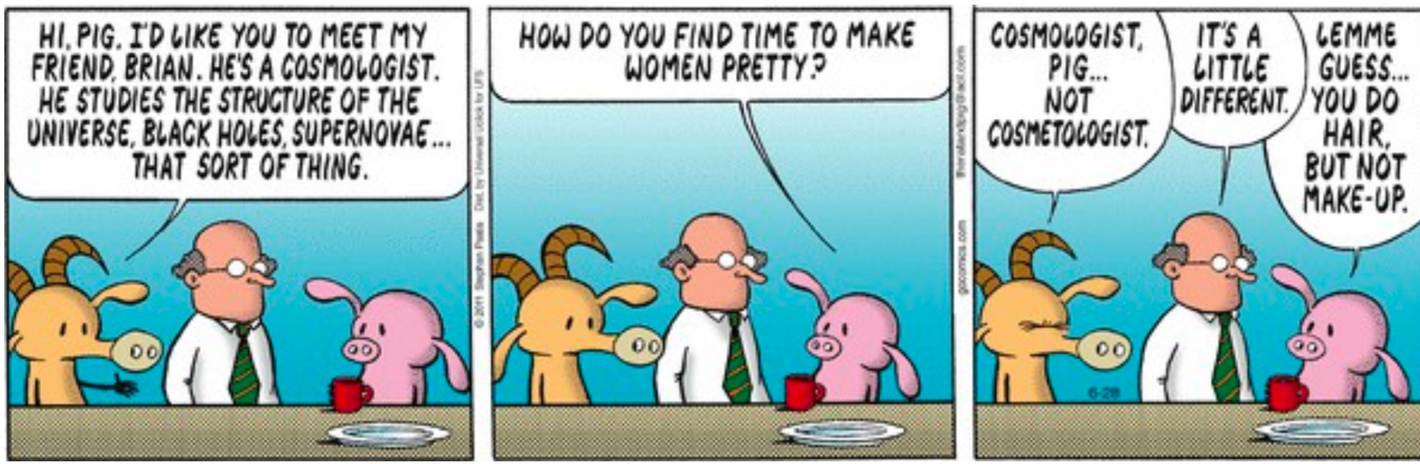
Neutrinos in cosmology



Yvonne Y. Y. Wong, UNSW Sydney

Neutrino Frontiers Training Week, Galileo Galilei Institute, Florence,
June 25 – 28, 2024

Part 2: Neutrinos in inhomogeneous universe

1. Theory of inhomogeneities
2. Neutrinos and structure formation
3. Relativistic neutrino free-streaming and non-standard interactions

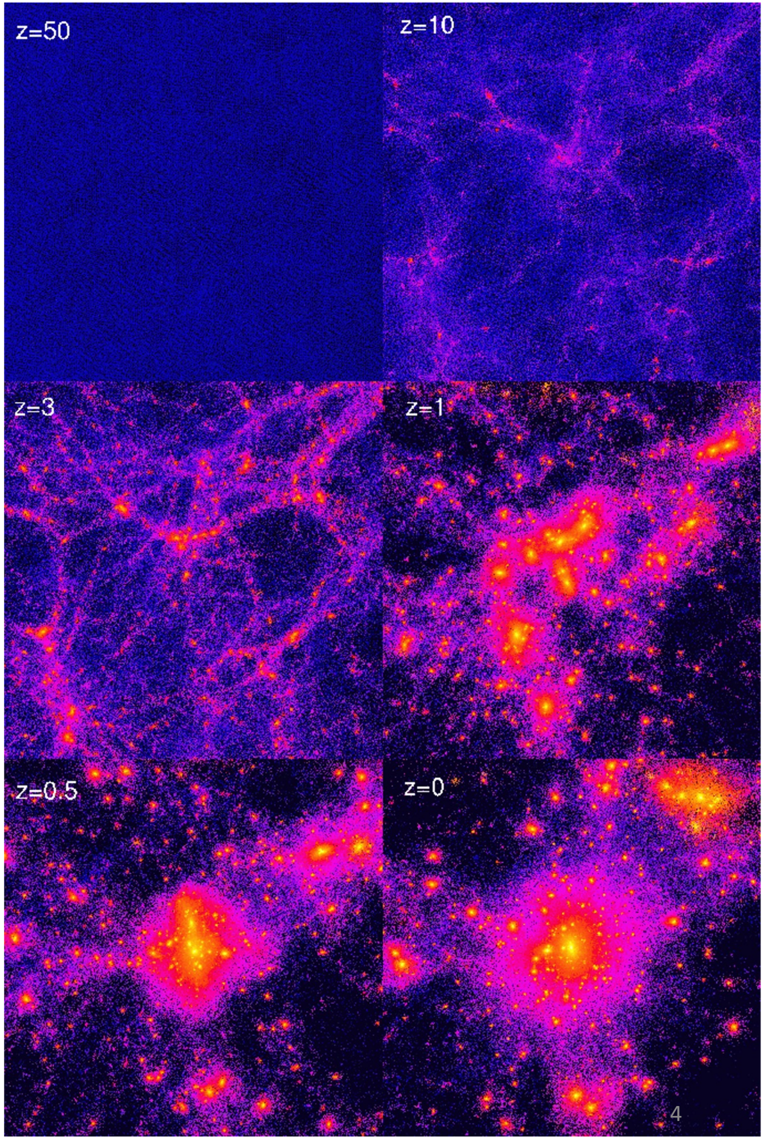
2. Neutrinos and structure formation…

How structures form…

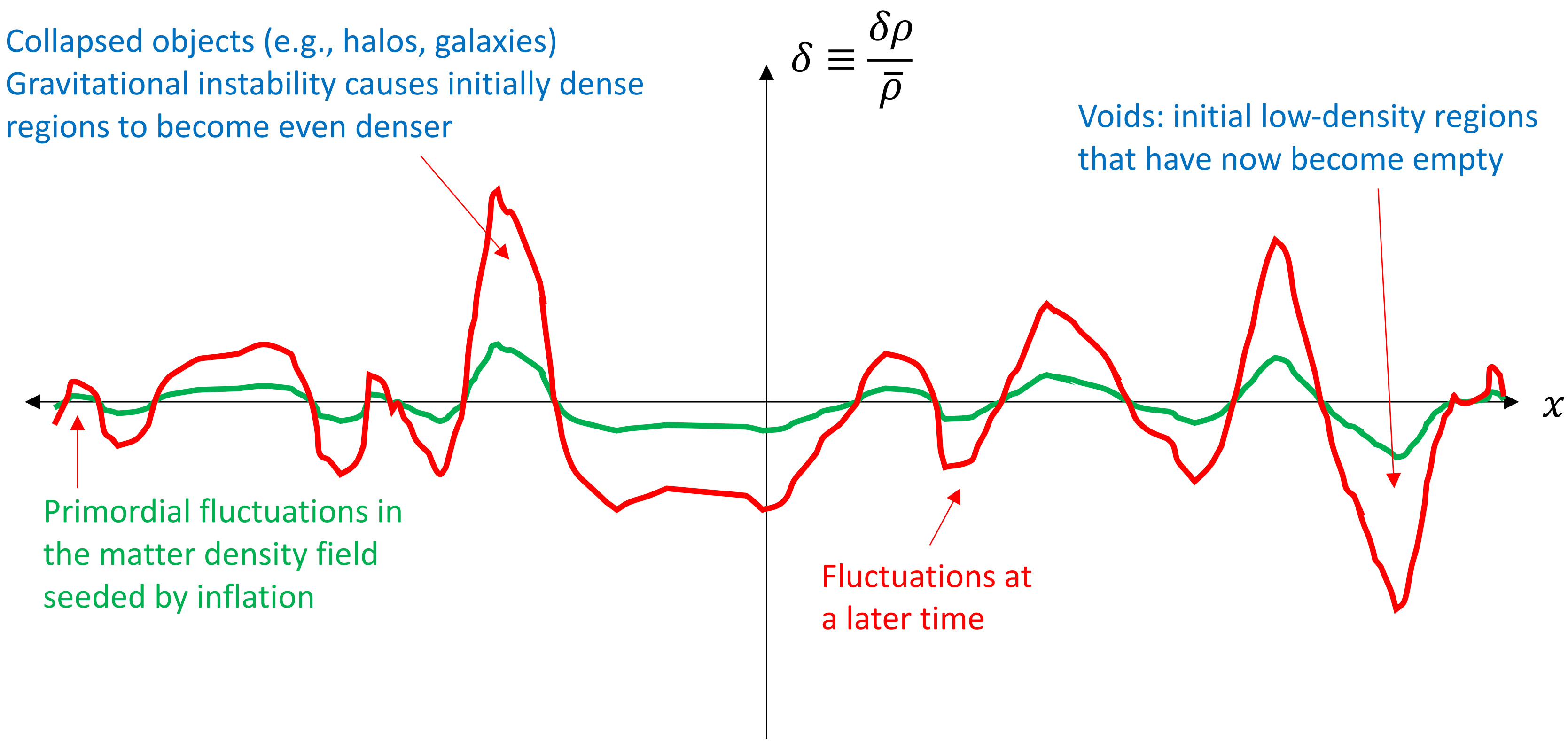
In the standard inflationary paradigm, the early universe is filled with an almost homogeneous matter density field with tiny **random fluctuations**:

Density contrast → $$\delta \equiv \frac{\delta\rho}{\bar{\rho}}$$ ← Mean density

- These fluctuations "grow" via **gravitational instability** and eventually **collapse** to form galaxies and clusters, etc.



How structures form…



Neutrino dark matter…

Standard hot big bang predicts a relic neutrino background with present-day properties:

- **Temperature**: $T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_{\mathrm{CMB},0} = 1.95\ \mathrm{K} = 1.7{\times}10^{-4}\ \mathrm{eV}$
- **Number density per family**: $n_{\nu,0} = \frac{6}{4}\frac{\zeta(3)}{\pi^2} T_{\nu,0}^3 = 112\ \mathrm{cm}^{-3}$
- **Total non-relativistic energy density:** $\Omega_{\nu,0} = \sum \frac{m_\nu}{94\ h^2 \mathrm{eV}}$

- Observations indicate are DM abundance of $\Omega_{\mathrm{DM}} \approx 0.25$.

Can standard-model neutrinos be the dark matter?

Neutrino dark matter…

Neutrinos cannot make up all of the dark matter.

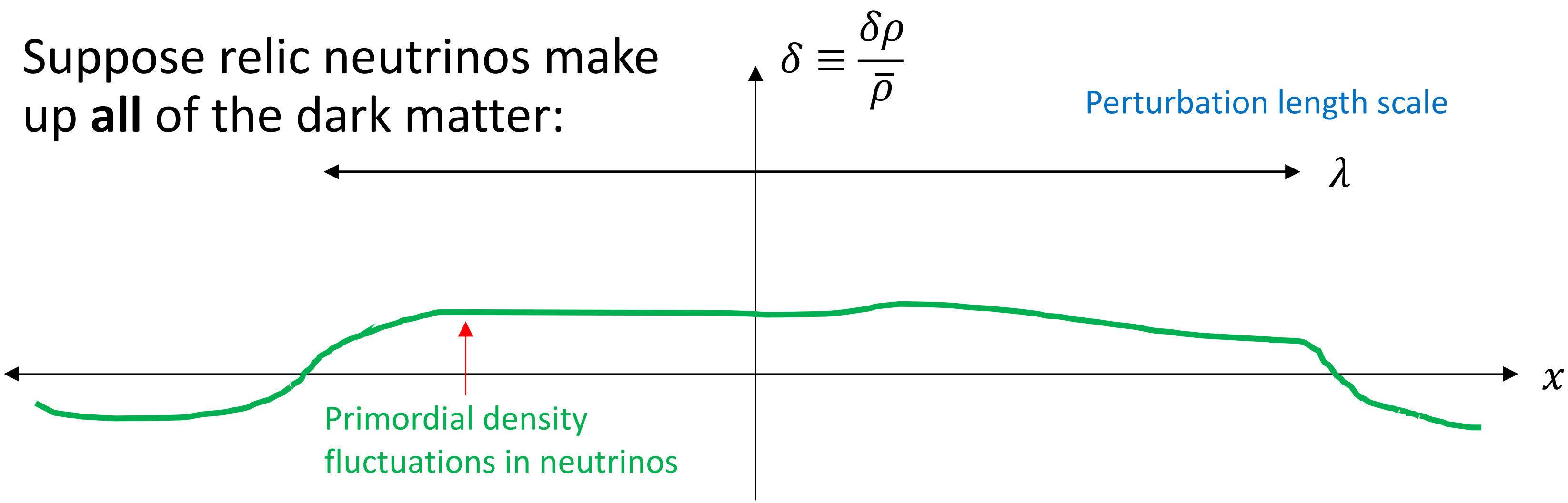
- The **obvious reason**: a neutrino mass of $\sim 10\ \mathrm{eV}$ is required to give $\Omega_{\nu,0} = \Omega_{\mathrm{DM}}$, which is not allowed by the KATRIN limit $m_e \lesssim 0.9\ \mathrm{eV}$.
- The **deeper reason**: the CνB has a lot of kinetic energy. The average speed of a NR relic neutrino is

$$v_\nu = \frac{p_\nu}{m_\nu} = \frac{3T_\nu}{m_\nu} \approx 150\,(1+z)\left(\frac{\mathrm{eV}}{m_\nu}\right)\mathrm{km\ s^{-1}}$$

- Typical velocity dispersions: galaxy cluster $O(1000)\ \mathrm{km\ s^{-1}}$, galaxy $O(100)\ \mathrm{km\ s^{-1}}$, dwarf galaxy $< 100\ \mathrm{km\ s^{-1}}$.
- An eV-mass relic neutrino has too much kinetic energy to have formed some of these objects.

Instantaneous free-streaming length…

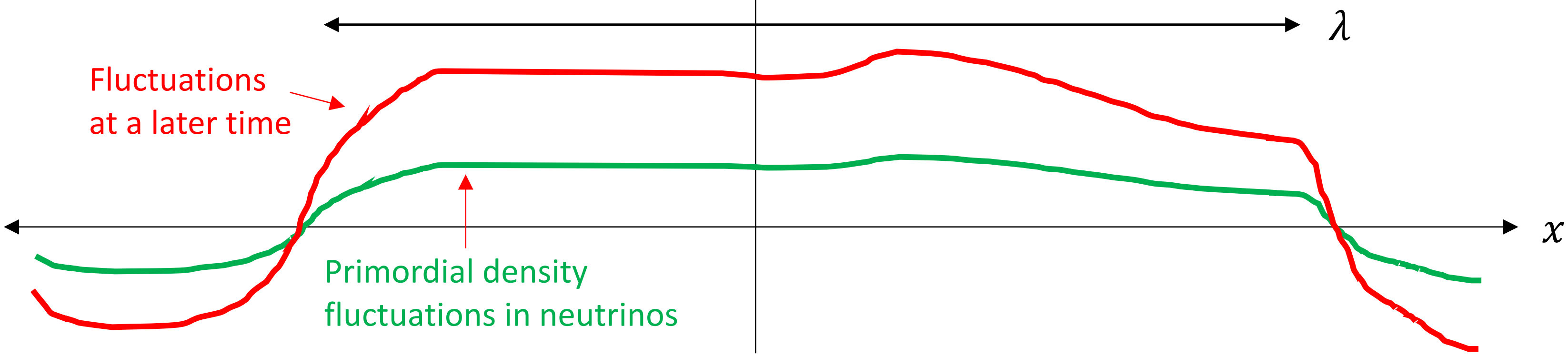
Suppose relic neutrinos make up **all** of the dark matter:



- **Instantaneous collapse time scale**: $\Delta t_{\text{collapse}} \equiv (4\pi G\bar{\rho}a^2)^{-1/2}$ How long does it take an overdense region to collapse to a point
- **Instantaneous escape time scale**: $\Delta t_{\text{escape}} \equiv \frac{\lambda}{v_\nu}$ How long does it take a neutrino to fly out of the region

Instantaneous free-streaming length…

Suppose relic neutrinos make up **all** of the dark matter:



- **Instantaneous collapse time scale:** $\Delta t_{\text{collapse}} \equiv (4\pi G \bar{\rho} a^2)^{-1/2}$
- **Instantaneous escape time scale:** $\Delta t_{\text{escape}} \equiv \frac{\lambda}{v_\nu}$

Limit 1: Growth
Collapse happens **faster** than escape

$$\Delta t_{\text{collapse}} \ll \Delta t_{\text{escape}}$$

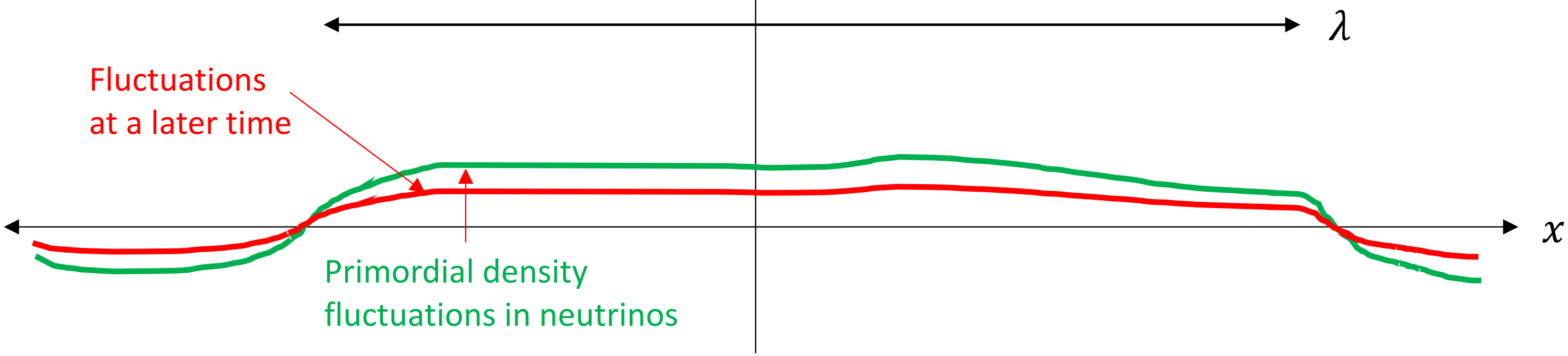
→ Perturbation grows.

Instantaneous free-streaming length…

Suppose relic neutrinos make up **all** of the dark matter:

$$\delta \equiv \frac{\delta\rho}{\bar{\rho}}$$

Perturbation length scale



- **Instantaneous collapse time scale:** $\Delta t_{\text{collapse}} \equiv (4\pi G\bar{\rho}a^2)^{-1/2}$
- **Instantaneous escape time scale:** $\Delta t_{\text{escape}} \equiv \dfrac{\lambda}{v_\nu}$

Limit 2: Erasure

Collapse happens **slower** than escape

$$\Delta t_{\text{collapse}} \gg \Delta t_{\text{escape}}$$

→ Perturbation is erased.

Instantaneous free-streaming length…

Growth or erasure? Define the instantaneous free-streaming length λ_{FS} to be the scale at which $\Delta t_{\mathrm{collapse}} = \Delta t_{\mathrm{escape}}$, i.e.,

Using the NR speed from earlier and assuming matter domination

$$\boxed{\begin{aligned}\lambda_{\mathrm{FS}}(z) &\equiv v_\nu \Delta t_{\mathrm{collapse}} \\ &\approx 1.2\ \Omega_{m,0}^{-1/2}(1+z)^{1/2}\left(\frac{\mathrm{eV}}{m_\nu}\right)\ h^{-1}\mathrm{Mpc}\end{aligned}}$$

→ Unless density fluctuations are regenerated by other means, at any redshift z relic neutrinos **cannot form structures** of length scale $\lambda < \lambda_{\mathrm{FS}}(z)$.

Instantaneous free-streaming length…

The **maximum instantaneous free-streaming length** is that at the time neutrinos just become non-relativistic:

$$\boxed{\lambda_{\mathrm{FS,max}} \equiv \lambda_{\mathrm{FS}}(z_{\mathrm{nr}}) \approx 55\ \Omega_{m,0}^{-1/2} \left(\frac{\mathrm{eV}}{m_\nu}\right)^{1/2} \ h^{-1}\mathrm{Mpc}}$$

Using

$$1 + z_{nr} \approx \frac{m_\nu}{3\ T_{\nu,0}}$$

→ $\lambda_{\mathrm{FS,max}}$ corresponds to the maximum size of objects that could not have been formed in a neutrino dark matter-only universe.

→ If a 10 eV-mass neutrino was the dark matter, $\lambda_{\mathrm{FS,max}} \sim 45\ \mathrm{Mpc}$, we **would not have** galaxies ($\lambda \sim 10\ \mathrm{kpc}$) and galaxy clusters ($\lambda \sim 1\ \mathrm{Mpc}$)

Neutrino masses & perturbation growth...

Cold dark matter only

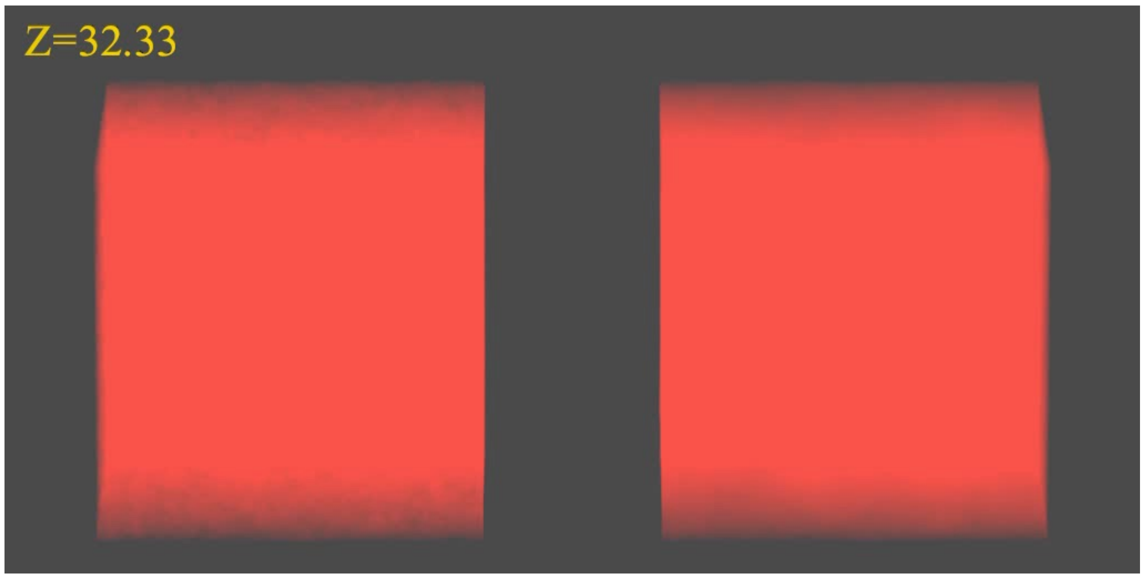
$\Omega_{\mathrm{CDM}} \approx 25\%$

Cold dark matter + neutrinos ($\sum m_\nu = 6.9\ \mathrm{eV}$)

$\Omega_{\mathrm{CDM}} \approx 10\%$

$\Omega_\nu = \dfrac{\sum m_\nu}{93\ h^2} \approx 15\%$

$256\ h^{-1}\mathrm{Mpc}$



Observable universe $\sim O(10)\mathrm{Gpc}$

Simulations by Troels Haugbølle

Neutrino masses & perturbation growth...

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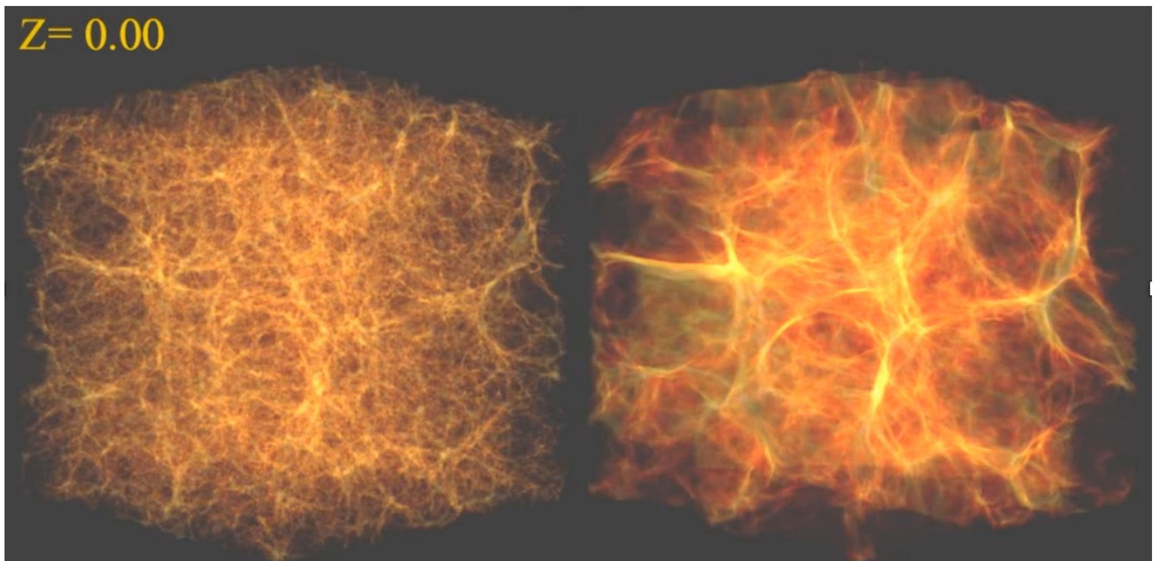


Observable universe
$\sim O(10)$Gpc

Simulations by Troels Haugbølle

Why study neutrino dark matter then?

Because the CνB is a prediction of standard cosmology.

- **Neutrino oscillations** provide a lower limit that at least one neutrino mass eigenstate has a mass $> 0.05\ \mathrm{eV}$.
- KATRIN (**tritium $\boldsymbol{\beta}$-decay**) provides an upper limit on the effective ν_e mass of $< 0.9\ \mathrm{eV}$.

Prediction →

$$\boxed{0.1\% < \Omega_{\nu,0} = \sum \frac{m_\nu}{94\, h^2} < 6\%}$$

- Although only a subdominant DM component, the free-streaming behaviour of neutrino DM still leaves an **imprint on large-scale structures**.

→ Can be used to establish $\Omega_{\nu,0}$ and hence the neutrino mass.

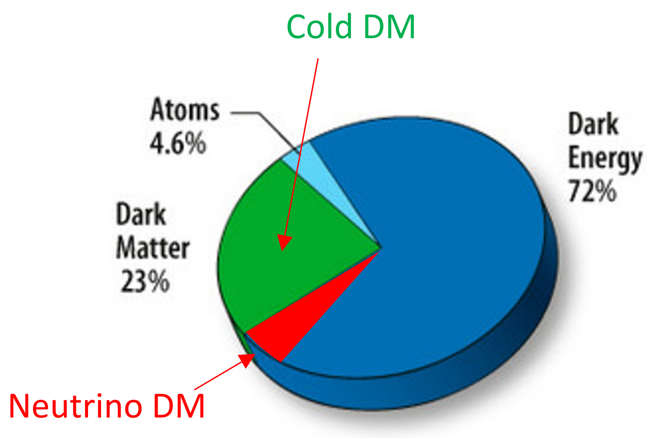
Subdominant neutrino DM…

If neutrino DM is subdominant to CDM, the presence of CDM acts as a **source of density perturbations**.

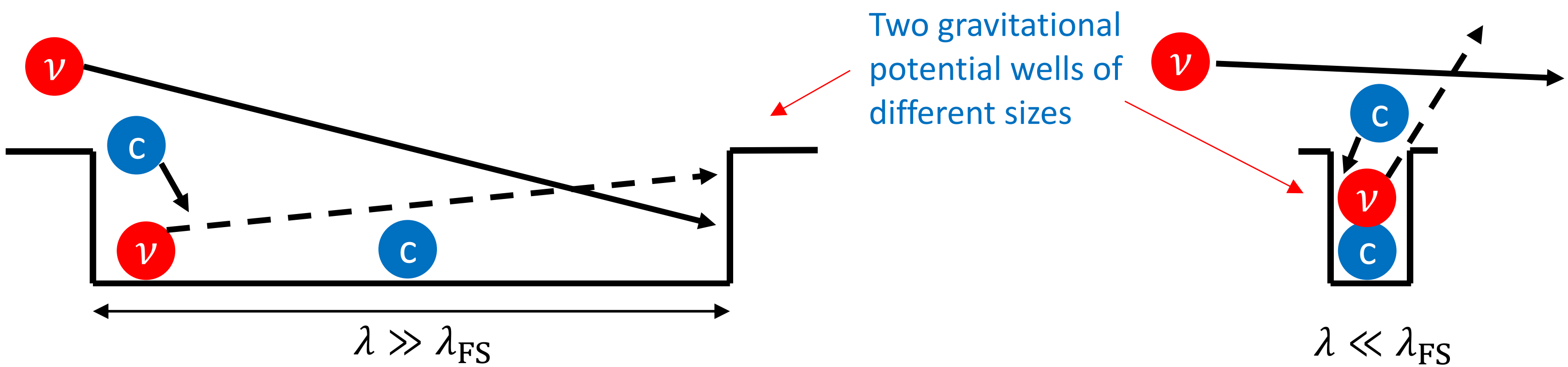
→ Density fluctuations on length scales below the instantaneous free-streaming scale λ_{FS} are **not completely erased**.

- However, the neutrinos' kinetic energy still makes gravitational clustering very difficult.

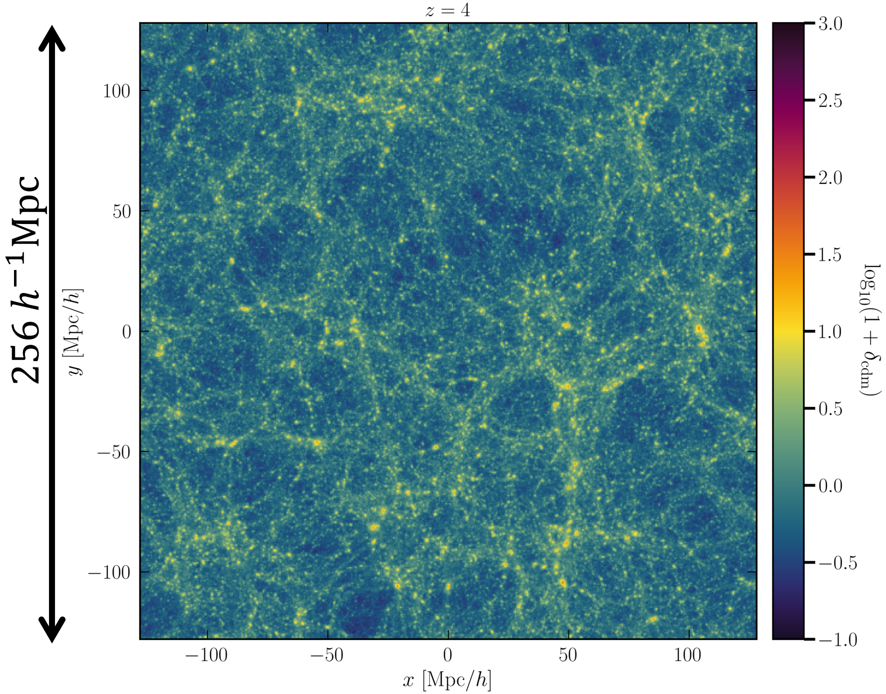
→ Expect a suppression in the abundance of structures on scales below λ_{FS} through free-streaming-induced **potential decay**.



Free-streaming-induced potential decay…

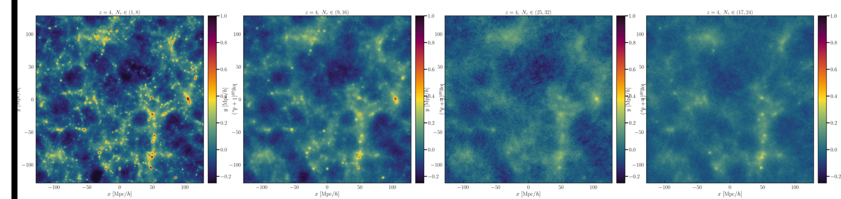


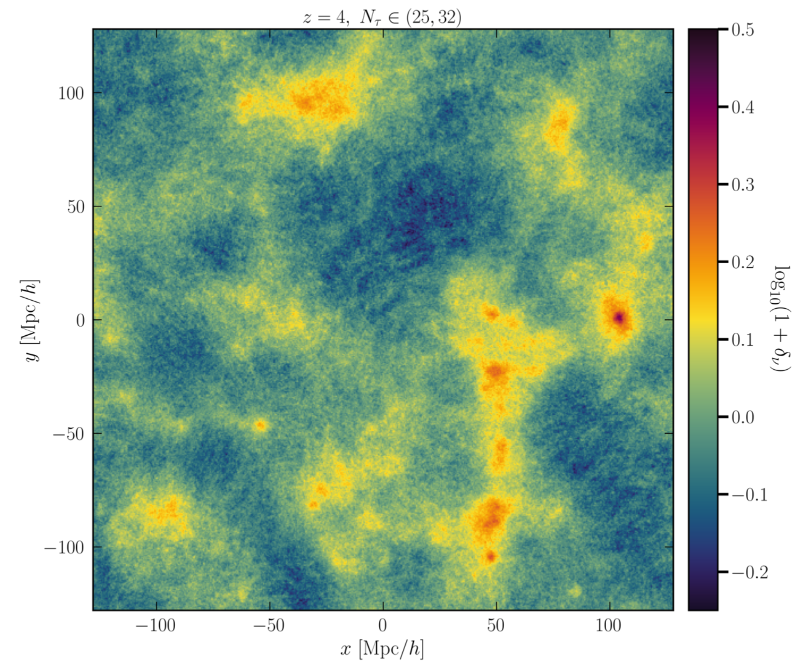
Cold dark matter component



N-body code: Gadget4-Hybrid:
Chen, Mosbech, Upadhye & Y[3]W 2023
Post-processing/graphics: G. Pierobon

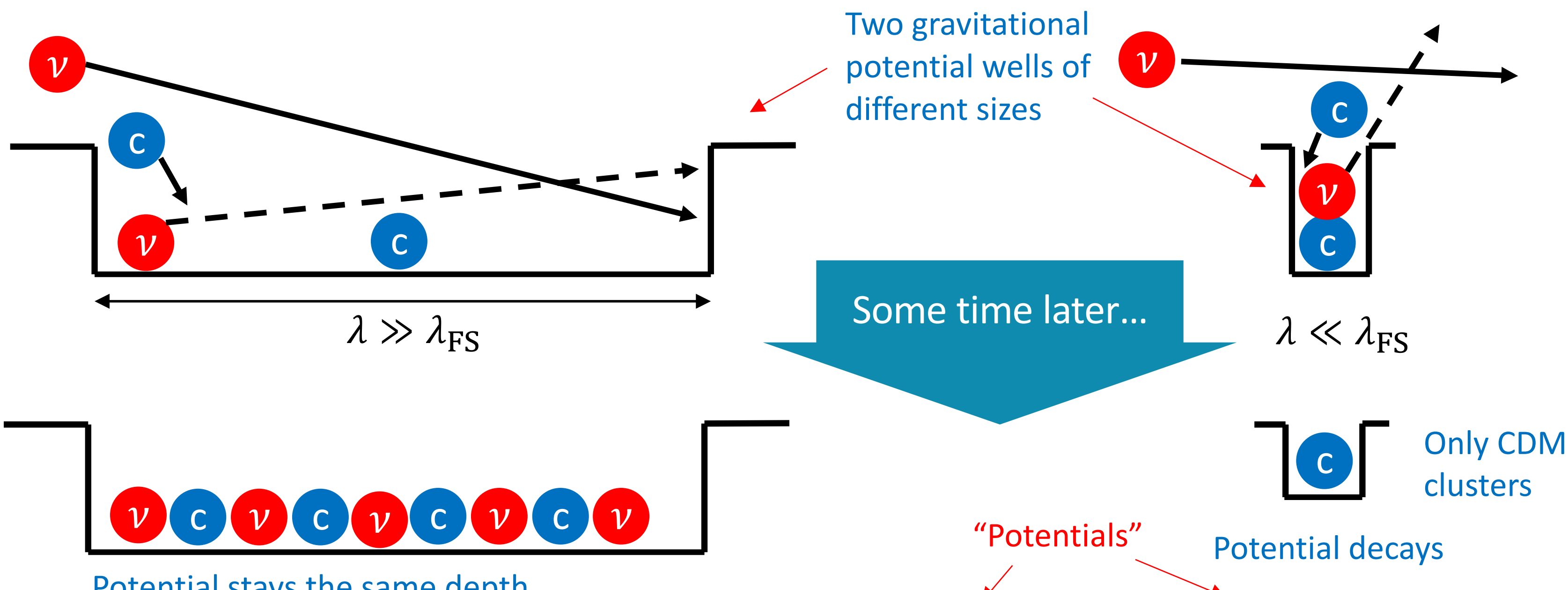
Increasing neutrino momentum





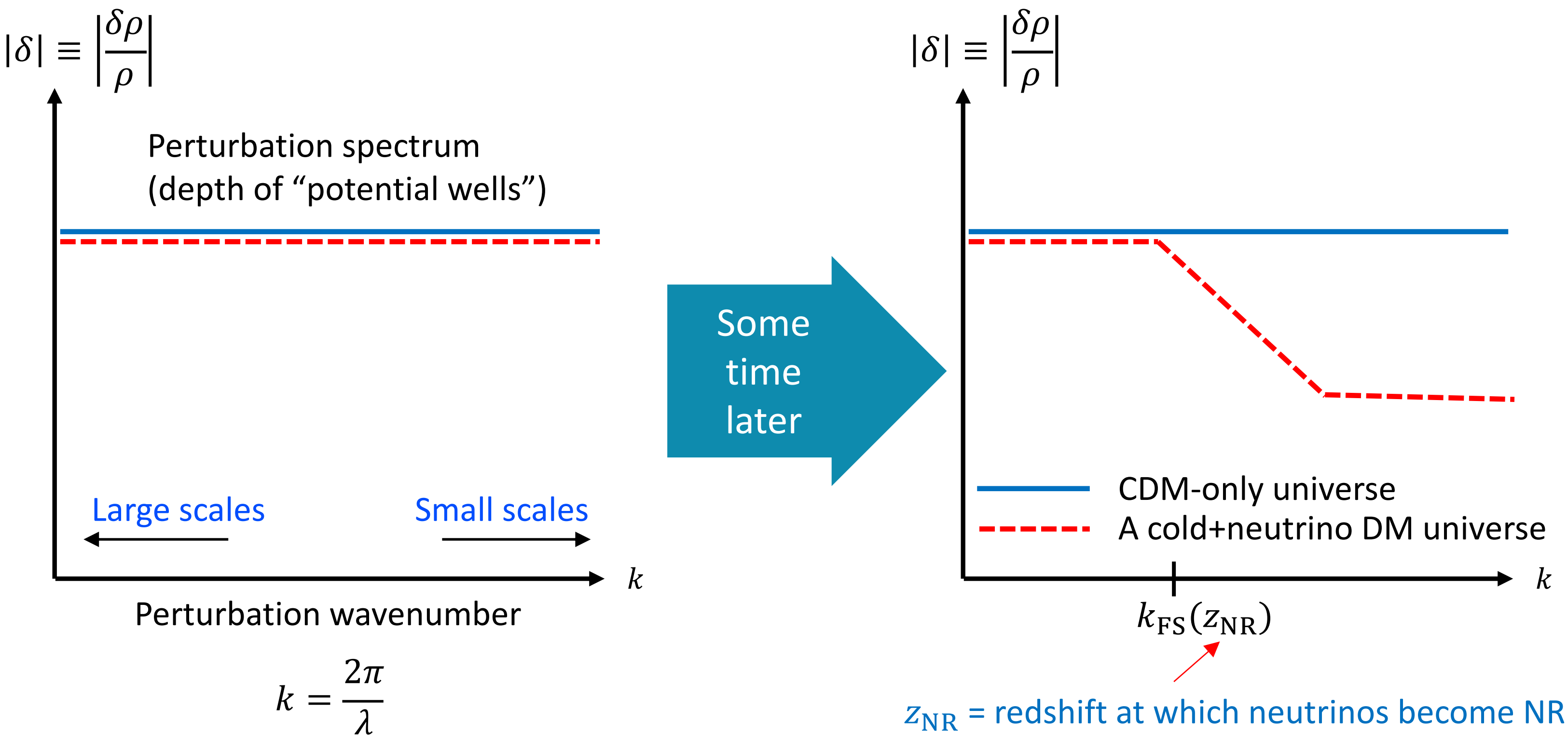
Neutrino component ($\sum m_\nu = 0.5\mathrm{eV}$)

Free-streaming-induced potential decay…



$$ds^2 = a^2(\eta)[-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j]$$

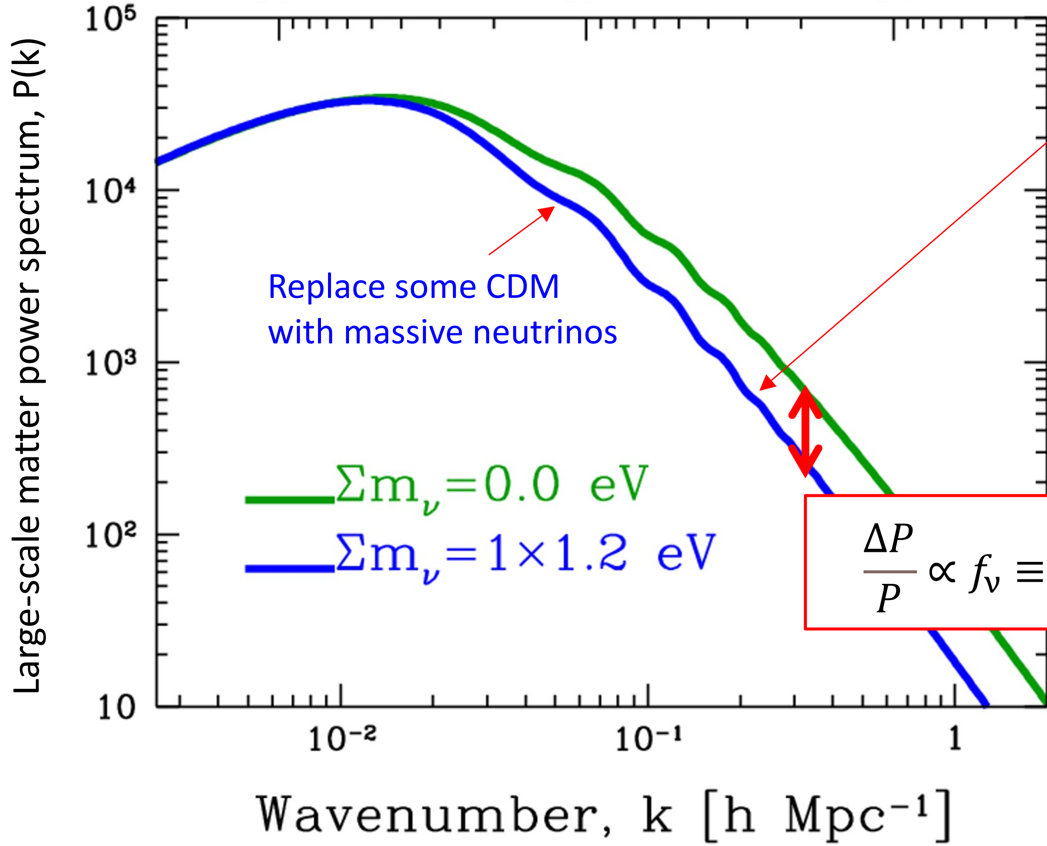
Cosmological neutrino mass "measurement" is based on observing this potential decay at $\lambda \ll \lambda_{\mathrm{FS}}$.



The presence of neutrino dark matter induces a step-like feature in the spectrum of gravitational potential wells.

Large-scale matter power spectrum…

From linear perturbation theory



Suppression of power due to free-streaming-induced potential decay

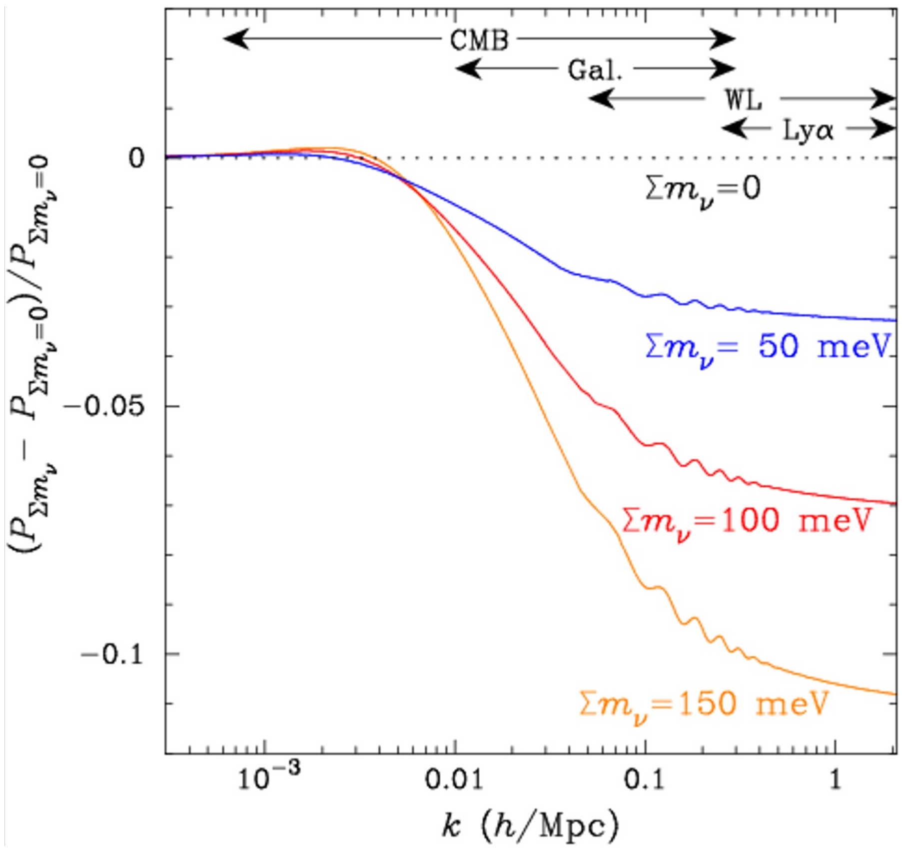
f_ν = Neutrino fraction

$$\frac{\Delta P}{P} \propto f_\nu \equiv \frac{\Omega_\nu}{\Omega_m}$$

$$\Omega_\nu h^2 = \sum \frac{m_\nu}{94\mathrm{eV}}$$

Large-scale matter power spectrum...

Relative power spectrum



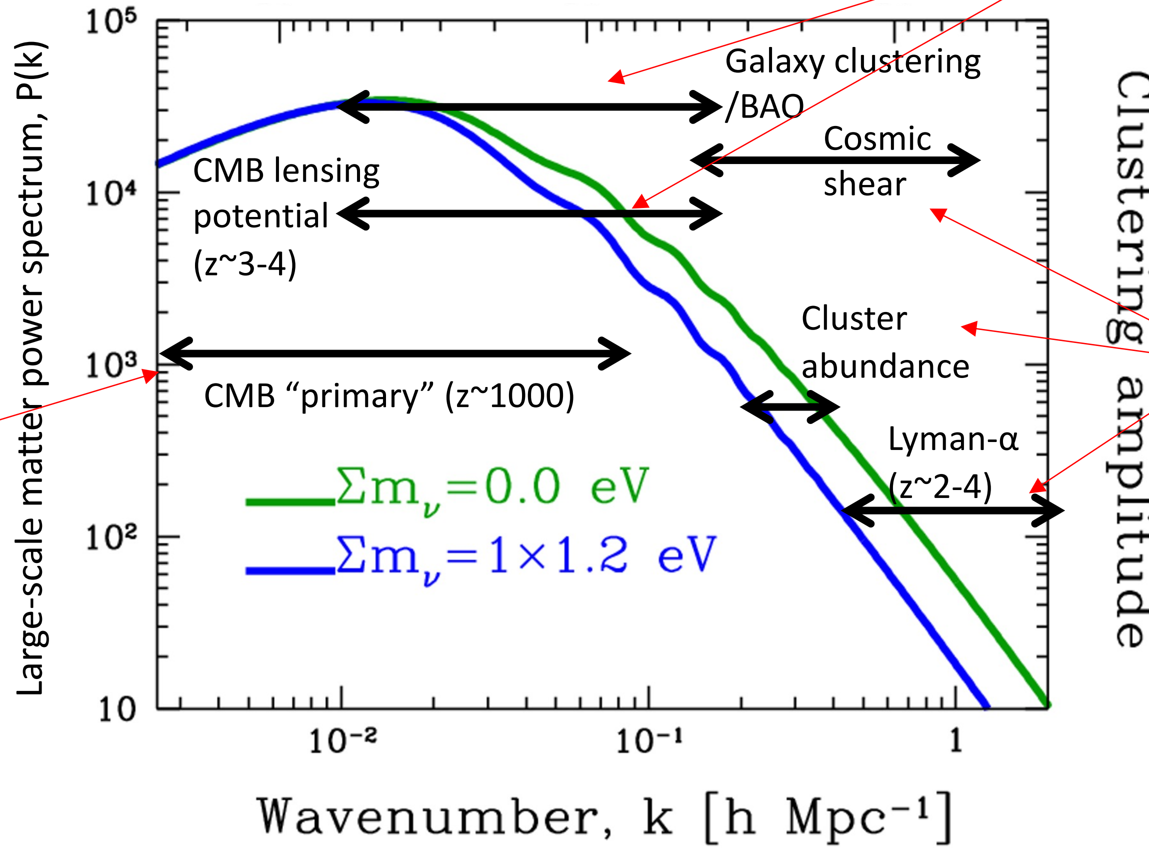
The larger the mass sum, the larger the suppression.

Who can measure it?

ω_m + geometry

Physical matter density

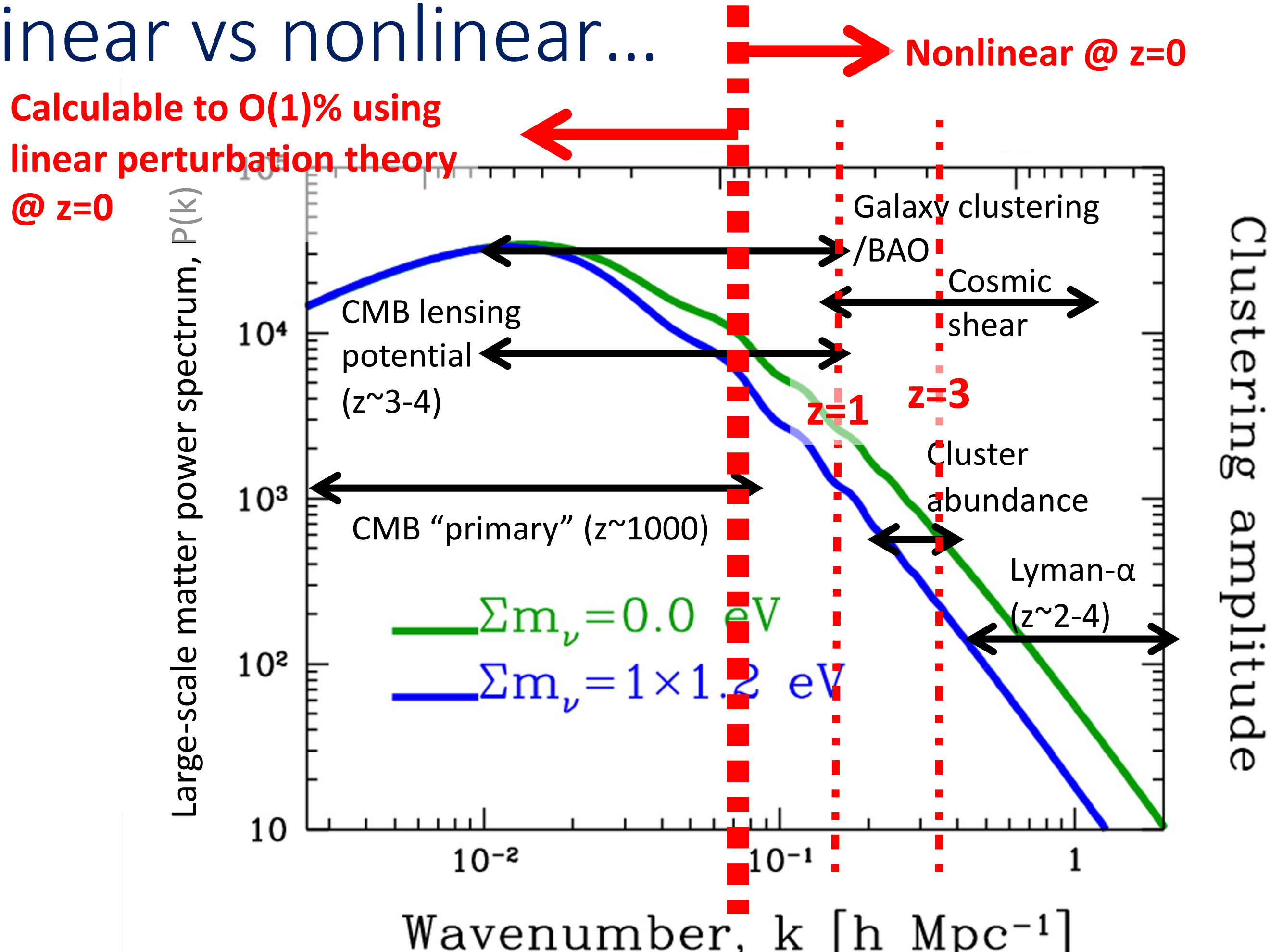
"Anchor" ΛCDM model parameters $\omega_m, \omega_b, n_s, A_s, \tau, h$



σ_8, Ω_m

Small-scale fluctuation amplitude

Linear vs nonlinear…

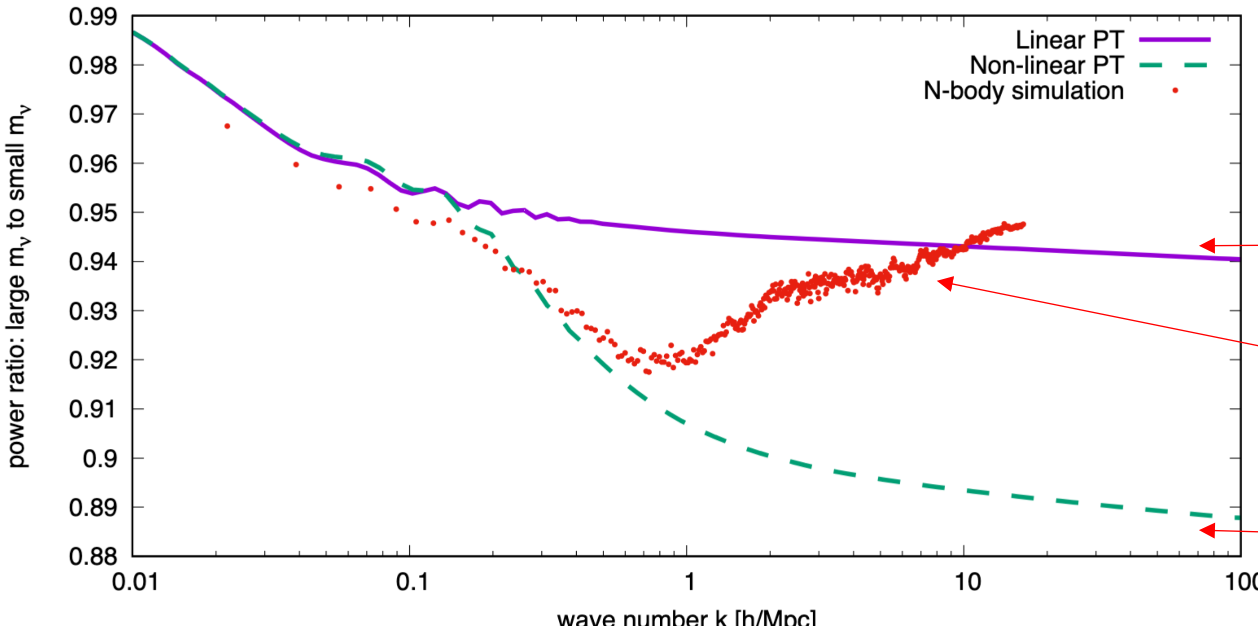


There are nonlinearities and nonlinearities…

	Nonlinear Dark matter (collisionless)	**Baryonic astrophysics @ k ~ 1/Mpc**	**Empirical tracers or proxies**
CMB	No	No	No
BAO	Mild	No	Mild
Cosmic shear	Yes	No	No
Galaxy power spectrum	Yes	No	Assume galaxy number density tracks DM density
Cluster abundance	Yes	No	X-ray temperature, cluster richness as proxies for mass
Lyman alpha	Yes	Hydrogen distribution	No
Calculable from first principles (i.e., described by a Lagrangian)?	**Yes**	**No**	**No**

“Fairly easily” calculable nonlinearities…

Collisionless nonlinearities concern only the gravitational interactions of the cold dark matter and neutrinos.



Linear perturbation theory

+ Nonlinear corrections (N-body simulation)

+ Nonlinear corrections (perturbation theory)

Hannestad, Upadhye & Y³W 2020

N-body simulations…

Standard method for computing **nonlinear CDM dynamics**.

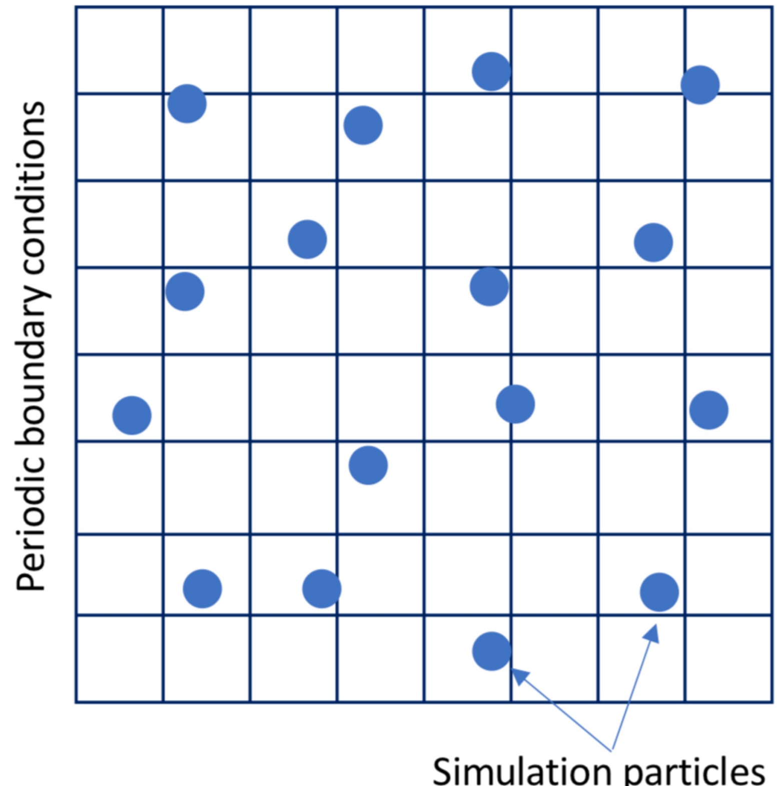
- Discretise CDM fluid into particles (10M to 10B, depending on what you want to do)
- Solve equations of motion for each particle under gravity.

Equations of motion

$$\frac{d\vec{x}}{d\tau} = \frac{p}{am} \qquad \frac{d\vec{p}}{d\tau} = -am\nabla\Phi$$

Poisson equation

$$\nabla^2\Phi = 4\pi G a^2 \delta\rho$$



Review in Angulo & Hahn arXiv:2112.05165

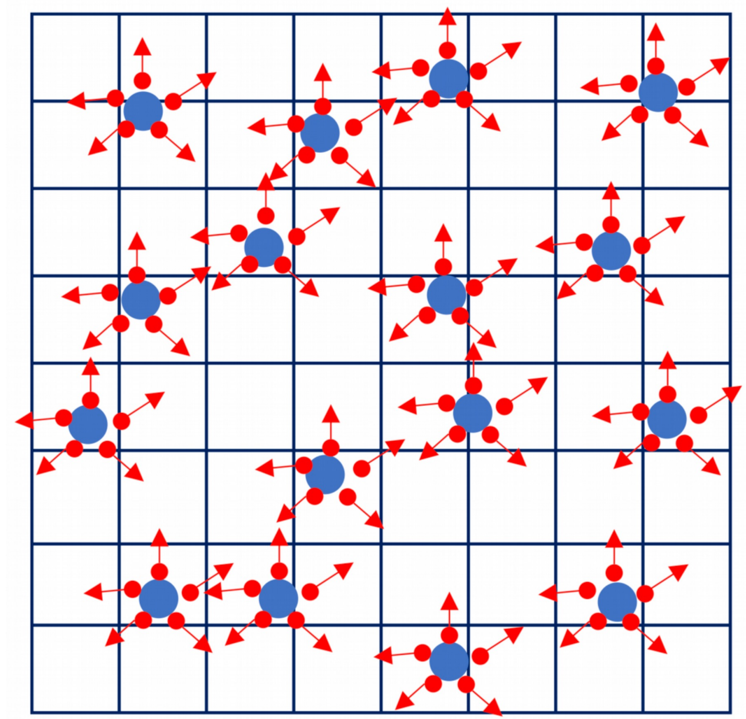
N-body simulations with neutrinos…

We can in principle do the same thing with the CνB. But…

- Need several neutrino particles per CDM particles, sampled from the FD distribution, to model free-streaming.
- Neutrino particles have very large initial velocities.

→ In practice, this type of simulations very computationally demanding because of **shot-noise** and **long run time**.

→ Finding cleverer ways to do these simulations is an active area of research.



Partial review in Angulo & Hahn arXiv:2112.05165

Currents bounds on the neutrino mass sum…

There is **no** cosmological measurement of the neutrino mass sum yet.

- Current constraints on $\sum m_\nu$ are typically $O(0.1 - 0.3)$ eV, depending on exactly how you do the analysis → Model dependence.

6+1 fit parameters

Primordial tensors

Dynamical dark energy

Spatial curvature

Model	Degenerate	Normal	Inverted
Baseline ΛCDM+Σm_ν	**0.121**	**0.146**	**0.172**
+ r	0.115	0.142	0.167
+ w	0.186	0.215	0.230
+ $w_0 w_a$	0.249	0.256	0.276
+ $w_0 w_a$, $w(z) > -1$	0.096	0.129	0.157
+ Ω_k	0.150	0.173	0.198

Factor of 3 variation between min and max.

Roy Choudhury & Hannestad 2019

It is possible to relax the bound further…

You can also **alter the physics and properties of the CνB** itself to physically relax cosmological constraints.

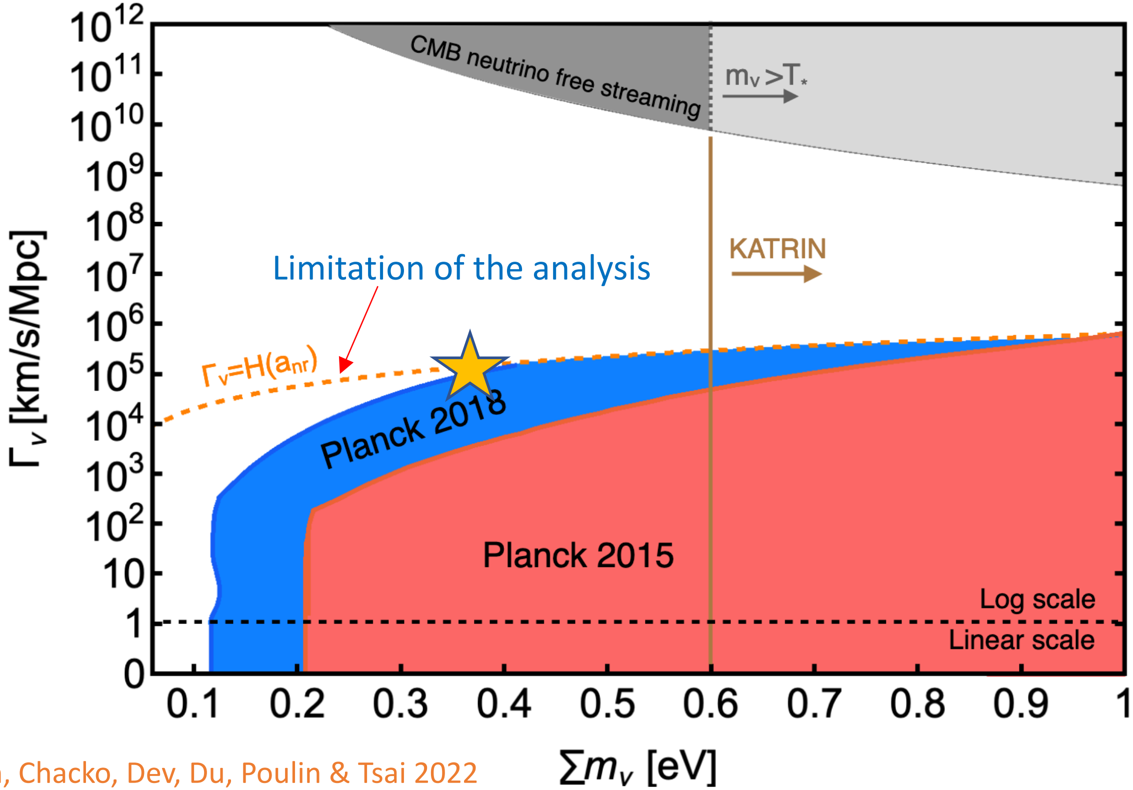
- Neutrino decay
- Neutrino spectral distortion
- Late-time neutrino mass generation
- …

These “physics” games can usually buy you more room for play, provided you are happy to accept the non-standard neutrino physics.

Non-relativistic neutrino decay…

… into dark radiation

Official Planck benchmark:
$\sum m_\nu < 0.12$ eV



Abellán, Chacko, Dev, Du, Poulin & Tsai 2022

If neutrinos decay with a **lifetime**

$$\tau_\nu \sim 0.1 \text{ Myr}$$

then it is possible to accommodate

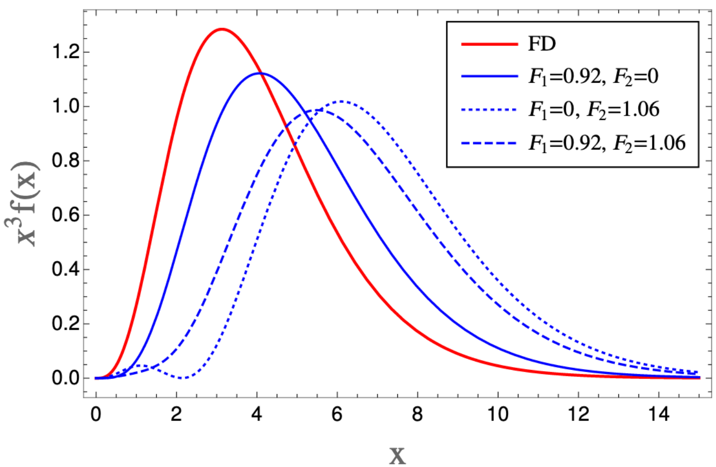
$$\sum m_\nu \lesssim 0.42 \text{ eV}$$

Planck+BAO+SN

Neutrino spectral distortion…

Enhancing the average momentum (via decay, interaction, etc.) while maintaining the early-time neutrino energy density (i.e., $N_{\rm eff}$) relaxes the neutrino mass bound.

Planck 2015



	TT+lowP (95 % CL)	TT+lowP+BAO (95 % CL)
FD	$\sum m_\nu < 0.73$ eV	$\sum m_\nu < 0.18$ eV
$F_1 = 0.92, F_2 = 0$	$\sum m_\nu < 0.95$ eV	$\sum m_\nu < 0.26$ eV
$F_1 = 0, F_2 = 1.06$	$\sum m_\nu < 1.45$ eV	$\sum m_\nu < 0.37$ eV
$F_1 = 0.92, F_2 = 1.06$	$\sum m_\nu < 1.34$ eV	$\sum m_\nu < 0.32$ eV

Oldengott, Barenboim, Kahlen, Salvado & Schwarz 2019

- If you're adventurous and take a Gaussian momentum distribution, you could even relax the bound to $\sum m_\nu \lesssim 3$ eV. Alvey, Escudero & Sabti 2022

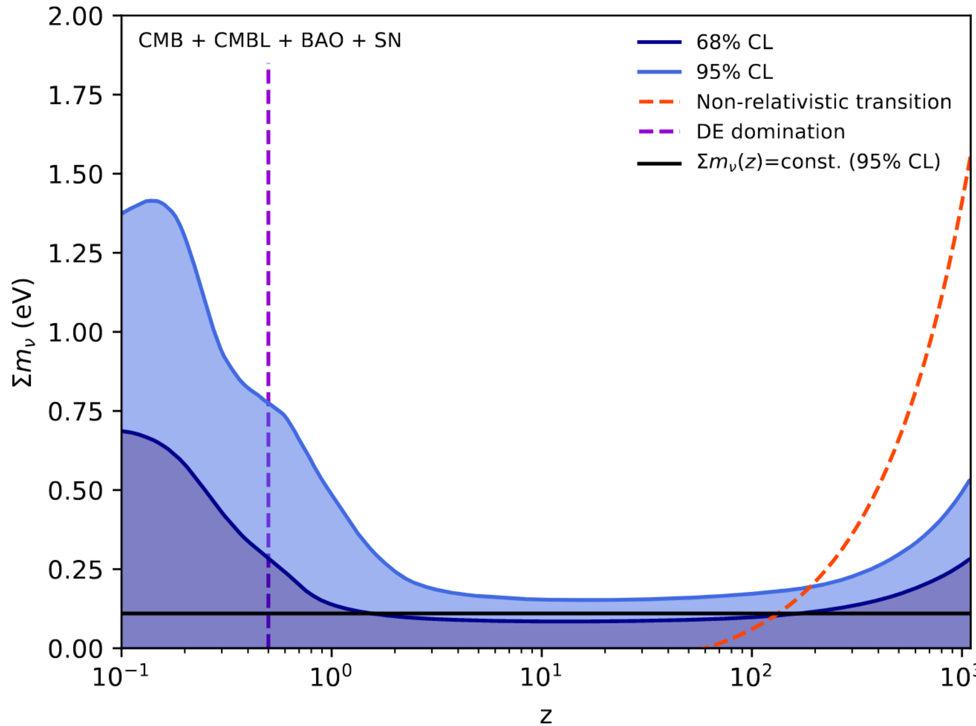
Late-time ν mass generation...

Official Planck benchmark:
$\sum m_\nu < 0.12$ eV

Late-time mass through a phase transition at $T \sim \text{meV}$. Dvali & Funcke 2016

- But phenomenologically, if neutrinos pick up masses only after $z \sim 1$, then this is allowed:

$$\sum m_\nu \lesssim 1.46 \text{ eV}$$



Lorenz, Funcke, Löffler & Calabrese 2021

Take-home message so far…

Massive neutrinos leave an imprint on the cosmic large-scale structure.

- We can use this to measure/constrain neutrino masses with cosmological observations.
- Current constraint the **neutrino mass sum is** conservatively $\sum m_\nu \lesssim O(0.1 - 0.3)\ \mathrm{eV}$.
    - The range comes from how exactly you do the analysis, e.g., what background cosmology you use, etc.
    - You can evade the tightest constraints to a good extent with very non-standard neutrino physics.

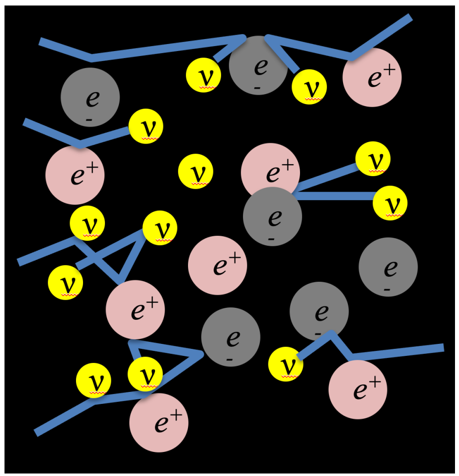
3. Relativistic neutrino free-streaming and non-standard interactions...

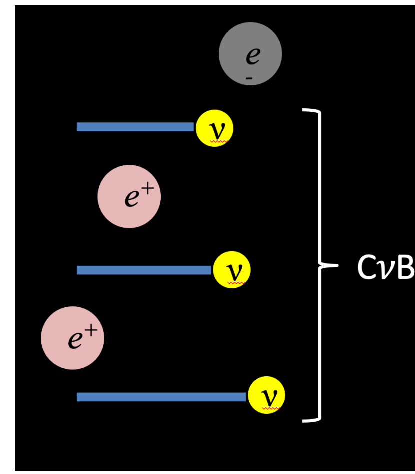
Cosmic neutrino background …

Interaction rate: $\Gamma_{\text{weak}} \sim G_F^2 T^5$

Expansion rate: $H \sim M_{\text{pl}}^{-2} T^2$

The CνB is formed when neutrinos decouple from the cosmic plasma.





Neutrinos “free-stream” to infinity.

($T_{\odot\text{core}} \sim 1$ keV)

Above $T \sim 1$ MeV, even weakly-interacting neutrinos can be produced, scatter off e^+e^- and other neutrinos, and attain thermodynamic equilibrium

Below $T \sim 1$ MeV, expansion dilutes plasma, and reduces interaction rate: the universe becomes transparent to neutrinos.

Relativistic neutrino free-streaming..

Fundamentally **the same** as non-relativistic neutrino free-streaming.

- But relativistic = can develop significant pressure and anisotropic stress.

~ 0 if neutrinos are NR.

Unperturbed part

$$T^{\mu}{}_{\nu} = \begin{bmatrix} -\bar{\rho} & 0 & 0 & 0 \\ 0 & \bar{P} & 0 & 0 \\ 0 & 0 & \bar{P} & 0 \\ 0 & 0 & 0 & \bar{P} \end{bmatrix}$$

Velocity perturbations

Density perturbations

Perturbed part

$$+ \begin{bmatrix} -\delta\rho & (\bar{\rho}+\bar{P})v_{\parallel} & (\bar{\rho}+\bar{P})v_{\parallel} & (\bar{\rho}+\bar{P})v_{\parallel} \\ -(\bar{\rho}+\bar{P})v_{\parallel} & \delta P & \Sigma_2^1 & \Sigma_3^1 \\ -(\bar{\rho}+\bar{P})v_{\parallel} & \Sigma_1^2 & \delta P & \Sigma_3^2 \\ -(\bar{\rho}+\bar{P})v_{\parallel} & \Sigma_1^3 & \Sigma_2^3 & \delta P \end{bmatrix}$$

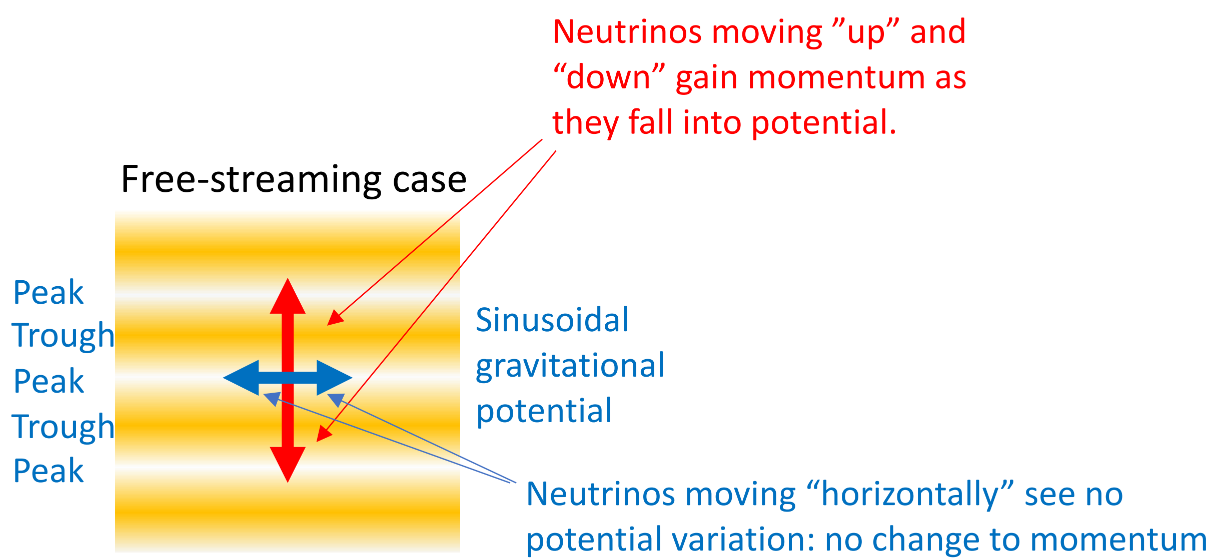
Anisotropic stress

Pressure perturbations

Free-streaming and anisotropic stress…

Standard-model neutrinos free-stream.

- Free-streaming in an inhomogeneous background induces anisotropic stress (aka momentum anisotropy).



Neutrino anisotropic stress and the metric...

Neutrino anisotropic stress (or lack thereof) leaves distinct imprints on the spacetime metric perturbations.

Conformal Newtonian gauge

Scale factor

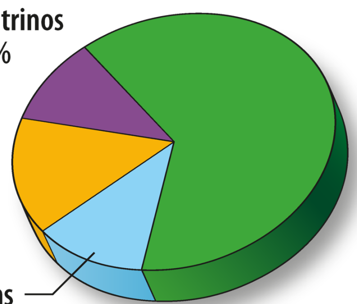
$$\mathrm{d}s^2 = a^2(\tau)[-(1+2\psi)\mathrm{d}\tau^2 + (1-2\phi)\mathrm{d}x^i\mathrm{d}x_i]$$

where $$k^2(\phi - \psi) = 12\pi G a^2(\bar{\rho} + \bar{P})\sigma$$

Mean energy density & pressure

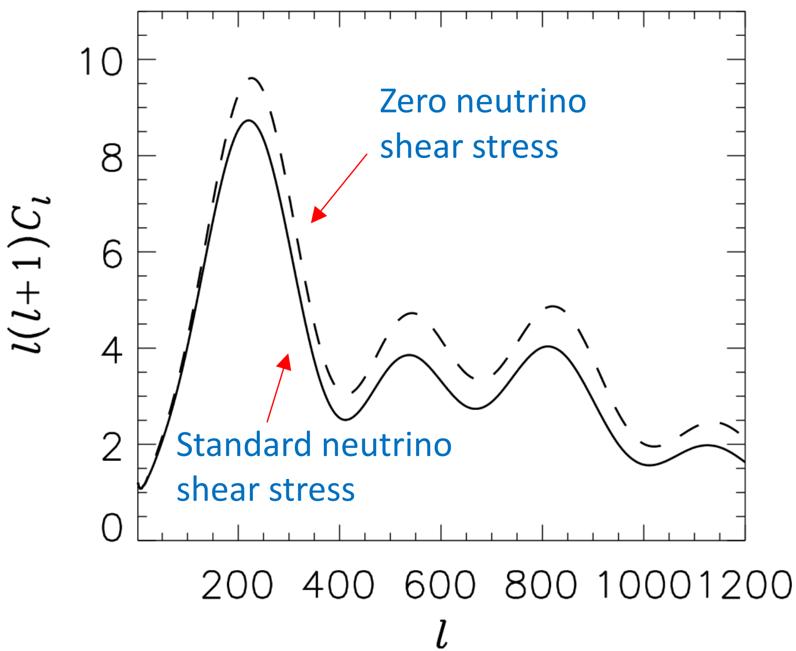
Anisotropic stress

In ΛCDM, mainly from ultra-relativistic neutrinos and photons.



- Changes to $(\phi - \psi)$ around CMB times $(t \sim 400\ \mathrm{kyr})$ affect the evolution of CMB perturbations and are observable in the **CMB TT power spectrum**.

Neutrino anisotropic stress & the CMB…

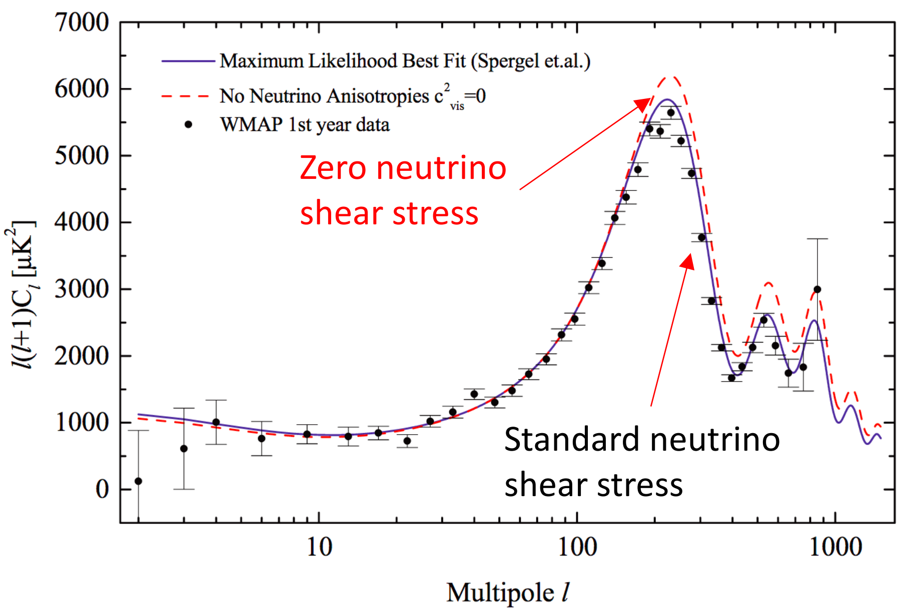


Hannestad 2005

Removing neutrino anisotropic stress enhances power at multipoles $\ell \gtrsim 200$ in the **CMB TT spectrum**.

- Effect is mildly degenerate with the primordial fluctuation amplitude and spectral tilt.
- But even with **WMAP-1st year data**, it was already possible to exclude zero neutrino anisotropic stress at $\gtrsim 2\sigma$.

Neutrino anisotropic stress & the CMB…



Melchiorri & Trotta 2005

So what can we do with this??

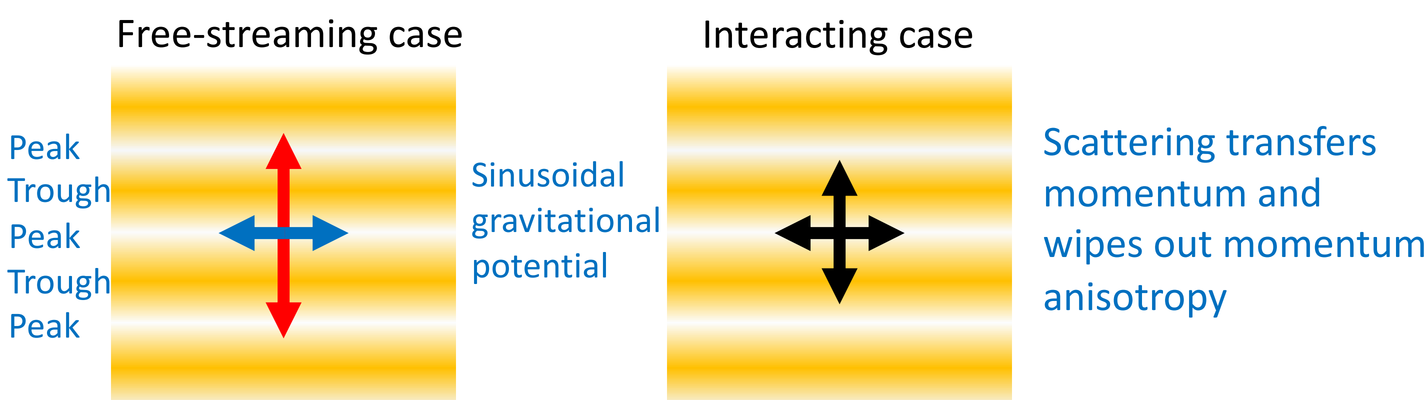
Removing neutrino anisotropic stress enhances power at multipoles $\ell \gtrsim 200$ in the **CMB TT spectrum**.

- Effect is mildly degenerate with the primordial fluctuation amplitude and spectral tilt.
- But even with **WMAP-1st year data**, it was already possible to exclude zero neutrino anisotropic stress at $\gtrsim 2\sigma$.

Free-streaming vs interacting…

Standard-model neutrinos free-stream.

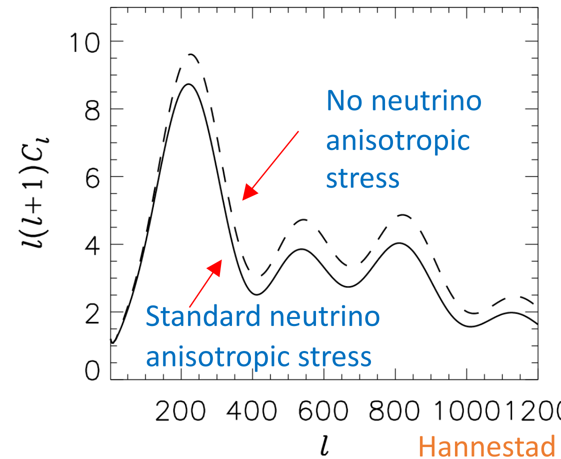
- Free-streaming in an inhomogeneous background induces anisotropic stress (aka momentum anisotropy).
- Conversely, interactions transfer momentum and, if sufficiently efficient, can wipe to out anisotropy.



Using anisotropic stress to test ν interactions.

Demanding that neutrinos free-streaming at CMB times ($t \sim 400$ kyr), we can constrain non-standard neutrino interactions in that epoch.

- Neutrino self-interaction
- Relativistic neutrino decay
- (Neutrino-DM interactions wipe anisotropic stress too. But because it involves DM, the phenomenology is slightly different.)



Hannestad 2005

To do so, we need to figure out the **isotropisation timescale** $T_{\text{isotropise}}$ given an interaction.

Tracking neutrino perturbations…

The standard approach is to use the **relativistic Boltzmann equation** to describe the neutrino phase space distribution $f_i(x^\mu, P^i)$.

Liouville operator

$$P^\mu \frac{\partial f_i}{\partial x^\mu} - \Gamma^\nu_{\rho\sigma} P^\rho P^\sigma \frac{\partial f_i}{\partial P^\nu} = 0$$

Gravitational effects

Integrate in momentum:
$\ell = 0 \rightarrow$ density and pressure perturbations
$\ell = 1 \rightarrow$ velocity perturbations
$\ell \geq 2 \rightarrow$ anisotropies

- **Split** into $f_i(x^\mu, P^i) = \bar{f}_i(x^0, |P^i|) + F_i(x^\mu, P^i)$
- **Linearise** and go to Fourier space $x^i \leftrightarrow k^i$
- **Decompose** $F_i(x^o, k^i, P^i)$ into a Legendre series in $k \cdot P$.

Ma & Bertschinger 1995

Adding a short-range particle interaction…

To describe a **short-range interaction**, add a collision integral to the RHS of the relativistic Boltzmann equation for $f_i(x^\mu, P^i)$.

Liouville operator

$$P^\mu \frac{\partial f_i}{\partial x^\mu} - \Gamma^\nu_{\rho\sigma} P^\rho P^\sigma \frac{\partial f_i}{\partial P^\nu} = C[f]$$

Collision integral

Gravitational effects

Integrate in momentum:
$\ell = 0 \rightarrow$ density and pressure perturbations
$\ell = 1 \rightarrow$ velocity perturbations
$\ell \geq 2 \rightarrow$ anisotropies

- **Split** into $f_i(x^\mu, P^i) = \bar{f}_i(x^0, |P^i|) + F_i(x^\mu, P^i)$
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- **Decompose** $F_i(x^o, k^i, P^i)$ into a Legendre series in $k \cdot P$.

Ma & Bertschinger 1995

Collision integral and the isotropisation rate…

Given an interaction Lagrangian, the collision integral for $f_i(x^\mu, P^i)$ is

$$C[f] = \frac{1}{2}\left(\prod_j^N \int g_j \frac{\mathrm{d}^3\mathbf{n}_j}{(2\pi)^3 2E_j(\mathbf{n}_j)}\right)\left(\prod_k^M \int g_k \frac{\mathrm{d}^3\mathbf{n}_k}{(2\pi)^3 2E_k(\mathbf{n}_k)}\right)$$
$$\times (2\pi)^4\, \delta_D^{(4)}\left(p + \sum_j^N n_j - \sum_k^M n_k'\right) |\mathcal{M}_{i+j_1+\cdots+j_N \leftrightarrow k_1+\cdots+k_M}|^2$$
$$\times [f_{k_1}\cdots f_{k_N}(1\pm f_i)(1\pm f_{j_1})\cdots(1\pm f_{j_N}) - f_i f_{j_1}\cdots f_{j_N}(1\pm f_{k_1})\cdots(1\pm f_{k_M})]$$

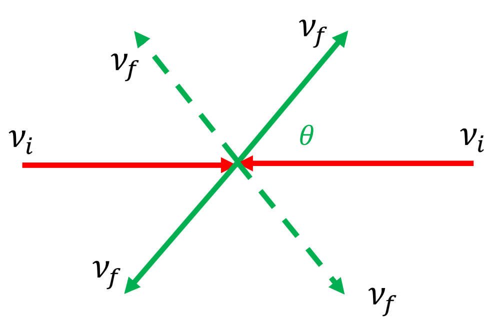
- **To compute the isotropisation rate**, follow the previous procedure of linearisation and decomposition into a Legendre series.

→ The damping rate of the quadrupole ($\ell = 2$) moment represents the lowest-order isotropisation rate of the neutrino ensemble.

Tedious stuff, but this is really the only correct way to calculate these things, else you can get it very wrong…
However, the result can usually be understood in simple terms. → **Next slide**

Isotropisation from ν self-interaction…

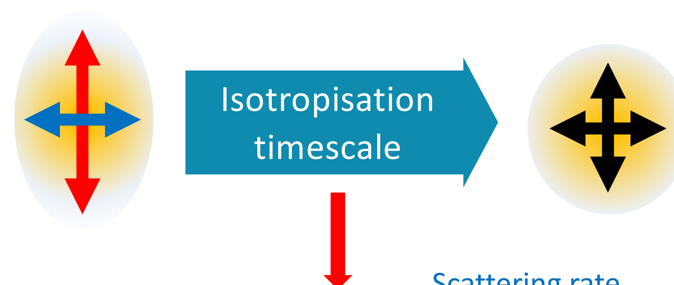
Consider a $2 \rightarrow 2$ scattering event $\nu_i + \nu_i \rightarrow \nu_f + \nu_f$.



- The probability of ν_f emitted at any angle θ is the same for all $\theta \in [0, \pi]$.

Cyr-Racine & Sigurdson 2014; Oldengott, Rampf & Y[3]W 2015; Lancaster, Cyr-Racine, Knox & Pan 2017; Oldengott, Tram, Rampf & Y[3]W 2017; Kreisch, Cyr-Racine & Dore 2019; Forastieri et al. 2019; Camarena & Cyr-Racine 2023, etc.

→ Particles in two head-on ν_i beams need only scatter once to transfer their momenta equally in all directions.



Scattering rate

$$T_{\text{isotropise}} \sim 1/\Gamma_{\text{scattering}} \gtrsim 400\ \text{kyr}$$

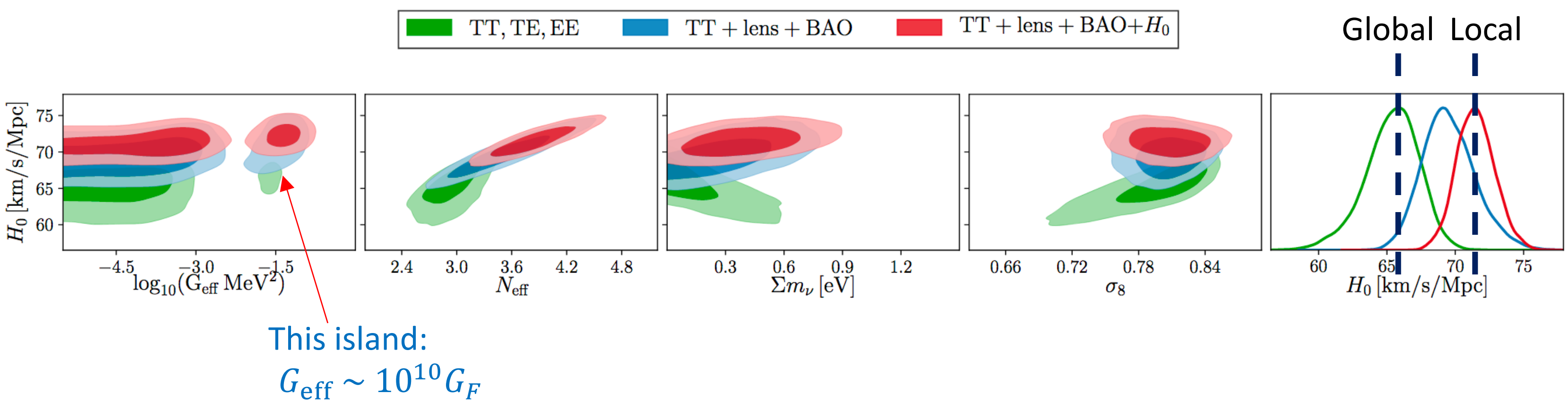
→ Upper limit on $\Gamma_{\text{scattering}}$ (hence coupling).

ν self-interaction and the H_0 tension...

Kreisch, Cyr-Racine & Dore 2019

Recent claim that self-interaction **alleviates the Hubble tension**.

- **Local/late time**: Cepheid-calibrated SNIa (SH0ES) and strong-lensing time delays (H0liCOW); $H_0 = (73.5 \pm 1.4)\ \mathrm{km/s/Mpc}$
- **Global/early time**: Statistical inference from CMB anisotropies (Planck), weak lensing, BAO; $H_0 = (67.4 \pm 0.5)\ \mathrm{km/s/Mpc}$



Isotropisation from invisible neutrino decay…

Invisible here means the decay products do **not** include a photon.

- **SM 1 → 3 decay:** $\nu_j \to \nu_i \nu_k \bar{\nu}_k$, but the rate is proportional to m_ν^6.

  → For sub-eV neutrino masses, the neutrino lifetime would be $> 10^{10}$ longer than the present age of the universe, i.e., not very interesting.

  Bahcall, Cabibbo & Yahil 1972

- **Beyond SM**: generically one could consider

$$\nu_H \to \nu_l + \phi$$

SM neutrinos (sub-eV masses)

Some almost massless boson (scalar, pseudo-scalar, vector)

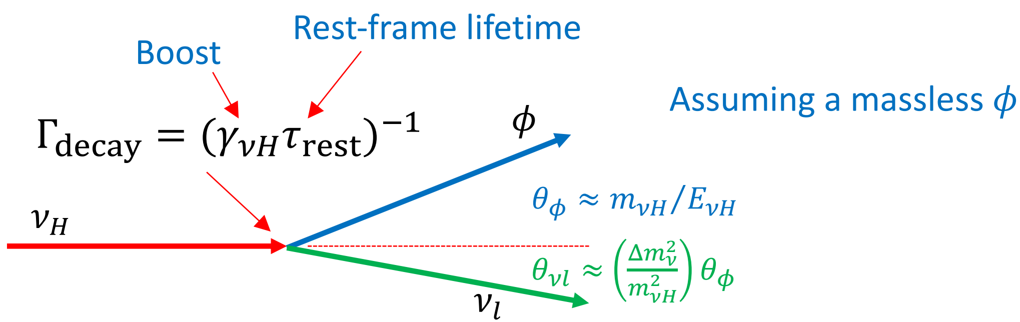
  - More freedom with the coupling strength and hence lifetime.
  - Predicted by a many extensions to the SM (mostly linked to neutrino mass generation or dark matter).

Gelmini & Roncadelli 1981; Chikashige, Mohapatra & Peccei 1981; Schechter & Valle 1982; Dror 2020; Ekhterachian, Hook, Kumar & Tsai 2021; etc.

Isotropisation from relativistic $1 \to 2$ decay…

How long does it take $\nu_H \to \nu_l + \phi$ and its inverse process to wipe out momentum anisotropies? (Hint: it's not the lifetime of ν_H.)

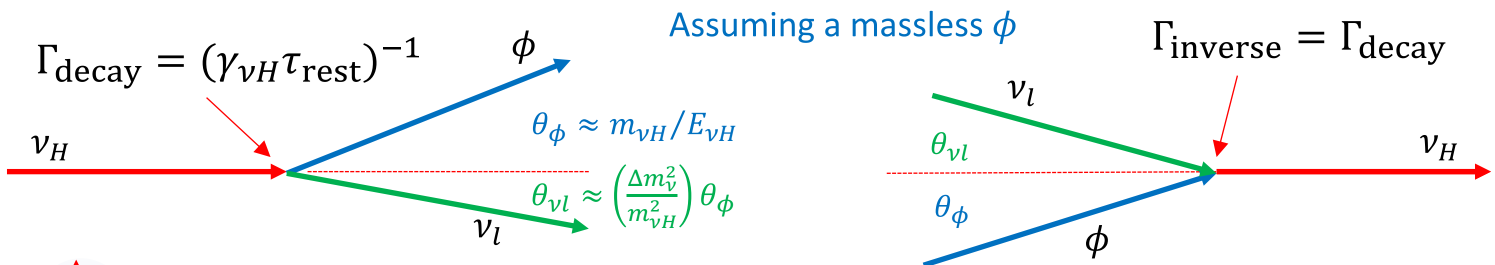
- In relativistic decay, the decay products are **beamed**.



Isotropisation from relativistic $1 \rightarrow 2$ decay…

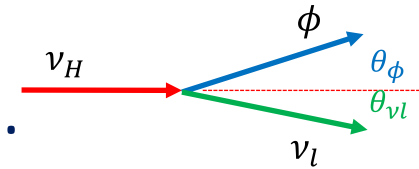
How long does it take $\nu_H \rightarrow \nu_l + \phi$ and its inverse process to wipe out momentum anisotropies? (Hint: it's not the lifetime of ν_H.)

- In relativistic decay, the decay products are **beamed**.
- Inverse decay also only happens when the daughter particles meet **strict momentum/angular requirements**.



→ Isotropisation is going to take a loooong time compared with the ν_H lifetime.

The isotropisation rate is calculable…



With some reasonable approximations (e.g., separation of scales), we have calculated the damping rate of the ℓth neutrino kinetic moment from relativistic $\nu_H \to \nu_l + \phi$ and its inverse process:

T_0 = comoving neutrino temperature

$$\frac{d\mathcal{F}_{\ell\geq 2}}{dt} = -\alpha_\ell \tilde{\Gamma}_{\mathrm{dec}} \overbrace{\left(\frac{a m_{\nu H}}{T_0}\right)^4 \Phi\left(\frac{m_{\nu l}}{m_{\nu H}}\right)}^{\sim(\theta_\phi \theta_{\nu l})^2} \mathfrak{F}\left(\frac{a m_{\nu H}}{T_0}\right) \mathcal{F}_{\ell\geq 2}$$

$O(1)$ prefactor

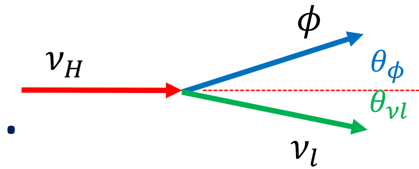
Boosted decay rate, $\sim(\gamma_{\nu H}\tau_{\mathrm{rest}})^{-1}$

Phase space factor $\sim \frac{1}{3}\left(\frac{\Delta m_\nu^2}{m_{\nu H}^2}\right)^2$

Bonus: Relativistic to non-relativistic transition:
~ 1-10 when relativistic; drops to 0 when non-relativistic

Barenboim, Chen, Hannestad, Oldengott, Tram & Y[3]W 2021
Chen, Oldengott, Pierobon & Y[3]W 2022

The isotropisation rate is calculable…



With some reasonable approximations (e.g., separation of scales), we have calculated the damping rate of the ℓth neutrino kinetic moment from relativistic $\nu_H \to \nu_l + \phi$ and its inverse process:

$$\frac{d\mathcal{F}_{\ell\geq 2}}{dt} \sim -\, T_{\text{isotropisation}}^{-1}\, \mathcal{F}_{\ell\geq 2}$$

Isotropisation timescale from relativistic decay/inverse decay

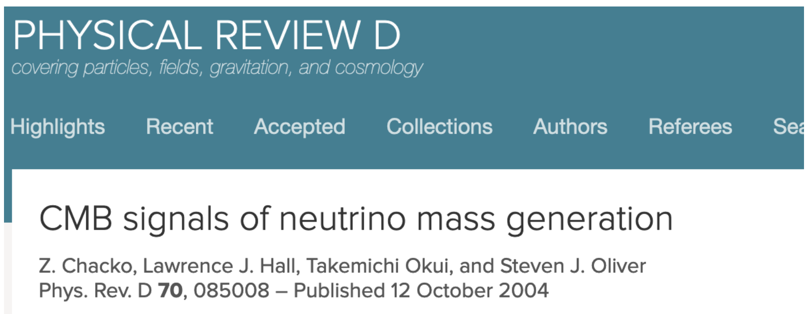
$$\boxed{T_{\text{isotropisation}} \sim \left(\theta_\phi \theta_{\nu_l}\right)^{-2} \gamma_{\nu H}\, \tau_{\text{rest}}}$$

It's model-independent; any dependence on the interaction structure is contained in τ_{rest}; the rest is just kinematics.

Barenboim, Chen, Hannestad, Oldengott, Tram & Y[3]W 2021
Chen, Oldengott, Pierobon & Y[3]W 2022

Comparison with older works…

Two works in the 2000s that considered how long it would take relativistic $1 \to 2$ decay and inverse to isotropise a neutrino ensemble.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CMB signals of neutrino mass generation

Z. Chacko, Lawrence J. Hall, Takemichi Okui, and Steven J. Oliver
Phys. Rev. D **70**, 085008 – Published 12 October 2004


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Constraining invisible neutrino decays with the cosmic microwave background

Steen Hannestad and Georg G. Raffelt
Phys. Rev. D **72**, 103514 – Published 14 November 2005

- **Neither** work actually calculated it… But this is the isotropisation timescale they guessed:

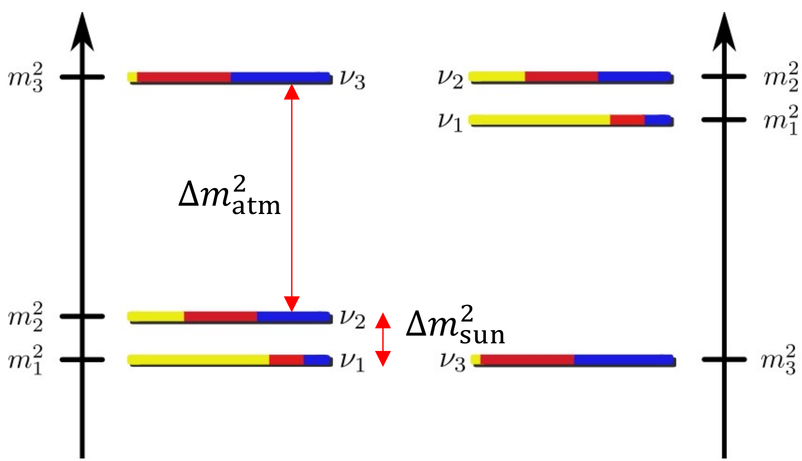
$$T \sim (\theta_{\nu l}\theta_{\phi})^{-1}\gamma_{\nu H}\,\tau_{\mathrm{rest}}$$

cf our first-principles rate:

$$T \sim \left(\theta_{\phi}\theta_{\nu_l}\right)^{-2}\gamma_{\nu H}\,\tau_{\mathrm{rest}}$$

Bounds on the neutrino lifetime: scenarios...

Global neutrino oscillation data currently point to two possible orderings of neutrino masses → **several possible decay/free-streaming patterns**.



Normal mass ordering

Inverted mass ordering

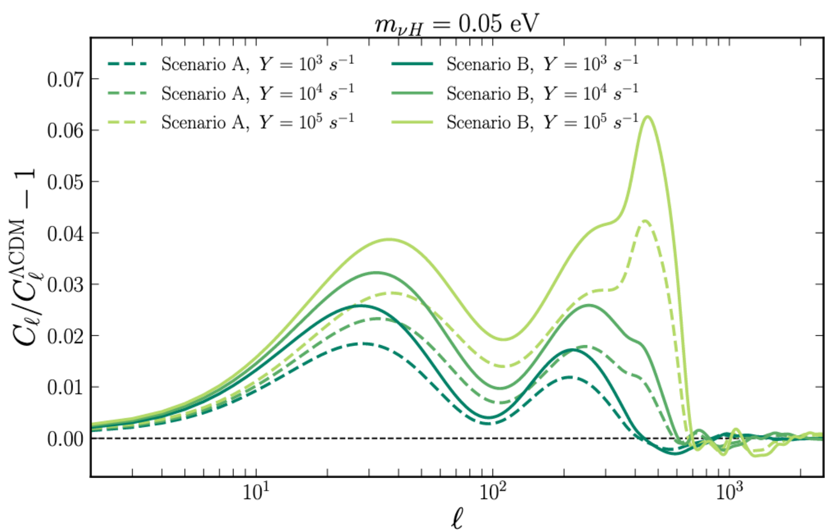
		FS	Decay	Gap	Min $m^2_{\nu H}$
Scenario A: one decay channel					
A1	NO	ν_1	$\nu_3 \to \nu_2$	$\Delta m^2_{32}\|_{\mathrm{N}}$	$\|\Delta m^2_{31}\|_{\mathrm{N}}$
		ν_2	$\nu_3 \to \nu_1$	$\|\Delta m^2_{31}\|_{\mathrm{N}}$	
	IO	ν_2	$\nu_1 \to \nu_3$	$\|\Delta m^2_{31}\|_{\mathrm{I}}$	$\|\Delta m^2_{31}\|_{\mathrm{I}}$
		ν_1	$\nu_2 \to \nu_3$	$\Delta m^2_{23}\|_{\mathrm{I}}$	$\Delta m^2_{23}\|_{\mathrm{I}}$
A2	NO	ν_3	$\nu_2 \to \nu_1$	Δm^2_{21}	Δm^2_{21}
A3	IO	ν_3	$\nu_2 \to \nu_1$		$\Delta m^2_{23}\|_{\mathrm{I}}$
Scenario B: two decay channels					
B1	NO	–	$\nu_3 \to \nu_2, \nu_1$	$\|\Delta m^2_{31}\|_{\mathrm{N}}$	$\|\Delta m^2_{31}\|_{\mathrm{N}}$
B2	IO	–	$\nu_1, \nu_2 \to \nu_3$	$\|\Delta m^2_{31}\|_{\mathrm{I}}$	$\|\Delta m^2_{31}\|_{\mathrm{I}}$

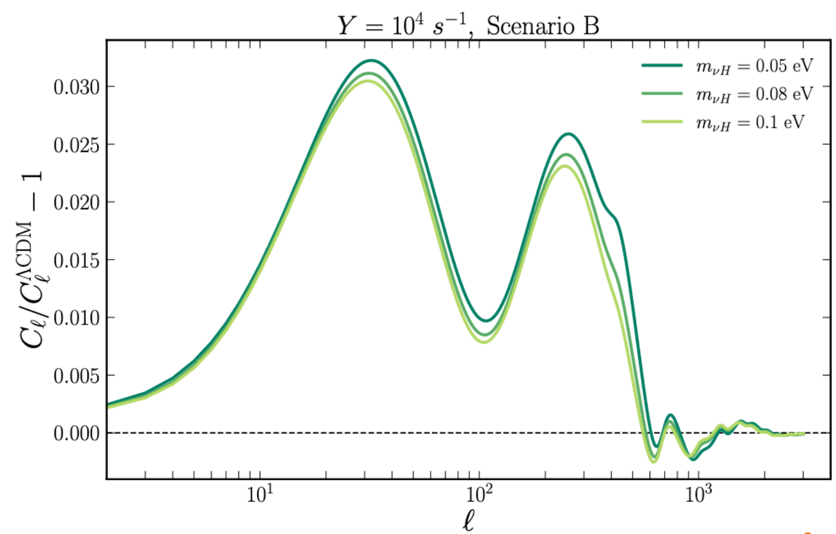
Free-streaming

Decay pairs

Signatures in the CMB TT power spectrum…

Fractional deviations in the CMB TT power spectrum from ΛCDM for various the effective isotropisation rate Y and ν_H masses.





Chen, Oldengott, Pierobon & Y[3]W 2022

Effective isotropisation rate: $Y = 6.55C\times10^{10}\Phi(m_{\nu l}/m_{\nu H})\left(\frac{m_{\nu H}}{0.05eV}\right)^5 \tau_{\mathrm{rest}}^{-1}$

Scenario A = 2 neutrinos participate in decay/inverse decay; Scenario B = all 3 participate

CMB lower bounds on the neutrino lifetime...

Implementing the isotropisation rate in CLASS and using the Planck 2018 CMB TTTEEE+low+lensing data, **our lifetime constraint** is:

Rel to non-rel factor

$$\tau_{\rm rest} \gtrsim 1.2\times10^6\, \mathfrak{F}\left[0.12\left(\frac{m_{\nu H}}{0.05\ \rm eV}\right)\right]\Phi\left(\frac{m_{\nu l}}{m_{\nu H}}\right)\left(\frac{m_{\nu H}}{0.05\ \rm eV}\right)^5 \rm s$$

Phase space factor $\sim \frac{1}{3}\left(\frac{\Delta m_\nu^2}{m_{\nu H}^2}\right)^2$

Chen, Oldengott, Pierobon & Y[3]W 2022

- Or equivalently:

$$\left.\begin{array}{l} \nu_3 \to \nu_{1,2} + \phi \text{ (NO)} \\ \nu_{1,2} \to \nu_3 + \phi \text{ (IO)} \end{array}\right\} \quad \tau_{\rm rest} \gtrsim (6\ - 10)\times10^5 \rm s$$

$$\nu_2 \to \nu_1 + \phi \qquad \tau_{\rm rest} \gtrsim (400 - 500) \rm s$$

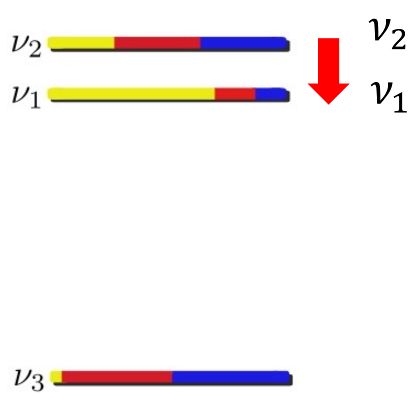
Cf old constraints (using a guesstimated $T_{\rm isotropise}$):

$$\tau_{\rm rest} \gtrsim 10^9\left(\frac{m_{\nu H}}{0.05\ \rm eV}\right)^3 \rm s$$

Hannestad & Raffelt 2005

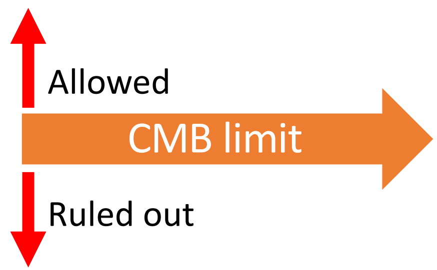
CMB lower bounds on the neutrino lifetime…

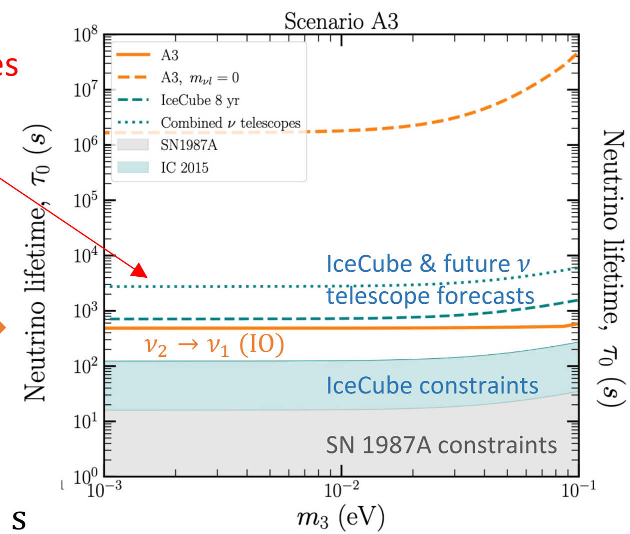
… currently the best limits on invisible neutrino decay $\nu_H \to \nu_l + \phi$.



Inverted mass ordering

However, depending on the exact decay scenario, neutrino telescopes may become competitive in the future! Watch this space!





BBN: $\tau_0 \gtrsim 10^{-2} \to 10^{-1}$ s

Solar ν: $\tau_0 \gtrsim 10^{-5} \to 10^{-4}$ s

Lab ν: $\tau_0 \gtrsim 10^{-13} \to 10^{-11}$ s

Chen, Oldengott, Pierobon & Y[3]W 2022

* IceCube constraints & forecasts from Song et al. 2021

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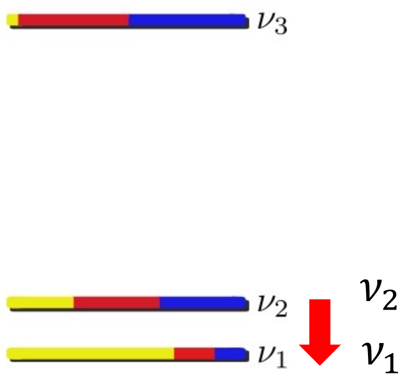

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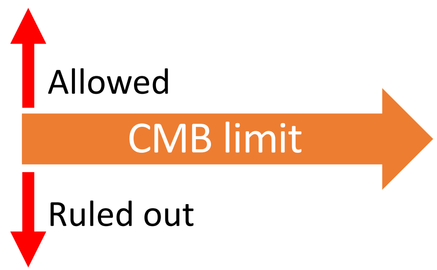

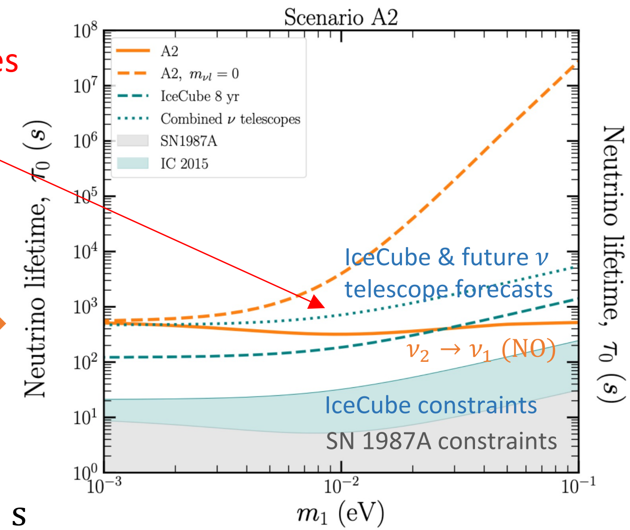

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Chen, Oldengott, Pierobon & Y^{3}W 2022

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Summary: Part 2...

- The **cosmic neutrino background** is a fundamental prediction of standard hot big bang cosmology.

- Given this, we can contemplate using **precision cosmological observables** to measure/constrain
  - Neutrino masses
  - Non-standard neutrino properties like self-interaction and invisible decay.

- Current cosmological data constrain the **neutrino mass sum** conservatively to $\sum m_\nu \lesssim O(0.1-0.3)$ eV.
  - You can get around these to an extent with non-standard neutrino physics.

- We have calculated the isotropisation rate from first-principles and revised the CMB constraint on the neutrino lifetime by many orders of magnitude.