Neutrinos in cosmology

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Neutrino Frontiers Training Week, Galileo Galilei Institute, Florence,
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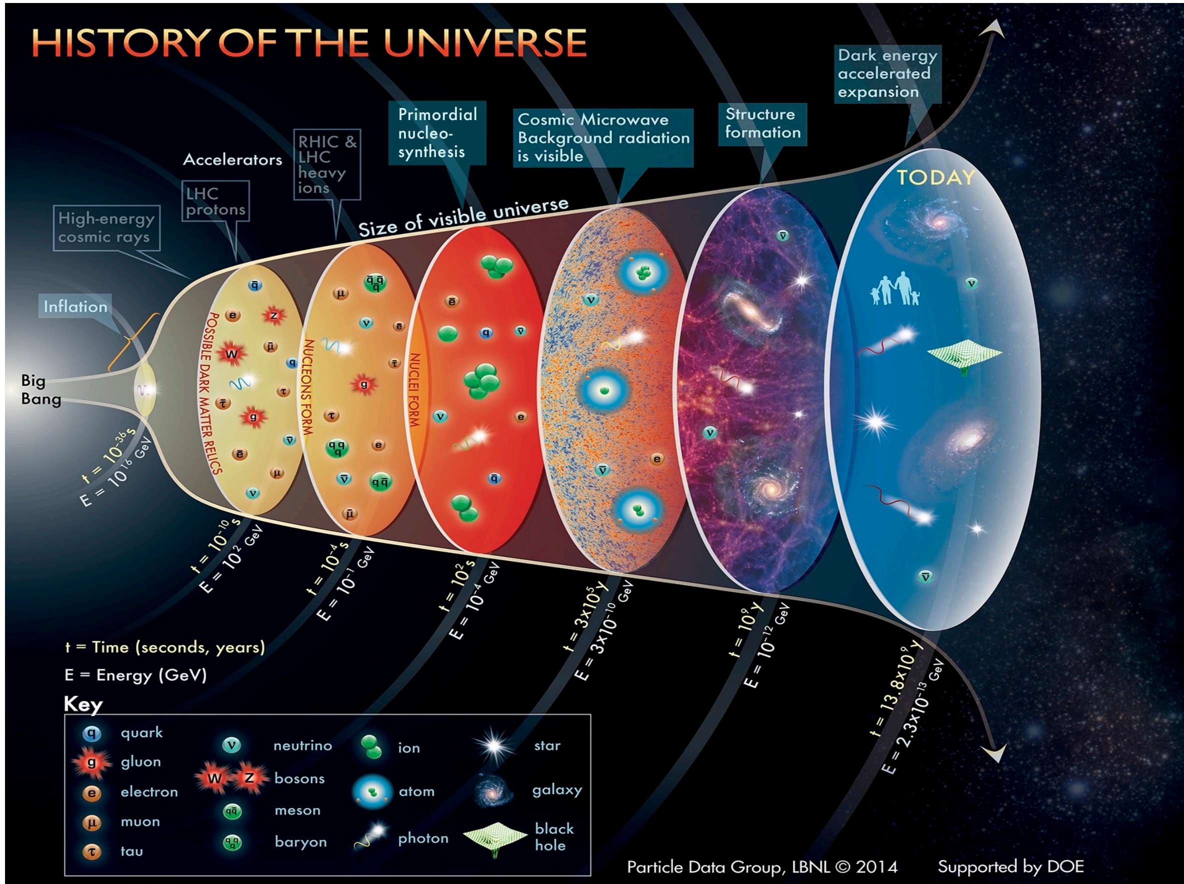
Or, how neutrinos fit into this grand scheme?



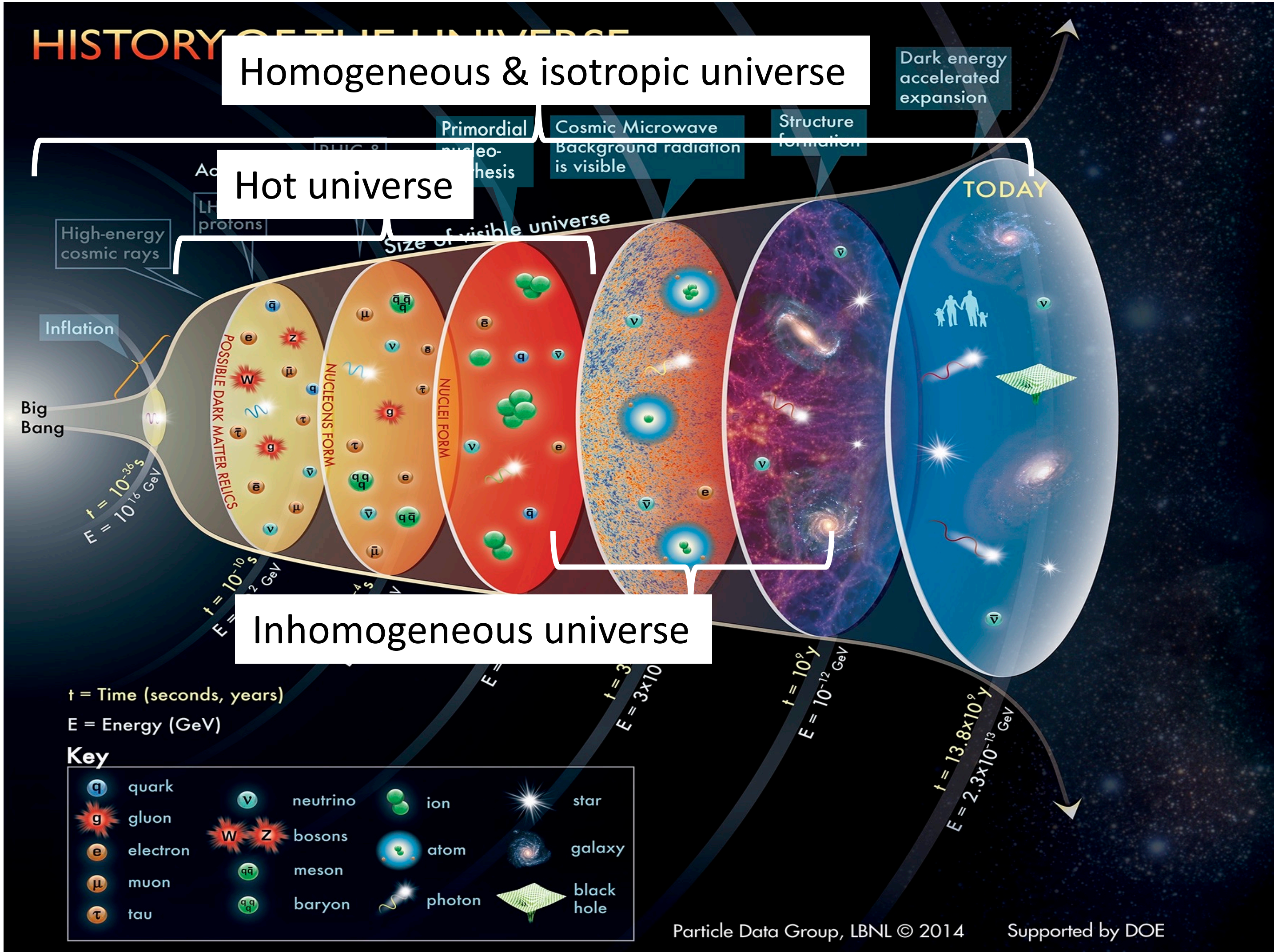
The grand lecture plan...

Part 1: Neutrinos in homogeneous cosmology

1. The homogeneous and isotropic universe
2. The hot universe and the cosmic neutrino background
3. Precision CνB

Part 2: Neutrinos in inhomogeneous cosmology

1. Theory of inhomogeneities
2. Neutrinos and structure formation
3. Relativistic neutrino free-streaming and non-standard interactions


HISTORY
Homogeneous & isotropic universe
Hot universe
Inhomogeneous universe
Dark energy accelerated expansion
Primordial
Cosmic Microwave Background radiation is visible
Structure
High-energy cosmic rays
Inflation
Big Bang
Size of visible universe
TODAY
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
t = Time (seconds, years)
E = Energy (GeV)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL © 2014
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Useful references…

- **Textbook**
  - J. Lesgourgues, G. Mangano, G. Miele & S. Pastor, *Neutrino cosmology*

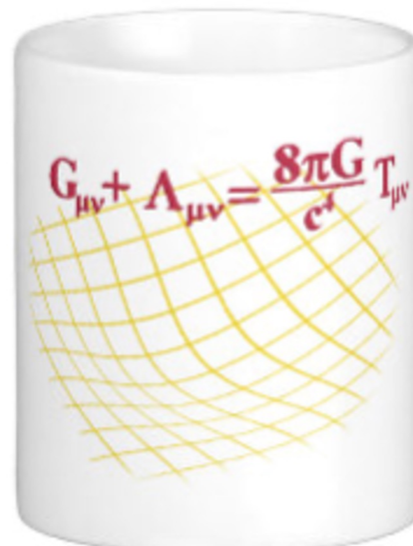
- **Lecture notes**
  - Baumann, *Cosmology* (many different versions)
  - Seljak, *Lectures on dark matter*, ICTP Lect. Notes Ser. **4** (2001) 33
  - Hu, *Covariant linear perturbation formalism*

- **Reviews**
  - A. D. Dolgov, *Neutrinos in cosmology*, Phys. Rept. **370** (2002) 333 [hep-ph/0202122]
  - J. Lesgourgues & S. Pastor, *Massive neutrinos and cosmology*, Phys. Rept. **429** 307 [astro-ph/0603494]

Part 1: Neutrinos in homogeneous cosmology

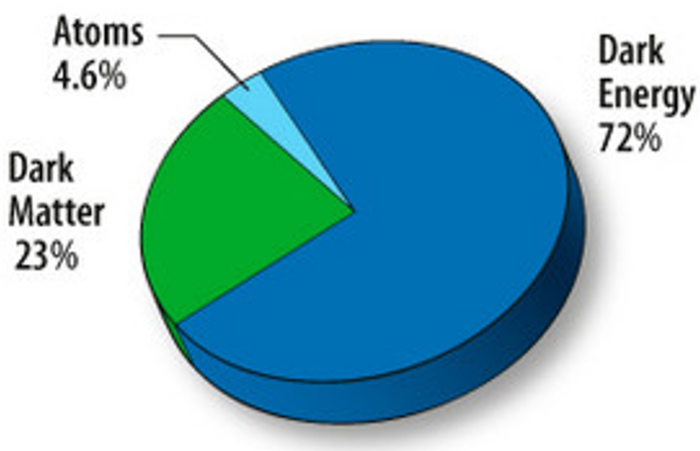
1. The homogeneous and isotropic universe
2. The hot universe and the cosmic neutrino background
3. Precision CνB

1. The homogeneous and isotropic universe...

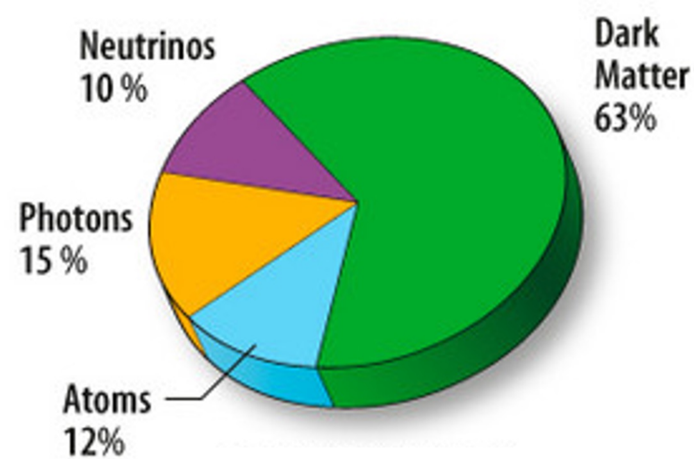


The concordance flat ΛCDM model...

The simplest model consistent with **present observations**.



Composition today



13.4 billion years ago
(at photon decoupling)

Plus flat spatial geometry+initial conditions
from single-field inflation

FLRW universe…

FLRW = Friedmann-Lemaître-Robertson-Walker

Cosmological principle: our universe is spatially homogeneous and isotropic on sufficiently large length scales (i.e., we are not special).

- Homogeneous → same everywhere
- Isotropic → same in all directions
- Sufficiently large scales → $> O(100) Mpc$

Isotropic but not homogeneous

Homogeneous but not isotropic

Homogeneous and isotropic

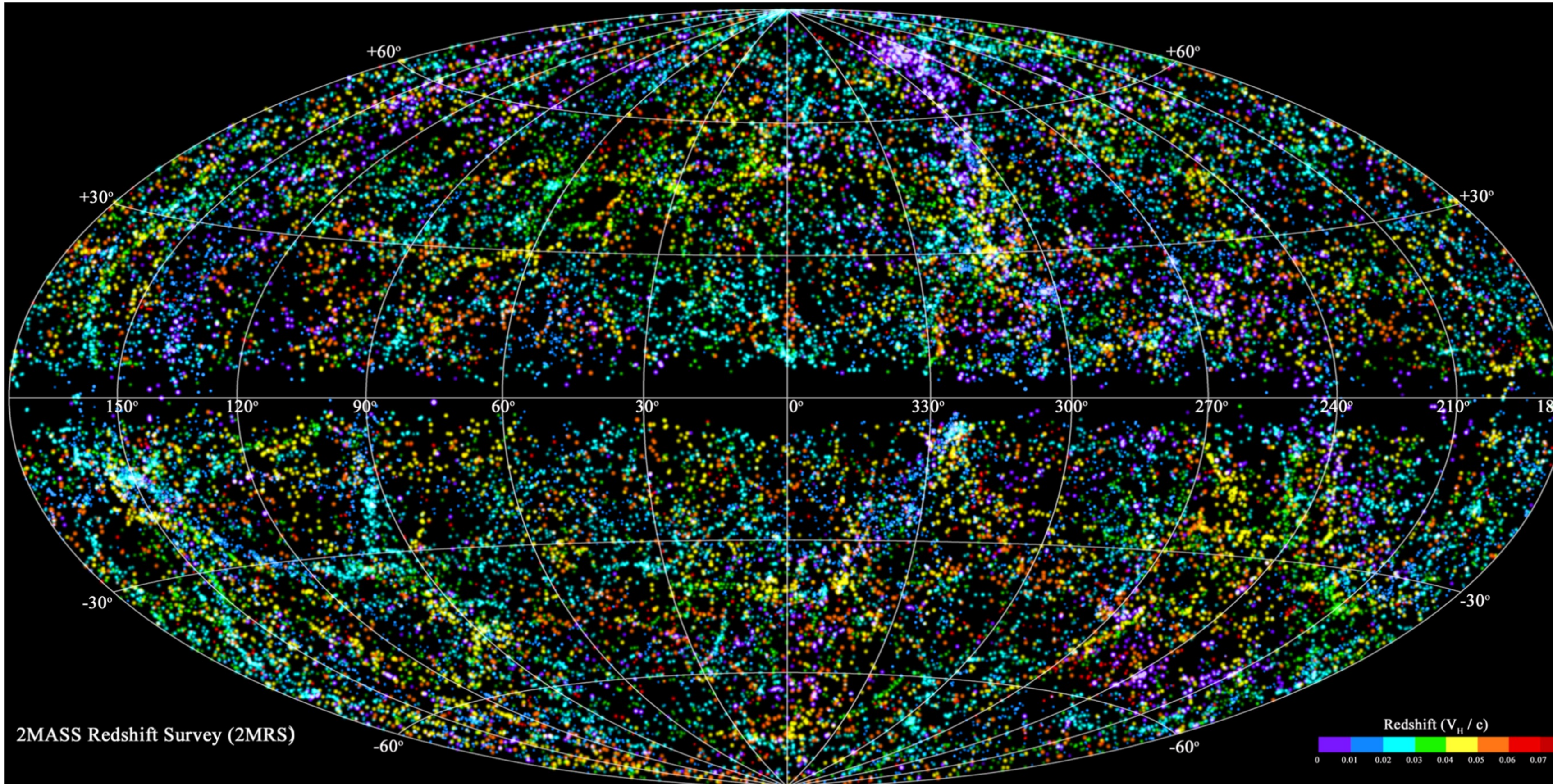
FLRW universe…

Cosmological principle: our universe is spatially homogeneous and isotropic on sufficiently large length scales (i.e., we are not special).

- Homogeneous → same everywhere
- Isotropic → same in all directions
- Sufficiently large scales → $> O(100)Mpc$

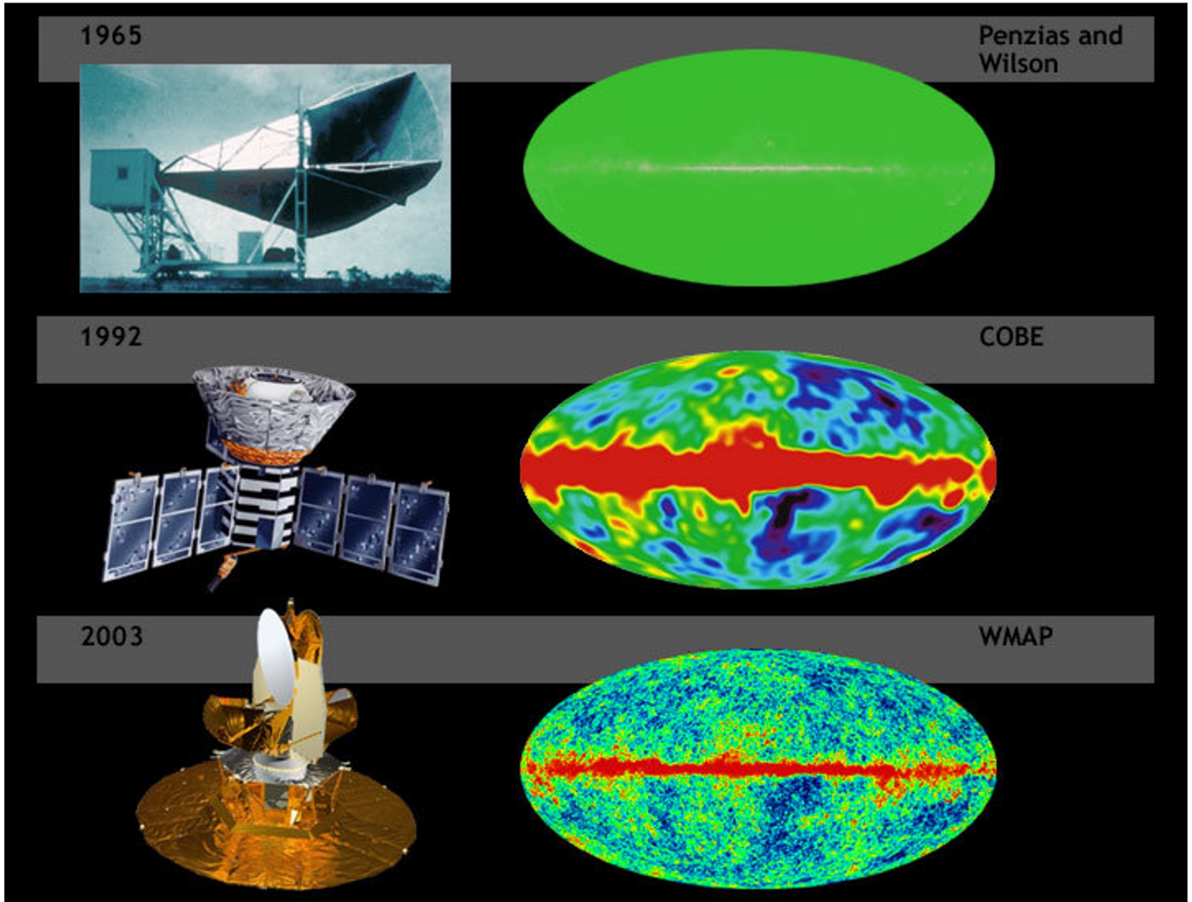
- $1 \text{ pc} = 1 \text{ parsec} = 3.0856 \times 10^{18} \text{ cm}$
  - Distance from Sun to Galactic centre $\sim 10 \text{ kpc}$
  - Distance to the Virgo cluster $\sim 20 \text{ Mpc}$
  - Size of the visible universe $\sim O(10 \text{ Gpc})$

Evidence for large-scale homogeneity and isotropy:



Local galaxy distribution as measured by the 2Mass Redshift Survey

Evidence for large-scale homogeneity and isotropy:



2.73 K background

 (1978)

$\sim 10^{-5}$ temperature fluctuations
($\sim 7°$ resolution)

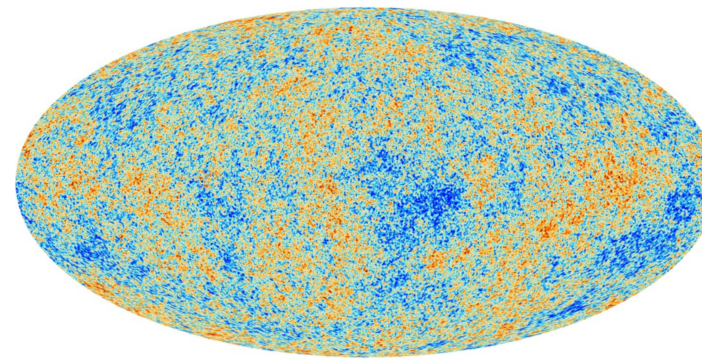
 (2006)

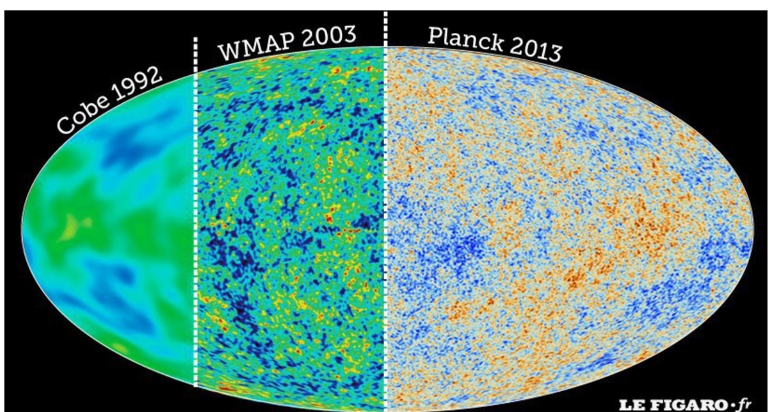
$\sim 0.2°$ resolution

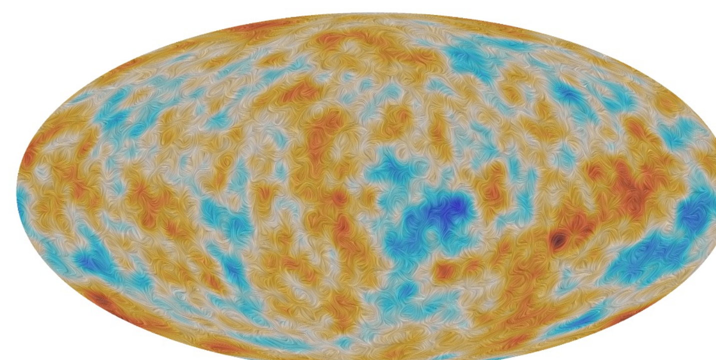
Cosmic microwave background (temperature)

State-of-the-art: Temperature and polarisation fluctuations in the **cosmic microwave background** as seen by Planck. (Latest results 2018)

Temperature







Polarisation

FLRW universe…

Homogeneity and isotropy imply maximally symmetric 3-spaces (3 translational and 3 rotational symmetries).

- A spacetime geometry that satisfies these requirements is the Friedmann-Lemaître-Robertson Walker (FLRW) geometry:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta \, d\phi^2)\right]$$

FLRW metric

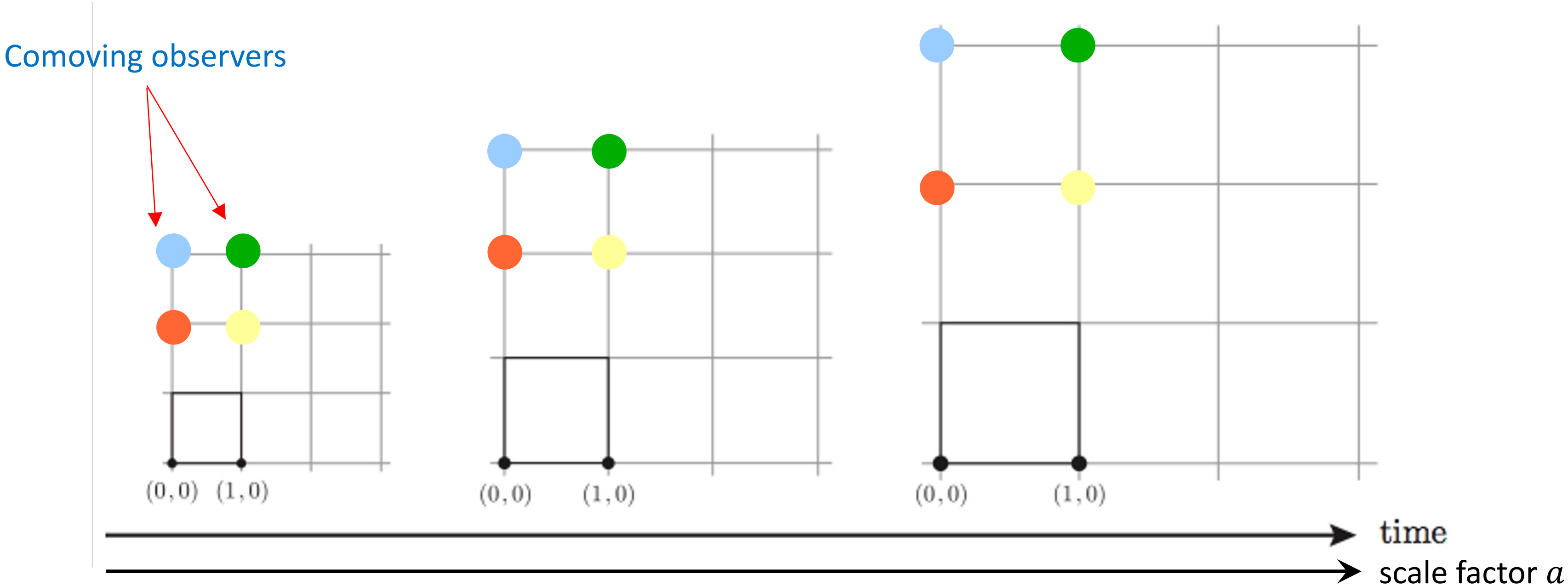
$a(t)$ = scale factor

Spatial geometry
$K = -1$ (hyperbolic), 0 (flat), $+1$ (spherical)

- $\frac{a(t_2)}{a(t_1)} =$ factor by which a physical length scale increases between time t_1 and t_2.

An observer at rest with the FLRW spatial coordinates is a **comoving observer**.

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

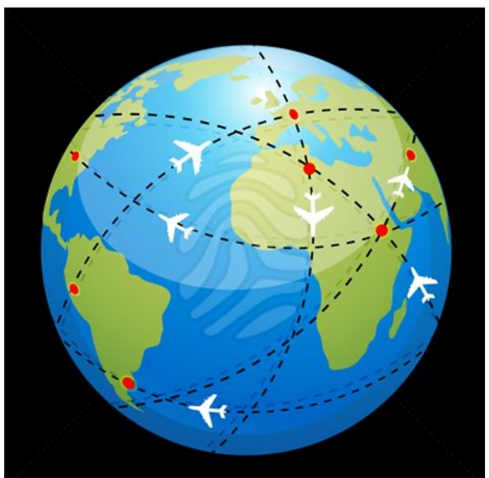


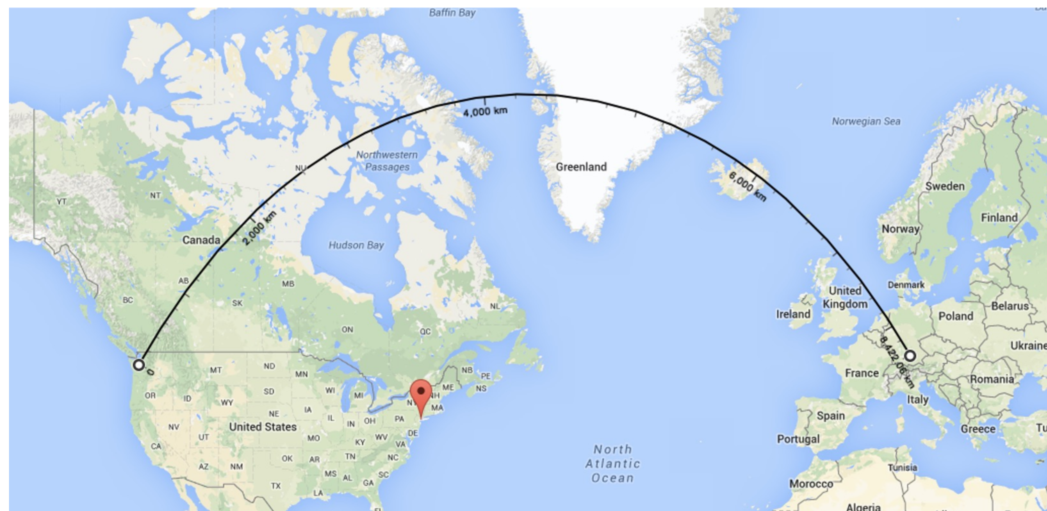
→ The **physical distance** between two comoving observers increases with time, but the coordinate distance between them remains unchanged.

Geodesics…

In the **absence of other forces**, test particles move on the geodesics of a spacetime geometry, i.e., the “straight lines” of a curved spacetime.

- It's like flight paths, which follow (more or less) the geodesics on the surface of the Earth.





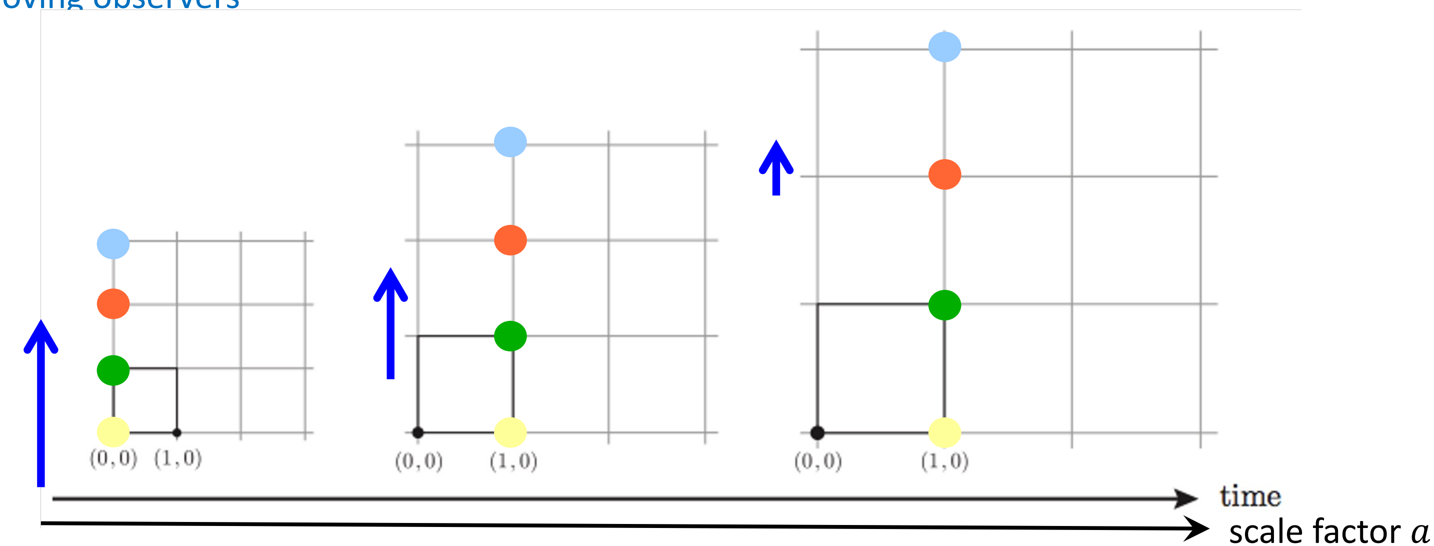
Geodesics and cosmological redshift…

All test particles (massive or massless) moving on geodesics of an FLRW universe suffer cosmological redshift of its momentum:

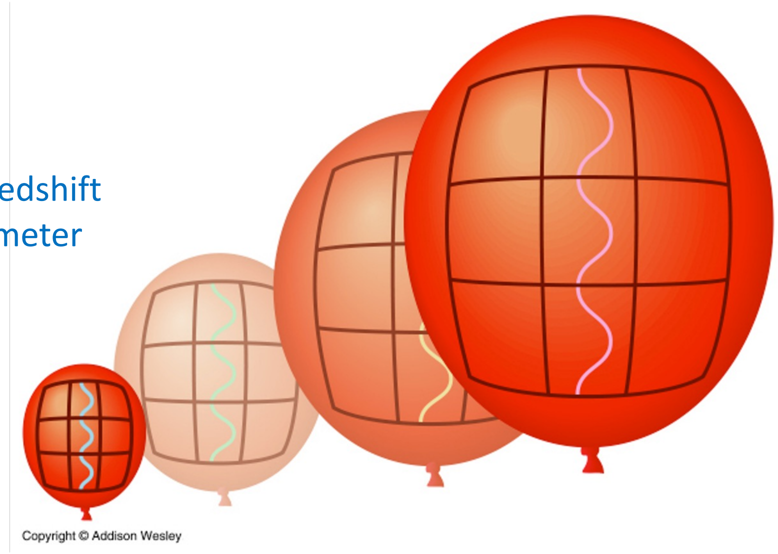
Momentum of a point particle measured by comoving observers

$$|\vec{p}| \propto a^{-1}$$

Momentum of a particle decreases with expanding space.



Or in terms of wavelength: $\lambda \propto 1/|\vec{p}|$



Wavelength measured by comoving observer

t_0= today

z = Redshift parameter

$$\frac{\lambda_0}{\lambda_e} = \frac{a(t_0)}{a(t_e)} \equiv 1 + z$$

Wavelength of particle (usually photon) emitted by comoving emitter

- A particle emitted at a very early time t when the scale factor a was very small would be observed today with a very large redshift z

→ There is a one-to-one correspondence between t, a, and z:

redshift z ←

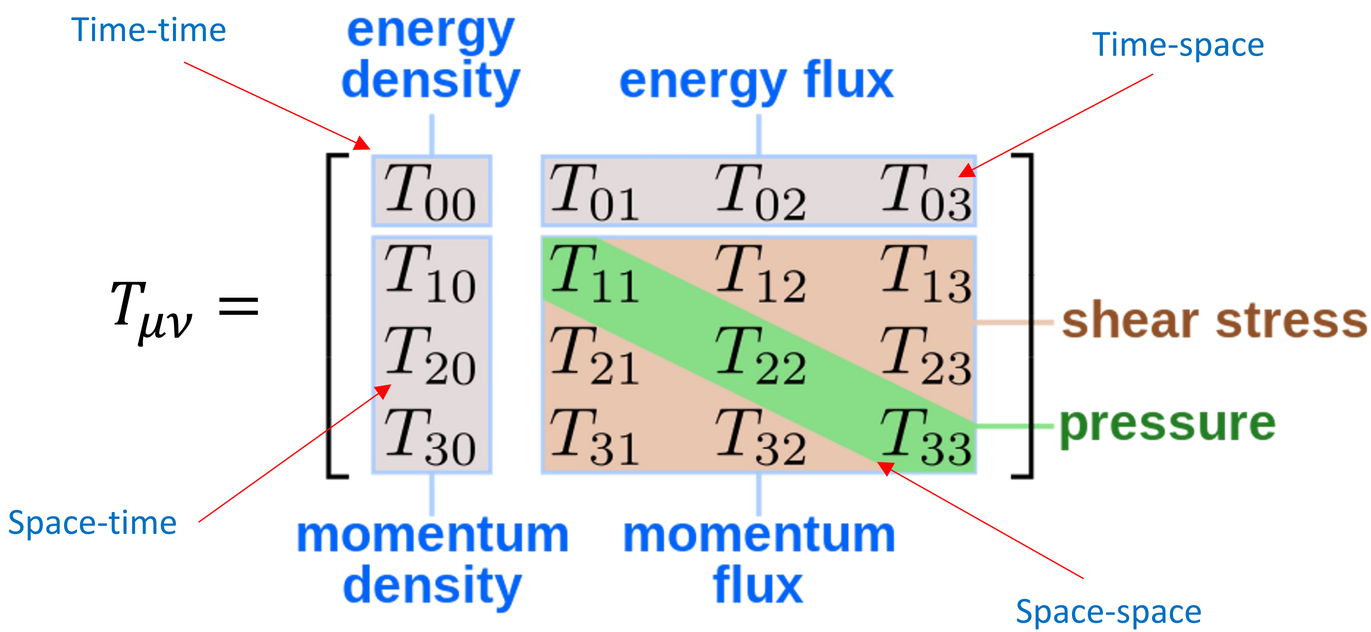
→ time t

→ scale factor a

→ We use them interchangeably as a measure of time.

Matter/energy content (stuff in the universe).

In GR, the stress-energy tensor $T_{\mu\nu}$ encodes the matter/energy content.



Matter/energy content (stuff in the universe).

In GR, the stress-energy tensor $T_{\mu\nu}$ encodes the matter/energy content.

- Homogeneity and isotropy imply **only one viable form**:

ρ_α = Energy density (energy per unit volume) of substance α in its rest frame

$$T^{\mu}{}_{\nu(\alpha)} = \begin{pmatrix} -\rho_\alpha(t) & 0 & 0 & 0 \\ 0 & P_\alpha(t) & 0 & 0 \\ 0 & 0 & P_\alpha(t) & 0 \\ 0 & 0 & 0 & P_\alpha(t) \end{pmatrix}$$

P_α = Pressure of substance α in its rest frame

- $\rho(t)$ and $P(t)$ can depend on time, but **not** on the spatial coordinates.

→ How do they evolve with time?

Matter/energy content: conservation law…

Local conservation of energy-momentum in an FLRW universe implies:

Energy density

Pressure

$$\frac{d\rho_\alpha}{dt} + 3\frac{\dot{a}}{a}(\rho_\alpha + P_\alpha) = 0$$

Continuity equation (from $\nabla_\mu T^{\mu\nu}_{(\alpha)} = 0$)

- There is **one such continuity equation for each substance α**.
- We need in addition to specify a relation between $\rho(t)$ and $P(t)$, i.e., the **equation of state** of the substance α, which is a property of the substance.
  - It's common to use an equation of state parameter w: $w_\alpha(t) \equiv \dfrac{P_\alpha(t)}{\rho_\alpha(t)}$
  - Assuming a constant w: $\rho_\alpha(t) \propto a^{-3(1+w_\alpha)}$ → How energy density evolves with the scale factor.

Matter/energy content: what's there?

$$\rho_\alpha(t) \propto a^{-3(1+w_\alpha)}$$

- **Non-relativistic** matter
  - Atoms (or constituents thereof)
  - Dark matter (does not emit light but feels gravity); GR people call it "dust"

$w_m \simeq 0$

$\Rightarrow \rho_m \propto a^{-3}$

Volume expansion

- **Ultra-relativistic** radiation
  - Photons (main the CMB)
  - Relic neutrinos (at early times at least)
  - Gravitational waves

$w_r = 1/3$

$\Rightarrow \rho_r \propto a^{-4}$

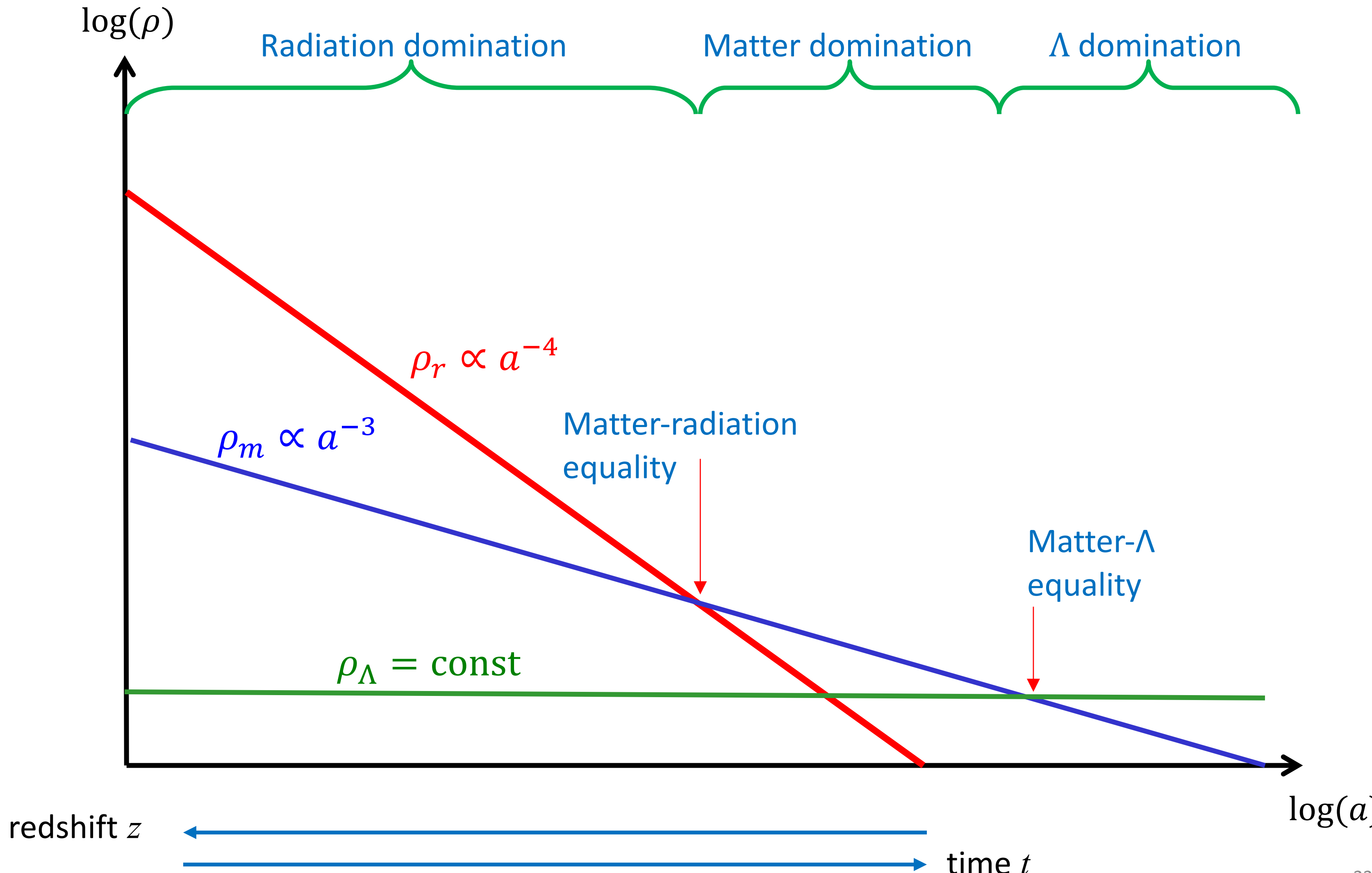
Volume expansion
+ momentum redshift

- **Other** funny things
  - Cosmological constant/vacuum energy
  - ??

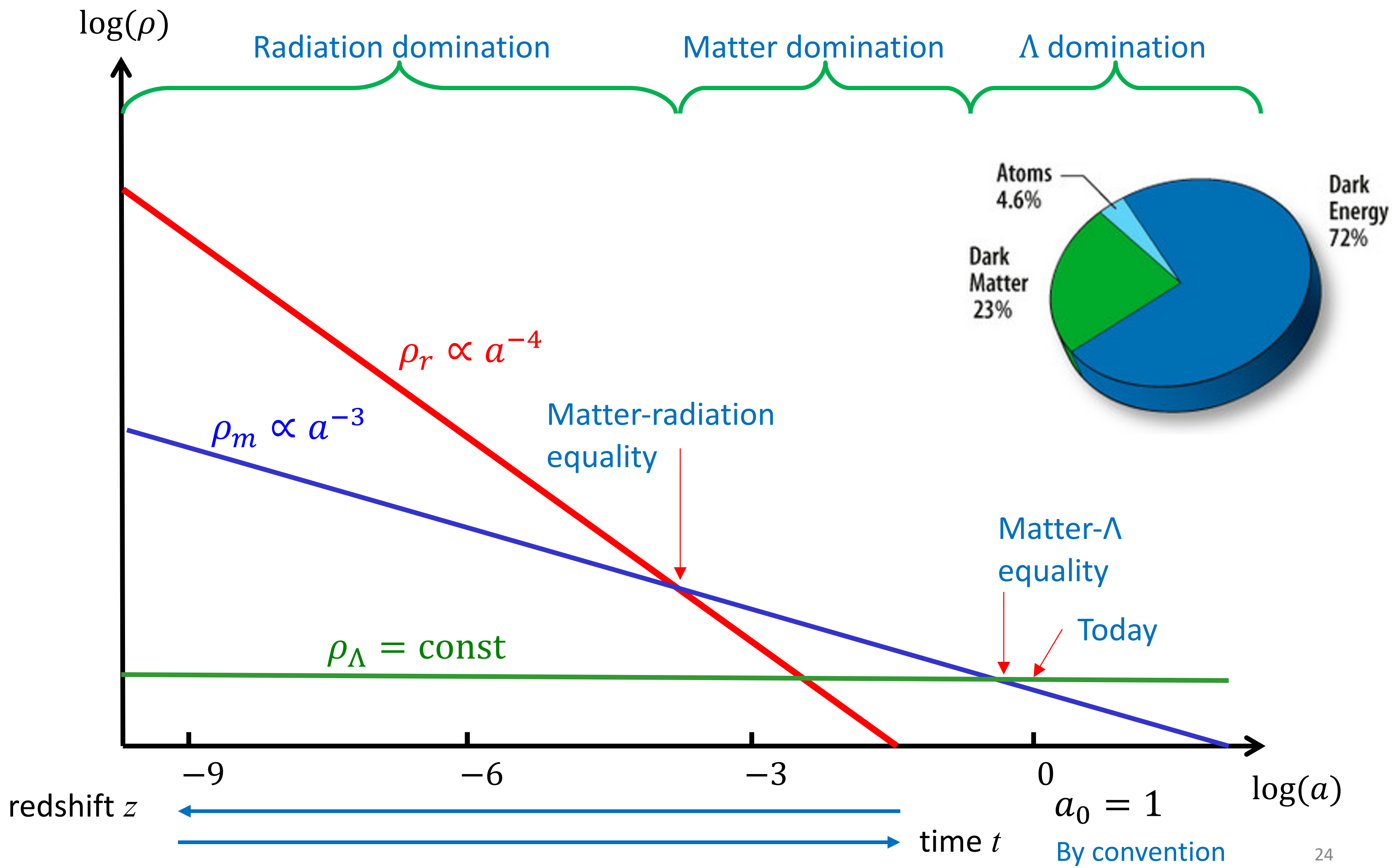
$w_\Lambda = -1$

$\Rightarrow \rho_\Lambda \propto \text{constant}$

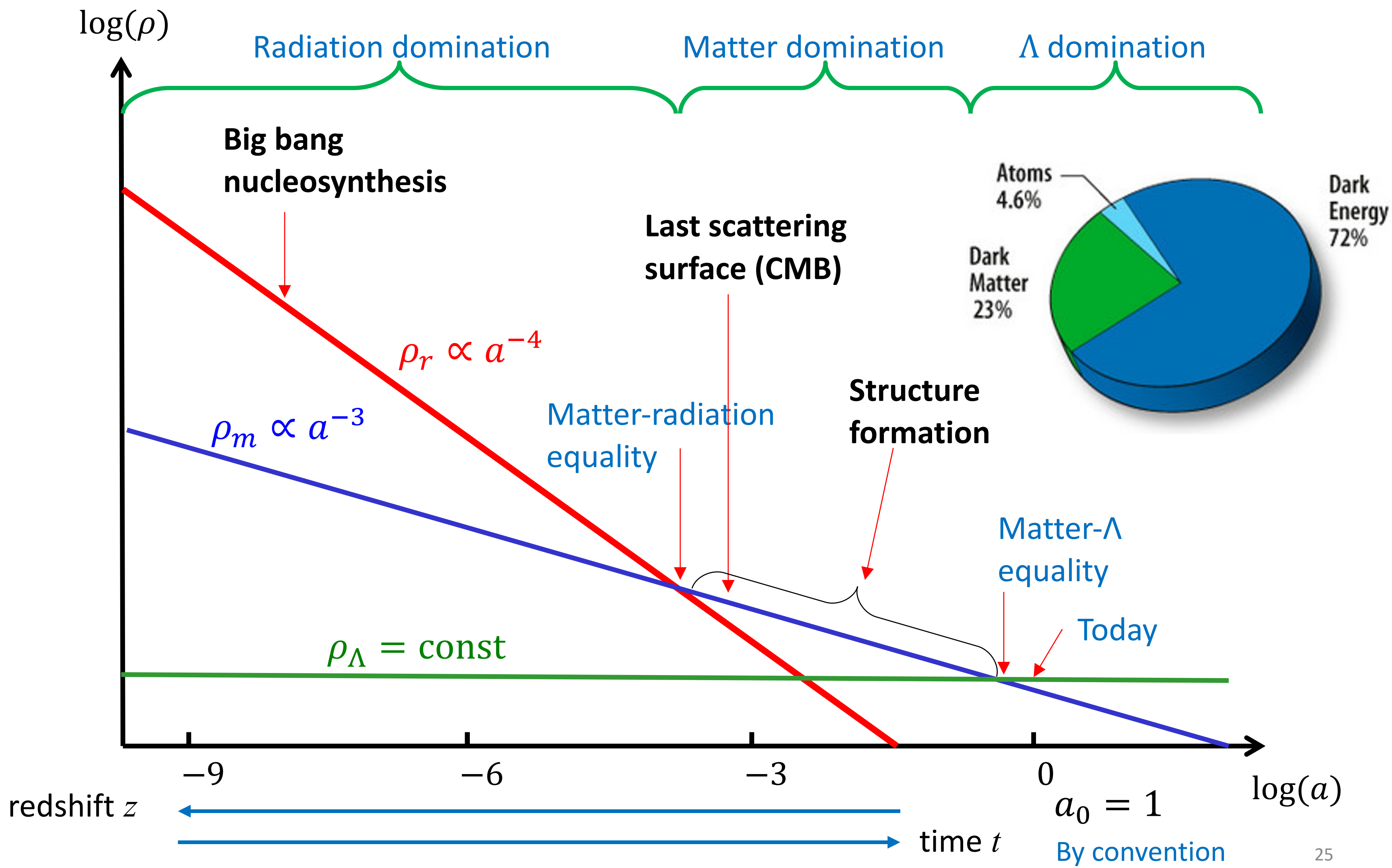
More space,
more energy



$\log(\rho)$
Radiation domination
Matter domination
Λ domination
$\rho_r \propto a^{-4}$
$\rho_m \propto a^{-3}$
Matter-radiation equality
Matter-Λ equality
$\rho_\Lambda = \text{const}$
$\log(a)$
redshift z
time t

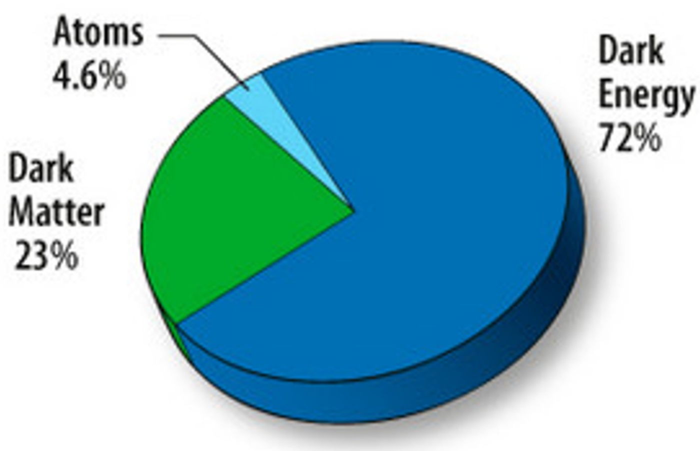


$\log(\rho)$
Radiation domination
Matter domination
Λ domination
Atoms 4.6%
Dark Energy 72%
Dark Matter 23%
$\rho_r \propto a^{-4}$
$\rho_m \propto a^{-3}$
Matter-radiation equality
Matter-Λ equality
Today
$\rho_\Lambda = \text{const}$
-9
-6
-3
0
$\log(a)$
redshift z
time t
$a_0 = 1$
By convention

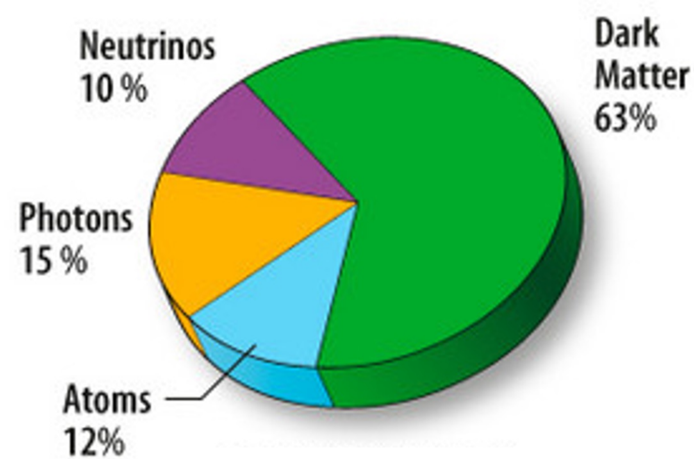


$\log(\rho)$
Radiation domination
Matter domination
Λ domination
Big bang nucleosynthesis
Last scattering surface (CMB)
Atoms 4.6%
Dark Energy 72%
Dark Matter 23%
$\rho_r \propto a^{-4}$
$\rho_m \propto a^{-3}$
Matter-radiation equality
Structure formation
Matter-Λ equality
Today
$\rho_\Lambda = \text{const}$
-9
-6
-3
0
$\log(a)$
redshift z
$a_0 = 1$
time t
By convention

Different evolution for different forms of energy densities means that radiation dominated in the early universe, while dark energy was unimportant.



Composition today



13.4 billion years ago
(at photon decoupling)

Friedmann equation...

The Friedmann equation describes the evolution of the scale factor $a(t)$.

G = Gravitational constant

$$H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \sum_\alpha \rho_\alpha - \frac{K}{a^2}$$

$H(t)$ = Hubble parameter

Spatial curvature: $K = 0, +1, -1$

Some over all forms of energy density

- The Friedman equation is itself derived from Einstein's equation:

R = Ricci scalar and tensor (nonlinear functions of the 2nd derivative of the spacetime metric)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

Stress-energy tensor

Friedmann equation...

You may also have seen the Friedmann equation in this form:

$$H^2(t) = H^2(t_0)[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_\Lambda + \Omega_K a^{-2}]$$

Present-day reduced energy density

$$\Omega_\alpha = \frac{\bar{\rho}_\alpha(t_0)}{\rho_{\rm crit}(t_0)}, \qquad \rho_{\rm crit}(t) \equiv \frac{3H^2(t)}{8\pi G}, \qquad \Omega_K \equiv -\frac{K}{H^2(t_0)}$$

Critical density

- A **flat universe** means

$$\Omega_K = 0 \quad \Rightarrow \quad \Omega_m + \Omega_r + \Omega_\Lambda \simeq \Omega_m + \Omega_\Lambda = 1$$

Radiation energy density is negligibly small today:

From measuring the CMB temperature a and energy spectrum:

$$\Omega_r \sim 10^{-5}$$

Friedmann equation…

You may also have seen the Friedmann equation in this form:

$$H^2(t) = H^2(t_0)[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_\Lambda + \Omega_K a^{-2}]$$

Present-day reduced energy density

$$\Omega_\alpha = \frac{\bar{\rho}_\alpha(t_0)}{\rho_{\rm crit}(t_0)}, \qquad \rho_{\rm crit}(t) \equiv \frac{3H^2(t)}{8\pi G}, \qquad \Omega_K \equiv -\frac{K}{H^2(t_0)}$$

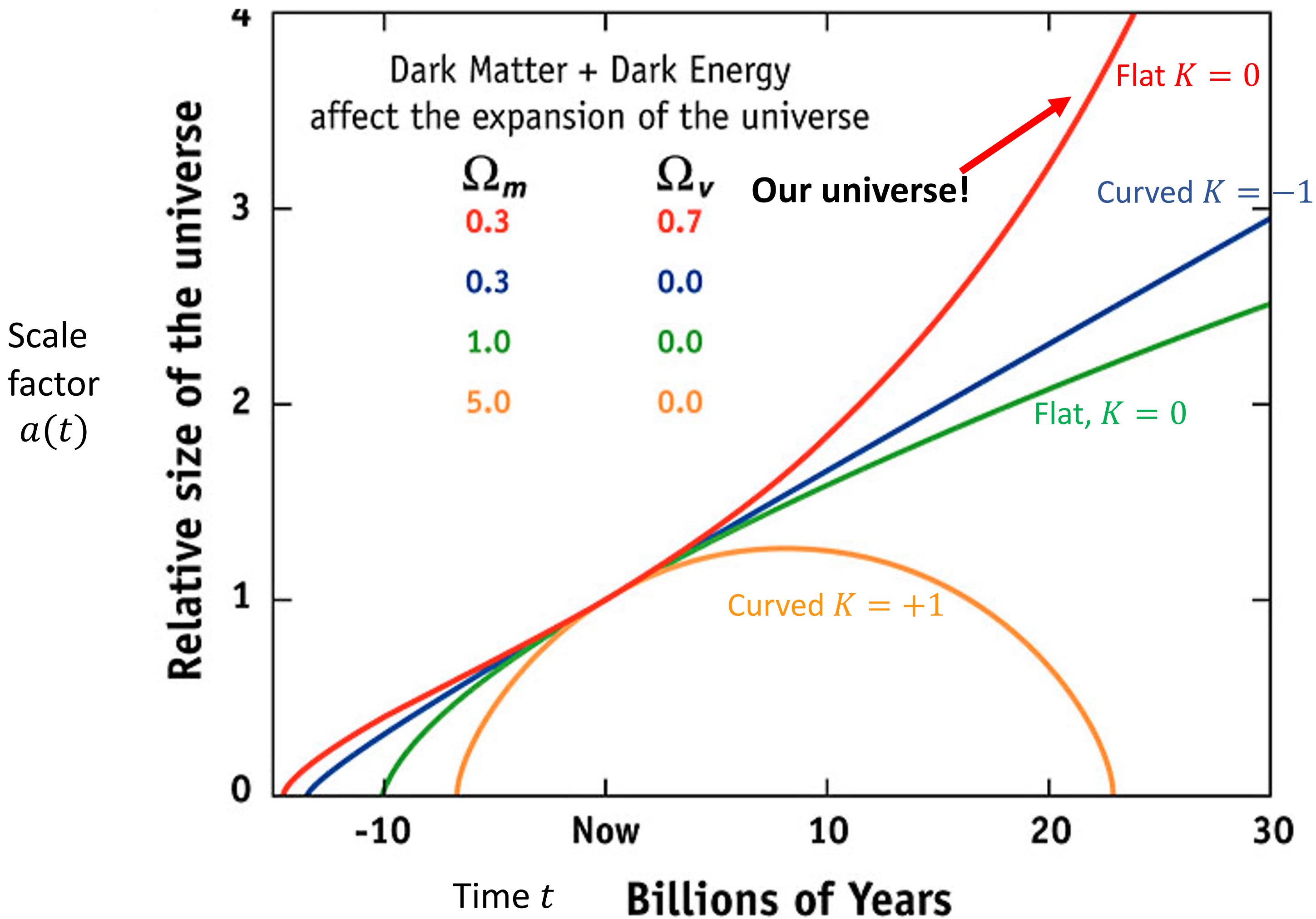
Critical density

- From **current observations**:

$$\Omega_m \sim 0.3, \qquad \Omega_\Lambda \sim 0.7, \qquad |\Omega_K| < 0.01$$

$$H_0 \equiv H(t_0) \sim 70 \ \mathrm{kms^{-1}Mpc^{-1}}$$

e.g., Aghanim et al. [Planck collaboration] 2019


Dark Matter + Dark Energy
affect the expansion of the universe
Ω_m Ω_v
0.3 0.7
0.3 0.0
1.0 0.0
5.0 0.0
Our universe!
Flat $K = 0$
Curved $K = -1$
Flat, $K = 0$
Curved $K = +1$
Relative size of the universe
Scale factor $a(t)$
0 1 2 3 4
-10 Now 10 20 30
Time t
Billions of Years

Friedmann equation: accelerated expansion…

Yet another form of the Friedmann equation:

Obtained by combining the usual Friedmann equation for $H(t)$ and the continuity equation.

Acceleration of the scale factor

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_{\alpha}(\rho_\alpha + 3P_\alpha)$$

Compare with

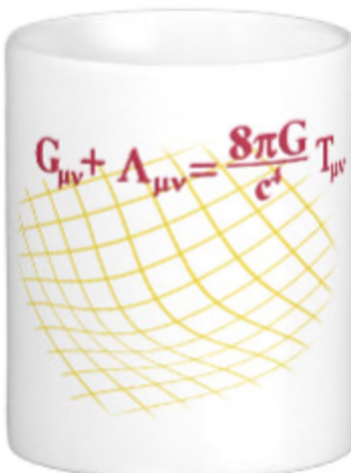
$$H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\sum_{\alpha}\rho_\alpha - \frac{K}{a^2}$$

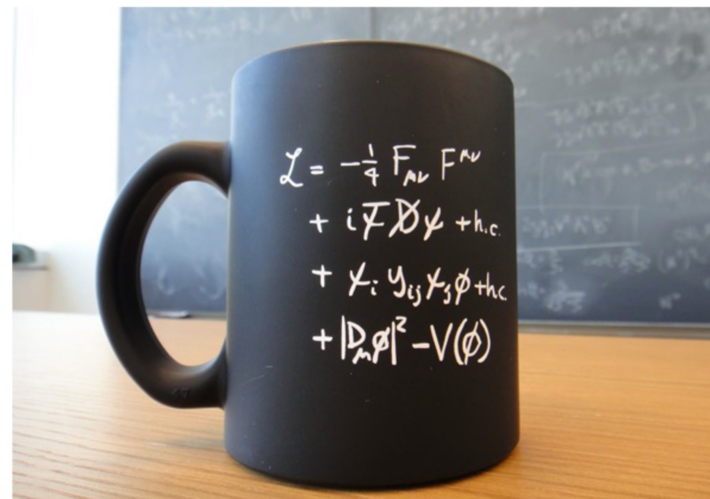
- **Accelerated or decelerated expansion** happens when:

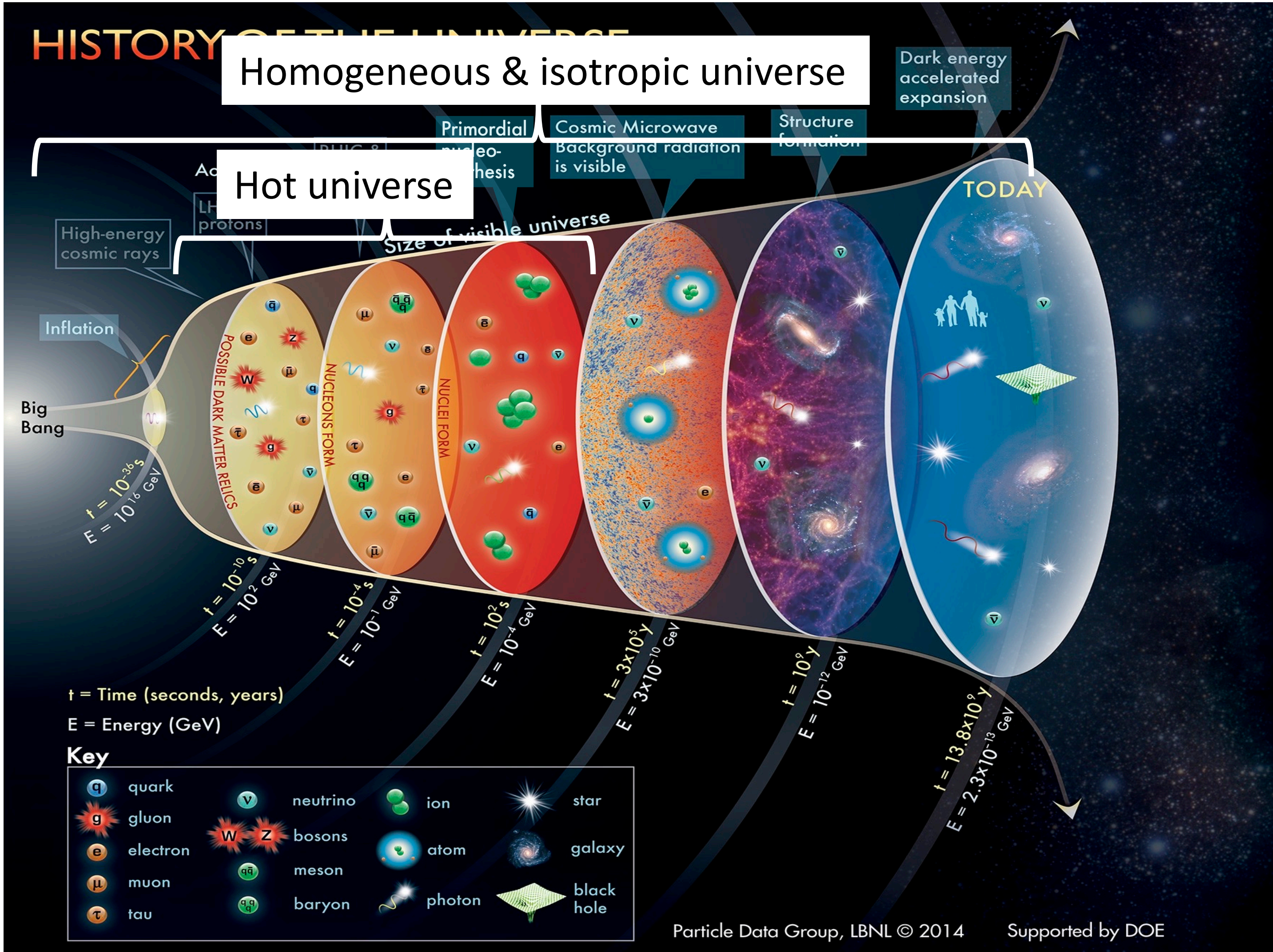
Acceleration $\sum_{\alpha}(\rho_\alpha + 3P_\alpha) < 0 \Rightarrow w_{\text{eff}} = \frac{\sum_\alpha P_\alpha}{\sum_\alpha \rho_\alpha} < -\frac{1}{3}$

Deceleration $\sum_{\alpha}(\rho_\alpha + 3P_\alpha) > 0 \Rightarrow w_{\text{eff}} = \frac{\sum_\alpha P_\alpha}{\sum_\alpha \rho_\alpha} > -\frac{1}{3}$

2. The hot universe and the cosmic neutrino background...







Homogeneous & isotropic universe
Hot universe
Dark energy accelerated expansion
Cosmic Microwave Background radiation is visible
Primordial
High-energy cosmic rays
Inflation
Big Bang
TODAY
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
$t = 10^{-36}$ s
$E = 10^{16}$ GeV
$t = 10^{-10}$ s
$E = 10^{2}$ GeV
$t = 10^{-4}$ s
$E = 10^{-1}$ GeV
$t = 10^{2}$ s
$E = 10^{-4}$ GeV
$t = 3\times10^{5}$ y
$E = 3\times10^{-10}$ GeV
$t = 10^{9}$ y
$E = 10^{-12}$ GeV
$t = 13.8\times10^{9}$ y
$E = 2.3\times10^{-13}$ GeV
t = Time (seconds, years)
E = Energy (GeV)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
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Supported by DOE

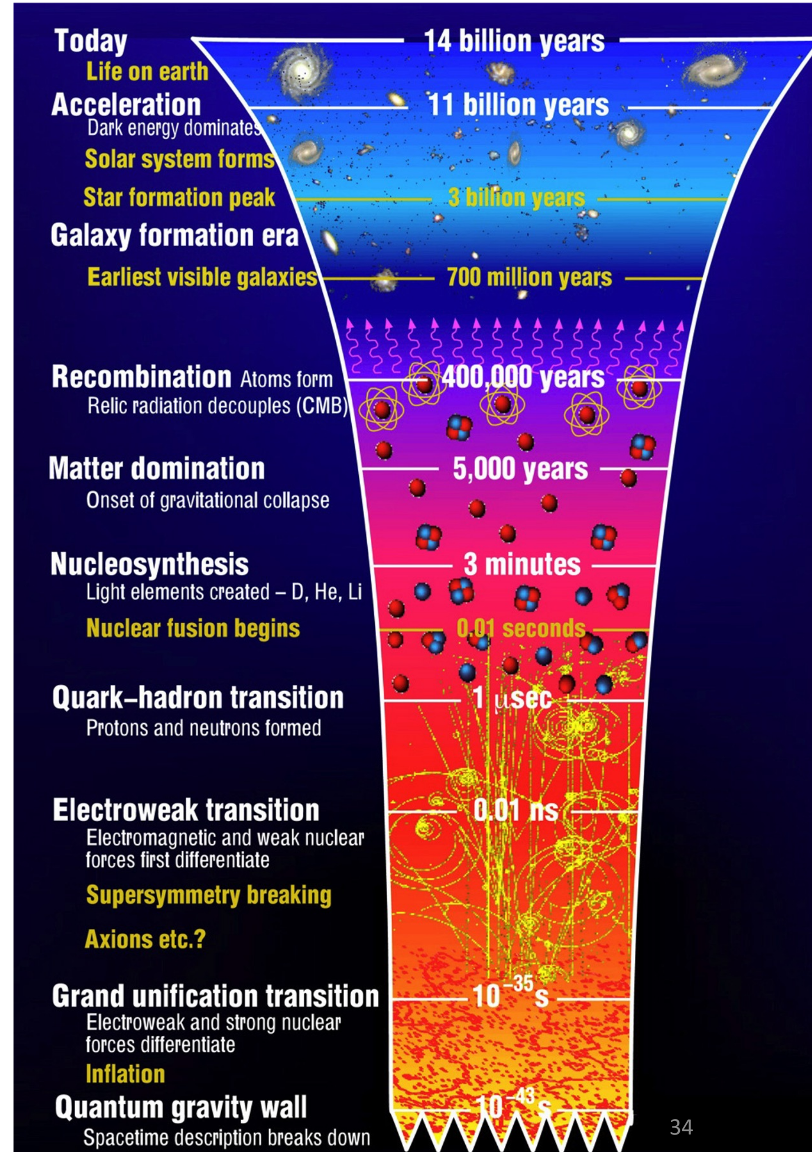
The hot universe…

The early universe was a **very hot and dense place**.

- Particle interactions (e.g., scattering) can happen very frequently.
- What interactions are available depends on the particle physics theory.
- But if an interaction rate (per particle) far exceeds the Hubble expansion rate,

$$\Gamma_{\rm int} \gg H$$

the interaction can be taken to be in a **state of equilibrium**.



Classic example: weak interaction...

Also the most relevant example for neutrinos!

Say you have a gas of ultra-relativistic particles with **temperature** T.

- The Weak interaction rate per particle is estimated to be

$$\Gamma_{\text{int}} = n\langle\sigma v\rangle \sim G_F^2 T^5$$

Fermi constant

Number density of scattering centres $n \sim T^3$

Relative velocity $v \sim 1$

Cross-section $\sigma \sim G_F^2 T^2$

- The Hubble expansion rate is

$$H = \sqrt{\frac{8\pi G}{3}\sum_{\alpha}\rho_\alpha} \sim \frac{T^2}{m_{\text{planck}}}$$

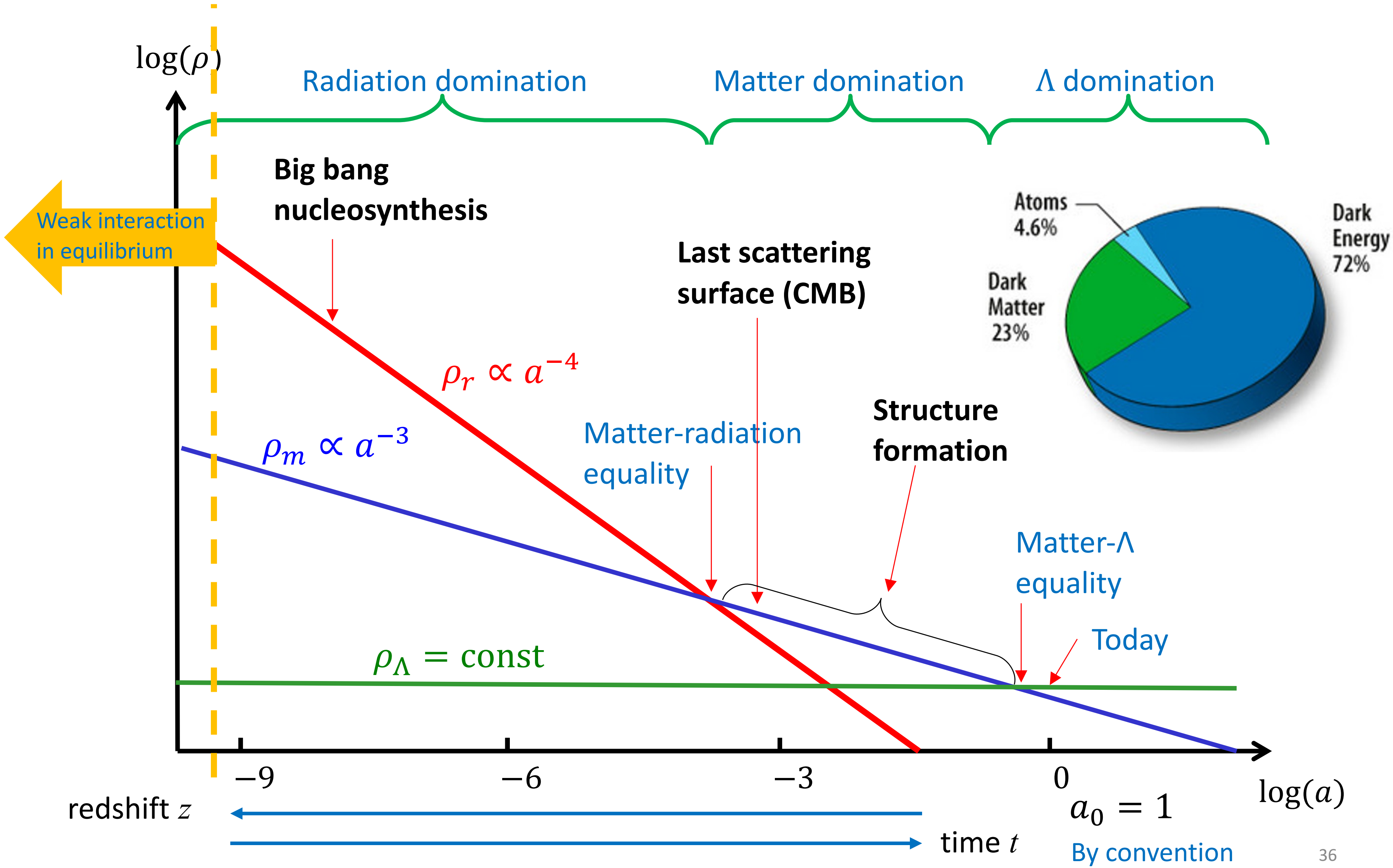
Planck mass

$$G_F \sim 10^{-5}\ \text{GeV}^{-2}$$
$$m_{\text{pl}} \sim 10^{19}\ \text{GeV}$$

Ratio increases with temperature

$$\frac{\Gamma_{\text{int}}}{H} \sim m_{\text{planck}} G_F^2 T^3 \sim \left(\frac{T}{1\ \text{MeV}}\right)^3$$

Weak interactions are in equilibrium at $T \gg 1$ MeV.



log(ρ)
Radiation domination
Matter domination
Λ domination
Big bang
nucleosynthesis
Weak interaction
in equilibrium
Last scattering
surface (CMB)
Atoms
4.6%
Dark
Energy
72%
Dark
Matter
23%
$\rho_r \propto a^{-4}$
$\rho_m \propto a^{-3}$
Matter-radiation
equality
Structure
formation
Matter-Λ
equality
Today
$\rho_\Lambda = \text{const}$
−9
−6
−3
0
log(a)
redshift z
$a_0 = 1$
time t
By convention

Equilibrium thermodynamics...

In the ideal gas limit, when an interaction is in equilibrium, all participating particles have phase space distributions described by one of the equilibrium forms:

$f(p)$ = Phase space distribution

$$f_{\mathrm{eq}}(p) = \frac{1}{\exp[(E(p) - \mu)/T] \pm 1}$$

$+$ Fermi-Dirac
$-$ Bose-Einstein

μ = Chemical potential

T = Temperature

- All participating particles in that interaction have the same temperature T.
- Their chemical potentials satisfy $\sum_{\mathrm{initial}} \mu_i = \sum_{\mathrm{final}} \mu_i$.
- In standard cosmology μ is generally related to the $\sim 10^{-10}$ matter-antimatter asymmetry; for most applications, it suffices to set $\mu = 0$.

Equilibrium thermodynamics…

Given its phase space distribution $f(p)$, it is straightforward to find a particle species' bulk properties:

Number density: $$n_\alpha = \frac{g_\alpha}{(2\pi)^3}\int d^3p\, f_\alpha(\vec{p})$$

Internal d.o.f.

Energy density: $$\boxed{\rho_\alpha = \frac{g_\alpha}{(2\pi)^3}\int d^3p\, E f_\alpha(\vec{p})} \left\{ \begin{array}{ll} \sim T^4 & \text{Ultra-relativistic } T \gg m \\ \sim m(mT)^{3/2}e^{-m/T} & \text{Non-relativistic } T \ll m \end{array} \right.$$

The energy density of a non-relativistic particle species is highly suppressed!

Pressure: $$P_\alpha = \frac{g_\alpha}{(2\pi)^3}\int d^3p\, \frac{|\vec{p}|^2}{3E} f_\alpha(\vec{p})$$

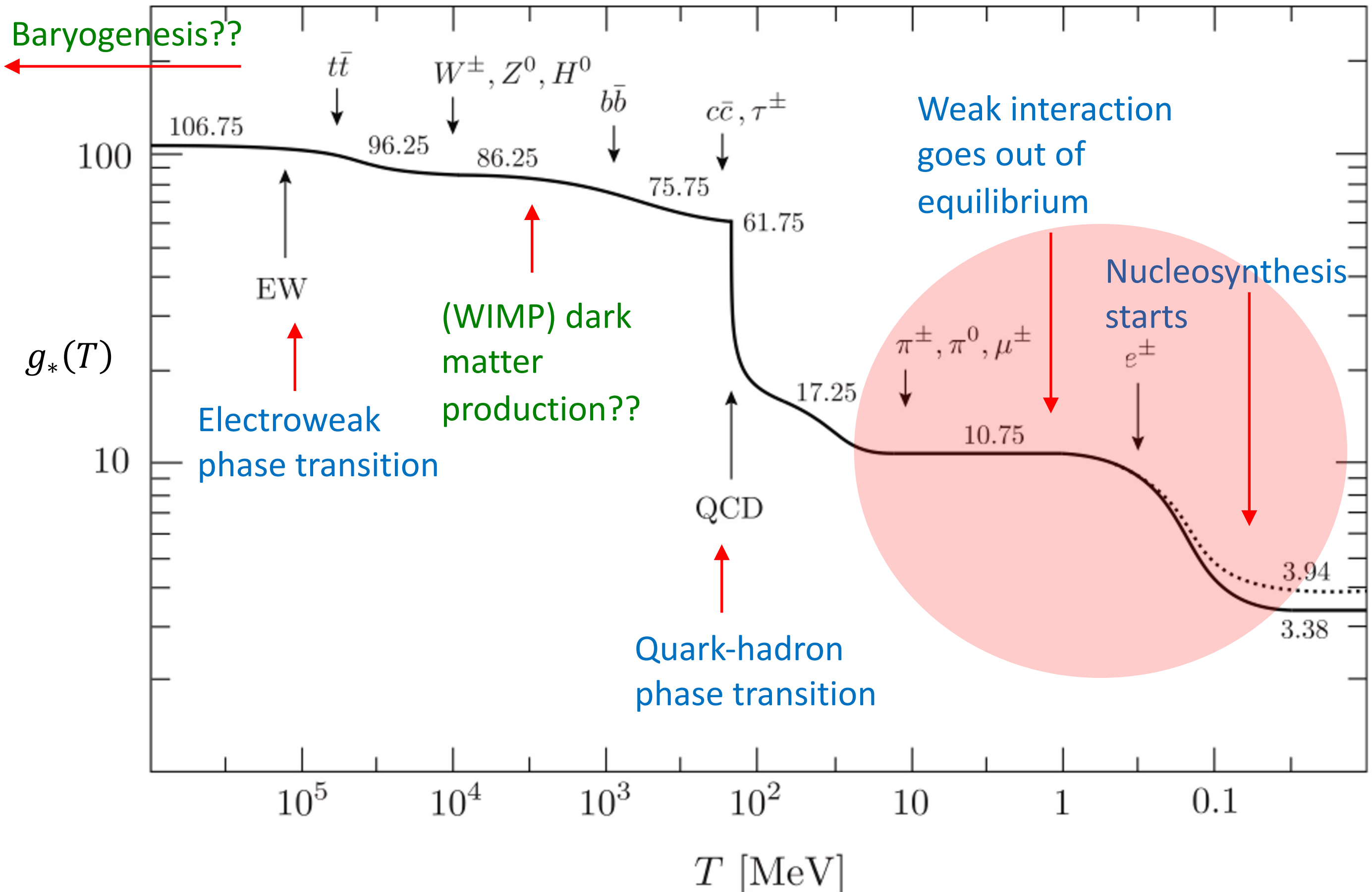
- We can therefore express the Hubble expansion rate in the early universe as:

$$\boxed{H^2(t) = \frac{8\pi G}{3}\sum_\alpha \rho_\alpha \equiv \frac{8\pi G}{3}\frac{\pi^2}{30} g_*(T_\gamma) T_\gamma^4}$$

Photon temperature

g_* is a temperature-dependent function, dominated by relativistic species, specific to a particle physics theory.

g_* of the standard model of particle physics:



Photon temperature

Time

What's left?

Mainly

- Photons
- Neutrinos

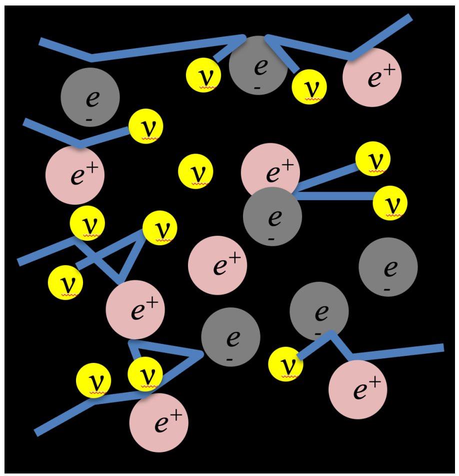
Small amounts* of

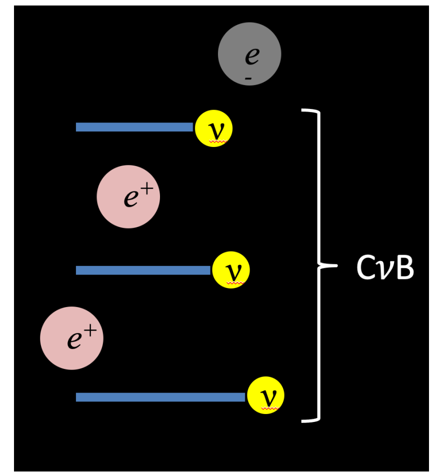
- Electrons
- Nucleons
- Nuclei

* Small means $< 10^{-9} n_\gamma$

Cosmic neutrino background …

The CνB is formed when neutrinos decouple from the cosmic plasma.



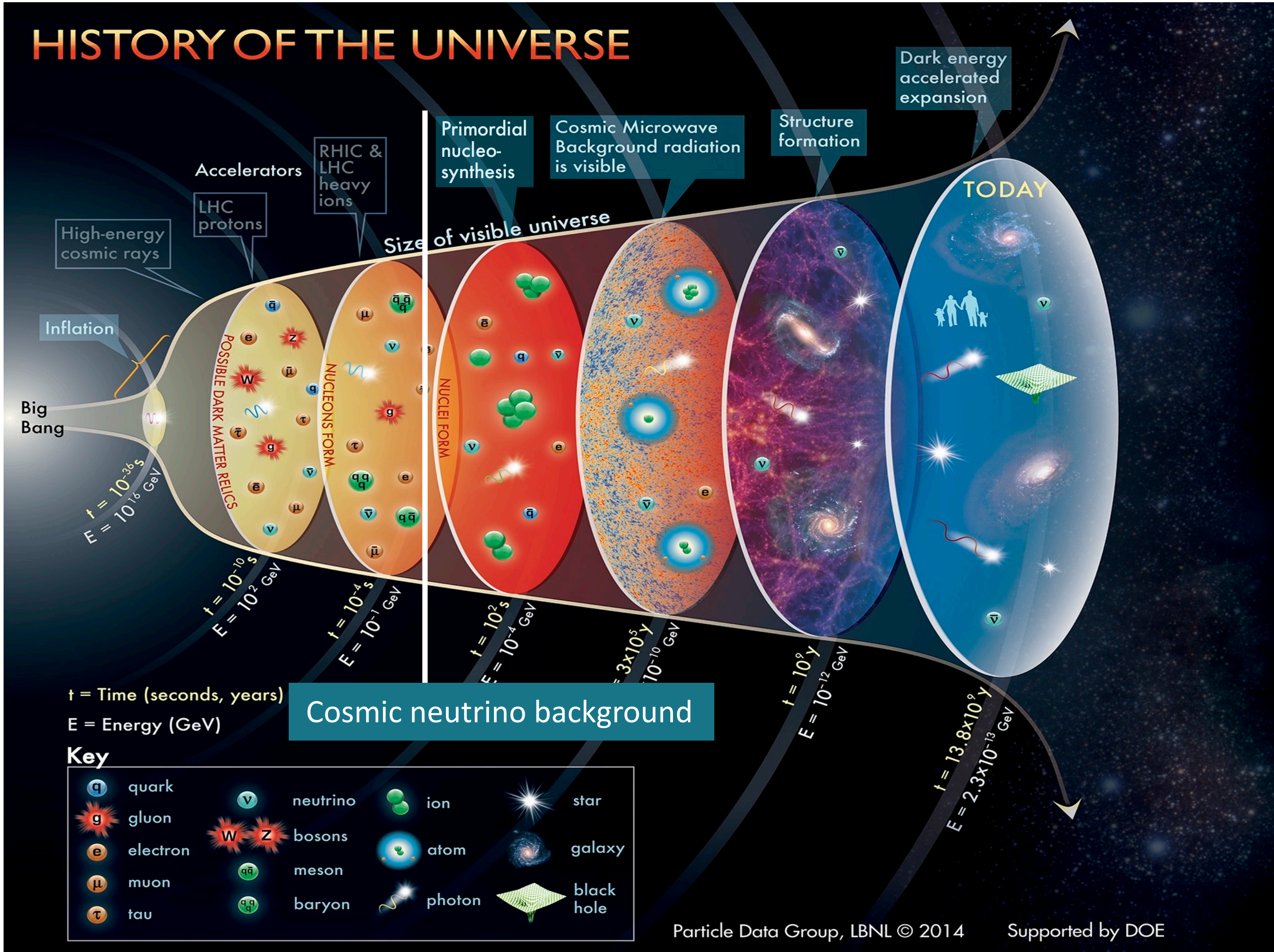


Neutrinos "free-stream" to infinity.

($T_{\odot\text{core}} \sim 1\ \text{keV}$)

Above $T \sim 1\ \text{MeV}$, even weakly-interacting neutrinos can be produced, scatter off e^+e^- and other neutrinos, and attain thermodynamic equilibrium

Below $T \sim 1\ \text{MeV}$, expansion dilutes plasma, and reduces interaction rate: the universe becomes transparent to neutrinos.



HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Primordial nucleo-synthesis
Cosmic Microwave Background radiation is visible
Structure formation
Accelerators
RHIC & LHC heavy ions
LHC protons
High-energy cosmic rays
Size of visible universe
TODAY
Inflation
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
$t = 10^{-36}$ s
$E = 10^{16}$ GeV
$t = 10^{-10}$ s
$E = 10^{2}$ GeV
$t = 10^{-4}$ s
$E = 10^{-1}$ GeV
$t = 10^{2}$ s
$E = 10^{-4}$ GeV
$t = 3\times10^{5}$ y
$E = 10^{-10}$ GeV
$t = 10^{9}$ y
$E = 10^{-12}$ GeV
$t = 13.8\times10^{9}$ y
$E = 2.3\times10^{-13}$ GeV
t = Time (seconds, years)
E = Energy (GeV)
Cosmic neutrino background
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL © 2014
Supported by DOE

Particle content at $0.1 < T < 10$ MeV…

The particle content and interactions at $0.1 < T < 10$ MeV determine the properties of the CνB.

- **QED plasma**: $e^{\pm}, \gamma$
- **3 families of $\boldsymbol{\nu} + \bar{\boldsymbol{\nu}}$**: $\nu_e, \bar{\nu}_e,$ $\nu_\mu, \bar{\nu}_\mu,$ $\nu_\tau, \bar{\nu}_\tau$

EM interactions (always in equilibrium @ $0.1 < T < 10$ MeV):

$$e^+e^- \leftrightarrow \gamma\gamma$$
$$e^+e^- \leftrightarrow e^+e^-$$
$$e^{\pm}e^{\mp} \leftrightarrow e^{\pm}e^{\mp}$$
$$e^{\pm}e^{\pm} \leftrightarrow e^{\pm}e^{\pm}$$
$$\gamma e^{\pm} \leftrightarrow \gamma e^{\pm}$$

Coupled @ $T > O(1)\mathrm{MeV}$

$$\nu_\alpha e^{\pm} \leftrightarrow \nu_\alpha e^{\pm}$$
$$\nu_\alpha \bar{\nu}_\alpha \leftrightarrow e^+e^-$$

Weak interactions (in equilibrium @ $T > O(1)\mathrm{MeV}$)

Weak interactions (in equilibrium @ $T > O(1)\mathrm{MeV}$):

$$\nu_\alpha \nu_\beta \leftrightarrow \nu_\alpha \nu_\beta$$
$$\nu_\alpha \bar{\nu}_\beta \leftrightarrow \nu_\alpha \bar{\nu}_\beta \qquad \alpha, \beta = e, \mu, \tau$$
$$\bar{\nu}_\alpha \bar{\nu}_\beta \leftrightarrow \bar{\nu}_\alpha \bar{\nu}_\beta$$

Particle content at $0.1 < T < 10$ MeV…

The particle content and interactions at $0.1 < T < 10$ MeV determine the properties of the CνB.

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$$e^{\pm}e^{\mp} \leftrightarrow e^{\pm}e^{\mp}$$
$$e^{\pm}e^{\pm} \leftrightarrow e^{\pm}e^{\pm}$$
$$\gamma e^{\pm} \leftrightarrow \gamma e^{\pm}$$

Decoupled @ $T \ll O(1)$MeV

$$\cancel{\nu_\alpha e^{\pm} \leftrightarrow \nu_\alpha e^{\pm}}$$
$$\cancel{\nu_\alpha \bar{\nu}_\alpha \leftrightarrow e^+e^-}$$

Weak interactions (**not** in equilibrium @ $T \ll O(1)$MeV)

Weak interactions (**not** in equilibrium @ $T \ll O(1)$MeV):

$$\cancel{\nu_\alpha \nu_\beta \leftrightarrow \nu_\alpha \nu_\beta}$$
$$\cancel{\nu_\alpha \bar{\nu}_\beta \leftrightarrow \nu_\alpha \bar{\nu}_\beta}$$
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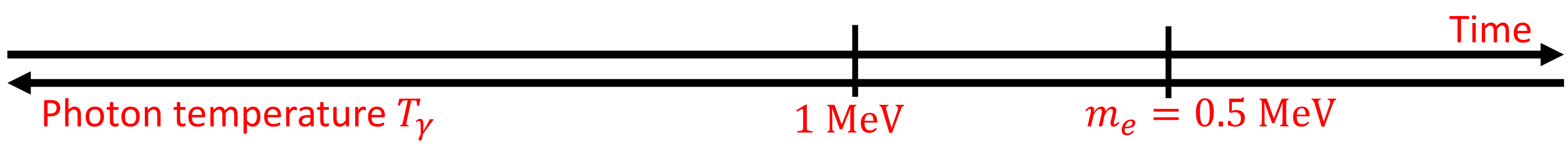
$\alpha, \beta = e, \mu, \tau$

Thermal history of neutrinos…

Events



Neutrino temperature

Phase space distribution

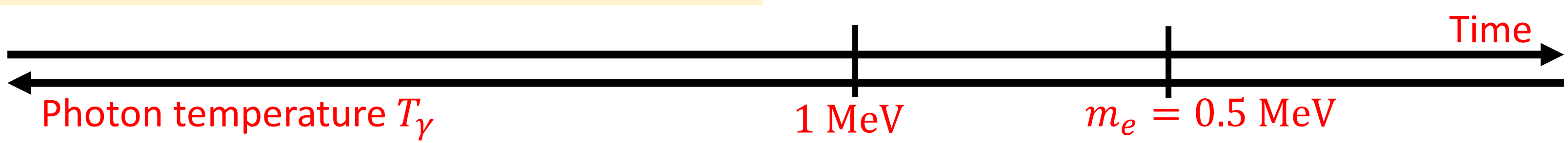
Thermal history of neutrinos...

Events	Neutrinos coupled to QED plasma

Weak interaction: $\Gamma_{\text{int}} \sim G_F^2 T^5$

Expansion: $H \sim \dfrac{T^2}{m_{\text{planck}}}$

$\Rightarrow \Gamma_{\text{int}} > H$



Neutrino temperature	$T_\nu = T_\gamma$	
Phase space distribution	$f_\nu(p) \approx \dfrac{1}{\exp[p/T_\nu] + 1}$	Relativistic Fermi-Dirac

Thermal history of neutrinos...

Events

Neutrinos coupled to QED plasma | Neutrinos decoupling

Weak interaction: $\Gamma_{\text{int}} \sim G_F^2 T^5$

Expansion: $H \sim \dfrac{T^2}{m_{\text{planck}}}$

$\Big\} \; \Gamma_{\text{int}} > H$

$\Gamma_{\text{int}} \sim H$

$\Gamma_{\text{int}} < H \rightarrow$ Neutrinos/QED decoupled

Time →

← Photon temperature T_γ

$1\ \text{MeV}$ | $m_e = 0.5\ \text{MeV}$

Neutrino temperature

$$T_\nu = T_\gamma$$

Phase space distribution

$$f_\nu(p) \approx \frac{1}{\exp[p/T_\nu] + 1}$$

Relativistic Fermi-Dirac

Thermal history of neutrinos...

Events	Neutrinos coupled to QED plasma	Neutrinos decoupling	e^+e^--annihilation	Kinematically forbidden $e^+e^- \leftarrow \gamma\gamma$; $e^+e^- \rightarrow \gamma\gamma$

Weak interaction: $\Gamma_{\text{int}} \sim G_F^2 T^5$

Expansion: $H \sim \dfrac{T^2}{m_{\text{planck}}}$

$\Big\}\ \Gamma_{\text{int}} > H$

$\Gamma_{\text{int}} \sim H$

$\Gamma_{\text{int}} < H$ → Neutrinos/QED decoupled

Time →

← Photon temperature T_γ — $1\ \text{MeV}$ — $m_e = 0.5\ \text{MeV}$

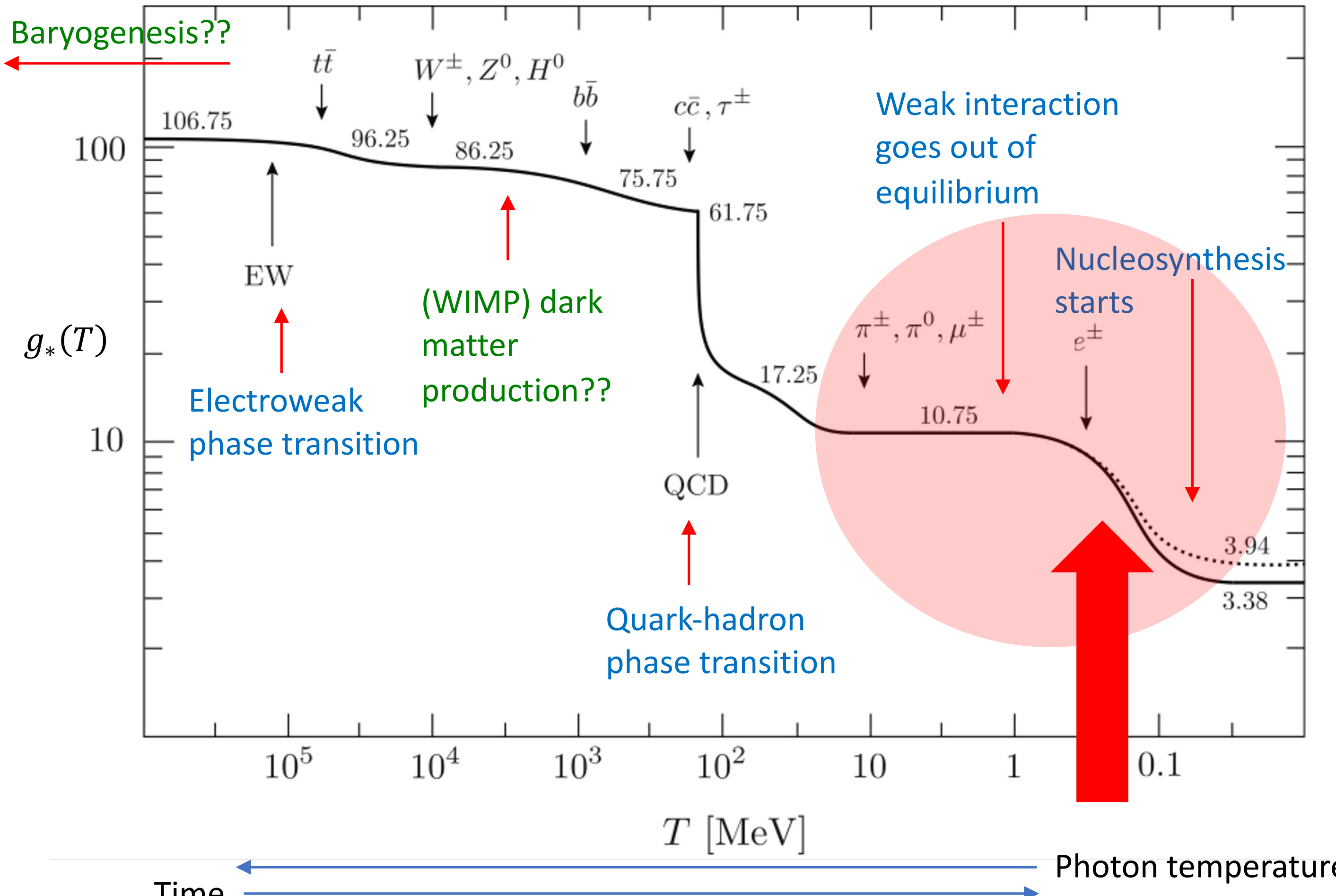
Neutrino temperature	$T_\nu = T_\gamma$	$T_\nu = \left(\dfrac{4}{11}\right)^{1/3} T_\gamma$

Phase space distribution

$$f_\nu(p) \approx \frac{1}{\exp[p/T_\nu] + 1}$$

Relativistic Fermi-Dirac

g_* of the standard model of particle physics:



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Thermal history of neutrinos…

Events

Neutrinos coupled to QED plasma

Weak interaction: $\Gamma_{\text{int}} \sim G_F^2 T^5$

Expansion: $H \sim \dfrac{T^2}{m_{\text{planck}}}$

$\Gamma_{\text{int}} > H$

Neutrinos decoupling

$\Gamma_{\text{int}} \sim H$

e^+e^--annihilation

Kinematically forbidden

$\cancel{e^+e^- \leftarrow \gamma\gamma}$

$e^+e^- \rightarrow \gamma\gamma$

$\Gamma_{\text{int}} < H$ → Neutrinos/QED decoupled

Time

Photon temperature T_γ

1 MeV

$m_e = 0.5$ MeV

Neutrino temperature

$$T_\nu = T_\gamma$$

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

Phase space distribution

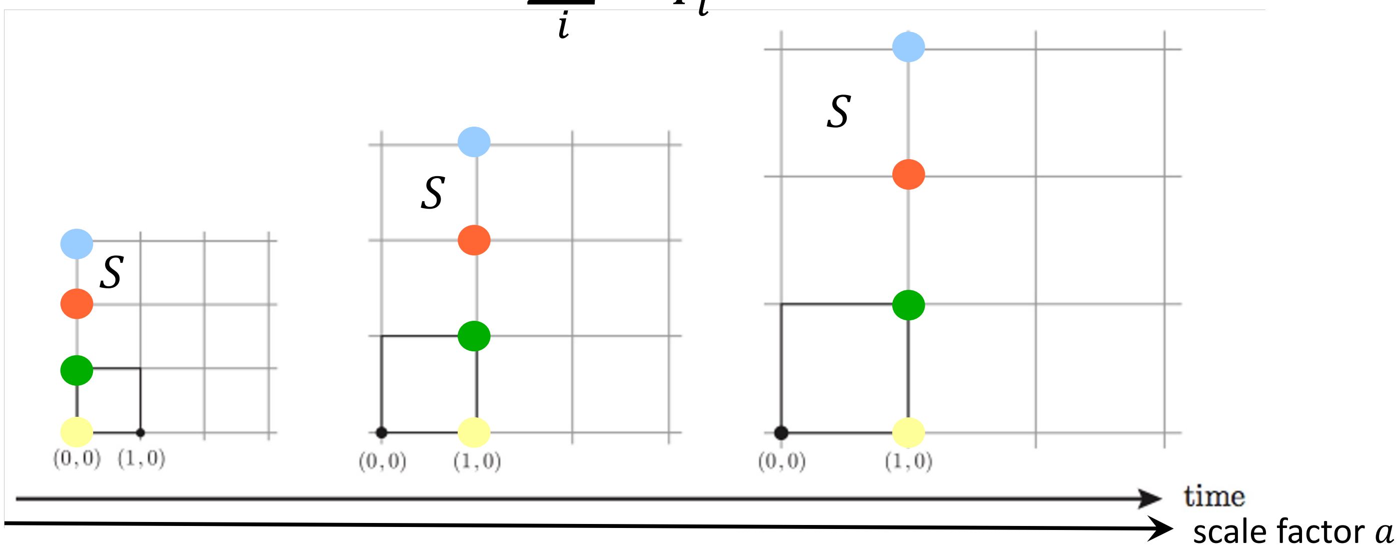
$$f_\nu(p) \approx \frac{1}{\exp[p/T_\nu] + 1}$$

Relativistic Fermi-Dirac

Comoving entropy density & conservation…

Where expansion is **quasi-static** so that equilibrium is maintained, the comoving entropy density S is approximately conserved.

Comoving entropy density $$S \equiv a^3 \sum_i \frac{\rho_i + P_i}{T_i}$$



T_ν from entropy conservation...

Decoupling

1 MeV

QED plasma

3 families of neutrinos + antineutrinos

Same temperature pre-e^+e^--annihilation

$e^\pm, \gamma$

$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$

$$\boxed{S_{\mathrm{QED}} = \frac{11}{45}\pi^2 T_\gamma^3 a^3}$$

$$\boxed{S_\nu = \frac{7}{30}\pi^2 T_\nu^3 a^3}$$

$T_\gamma = m_e$

Pre-e^+e^--annihilation

Post-e^+e^--annihilation

Different temperature post-e^+e^--annihilation

γ

$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$

$$\boxed{S_{\mathrm{QED}} = \frac{4}{45}\pi^2 T_\gamma^3 a^3}$$

$$\boxed{S_\nu = \frac{7}{30}\pi^2 T_\nu^3 a^3}$$

Time

T_ν from entropy conservation...

Decoupling

1 MeV

$T_\gamma = m_e$

QED plasma

Same temperature pre-e^+e^--annihilation

3 families of neutrinos + antineutrinos

$e^\pm, \gamma$

$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$

$$\boxed{S_{\mathrm{QED}} = \frac{11}{45}\pi^2 T_\gamma^3 a^3}$$

$$\boxed{S_\nu = \frac{7}{30}\pi^2 T_\nu^3 a^3}$$

Pre-e^+e^--annihilation

Post-e^+e^--annihilation

Different temperature post-e^+e^--annihilation

γ

$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$

$$\boxed{S_{\mathrm{QED}} = \frac{4}{45}\pi^2 T_\gamma^3 a^3}$$

$$\boxed{S_\nu = \frac{7}{30}\pi^2 T_\nu^3 a^3}$$

Time

Conservation: $S_\nu^{\mathrm{post}} = S_\nu^{\mathrm{pre}}$ $\Rightarrow$ $\boxed{a^{\mathrm{post}} T_\nu^{\mathrm{post}} = a^{\mathrm{pre}} T_\nu^{\mathrm{pre}}}$

T_ν from entropy conservation...

Decoupling

1 MeV

QED plasma

$e^\pm, \gamma$

$$S_{\text{QED}} = \frac{11}{45}\pi^2 T_\gamma^3 a^3$$

Same temperature pre-e^+e^--annihilation

3 families of neutrinos + antineutrinos

$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$

$$S_\nu = \frac{7}{30}\pi^2 T_\nu^3 a^3$$

$T_\gamma = m_e$

Pre-e^+e^--annihilation

Post-e^+e^--annihilation

γ

$$S_{\text{QED}} = \frac{4}{45}\pi^2 T_\gamma^3 a^3$$

Conservation: $S_{\text{QED}}^{\text{post}} = S_{\text{QED}}^{\text{pre}}$

$$T_\gamma^{\text{post}} = \left(\frac{11}{4}\right)^{1/3} \frac{a^{\text{pre}}}{a^{\text{post}}} T_\gamma^{\text{pre}}$$

$T_\gamma^{\text{pre}} = T_\nu^{\text{pre}}$

Time

T_ν from entropy conservation…

Decoupling

1 MeV

$T_\gamma = m_e$

Time

QED plasma

$e^\pm, \gamma$

Same temperature
pre-e^+e^--annihilation

$$\boxed{S_{\mathrm{QED}} = \frac{11}{45}\pi^2 T_\gamma^3 a^3}$$

3 families of neutrinos + antineutrinos

$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$

$$\boxed{S_\nu = \frac{7}{30}\pi^2 T_\nu^3 a^3}$$

Pre-e^+e^--annihilation

Post-e^+e^--annihilation

γ

$$\boxed{S_{\mathrm{QED}} = \frac{4}{45}\pi^2 T_\gamma^3 a^3}$$

Conservation: $S_{\mathrm{QED}}^{\mathrm{post}} = S_{\mathrm{QED}}^{\mathrm{pre}}$

$$T_\gamma^{\mathrm{post}} = \left(\frac{11}{4}\right)^{1/3} \frac{a^{\mathrm{pre}}}{a^{\mathrm{post}}} T_\gamma^{\mathrm{pre}}$$

$T_\gamma^{\mathrm{pre}} = T_\nu^{\mathrm{pre}}$

$a^{\mathrm{post}} T_\nu^{\mathrm{post}} = a^{\mathrm{pre}} T_\nu^{\mathrm{pre}}$

$$= \left(\frac{11}{4}\right)^{1/3} T_\nu^{\mathrm{post}}$$

CνB-CMB temperature relation

$$\boxed{T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma}$$

Evolution of the CνB...

At formation ($T \sim O(1)\ MeV \gg m_\nu$), the **C$\nu$B phase space distribution** is well described by the relativistic Fermi-Dirac distribution:

$$f_\nu(p) \approx \frac{1}{\exp[p/T_\nu] + 1}$$

with $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$

→ $T_{\nu,0} = 1.95\ \mathrm{K} = 1.7\times 10^{-4}\ \mathrm{eV}$

Present-day temperature

- Since the CνB do not scatter anymore, only the following can happen:
  - **Temperature redshift**: $T_\nu \propto a^{-1}$
  - **Momentum redshift**: $p \propto a^{-1}$

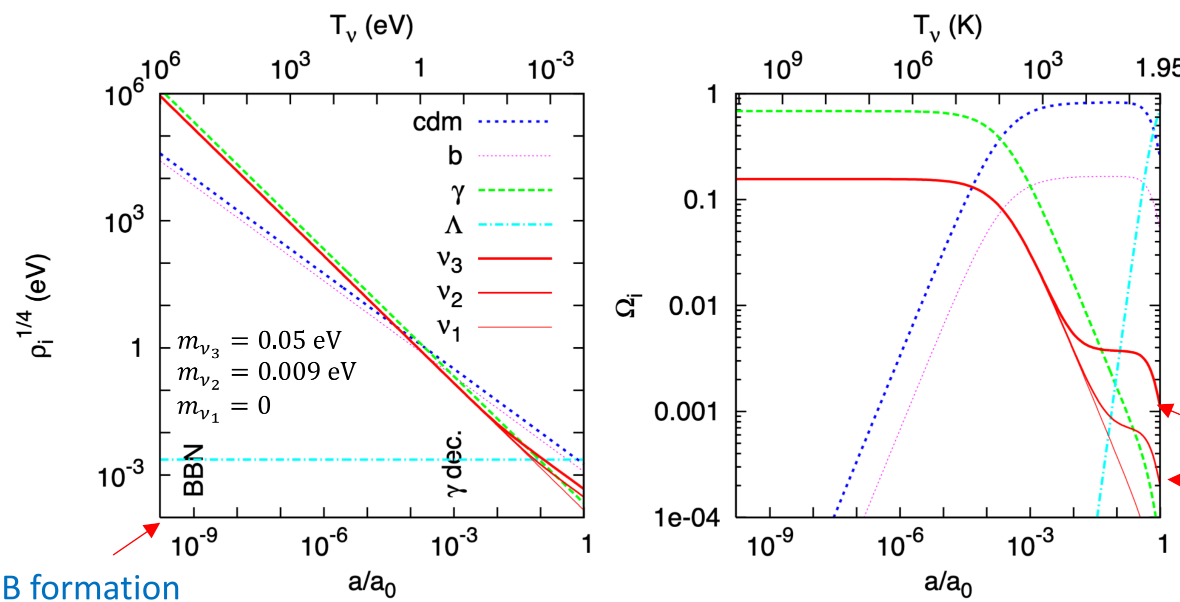
→ $f_\nu(p)$ must always take the same relativistic FD form, *even after* the CνB has become NR with redshift (a consequence of Liouville's theorem).

→ The **comoving number density** remains constant: $n_{\nu_\alpha,0} = \frac{6}{4}\frac{\zeta(3)}{\pi^2} T_{\nu,0}^3 = 112\ \mathrm{cm}^{-3}$

Present-day number density for one family

Evolution of the CνB...

But the CνB **energy density** depends on kinematics, scaling as $\rho \propto a^{-4}$ when the neutrinos are relativistic, and like $\rho \propto a^{-3}$ when NR.



CνB formation

Lesgourgues & Pastor 2006

- **If $T_\nu \gg m_\nu$:**

$$\rho_{\nu_\alpha} = \frac{7}{8}\left(\frac{4}{11}\right)^{\frac{4}{3}} \rho_\gamma$$

- **If $T_\nu \ll m_\nu$:**

$$\rho_{\nu_\alpha} = m_{\nu_\alpha} n_{\nu_\alpha}$$

$$\Omega_{\nu_\alpha,0} = \frac{m_{\nu_\alpha}}{94\ h^2}$$

Present-day reduced energy density

Summary of the CνB...

Standard hot big bang predicts a cosmic neutrino background with the properties:

- **Temperature**: $T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_{\mathrm{CMB},0} = 1.95\ \mathrm{K} = 1.7\times10^{-4}\ \mathrm{eV}$
- **Present-day number density per family**: $n_{\nu,0} = \frac{6}{4}\frac{\zeta(3)}{\pi^2} T_{\nu,0}^3 = 112\ \mathrm{cm}^{-3}$
- **Energy density**:
  - If neutrinos are relativistic: $\rho_{\nu_\alpha} = \frac{7}{8}\left(\frac{4}{11}\right)^{\frac{4}{3}} \rho_\gamma$
  - If neutrinos are non-relativistic: $\Omega_{\nu,0} = \sum \frac{m_\nu}{94\ h^2 \mathrm{eV}}$ Present-day reduced energy density