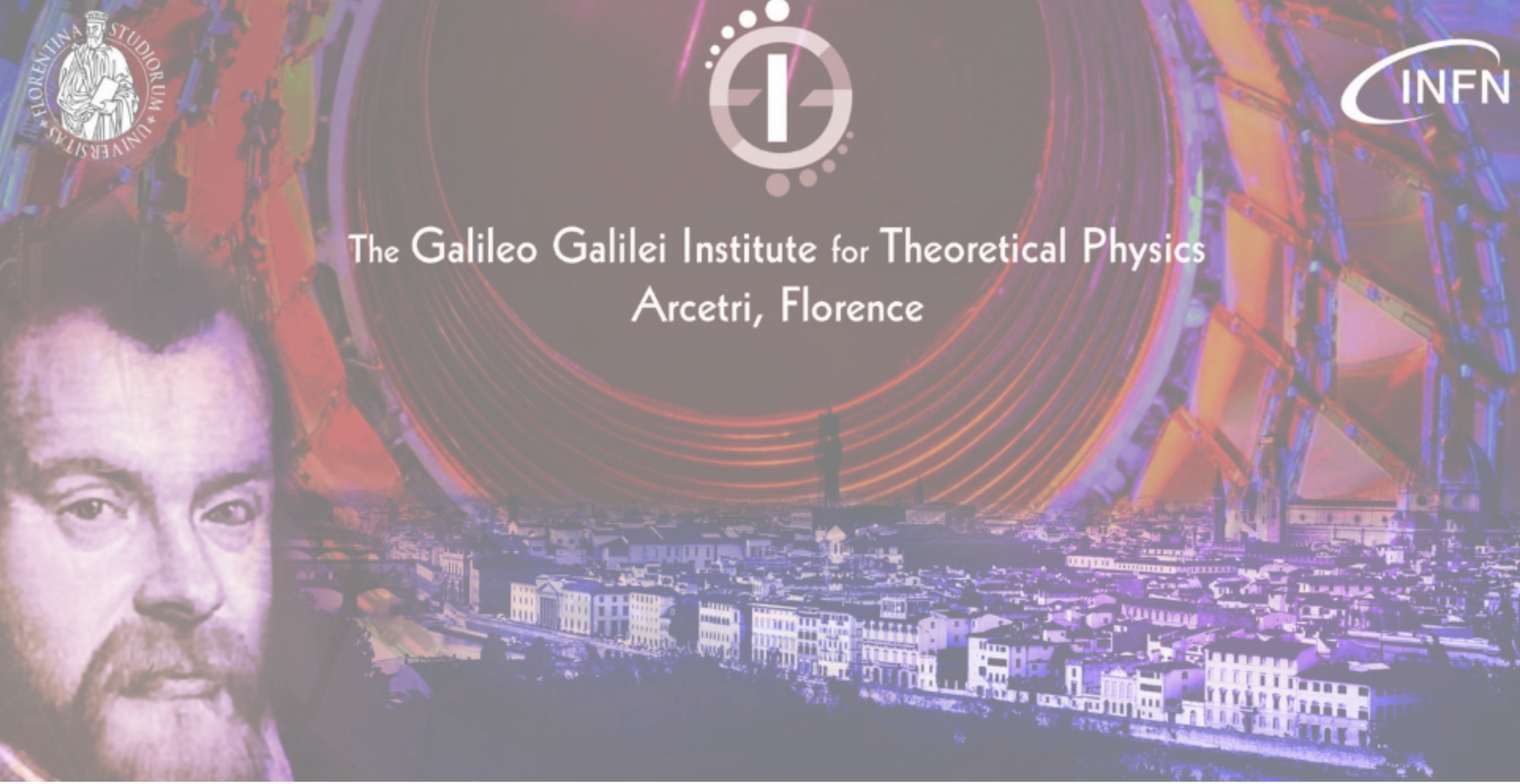


THE STRUCTURE OF THE PROTON (I)

Maria Ubiali, University of Cambridge

29th September 2023

GGI, Florence - Theoretical challenges in the precision era of the LHC

References

- G. Ridolfi "Notes on deep-inelastic scattering and the Parton model"
- Ellis, Stirling and Webber "QCD and collider physics"
- Dissertori, Knowles, Schmelling "Quantum Chromo Dynamics"
- "Proton structure at the precision frontier" Snowmass 2021 Whitepaper arXiv:2203.13923
- Kovarik, Nadolsky, Soper, arXiv:1905.06957
- Gao, Harland-Lang, Rojo - *Phys.Rept. 742 (2018) 1-121*
- Forte, Watt - *Ann.Rev.Nucl.Part.Sci. 63 (2013)*
- Perez, Rizvi, *Rep.Prog.Phys. 76 (2013) 046201.*
- Accardi, et al., *Eur. Phys. J. C76 (8) (2016) 471*
- http://pdg.lbl.gov/2021/reviews/rpp2021-rev-structure-functions.pdf

List of references complemented by specific references during the lectures

Goal of the lectures

➡ Give an overview on our understanding on the structure of the proton: from Feynman parton model to modern QCD picture

➡ Introduce basic concepts and techniques behind PDF global fits

➡ Wealth of ingredients involved from low to high energy: non-perturbative effects, perturbative QCD, experimental measurements, statistical and mathematical problems, higher order predictions, phenomenology tools, machine learning.

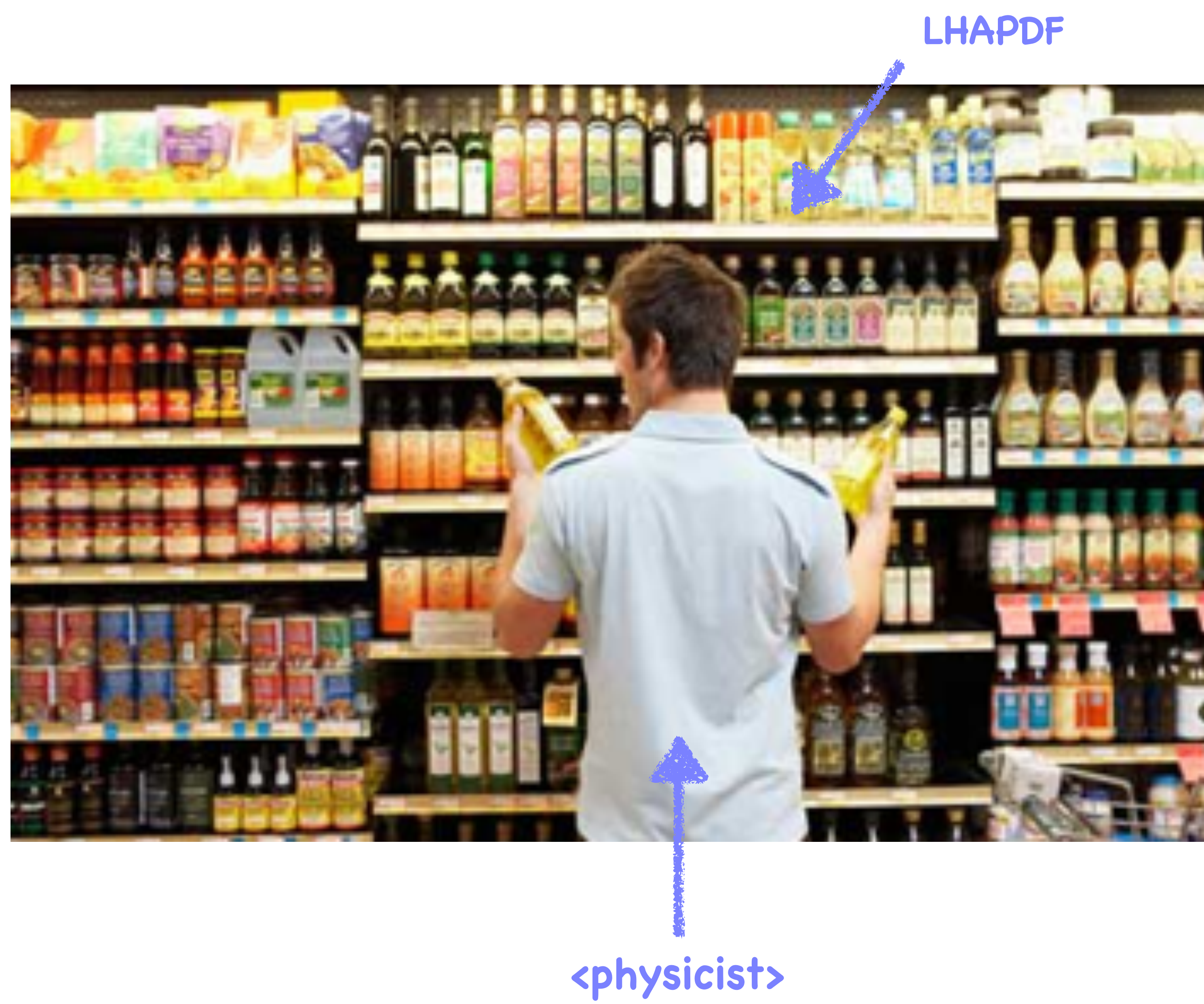
➡ Discuss PDF-related phenomenology at the LHC (mostly), EIC and beyond

➡ Discuss current frontiers and challenges

Disclaimer: these lectures are far from providing a complete picture of the topic. You can find complementary information in excellent lectures on PDFs from W. Giele, G. Salam, A. Martin, P. Nadolsky, S. Forte, D. Stump, W. Melnitchouk, D. Stump, A. Guffanti, J. Rojo ... at recent graduate schools

A trip inside the PDF sausage factory

- What do PDF uncertainties mean? What effects do they include? How reliable are they?
- How do we interpret the difference predictions using different PDF sets?
- Shall we just pick a set out of the PDFs "supermarket" shelf or take the envelope of ALL predictions?



Outline

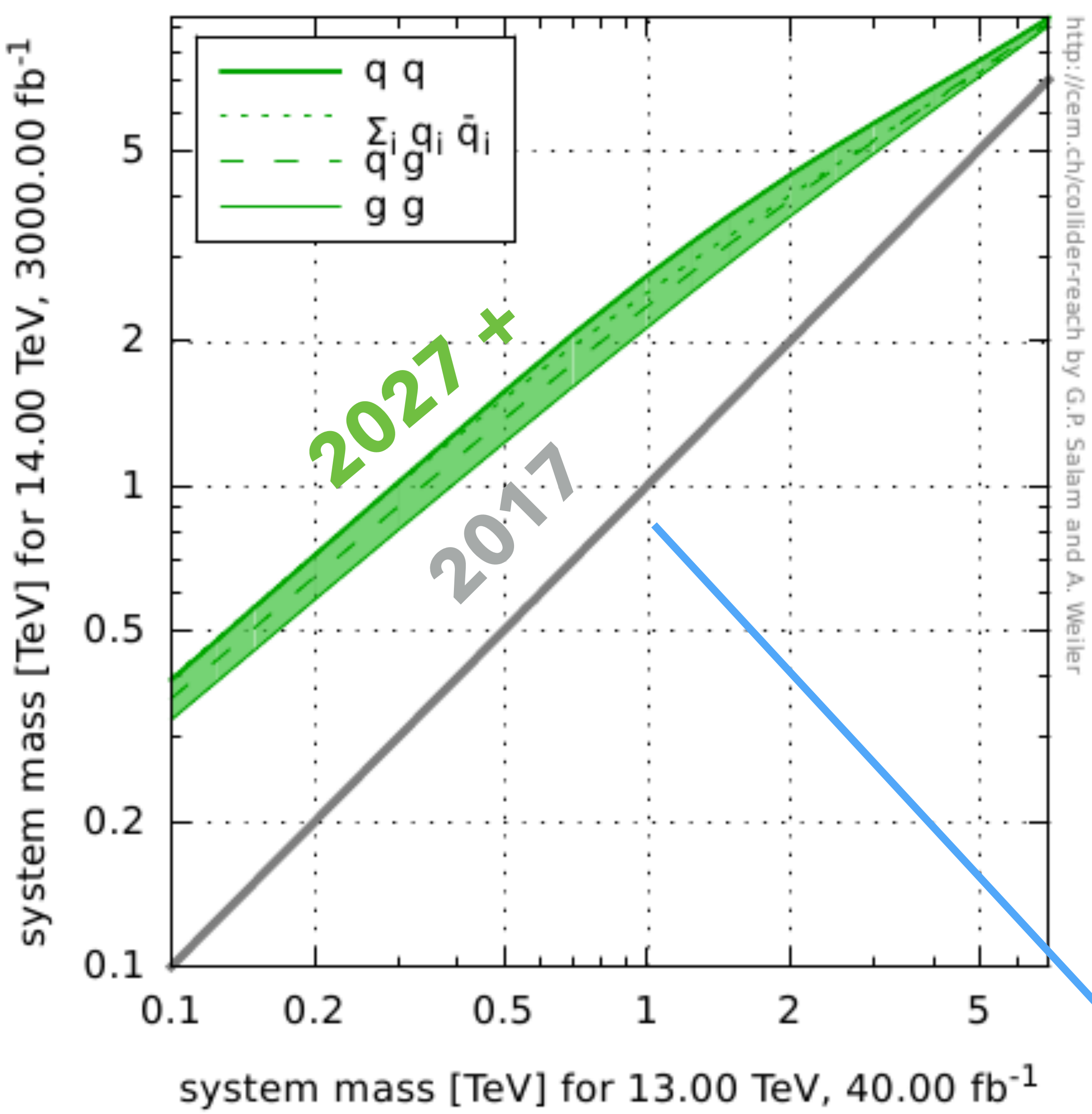
- **Lecture 1 (This morning)**
- Lecture 2 (This afternoon)
    - Methodological aspects
    - Theoretical aspects
    - New frontiers and challenges

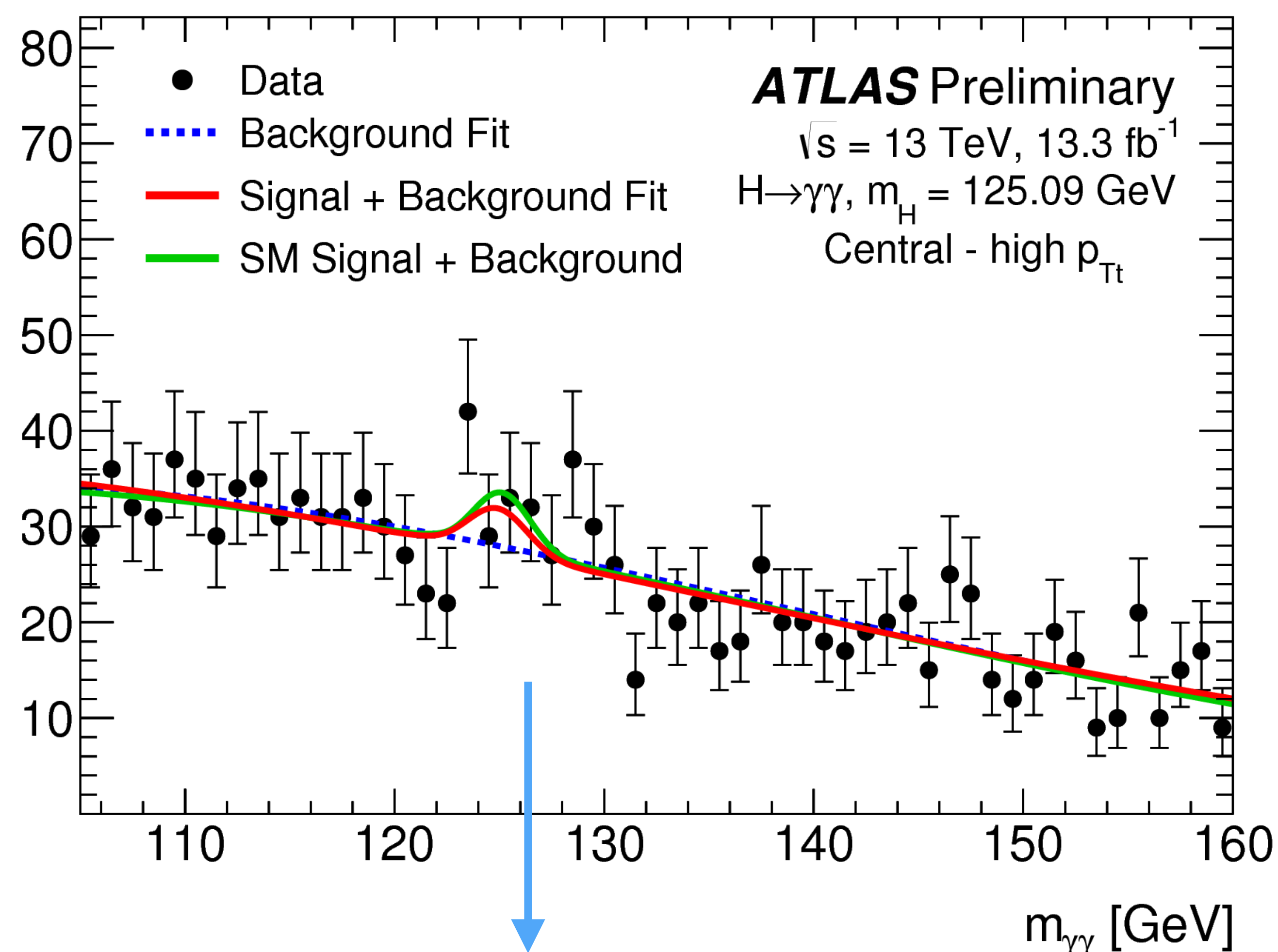
Lecture 1:

- Motivation: the high energy big picture
- Improved QCD parton model and DGLAP evolution equations
- Ingredients of a PDF global analysis
- Experimental input

The LHC precision era

- The Large Hadron Collider at CERN most powerful accelerator ever built
- Extremely successful Run I (7-8 TeV) and great performance at Run II (13-14 TeV)
- As luminosity increases, stronger probe on known processes (Higgs, Flavour anomalies…) & larger mass reach





Higgs rediscovery at Run II

With LHC upgrade in ten years
mass reach increase by ~2

The LHC precision era

- The Large Hadron Collider at CERN most powerful accelerator ever built
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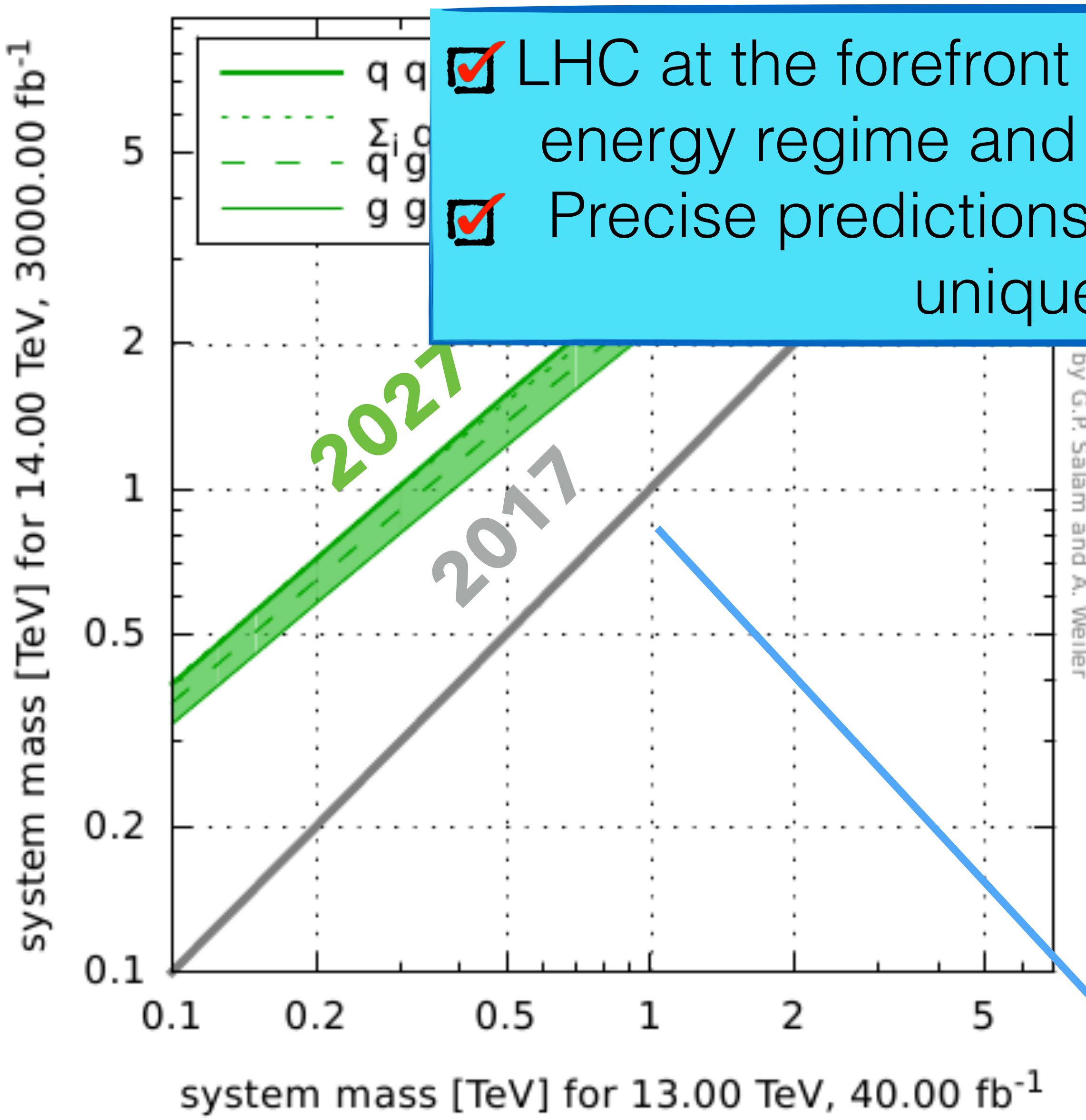
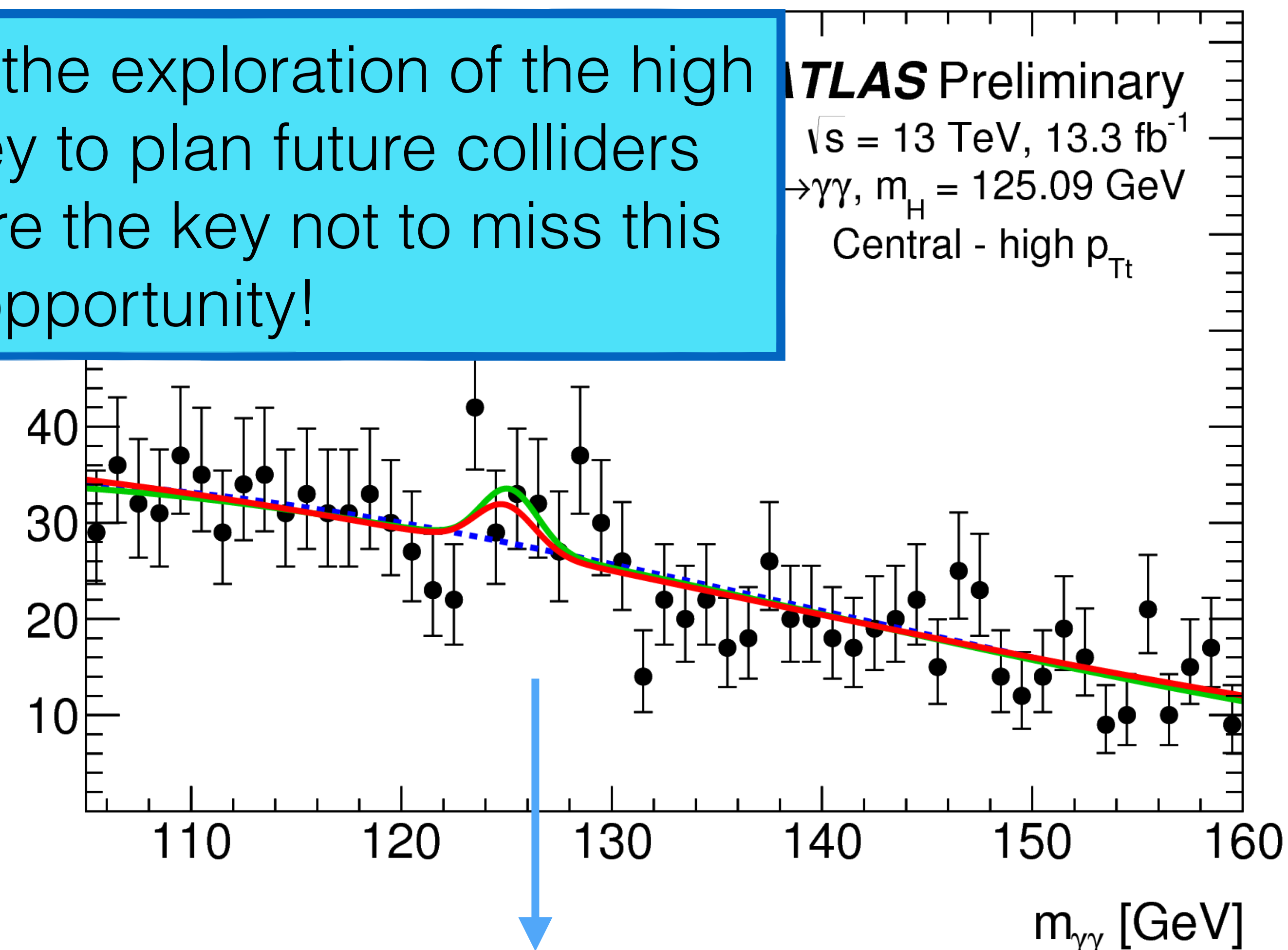


☑ LHC at the forefront of the exploration of the high energy regime and key to plan future colliders

☑ Precise predictions are the key not to miss this unique opportunity!

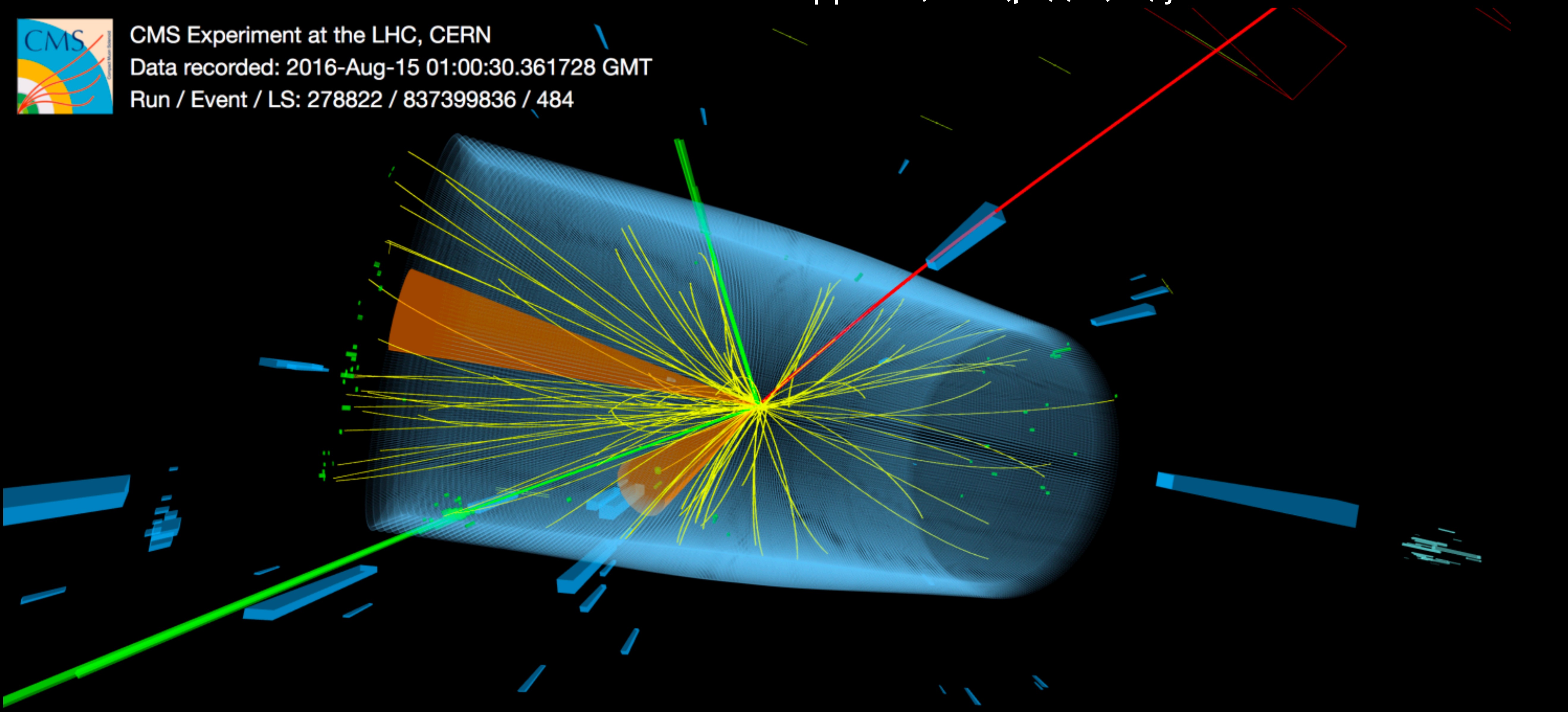


With LHC upgrade in ten years mass reach increase by ~2

Higgs rediscovery at Run II

Collider events: an experimental view

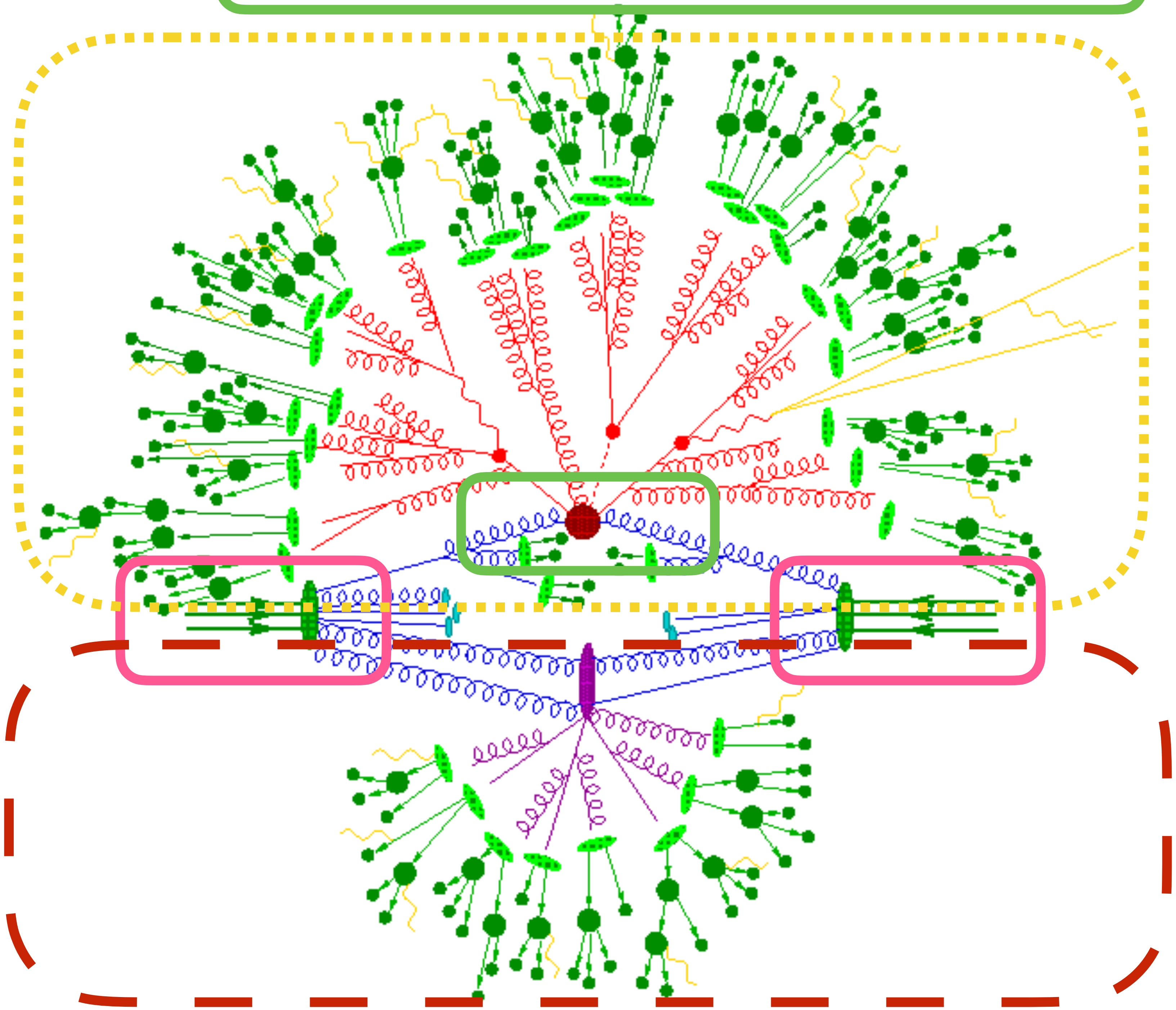
pp -> t (b W ($\mu\nu$)) Z(ee) j



A top + Z candidate collision recorded by CMS. The tZj state is characterised by three leptons (in this case two electrons and one muon), a jet produced from decay of a bottom quark, and a forward jet that is close to the LHC beam direction (Image: CMS/CERN)

Collider events: a theory view

$$\sigma^{\text{th}} = \hat{\sigma}[\mathcal{O}(1) + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^2) + \ldots] \otimes f_1 \otimes f_2 + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$



- Hard scattering of partons or Partonic cross section (Perturbative QCD+EW)
- Parton Distribution Functions
- Parton Showering and Hadronization
- Multiple Parton Interaction, Underlying Events

Collider events: a theory view

- Collinear factorisation: the LHC master formula
- Divide et impera!

$$\sigma^{pp\to ab} = \sum_{i,j=-n_f}^{n_f} \int dz_1\, dz_2\, f_i(z_1,\mu_F)\, f_j(z_2,\mu_F)\, \hat{\sigma}^{ij\to ab}(z_1 z_2 S, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

PDFs: universal functions that parametrise the proton structure

Partonic cross section computed in perturbative QCD + EW corrections

Parton showers, hadronisation

Soft stuff: higher twists, multiple parton interaction, underlying event

Motivation:
why we care about PDF uncertainties

From test of QCD factorisation to precision physics

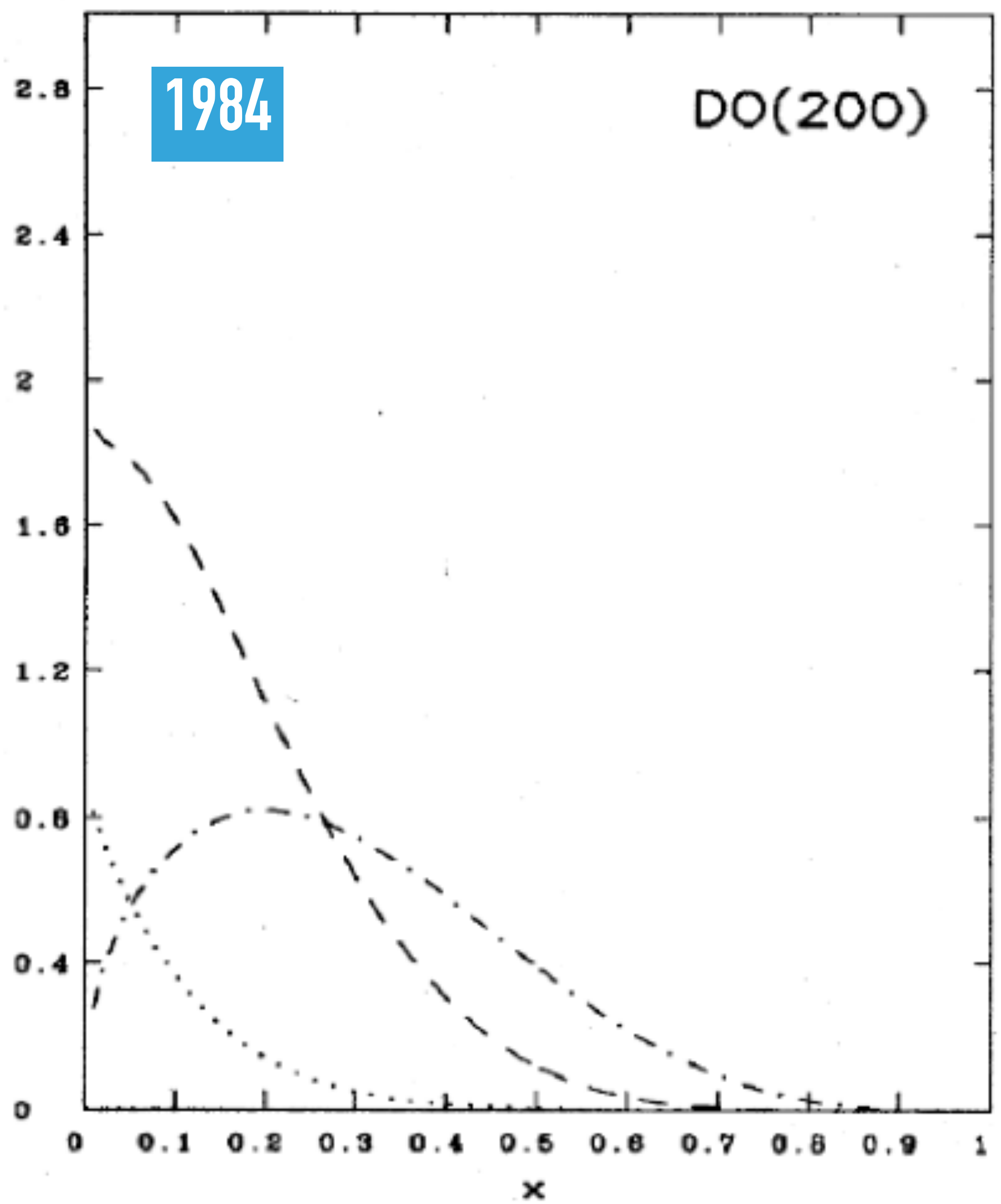
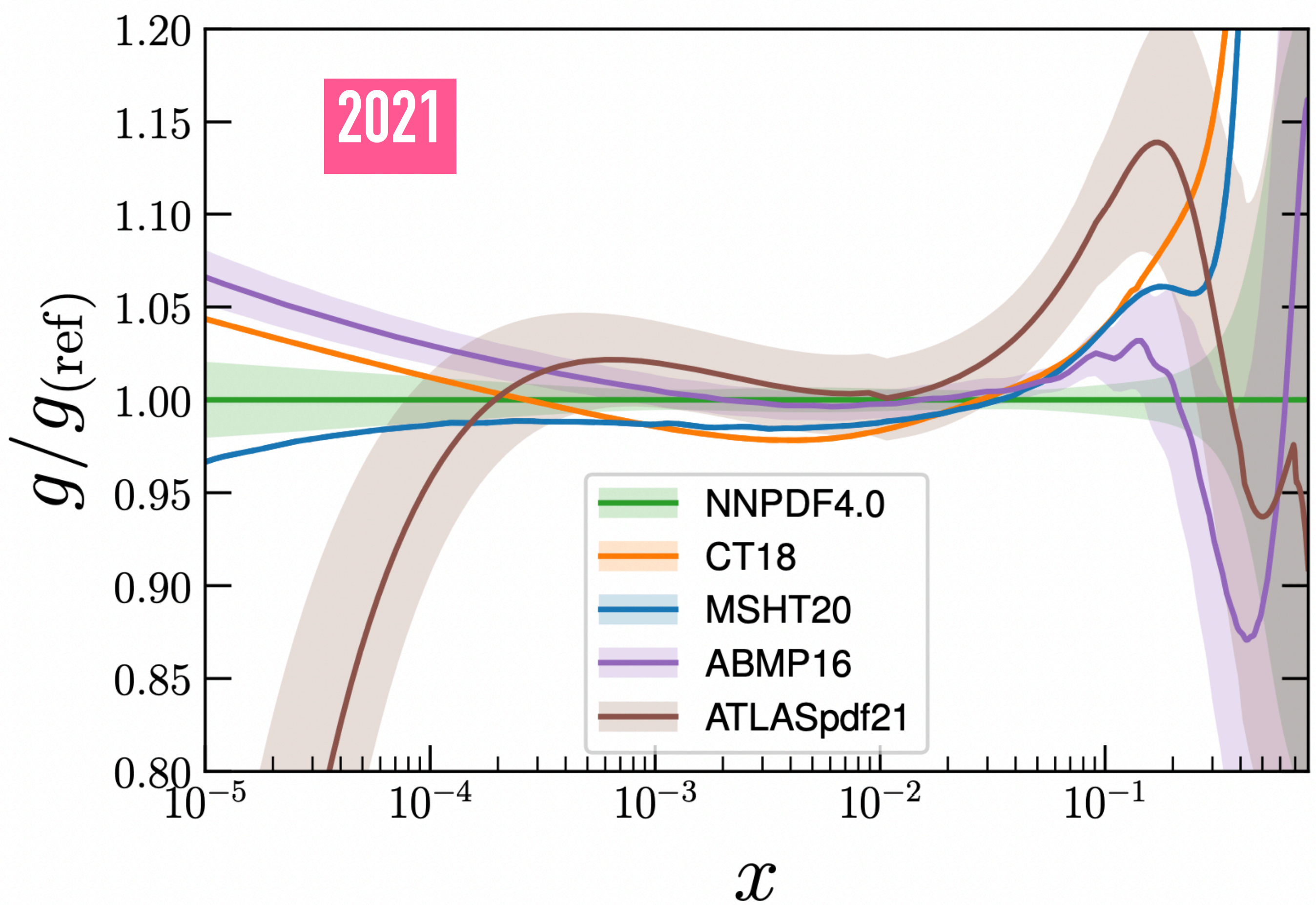


FIG. 27. "Soft-gluon" ($\Lambda=200$ MeV) parton distributions of Duke and Owens (1984) at $Q^2=5$ GeV^2: valence quark distribution $x[u_v(x)+d_v(x)]$ (dotted-dashed line), $xG(x)$ (dashed line), and $q_v(x)$ (dotted line).

Rev.. Mod. Phys. 1984

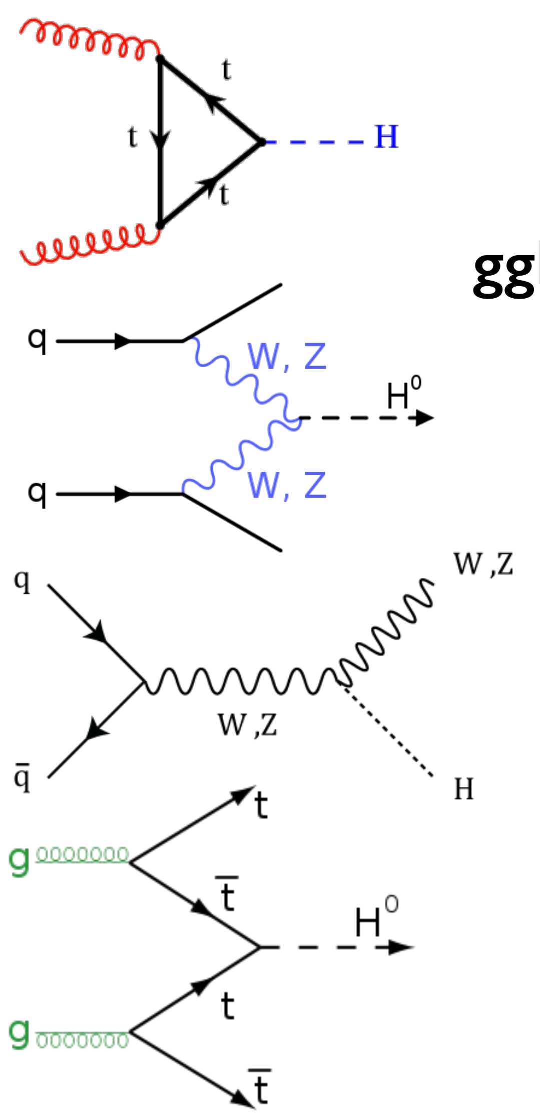


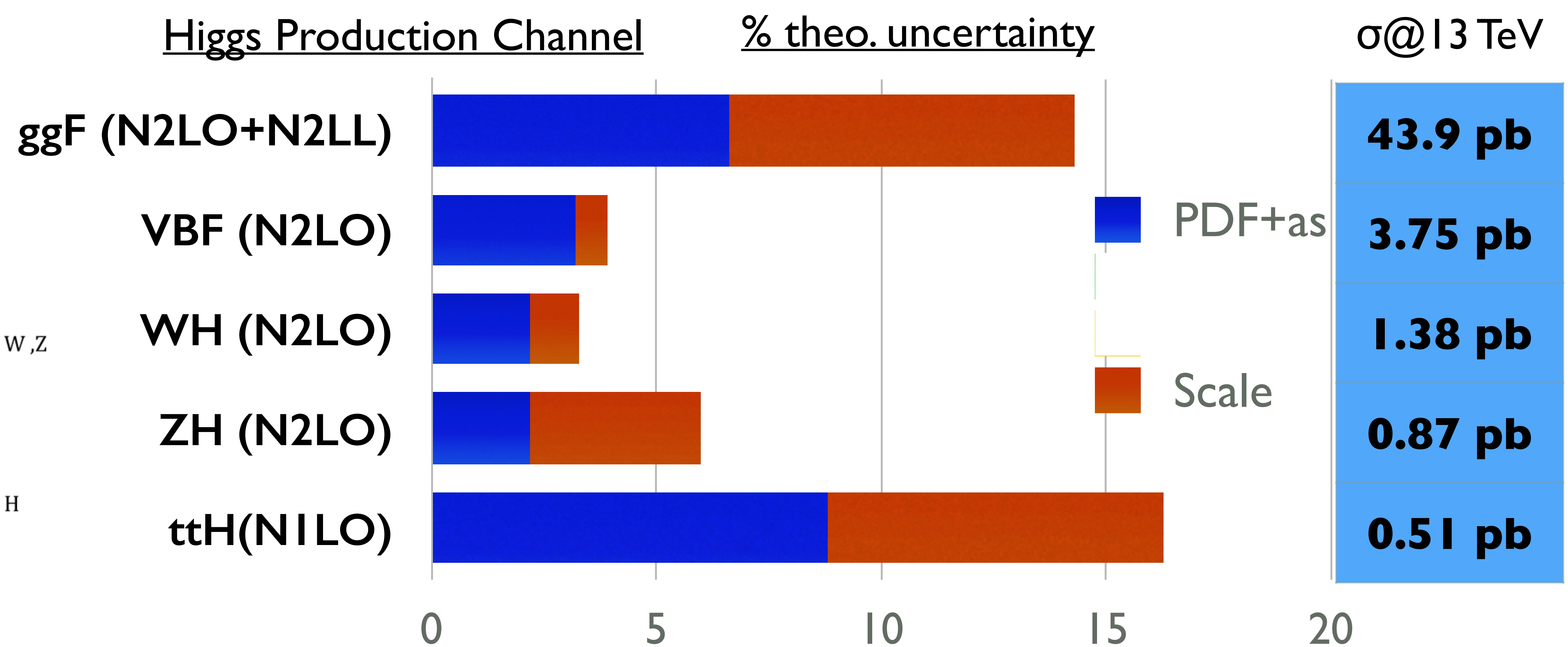
Snowmass 2021 Whitepaper

While at the beginning PDF analyses were performed as a test of QCD collider factorisation and no uncertainties were associated to functions, now PDF uncertainties and differences among different determinations are crucial

PDF uncertainties

Yellow Report 3 (2013)



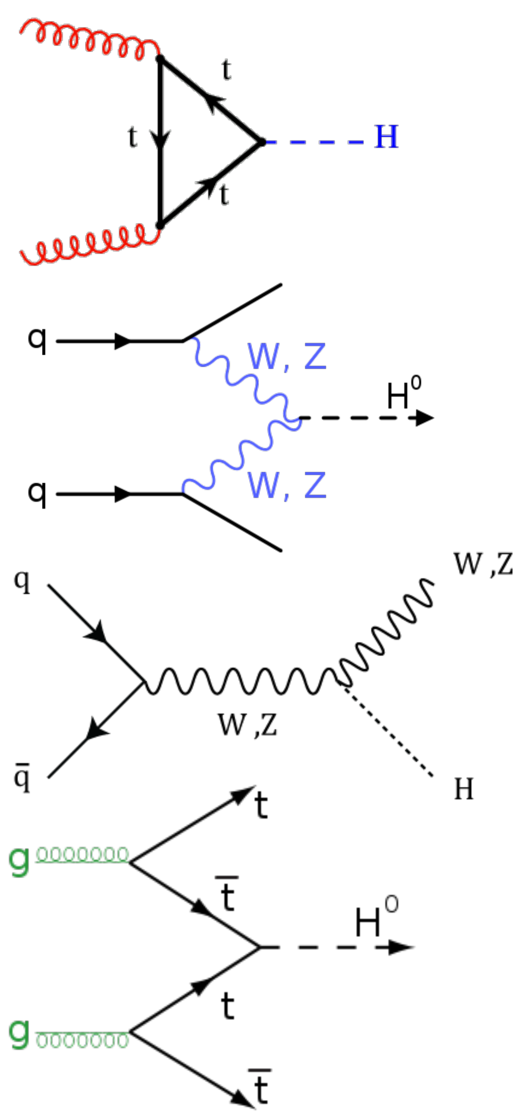


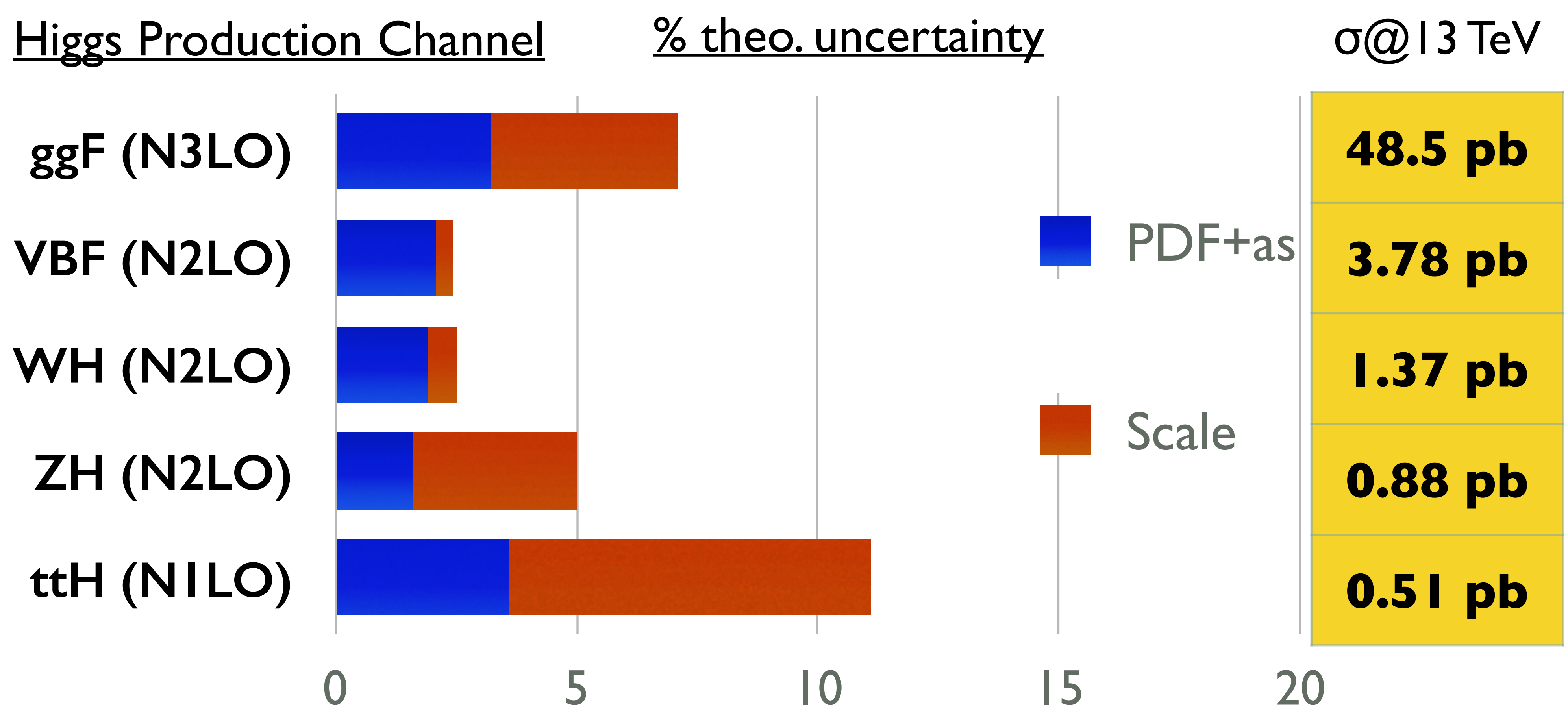
Higgs physics

PDF uncertainties limiting factor in the accuracy of theoretical predictions

PDF uncertainties

Yellow Report 4 (2016)

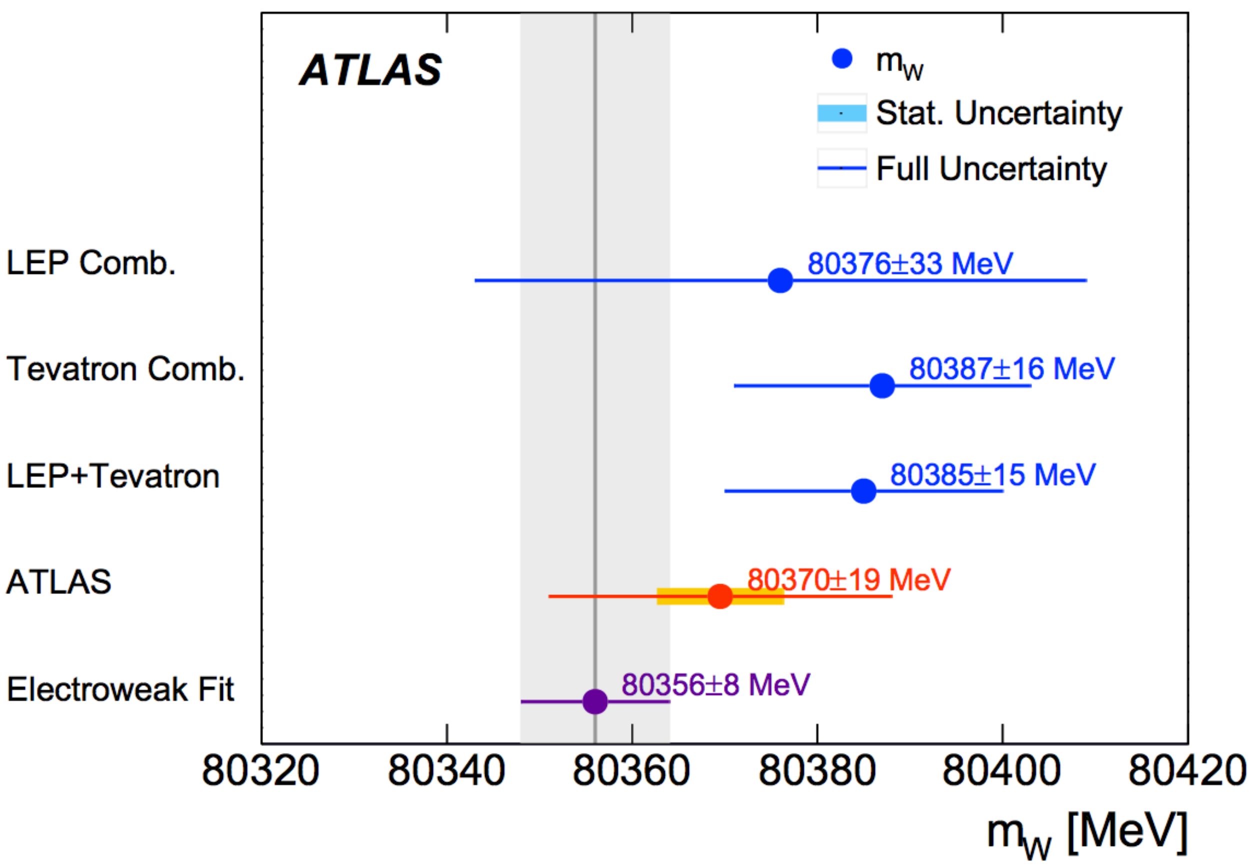


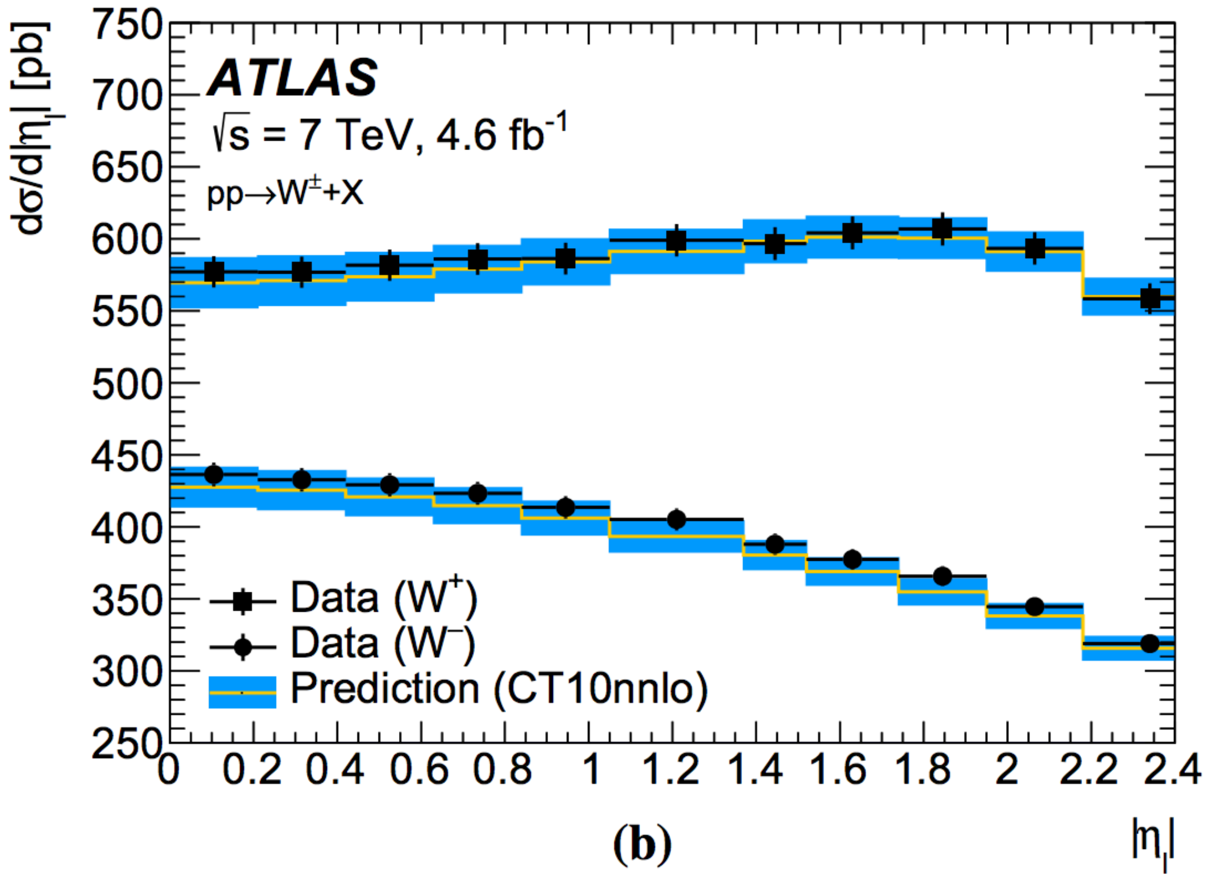


Reduced (still often dominant) PDF uncertainties

PDF uncertainties

Determination of SM parameters



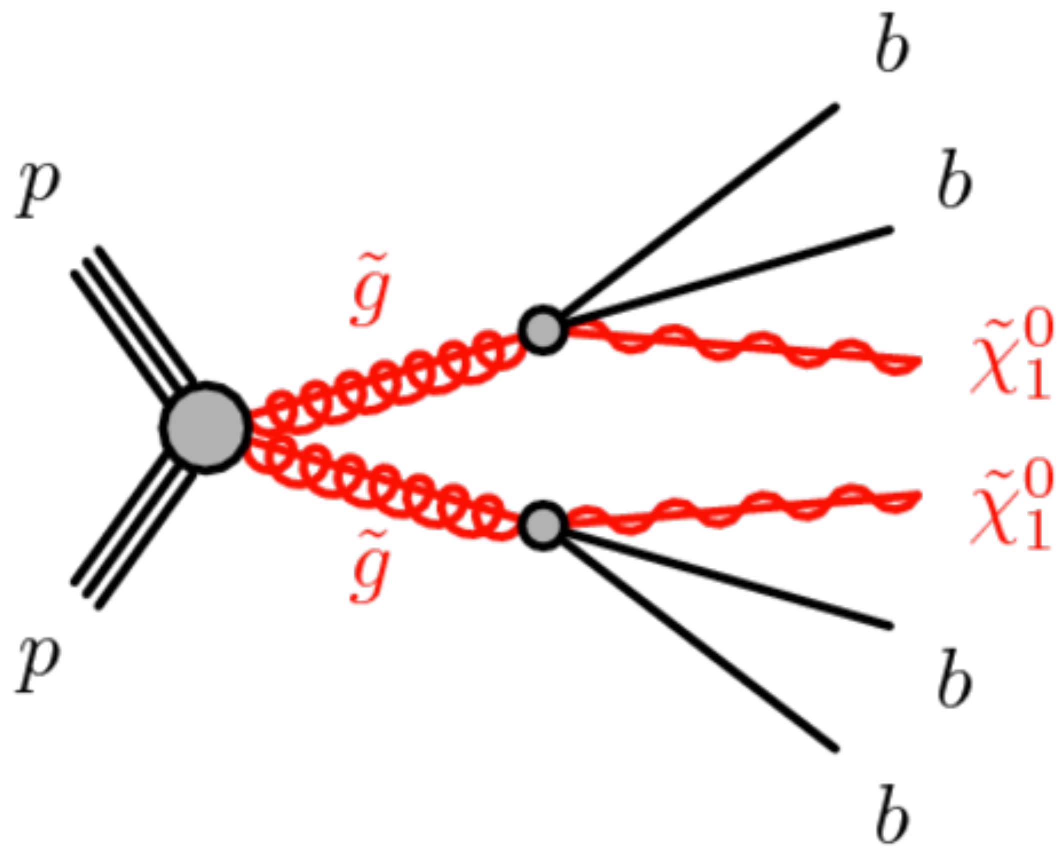


ATLAS collaboration, EPJC 78 (2018) 110

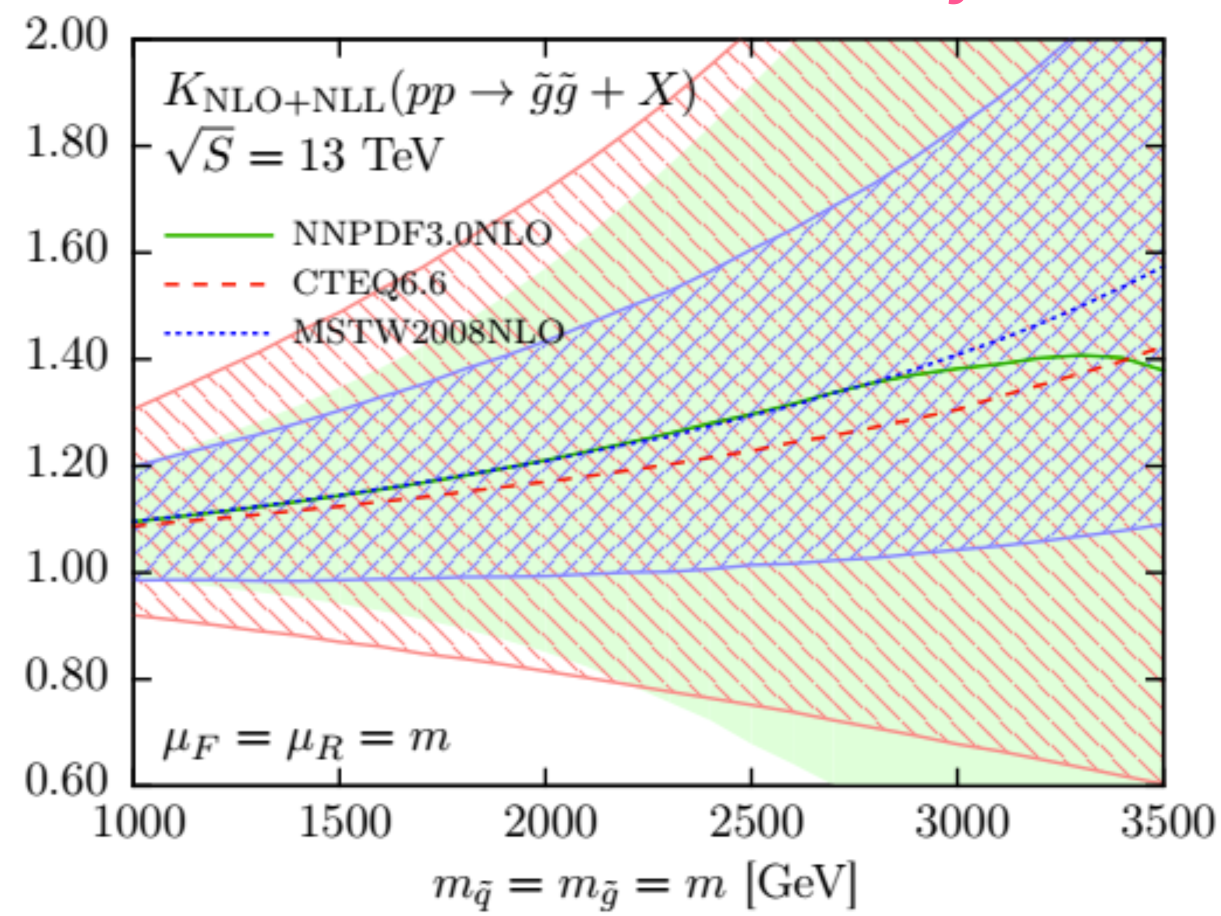
$$\eta = -\ln\tan(\theta/2)$$

Channel	$m_{W^+} - m_{W^-}$ [MeV]	Stat. Unc.	Muon Unc.	Elec. Unc.	Recoil Unc.	Bckg. Unc.	QCD Unc.	EW Unc.	PDF Unc.	Total Unc.
$W \to e\nu$	−29.7	17.5	0.0	4.9	0.9	5.4	0.5	0.0	24.1	30.7
$W \to \mu\nu$	−28.6	16.3	11.7	0.0	1.1	5.0	0.4	0.0	26.0	33.2
Combined	−29.2	12.8	3.3	4.1	1.0	4.5	0.4	0.0	23.9	28.0

PDF uncertainties



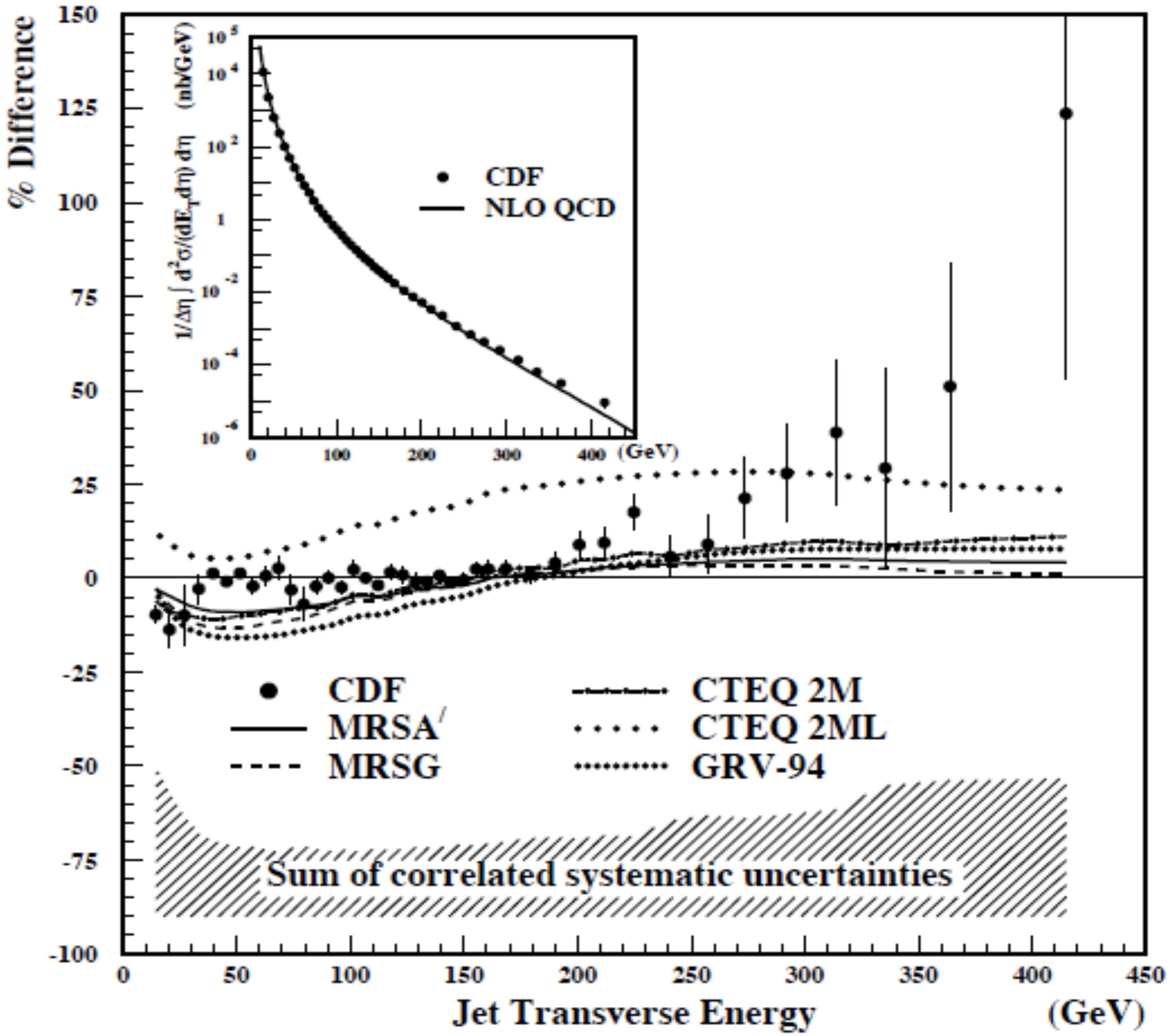
New Physics



Beenakker et al.
EPJC76 (2016)2, 53

PDF uncertainties are a limiting factor in the accuracy of theoretical predictions, both within **SM** and **beyond**

PDF uncertainties

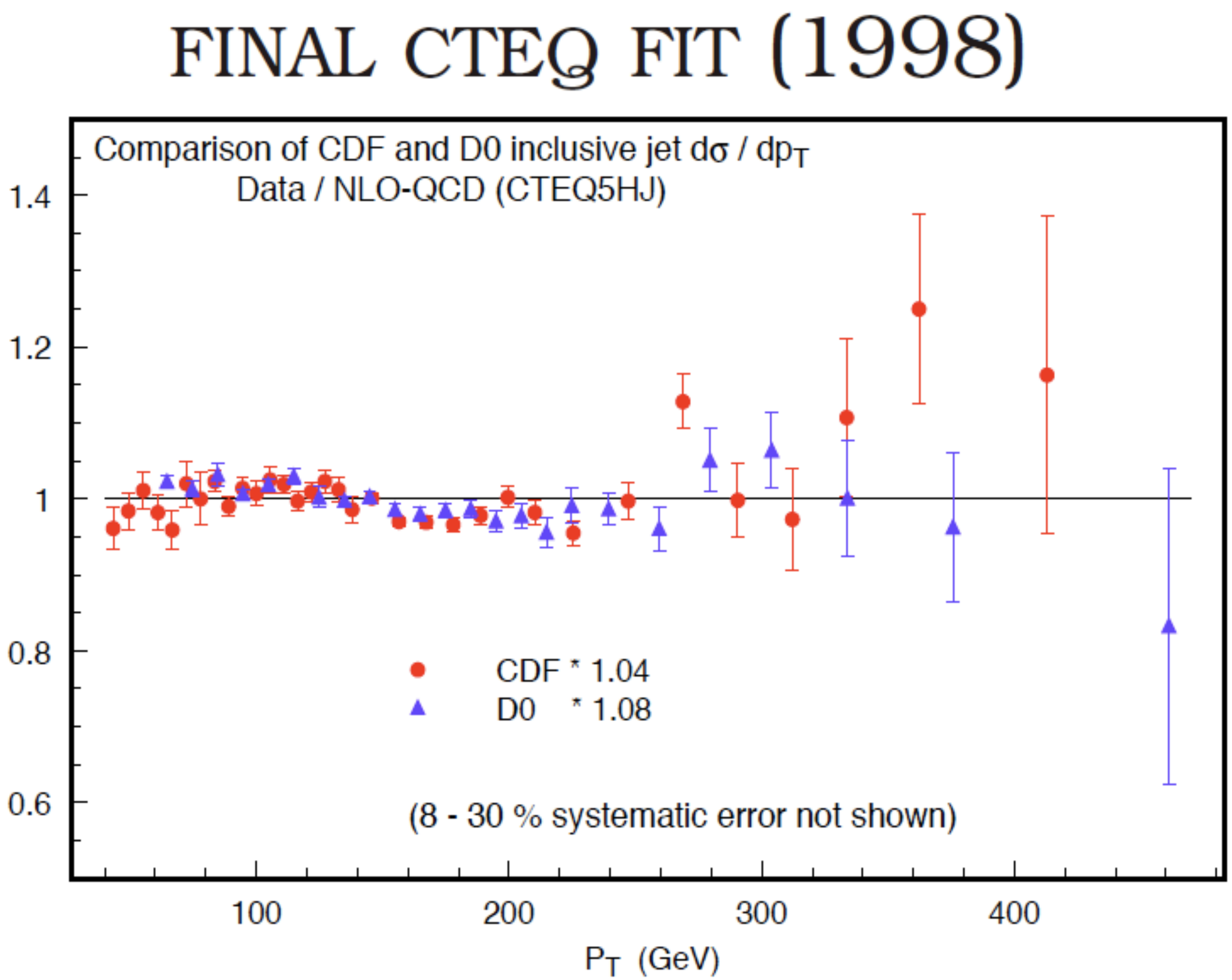


Historia magistra vitae est

Discrepancy between QCD calculations and CDF jet data (1995)

At that time there was no information on PDF uncertainties
and the theoretical prediction strongly depends on gluon shape at x>0.1

PDF uncertainties



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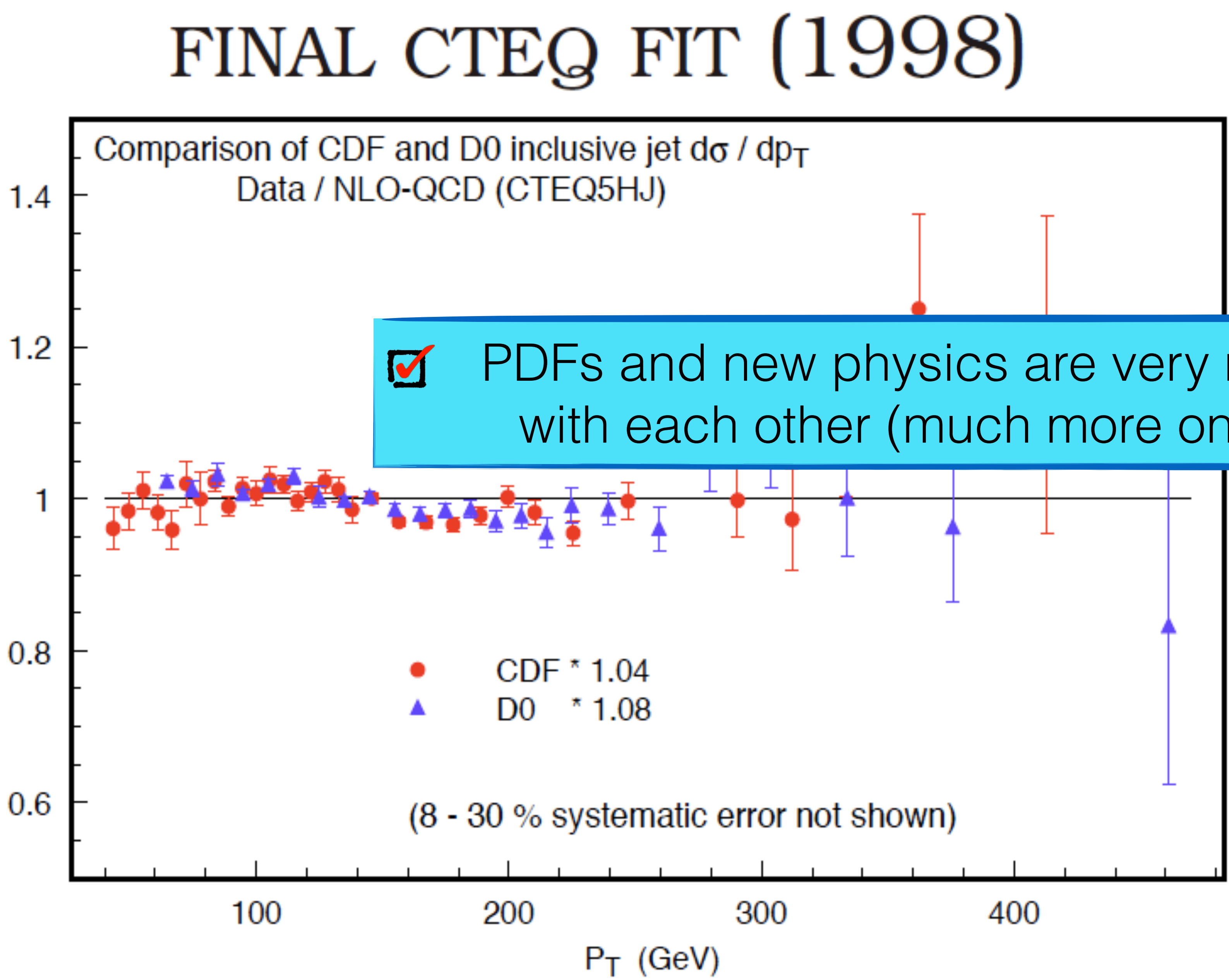
CTEQ re-performed the parton fit by including the jet data and the discrepancy was removed.

PDF uncertainties



☑ PDFs and new physics are very much related with each other (much more on lecture 5)

Historia magistra vitae est

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At that time there was no information on PDF uncertainties
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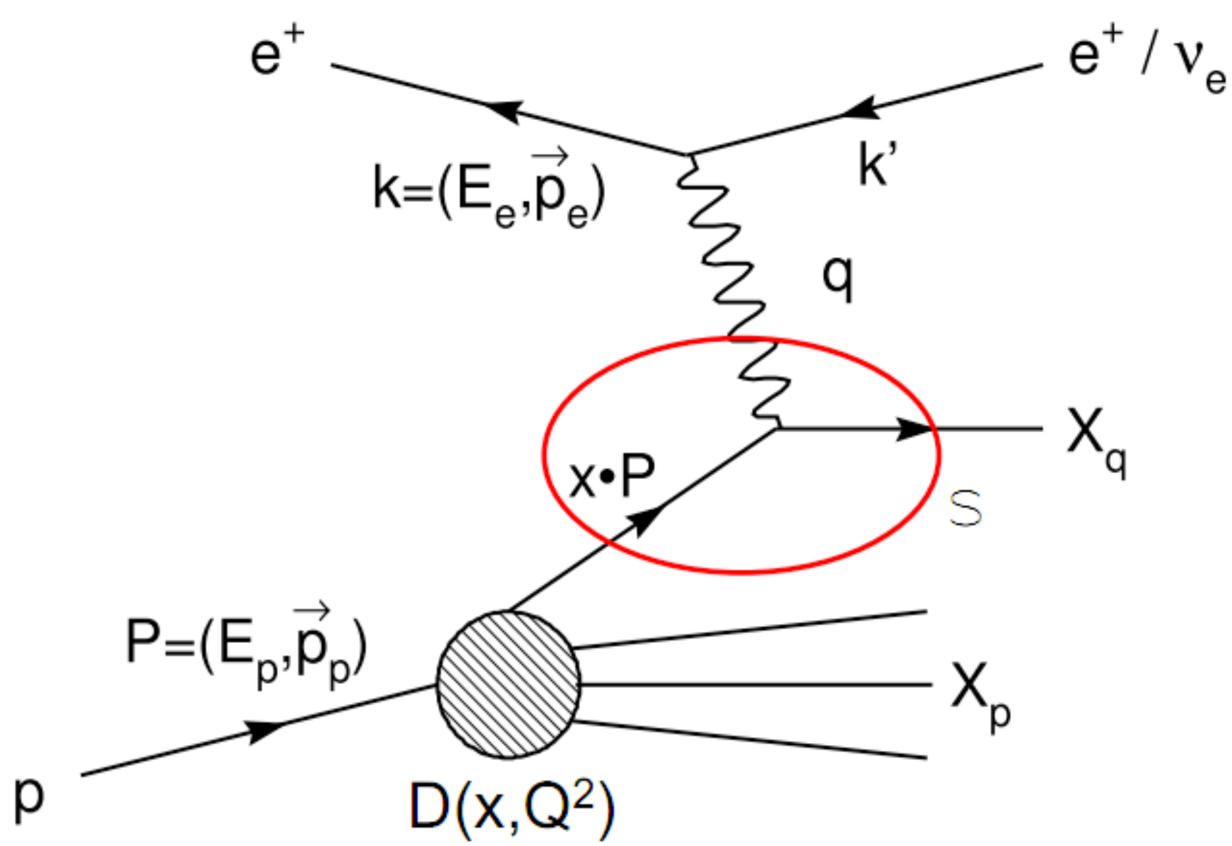
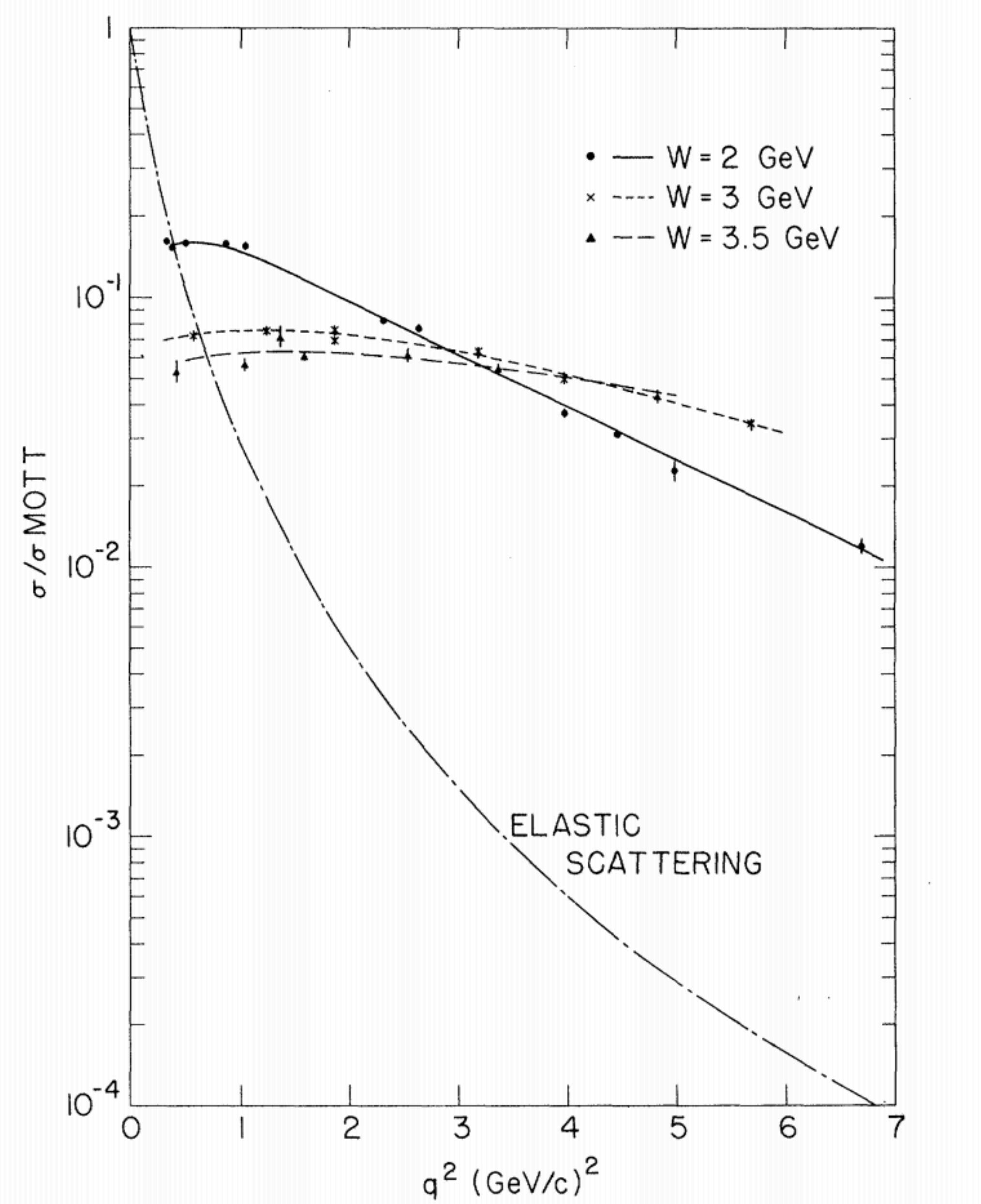
Parton model and QCD

Historic overview

- **1967**: First deep-inelastic scattering experiments at SLAC 20 GeV linear collider gave first evidence of point-like elementary constituents which were later identified as quarks (Bjorken scaling)

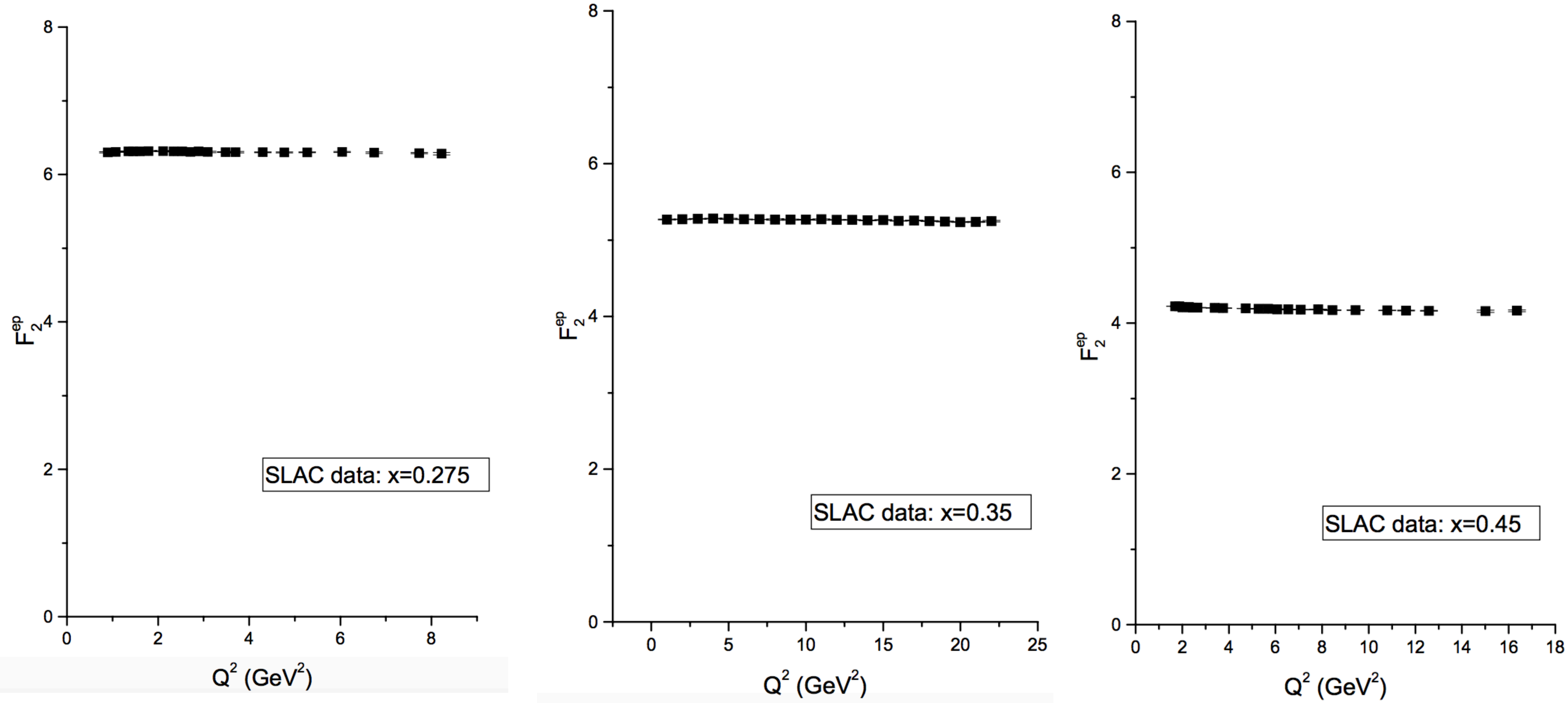




See G. Sterman's lectures

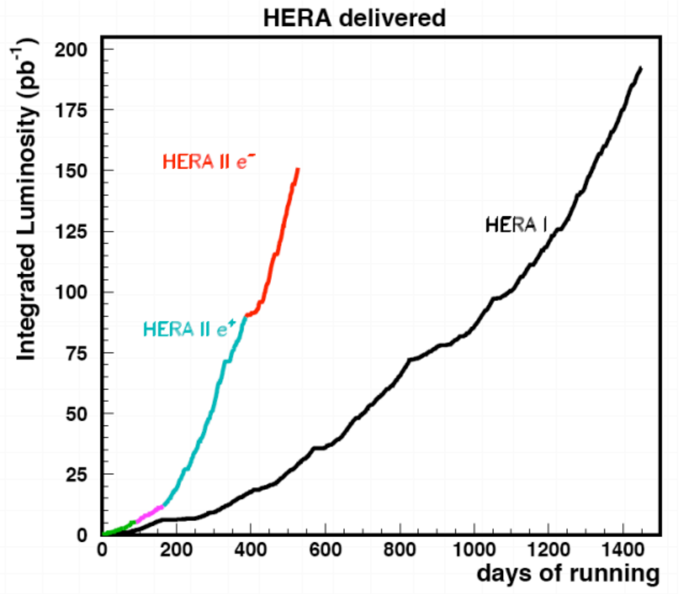
Deep Inelastic Scattering

The surprising results at SLAC was that the structure functions $F_{1,2}$ did not vanish as Q^2 increased, rather they remained finite and constant and depended only on x_B - Bjorken scaling (1969)



Such scaling pointed to the fact that the exchanged vector boson (photon) scatters elastically off point-like objects that have no mass or scale associated. The lepton scatters off charged spin 1/2 constituents (partons) that carry a fraction x of proton momentum **(Feynman's Parton model)**

The HERA collider



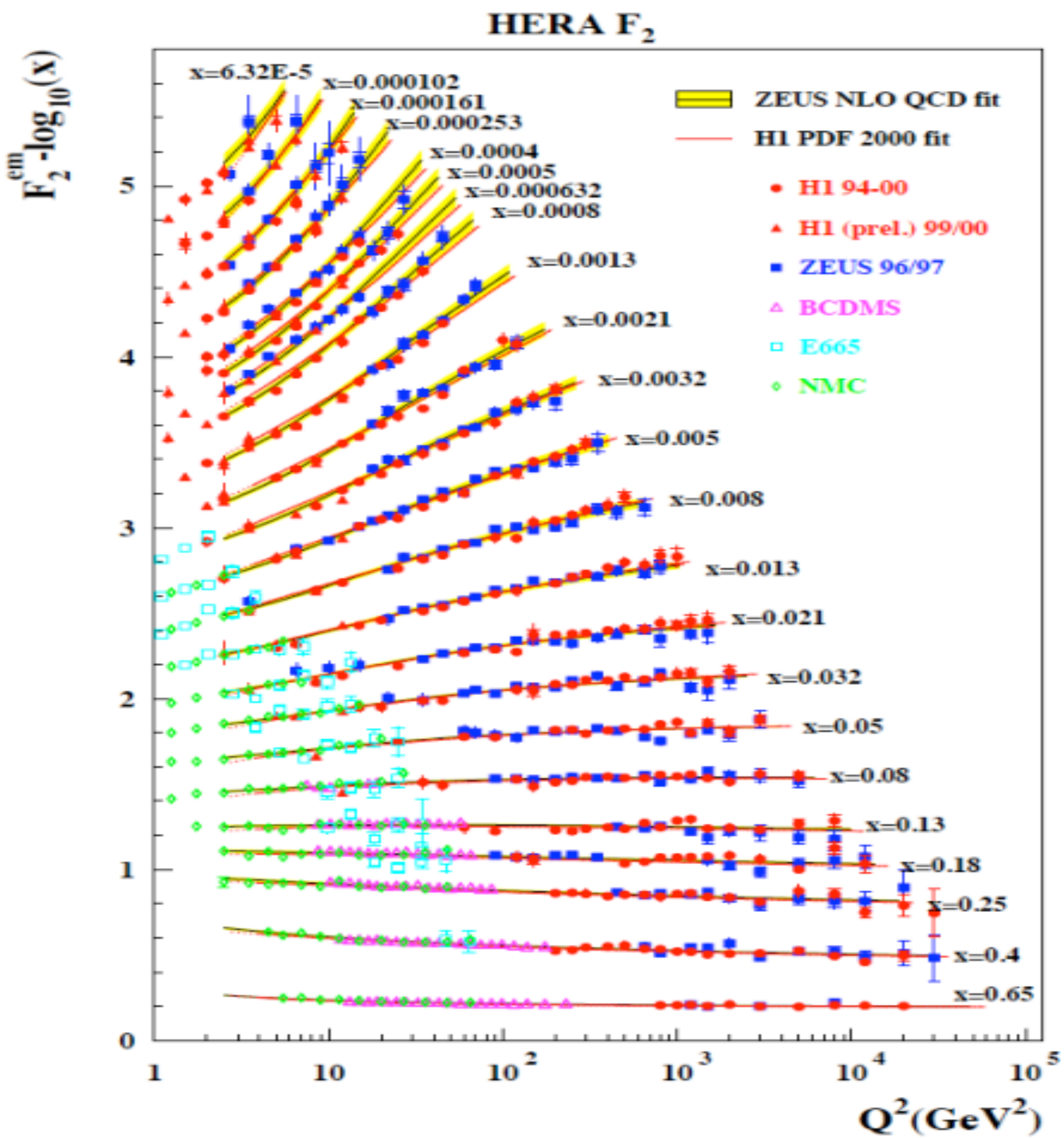
1992-2007

$$\sqrt{S} = 318\,\text{GeV}$$

$$E_e = 27.5\,\text{GeV}$$

$$E_p = 920\,\text{GeV}$$

Scaling violation

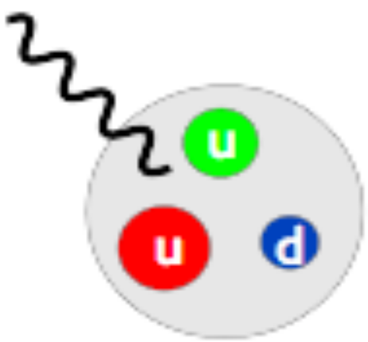


scaling violation

approx. scaling

QCD and improved parton model

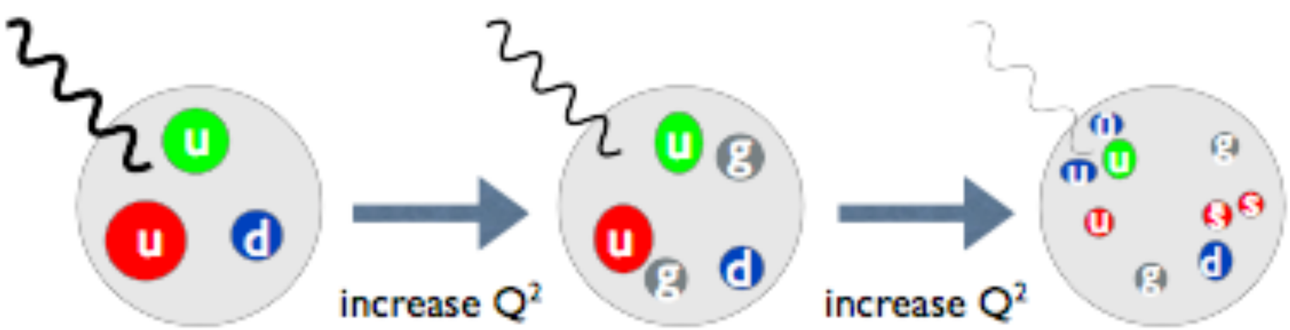
Parton model picture



$$\sigma_{eh}(p,q) = \sum_a \int_0^1 d\xi\, \hat{\sigma}_{ea}(\xi p, q)\, \Phi_{a/h}(\xi)$$

QCD-improved parton model

$$\sigma_{eh}(p,q,\mu_F) = \sum_a \int_0^1 d\xi\, \hat{\sigma}_{ea}(\xi p, q, \mu_F)\, \Phi_{a/h}(\xi, \mu_F)$$



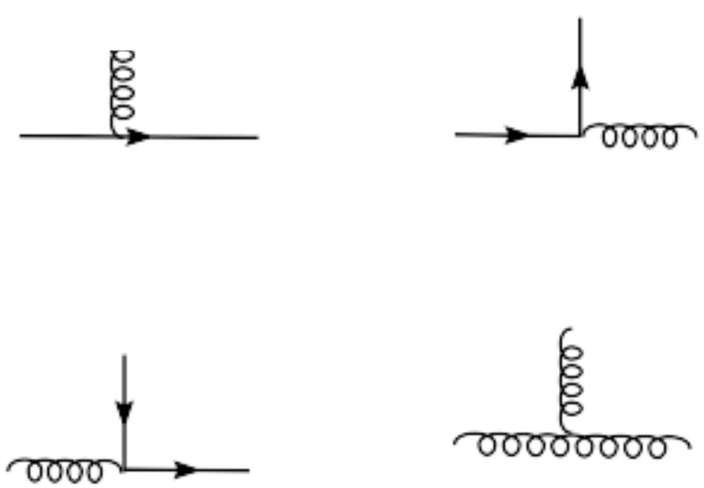
In QCD, collinear singularities that arise when partons interact via QCD interactions are absorbed into a re-definition of the Parton Distribution Functions, which acquire a dependence on the scale.

DGLAP evolution equations

When you put all flavours in, get 13 coupled integro-differential equations, which can be reduced to 11 decoupled and 2 coupled equation (with a change of basis in the space of PDFs)

Dokshitzer, Gribov, Lipatov, Altarelli, Parisi equations

$$\mu_F \frac{\partial}{\partial \mu_F} \begin{pmatrix} \Phi_{Q/h} \\ \Phi_{G/h} \end{pmatrix} (\xi, \mu_F) = \frac{\alpha_s}{2\pi} \int_\xi^1 \frac{dy}{y} \begin{pmatrix} P_{QQ}\left(\frac{\xi}{y}, \alpha_s\right) & P_{QG}\left(\frac{\xi}{y}, \alpha_s\right) \\ P_{GQ}\left(\frac{\xi}{y}, \alpha_s\right) & P_{GG}\left(\frac{\xi}{y}, \alpha_s\right) \end{pmatrix} \begin{pmatrix} \Phi_{Q/h} \\ \Phi_{G/h} \end{pmatrix} (y, \mu_F)$$



- Splitting functions known up to NNLO:

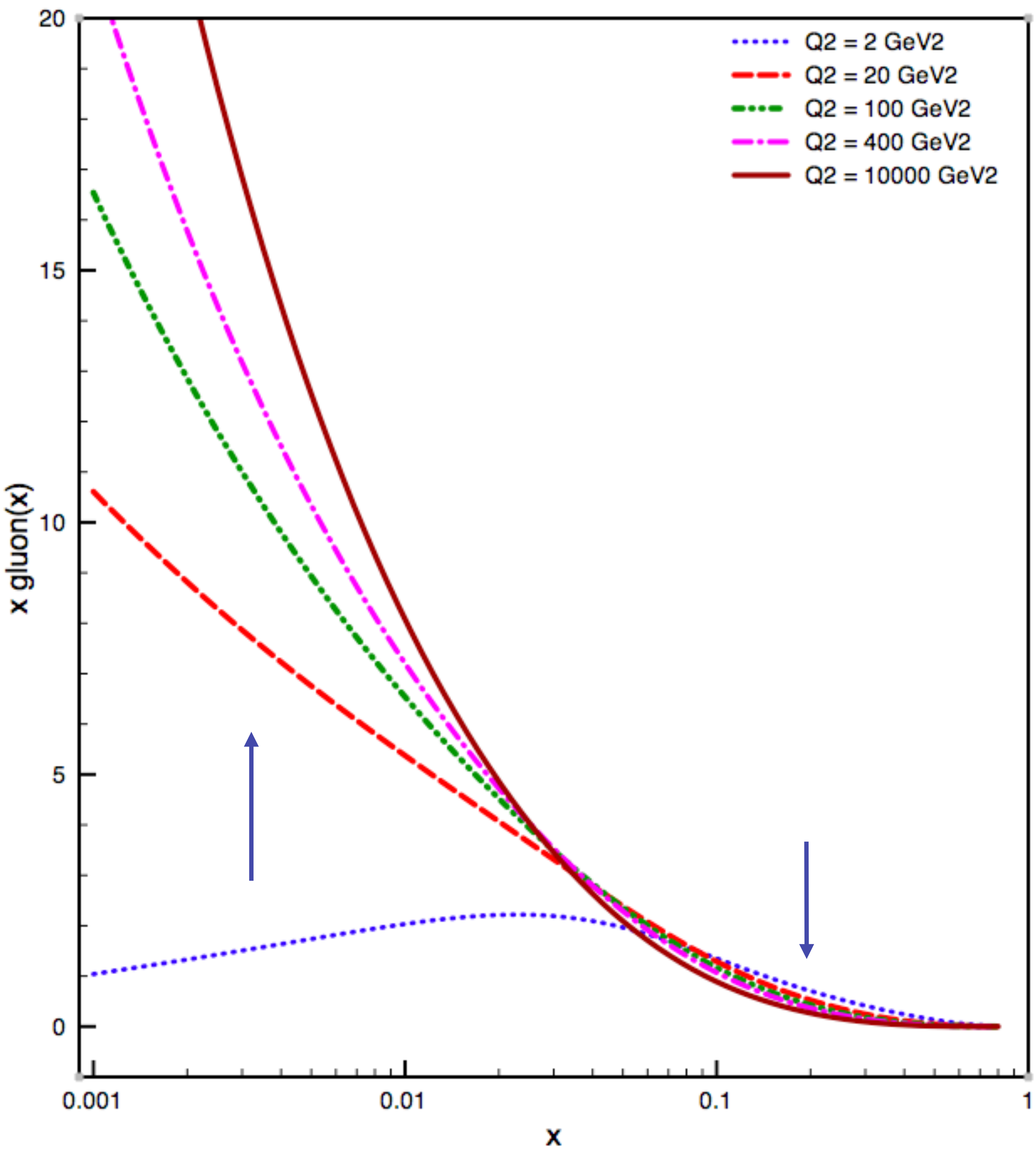
  LO Dokshitzer; Gribov, Lipatov; Altarelli, Parisi (1977)

  NLO Floratos,Ross,Sachrajda; Floratos, Lacaze, Kounnas, Gonzalez-Arroyo,Lopez,Yndurain; Curci,Furmanski Petronzio, (1981)

  NNLO - Moch, Vermaseren, Vogt, 2004

  N3LO - Moch, Vermaseren, Bluemlein 2020-

DGLAP evolution equations



Gluon evolution

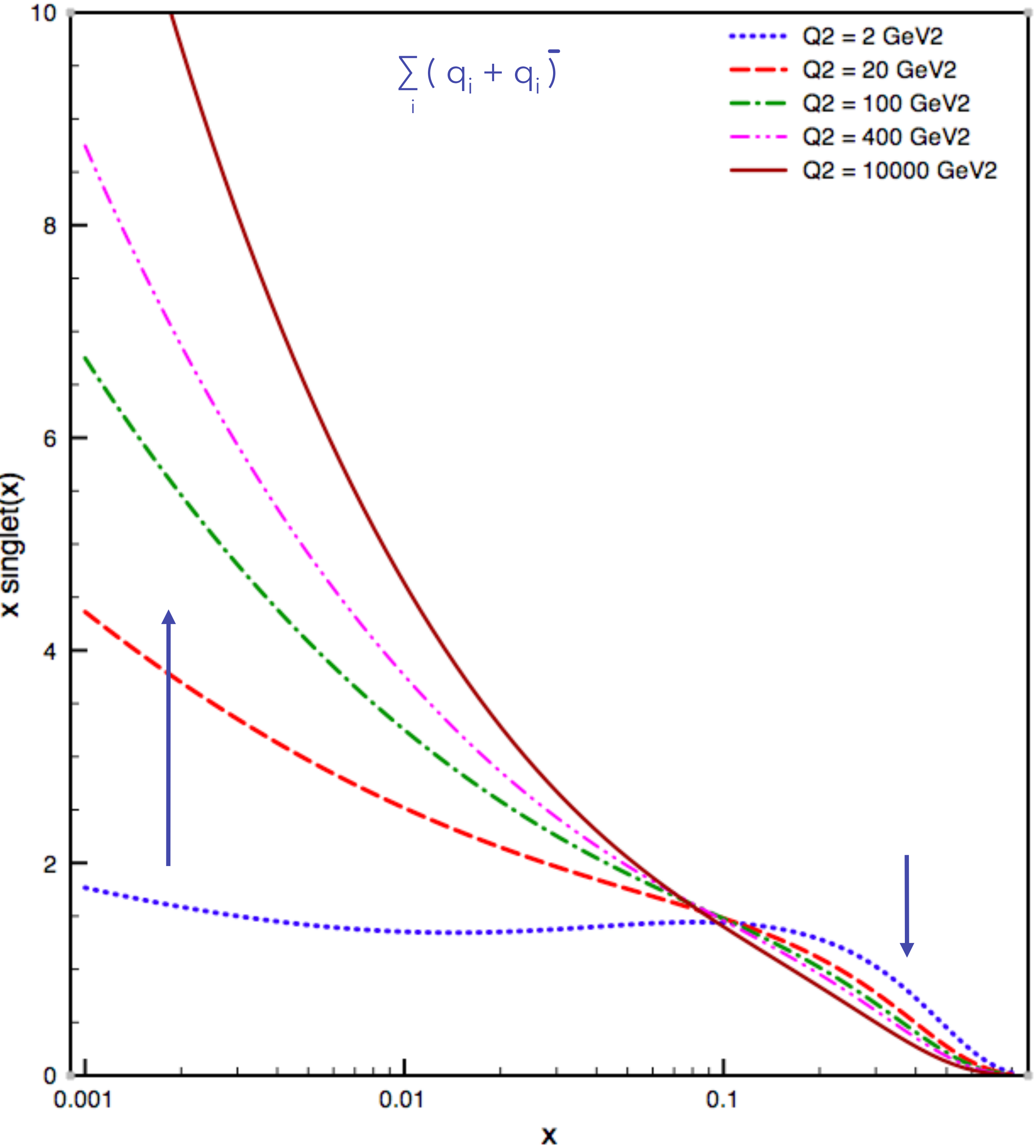
$$g(x,\mu^2) = \Gamma_{gq} \otimes \Sigma(x,\mu_0^2) + \Gamma_{gg} \otimes g(x,\mu_0^2)$$

$$P_{gq}^{(0)}(x) = C_F \left[\frac{1+(1-x)^2}{x}\right]$$

$$P_{gg}^{(0)}(x) = 2N \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right] + \delta(1-x)\frac{(11N - 4n_f T_R)}{6}$$

- Both P_{gg} and P_{gq} diverge for $x \to 0$
- Gluon is depleted at large x

DGLAP evolution equations



Singlet evolution

$$\Sigma(x,\mu^2) = \Gamma_{qq} \otimes \Sigma(x,\mu_0^2) + \Gamma_{qg} \otimes g(x,\mu_0^2)$$

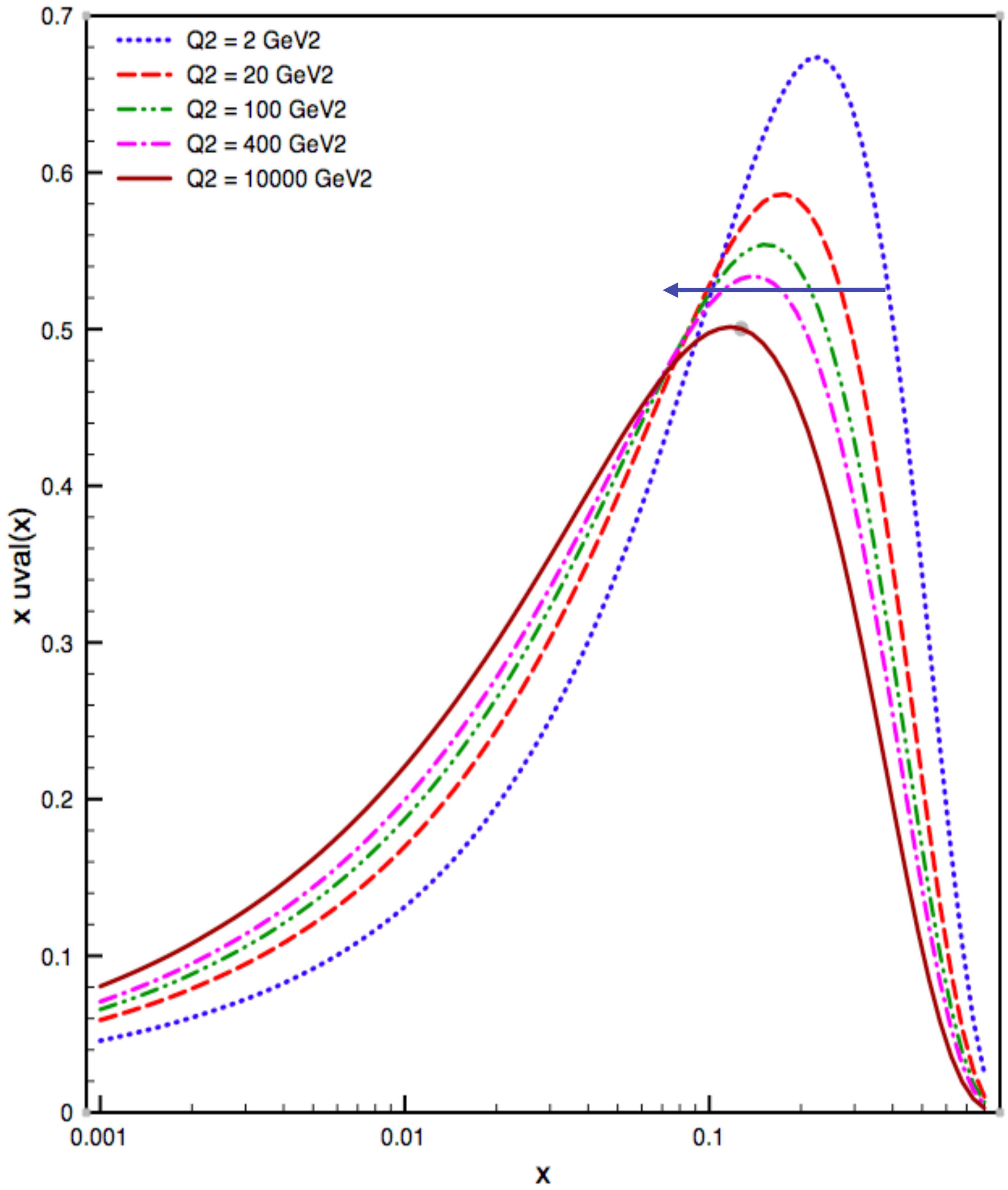
$$P_{qq}^{(0)}(x) = C_F \left[\frac{(1+x^2)}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right]$$

$$P_{qg}^{(0)}(x) = T_R \left[x^2 + (1-x)^2 \right]$$

- High-x gluon feeds growth of small-x gluon and quark

- Gluons can be seen because they help drive the quark evolution

DGLAP evolution equations



Non-singlet valence evolution

$$u_v(x, \mu^2) = \Gamma^v_{NS} \otimes u_v(x, \mu_0^2)$$

$$P^{(0),v}_{NS} = P^{(0)}_{qq}(x) = C_F \left[\frac{(1+x^2)}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right]$$

- As Q^2 increases partons lose longitudinal momentum; distributions all shift to lower x

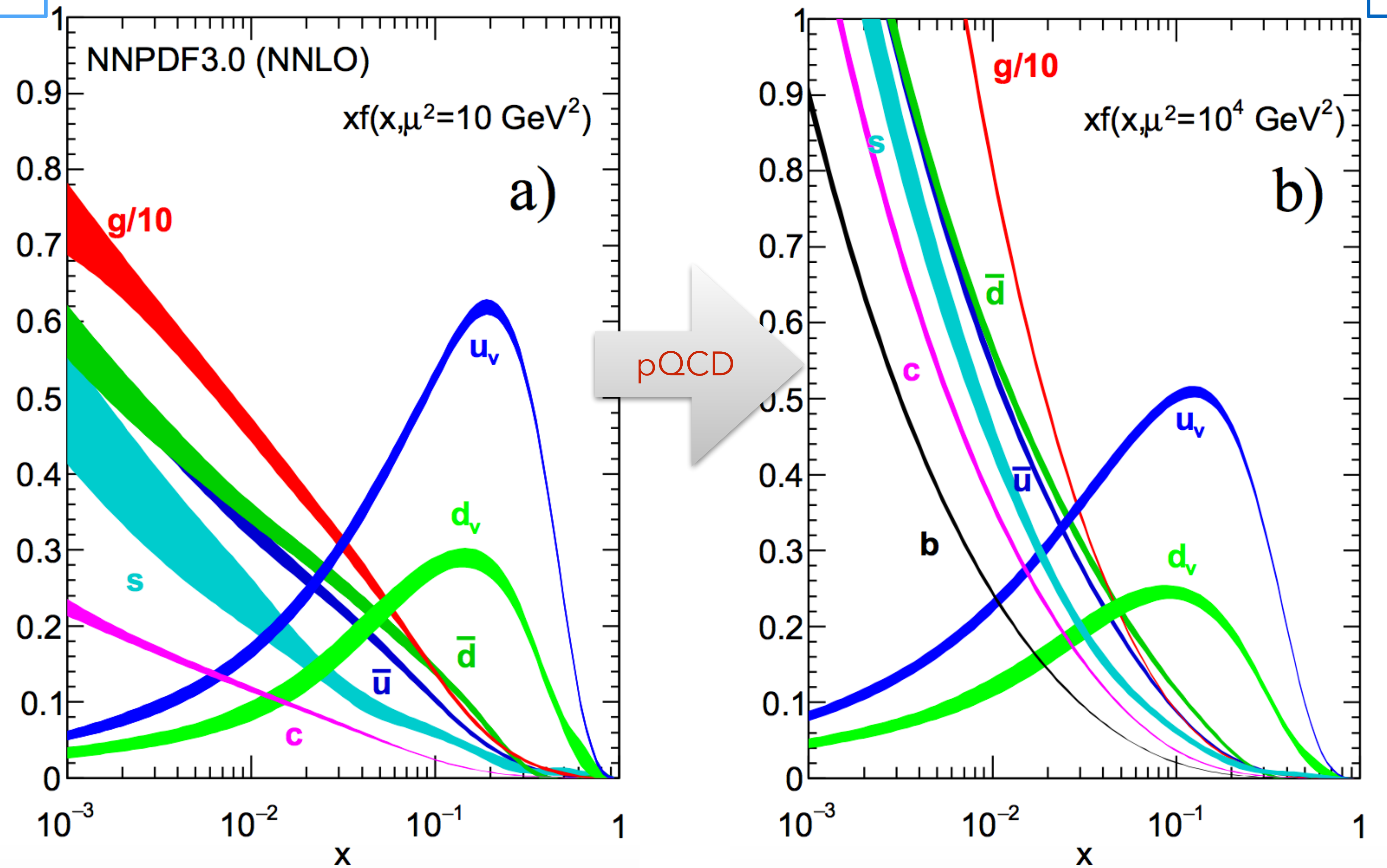
- Gluons can be seen because they help drive the quark evolution

DGLAP evolution equations

Functional dependence of PDFs on the scale is totally predicted up to NNLO (shortly up to full N3LO) accuracy by solving DGLAP evolution equations

Hadronic scale: global fit of PDFs

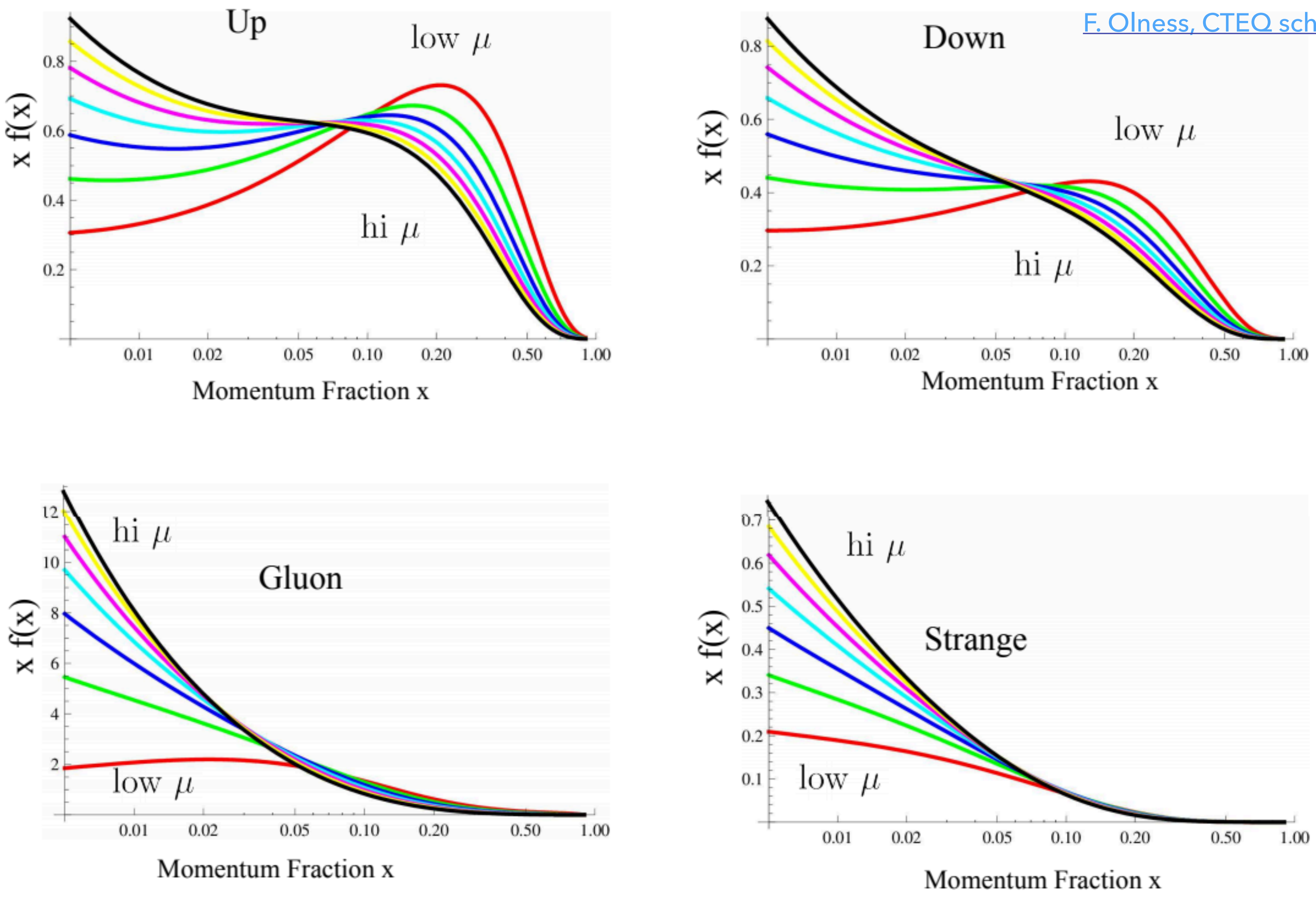
High scale: input to the LHC



PDG 2016

DGLAP evolution equations

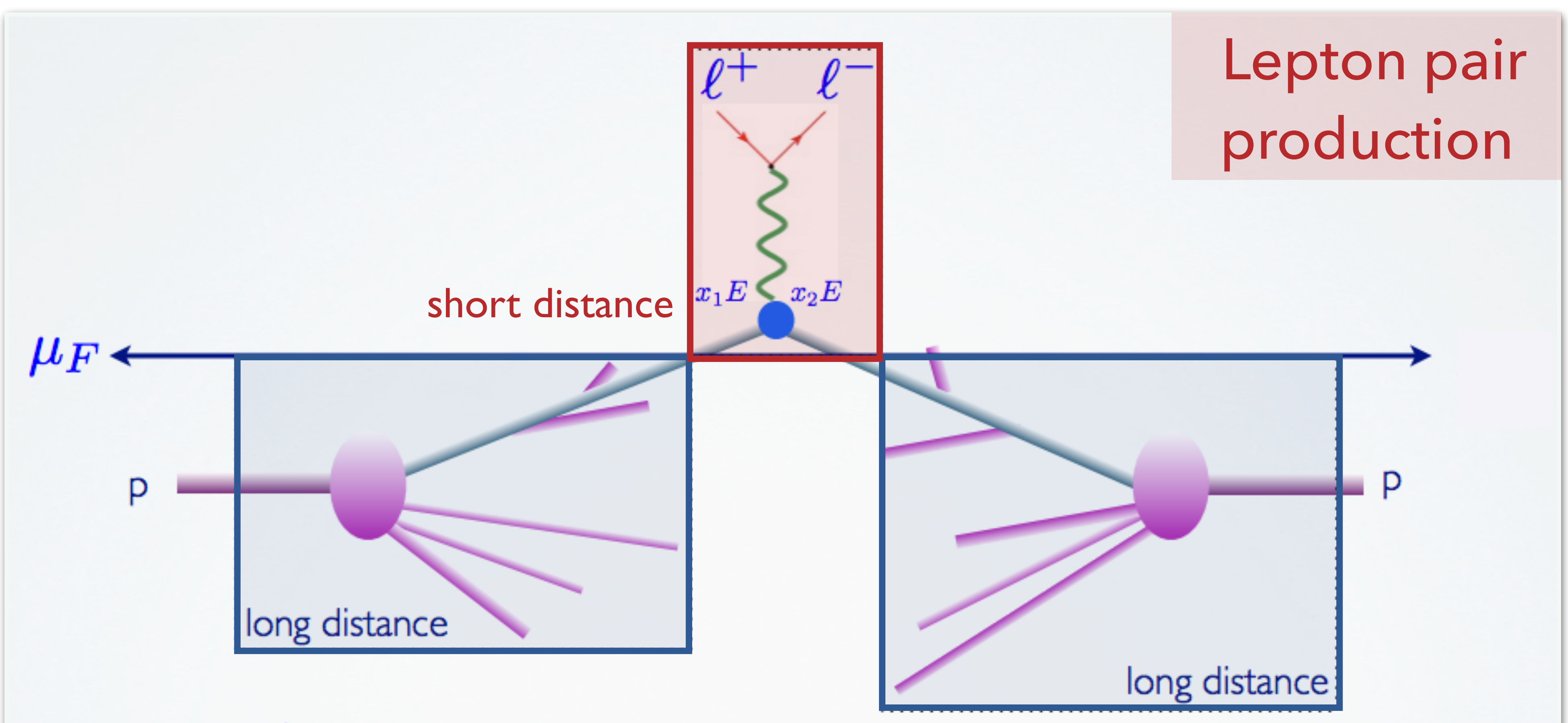
F. Olness, CTEQ school 2017



Collinear Factorisation Theorem

$$\frac{d\sigma_H^{ep\to ab}}{dX} = \sum_{i=-n_f}^{+n_f} \int_{x_B}^{1} \frac{dz}{z} f_i(z,\mu_F) \frac{d\hat{\sigma}_i^{ei}}{dX}(zS, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

$$\frac{d\sigma_H^{pp\to ab}}{dX} = \sum_{i,j=-n_f}^{+n_f} \int_{\tau_0}^{1} \frac{dz_1}{z_1} \frac{dz_2}{z_2} f_i(z_1,\mu_F) f_j(z_2,\mu_F) \frac{d\hat{\sigma}_i^{ij}}{dX}(zS, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$



Collinear Factorisation Theorem

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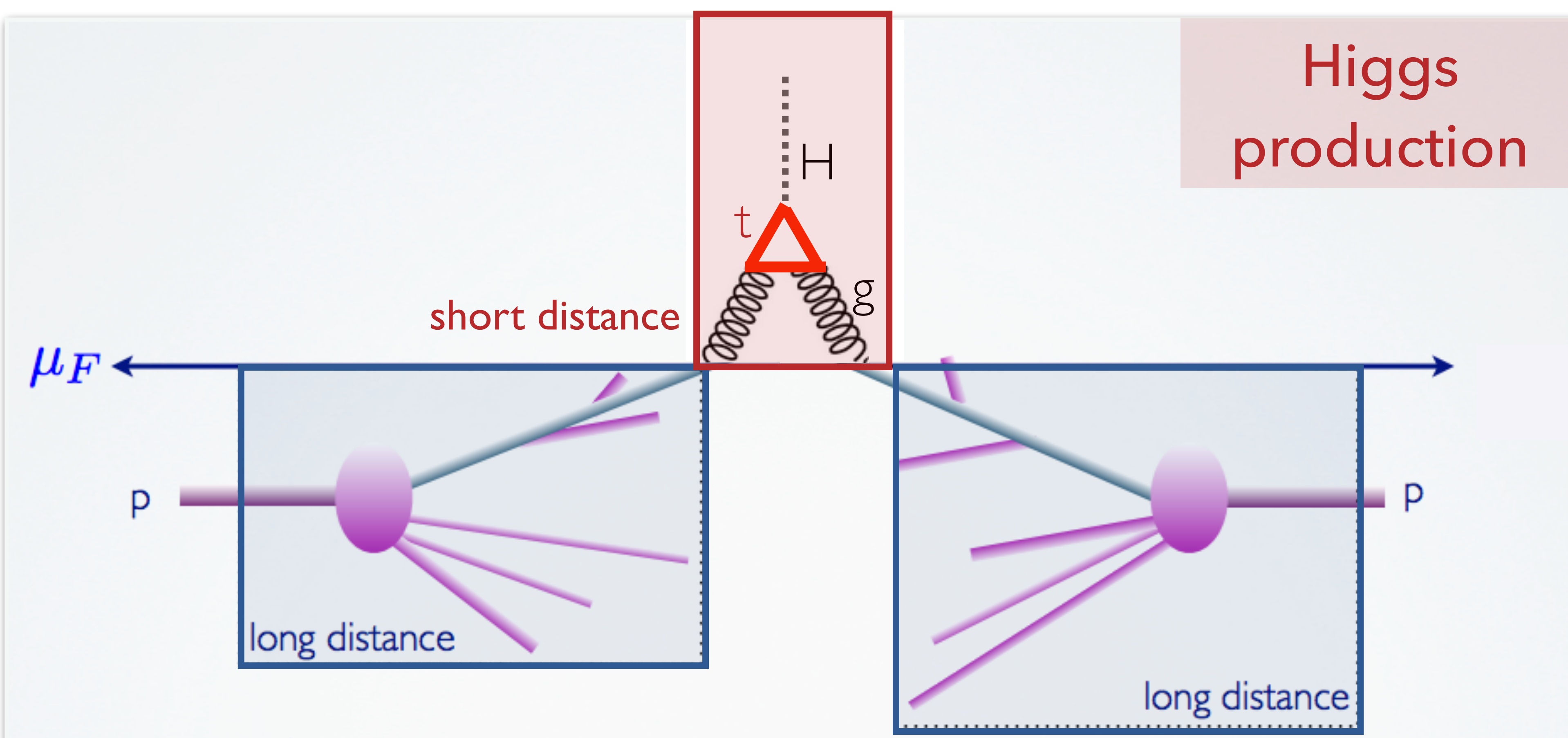

PDF determination

Thanks to **PDF universality** in the collinear factorisation picture and the **perturbative evolution** of PDFs we can extract the non-perturbative x-dependence of PDF from a wide set of data that involve initial state protons (hadrons)

$$f_i(x, \mu)$$

Data

Perturbative QCD

Ingredients of a PDF global fits

The ingredients

- Choose **experimental data** to fit and include all info on correlations
- **Theory settings**: perturbative order, heavy quark mass scheme, EW corrections, intrinsic heavy quarks, α_s, quark masses value and scheme
- Choose a starting scale Q_0 where pQCD applies
- **Parametrise** independent quarks and gluon distributions at the starting scale
- Solve **DGLAP equations** from initial scale to scales of experimental data and build up observables
- **Fit** PDFs to data
- Provide **error sets** to compute PDF uncertainties

The ingredients

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The ingredients

- Choose **experimental data** to fit and include all info on correlations
- **Theory settings**: perturbative order, heavy quark mass scheme, EW corrections, intrinsic heavy quarks, α_s, quark masses value and scheme
- Choose a starting scale Q_0 where pQCD applies
- **Parametrise** independent quarks and gluon distributions at the starting scale
- Solve **DGLAP equations** from initial scale to scales of experimental data and build up observables
- **Fit** PDFs to data
- Provide **error sets** to compute PDF uncertainties

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The ingredients

$$\sigma_{\mathcal{F}} = \left(\sum_{k=1}^{N_{\rm set}} \left(\mathcal{F}[\{f^{(k)}\}] - \mathcal{F}[\{f^{(0)}\}] \right)^2 \right)^{1/2}$$

error sets
mem > 1

central set
mem = 0

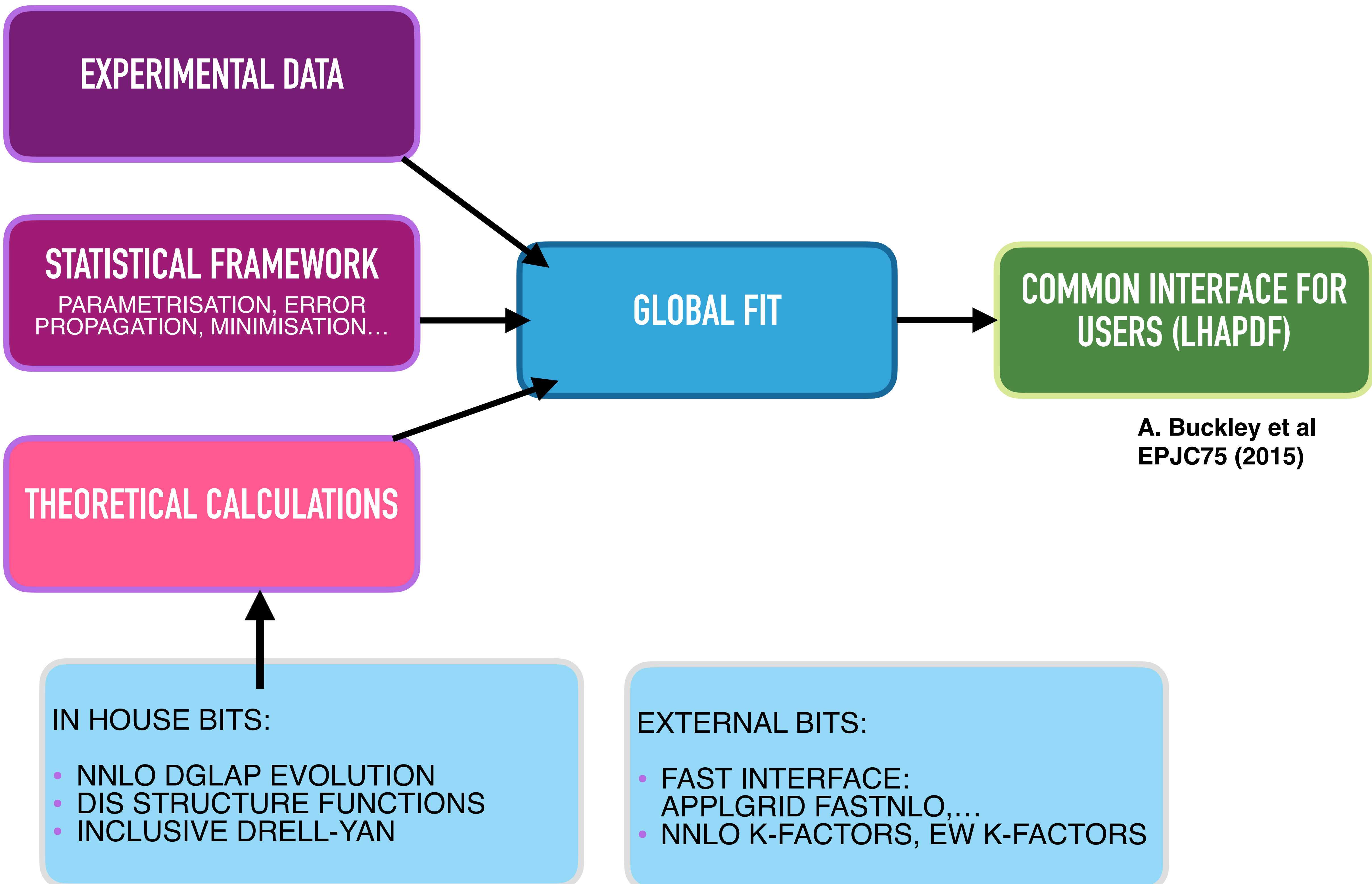
```
call InitPDF(mem)

call evolvePDF(x,Q,f)
```

LHAPDF interface
http://lhapdf.hepforge.org

- Provide PDF **error sets** to compute PDF uncertainties

	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
Parton	tbar	bbar	cbar	sbar	ubar	dbar	g	d	u	s	c	b	t

A complex machinery



Experimental input

Experimental data

- PDFs are not measurable, we measure observables that convolute PDFs with partonic cross sections

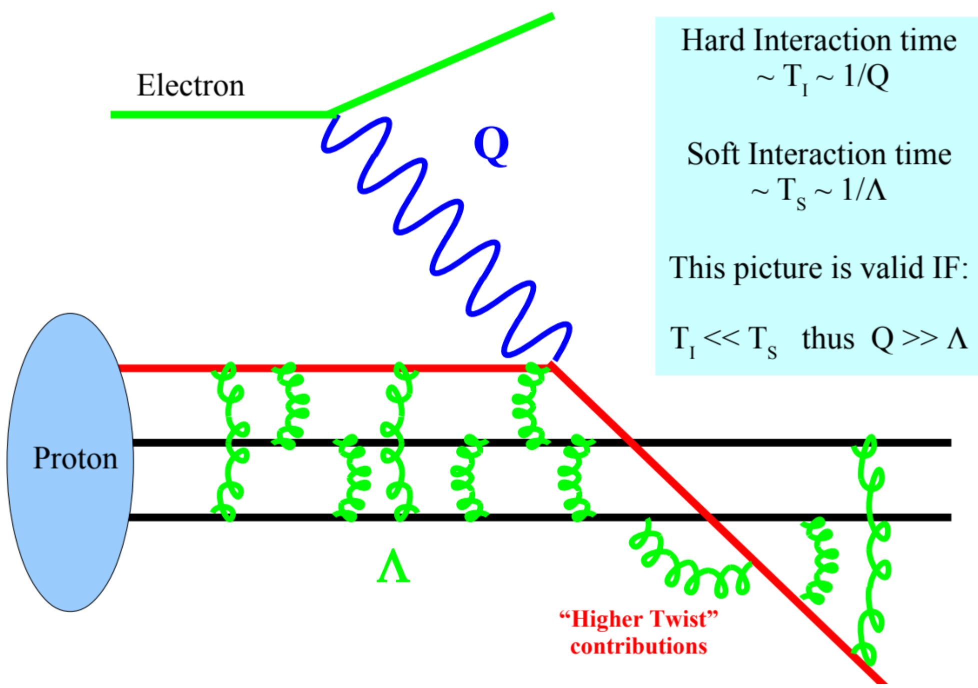
$$\frac{d\sigma_H^{ep\to ab}}{dX} = \sum_{i=-n_f}^{+n_f} \int_{x_B}^{1} \frac{dz}{z} f_i(z,\mu_F) \frac{d\hat{\sigma}_i^{ei}}{dX}(zS, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

$$\frac{d\sigma_H^{pp\to ab}}{dX} = \sum_{i,j=-n_f}^{+n_f} \int_{\tau_0}^{1} \frac{dz_1}{z_1} \frac{dz_2}{z_2} f_i(z_1,\mu_F) f_j(z_2,\mu_F) \frac{d\hat{\sigma}_i^{ij}}{dX}(zS, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

- Most fits exclude regions where factorisation fails to apply (low Q^2 and large x). Typically

$$Q^2_{\min} = 2\,\mathrm{GeV}^2$$

$$W^2_{\min} = \left(Q^2 \frac{1-x}{x}\right)_{\min} = 12.5\,\mathrm{GeV}^2$$



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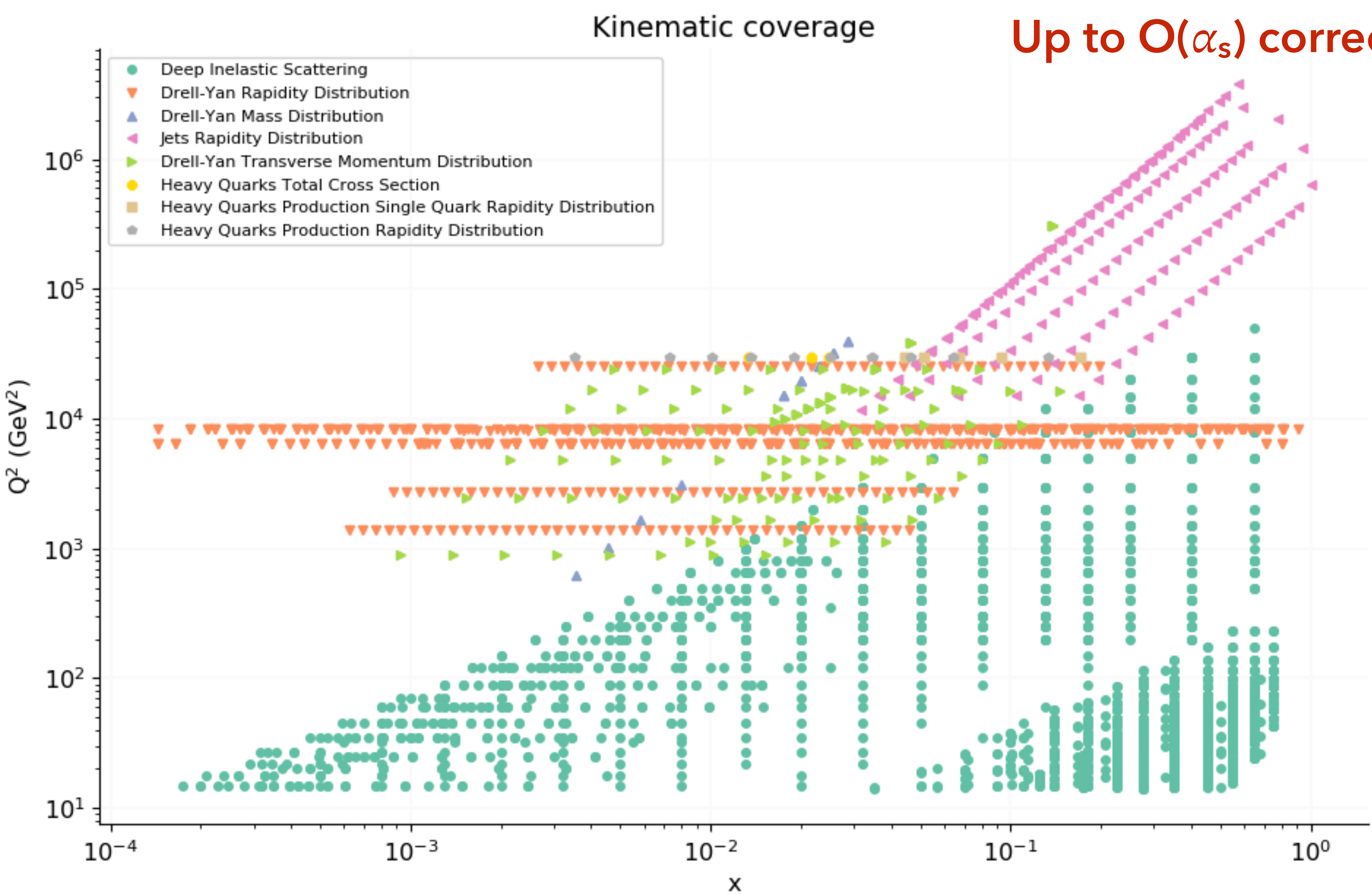
- Different data constrain different PDF combinations in different regions

➡ DIS data on proton abundant and precise (HERA)
➡ In principle F_2, F_3 CC provide 4 light quark combinations
F_2, F_3 NC provide 2 extra light quark combinations
➡ HERA data only determine four combinations of PDFs
➡ Old DIS and Drell-Yan data still used because of isospin symmetry
➡ W,Z boson final state provide lot of information, gluon from scale dependence
➡ Processes with jets and/or heavy quark in final states direct handle on the gluon

Disentangling PDFs

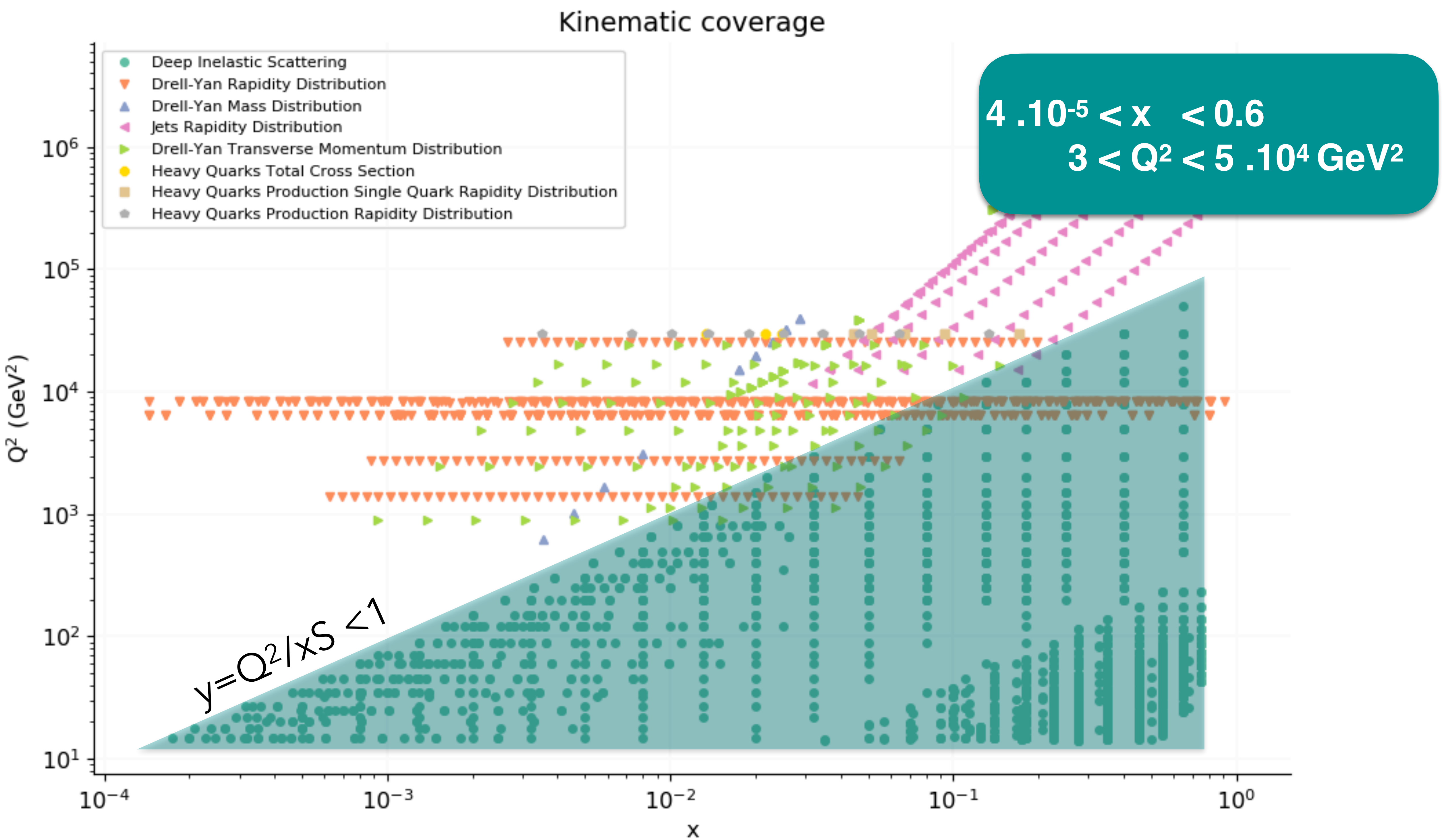


Up to $O(\alpha_s)$ corrections

Q-dependence: pert. theory

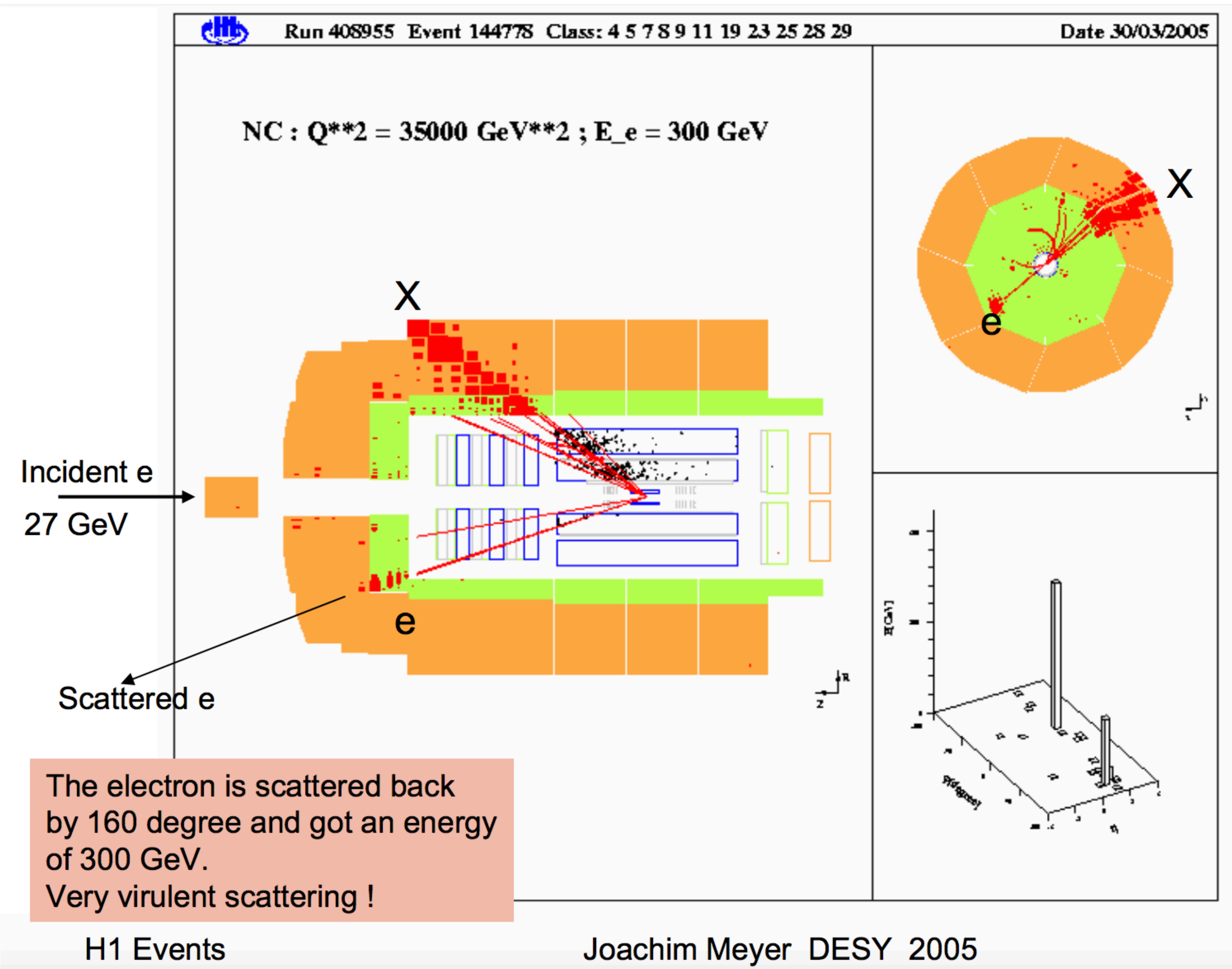
x-dependence: from data

HERA data



Kinematic coverage
Deep Inelastic Scattering
Drell-Yan Rapidity Distribution
Drell-Yan Mass Distribution
Jets Rapidity Distribution
Drell-Yan Transverse Momentum Distribution
Heavy Quarks Total Cross Section
Heavy Quarks Production Single Quark Rapidity Distribution
Heavy Quarks Production Rapidity Distribution
Q^2 (GeV^2)
x
10^1
10^2
10^3
10^4
10^5
10^6
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
$y=Q^2/xS<1$
$4 .10^{-5} < x < 0.6$
$3 < Q^2 < 5 .10^4$ GeV^2

HERA data



Neutral Current event

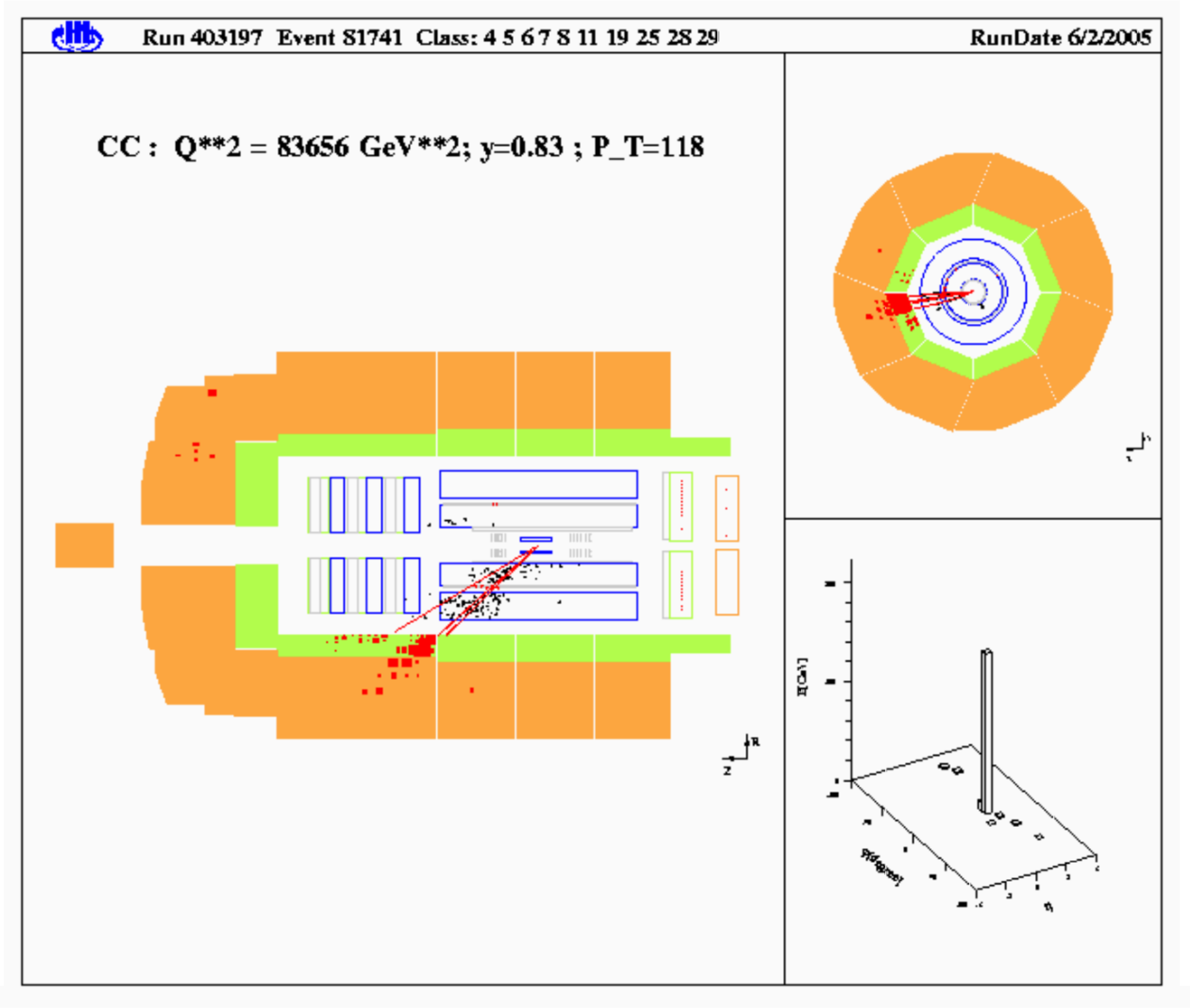
$$ep \to e\,X$$

$$Q = 180\,\mathrm{GeV}$$

$$y = 0.66$$

$$x_B = 0.47$$

HERA data



Charged Current event

$$ep \to \nu X$$

$$Q = 289\,\mathrm{GeV}$$

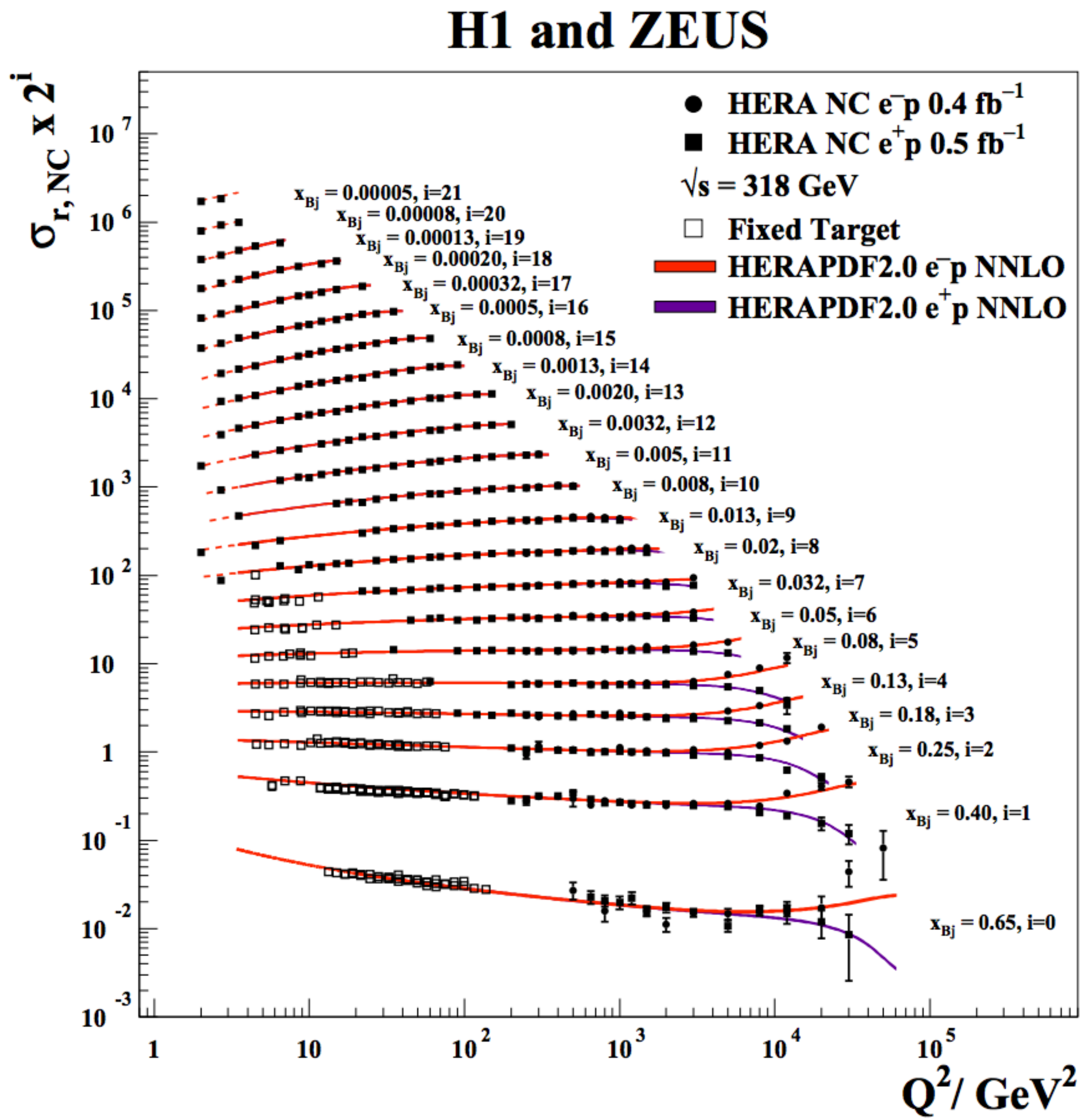
$$y = 0.83$$

$$x_B = 0.91$$

Joachim Meyer DESY 2005

HERA data

- Combination of Run I + Run II data led to very precise measurements of reduced xsec
- F3 contribution visible at larger x and Q~Mz



arXiv: 1506.06042

HERA data

Neutral Current

$$\left[F_2^\gamma, F_2^{\gamma Z}, F_2^Z\right] = x \sum_{i=1}^{n_f} \left[e_i^2, 2e_i g_V^i, (g_V^i)^2 + (g_A^i)^2\right] (q_i + \bar{q}_i)$$

③ ④

$$\left[F_3^\gamma, F_3^{\gamma Z}, F_3^Z\right] = x \sum_{i=1}^{n_f} \left[0, 2e_i g_A^i, 2g_V^i g_A^i\right] (q_i - \bar{q}_i)$$

Charged Current

①

$$F_2^{W^-} = 2x\left(u + \bar{d} + \bar{s} + c\right),$$

$$F_3^{W^-} = 2x\left(u - \bar{d} - \bar{s} + c\right),$$

②

$$F_2^{W^+} = 2x\left(d + \bar{u} + \bar{c} + s\right),$$

$$F_3^{W^+} = 2x\left(d - \bar{u} - \bar{c} + s\right),$$

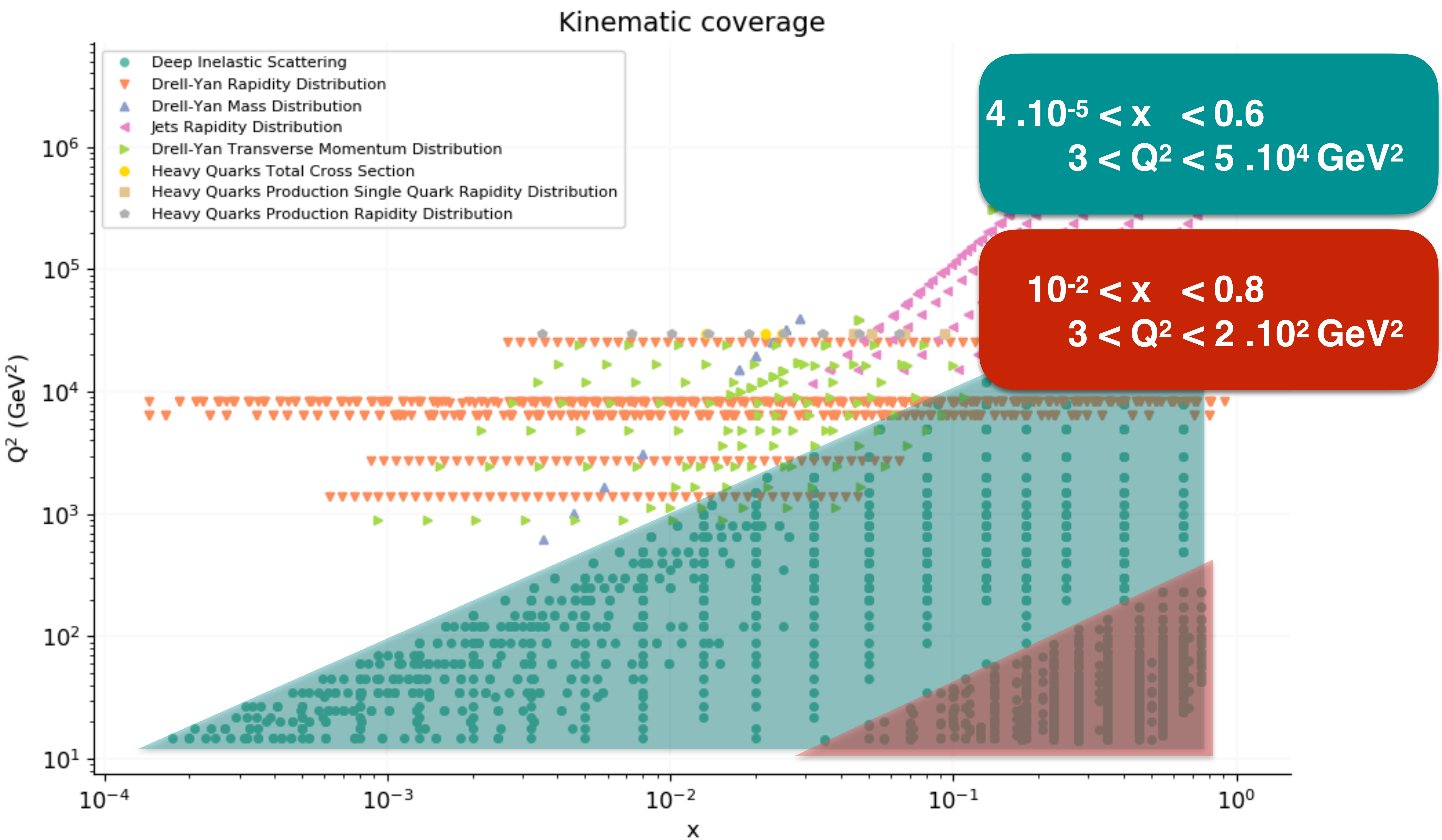
$$\frac{d^2\sigma}{dx\,dQ^2} \propto Y_+ F_2(x, Q^2) \mp Y_- x F_3(x, Q^2) - y^2 F_L(x, Q^2)$$

Longitudinal Structure function

$$F_L(x, Q^2) = \frac{\alpha_s(Q^2)}{\pi}\left[\frac{4}{3}\int_x^1 \frac{dy}{y}\left(\frac{x}{y}\right)^2 F_2(y, Q^2) + 2\sum_i e_i^2 \int_x^1 \frac{dy}{y}\left(\frac{x}{y}\right)^2\left(1 - \frac{x}{y}\right) g(y, Q^2)\right]$$

Altarelli, Martiinelli Phys. Lett. 76B (1978)

Fixed target DIS data



Fixed target DIS data

- Experimentally measured is deuteron structure function

$$F_2^d = (F_2^p + F_2^n)/2$$

- Assumption (SU(2) isospin): neutron is just like proton with u ⇔ d

proton = uud
neutron = ddu

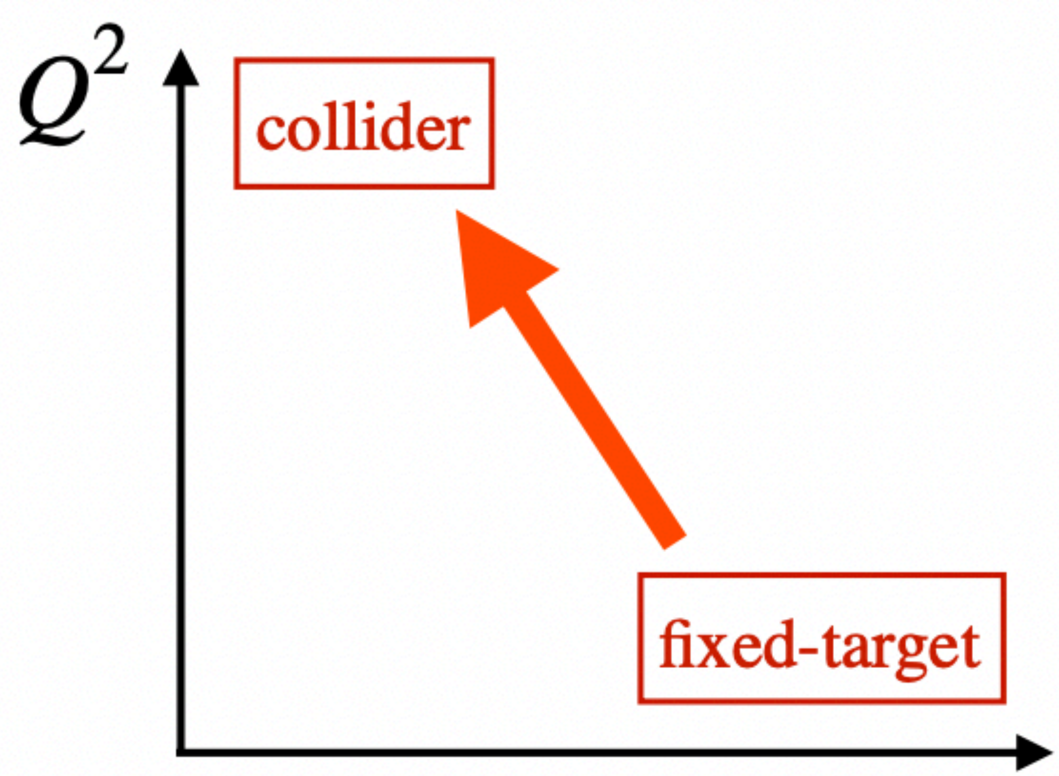
⇒ $\mathbf{u_n(x)=d_p(x)}$ and $\mathbf{d_n(x) = u_p(x)}$

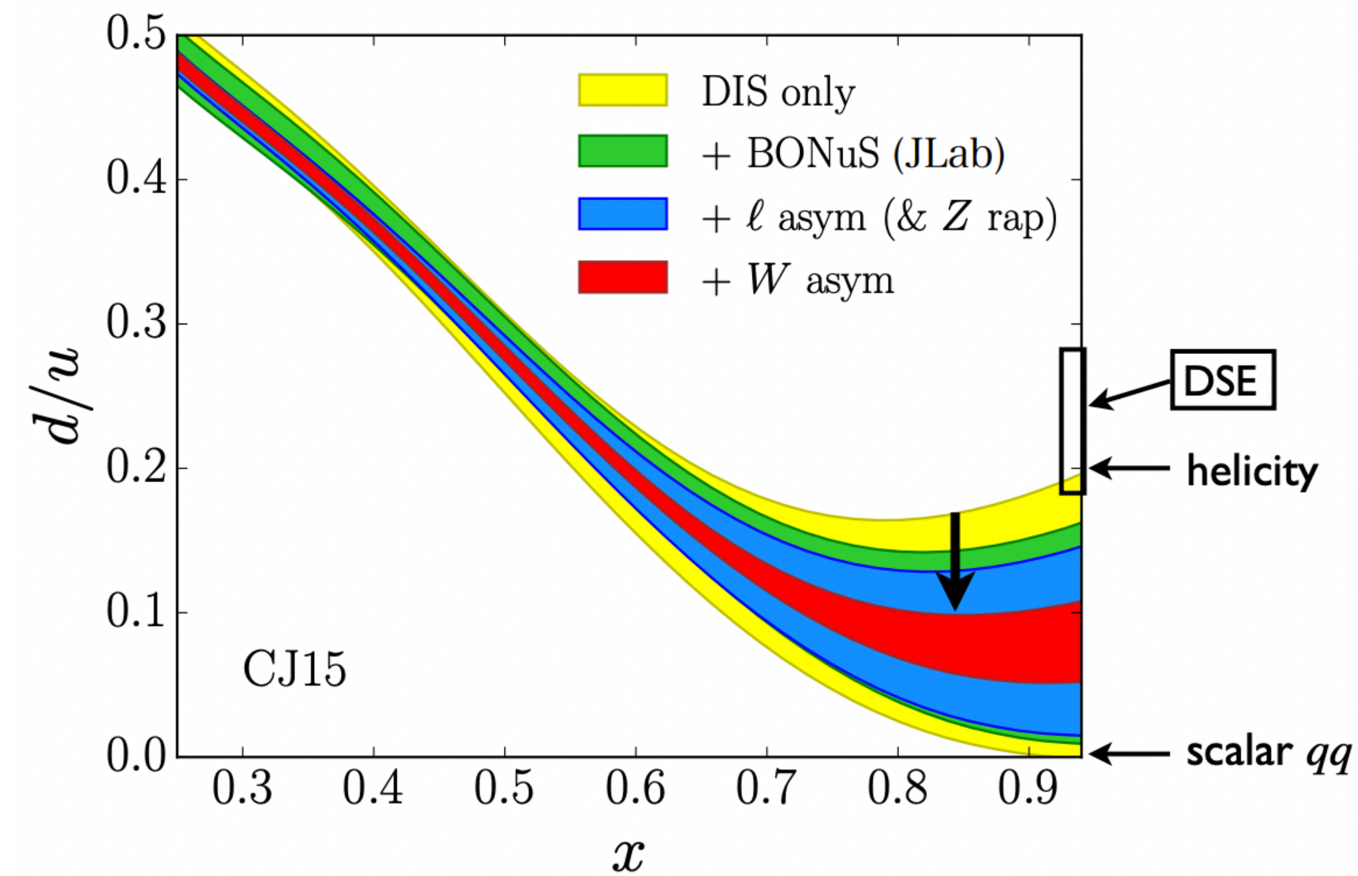
- Linear combinations of F_2^p and F_2^n give separately $u_p(x)\equiv u(x)$ and $d_p(x)\equiv d(x)$

$$F_2^p(x,Q^2) - F_2^d(x,Q^2) = \frac{1}{3}(u + \bar{u} - d - \bar{d})$$

$$\frac{F_2^d(x)}{F_2^p(x)} \sim \frac{u}{d}$$

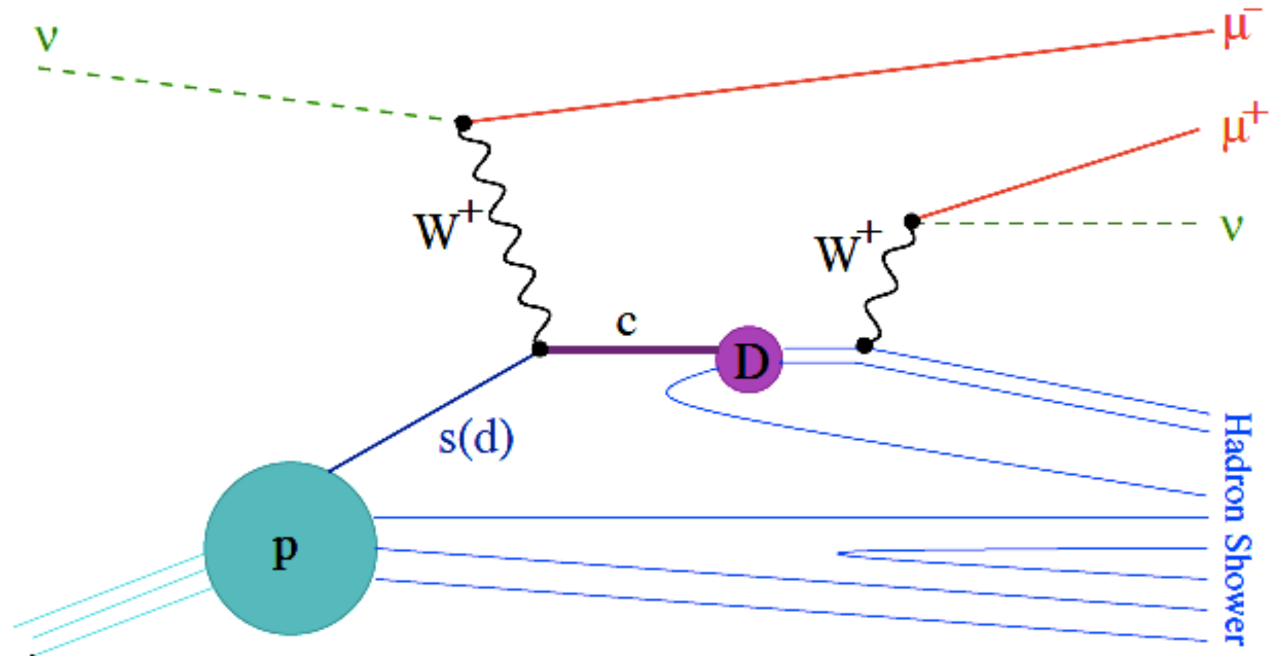
Fixed target DIS data





- Valence d/u ratio at high-x accessible thanks to low-Q2 fixed-target experiments (SLAC, BCDMS, NMC, CHORUS, NuTeV, JLAB)
- Testing ground for nucleon models in the x → 1 limit
- At high-x nuclear corrections are very important (deuteron target)
- Tevatron W asymmetry data and JLab tagged neutron help constraining d/u ratio up to large values of x ~ 0.85

Fixed target DIS data

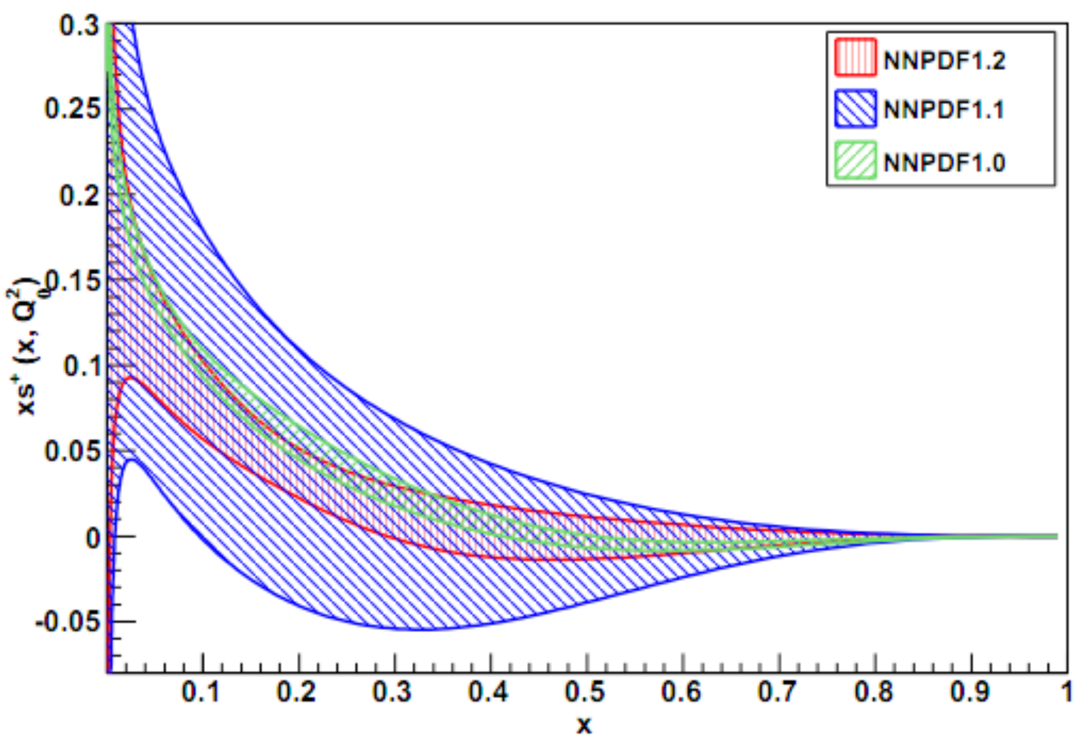


$$\tilde{\sigma}^{\nu(\bar{\nu}),c} \propto (F_2^{\nu(\bar{\nu}),c}, F_3^{\nu(\bar{\nu}),c}, F_L^{\nu(\bar{\nu}),c})$$

$$F_2^{\nu,c} = x\left[C_{2,q} \otimes 2|V_{cs}|^2 s + \frac{1}{n_f} C_{2,g} \otimes g\right]$$

$$F_2^{\bar{\nu},c} = x\left[C_{2,q} \otimes 2|V_{cs}|^2 \bar{s} + \frac{1}{n_f} C_{2,g} \otimes g\right]$$

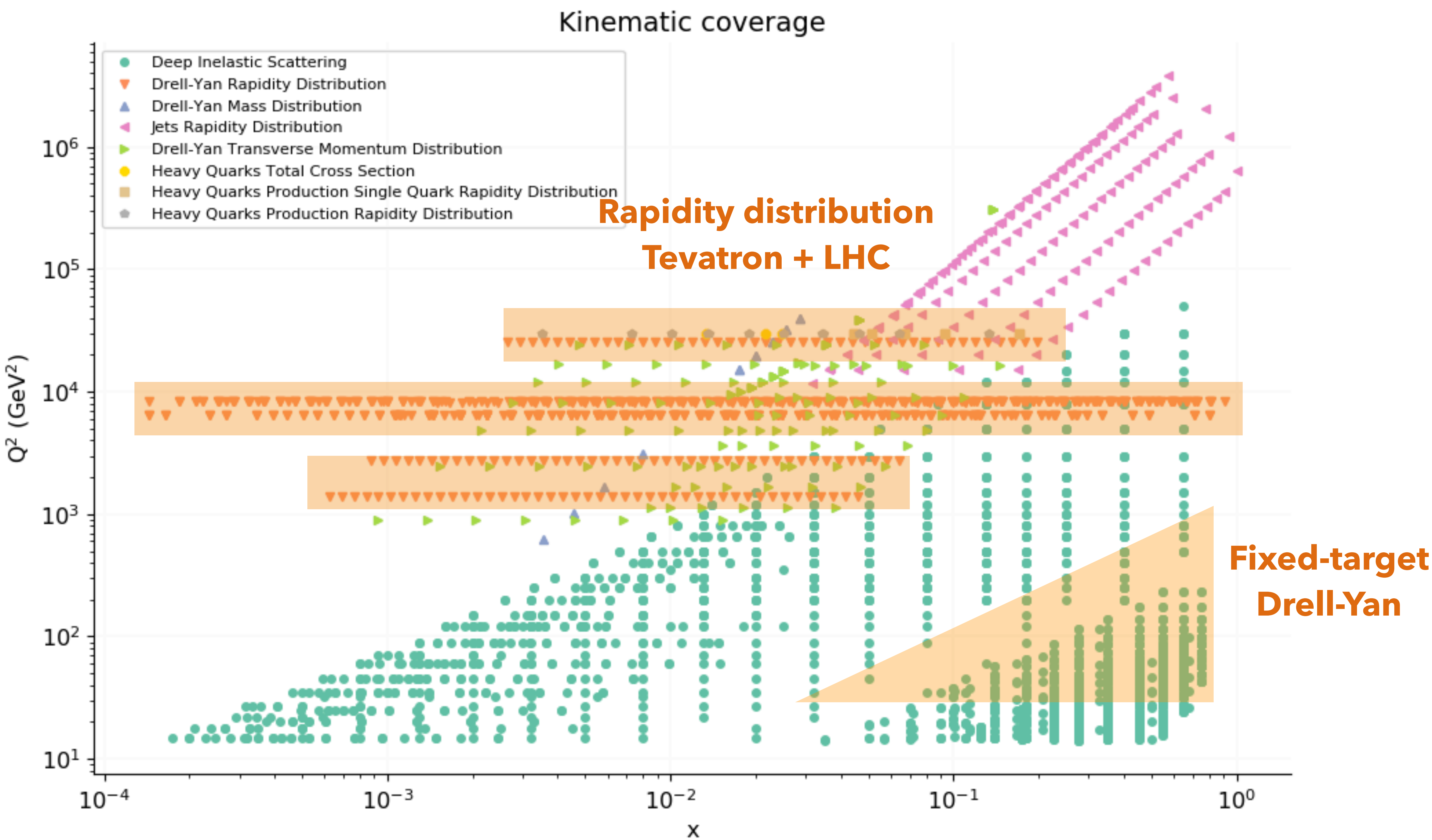
V_{cs} enhancement



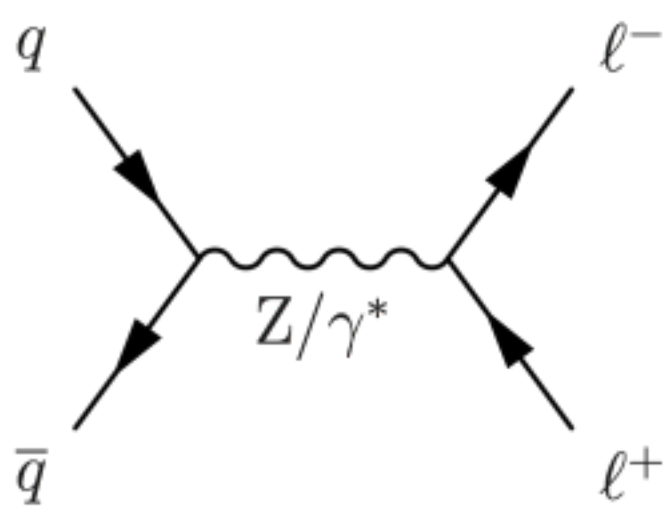
$$x = \frac{Q^2}{2M_n E_\nu y}$$

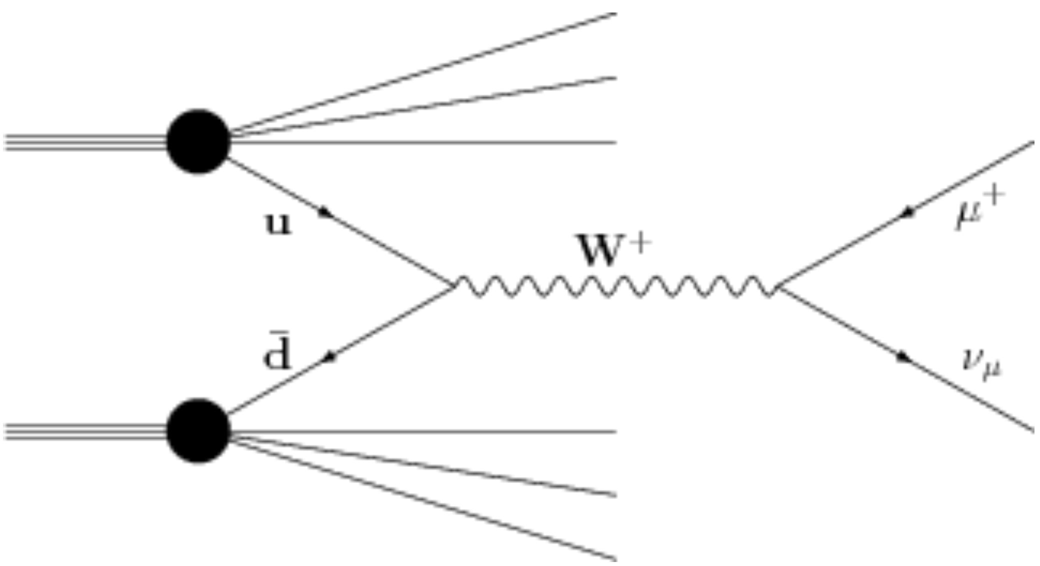
- NuTeV structure function data on isoscalar iron target provide strong constraints on strangeness inside the proton
- Some (mild) tension between fixed target data and W+c data at the LHC

Drell-Yan/V production data



Drell-Yan/V production data





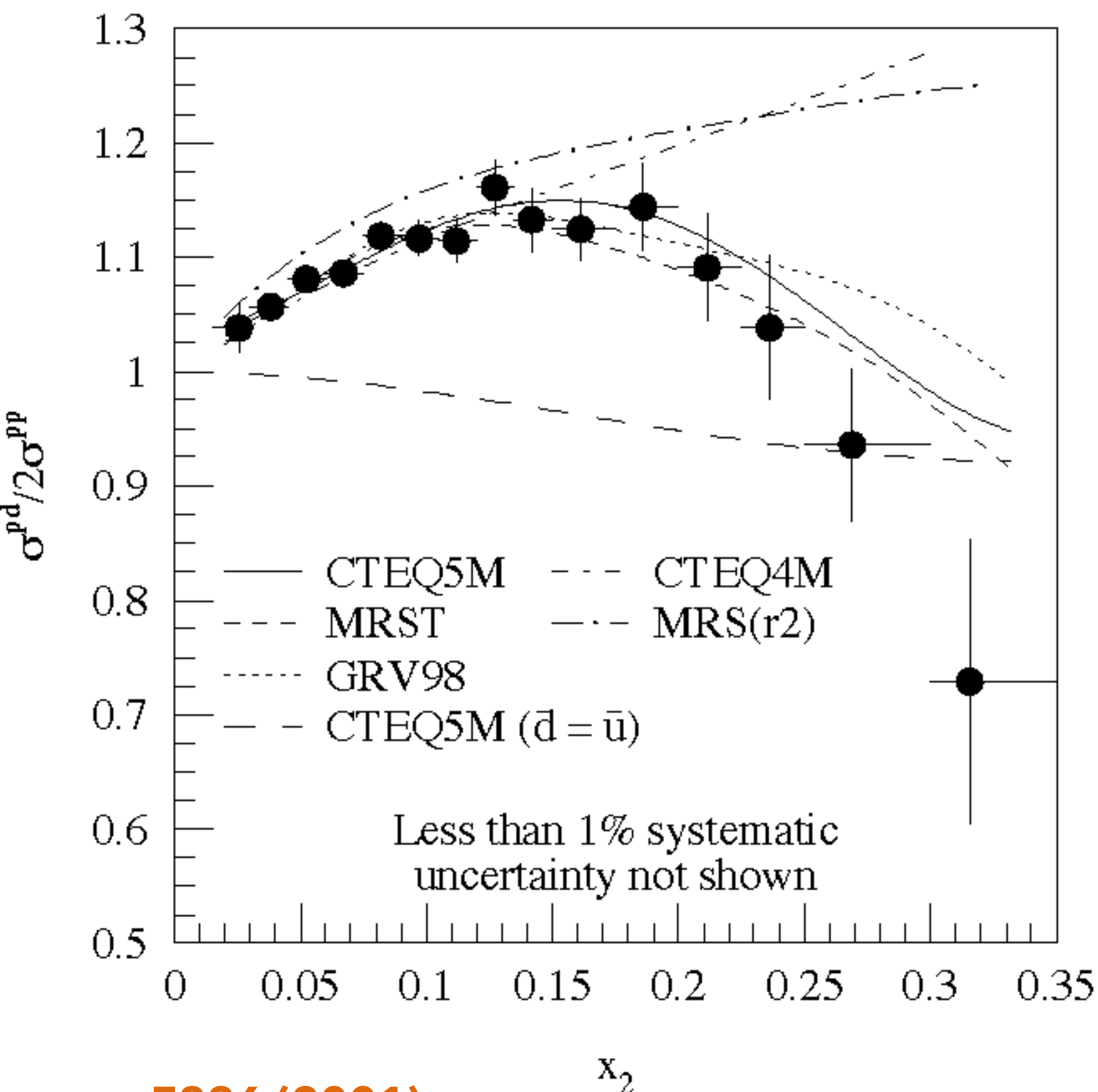
$$L_{ij}(x_1, x_2) = q_i(x_1)\bar{q}_j(x_2)$$

$$\gamma^* : \quad \frac{d\sigma}{dydM^2} = \frac{4\pi\alpha^2}{9M^2S}\sum_i e_i^2 L_{ij}(x_1, x_2)$$

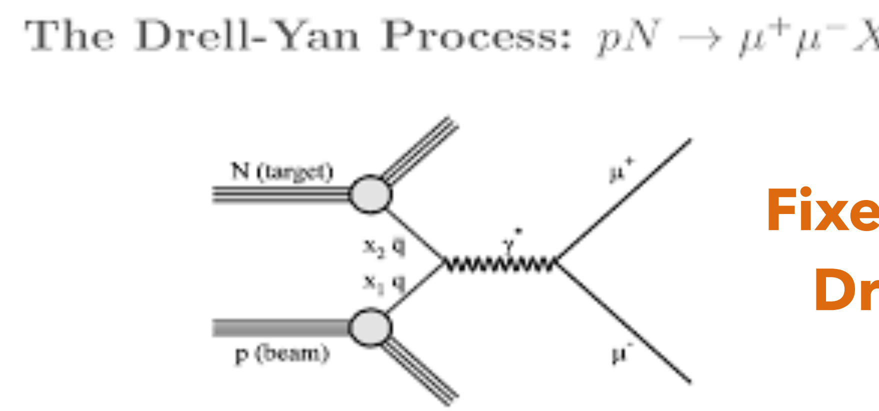
$$Z : \quad \frac{d\sigma}{dy} = \frac{\pi G_F M_V^2 \sqrt{2}}{3S}\sum_i (v_{iZ}^2 + a_{iZ}^2) L_{ij}(x_1, x_2)$$

$$W : \quad \frac{d\sigma}{dydM^2} = \frac{\pi G_F M_V^2 \sqrt{2}}{3S}\sum_{ij} |V_{ij}^{\mathrm{CKM}}|^2 L_{ij}(x_1, x_2)$$

Drell-Yan data

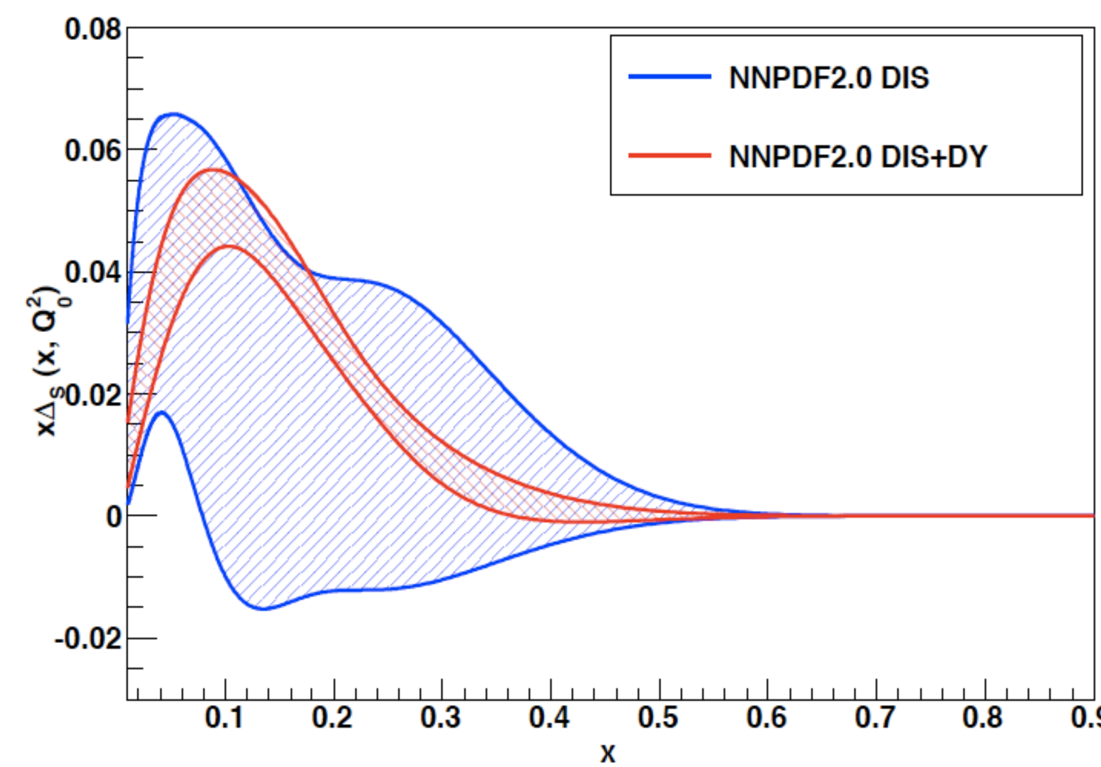


E886 (2001)

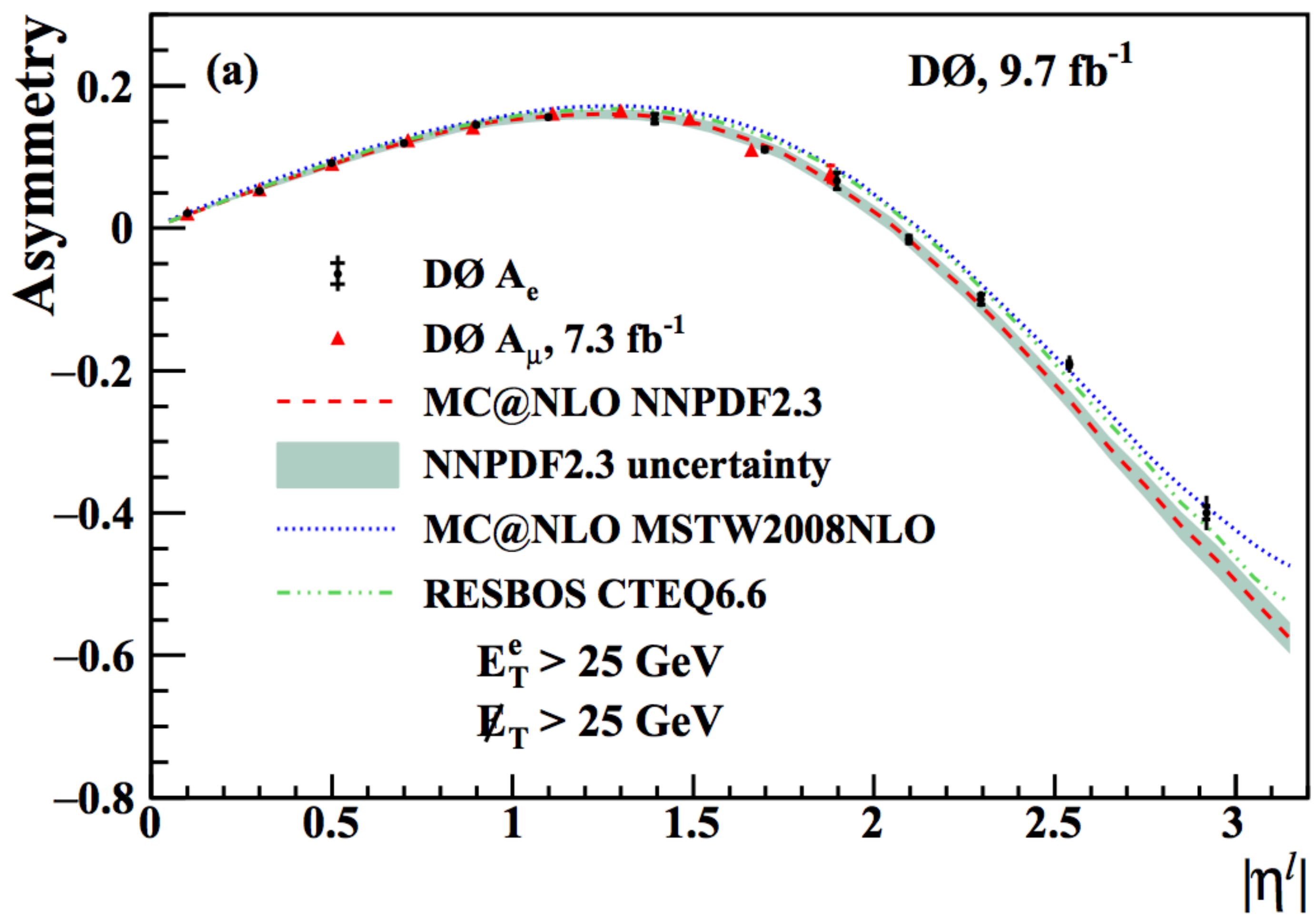


Fixed-target Drell-Yan

$$\frac{\sigma(pd \to \mu^+\mu^-)}{\sigma(pp \to \mu^+\mu^-)} = \frac{\frac{4}{9}u\bar{d} + \frac{1}{9}d\bar{u}}{\frac{4}{9}u\bar{u} + \frac{1}{9}d\bar{d}} \sim \frac{\bar{d}}{\bar{u}}$$



Z/W production data



W asymmetry at Tevatron

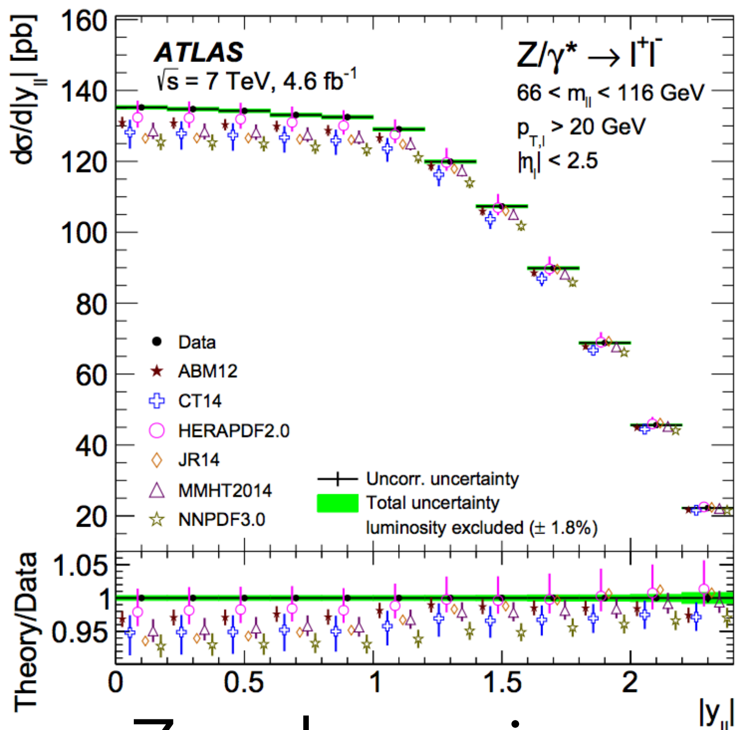
$$u^{\bar{p}} = \bar{u}^p$$
$$d^{\bar{p}} = \bar{d}^p$$

Charge conjugation

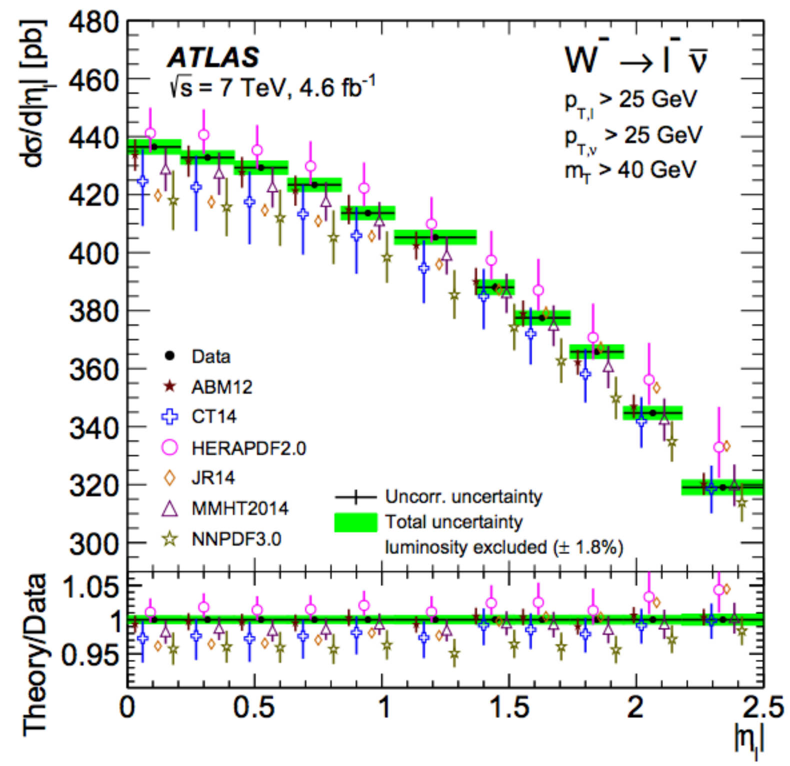
$$\frac{\sigma(p\bar{p} \to W^+)}{\sigma(p\bar{p} \to W^-)} = \frac{u(x_1)d(x_2) + \bar{u}(x_1)\bar{d}(x_2)}{d(x_1)u(x_2) + \bar{d}(x_1)\bar{u}(x_2)} \sim \frac{u}{d}(x_1)\frac{u}{d}(x_2)$$

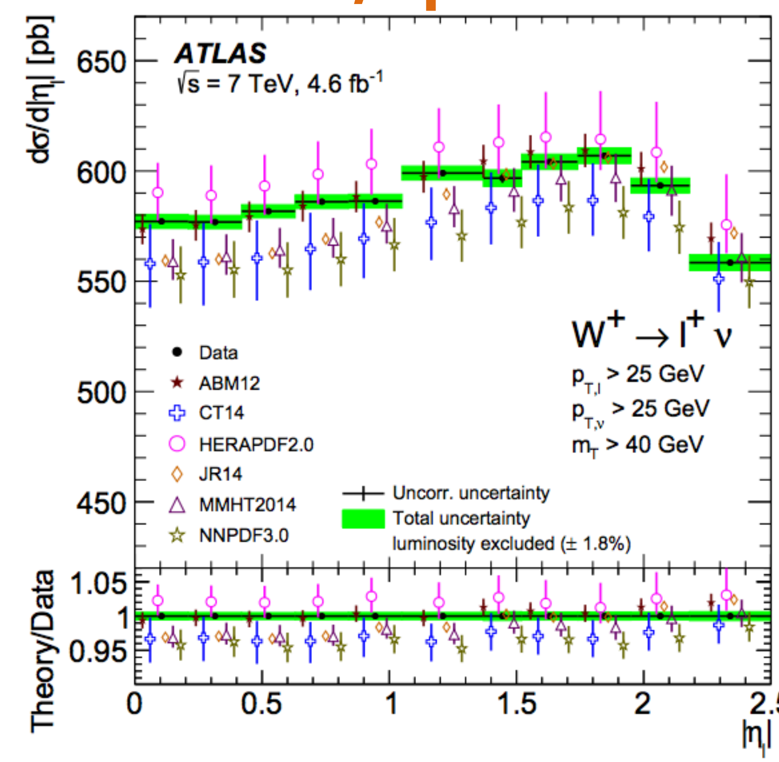
Z/W production data

ATLAS W,Z production 7 TeV



Z pole region





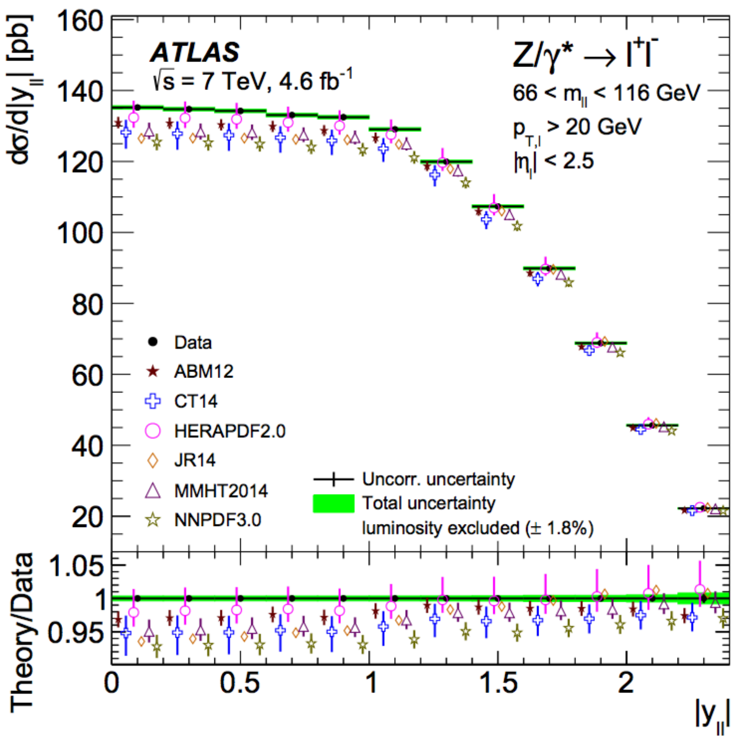
$$\sigma(pp \to Z) = u\bar{u} + d\bar{d} + s\bar{s}$$

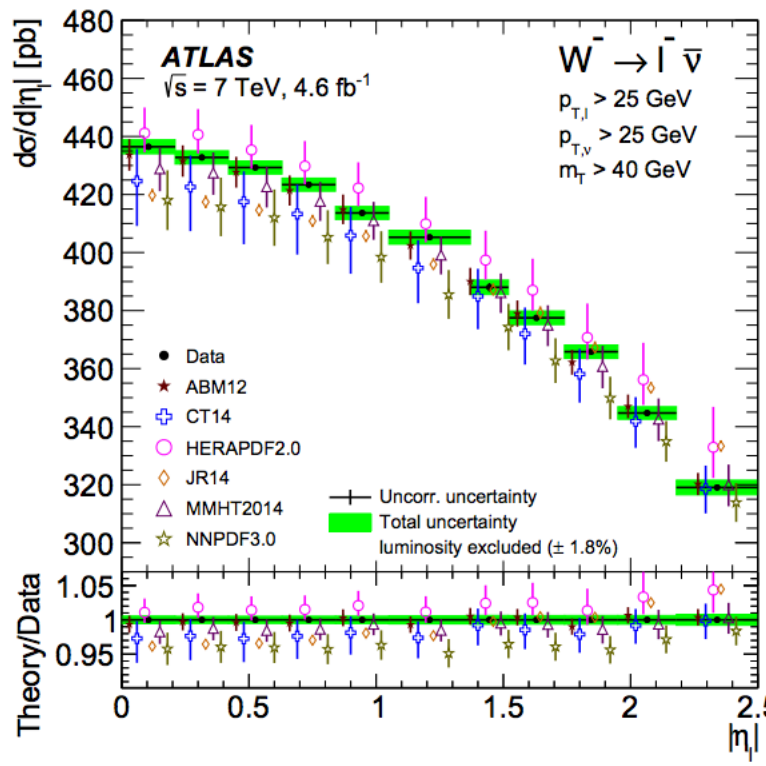
$$\sigma(pp \to W^+) = u\bar{d} + c\bar{s}$$

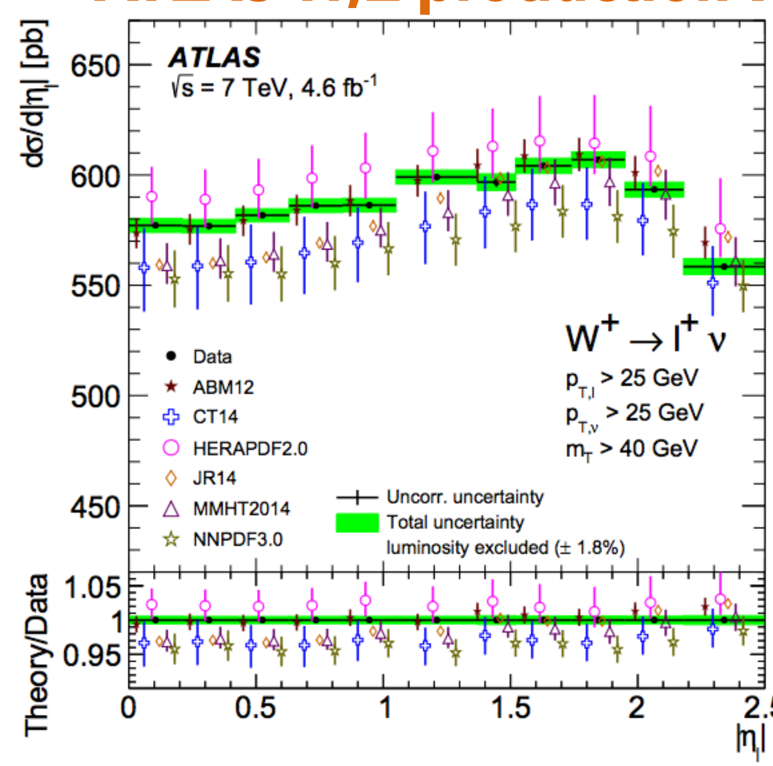
$$\sigma(pp \to W^-) = d\bar{u} + s\bar{c}$$

Z/W production data

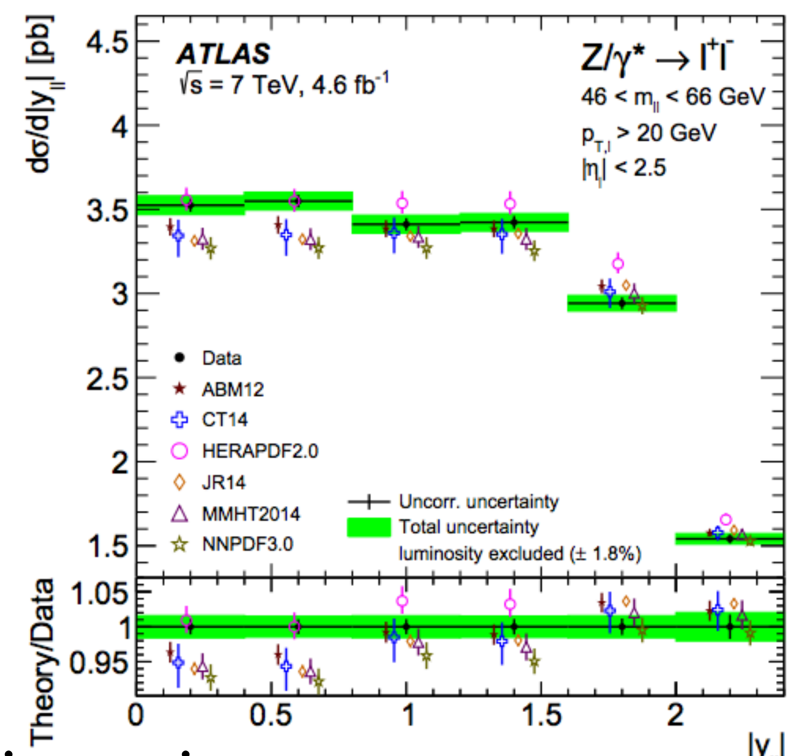
ATLAS W,Z production 7 TeV

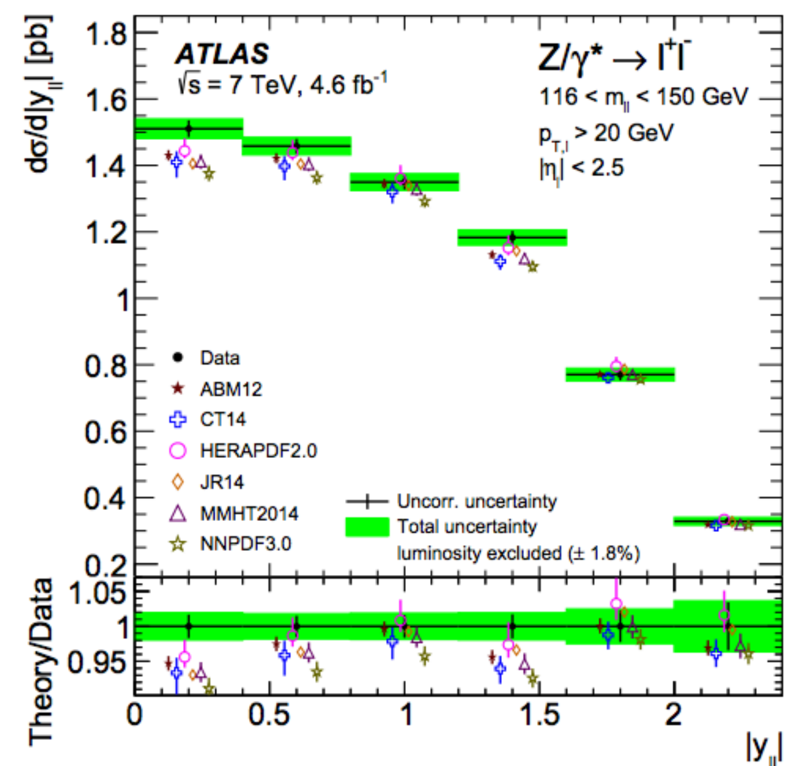






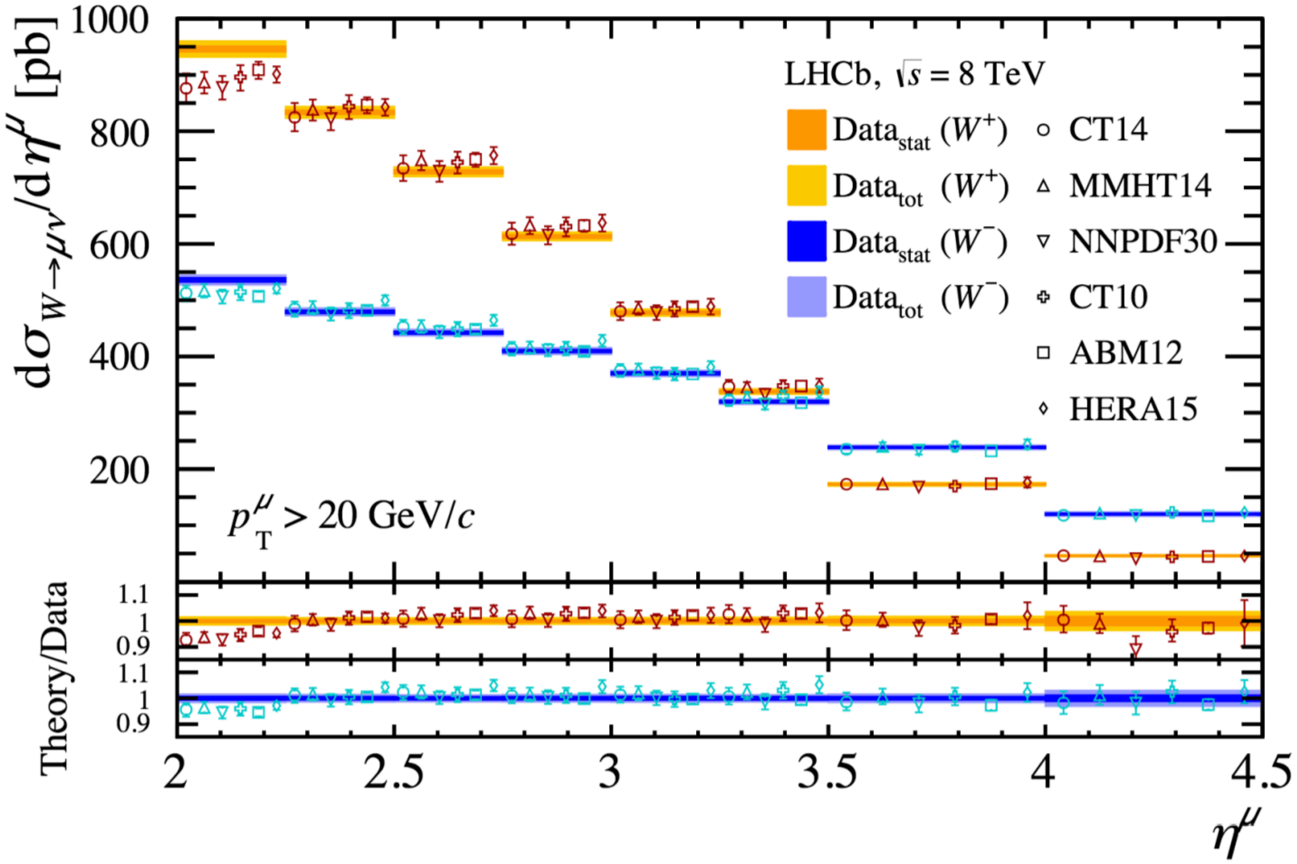
$$x_{1,2} = \frac{M}{\sqrt{S}} e^{\pm y}$$

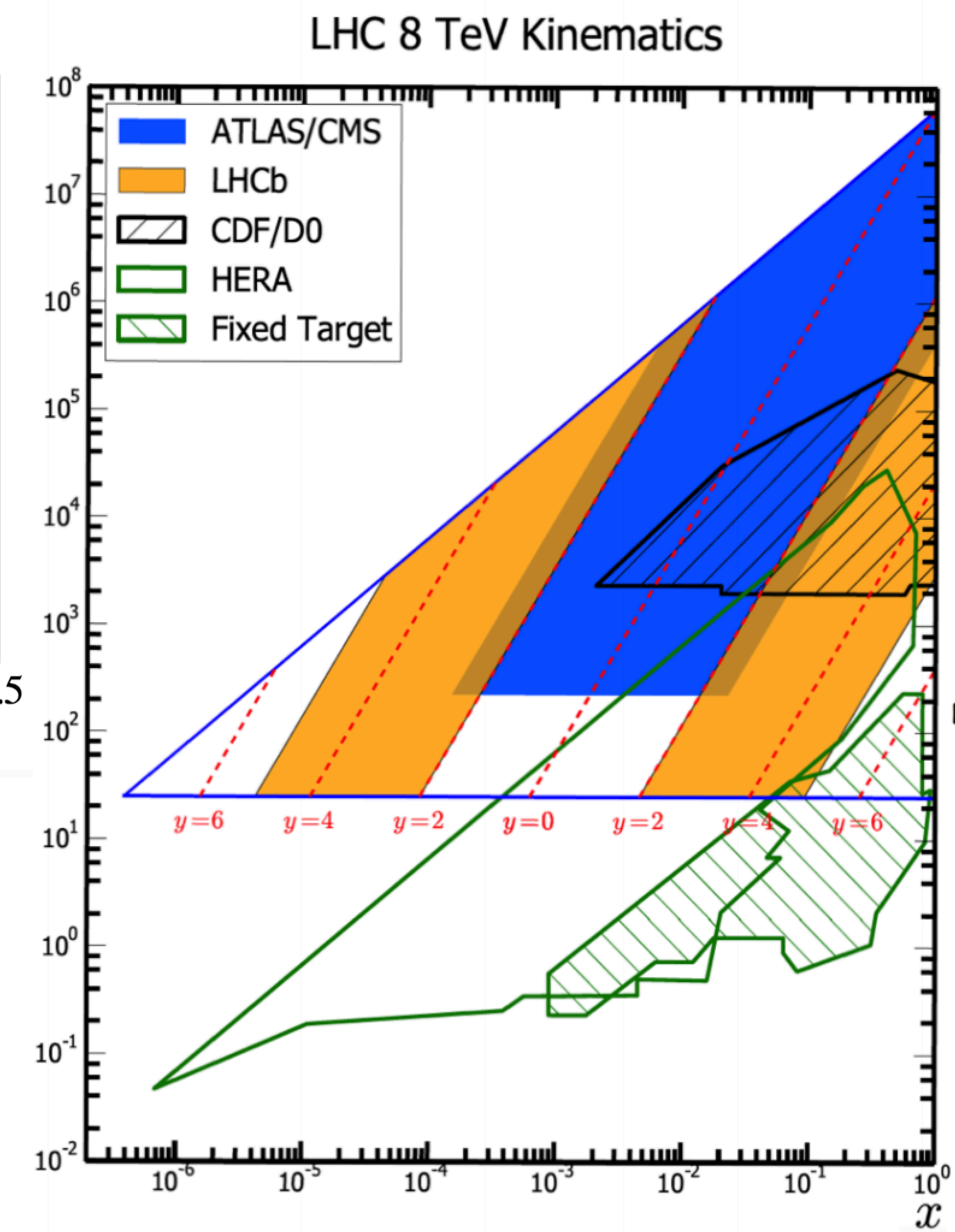




<u>high/low invariant mass</u>

Z/W production data

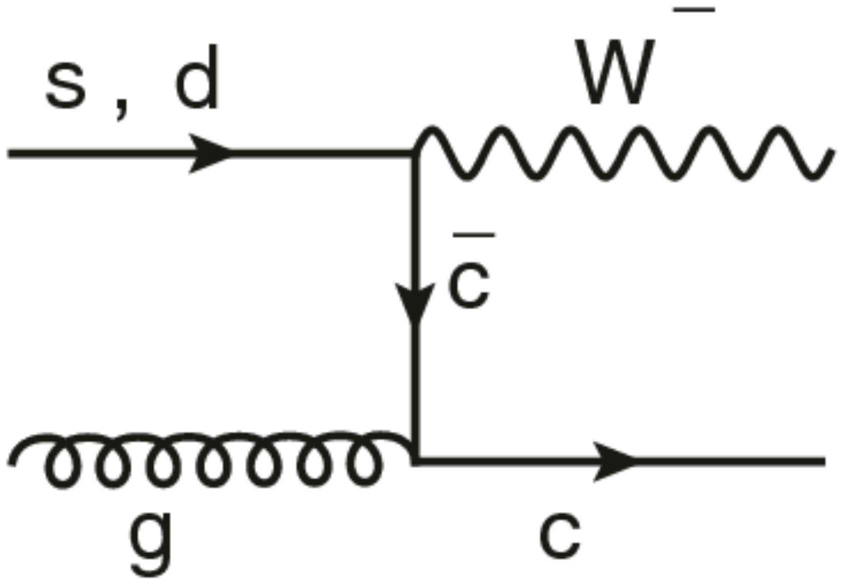


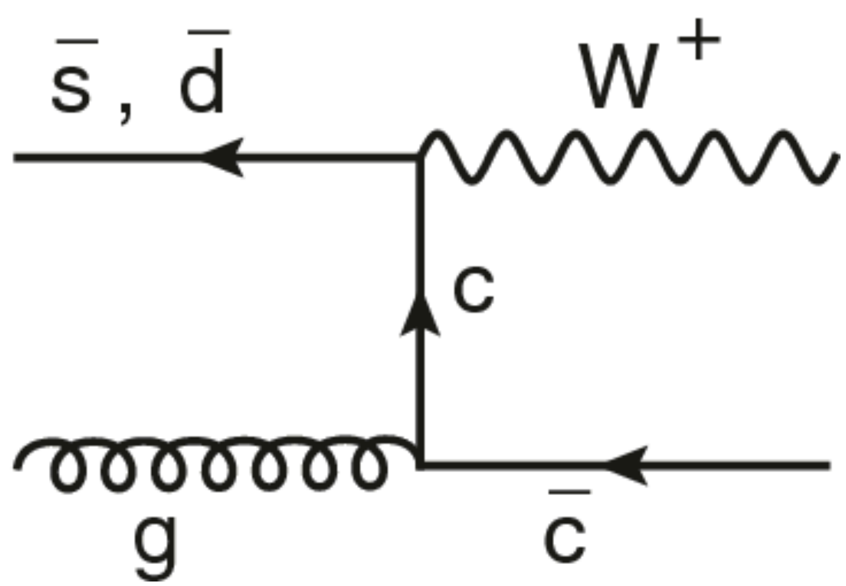


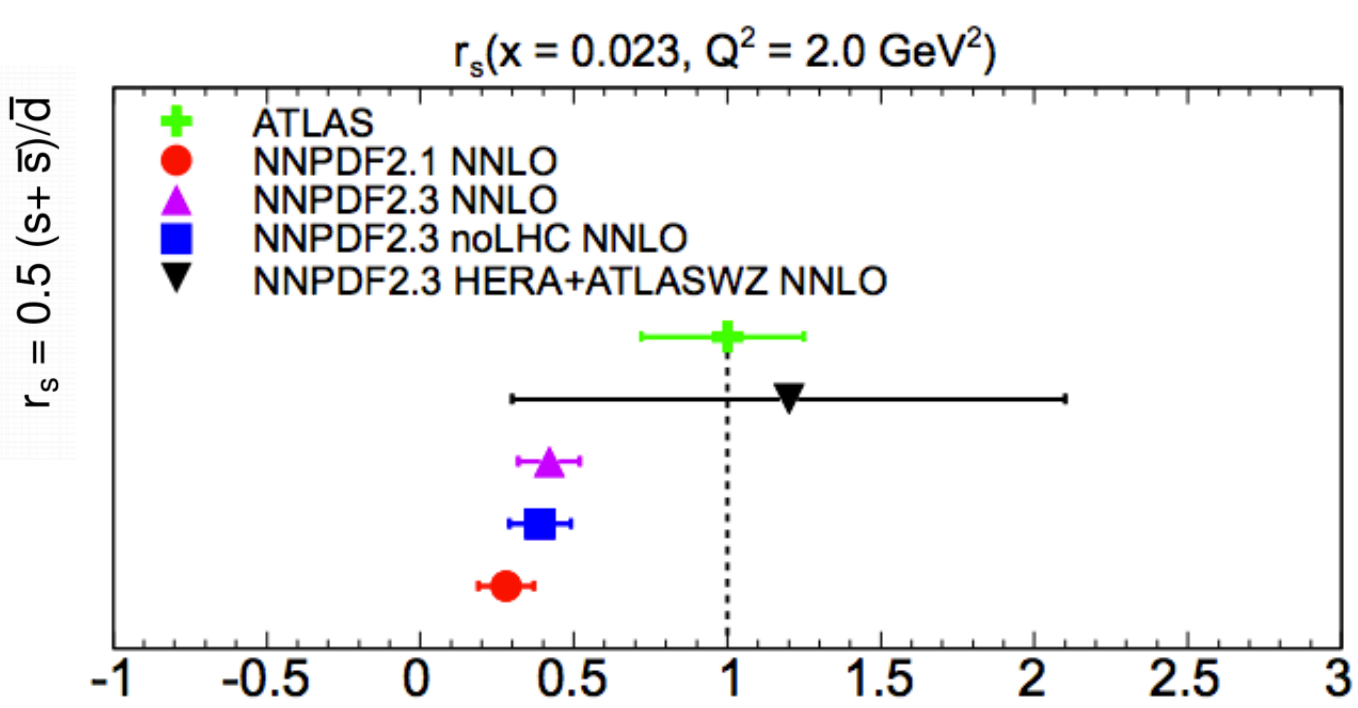
$$x_{1,2} = \frac{M^2}{s} e^{\pm y}$$

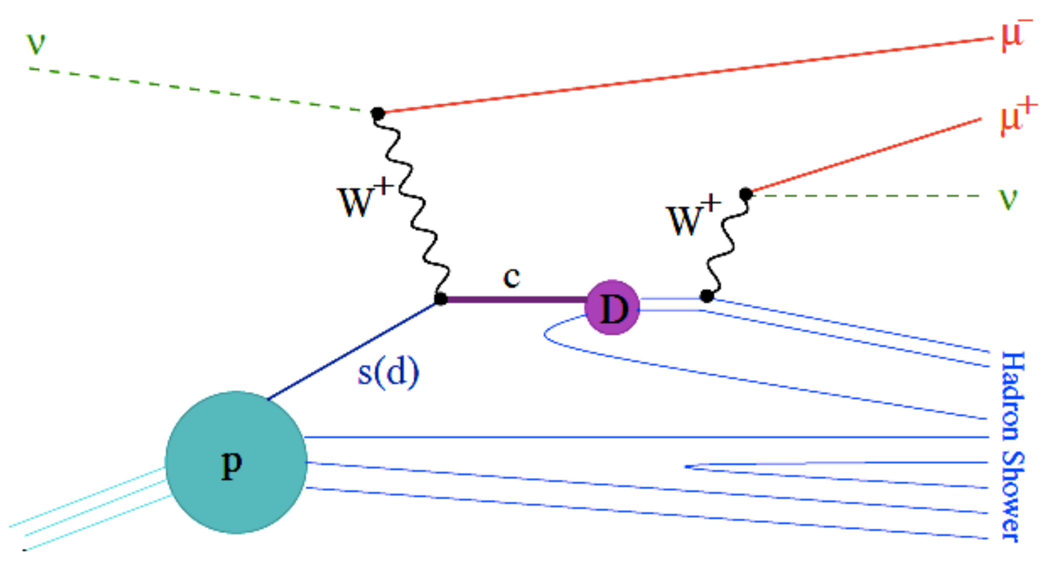
LHCb: forward region

W+charm data


s , d
W⁻
c̄
g
c


s̄ , d̄
W⁺
c
g
c̄

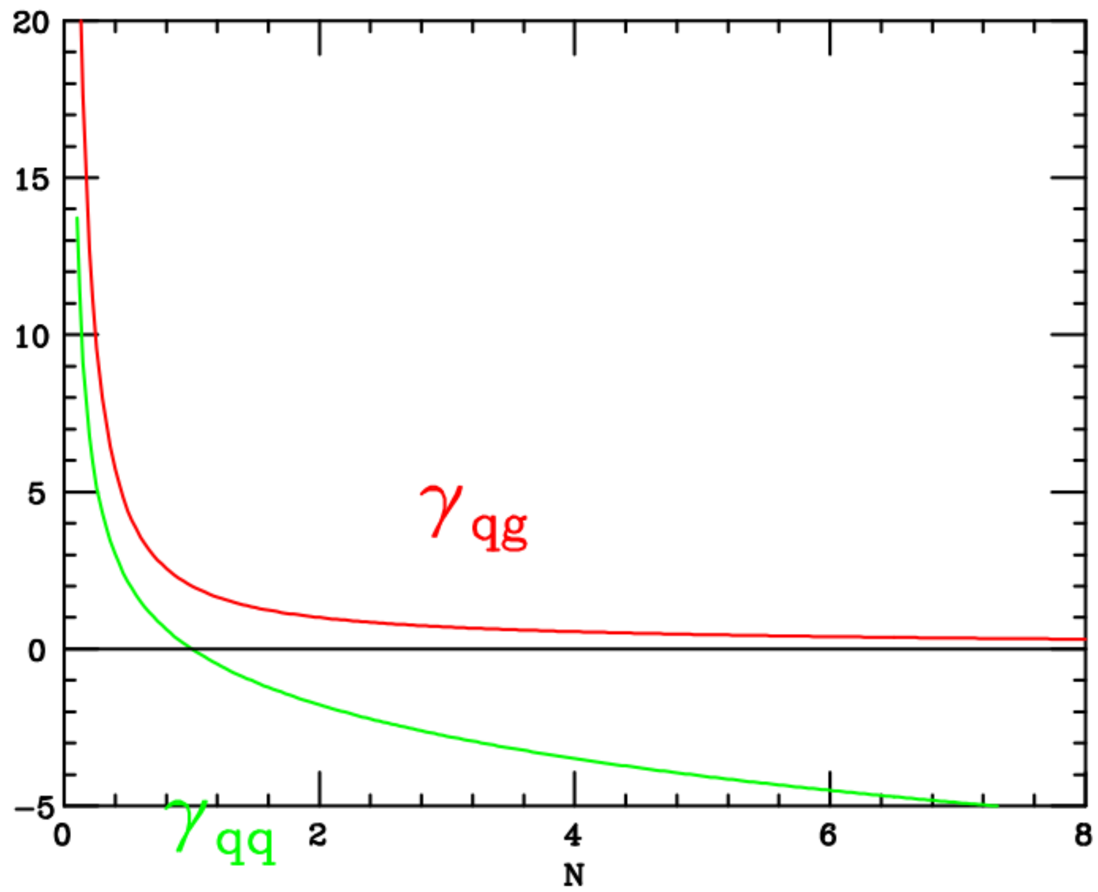

$r_s(x = 0.023, Q^2 = 2.0\ GeV^2)$
ATLAS
NNPDF2.1 NNLO
NNPDF2.3 NNLO
NNPDF2.3 noLHC NNLO
NNPDF2.3 HERA+ATLASWZ NNLO
$r_s = 0.5\ (s+\bar{s})/\bar{d}$
-1
-0.5
0
0.5
1
1.5
2
2.5
3


ν
μ⁻
μ⁺
W⁺
W⁺
ν
c
D
s(d)
p
Hadron Shower

Gluon: indirect handle

- Gluon is partially determined by scale dependence of DIS structure functions and Drell-Yan/Vector Boson production

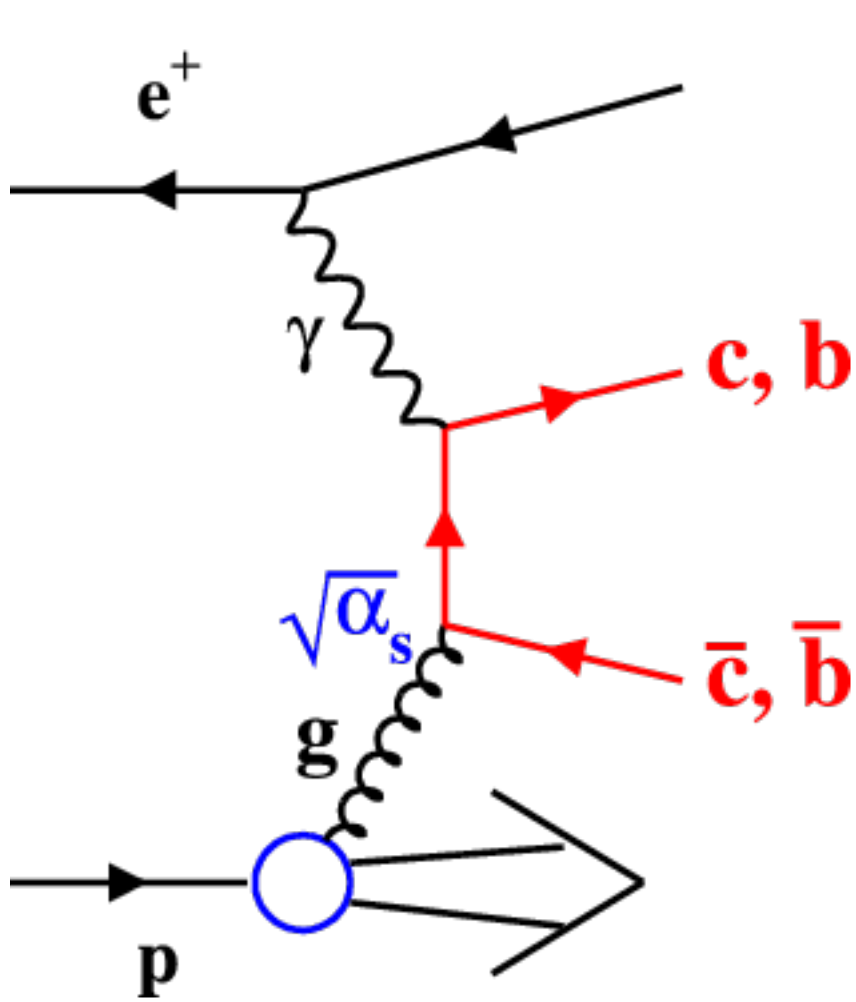
$$\frac{d}{d\log\mu^2}F_2(x,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi}\left[P_{qq}\otimes F_2(x,\mu^2) + 2n_f P_{qg}\otimes g(x,\mu^2)\right]$$



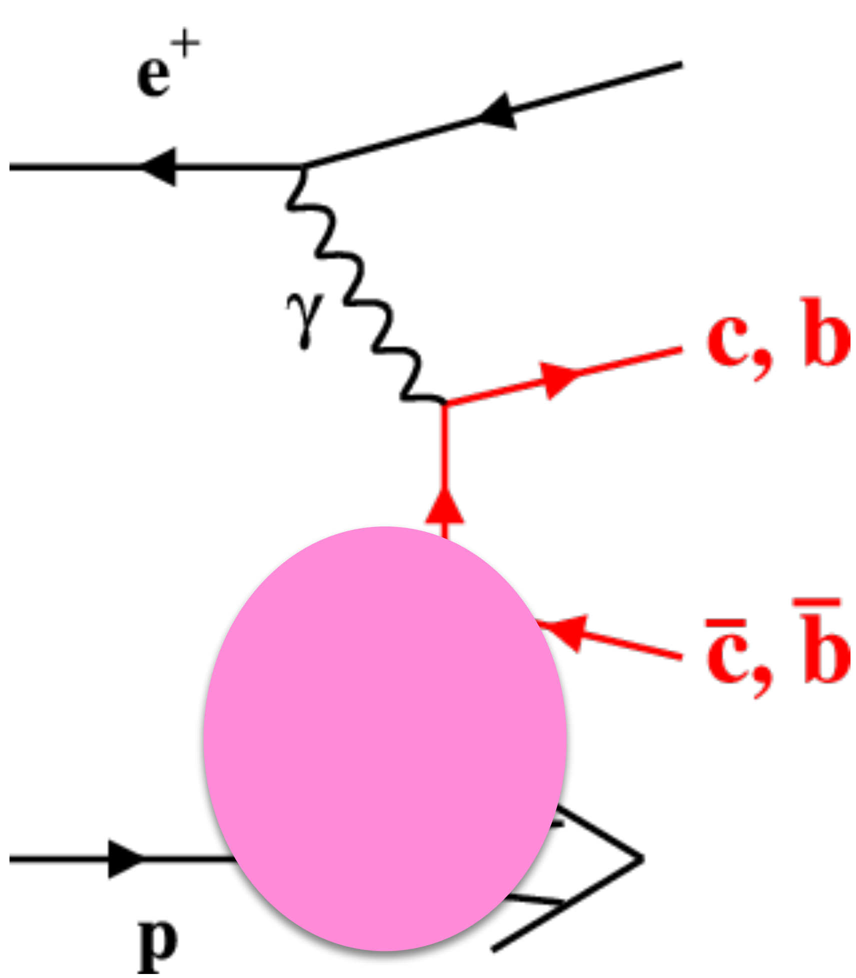
- Mostly determine small-x gluon, large-x gluon hard to determine from DIS+DY only data

Gluon: indirect handle

- Heavy quarks are produced at threshold inside proton
- Heavy quark production process (at ep and pp colliders) probe gluon
- Dependence on heavy flavour scheme adopted in PDF fitting



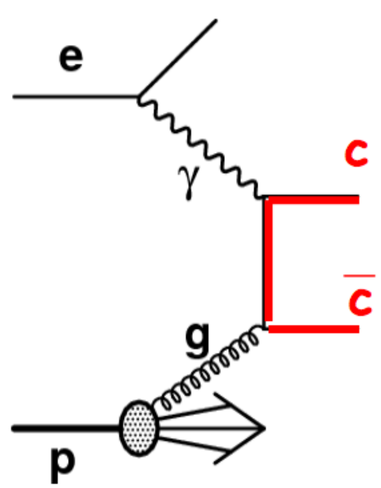
Nf = 3,4

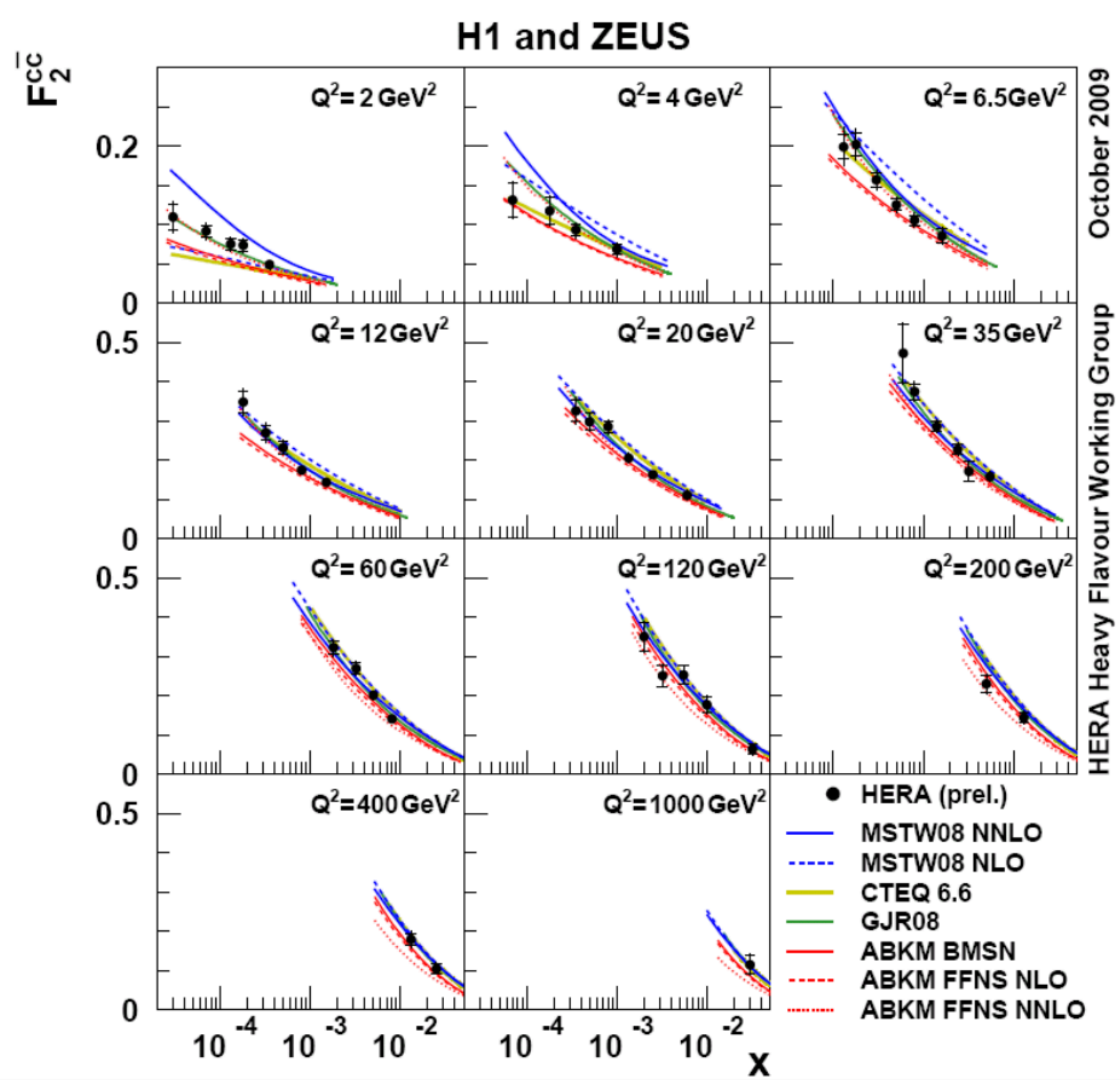


Nf = 5

Gluon: indirect handle

- Heavy quarks are produced at threshold inside proton
- Heavy quark production process (at ep and pp colliders) probe gluon
- Dependence on heavy flavour scheme adopted in PDF fitting





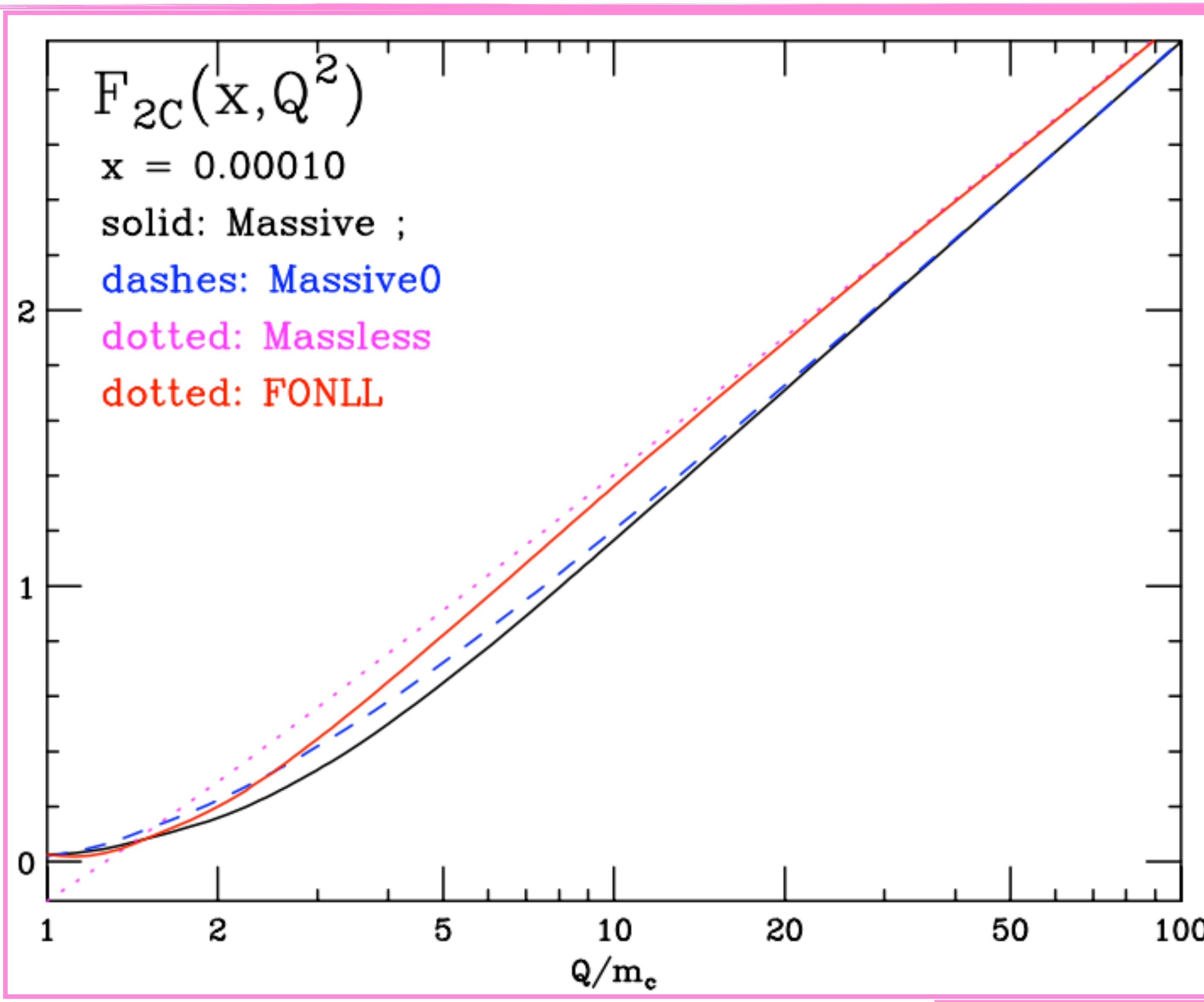
Intermission: heavy flavour schemes

- Charm, Bottom and Top have mass >> Λ_{QCD} - heavy quarks (HQ)
- The presence of a new scale, m_Q, makes pert QCD calculations more challenging
- Two well understood schemes:
  - Assume heavy quark effectively massless for $Q > m_Q$
    HQ becomes active massless parton above threshold
  - Heavy quarks retain their mass for all Q
    HQ is not a parton, it is a final state particle
- However in PDF fits we have all scales. General-Mass Variable-Flavor-Number schemes allow to match between the zero-mass and the massive scheme
- Many schemes available

e.g. FONLL

$$\begin{aligned}\sigma^{(FONLL)} &= \sigma^{(4)} + \sigma^{(5)} - \text{double counting} \\ &= \mathcal{L}_{ij}(x_1, x_2, \mu^2) \otimes \sum_p^N \left(\alpha_S^{(5)}(\mu^2)\right)^p \\ &\times \left\{ \mathcal{B}_{ij}^{(p)}\left(x_1, x_2, \frac{\mu^2}{m_b^2}\right) + \sum_{k=0}^{\infty} \mathcal{A}_{ij}^{(p),(k)}(x_1, x_2)\left(\alpha_S^{(5)}(\mu^2) L\right)^k \right\} \\ &- \text{double counting}\end{aligned}$$

Intermission: heavy flavour schemes



- heavy quarks (HQ)

D calculations more challenging

less for Q > mQ
above threshold

Q2
article

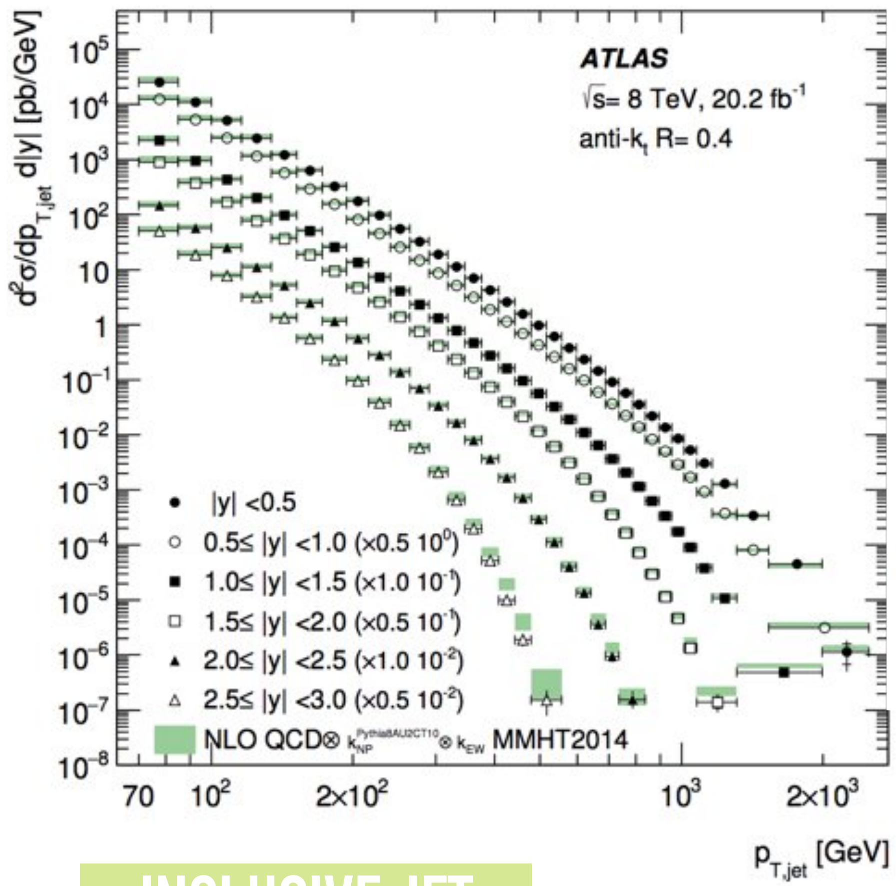
l-Mass Variable-Flavor-Number
and the massive scheme

- Many schemes available

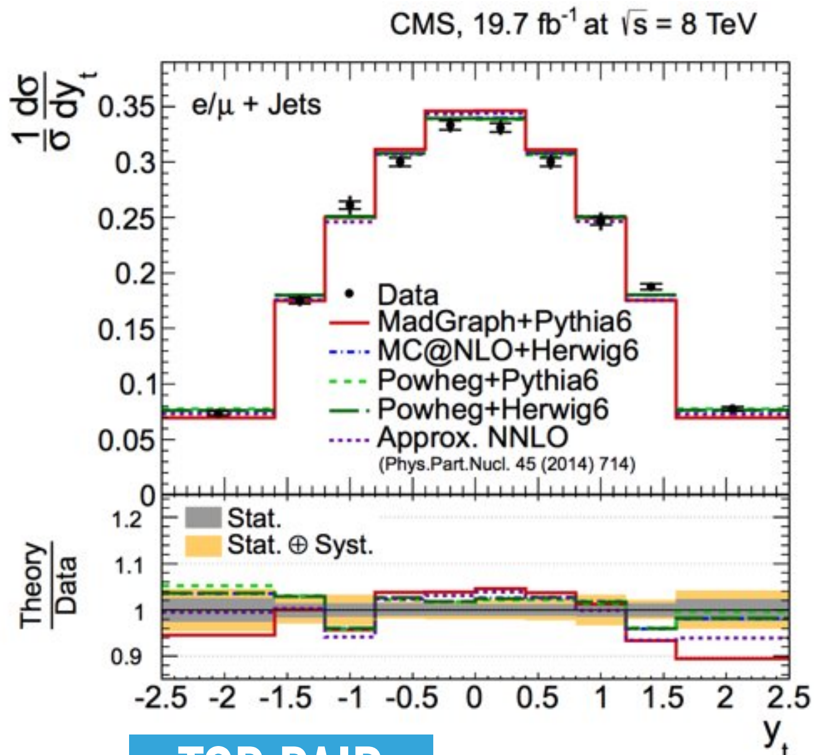
e.g. FONLL

$$
\begin{aligned}
\sigma^{(FONLL)} &= \sigma^{(4)} + \sigma^{(5)} - \text{double counting} \\
&= \mathcal{L}_{ij}(x_1, x_2, \mu^2) \otimes \sum_{p}^{N} \left(\alpha_S^{(5)}(\mu^2)\right)^p \\
&\times \left\{ \mathcal{B}_{ij}^{(p)}\left(x_1, x_2, \frac{\mu^2}{m_b^2}\right) + \sum_{k=0}^{\infty} \mathcal{A}_{ij}^{(p),(k)}(x_1, x_2)\left(\alpha_S^{(5)}(\mu^2) L\right)^k \right\} \\
&- \text{double counting}
\end{aligned}
$$

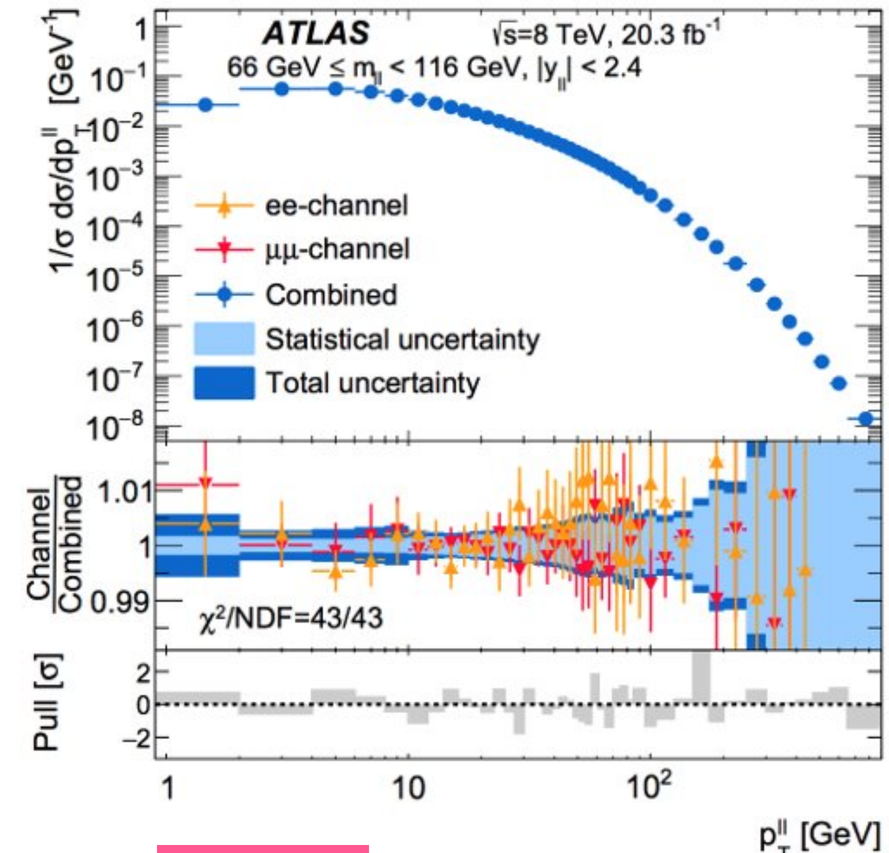
Gluon: direct handle



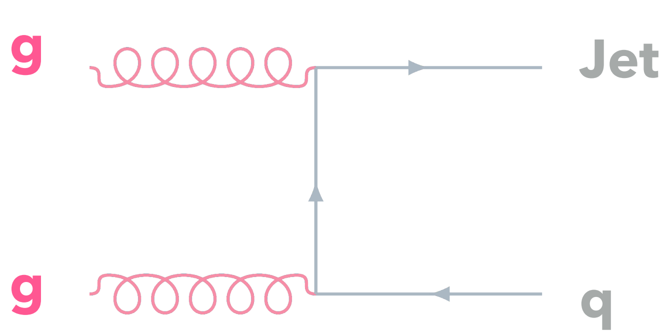
INCLUSIVE JET

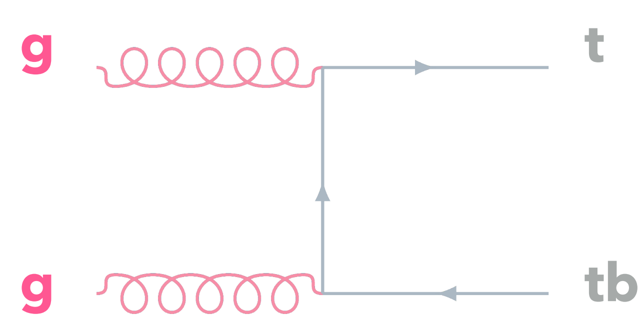


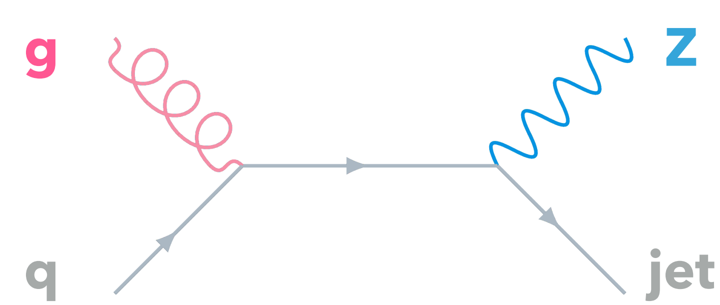
TOP PAIR



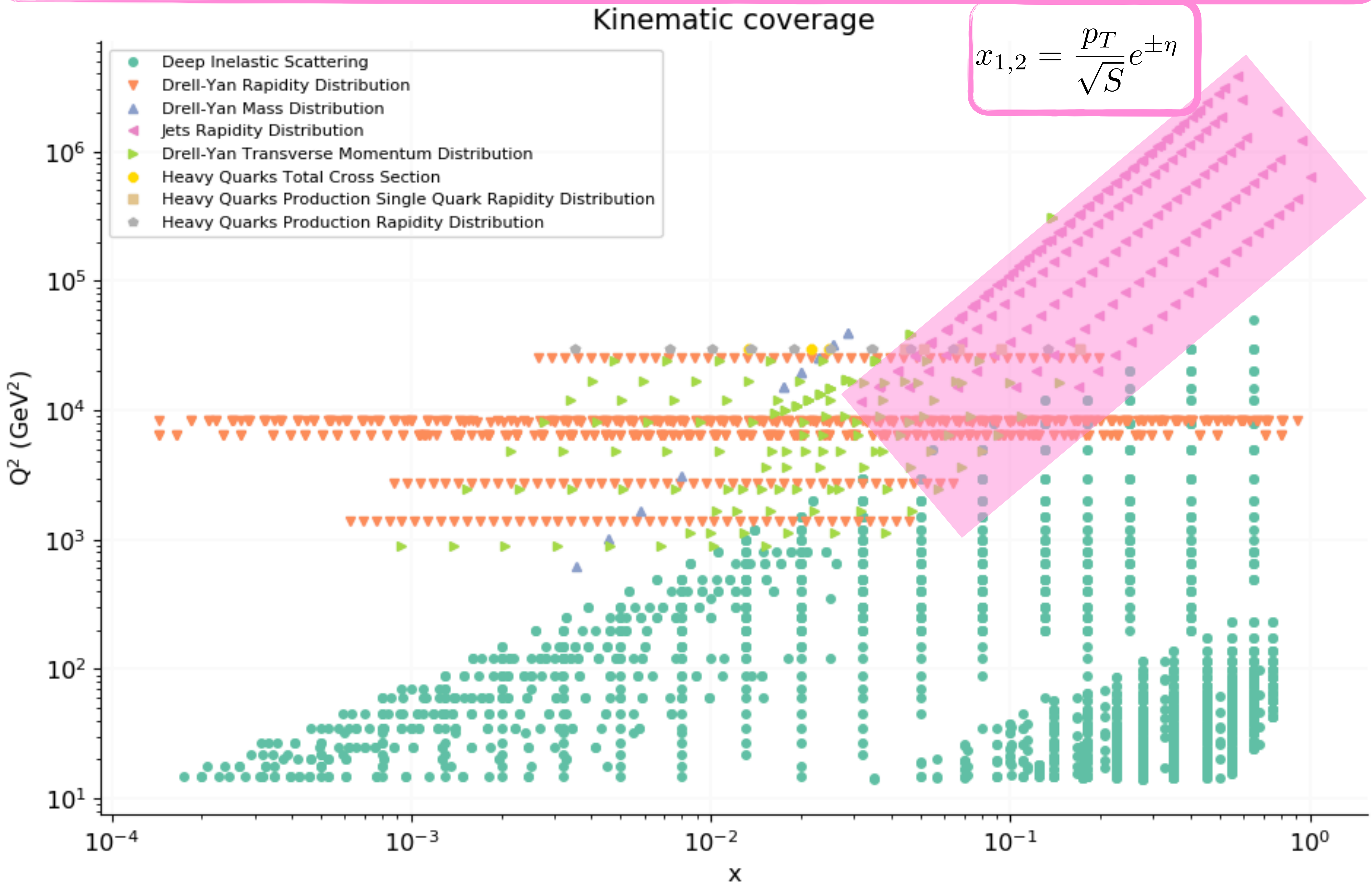
Z P_T





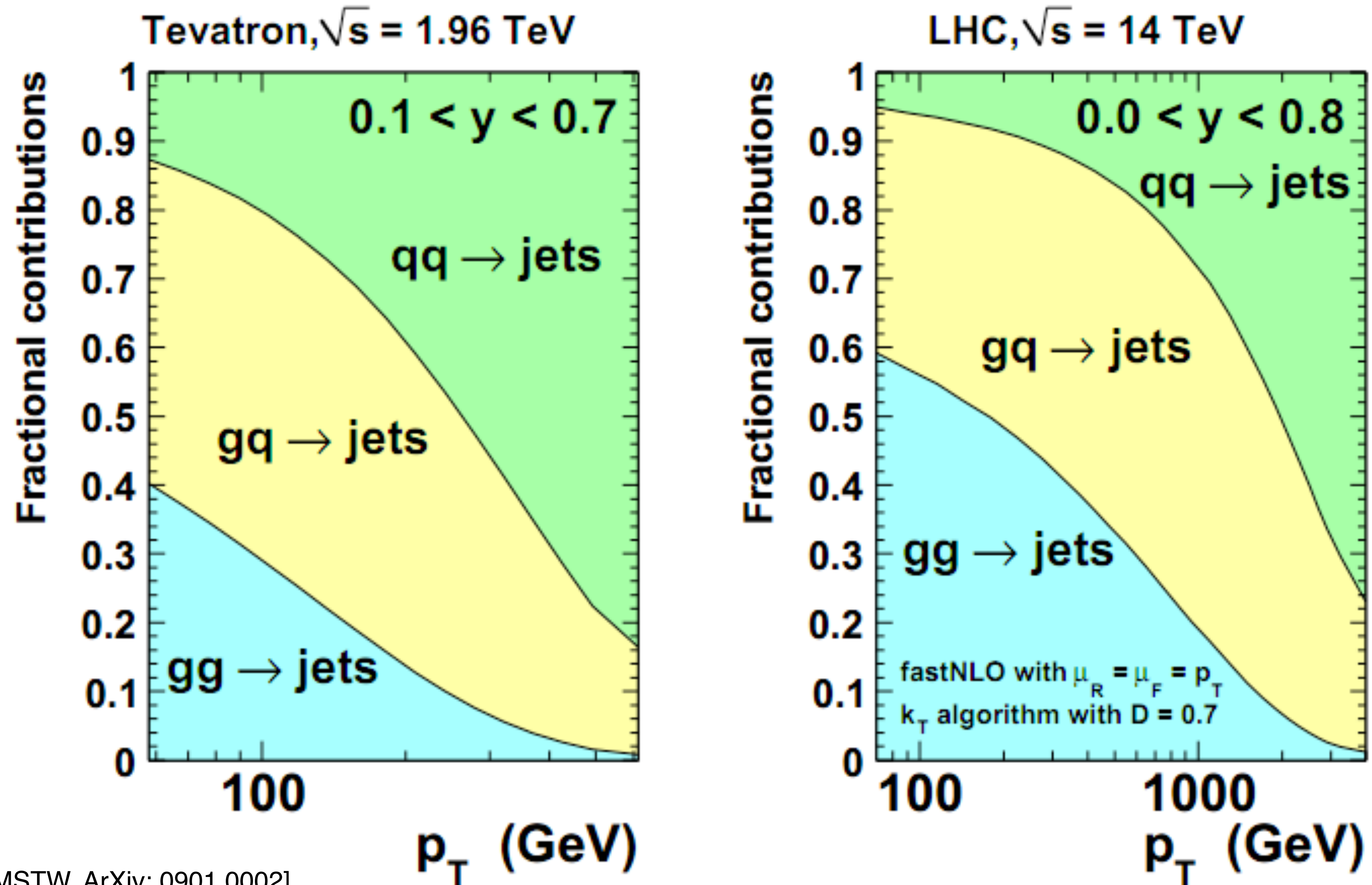


Gluon: direct handle



$$x_{1,2} = \frac{p_T}{\sqrt{S}} e^{\pm\eta}$$

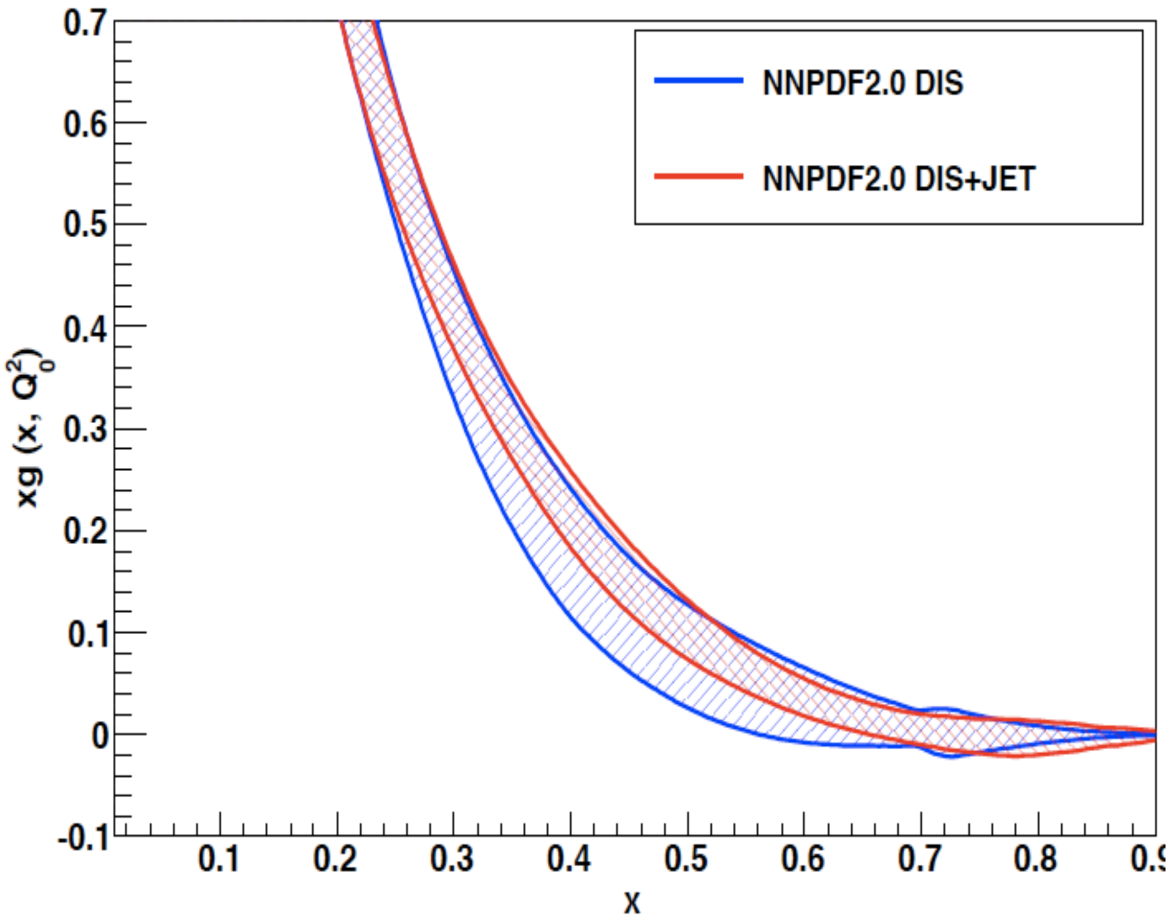
Gluon: jets data

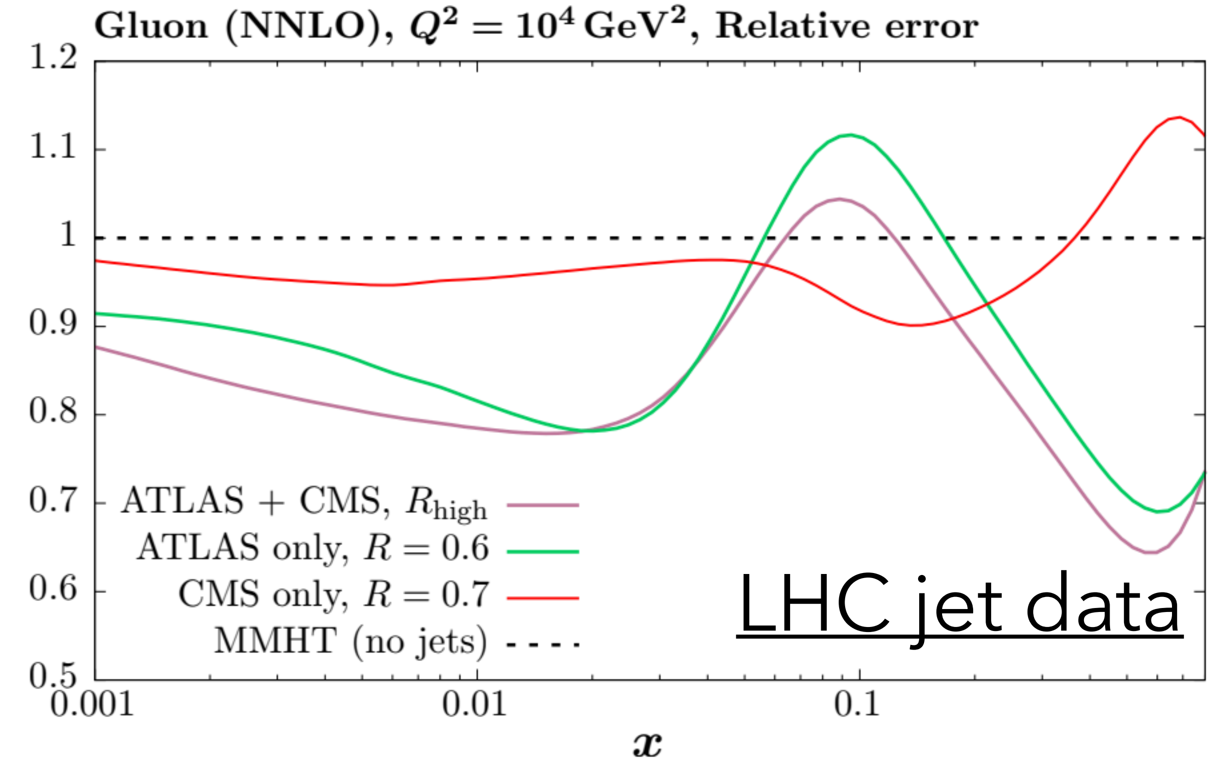


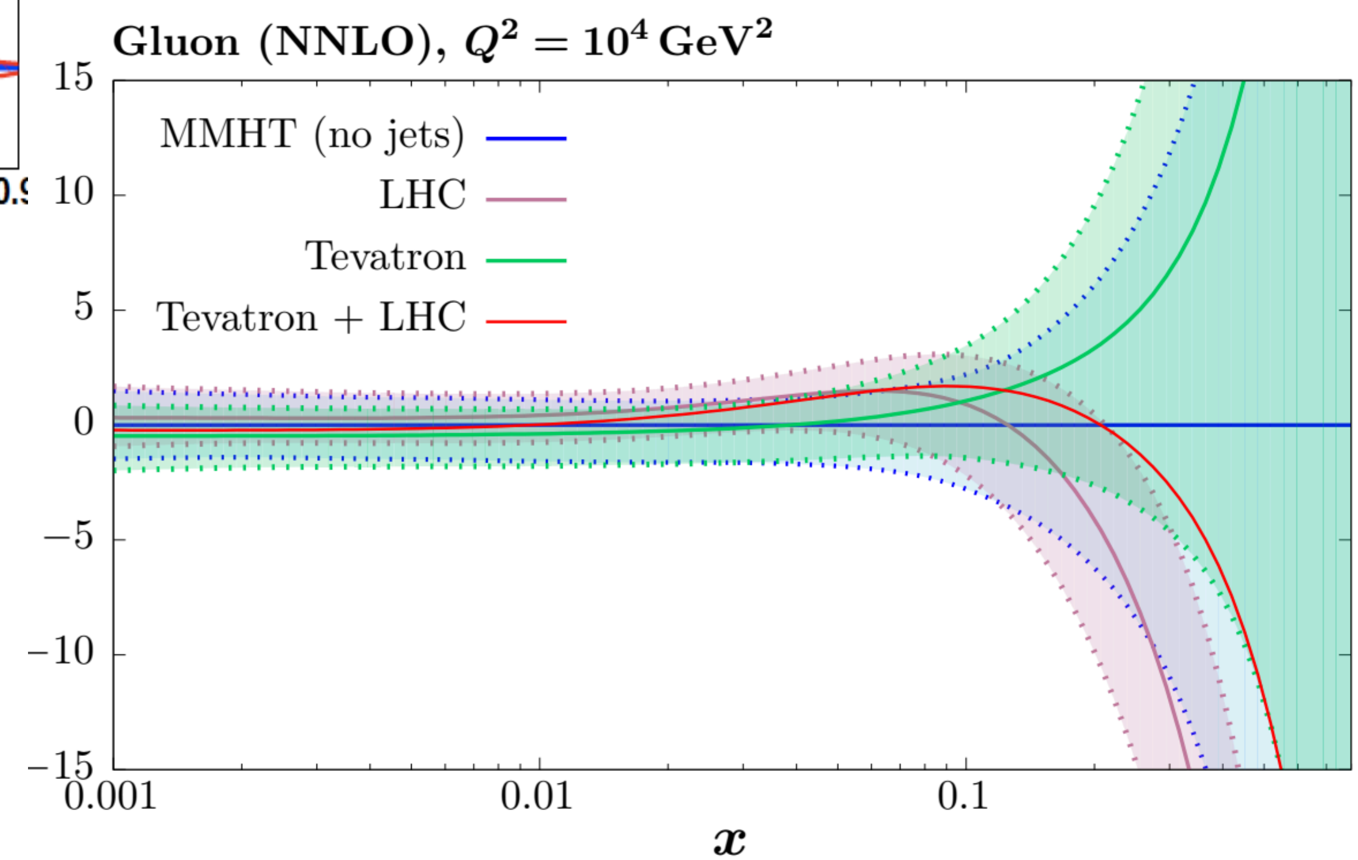
[MSTW, ArXiv: 0901.0002]

Gluon: jets data

Tevatron jet data





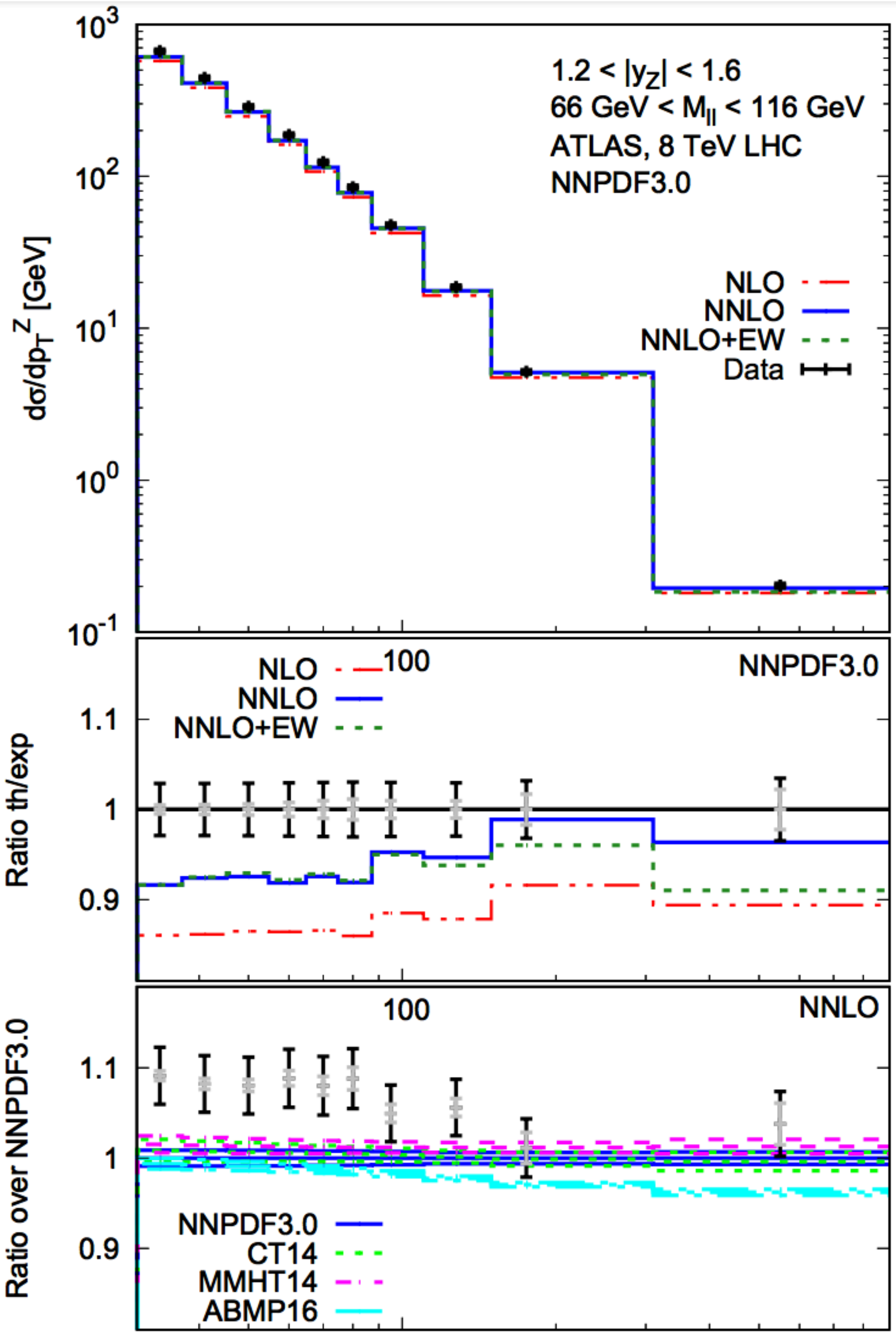


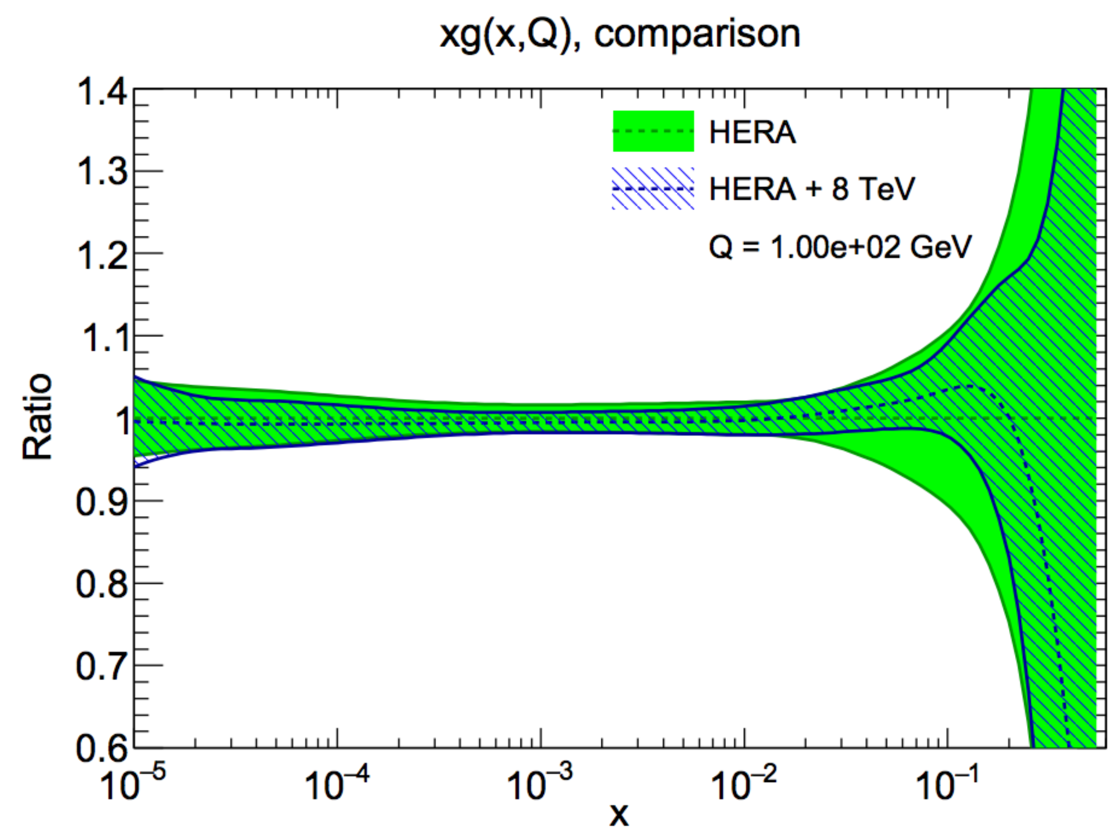
[Ball et al, ArXiv: 1002.4407]

[Harland-Lang et al, ArXiv: 1711.05757]

Gluon: Z transverse momentum

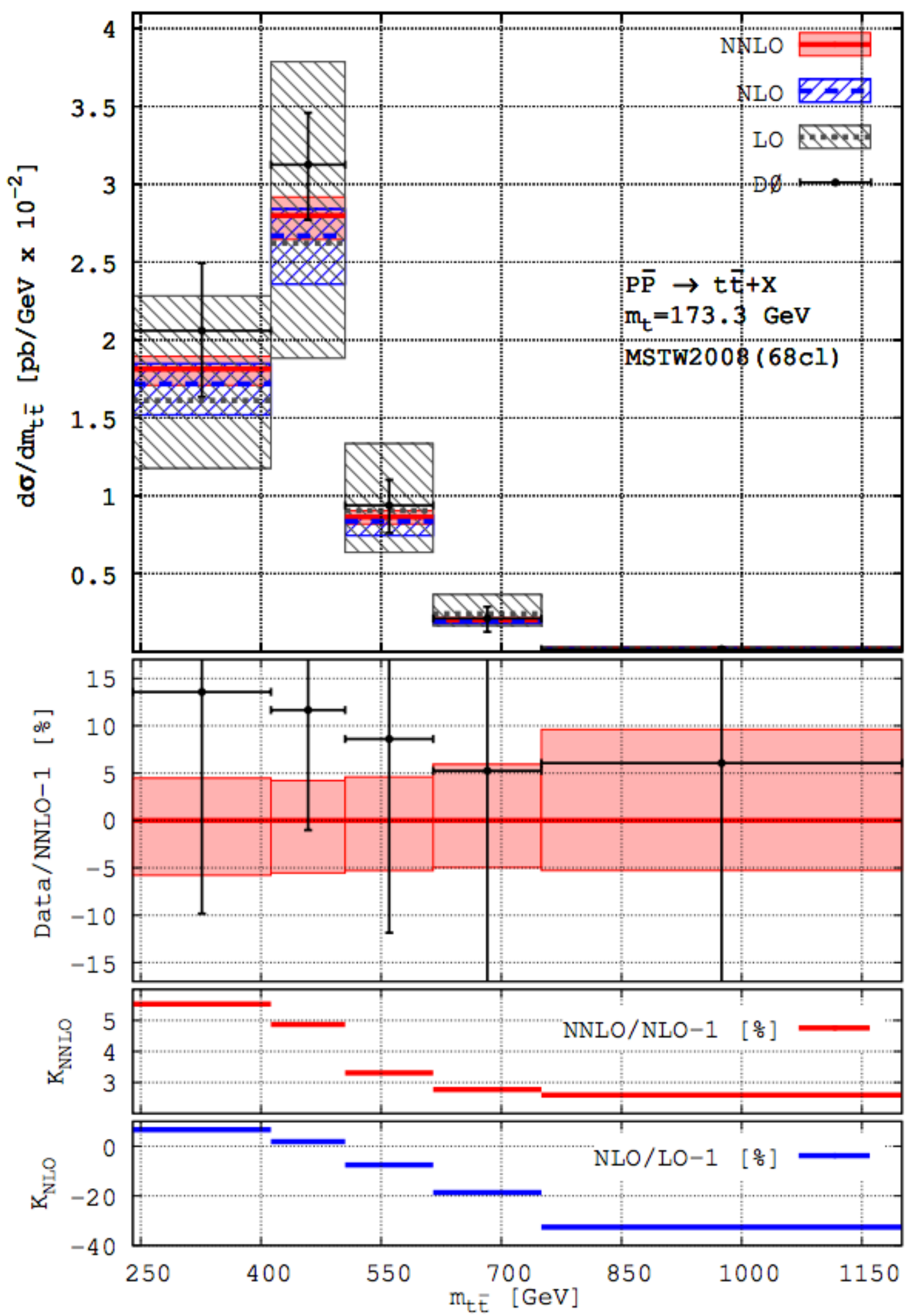
- Experimental precision < 1% up to pT~200 GeV
- Data hugely dominate by correlated systematic uncertainties
- Interesting case-study to probe current theory-experiment frontier

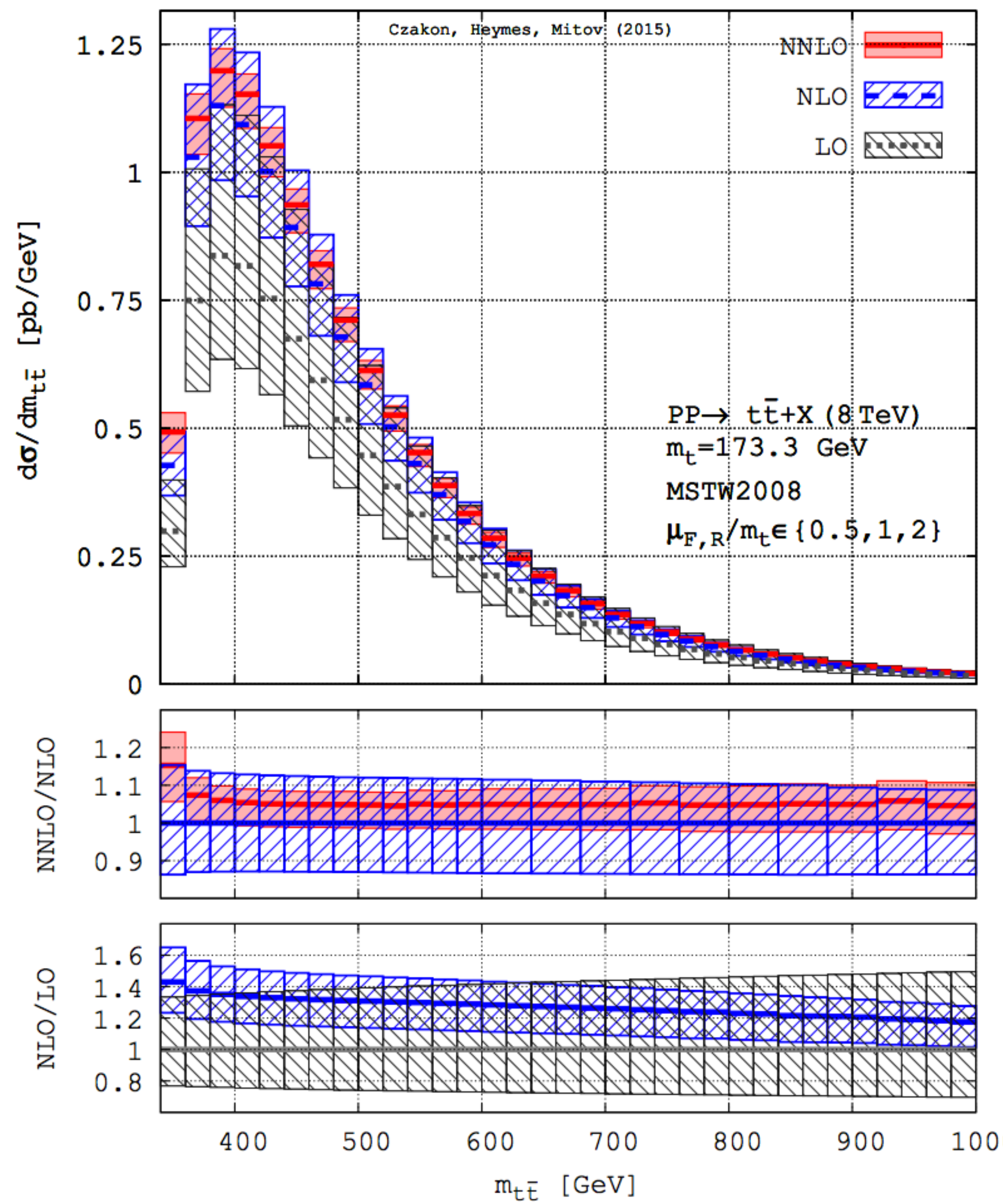




- Data/Theory comparison not so intuitive for correlation-dominated data
- Fluctuation in NNLO predictions (0.5 - 1%) had to be accounted for as extra nuisance parameter to get a good fit of such precise data

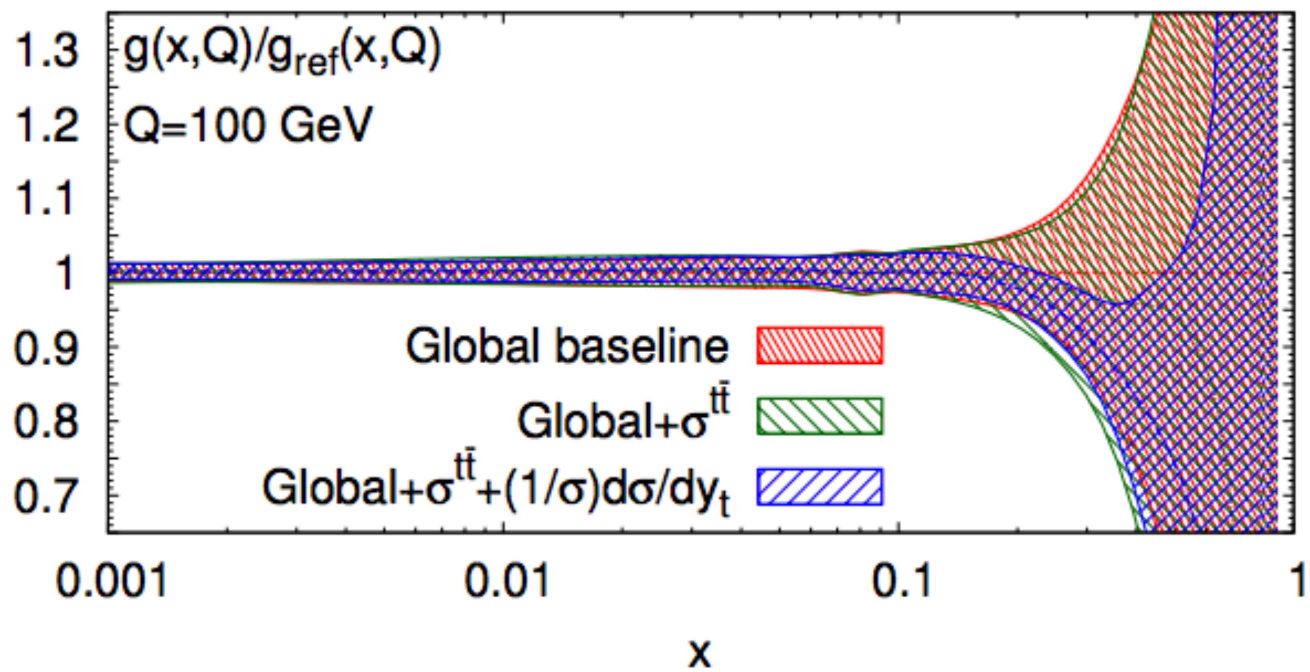
Gluon: top pair production

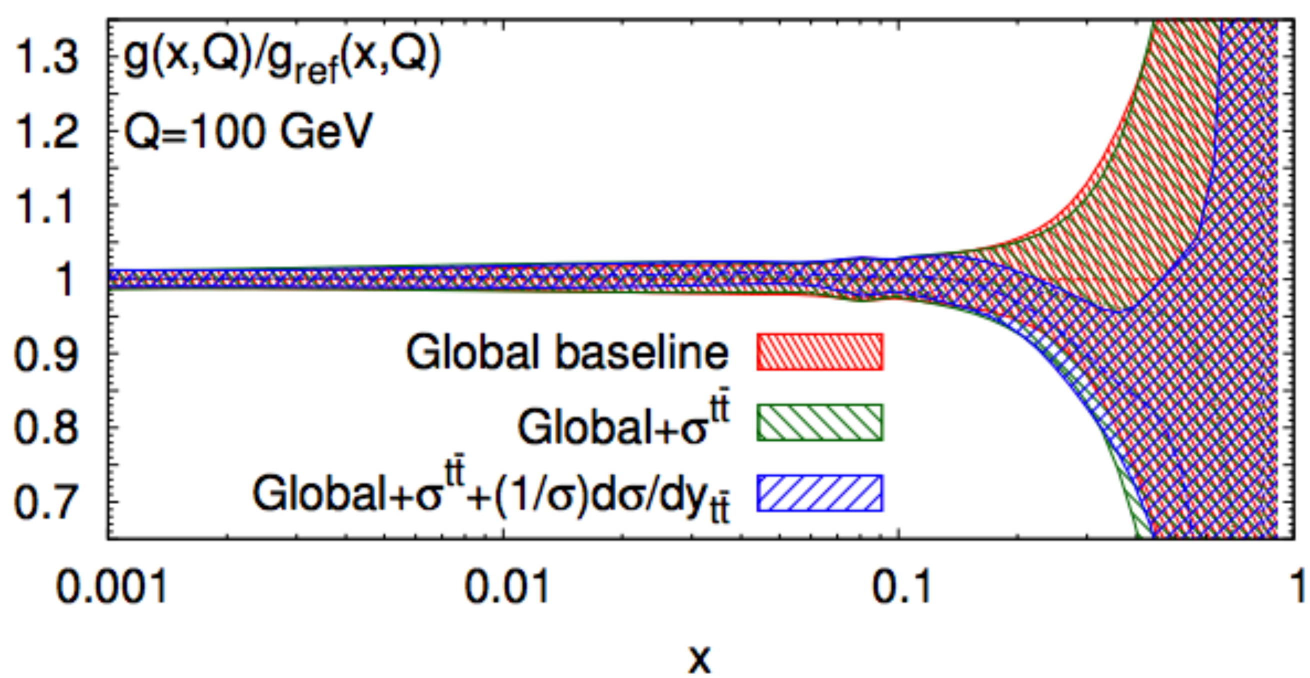




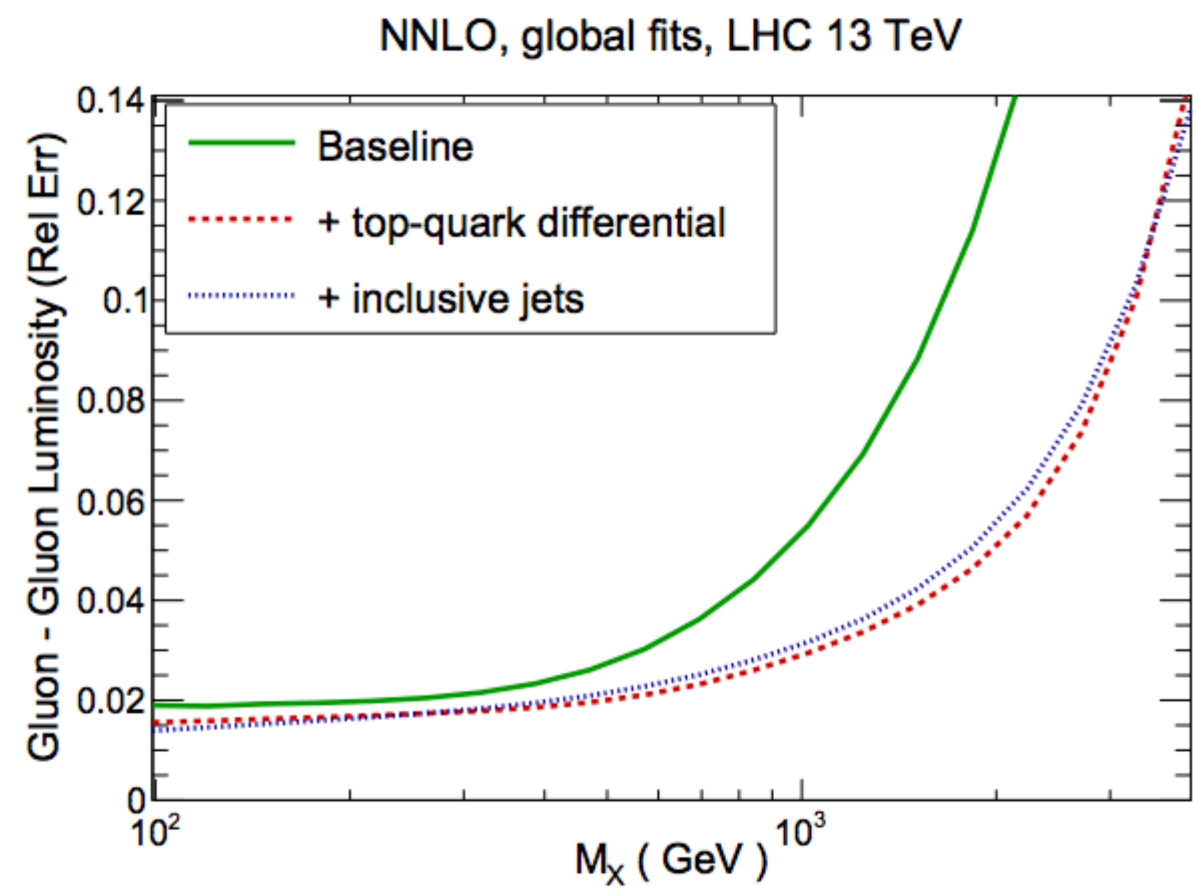
Czakon, Fiedler, Mitov [PRL 116(2016) 082003]

Gluon: top pair production





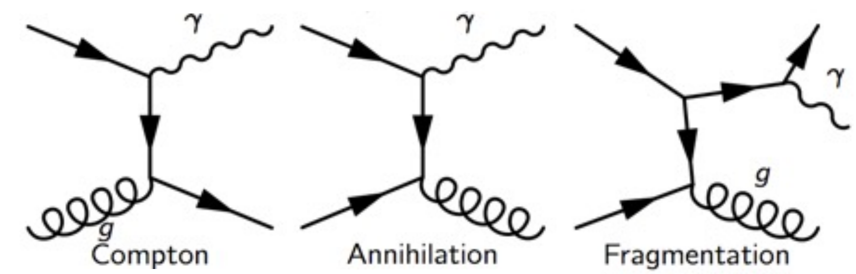
Czakon, Hartland, Mitov, Nocera and Rojo, arXiv: 1611.08609

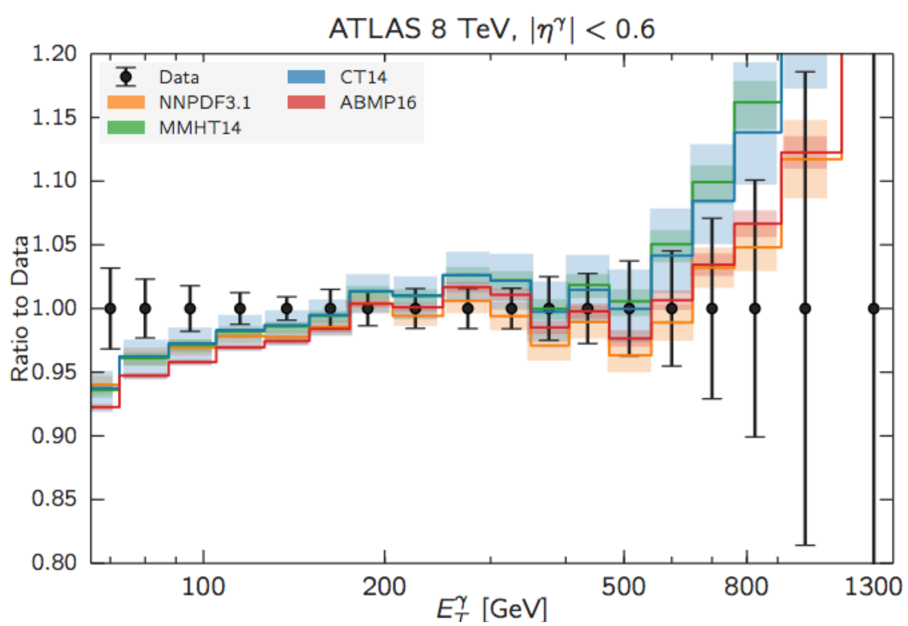


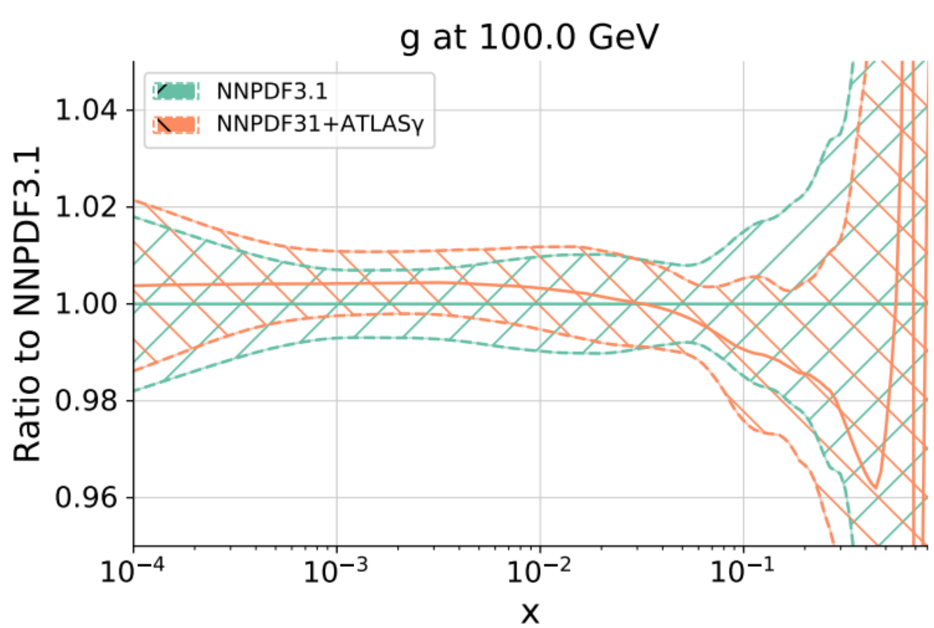
- Most constraining is inclusion of y_t list from ATLAS and y_{tt} from CMS jointly with total xsec
- Competitive reduction of gluon uncertainty with jets measurement
- Slight tension between ATLAS and CMS in NNPDF3.1 ($\chi^2_{ATLAS} \sim 1.6$, $\chi^2_{CMS} \sim 0.9$)

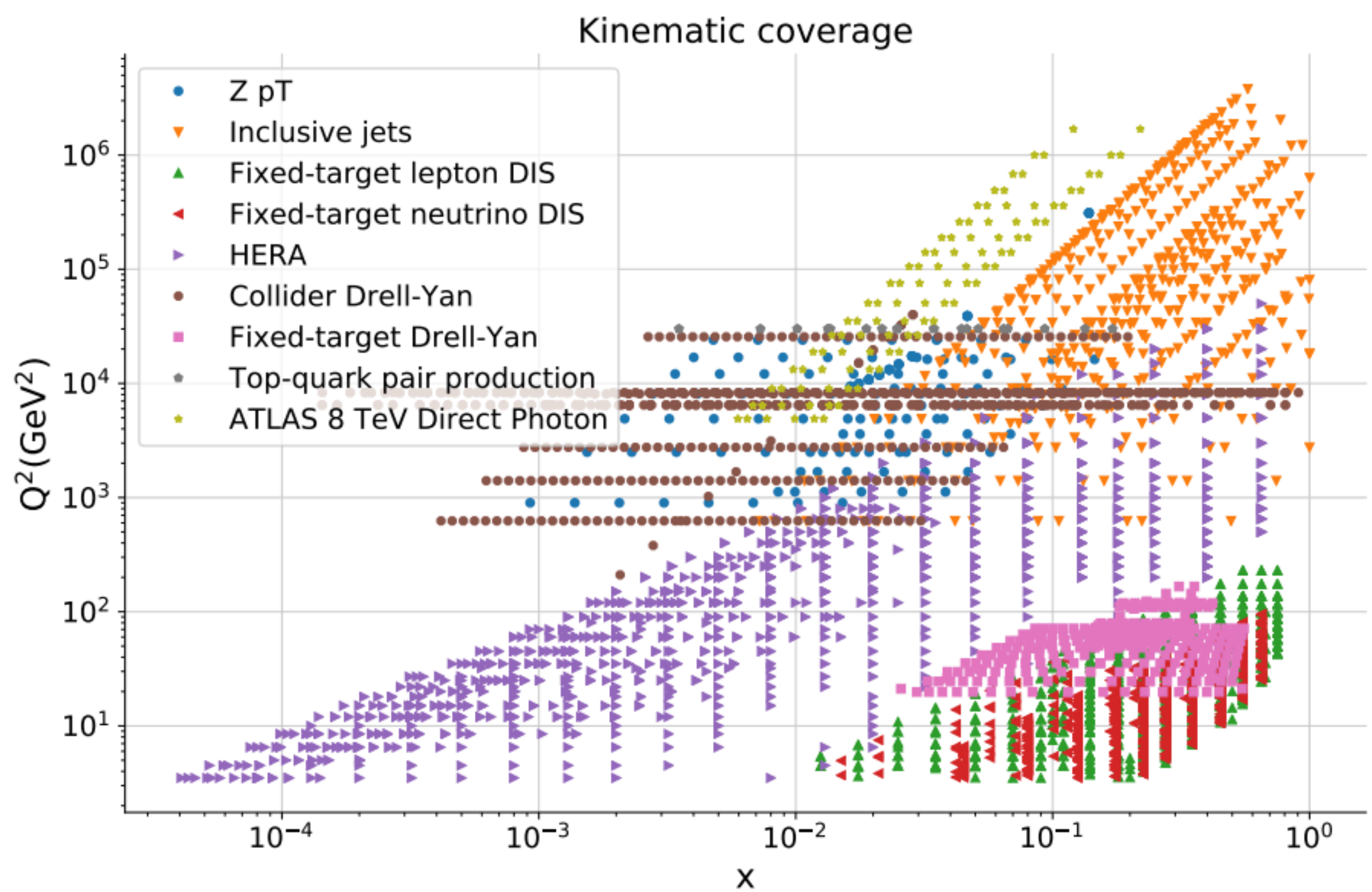
Gluon: direct photon production

- Prompt photon production directly sensitive to the gluon-quark luminosity via Compton scattering
- Isolated prompt photon data known at NNLO [Campbell et al 1612.04333] and accurately measured by ATLAS









Campbell et al 1802.03021

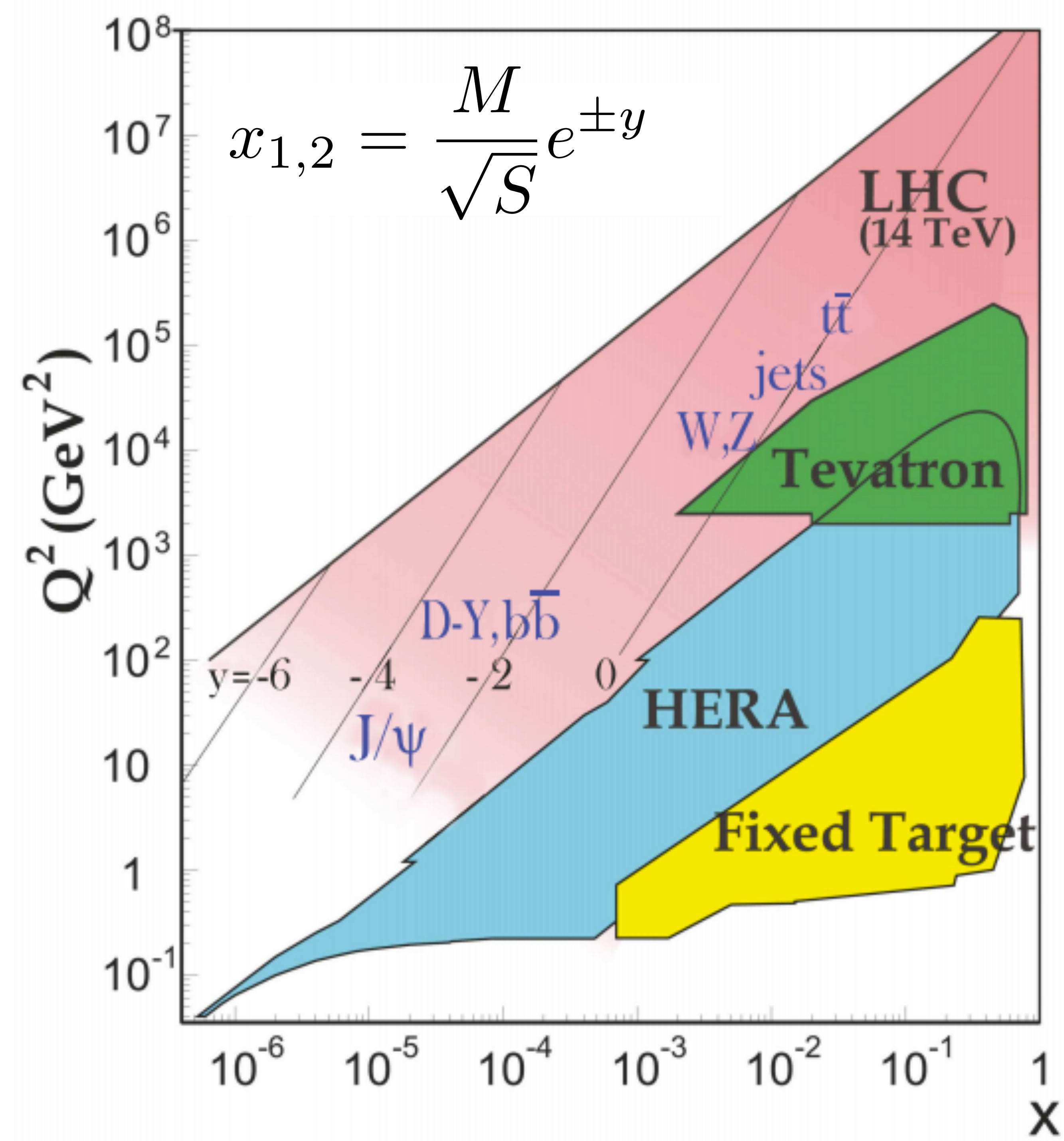
To summarise

GLUON

- Inclusive jets and dijets **(medium/large x)**
- Isolated photon and γ+jets **(medium/large x)**
- Top pair production **(large x)**
- High p_T V(+jets) distribution **(medium x)**

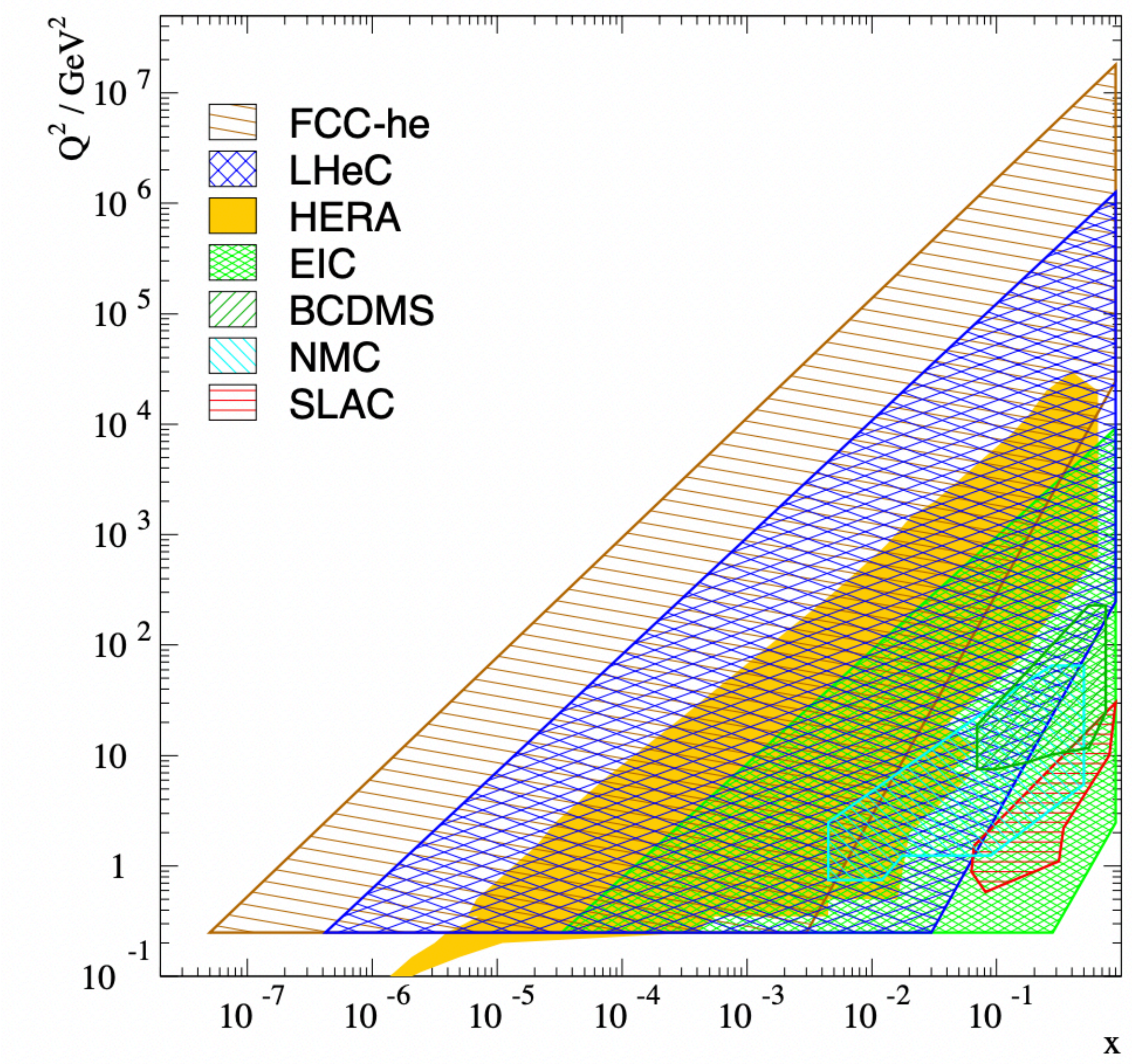
QUARKS

- High p_T V(+jets) ratios **(medium x)**
- W and Z production **(medium x)**
- Low and high mass Drell-Yan **(small and large x)**
- Wc **(strangeness at medium x)**



PDG 2016

Looking forward



T. Hobbs, SciPost(2022)

Wrap-up

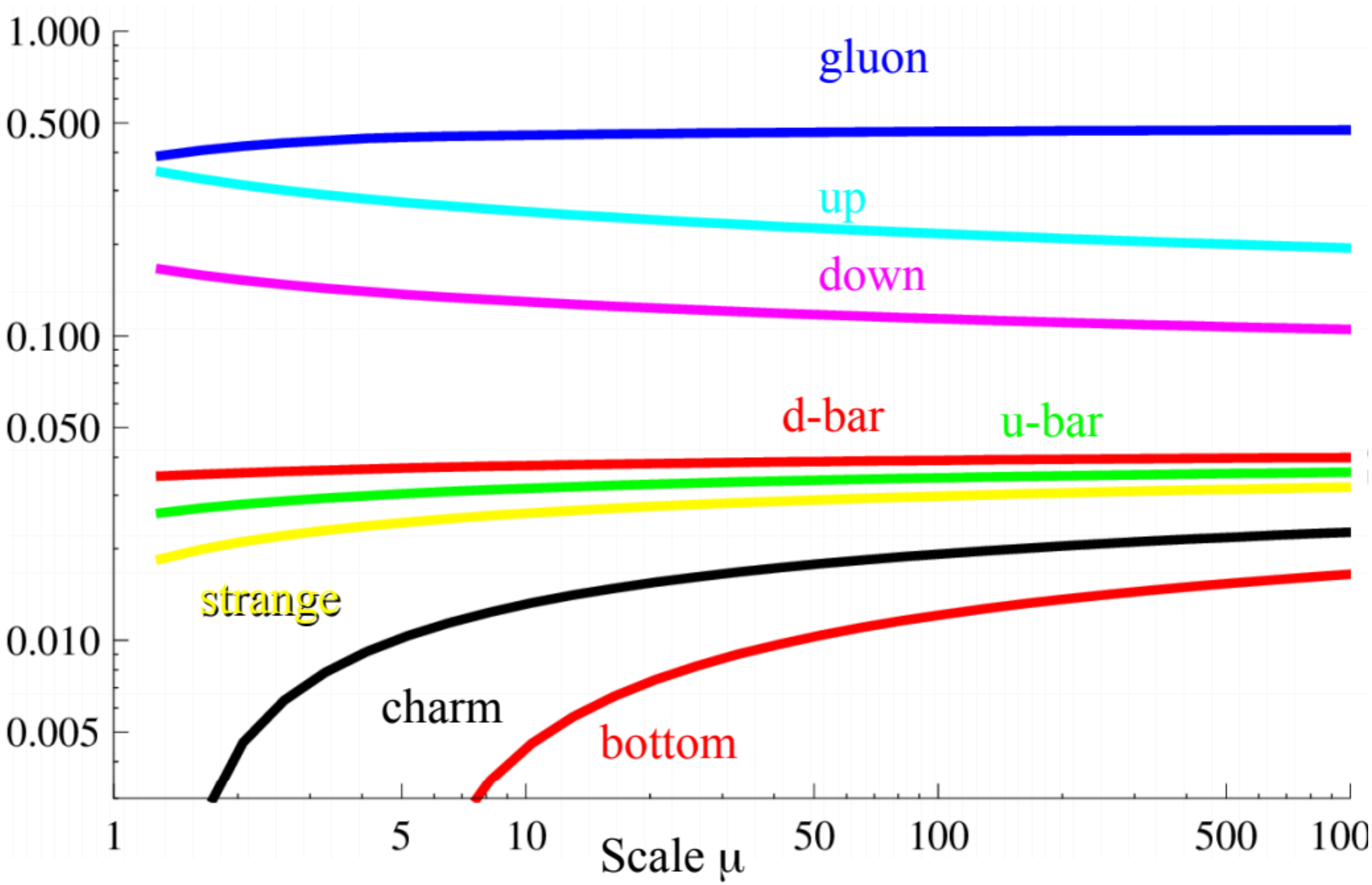
➡ The structure of the proton has been a crucial ingredient to test and verify perturbative QCD and it is now key to the precision challenge that we are facing at the LHC

➡ Today's lecture

- ✓ QCD - Improved parton model
- ✓ DGLAP evolution equations
- ✓ Collinear Factorisation Theorem
- ✓ Ingredients in a PDF determination
- ✓ Experimental input

DGLAP evolution equations

F. Olness, CTEQ school 2017



Parton Luminosities

- A quick and easy way to assess the mass and the collider dependence of production cross sections at hadron-hadron colliders is to use Parton Luminosities
- At leading order in QCD (parton model)

$$\hat{\sigma}_{ab\to X} = C_X \delta(x_a x_b S - M^2)$$

$$\sigma_{pp\to X} = \int_0^1 dx_a dx_b f_a(x_a, M^2) f_b(x_b, M^2) \hat{\sigma}_{ab\to X}$$

- Thus

$$\sigma_{pp\to X} = C_X \int_0^1 dx_a dx_b f_a(x_a, M^2) f_b(x_b, M^2) \delta(x_a x_b S - M^2)$$

$$= \frac{C_X}{S} \frac{\partial \mathcal{L}_{ab}}{\partial \tau}$$

with

$$\tau = \frac{M^2}{\tau}$$

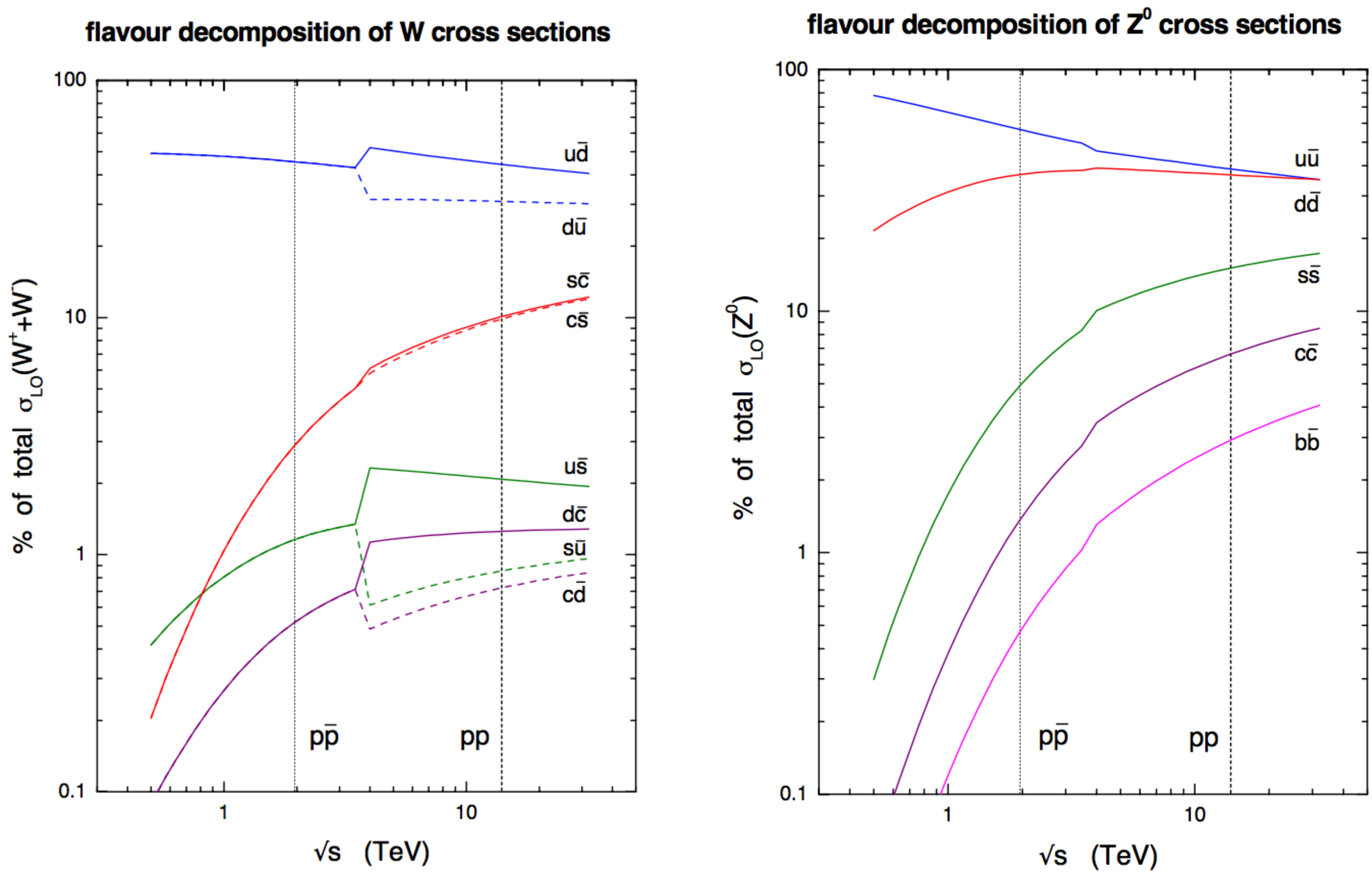
- Define

$$\Phi_{ab}(M^2) = \frac{\partial \mathcal{L}_{ab}}{\partial \tau}$$

$$= \int_0^1 dx_a dx_b f_a(x_a, M^2) f_b(x_b, M^2) \delta(x_a x_b - \tau)$$

$$= \frac{1}{S} \int_\tau^1 \frac{dy}{y} f_a(y, M^2) f_b\left(\frac{\tau}{y}, M^2\right)$$

Parton Luminosities



Parton Luminosities