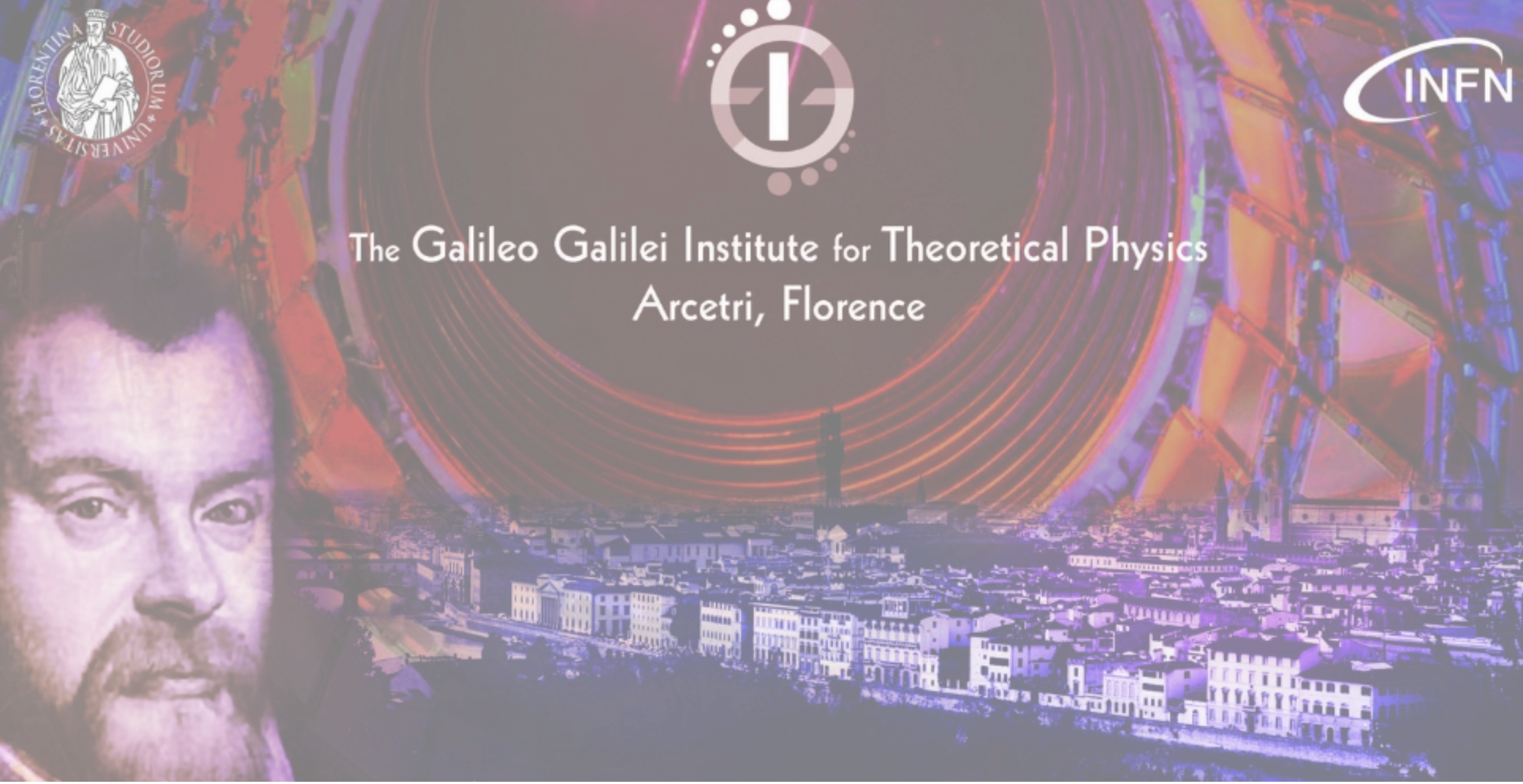


THE STRUCTURE OF THE PROTON (II)

Maria Ubiali, University of Cambridge

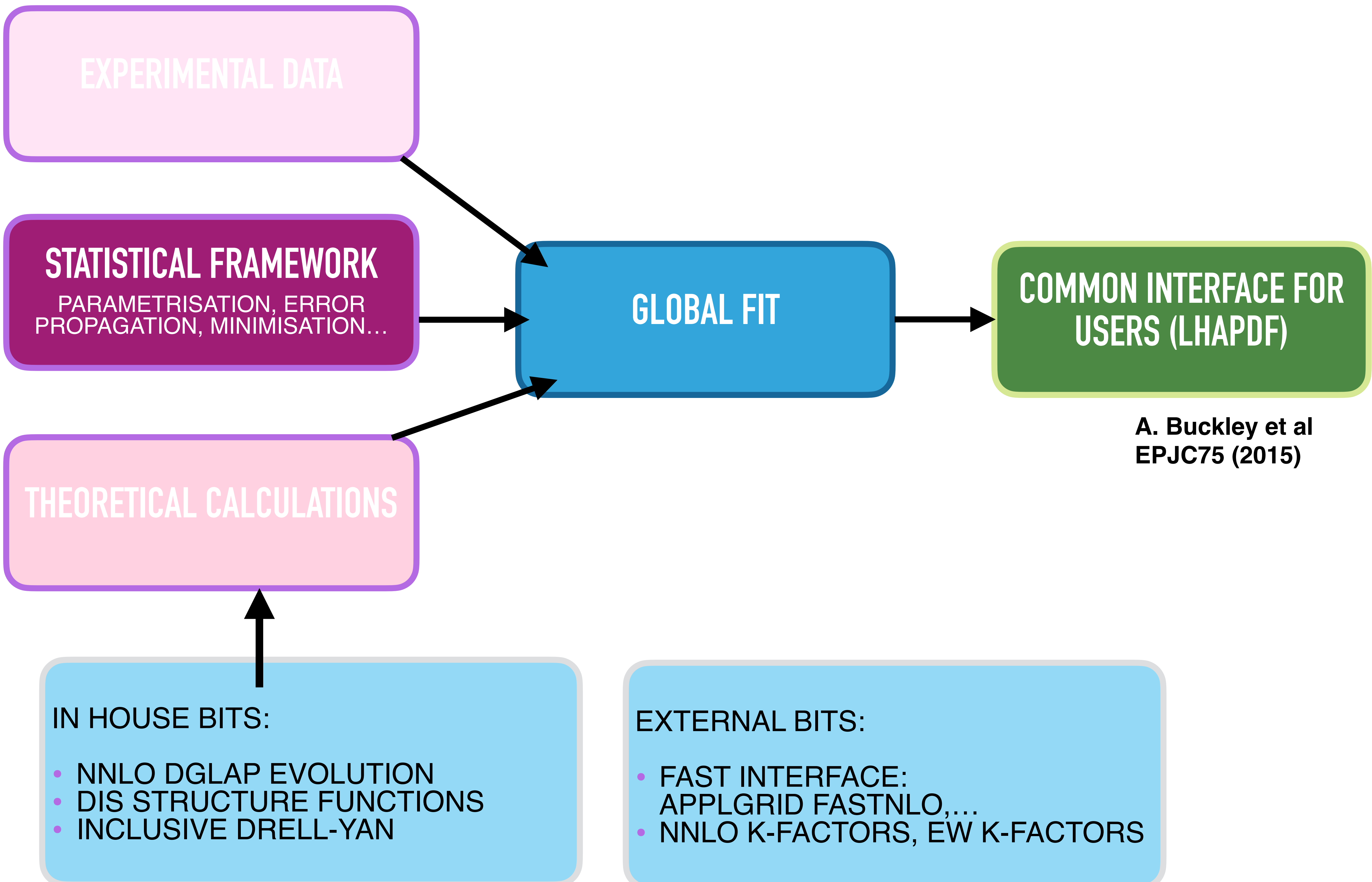
29th September 2023

GGI, Florence - Theoretical challenges in the precision era of the LHC

Outline

- Lecture 1 (this morning)
  - Motivation:
    the high energy big picture
  - Parton Model and QCD
  - Collinear Factorisation
- **<u>Lecture 2</u> (this afternoon)**
  - Methodological aspects
  - Theoretical aspects
  - New frontiers and challenges

A complex machinery



Methodological aspects

A quite complicated game

- A single quantity: 1σ error

- Multiple quantities: 1σ contours

- Functions: 1σ "error band" in the space of functions
  = find the probability density in the space of functions f(x)
  Expectation values are functional integrals

Not as simple as it may look…

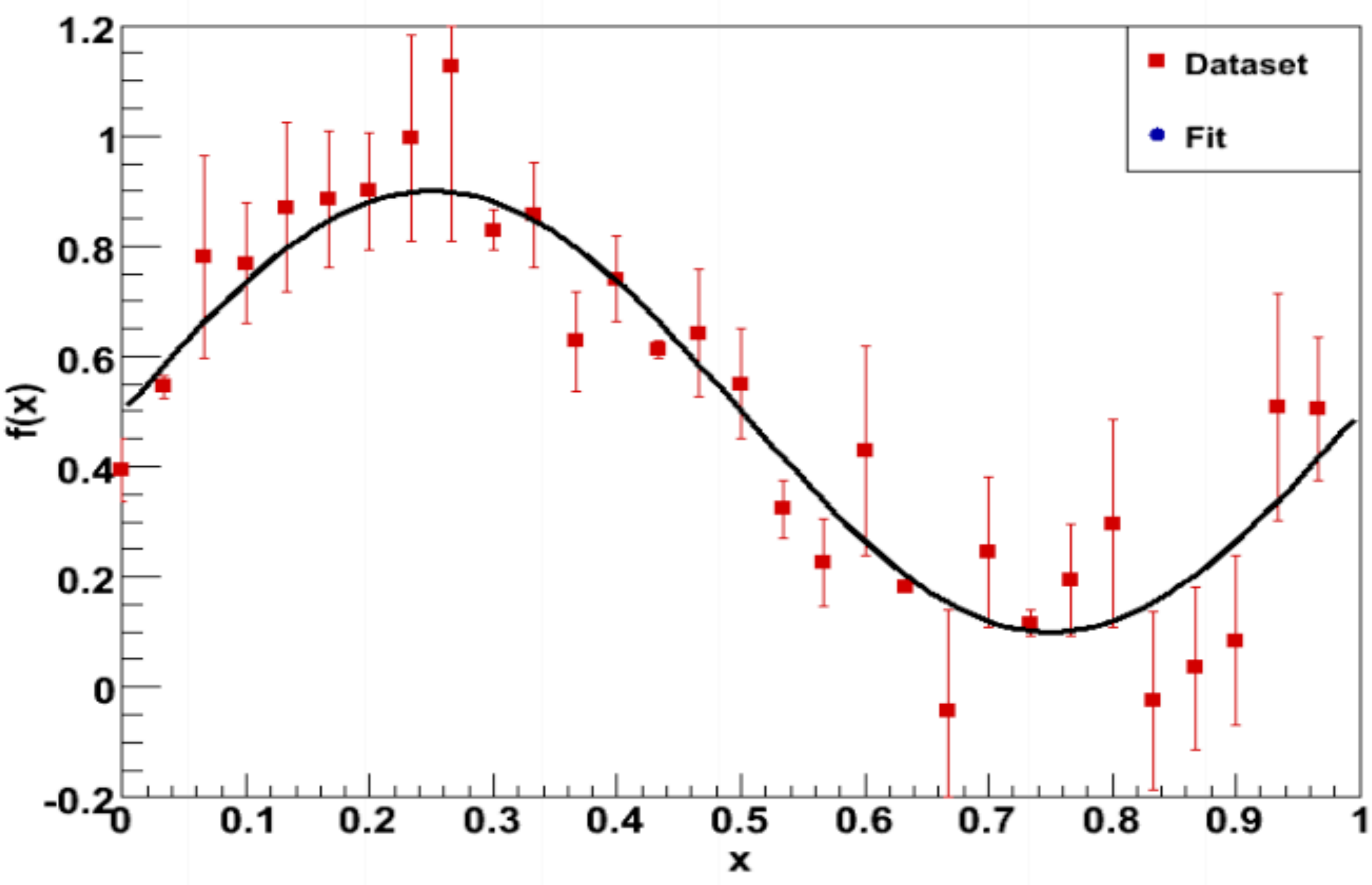
$$\langle \mathcal{O}[\{f\}]\rangle = \int [\mathcal{D}f]\, \mathcal{O}[\{f\}]\mathcal{P}[\{f\}]$$

- Given a finite number of experimental data points want a set of functions
- Want to find a infinite-dimensional object from a finite number of information

A toy model

A toy-model:

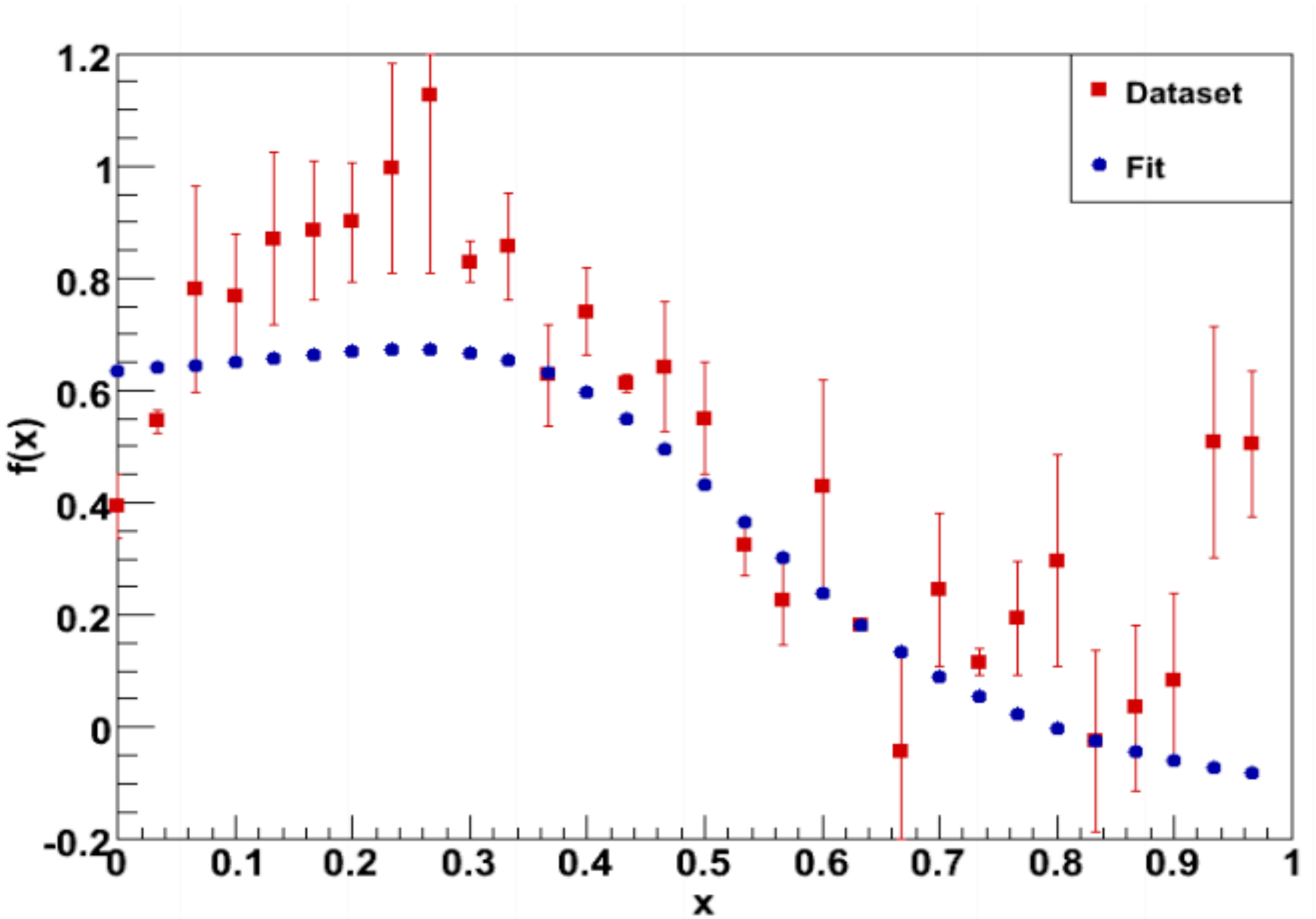
1) Imagine that we have a set of uncorrelated measurements of a quantity f(x) at different x. The underlying law that Nature established for this quantity is a sinusoidal, but we don't know anything about that and try to guess it with a fit.



A toy model

A toy-model:

2) Choose a parametrisation for f(x) and perform a fit by minimising a function, a figure of merit, like the χ^2



$$\chi^2 = \sum_{i=1}^{N_{\rm data}} \frac{(D_i - T_i)^2}{\sigma_i^2}$$

χ^2/d.o.f. » 1

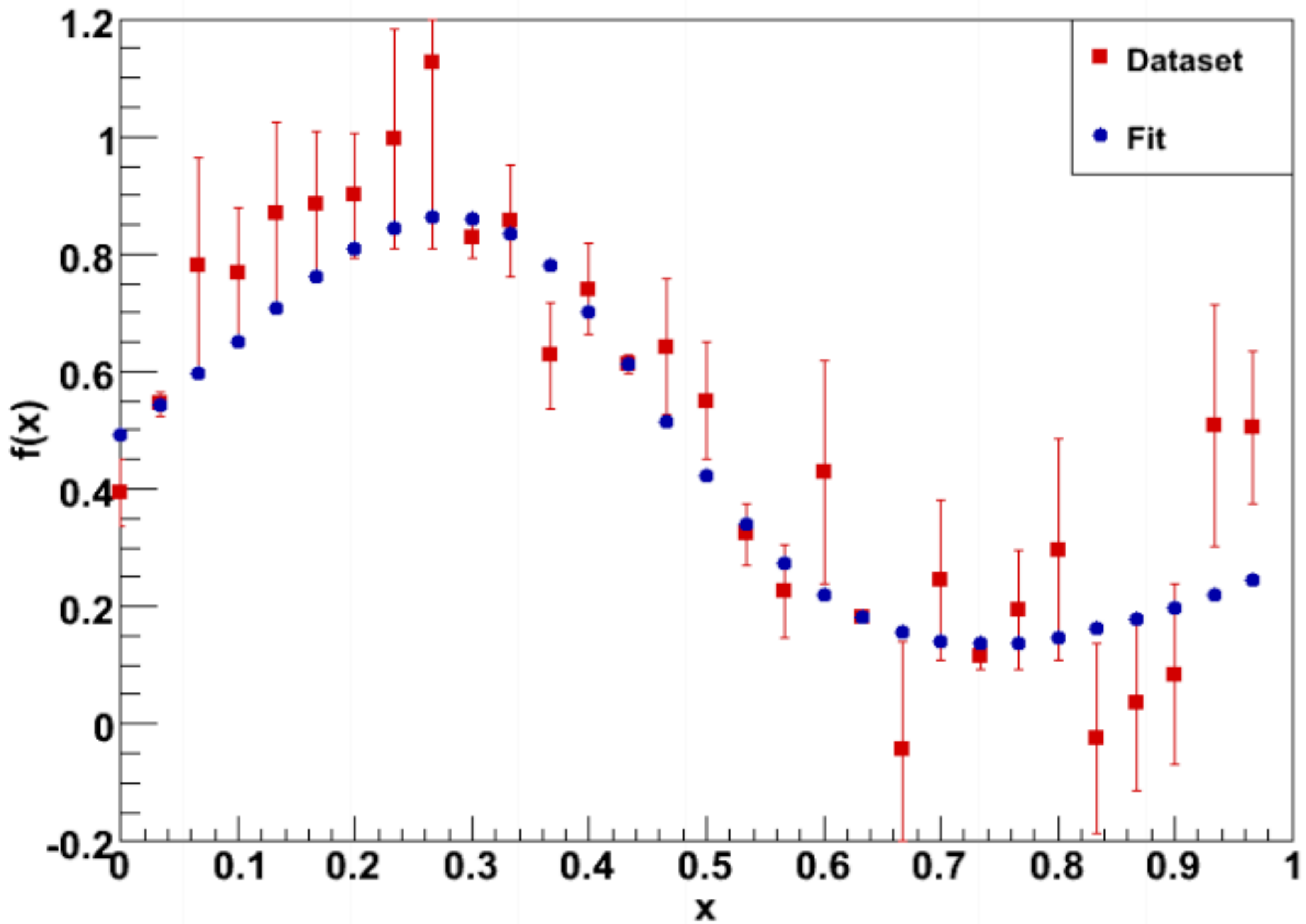
We are not quite there…

under-learning

A toy model

A toy-model:

2) Choose a parametrisation for f(x) and perform a fit by minimising a function, a figure of merit, like the χ^2



$$\chi^2 = \sum_{i=1}^{N_{\text{data}}} \frac{(D_i - T_i)^2}{\sigma_i^2}$$

χ^2/d.o.f. ~ 1

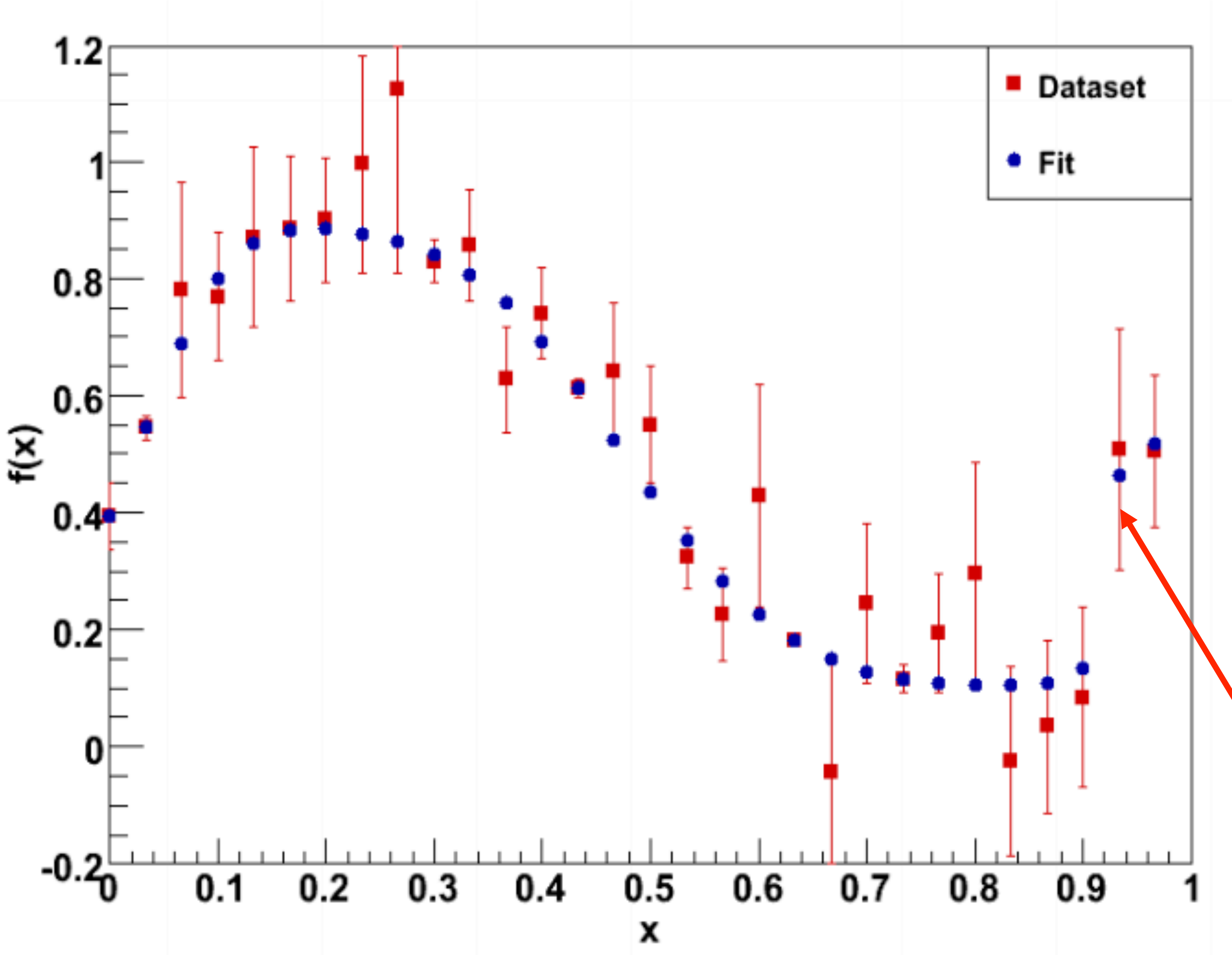
We are there...

proper learning

A toy model

A toy-model:

2) Choose a parametrisation for f(x) and perform a fit by minimising a function, a figure of merit, like the χ^2



$$\chi^2 = \sum_{i=1}^{N_{\text{data}}} \frac{(D_i - T_i)^2}{\sigma_i^2}$$

χ^2/d.o.f. → 0

We went too far…

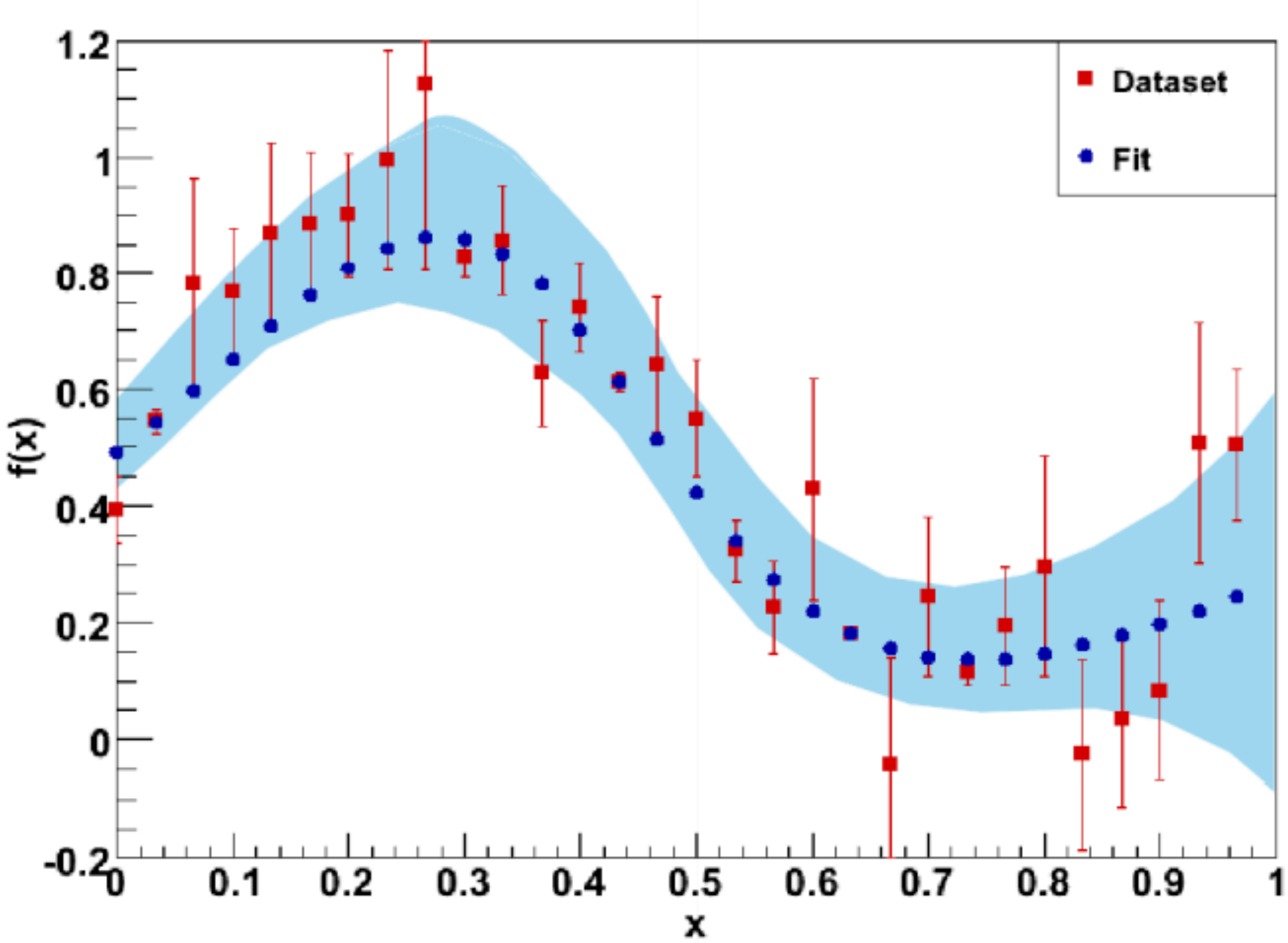
over-learning

start fitting the statistical noise

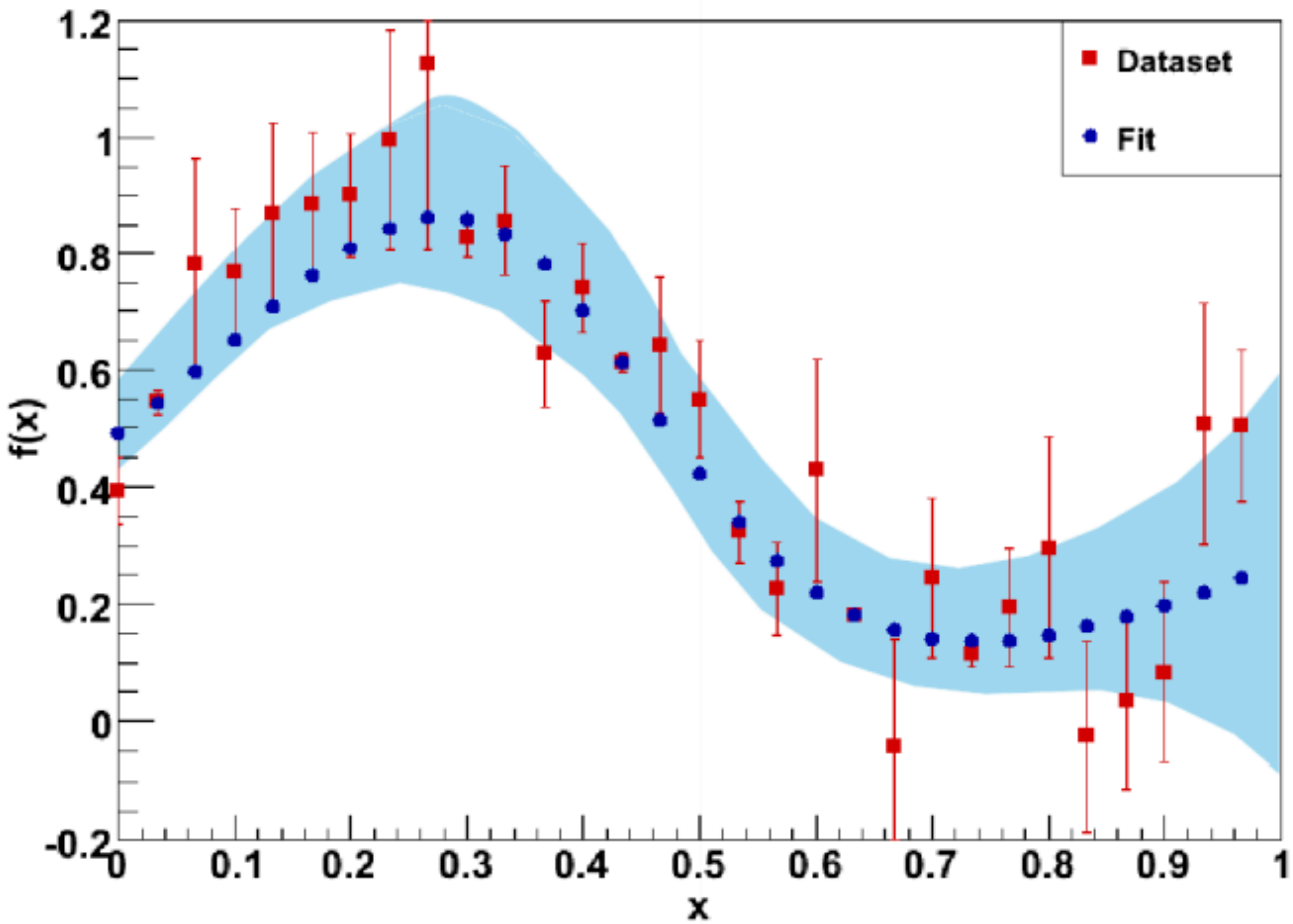
A toy model

A toy-model:

3) Determine the error of our fit, which corresponds to the lack of information that the data provide. In the limit of infinite and infinitely-precise and compatible data, the error band tends to 0



The actual game



The actual games is more complicated since we have 6+6+1 functions (actually 3+3+1+1) and errors to determine which are not directly measured. They enter in the measured observables according to different combinations. But still…

- ✓ Need to choose a clever and flexible parametrisation
- ✓ Need a way to stop the fit before over-learning sets in to avoid fitting statistical noise
- ✓ Need a reliable error estimate

(i) Choice of parametrisation

Usually one parametrises independently the gluon, light quarks and anti-quarks, strange and anti-strange (+ intrinsic charm), while heavy quarks are generated perturbatively from light quarks and gluons*

The ideal parametrisation

- Too rigid

Global fit might not have flexibility to describe data or inadequate small uncertainties where there are no data

- Too flexible

Difficult minimisation and it might develop artefacts driven by statistical fluctuations of the data

Sum rules

- From baryon number conservation → Valence Sum Rules

$$\int_0^1 dx\,(u(x,Q^2) - \bar{u}(x,Q^2)) = 2$$

$$\int_0^1 dx\,(d(x,Q^2) - \bar{d}(x,Q^2)) = 1$$

$$\int_0^1 dx\,(s(x,Q^2) - \bar{s}(x,Q^2)) = 0$$

- From momentum conservation → Momentum Sum Rule

$$\int_0^1 dx\,(x\Sigma(x,Q^2) + xg(x,Q^2)) = 1$$

$$\text{with } \Sigma = \sum_{i=1}^{n_F} q_i + \bar{q}_i$$

Traditional (parametrical) approach

- Introduce a simple functional form with enough free parameters

$$f_i(x, Q_0^2) = a_0\, x^{a_1}\, (1-x)^{a_2} P(x, a_3, a_4, \ldots)$$

- Typically about 20-25 free parameters for 7 independent functions

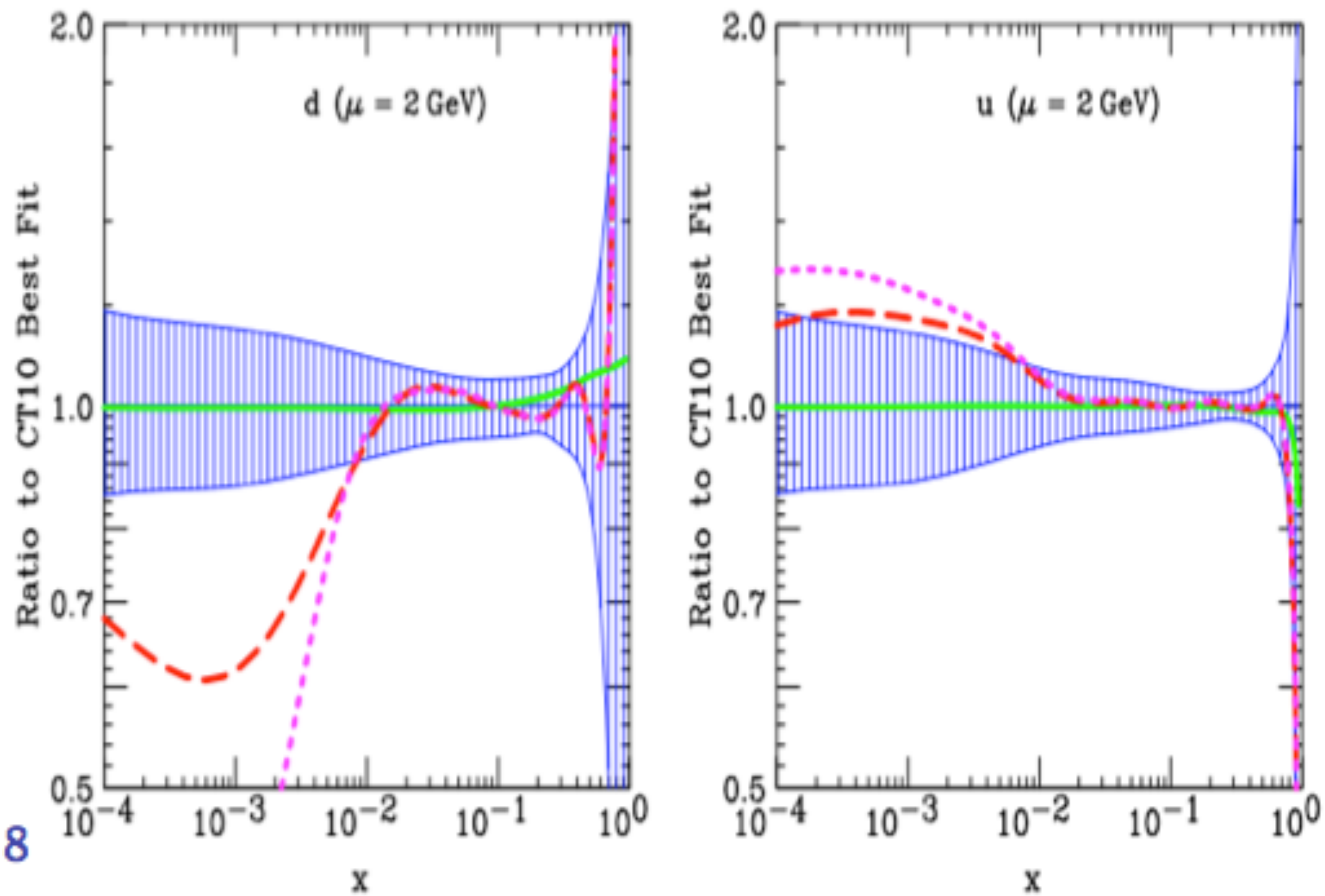
MSTW2008

20 free parameters

$$\begin{aligned}
xu_v(x, Q_0^2) &= A_u\, x^{\eta_1} (1-x)^{\eta_2} (1 + \epsilon_u \sqrt{x} + \gamma_u\, x), \\
xd_v(x, Q_0^2) &= A_d\, x^{\eta_3} (1-x)^{\eta_4} (1 + \epsilon_d \sqrt{x} + \gamma_d\, x), \\
xS(x, Q_0^2) &= A_S\, x^{\delta_S} (1-x)^{\eta_S} (1 + \epsilon_S \sqrt{x} + \gamma_S\, x), \\
x\Delta(x, Q_0^2) &= A_\Delta\, x^{\eta_\Delta} (1-x)^{\eta_S+2} (1 + \gamma_\Delta\, x + \delta_\Delta\, x^2), \\
xg(x, Q_0^2) &= A_g\, x^{\delta_g} (1-x)^{\eta_g} (1 + \epsilon_g \sqrt{x} + \gamma_g\, x) + A_{g'}\, x^{\delta_{g'}} (1-x)^{\eta_{g'}}, \\
x(s+\bar{s})(x, Q_0^2) &= A_+\, x^{\delta_S} (1-x)^{\eta_+} (1 + \epsilon_S \sqrt{x} + \gamma_S\, x), \\
x(s-\bar{s})(x, Q_0^2) &= A_-\, x^{\delta_-} (1-x)^{\eta_-} (1 - x/x_0),
\end{aligned}$$

Traditional (parametrical) approach

- Possible issues:
  What is the error associated to a given functional form?

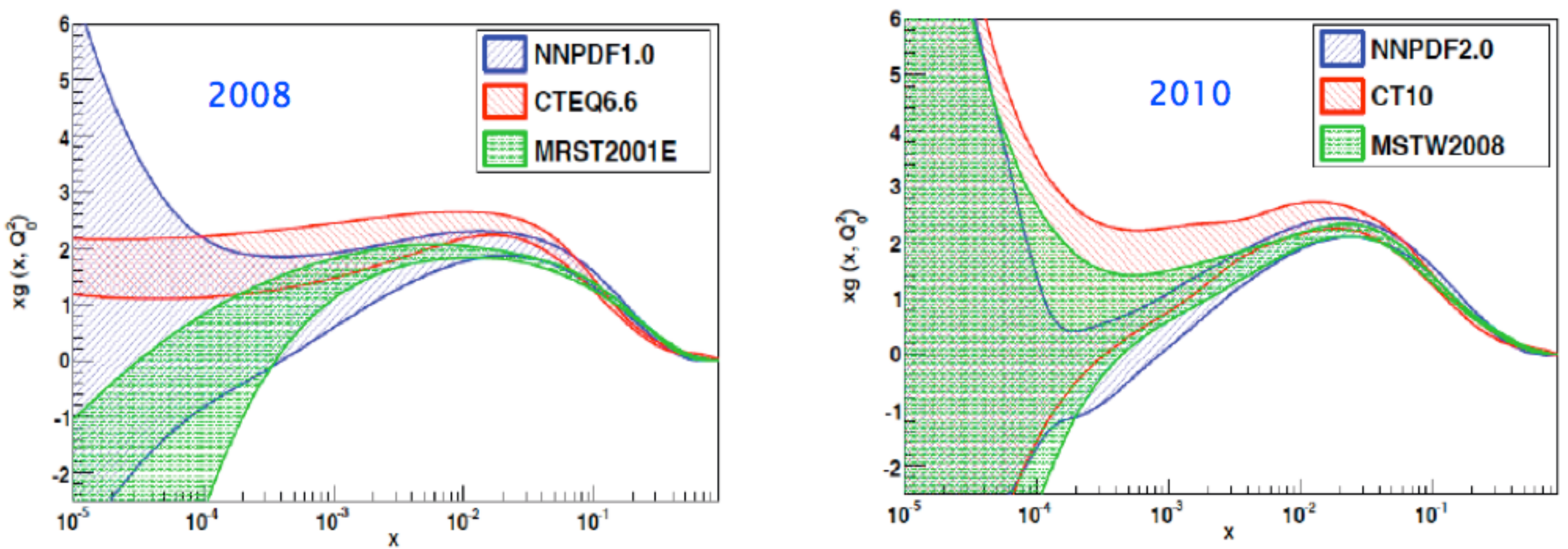


Pink and red curves give same good description of data but outside error bar

J. Pumplin ArXiv:0909.0268

Traditional (parametrical) approach

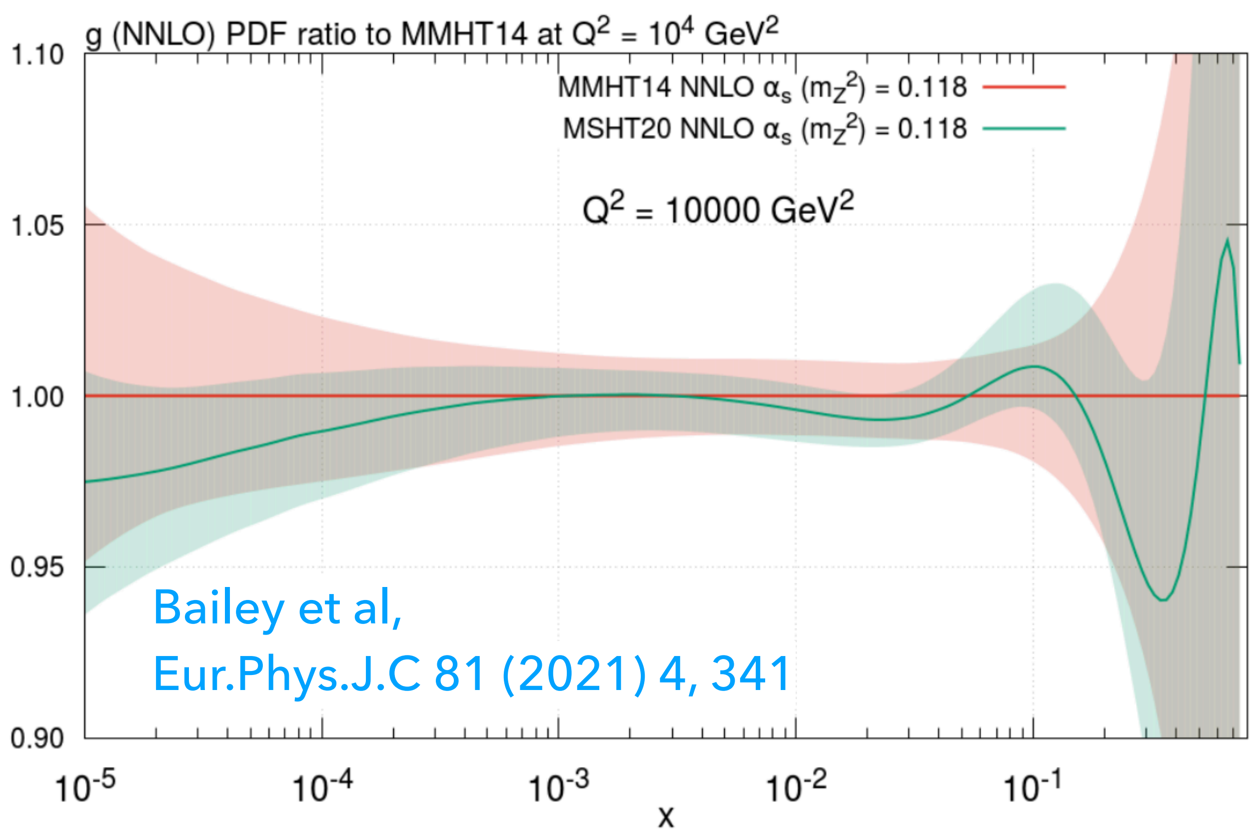
- Possible issues:
  If functional form not flexible enough PDFs may present unrealistically small errors where data do not constrain PDF uncertainties

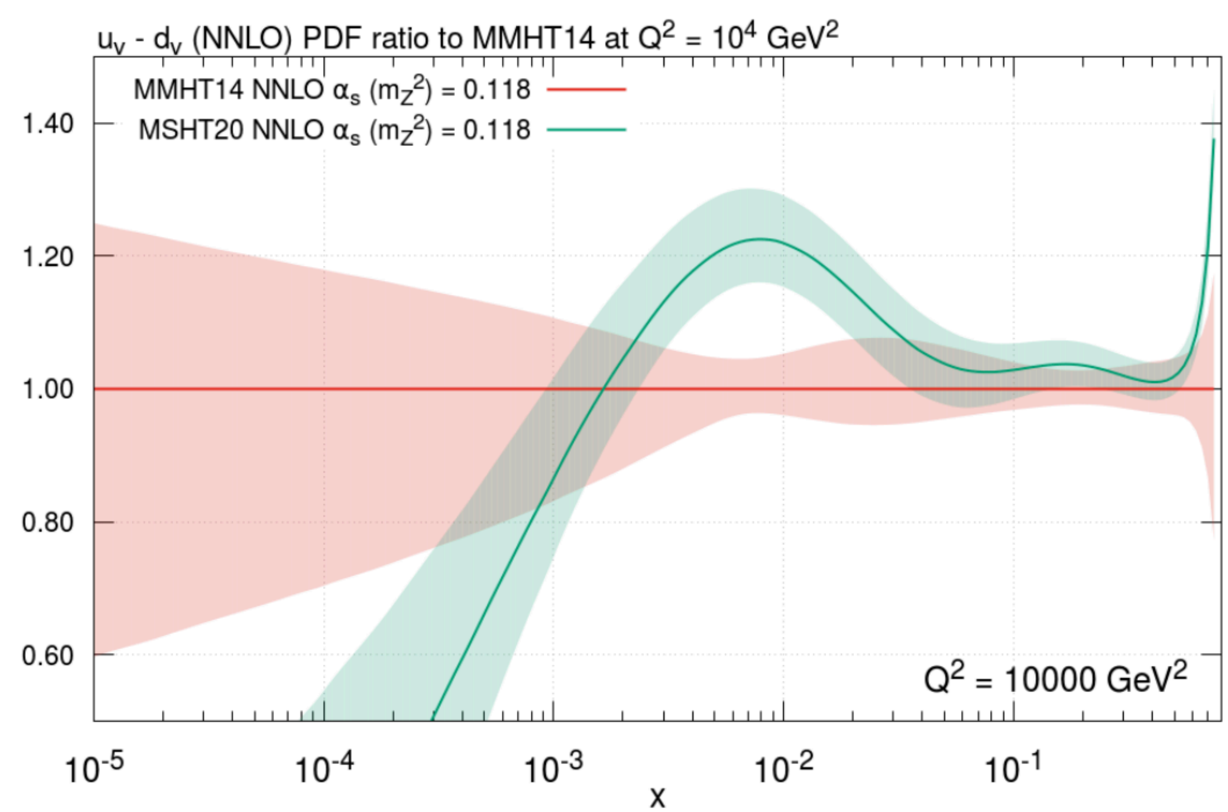


$$xg = A_g\, x^{\delta_g} (1-x)^{\eta_g} (1 + \epsilon_g \sqrt{x} + \gamma_g\, x) + A_{g'}\, x^{\delta_{g'}} (1-x)^{\eta_{g'}}$$

Traditional (parametrical) approach

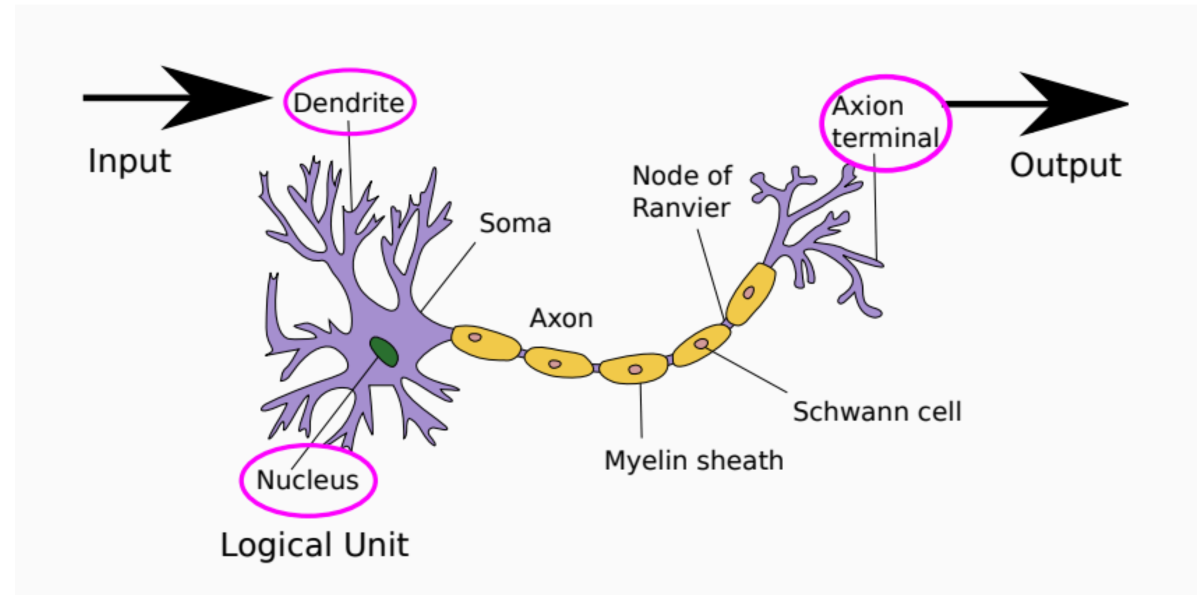
- Possible issues:
  If functional form not flexible enough PDFs may be not able to adapt to new data





- In recent updates from a global PDF fitting collaborations (MSHT20) the effect of LHC data required big change in the parametrization which makes PDF uncertainty increase (data-driven parametrization)

Neural networks and ML



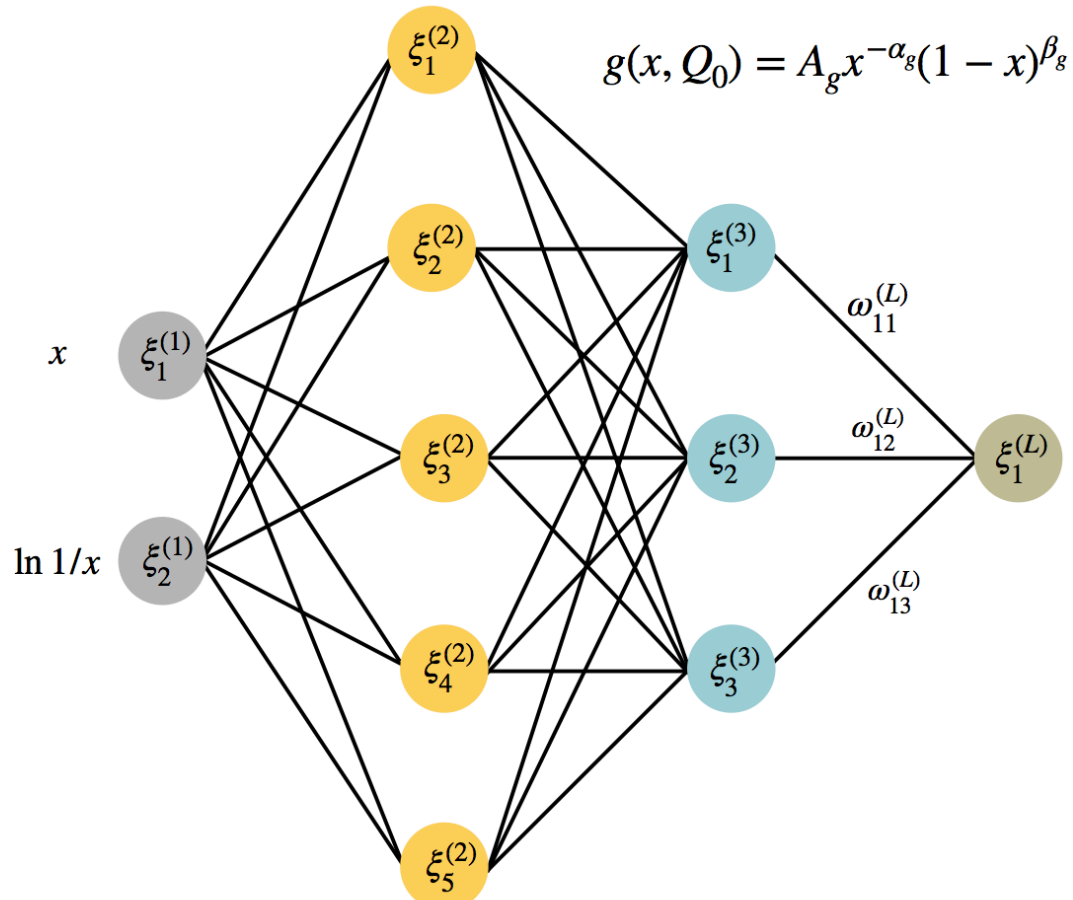
S. Carrazza, Colloquium, S. Paolo, N3PDF

- Artificial neural networks are computer systems inspired by the biological neural networks in the brain
- Data communication pattern
- Currently state-of-the-art for several Machine Learning Applications

Neural network parametrisation

Fully connected multi-layer perceptron

$$g(x, Q_0) = A_g x^{-\alpha_g}(1-x)^{\beta_g}\, \xi_L^{(1)}(x)$$



For a 1-2-1 feedforward neural network can write explicitly functional form

$$\xi_1^{(3)}(\xi_1^{(1)}) = \frac{1}{1 + e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1+e^{\theta_1^{(2)} - \xi_1^{(1)}\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1+e^{\theta_2^{(2)} - \xi_1^{(1)}\omega_{21}^{(1)}}}}}$$

- Neural Networks: all independent PDFs are associated to an unbiased and flexible parametrisation: O(300) parameters versus O(30) in polynomial parametrisation

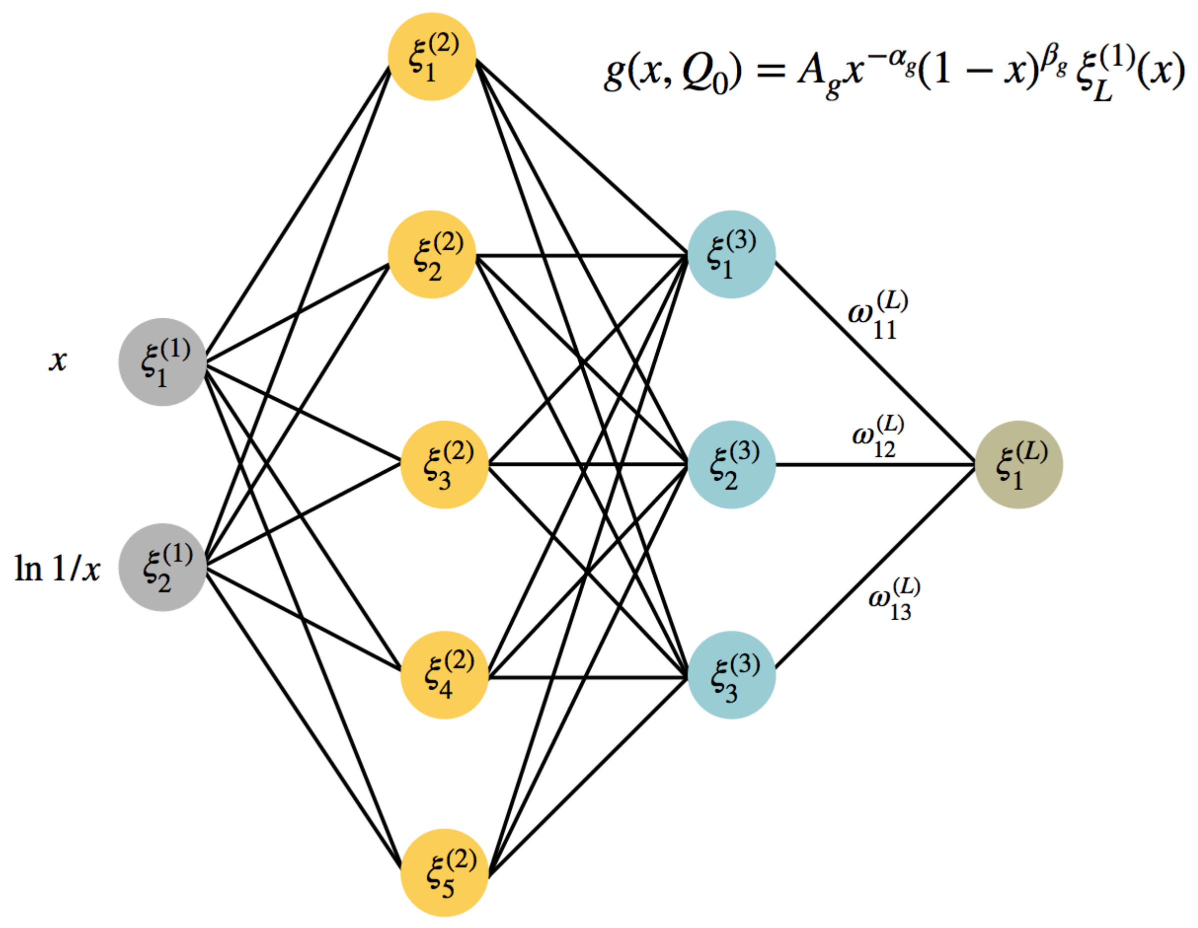
- 2-5-3-1 Neural network associated to each independent PDF (gluon, up, anti-up, down, anti-down, strange, anti-strange and charm)

$$\xi_i = g\left(\sum_j \omega_{ij}\xi_j - \theta_i\right)$$

$$g(x) = \frac{1}{1+e^{-x}}$$

Neural network training

Fully connected multi-layer perceptron



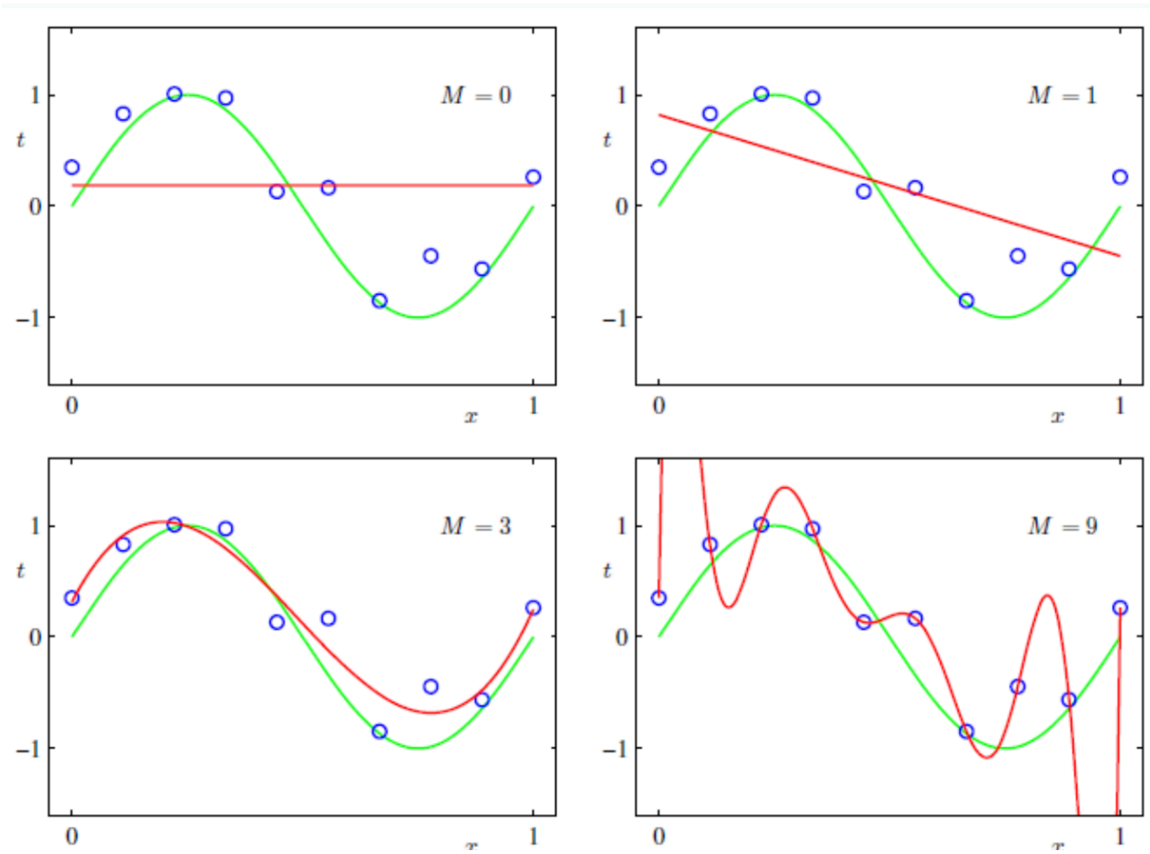
How do we train the 7(8) independent NN?

Minimise the cost function:

$$\chi^2 = \sum_{i,j=1}^{N_{\mathrm{dat}}} (D_i - T_i)(\mathrm{cov})^{-1}_{ij}(D_j - T_j)$$

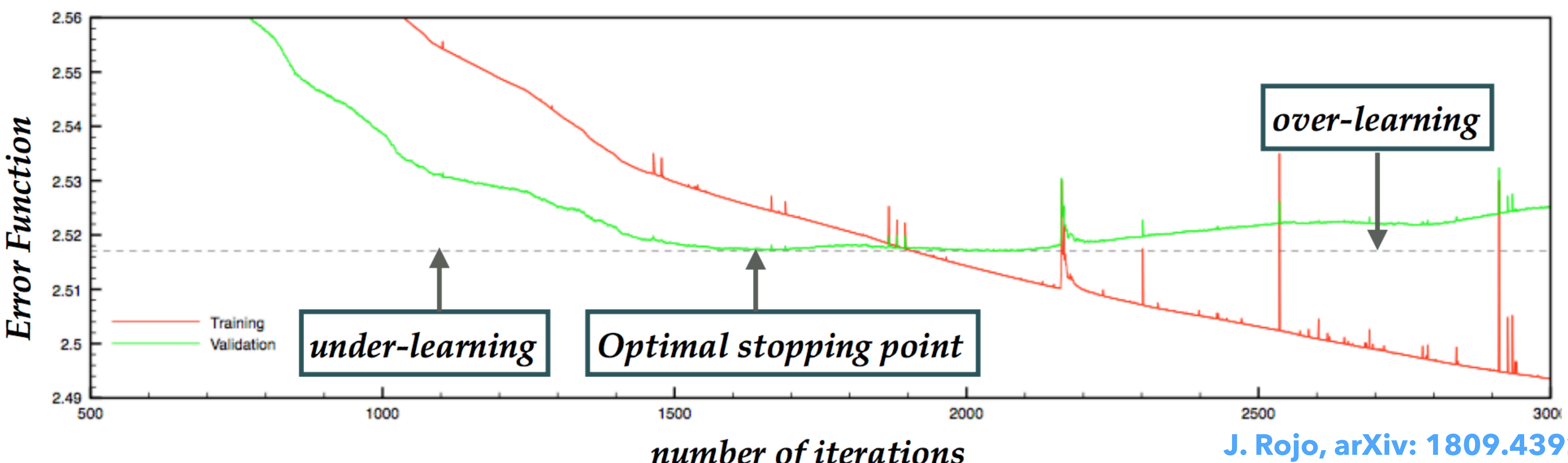
- D_i experimental measurement for the point i
- T_i theoretical prediction for the point i(depending on PDF parameters $\sigma_h = \sigma_{12} \otimes f_1 \otimes (f_2)$)
- $(\mathrm{cov})_{ij}$ is the covariance matrix between point i and j with corrections for normalisation uncertainties
- Supplemented by additional penalty for positive observables

Neural network training



- Large parameter space: need an algorithm that is able to explore it without getting trapped in local minima such as genetic algorithm

- Redundant parametrization: risk of over-fitting. Cross-validation necessary.



J. Rojo, arXiv: 1809.4392

(ii) Error propagation

$$\langle \mathcal{O}[\{f\}] \rangle = \int [\mathcal{D}f]\, \mathcal{O}[\{f\}] \mathcal{P}[\{f\}]$$

- Given a finite number of experimental data points want a set of functions
- Want to find a infinite-dimensional object from a finite number of information

Option a) Project into a n-dimensional space of parameters which parametrise PDFs and use linear approximation around minimum χ^2

$$\langle \mathcal{O}[\{f\}] \rangle \simeq \int da_1 da_2 ... da_{N_{par}}\, \mathcal{O}[\vec{a}]\, \mathcal{P}[\vec{a}]$$

Hessian Method

Option b) Choose a parametrisation and perform a Monte Carlo sampling of probability density in functional space

$$\langle \mathcal{O}[\{f\}] \rangle \simeq \frac{1}{N_{\rm rep}} \sum_{i=1}^{N_{\rm rep}} \mathcal{O}[f_i]$$

Monte Carlo Method

Hessian method

➡ Used by most PDF fitters (CTEQ/TEA, MSTW/MMHT, HERAPDF, ABM)

➡ Pick a functional form and project problem in the N_{par}-dimensional space of parameters (typically 15 - 25)

➡ Determine best fit values of parameters $\{\vec{a}_0\}$

➡ Shift $\vec{a} \to \vec{a} - \vec{a}_0$

➡ Determine error on PDFs and any observable depending on PDFs (all denoted by X) by propagation of the error in the parameter space

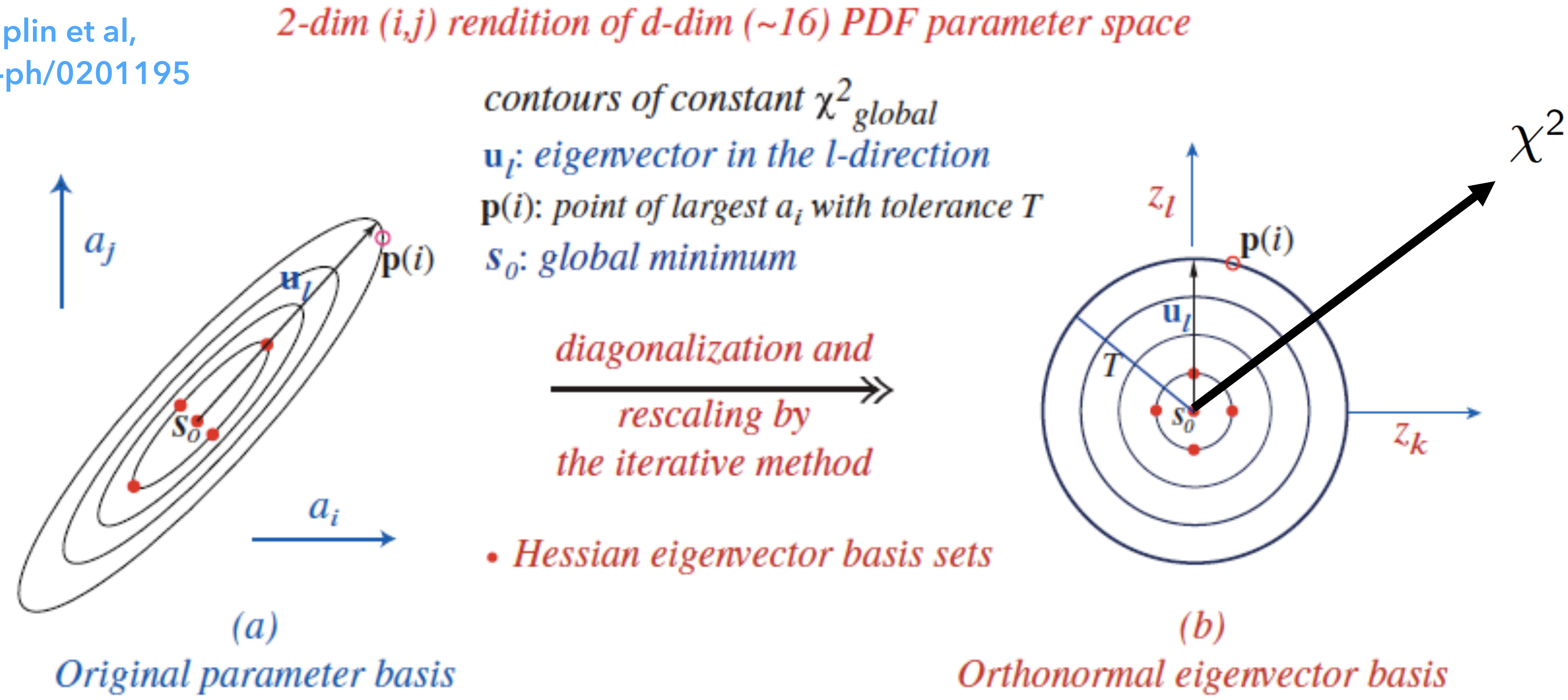
Assuming linear prop: $X(\vec{a}) \simeq X(\vec{0}) + a_i \partial_i X(\vec{a})\big|_{\vec{a}=\vec{0}}$

Variance: $\sigma_X^2 = (\mathrm{cov})_{ij}\, \partial_i X \partial_j X$ $(\mathrm{cov})_{IJ}$ covariance matrix in param, space

Maximum likelihood: $(\mathrm{cov})_{ij} = (H)_{ij} = \left.\dfrac{\partial^2 \chi^2(\vec{a})}{\partial_i a \partial_j a}\right|_{\vec{a}=\vec{0}}$ cov $\Longleftrightarrow$ Hessian at the minimum of χ^2

Hessian method

Pumplin et al, hep-ph/0201195



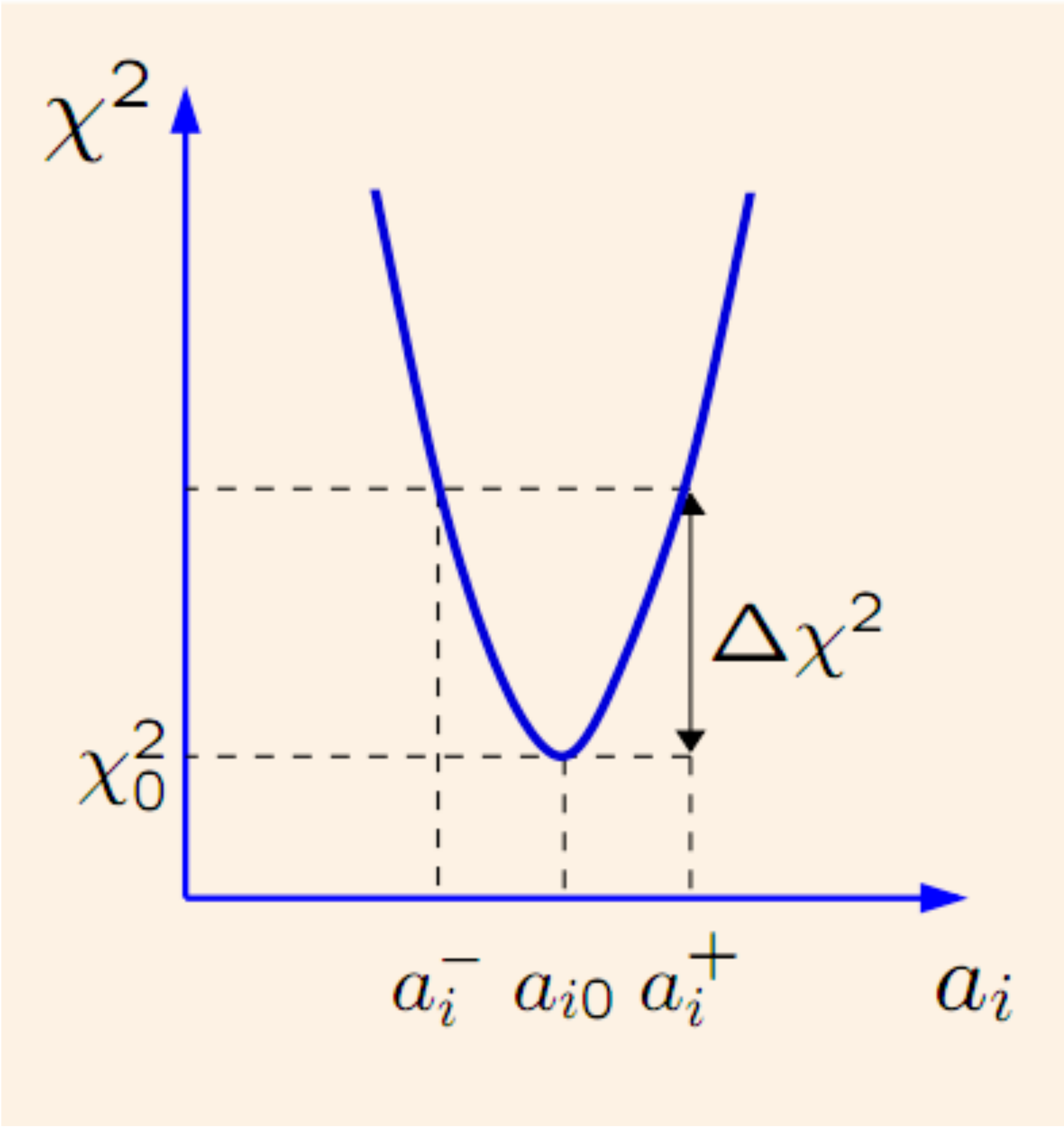
$$\sigma_X^2 = (H)_{ij}\, \partial_i X\, \partial_j X \quad \xrightarrow{\text{diagonalisation}} \quad \sigma_X^2 = |\vec{\nabla} X|^2$$

z_i eigenvectors of H with unit eigenvalues

➡ The total uncertainty is the sum in quadrature of the uncertainties due to each parameter

➡ $\Delta\chi^2 = \sum z_i^2$ the surfaces of constant χ^2 are spheres in the z space of radius $\sqrt{\Delta\chi^2}$

Hessian method



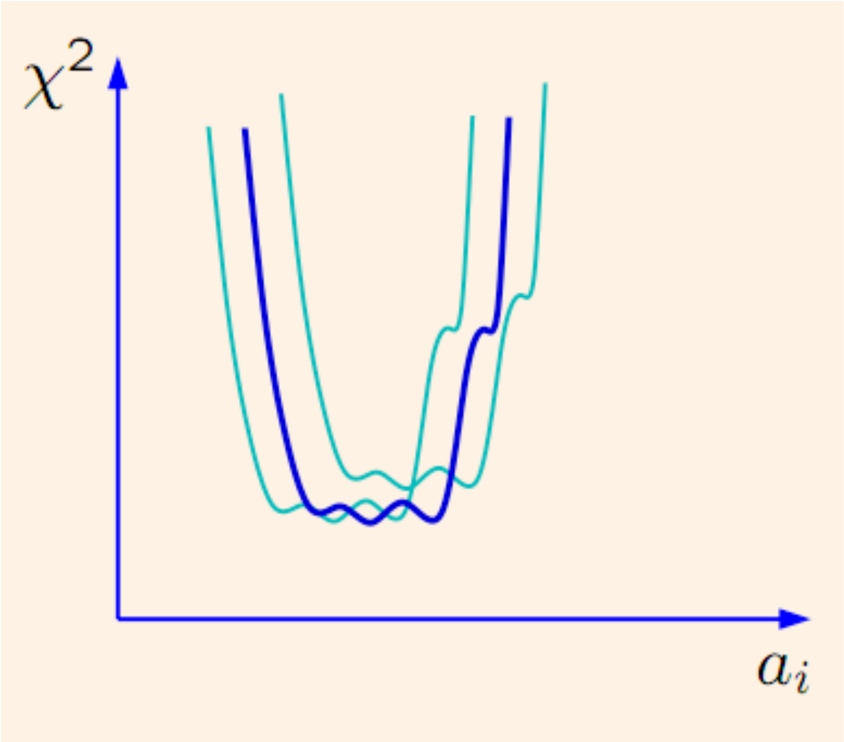
P. Nadolsky, CTEQ summer school 2009

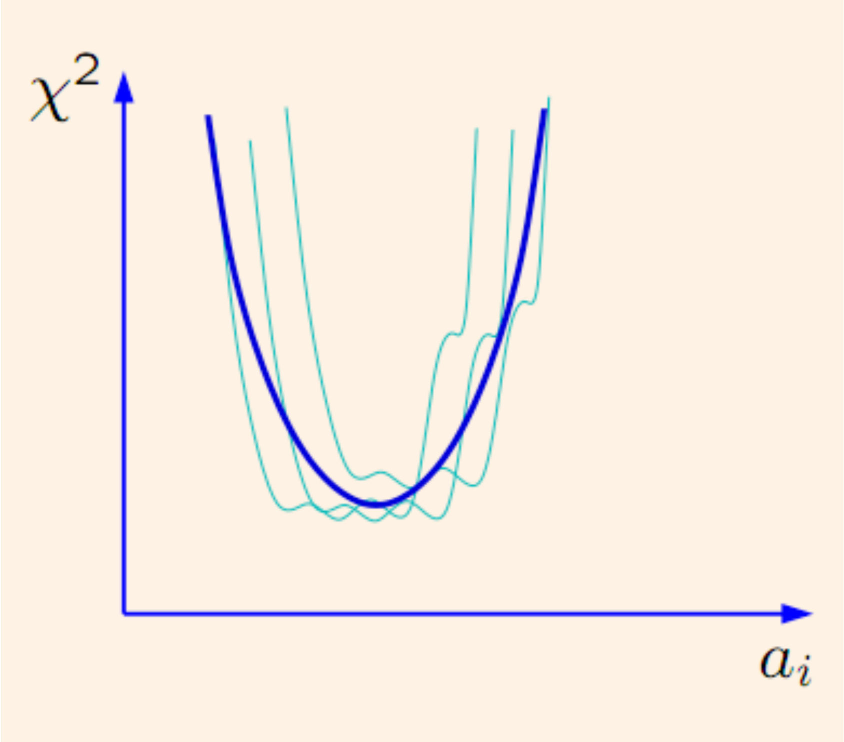
- According to textbook statistics, the 1σ contour in parameter space is given by

$$\Delta\chi^2 = 1$$

- Projection of the radius one sphere would give the uncertainty on parameters and on the PDFs, observables…
- The textbook statistics should work in case of perfectly compatible Gaussian errors
- But in practice, for global fits a tolerance is introduced
- NB: introducing a tolerance corresponds to blow up uncertainties by a factor $\sqrt{\Delta\chi^2}$

Hessian method





The actual χ^2 function displays

- A well pronounced global minimum $\chi_0{}^2$
- Some tensions between datasets in the vicinity of the minimum
- Some dependence on assumptions about flat directions (= unconstrained combinations of PDF parameters)

The likelihood is approximately described by a quadratic χ^2 with a revised tolerance condition

$$\Delta\chi^2 \leq T^2$$

P. Nadolsky, CTEQ summer school 2009

Monte Carlo method

➡ First idea by Giele Keller Kosover (hep-ph/0104052)
➡ Monte Carlo in parameter space

$$X(\vec{a})$$

MC sampling ➡

$$\langle X \rangle = \int d\vec{a} X[\vec{a}]\mathcal{P}[\vec{a}]$$

P probability of parameter values

MC sampling in **parameter** space

$$\langle X \rangle \sim \frac{1}{N_{\rm rep}} \sum_{i=1}^{N_{\rm rep}} X(\vec{a}_i)$$

$$\sigma_X^2 = \langle X^2 \rangle - \langle X \rangle^2$$

Problem

How many replicas are needed?
Three bins per parameter ⇒ $3^{N_{par}}$ bins

E.g. for 23 parameters need more than 10^{11} replicas!!!

Monte Carlo method

- ➡ Forte, J. I Latorre, Piccione (hep-ph/0701127)
- ➡ First applied to structure functions then to PDFs

$$X(\vec{a})$$

MC sampling →

$$\langle X \rangle = \int d\vec{a} X[\vec{a}] \mathcal{P}[\vec{a}]$$

P probability of parameter values

MC sampling in **data** space

$$\langle X \rangle \sim \frac{1}{N_{\rm rep}} \sum_{i=1}^{N_{\rm rep}} X(\vec{a}_i)$$

$$\sigma_X^2 = \langle X^2 \rangle - \langle X \rangle^2$$

Idea

Choose parameters along ∇X ⟺ Choose replicas of the data, i.e. work in the space of data and project back into PDF space

How many replicas does one need? 1-dim average of N_{rep} converges to true average with standard deviation $\sigma/\sqrt{N_{rep}}$

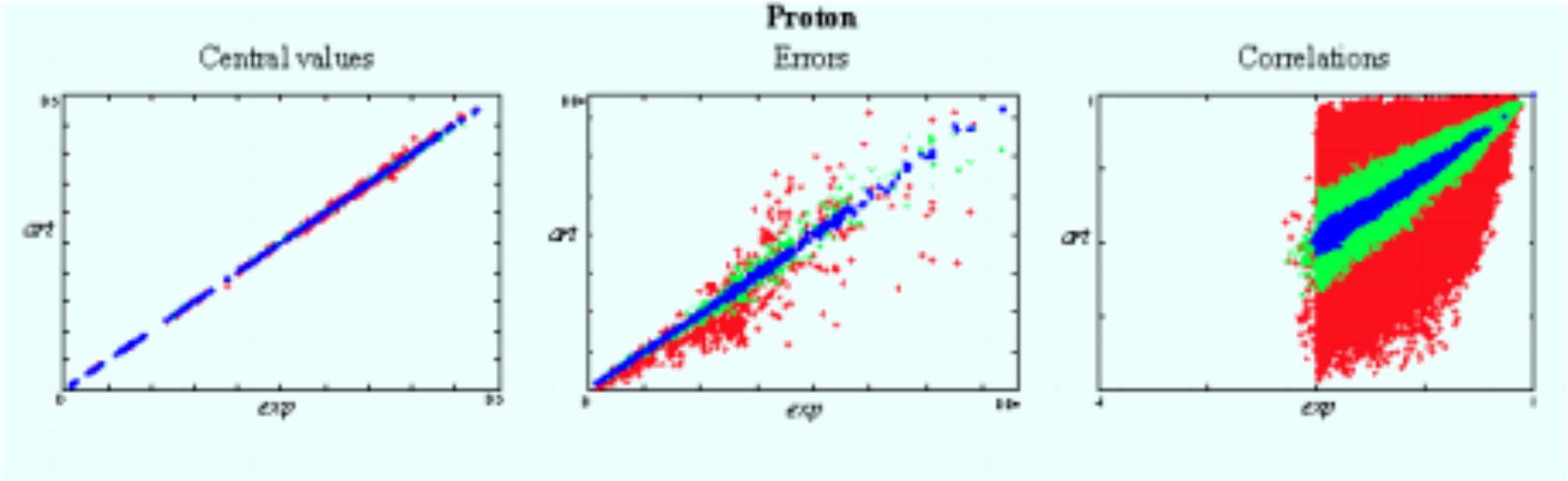
E.g. 10 replicas are enough for getting "true" central value with σ/3 accuracy

Monte Carlo method

- Generate artificial data according to distribution

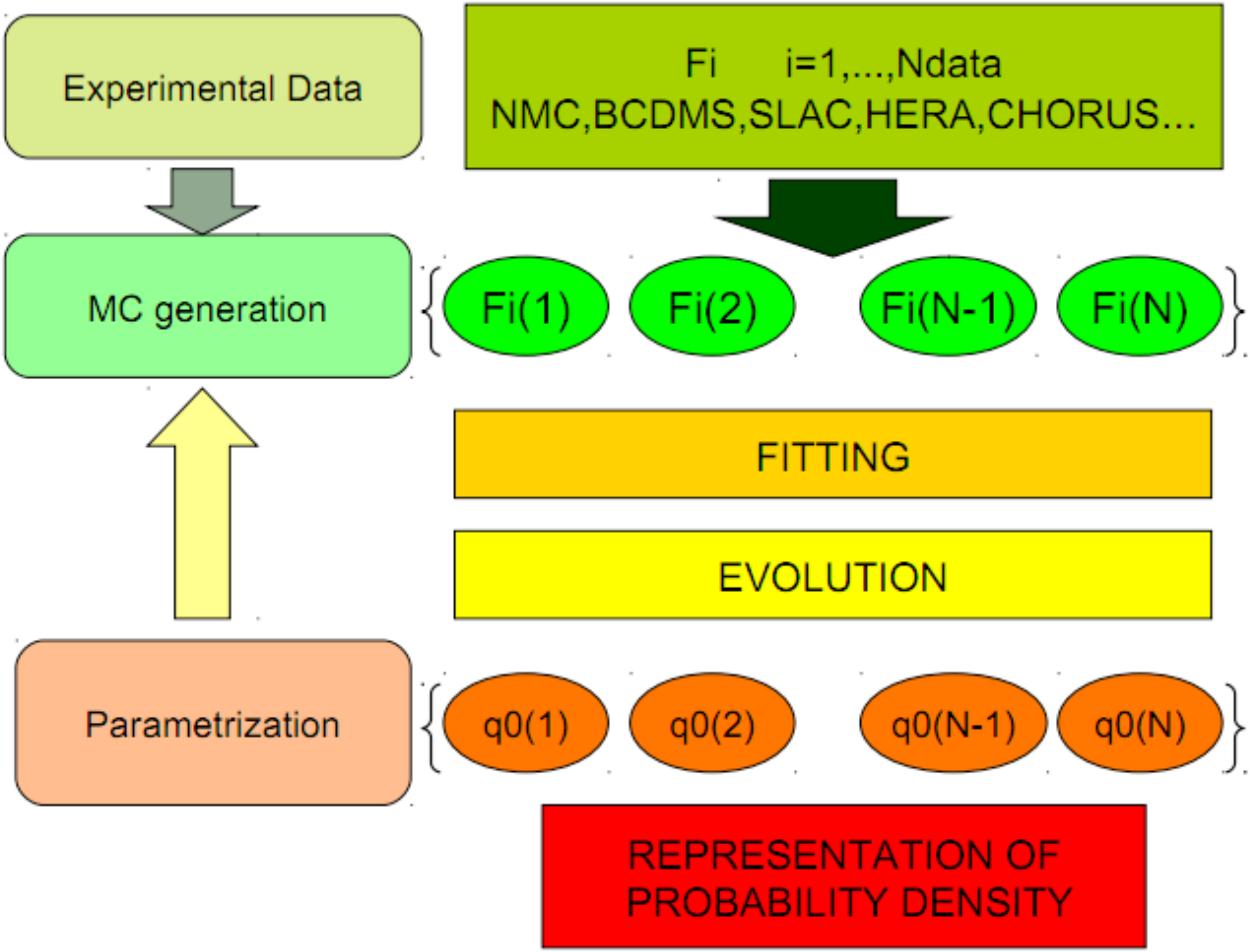
$$F_p^{(\mathrm{art})(k)} = S_{p,N}^{(k)} F_p^{(exp)} \left(1 + \sum_{l=1}^{N_c} r_{p,l}^{(k)} \sigma_{p,l} + r_p^{(k)} \sigma_{p,s} \right)$$

- r_i are univariate Gaussian random numbers such that if two points have correlated systematic uncertainties, they oscillate in the same directions
- S normalisation factors
- Validate Monte Carlo replicas against experimental data

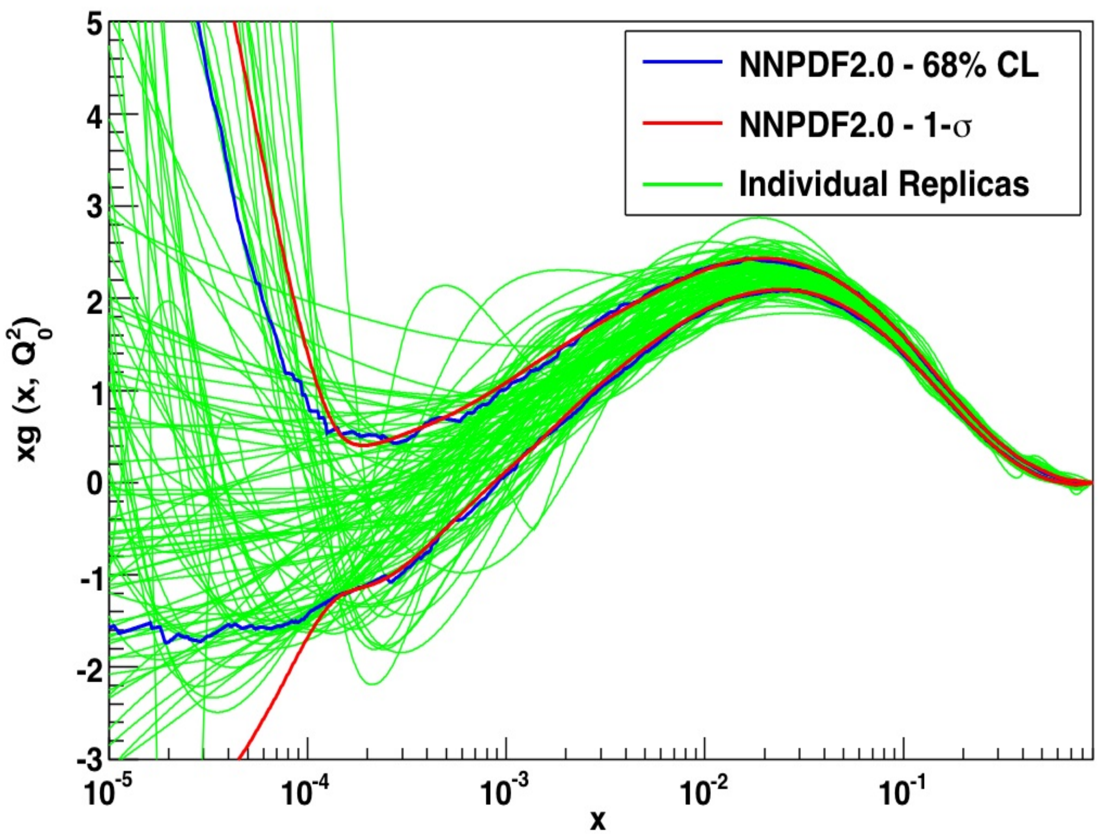


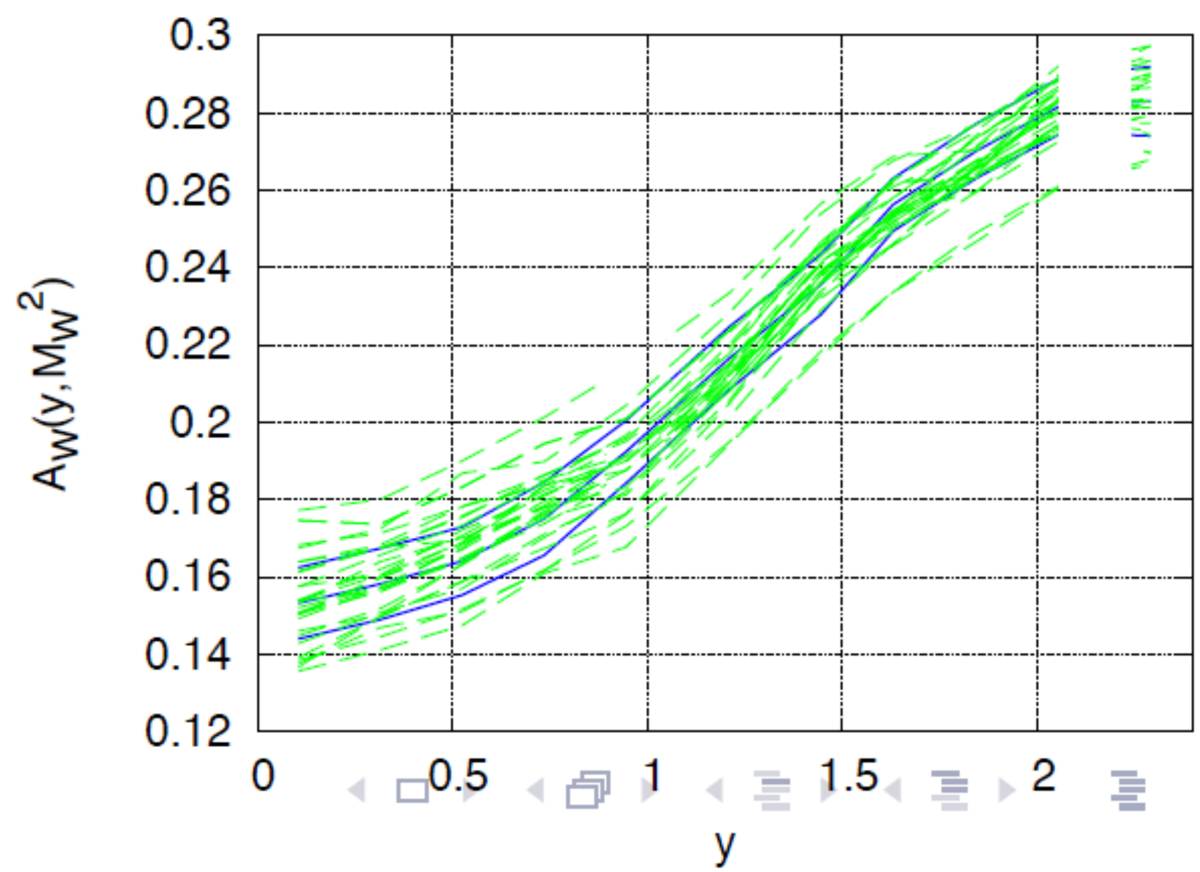
- Convergence rate increases with N_{rep}
- Correlations reproduced to % accuracy with 1000 reps

Monte Carlo method



Monte Carlo method





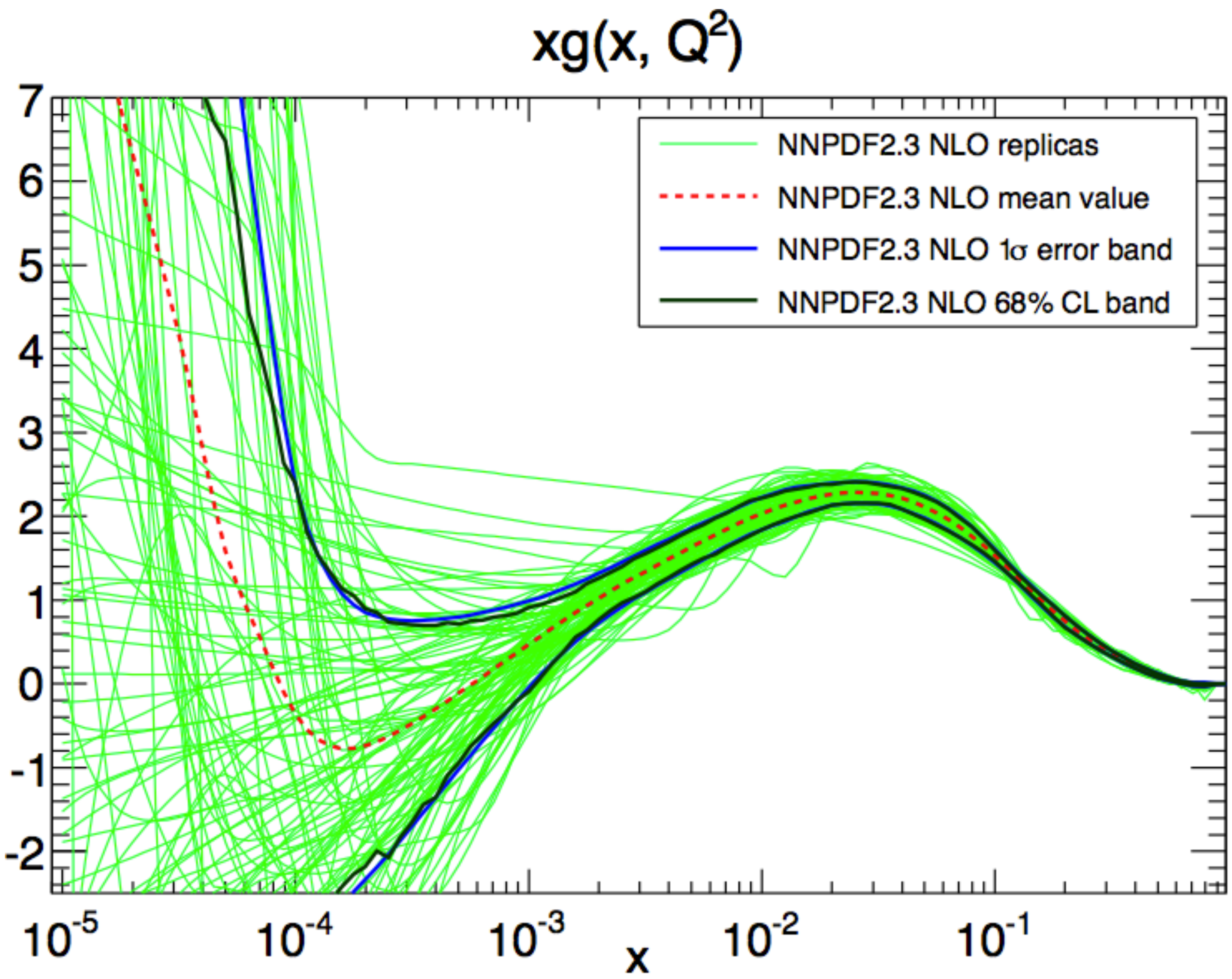
$$\langle f_J \rangle = \frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} f_J^{(k)}$$

$$\langle A(\{f\}) \rangle = \frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} A(\{f^{(k)}\})$$

$$\sigma_X^2 = \langle X^2 \rangle - \langle X \rangle^2$$

Individual replicas may fluctuate significantly, average quantities such as central values and 1σ error bands are smooth inasmuch as stability is reached due to the dimension of the ensemble increasing

The NNPDF solution



The N(eural)N(etwork)PDFs:

- Monte Carlo techniques: sampling the probability measure in PDF functional space
- Neural Networks: all independent PDFs are associated to single NN

Summary for the user

Hessian method (CT, CJ, MSTW, ABKM, HERAPDF)

$$\langle \mathcal{F} \rangle = \mathcal{F}[q^{(0)}]$$

$$\sigma_{\mathcal{F}}^{\mathrm{Hess}} = \frac{1}{2} \left(\sum_{k=1}^{N_{\mathrm{set}}/2} \left(\mathcal{F}[\{q^{(2k-1)}\}] - \mathcal{F}[\{q^{(2k)}\}] \right)^2 \right)^{1/2}$$

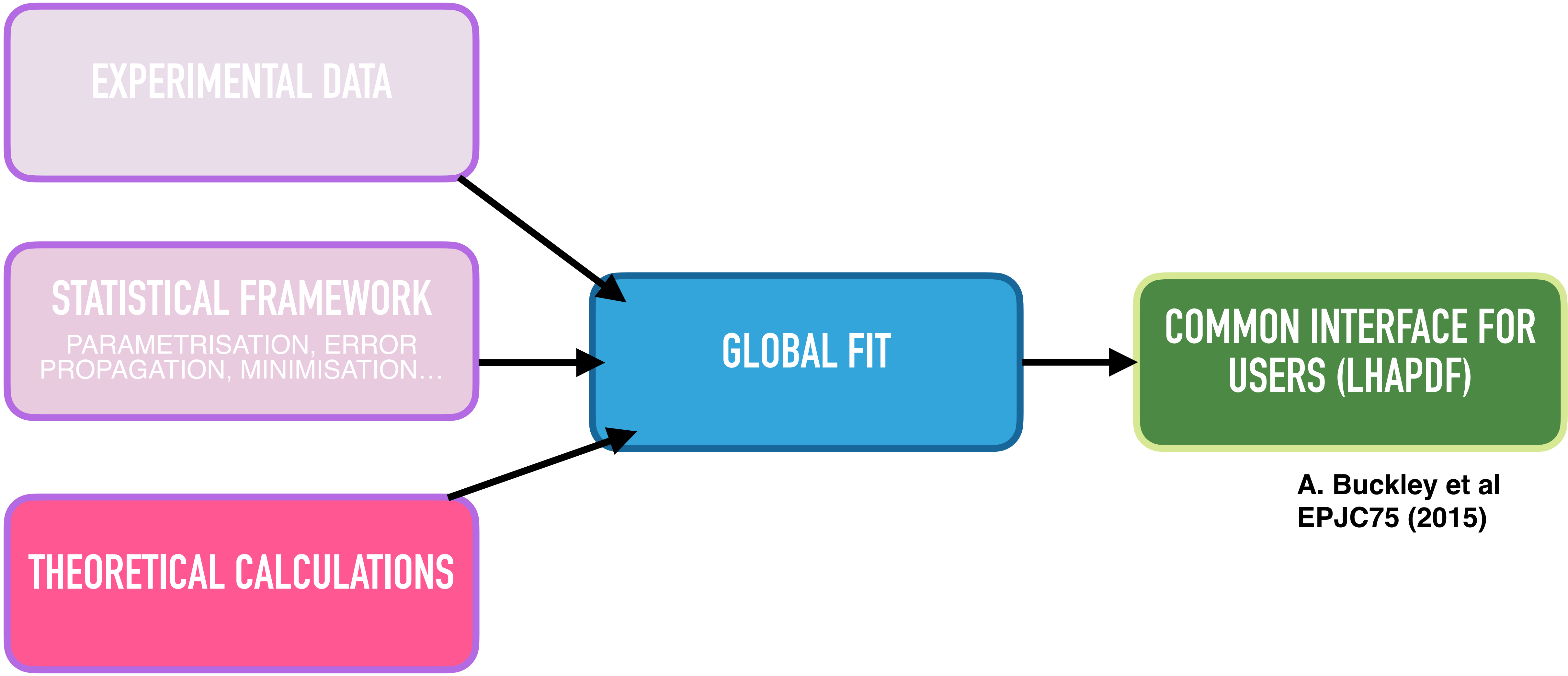
Monte Carlo method (NNPDF, JAM)

$$\langle \mathcal{F} \rangle = \frac{1}{N_{\mathrm{set}}} \sum_{i=1}^{N_{\mathrm{set}}} \mathcal{F}[q^{(i)}]$$

$$\sigma_{\mathcal{F}}^{\mathrm{MC}} = \left(\frac{1}{N_{\mathrm{N_{set}}}} \sum_{k=1}^{N_{\mathrm{set}}} \left(\mathcal{F}[\{q^{(k)}\}] - \langle \mathcal{F}[\{q\}] \rangle \right)^2 \right)^{1/2}$$

Theoretical aspects & theory frontiers

A complex machinery



A. Buckley et al EPJC75 (2015)

Theory predictions in PDF fits

$$\chi^2 = \sum_{i,j=1}^{N_{\rm dat}} (T_i - D_i)\,({\rm cov}_{\rm exp})^{-1}_{ij}\,(T_j - D_j)$$

PDF parameters determined by minimising figure of merit

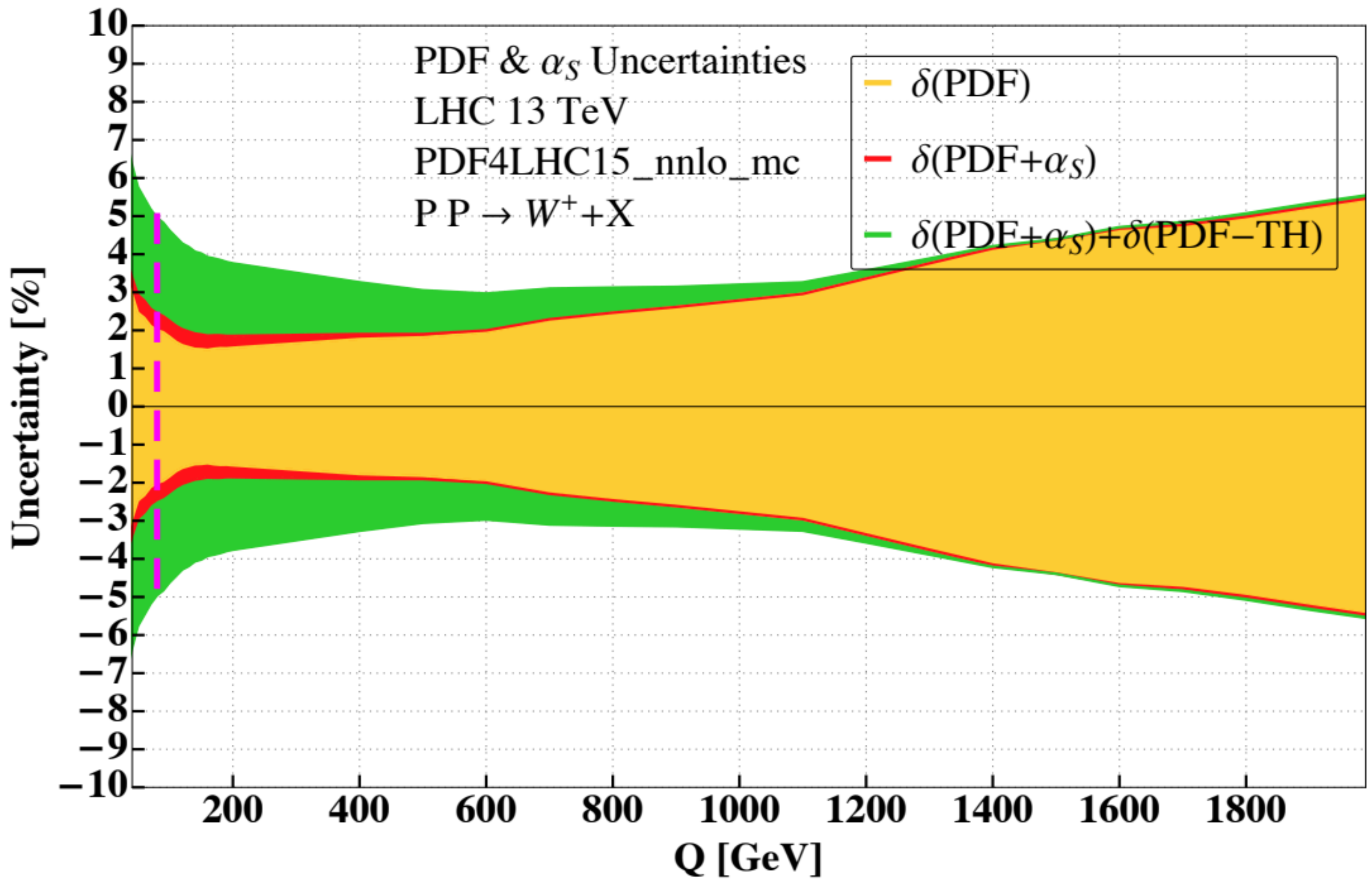
$$T = f_1\,(\otimes f_2)\,\otimes\,\hat{\sigma}$$

$$\hat{\sigma} = \alpha_s^p \sigma_0 + \alpha_s^{p+1}\sigma_1 + \alpha_s^{p+2}\sigma_2 + \mathcal{O}(\alpha_s^{p+3})$$

- Standard global PDF fits based on fixed-order NNLO QCD calculations
- Standard global PDF fits set specific values for
  - $\alpha_s(M_z)$
  - M_w, M_z, $\alpha_e(M_z)$
  - CKM matrix elements
  - Heavy quark mass thresholds
  - Branching ratios…
- MHOUs in perturbative expansion, the uncertainty on the value of the parameters that enter a PDF fit are <u>NOT</u> included in PDF error bars

Mismatch between pert. orders

- PDF fits performed at given perturbative order - NNLO
- PDF uncertainties only reflect lack of information from data
- Theoretical uncertainties (dominated by MHOU) ignored so far
- New frontier for partonic cross sections is N3LO
- Mismatch between perturbative order of partonic cross section and PDFs becoming significant source of uncertainty



$$\delta(PDF-TH) = \frac{1}{2}\left|\frac{\sigma^{(2)}_{\mathrm{NNLO-PDFs}} - \sigma^{(2)}_{\mathrm{NLO-PDFs}}}{\sigma^{(2)}_{\mathrm{NNLO-PDFs}}}\right|$$

C. Duhr et al, *JHEP* 11 (2020) 143

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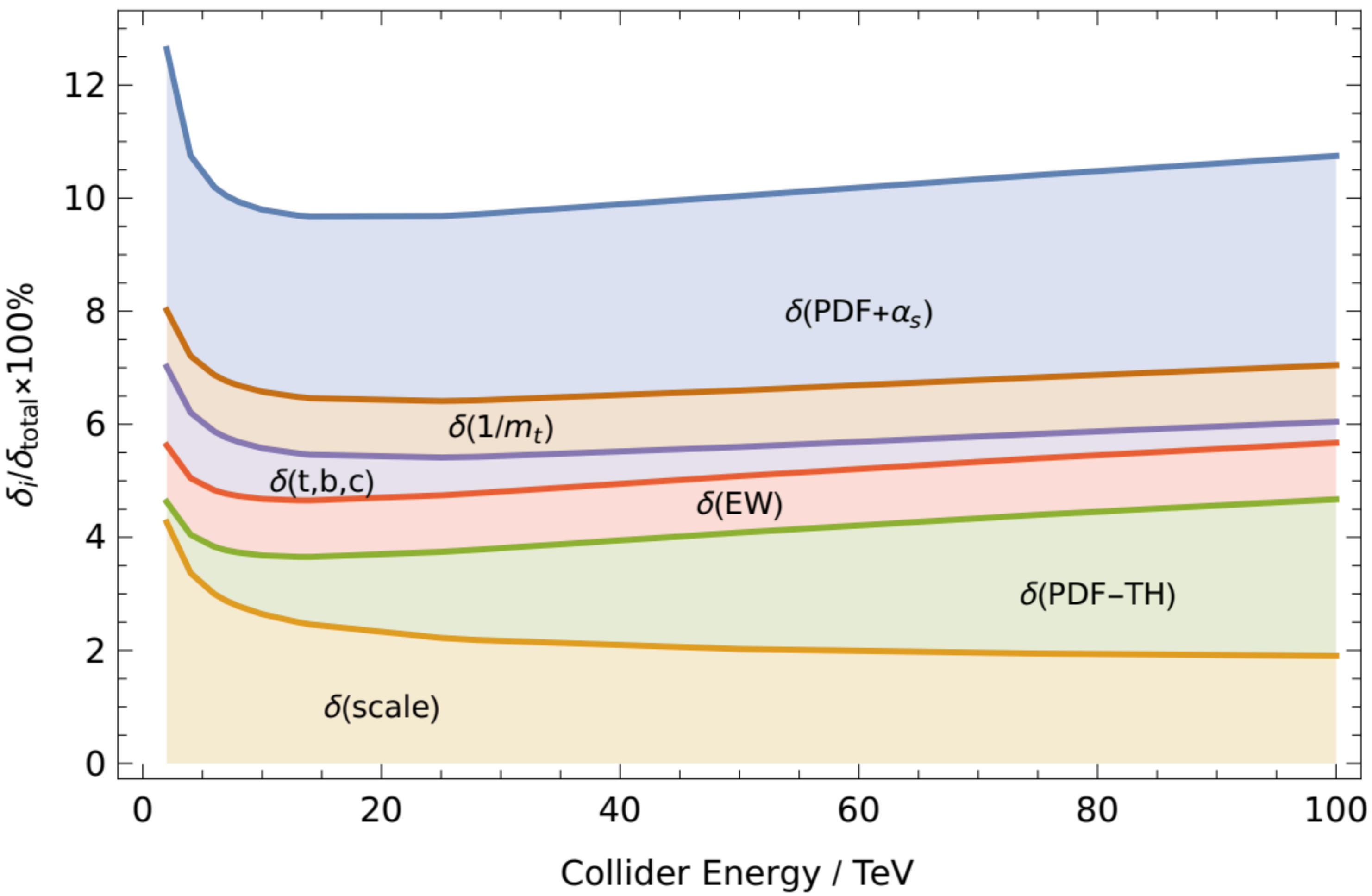
- Mismatch between perturbative order of partonic cross section and PDFs becoming significant source of uncertainty

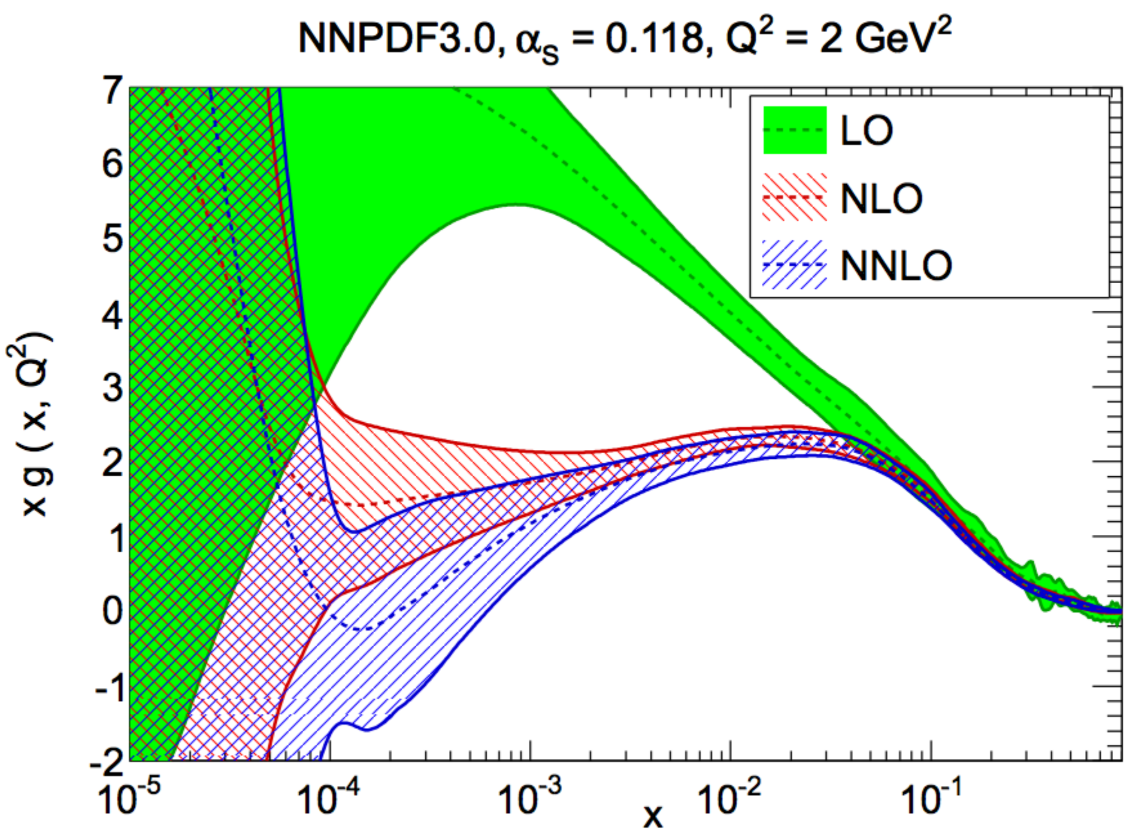


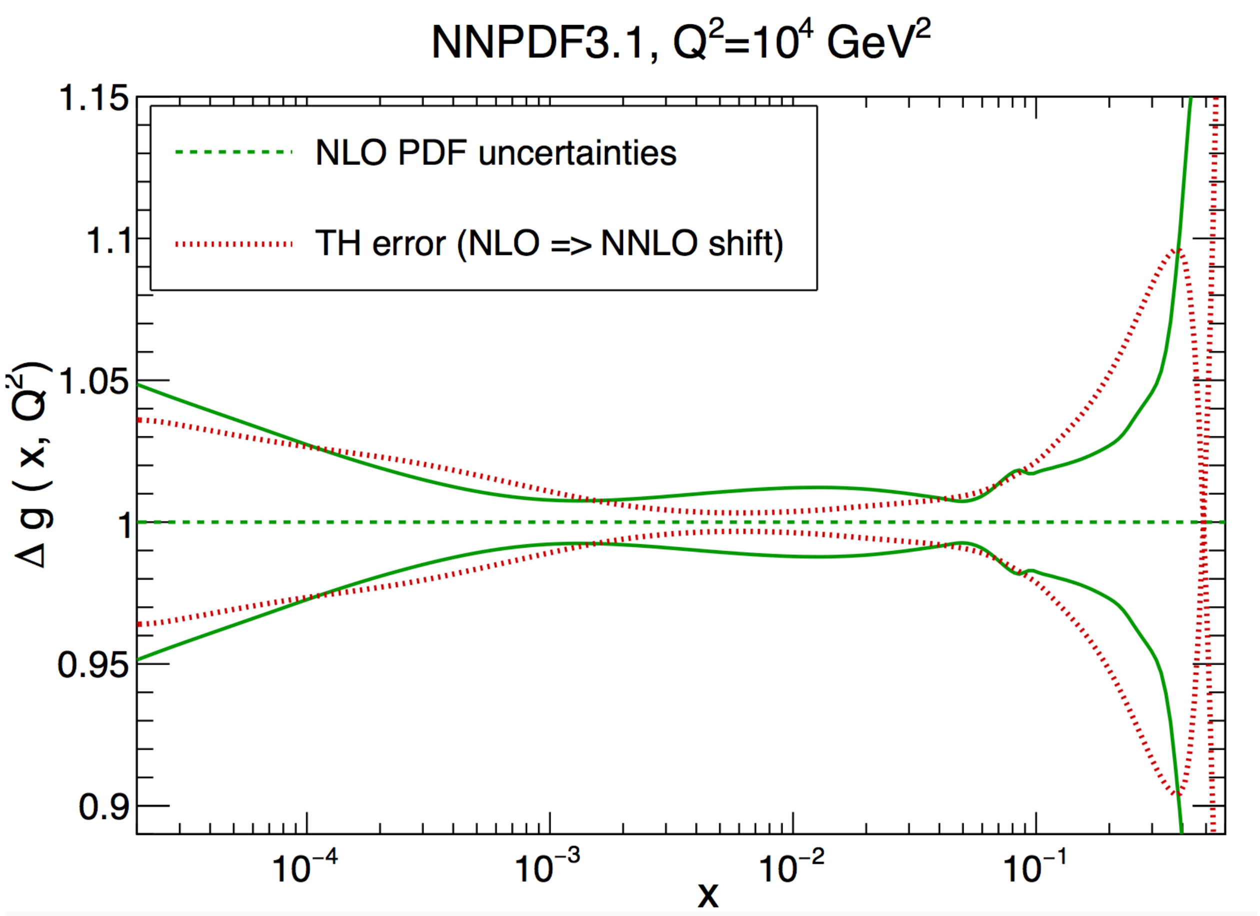
$$\delta(PDF-TH) = \frac{1}{2}\left|\frac{\sigma^{(2)}_{\mathrm{NNLO-PDFs}} - \sigma^{(2)}_{\mathrm{NLO-PDFs}}}{\sigma^{(2)}_{\mathrm{NNLO-PDFs}}}\right|$$

M. Cepeda et al. [HL/HE WG2 group], arXiv:1902.00134

MHOU in PDF fits

- In a fit based on NLO theoretical predictions the theory error is already comparable to experimental error. What about a NNLO fit?
- How to include MHOUs in PDF error bands at NNLO?





Ball et al, EPJC 77 (2017)

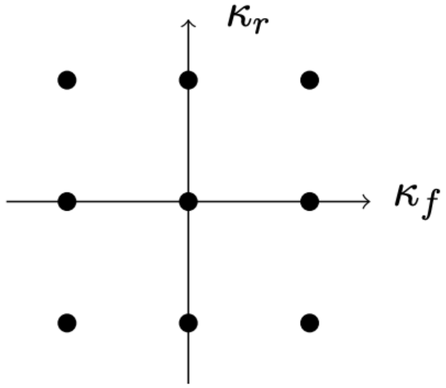
MHOU in PDF fits

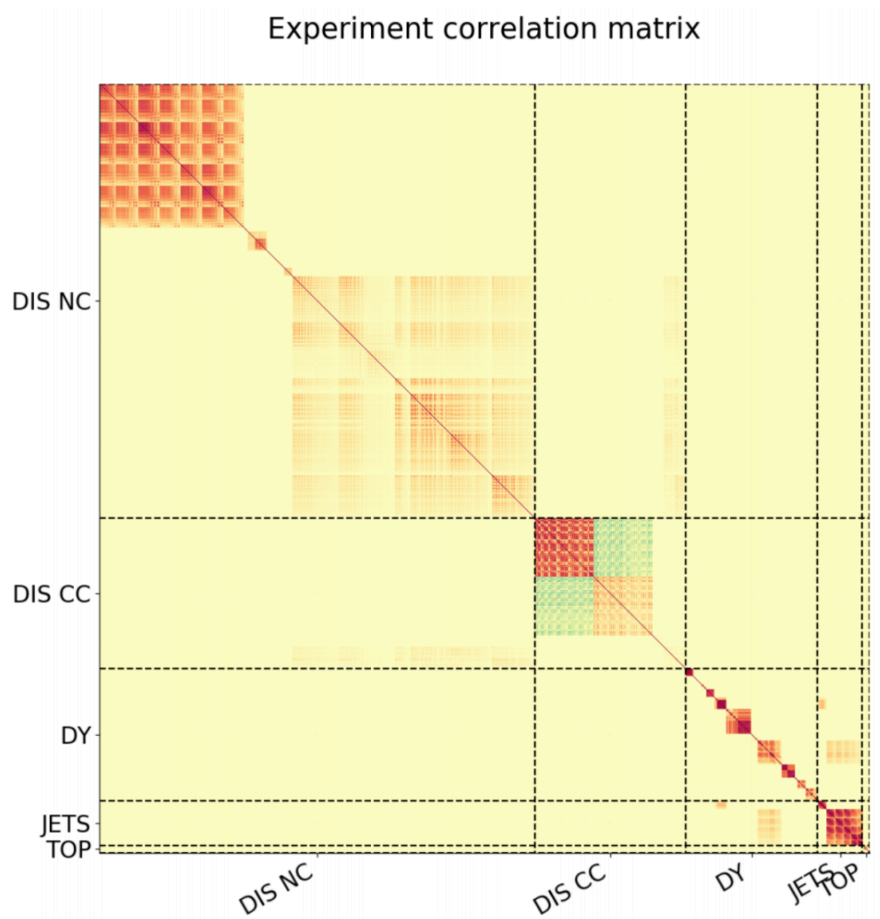
Option 1 - theory covmat [NNPDF: 1906.10698]

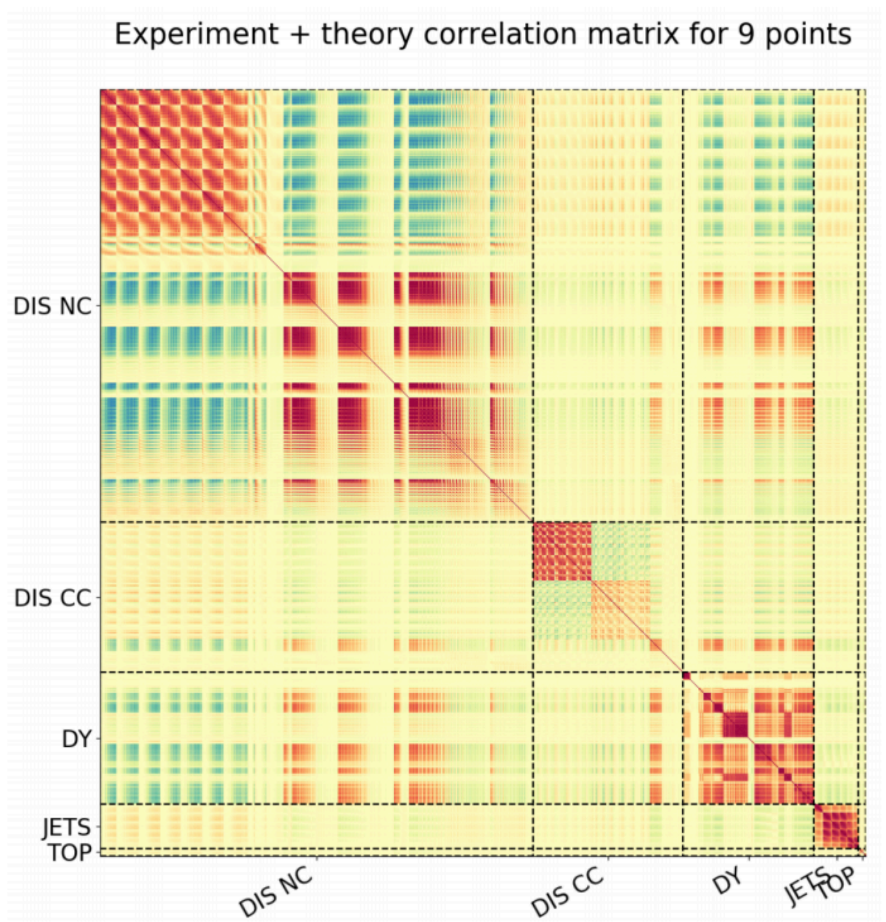
Construct a theory covariance matrix from scale-varied cross sections and combine it with the experimental covariance matrix

$$\chi^2 = \sum_{i,j=1}^{N_{\rm dat}} (T_i - D_i)\,(\mathrm{cov}_{\rm exp} + \mathrm{cov}_{\rm th})^{-1}_{ij}\,(T_j - D_j)$$

- Assumptions: experimental and theoretical errors independent and Gaussian
- Assumptions on correlation of scales and scale ratio will determine the specific form of the covariance matrix







MHOU in PDF fits

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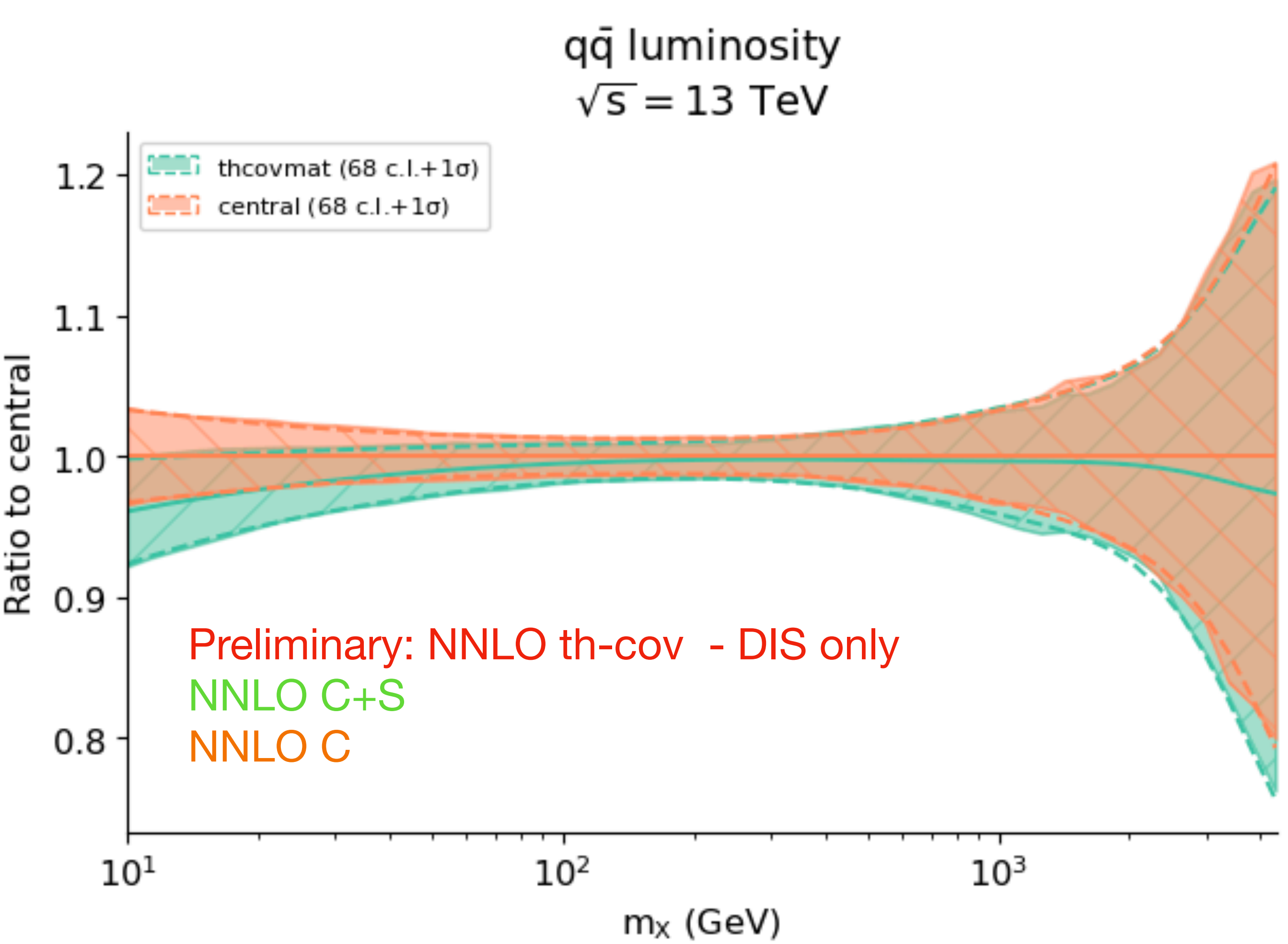
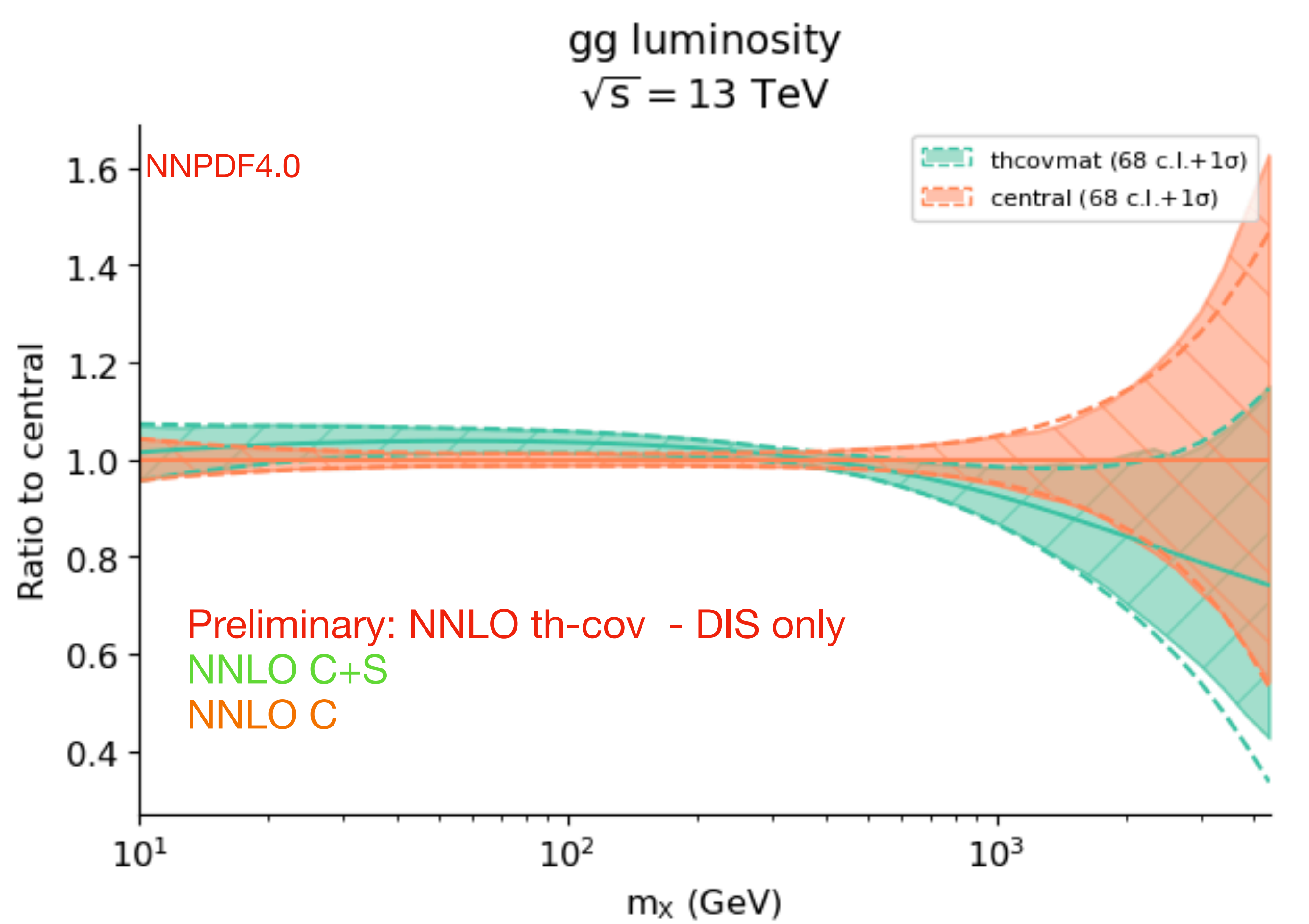


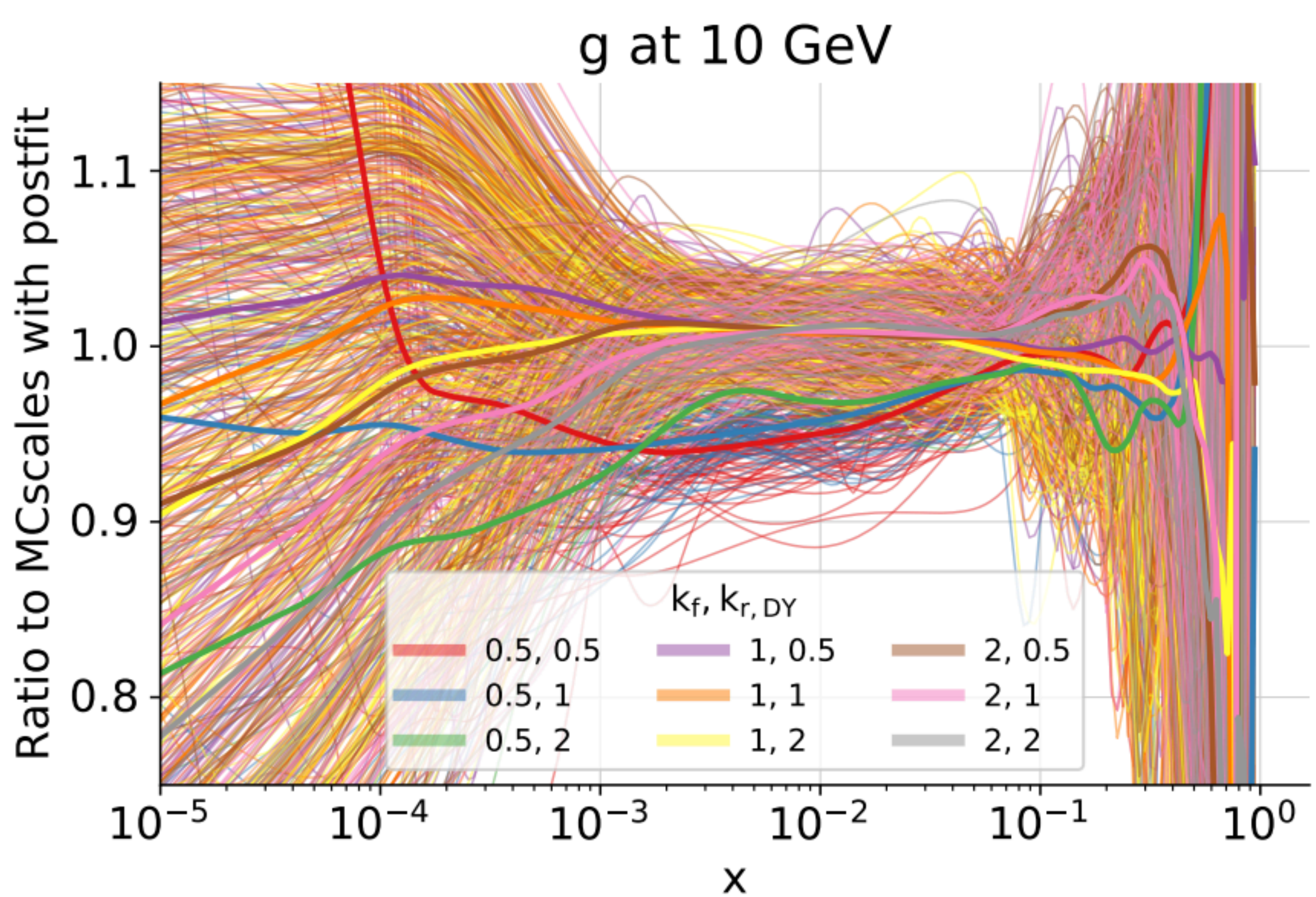


MHOU in PDF fits

Option 2 - MCscales [Kassabov et al: 2207.07616]

Main idea: renorm. and fact. scales are free parameters of the fixed-order theory, that induce an uncertainty on theory predictions included in a PDF fit & need to be propagated

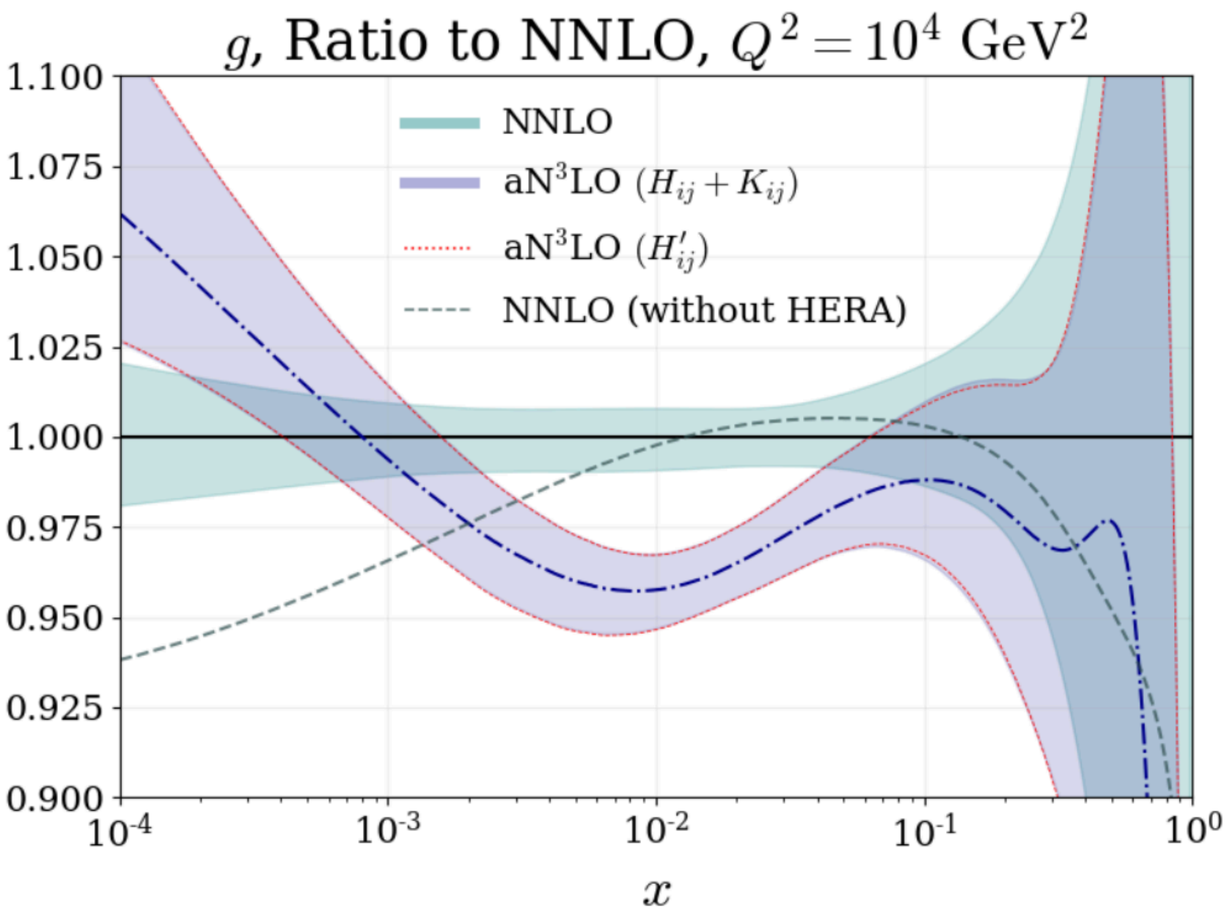
- Joint sampling of experimental uncertainty (propagated to PDF uncertainty by Monte Carlo) by specifying a suitable prior probability distribution of all possible scale choices and an a-posteriori criterion based on agreement with the data.

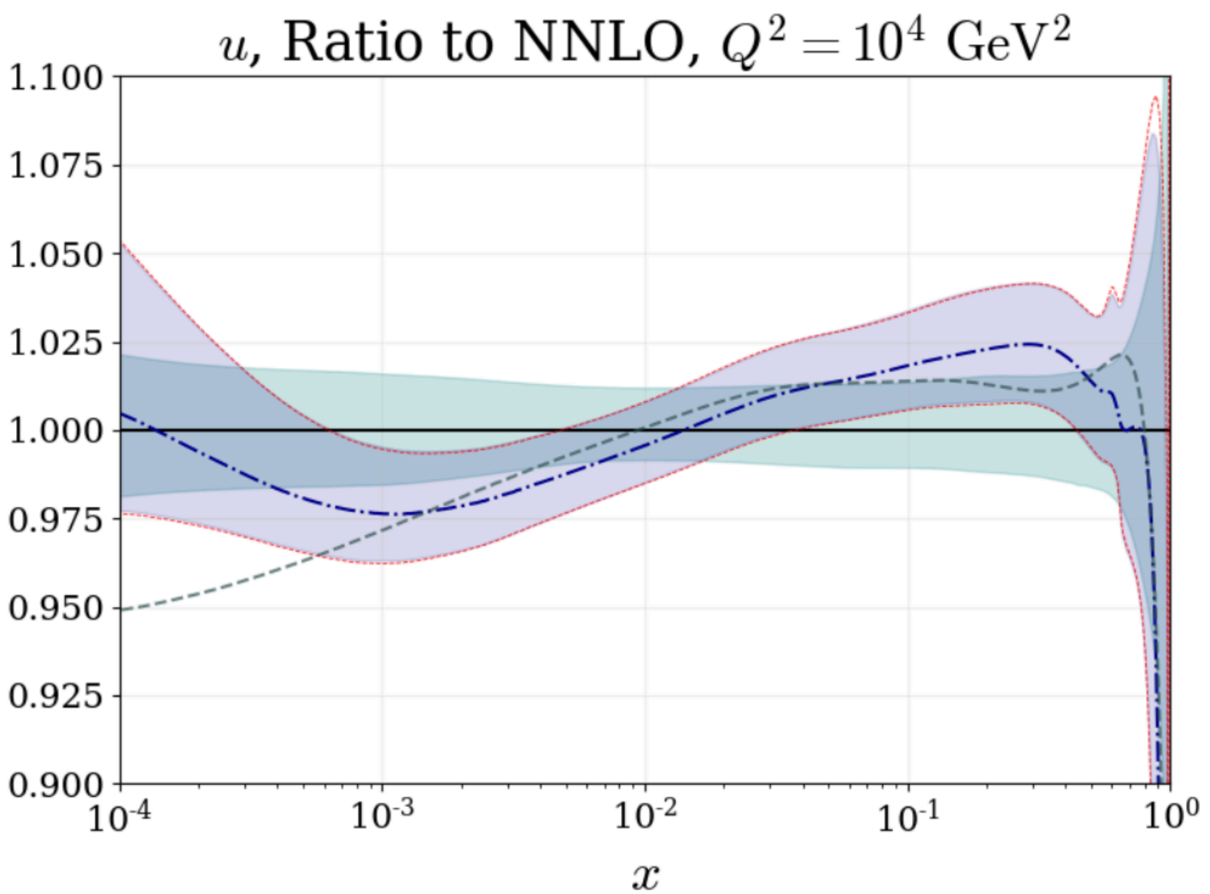


MHOU in PDF fits

Option 3 - theory nuisance parameters [MMHT: 2207.04739]

Main idea: add MHOU as nuisance parameters and fit nuisance parameters from data. Unknown terms in N3LO theory added via a Gaussian prior and parameters fitted from the data give rise to approximated N3LO MSHT PDFs



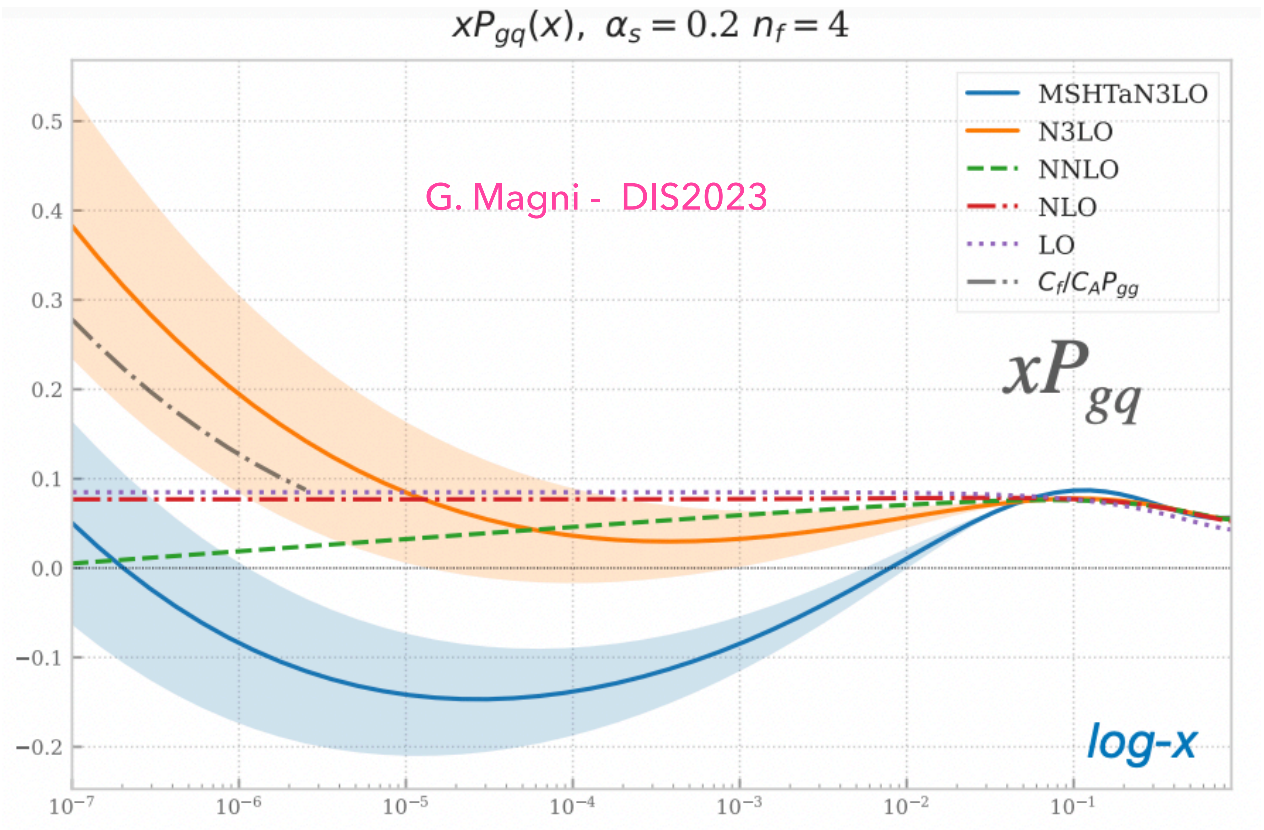


The N3LO frontier

- MSHT & NNPDF: inclusion of available theoretical ingredients at N3LO (non-singlet splitting functions, singlet splitting function in the large nf limit, small-x limit, large-x limit, Mellin moments + DIS structure functions in the massless limit and approximate heavy flavour structure functions between known limits + hadronic N3LO K-factors)

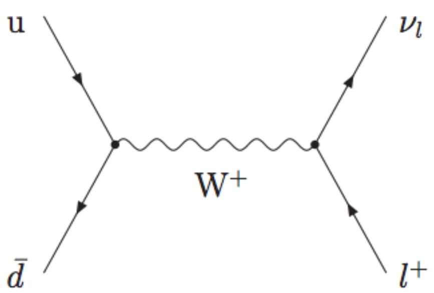
- MSHT: MHOU and IHOU (incomplete higher order uncertainty) both added as nuisance parameters and fitted from the data

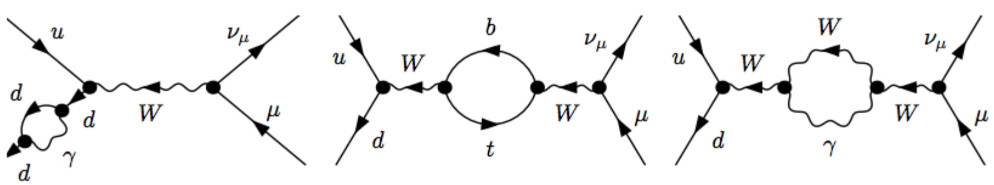
- NNPDF: MHOU added via theory covariance matrix, IHOU added as extra additional theory uncertainties computed by varying each of the possible parametrisation that interpolate the known ingredients



Electroweak corrections

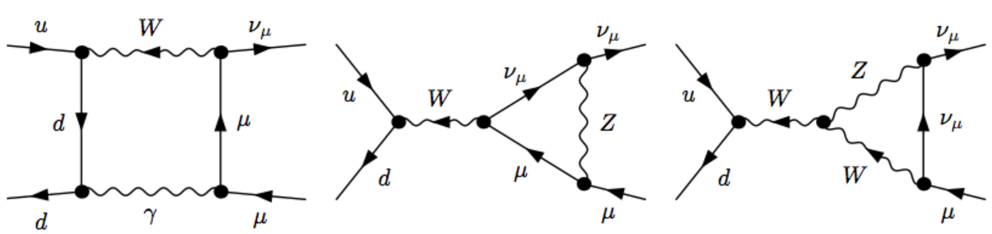
- Given that $\alpha(M_Z) \sim \alpha_S(M_Z)/10 \Longrightarrow$ NLO EW corrections ~ NNLO QCD corrections

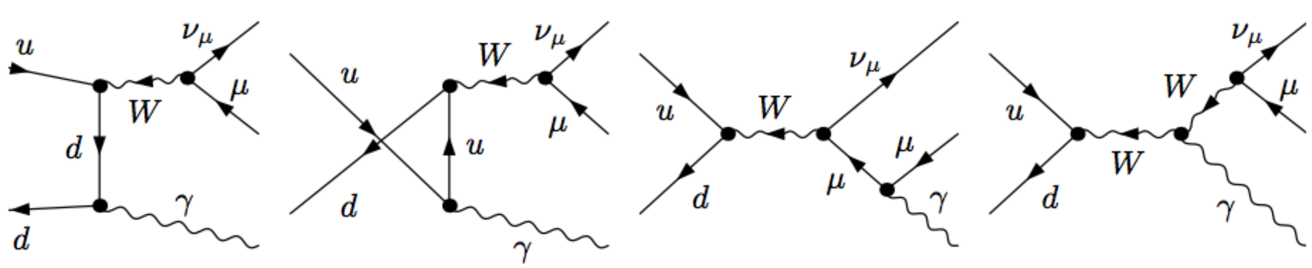




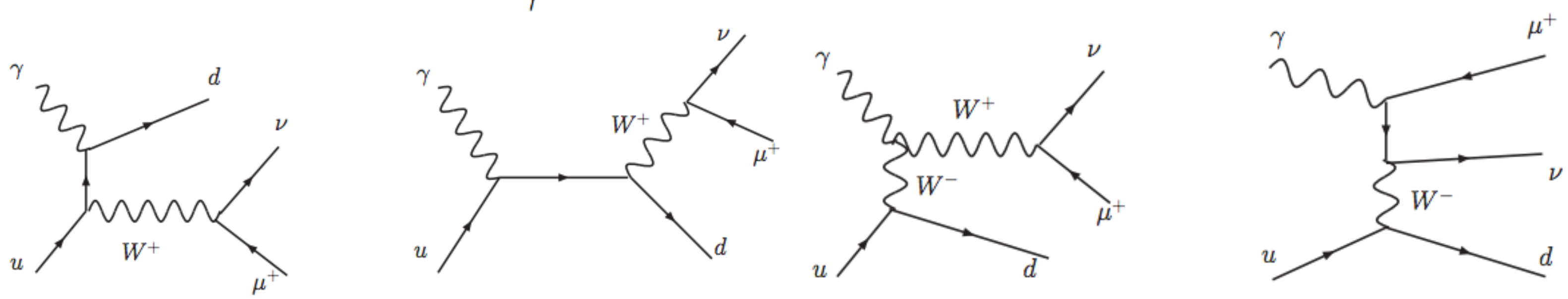
(a) (b) (c)

Virtual EW corrections





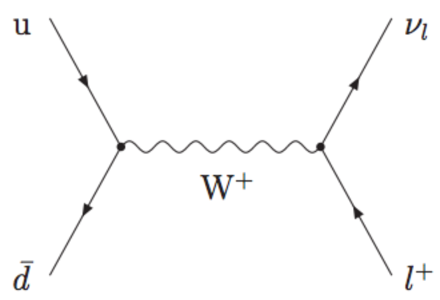
Real EW corrections - quark initiated

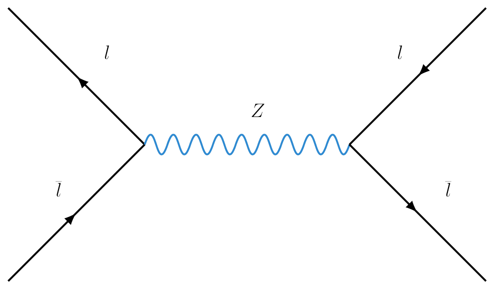


Real EW corrections - photon initiated

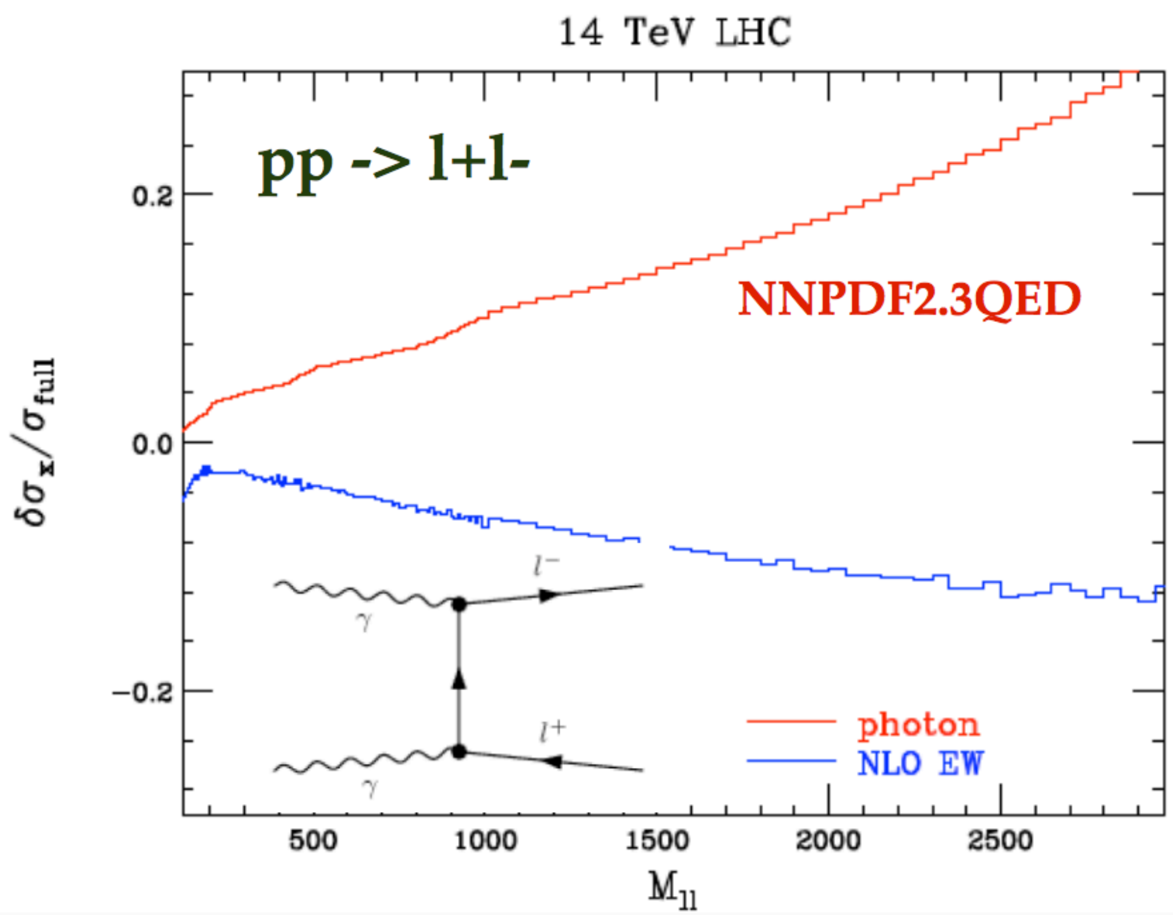
Electroweak corrections

- Given that $\alpha(M_Z) \sim \alpha_S(M_Z)/10 \Longrightarrow$ NLO EW corrections ~ NNLO QCD corrections





- NLO virtual EW corrections become large in the large p_T region of lepton but partially compensated by photon-initiated real corrections



Boughezal et al Phys.Rev. D89 (2014)3, 034030

Photon-modified DGLAP

- How are PDFs modified by inclusion of initial photon PDF?

$$Q^2 \frac{\partial}{\partial Q^2} g(x,Q^2) = \sum_{q,\bar{q},g} P_{ga}(x,\alpha_s(Q^2)) \otimes f_a(x,Q^2) + P_{g\gamma}(x,\alpha_s(Q^2)) \otimes \gamma(x,Q^2),$$

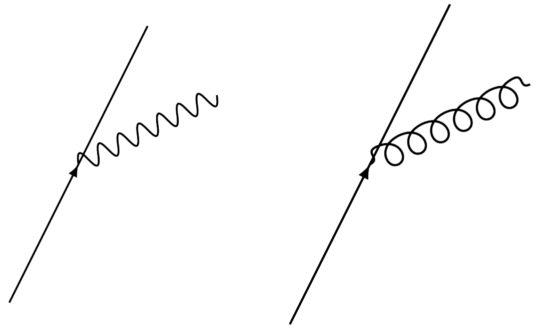
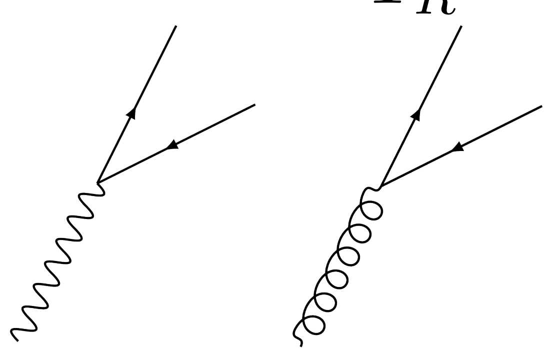
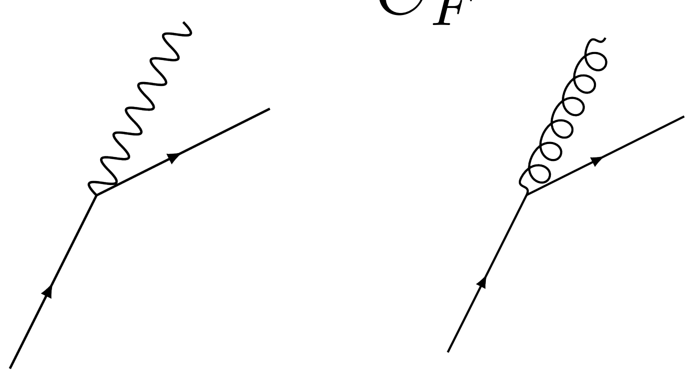
$$Q^2 \frac{\partial}{\partial Q^2} q(x,Q^2) = \sum_{q,\bar{q},g} P_{qa}(x,\alpha_s(Q^2)) \otimes f_a(x,Q^2) + P_{q\gamma}(x,\alpha_s(Q^2)) \otimes \gamma(x,Q^2),$$

$$Q^2 \frac{\partial}{\partial Q^2} \gamma(x,Q^2) = P_{\gamma\gamma} \otimes \gamma(x,Q^2) + \sum_{q,\bar{q},g} P_{\gamma a}(x,\alpha_s(Q^2)) \otimes f_a(x,Q^2).$$

- DGLAP splitting functions expanded in powers of α_s and $\boldsymbol{\alpha}$

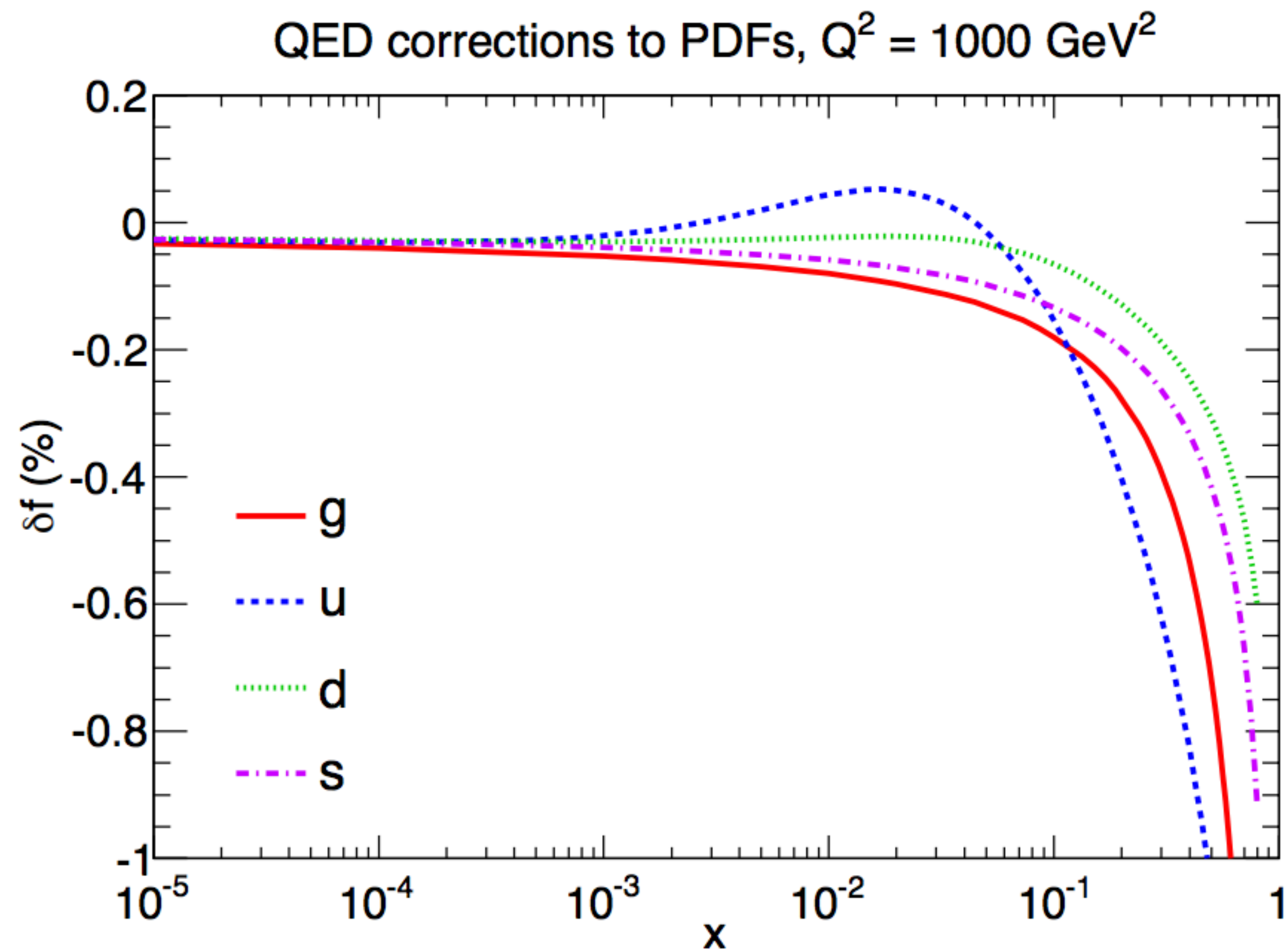
$$P_{ij} = \sum_{m,n} \left(\frac{\alpha_S}{2\pi}\right)^m \left(\frac{\alpha}{2\pi}\right)^n P_{ij}^{(m,n)}$$

$$P_{qq}^{(0,1)} = \frac{e_q^2}{C_F} P_{qq}^{(1,0)} \qquad P_{q\gamma}^{(0,1)} = \frac{e_q^2}{T_R} P_{qg}^{(1,0)} \qquad P_{\gamma q}^{(0,1)} = \frac{e_q^2}{C_F} P_{gq}^{(1,0)}$$

Photon-modified DGLAP

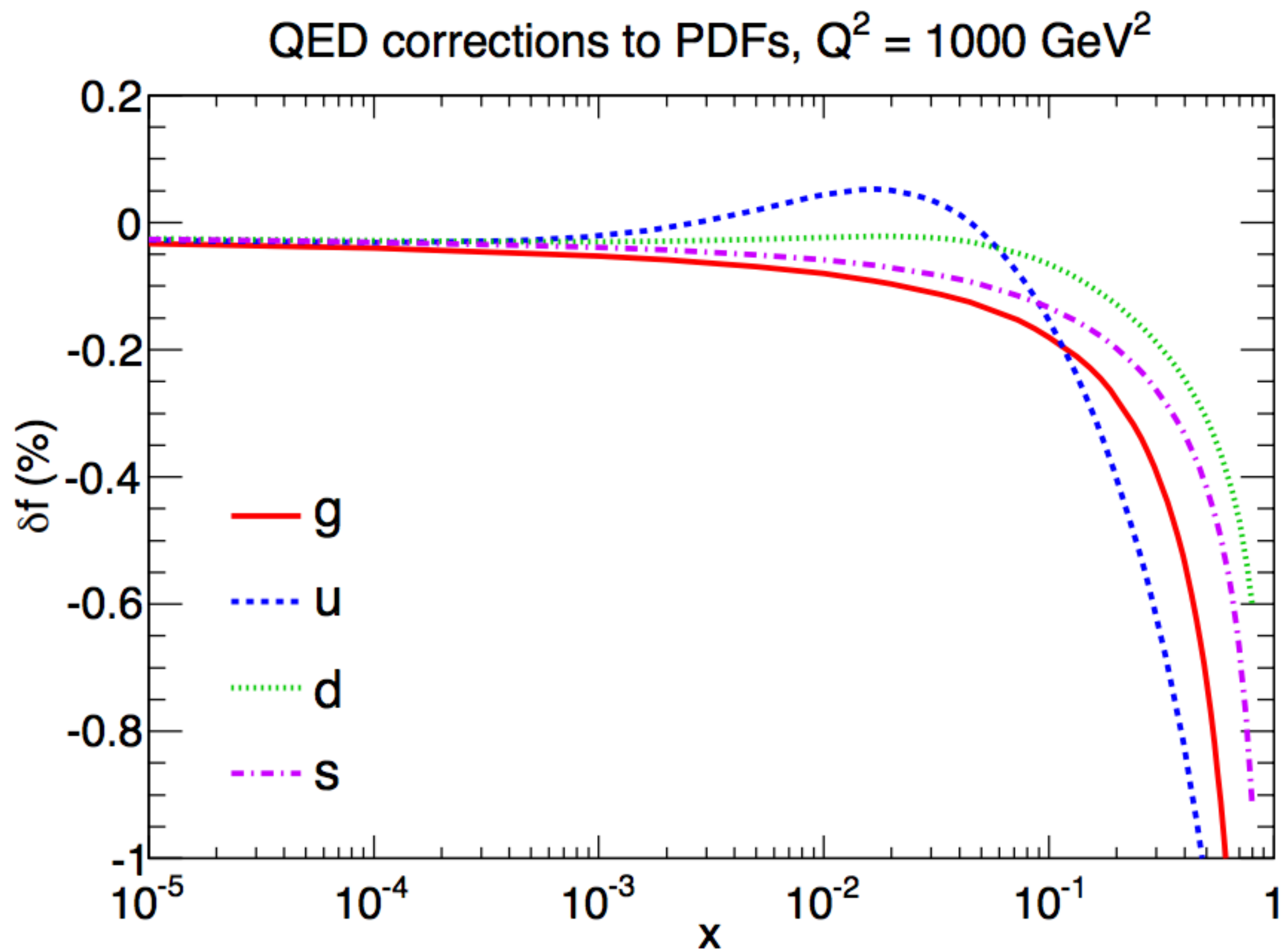
- Quark and gluon PDFs change up to 1% at large x



Ball et al, Nucl.Phys. B877 (2013) 290-320

Photon-modified DGLAP

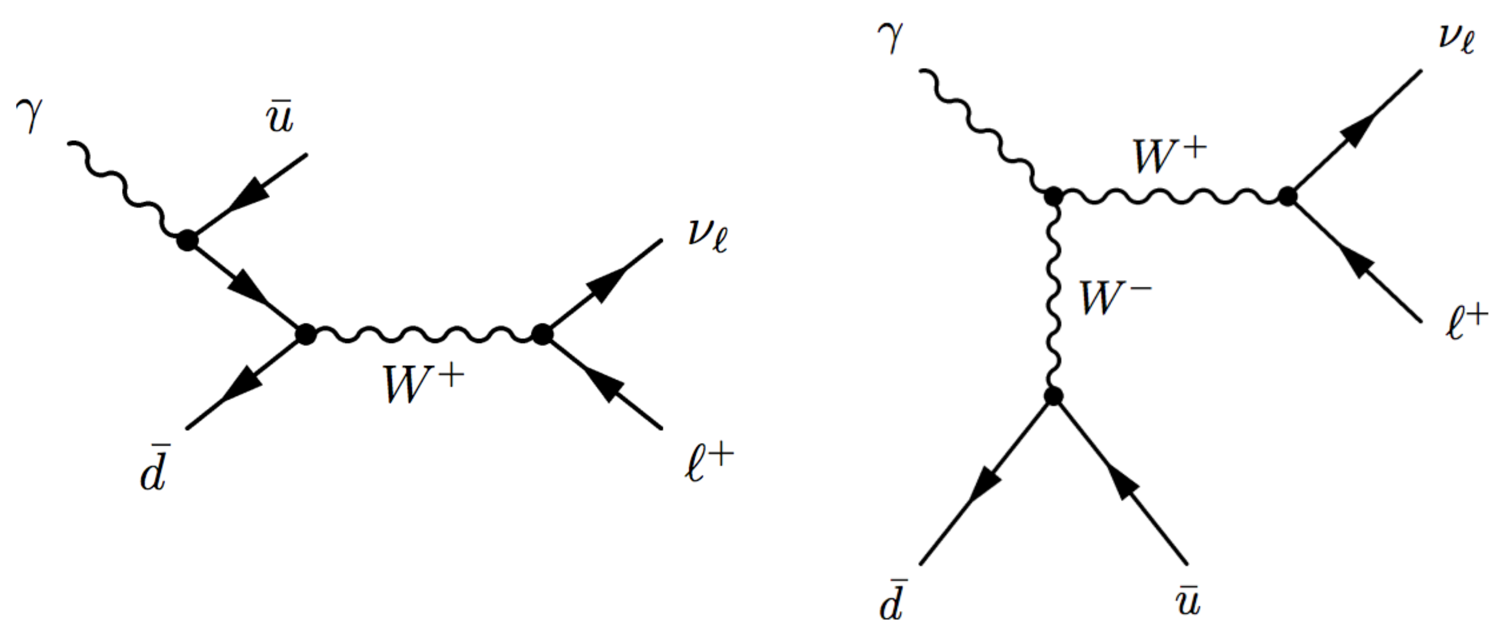
- Quark and gluon PDFs change up to 1% at large x
- How do we determine the photon PDF?
- Two ways in the next slides: from data or from theory
- In the best possible world: theory input and data input together



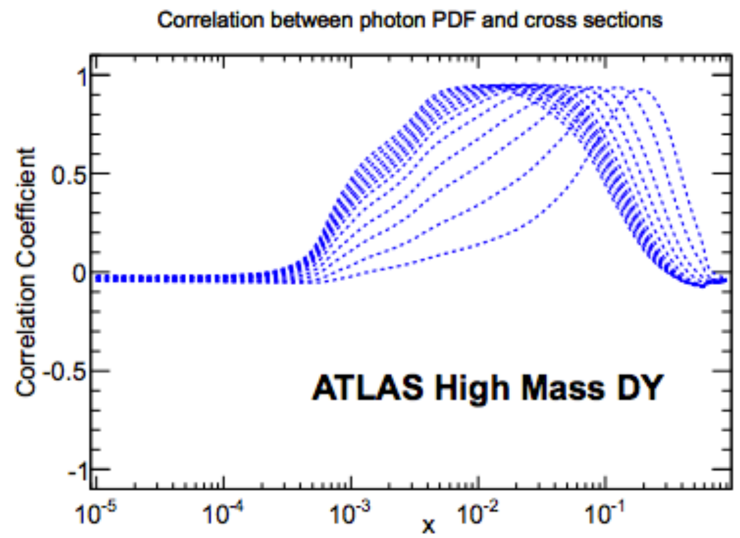
Ball et al, Nucl.Phys. B877 (2013) 290-320

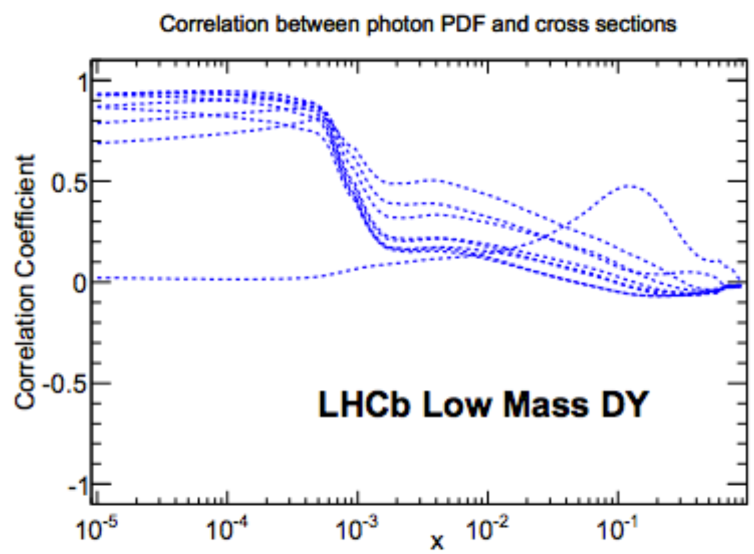
Photon PDFs

- Largest correlations between photon PDFs and pp cross sections are for Drell-Yan processes, but also for top pair production and VV production



$Q = 100\,\mathrm{GeV}$



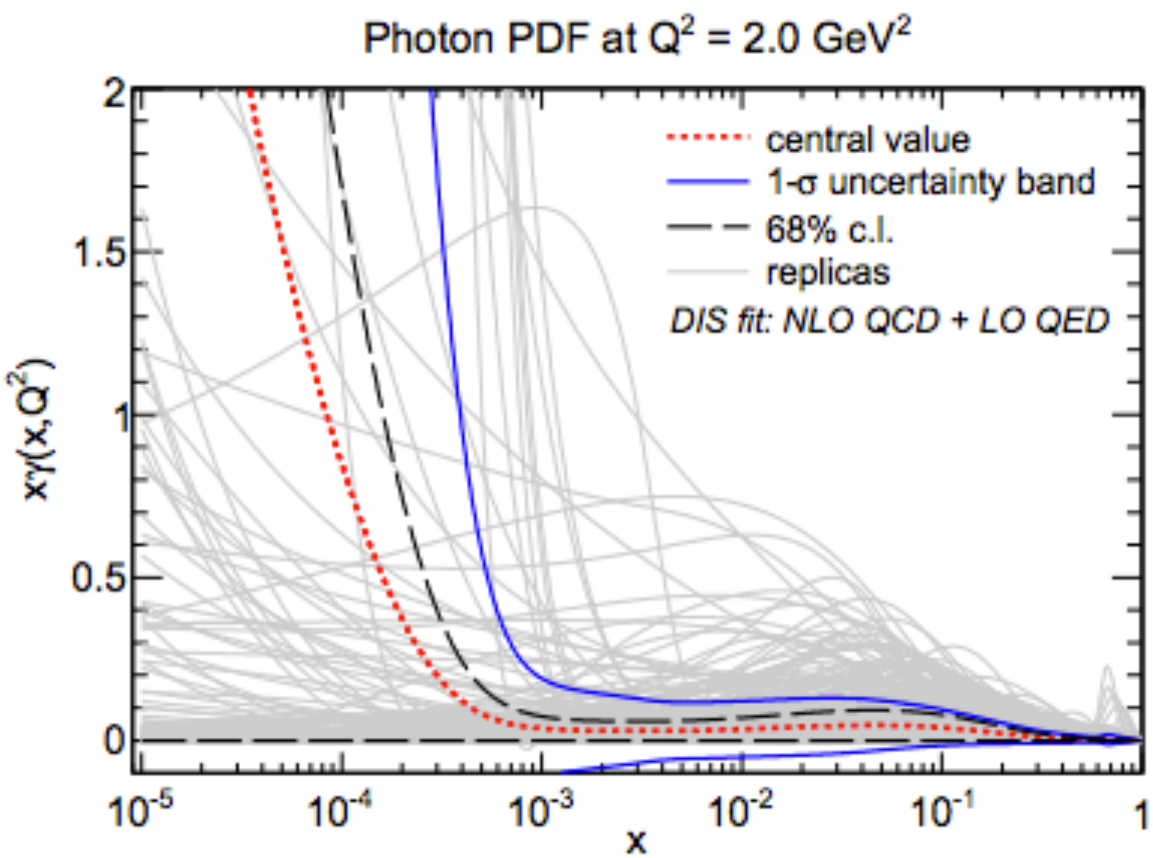


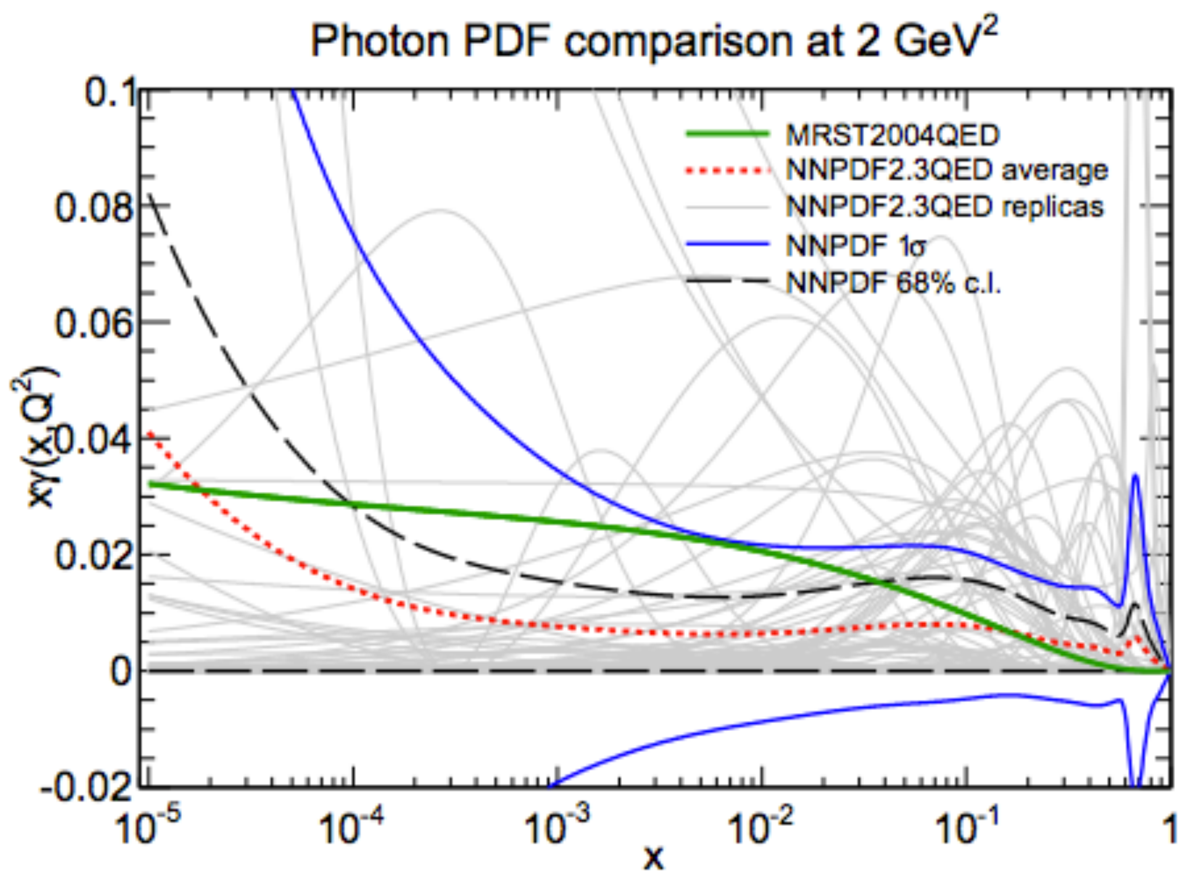
Photon-induced Drell-Yan

Ball et al, Nucl.Phys. B877 (2013) 290-320

Photon PDFs

Data-driven knowledge





Ball et al, Nucl.Phys. B877 (2013) 290-320

- Data-driven approach associated with a large uncertainty on photon PDF
- Theory breakthrough: LUX PDF [Manohar, Nason, Salam, Zanderighi,1607.04266]

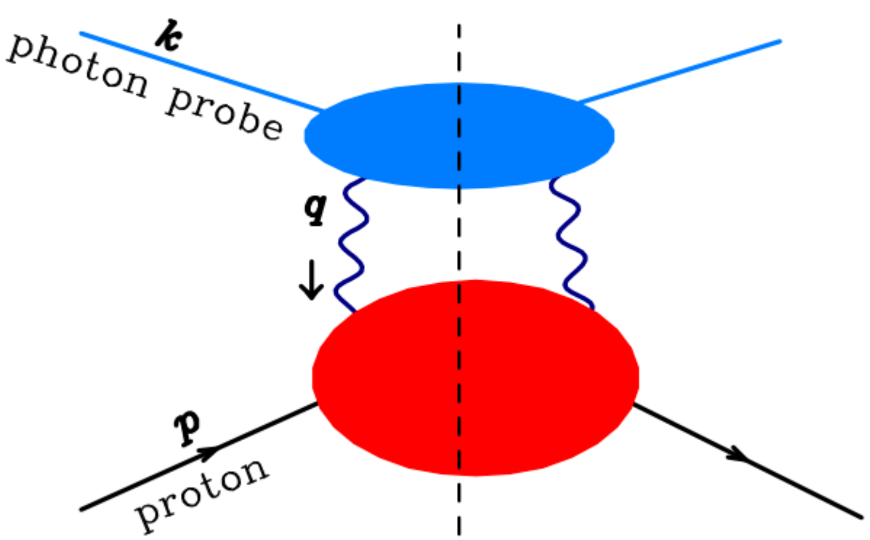
Photon PDFs

- QED is perturbative down to low scales $\Longrightarrow$ The photon must be computable if the input substructure is known
- Manohar et al: write the cross section for a chosen BSM process, e.g. production of heavy supersymmetric lepton L in ep collision (Drees, Zeppenfeld 1989)

$$\sigma = \frac{1}{4p\cdot k}\int \frac{d^4q}{(2\pi)^4 q^4} e_{\rm ph}^2(q^2)\left[4\pi W_{\mu\nu}(p,q)\, L^{\mu\nu}(k,q)\right]\; 2\pi\delta((k-q)^2 - M^2)$$

$$l(k) + p(p) \to L(k') + X$$

$$\sigma = c_0 \sum_a \int_x^1 \frac{dz}{z}\, \hat{\sigma}_a(z,\mu^2)\, \frac{M^2}{zs} f_{a/p}\left(\frac{M^2}{zs},\mu^2\right)$$



$$\sigma = \frac{c_0}{2\pi}\int_x^{1-\frac{2xm_p}{M}} \frac{dz}{z}\int_{Q^2_{\rm min}}^{Q^2_{\rm max}} \frac{dQ^2}{Q^2}\alpha_{\rm ph}^2(-Q^2)\Bigg[\Bigg(2 - 2z + z^2 + \frac{2x^2m_p^2}{Q^2} + \frac{z^2Q^2}{M^2} - \frac{2zQ^2}{M^2} - \frac{2x^2Q^2m_p^2}{M^4}\Bigg)F_2(x/z,Q^2) + \left(-z^2 - \frac{z^2Q^2}{2M^2} + \frac{z^2Q^4}{2M^4}\right)F_L(x/z,Q^2)\Bigg], \quad (3)$$

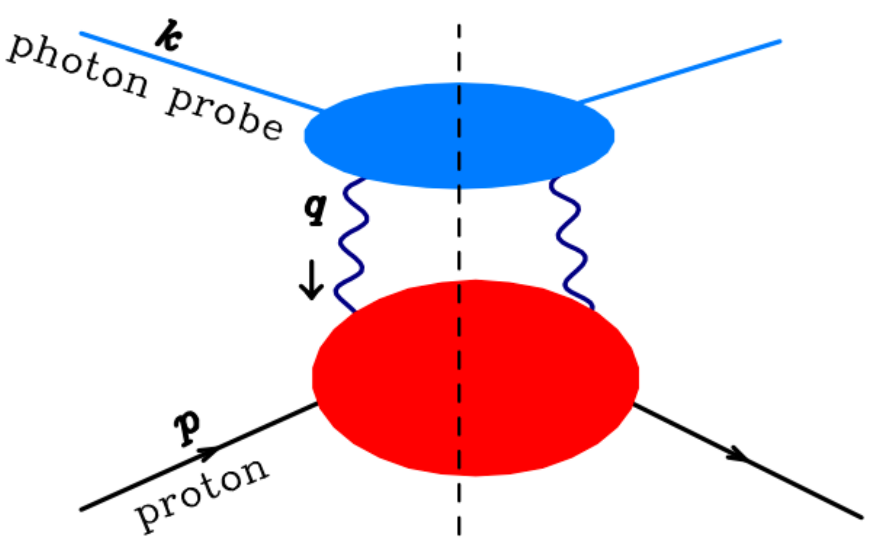
Manohar et al 1607.04266

Theory-driven knowledge

Photon PDFs

- QED is perturbative down to low scales ⟹ The photon must be computable if the input substructure is known
- Manohar et al: write the cross section for a chosen BSM process, e.g. production of heavy supersymmetric lepton L in ep collision (Drees, Zeppenfeld 1989)
- Equate the two expressions and find analytically the PDF of the photon

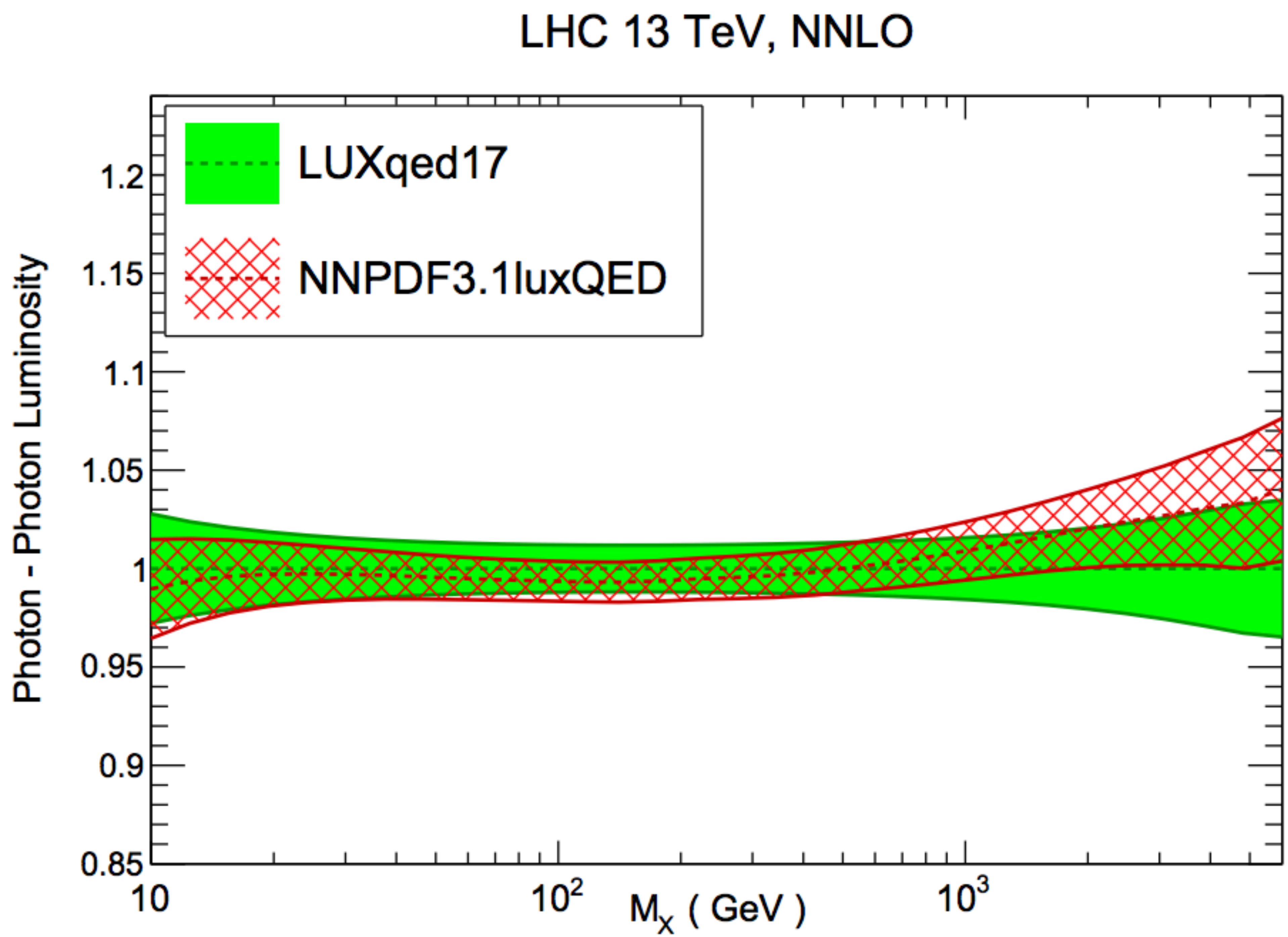
⇒ PDFs expressed in terms of the structure functions integrated over all scales, including elastic form factors (in the x →1 region)



$$
\begin{aligned}
&x f_{\gamma/p}(x,\mu^2) = \\
&\frac{1}{2\pi\alpha(\mu^2)}\int_x^1 \frac{dz}{z}\Bigg\{\int_{\frac{x^2 m_p^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2}\alpha^2(Q^2) \\
&\left[\left(z p_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2}\right) F_2(x/z,Q^2) - z^2 F_L\left(\tfrac{x}{z},Q^2\right)\right] \\
&- \alpha^2(\mu^2) z^2 F_2\left(\tfrac{x}{z},\mu^2\right)\Bigg\},
\end{aligned}
$$

<u>Theory-driven knowledge</u>

Photon PDFs



Bertone et al, 1712.07053

<u>(Data+Theory)-driven knowledge</u>

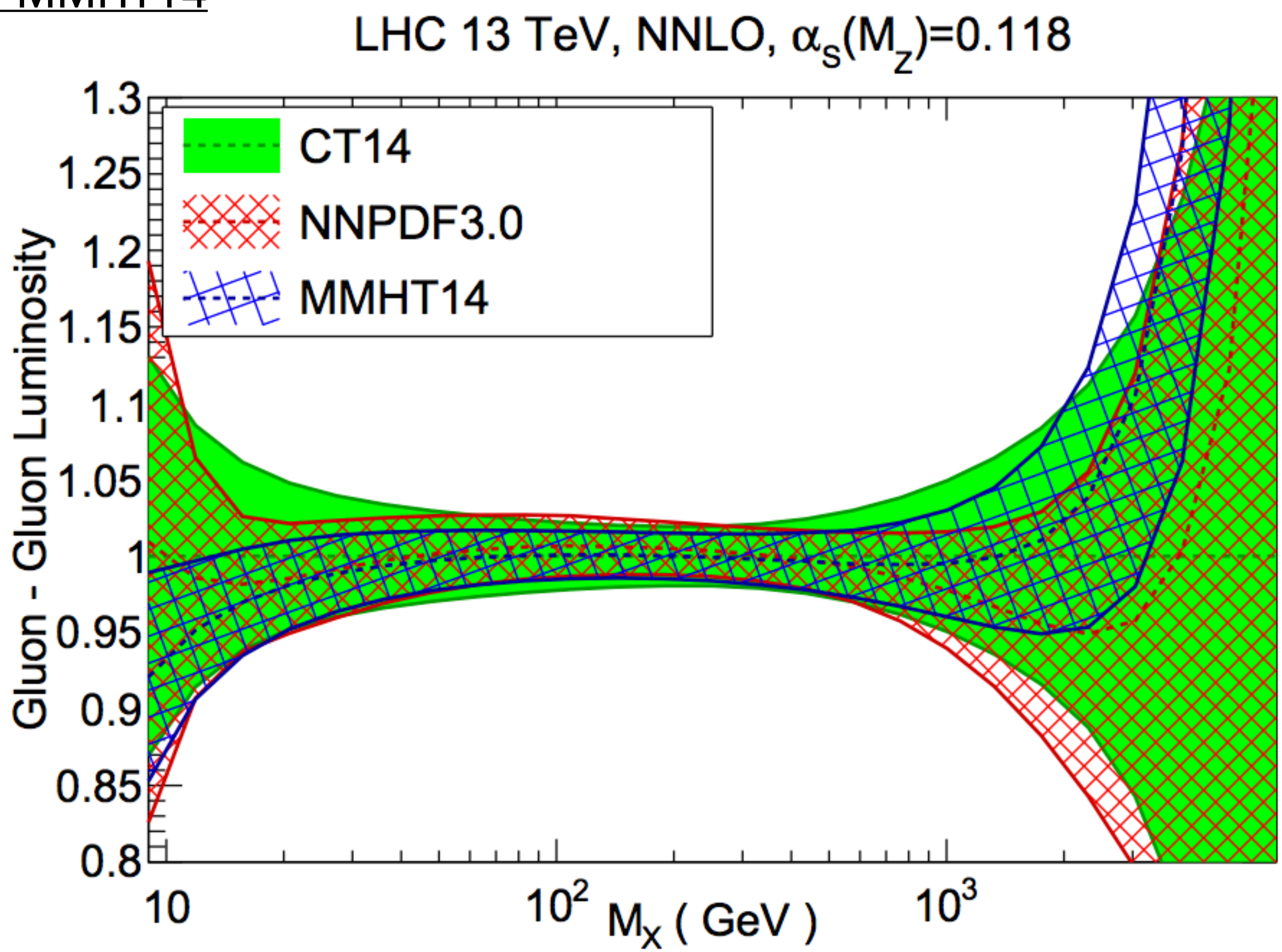
State-of-the-art PDFs

The players

June 2022	NNPDF4.0	MSHT20	CT18	ABMP16	CJ15	JAM
Fixed Target DIS	✔	✔	✔	✔	✔	✔
HERA I+II	✔	✔	✔	✔	✔	✔
HERA jets	✔	✔	✘	✘	✘	✘
Fixed Target DY	✔	✔	✔	✔	✔	✔
Compass SIDIS	✘	✘	✘	✘	✔	✔
Tevatron W,Z	✔	✔	✔	✔	✔	✘
Tevatron jets	✔	✔	✔	✘	✘	✘
LHC jets	✔	✔	✔	✘	✘	✘
LHC vector boson	✔	✔	✔	✔	✘	✘
LHC top	✔	✔	✔	✔	✘	✘
Stat. treatment	Monte Carlo	Hessian $\Delta\chi^2$ dynamical	Hessian $\Delta\chi^2$ dynamical	Hessian $\Delta\chi^2=1$	Hessian $\Delta\chi^2=1,10$	Monte Carlo Baysian
Parametrization	Neural Networks	Polynomial (Chebyshev)	Polynomial (Bernstein)	Polynomial	Polynomial	Polynomial (50 pars)
HQ scheme	FONLL	TR'	ACOT-χ	FFN (+BMST)	ACOT	ACOT
Order	NNLO	NNLO	NNLO	NNLO	NLO	NLO

Gluon luminosity

NNPDF3.0 / CT14 / MMHT14

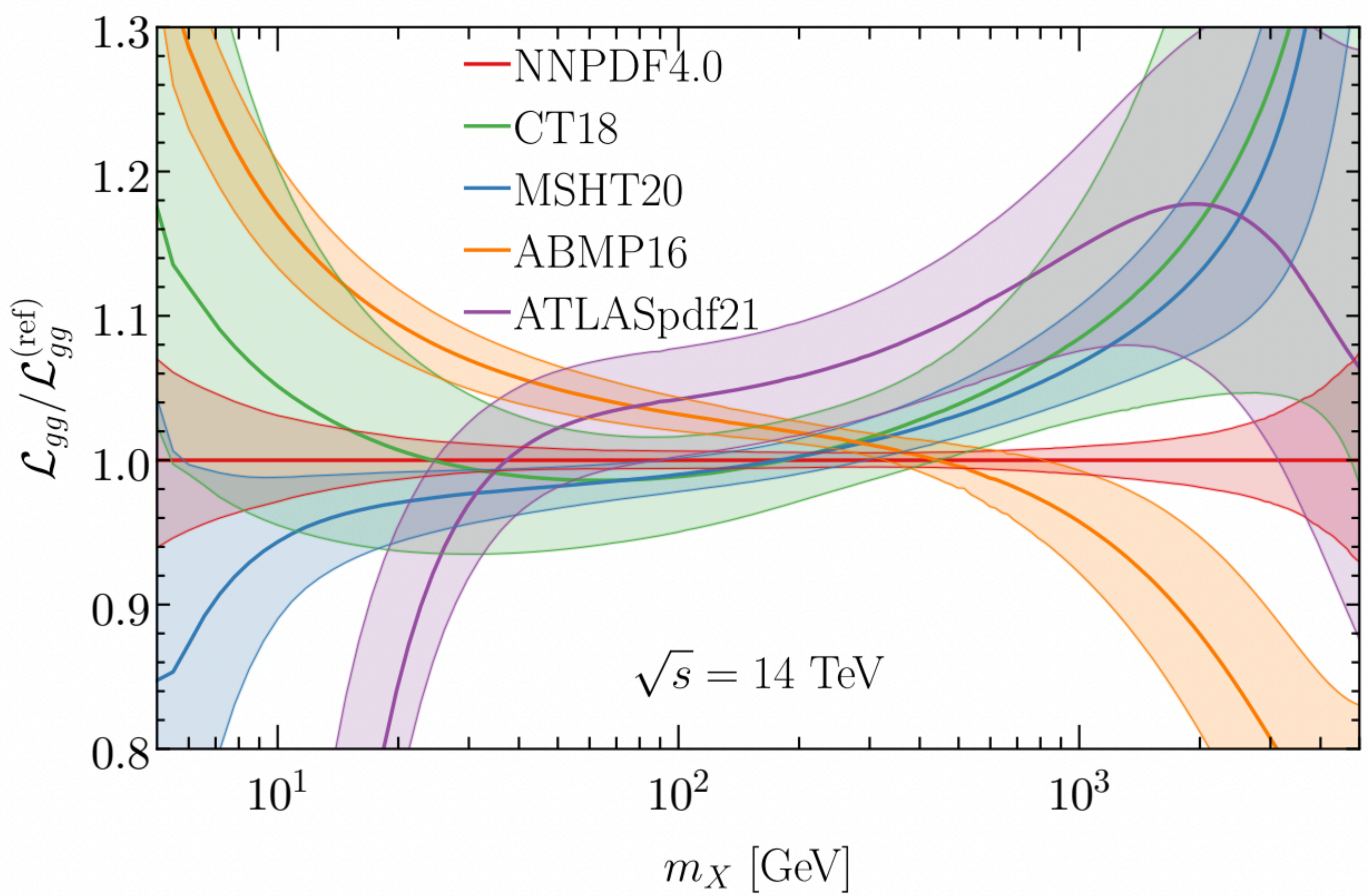


(2016)

J. Butterworth et al, J.Phys. G43 (2016) 023001

Gluon luminosity

NNPDF4.0/ CT18/MSHT20/ABMP16/ATLASpdf21

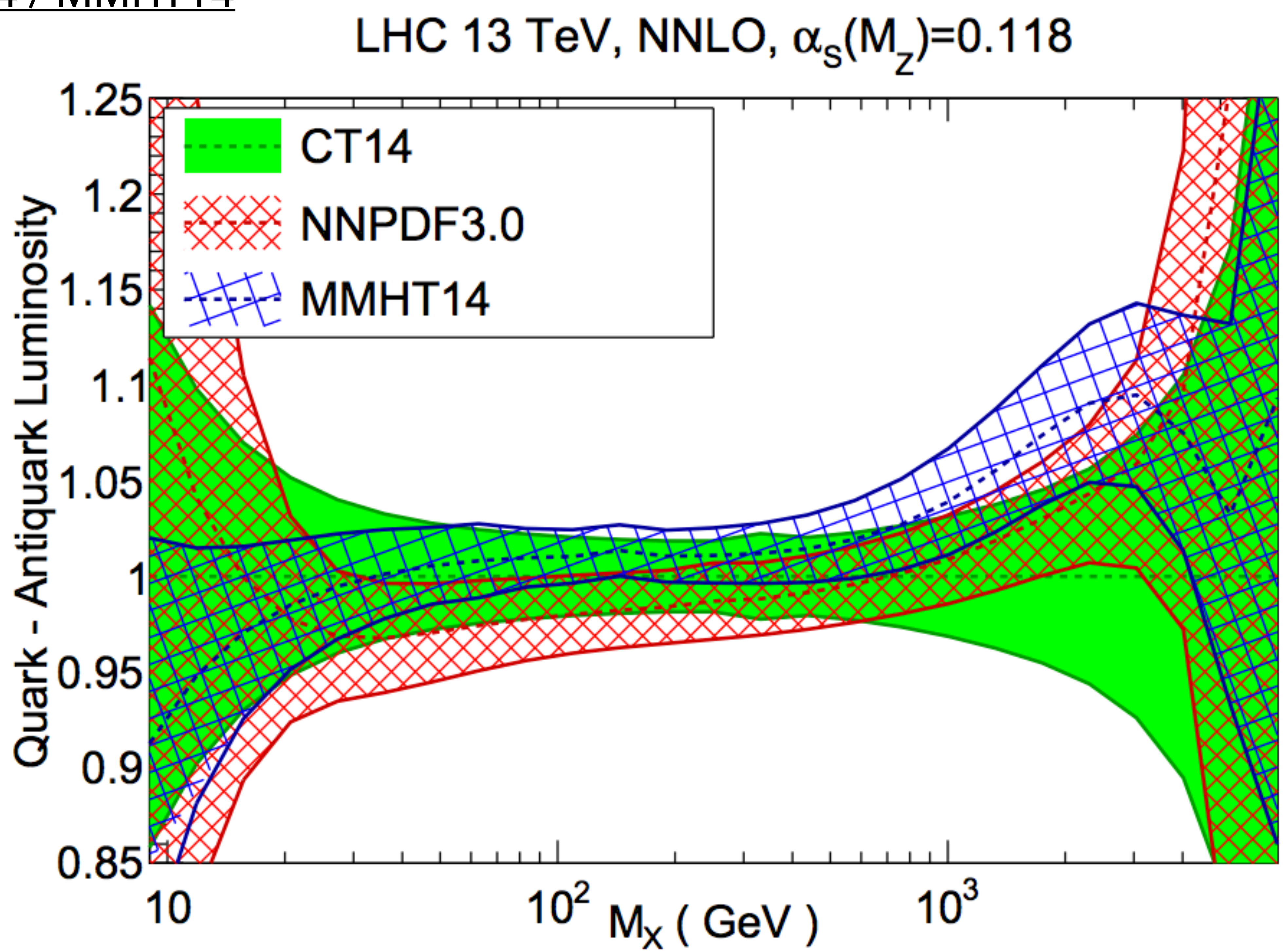


(2022)

Snowmass 2022 white paper, arXiv: 2203.13923

Quark-Antiquark luminosity

NNPDF3.0 / CT14 / MMHT14

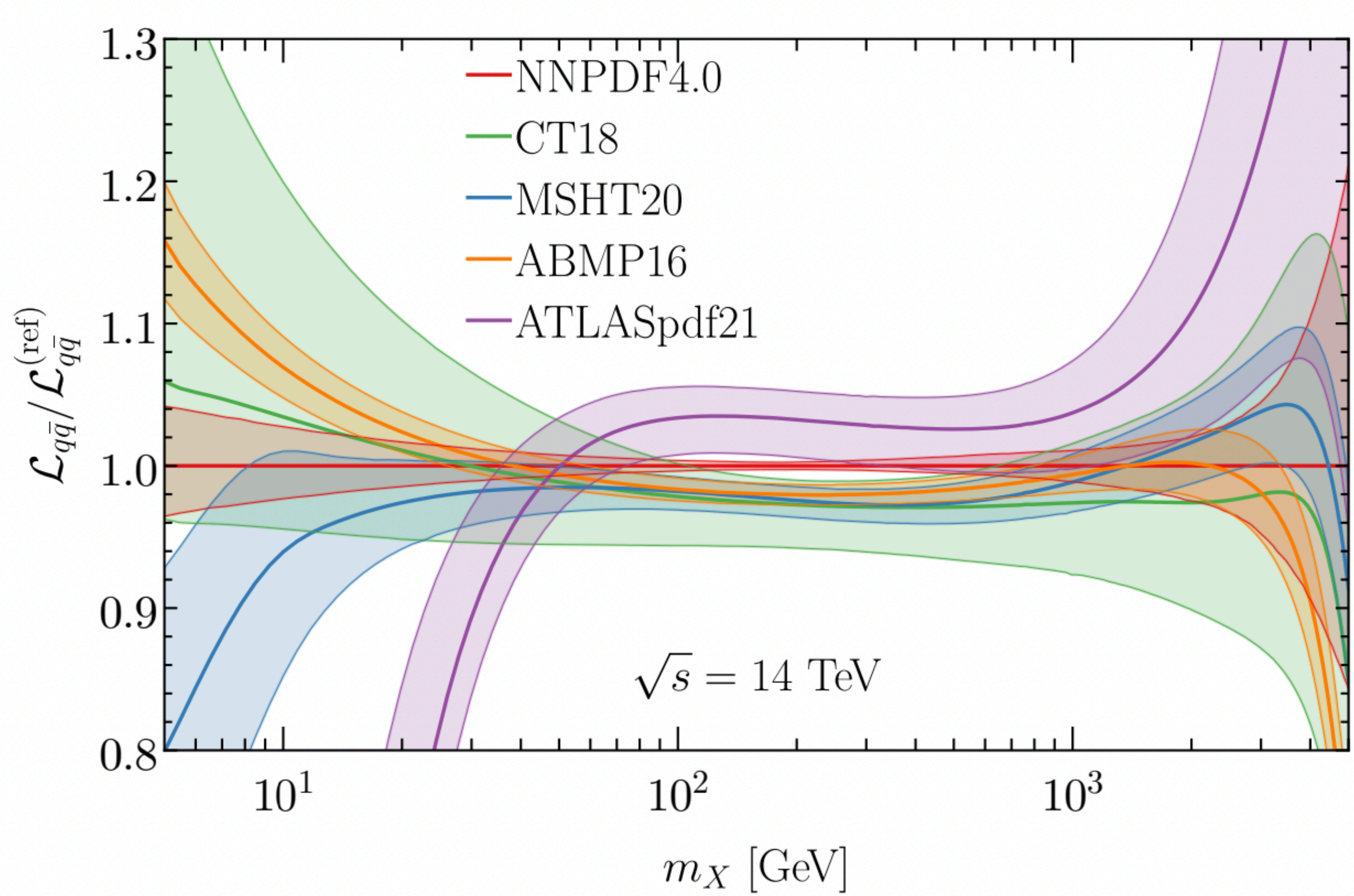


(2016)

J. Butterworth et al, J.Phys. G43 (2016) 023001

Quark-Antiquark luminosity

NNPDF4.0/ CT18/MSHT20/ABMP16/ATLASpdf21

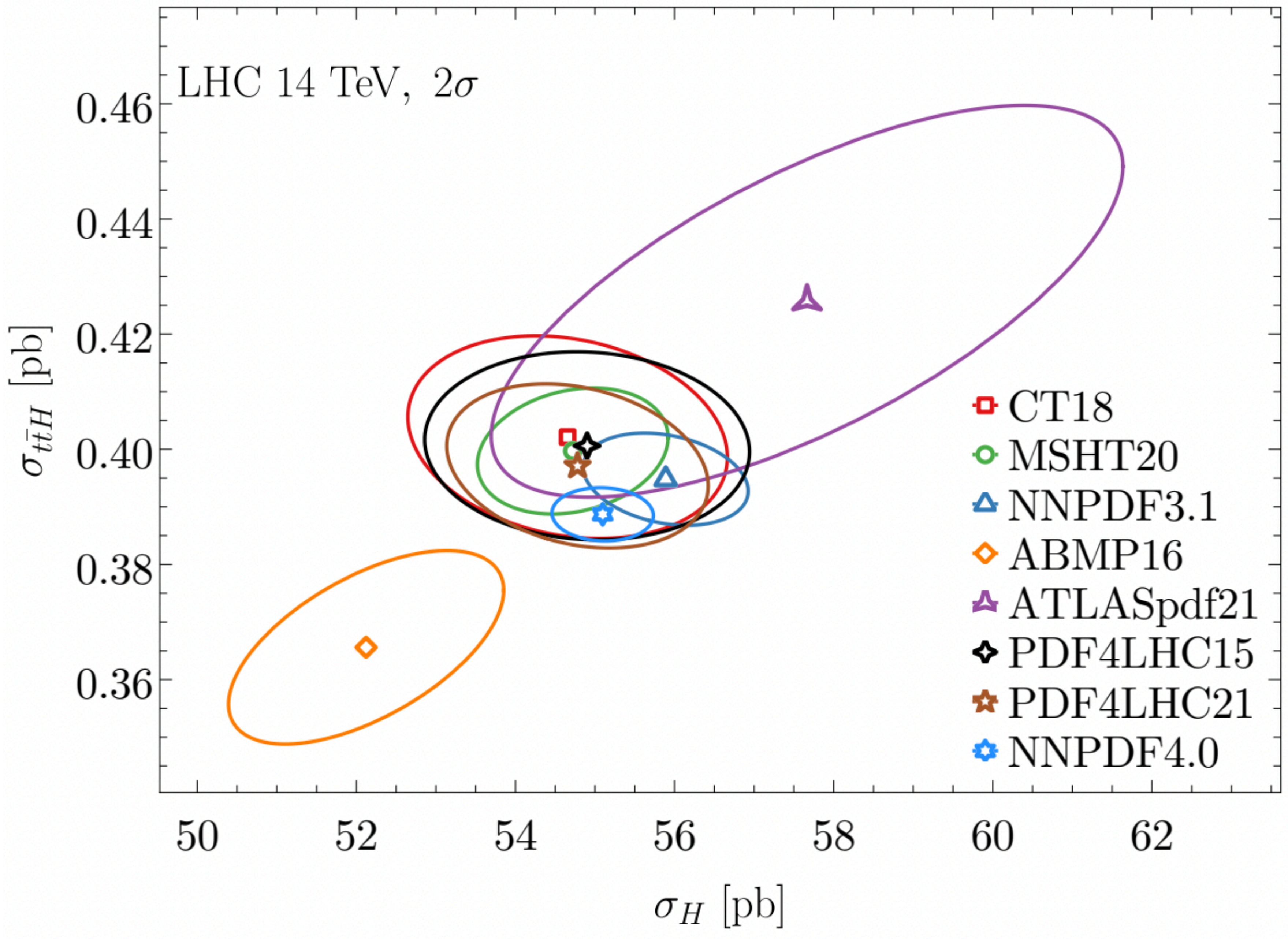


(2022)

Snowmass 2022 white paper, arXiv: 2203.13923

The choice of PDFs matters

- What does PDF uncertainty include? How reliable it is?
- How do we interpret the difference predictions using different PDF sets?
- Shall we just pick a set out of the PDFs "supermarket" shelf or take the envelope of ALL predictions?



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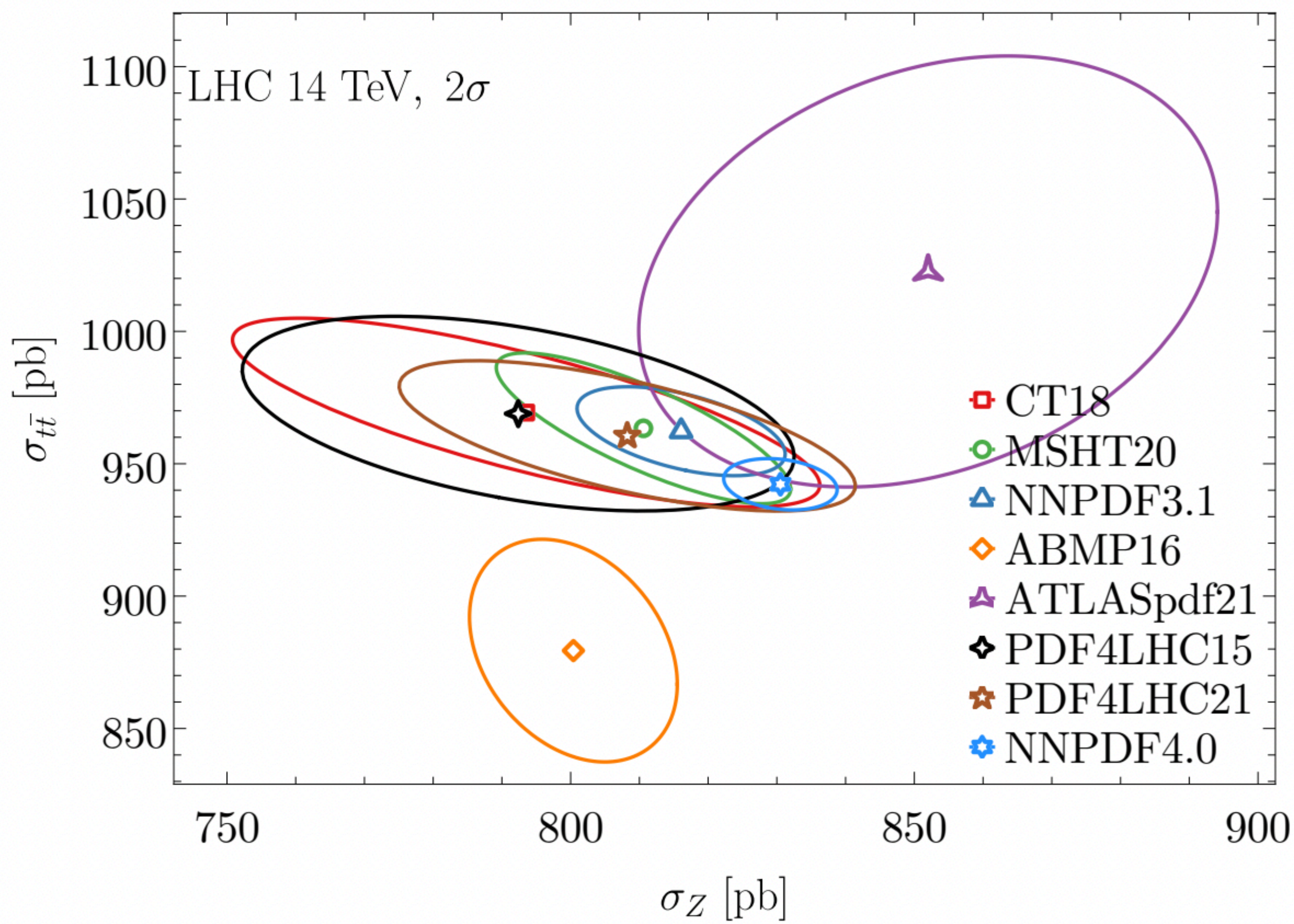
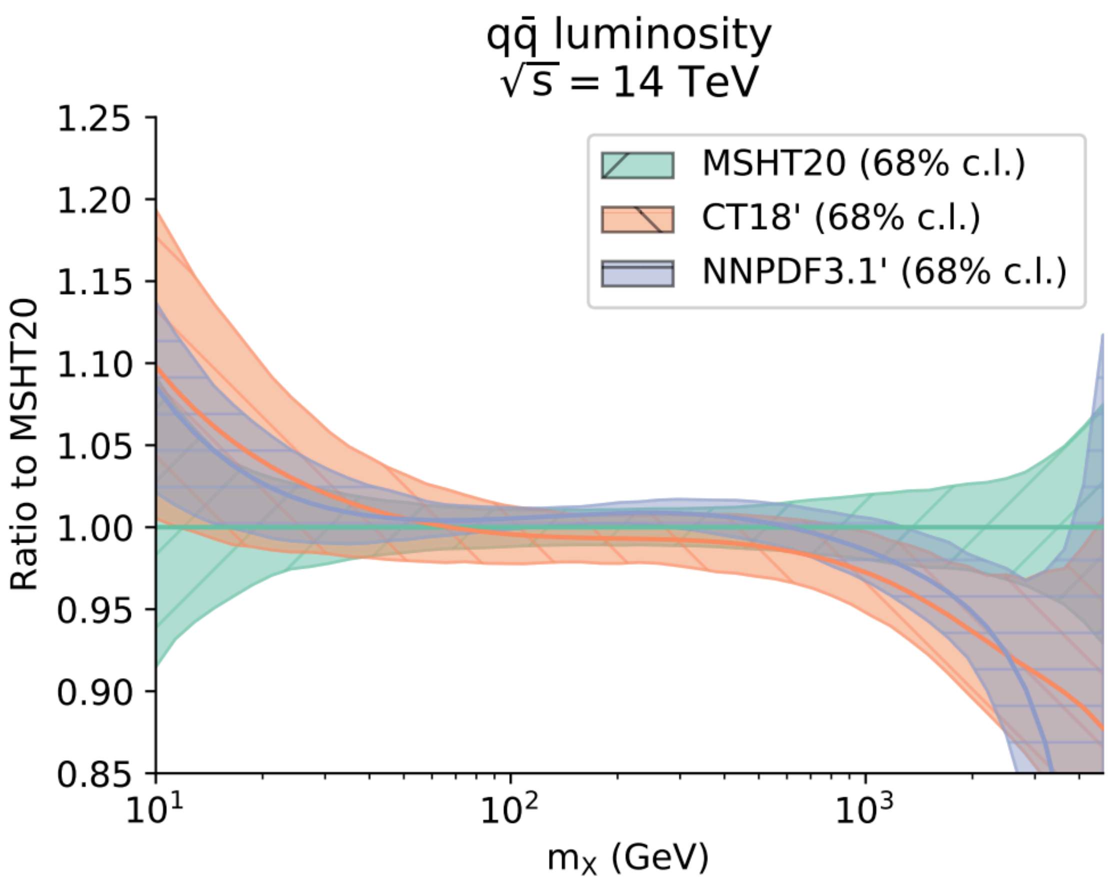
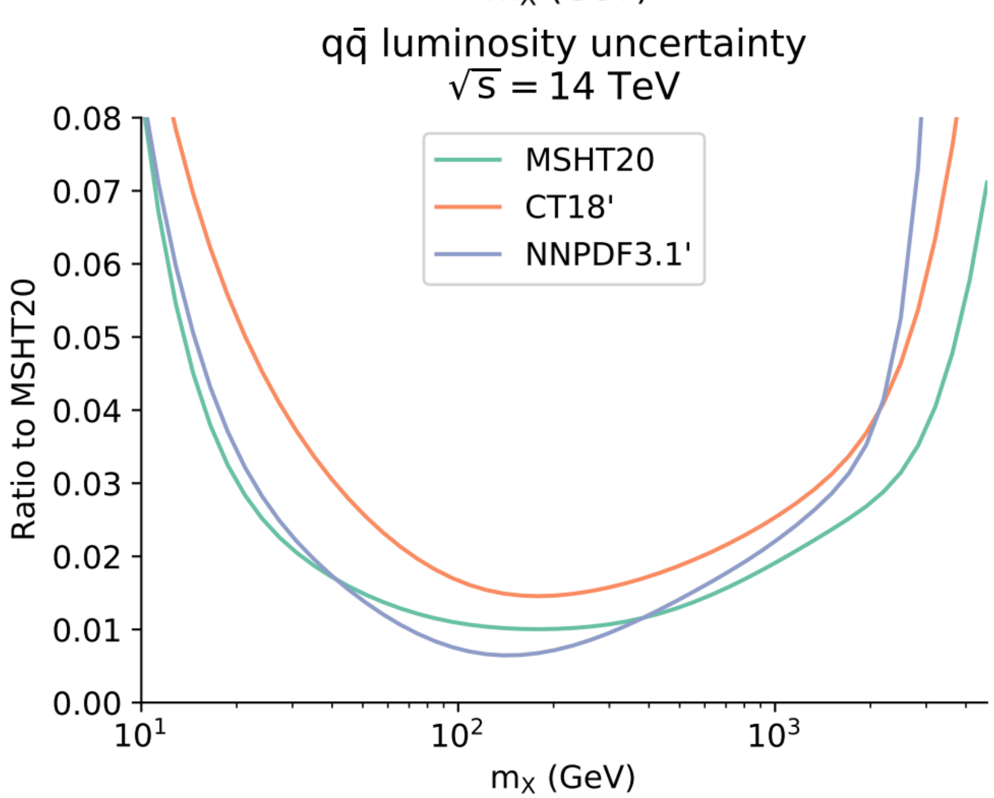


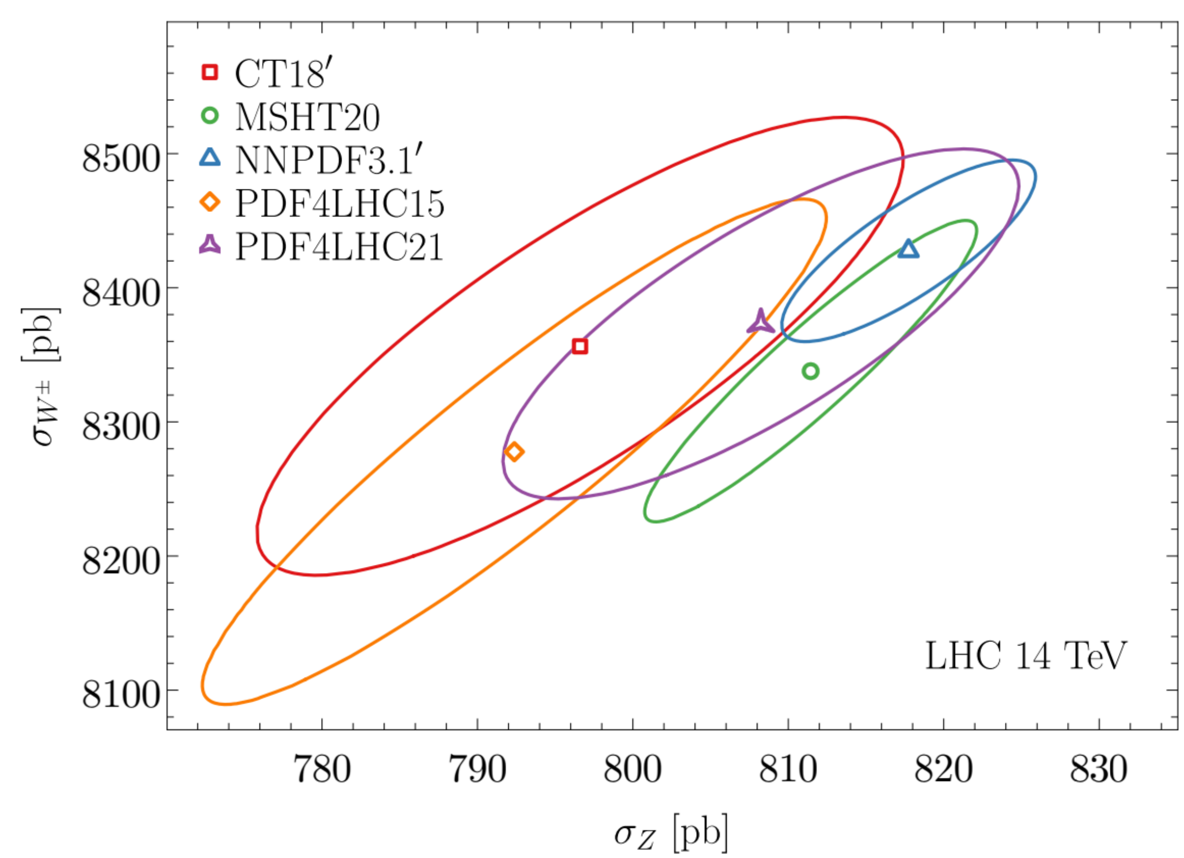
Snowmass 2022 white paper, arXiv: 2203.13923

Ongoing benchmarks



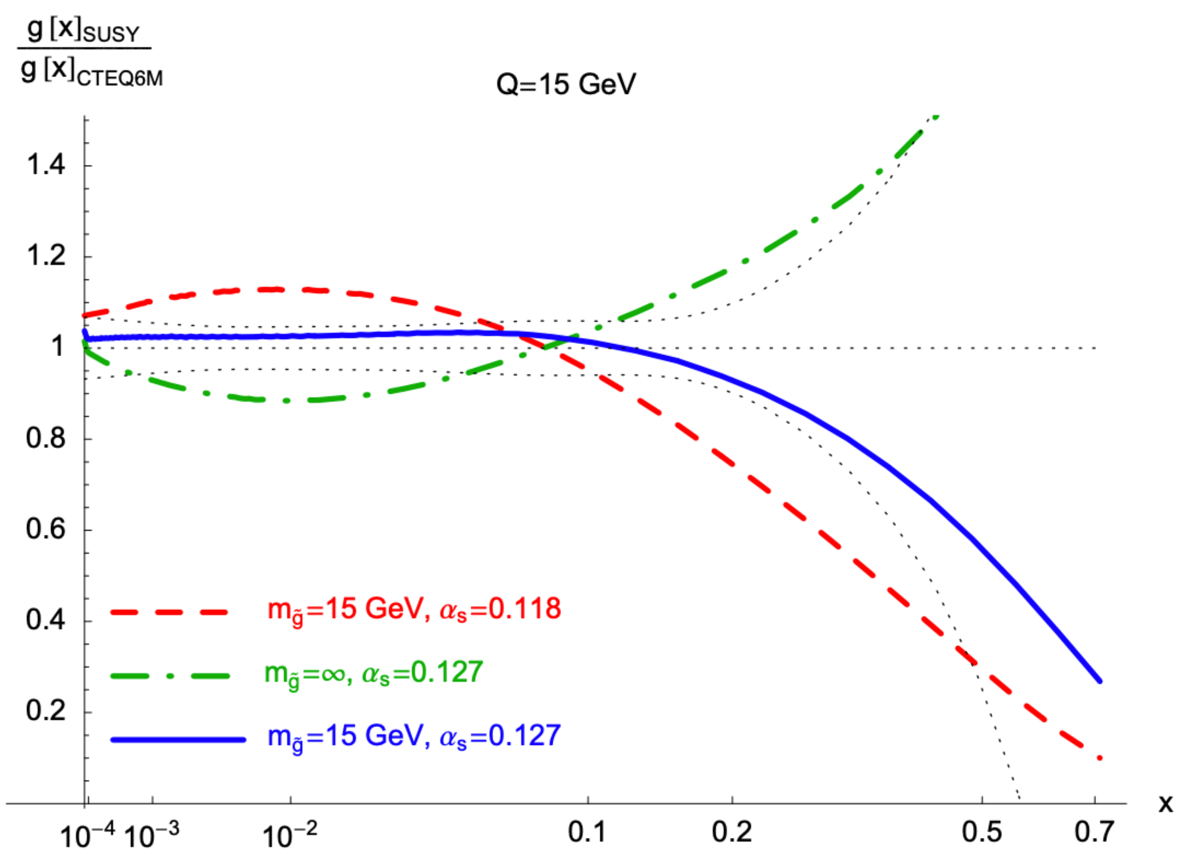


- Benchmark exercise among NNPDF3.1, MSHT20 and CT18 at the basis of PDF4LHC combination
- Overall agreement, which improves once common dataset is used, differences in uncertainties with ΔCT ≳ ΔMHST ≳ ΔNN due to methodology

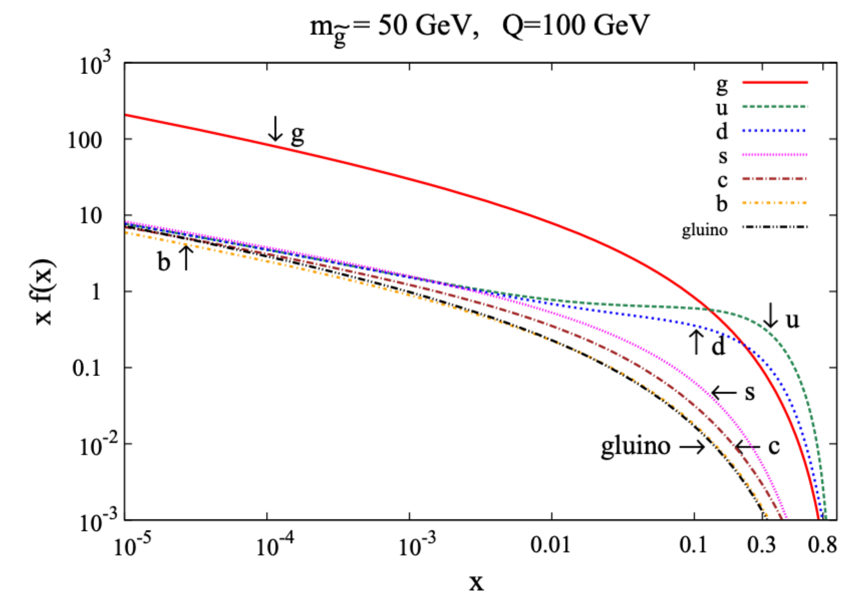


PDF fits and New Physics interplay

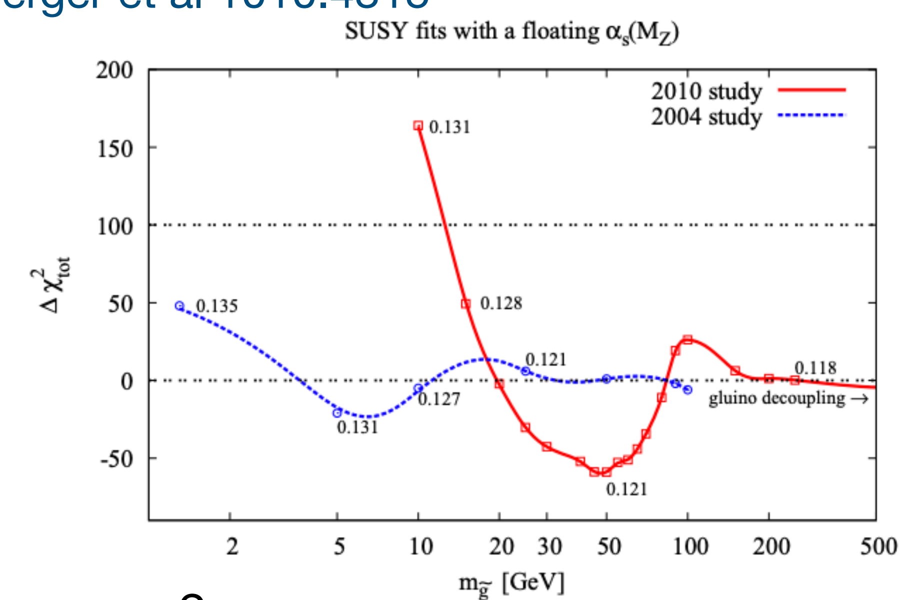
1. New partons



Berger et al hep-ph/0406143



Berger et al 1010.4315



- Pre-LHC studies: what is there was a light SUSY coloured partner?
- A light SUSY Parton would modify DGLAP equation and running of α_s
- Comparison to data excludes any light coloured parton on increasing mass range as more (and more precise) data are included in the global PDF fit

1. New partons

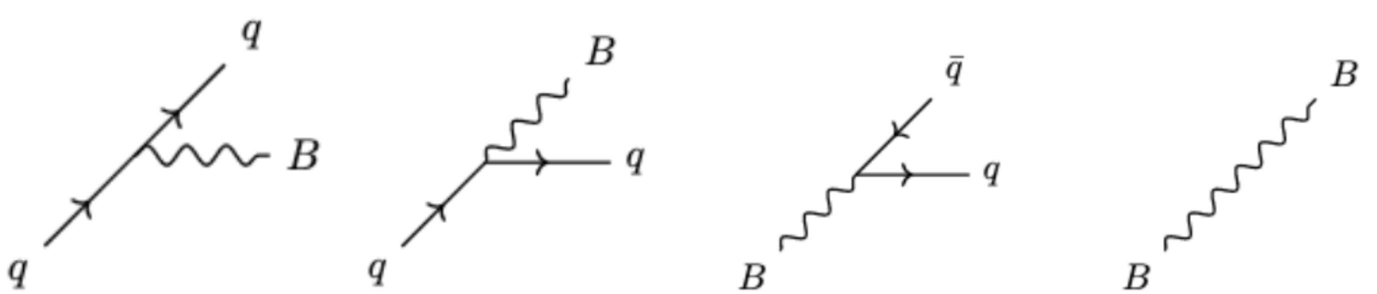
M. McCullough, J. Moore, MU, arXiv:2203.12628

- Idea: now PDFs are known very precisely, and their uncertainties will continue to reduce in the near future with the HL-LHC, could we do the same for a colourless particle too?
- If there was a lepto-phobic dark photon weakly coupled to quarks via effective Lagrangian

$$\mathcal{L}_{\rm int} = \frac{1}{3} g_B \bar{q} \not{B} q \qquad\qquad m_B \in [2,80]\ \text{GeV}$$

it would appear among the partons of the proton.

- To include the dark photon as a constituent of the proton: compute the dark photon splitting functions, and add them to DGLAP evolution. Starting from an appropriate initial-scale ansatz (dark photon generated dynamically off quarks and antiquarks at threshold) and a reference PDF set, evolve using the modified DGLAP equations

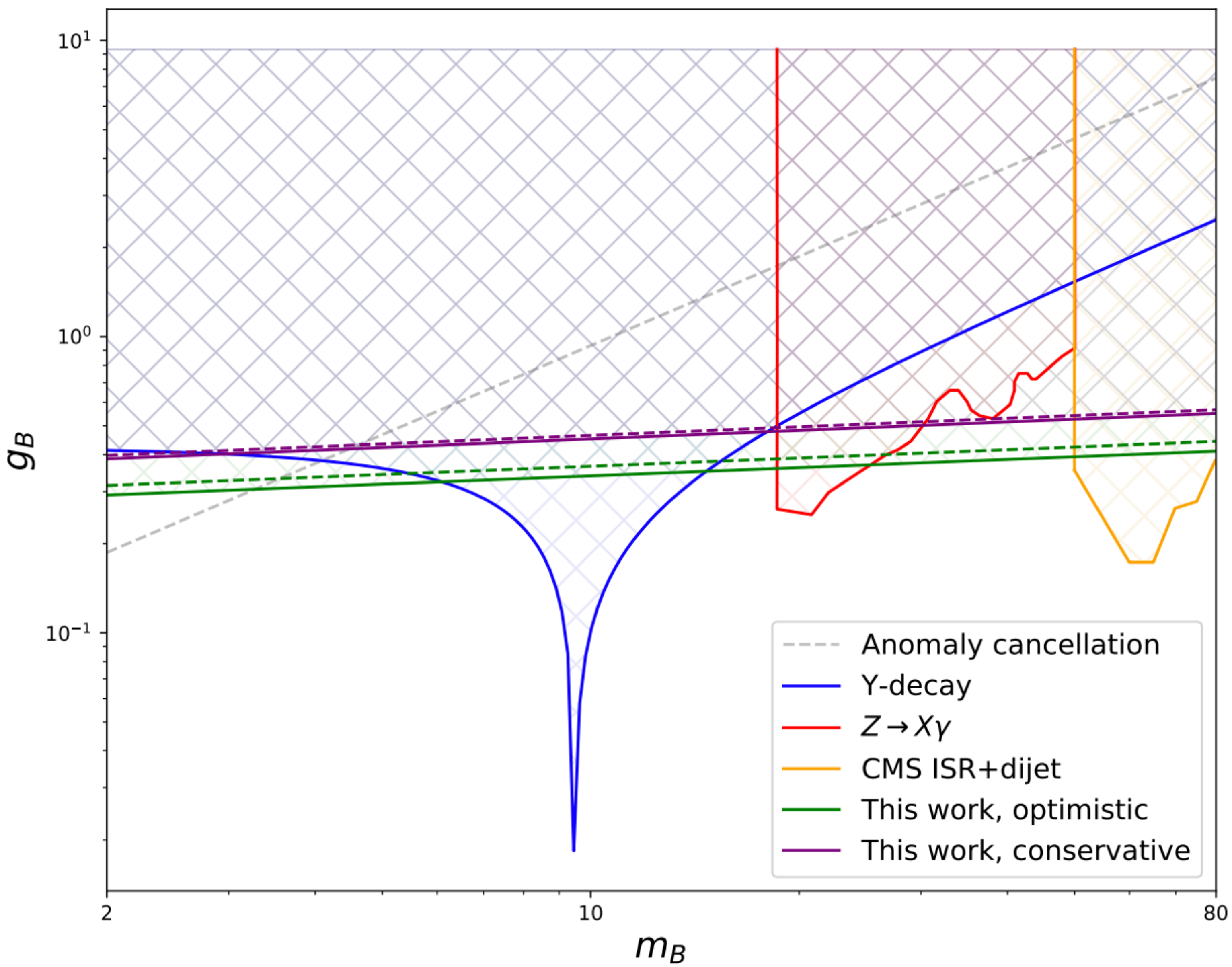


$$P_{ij} = \left(\frac{\alpha_s}{2\pi}\right) P_{ij}^{(1,0,0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ij}^{(2,0,0)} + \left(\frac{\alpha_s}{2\pi}\right)^3 P_{ij}^{(3,0,0)} + \left(\frac{\alpha}{2\pi}\right) P_{ij}^{(0,1,0)} + \left(\frac{\alpha_s}{2\pi}\right)\left(\frac{\alpha}{2\pi}\right) P_{ij}^{(1,1,0)} + \left(\frac{\alpha}{2\pi}\right)^2 P_{ij}^{(0,2,0)} + \left(\frac{\alpha_B}{2\pi}\right) P_{ij}^{(0,0,1)} + \cdots,$$

$$\alpha_B \sim 0.001$$

1. New partons

M. McCullough, J. Moore, MU, arXiv:2203.12628



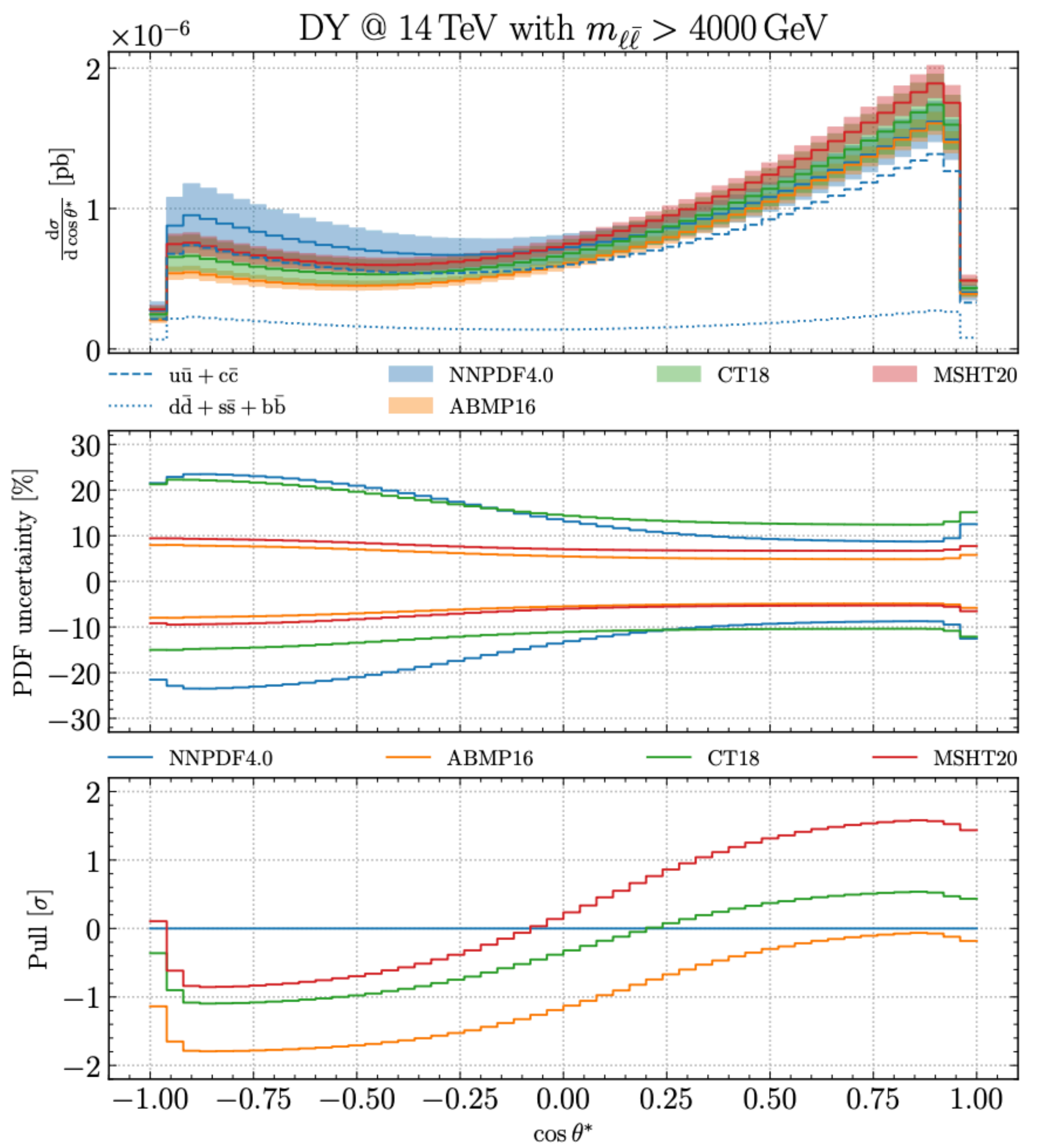
- The presence of the dark Parton would modify the evolution of standard quarks and gluon.
- Interesting to combine with dark photon effects in DIS structure functions [N. T. Hunt-Smith et al arXiv:2302.11126]

Precise LHC data can indirectly constrain parameter space of the dark photon in a competitive way compared to direct searches

2. Large-x PDFs and new physics

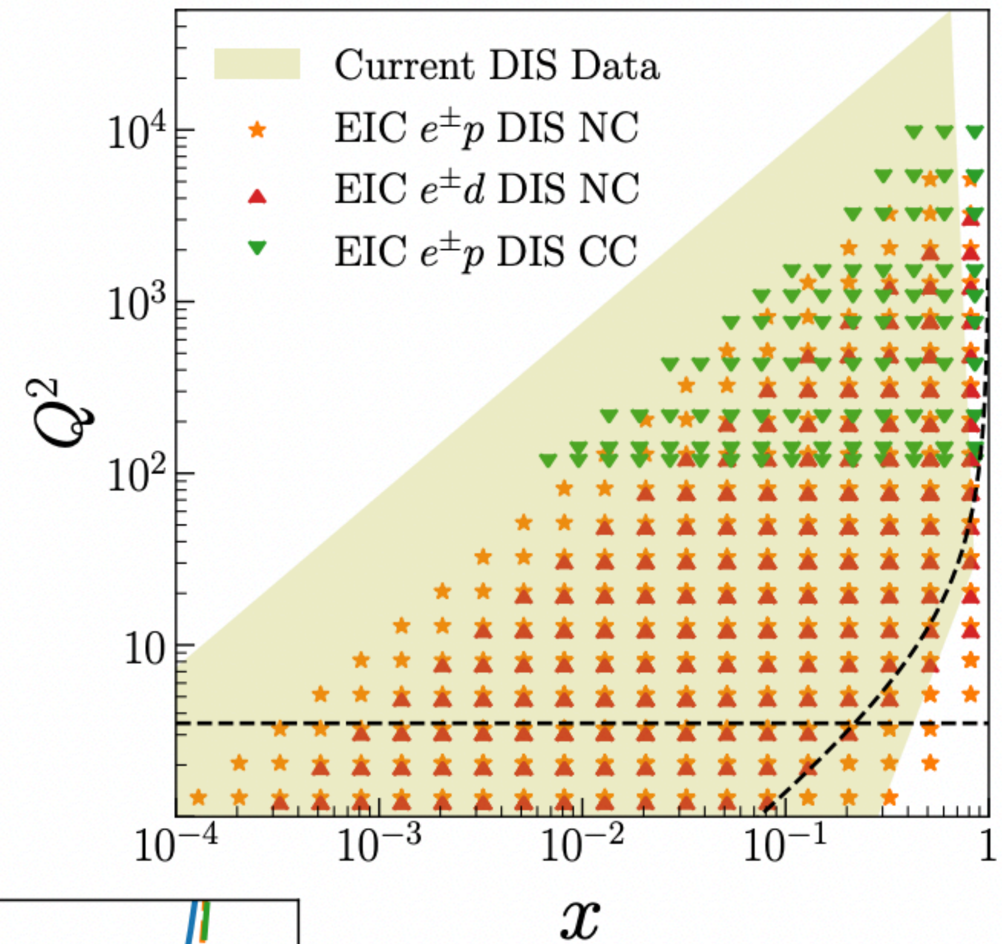
✓ High mass Drell-Yan tails affected by large PDF uncertainties
✓ This affects searches for new physics, for example in forward-backward asymmetry
✓ Need data constraining large-x to see and characterise new physics
(at the LHC high energy - high-x)

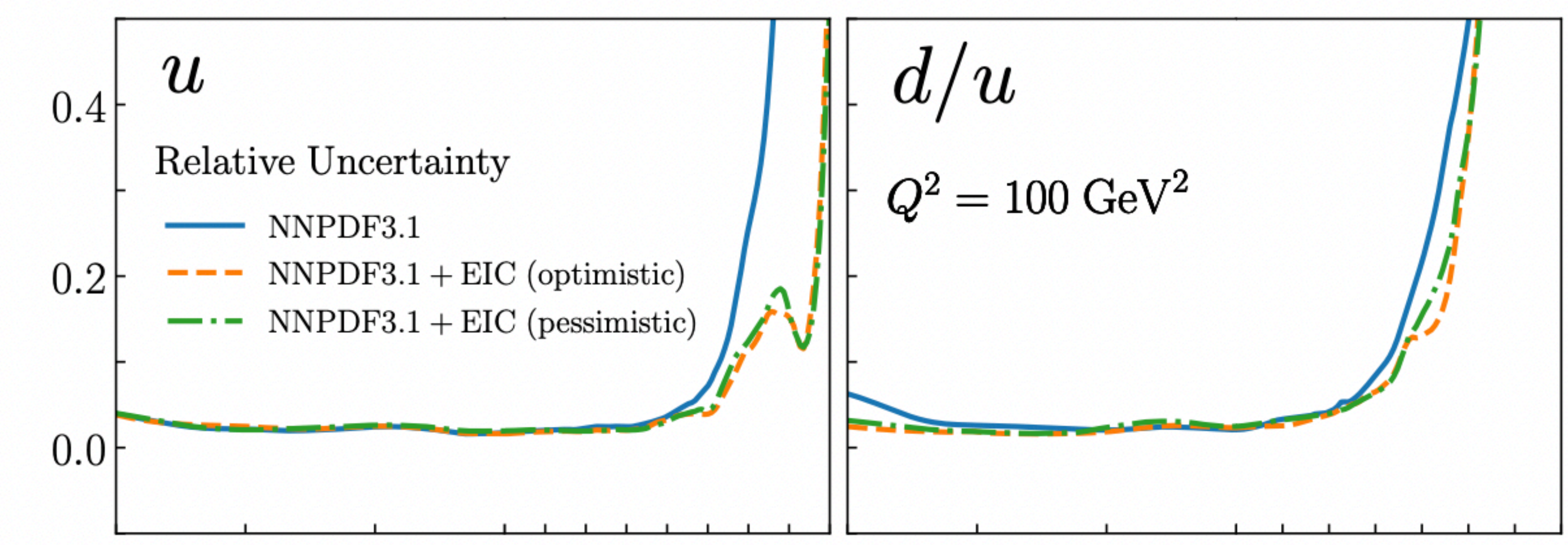
$$\frac{d\sigma}{d\cos\theta^*} = \int_{m_{\ell\bar{\ell}}^{\min}}^{\sqrt{s}} dm_{\ell\bar{\ell}} \int_{\ln(m_{\ell\bar{\ell}}/\sqrt{s})}^{\ln(\sqrt{s}/m_{\ell\bar{\ell}})} dy_{\ell\bar{\ell}} \frac{d^3\sigma}{dm_{\ell\bar{\ell}}\, dy_{\ell\bar{\ell}}\, d\cos\theta^*}$$



2. Large-x PDFs and new physics

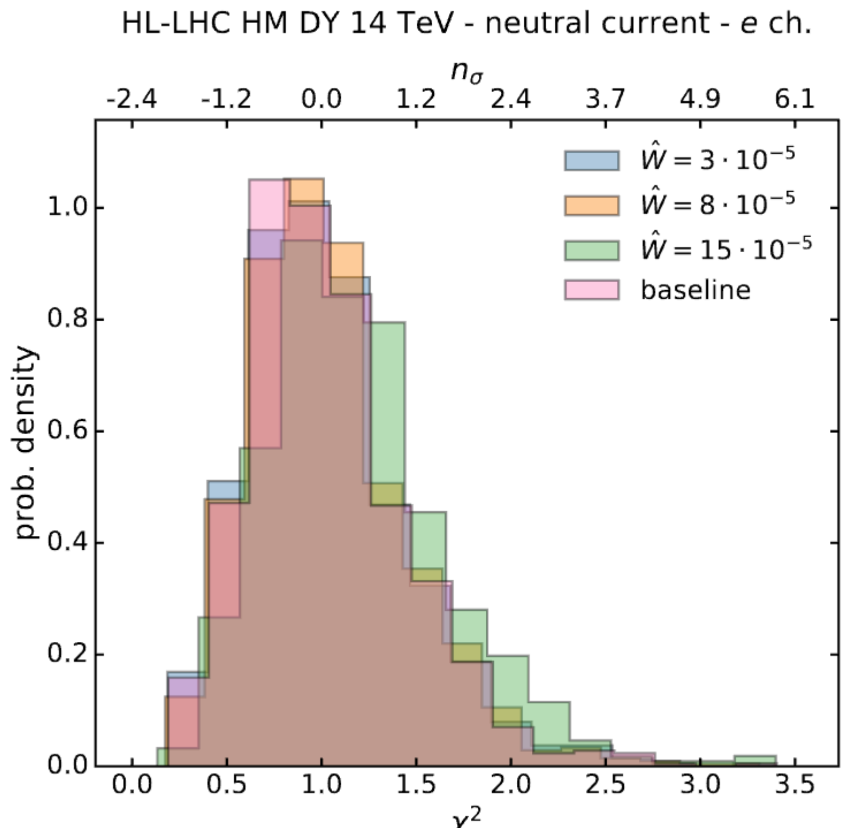
- ✓ High mass Drell-Yan tails affected by large PDF uncertainties
- ✓ This affects searches for new physics, for example in forward-backward asymmetry
- ✓ Need data constraining large-x to see and characterise new physics (at the LHC high energy - high-x)
- ✓ EIC crucial to give complementary constraints

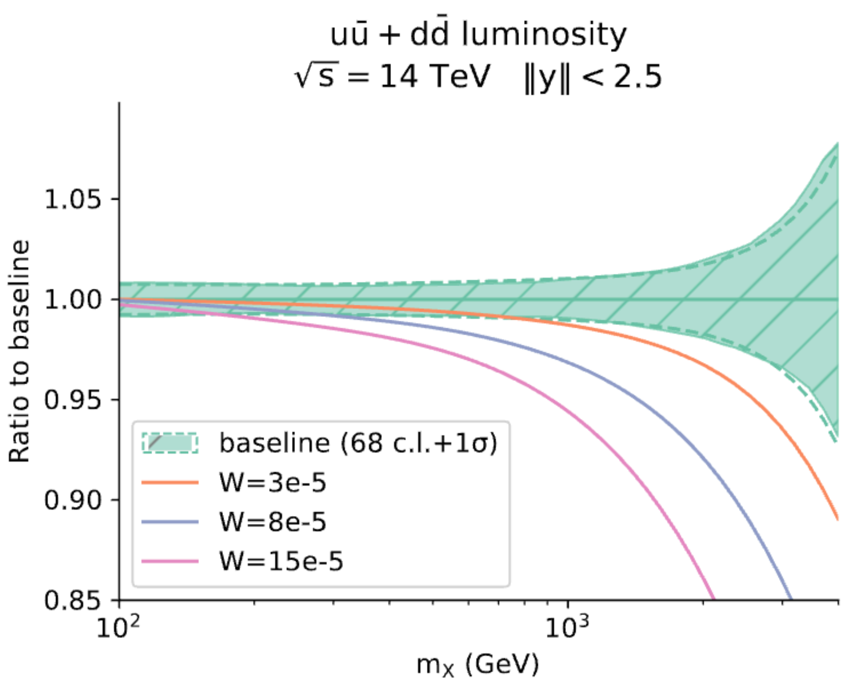




Rabah Abdul Khalek et al, arXiv:2102.00018

2. Large-x PDFs and new physics





- Imagine that UV completion of SM includes a W′ with Mw′ = 14 TeV
- This would show in HL-LHC Drell-Yan distributions at large invariant mass, but we would not see it due to PDF uncertainties.
- We then put this data in a PDF fit to constrain large-x PDFs but use SM to generate NNLO QCD predictions in the fit.
- We get the same good data/theory agreement, a good fit, hence no signs of inconsistency due to New Physics.
- Anti-quark PDFs at large-x compensate or “fit away” the effect of New Physics and we would not know in a real fit.
- **Low E / High E programs complementary and necessary!**

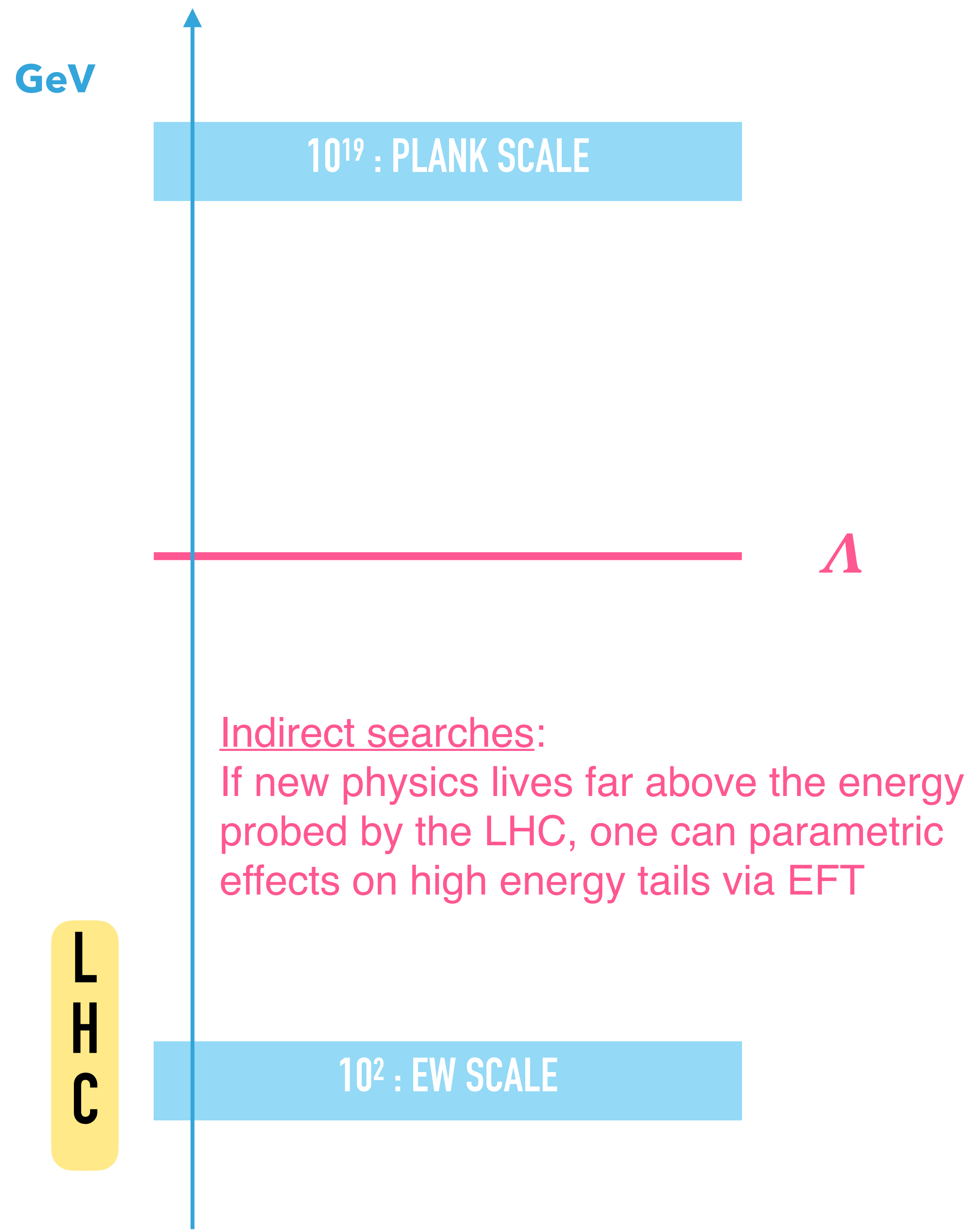
Hammou et al, 2307.10370

3. Indirect searches for New Physics

- EFT is a well-defined theoretical approach for indirect searches
- Assumption: new physics states are heavy
- Write the Lagrangian with only light SM particles
- BSM effects can be incorporated as a momentum expansion
- SMEFT: assume SM field content and gauge symmetries (apart from accidental)

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i^{N_{d6}} \frac{c_i}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_j^{N_{d8}} \frac{b_j}{\Lambda^4}\mathcal{O}_j^{(8)} + \dots$$

- Full dim-6 basis of operators under SMEFT assumptions includes **2499** operators **[Grzadkowski et al, arXiv:1008.4884]**



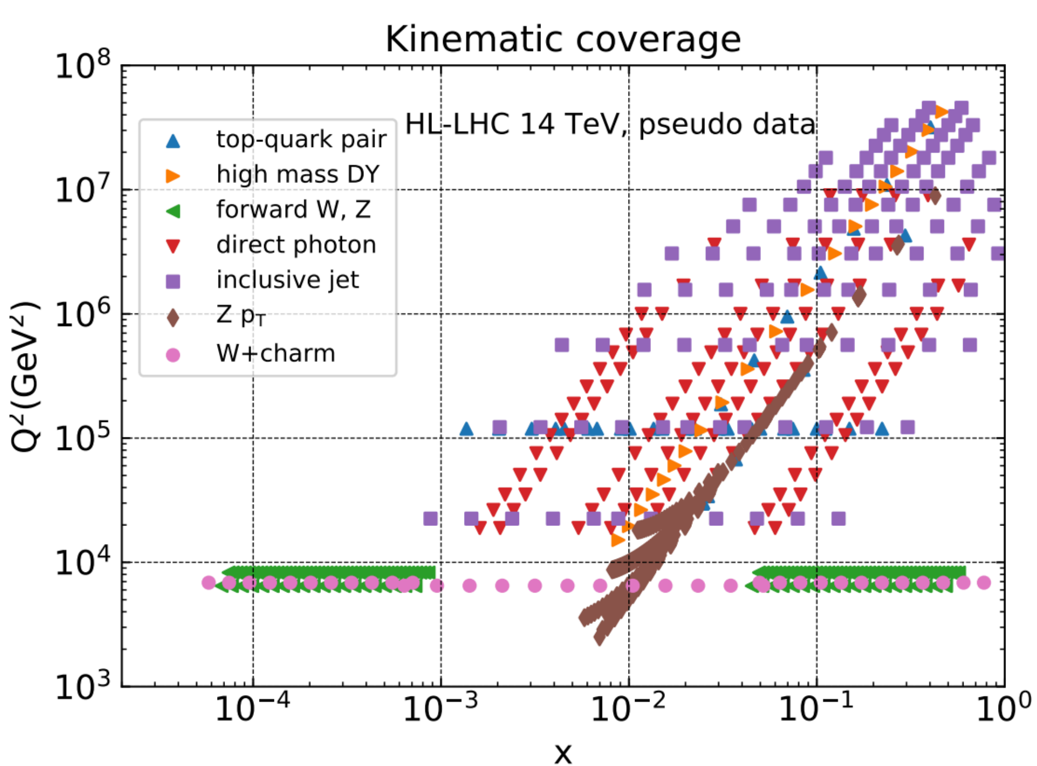
➡ Top pair production and single top data included in SMEFT analysis [Hartland et al 1901.05965] [Ellis et al 2012.02779]

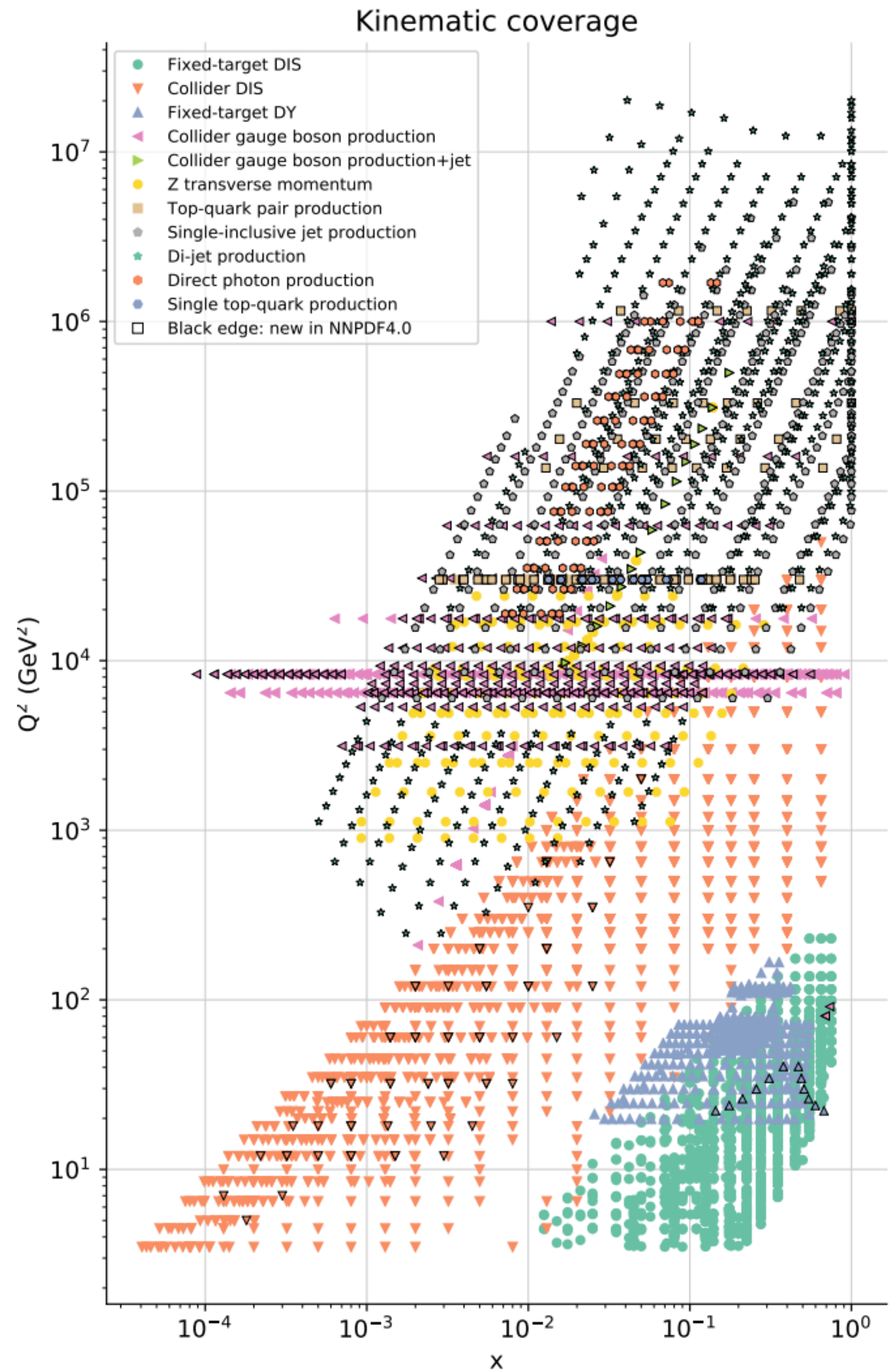
➡ Dijets data [Bordone et al 2103.10332] [Alioli et al 1706.03068]

➡ Drell-Yan data in [Farina et al 1609.08157] [Torre et al 2008.12978]

➡ Inclusive jets in [Alte et al 1711.07484]

➡ Overlap enhanced in HL-LHC projections [Abdul Khalek et al,1810.03639]





3. Indirect searches for New Physics

$$\chi^2 = \frac{1}{N_{\rm dat}} \sum_{i=1}^{N_{\rm dat}} (T_i(\{\theta\},\{c\}) - D_i)\, {\rm cov}^{-1}_{ij}\, (T_j(\{\theta\},\{c\}) - D_j)$$

$$T_i(\{\theta\},\{c\}) = {\rm PDFs}(\{\theta\},\{c\}) \otimes \hat{\sigma}_i(\{c\})$$

(B)SM parameters: $\alpha_s(M_z)$, M_w, θ_w ,**SMEFT WCs**……

Parameters determining PDFs at initial scale

✓ In a PDF fit typically

$$T_i(\{\theta\}) = {\rm PDFs}(\{\theta\},\{c=0\}) \otimes \hat{\sigma}_i(\{c=0\})$$

✓ In a fit of SMEFT Wilson Coefficients

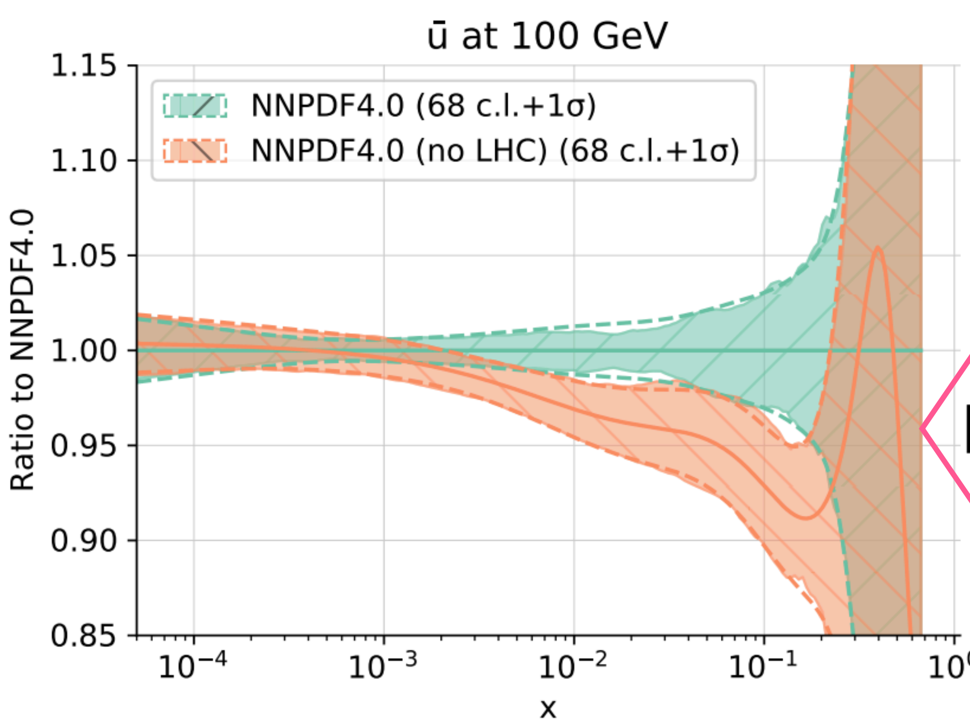
$$T_i(\{c\}) = {\rm PDFs}(\{\theta=\bar{\theta}\},\{c=0\}) \otimes \hat{\sigma}_i(\{c\})$$

3. Indirect searches for New Physics

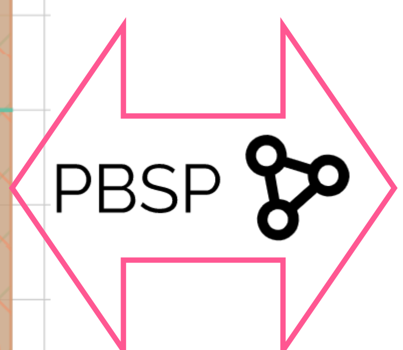
- In principle low-scale physics is separable from high-scale physics, BUT the complexity of the LHC environment might well intertwine them.

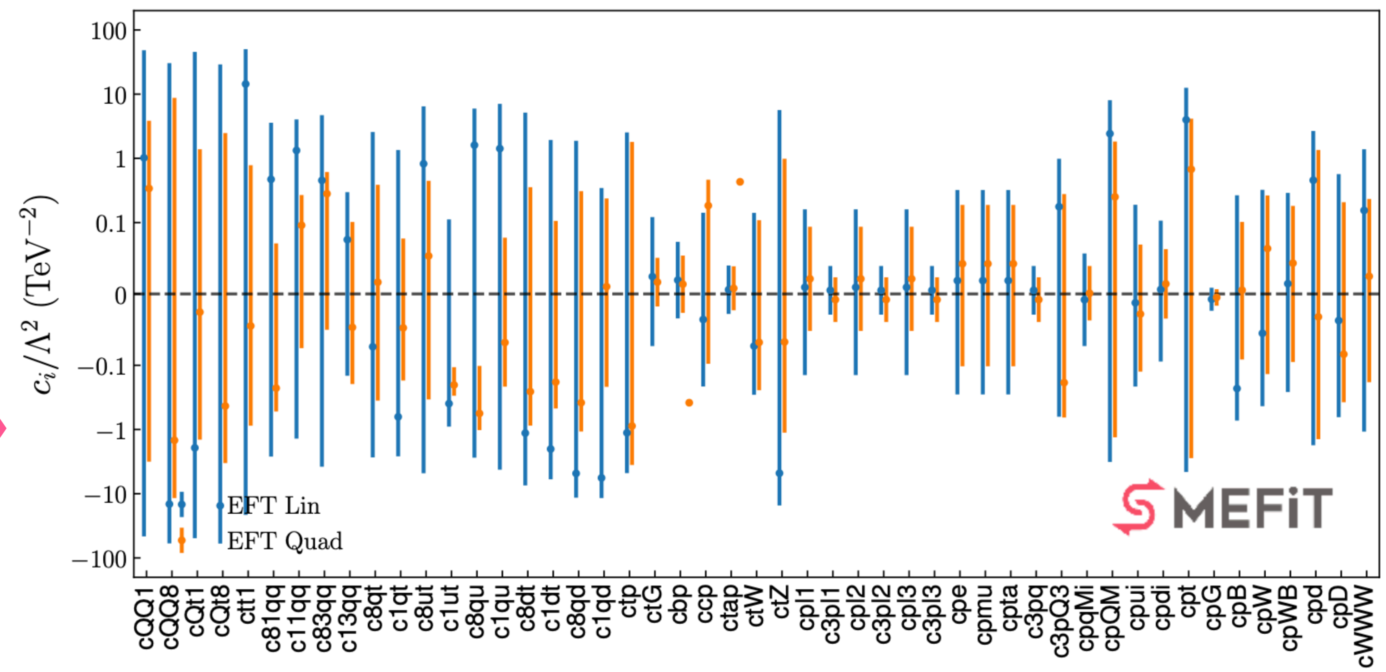
- PDFs are low-scale quantities extracted from experimental data at all scales, without considering any potential high-scale contamination due to new physics.

- (SM)EFT fits are performed by assuming a priori that PDFs are SM-like.



Ball et al, arXiv:2109.02653





Ethier et al, arXiv: 2105.00006

3. Indirect searches for New Physics

- From the point of view of PDF fits:
  - How to make sure that new physics effects are not inadvertently fitted away in a PDF fit?

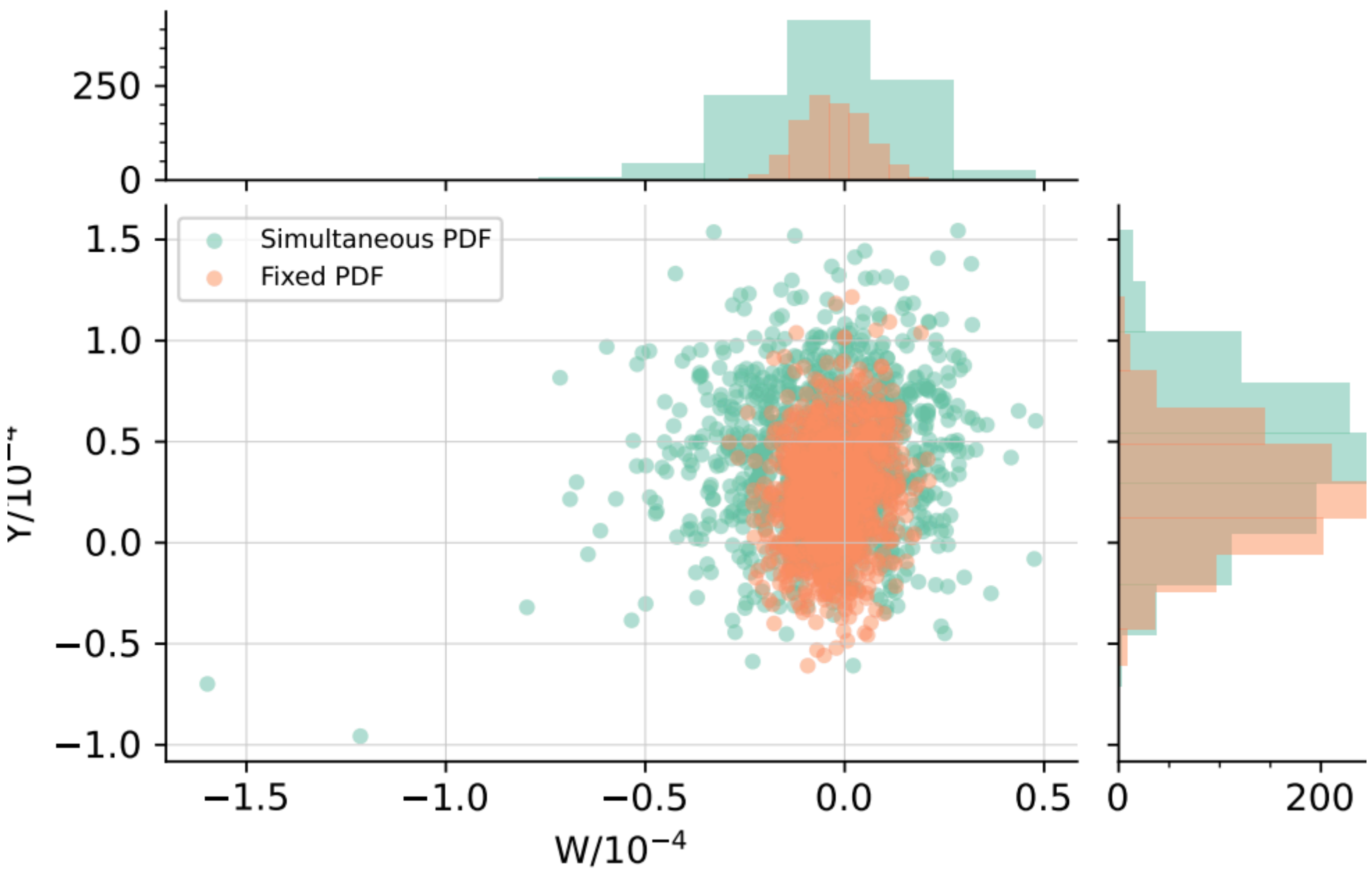
- From the point of view of SMEFT fits:
  - Should I make sure I am using a clean set of PDFs in a SMEFT analysis? How to define it? Is it enough?
  - How would the bounds change if I was consistently using PDFs that include in the fit theory predictions computed adding the same operators that I am fitting?

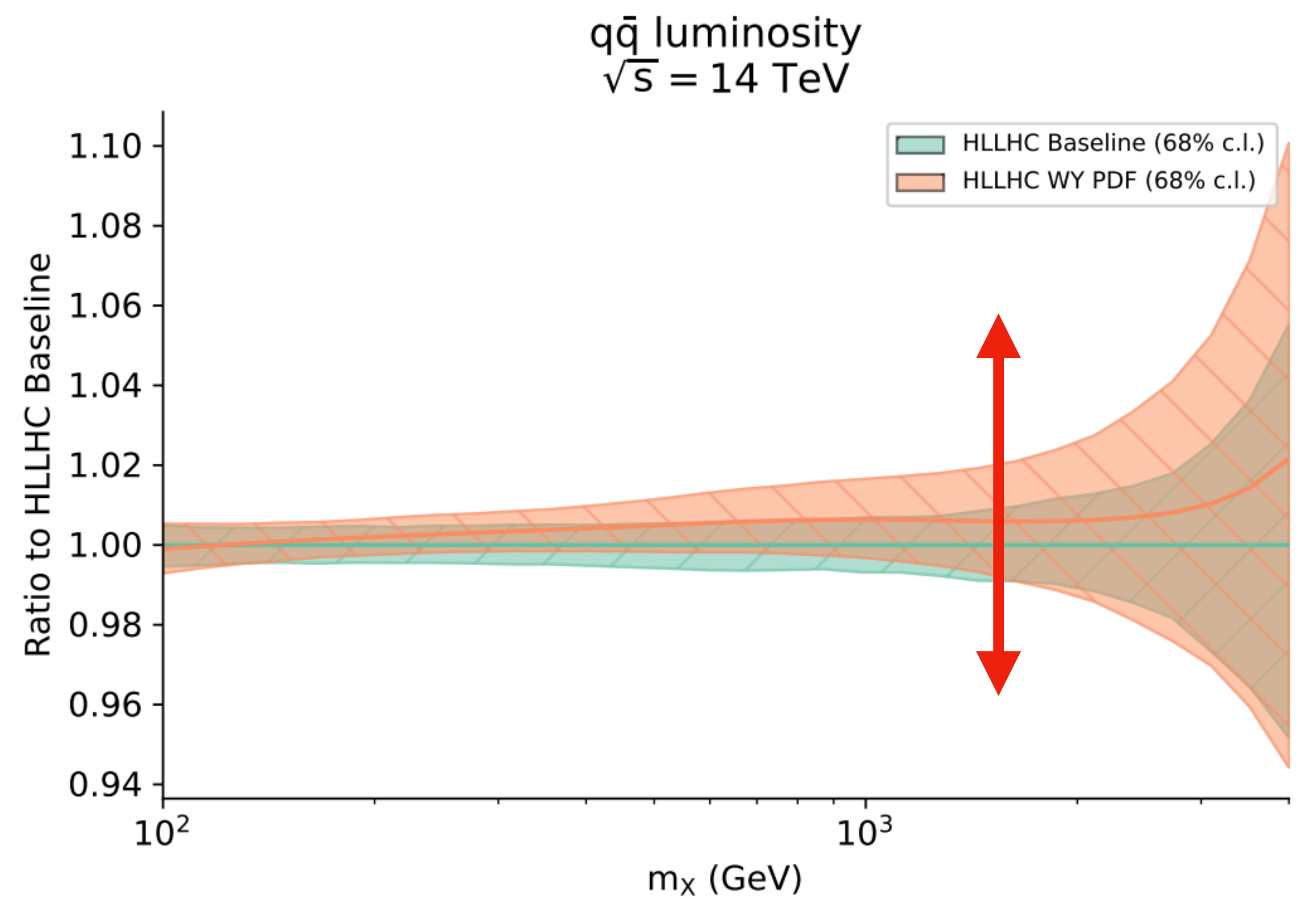
$$T_i(\{\theta\},\{c\}) = \mathrm{PDFs}(\{\theta\},\{c\}) \otimes \hat{\sigma}_i(\{c\})$$

Simultaneous fits can shed light on their interplay

3. Test-ground: DY data at HL-LHC

x 2.3 broadening of bounds for W
x 1.3 broadening of bounds for Y





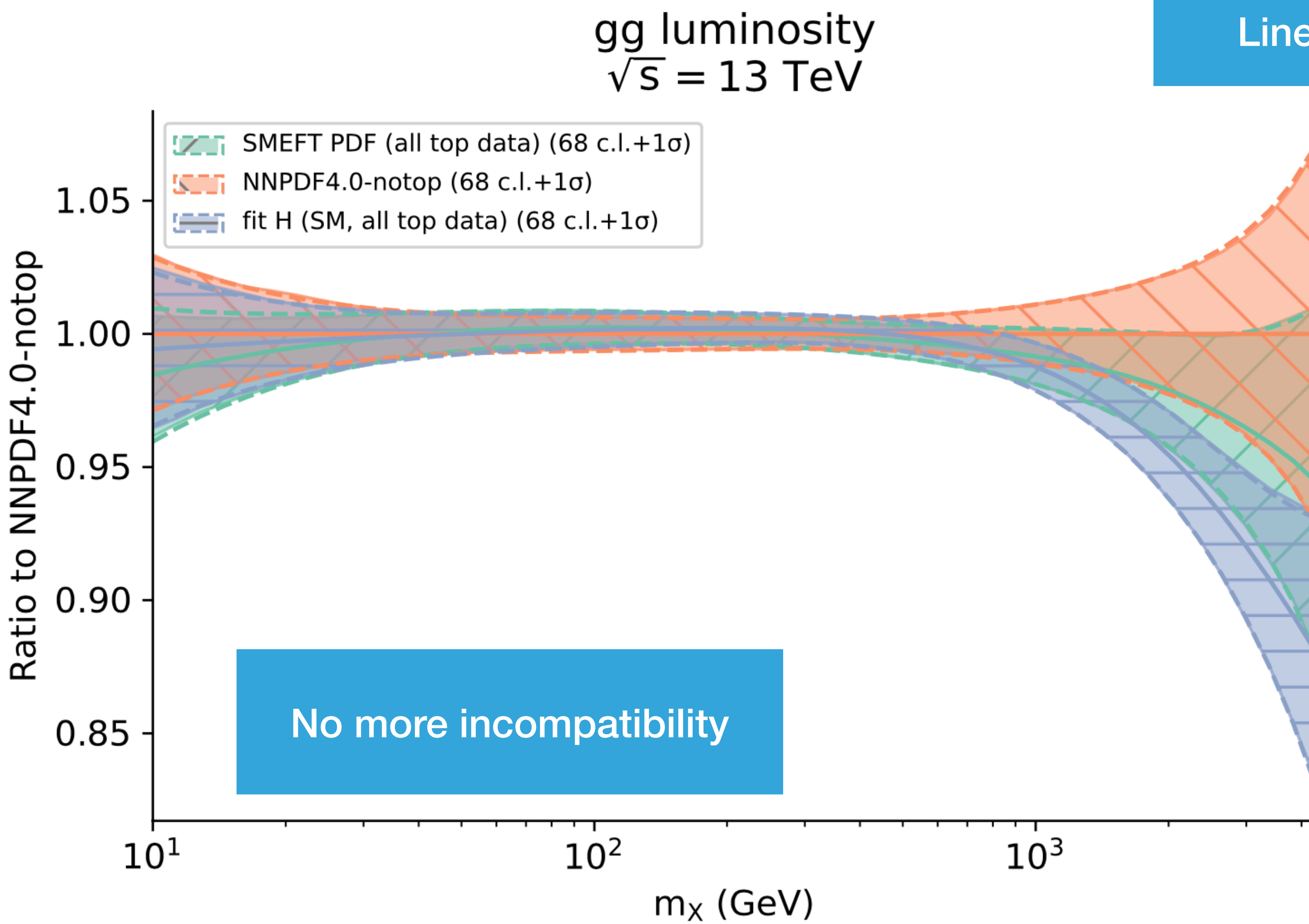
✓ Simultaneous fit shows that at HL-LHC the effect of interplay between SMEFT fits and PDF fits becomes important as bounds on Wilson Coefficients that affect high-mass invariant tails broaden

✓ Also PDF uncertainties broaden significantly once SMEFT effects allowed in theory predictions entering PDF fit

S. Iranipour, MU - arXiv: 2201.07240

3. Test-ground: top data at LHC

Linear SMEFT



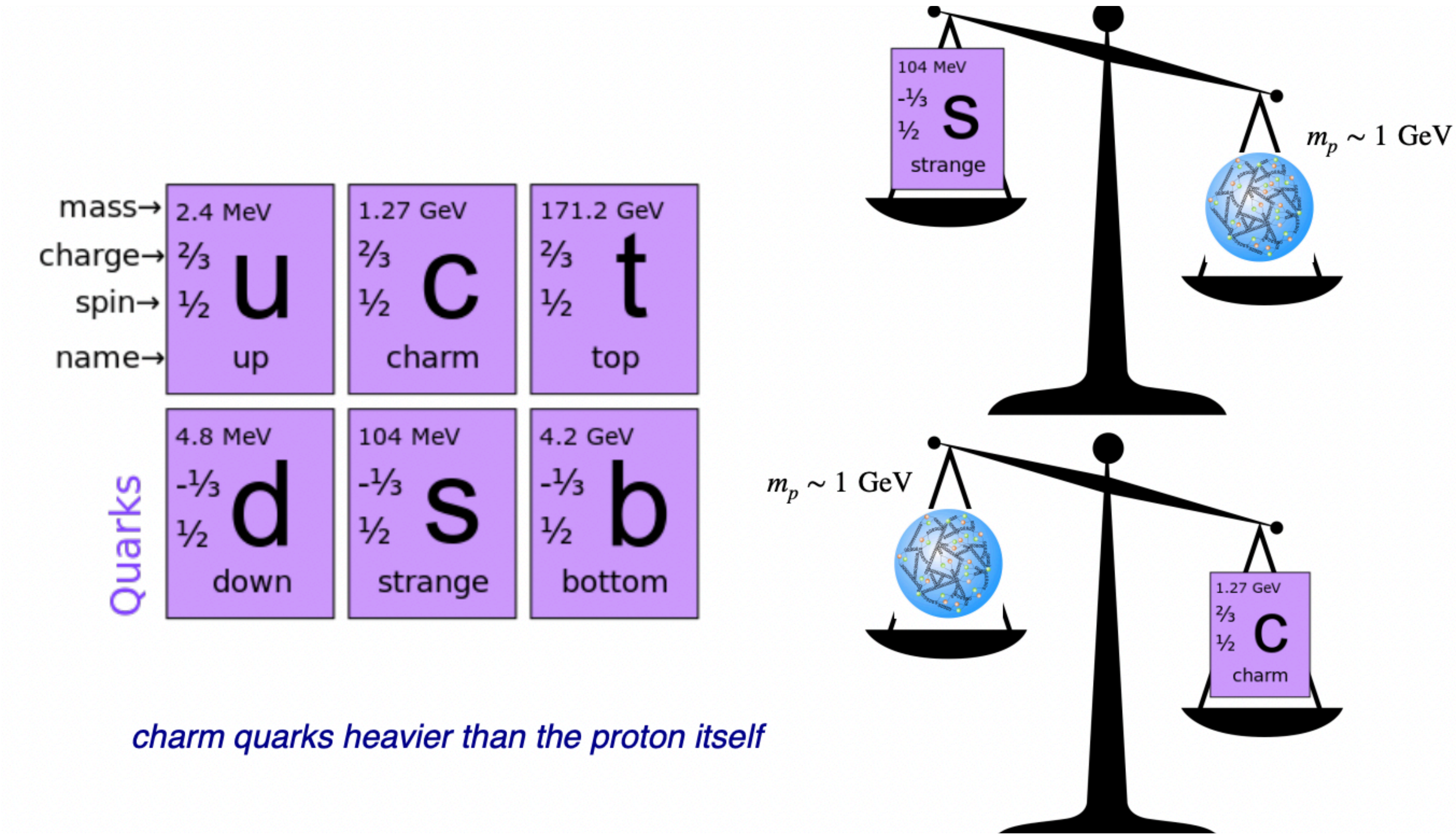
No more incompatibility

Z. Kassabov, M. Madigan, L. Mantani, J. Moore, M. Morales, J. Rojo, MU - arXiv: 2303.06159

Intrinsic charm

The charm of the proton

✓ Common assumption in PDF fits: the static proton wavefunction does not contain charm quarks: the proton contains intrinsic up, down, strange quarks and anti quarks but no intrinsic charm quarks



From J. Rojo's talk at KIT

The charm of the proton

✓ Common assumption in PDF fits: the static proton wavefunction does not contain charm quarks: the proton contains intrinsic up, down, strange quarks and anti quarks but no intrinsic charm quarks

✓The charm PDF is generated perturbatively (DGLAP evolution from radiation off gluons and quarks)

$$f_c^{(n_f)} = 0 \quad \rightarrow \quad f_c^{(n_f+1)} \propto \alpha_s \ln \frac{Q^2}{m_c^2} \left(P_{qg} \otimes f_g^{(n_f+1)} \right) + \mathcal{O}\left(\alpha_s^2\right)$$

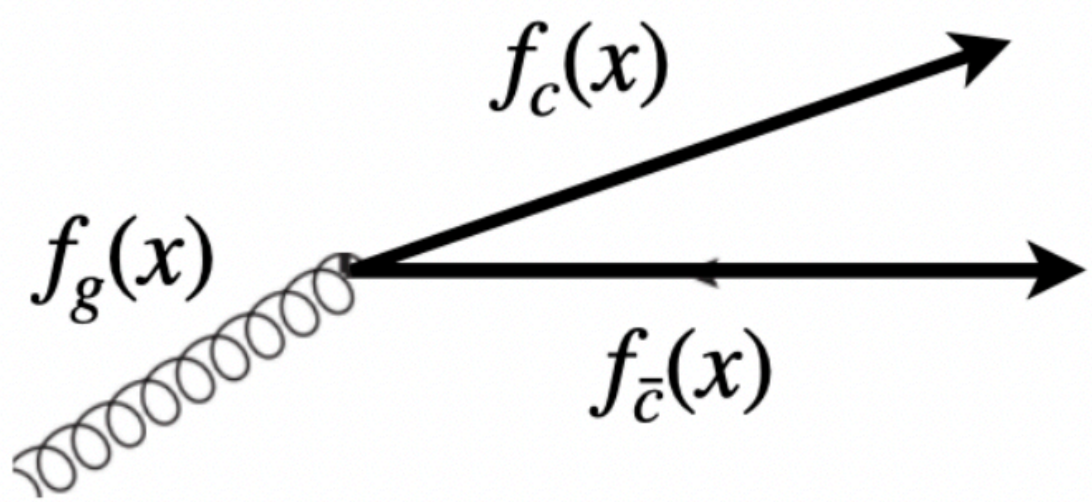
NLO matching

3FNS charm — 4FNS charm — 4FNS gluon

4 flavour scheme, $Q > m_c$
$u^{(4)}, d^{(4)}, s^{(4)}, c^{(4)} g^{(4)}$

↕ ↕

3 flavour scheme, $Q < m_c$
$u^{(3)}, d^{(4)}, s^{(3)}, g^{(3)}$



If charm is **perturbatively generated**, the charm PDF is **trivial**

From J. Rojo's talk at KIT

The charm of the proton

- Common assumption in PDF fits: the static proton wavefunction does not contain charm quarks: the proton contains intrinsic up, down, strange quarks and anti quarks but no intrinsic charm quarks
- It does not need to be so, as an intrinsic charm component is predicted in many models.

THE INTRINSIC CHARM OF THE PROTON

S.J. BRODSKY [1]

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and

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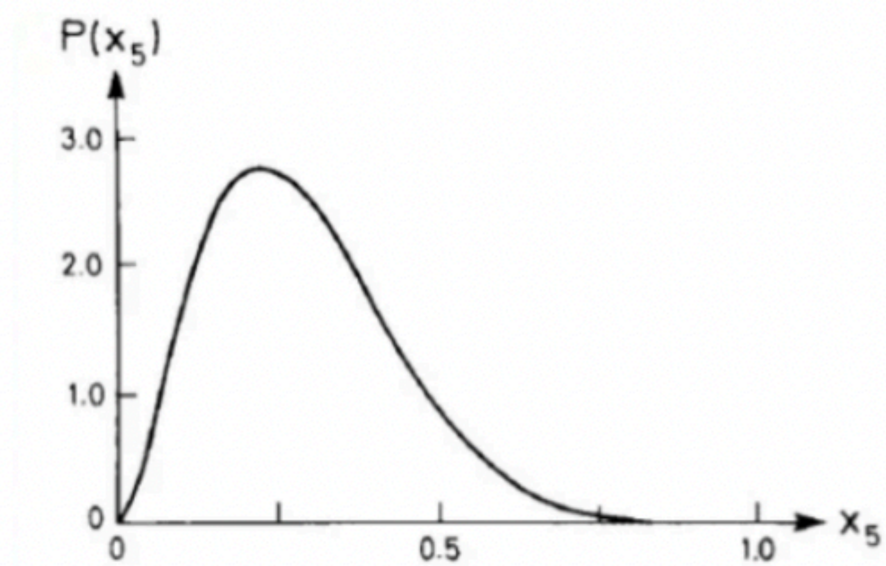
NORDITA, Copenhagen, Denmark

Received 22 April 1980



$$|p\rangle = \mathcal{P}_{3q}|uud\rangle + \mathcal{P}_{5q}|uudc\bar{c}\rangle + \ldots$$

Recent data give unexpectedly large cross-sections for charmed particle production at high x_F in hadron collisions. This may imply that the proton has a non-negligible $uudc\bar{c}$ Fock component. The interesting consequences of such a hypothesis are explored.

The charm of the proton

- ✓ Common assumption in PDF fits: the static proton wavefunction does not contain charm quarks: the proton contains intrinsic up, down, strange quarks and anti quarks but no intrinsic charm quarks
- ✓ It does not need to be so, as an intrinsic charm component is predicted in many models.

in this scenario, the charm PDF extracted from data in the global fit is the combination of the **perturbative** (DGLAP) and the **intrinsic** components

$$c^{(n_f=4)}(x,Q) \simeq c^{(n_f=4)}_{(\mathrm{pert})}(x,Q) + c^{(n_f=4)}_{(\mathrm{intr})}(x,Q)$$

Extracted phenomenologically from data

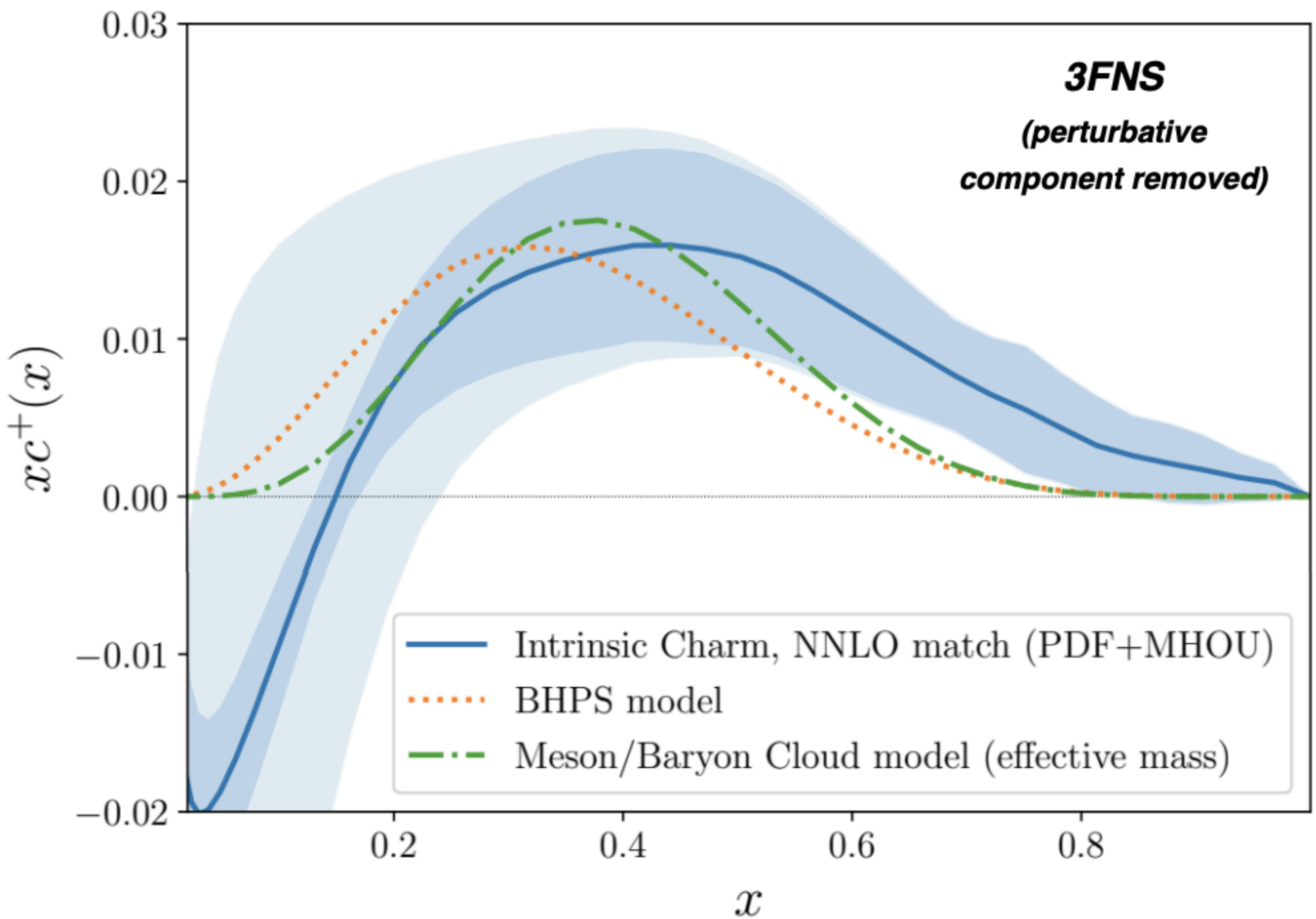
from QCD evolution and matching

from intrinsic component $c^{(n_f=3)}_{(\mathrm{intr})}(x) \neq 0$

How to **disentangle perturbative** from **intrinsic components?**

nb we ***define IC*** *as the charm PDF once know perturbative component is removed*

The charm of the proton



R. D. Ball, A. Candido, J. Cruz-Martinez, S. Forte, T. Giani, F. Hekhorn, K. Kudashkin, G. Magni & J. Rojo, Nature 608 (2022)

Conclusions

[…] Global QCD Analysis of available hard processes critically tests the validity of the PQCD framework, allows the determination of the non-perturbative parton distribution functions, thereby provides the necessary input to calculate and predict most Standard Model and New Physics processes for future, higher, energy interactions. **After two decades of steady progress in this venture, has global QCD analysis of parton distributions reached the End of the Road (as some have proclaimed); or, will the physics challenges of the next generation of colliders usher in the Dawn of a New Era, with fresh ideas and more powerful methodology (as some have promised)? That, is the question.**

Wu-Ki Tung - CERN-TH colloquium 2000

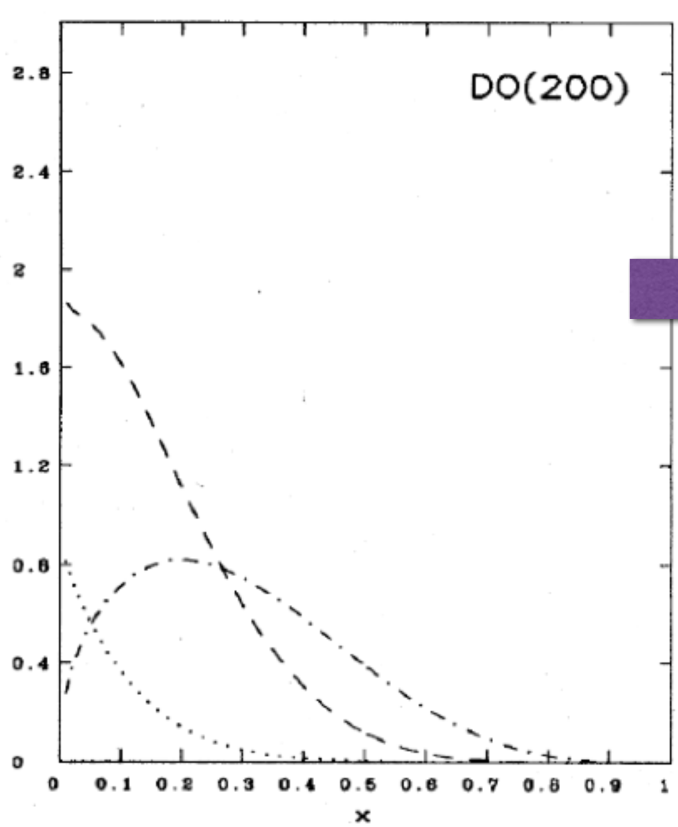
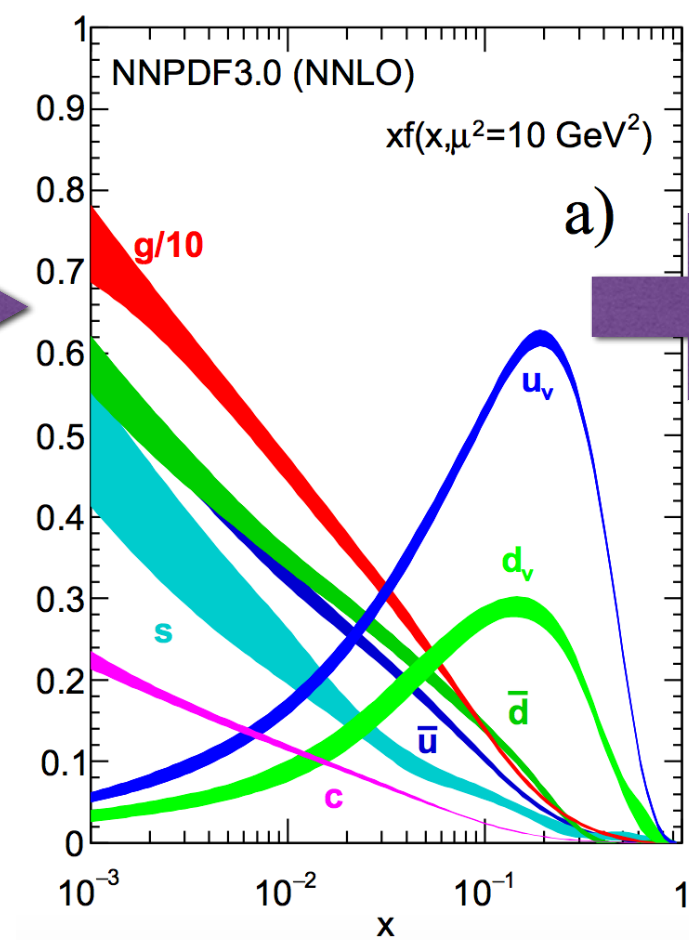


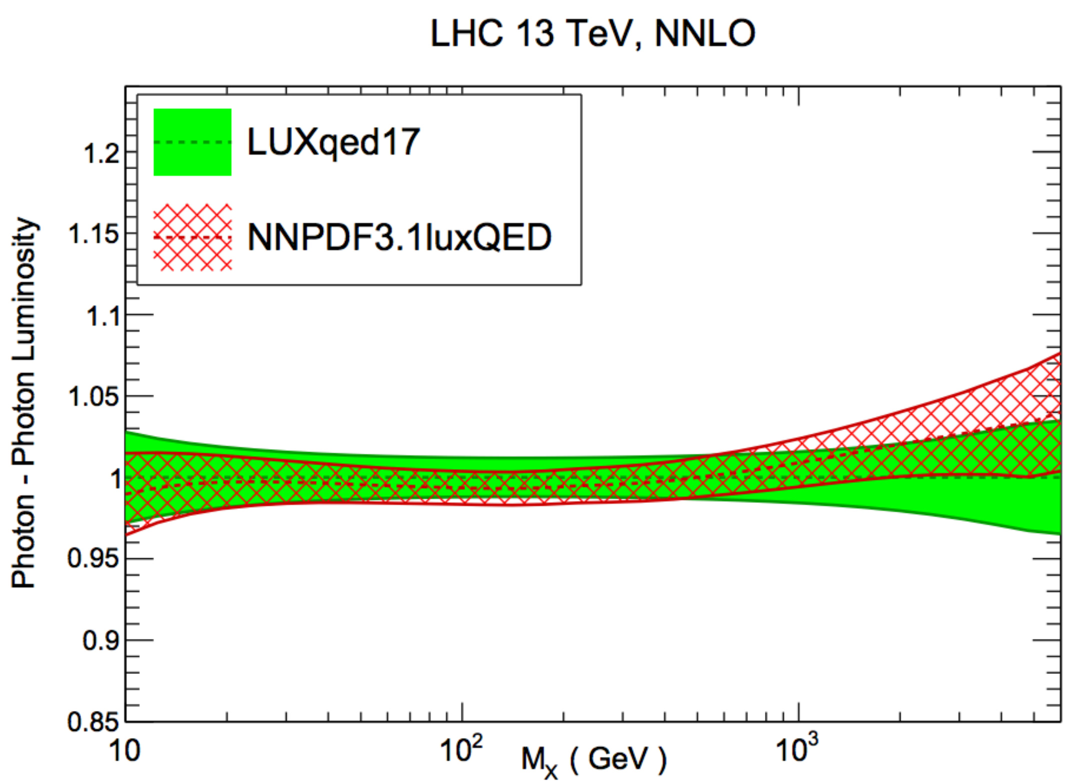
FIG. 27. "Soft-gluon" ($\Lambda=200$ MeV) parton distributions of Duke and Owens (1984) at $Q^2=5$ GeV^2: valence quark distribution $x[u_v(x)+d_v(x)]$ (dotted-dashed line), $xG(x)$ (dashed line), and $q_v(x)$ (dotted line).

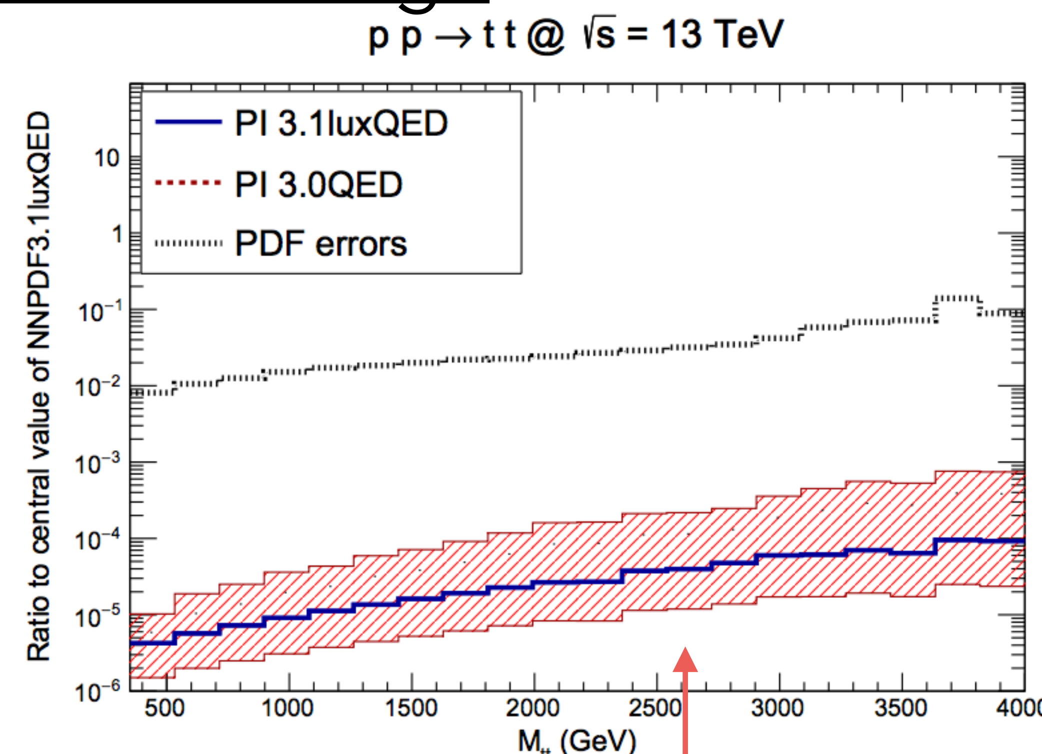


Extra material

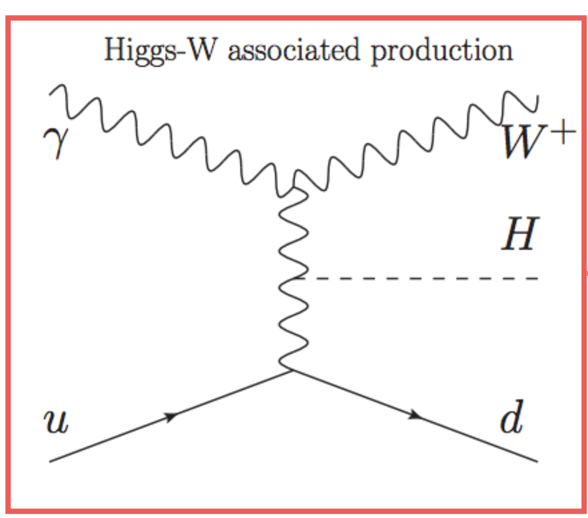
Photon PDFs

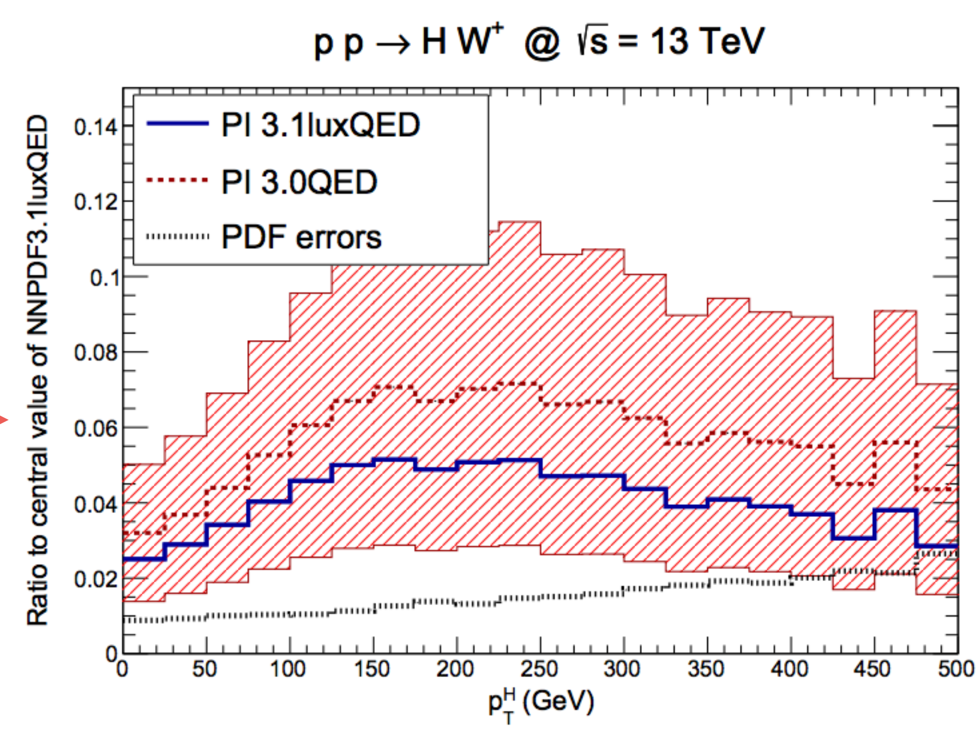
(Data+Theory)-driven knowledge

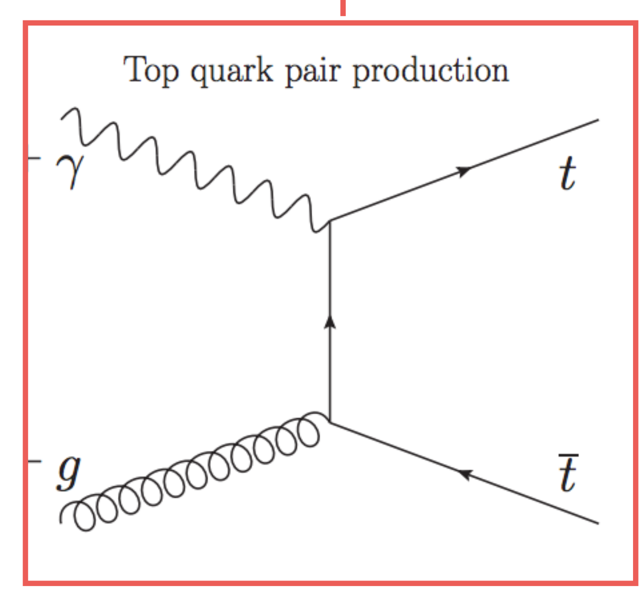




Bertone et al, 1712.07053

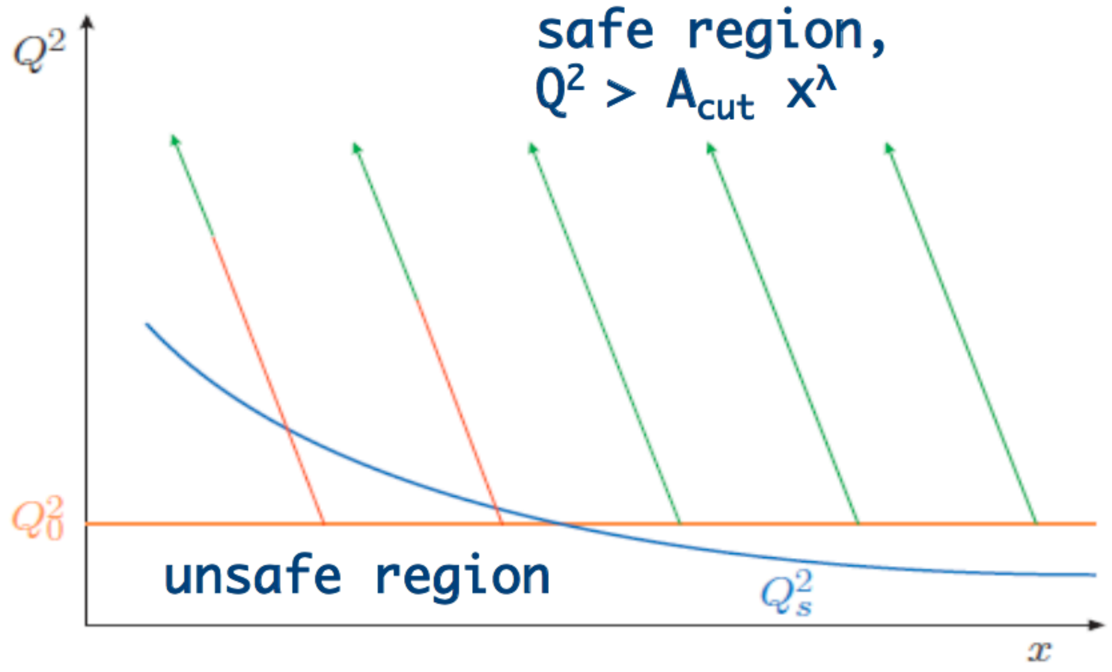






Beyond DGLAP

- In DGLAP formalism there is an implicit approximation: the transverse momentum of the emitted partons in the initial state is much smaller than hard scale
- It works well for inclusive processes with one hard scale and for not-too-small x



Possible effects beyond DGLAP

(i) Leading-twist small-x perturbative resummation

(ii) Non-linear evolution and saturation

(iii) Higher twist effects

Beyond DGLAP

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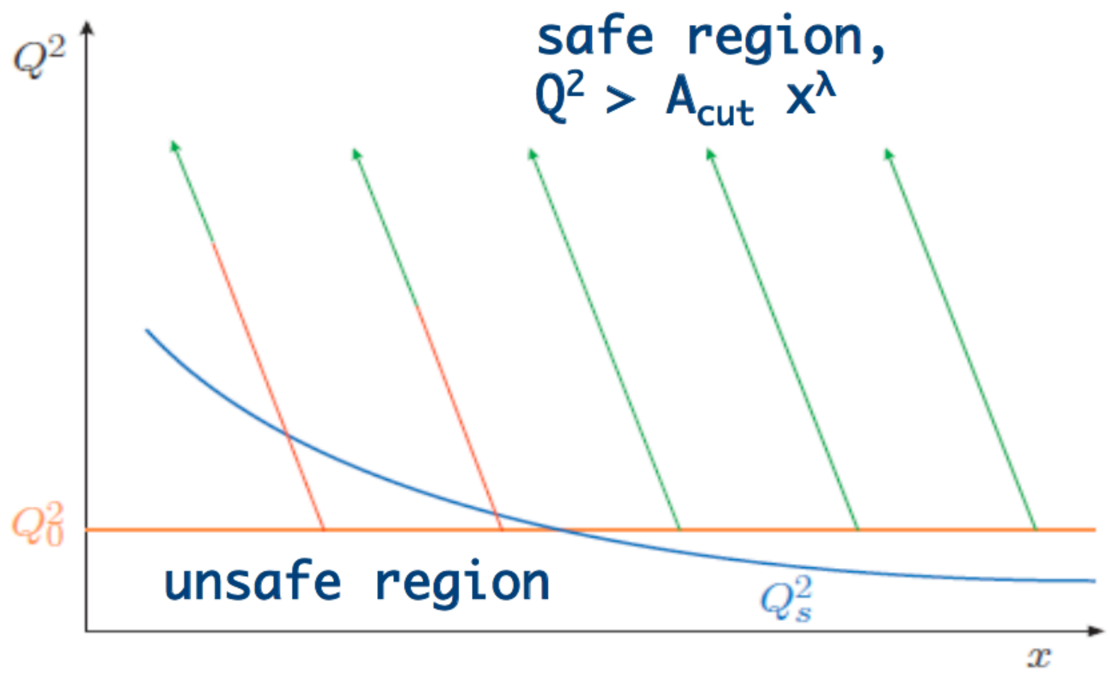


Possible effects beyond DGLAP

➜ (i) Leading-twist small-x perturbative resummation

➜ (ii) Non-linear evolution and saturation

(iii) Higher twist effects

(i) Small-x resummation

- In DGLAP formalism there is an implicit approximation: the transverse momentum of the emitted partons in the initial state is much smaller than hard scale
- It works well for inclusive processes with one hard scale and for not-too-small x
- If s >> M^2 (the high-energy limit or small-x limit) then there are enhanced small-x logarithms in the DGLAP Pqg and Pgg splitting functions that spoil the perturbative expansion in α_s
- These large logs are resumed by BFKL evolution equations

DGLAP Evolution in Q^2

$$\frac{\partial}{\partial \ln Q^2} f_i(x, Q^2) = \int_x^1 \frac{dz}{z} P_{ij}\left(\frac{x}{z}, \alpha_s(Q^2)\right) f_j(z, Q^2)$$

BFKL Evolution in x

$$\frac{\partial}{\partial \ln 1/x} f_+(x, Q^2) = \int_0^\infty \frac{d\nu^2}{\nu^2} K\left(\frac{Q^2}{\nu^2}, \alpha_s(Q^2)\right) f_+(x, \nu^2)$$

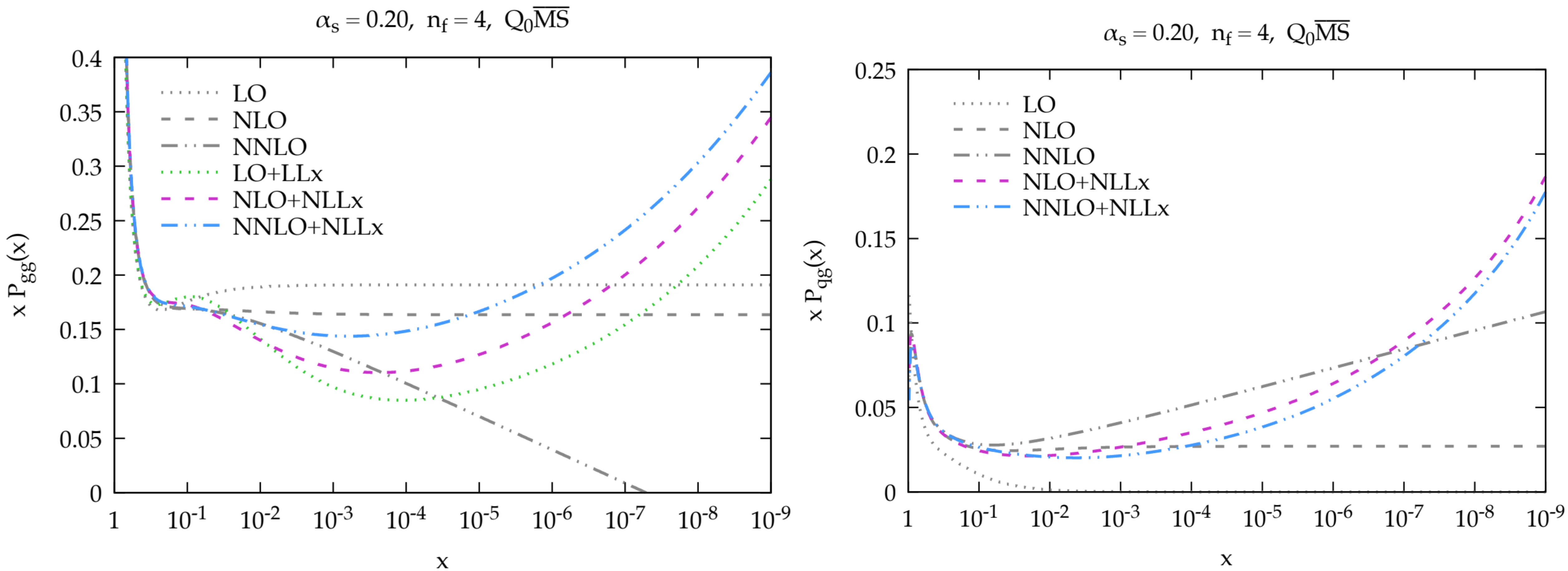
Valid only at small-x

- There are way of combining DGLAP & BFKL - what are the effects?

Balitsky Fadin Kuraev Lipatov 1978, Lipatov Fadin 1998

(i) Small-x resummation

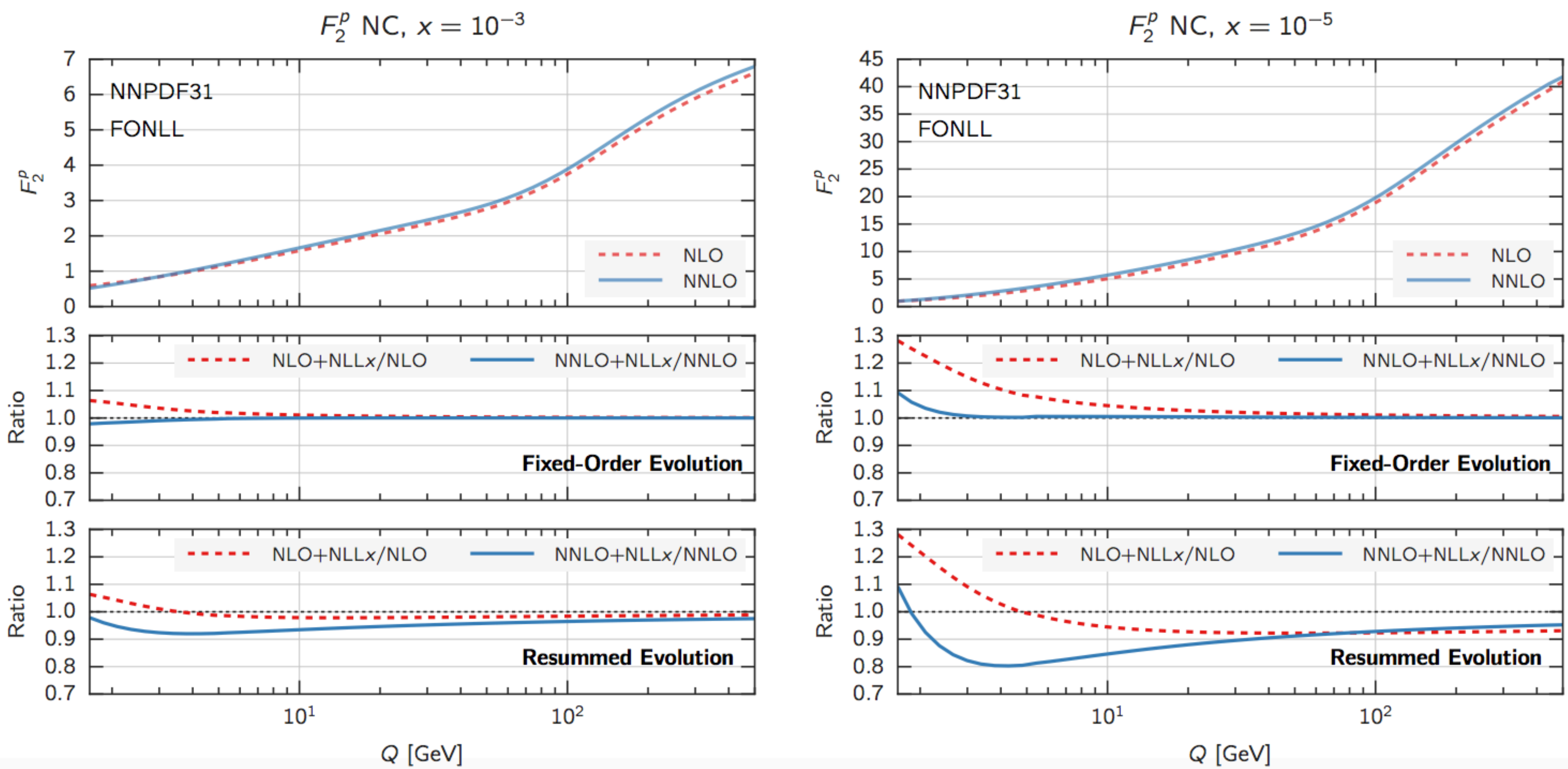
- Small-x resummation stabilises splitting function behaviour at small x



Ball et al, 1710.05935

(i) Small-x resummation

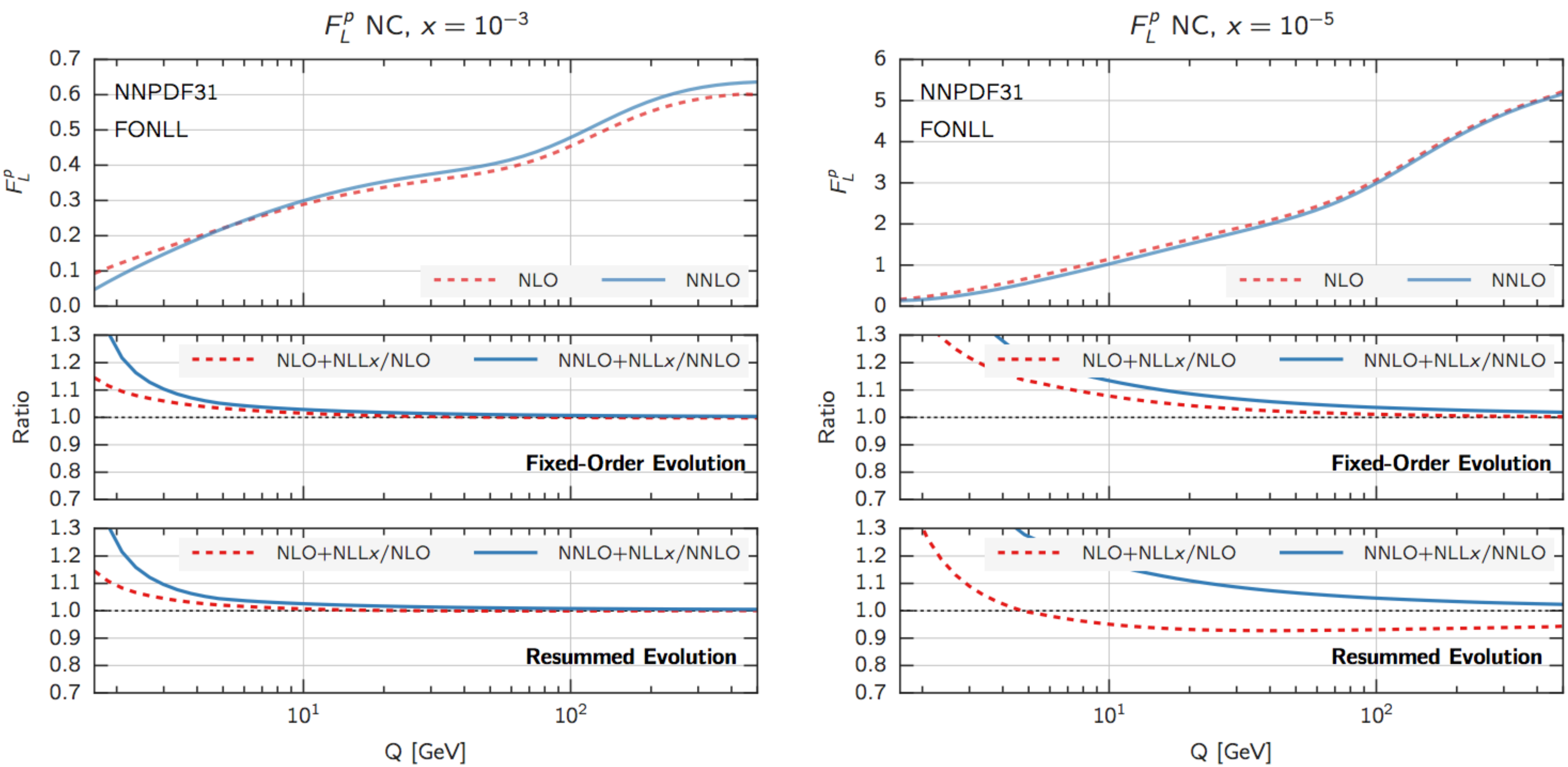
- Large corrections at small Q2 and small-x



Ball et al, 1710.05935

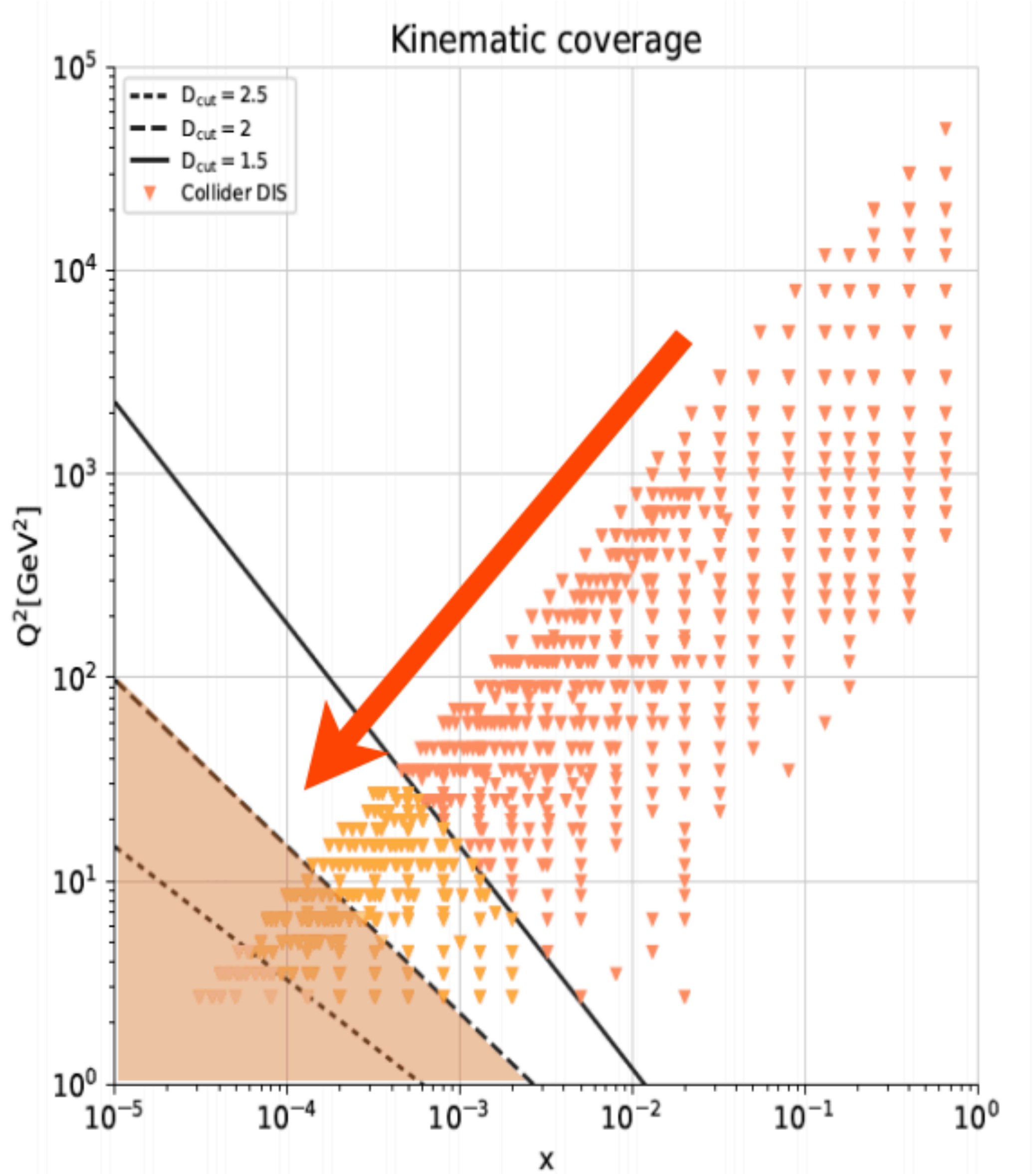
(i) Small-x resummation

- Large corrections at small Q2 and small-x, especially for FL

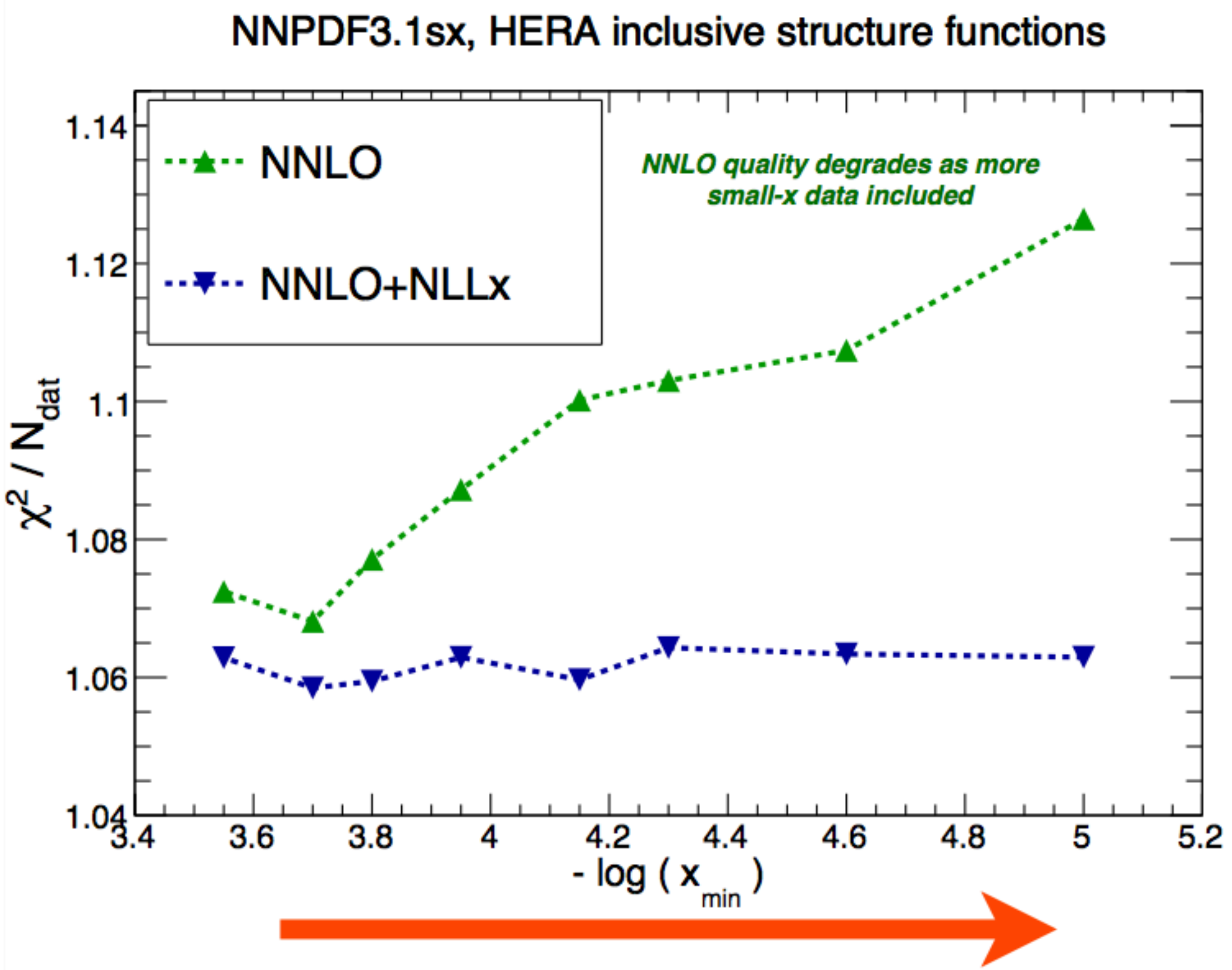


Ball et al, 1710.05935

(i) Small-x resummation



Monitor the **fit quality** as one includes more data from the **small-*x* region**



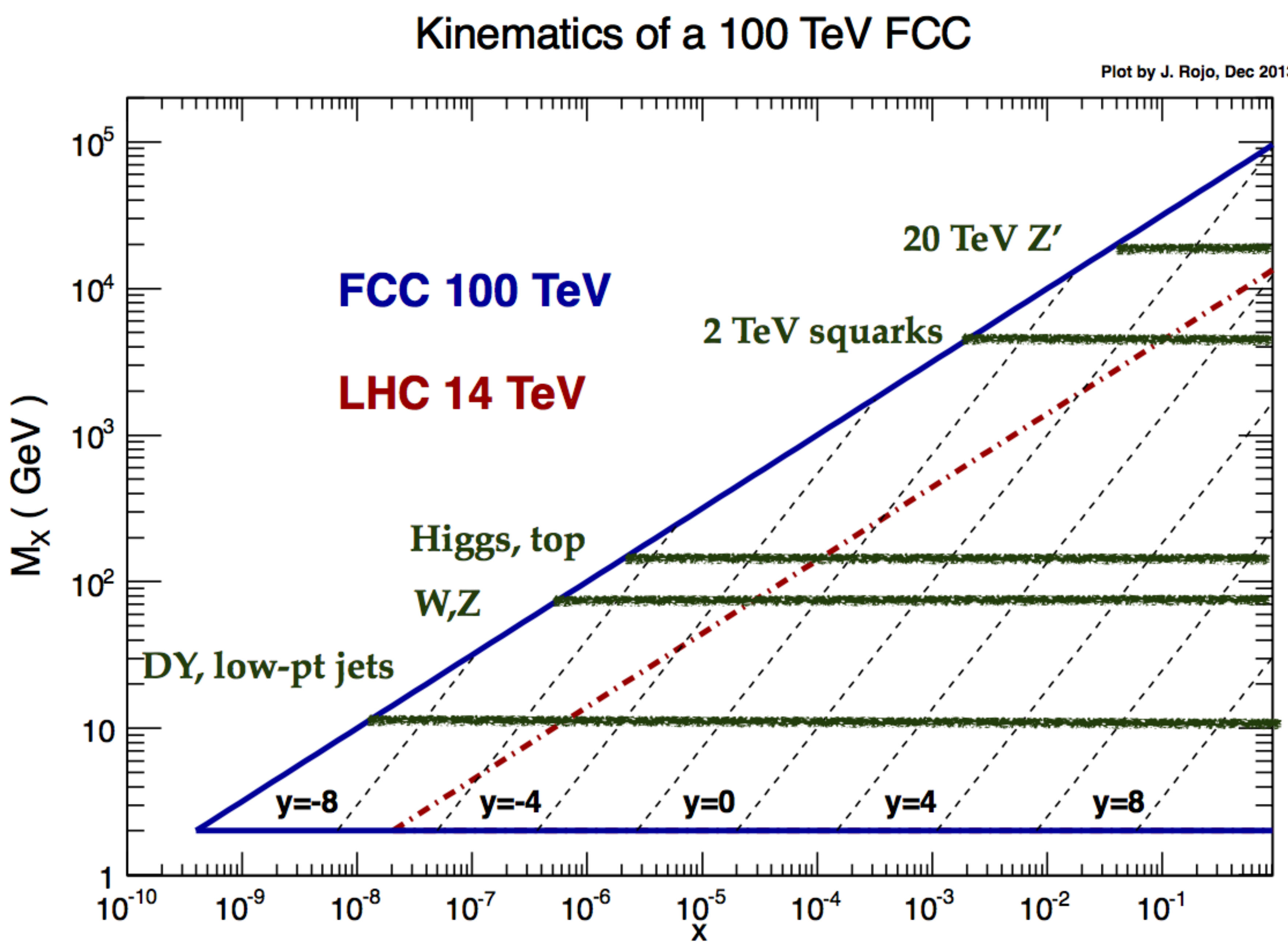
Best description of **small-*x* HERA data** only possible with **BFKL effects!**

J Rojo, BNL talk

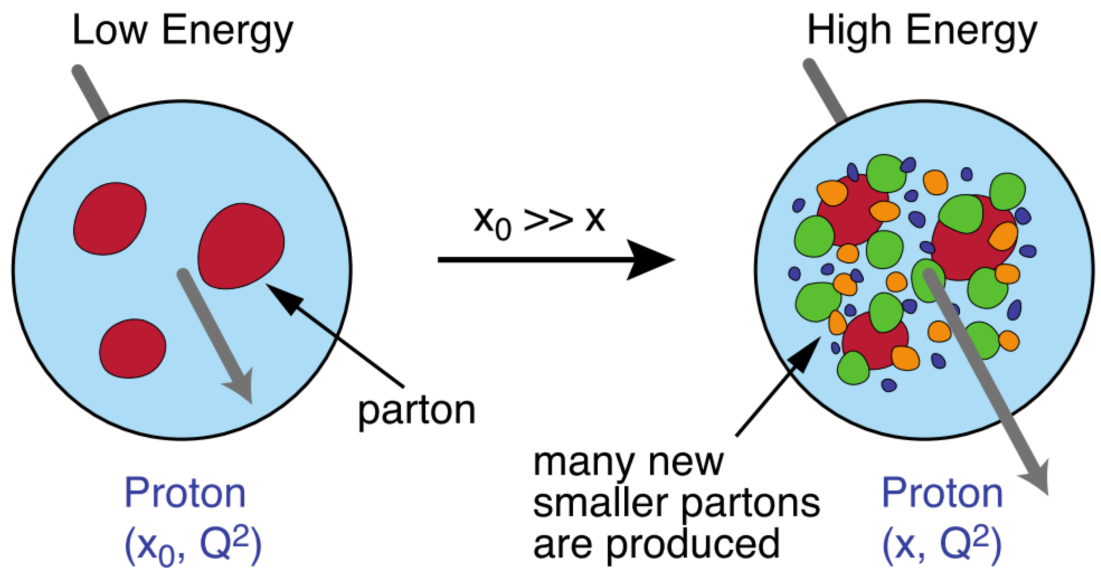
Ball et al, 1710.05935

(i) Small-x resummation

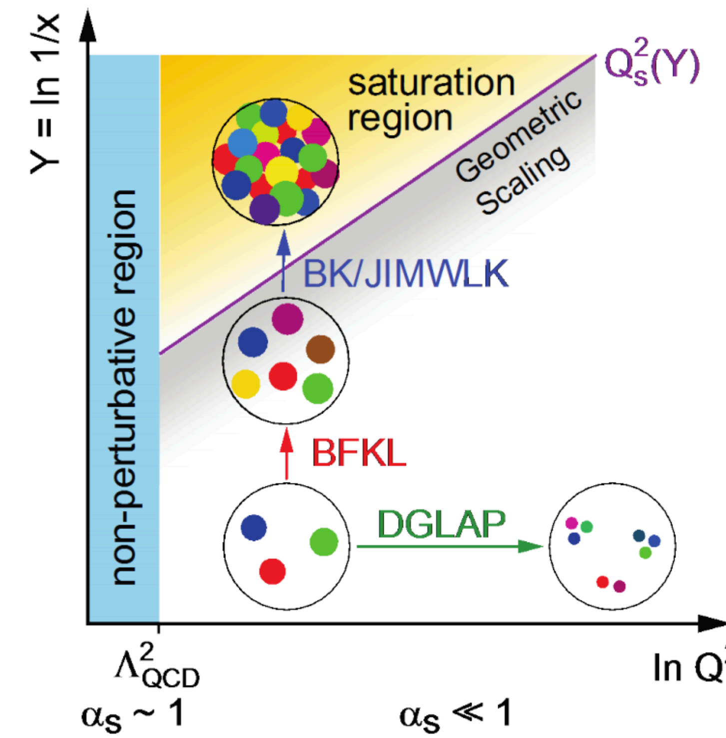
- Will this be enough when we will reach even smaller values of x?



(ii) Non-linear evolution and saturation



"Color Glass Condensate"

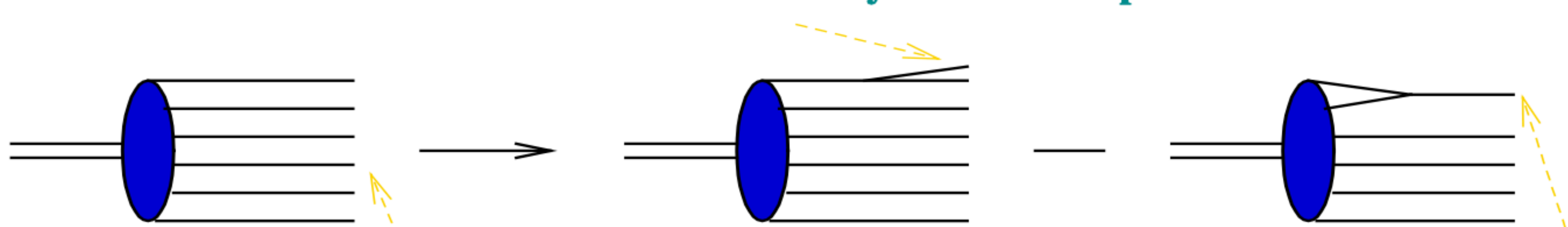


$Q_s^2 \sim A^{1/3} s^\lambda$

Important for ion-ion collisions

new parton is emitted as energy increases

it could be emitted off anyone of the N partons



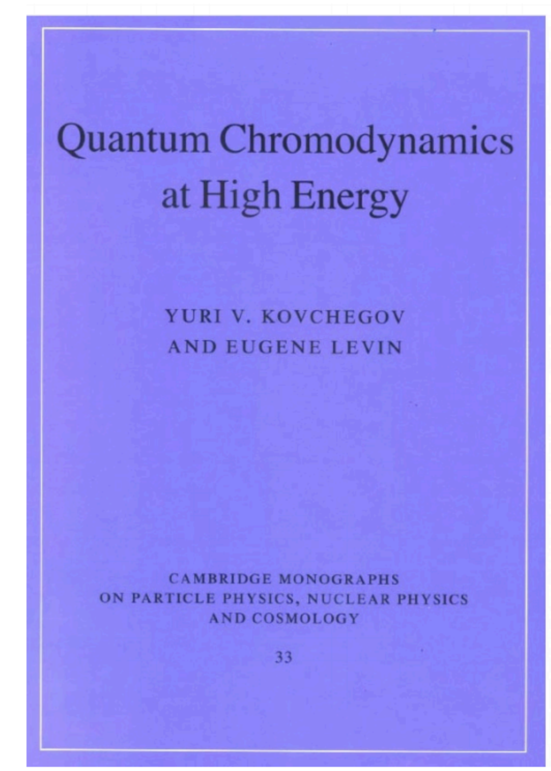
N partons

any two partons can recombine into one

$$\frac{\partial}{\partial Y} N(x, k_T^2) = \alpha_s \, K_{BFKL} \otimes N(x, k_T^2) - \alpha_s \, [N(x, k_T^2)]^2$$

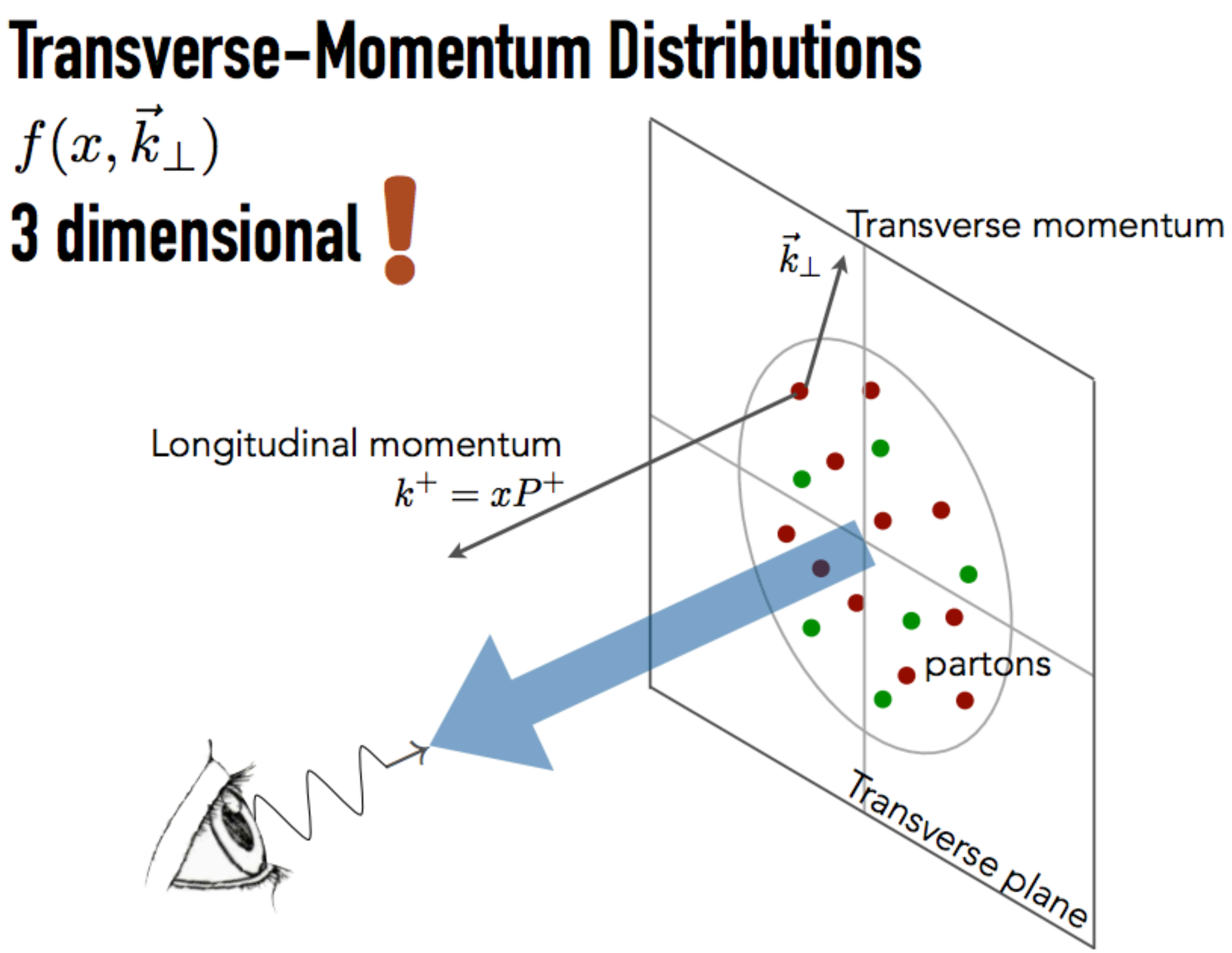
Number of parton pairs ~ N^2

I. Balitsky ' 96 (effective Lagrangian)
Yu. K. ' 99 (large N_c QCD)
JIMWLK '98-'01 (beyond large-N_c)



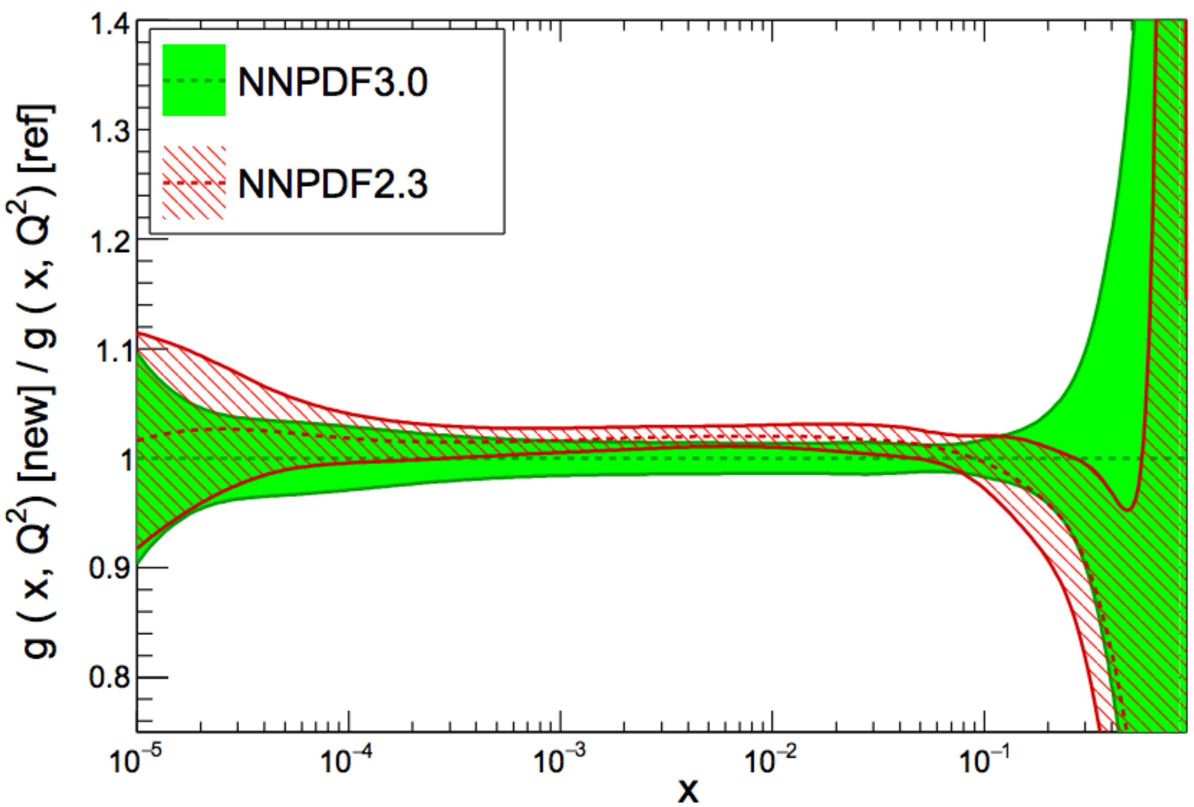
Beyond DGLAP: TMDs

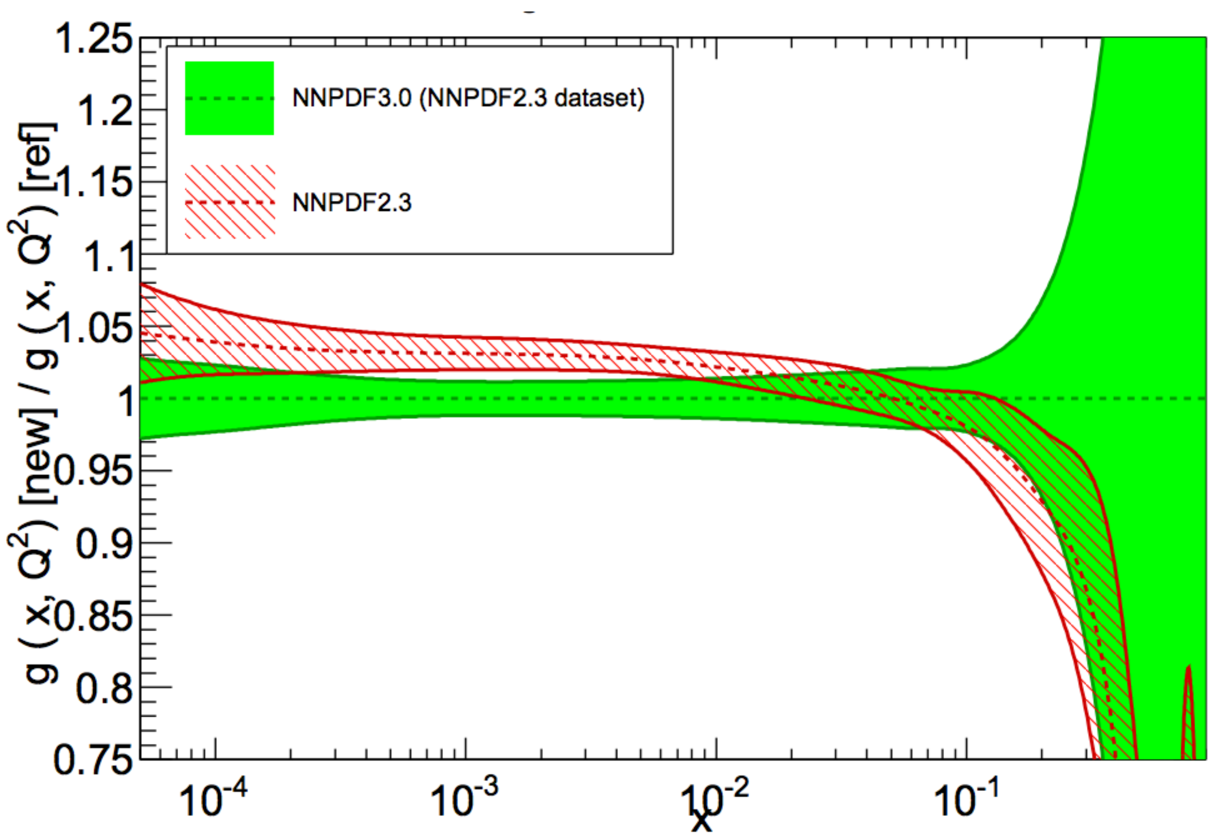
- Much less mature field (universality and factorisation not well established but lots of interesting developments)

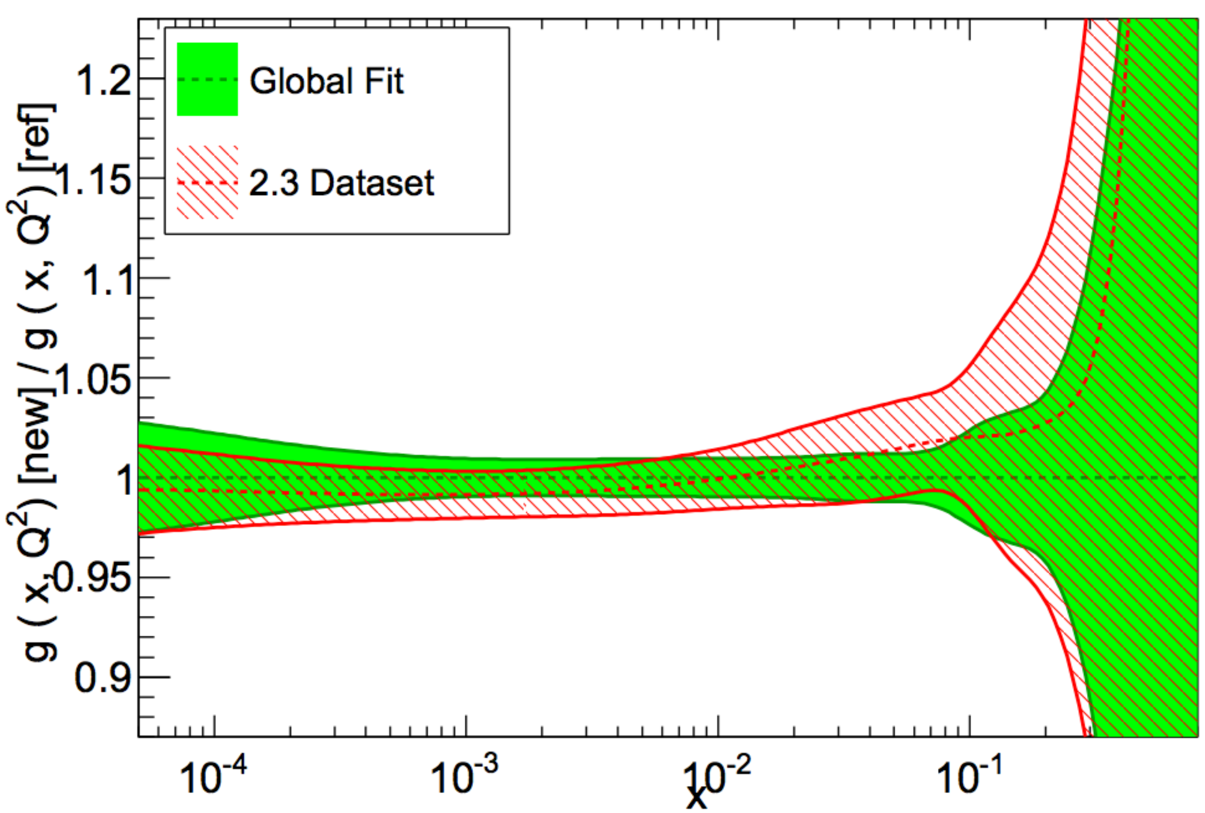


A. Bacchetta, talk at DIS2017

Key issue: methodology





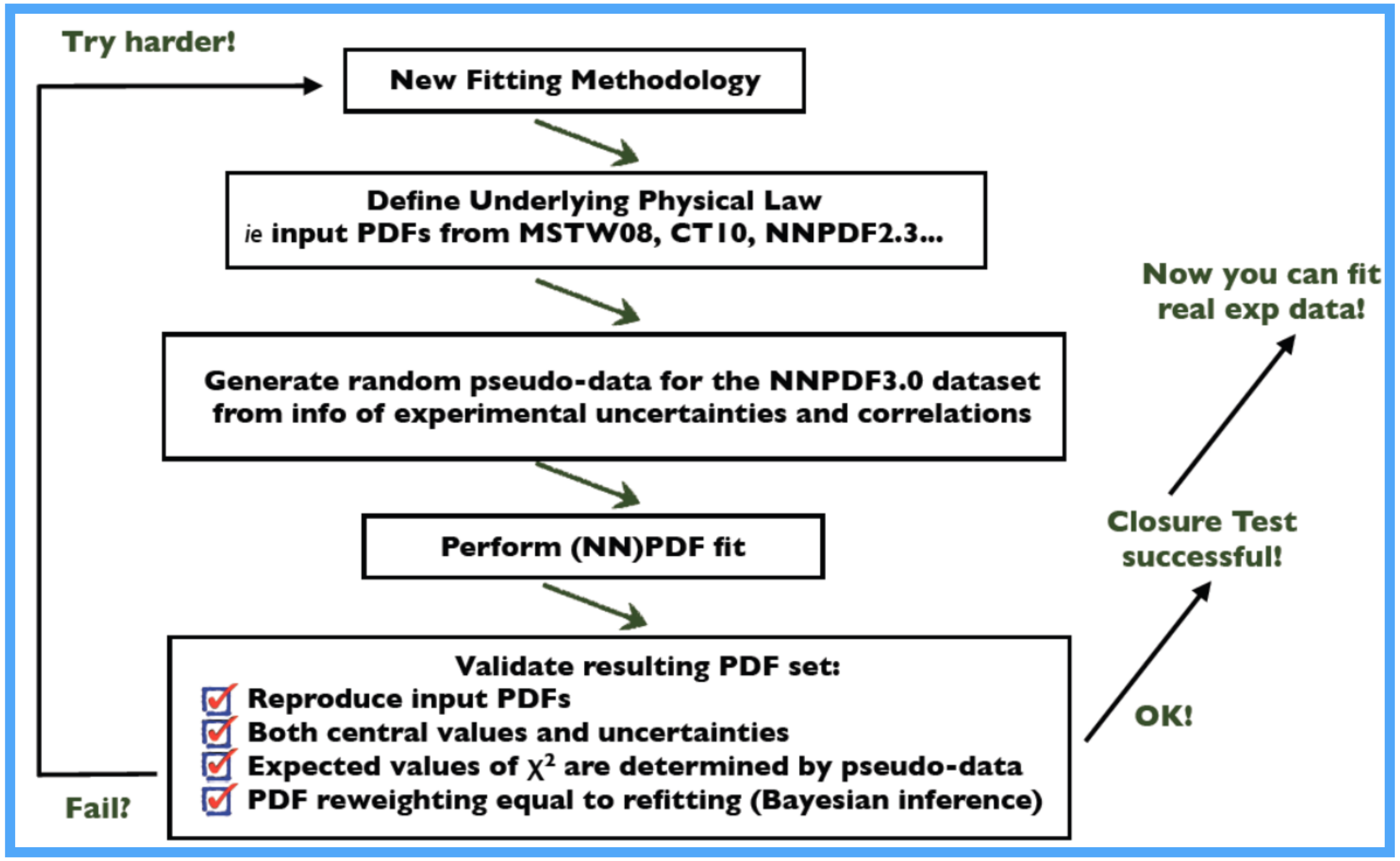


- NNPDF2.3 -> NNPDF3.0: included many new data (LHC and combined HERA) & change in fitting methodology (genetic algorithm and stopping criterion)
- Main changes in the gluon are due to the change in methodology
- How to make sure that we have a "perfect" methodology?

Statistical validation

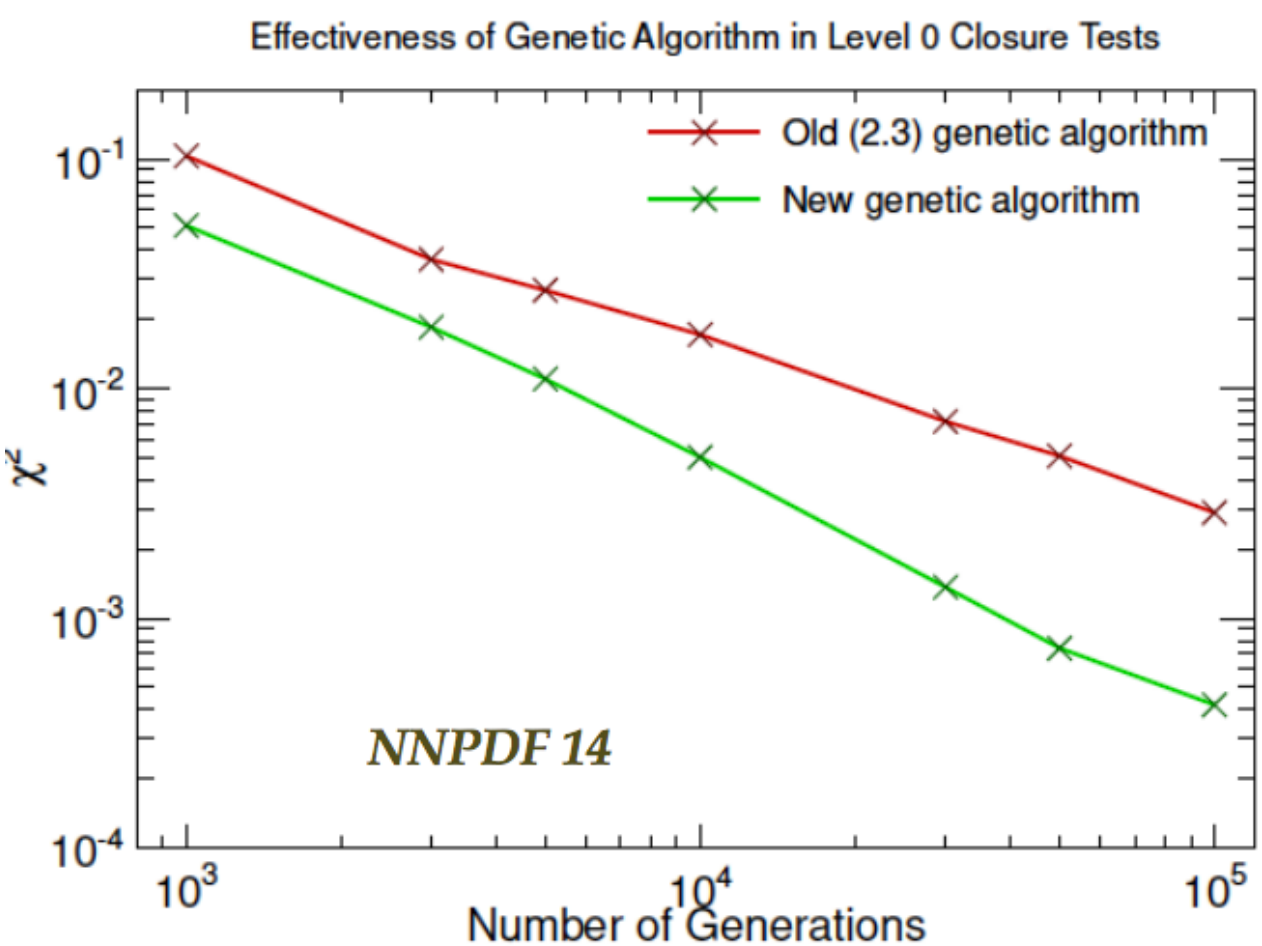
Closure test: the ultimate check of PDF fitting

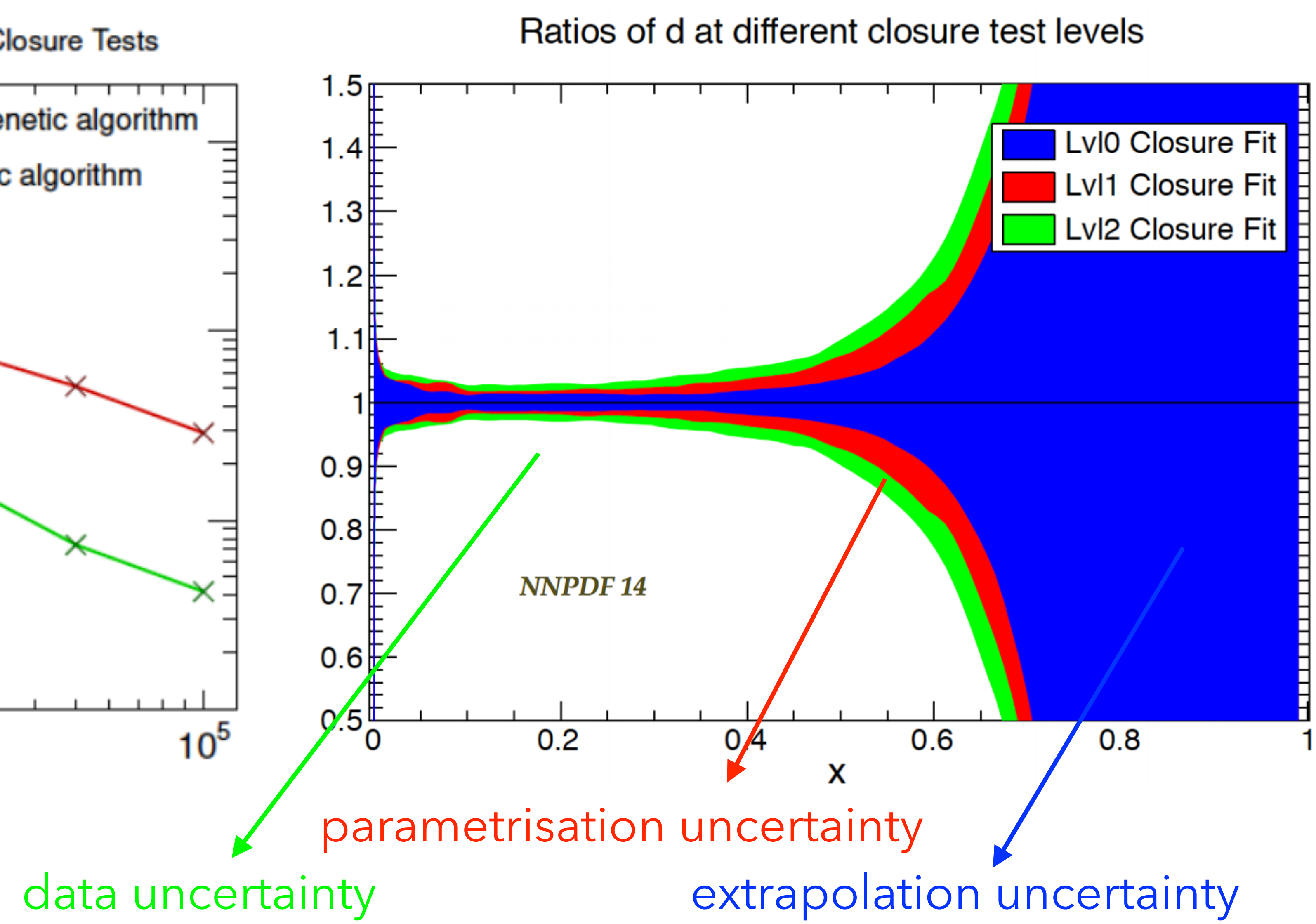
- Assume PDFs known: generate fake experimental data with them and th predictions
- Can decide data uncertainty (zero uncertainty - level 1-2, or as in real data - level 3)
- Fit PDFs to fake data
- Check whether fit reproduces the underlying "truth"
  - ➡ Check whether true values are gaussianly distributed about the fit
  - ➡ Check whether uncertainties are faithful
  - ➡ Trace different sources of uncertainty



Statistical validation

- **Level-0:** if pseudo-data are identical to the input theory, then agreement with theory should be arbitrarily good, i.e. $\chi^2 \rightarrow 0$ but PDF uncertainty $\rightarrow 0$ only in the region where there are enough data
- **Level-1:** add uncertainty to pseudo data equal to actually experimental uncertainties: replicas fit same data over and over again, then $\chi^2 \rightarrow 1$ and test equivalent minima (parametrisation Δ)
- **Level-2:** generate Monte Carlo replicas of pseudo-data with fluctuations, then $\chi^2 \rightarrow 2$ (data Δ)

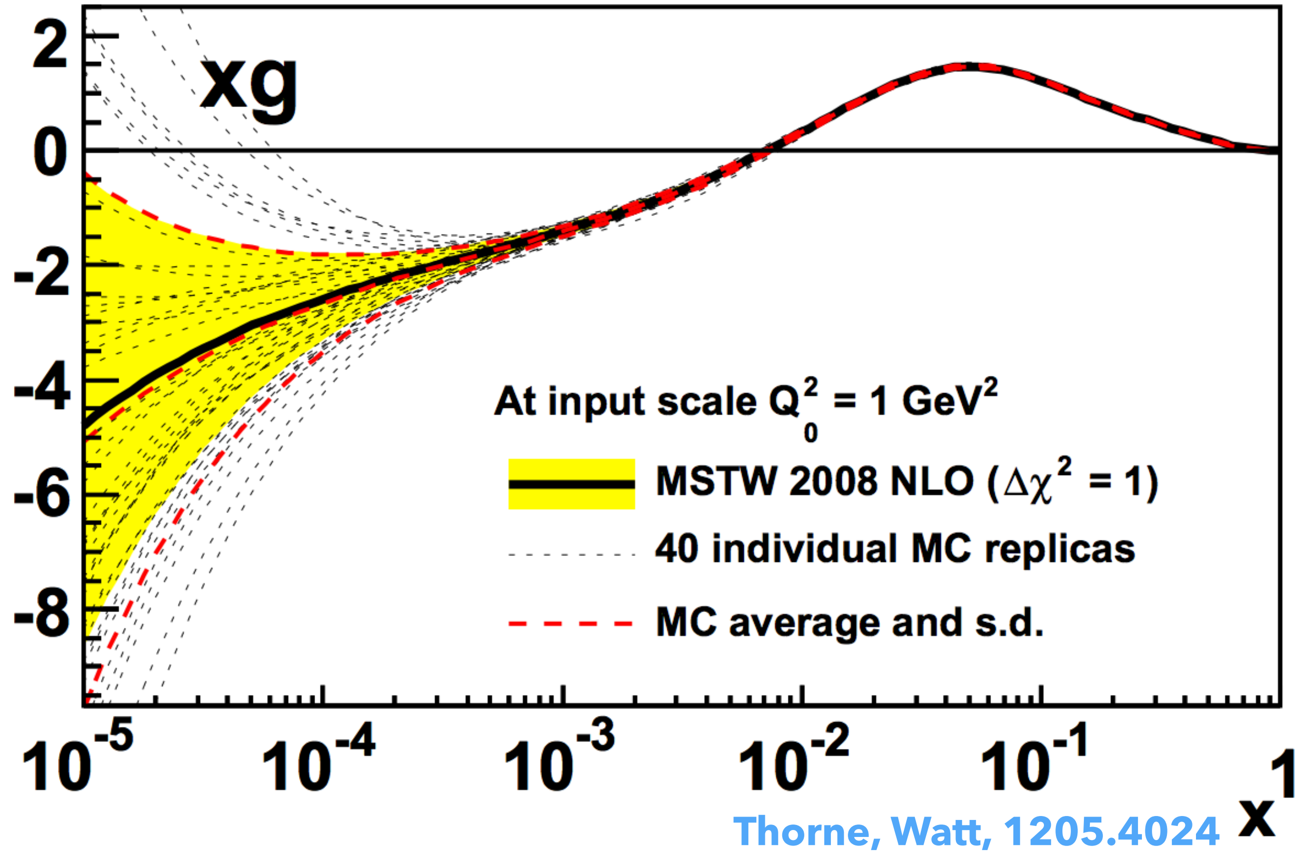




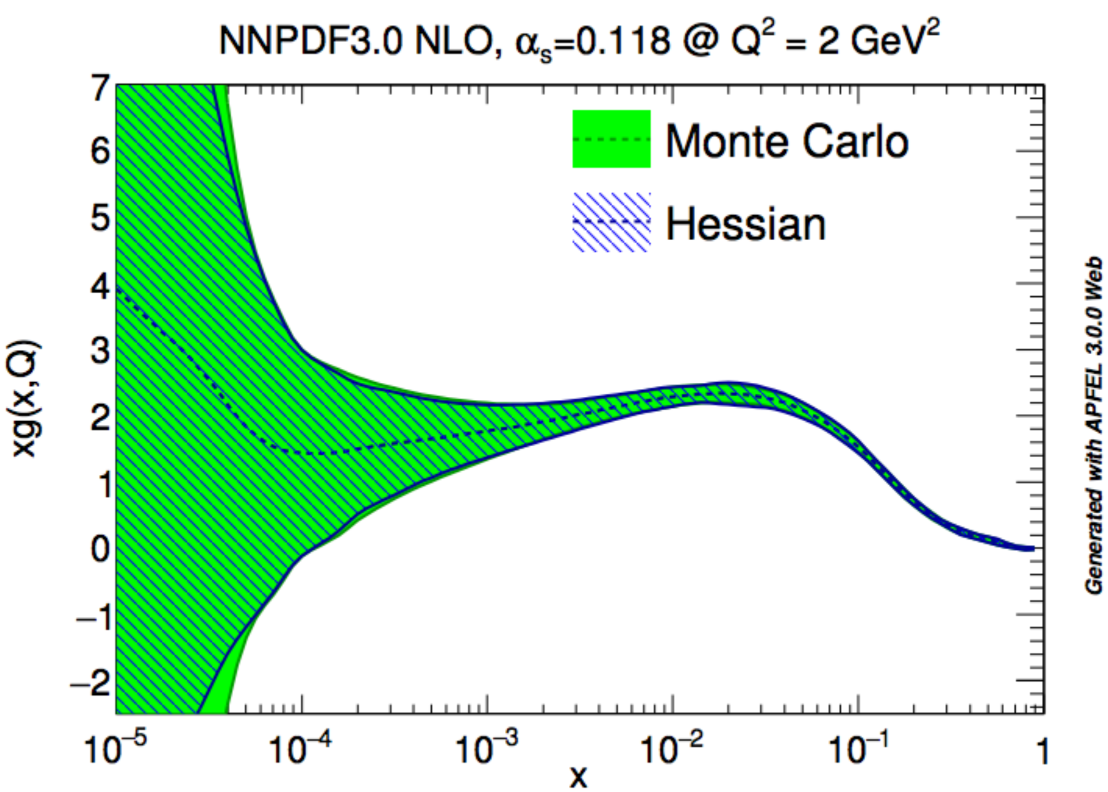
3 Δs comparable in data region

Hessian ⟺ Monte Carlo

- To convert Hessian into Monte Carlo, generate multi-gaussian replicas in the fitted parameters space
- Accurate when the number of replicas similar to that that reproduces the data



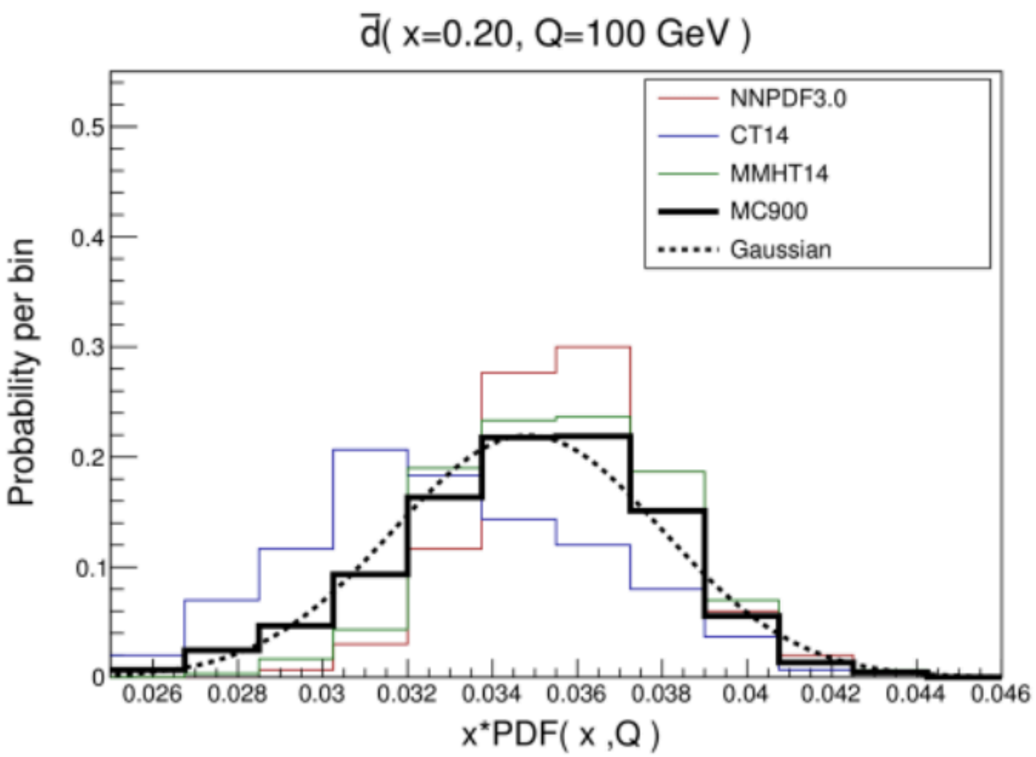
Thorne, Watt, 1205.4024



Carrazza et al 1505.06736

- To convert Monte Carlo into Hessian, sample the replicas f(x) at discrete set of points and construct the ensuing covariance matrix
- Eigenvectors of the covariance matrix as a basis in the vector space spanned by the replicas by the singular-value decomposition
- Number of dominant eigenvectors similar to numbers of replicas for accurate representation

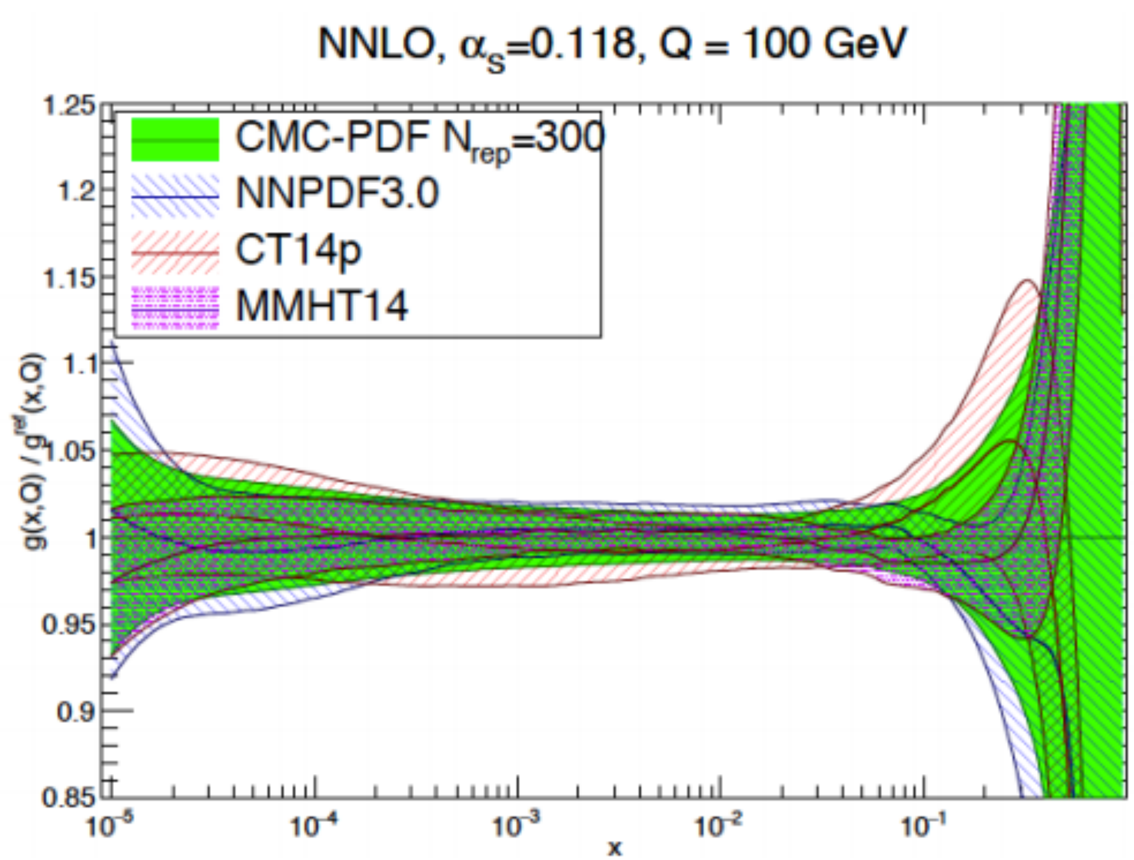
Hessian ⟺ Monte Carlo



- Using Monte Carlo conversion of Hessian sets, can combine different PDF sets, combining MC replicas into a single set
- Useful for conservative estimate
- Combined set approximatively Gaussian

PDF4LHC15 recipe

- Monte Carlo combination of most recent global PDF sets **[Forte, Watt]**
- Each replica receives the same weight: uncertainty smaller than in the envelope, as in the latter outliers are given a larger weight
- New compression studies: N=40 replicas are virtually identical to the original 300 replicas from the point of view of correlation, standard deviation, observables **[Carrazza et al.]**



Statistics and methodology summary

➡ PDF determination: Hessian Method
- Simple linear error propagation
- Tolerance required for realistic uncertainties
- Parametrisation bias possible

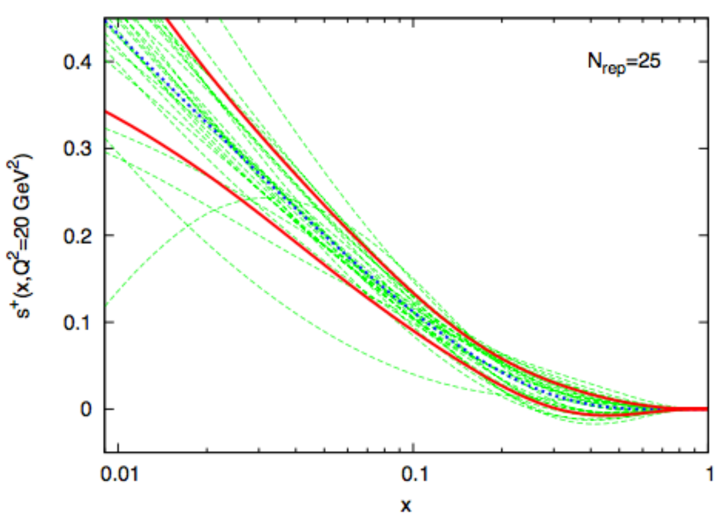
➡ PDF determination: Monte Carlo method
- Two-step procedure: data MC -> PDF MC
- Very general parametrisation allowed
- Need optimal fit determination method (cross-validation)

➡ PDF representation: Hessian vs Monte Carlo
- Conversion possible either way
- Compression method available either way
- MC very flexible, Hessian very efficient

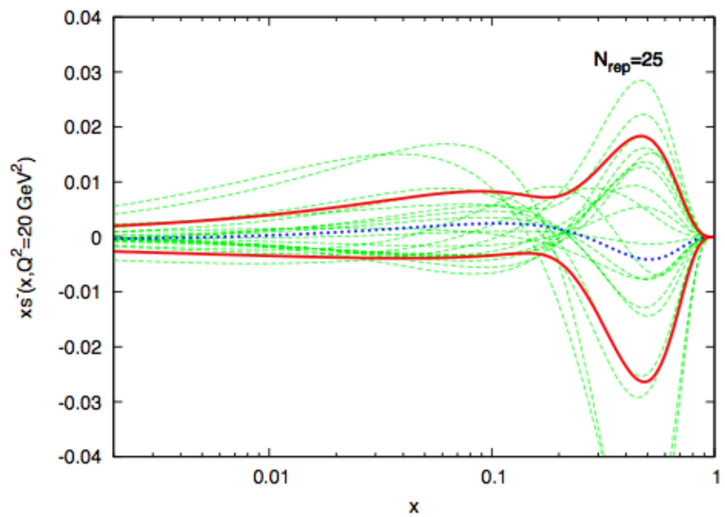
➡ PDF validation: the closure test
- Performed in the MC approach (so far)
- Interpolation and functional uncertainties significant

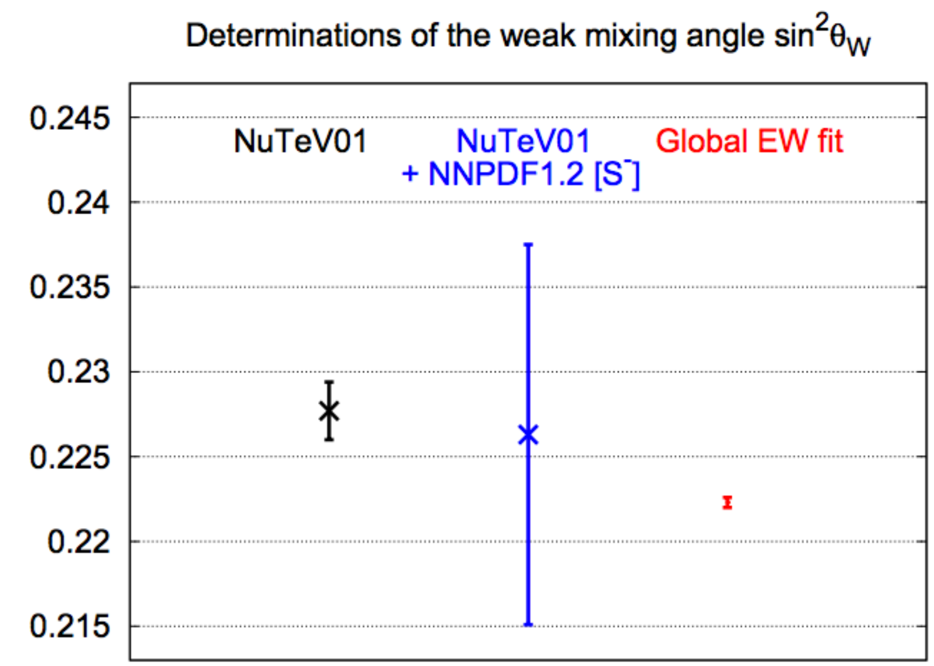
E.g. the NuTeV anomaly

Total strangeness ↓



Strange valence ↓





EW fit

$\sin^2\theta_W = 0.2223 \pm 0.0002$

NuTeV

$\sin^2\theta_W = 0.2276 \pm 0.0014$

$$\sin^2\theta_W\Big|_{\rm NuTeV} - \sin^2\theta_W\Big|_{\rm EW} = 0.0053$$

$$[F] = \int_0^1 dx\, x\, f(x, Q^2).$$

- $>3\sigma$ discrepancy between EW fits and NuTeV measurements
- Unbiased parametrisation of strangeness (2010) solved NuTeV anomaly

$$\delta_s \sin^2\theta_W \sim -0.240\,\frac{[S^-]}{[Q^-]}$$

$$\delta_s \sin^2\theta_W = -0.0005 \pm 0.0096^{\rm PDFs} \pm sys$$

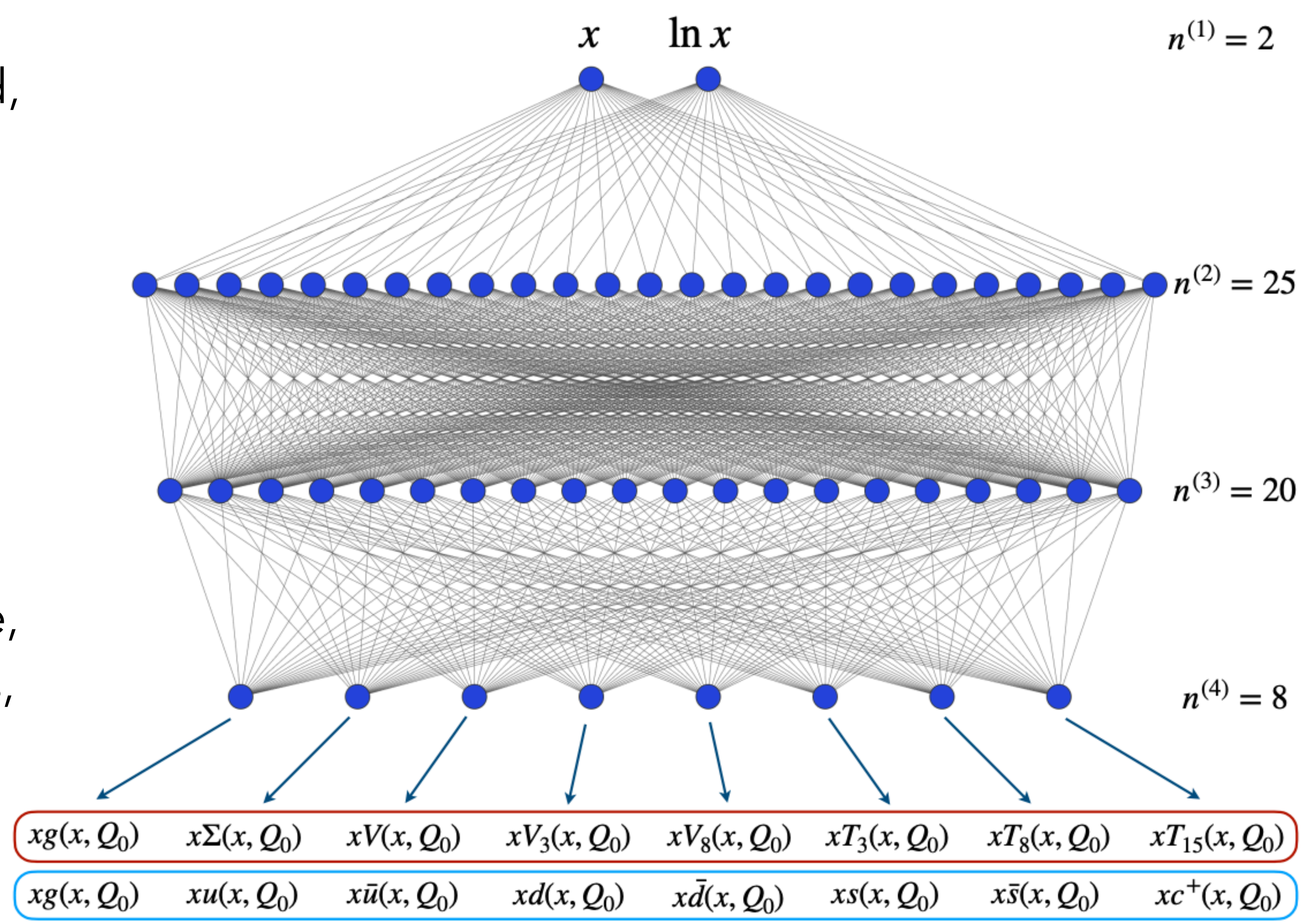
Ball et al, 0906.1958

A deep-learning based fit

- Single neural network to parametrise 8 independent PDF combinations (g, u, d, s, u~, d~, s~, c=c~)
- New optimisation strategy based on gradient descent rather than genetic algorithm
- Hyper-optimised methodology: scan of the hyper parameter space to find optimal minimisation settings (optimiser, initialiser, stopping patience, number of layers, learning rate, epochs, activation function) by minimising χ^2_{val}
  [Carrazza et al, Eur.Phys.J.C 79 (2019) 8, 676]
- Statistical validation of PDF uncertainties via closure tests (data region)

  [Del Debbio et al, Eur.Phys.J.C 82 (2022) 4, 330]

  and future test (extrapolation region)
  [J. Cruz-Martinez et al, Acta Phys.Polon.B 52 (2021) 243]



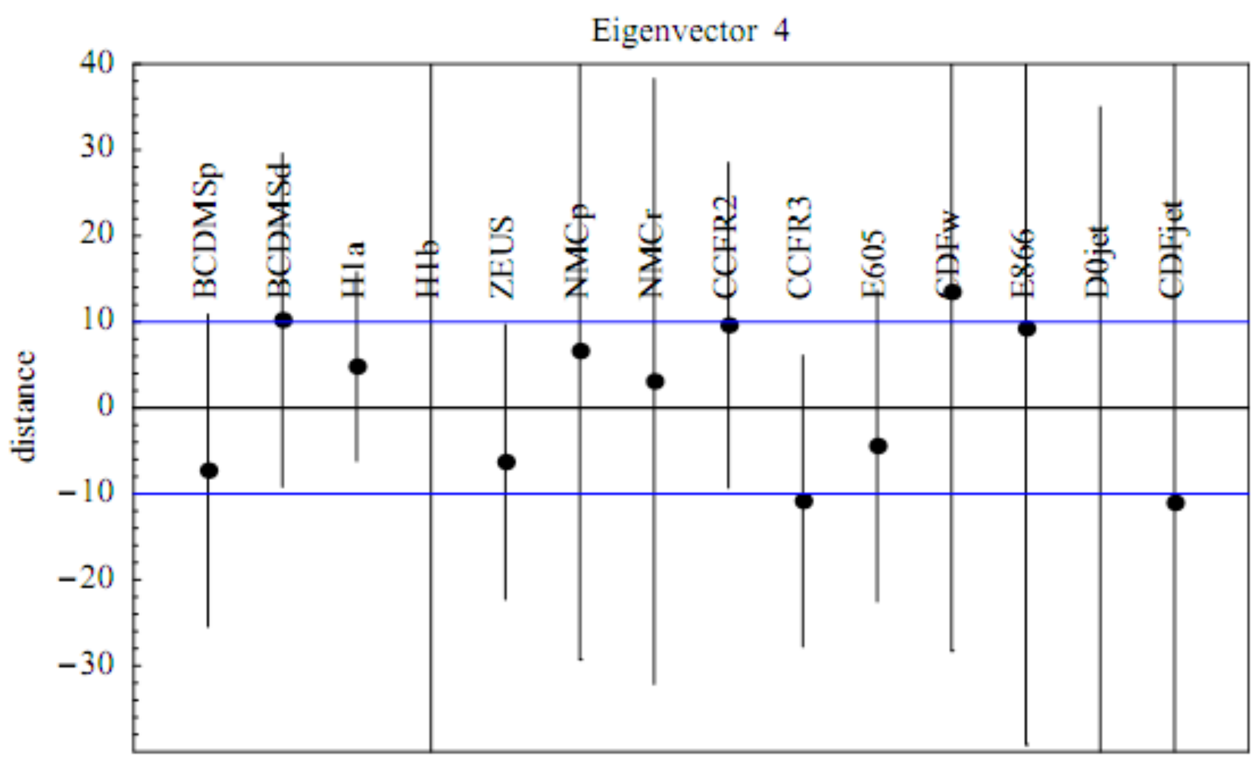
NNPDF4.0, arXiv: 2109.02653

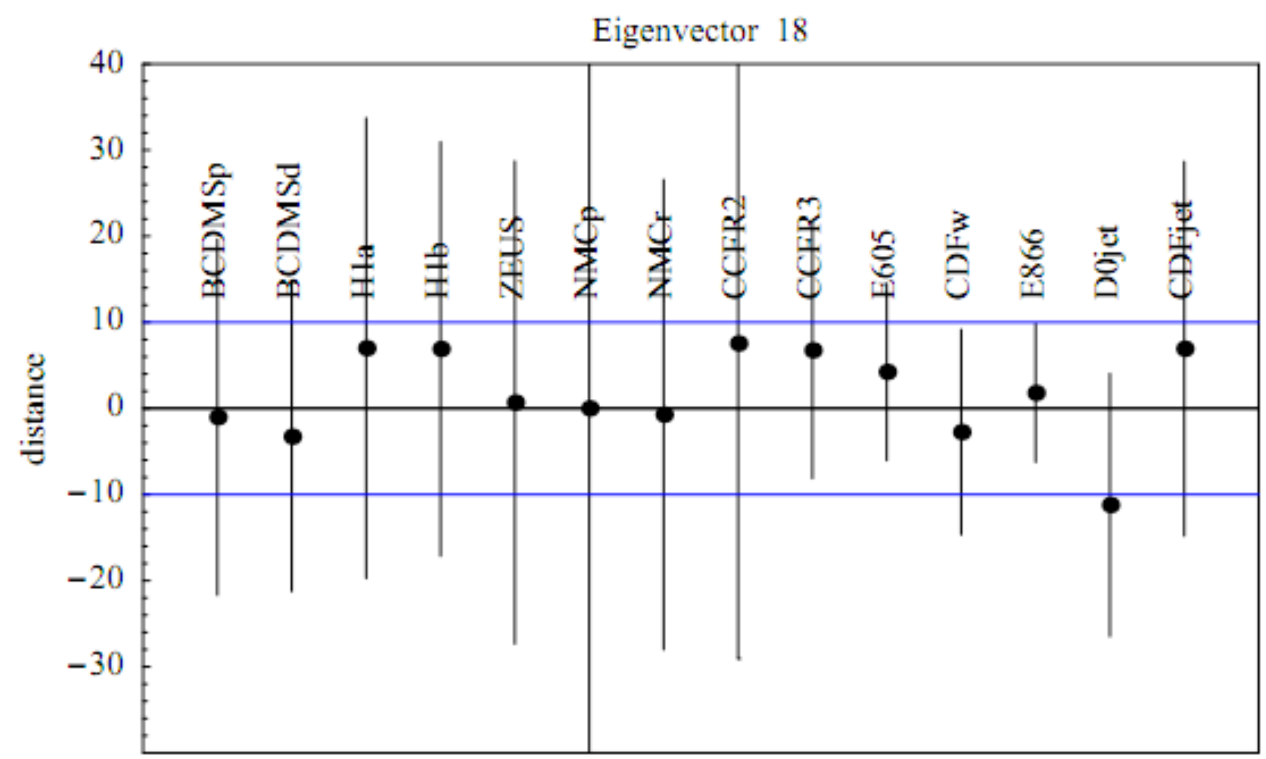
Hessian method

CTEQ6 tolerance criterion

- Acceptable values of PDF parameters must agree at ~ 90% C.L. with all experiments included in the fit, for a plausible range of assumptions about the PDF parametrisation, scale dependence, systematic uncertainty

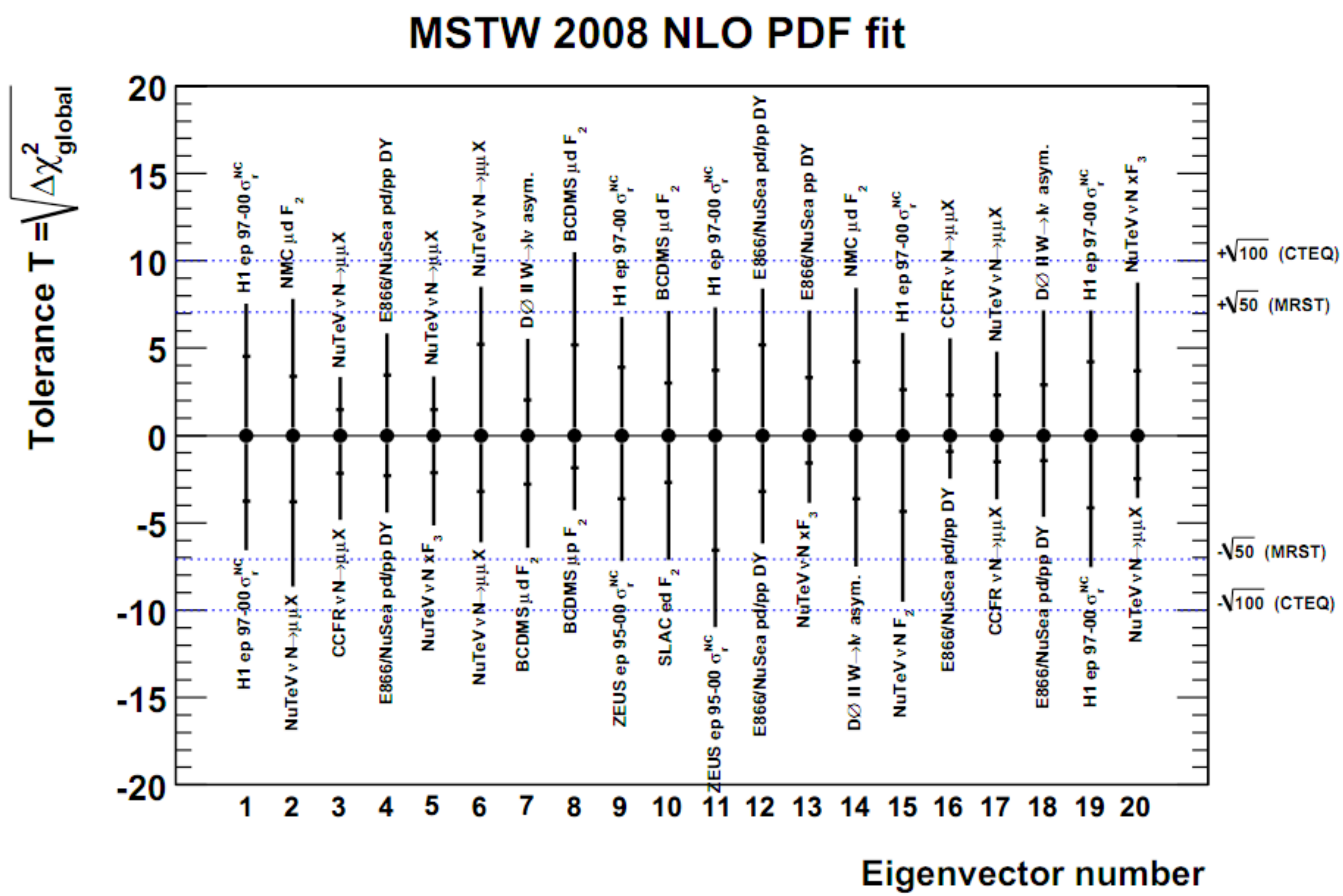
- Can be crudely approximated by assuming T ~ 10 for all PDF parameters





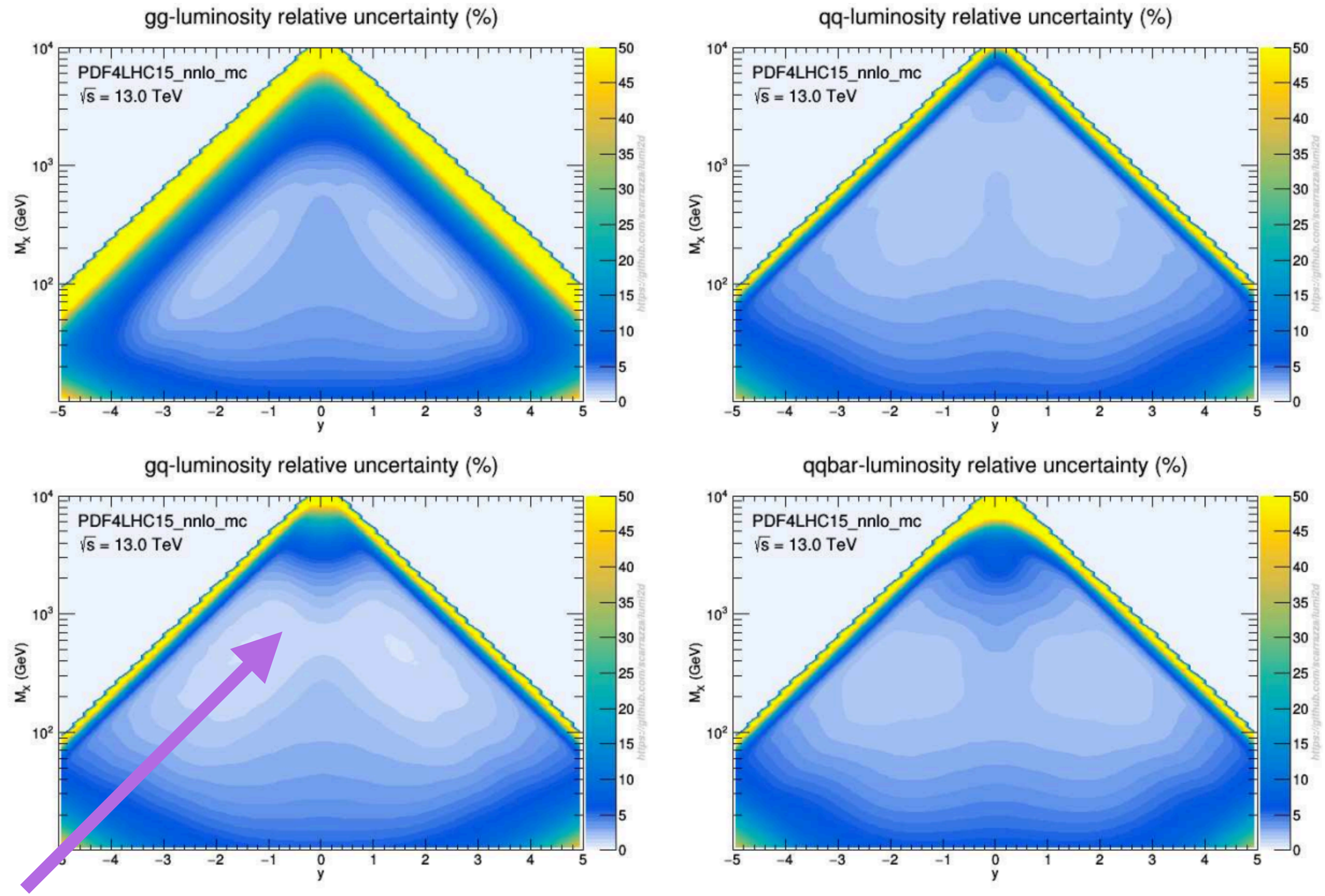
Hessian method

MSTW08 tolerance criterion



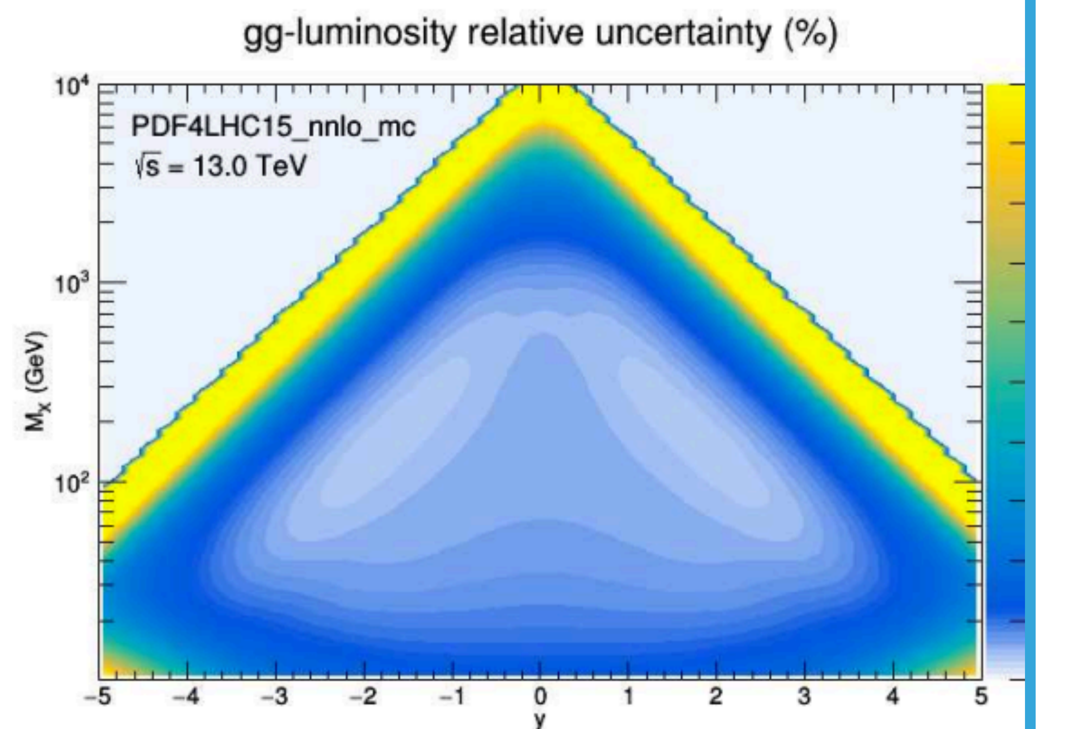
A dynamical tolerance, which varies according to the considered parameter

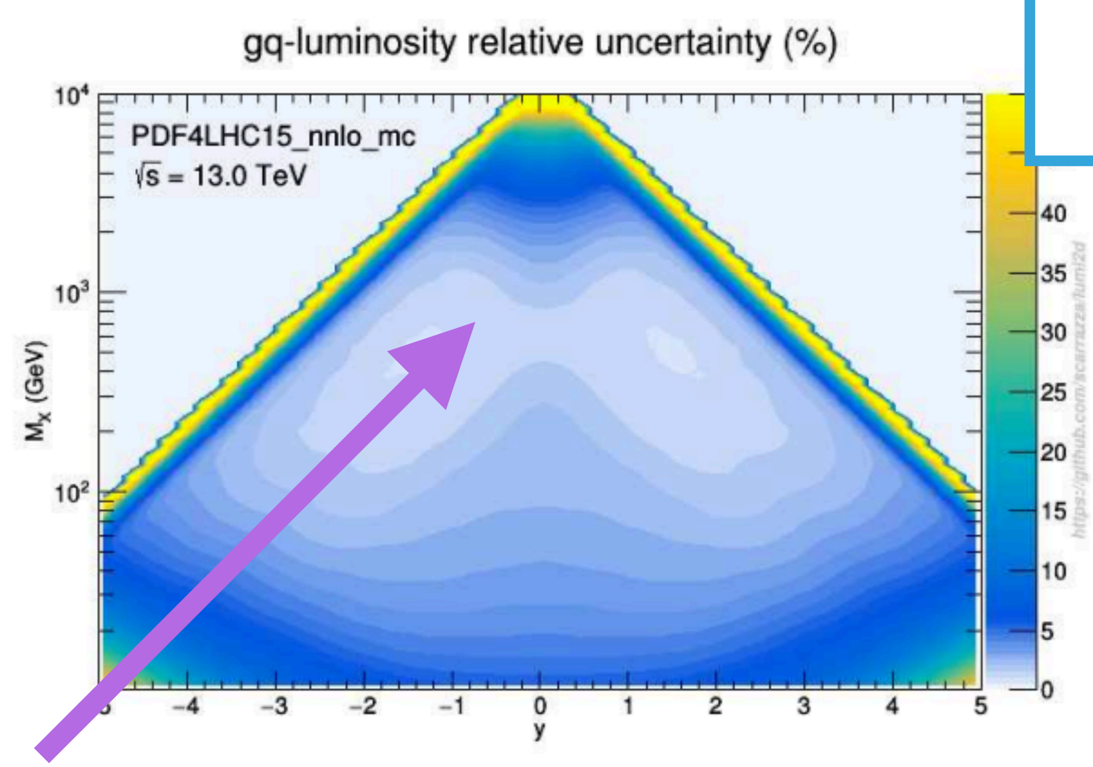
The precision frontier

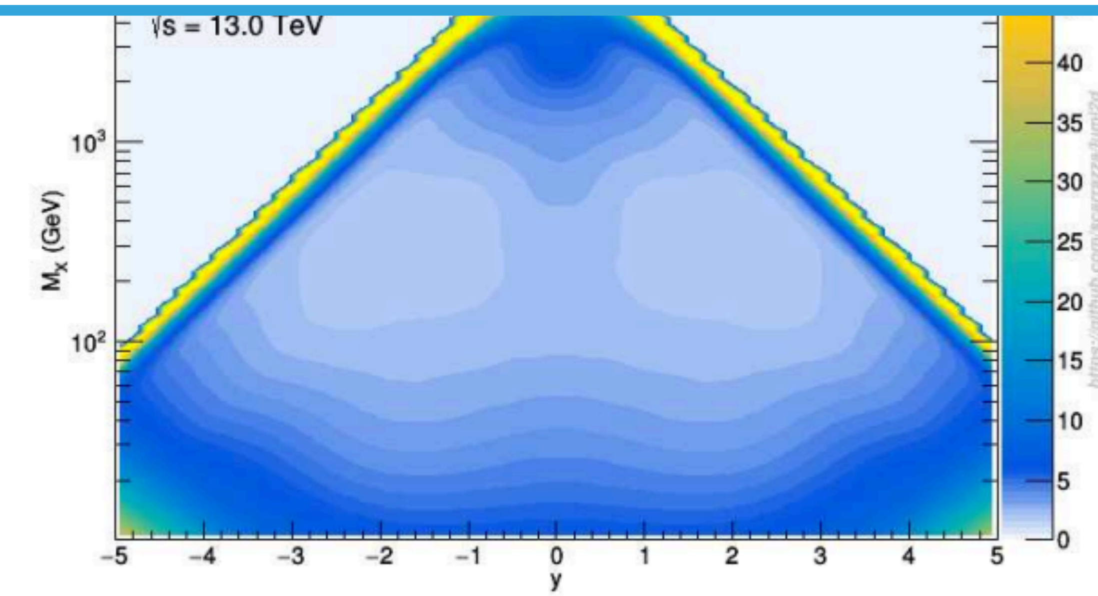


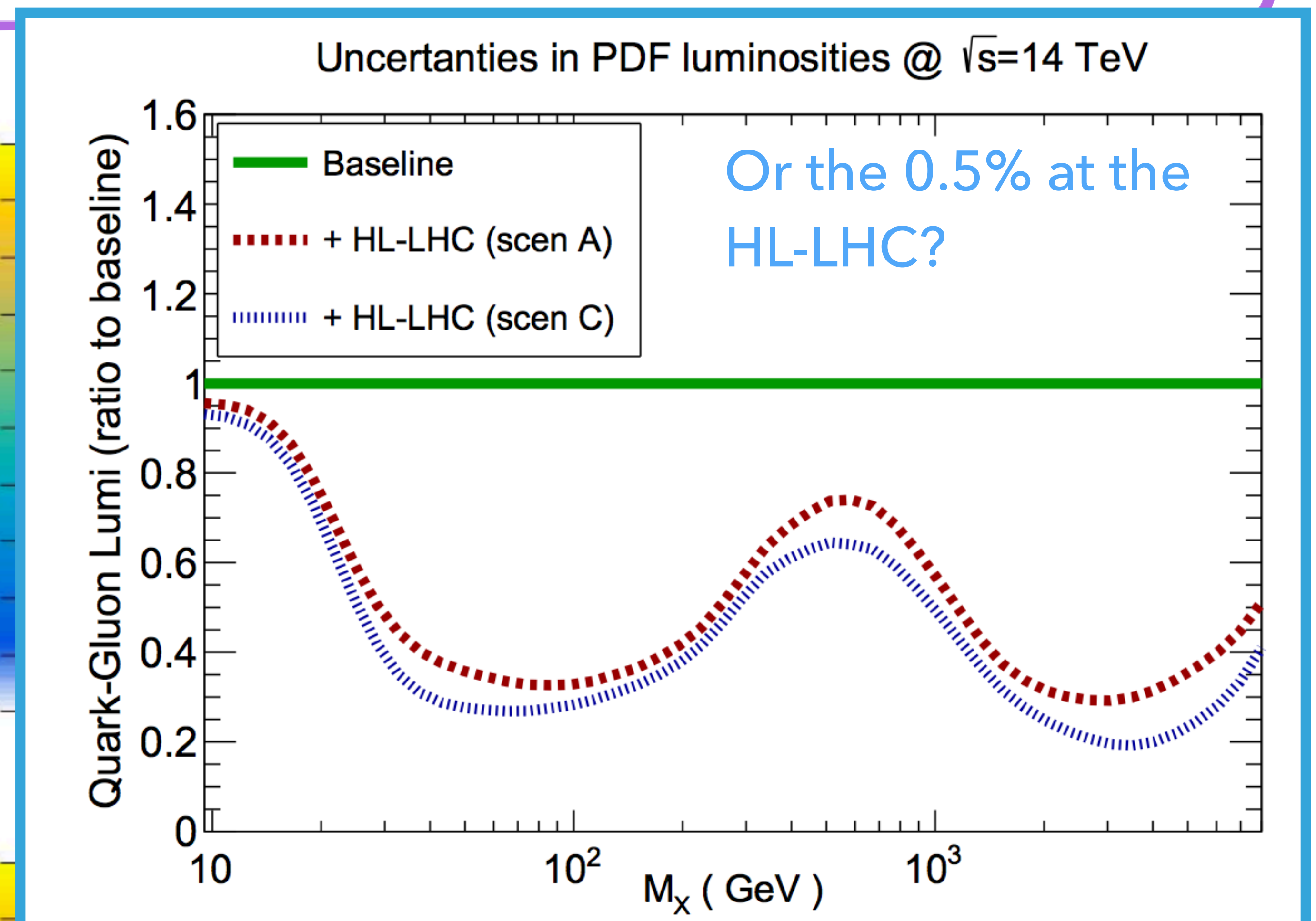
Can we trust 1% accuracy?

The precision frontier







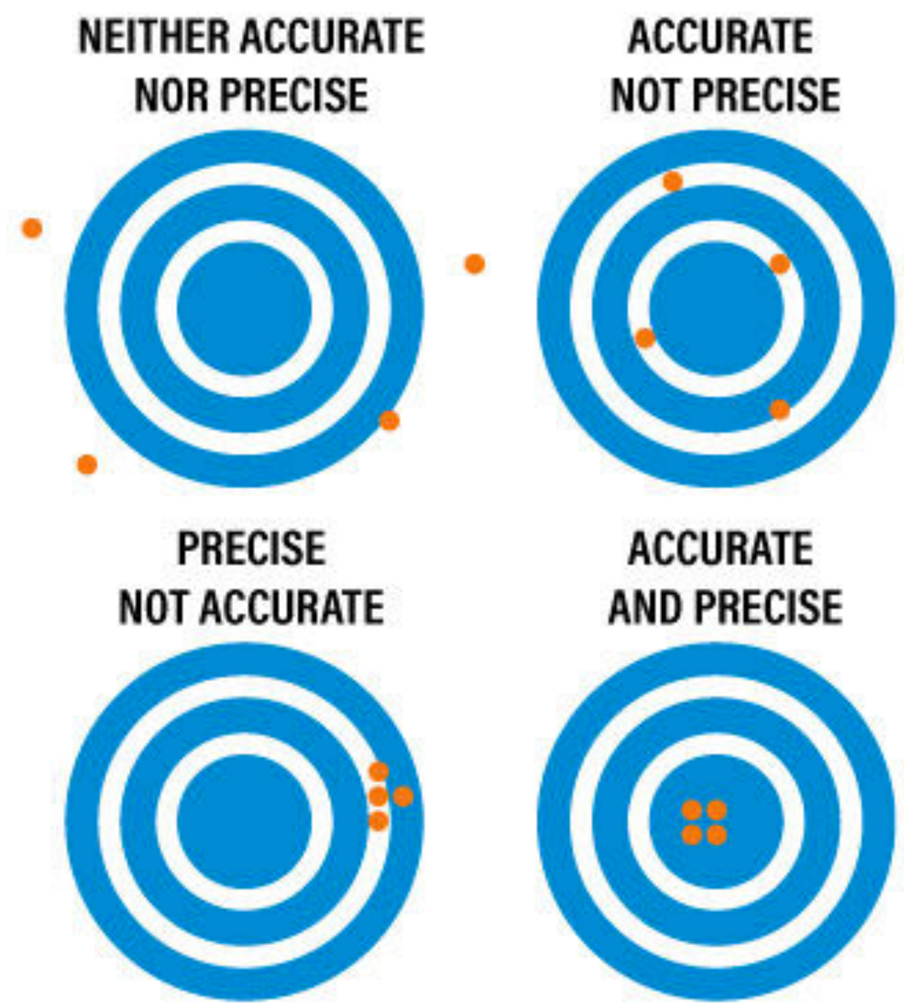


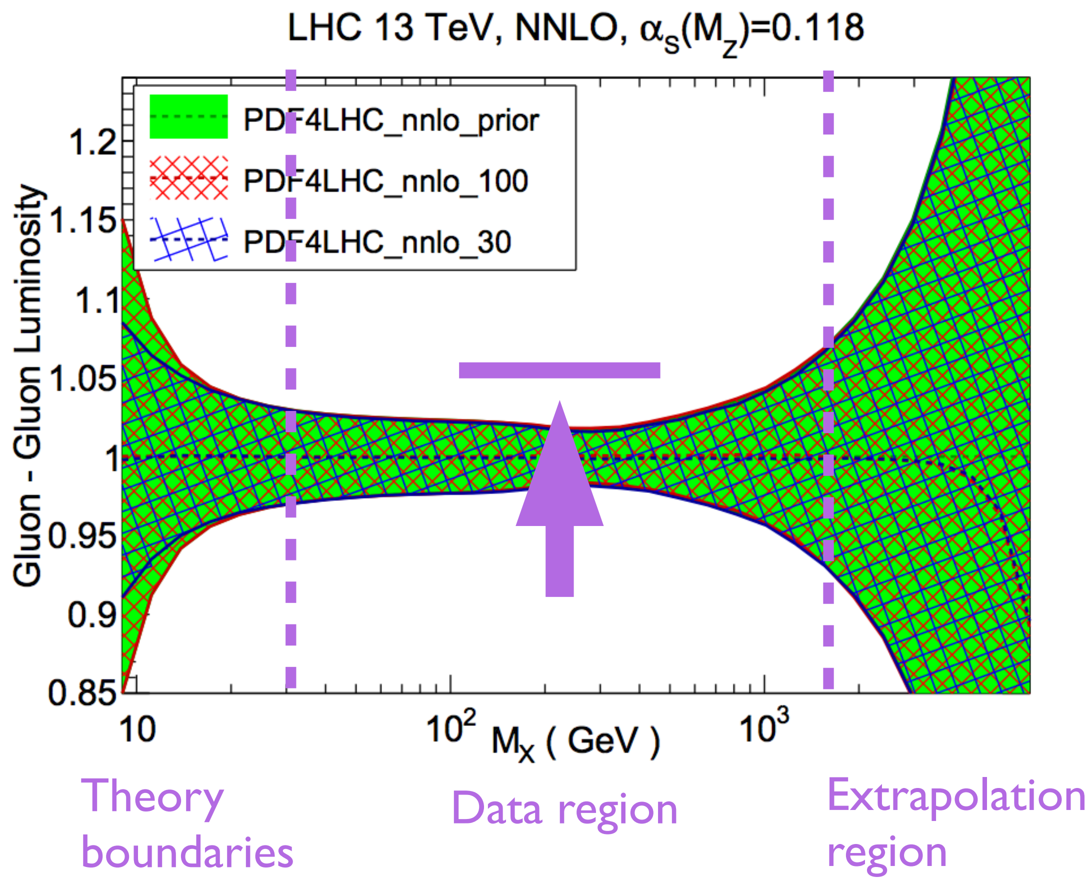
Or the 0.5% at the HL-LHC?

Can we trust 1% accuracy?

Theory uncertainties

On top of benchmarking different PDF sets, each set must deal with inconsistencies in updated determinations.





- Inconsistent data → Tolerance/ Statistical estimators
- Updated parametrization
- Differences in fitting methodology/minimisation? → Closure Test
- Changes in theory? → MHOU/New Physics