Electroweak logarithms in OpenLoops

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Theory Challenges in the Precision Era of the LHC

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Artwork by C.Z.

Introduction

- In the energy range above the EW scale ($\sqrt{s} \gg M_W$), *Sudakov logs* represent the leading contribution of EW radiative corrections

- *Sudakov logarithms* from N^{n}LO EW corrections

$$\alpha^n \log^k \frac{s}{M_W^2}, \qquad 1 \leq k \leq 2n$$

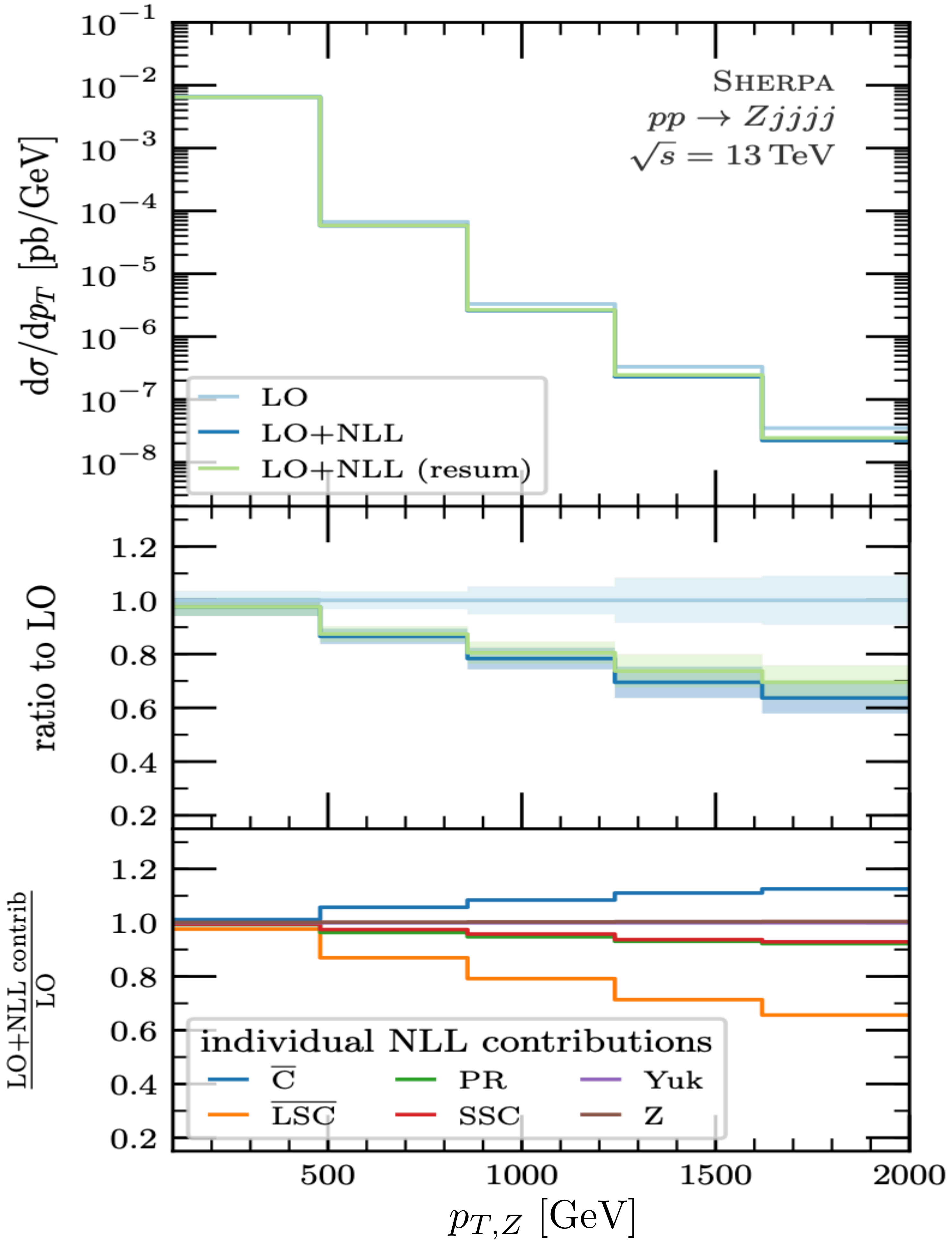
- At NLO

Double logs: $$L(s) = \frac{\alpha}{4\pi} \log^2 \frac{s}{M_W^2},$$

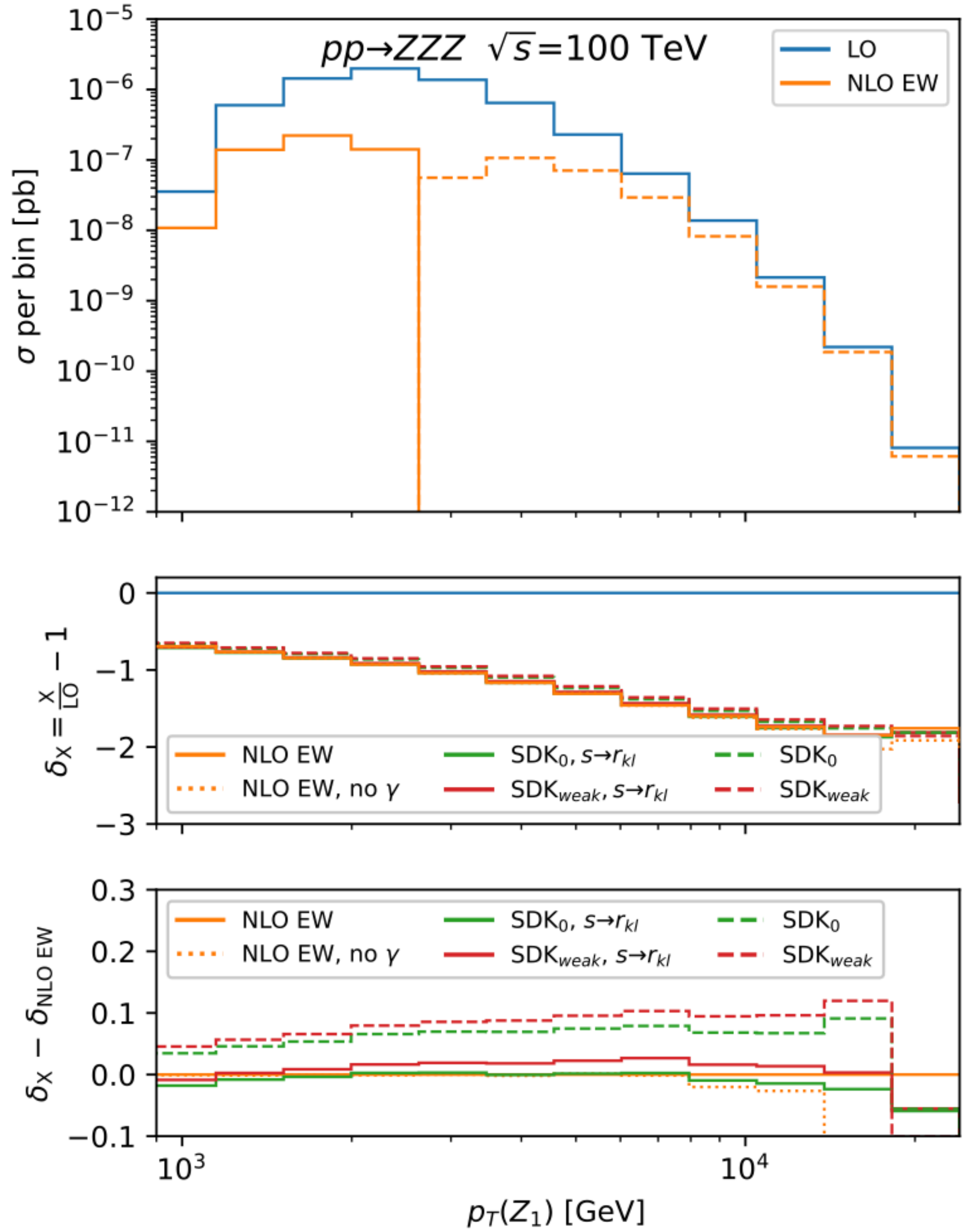
Single logs: $$l(s) = \frac{\alpha}{4\pi} \log \frac{s}{M_W^2}$$

Introduction

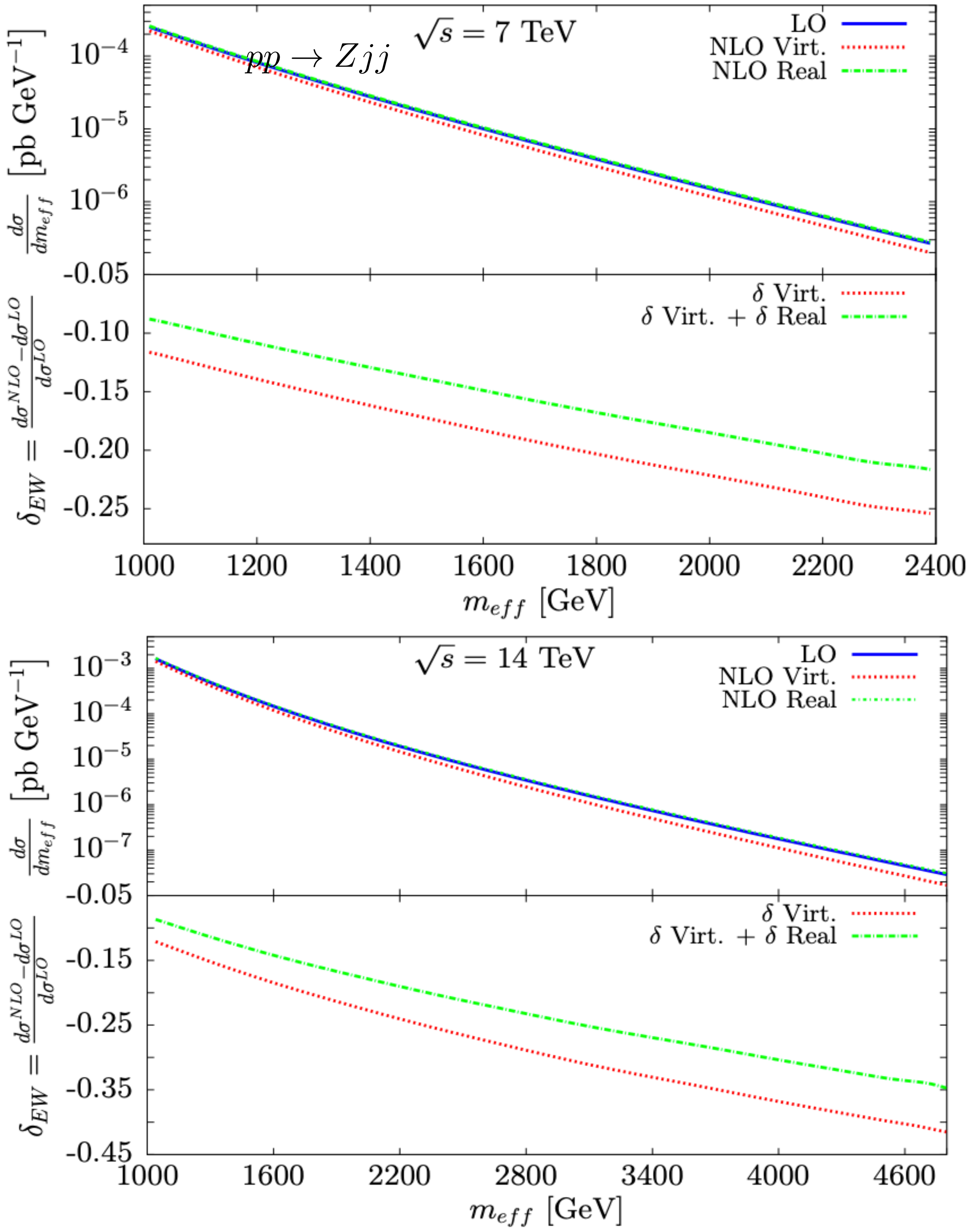
- Significant enhancement of tails of kinematic distributions up to several tens percent



[Bothmann, Napoletano 2006.14635; 2020]



[Pagani, Zaro 2110.03714; 2021]



[Chiesa *et al.* 1305.6837; 2013]

Framework: notation & conventions

- $n \to 0$ process

$$\varphi_{i_1}(p_1)\dots\varphi_{i_n}(p_n) \to 0$$

with not mass suppressed Born matrix-element, i.e $\mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i_n}} \sim E^d$

- DP algorithm based on logarithmic approximation (LA):

→ Hierarchy scales

$$\mu^2 = s \sim (p_k + p_l)^2 \gg m_{\mathrm{t}}^2, M_{\mathrm{H}}^2 > M_{\mathrm{Z,W}}^2 \gg m_f^2 \gg \lambda^2, \quad \forall\, k, l$$

→ At one-loop keep only double and singular logarithmic corrections

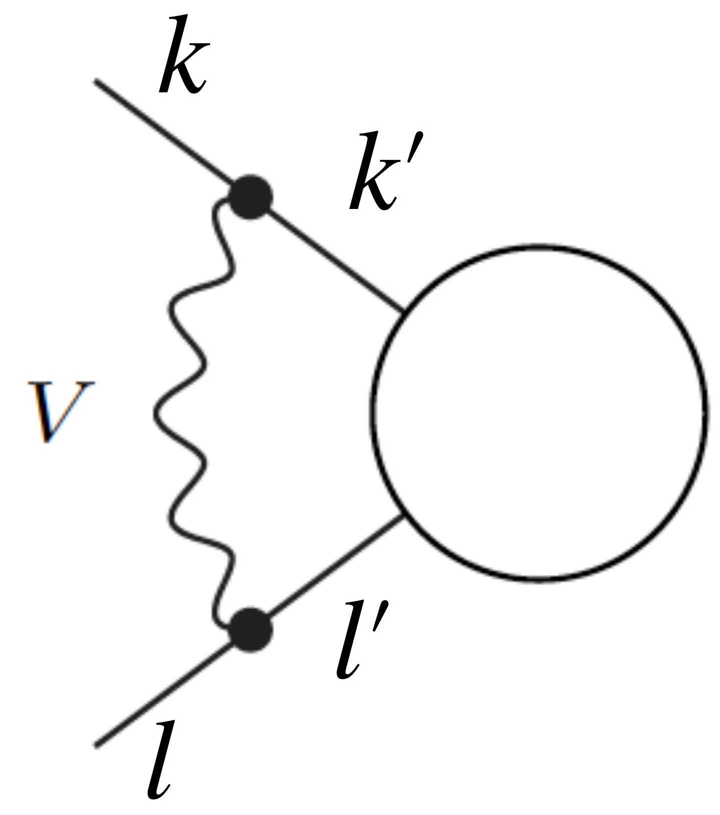
$$\delta^{\mathrm{DL}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} \sim E^d L \qquad \delta^{\mathrm{SL}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} \sim E^d l$$

neglecting constant $(\sim \alpha E^d)$ and mass suppressed $(\sim M^n E^{d-n} L)$ contributions

[Denner and Pozzorini 0010201; *2001*]

Double Logs (DL)

- DL originate from triangle diagrams where two external legs exchange a ***soft and collinear* (SC)** gauge boson V



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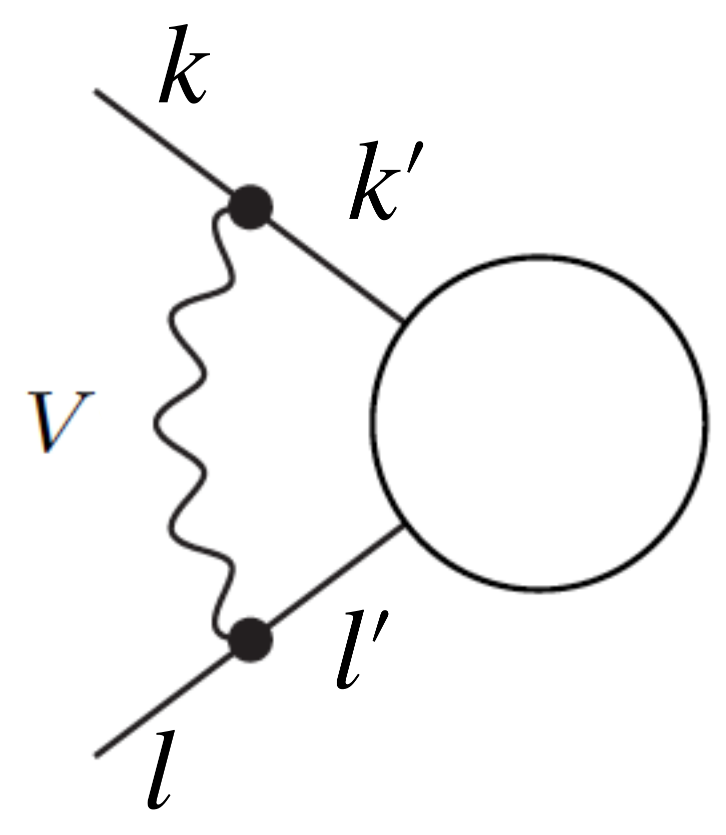

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- In the *Eikonal approximation*, the loop integral reduces to the scalar three-point function C_0, which ***factorises***

$$\delta^{\mathrm{DL}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} \stackrel{\mathrm{LA}}{=} \sum_k \sum_{l<k} \sum_V \sum_{k',l'} \frac{\alpha}{4\pi} I^V_{kk'} I^{\bar{V}}_{ll'} \underbrace{\left[\log^2 \frac{|r_{kl}|}{M_V^2} - 2i\pi\Theta(r_{kl}) \log \frac{|r_{kl}|}{M_V^2}\right]}_{\propto\, C_0|_{\mathrm{LA}}} \mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i'_l}\cdots\varphi_{i_n}}$$

with $r_{kl} = (p_k + p_l)^2$

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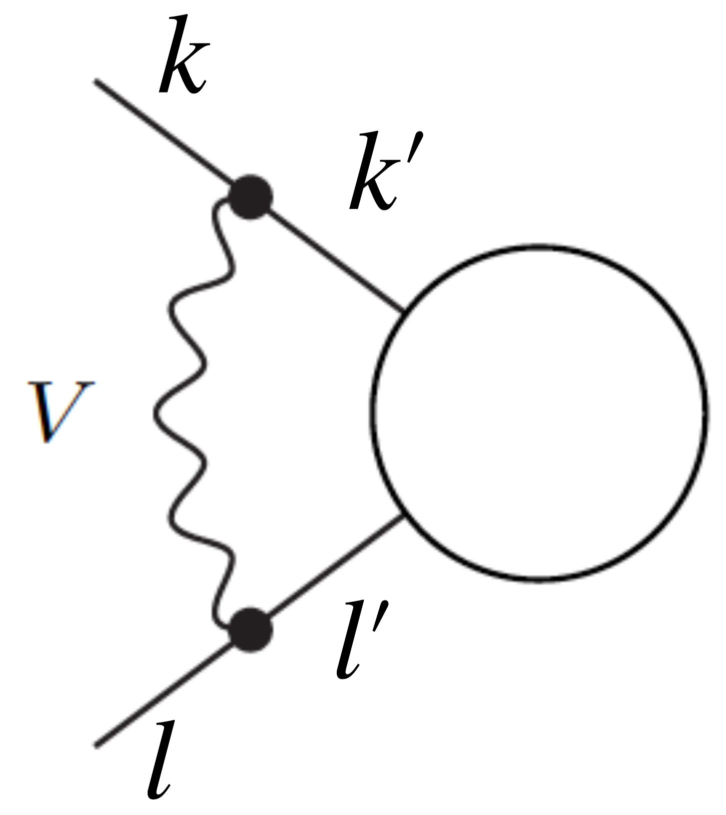

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- Consequence of C_0 ***factorisation***: DL are ***universal***, i.e. process independent

Double Logs: LSC, SSC, S-SSC

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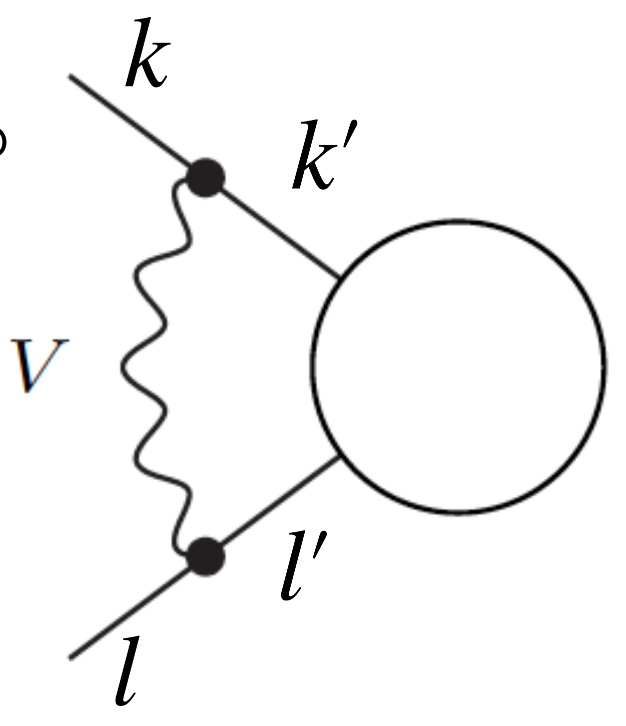

- DL can be split into

→ **Leading *Soft-Collinear* (LSC)**: angular independent, single sum over external legs

$$\delta^{\mathrm{LSC}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} = \sum_{k}\sum_{k'}\sum_{V}\delta^{\mathrm{LSC,V}}_{kk'}\mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i_n}}, \qquad \delta^{\mathrm{LSC,\ V}}_{kk'} \sim \frac{\alpha}{4\pi}I^V I^{\bar{V}}\log^2\left(\frac{s}{M_V^2}\right)$$

→ **Subleading *Soft-Collinear* (SSC)** and **Sub-SSC**: angular dependent, double sum over external legs

$$\delta^{\mathrm{(S-)SSC}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} = \sum_{k}\sum_{l<k}\sum_{k',l'}\sum_{V}\delta^{\mathrm{(S-)SSC,V}}_{kk'll'}\mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i'_l}\cdots\varphi_{i_n}}$$

$$\delta^{\mathrm{SSC,\ V}}_{kk'll'} \sim \frac{\alpha}{4\pi}I^V I^{\bar{V}}\log\left(\frac{s}{M_V^2}\right)\log\left(\frac{|r_{kl}|}{s}\right), \qquad \delta^{\mathrm{S\text{-}SSC,\ V}}_{kk'll'} \sim \frac{\alpha}{4\pi}I^V I^{\bar{V}}\log^2\left(\frac{|r_{kl}|}{s}\right) \qquad r_{kl} = (p_k+p_l)^2$$

[Denner and Pozzorini 0010201; *2001*]

Single Logs (SL)

- SL have a triple origin

Single Logs (SL): PR

$$\mu_{i,0}^2 = \mu_i^2 + \delta\mu_i^2$$

$$\varphi_{i,0} = \left(1 + \frac{1}{2}\delta Z_{\varphi_i\varphi_j}\right)\varphi_j$$

$$g_{i,0} = g_i + \delta g_i = (1 + \delta Z_{g_i})\, g_i$$

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yields to the ***factorised*** correction

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Renormalisation of masses and couplings with mass dimensions brings only mass-suppressed corrections

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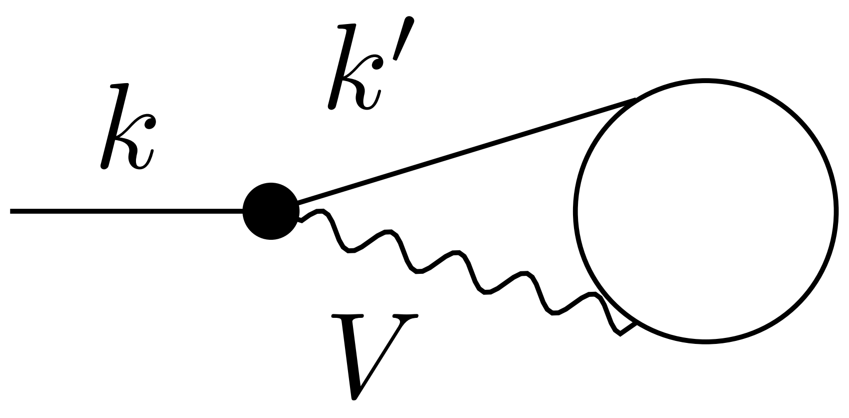
→ **Coll**: external leg emission of a collinear gauge boson



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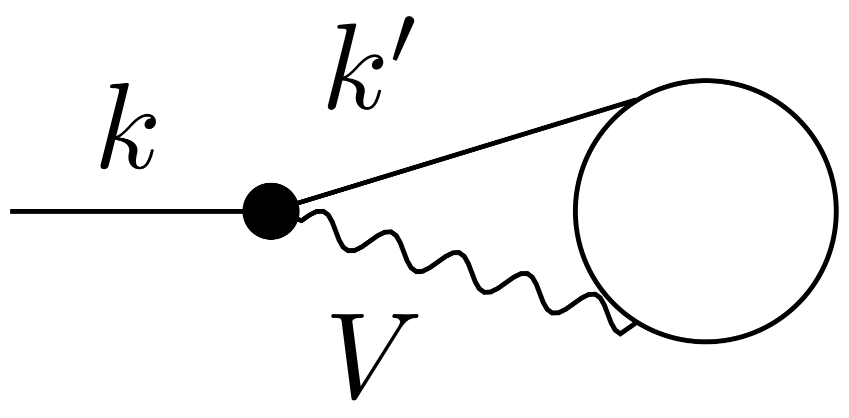

Its evaluation in *Eikonal approximation* leads to the ***factorised*** contribution

$$\delta^{\text{coll}} \mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} \stackrel{\text{LA}}{=} \sum_k \sum_{k'} \delta^{\text{coll}}_{kk'} \mathcal{M}_0^{\varphi_{i_1}\dots\varphi_{i'_k}\dots\varphi_{i_n}}, \qquad \delta^{\text{coll}}_{kk'} \sim \frac{\alpha}{4\pi} \sum_V I^V I^{\bar{V}} \log\left(\frac{s}{M_V^2}\right)$$

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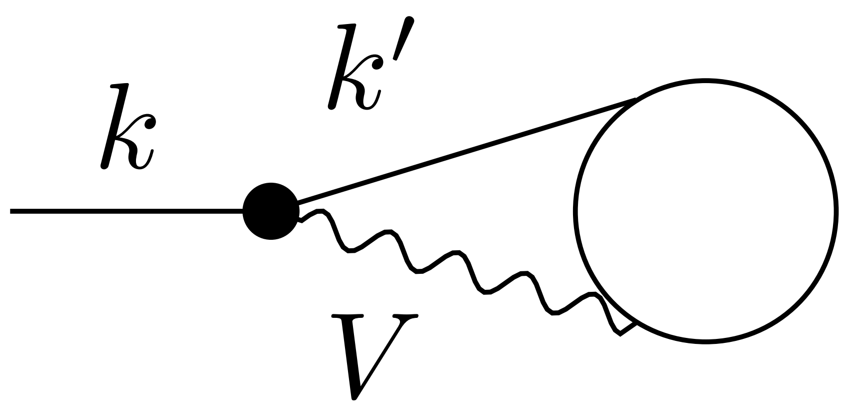

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→ C: Full gauge-invariant SL correction associated to external fields:

$$\delta^{\mathrm{C}} \mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} \overset{\mathrm{LA}}{=} \sum_{k} \sum_{k'} \delta^{\mathrm{C}}_{kk'} \mathcal{M}_0^{\varphi_{i_1}\dots\varphi_{i'_k}\dots\varphi_{i_n}}, \qquad \delta^{\mathrm{C}}_{kk'} = \left(\delta^{\mathrm{coll}}_{kk'} + \delta^{\mathrm{WF}}_{kk'}\right)\Big|_{\mu^2=s}$$

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However:

- Even if automated, one-loop computations can be very complicated (e.g. high multiplicity processes)
- No NNLO/two-loop level automation available
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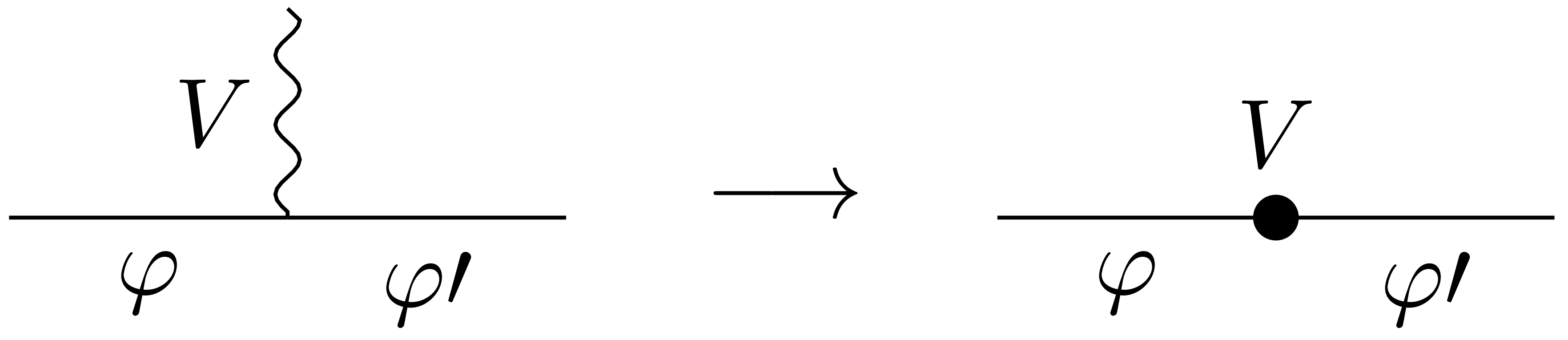
- OpenLoops (OL): automated tool for the calculation of tree and one-loop amplitudes [Buccioni *et al*, 1907.13071*; 2019*]

- Goal of the implementation: evaluate NLO EW *Sudakov* corrections via tree amplitudes (w/o loop computations) and make them available to any MC with OL interface

Implementation in OpenLoops: how

- Representation of Denner-Pozzorini algorithm via effective CT vertices



$$= ieI^{V}_{\varphi\varphi'}K^{V}_{\mathrm{ew}}$$

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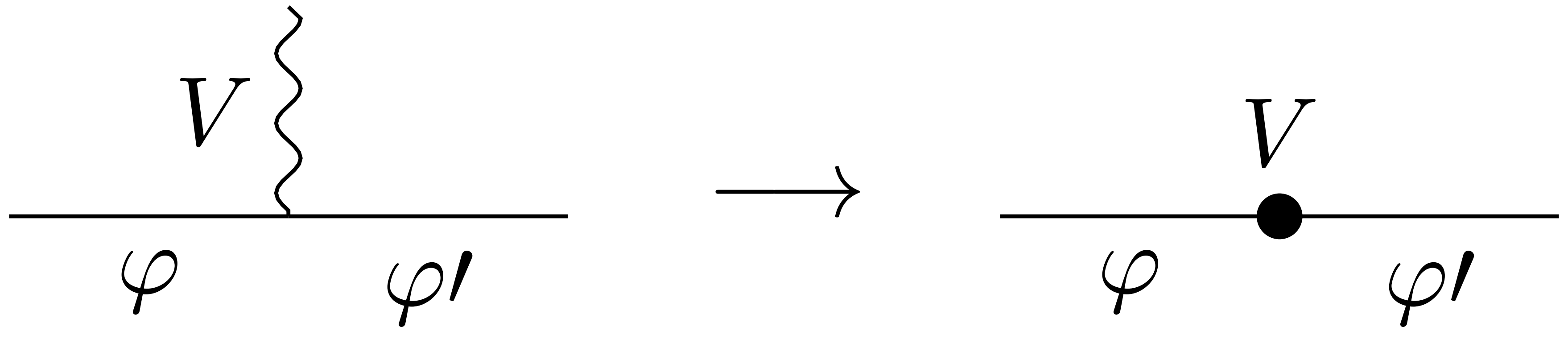

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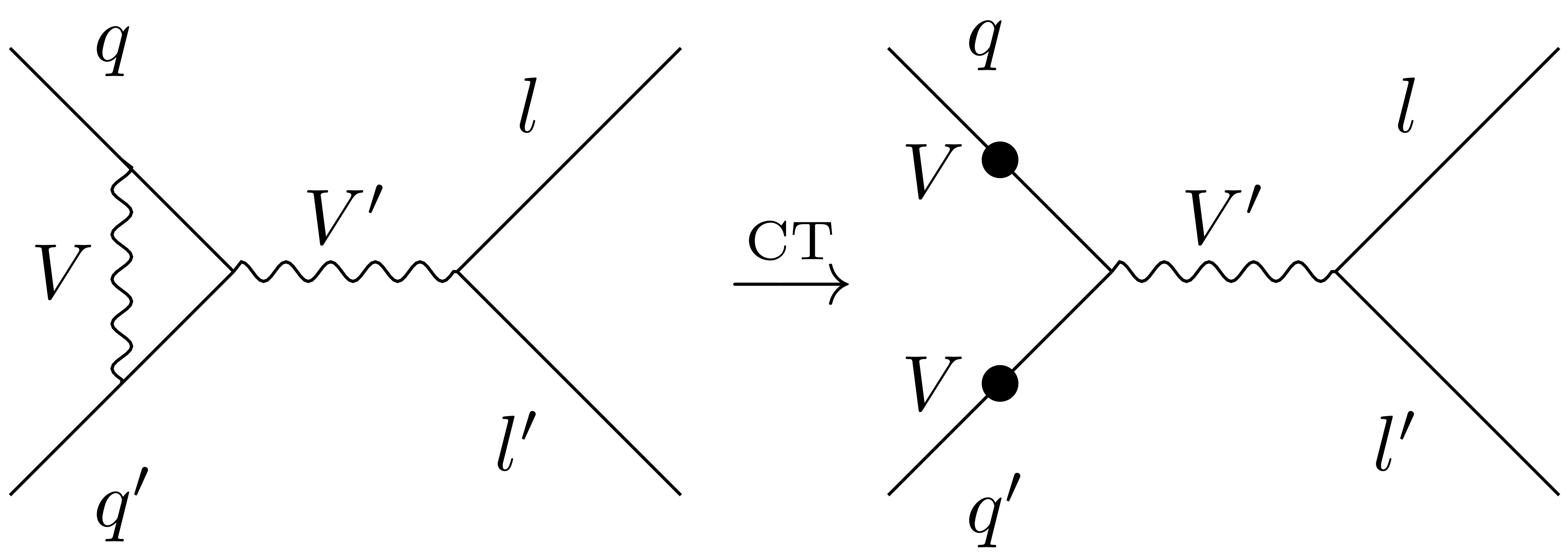
reducing one-loop amplitudes to tree-level ones via double CT insertions

Eg.: Drell-Yann



$$\Rightarrow \mathcal{M} \sim e^2 \sum_{V=A,Z,W^{\pm}} I^{V}_{q} I^{V}_{q'} \left(K^{V}_{\mathrm{ew}}\right)^2 \mathcal{M}_0$$

$$\sim \delta^{\mathrm{DL,\ SL}}$$

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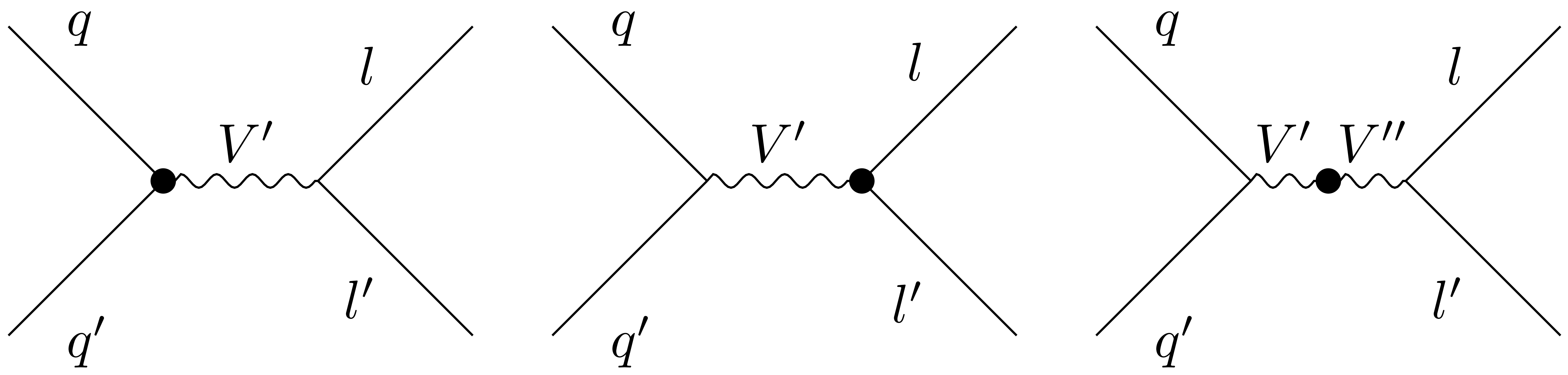
- Single logs coming from **PR** contributions can be evaluated via generation of standard UV counterterms, e.g.



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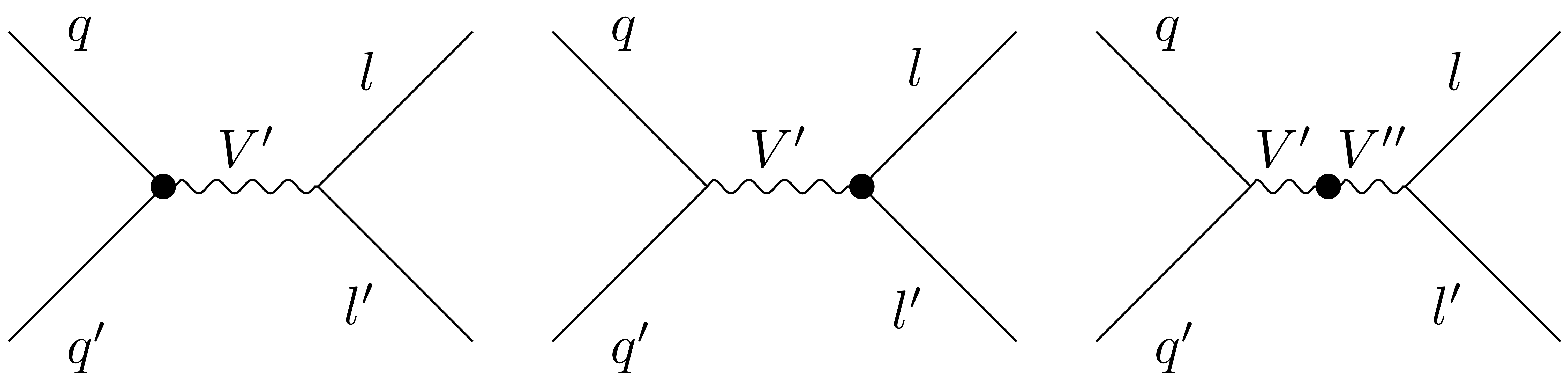

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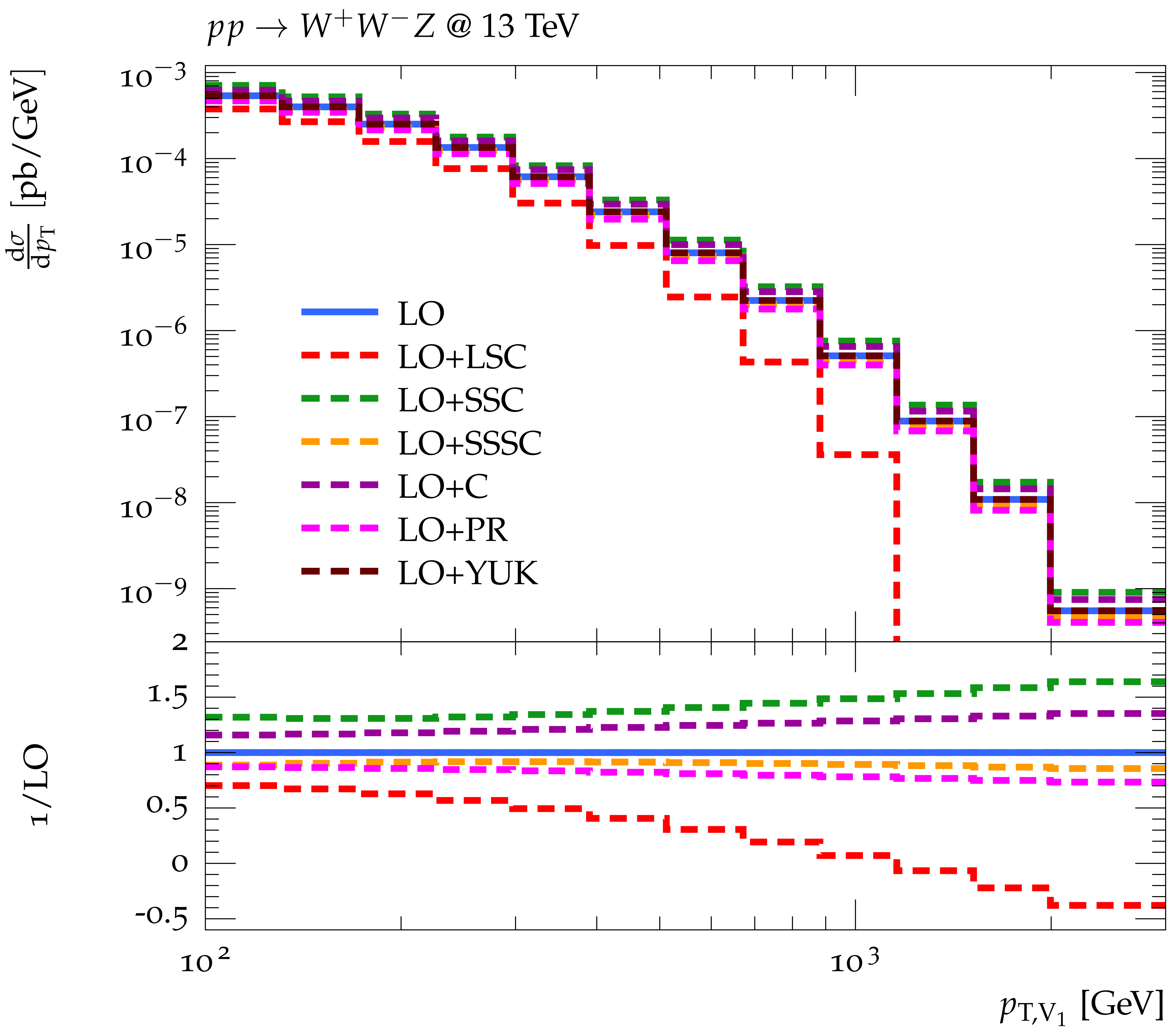
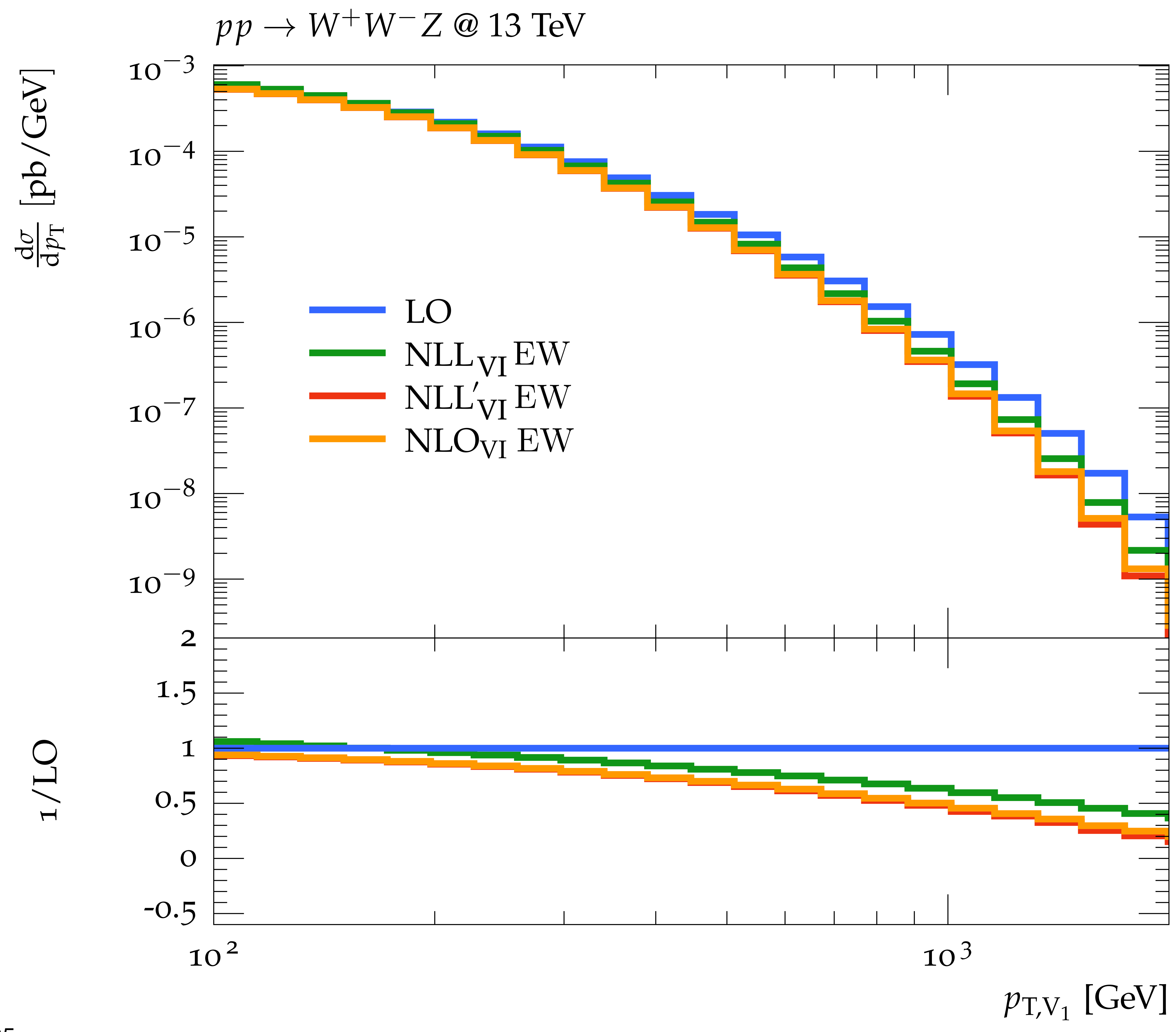
- Alternative way: set $\delta^{\mathrm{WF}}_{kk'}$ to zero and evaluate **WF** + **PR** via standard UV counterterms

Results: $pp \to W^+W^-Z$





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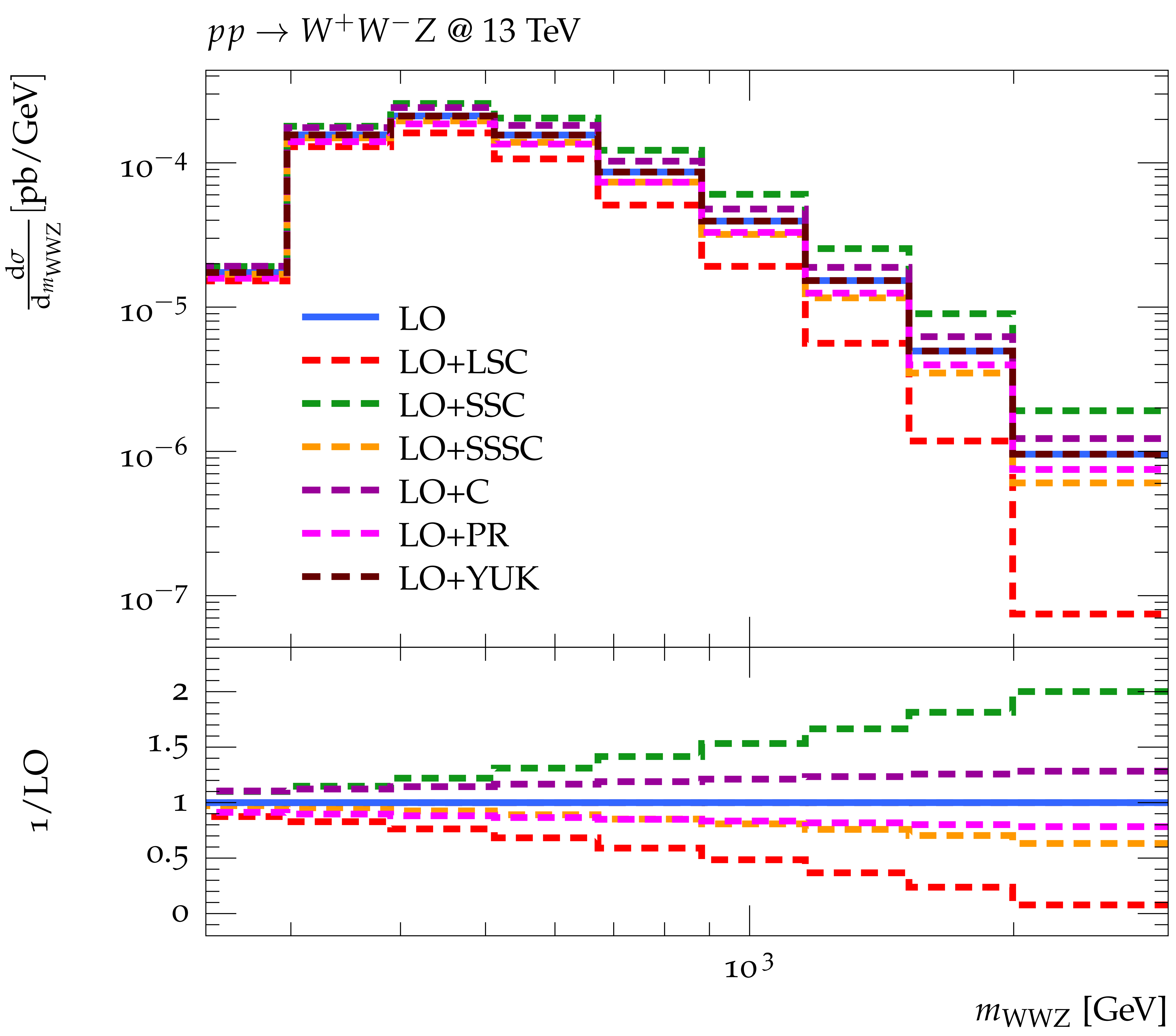

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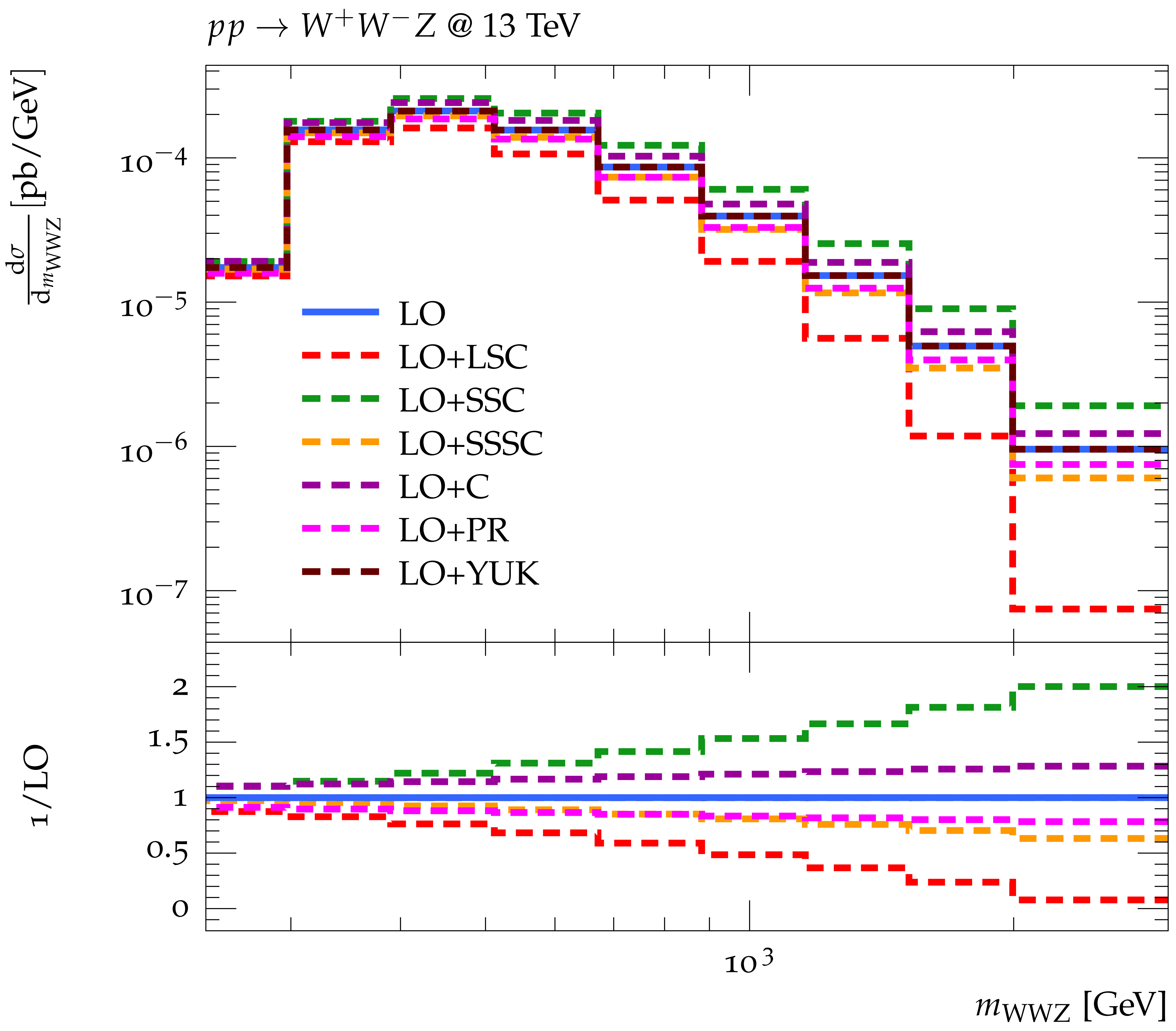

SSC and **S-SSC** become very sizeable for PS regions where Sudakov condition

$$s \sim (p_k + p_l)^2 \gg M^2_{\mathrm{Z,W}} \quad \forall\, k, l$$

is violated, with hierarchy among invariants

$$s \sim (p_k + p_l)^2 \gg (p_{k'} + p_{l'})^2 \gg M^2_{\mathrm{Z,W}}$$

$$\delta^{\mathrm{SSC,\ V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log\left(\frac{s}{M_V^2}\right) \log\left(\frac{|r_{kl}|}{s}\right)$$

$$\delta^{\mathrm{S\text{-}SSC,\ V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log^2\left(\frac{|r_{kl}|}{s}\right)$$

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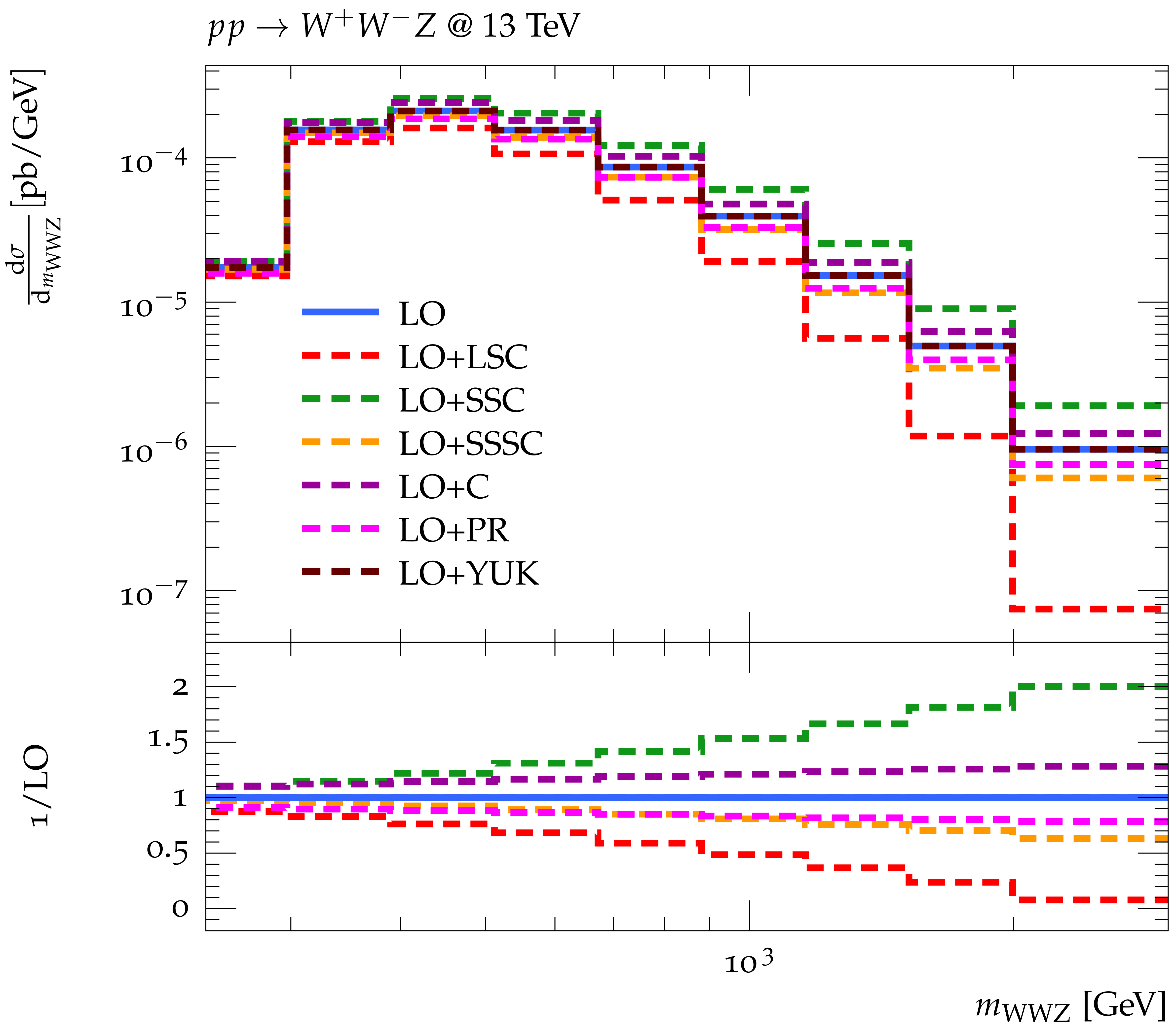

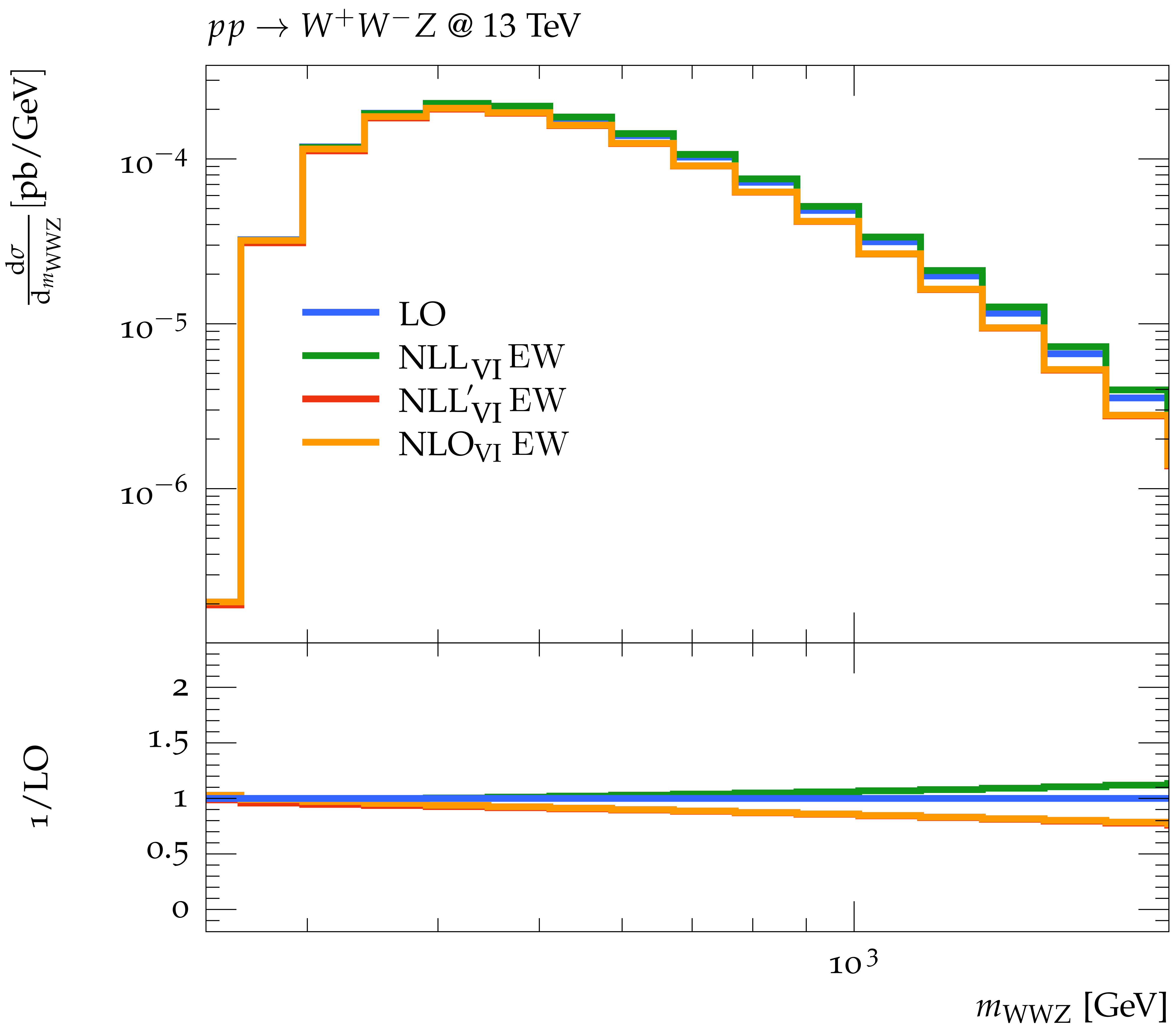

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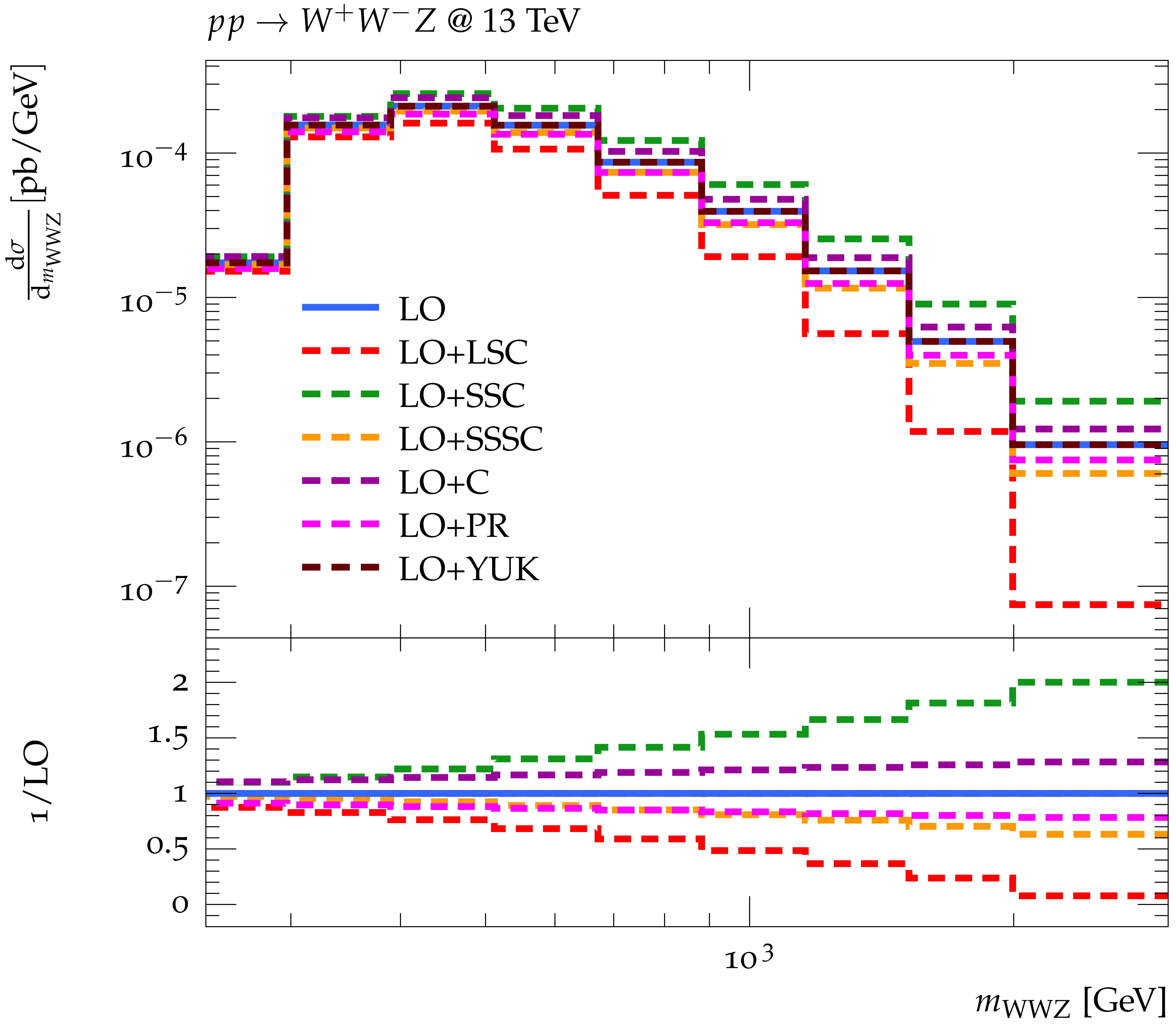
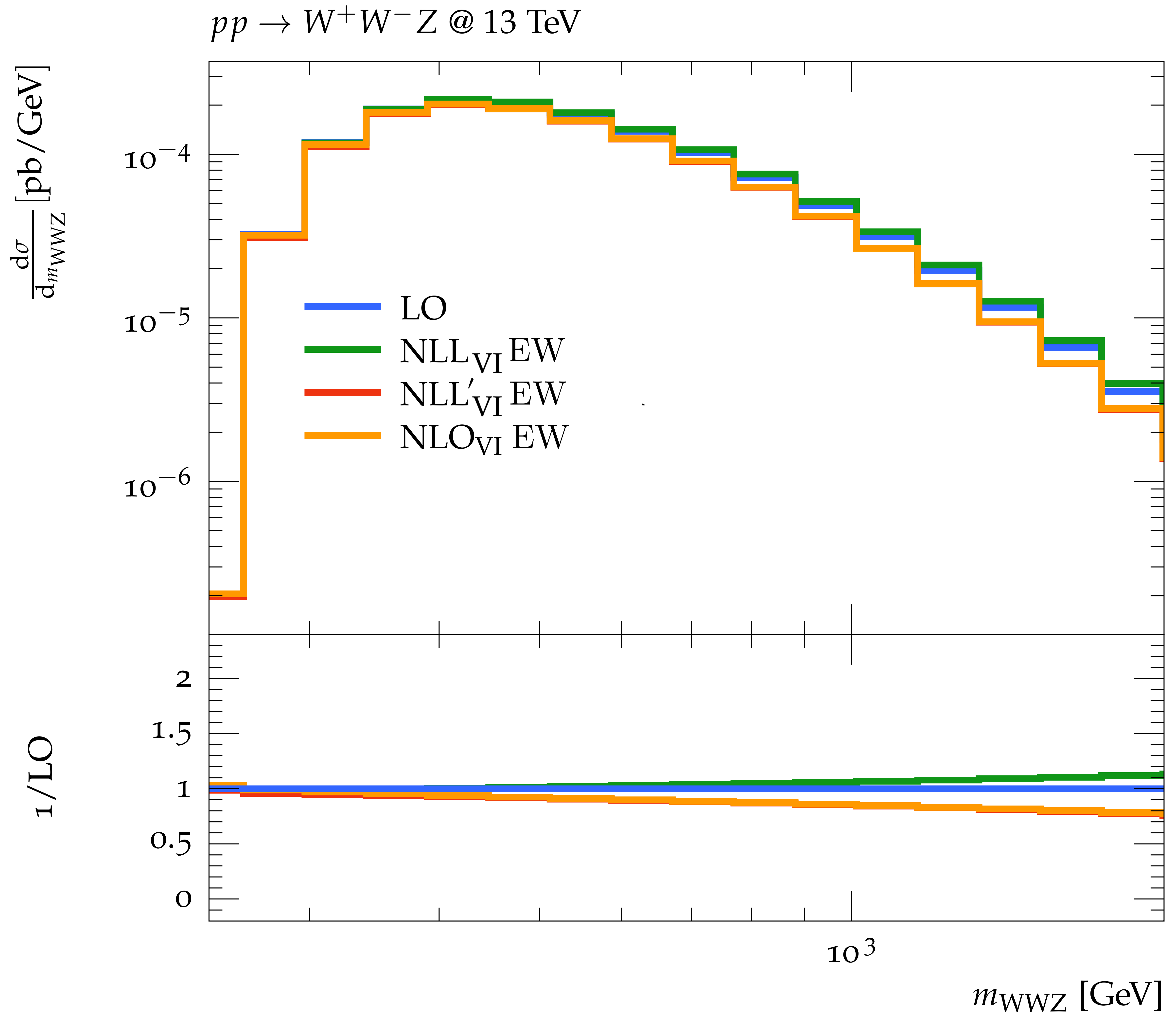
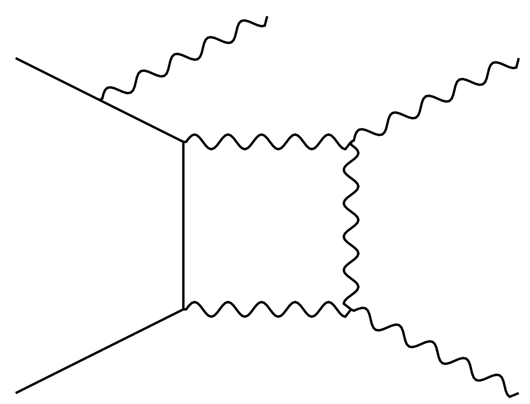




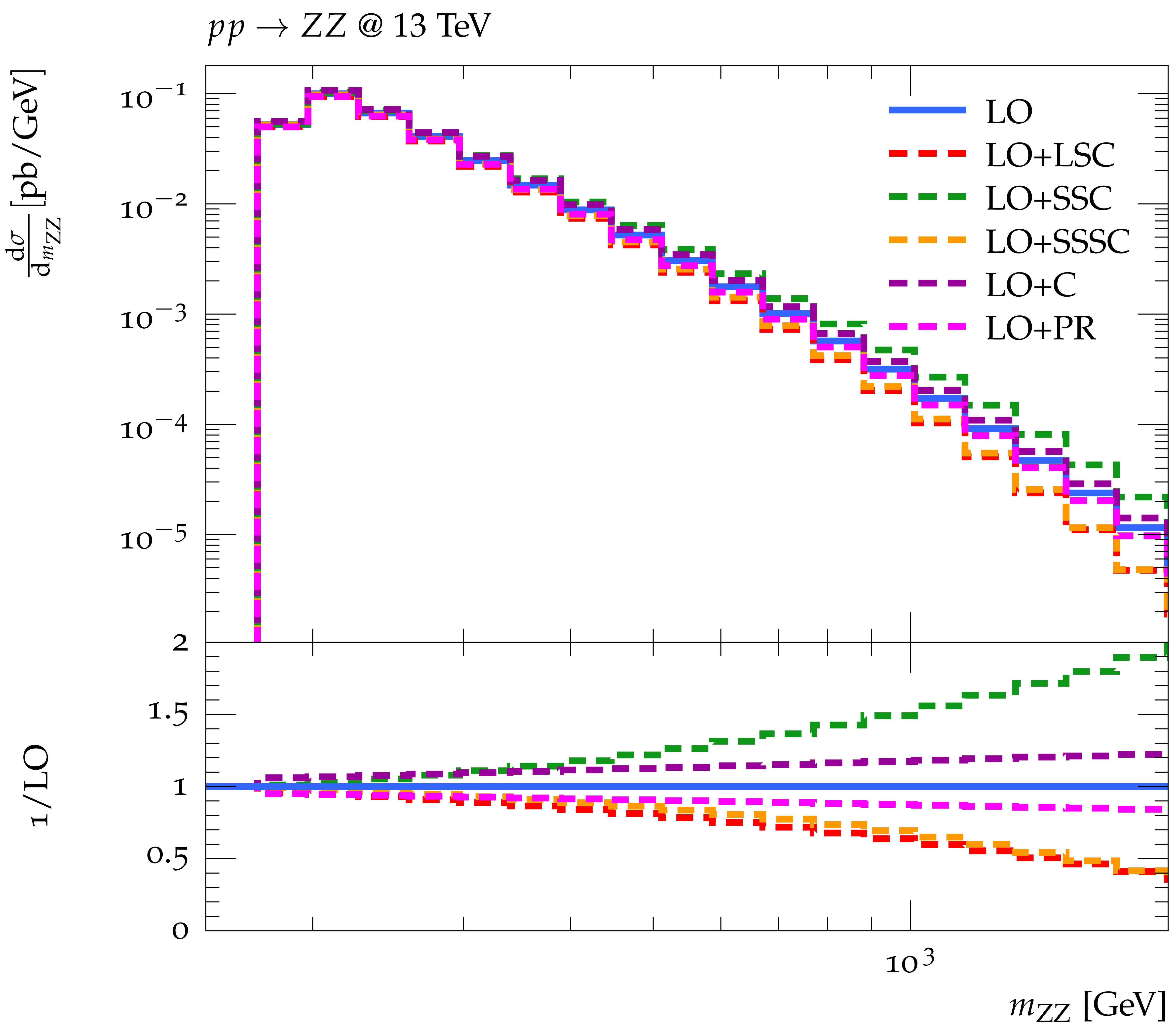
However, no full control on S-SSC term!

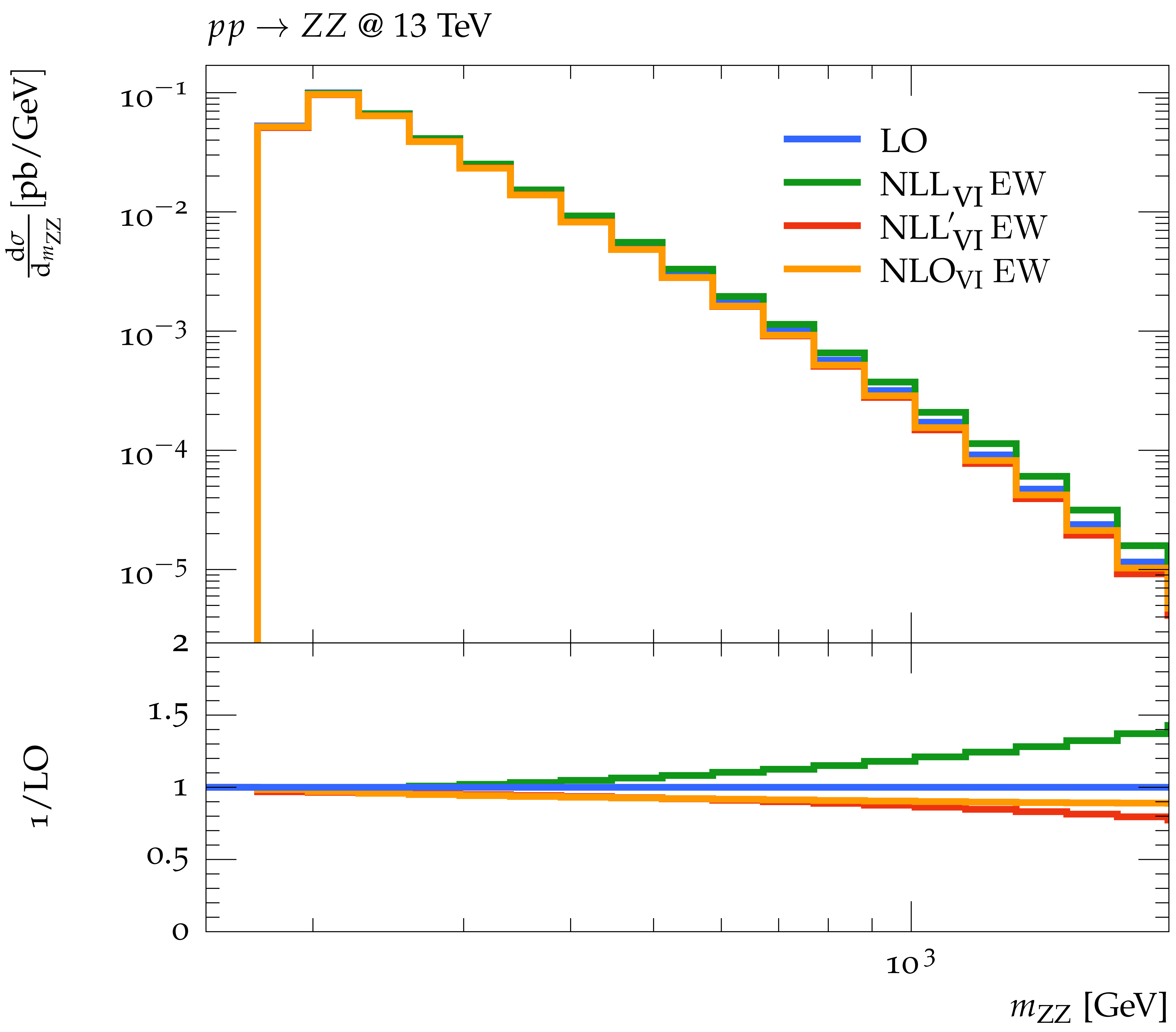
S-SSC-like terms arise also from box diagrams



$\sim D_0 \sim \log\left(\frac{|r_{kl}|}{s}\right)$

Results: $pp \to ZZ$





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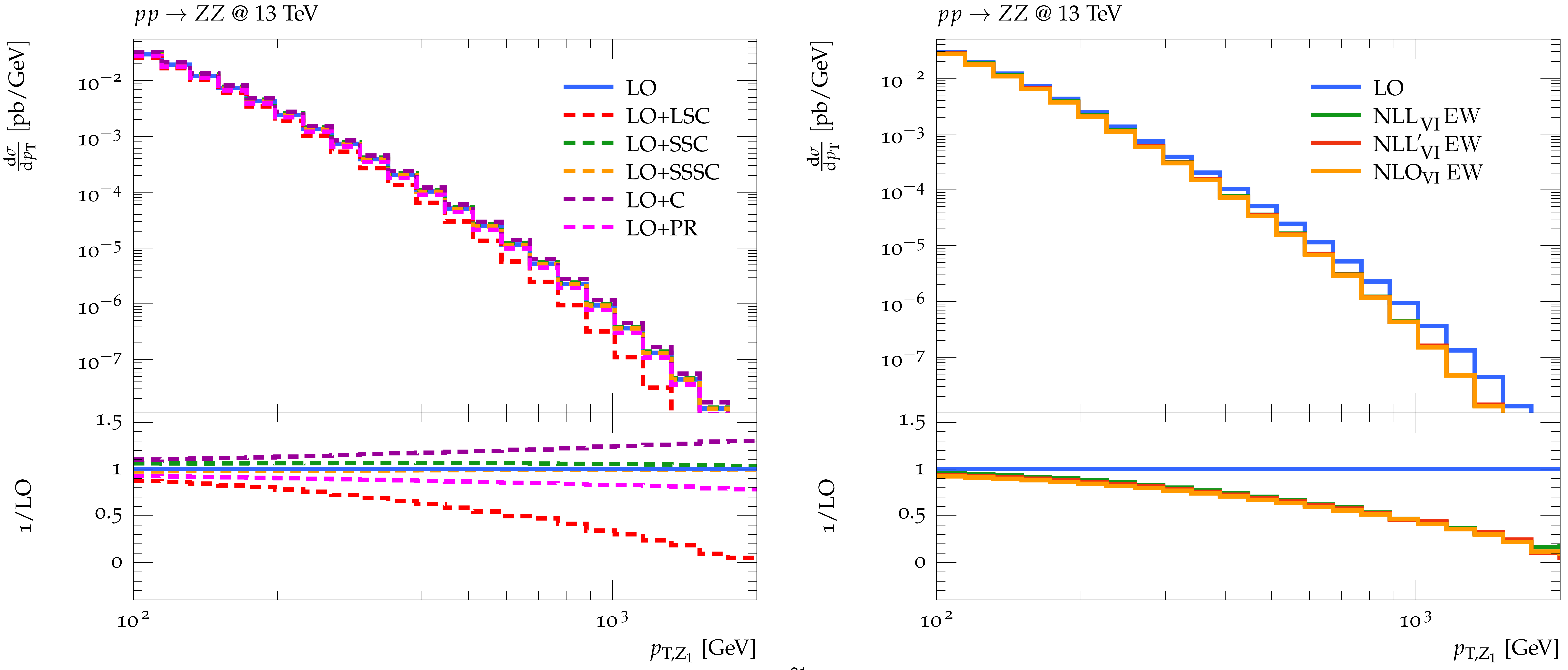

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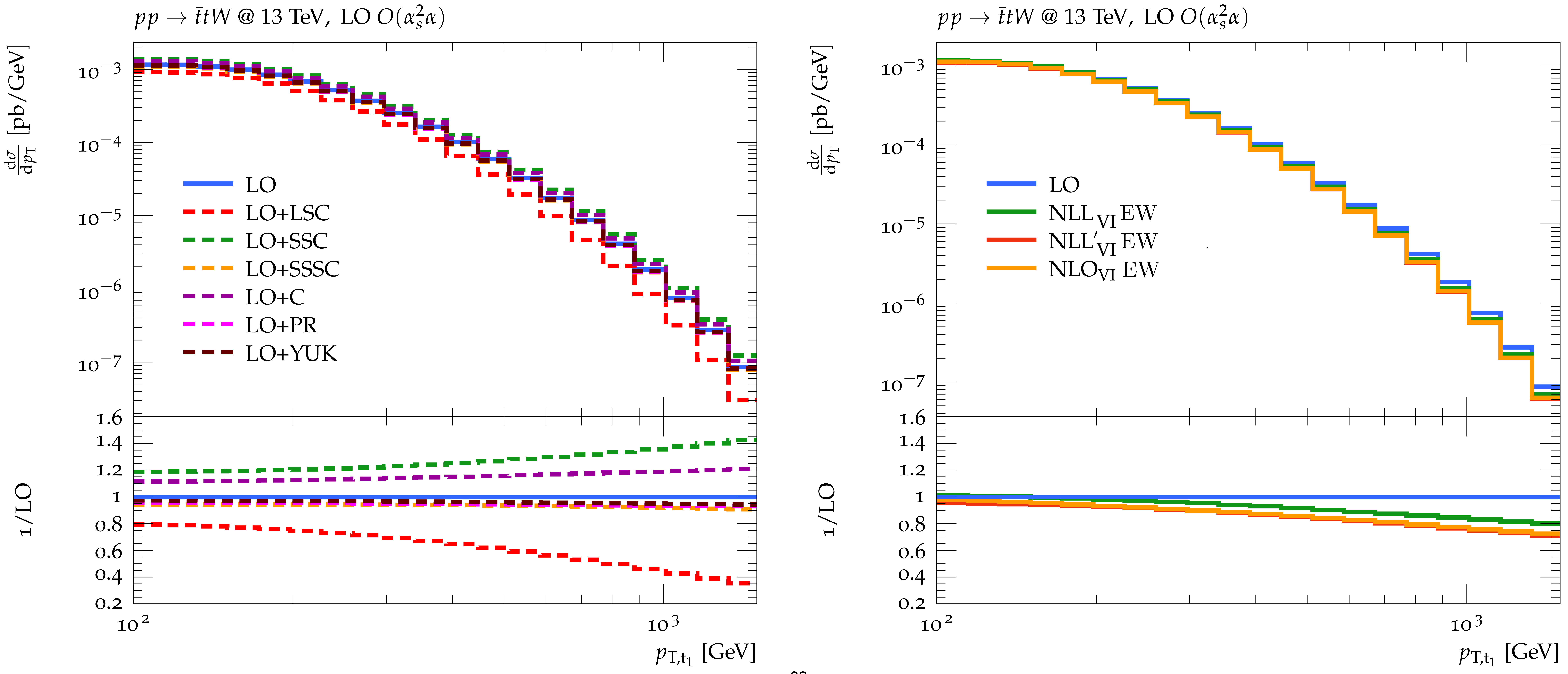

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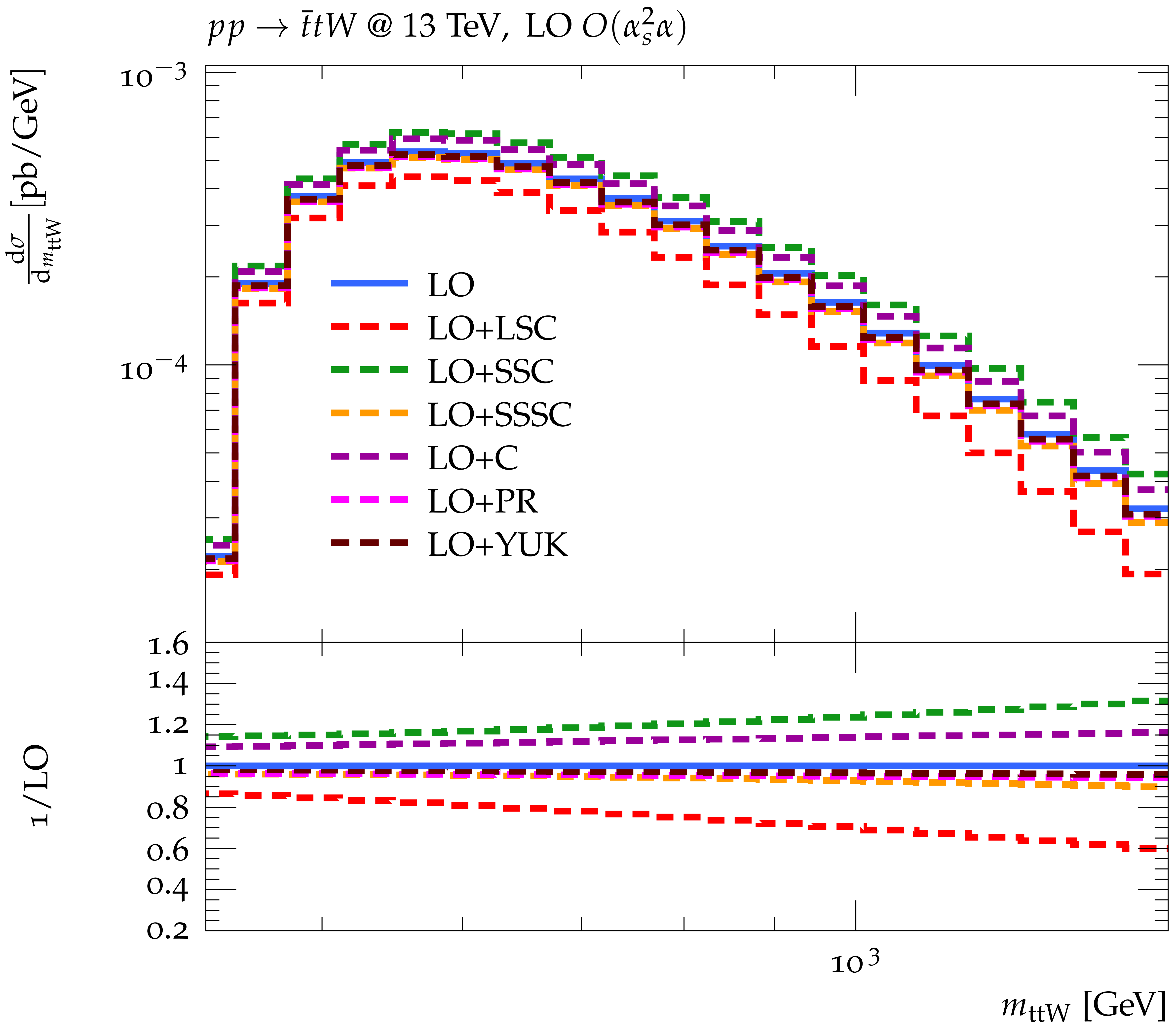

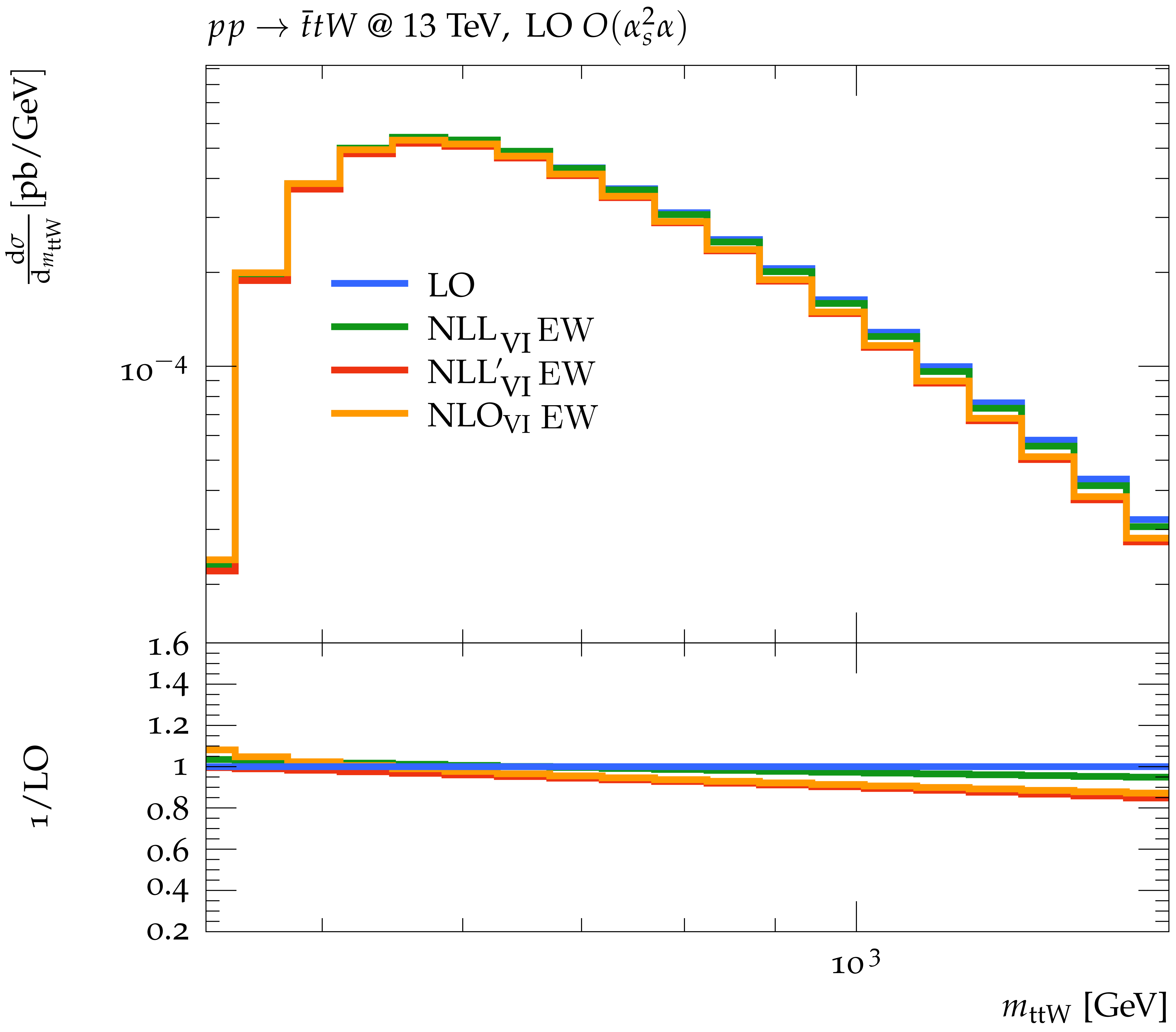

Implementation in OpenLoops: resonances

- DP algorithm:
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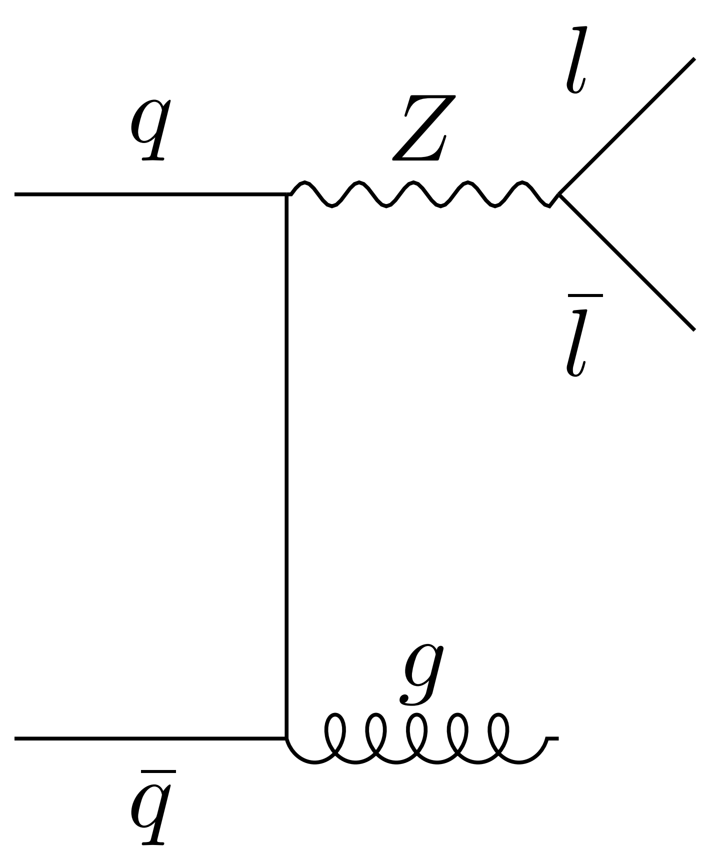

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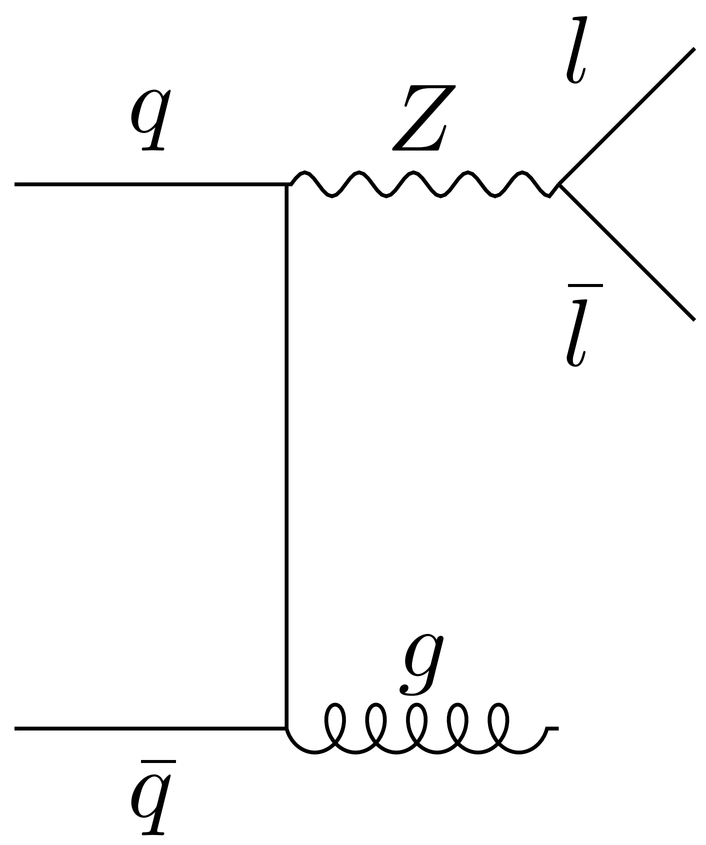


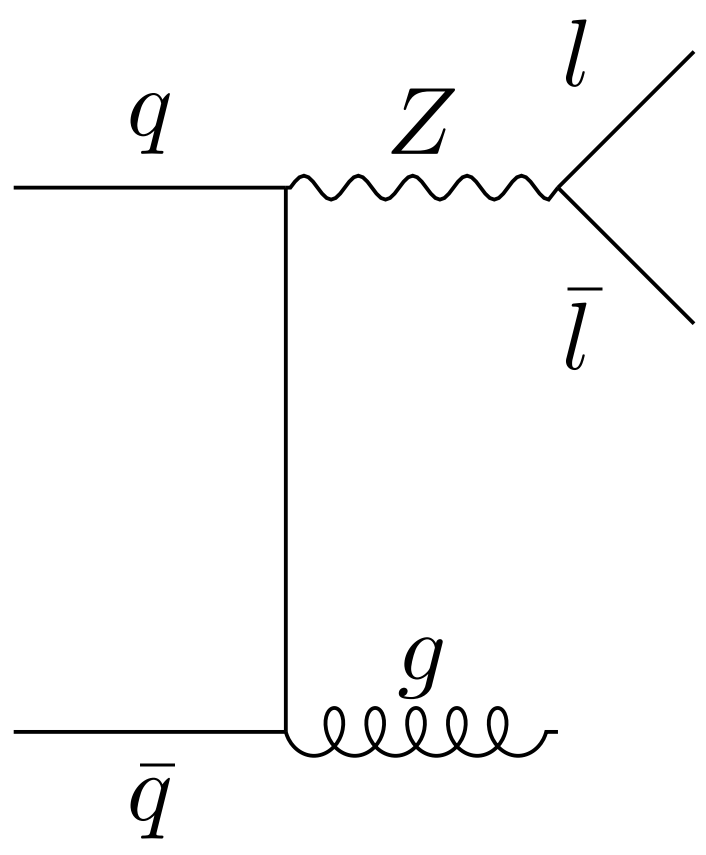
LSC, C: $\delta_{kk}^{\mathrm{LSC,C}}$, $k \in \{q, \bar{q}, l, \bar{l}\}$

SSC, S-SSC: $\delta_{kl}^{\mathrm{(S\text{-})SSC}}$, $k \neq l$ and $k, l \in \{q, \bar{q}, l, \bar{l}\}$

PR: CTs for $Z\bar{q}q, Z\bar{l}l$ vertices

Implementation in OpenLoops: resonances

- E.g.: partonic channel of $pp \to Zj \to \bar{l}lj$



LSC, **C**: $\delta^{\mathrm{LSC,C}}_{kk}$, $k \in \{q, \bar{q}, l, \bar{l}\}$

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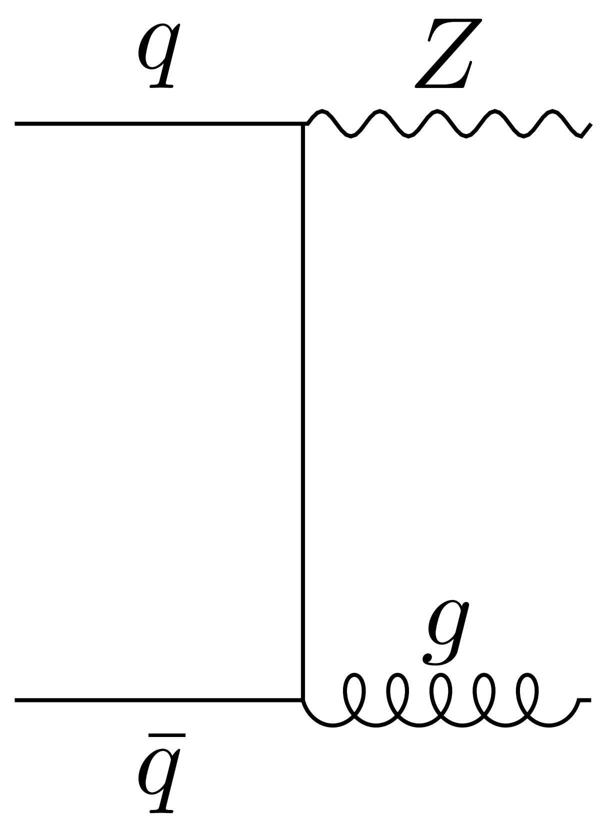
PR: CTs for $Z\bar{q}q$, $Z\bar{l}l$ vertices

- In the kinematic region where the Z boson is nearly on shell



LSC, **C**: $\delta^{\mathrm{LSC,C}}_{kk}$, $k \in \{q, \bar{q}, Z\}$

SSC, **S-SSC**: $\delta^{\mathrm{(S\text{-})SSC}}_{kl}$, $k \neq l$ and $k, l \in \{q, \bar{q}, Z\}$

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Implementation in OpenLoops: resonances

- Solution: evaluation of _Sudakov_ corrections associated to both Z and $\{l, \bar{l}\}$ with different weights $P_i(k_i)$

Implementation in OpenLoops: resonances

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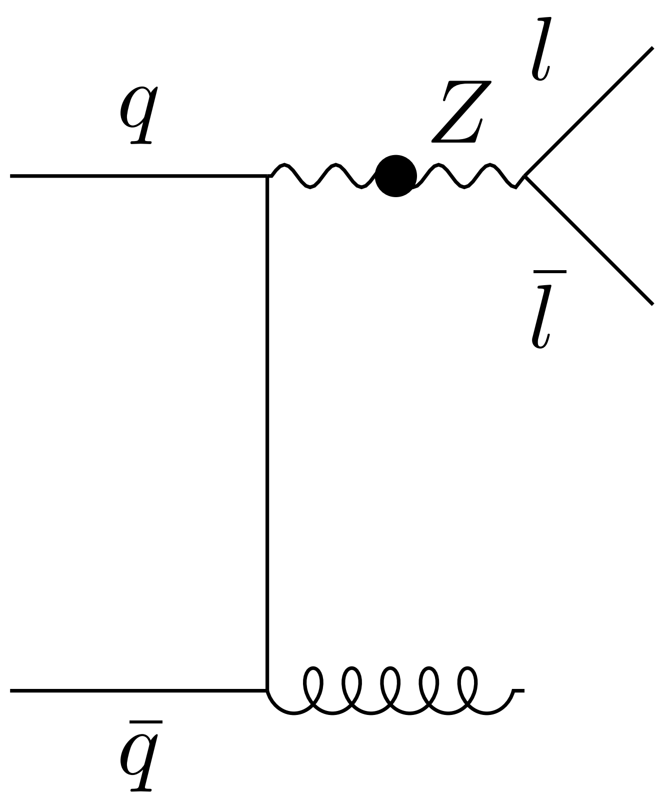
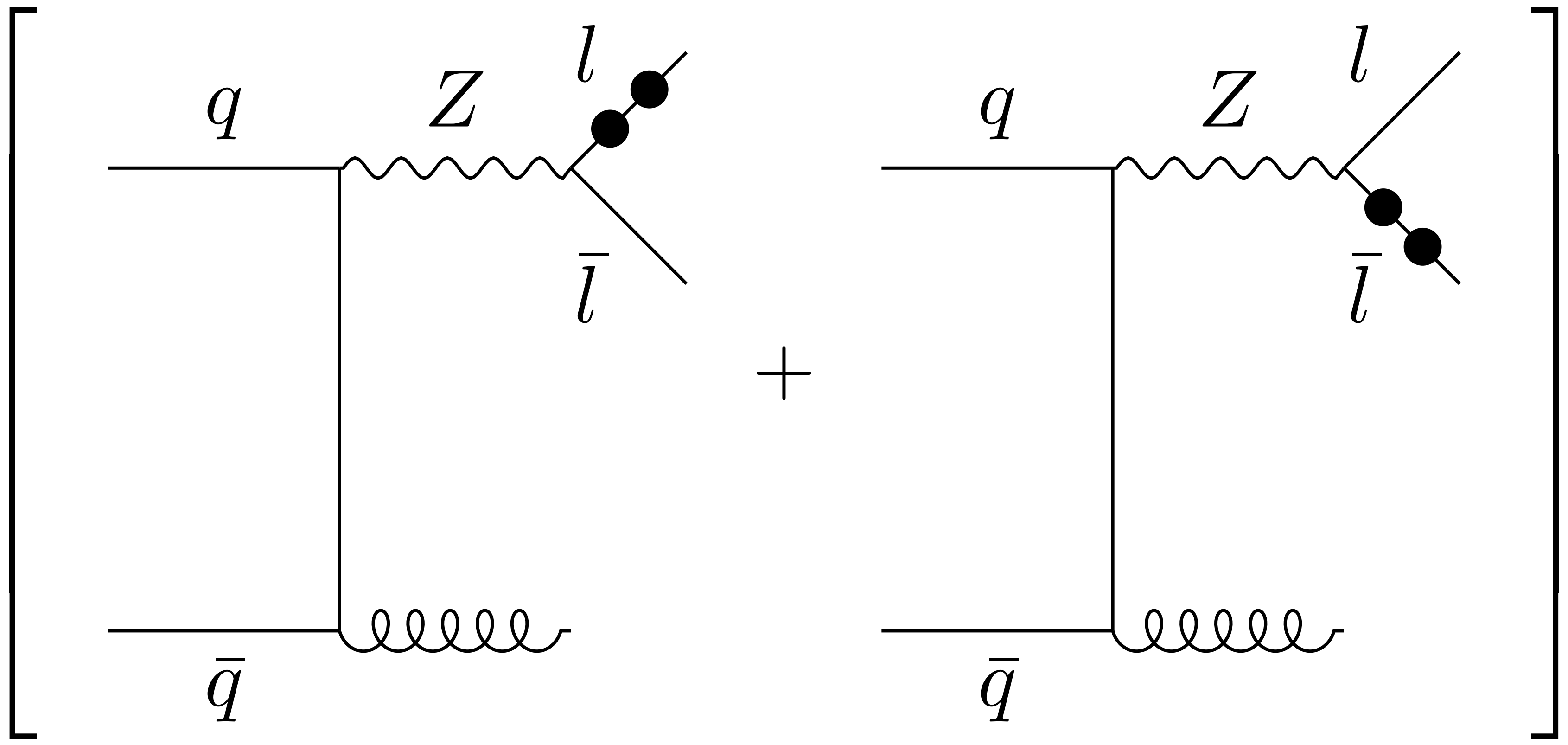


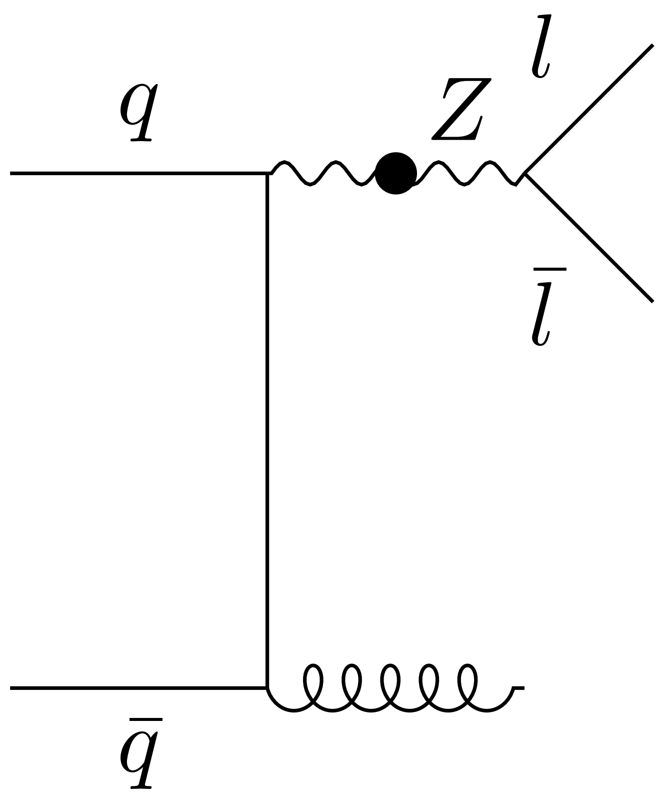
$\times P_Z(k)$



$\times P_{\{l,\bar{l}\}}(k) = 1 - P_Z(k)$

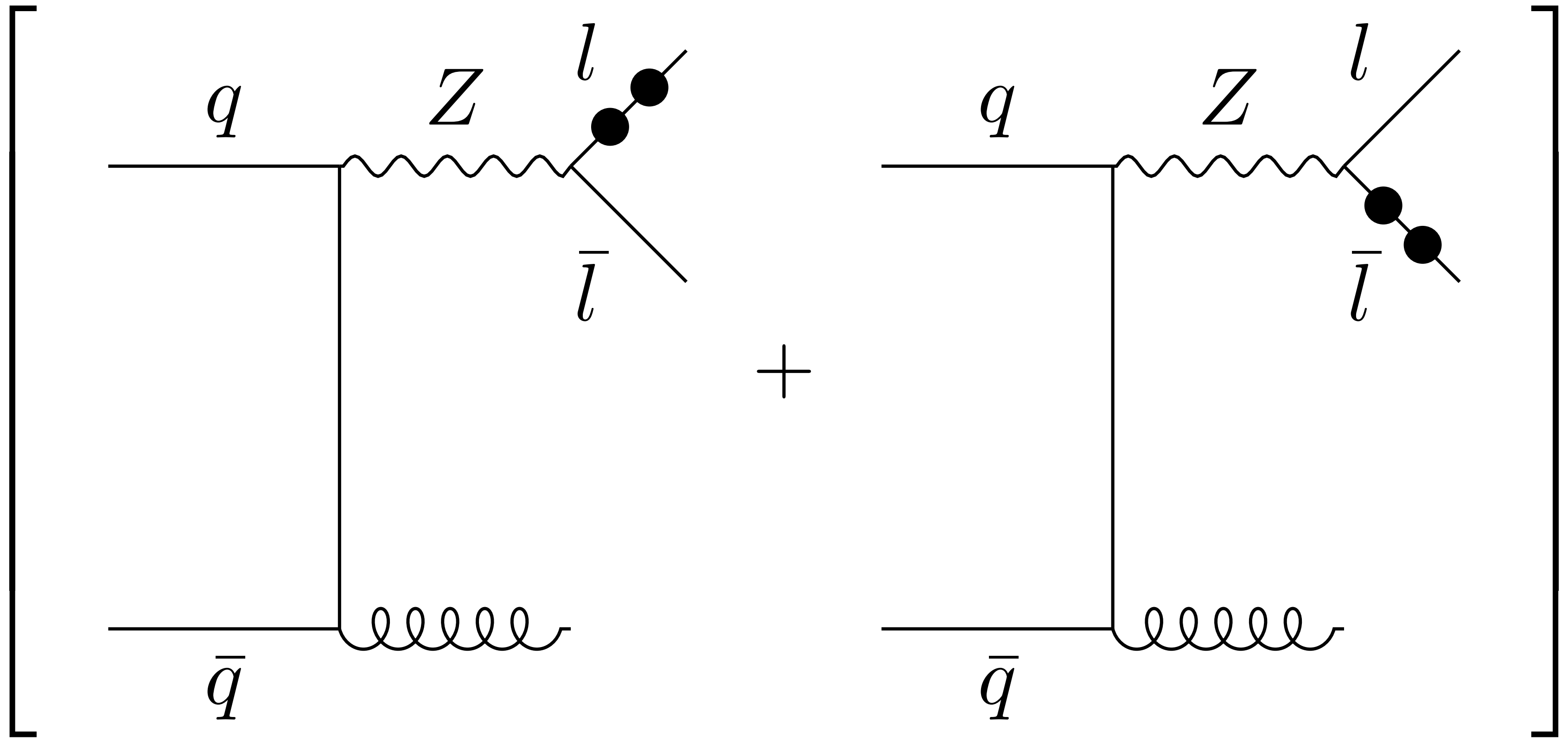
Implementation in OpenLoops: resonances

- Solution: evaluation of ***Sudakov*** corrections associated to both Z and $\{l, \bar{l}\}$ with different weights $P_i(k_i)$



$\times P_Z(k)$

$$P_{X_i}(k_i) = \left| \frac{\mu^2_{X_i} - M^2_{X_i}\Gamma^2_{X_i}}{(k_i^2 - \mu^2_{X_i})^2 + \mu^2_{X_i}} \right| = \begin{cases} 1 & \text{if } k_i^2 \to M^2_{X_i} \\ 0 & \text{if } k_i^2 \to \infty \end{cases}$$



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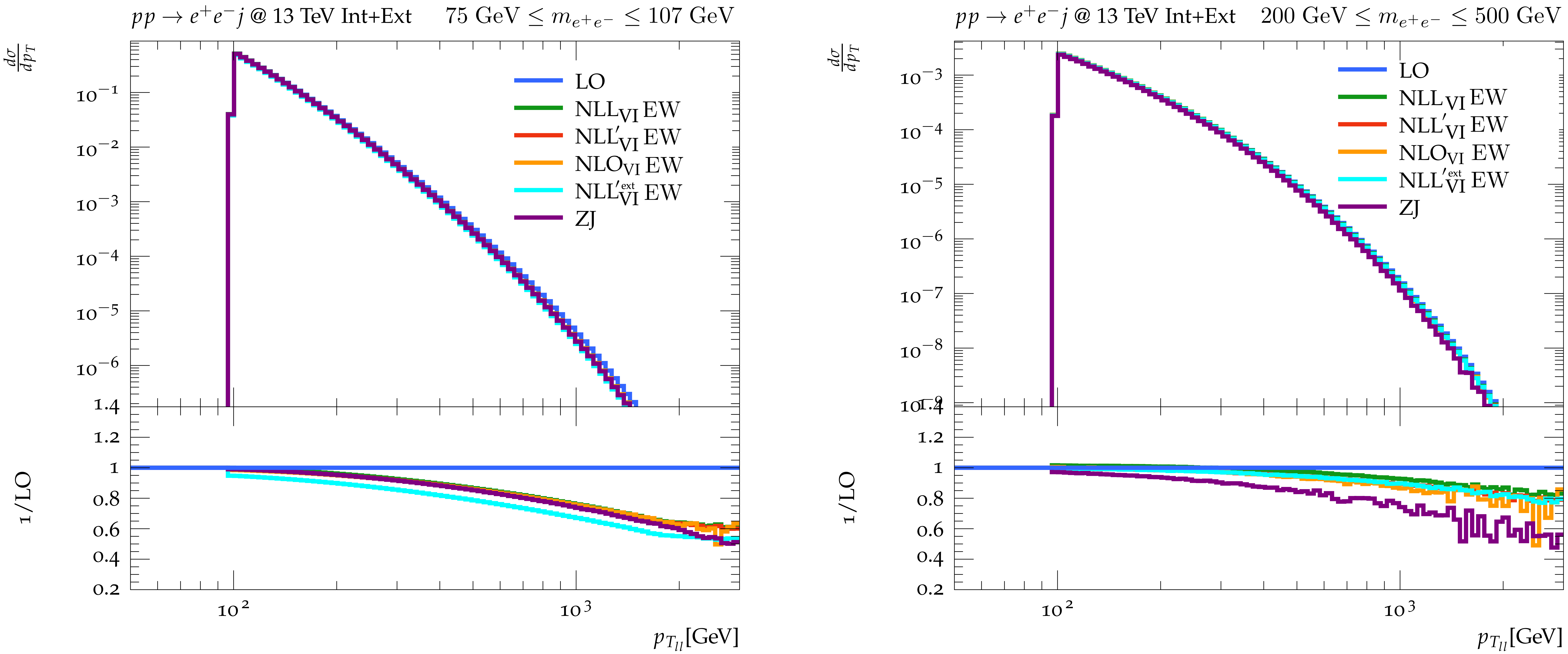
Results: $pp \to e^+e^-j$



- External insertions approach (as well Sudakov corrections to the hard process only) fail in reproducing the full $\mathrm{NLO_{VI}}$ prediction for $m_{e^+e^-}$ range "capturing" the resonance
- Issue naturally solved with internal insertions technique via projectors
- Automatic recover of standard algorithm when far from the resonance

Conclusions and outlook

- In the EW sector, radiative corrections at high energies are dominated by *Sudakov logarithms* which significantly enhance tails of kinematic distributions ($>$ 10%)
- Exploiting the ***universality*** of *Sudakov logs* we developed an effective CT vertex approach for the DP algorithm and implemented it in OpenLoops

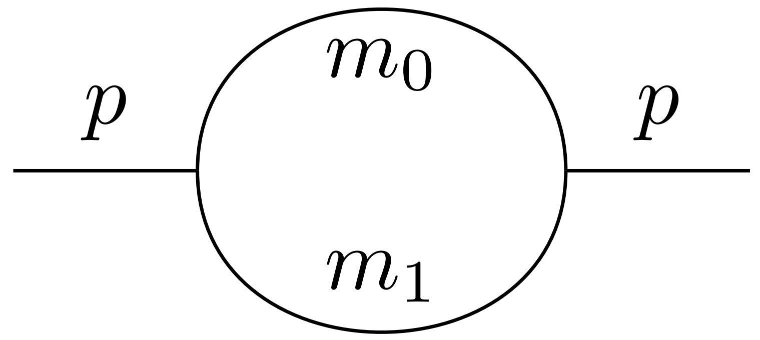
Reduction of one-loop EW corrections to a tree-level problem with percent level of accuracy

- Additional aspects of the implementation:
  - ▶ Model independent
  - ▶ Direct employment in PS Event Generators with OL interface
  - ▶ Can be used together with differential QED radiation at NLO (both mass and dim reg are available)
  - ▶ Support EW corrections for resonant processes
- Outlook:
  - ▶ Understand when and how to do resummation
  - ▶ Suitable for NNLO/two-loop extension

Backup

Single Logs: PR

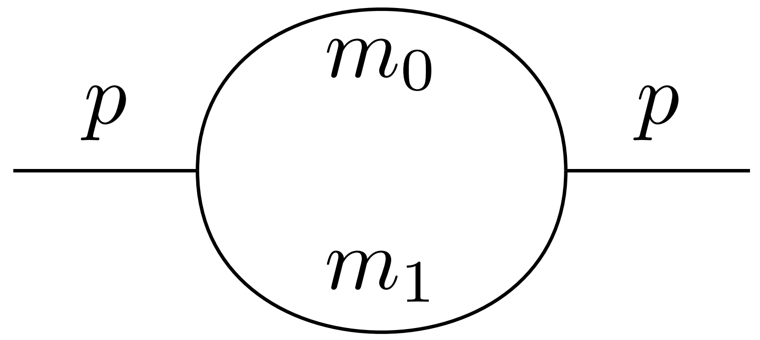
- Generic two-point function



$$\frac{\mathrm{i}}{(4\pi)^2} B_{\{0,\mu,\mu\nu\}}\left(p, m_0, m_1\right) := \mu^{4-D} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{\{1, q_\mu, q_\mu q_\nu\}}{\left(q^2 - m_0^2 + \mathrm{i}\varepsilon\right)\left[(q+p)^2 - m_1^2 + \mathrm{i}\varepsilon\right]}$$

Single Logs: PR

- Generic two-point function



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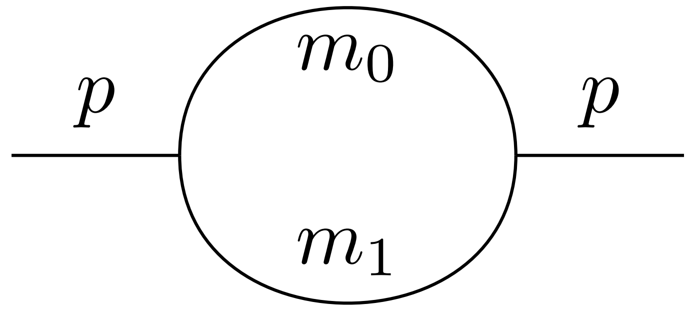
- In LA $\mu^2 = s \gg p^2, m_0^2, m_1^2 \Rightarrow$ four possible hierarchy of masses

$$\begin{aligned}
&(a) \quad m_i^2 \ll p^2 \text{ and } p^2 - m_{1-i}^2 \ll p^2 \quad \text{for } i = 0 \text{ or } \quad i = 1, \\
&(b) \quad \text{not} \quad (a) \text{ and } m_i^2 \not> p^2 \text{ for } i = 0, 1, \\
&(c) \quad m_0^2 = m_1^2 \gg p^2 \\
&(d) \quad m_i^2 \gg p^2 \not> m_{1-i}^2 \text{ for } i = 0 \text{ or } i = 1
\end{aligned}$$

Single Logs: PR



- Generic two-point function

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- Results for two point functions and their derivatives

$$B_0\left(p, m_0, m_1\right) \stackrel{\mathrm{LA}}{=} \log \frac{\mu^2}{M^2},$$

$$B_1\left(p^2, m_0, m_1\right) \stackrel{\mathrm{LA}}{=} -\frac{1}{2} \log \frac{\mu^2}{M^2},$$

$$\frac{1}{p^2} B_{00}\left(p^2, m_0, m_1\right) \stackrel{\mathrm{LA}}{=} \frac{3m_0^2 + 3m_1^2 - p^2}{12p^2} \log \frac{\mu^2}{M^2},$$

$$\frac{1}{p^2} g^{\mu\nu} B_{\mu\nu}\left(p^2, m_0, m_1\right) \stackrel{\mathrm{LA}}{=} \frac{m_0^2 + m_1^2}{p^2} \log \frac{\mu^2}{M^2}$$

$$p^2 B_0'\left(p, m_0, m_1\right) \stackrel{\mathrm{LA}}{=} \frac{1}{2} \log \frac{m_{1-i}^2}{m_i^2} = \frac{1}{2} \log \frac{p^2}{\lambda^2},$$

$$p^2 B_1'\left(p^2, m_0, m_1\right) + \frac{1}{2} p^2 B_0'\left(p, m_0, m_1\right) \stackrel{\mathrm{LA}}{=} -\frac{1}{4} \log \frac{m_0^2}{m_1^2}$$

Implementation in OpenLoops: projectors

- Explicit expression of the projectors for unstable particles X

$$P_{X_i}(k_i) = \left| \frac{\mu_{X_i}^2 - M_{X_i}^2 \Gamma_{X_i}^2}{(k_i^2 - \mu_{X_i}^2)^2 + \mu_{X_i}^2} \right| = \begin{cases} 1 & \text{if } k_i^2 \to M_{X_i}^2 \\ 0 & \text{if } k_i^2 \to \infty \end{cases}$$

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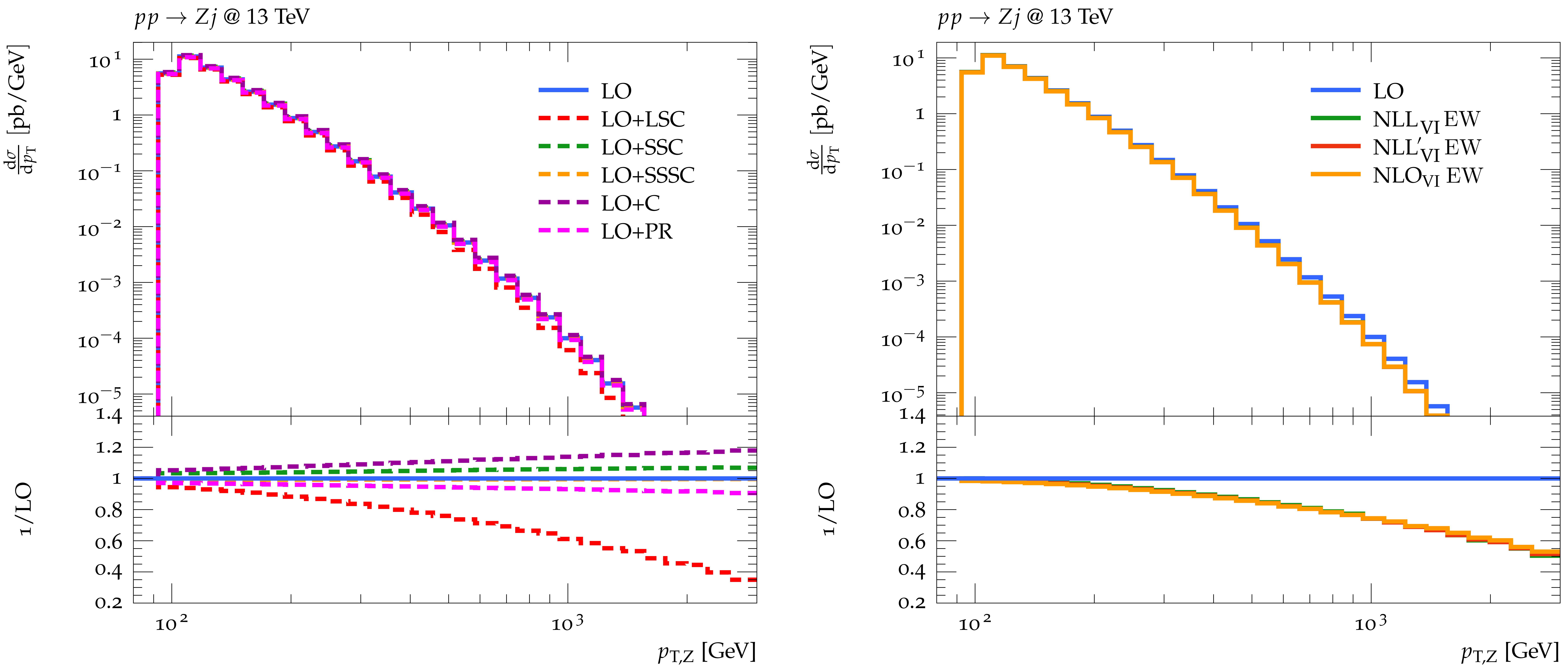
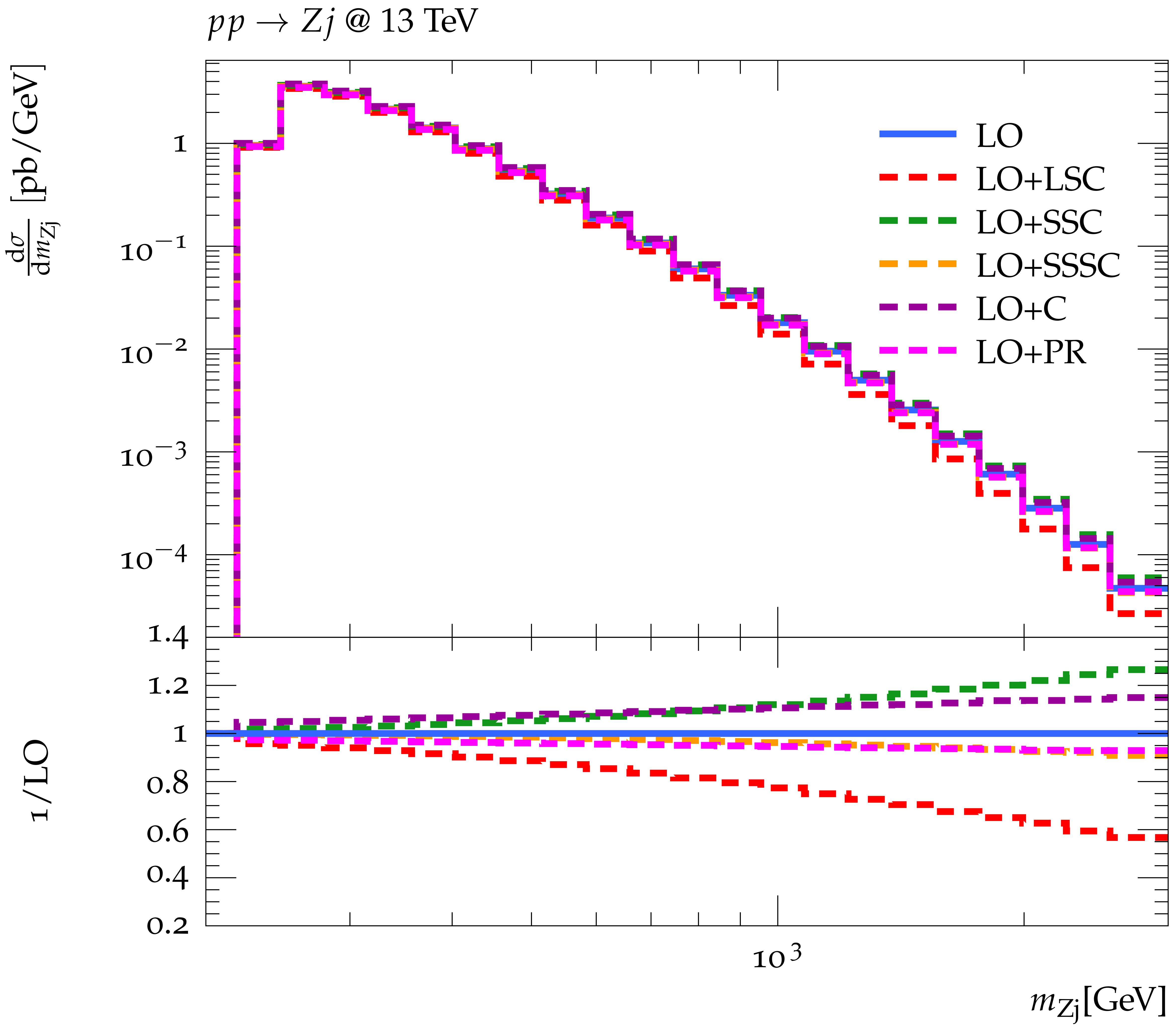
- Unitarity is violated but it can be restored:
  - Evaluation of $P_{X_i}(k_i)$ for a given psp
  - Generation of random number $0 \leq a \leq 1$
  - Choice $P_{X_i} = \begin{cases} 1 & \text{if } P_{X_i} \geq a \\ 0 & \text{if } P_{X_i} \leq a \end{cases}$

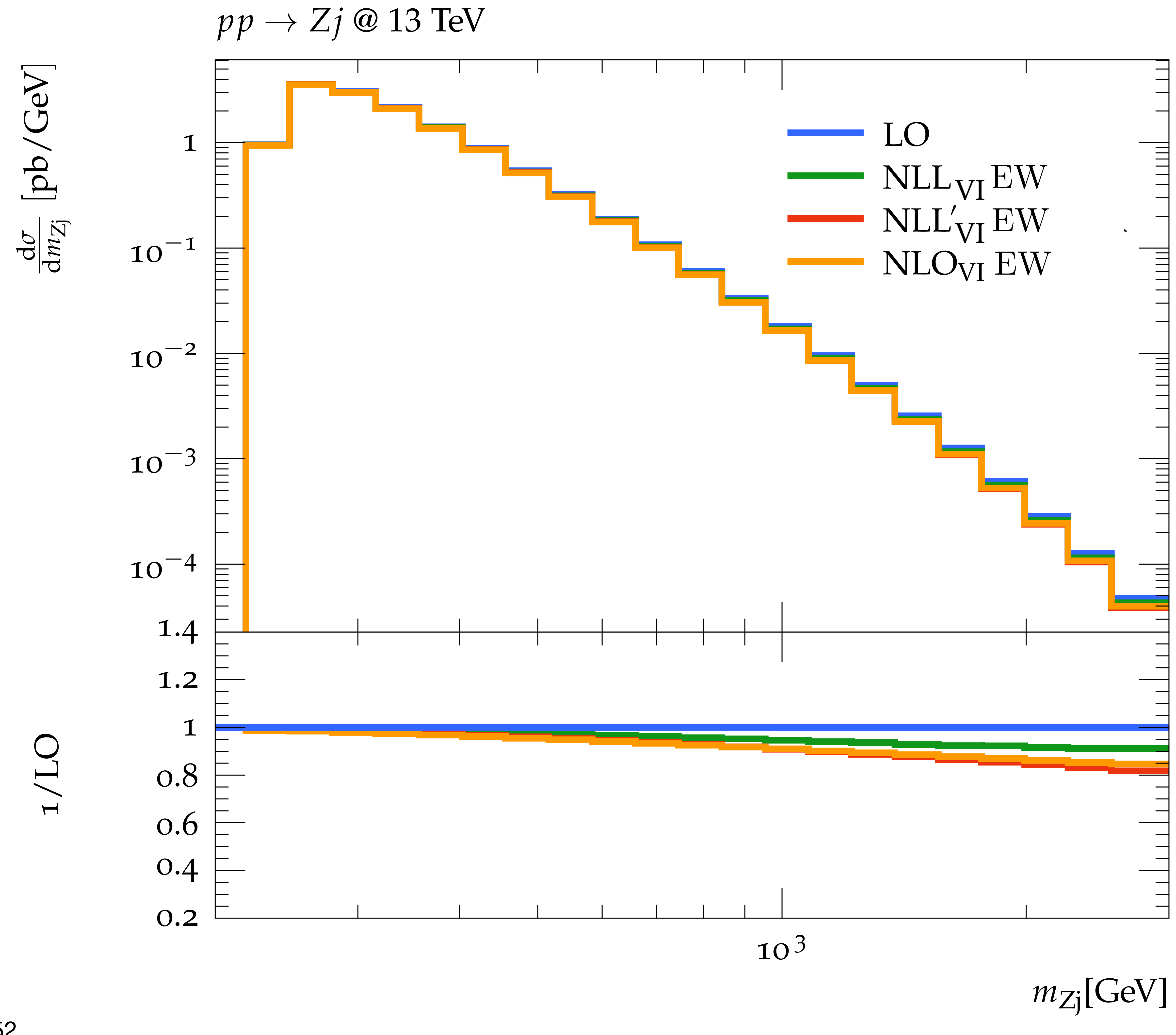
Additional results

Results: $pp \to Z + j$

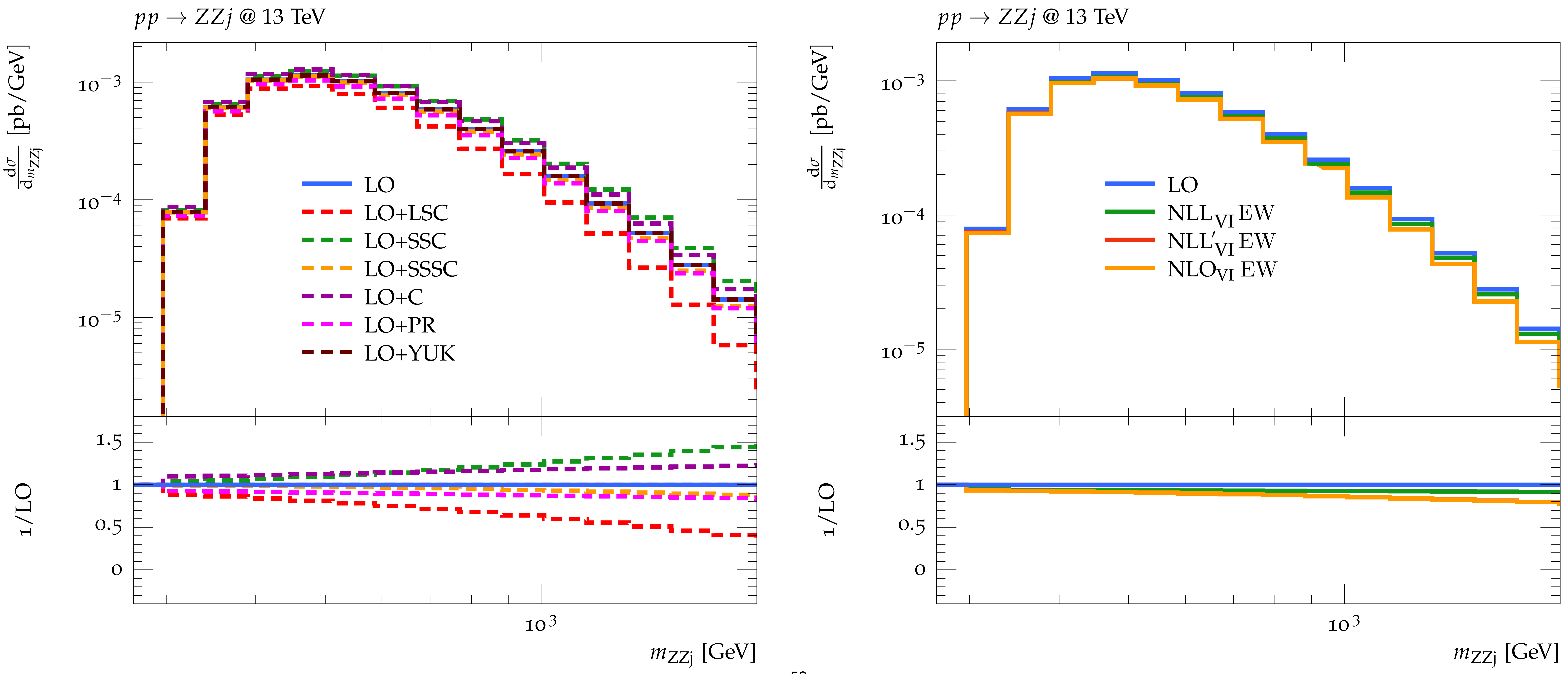


Results: $pp \to Z + j$





Results: $pp \to ZZj$



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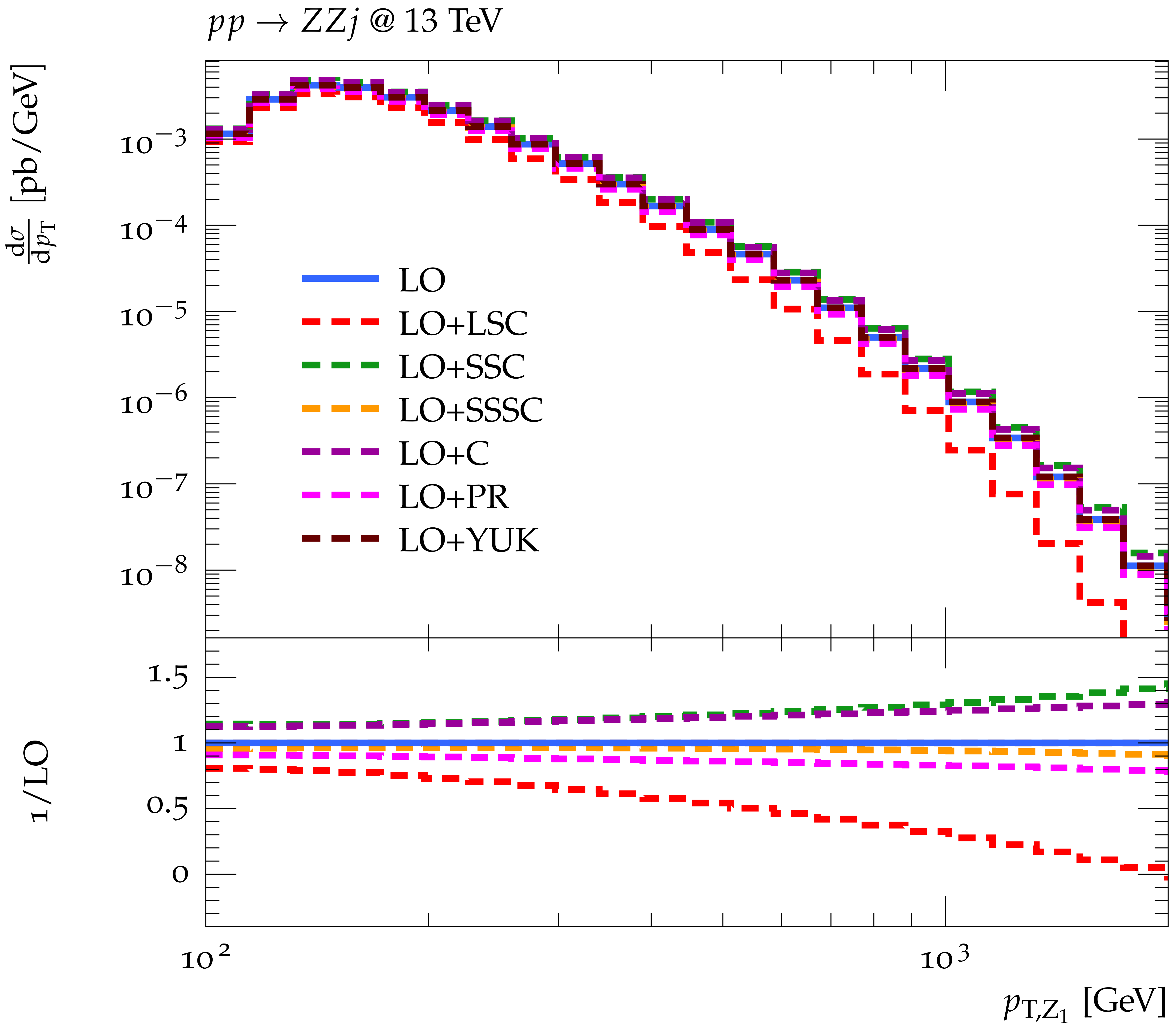
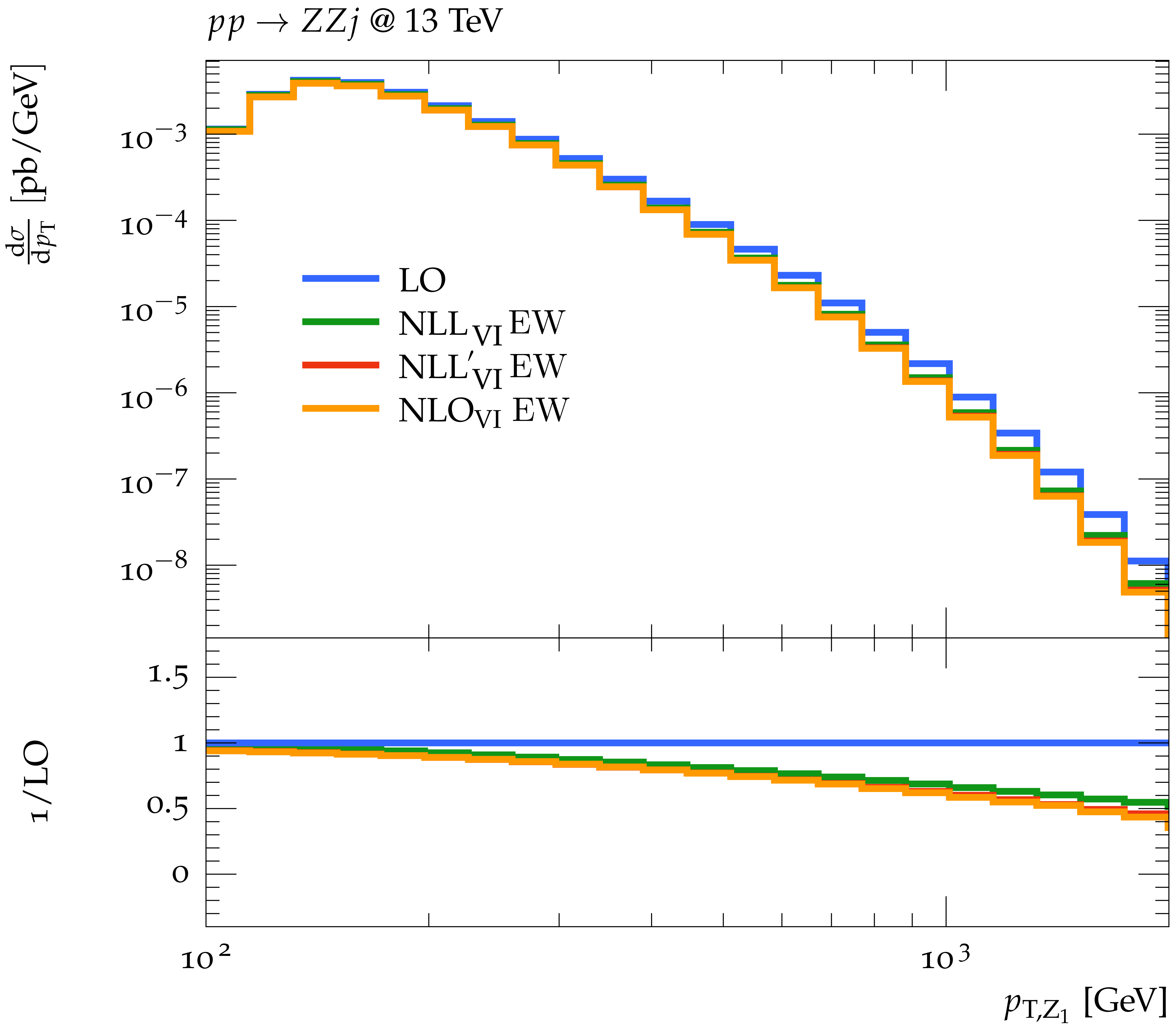
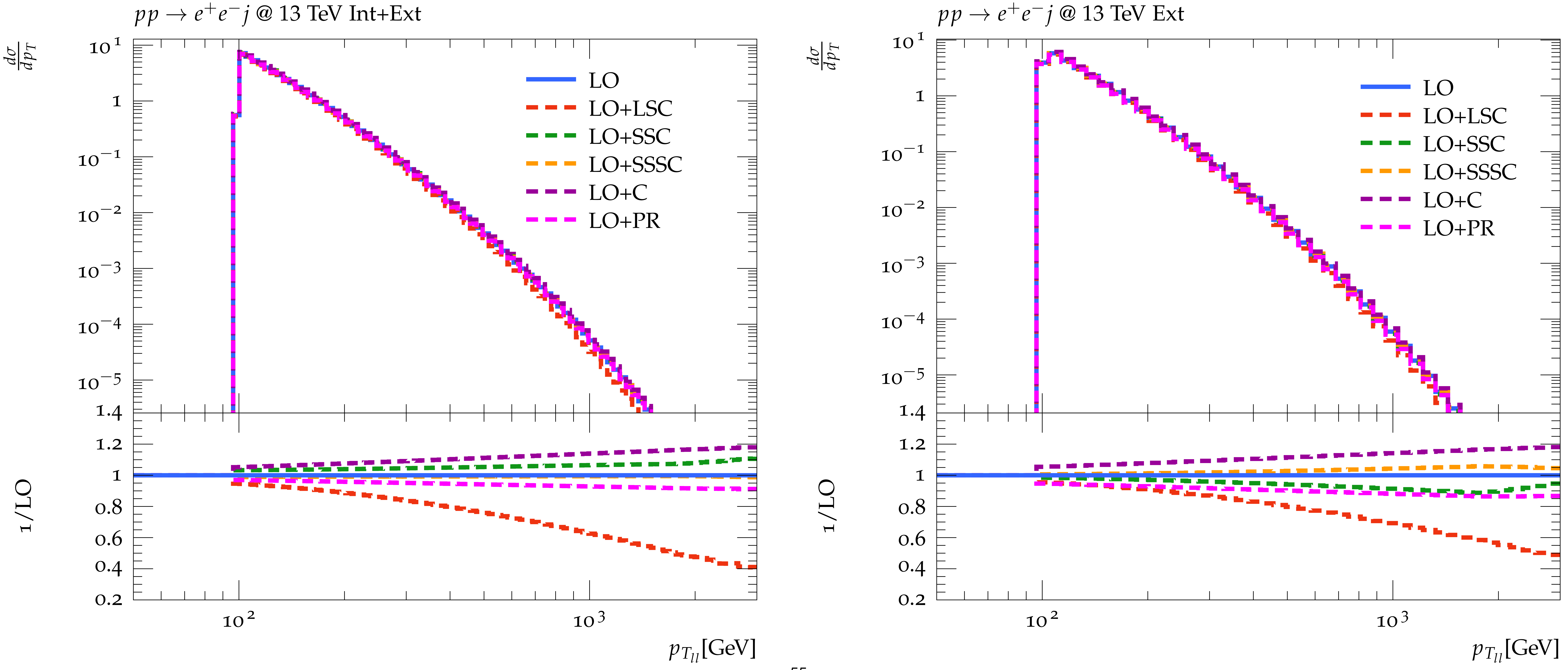




Results: $pp \to e^+e^-j$

$75 \text{ GeV} \leq m_{e^+e^-} \leq 107 \text{ GeV}$



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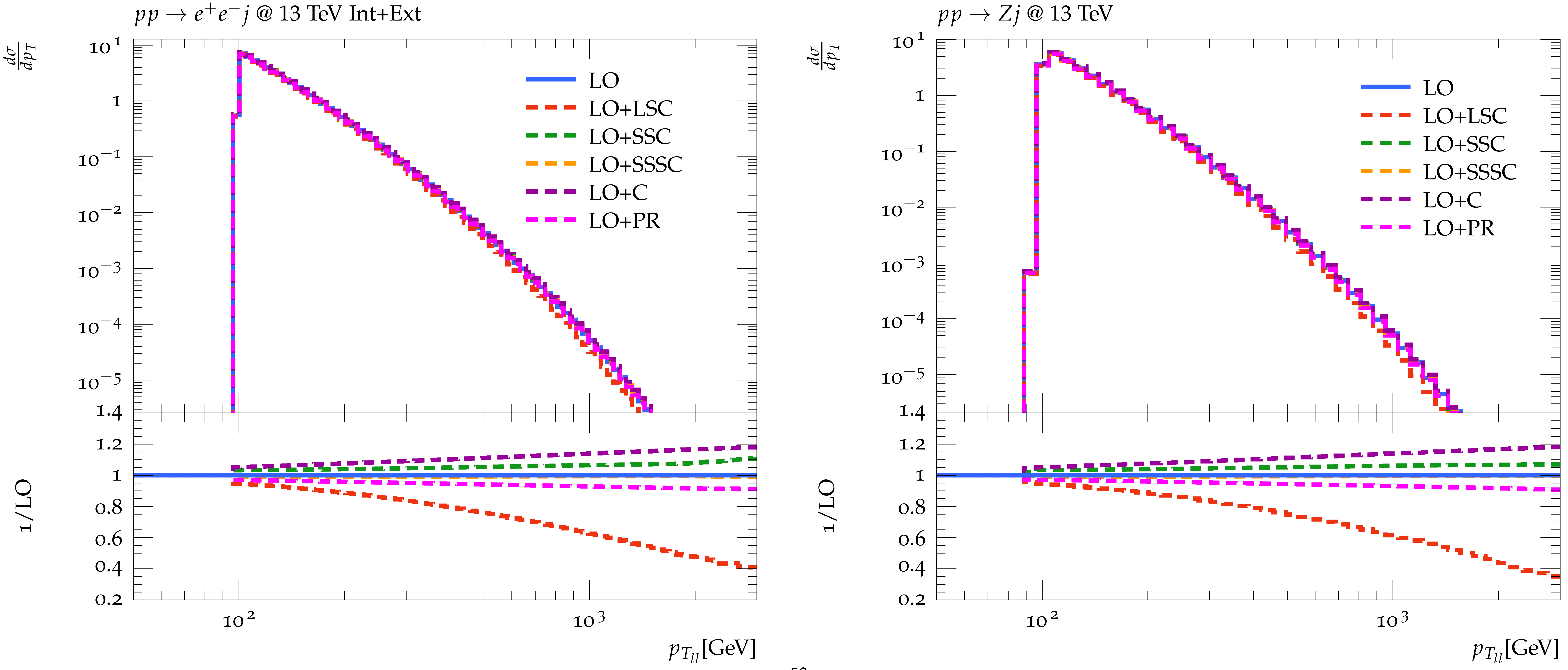

Results: $pp \to e^+e^-j$

$200 \text{ GeV} \leq m_{e^+e^-} \leq 500 \text{ GeV}$