s)/s\}$ and thus of δZ_e

via dispersion relation to $R(s) \; = \; \dfrac{\sigma(\mathrm{e}^+\mathrm{e}^- \to \text{hadrons})}{\sigma(\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-)}$ Jegerlehner et al.

$\Rightarrow$ Running elmg. coupling: $\quad \alpha(s) \; = \; \dfrac{\alpha(0)}{1 - \Delta\alpha_{\mathrm{ferm}\neq\mathrm{top}}(s)}$

Universal contribution of $\Delta\alpha(M_\mathrm{Z}^2)$ to renormalization constants:

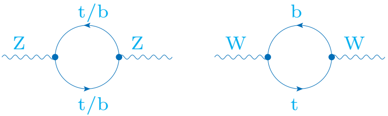
$$\delta Z_e = \tfrac{1}{2}\,\Delta\alpha(M_\mathrm{Z}^2) + \dots, \qquad \delta Z_{AA} = -\Delta\alpha(M_\mathrm{Z}^2) + \dots$$

Leading correction to the ρ-parameter:

mass differences in fermion doublets break custodial SU(2) symmetry

$\hookrightarrow$ large effects from bottom–top loops in W/Z self-energies Veltman '77

- large corrections $\propto m_\mathrm{t}^2$ in $\Sigma_\mathrm{T}^{VV}(s)$, $V = W, Z$



$$\frac{\Sigma_\mathrm{T}^{ZZ}(s)}{M_\mathrm{Z}^2} - \frac{\Sigma_\mathrm{T}^{WW}(s)}{M_\mathrm{W}^2} \underset{|s| \ll m_\mathrm{t}^2}{\widetilde{\quad\quad}} \frac{3 G_\mu m_\mathrm{t}^2}{8\sqrt{2}\pi^2} \equiv \Delta\rho_\mathrm{top}$$

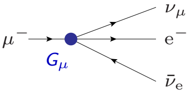
- m_t^2-enhanced terms show up in δs_W, δc_W,
  but cancel in $\Sigma_\mathrm{T,R}^{VV}(s)$
- leading terms to $\Delta\rho$ known beyond NLO

Universal contribution of $\Delta\rho$ to renormalization constants:

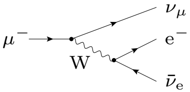
$$\frac{\delta c_\mathrm{W}^2}{c_\mathrm{W}^2} = -\Delta\rho_\mathrm{top} + \ldots, \qquad \frac{\delta s_\mathrm{W}^2}{s_\mathrm{W}^2} = \underbrace{\frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}}_{\text{major effect due to } 1/s_\mathrm{W}^2 \text{ enhancement}} \Delta\rho_\mathrm{top} + \ldots$$

Fermi constant G_μ as input parameter – the quantity Δr

μ decay including higher-order corrections



+ QED corrections $\Longleftrightarrow$



+ EW corrections

$\hookrightarrow$ Relation between G_μ, $\alpha(0)$, M_W, and M_Z including corrections:

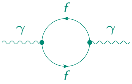
$$\alpha_{G_\mu} \equiv \frac{\sqrt{2}}{\pi} G_\mu M_\mathrm{W}^2 \left(1 - \frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right) = \alpha(0)(1 + \Delta r)$$

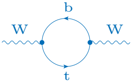
Δr comprises quantum corrections to μ decay
(beyond electromagnetic corrections in Fermi model)

Sirlin '80, Marciano, Sirlin '80

$$\Delta r_{1-\mathrm{loop}} = \Delta\alpha(M_\mathrm{Z}^2) - \frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\Delta\rho_\mathrm{top} + \Delta r_\mathrm{rem}(M_\mathrm{H})$$

$\Delta\alpha(M_\mathrm{Z}^2)$	$\frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\Delta\rho_\mathrm{top}$	$\Delta r_\mathrm{rem}(M_\mathrm{H})$
$\sim 6\%$	$\sim 3\%$	$\sim 1\%$
$\alpha \ln(m_f/M_\mathrm{Z})$	$G_\mu m_\mathrm{t}^2$	$\alpha \ln(M_\mathrm{H}/M_\mathrm{Z})$





Excursion: predicting M_W from muon decay

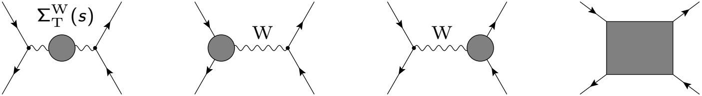
Measure G_μ in μ decay and trade M_W for G_μ as input in

$$\frac{\sqrt{2}}{\pi}\, G_\mu\, M_\mathrm{W}^2 \left(1 - \frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right) \;=\; \alpha(0)(1+\Delta r) \qquad \to \text{ solve for } M_\mathrm{W}$$

Δr depends on all input parameters $\quad\to\quad$ sensitivity to m_t, M_H in SM fit

Contributions to Δr:

$+$ virtual corrections:



W self-energy $\qquad$ $\mathrm{W}l\nu_l$ vertex correction $\qquad$ box diagrams

$+$ photonic bremsstrahlung in the SM

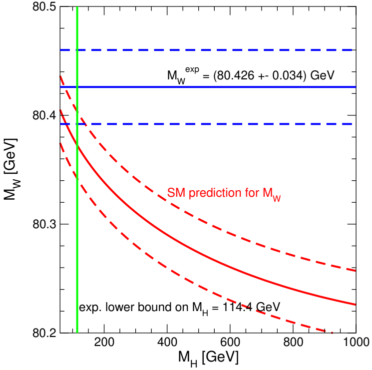
$-$ photonic bremsstrahlung in the Fermi model

$+$ full two-loop contributions $+$ higher-order corrections to ρ-parameter

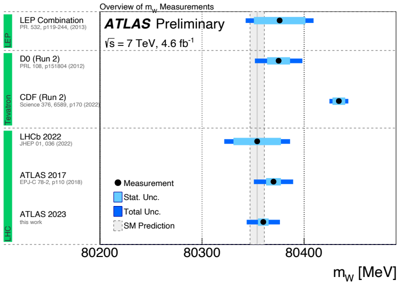
v.Ritbergen,Stuart '98; Seidensticker,Steinhauser '99; Freitas et al. '00-'02; Awramik,Czakon '02/'03; Onishchenko,Veretin '02

Confronting predicted and measured values of M_W

Hollik et al. '03



ATLAS '23



- Current theoretical precision: $\Delta M_W \sim 0.003\,\mathrm{GeV}$
- Most precise measurements:

CDF '22: $(80.4335 \pm 0.0094)\,\mathrm{GeV}$ (controversial analysis)

ATLAS '23: $(80.360 \pm 0.016)\,\mathrm{GeV}$

Adaption of input parameter schemes for cross-section predictions

Aim: absorb universal corrections from $\Delta\alpha$ and $\Delta\rho$ into leading-order (LO) predictions as much as possible

- $\Delta\alpha^n$ terms can be absorbed to all orders
- $\Delta\rho^n$ terms can be absorbed at least to two-loop order
- factor α in δ_{EW} can still be adjusted appropriately
  (e.g. $\alpha \to \alpha(0)$ if γ radiation dominates, $\alpha \to \alpha_{G_\mu}$ if weak corrections dominate)

Consider NLO cross section:

$$\sigma_{\mathrm{NLO}} = \alpha^N A_{\mathrm{LO}} \, (1 + \delta_{\mathrm{EW}}), \qquad \delta_{\mathrm{EW}} = \mathcal{O}(\alpha)$$

- for process at some generic energy scale $Q \gtrsim M_{\mathrm{W}}$
- with N_γ external photons (separable from $\gamma^* \to f\bar{f}$)
- with N_W couplings of $\mathrm{W/Z}$ in dominating LO diagrams
  ($\Delta\rho$ effects from c_{W} from difference between $\mathrm{W/Z}$ ignored)
  $\hookrightarrow$ N_W factors of $g_2^2 \propto 1/s_{\mathrm{W}}^2$ in LO cross section

$\alpha(0)$-scheme: $\quad \sigma_{\mathrm{LO}} = \alpha(0)^N A_{\mathrm{LO}}$

$$\delta_{\mathrm{EW}}^{\alpha(0)} = 2N\,\delta Z_e + N_\gamma\,\delta Z_{AA} - N_W \frac{\delta s_{\mathrm{W}}^2}{s_{\mathrm{W}}^2} + \dots$$

$\alpha(0)$-scheme: $\quad \sigma_{\mathrm{LO}} = \alpha(0)^N \, A_{\mathrm{LO}}$

$$\delta_{\mathrm{EW}}^{\alpha(0)} = (N - N_\gamma)\,\Delta\alpha(M_\mathrm{Z}^2) - N_W \frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\Delta\rho_{\mathrm{top}} + \dots$$

$\Rightarrow$ cancellation of $\Delta\alpha$, $\Delta\rho$ for $N_\gamma = N$, $N_W = 0$,

i.e. for processes such as $\gamma\gamma \to \ell^+\ell^-, \mathrm{W}^+\mathrm{W}^-$, $\mathrm{e}\gamma \to \mathrm{e}\gamma$, etc.

$\alpha(M_\mathrm{Z})$-scheme: $\quad \sigma_{\mathrm{LO}} = \alpha(M_\mathrm{Z}^2)^N \, A_{\mathrm{LO}}$

$$\delta_{\mathrm{EW}}^{\alpha(M_\mathrm{Z})} = \delta_{\mathrm{EW}}^{\alpha(0)} - N\Delta\alpha(M_\mathrm{Z}) + \dots = -N_\gamma\,\Delta\alpha(M_\mathrm{Z}^2) - N_W \frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\Delta\rho_{\mathrm{top}} + \dots$$

$\Rightarrow$ cancellation of $\Delta\alpha$, $\Delta\rho$ for $N_\gamma = 0$, $N_W = 0$,

which is not possible, since there is at least one Z exchange for $N_\gamma = 0$.

But: $\quad \gamma$ exchange dominates over Z exchange for $Q \ll M_\mathrm{W}$ $(N_W \to 0)$

$\Rightarrow$ "$\alpha(Q)$ scheme" for neutral-current processes appropriate, $\mathrm{e}^+\mathrm{e}^-/q\bar{q} \to \ell^+\ell^-$, etc.

G_μ-scheme: $\quad \sigma_{\mathrm{LO}} = \alpha_{G_\mu}^N \, A_{\mathrm{LO}}$

$$\delta_{\mathrm{EW}}^{G_\mu} = \delta_{\mathrm{EW}}^{\alpha(0)} - N\Delta r + \dots = -N_\gamma\,\Delta\alpha(M_\mathrm{Z}^2) + (N - N_W)\frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\Delta\rho_{\mathrm{top}} + \dots$$

$\Rightarrow$ cancellation of $\Delta\alpha$, $\Delta\rho$ for $N_\gamma = 0$, $N_W = N$,

i.e. for W/Z decays, all EW processes without external γ at $Q \gtrsim M_\mathrm{W}$

Mixed scheme: $\sigma_{\rm LO} = \alpha(G_\mu)^n \alpha(0)^m A_{\rm LO}$

$$\delta_{\rm EW}^{\rm mix} = \delta_{\rm EW}^{\alpha(0)} - n\,\Delta r + \ldots \;=\; (m - N_\gamma)\,\Delta\alpha(M_{\rm Z}^2) + (n - N_W)\,\frac{c_{\rm W}^2}{s_{\rm W}^2}\Delta\rho_{\rm top} + \ldots$$

$\Rightarrow$ cancellation of $\Delta\alpha$, $\Delta\rho$ for $N_\gamma = m$, $N_W = n$,

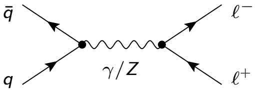
i.e. for all EW processes with m external γ at $Q \gtrsim M_{\rm W}$

Note: m does not include γ as parton from $\mathrm{p}/\bar{\mathrm{p}}$, because processes induced by $\gamma \to q\bar{q}, \ell\bar{\ell}$ cannot be separated form pure γ processes

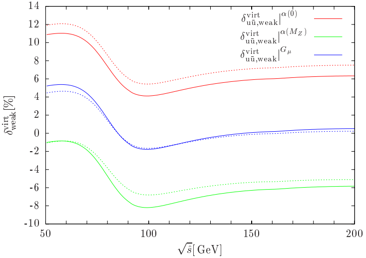
Harland-Lang et al. '16

Example: weak corrections to Z production

(partonic cross sections, no photonic corrections)



S.D., Huber '09



- expected off-sets between NLO EW corrections in different schemes
- most suited EW input parameter schemes:

  $\sqrt{\hat s} \gtrsim M_{\rm Z}$: $\quad G_\mu$ scheme

  $\sqrt{\hat s} \lesssim 70\,{\rm GeV}$: $\quad \alpha(M_{\rm Z})$ scheme scheme $\quad (\alpha(Q)$ scheme for $Q = \sqrt{\hat s} \ll M_{\rm Z})$
- dashed lines include leading 2-loop effects from $\Delta\alpha$ and $\Delta\rho$
  $\hookrightarrow$ highest stability against h.o. corrections in recommended schemes

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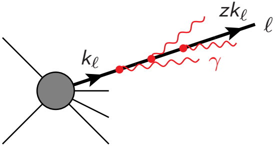
Collinear final-state radiation (FSR) off leptons

Leading logarithmic effect is universal:



$$\sigma_{\mathrm{LL,FSR}} = \int \underbrace{\mathrm{d}\sigma^{\mathrm{LO}}(k_l)}_{\text{hard scattering}} \int_0^1 \mathrm{d}z \underbrace{\Gamma_{\ell\ell}^{\mathrm{LL}}(z, Q^2)}_{\substack{\text{leading-log structure} \\ \text{function, } Q\,=\,\text{typ. scale}}} \Theta_{\mathrm{cut}}(zk_l)$$

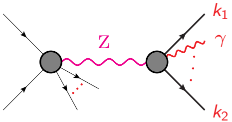
- $\Gamma_{\ell\ell}^{\mathrm{LL}}(z, Q^2)$ known to $\mathcal{O}(\alpha^5)$ + soft exponentiation, equivalent description by QED parton showers
- $\mathcal{O}(\alpha)$ approximation: $\quad \Gamma_{\ell\ell}^{\mathrm{LL},1}(z, Q^2) = \dfrac{\alpha(0)}{2\pi}\left[\ln\left(\dfrac{Q^2}{m_\ell^2}\right) - 1\right]\left(\dfrac{1+z^2}{1-z}\right)_+$
- Alternative approach: QED parton shower
  $\hookrightarrow$ advantage: photons described with finite p_{T} and definite multiplicity

Impact on predictions:

- log-enhanced corrections for "bare" leptons (muons) $\rightarrow$ large radiative tails
- KLN theorem:
  mass-singular FSR effects cancel if $(\ell\gamma)$ system is inclusive
  (full integration over z)
- full FSR not universal,
  in general not even separable from other EW corrections
  (possible only if LO amplitudes do not include W bosons)

Radiative tail from final-state radiation

occurs if resonances reconstructed from decay products



Typical situations: $\mathrm{e^+e^-} \to \mathrm{WW/ZZ} \to 4f$,

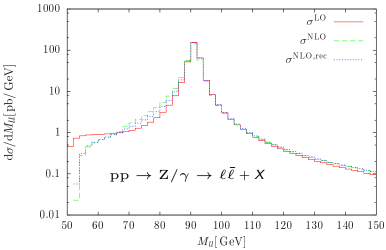
$\mathrm{pp} \to \mathrm{Z}/\gamma \to \ell\bar{\ell} + X$

Final-state radiation:

resonance for

$$M^2 = (k_1+k_2)^2 < (k_1+k_2+k_\gamma)^2 \sim M_\mathrm{Z}^2$$

$\hookrightarrow$ radiative tail in distribution $\frac{\mathrm{d}\sigma}{\mathrm{d}M}$
of reconstructed invariant mass M
for $M < M_\mathrm{Z}$

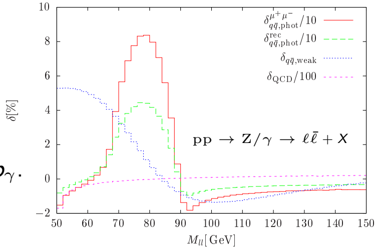


S.D., Huber '09

Example: Single-Z production

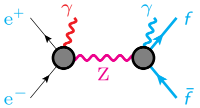
- radiative tail with corrections up to $\sim 80\%$
- FSR effect drastically reduced
  by photon recombination ("rec"):

  If $R_{l\gamma} < 0.1$ then $(l\gamma) \to \tilde{l}$ with $p_{\tilde{l}} = p_l + p_\gamma$.



Comparison with radiative tail from initial-state radiation

occurs if initial state is fixed



Typical situations: $\quad e^+e^- \to Z/\gamma \to f\bar{f}$,
$\quad\quad\quad\quad\quad\quad\quad \mu^+\mu^- \to Z, H, ? \to f\bar{f}$

$\hookrightarrow$ scan over s-channel resonance in $\sigma_{\mathrm{tot}}(s)$ by changing CM energy $\sqrt{s}$

Initial-state radiation:

Z can become resonant for $s = (p_+ + p_-)^2 \; > \; (p_+ + p_- - k_\gamma)^2 \sim M_Z^2$

$\hookrightarrow$ radiative tail for $s > M_Z^2$ due to "radiative return"

Final-state radiation:

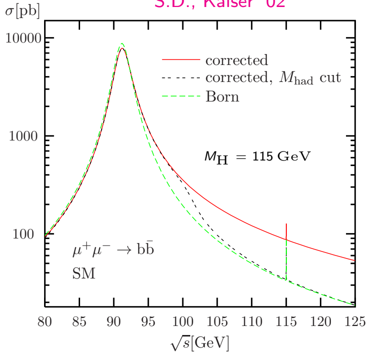
$s = k_Z^2 \sim M_Z^2$ for FSR

$\hookrightarrow$ only rescaling of resonance

Example:

cross section for $\mu^-\mu^+ \to b\bar{b}$ in lowest order, including photonic and QCD corrections, with and without invariant-mass cut $\sqrt{s} - M(b\bar{b}) < 10\,\mathrm{GeV}$

S.D., Kaiser '02



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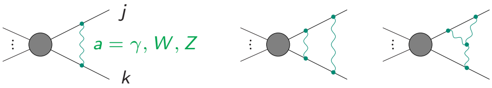
Electroweak corrections at high energies

Sudakov logarithms induced by soft gauge-boson exchange



etc.

$+$ sub-leading logarithms from collinear singularities

Typical impact on $2 \to 2$ reactions at $\sqrt{s} \sim 1\,\mathrm{TeV}$:

$$\delta_{\mathrm{LL}}^{1-\mathrm{loop}} \sim -\frac{\alpha}{\pi s_{\mathrm{W}}^2} \ln^2\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq -26\%, \qquad \delta_{\mathrm{NLL}}^{1-\mathrm{loop}} \sim +\frac{3\alpha}{\pi s_{\mathrm{W}}^2} \ln\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq 16\%$$

$$\delta_{\mathrm{LL}}^{2-\mathrm{loop}} \sim +\frac{\alpha^2}{2\pi^2 s_{\mathrm{W}}^4} \ln^4\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq 3.5\%, \qquad \delta_{\mathrm{NLL}}^{2-\mathrm{loop}} \sim -\frac{3\alpha^2}{\pi^2 s_{\mathrm{W}}^4} \ln^3\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq -4.2\%$$

$\Rightarrow$ Corrections still relevant at 2-loop level

Note: differences to QED/QCD where Sudakov logs cancel

- massive gauge bosons W, Z can be reconstructed
  $\hookrightarrow$ no need to add "real W, Z radiation"
- non-Abelian charges of W, Z are "open" $\rightarrow$ Bloch–Nordsieck theorem not applicable

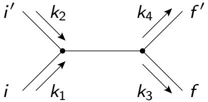
Extensive theoretical studies at fixed perturbative (1-/2-loop) order and suggested resummations via evolution equations

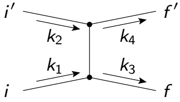
Beccaria et al.; Beenakker, Werthenbach; Ciafaloni, Comelli; Denner, Pozzorini; Fadin et al.; Hori et al.; Melles; Kühn et al., Denner et al.; Manohar et al. '00–

High-energy limit – Sudakov versus Regge regime

Sudakov regime: all invariants $k_i \cdot k_j \gg M_\mathrm{W}^2$!

Example:
$2 \to 2$ particle process





Kinematic variables in centre-of-mass frame in high-energy limit ($k_j^2 \to 0$):

$$
\begin{aligned}
s &= (k_1+k_2)^2 \sim 4E^2, && E = \text{beam energy}, \\
t &= (k_1-k_3)^2 \sim -4E^2\sin^2(\theta/2), && \theta = \text{scattering angle}, \\
M_{34} &= \sqrt{s} \sim 2E, \\
k_\mathrm{T} &= k_{3,\mathrm{T}} \sim E\sin\theta
\end{aligned}
$$

High-energy limits in distributions:

- $\dfrac{\mathrm{d}\sigma}{\mathrm{d}k_\mathrm{T}}$: $k_\mathrm{T} \gg M_\mathrm{W} \quad \Rightarrow \quad s, |t| \gg M_\mathrm{W}^2 \quad \Rightarrow$ Sudakov domination
- $\dfrac{\mathrm{d}\sigma}{\mathrm{d}M_{34}}$: $M_{34} \gg M_\mathrm{W} \;\Rightarrow$ small $|t|$ possible $\;\Rightarrow$ in general no Sudakov domination
(i.e. typically smaller corrections)

Example: Drell–Yan production

Neutral current: $\mathrm{pp} \to \ell^+\ell^-$ at $\sqrt{s} = 14\,\mathrm{TeV}$ (based on S.D./Huber arXiv:0911.2329)

$M_{\ell\ell}/\mathrm{GeV}$	$50-\infty$	$100-\infty$	$200-\infty$	$500-\infty$	$1000-\infty$	$2000-\infty$
σ_0/pb	738.733(6)	32.7236(3)	1.48479(1)	0.0809420(6)	0.00679953(3)	0.000303744(1)
$\delta^{\mathrm{rec}}_{\mathrm{q\bar{q},phot}}/\%$	-1.81	-4.71	-2.92	-3.36	-4.24	-5.66
$\delta_{\mathrm{q\bar{q},weak}}/\%$	-0.71	-1.02	-0.14	-2.38	-5.87	-11.12
$\delta^{(1)}_{\mathrm{Sudakov}}/\%$	0.27	0.54	-1.43	-7.93	-15.52	-25.50
$\delta^{(2)}_{\mathrm{Sudakov}}/\%$	-0.00046	-0.0067	-0.035	0.23	1.14	3.38

no Sudakov domination!

Charged current: $\mathrm{pp} \to \ell^+\nu_\ell$ at $\sqrt{s} = 14\,\mathrm{TeV}$ (based on Brensing et al. arXiv:0710.3309)

$M_{\mathrm{T},\nu_\ell\ell}/\mathrm{GeV}$	$50-\infty$	$100-\infty$	$200-\infty$	$500-\infty$	$1000-\infty$	$2000-\infty$
σ_0/pb	4495.7(2)	27.589(2)	1.7906(1)	0.084697(4)	0.0065222(4)	0.00027322(1)
$\delta^{\mu^+\nu_\mu}_{\mathrm{q\bar{q}}}/\%$	$-2.9(1)$	$-5.2(1)$	$-8.1(1)$	$-14.8(1)$	$-22.6(1)$	$-33.2(1)$
$\delta^{\mathrm{rec}}_{\mathrm{q\bar{q}}}/\%$	$-1.8(1)$	$-3.5(1)$	$-6.5(1)$	$-12.7(1)$	$-20.0(1)$	$-29.6(1)$
$\delta^{(1)}_{\mathrm{Sudakov}}/\%$	0.0005	0.5	-1.9	-9.5	-18.5	-29.7
$\delta^{(1)}_{\mathrm{EWslog}}/\%$	0.008	0.9	2.3	3.8	4.8	5.9
$\delta^{(2)}_{\mathrm{Sudakov}}/\%$	-0.0002	-0.023	-0.082	0.21	1.3	3.8

Sudakov domination!

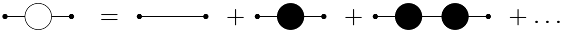
Table of contents

Problem of unstable particles:

description of resonances requires resummation of propagator corrections

$\hookrightarrow$ mixing of perturbative orders potentially violates gauge invariance

Dyson series and propagator poles (scalar example)



$$G_{\mathrm{R}}^{\phi\phi}(p) = \frac{\mathrm{i}}{p^2-m^2} + \frac{\mathrm{i}}{p^2-m^2}\,\mathrm{i}\Sigma_{\mathrm{R}}(p^2)\,\frac{\mathrm{i}}{p^2-m^2} + \ldots = \frac{\mathrm{i}}{p^2-m^2+\Sigma_{\mathrm{R}}(p^2)}$$

$\Sigma_{\mathrm{R}}(p^2) =$ renormalized self-energy, $m =$ ren. mass

stable particle: $\mathrm{Im}\{\Sigma_{\mathrm{R}}(p^2)\} = 0$ at $p^2 \sim m^2$

$\hookrightarrow$ propagator pole for real value of p^2,
renormalization condition for physical mass m: $\Sigma_{\mathrm{R}}(m^2) = 0$

unstable particle: $\mathrm{Im}\{\Sigma_{\mathrm{R}}(p^2)\} \neq 0$ at $p^2 \sim m^2$

$\hookrightarrow$ location μ^2 of propagator pole is complex,
possible definition of mass M and width Γ: $\mu^2 = M^2 - \mathrm{i}M\Gamma$

Commonly used mass/width definitions:

- "on-shell mass/width" $M_{\mathrm{OS}}/\Gamma_{\mathrm{OS}}$: $\quad M_{\mathrm{OS}}^2 - M_0^2 + \mathrm{Re}\{\Sigma(M_{\mathrm{OS}}^2)\} \stackrel{!}{=} 0$

  $$\hookrightarrow \quad G^{\phi\phi}(p) \underset{p^2 \to M_{\mathrm{OS}}^2}{\widetilde{\quad\quad}} \frac{1}{(p^2 - M_{\mathrm{OS}}^2)(1 + \mathrm{Re}\{\Sigma'(M_{\mathrm{OS}}^2)\}) + \mathrm{i}\,\mathrm{Im}\{\Sigma(p^2)\}}$$

  comparison with form of Breit–Wigner resonance $\quad \dfrac{R_{\mathrm{OS}}}{p^2 - m^2 + \mathrm{i}m\Gamma}$

  yields: $\quad M_{\mathrm{OS}}\Gamma_{\mathrm{OS}} \equiv \mathrm{Im}\{\Sigma(M_{\mathrm{OS}}^2)\} \,/\, (1 + \mathrm{Re}\{\Sigma'(M_{\mathrm{OS}}^2)\}), \quad \Sigma'(p^2) \equiv \dfrac{\partial \Sigma(p^2)}{\partial p^2}$

- "pole mass/width" M/Γ: $\quad \mu^2 - M_0^2 + \Sigma(\mu^2) \stackrel{!}{=} 0$

  complex pole position: $\quad \mu^2 \equiv M^2 - \mathrm{i}M\Gamma$

  $$\hookrightarrow \quad G^{\phi\phi}(p) \underset{p^2 \to \mu^2}{\widetilde{\quad\quad}} \frac{1}{(p^2 - \mu^2)[1 + \Sigma'(\mu^2)]} = \frac{R}{p^2 - M^2 + \mathrm{i}M\Gamma}$$

Note:

μ = gauge independent for any particle (pole location is property of S-matrix)

M_{OS} = gauge dependent at 2-loop order Sirlin '91; Stuart '91; Gambino, Grassi '99; Grassi, Kniehl, Sirlin '01

Relation between "on-shell" and "pole" definitions:

Subtraction of defining equations yields:

$$M_{\mathrm{OS}}^2 + \mathrm{Re}\{\Sigma(M_{\mathrm{OS}}^2)\} = M^2 - \mathrm{i}M\Gamma + \Sigma(M^2 - \mathrm{i}M\Gamma)$$

Equation can be uniquely solved via recursion in powers of coupling α:

$$\begin{aligned} \text{ansatz:} \quad M_{\mathrm{OS}}^2 &= M^2 + c_1\alpha^1 + c_2\alpha^2 + \dots \\ M_{\mathrm{OS}}\Gamma_{\mathrm{OS}} &= M\Gamma + d_2\alpha^2 + d_3\alpha^3 + \dots, \qquad c_i, d_i = \text{real} \end{aligned}$$

counting in α: $\quad M_{\mathrm{OS}}, M = \mathcal{O}(\alpha^0), \quad \Gamma_{\mathrm{OS}}, \Gamma, \Sigma(p^2) = \mathcal{O}(\alpha^1)$

Result:

$$\begin{aligned} M_{\mathrm{OS}}^2 &= M^2 + \mathrm{Im}\{\Sigma(M^2)\}\,\mathrm{Im}\{\Sigma'(M^2)\} + \mathcal{O}(\alpha^3) \\ M_{\mathrm{OS}}\Gamma_{\mathrm{OS}} &= M\Gamma + \mathrm{Im}\{\Sigma(M^2)\}\,\mathrm{Im}\{\Sigma'(M^2)\}^2 \\ &\quad + \tfrac{1}{2}\,\mathrm{Im}\{\Sigma(M^2)\}^2\,\mathrm{Im}\{\Sigma''(M^2)\} + \mathcal{O}(\alpha^4) \end{aligned}$$

i.e. $\{M_{\mathrm{OS}}, \Gamma_{\mathrm{OS}}\} = \{M, \Gamma\}$ + gauge-dependent 2-loop corrections

Important examples: W and Z bosons

In good approximation: $\mathrm{W} \to f\bar{f}'$, $\mathrm{Z} \to f\bar{f}$ with masses fermions f, f'

so that: $$\mathrm{Im}\{\Sigma_{\mathrm{T}}^{\mathrm{V}}(p^2)\} = p^2 \times \frac{\Gamma_{\mathrm{V}}}{M_{\mathrm{V}}}\,\theta(p^2), \qquad \mathrm{V} = \mathrm{W}, \mathrm{Z}$$

$$\hookrightarrow \quad M_{\mathrm{OS}}^2 = M^2 + \Gamma^2 + \mathcal{O}(\alpha^3) \qquad M_{\mathrm{OS}}\Gamma_{\mathrm{OS}} = M\Gamma + \frac{\Gamma^3}{M} + \mathcal{O}(\alpha^4)$$

In terms of measured numbers:

W boson: $M_{\mathrm{W}} \approx 80\,\mathrm{GeV}$, $\Gamma_{\mathrm{W}} \approx 2.1\,\mathrm{GeV}$

$\hookrightarrow \quad M_{\mathrm{W,OS}} - M_{\mathrm{W,pole}} \approx 28\,\mathrm{MeV}$

Z boson: $M_{\mathrm{Z}} \approx 91\,\mathrm{GeV}$, $\Gamma_{\mathrm{Z}} \approx 2.5\,\mathrm{GeV}$

$\hookrightarrow \quad M_{\mathrm{Z,OS}} - M_{\mathrm{Z,pole}} \approx 34\,\mathrm{MeV}$

Exp. accuracy: $\Delta M_{\mathrm{W,exp}}^{\mathrm{ATLAS}} = 16\,\mathrm{MeV}$, $\Delta M_{\mathrm{Z,exp}} = 2.1\,\mathrm{MeV}$

$\hookrightarrow$ Difference in definitions phenomenologically important !

Example of W and Z bosons continued:

Approximation of massless decay fermions:

$$\Gamma_{\mathrm{V,OS}}(p^2) \;=\; \Gamma_{\mathrm{V,OS}} \times \frac{p^2}{M_{\mathrm{V,OS}}^2}\,\theta(p^2), \qquad \mathrm{V} = \mathrm{W}, \mathrm{Z}$$

Fit of W/Z resonance shapes to experimental data:

- ansatz $\left|\frac{R'}{p^2 - m'^2 + \mathrm{i}\gamma' p^2/m'}\right|^2$ yields: $m' = M_{\mathrm{V,OS}}, \quad \gamma' = \Gamma_{\mathrm{V,OS}}$
- ansatz $\left|\frac{R}{p^2 - m^2 + \mathrm{i}\gamma m}\right|^2$ yields: $m = M_{\mathrm{V,pole}}, \quad \gamma = \Gamma_{\mathrm{V,pole}}$

Note: The two forms are equivalent:

$$R = \frac{R'}{1+\mathrm{i}\gamma'/m'}, \quad m^2 = \frac{m'^2}{1+\gamma'^2/m'^2}, \quad m\gamma = \frac{m'\gamma'}{1+\gamma'^2/m'^2}$$

$\hookrightarrow$ consistent with relation between “on-shell” and “pole” definitions !

The issue of gauge invariance

Preliminary remarks:

The issue of gauge invariance goes

- beyond the definition of M and Γ and also
- beyond the question of parametrizing the resonance!

It is about the consistency of amplitudes everywhere in phase space, i.e.

- on resonance,
- in off-shell regions, and
- in the transition region between on-/off-shell domains.

Gauge-invariance requirements in amplitude calculations:

- proper cancellation of gauge-parameter dependences
  (relations between self-energies, vertex corrections, boxes, etc.)
- validity of (internal) Ward identities
  (e.g. ruling cancellations for forward scattering of $\mathrm{e}^{\pm}$ or at high energies)

$\Rightarrow$ Required: schemes to introduce width Γ

- without breaking gauge invariance
- maintaining (at least) NLO accuracy everywhere in phase space

Width schemes for LO calculations:

Naive propagator substitutions in full tree-level amplitudes:

$$\frac{1}{k^2 - m^2} \to \frac{1}{k^2 - m^2 + \mathrm{i} m \Gamma(k^2)}$$ for resonant or all propagators

- constant width $\Gamma(k^2) = \text{const.}$ $\to$ U(1) respected (if all propagators dressed), SU(2) "mildly" violated
- step width $\Gamma(k^2) \propto \theta(k^2)$ $\to$ U(1) and SU(2) violated
- running width $\Gamma(k^2) \propto \theta(k^2) \times k^2$ $\to$ U(1) and SU(2) violated
  $\hookrightarrow$ results can be totally wrong !

Complex-mass scheme

Denner et al. '99

Complex masses for $V = \mathrm{W}, \mathrm{Z}$ from

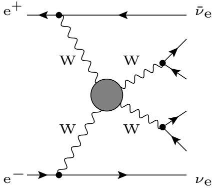
$$\mu_V^2 = M_V^2 - \mathrm{i} M_V \Gamma_V = \text{location of complex poles in } V \text{ propagators}$$

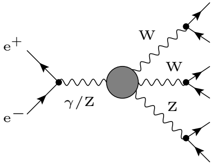
Complex (on-shell) weak mixing angle via $c_\mathrm{W} = \mu_\mathrm{W} / \mu_\mathrm{Z}$

$\Rightarrow$ All algebraic relations expressing gauge invariance hold exactly (gauge-parameter cancellation, Ward identities).

Major benefit: Generalization to NLO Denner et al. '05; Denner, SD '19
provides NLO accuracy everywhere in phase space!

LO example from $\mathrm{e^+e^-}$ physics: $\sigma[\mathrm{fb}]$ for $\mathrm{e^+e^-} \to \nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mu^-\bar{\nu}_\mu \mathrm{u}\bar{\mathrm{d}}$ (with cuts)





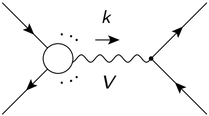
$+$ etc.

$\sqrt{s}$	500 GeV	800 GeV	2 TeV	10 TeV
constant width	1.633(1)	4.105(4)	11.74(2)	26.38(6)
running width	1.640(1)	4.132(4)	12.88(1)	12965(12)
complex mass	1.633(1)	4.104(3)	11.73(1)	26.39(6)

S.D., Roth '02

$\leftarrow$ totally wrong!

High-energy behaviour of longitudinal $V = \mathrm{W/Z}$ bosons:



$$\frac{1}{k^2 - M_V^2}\, k^\mu\, T_\mu^V \underset{k^0 \gg M_V}{\sim} \;=\; \frac{1}{k^2 - M_V^2}\, c_V M_V T^S$$

(S = Goldstone partner of V)

SU(2) Ward identity $k^\mu T_\mu^V = c_V M_V T^S$ essential to cancel factor k^0,
otherwise gauge-invariance/unitarity-breaking terms enhanced by k^0/M_V

Width schemes for higher-order calculations:

- Pole Scheme (PS) Stuart '91; Aeppli et al. '93, '94; etc.

  Isolate resonance in a gauge-invariant way and introduce Γ only there:

$$\mathcal{M} = \frac{R(p^2)}{p^2-M^2} + N(p^2) = \frac{R(M^2)}{p^2-M^2} + \frac{R(p^2)-R(M^2)}{p^2-M^2} + N(p^2)$$

$$\to \underbrace{\frac{\tilde{R}(M^2-\mathrm{i}M\Gamma)}{p^2-M^2+\mathrm{i}M\Gamma}}_{\text{resonant}} + \underbrace{\frac{R(p^2)-R(M^2)}{p^2-M^2}}_{\text{non-res./non-fact. corrs.}} + \underbrace{\tilde{N}(p^2)}_{\text{non-resonant}}$$

  $\hookrightarrow$ consistent, gauge invariant, NLO everywhere possible,
  but subtle and cumbersome in practice (complex kinematics, pole location is branch point rather than pole, IR structure of radiation)

- Leading pole approximation (PA)

  Take term with highest resonance enhancement of pole expansion
  = leading term of Pole Scheme

  $\hookrightarrow$ consistent, gauge invariant, straightforward,
  but valid only in resonance neighbourhood,
  rel. uncertainty for EW corrections $= \frac{\alpha}{\pi} \times \mathcal{O}(\Gamma/M)$

- Complex-mass scheme at NLO   Denner et al. '05; Denner, S.D. '19

mass^2 = location of propagator pole in complex p^2 plane

$\hookrightarrow$ complex mass renormalization: $\underbrace{M^2_{\mathrm{W},0}}_{\text{bare mass}} = \mu^2_{\mathrm{W}} + \underbrace{\delta\mu^2_{\mathrm{W}}}_{\text{ren. constant}}$, etc.

Gauge invariance by complex weak mixing angle:

$$c_{\mathrm{W}} = \frac{\mu_{\mathrm{W}}}{\mu_{\mathrm{Z}}}, \qquad \frac{\delta c^2_{\mathrm{W}}}{c^2_{\mathrm{W}}} = \frac{\delta\mu^2_{\mathrm{W}}}{\mu^2_{\mathrm{W}}} - \frac{\delta\mu^2_{\mathrm{Z}}}{\mu^2_{\mathrm{Z}}}$$

Features of the complex-mass scheme:

- ⊕ perturbative calculations as usual (with complex masses and couplings)
- ⊕ no double counting of contributions (bare Lagrangian unchanged!)
- ⊕ gauge invariance (ST identities, gauge-parameter independence)
- ⊕ NLO accuracy everywhere in phase space
- ▶ spurios terms are beyond NLO, but spoil unitarity
- ▶ complex gauge-boson masses also in loop integrals (all known)
- ⊖ unstable particles only allowed as resonances (not as external states)
- ⊖ generalization to NNLO not yet known (but expected to work)

Technical details, exemplified for W bosons:

OS renormalization conditions for renormalized (transverse) self-energy

$$\Sigma^{W}_{\mathrm{T,R}}(\mu_{\mathrm{W}}^2) = 0, \quad \Sigma'^{W}_{\mathrm{T,R}}(\mu_{\mathrm{W}}^2) = 0$$

$\hookrightarrow$ μ_{W}^2 is location of propagator pole, and residue $= 1$

Solution of renormalization conditions:

$$\delta\mu_{\mathrm{W}}^2 = \Sigma^{W}_{\mathrm{T}}(\mu_{\mathrm{W}}^2), \quad \delta\mathcal{Z}_W = -\Sigma'^{W}_{\mathrm{T}}(\mu_{\mathrm{W}}^2)$$

Note: Evaluation of $\Sigma^{W}_{\mathrm{T}}(p^2)$ at complex p^2 can be avoided

$$\Sigma^{W}_{\mathrm{T}}(\mu_{\mathrm{W}}^2) = \Sigma^{W}_{\mathrm{T}}(M_{\mathrm{W}}^2) + (\mu_{\mathrm{W}}^2 - M_{\mathrm{W}}^2)\Sigma'^{W}_{\mathrm{T}}(M_{\mathrm{W}}^2) + \underbrace{\frac{\alpha}{\pi}\mathrm{i}M_{\mathrm{W}}\Gamma_{\mathrm{W}}}_{\substack{\text{from non-analyticity}\\ \text{at } p^2 = M_{\mathrm{W}}^2}} + \underbrace{\mathcal{O}(\alpha^3)}_{\substack{\text{beyond one loop}\\ \text{and finite}}}$$

$\Rightarrow$ Renormalized W self-energy:

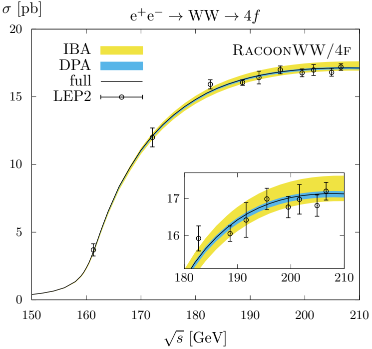
$$\Sigma^{W}_{\mathrm{T,R}}(p^2) = \Sigma^{W}_{\mathrm{T}}(p^2) - \delta M_{\mathrm{W}}^2 + (p^2 - M_{\mathrm{W}}^2)\delta Z_W$$

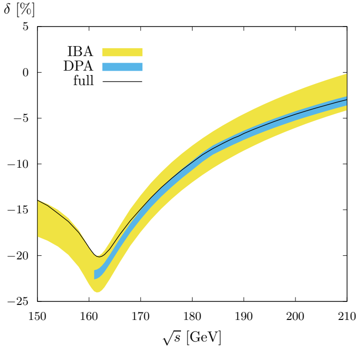
with $\delta M_{\mathrm{W}}^2 = \Sigma^{W}_{\mathrm{T}}(M_{\mathrm{W}}^2) + \frac{\alpha}{\pi}\mathrm{i}M_{\mathrm{W}}\Gamma_{\mathrm{W}}, \quad \delta Z_W = -\Sigma'^{W}_{\mathrm{T}}(M_{\mathrm{W}}^2)$

Differences to the usual on-shell scheme:

- no real parts taken from Σ^{W}_{T}
- Σ^{W}_{T} evaluated with complex masses and couplings

Example: predictions for σ_{WW} in the LEP2 energy range





- IBA = based on leading-log ISR and universal EW corrections ($\Delta \sim 2\%$)
  $\hookrightarrow$ shows large ISR impact near threshold (also by GENTLE)
- DPA = "Double-Pole Approximation" (leading term of resonance expansion)
  $\hookrightarrow$ $\Delta \sim 0.5\%$ above threshold, not applicable at threshold RacoonWW, YFSWW
- "full" = full NLO prediction for $\mathrm{e}^+\mathrm{e}^- \to 4f$ via charged current Denner at al. '05
  based on complex-mass scheme
  + leading-log improvements for ISR beyond NLO

$\hookrightarrow$ $\Delta \sim 0.5\%$ everywhere

Literature

$\hookrightarrow$ See Lecture 1 !