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Collinear fragmentation at NNLL: generating functionals, groomed correlators and angularities

P. Monni (CERN)

based on work with
M. van Beekveld, M. Dasgupta, B. El-Menoufi, J. Helliwell, G. Salam
2307.15734 + ongoing work

Galileo Galilei Institute - September 2023

Motivation

- Broad classes of resummations do not admit a closed analytic solution (or very hard to derive): non-linearity of evolution equations (e.g. NGLs, micro jets, …) or lack of analytic form in multi-particle limit (e.g. complex event shapes or jet rates). Numerical methods are effective in these problems

- Large ongoing efforts to improve parton shower's perturbative (logarithmic) accuracy. Solutions at NLL now exist for rIRC safe global and classes of non-global observables: based on constraints inferred from QCD (multi-parton squared amplitudes, consistency with QCD resummations)

  ➡ Higher orders (e.g. NNLL) present additional subtleties, e.g. treatment of virtual corrections in dimensional regularisation and cancellation of IR singularities

GOAL: work towards a solid framework to bridge between resummations and parton showers. Crucial to study features of shower evolution (e.g. IR cutoffs) and develop algorithmic solutions beyond NLL

Outline of the talk

- Focus on collinear fragmentation

$$\ln \Sigma(v) \sim \alpha_s L + \alpha_s^2 L^2 + \ldots \quad \rightarrow \text{SL (NLL in DL obs.)}$$
$$+ \alpha_s + \alpha_s^2 L + \alpha_s^3 L^2 + \ldots \quad \rightarrow \text{NSL (NNLL in DL obs.)}$$
$$+ \ \ldots$$

- Generating functional method
- Application to fractional moments of EEC (FC_x) and angularities (λ_x) measured on mMDT/SD groomed jets
  - ➡ analytic solution at SL & Markov chain algorithm
- Formulation at NSL: application to FC_x and λ_x
- Outlook

Disclaimer: slides mainly prepared on a train ride from Geneva to Florence, apologies for the poor quality and the omission of some references

SL fragmentation

Generating functionals: definitions

see e. g. [Konishi, Ukawa, Veneziano '79; Dokshitzer, Khoze, Mueller, Troyan '91]

- GFs method postulates the existence of $2\,n_f + 1$ generating functionals $G_f(x, t)$, which describe the (time-like) fragmentation of a parton of flavour f, carrying a fraction x of the initial energy E, and starting at an initial evolution "time" t, function of the emission's kinematics (e.g. angle)

$$t_i = \int_{\theta_i^2}^{1} \frac{d\theta^2}{\theta^2} \frac{\alpha_s(E^2 g^2(z)\,\theta^2)}{2\pi} = \frac{\alpha_s}{2\pi} \ln\frac{1}{\theta_i^2} + \mathcal{O}(\alpha_s^2)$$

- The cross section for the production of a final state with exactly m final state partons originating from the above fragmentation reads

$$\int dP_m^{(f)} = \frac{1}{m!} \frac{\delta^m}{\delta u^m} G_f(x,t)\Big|_{\{u\}=0}.$$

probing function (source) $u = u(x, t; f)$

- Physical observables then computed as

$$d\sigma^{(f)} = \sigma_0\, C(\alpha_s) \otimes J^{(f)}(\alpha_s, v)$$

perturbative matching coefficient

$$J^{(f)}(\alpha_s, v) = \sum_{m=1}^{\infty} \int dP_m^{(f)}\, \delta(v - V(\{k\}_m))$$

Generating functionals: evolution equations

- Evolution of GFs with time t is governed by a system of equations (anti-quark GF by charge conjugation)

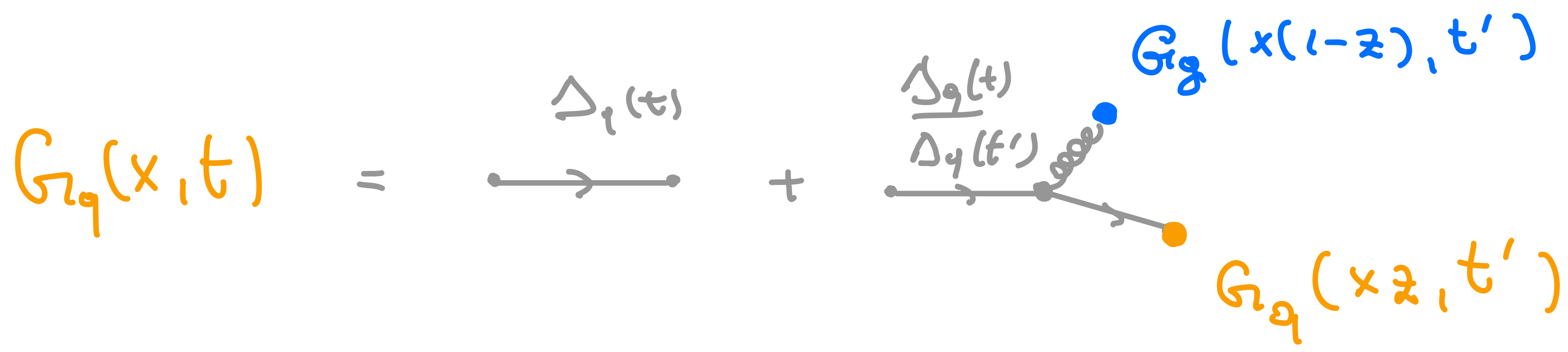
Sudakov form factor = no-emission prob.

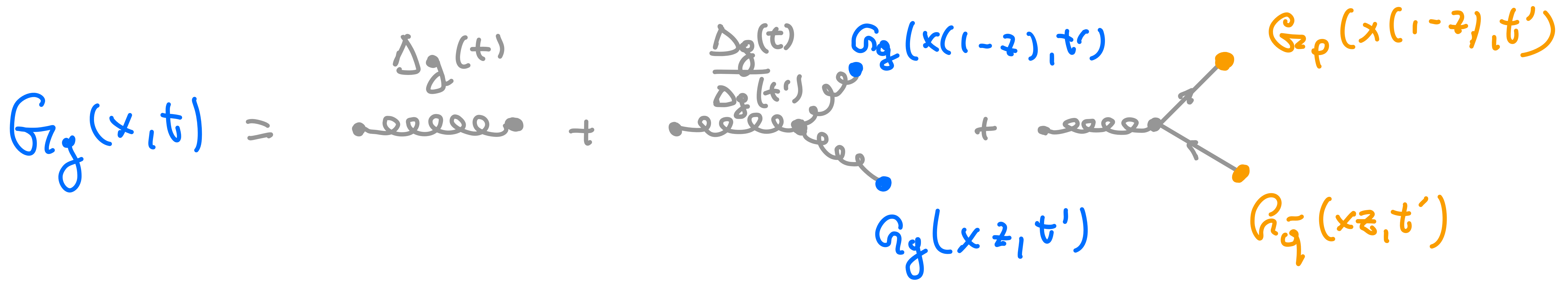
$$G_q(x,t) = u\,\Delta_q(t) + \int_t^{t_0} dt' \int_{z_0}^{1-z_0} dz\, P_{qq}(z)\, G_q(x\,z,t')\, G_g(x\,(1-z),t')\,\frac{\Delta_q(t)}{\Delta_q(t')}$$

$$G_g(x,t) = u\,\Delta_g(t) + \int_t^{t_0} dt' \int_{z_0}^{1-z_0} dz \Big[P_{gg}(z)\, G_g(x\,z,t')\, G_g(x\,(1-z),t') + P_{qg}(z)\, G_q(x\,z,t')\, G_q(x\,(1-z),t')\Big]\frac{\Delta_g(t)}{\Delta_g(t')}\,,$$

see e. g. [Dasgupta, Dreyer, Salam, Soyez '14]

- Or in graphic form





Remarks

- **Sudakov form factor**: defined by requiring unitarity of GFs (i.e. total XS unaffected by inclusive collinear radiation)

$$G_f(x,t)\big|_{u=1} = 1 \quad \rightarrow \quad \begin{aligned} \ln \Delta_q(t) &= -\int_t^{t_0} dt' \int_{z_0}^{1-z_0} dz\, P_{qq}(z)\,, \\ \ln \Delta_g(t) &= -\int_t^{t_0} dt' \int_{z_0}^{1-z_0} dz\, (P_{gg}(z) + P_{qg}(z)) \end{aligned}$$

- **Regularisation scheme**: IRC singularities could be consistently regulated in dim. reg., but the use of IR cutoffs allows for algorithmic solution in a computer (Monte Carlo). Important for connection with PS. Physical results are always obtained in the limit $t_0 \to \infty\,, z_0 \to 0$ (modulo Landau pole regularisation)

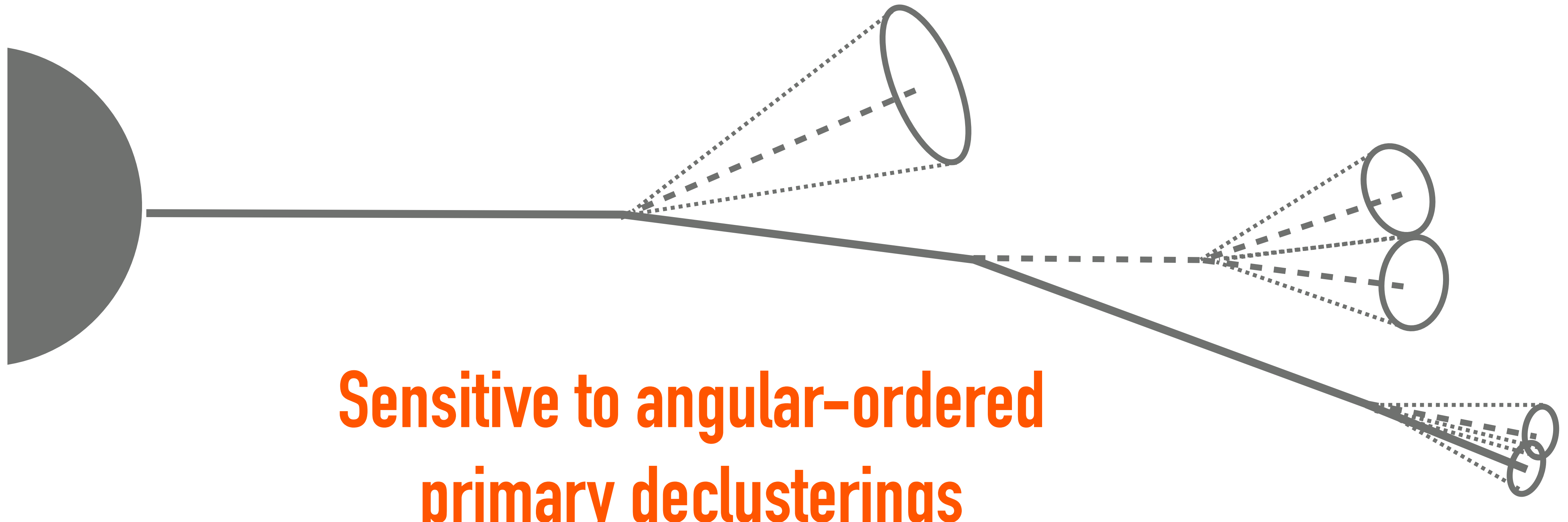
- **Ordering**: choice of ordering (definition of t) such that multi-parton squared amplitudes are reconstructed recursively order by order. Options in the collinear limit (angle, transverse momentum, …). Crucially, physical results not affected by this choice

An example:
fractional moments of EEC and angularities

Moments of EEC and angularities on groomed jets

- Consider a simple observables that admit an analytic solution
  - ➡ Consider a mMDT/SD ($\beta = 0$) groomed jet, and measure

These observables are naturally double logarithmic, though grooming makes them single logarithmic by eliminating soft logs



$$FC_x^{\mathcal{H}} = \frac{2^{-x}}{E^2} \sum_{i \neq j} E_i E_j |\sin\theta_{ij}|^x (1 - |\cos\theta_{ij}|)^{1-x}$$

$$\lambda_x^{\mathcal{H}} = \frac{2^{1-x}}{E} \sum_{i} E_i \, |\sin\theta_i|^x (1 - |\cos\theta_i|)^{1-x}$$

- This class of event shapes is insensitive to the full non-linear structure of the fragmentation. At SL we can ignore the (secondary) fragmentation of primary radiation, e.g. for quark jets $G_g(x,t) \simeq u$

$$G_q(x,t) = u\,\Delta_q(t) + \int_t^{t_0} dt' \int_{z_0}^{1-z_0} dz\, P_{qq}(z)\, G_q(x\,z, t')\, \overset{u}{\cancel{G_g(x(1-z), t')}}\, \frac{\Delta_q(t)}{\Delta_q(t')}$$

Analytic solution

- Resummation can be worked out analytically

$$J^{(f)}(\alpha_s, v) = \sum_{m=1}^{\infty} \int dP_m^{(f)}\, \delta(v - V(\{k\}_m))$$

→ Weigh each probability with measurement function, e.g. for FC_x (using $\Theta_{z_{\rm cut}}(z) \equiv \Theta(z - z_{\rm cut})\,\Theta(1 - z - z_{\rm cut})$)

$$\int dP_1^{(q)} = \Delta_q(t) \qquad \times\, \delta(FC_x)$$

$$\int dP_2^{(q)} = \Delta_q(t) \int_t^{t_0} dt_1 \int_{z_0}^{1-z_0} dz_1\, P_{qq}(z_1) \qquad \times \left[\delta(FC_x - z_1(1-z_1)\theta_{g_1q}^{2-x})\,\Theta_{z_{\rm cut}}(z_1) + \delta(FC_x)\,\left(1 - \Theta_{z_{\rm cut}}(z_1)\right)\right]$$

$$\int dP_3^{(q)} = \Delta_q(t) \int_t^{t_0} dt_1 \int_{t_1}^{t_0} dt_2 \int_{z_0}^{1-z_0} dz_1 dz_2\, P_{qq}(z_1)\, P_{qq}(z_2) \quad \times \Big[\delta(FC_x - z_1(1-z_1)\theta_{g_1q}^{2-x})\,\Theta_{z_{\rm cut}}(z_1) + \delta(FC_x)\,(1 - \Theta_{z_{\rm cut}}(z_1))\,(1 - \Theta_{z_{\rm cut}}(z_2))$$

$$\ldots = \ldots \qquad + \delta(FC_x - z_2(1-z_2)\theta_{g_2q}^{2-x})\,\Theta_{z_{\rm cut}}(z_2)\,(1 - \Theta_{z_{\rm cut}}(z_1))\Big]$$

$$\Sigma(FC_x) = \frac{1}{\sigma_0}\int_0^{FC_x} \frac{d\sigma}{dO_x} dO_x = \exp\left\{ - \int dt' \int_{z_{\rm cut}}^{1-z_{\rm cut}} dz\, P_{qq}(z)\Theta(z(1-z)\theta^{2-x} - FC_x) \right\}$$

Monte Carlo solution

- Generate m-particles states with a Markov chain MC
  - Measure observable only at the end of the evolution (PS like)

$$-d\left(\Delta_q(t)/\Delta_q(t_1)\right)$$

$$\int dP_1^{(q)} = \Delta_q(t) = \frac{\Delta_q(t)}{\Delta_q(t_0)}$$

$$\int dP_2^{(q)} = \Delta_q(t)\int_t^{t_0} dt_1 \int_{z_0}^{1-z_0} dz_1\, P_{qq}(z_1) = \int_t^{t_0} dt_1 \int_{z_0}^{1-z_0} dz_1 \frac{\Delta_q(t)}{\Delta_q(t_1)} P_{qq}(z_1) \frac{\Delta_q(t_1)}{\Delta_q(t_0)}$$

$$\int dP_3^{(q)} = \Delta_q(t)\int_t^{t_0} dt_1 \int_{t_1}^{t_0} dt_2 \int_{z_0}^{1-z_0} dz_1 dz_2\, P_{qq}(z_1)\, P_{qq}(z_2) = \int_t^{t_0} dt_1 \int_{t_1}^{t_0} dt_2 \int_{z_0}^{1-z_0} dz_1 dz_2 \frac{\Delta_q(t)}{\Delta_q(t_1)} P_{qq}(z_1) \frac{\Delta_q(t_1)}{\Delta_q(t_2)} P_{qq}(z_1) \frac{\Delta_q(t_2)}{\Delta_q(t_0)}$$

$$\ldots = \ldots$$

$$\frac{\Delta_q(t_i)}{\Delta_q(t_{i+1})} = \chi \in [0,1]$$

recursively solve for next evolution time until $t_{i+1} > t_0$

NSL fragmentation

Anatomy of NSL formulation

$$d\sigma^{(f)} = \sigma_0\, C(\alpha_s) \otimes J^{(f)}(\alpha_s, v)$$

Matching coefficient at one loop
(coupling at the hard scale for IRC safe obs.)

GFs evolution with two loop kernels

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GFs evolution with two loop kernels

- Two loop corrections to evolution equation (e.g. quark fragmentation in NS channel)

Virtual corrections
(for free from unitarity)

$$G_q(x,t) = u\,\Delta_q(t) + \int_t^{t_0} dt' \int_{z_0}^{1-z_0} dz\; G_q(x\,z, t')\, G_g(x\,(1-z), t')\, \frac{\Delta_q(t)}{\Delta_q(t')}\, \mathcal{P}_q(z,\theta) + \mathbb{K}_q^{\rm finite}[G_q, G_g]\,.$$

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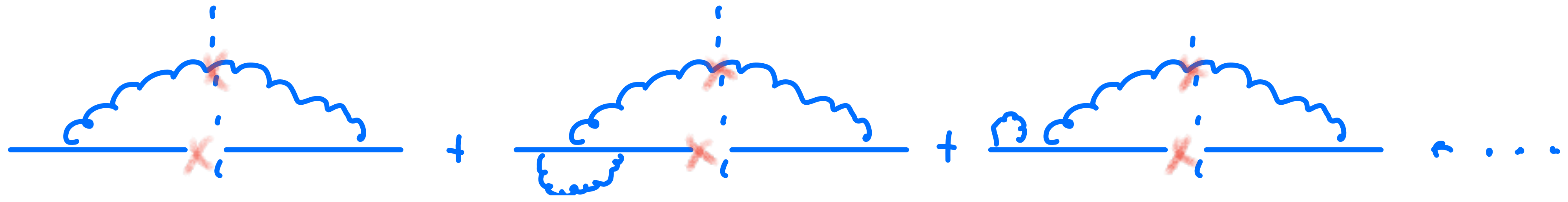

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$$+\, \mathbb{K}_q^{\text{finite}}[G_q, G_g]\,.$$

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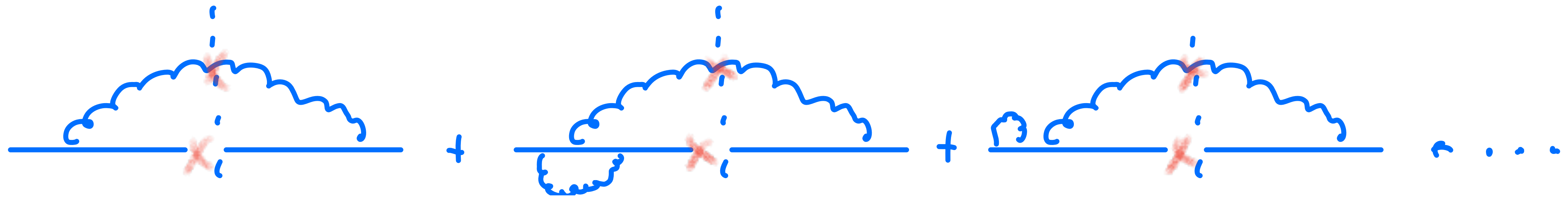

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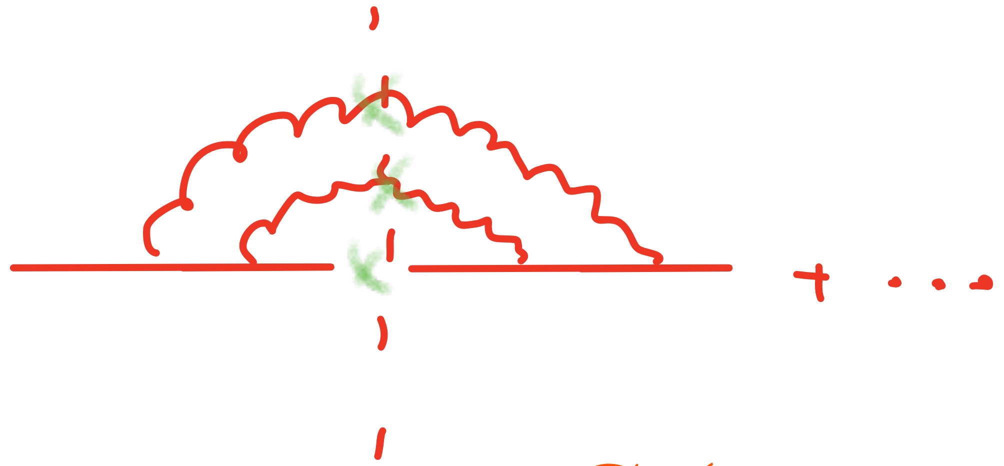
$$G_q(x,t) = u\,\Delta_q(t) + \int_t^{t_0} dt' \int_{z_0}^{1-z_0} dz\; G_q(x\,z, t')\, G_g(x\,(1-z), t')\, \frac{\Delta_q(t)}{\Delta_q(t')}\, \mathcal{P}_q(z,\theta)$$

$$+\, \mathbb{K}_q^{\rm finite}[G_q, G_g]\,.$$

$$\mathbb{K}_q^{\rm finite}[G_q, G_g] \equiv \mathbb{K}_q^{\rm R}[G_q, G_g] - \mathbb{K}_q^{\rm DC}[G_q, G_g]$$



$$\supset G_g(x_1, t_{1,23})\, G_g(x_2, t_{2,3})\, G_q(x_3, t_{2,3})$$

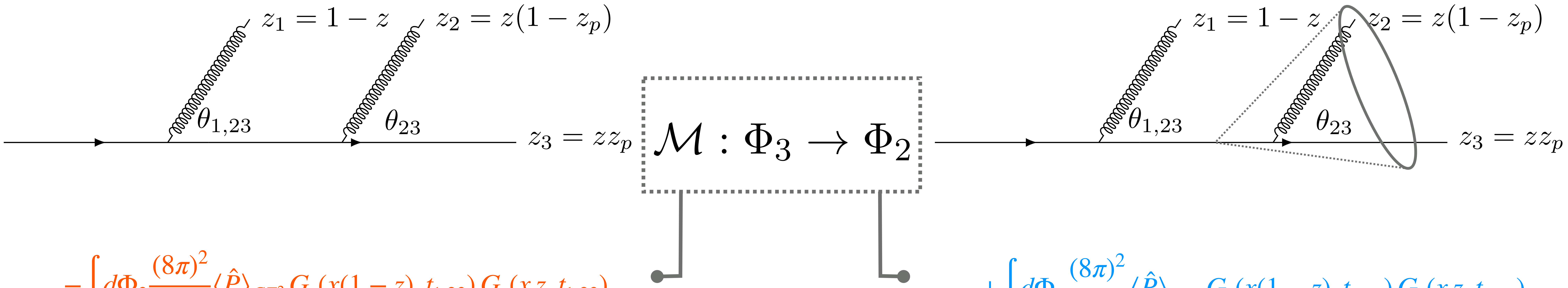
Subtract iteration of
one-loop evolution
operator

Cancellation of IRC divergences

- Local counter-term to make cancellation manifest and evaluate numerically (e.g. quark NS C_F^2 channel)

$$\int d\Phi_3 \frac{(8\pi)^2}{s_{123}^2} \langle \hat{P} \rangle_{CF^2} \, G_g(x(1-z), t_{1,23}) \, G_g(x\,z(1-z_p), t_{2,3}) \, G_q(x\,z\,z_p, t_{2,3})$$

$$\int d\Phi_2 \mathscr{V}^{(1)}_{CF^2}(z,\epsilon) \, G_g(x(1-z), t_{1,2}) \, G_q(x\,z, t_{1,2})$$



$$-\int d\Phi_3 \frac{(8\pi)^2}{s_{123}^2} \langle \hat{P} \rangle_{CF^2} \, G_g(x(1-z), t_{1,23}) \, G_q(x\,z, t_{1,23})$$

$$+\int d\Phi_3 \frac{(8\pi)^2}{s_{123}^2} \langle \hat{P} \rangle_{CF^2} \, G_g(x(1-z), t_{1,23}) \, G_q(x\,z, t_{1,23})$$

Cancellation of IRC divergences

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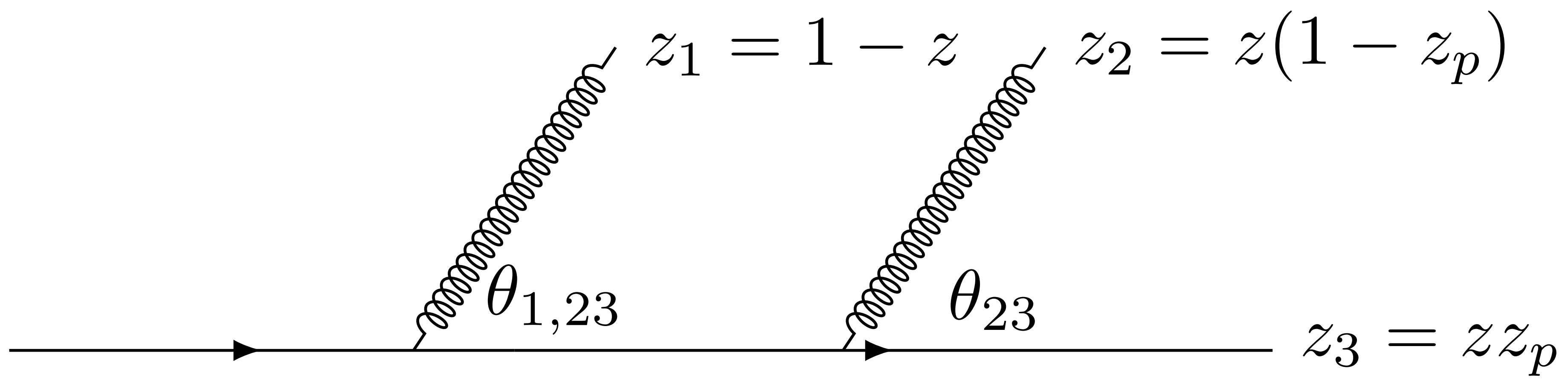
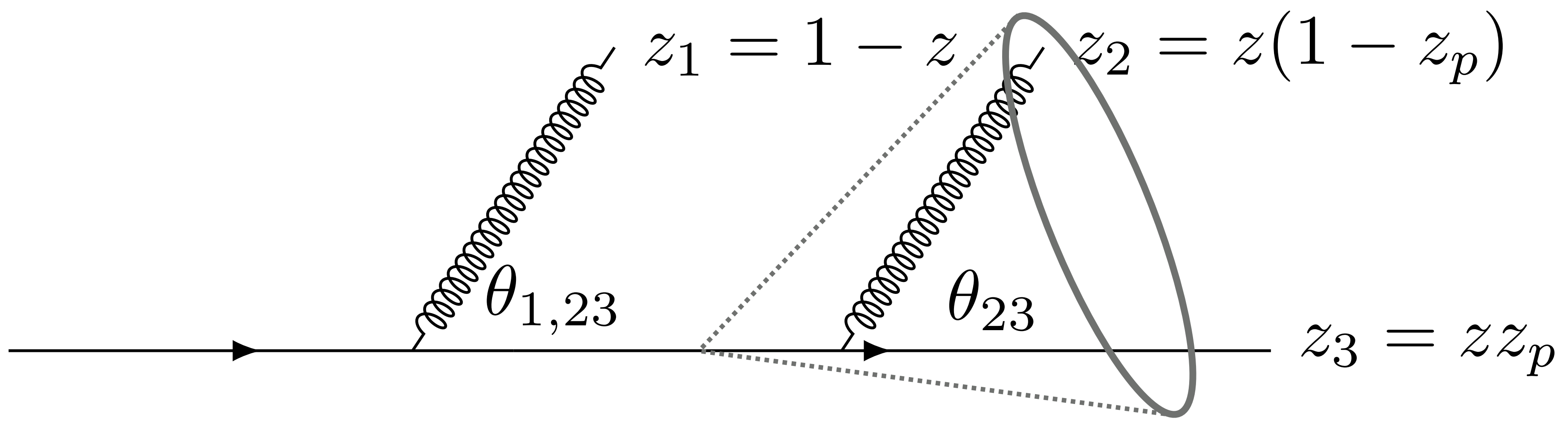
$$\int d\Phi_3 \frac{(8\pi)^2}{s_{123}^2} \langle \hat{P} \rangle_{CF^2} G_g(x(1-z), t_{1,23})\, G_g(x\,z(1-z_p), t_{2,3})\, G_q(x\,z\,z_p, t_{2,3})$$



$$-\int d\Phi_3 \frac{(8\pi)^2}{s_{123}^2} \langle \hat{P} \rangle_{CF^2} G_g(x(1-z), t_{1,23})\, G_q(x\,z, t_{1,23})$$

calculate in D=4
(e.g. via MC)

$$\int d\Phi_2 \mathscr{V}^{(1)}_{CF^2}(z, \epsilon)\, G_g(x(1-z), t_{1,2})\, G_q(x\,z, t_{1,2})$$



$$+\int d\Phi_3 \frac{(8\pi)^2}{s_{123}^2} \langle \hat{P} \rangle_{CF^2} G_g(x(1-z), t_{1,23})\, G_q(x\,z, t_{1,23})$$

calculate in D=4-2ϵ
analytically at fixed Φ_2

Kinematic map and $\mathscr{B}_2^f(z)$

- Map $\mathscr{M} : \Phi_3 \to \Phi_2$ obtained in general following a Cambridge-Aachen like clustering sequence:

☑ Easy to map out the collinear singularities in each of the colour/flavour channels

☒ Phase space angular constraints lead to complicated integrals (especially for gluon fragmentation)

- Result leads to finite integral operator in D=4. Sudakov form factor defined via unitarity i.e. $G_q(x,t)\Big|_{u=1} = 1$

$$\ln \Delta_q(t) = -\int_t^{t_0} dt' \int_{z_0}^{1-z_0} dz\, \mathcal{P}_q(z,\theta)$$

$$\mathcal{P}_q(z,\theta) \equiv \frac{2\,C_F}{1-z}\left(1 + \frac{\alpha_s(E^2 g^2(z)\theta^2)}{2\pi} K^{(1)}\right) + \mathcal{B}_1^q(z) + \frac{\alpha_s(E^2 g^2(z)\theta^2)}{2\pi}\left(\mathcal{B}_2^q(z) + \mathcal{B}_1^q(z) b_0 \ln g^2(z)\right)$$

Two loop cusp AD

New anomalous dimension. Note:

z is momentum fraction after the first splitting!

for quark jets obtained in [Dasgupta, El-Menoufi '21]

for gluon jets obtained in [v. Beekveld, Dasgupta, El-Menoufi, Helliwell, PM '23]

An application: FC_x and λ_x on groomed jets at NSL

for λ_x in quark jets see also [Dasgupta, El-Menoufi, Helliwell '22]

- One readily gets analytic results for FC_x and λ_x at NSL for quark and gluon jets, usable at hadron colliders

e.g. FC_x

Radiator originating from NSL Sudakov FFs

$$R^q_{FC_x}(v, z_{\rm cut}) = \int dt' \int_{z_{\rm cut}}^{1-z_{\rm cut}} dz\, \mathscr{P}_q(z,\theta)\Theta(z(1-z)\theta^{2-x} - FC_x)$$

$$R^g_{FC_x}(v, z_{\rm cut}) = \int dt' \int_{z_{\rm cut}}^{1-z_{\rm cut}} dz\, \Big(\mathscr{P}_{qg}(z,\theta) + \mathscr{P}_{gg}(z,\theta)\Big)\Theta(z(1-z)\theta^{2-x} - FC_x)$$

$$\Sigma^q(v) = \sigma_0^{Z\to q\bar q}\left(1 + \frac{\alpha_s(E^2)}{2\pi} C_v^{q(1)}(z_{\rm cut})\right) e^{-2R_v^q(v,z_{\rm cut})}\left(1 + \frac{\alpha_s^2(E^2)}{(2\pi)^2}\, 2\mathcal{F}^q_{\rm clust}(v)\right)$$

$$\Sigma^g(v) = \sigma_0^{H\to gg}\left(1 + \frac{\alpha_s(E^2)}{2\pi} C_v^{g(1)}(z_{\rm cut})\right) e^{-2R_v^g(v,z_{\rm cut})}\left(1 + \frac{\alpha_s^2(E^2)}{(2\pi)^2}\, 2\mathcal{F}^g_{\rm clust}(v)\right)$$

One-loop matching coefficients for $Z \to q\bar q$ and $H \to gg$ (only process dependent piece)

$$C_v^{q(1)}(z_{\rm cut}) = H^{q(1)} - 2X_v^q + C_F\left(8\ln 2 \ln z_{\rm cut} + 6\ln 2 - \frac{\pi^2}{3}\right),$$

$$C_v^{g(1)}(z_{\rm cut}) = H^{g(1)} - 2X_v^g + C_A\left(8\ln 2 \ln z_{\rm cut} - \frac{\pi^2}{3}\right),$$

Clustering corrections originating from soft limit of $\mathbb{K}_q^{\rm finite}[G_q, G_g]$

$$\mathcal{F}^q_{\rm clust.}(v) = C_F\left(C_F\frac{4\pi}{3}{\rm Cl}_2\left(\frac{\pi}{3}\right) + C_A\, h^{C_A}_{\rm clust.} + T_R n_f\, h^{T_R n_f}_{\rm clust.}\right)\frac{\ln v}{2 - x - 2\lambda_v}$$

$$\mathcal{F}^g_{\rm clust.}(v) = C_A T_R n_f\, h^{T_R n_f}_{\rm clust.}\frac{\ln v}{2 - x - 2\lambda_v},$$

Conclusions & outlook

- Formulation of jet calculus to NSL for collinear fragmentation
  - New angle on resummation of collinear sensitive observables (e.g. micro jets fragmentation, groomed event shapes, correlators)
  - Direct link to parton shower algorithms. Essential insight on inclusion of higher order corrections, treatment of IR cutoffs, …
- Next steps:
  - Numerical algorithm for collinear fragmentation (many subtleties) & applications
  - Consistent simultaneous description of soft evolution at wide angles (at least in planar limit)
  - Explore implications for building NNLL parton shower algorithms

Backup

Gluon jets: structure of NLL evolution equation

$$\mathcal{P}_{gg}(z,\theta) \equiv \frac{C_A}{1-z}\left(1+\frac{\alpha_s(E^2(1-z)^2\theta^2)}{2\pi}K^{(1)}\right)$$
$$+\frac{\alpha_s(E^2z^2\theta^2)}{\alpha_s(E^2(1-z)^2\theta^2)}\frac{C_A}{z}\left(1+\frac{\alpha_s(E^2z^2\theta^2)}{2\pi}K^{(1)}\right)$$
$$+\mathcal{B}_1^{gg}(z)+\frac{\alpha_s(E^2(1-z)^2\theta^2)}{2\pi}\left(\mathcal{B}_2^{gg}(z)+\mathcal{B}_1^{gg}(z)b_0\ln(1-z)^2\right)$$
$$\mathcal{P}_{qg}(z,\theta) \equiv \mathcal{B}_1^{qg}(z)+\frac{\alpha_s(E^2(1-z)^2\theta^2)}{2\pi}\left(\mathcal{B}_2^{qg}(z)+\mathcal{B}_1^{qg}(z)b_0\ln(1-z)^2\right)$$

$$G_g(x,t) = u\,\Delta_g(t) + \int_t^{t_0} dt' \int_{z_0}^{1-z_0} dz \Big[\mathcal{P}_{gg}(z,\theta)\,G_g(x\,z,t')\,G_g(x\,(1-z),t')$$
$$+\mathcal{P}_{qg}(z,\theta)\,G_q(x\,z,t')\,G_q(x\,(1-z),t')\Big]\frac{\Delta_g(t)}{\Delta_g(t')} + \mathbb{K}_g^{\text{finite}}[G_q,G_g]\,,$$

$$\ln\Delta_g(t) = -\int_t^{t_0} dt' \int_{z_0}^{1-z_0} dz\ \left(\mathcal{P}_{gg}(z,\theta)+\mathcal{P}_{qg}(z,\theta)\right)$$

$$\mathbb{K}_g^{\text{R}}[G_q,G_g] = \mathbb{K}_g^{\text{R, C}_\text{A}\,\text{T}_\text{R}}[G_q,G_g] + \mathbb{K}_g^{\text{R, C}_\text{F}\,\text{T}_\text{R}}[G_q,G_g] + \mathbb{K}_g^{\text{R, C}_\text{A}^2}[G_q,G_g]$$

$$\mathbb{K}_g^{\text{DC}}[G_q,G_g] = \mathbb{K}_g^{\text{DC, C}_\text{A}\,\text{T}_\text{R}}[G_q,G_g] + \mathbb{K}_g^{\text{DC, C}_\text{F}\,\text{T}_\text{R}}[G_q,G_g] + \mathbb{K}_g^{\text{DC, C}_\text{A}^2}[G_q,G_g]$$

Quark jets: double real corrections

$$\mathbb{K}_q^{\text{finite}}[G_q, G_g] \equiv \mathbb{K}_q^{\text{R}}[G_q, G_g] - \mathbb{K}_q^{\text{DC}}[G_q, G_g]$$

$$\mathbb{K}_q^{\text{R}}[G_q, G_g] = \sum_{(A)} \frac{1}{S_2} \int d\Phi_3^{(A)} \, P_{1\to 3}^{(A)} \Big\{ G_{f_1}(x\, z_p\, (1-z), t_{1,2})\, G_{f_2}(x\, (1-z_p)\, (1-z), t_{1,2}) \times G_q(x\, z, t_{12,3}) - G_{f_{12}}(x\,(1-z), t_{12,3})\, G_q(x\, z, t_{12,3}) \Big\} \frac{\Delta_q(t)}{\Delta_q(t_{1,2})} + \int d\Phi_3^{(B)} \, P_{1\to 3}^{(B)} \Big\{ G_g(x\,(1-z), t_{1,23})\, G_g(x\, z\,(1-z_p), t_{2,3}) \times G_q(x\, z\, z_p, t_{2,3}) - G_g(x\,(1-z), t_{1,23})\, G_q(x\, z, t_{1,23}) \Big\} \frac{\Delta_q(t)}{\Delta_q(t_{2,3})} \, \Theta(t_{2,3} - t_{1,3})\,, \qquad \text{(C.2)}$$

$$\mathbb{K}_q^{\text{DC}}[G_q, G_g] = \sum_{g\to f\bar f} \int_t^{t_0} dt_{12,3}\, dt_{1,2} \int_{z_0}^{1-z_0} dz\, dz_p P_{qq}(z) P_{fg}(z_p) \Big\{ G_f(x\, z_p\,(1-z), t_{1,2}) \times G_f(x\,(1-z_p)\,(1-z), t_{1,2}) G_q(x\, z, t_{12,3}) - G_g(x\,(1-z), t_{12,3})\, G_q(x\, z, t_{12,3}) \Big\} \times \frac{\Delta_q(t)}{\Delta_q(t_{1,2})}\, \Theta(t_{1,2} - t_{12,3}) + \int_t^{t_0} dt_{1,23}\, dt_{2,3} \int_{z_0}^{1-z_0} dz\, dz_p P_{qq}(z) P_{qq}(z_p) \Big\{ G_g(x\,(1-z), t_{1,23})\, G_g(x\, z\,(1-z_p), t_{2,3}) \times G_q(x\, z\, z_p, t_{2,3}) - G_g(x\,(1-z), t_{1,23})\, G_q(x\, z, t_{1,23}) \Big\} \frac{\Delta_q(t)}{\Delta_q(t_{2,3})}\, \Theta(t_{2,3} - t_{1,23})\,, \qquad \text{(C.5)}$$

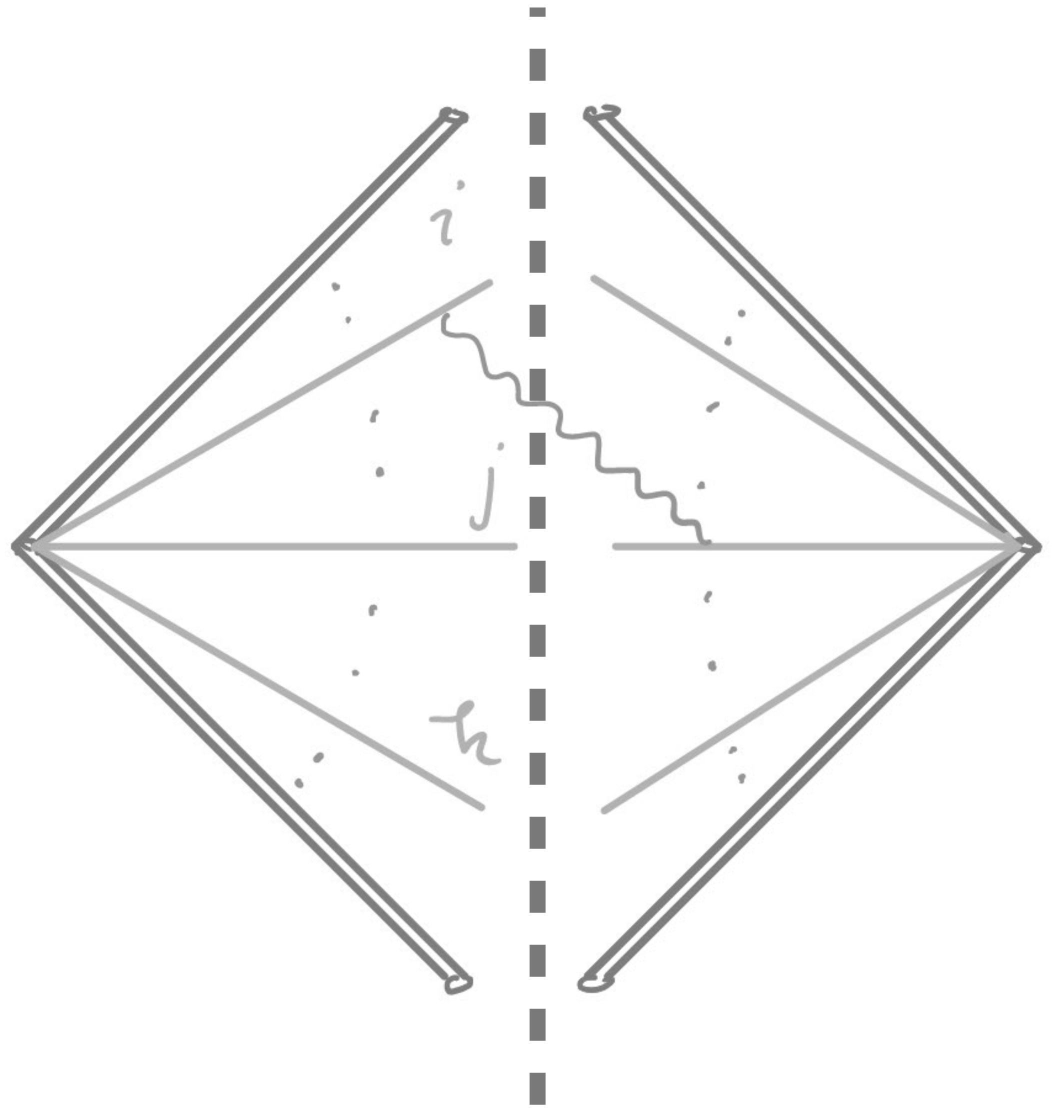
Soft gluons on the celestial sphere & NGLs

- Similar formalism (albeit for colour dipoles in planar limit) was used for first NSL calculation of NGLs (not considered further in this talk)
- e.g. GFs evolution at SL

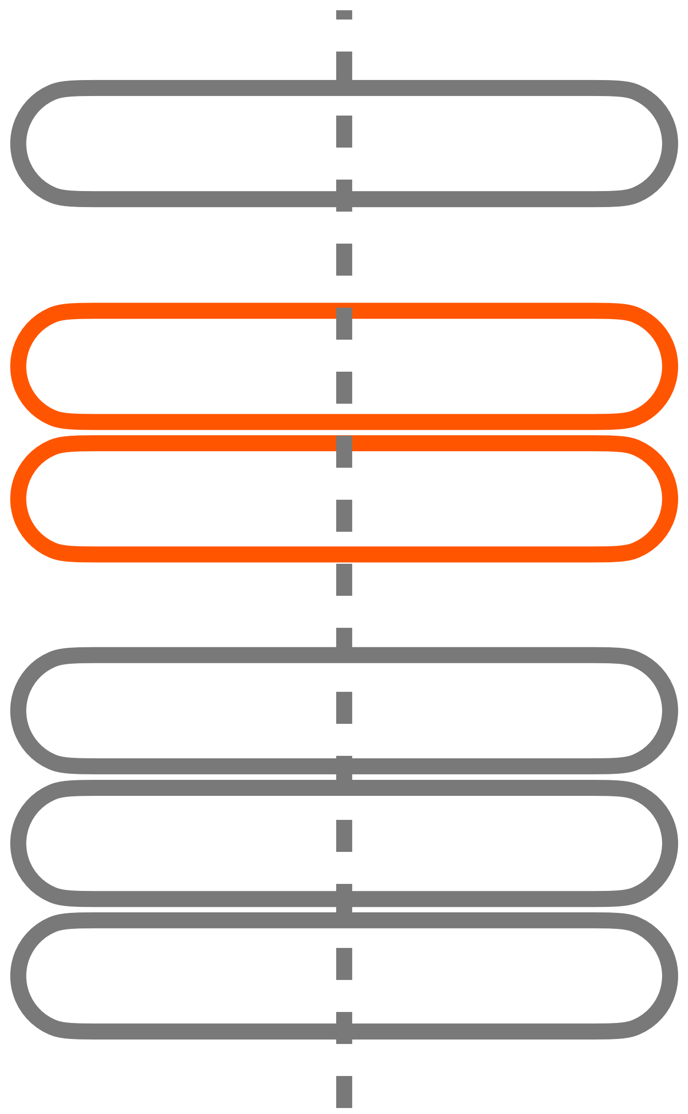
$$Z_{12}[Q;\{u\}] = \Delta_{12}(Q) + \int [dk_a]\bar{\alpha}(k_{ta})w_{12}^{(0)}(k_a)\,\frac{\Delta_{12}(Q)}{\Delta_{12}(k_{ta})} \times Z_{1a}[k_{ta};\{u\}]Z_{a2}[k_{ta};\{u\}]u(k_a)\Theta(Q-k_{ta})$$

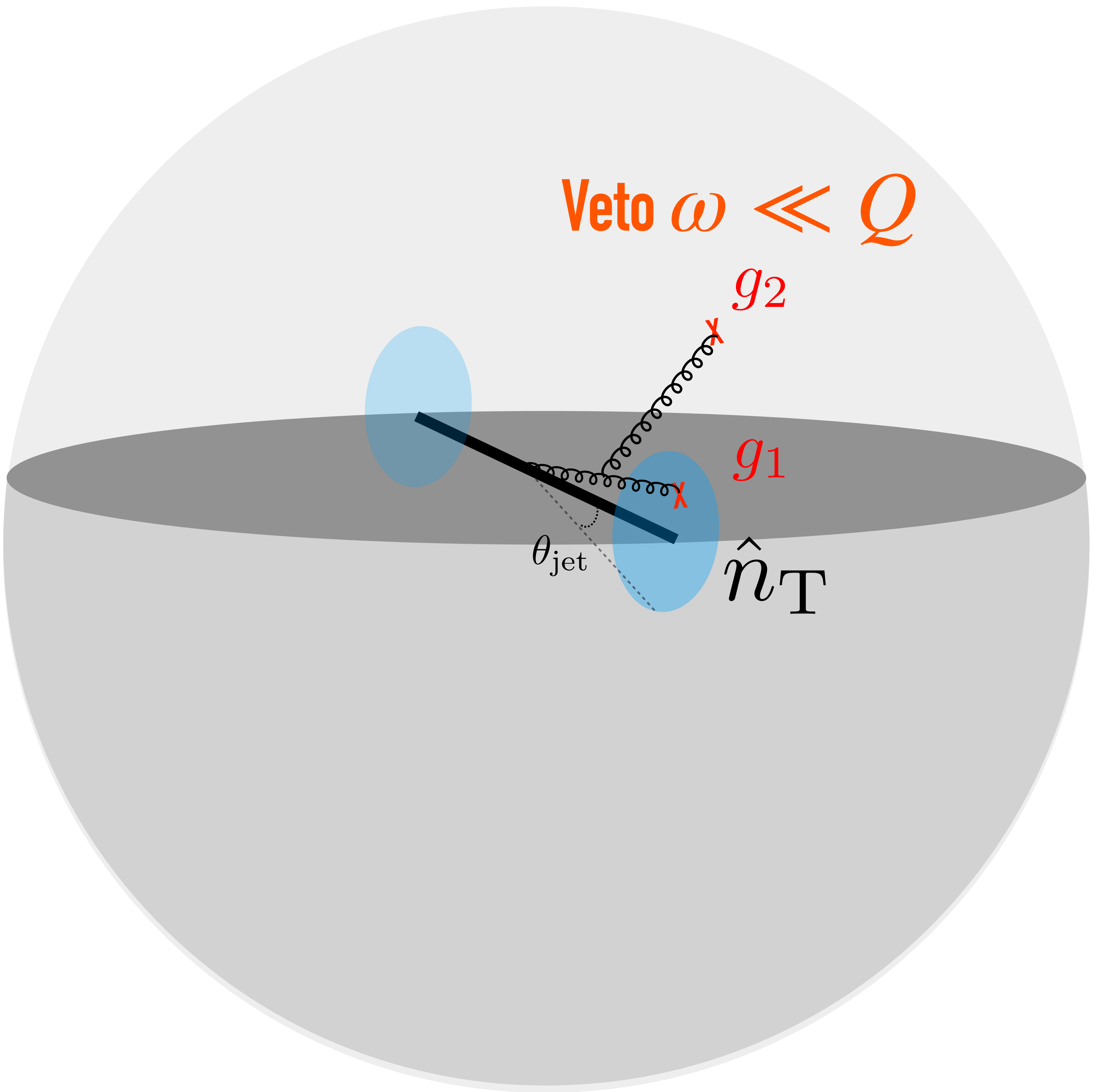
defined by $Z_{12}[Q;\{u=1\}] = 1$

$\mathbf{T}_i \cdot \mathbf{T}_j \sim N_c\,\delta_{j,i\pm 1}$



$\Rightarrow$ ['t Hooft '73]





Fraction of events passing the veto is affected by large logarithms L=ln(Q/ω). All order resummation requires distribution of soft gluons on the sphere