

Recent Progress on Track Functions

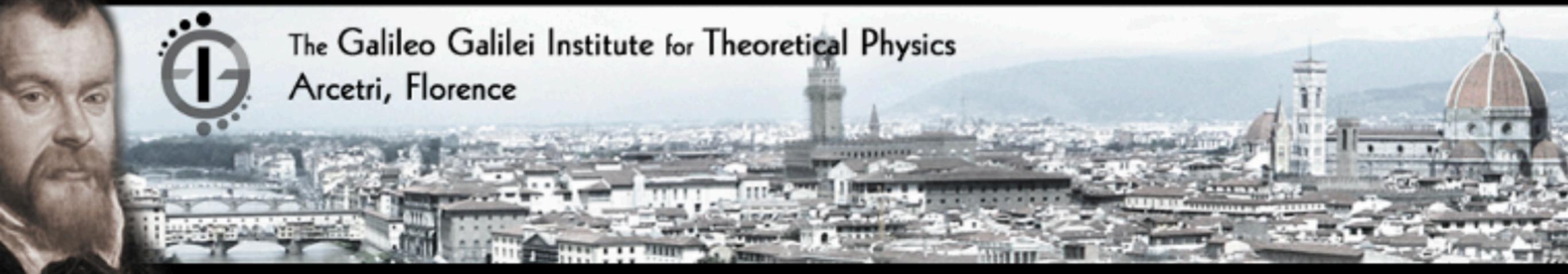
Wouter Waalewijn



UNIVERSITY OF AMSTERDAM

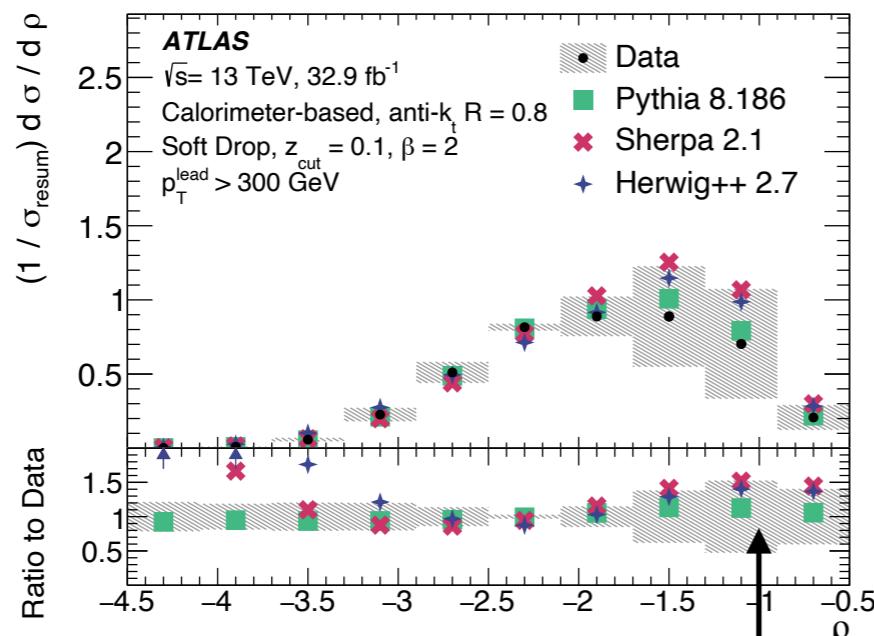
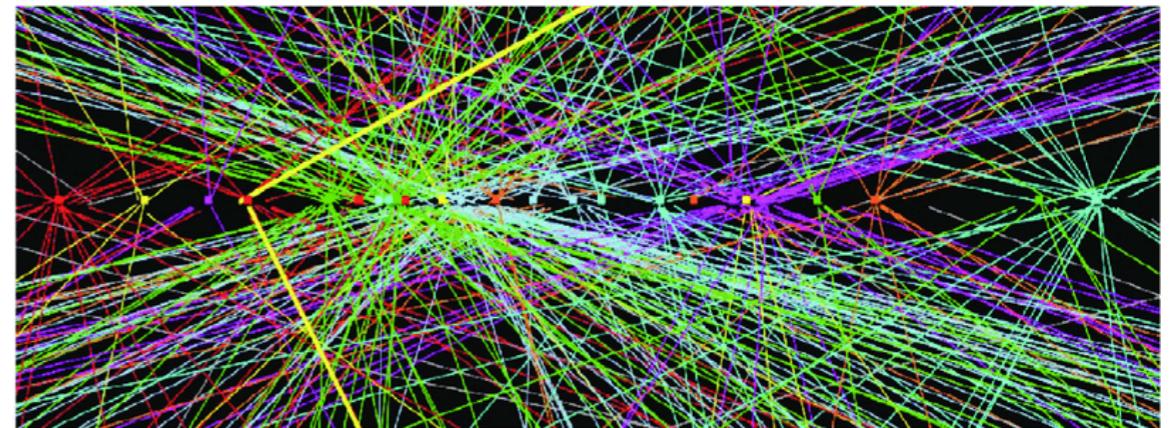


The Galileo Galilei Institute for Theoretical Physics
Arcetri, Florence

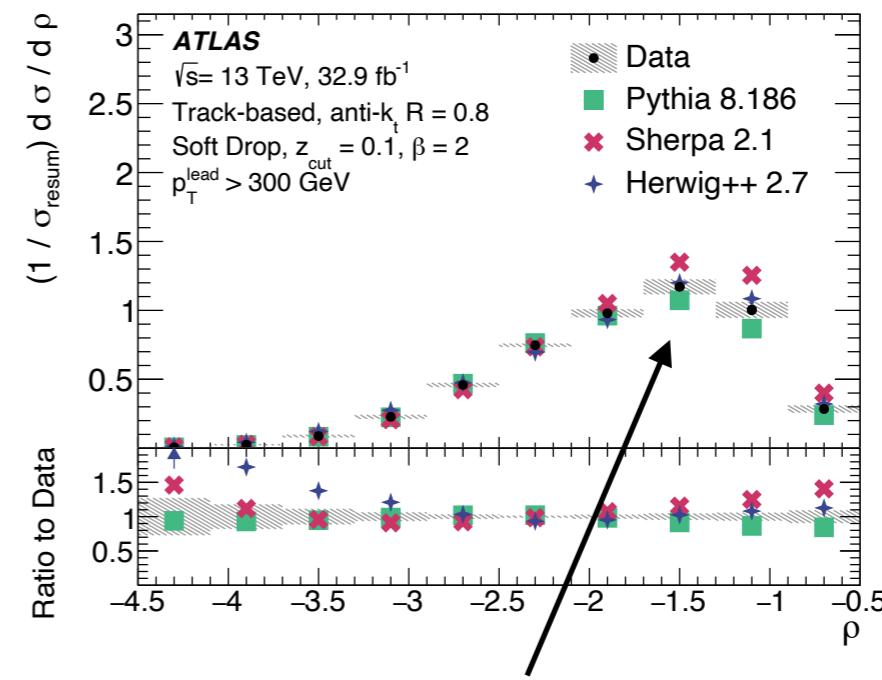


Motivation for track-based measurements

- Superior angular resolution
→ good for jet substructure
- Pile-up removal
- E.g. for groomed $\rho = \ln(m^2/p_T^2)$



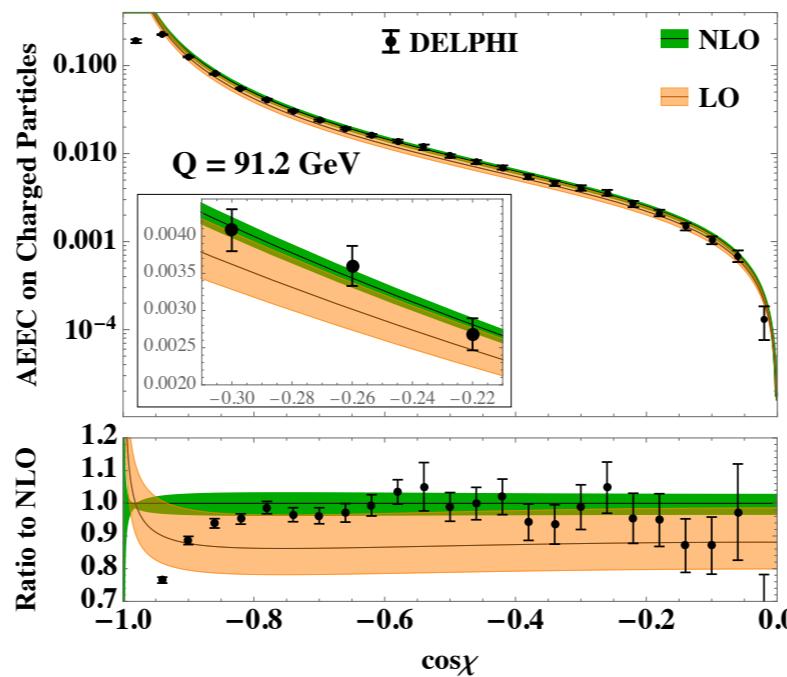
Pile up



Difference of parton showers
larger than exp. uncertainty

Introduction

- Track-based measurements sensitive to hadronization
→ can be modeled in parton showers.
- Track functions offer **systematically improvable** framework.
- Recently extended to $\mathcal{O}(\alpha_s^2)$ → high precision possible!
- Track functions are being extracted by ATLAS.
- Particularly easy to use for energy correlators

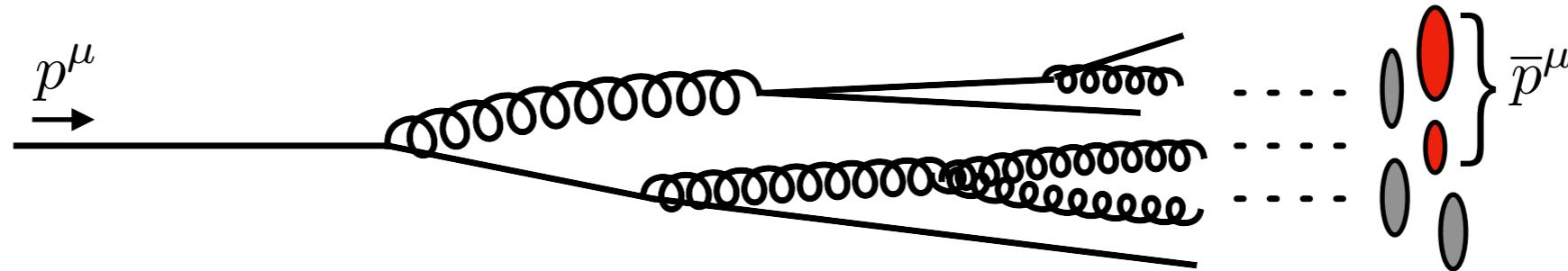


Outline

1. Track function basics
2. Track functions at α_s^2
3. Phenomenology with track functions

1. Track functions basics

1. Track function



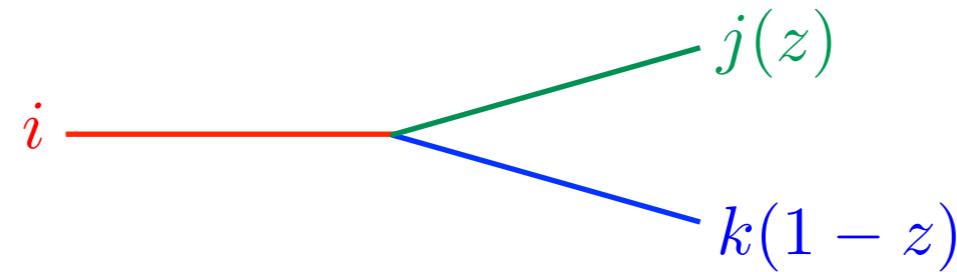
[Chang, Procura, Thaler, WW]

- $T_i(x, \mu)$ describes **total** momentum fraction x of initial parton i converted to **tracks**, i.e. $\bar{p}^\mu = \textcolor{red}{x} p^\mu + \mathcal{O}(\Lambda_{\text{QCD}})$
- Nonperturbative, process-independent function.
- Conservation of probability: $\int_0^1 dx T_i(x) = 1$
- Definition in light-cone gauge

$$T_q(x) = \int dy^+ d^2 y_\perp e^{ik^- y^+/2} \sum_X \delta\left(x - \frac{\bar{p}^-}{k^-}\right) \times \frac{1}{2N_c} \text{tr} \left[\frac{\gamma^-}{2} \langle 0 | \psi(y^+, 0, y_\perp) | X \rangle \langle X | \bar{\psi}(0) | 0 \rangle \right]$$

all charged hadrons in X

1. Track function at order α_s

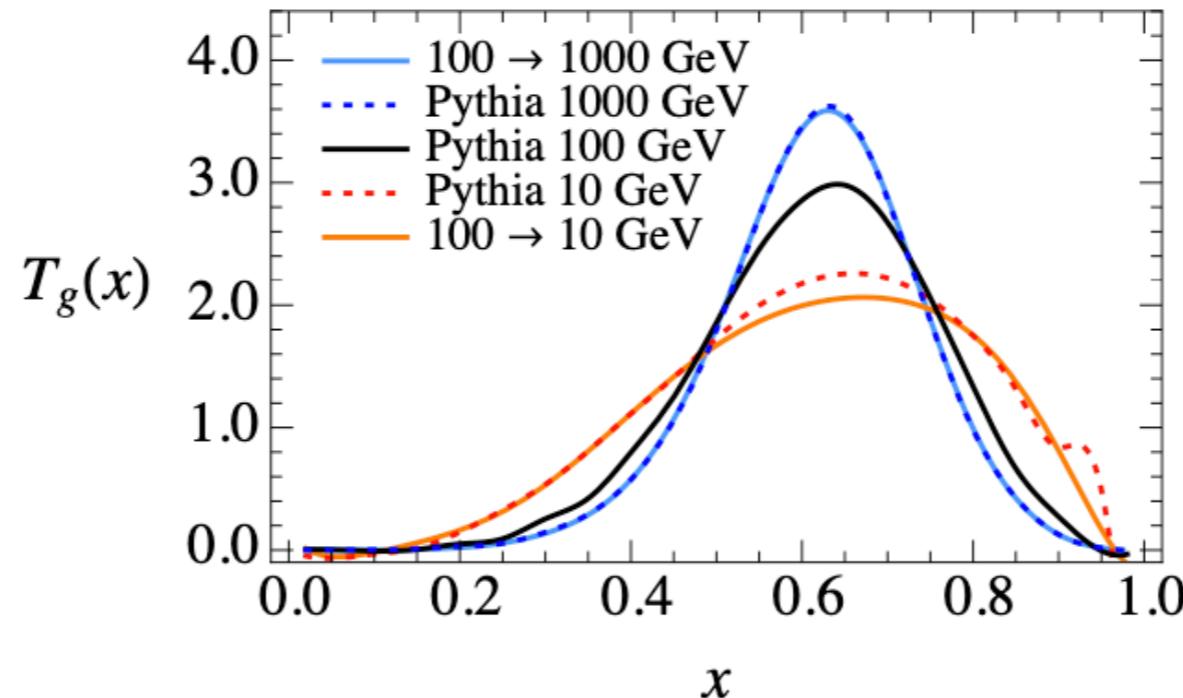


$$T_{i,\text{bare}}^{(1)}(x) = \sum_j \int dz \left[\frac{\alpha_s}{4\pi} \left(\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}} \right) P_{ji}(z) \right] \int dx_1 T_j^{(0)}(x_1, \mu) \times \int dx_2 T_k^{(0)}(x_2, \mu) \delta[x - zx_1 - (1-z)x_2]$$

splitting function
summing contribution of branches

- $1/\epsilon_{\text{IR}}$ cancels against IR pole in partonic cross section.
- $1/\epsilon_{\text{UV}}$ is renormalized, leads to evolution of track function

1. Evolution at order α_s



$$\frac{d}{d \ln \mu^2} T_i(x, \mu) = \sum_{j,k} \int dz \frac{\alpha_s}{4\pi} P_{ji}(z) \int dx_1 T_j(x_1, \mu) \int dx_2 T_k(x_2, \mu) \\ \times \delta[\textcolor{red}{x} - \textcolor{green}{z}x_1 - (\textcolor{blue}{1}-z)x_2] \quad [\text{Chang, Procura, Thaler, WW}]$$

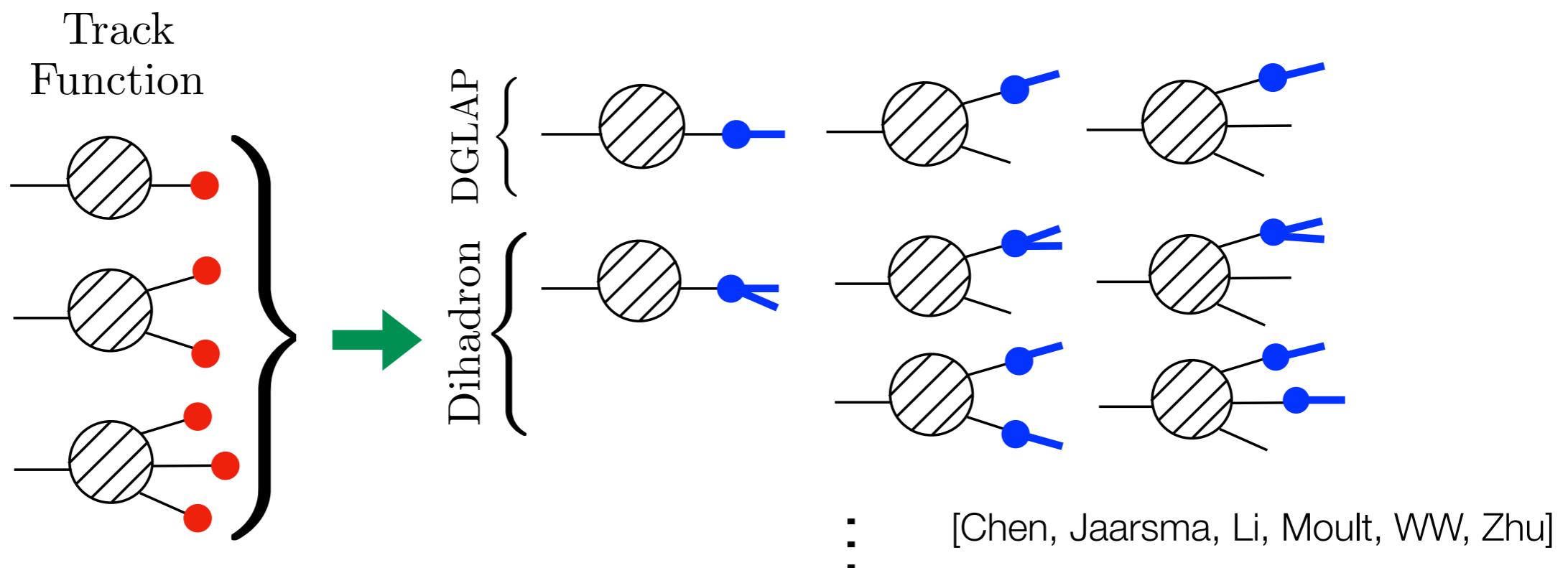
- Consistent with extraction from Pythia at different energies.
- Simplifies for integer moments: $[\textcolor{red}{x} - \textcolor{green}{z}x_1 - (\textcolor{blue}{1}-z)x_2]^N$
binomial expansion

1. Relation to (multi-)hadron fragmentation

- Track vs. fragmentation functions:

$$\delta[\textcolor{red}{x} - zx_1 - (1-z)x_2] \rightarrow \delta[\textcolor{red}{x} - zx_1] + \delta[\textcolor{red}{x} - (1-z)x_2]$$

- Evolution for (multi-)hadron fragmentation follows from tracks:



1. Using track functions

- Consider a cross section differential in observable e

$$\frac{d\sigma}{de} = \sum_N \int d\Pi_N \frac{d\sigma_N}{d\Pi_N} \delta[e - \hat{e}(\{p_i^\mu\})]$$

- At leading order, for track-based measurement \bar{e}

$$\frac{d\sigma}{d\bar{e}} = \sum_N \int d\Pi_N \frac{d\sigma_N}{d\Pi_N} \underbrace{\int_{i=1}^N dx_i T_i(x_i)}_{\text{hadronization}} \delta[e - \hat{e}(\{\cancel{x}_i p_i^\mu\})]$$

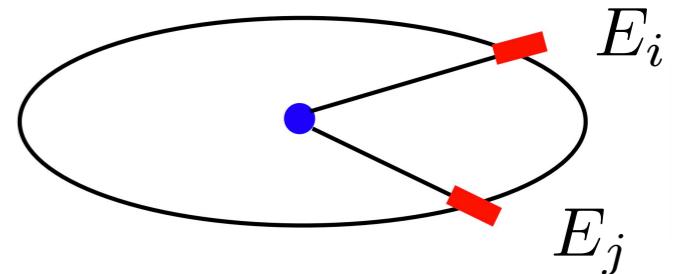
- Beyond leading order, there is a cancellation of IR divergences, $d\sigma_N \rightarrow d\bar{\sigma}_N$, similar to fragmentation functions/PDFs.

1. Track-based energy correlators

- Weighted cross section in e^+e^- collisions

$$\frac{d\sigma}{d \cos \chi} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta(\cos \chi - \cos \theta_{ij})$$

[Basham, Brown, Ellis, Love]



- Tracks are essential for small χ
- Conversion to tracks is simple:

$$E_i \rightarrow \int dx_i T_i(x_i) x_i E_i = T_i(1) E_i$$

[Chen, Moult, Zhang, Zhu]

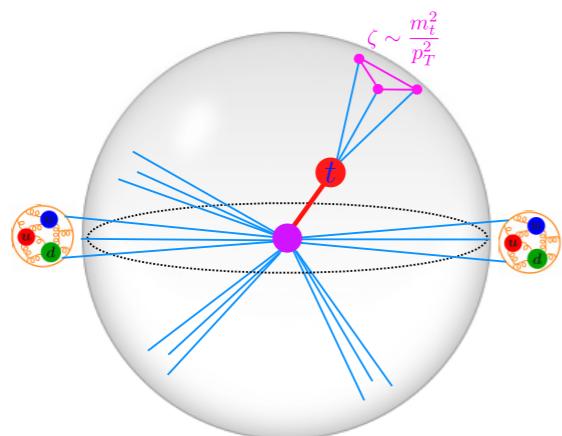
- Contact term $\chi = 0$ involves $T_i(2)$.

1. Track-based weighted energy correlators

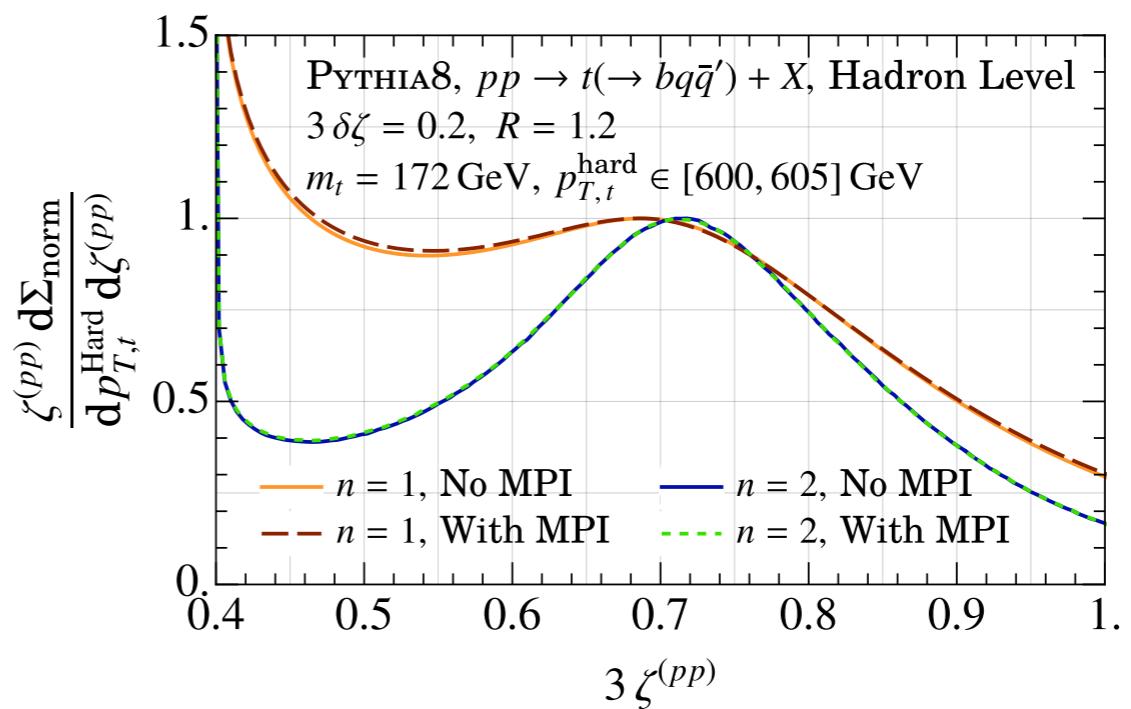
- N -point energy correlators involve at most the N th moment
- **Modifying** weights to suppress soft radiation is easy for tracks:

$$\frac{d\sigma}{d \cos \chi} = \sum_{i,j} \int d\sigma \frac{E_i^n E_j^n}{Q^{2n}} \delta(\cos \chi - \cos \theta_{ij})$$

- Advantageous for top quark mass determination:



[Holguin, Moult, Pathak, Procura]



2. Track functions at α_s^2

2. Calculation in a nut shell

- $T_i(x) \propto 1/\epsilon_{\text{UV}} - 1/\epsilon_{\text{IR}} = 0 \rightarrow$ need a scale.
- Extract it from $J_i(s, x)$, with s invariant mass of all particles.
- Consistency of factorization implies same UV poles as $J_i(s)$
[Becher, Neubert; Becher, Bell]

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- Consistency of factorization implies same UV poles as $J_i(s)$
[Becher, Neubert; Becher, Bell]
- Remaining $1/\epsilon$ poles are infrared and cancel in

want to extract

$$J(s, x, \mu) = \delta(s) \mathbf{T}(x, \mu) + a_s \delta(s) \left[\mathcal{J}_{1 \rightarrow 1}^{(1)} T(x, \mu) + \mathcal{J}_{1 \rightarrow 2}^{(1)} \otimes TT(x, \mu) \right] \\ + a_s^2 \delta(s) \left[\mathcal{J}_{1 \rightarrow 1}^{(2)} T(x, \mu) + \mathcal{J}_{1 \rightarrow 2}^{(2)} \otimes TT(x, \mu) + \mathcal{J}_{1 \rightarrow 3}^{(2)} \otimes TTT(x, \mu) \right] + \dots$$

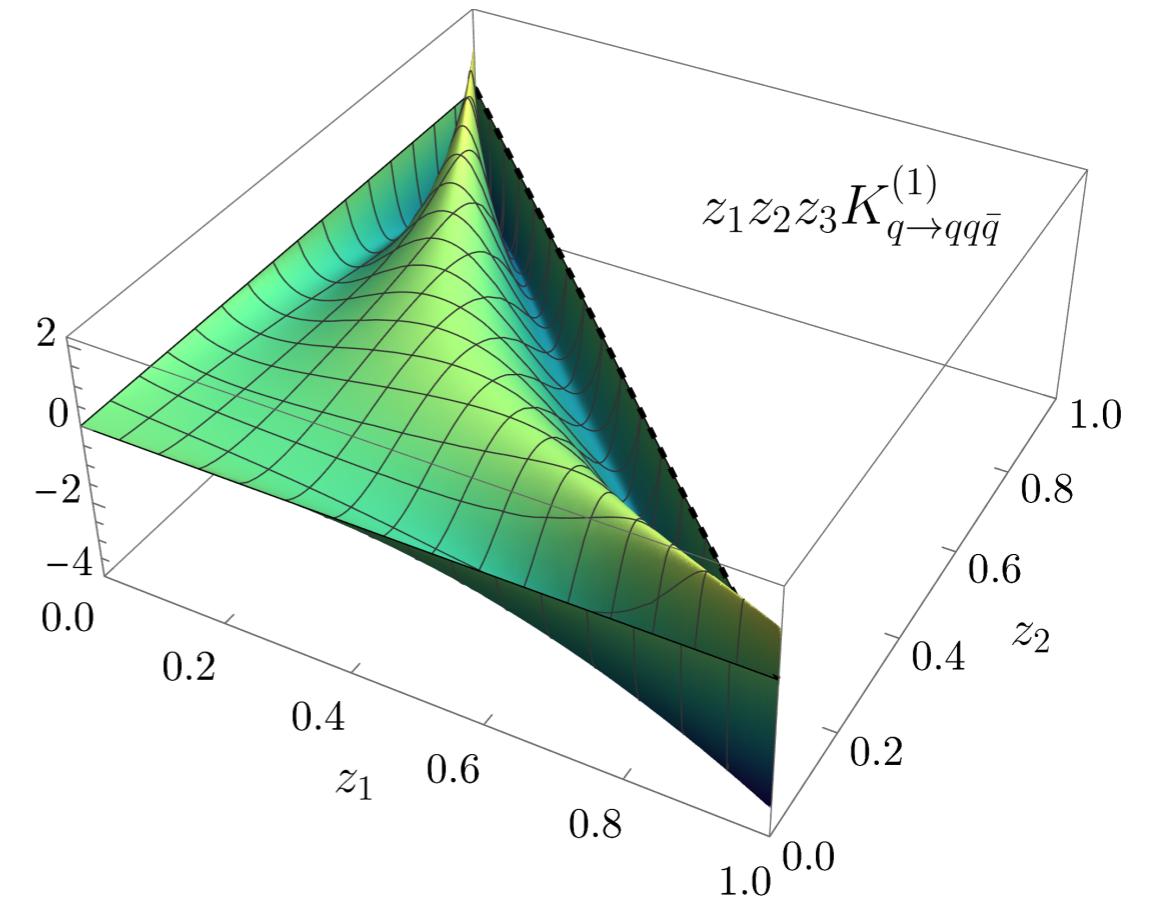
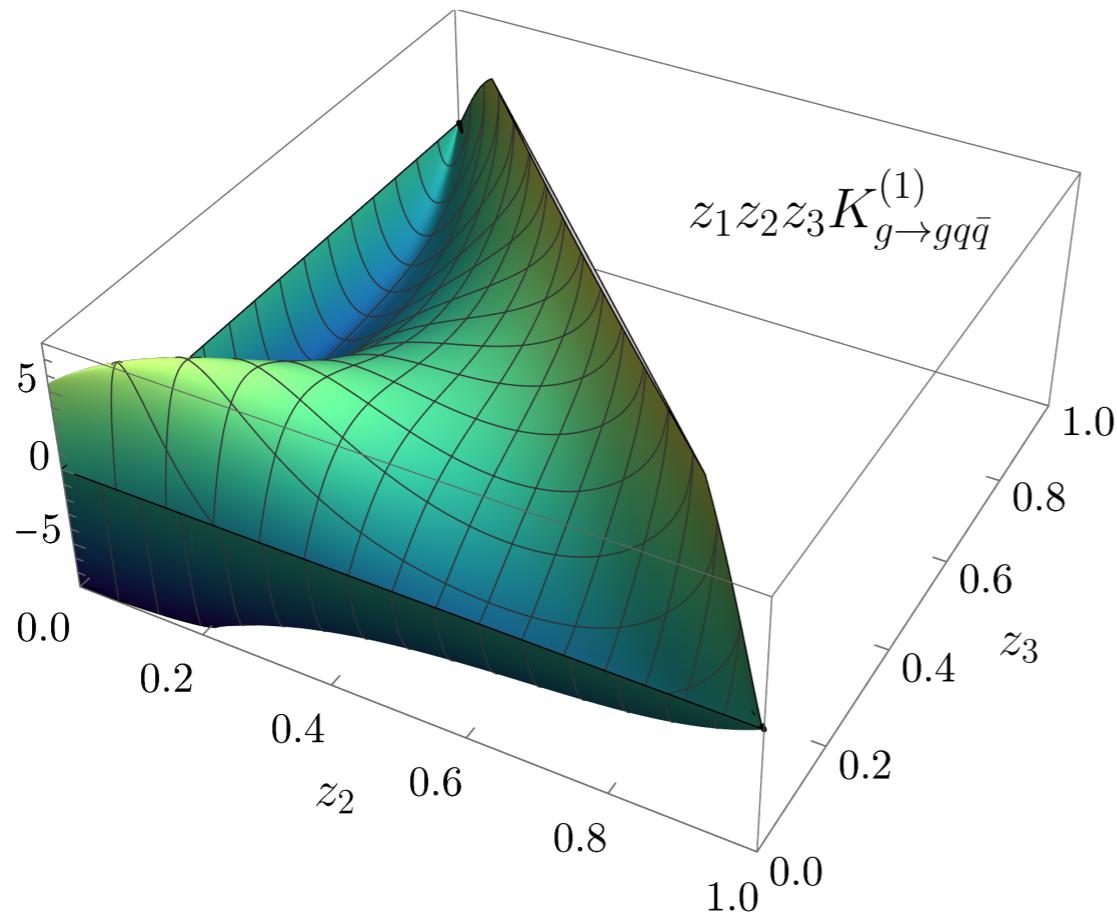
- Only need $\delta(s)$ term and IR poles (i.e. drop second line).
- To isolate poles in $J_i(s, x, \mu)$, use sector decomposition.

[Li, Moult, Schrijnder van Velzen, WW, Zhu; Jaarsma, Li, Moult, WW, Zhu; Chen, Jaarsma, Li, Moult, WW, Zhu]

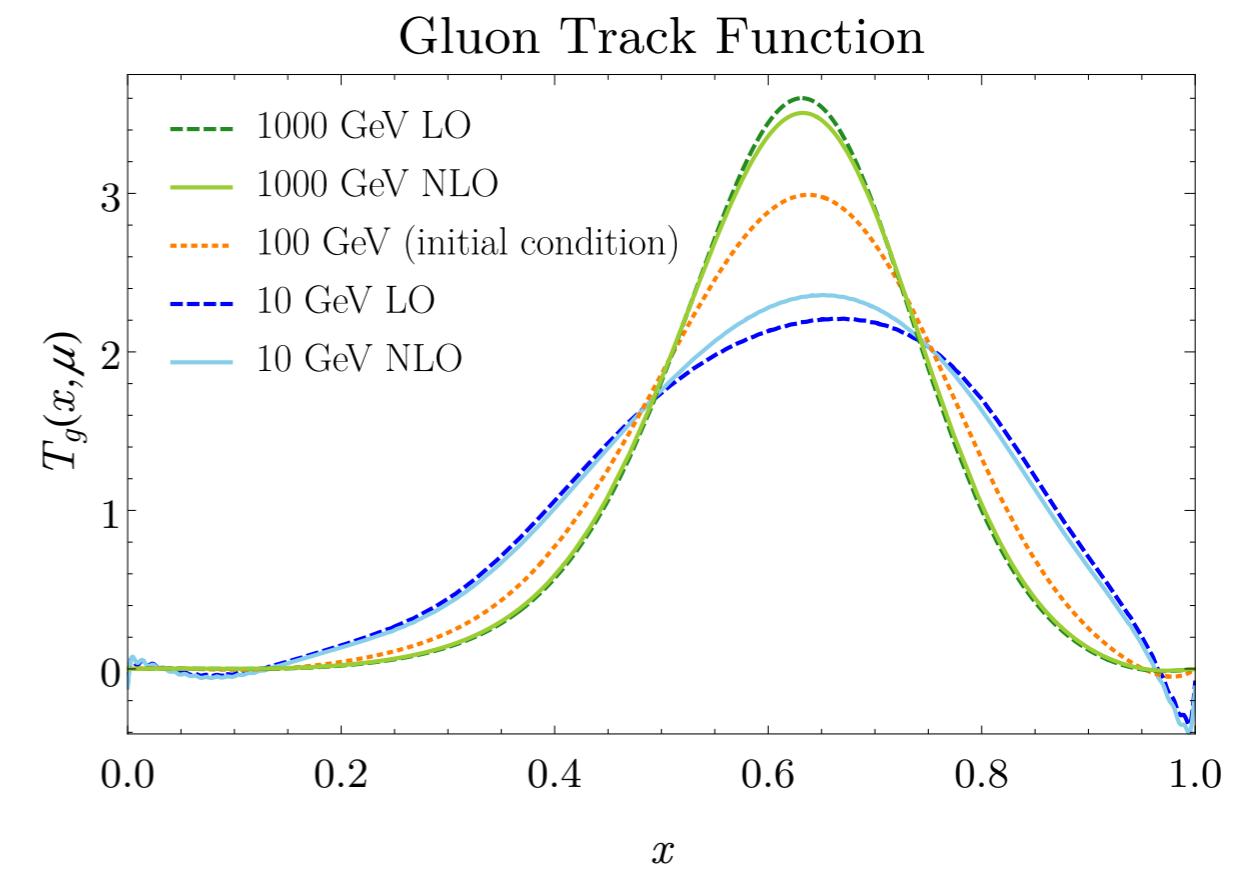
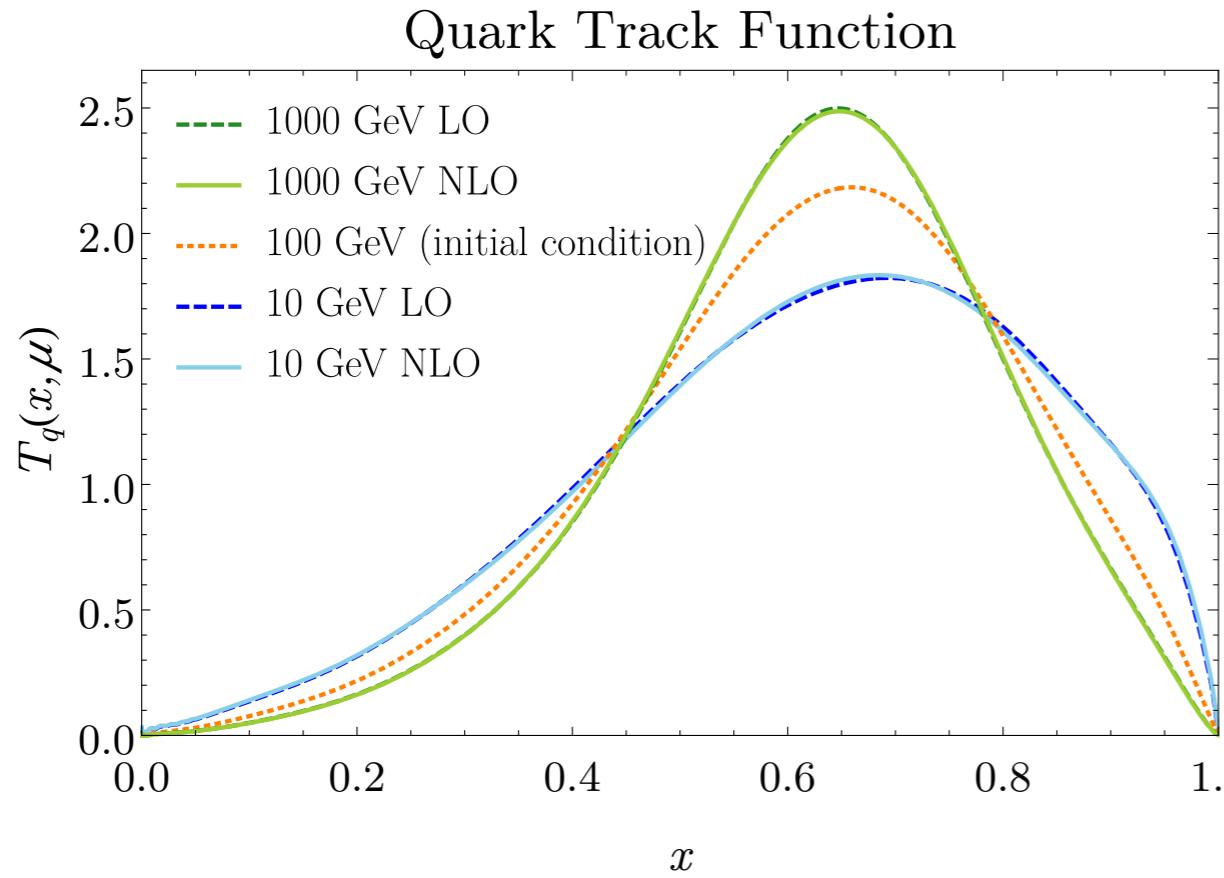
2. Track function evolution at NLO

$$\begin{aligned}\frac{d}{d \ln \mu^2} T(x, \mu) = & a_s \left[K_{1 \rightarrow 1}^{(0)} \otimes T(x, \mu) + K_{1 \rightarrow 2}^{(0)} \otimes TT(x, \mu) \right] \\ & + a_s^2 \left[K_{1 \rightarrow 1}^{(1)} \otimes T(x, \mu) + K_{1 \rightarrow 2}^{(1)} \otimes TT(x, \mu) + K_{1 \rightarrow 3}^{(1)} \otimes TTT(x, \mu) \right]\end{aligned}$$

- Kernels are lengthy and available electronically.



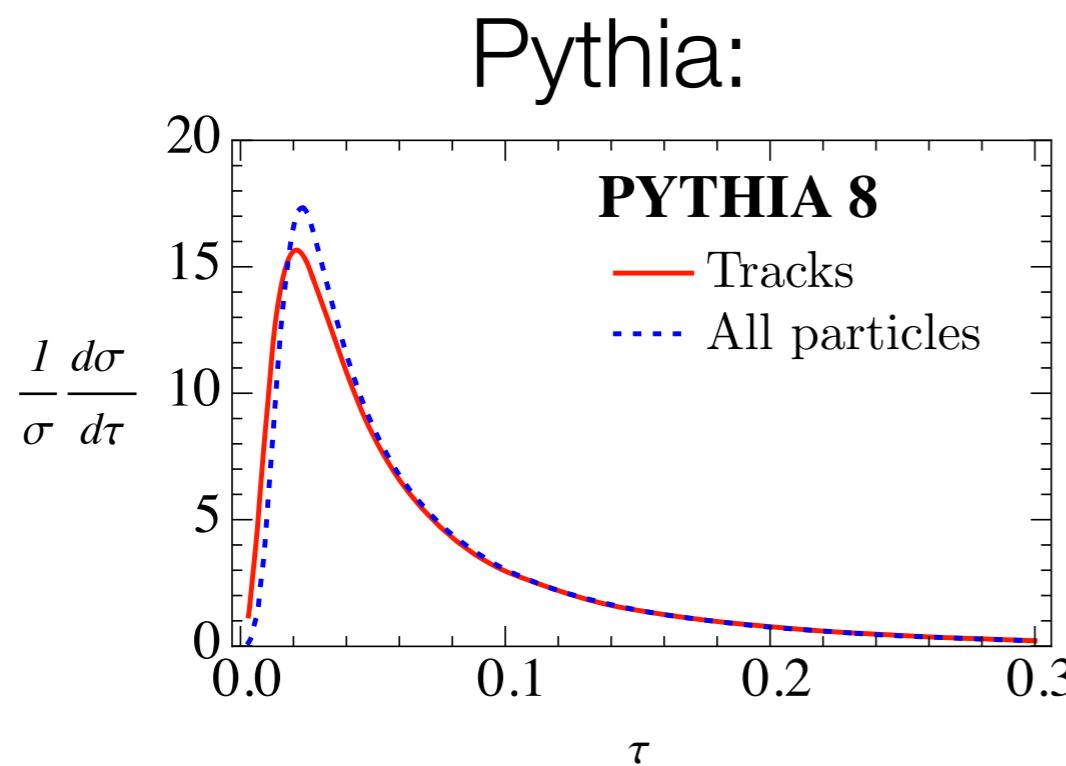
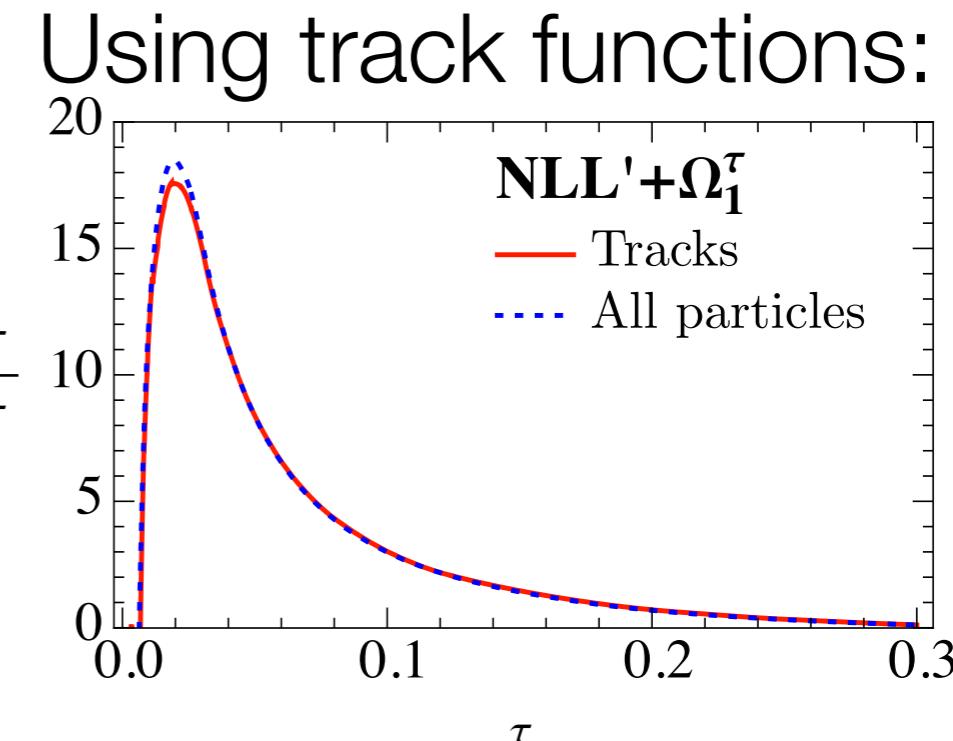
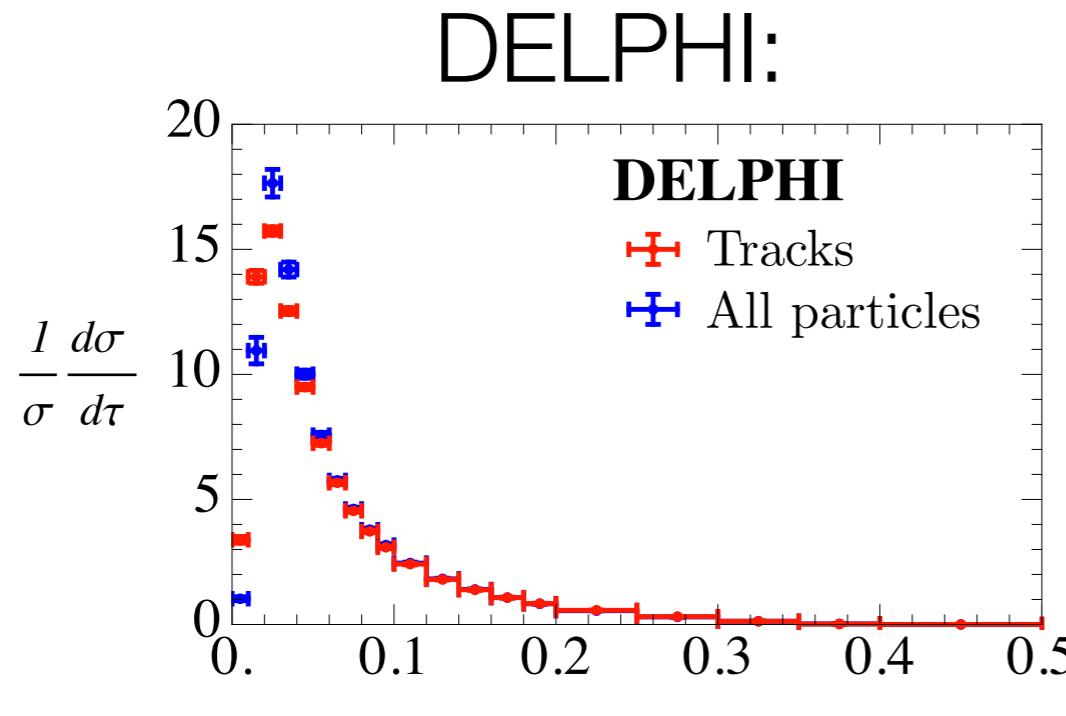
2. Results for track function evolution



- Evolution is milder than for fragmentation functions or PDFs.
- NLO evolution is a small correction, especially for quarks.
- Evolution code is available (based on moments, Fourier or Legendre wavelets).

3. Phenomenology with track functions

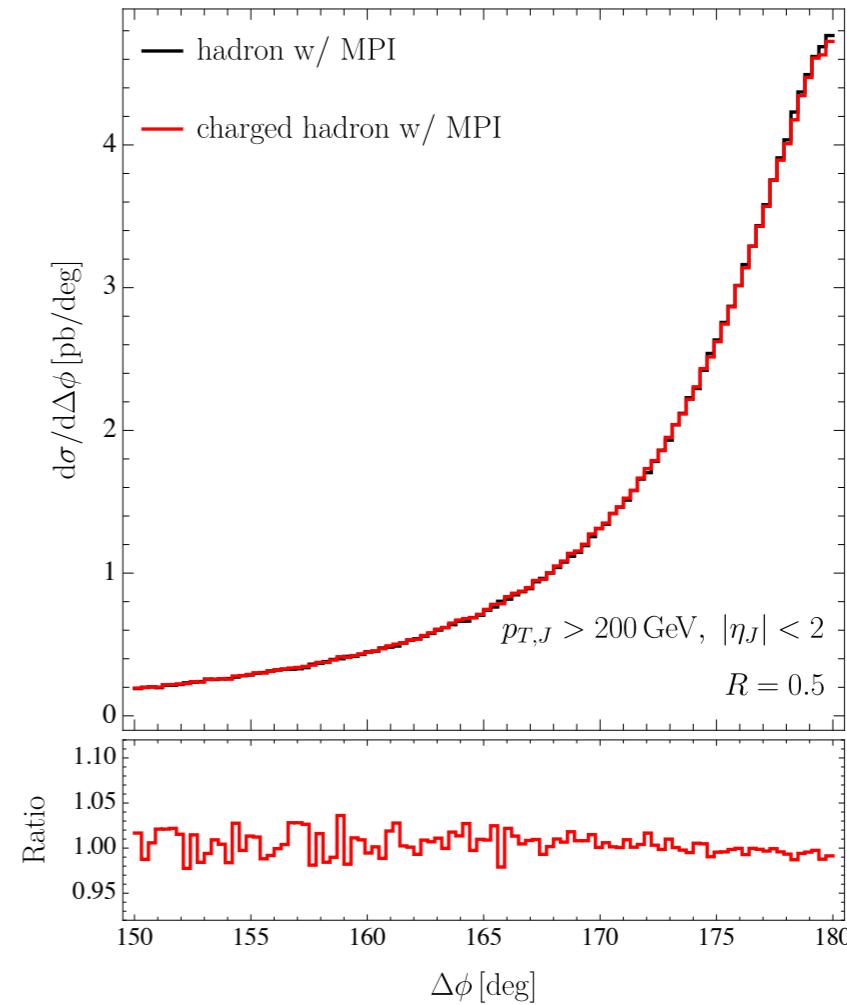
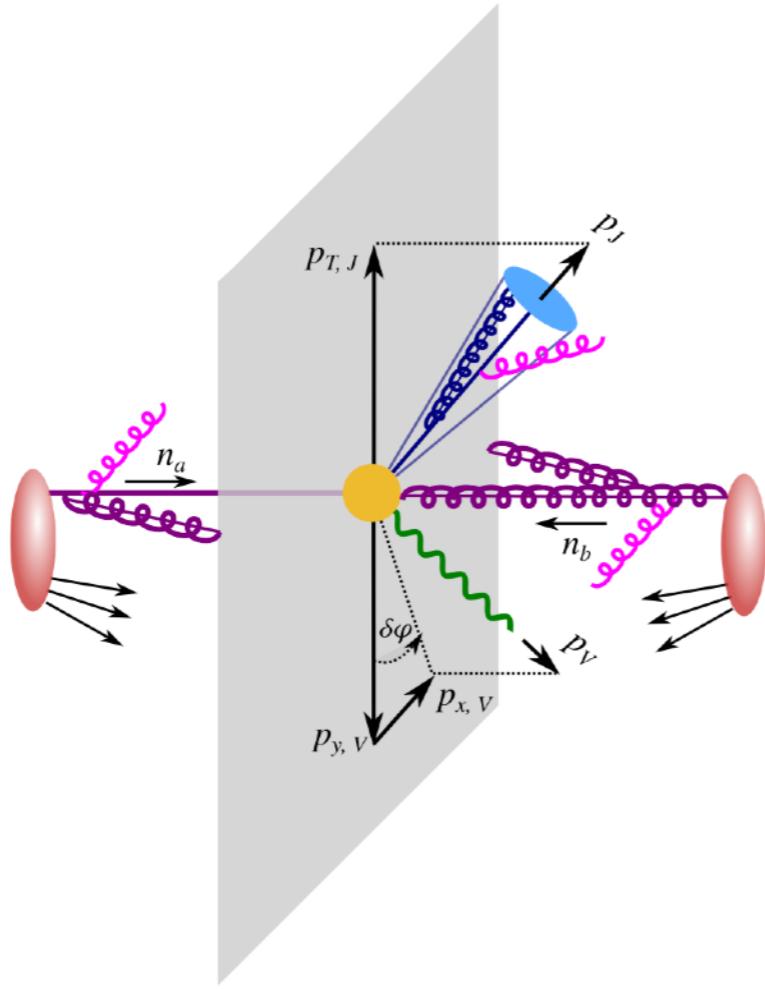
3. Track thrust



Good agreement, but...

- Not easy calculation + effect of tracks accidentally small.
- Differences in peak from nonperturbative effects.

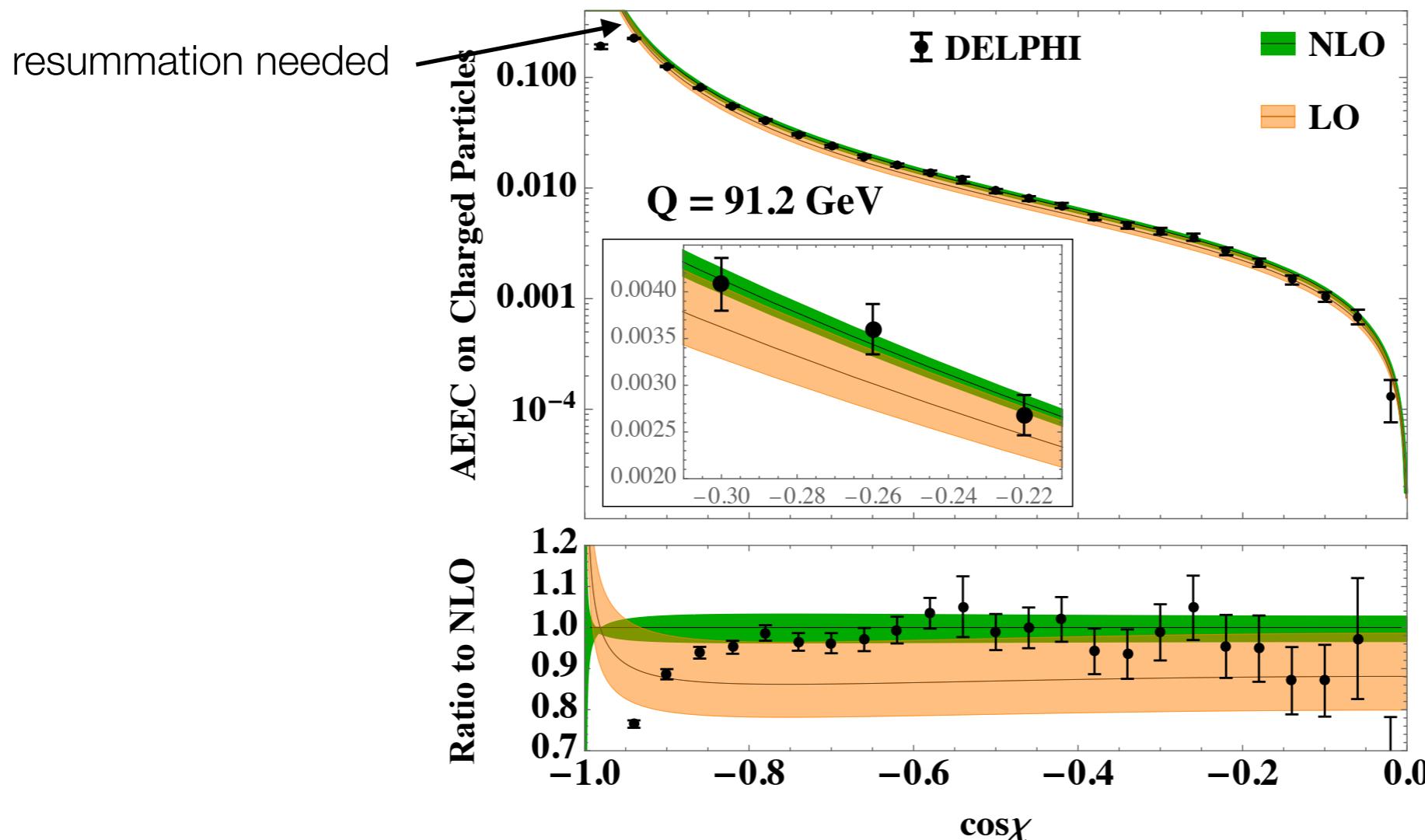
3. Azimuthal decorrelation in V +jet on tracks



- Azimuthal angle $\Delta\phi$ between vector boson and jet requires resummation in back-to-back limit.
- Using **recoil-free** recombination scheme, tracks only matter for collinear (not soft) radiation \rightarrow beyond NLL, so very small.

3. Track-based EEC

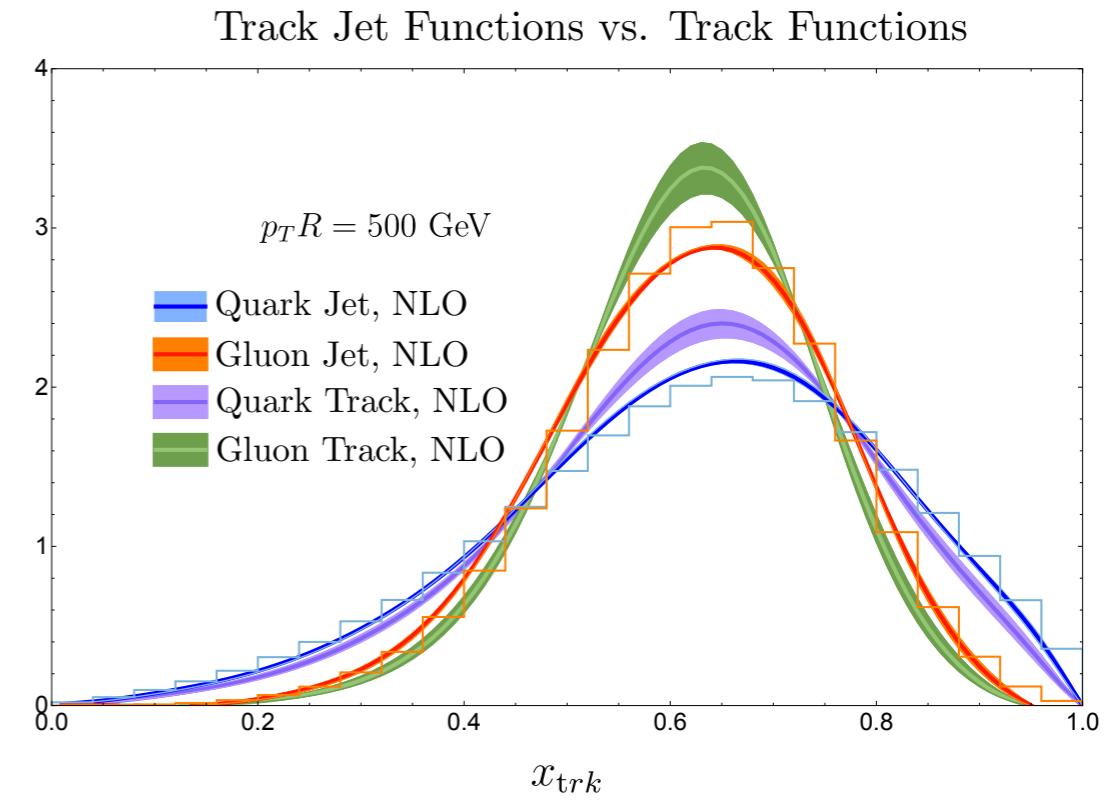
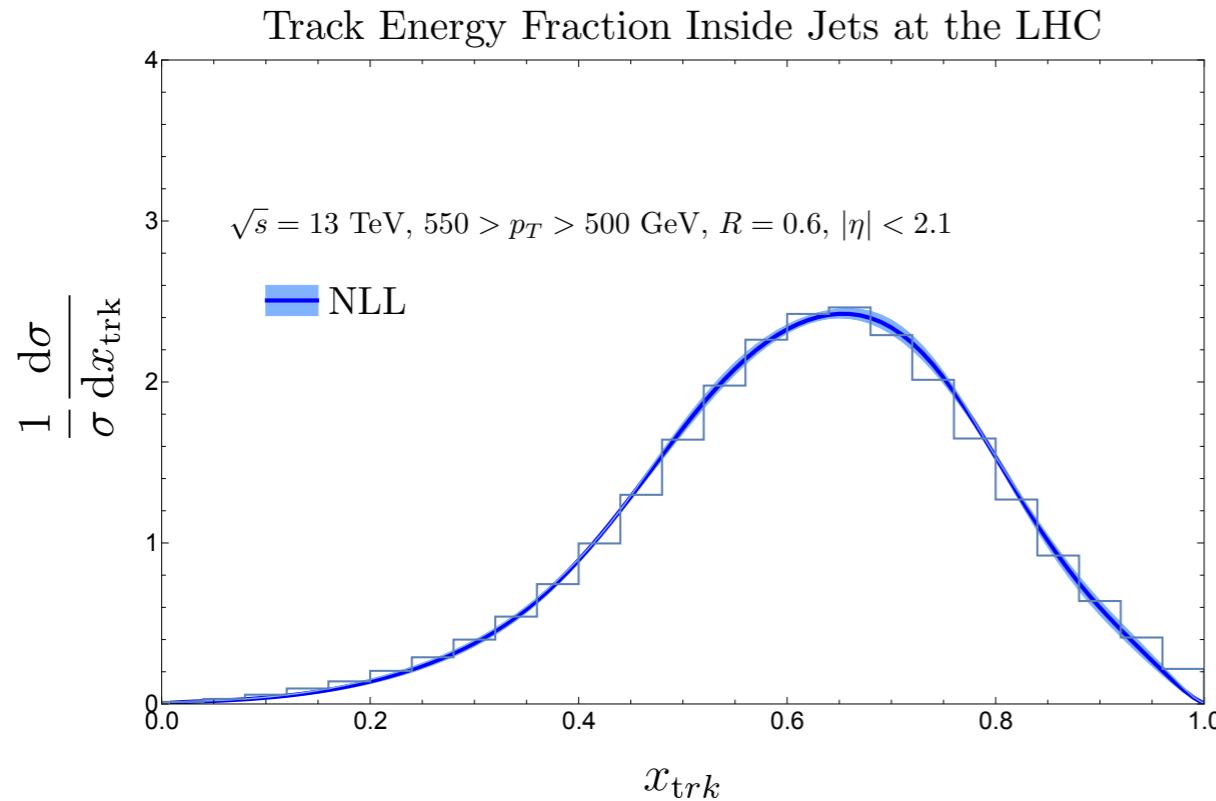
- First $\mathcal{O}(\alpha_s^2)$ result for track-based measurement:



$$\text{AEEC}(\cos \chi) = \text{EEC}(\cos \chi) - \text{EEC}(-\cos \chi)$$

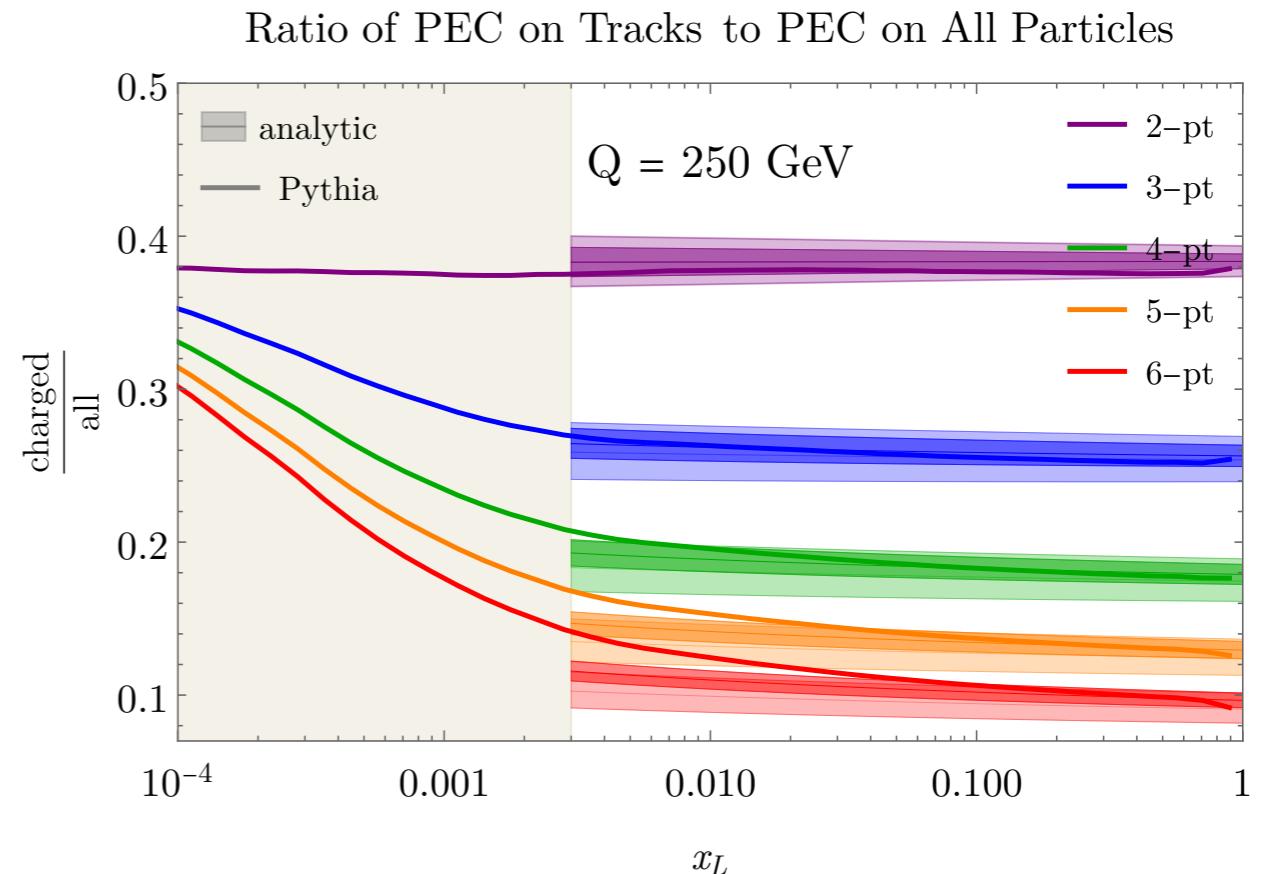
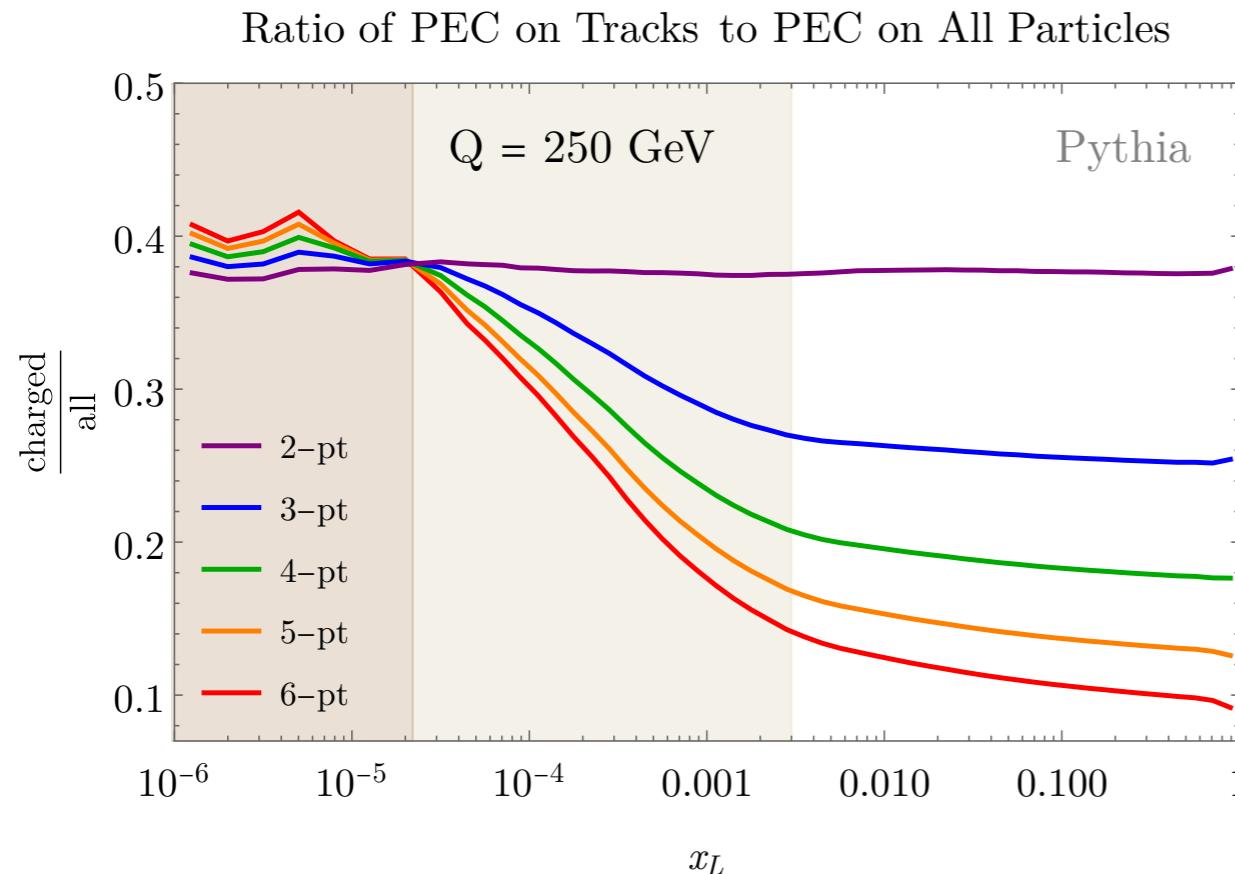
- Uncertainty reduced at NLO, tantalizing agreement with data.

3. Charged energy fraction in jets at LHC



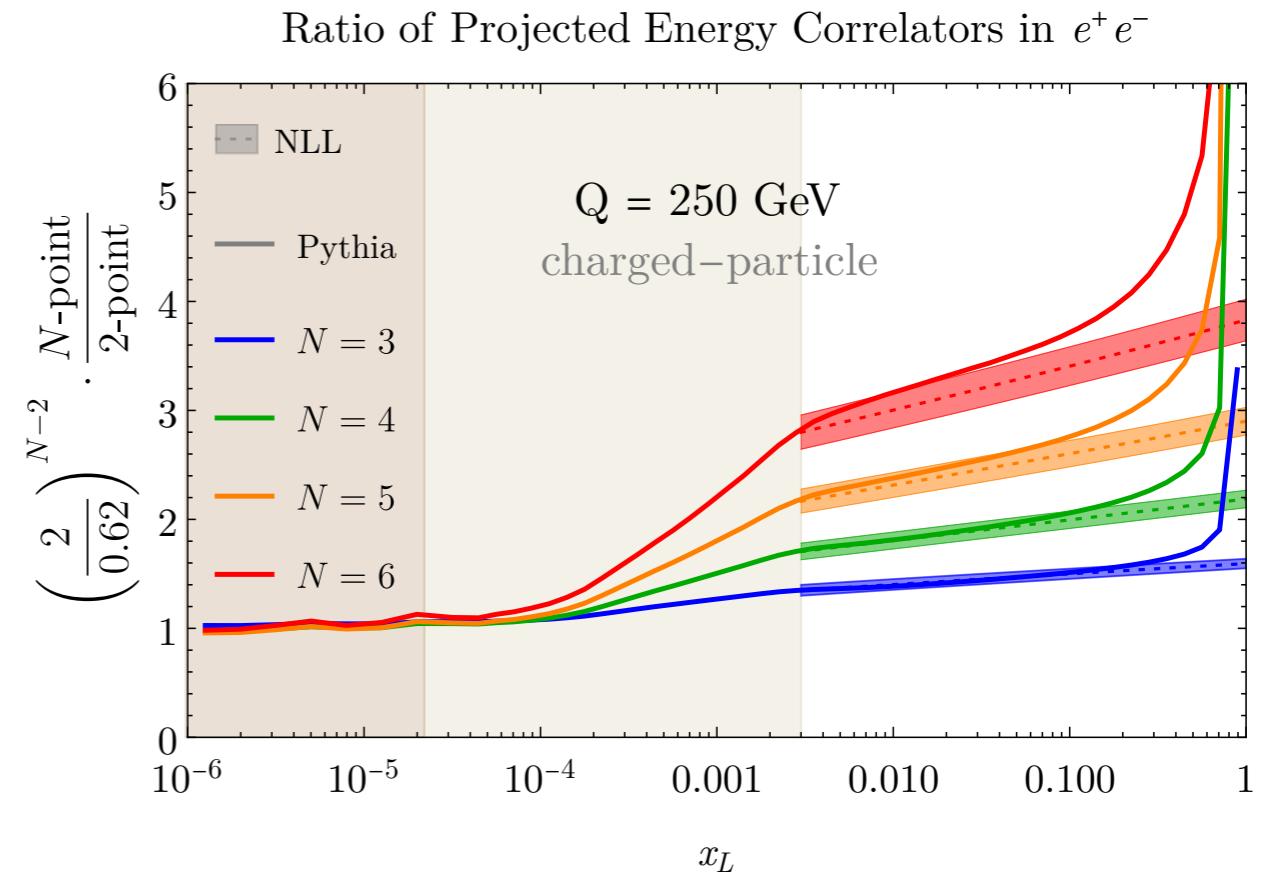
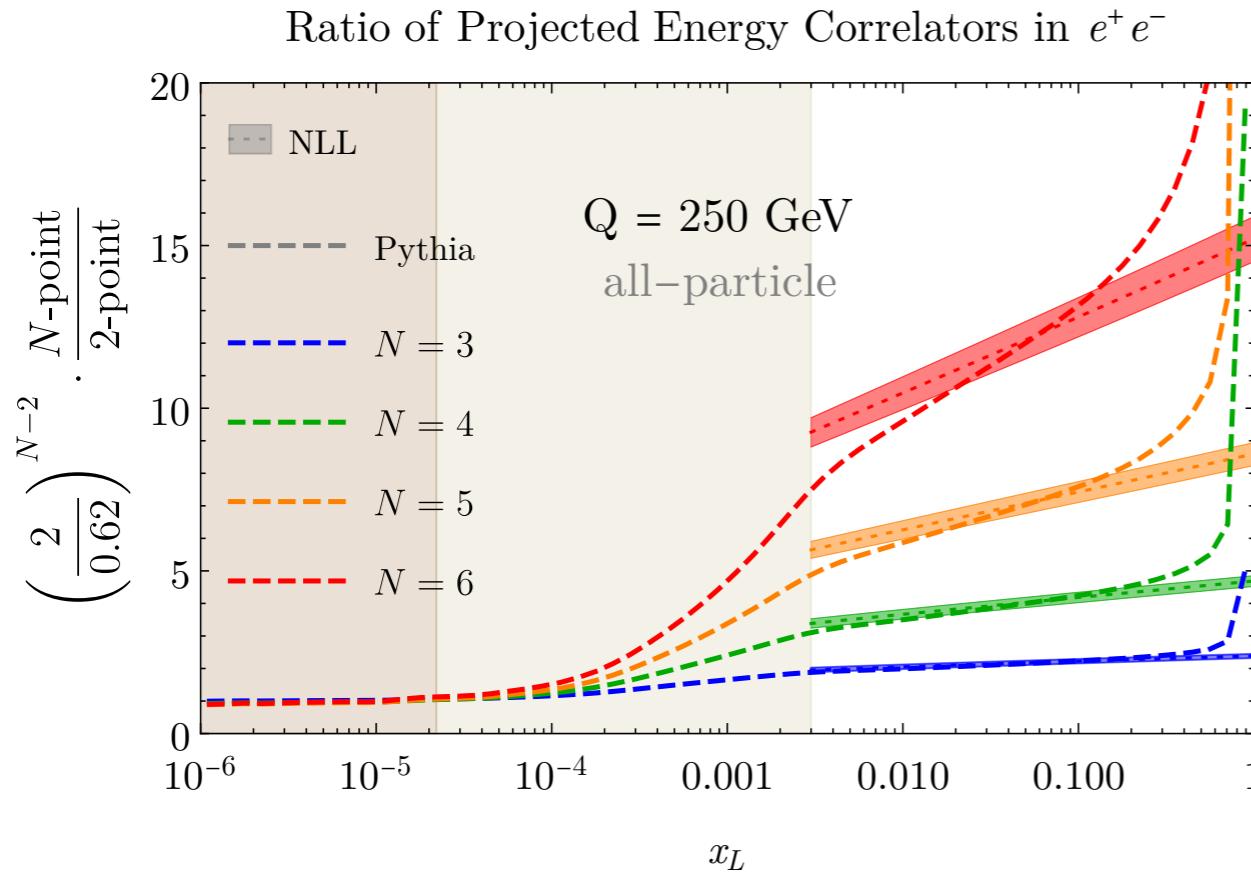
- At LO this is the track function \rightarrow use this to extract it!
- We also include effects of hard scattering (quark vs. gluon) and jet formation (nontrivial, see right panel).

3. Projected N -point energy correlator



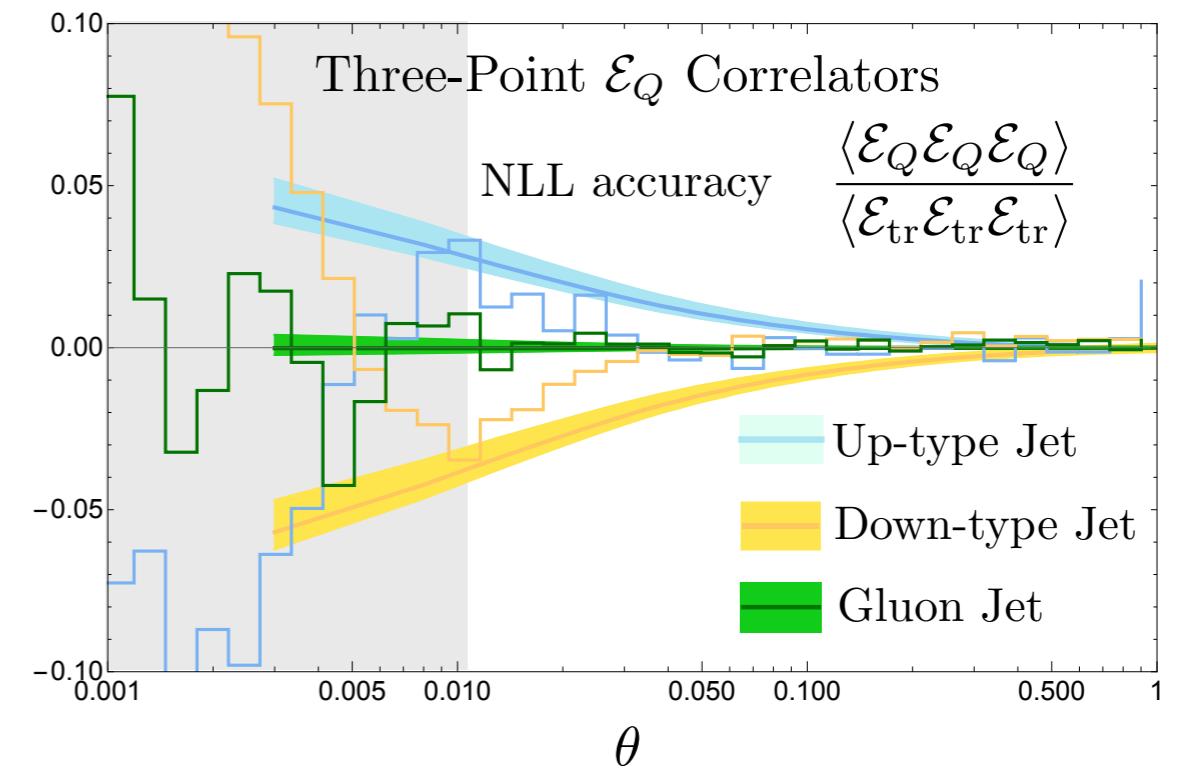
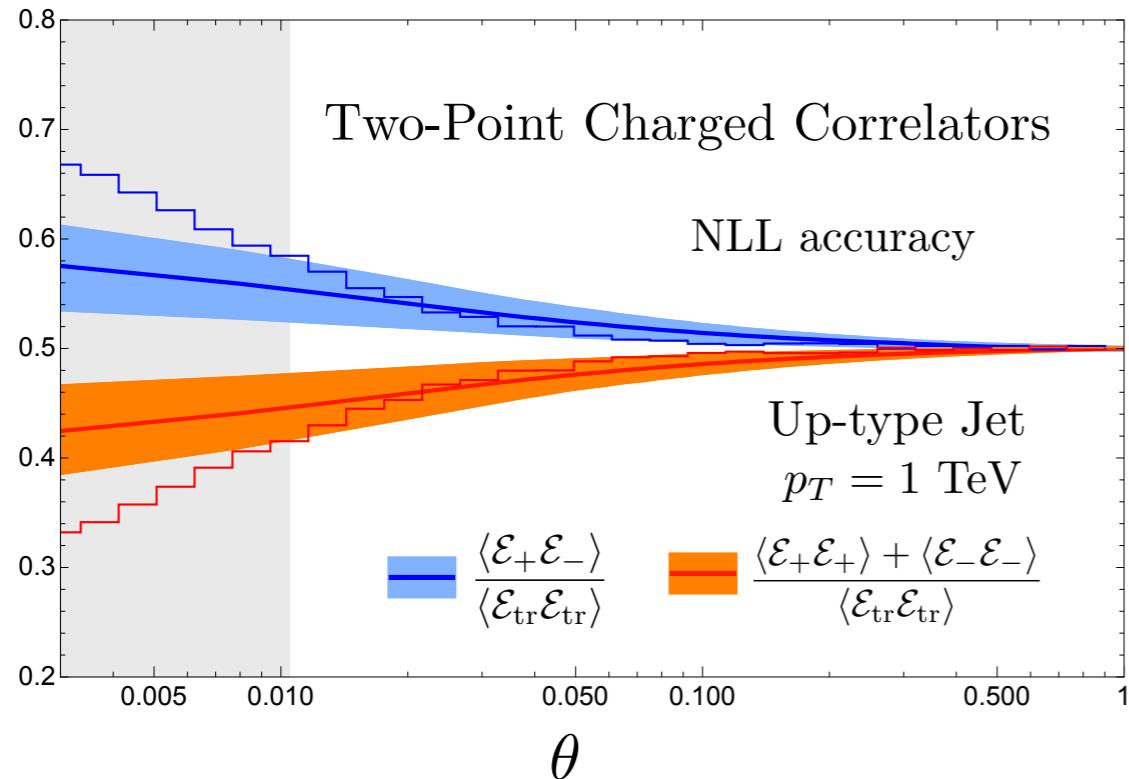
- Projected means: fix largest angle, $x_L = (1 - \cos \theta)/2$
- Ratio of charged to all particles is constant for 2-point, but not higher point.

3. Projected N -point energy correlator



- All vs. charged particles are qualitatively similar, e.g. slope increases with N
- Quantitative difference is **calculable** with track functions!

3. Energy correlators taking charge

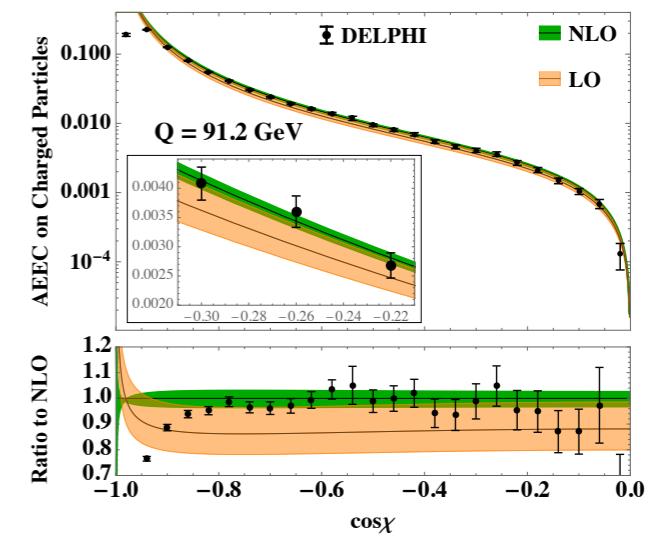
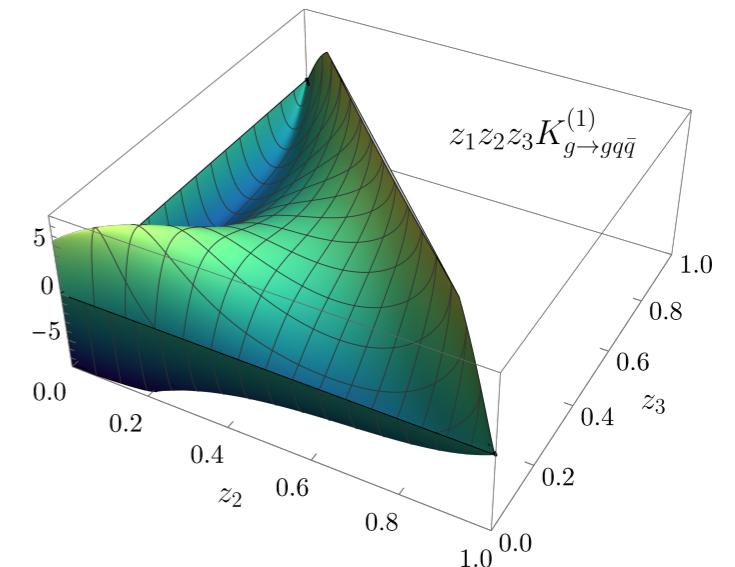
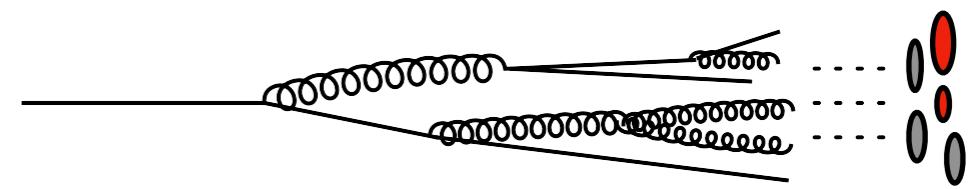


Can also consider correlators on:

- \mathcal{E}_\pm : energy of positively/negatively charged particles
- \mathcal{E}_Q : charged-weighted energy

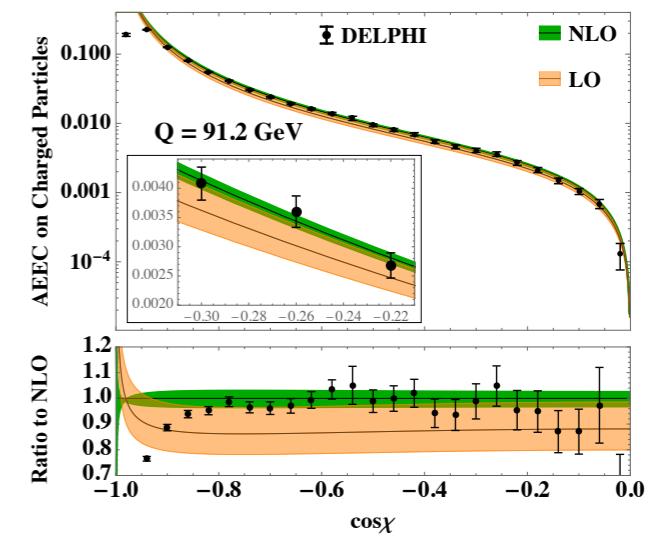
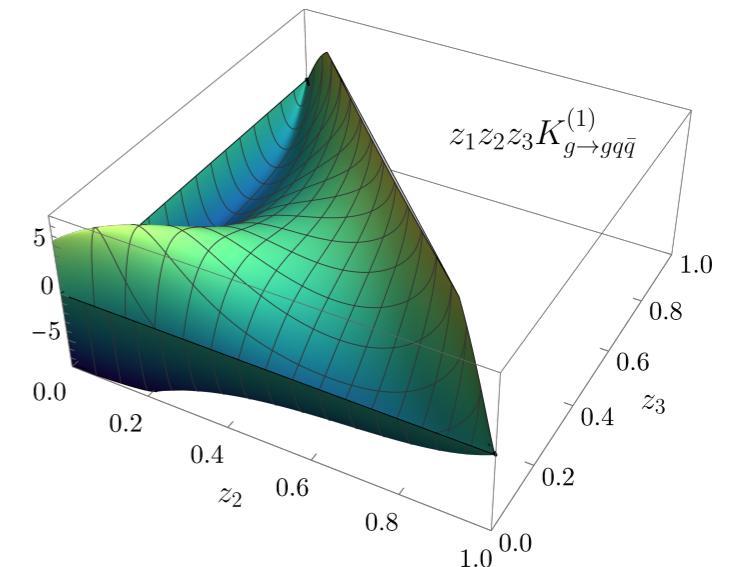
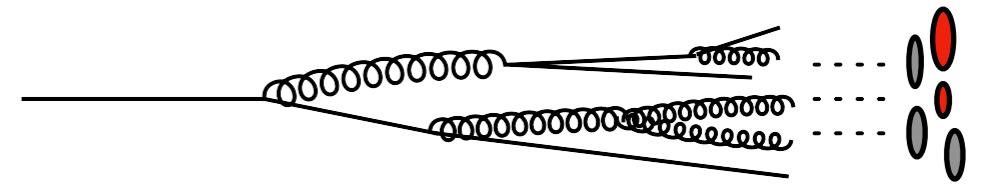
Conclusions and outlook

- Tracks offer superior angular resolution and reduce pile-up.
- Track functions framework at order α_s^2
- It's evolution contains that of multi-hadron fragmentation functions
- Applications: thrust, azimuthal decorrelation. energy correlators, ...
- Energy correlators are particular simple as they only use moments.



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Thank you!