

Neutrinos and physics beyond the Standard Model

Stéphane Lavignac (IPhT Saclay)

- Dirac versus Majorana neutrinos
- neutrino masses as evidence for physics beyond the SM ?
- models of neutrino mass generation
- light sterile neutrinos
- heavy neutral leptons and their experimental signatures
- (low-scale) leptogenesis

Neutrino Frontiers training week
Galileo Galilei Institute, Florence, 25-28 June 2024

Neutrinos and physics beyond the Standard Model

Lecture 1

Stéphane Lavignac (IPhT Saclay)

- Dirac versus Majorana neutrinos
- neutrino masses as evidence for physics beyond the SM ?
- models of neutrino mass generation

Neutrino Frontiers training week
Galileo Galilei Institute, Florence, 25-28 June 2024

Introduction

Neutrino physics has made spectacular progress over the past 25 years, with the discovery of oscillations of atmospheric (Super-Kamiokande 1998), solar (SNO 2001) and reactor neutrinos (Daya Bay 2012)

→ evidence that neutrinos have (nondegenerate) masses and mix

$$m_\nu \neq 0 \quad U_{\text{PMNS}} \neq \mathbf{1} \quad [\text{PMNS} = \text{Pontecorvo-Maki-Nakagawa-Sakata}]$$

More and more precise measurement of oscillation parameters

$$|\Delta m_{31}^2| = (2.507^{+0.026}_{-0.027}) \times 10^{-3} \text{ eV}^2 \quad \Delta m_{21}^2 = (7.41^{+0.21}_{-0.20}) \times 10^{-5} \text{ eV}^2$$

$$\theta_{12}, \theta_{23} \text{ large} \quad (\theta_{13} \simeq 8.6^\circ \text{ a bit smaller}) \quad [\text{NO, Nu-FIT 5.2 (2022)}]$$

3σ uncertainties between 3 - 8% (except for θ_{23})

But still many open questions...

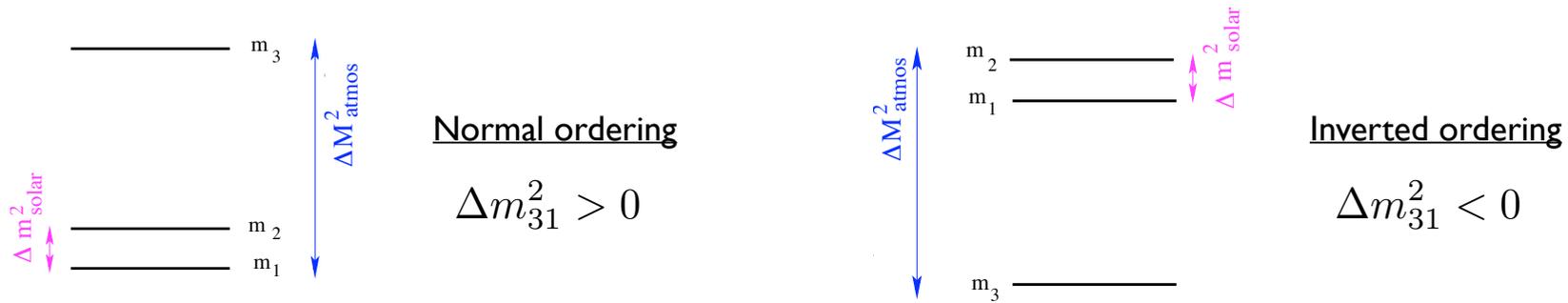
I) is CP violated in the lepton sector? [see Gabriela's lectures]

$$\delta \neq 0, \pi \text{ would imply } P(\nu_\mu \rightarrow \nu_e) \neq P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$$

Hint (90% C.L.) from the T2K experiment

[no constraint on the Majorana phases of U_{PMNS} (relevant only for Majorana neutrinos)]

2) what are the absolute neutrino mass scale and the mass ordering? [see Gabriela's and Yvonne's lectures]



$$m_{\text{heaviest}} \geq \sqrt{\Delta m_{\text{atm}}^2} \simeq 0.05 \text{ eV} \quad \text{from oscillations}$$

$$m_{\beta} \equiv \sqrt{\sum_i m_i^2 |U_{ei}|^2} < 0.8 \text{ eV} \quad (90\% \text{ C.L.}) \quad \text{from beta decay endpoint [KATRIN]}$$

$$\sum_i m_i < 0.12 \text{ eV} \quad (95\% \text{ C.L.}) \quad \text{from cosmology [Planck 2018 + BAO]}$$

3) are neutrinos Dirac or Majorana fermions? (“neutrino nature”)

$$\bar{\nu} \neq \nu \quad (\text{Dirac}) \quad \text{vs.} \quad \bar{\nu} = \nu \quad (\text{Majorana})$$

4) what is the origin of neutrino masses?

No answer within the Standard Model (predicts $m_{\nu} = 0$)

Fundamental question that is closely related to the one of the neutrino nature

Dirac versus Majorana neutrinos

Neutrinos are the only SM fermions that do not carry electric charge
 \Rightarrow can be their own antiparticles (Majorana fermions)

Experimentally, only ν_L (the “neutrino”) and its CP conjugate ν_R^c (the “antineutrino”) have been observed. We don’t know if the neutrino also has a RH component ν_R (which would be an SM gauge singlet, hence unobservable)

The observed (LH) neutrino and (RH) antineutrino can be described equally well by a Dirac or Majorana neutrino. The only difference is that the RH component of a Dirac neutrino, ν_R , is independent of ν_L and is an SM gauge singlet, while the RH component of a Majorana neutrino coincides with ν_R^c , the CP conjugate of its LH component

Dirac and Majorana mass terms

Dirac mass term

The simplest way to describe a massive neutrino is to add a ν_R to the SM and to write a Dirac mass term, as for the other fermions:

$$\mathcal{L}_{\text{mass}}^{\text{Dirac}} = -m_D (\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L) \equiv -m_D \bar{\nu}_D \nu_D \quad \nu_D \equiv \nu_L + \nu_R$$

The massive neutrino ν_D is a Dirac fermion (2 independent chiralities)

$$\begin{array}{c} \nu_R \qquad \nu_L \\ \longrightarrow \quad \text{X} \quad \longrightarrow \\ m_D \end{array} \quad \Delta L = 0 \quad \Delta T^3 = \frac{1}{2} \quad [\text{note: } \nu_R \text{ is an SM gauge singlet}]$$

not invariant under $SU(2)_L \times U(1)_Y$ but can be generated from a Yukawa coupling to the SM Higgs doublet (which has weak isospin 1/2)

$$\mathcal{L}_{\text{Yuk.}} = -y_D \bar{L} i\sigma^2 H^* \nu_R + \text{h.c.} \quad \longrightarrow \quad m_D = y_D \frac{v}{\sqrt{2}}$$

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \quad \langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad m_\nu \lesssim 1 \text{ eV} \quad \Rightarrow \quad y_D \lesssim 10^{-11}$$

caveat: possible to write a Majorana mass term for $\nu_R \Rightarrow$ end up with two Majorana neutrinos rather than one Dirac neutrino (see later)

Majorana mass term

Preliminary remark: can form a RH spinor from ν_L ($C\gamma_\mu^T C^{-1} = -\gamma_\mu$)

$$\nu_R^c \equiv C \bar{\nu}_L^T \sim \text{CP conjugate of } \nu_L \quad (\bar{\nu}_L \equiv \nu_L^\dagger \gamma^0)$$

C = charge conjugation matrix; enters the charge conjugate of a Dirac spinor

$\psi(x) \rightarrow \psi^c(x) \equiv C\bar{\psi}^T(x)$ describes the corresponding antifermion

\Rightarrow the existence of a LH neutrino (ν_L) implies the existence of a RH antineutrino ($C\bar{\nu}_L^T \equiv \nu_R^c \sim \bar{\nu}_R$)

Now, with ν_L and ν_R^c , can write a (Majorana) mass term :

$$\mathcal{L}_{\text{mass}}^{\text{Maj.}} = -\frac{1}{2} m_M (\bar{\nu}_L \nu_R^c + \bar{\nu}_R^c \nu_L) \equiv -\frac{1}{2} m_M \bar{\nu}_M \nu_M \quad \nu_M \equiv \nu_L + \nu_R^c$$

The massive neutrino $\nu_M = \nu_L + \nu_R^c$ satisfies the Majorana condition

$\nu_M = \nu_M^c \rightarrow$ Majorana fermion

$$\begin{array}{c} \nu_R^c \quad \nu_L \\ \longrightarrow \quad \text{X} \quad \longrightarrow \\ m_M \end{array} \iff \begin{array}{c} \nu_L \quad \nu_L \\ \longleftarrow \quad \text{X} \quad \longrightarrow \\ m_M \end{array} \quad \Delta L = 2 \quad \Delta T^3 = 1$$

A Majorana mass term violates lepton number (signature of a Majorana neutrino). It cannot be generated from a coupling to the SM Higgs doublet, which has a $T = 1/2 \Rightarrow$ neutrino masses require an extension of the SM

Dirac versus Majorana neutrino

A Dirac neutrino is different from its antiparticle ($\nu \neq \nu^c$)

\Rightarrow describes 4 degrees of freedom: $\nu\uparrow, \nu\downarrow, \bar{\nu}\uparrow, \bar{\nu}\downarrow$ [or $\nu_R, \nu_L, \bar{\nu}_R, \bar{\nu}_L$]

Described by a 4-component spinor $\nu_D = \begin{pmatrix} \nu_{D,L} \\ \nu_{D,R} \end{pmatrix}$, with independent LH and RH components $\nu_{D,L}$ and $\nu_{D,R}$

A Majorana neutrino satisfies the condition $\nu = \nu^c = C\bar{\nu}^T$

\Rightarrow describes only 2 degrees of freedom: $\nu\downarrow, \bar{\nu}\uparrow$ [or $\nu_L, \bar{\nu}_R$]

Can be described by a 4-component spinor $\nu_M = \begin{pmatrix} \nu_{M,L} \\ \nu_{M,R} \end{pmatrix}$, but its LH and RH components are not independent, as $\nu_M = \nu_M^c \Rightarrow \nu_{M,R} = \nu_{M,R}^c \equiv C\bar{\nu}_{M,L}^T$

The Majorana condition is inconsistent with any conserved additive quantum number: if ψ possesses a conserved quantum number q ,

$$\psi \rightarrow e^{i\theta q} \psi \quad \Rightarrow \quad \psi^c \rightarrow e^{-i\theta q} \psi^c$$

Thus only neutrinos (not quarks, charged leptons) can be Majorana fermions

For the same reason, one cannot rephase a Majorana neutrino

\Rightarrow 2 additional physical phases in the PMNS matrix wrt the Dirac case

How to distinguish Majorana from Dirac neutrinos?

Dirac and Majorana neutrinos have the same gauge interactions, since weak interactions only involve ν_L and its CP conjugate $\bar{\nu}_R$, which can be described either by a Dirac or a Majorana neutrino (ν_R and $\bar{\nu}_L$, if they exist, are SM gauge singlets and do not interact at all)

One commonly calls ν_L “neutrino” and $\bar{\nu}_R$ “antineutrino”, irrespective of whether they are degrees of freedom of a Dirac or Majorana neutrino. This terminology is motivated by the fact that, via the charged weak interaction, ν_L ($\bar{\nu}_R$) creates a negatively charged (positively charged) lepton

Similarly, oscillations probabilities are the same for Dirac and Majorana neutrinos (production and detection are weak interaction processes: only ν_L and $\bar{\nu}_R$ can be produced and detected) (*)

The only practical difference between Dirac and Majorana neutrinos lies in their mass term, which violates lepton number by 2 units in the Majorana case → the Majorana nature of neutrinos can be established in $\Delta L = 2$ processes

(*) note: $P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$ corresponds to CP violation, not C violation, and is possible both for Dirac and Majorana neutrinos, precisely because

$$\bar{\nu}_\alpha \equiv \bar{\nu}_{R\alpha} = \text{CP conjugate of } \nu_\alpha \equiv \nu_{L\alpha}$$

Neutrino masses as evidence for physics beyond the SM ?

Neutrinos are massless in the Standard Model as it was defined historically (i.e. without a ν_R , in agreement with observations): neither a Dirac nor a Majorana mass terms can be generated from electroweak symmetry breaking

Adding a RH neutrino to the SM may look like a minor modification that allows to generate a Dirac mass term, but it also makes it possible to write a Majorana mass term for the RH neutrino, thus allowing lepton number violation and introducing a new mass scale, potentially much larger than the electroweak scale

→ physics beyond the Standard Model !

Neutrino mass generation: adding a ν_R to the SM

Simplest possibility: add a RH neutrino N_R to the Standard Model

In addition to the Dirac mass term $-m_D \bar{\nu}_L N_R + \text{h.c.}$, must write a Majorana mass term for the RH neutrino, which is allowed by all (non-accidental) symmetries of the SM (or justify its absence):

$$-\frac{1}{2} M \bar{N}_L^c N_R + \text{h.c.} = -\frac{1}{2} M N_R^T C N_R + \text{h.c.} \quad \Delta L = 2 \quad \Delta T^3 = 0$$

[only lepton number, if imposed, can forbid this term]

Mass eigenstates: write the mass terms in a matrix form and diagonalize

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= -\frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \bar{N}_L^c \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix} + \text{h.c.} \\ &= -\frac{1}{2} \begin{pmatrix} \bar{\nu}_{L1} & \bar{\nu}_{L2} \end{pmatrix} \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} \nu_{R1}^c \\ \nu_{R2}^c \end{pmatrix} + \text{h.c.} \end{aligned}$$

$$\text{where } \begin{cases} \nu_{L1} = \cos \theta \nu_L - \sin \theta N_L^c \\ \nu_{L2} = \sin \theta \nu_L + \cos \theta N_L^c \end{cases}$$

Defining $\nu_{Mi} \equiv \nu_{Li} + \nu_{Ri}^c$ (such that $\nu_{Mi} = \nu_{Mi}^c$), one can see that the mass eigenstates are 2 Majorana neutrinos with masses m_1 and m_2 :

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} \sum_{i=1,2} m_i \bar{\nu}_{Li} \nu_{Ri}^c + \text{h.c.} = -\frac{1}{2} \sum_{i=1,2} m_i \bar{\nu}_{Mi} \nu_{Mi}$$

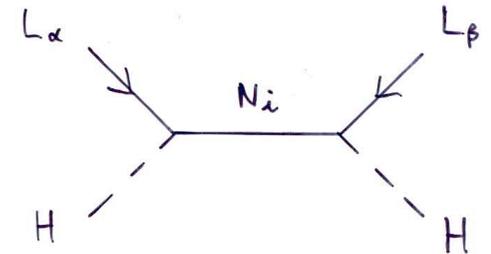
“Seesaw” limit: $M \gg M_W \gtrsim m_D$

Minkowski - Gell-Mann, Ramond, Slansky
Yanagida - Mohapatra, Senjanovic

($N_R = \text{gauge singlet} \Rightarrow M$ unconstrained by the electroweak symmetry)

$$m_1 \simeq -m_D^2/M \ll M_W \quad m_2 \simeq M \gg M_W$$

$$\sin \theta \simeq \frac{m_D}{M} \ll 1 \quad \Rightarrow \quad \nu_{L1} \simeq \nu_L, \quad \nu_{R2}^c \simeq N_R$$



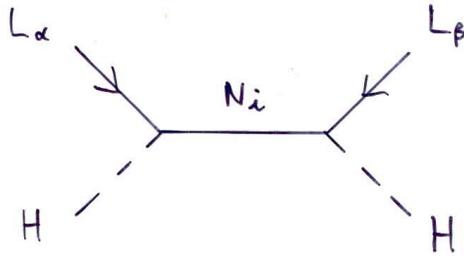
→ the light Majorana neutrino is essentially the SM neutrino

→ natural explanation of the smallness of neutrino masses

New physics interpretation: $M = \text{scale of the new physics responsible for lepton number violation}$ – can a priori lie anywhere between $\sim 10^{15}$ GeV (a larger M would give $m_\nu < \sqrt{|\Delta m_{31}^2|} \simeq 0.05$ eV, unless $y_D > 1$) and the weak scale (low-scale seesaw mechanism), or even below

3-generation (type I) seesaw mechanism ($i = 1, 2, 3; \alpha = e, \mu, \tau$)

$$\mathcal{L}_{\text{seesaw}}^I = -Y_{i\alpha} \bar{N}_{Ri} L_\alpha H - \frac{1}{2} M_i \bar{N}_{Ri} N_{Ri}^c + \text{h.c.}$$



$$\Rightarrow (M_\nu)_{\alpha\beta} = - \sum_i \frac{Y_{i\alpha} Y_{i\beta}}{M_i} v^2$$

Light neutrino mass matrix: $M_\nu = -Y^T M^{-1} Y v^2 = U^* D_\nu U^\dagger$

U = lepton mixing (PMNS) matrix

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i$$

$$D_\nu = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i(\sigma+\delta)} \end{pmatrix}$$

Natural realization of the seesaw mechanism in Grand Unified Theories (GUTs) based on the $SO(10)$ gauge group

- SM quarks and leptons fit in a single 16-dimensional representation of $SO(10)$, which also contains a right-handed neutrino:

$$\mathbf{16}_i = (Q_i, u_i^c, d_i^c, L_i, e_i^c, N_i^c) \quad (i = 1, 2, 3)$$

- the scale of RH neutrino masses is associated with the breaking of the B-L symmetry, which is a generator of $SO(10)$, and is typically broken at or a few orders of magnitude below the GUT scale M_{GUT}

$$M_i \longleftrightarrow M_{B-L} \longleftrightarrow SO(10) \text{ gauge symmetry breaking}$$

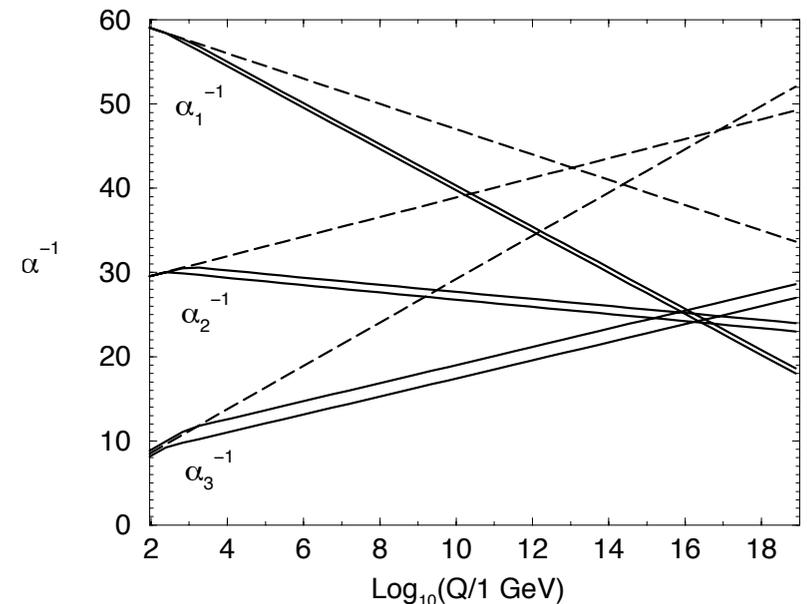
(\neq arbitrary scale, even if model dependent)

- natural values of the Dirac Yukawa coupling

$$y_D = \sqrt{2} m_D / v \quad (\text{i.e. } y_D \sim 1) \text{ give}$$

$$m_\nu = m_D^2 / M \sim 0.05 \text{ eV for } M \sim 10^{15} \text{ GeV,}$$

near the unification scale in supersymmetric extensions of the SM, $M_{\text{GUT}} \approx 2 \times 10^{16} \text{ GeV}$



Right-handed neutrinos imply a deep (even if minimal) modification of the SM

- without RHNs, gauge invariance and renormalizability imply that B and L are global symmetries of the SM, only broken by quantum effects (anomalies)
- with RHNs, this is no longer true: a $\Delta L = 2$ Majorana mass term is allowed both by gauge invariance and renormalizability

Dirac neutrinos remain a viable possibility, but lepton number has to be imposed: no longer automatic

Theoretical prejudices against Dirac neutrinos:

- must impose lepton number
- need very small Yukawa couplings: $m_\nu = y_\nu \langle H \rangle$ $\langle H \rangle = 174 \text{ GeV}$

$$m_\nu \lesssim 1 \text{ eV} \quad \Longrightarrow \quad y_\nu \lesssim 10^{-12} \quad (y_e \lesssim 10^{-6})$$

[this makes the SM flavour puzzle, i.e. the unexplained hierarchy of fermion masses / Yukawa couplings even stronger, but it might be explained by a theory of flavour]

Theoretical prejudices for Majorana neutrinos:

- lepton number violated in many extensions of the SM
- any mechanism generating neutrino masses without RHNs gives Majorana neutrinos
- natural in $SO(10)$ Grand Unified Theories (GUTs), left-right symmetric theories (based on the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ or larger), supersymmetry without R-parity
- possible explanation of the small neutrino masses (seesaw mechanism...)
- open the possibility of generating the baryon asymmetry of the Universe via leptogenesis (B-L violation and CP violation are necessary ingredients of baryogenesis)

While Majorana neutrinos are theoretically compelling, only experiment (neutrinoless double beta decay, or possibly some other $\Delta L = 2$ leptonic process) will tell whether neutrinos are Dirac or Majorana particles

Alternative mechanisms of neutrino (Majorana) mass generation

- other versions of the seesaw mechanism with heavy SU(2) triplets (scalar [type II seesaw] or fermionic [type III seesaw]). Can be realized at high or low energy (with possibly new states accessible at colliders in the latter case)
- radiative models: neutrino masses generated at the one-loop (Zee model, supersymmetry with trilinear R-parity violation), two-loop level (Babu-Zee model) or more. These are typically low-scale models, which can be tested at colliders and predict flavour-violating processes involving charged leptons
- more exotic: supersymmetric models with R-parity violation (in which lepton number is violated), extra spatial dimensions (*)...

(*) the minimal model with a flat extra dimensions, ν_L on the SM brane and ν_R in the bulk, predicted a mixing of ν_e with an infinite tower of sterile neutrinos, and has been excluded by Super-Kamiokande and SNO

Type II seesaw mechanism: heavy scalar triplet exchange

The Majorana mass term $\mathcal{L}_m^{\text{Maj.}} = -\frac{1}{2} m_\nu \nu_L^T C \nu_L + \text{h.c.}$ has $\Delta T_3 = 1$
 \Rightarrow can be generated from a coupling to a Higgs triplet: [Gelmini, Roncadelli]

$$-\frac{1}{2} f_{\alpha\beta} L_\alpha^T C i\sigma^2 \Delta_L L_\beta + \text{h.c.} \quad \Delta_L = \begin{pmatrix} \Delta^+ & \sqrt{2} \Delta^{++} \\ \sqrt{2} \Delta^0 & -\Delta^+ \end{pmatrix}$$

violation of lepton number in the scalar potential: $\frac{\mu}{2} H^T i\sigma^2 \Delta_L^\dagger H + \text{h.c.}$

need small vev v_Δ and/or small Yukawa coupling $f_{\alpha\beta}$

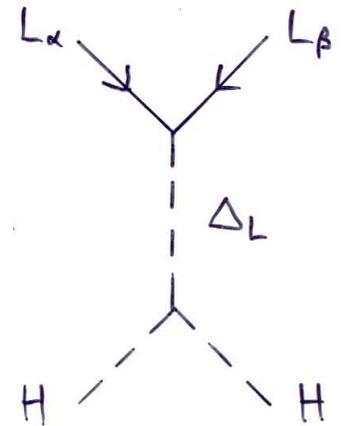
Natural limit: heavy Higgs triplet $\Rightarrow m_\nu$ suppressed by μ/M_Δ^2

\rightarrow type II seesaw mechanism

Magg, Wetterich - Lazarides, Shafi, Wetterich
 Mohapatra, Senjanovic - Schechter, Valle

$$(M_\nu)_{\alpha\beta} = f_{\alpha\beta} \frac{\mu}{M_\Delta^2} v^2$$

no need for small μ or $f_{\alpha\beta}$



The triplet mass is a priori not related to the lepton number breaking scale :
 low-scale lepton number breaking ($\mu \ll 1 \text{ GeV}$) possible

More economical in parameters than type I: $M_{\alpha\beta} \leftrightarrow f_{\alpha\beta}$

Type II seesaw can be realized in SO(10) GUTs, using the $SU(2)_L$ triplets present in the 54- and 126-dimensional Higgs representations

Type I and II can be simultaneously present in SO(10) models or in left-right symmetric theories with $SU(2)_L$ and $SU(2)_R$ triplets:

$$M_\nu = f_L v_L - Y f_R^{-1} Y \frac{v^2}{v_R} \quad v_L = \mu v^2 / M_\Delta^2$$

$f_L, f_R (v_L, v_R) = SU(2)_L$ and $SU(2)_R$ triplet couplings (vevs)

Often an underlying left-right symmetry ensures $f_L = f_R \equiv f$

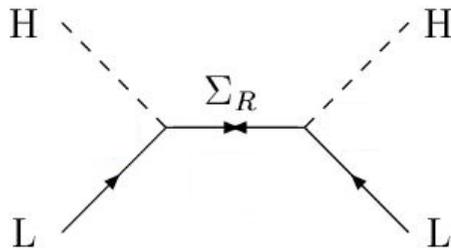
Type III seesaw mechanism: heavy fermion triplet exchange

The Majorana mass term $\mathcal{L}_m^{\text{Maj.}} = -\frac{1}{2} m_\nu \nu_L^T C \nu_L + \text{h.c.}$ can also be generated from a coupling to a fermion Higgs triplet:

$$-Y_\alpha^\Sigma \bar{L}_\alpha \Sigma i\sigma^2 H^* + \text{h.c.} \quad \Sigma = \begin{pmatrix} \Sigma^0 & \sqrt{2} \Sigma^+ \\ \sqrt{2} \Sigma^- & -\Sigma^0 \end{pmatrix}$$

Natural limit: heavy Higgs triplet $\Rightarrow m_\nu$ suppressed by $1/M_\Sigma$

\rightarrow type III seesaw mechanism Foot, Lew, He, Joshi - Ma



$$(M_\nu)_{\alpha\beta} = \frac{Y_\alpha^\Sigma Y_\beta^\Sigma}{M_\Sigma} v^2$$

With a single Σ , M_ν has rank one \Rightarrow a single massive neutrino

\Rightarrow at least two fermion triplets needed

Can be realized in SU(5) GUTs with a fermion in the adjoint representation

$$24_F \ni (1, 3)_0 \oplus (1, 1)_0 \quad \bar{5}_\alpha 24_F 5_H \quad \bar{5}_\alpha = (L_\alpha, d_\alpha^c), 5_H = (H, T)$$

\rightarrow type I+III seesaw mechanism

Radiative neutrino mass models

Zee model (1-loop)

Adding to the SM a second Higgs doublet Φ and a charged SU(2) singlet h^+ leads to the following leptonic Yukawa couplings + scalar trilinear coupling

$$\mathcal{L}_{Zee} \ni -Y_{\alpha\beta}^H \bar{L}_\alpha H e_{R\beta} - Y_{\alpha\beta}^\Phi \bar{L}_\alpha \Phi e_{R\beta} - f_{\alpha\beta} L_\alpha^T C^{-1} i\sigma^2 L_\beta h^+ - \mu H^\dagger i\sigma^2 \Phi^* h^+ + \text{h.c.}$$

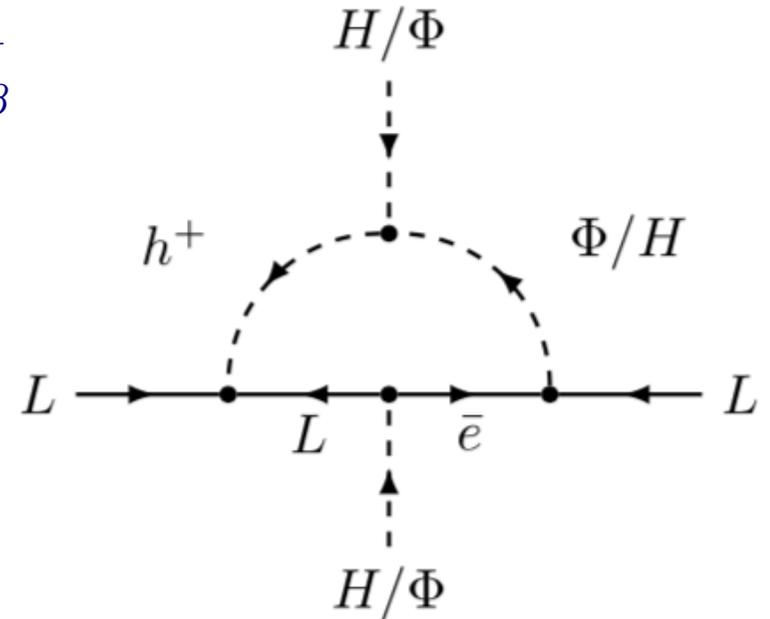
where $f_{\beta\alpha} = -f_{\alpha\beta}$ and both H and Φ acquire a vev

\Rightarrow charged lepton masses depend on both $Y_{\alpha\beta}^H$ and $Y_{\alpha\beta}^\Phi$, and neutrino masses arise at 1-loop

The testable signatures of this mechanism are exotic scalars and flavour-violating charged lepton decays such as $\mu \rightarrow e \gamma$

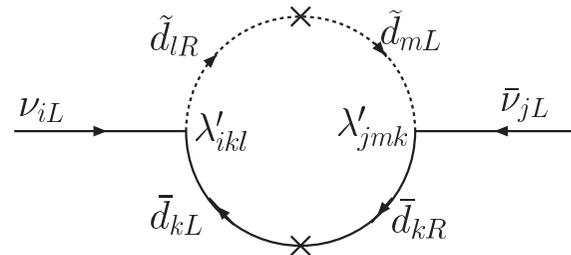
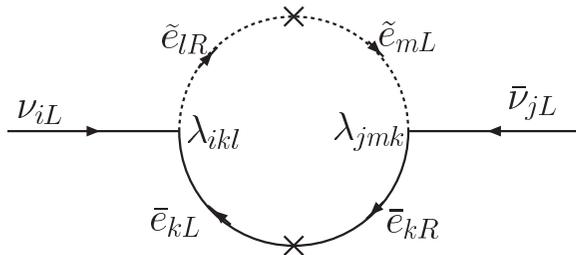
Note: the original Zee model had $Y_{\alpha\beta}^\Phi = 0$

and was predicting an inverted mass ordering with a near-maximal solar mixing angle, which is excluded by the data



Supersymmetry with trilinear R-parity breaking (1-loop)

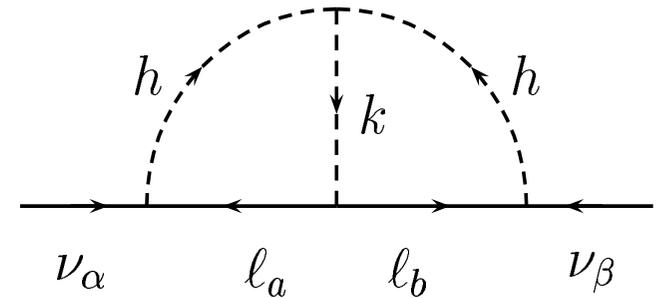
Neutrino masses arise from quark-squark and lepton-slepton loops



Zee-Babu model (2-loop)

introduce 2 charged SU(2) singlet scalars, h^+ and k^{++} , with couplings to leptons:

$$f_{\alpha\beta} L_{\alpha}^T C i \sigma^2 L_{\beta} h^+ + h'_{\alpha\beta} e_{R\alpha}^T C e_{R\beta} k^{++} + \text{h.c.}$$



Lepton number is violated by scalar couplings: $\mu h^+ h^+ k^{--} + \text{h.c.}$

Neutrino mass matrix: $(M_{\nu})_{\alpha\beta} \sim \frac{8\mu}{(16\pi^2)^2 m_h^2} f_{\alpha\gamma} m_{e_{\gamma}} h_{\gamma\delta} m_{e_{\delta}} f_{\delta\beta}$

In addition to new exotic scalars, this mechanism predicts flavour-violating processes involving charged leptons, such as $\mu \rightarrow e \gamma$