Neutrinos interactions

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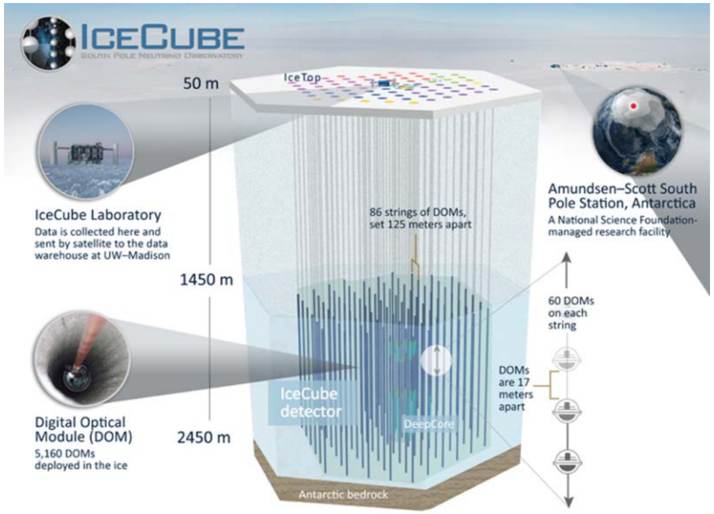
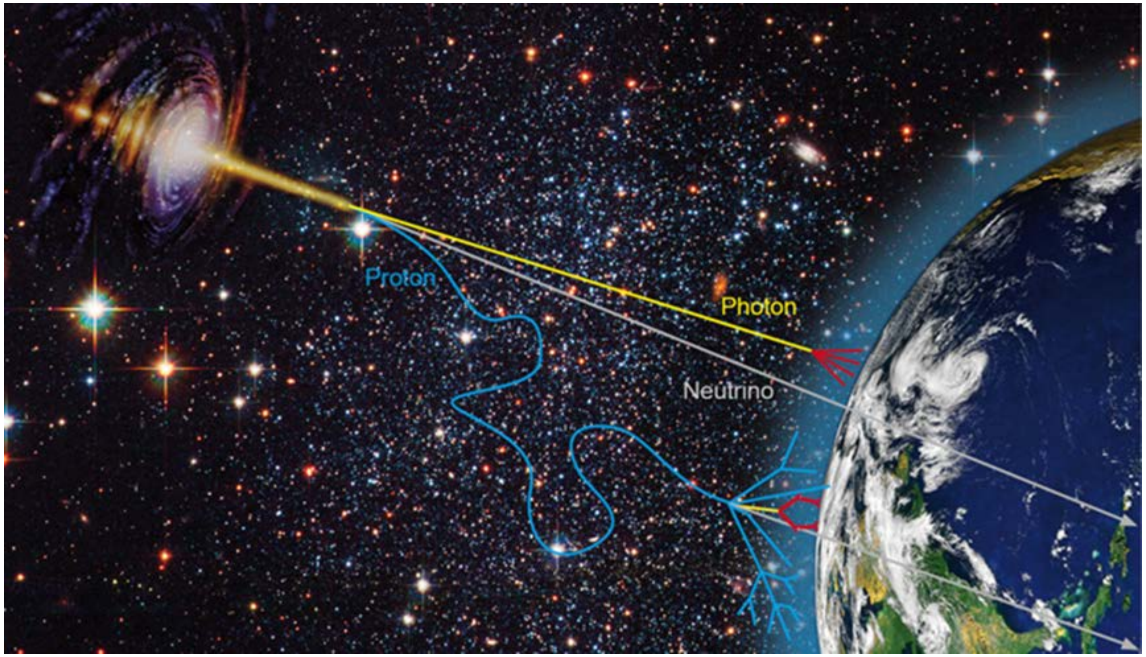
Introduction

- Neutrino interactions with matter are present in many interesting and relevant physical processes
  - Astrophysics
    - Dynamics of the core-collapse in supernovae
    - Nucleosynthesis (n/p ratio)
  - Neutrino astronomy
  - e.g. Ice Cube, KM3NeT



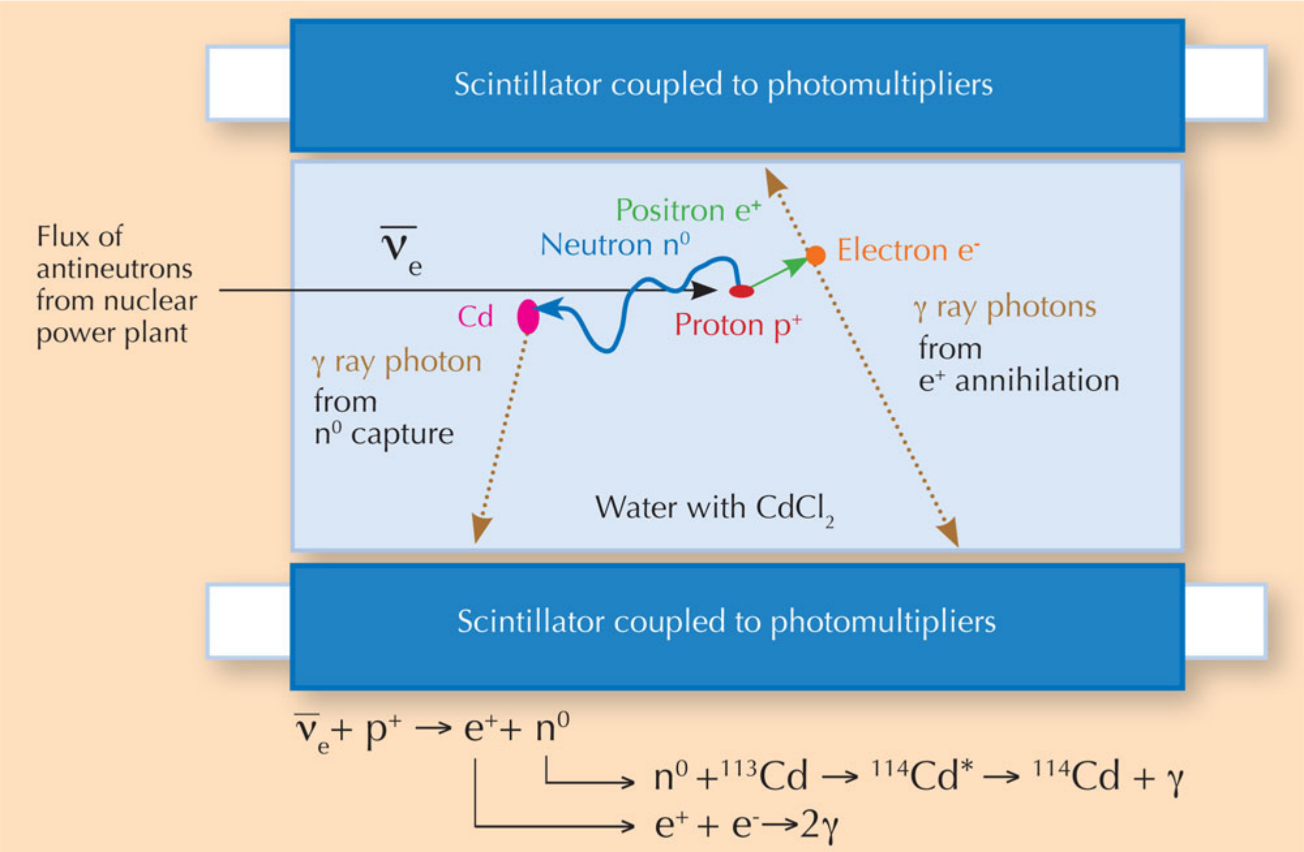


Introduction

- Neutrino interactions with matter are present in many interesting and relevant physical processes
  - Astrophysics
    - Dynamics of the core-collapse in supernovae
    - Nucleosynthesis (n/p ratio)
  - Neutrino astronomy
  - Hadron and Nuclear physics
    - Nucleon and Nucleon-Resonance (N-Δ, N-N*) axial form factors
    - Strangeness content of the nucleon spin
    - Nuclear correlations, giant resonances, ...
  - Beyond Standard Model physics
    - Non-standard ν interactions
    - Production of heavy-neutral leptons

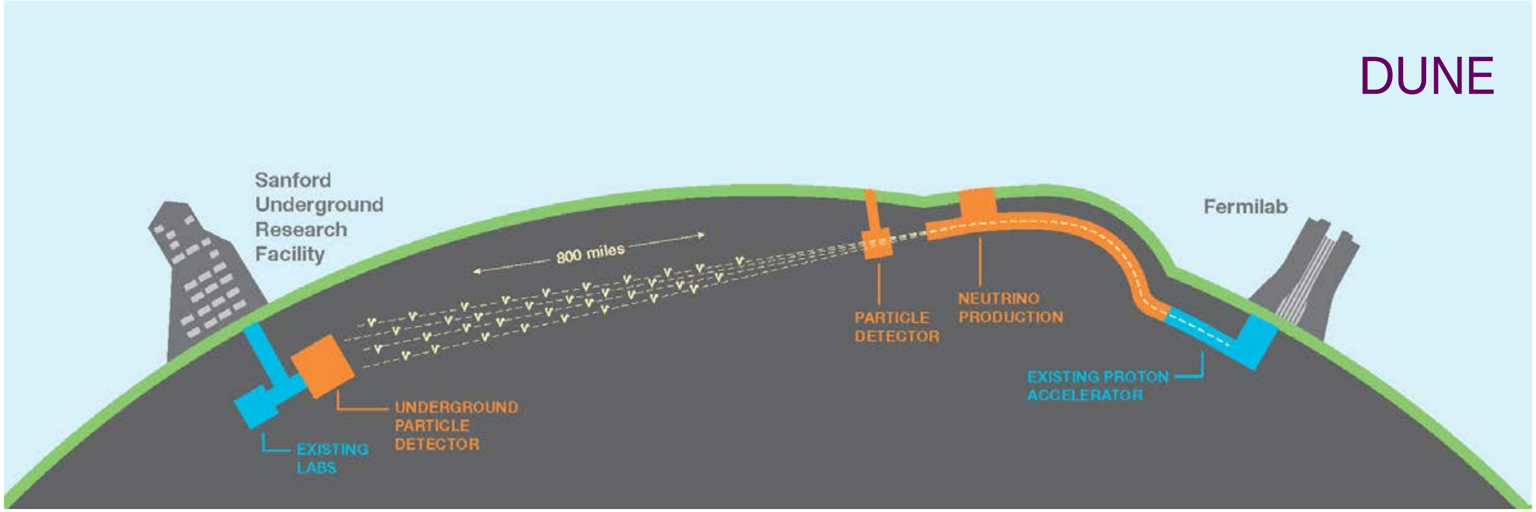
Introduction

- Neutrino interactions are our doorway to neutrino properties:
  - Detection
    - Discovery: $\bar{\nu}_e \, p \rightarrow n \, e^+$ Cowan & Reines



Introduction

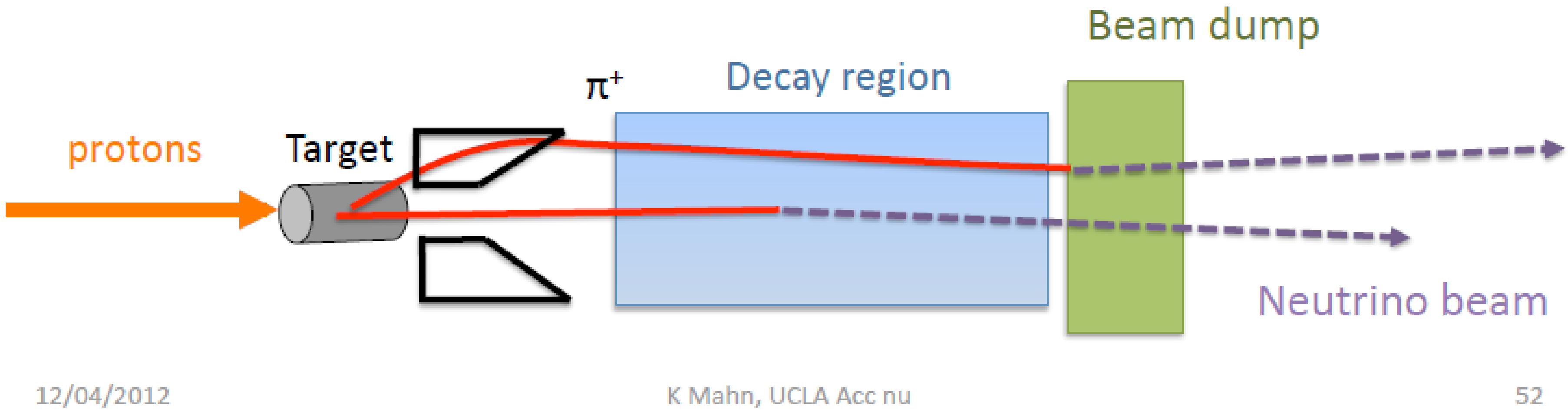
- Neutrino interactions are our doorway to neutrino properties:
  - Flavor ID: required to study oscillations
  - Neutrino propagation: matter effects



  - Relevant for oscillation experiments (with accelerator ν):
    - reduction of systematic errors
      - ν flux calibration
      - background determination
      - E_ν reconstruction

E_ν determination

- Neutrino beams are not monochromatic



$$\pi^- \rightarrow \mu^- \, \bar{\nu}_\mu$$ ← antineutrino mode

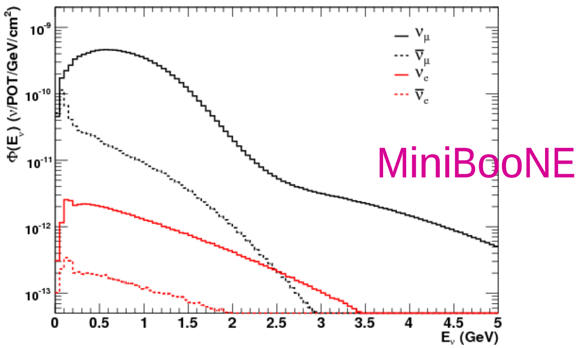
$$\pi^+ \rightarrow \mu^+ \, \nu_\mu$$ ← neutrino mode

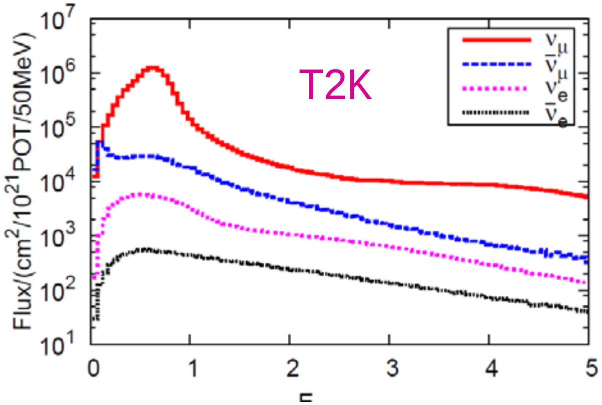
$$\mu^- \rightarrow e^- \, \bar{\nu}_e \, \nu_\mu$$

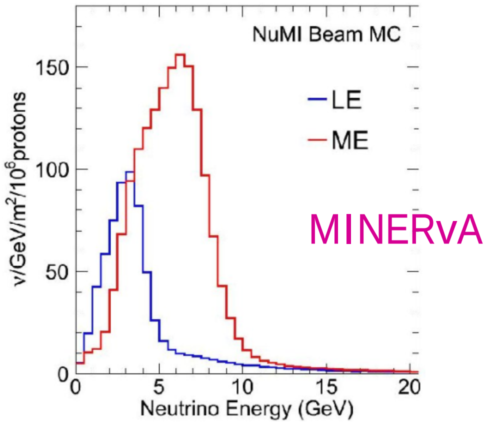
$$\mu^+ \rightarrow e^+ \, \nu_e \, \bar{\nu}_\mu$$ ← contamination

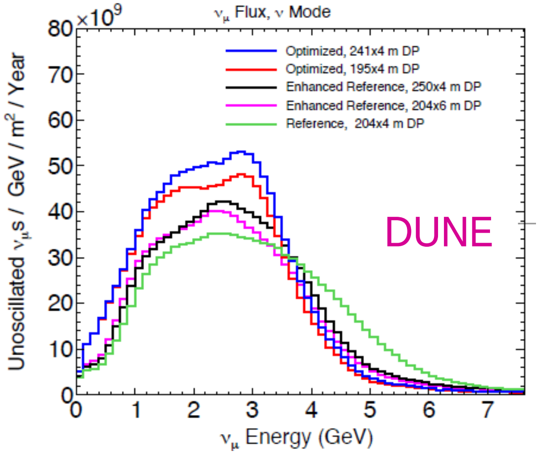
E_ν determination

- Neutrino beams are not monochromatic









- Important for oscillations: $P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta_{23}\, \sin^2 \frac{\Delta m^2_{23} L}{4E_\nu}$

E_ν determination

- (Calorimetric) E_ν reconstruction (e.g. NoVA, DUNE):
  - $E_\nu = E_{lep} + E_{had}$

but

  - There are **neutrons** or other undetected particles: $E_{vis} < E_{had}$

Example:

- For $\nu_l \, A \to l^- \, R \, (\text{spectator}) \, p$ E_ν reconstruction works well

but

- for $\nu_l \, A \to l^- \, R' \, (\text{spectator}) \, p \, n \, (\text{undetected})$ $E_\nu^{rec} < E_\nu \Rightarrow$ bias
- Migration from measured quantities → $E_\nu^{(rec)}$ → E_ν relies on Monte Carlo event generators (GENIE, NuWro, NEUT, ...)
  - tuned to (the best possible) data ← Near Detector
  - require a solid theoretical understanding and realistic modeling of neutrino interaction dynamics

Outline

- General Introduction/Motivation
- Electroweak and Strong interactions in the Standard Model
  - Approximate Symmetries
  - Chiral symmetry breaking and Chiral Perturbation Theory
- Charged pion decay
- Inclusive neutrino cross section
- Coherent elastic neutrino-nucleus cross section
- Nucleon EW current.
- The nucleon axial form factor: ChPT & LQCD
- Neutrino interactions on nuclei
- NN interaction and the nuclear ground state
- Two body currents
- Inelastic scattering
- Electroweak excitation of baryon resonances
- Pion production on nucleons and nuclei

Electroweak interactions in the SM

- Spontaneously broken SU(2) x U(1) gauge symmetry

$$\mathcal{L}_{\rm EW} = -e J^\mu_{em} A_\mu - \frac{g}{2\cos\theta_W} J^\mu_{nc} Z_\mu - \frac{g}{2\sqrt{2}} J^\mu_{cc} W^\dagger_\mu + h.c.$$

$$\sin\theta_W = \frac{e}{g} \qquad \cos\theta_W = \frac{M_W}{M_Z} \qquad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

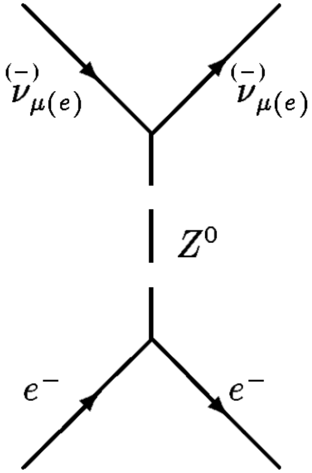
in the leptonic sector:

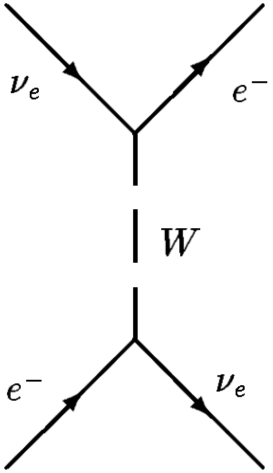
$$\begin{aligned} J^\mu_{em} &= \bar{l}_i \gamma^\mu l_i \qquad i = e, \mu, \tau \\ J^\mu_{cc} &= \bar{\nu}_i \gamma^\mu (1-\gamma_5) l_i \\ J^\mu_{nc} &= \frac{1}{2} \bar{l}_i \gamma^\mu (g_V - g_A \gamma_5) l_i + \frac{1}{2} \bar{\nu}_i \gamma^\mu (1-\gamma_5) \nu_i \end{aligned}$$

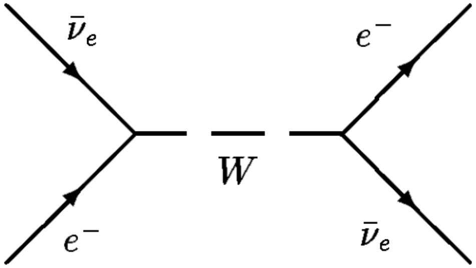
$$g_V = -1 + 4\sin^2\theta_W ,\; g_A = -1$$

$$|g_V| \approx 0.04 \ll |g_A|$$

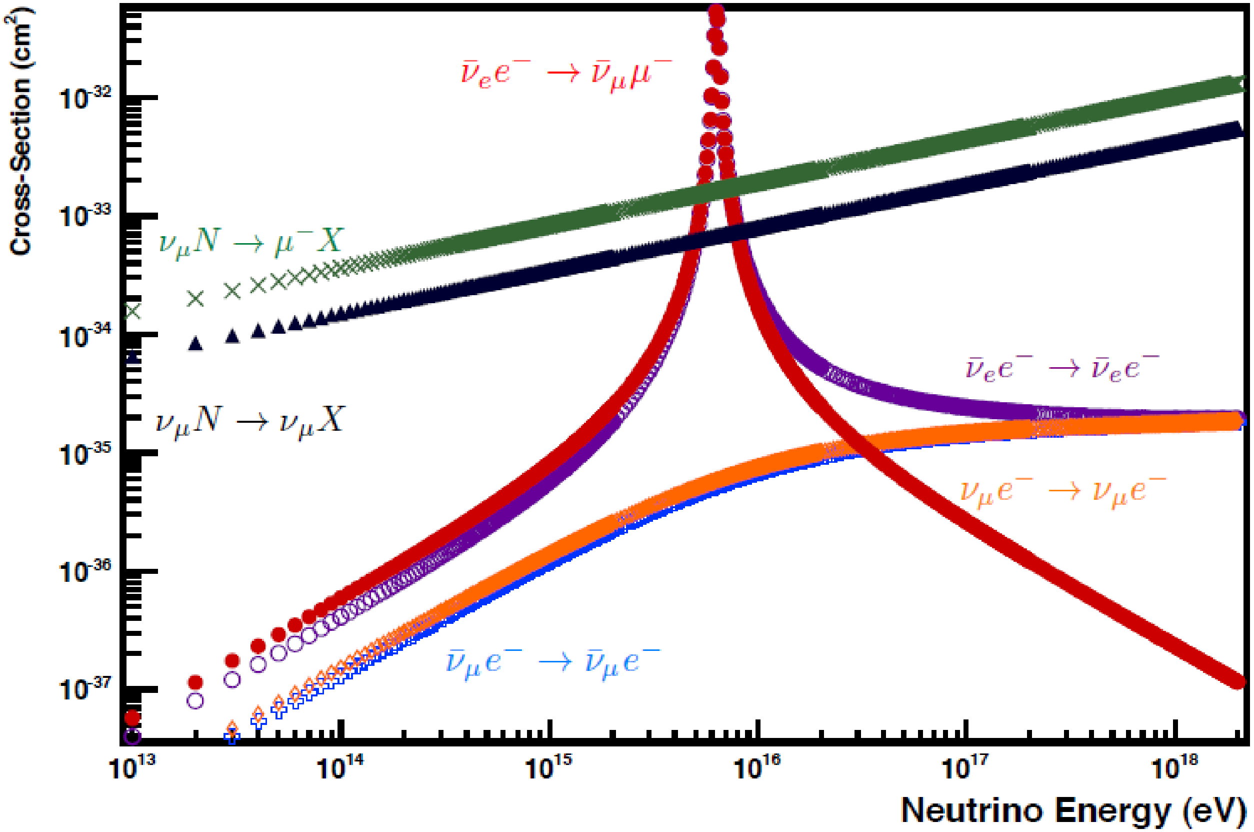
Neutrino-electron scattering

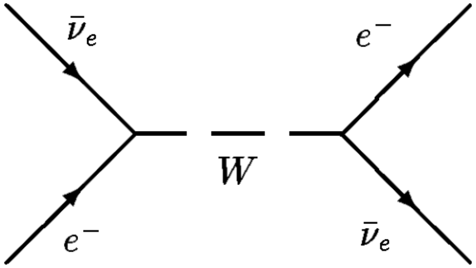






Neutrino-electron scattering





Formaggio, Zeller, Rev.Mod.Phys. 84 (2012)

Detection of a particle shower at the Glashow resonance with IceCube, Nature 591 (2021)

Electroweak interactions in the SM

- Spontaneously broken SU(2) x U(1) gauge symmetry

$$\mathcal{L}_{\mathrm{EW}} = -eJ^{\mu}_{em}A_{\mu} - \frac{g}{2\cos\theta_W}J^{\mu}_{nc}Z_{\mu} - \frac{g}{2\sqrt{2}}J^{\mu}_{cc}W^{\dagger}_{\mu} + h.c.$$

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in the quark sector:

$$J^{\mu}_{em} = Q_i\bar{q}_i\gamma^{\mu}q_i = \frac{2}{3}\bar{q}_u\gamma^{\mu}q_u - \frac{1}{3}(\bar{q}_d\gamma^{\mu}q_d + \bar{q}_s\gamma^{\mu}q_s) + \ldots$$

$$\begin{aligned} J^{\mu}_{nc} &= \bar{q}_u\gamma^{\mu}\left[\frac{1}{2} - \left(\frac{2}{3}\right)2\sin^2\theta_W - \frac{1}{2}\gamma_5\right]q_u + (u\to c) + (u\to t) \\ &+ \bar{q}_d\gamma^{\mu}\left[-\frac{1}{2} - \left(-\frac{1}{3}\right)2\sin^2\theta_W + \frac{1}{2}\gamma_5\right]q_d + (d\to s) + (d\to b) \end{aligned}$$

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in the quark sector:

$$J^{\mu}_{cc} = (\bar{q}_u\bar{q}_c\bar{q}_t)\,\gamma^{\mu}\,(1-\gamma_5)\,V\begin{pmatrix} q_d \\ q_s \\ q_b \end{pmatrix}$$

$V \leftarrow$ CKM matrix

Electroweak interactions in the SM

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$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

PDG

Electroweak interactions in the SM

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in the quark sector:

$$J^{\mu}_{cc} = (\bar{q}_u\bar{q}_c\bar{q}_t)\,\gamma^{\mu}\,(1-\gamma_5)\,V\begin{pmatrix} q_d \\ q_s \\ q_b \end{pmatrix} \qquad V \leftarrow \text{CKM matrix}$$

$$V \approx \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \qquad \theta_C \approx 13\text{ deg} \leftarrow \text{Cabibbo angle}$$

$$W^- p(uud) \rightarrow n(udd) \qquad \sim \cos^2\theta_c$$

$$\left.\begin{array}{l} W^- p(uud) \rightarrow \Lambda(usd) \\ W^- p(uud) \rightarrow p(uud)\,K^-(\bar{u}s) \end{array}\right\} \sim \sin^2\theta_c$$

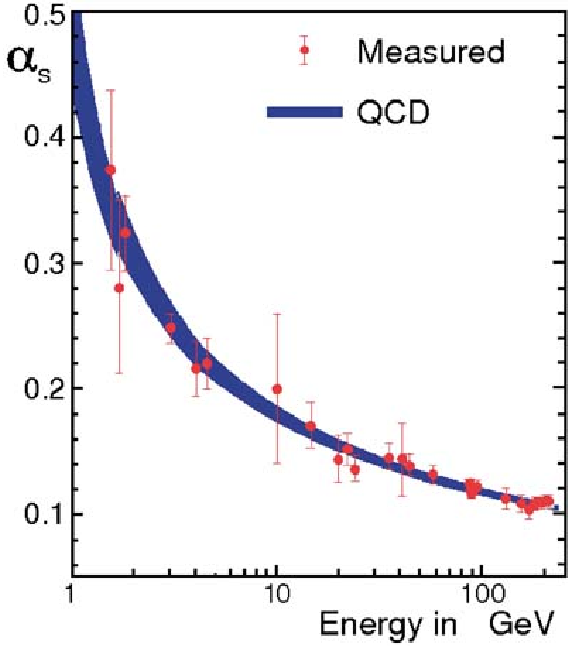
Strong interactions in the SM

- SU(3) (color) gauge symmetry: QCD

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_q \left(i\gamma^\mu D_\mu - m_q\right)\psi_q - \frac{1}{4} G_a^{\mu\nu} G_{a\mu\nu} \qquad q = u, d, s, \dots \;\; a = 1 - 8$$

$$D_\mu \psi = \left(\partial_\mu - i g \frac{\lambda_a}{2} A_\mu^a\right)\psi \qquad G_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_{abc} A_b^\mu A_c^\nu$$

- Asymptotically free ⇒ perturbative at high energies
- Nonperturbative at low energies
- Confining



$$\alpha_s = \frac{g^2}{4\pi}$$

Chiral symmetry

- Approximate symmetries of N_f =3 QCD
  - $m_u = m_d = m_s = 0 \Leftrightarrow$ Chiral $SU(3)_L \times SU(3)_R$ symmetry

m_u (1 GeV) = 1.7 - 3.3 MeV
m_d (1 GeV) = 4.1 - 5.8 MeV
m_s (1 GeV) = 80 - 130 MeV

$\ll$ rho meson, nucleon mass

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_{qL} i\gamma^\mu D_\mu \psi_{qL} + \bar{\psi}_{qR} i\gamma^\mu D_\mu \psi_{qR} + \ldots$$

- 8+8 conserved currents:

$$q = \begin{pmatrix} q_u \\ q_d \\ q_s \end{pmatrix}$$

$$\begin{aligned} R_a^\mu &= \bar{q}_R \gamma^\mu \frac{\lambda_a}{2} q_R \\ L_a^\mu &= \bar{q}_L \gamma^\mu \frac{\lambda_a}{2} q_L \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} V_a^\mu &= R_a^\mu + L_a^\mu = \bar{q}\gamma^\mu \frac{\lambda_a}{2} q \\ A_a^\mu &= R_a^\mu - L_a^\mu = \bar{q}\gamma^\mu \gamma_5 \frac{\lambda_a}{2} q \end{aligned}$$

$$\bar{q} = (\bar{q}_u \, \bar{q}_d \, \bar{q}_s)$$

$$\partial_\mu V_a^\mu = 0 \qquad \partial_\mu A_a^\mu = 0$$

Gell-Mann matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ +i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ +i & 0 & 0 \end{pmatrix},$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & +i & 0 \end{pmatrix}, \lambda_8 = \tfrac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Chiral symmetry breaking

- Approximate symmetries of N_f =3 QCD
  - $m_u = m_d = m_s = 0 \Leftrightarrow$ Chiral $SU(3)_L \times SU(3)_R$ symmetry

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$$\mathcal{L}_{\rm QCD} = \bar{\psi}_{qL} i\gamma^\mu D_\mu \psi_{qL} + \bar{\psi}_{qR} i\gamma^\mu D_\mu \psi_{qR} - m_q(\bar{\psi}_{qL}\psi_{qR} + \bar{\psi}_{qL}\psi_{qR}) + \ldots$$

  - 8+8 conserved currents:

$$\begin{aligned} R_a^\mu &= \bar{q}_R \gamma^\mu \frac{\lambda_a}{2} q_R \\ L_a^\mu &= \bar{q}_L \gamma^\mu \frac{\lambda_a}{2} q_L \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} V_a^\mu &= R_a^\mu + L_a^\mu = \bar{q}\gamma^\mu \frac{\lambda_a}{2} q \\ A_a^\mu &= R_a^\mu - L_a^\mu = \bar{q}\gamma^\mu \gamma_5 \frac{\lambda_a}{2} q \end{aligned}$$

$$q = \begin{pmatrix} q_u \\ q_d \\ q_s \end{pmatrix} \qquad \bar{q} = (\bar{q}_u\, \bar{q}_d\, \bar{q}_s)$$

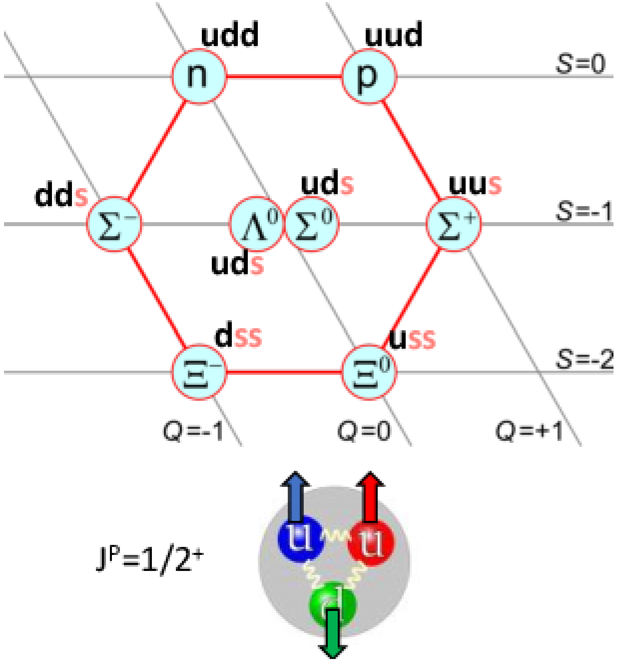
  - Explicit chiral symmetry breaking:

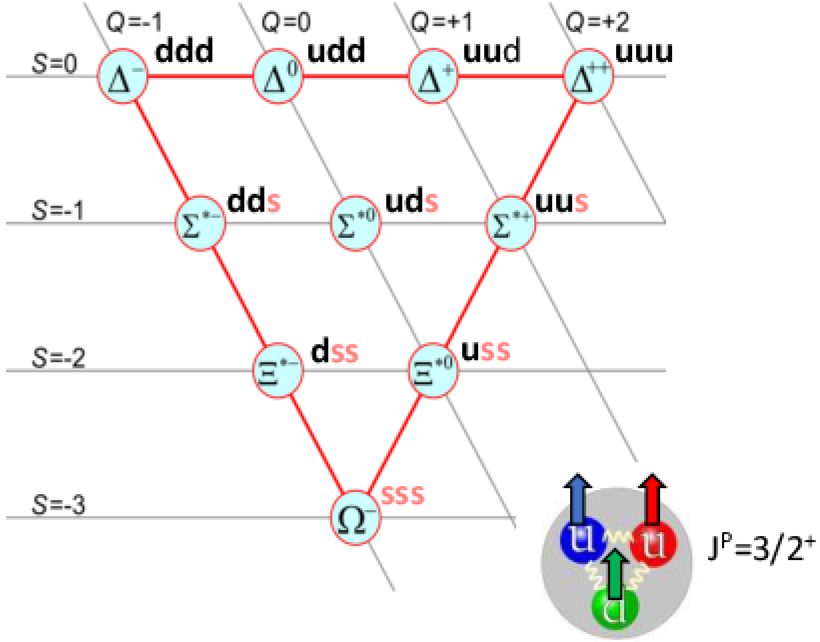
$$\partial_\mu V_a^\mu = \bar{q}\left[m, \frac{\lambda_a}{2}\right] q \qquad \partial_\mu A_a^\mu = i\bar{q}\left\{m, \frac{\lambda_a}{2}\right\}\gamma_5 q \qquad m = \mathrm{diag}(m_u, m_d, m_s)$$

Approximate global symmetries

$$\partial_\mu V^\mu_a = \bar{q}\left[m, \frac{\lambda_a}{2}\right] q \qquad \partial_\mu A^\mu_a = i\bar{q}\left\{m, \frac{\lambda_a}{2}\right\} \gamma_5 q$$

- The vector current is conserved (CVC)
  - chiral limit
  - $m_u = m_d = m_s \Leftrightarrow$ SU(3)$_{\text{flavor}}$ symmetry (broken at 20% level)





Approximate global symmetries

$$\partial_\mu V_a^\mu = \bar{q}\left[m, \frac{\lambda_a}{2}\right] q \qquad \partial_\mu A_a^\mu = i\bar{q}\left\{m, \frac{\lambda_a}{2}\right\}\gamma_5 q$$

- The vector current is conserved (CVC)
  - chiral limit
  - $m_u = m_d = m_s \Leftrightarrow$ SU(3)$_{\text{flavor}}$ symmetry (broken at 20% level)
  - $m_u = m_d \neq m_s \Leftrightarrow$ SU(2)$_{\text{flavor}}$ isospin symmetry (broken at 1% level)

$$V_a^\mu = \bar{q}\gamma^\mu \frac{\lambda_a}{2} q = \bar{q}'\gamma^\mu \frac{\tau_a}{2} q' \Leftrightarrow \partial_\mu V_a^\mu = 0 \qquad a = 1-3 \qquad q' = \begin{pmatrix} q_u \\ q_d \end{pmatrix}$$

- $\tau_{1\text{-}3} \leftarrow$ Pauli matrices

Approximate global symmetries

$$\partial_\mu V_a^\mu = \bar{q}\left[m, \frac{\lambda_a}{2}\right] q \qquad \partial_\mu A_a^\mu = i\bar{q}\left\{m, \frac{\lambda_a}{2}\right\}\gamma_5 q$$

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  - $m_u = m_d \neq m_s \Leftrightarrow$ SU(2)$_{\text{flavor}}$ isospin symmetry (broken at 1% level)

- The axial current is conserved in the chiral limit (PCAC)

Flavor symmetries

- Flavor structure of the EW quark currents:

$$\mathcal{L}_{\mathrm{EW}} = -eJ^{\mu}_{em}A_{\mu} - \frac{g}{2\cos\theta_W}J^{\mu}_{nc}Z_{\mu} - \frac{g}{2\sqrt{2}}J^{\mu}_{cc}W^{\dagger}_{\mu} + h.c.$$

$$\begin{aligned} J^{\mu}_{em} &= \frac{2}{3}\bar{q}_u\gamma^{\mu}q_u - \frac{1}{3}(\bar{q}_d\gamma^{\mu}q_d + \bar{q}_s\gamma^{\mu}q_s) = \bar{q}Q\gamma^{\mu}q \qquad Q = \mathrm{diag}\left(\tfrac{2}{3}\ -\tfrac{1}{3}\ -\tfrac{1}{3}\right) \\ &= \frac{1}{2}\bar{q}\frac{\lambda_8}{\sqrt{3}}\gamma^{\mu}q + \bar{q}\frac{\lambda_3}{2}\gamma^{\mu}q = \frac{1}{2}V^{\mu}_{Y} + V^{\mu}_{3} \end{aligned}$$

$$J^{\mu}_{cc} = \bar{q}_u\gamma^{\mu}(1-\gamma_5)(q_d\cos\theta_C + q_s\sin\theta_C)$$

$$V^{\mu}_{+} = \bar{q}_u\gamma^{\mu}q_d = \bar{q}\gamma^{\mu}\frac{\lambda_1 + i\lambda_2}{2}q = V^{\mu}_{1} + iV^{\mu}_{2}$$

$V_{1,2,3}$: components of the same (conserved) flavor vector current

$$J^{\mu}_{nc} = \bar{q}_u\gamma^{\mu}\left[\frac{1}{2} - \left(\frac{2}{3}\right)2\sin^2\theta_W - \frac{1}{2}\gamma_5\right]q_u + \bar{q}_d\gamma^{\mu}\left[-\frac{1}{2} - \left(-\frac{1}{3}\right)2\sin^2\theta_W + \frac{1}{2}\gamma_5\right]q_d + (d\to s)$$

$$V^{\mu}_{nc} = (1-2\sin^2\theta_W)V^{\mu}_{3} - 2\sin^2\theta_W\frac{1}{2}V^{\mu}_{Y} - \frac{1}{2}\bar{q}_s\gamma^{\mu}q_s$$

Flavor symmetries

- Flavor structure of the EW quark currents:

$$\mathcal{L}_{\mathrm{EW}} = -e J^\mu_{em} A_\mu - \frac{g}{2\cos\theta_W} J^\mu_{nc} Z_\mu - \frac{g}{2\sqrt{2}} J^\mu_{cc} W^\dagger_\mu + h.c.$$

$$J^\mu_{cc} = \bar{q}_u \gamma^\mu (1-\gamma_5)(q_d \cos\theta_C + q_s \sin\theta_C)$$

$$A^\mu_+ = \bar{q}_u \gamma^\mu \gamma_5 q_d = \bar{q}_u \gamma^\mu \gamma_5 \frac{\lambda_1 + i\lambda_2}{2} q_d = A^\mu_1 + i A^\mu_2$$

$$J^\mu_{nc} = \bar{q}_u \gamma^\mu \left[\frac{1}{2} - \left(\frac{2}{3}\right) 2\sin^2\theta_W - \frac{1}{2}\gamma_5\right] q_u + \bar{q}_d \gamma^\mu \left[-\frac{1}{2} - \left(-\frac{1}{3}\right) 2\sin^2\theta_W + \frac{1}{2}\gamma_5\right] q_d + (d \to s)$$

$$A^\mu_{nc} = A^\mu_3 + \frac{1}{2}\bar{q}_s \gamma^\mu \gamma_5 q_s$$

$A_{1,2,3}$: components of the same (partially conserved) flavor axial current

Chiral symmetry breaking

- $m_u = m_d = m_s = 0 \Leftrightarrow$ Chiral $SU(3)_L \times SU(3)_R$ symmetry
- Explicit chiral symmetry breaking:

$$\partial_\mu V_a^\mu = \bar{q}\left[m, \frac{\lambda_a}{2}\right] q \qquad \partial_\mu A_a^\mu = i\bar{q}\left\{m, \frac{\lambda_a}{2}\right\}\gamma_5 q$$

- The ground state does not have the chiral symmetry of the Lagrangian
- Parity doublets (with almost degenerate masses) are absent:
  - m_ρ = 770 MeV ($J^P=1^-$) ≠ m_{a_1} = 1230 MeV (1^+)
  - Light pseudoscalar mesons ($J^P=0^-$): m_π = 140 MeV , m_K = 494 MeV

    but heavy scalar mesons ($J^P=0^+$): m_{f_0} = 400-800 MeV, m_κ = 845 MeV
- Spontaneous chiral symmetry breaking:
  - $SU(3)_L \times SU(3)_R \to SU(3)_V$
  - Goldstone bosons: π, K, η

ChPT in a nutshell

- Chiral Perturbation Theory = EFT of QCD at low energies
  - order by order renormalizable
  - free parameters called low-energy constants (LEC)

$$\mathcal{L}_{\mathrm{QCD}} \to \mathcal{L}_{\mathrm{effective}}(\pi, K, \eta \ldots)$$

- $SU(3)_L \times SU(3)_R \to SU(3)_V$
- Goldstone boson fields ∊ quotient $SU(3)_L \times SU(3)_R$ / $SU(3)_V$
- Conveniently parametrized as:

$$U(x) = \exp\left(\frac{i}{f_\pi}\phi(x)\right) \qquad \phi = \sum_{a=1}^{8} \phi_a \lambda_a \equiv \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

$$\mathcal{L}_{\mathrm{eff}} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu U \partial^\mu U^\dagger\right] \qquad \mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots$$

ChPT in a nutshell

- Chiral Perturbation Theory = EFT of QCD at low energies
- $SU(3)_L \times SU(3)_R \to SU(3)_V$
- Pseudoscalar Mesons:

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$$\mathcal{L}_{\text{eff}} = \frac{f_\pi^2}{4}\text{Tr}\left[\partial_\mu U \partial^\mu U^\dagger\right] \qquad \mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots$$

- Baryon octet:

$$\mathscr{L}_{MB}^{(1)} = \text{Tr}[\bar{B}(i\not{D} - M_0)B] - \frac{D}{2}\text{Tr}\left(\bar{B}\gamma^\mu\gamma_5\{u_\mu, B\}\right) - \frac{F}{2}\text{Tr}\left(\bar{B}\gamma^\mu\gamma_5[u_\mu, B]\right)$$

$$D_\mu B = \partial_\mu B + [\Gamma_\mu, B]$$

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \ldots$$

$$B = \sum_{a=1}^{8} \frac{B_a \lambda_a}{\sqrt{2}} = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

  - Light $J^P=3/2^+$ decuplet can be added

- Heavier hadrons ⇒ LECs

ChPT in a nutshell

- Chiral Perturbation Theory = EFT of QCD at low energies
- $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$
- Pseudoscalar Mesons:

- Baryon octet:
  - Light $J^P=3/2^+$ decuplet can be added

- Heavier hadrons ⇒ LECs
- External currents (EM, weak, ...) and explicit chiral symmetry breaking
- Power counting:

$$p_i \rightarrow t p_i \, , \;\; m_q \rightarrow t^2 m_q$$

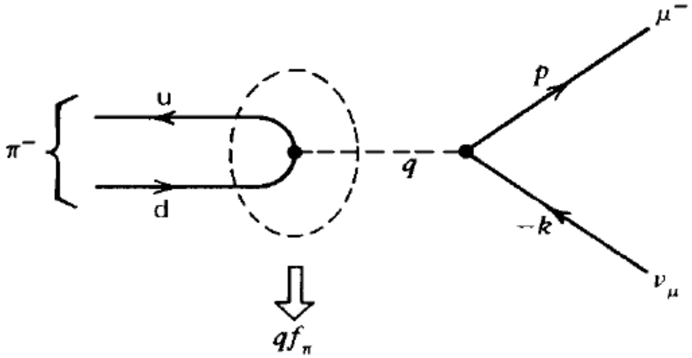
$$\mathcal{M}(t p_i, t^2 m_q) = t^n \mathcal{M}(p_i, m_q)$$

  - chiral order: $n = 4L - 2N_M - N_B + \sum_k V_k$

- ChPT predicts the light-quark mass dependence of amplitudes

Charged pion decay

- Example: $\pi^-(q) \to \mu^-(p) + \bar{\nu}_\mu(k)$



$$\mathcal{L}_{\mathrm{EW}} = -\frac{g}{2\sqrt{2}} J^\mu_{cc} W^+_\mu + h.c.$$

$$(-i)\mathcal{M} = (-i)\left\langle \mu^- \bar{\nu}_\mu \right| \left(-\frac{g}{2\sqrt{2}}\right) J^\mu_{cc} \left|0\right\rangle (-i) D_{\mu\nu}(q) (-i) \left\langle 0\right| \left(-\frac{g}{2\sqrt{2}}\right) J^\nu_{cc} \left|\pi^-\right\rangle$$

$$D_{\mu\nu} = \frac{1}{q^2 - M_W^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right) \approx -\frac{g_{\mu\nu}}{M_W^2} \qquad q^2 = m_\pi^2 \ll M_W^2$$

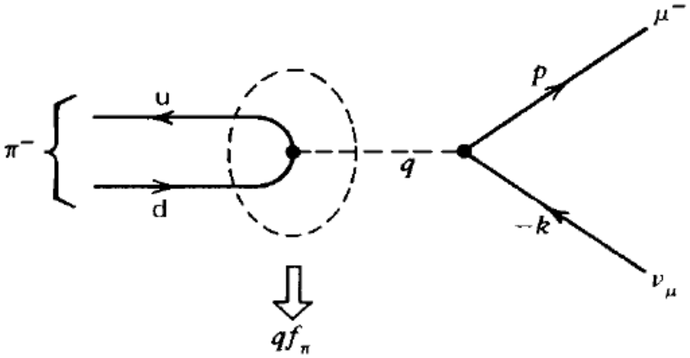
$$\left\langle \mu^- \bar{\nu}_\mu \right| J^\mu_{cc} \left|0\right\rangle = \bar{u}(p)\gamma_\mu (1-\gamma_5) v(k)$$

$$\left\langle 0\right| J^\nu_{cc} \left|\pi^-\right\rangle = \xcancel{\bar{v}_u \gamma^\nu (1-\gamma_5) u_{d'}} = \sqrt{2} f_\pi i q^\nu \qquad A^\mu_a = -f_\pi \partial^\mu \pi_a + \ldots$$

$$\left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{M_W^2} = \frac{G_F}{\sqrt{2}}$$

Charged pion decay

- Example: $\pi^-(q) \to \mu^-(p) + \bar{\nu}_\mu(k)$



$$\overline{|\mathcal{M}|^2} = \overline{\sum_{\text{polar.}}} |\mathcal{M}|^2 = 4G_F^2 L_{\mu\nu} H^{\mu\nu}$$

$$\text{Tr}\left[(\not{p} + m_\mu)\gamma_\mu(1-\gamma_5)\not{k}\gamma_\nu(1-\gamma_5)\right] = 8L_{\mu\nu}$$

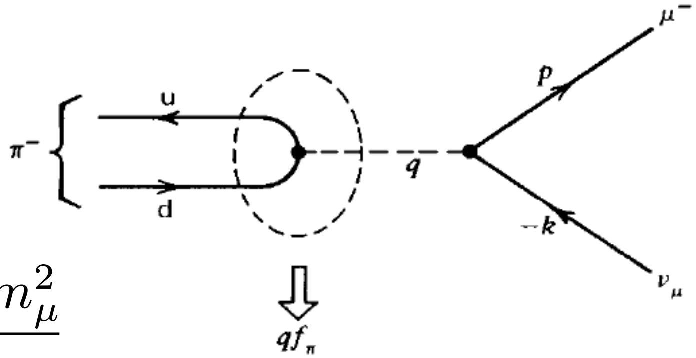
$$q = p + k$$

$$L_{\mu\nu} = p_\mu k_\nu + p_\nu k_\mu - g_{\mu\nu} k\cdot p + i\epsilon_{\mu\nu\alpha\beta} p^\alpha k^\beta$$

$$H^{\mu\nu} = 2f_\pi^2 q^\mu q^\nu$$

Charged pion decay

- Example: $\pi^-(q) \to \mu^-(p) + \bar{\nu}_\mu(k)$



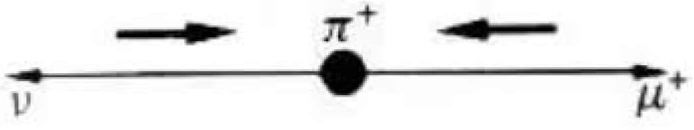
$$\overline{|\mathcal{M}|^2} = 8G_F^2 f_\pi^2 m_\mu^2 (p \cdot k)$$

$$q^2 = (p+k)^2 = m_\mu^2 + 2(p \cdot k) \Rightarrow (p \cdot k) = \frac{m_\pi^2 - m_\mu^2}{2}$$

Decay width, in the π rest frame:

$$\Gamma = \frac{1}{2m_\pi} \int \frac{d^3p}{2p^0(2\pi)^3} \frac{d^3k}{2k^0(2\pi)^3} (2\pi)^4 \delta^4(k+p-q) \overline{|\mathcal{M}|^2}$$

$$\Gamma = \frac{G_F^2}{4\pi} f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

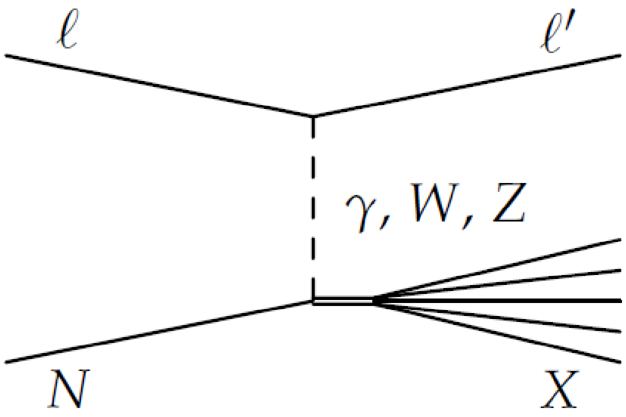


Pion rest frame

$$\tau = \frac{1}{\Gamma} = 2.6\, 10^{-8}\, s \Rightarrow f_\pi = 92.4\,\mathrm{MeV}$$

Inclusive cross section

$$l(k) + N(p) \to l'(k') + X(p')$$



$$k = (k_0, \vec{k}) \qquad p = (E, \vec{p})$$

$$k' = (k'_0, \vec{k}) \qquad p' = (E', \vec{p'})$$

$$q = k - k' = p' - p = (\omega, \vec{q}) \qquad q^2 = -Q^2 < 0$$

In Lab: $p = (M, \vec{0})$

For CC/NC: $$\frac{d\sigma}{dk'_0 d\Omega(\vec{k}')} = \frac{G_F^2}{(2\pi)^2} \frac{|\vec{k}'|}{k_0} L_{\mu\nu} W^{\mu\nu}$$

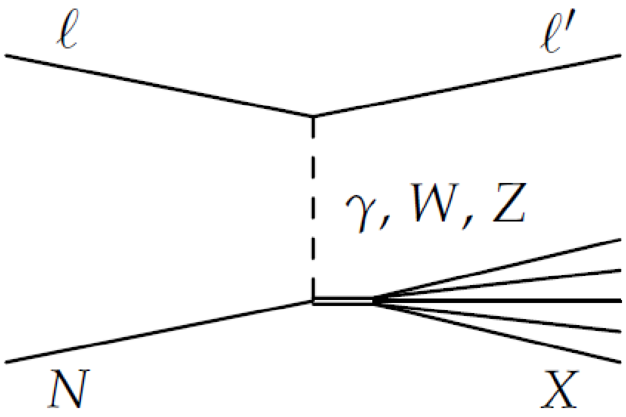
$$L_{\mu\nu} = k'_\mu k_\nu + k'_\nu k_\mu - g_{\mu\nu} k \cdot k' + i\epsilon_{\mu\nu\alpha\beta} k'^\alpha k^\beta$$

$$W^{\mu\nu} = \frac{1}{2M} \overline{\sum_{\text{polar.}}} \prod_i \left(\int \frac{d^3 p_i}{2E'_i (2\pi)^3} \right) (2\pi)^3 \delta^4(k' + p' - k - p) \langle X | J^\mu | N \rangle \langle X | J^\nu | N \rangle^*$$

For EM: $L_{\mu\nu} \to L_{\mu\nu}^{(\text{sym})} \qquad \frac{G_F^2}{(2\pi)^2} \to \frac{\alpha^2}{q^4}$

Inclusive cross section

$$l(k) + N(p) \to l'(k') + X(p')$$



$$k = (k_0, \vec{k}) \qquad p = (E, \vec{p})$$

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$$q = k - k' = p' - p = (\omega, \vec{q}) \qquad q^2 = -Q^2 < 0$$

General structure of the hadronic tensor:

Ingredients: $g^{\mu\nu}, \, q^\mu, \, p^\mu, \, \epsilon^{\alpha\beta\mu\nu}$ $\quad p' = p + q \leftarrow$ not independent

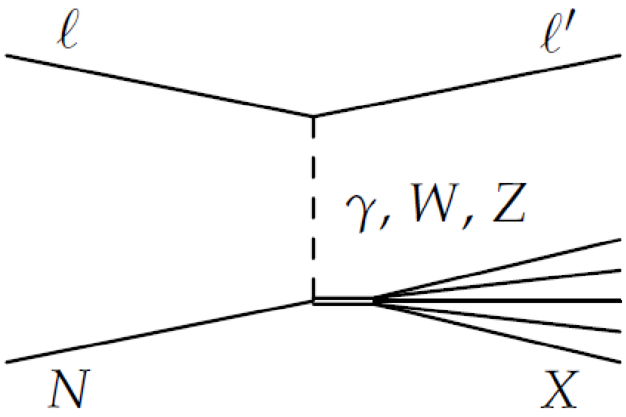
$$W^{\mu\nu} = -W_1 g^{\mu\nu} + W_2 \frac{p^\mu p^\nu}{M^2} + W_4 \frac{q^\mu q^\nu}{M^2} + W_5 \frac{p^\mu q^\nu + q^\mu p^\nu}{M^2}$$

$$+W_3 i\epsilon^{\mu\nu\alpha\beta} \frac{p_\alpha q_\beta}{2M^2} + W_6 \frac{p^\mu q^\nu - q^\mu p^\nu}{M^2}$$

Structure functions: $W_i = W_i(p^2 = M^2, q \cdot p = \omega M, q^2) = W_i(\omega, q^2)$

Inclusive cross section

$$l(k) + N(p) \to l'(k') + X(p')$$



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General structure of the hadronic tensor:

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + W_2 \frac{p^\mu p^\nu}{M^2} + W_4 \frac{q^\mu q^\nu}{M^2} + W_5 \frac{p^\mu q^\nu + q^\mu p^\nu}{M^2}$$
$$+W_3 i\epsilon^{\mu\nu\alpha\beta} \frac{p_\alpha q_\beta}{2M^2} + W_6 \frac{p^\mu q^\nu - q^\mu p^\nu}{M^2}$$

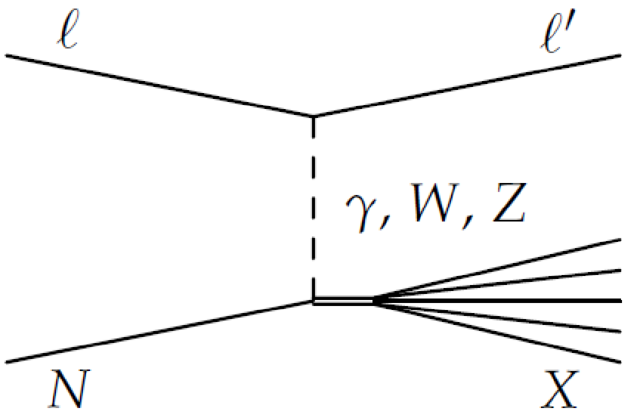
Structure functions: $W_i = W_i(\omega, q^2)$

For EM interactions: $q_\mu J^\mu = 0 \Rightarrow q_\mu W^{\mu\nu}_{em} = W^{\mu\nu}_{em} q_\nu = 0$

$$W^{\mu\nu}_{em} = W_1 \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right) + \frac{W_2}{M^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right)$$

Inclusive cross section

$$l(k) + N(p) \rightarrow l'(k') + X(p')$$



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In Lab: $p = (M, \vec{0})$

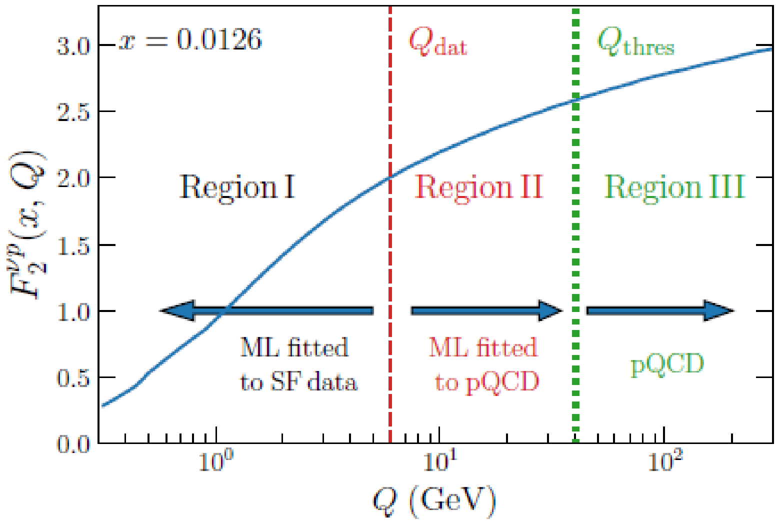
$$\frac{d\sigma}{dk'_0 d\Omega(\vec{k}')} = \frac{G_F^2}{(2\pi)^2} \frac{|\vec{k}'|}{k_0} \{W_1 2k \cdot k' + W_2 (2k'_0 k_0 - k \cdot k') + 2\frac{m_l^2}{M^2} [W_4 k \cdot k' - W_5 M k_0] + \frac{W_3}{M} [(k_0 + k'_0) k \cdot k' - k_0 m_l^2] \}$$

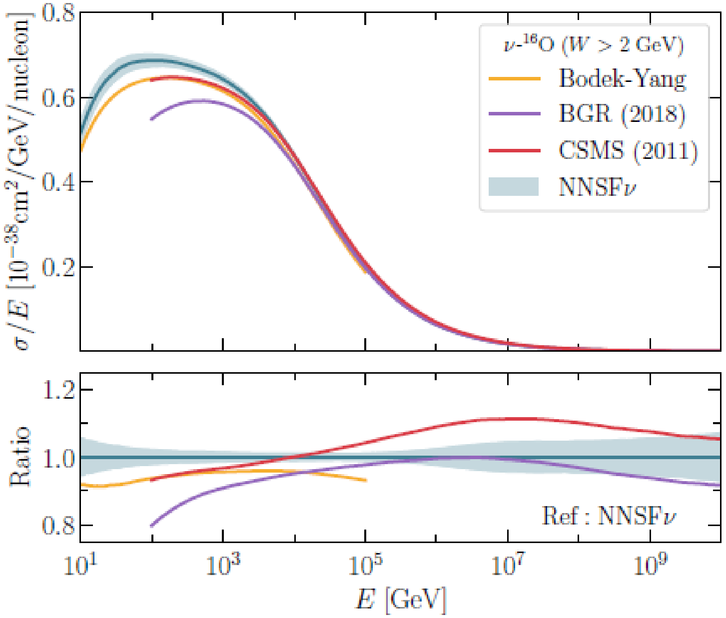
$m_l \rightarrow 0$

$$\frac{d\sigma}{dk'_0 d\Omega(\vec{k}')} = \frac{G_F^2}{2\pi^2} (k'_0)^2 \left[W_1 2 \sin^2 \frac{\theta'}{2} + W_2 \cos^2 \frac{\theta'}{2} \pm W_3 \frac{(k_0 + k'_0)}{M} \sin^2 \frac{\theta'}{2} \right]$$

Structure functions

- Determination of inelastic structure functions
- New approach NNSFν, Candido et al., JHEP 05 (2023)
  - W > 2 GeV and various targets
  - Machine learning parametrization
  - Implements a high Q region (II) for matching to pQCD





- Important for Ice Cube and KM3NeT

ν scattering on a scalar particle

$$\nu(k) + A(p) \to \nu(k') + A(p')$$

$$J^\mu_{\text{nc}} = 2(Ap^\mu + Bq^\mu)$$

CVC: $q_\mu J^\mu_{\text{nc}} = 0 \Rightarrow Ap \cdot q + Bq^2 = 0$

$$J^\mu_{\text{nc}} = 2Q_W \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \quad Q_W \equiv A$$

Using that: $p'^2 = M^2 = (q+p)^2 = q^2 + 2(p \cdot q) + M^2 \Rightarrow \frac{p \cdot q}{q^2} = \frac{M\omega}{q^2} = -\frac{1}{2}$

$$J^\mu_{\text{nc}} = Q_W \left(2p^\mu + q^\mu\right)$$

ν scattering on a scalar particle

$$\nu(k) + A(p) \to \nu(k') + A(p')$$

$$J^{\mu}_{\mathrm{nc}} = Q_W \left(2p^{\mu} + q^{\mu}\right)$$

$$W^{\mu\nu} = \frac{1}{2M} \int \frac{d^3p'}{2E'} \delta^4(k' + p' - k - p) H^{\mu\nu}$$

$$H^{\mu\nu} = Q_W^2 \left(2p^{\mu} + q^{\mu}\right)\left(2p^{\nu} + q^{\nu}\right)$$

Recall that: $$W^{\mu\nu} = -W_1 g^{\mu\nu} + W_2 \frac{p^{\mu}p^{\nu}}{M^2} + W_4 \frac{q^{\mu}q^{\nu}}{M^2} + W_5 \frac{p^{\mu}q^{\nu} + q^{\mu}p^{\nu}}{M^2} + W_3 i\epsilon^{\mu\nu\alpha\beta} \frac{p_{\alpha}q_{\beta}}{2M^2} + W_6 \frac{p^{\mu}q^{\nu} - q^{\mu}p^{\nu}}{M^2}$$

$$\frac{d\sigma}{dk_0' d\Omega(\vec{k}')} = \frac{G_F^2}{(2\pi)^2} \frac{|\vec{k}'|}{k_0} \left\{ W_1 2k\cdot k' + W_2 (2k_0' k_0 - k\cdot k') + 2\frac{m_l^2}{M^2}\left[W_4 k\cdot k' - W_5 M k_0\right] + \frac{W_3}{M}\left[(k_0 + k_0')k\cdot k' - k_0 m_l^2\right] \right\}$$

ν scattering on a scalar particle

$$\nu(k) + A(p) \to \nu(k') + A(p')$$

$$J^{\mu}_{\mathrm{nc}} = Q_W \left(2p^{\mu} + q^{\mu}\right)$$

$$W^{\mu\nu} = \frac{1}{2M} \int \frac{d^3p'}{2E'} \delta^4(k' + p' - k - p) H^{\mu\nu}$$

$$H^{\mu\nu} = Q_W^2 \left(2p^{\mu} + q^{\mu}\right)\left(2p^{\nu} + q^{\nu}\right)$$

Then: $W_2 = Q_W^2 \frac{M}{E'} \delta(k_0' + E' - k_0 - M)$

Using that: $\delta(E' + k_0' - M - k_0) = \frac{E'}{M} \delta\left(k_0' - k_0 - \frac{q^2}{2M}\right)$

one finds:

$$W_2 = \frac{Q_W^2}{\omega} \delta\left(1 + \frac{q^2}{2M\omega}\right) \Leftrightarrow \omega W_2 \equiv F_2(x) = Q_W^2 \delta(1 - x)$$

$$x = -\frac{q^2}{2M\omega}$$

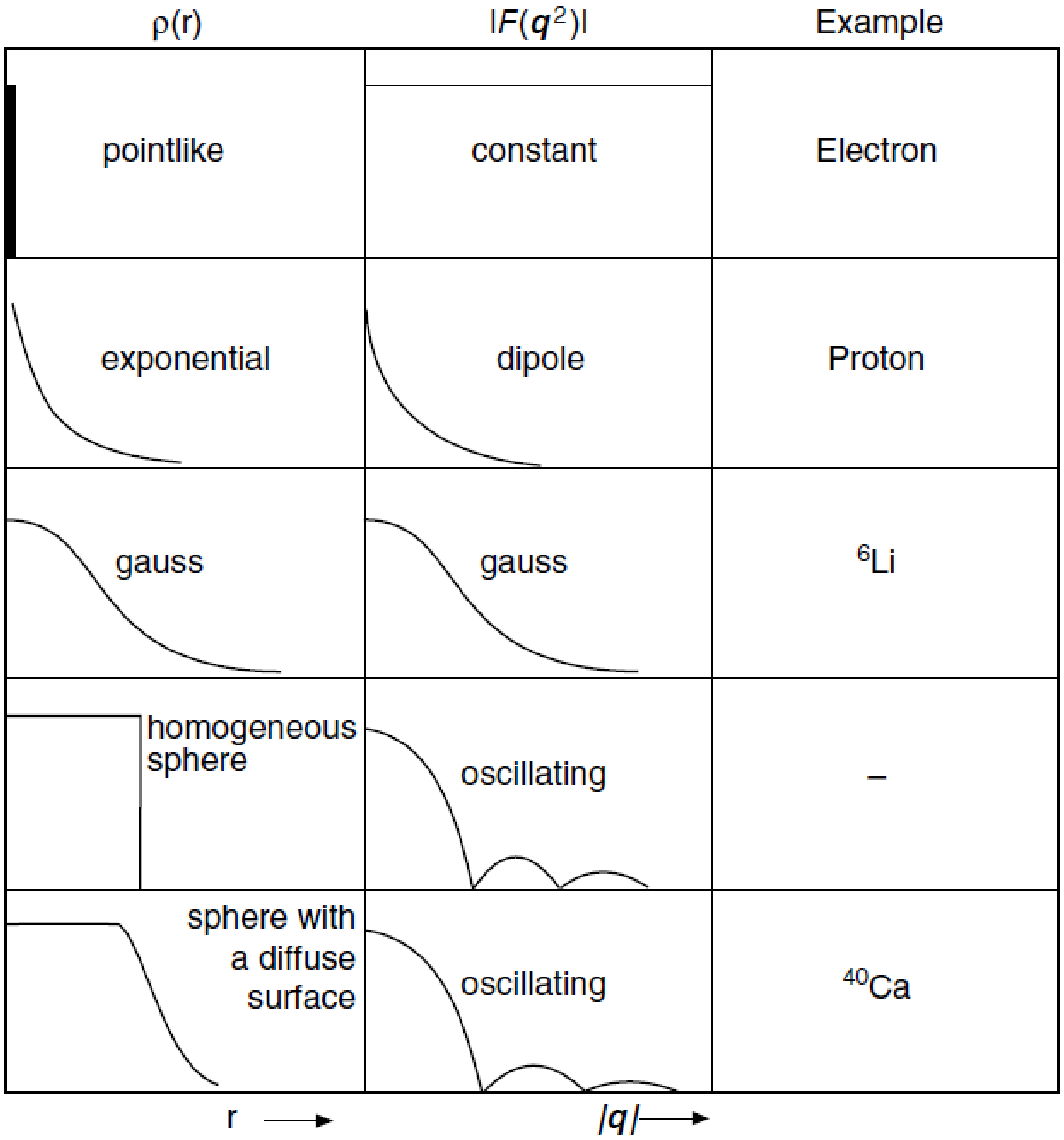
CEνNS

- A = (S = 0) nucleus

- For $|\vec{q}| > \frac{1}{r_A} \approx 70\ \mathrm{MeV}$ on ^{12}C

$$W_2 = \frac{Q_W^2}{\omega}\delta\left(1 + \frac{q^2}{2M\omega}\right) \to \frac{Q_W^2}{\omega} F^2(q^2)\delta\left(1 + \frac{q^2}{2M\omega}\right)$$

Form factors



$\rho(r)$	$\lvert F(\boldsymbol{q}^2)\rvert$	Example
pointlike	constant	Electron
exponential	dipole	Proton
gauss	gauss	^{6}Li
homogeneous sphere	oscillating	–
sphere with a diffuse surface	oscillating	^{40}Ca

CEνNS

- A = (S = 0) nucleus

- For $|\vec{q}| > \frac{1}{r_A} \approx 70 \text{ MeV}$ on ^{12}C

$$W_2 = \frac{Q_W^2}{\omega}\delta\left(1 + \frac{q^2}{2M\omega}\right) \rightarrow \frac{Q_W^2}{\omega}F^2(q^2)\delta\left(1 + \frac{q^2}{2M\omega}\right)$$

Substituting in:

$$\frac{d\sigma}{dk_0' d\Omega(\vec{k}')} = \frac{G_F^2}{(2\pi)^2}\frac{|\vec{k}'|}{k_0}\left\{W_1 2k\cdot k' + W_2(2k_0'k_0 - k\cdot k') + 2\frac{m_l^2}{M^2}\left[W_4 k\cdot k' - W_5 M k_0\right] + \frac{W_3}{M}\left[(k_0 + k_0')k\cdot k' - k_0 m_l^2\right]\right\}$$

CEνNS

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$$W_2 = \frac{Q_W^2}{\omega}\delta\left(1 + \frac{q^2}{2M\omega}\right) \rightarrow \frac{Q_W^2}{\omega}F^2(q^2)\delta\left(1 + \frac{q^2}{2M\omega}\right)$$

Results in:

$$\frac{d\sigma}{dk_0' d\Omega(\vec{k}')} = \frac{G_F^2}{(2\pi)^2}\frac{|\vec{k}'|}{k_0}\frac{Q_W^2}{\omega}F^2(q^2)\delta\left(1 + \frac{q^2}{2M\omega}\right)(2k_0' k_0 - k \cdot k')$$

Recoil energy distribution:

$$\frac{d\sigma}{dT} = \frac{G_F^2}{(2\pi)}Q_W^2 F^2(-2MT)M\left[2 - \frac{MT}{k_0^2}\left(1 - 2\frac{k_0}{M}\right)\right]$$

CEνNS

Q_W :

$$J^{\mu}_{nc} = \bar{q}_u \gamma^{\mu} \left[\underbrace{\frac{1}{2} - \left(\frac{2}{3}\right) 2 \sin^2 \theta_W}_{Q_W^u} - \frac{1}{2}\gamma_5 \right] q_u + \bar{q}_d \gamma^{\mu} \left[\underbrace{-\frac{1}{2} - \left(-\frac{1}{3}\right) 2 \sin^2 \theta_W}_{Q_W^d} + \frac{1}{2}\gamma_5 \right] q_d + (d \to s)$$

$$Q_W = Z(2Q_W^u + Q_W^d) + N(Q_W^u + 2Q_W^d)$$

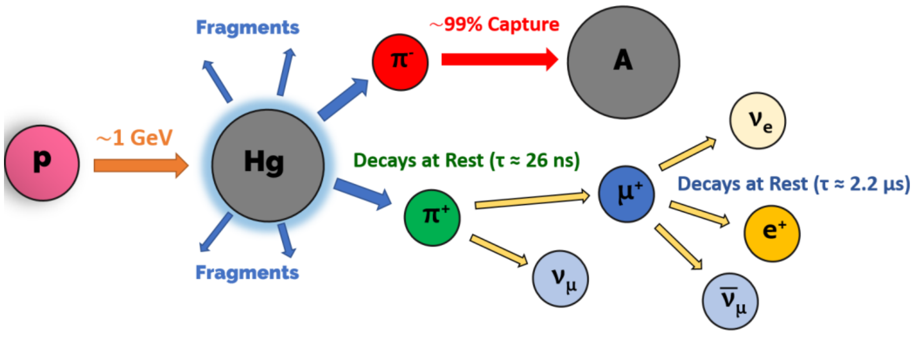
$$2Q_W = Z(1 - 4\sin^2\theta_W) - N$$

$$\frac{d\sigma}{dT} = \frac{G_F^2}{(2\pi)} \frac{\left[Z(1 - 4\sin^2\theta_W) - N\right]^2}{4} F^2(-2MT) M \left[2 - \frac{MT}{k_0^2} \left(1 - 2\frac{k_0}{M} \right) \right]$$

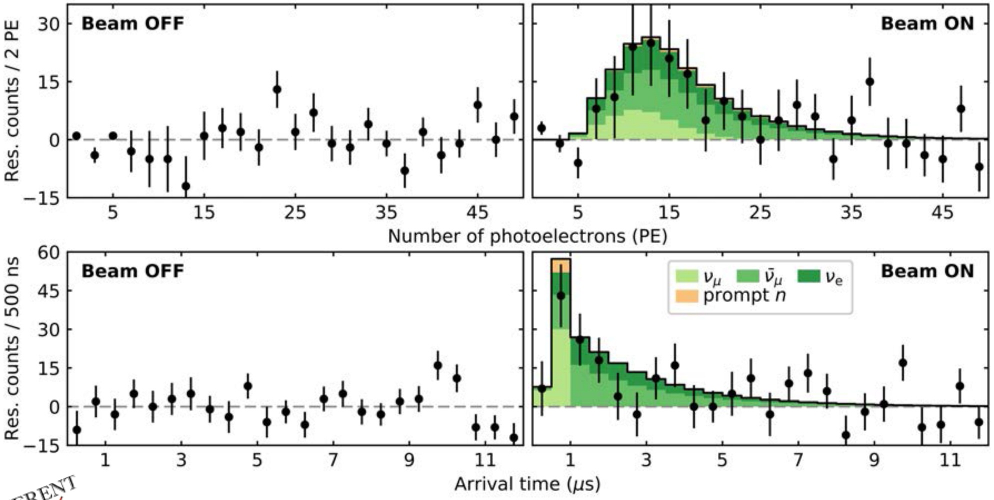
- Measurement requires a recoil energy
  - small enough: $F^2(-2\,M\,T) \sim 1$
  - large enough to be detected

CEνNS

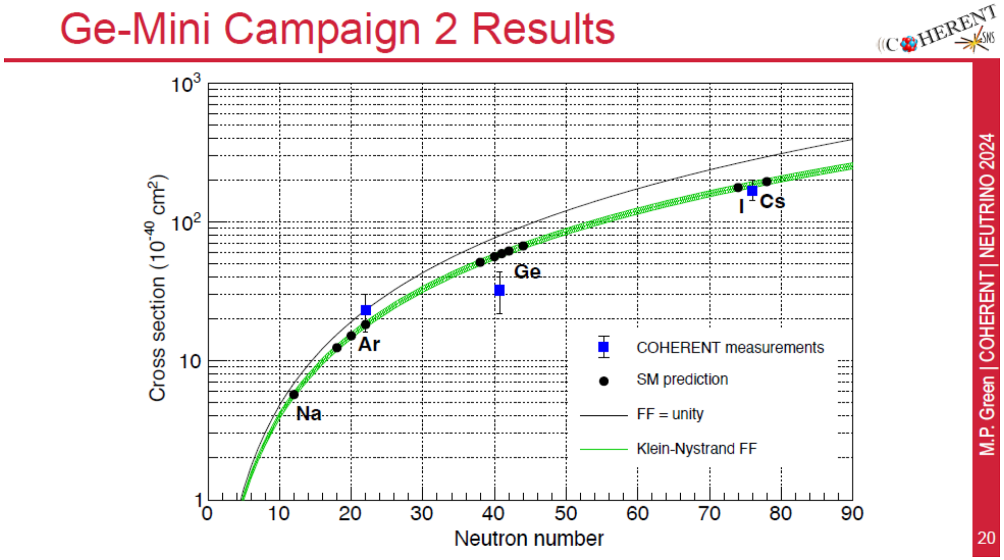
- Coherent
- Pulsed ν beam from pion decay at rest at Spallation Neutron Source



M. Green @ Neutrino 2024



Science 357 (2017)

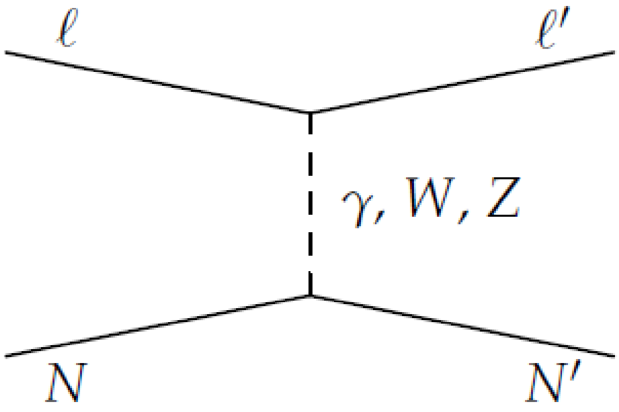


Ge-Mini Campaign 2 Results

- Several (reactor) experiments to follow.
- BSM searches
  - NSI, Z'
  - ν properties: magnetic moment, charge radius
- Neutron distributions in nuclei

QE scattering on the nucleon

$$\begin{aligned}
\textcolor{blue}{\mathrm{EM}:}\ l^{\pm}(k) + N(p) &\rightarrow l^{\pm}(k') + N(p') \\
\textcolor{blue}{\mathrm{CC}:}\ \nu(k) + n(p) &\rightarrow l^{-}(k') + p(p') \\
\bar{\nu}(k) + p(p) &\rightarrow l^{+}(k') + n(p') \\
\textcolor{blue}{\mathrm{NC}:}\ \nu(k) + N(p) &\rightarrow \nu(k') + N(p') \\
\bar{\nu}(k) + N(p) &\rightarrow \bar{\nu}(k') + N(p')
\end{aligned}$$



- QE kinematics:

$$(q+p)^2 = (p')^2$$

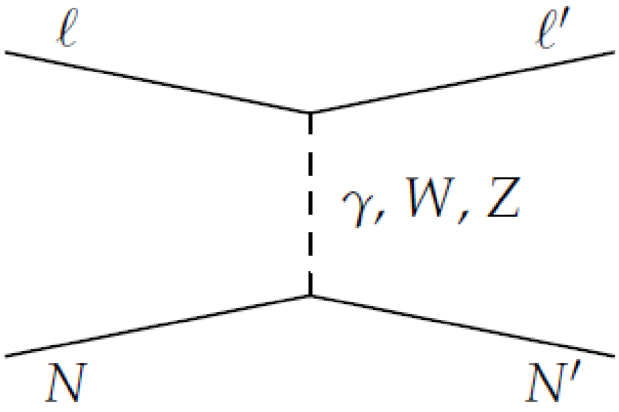
$$q^2 + 2M\omega + M^2 = M^2$$

$$\omega = -\frac{q^2}{2M}$$

$$x = 1$$

QE scattering on the nucleon

$$\begin{aligned}
\text{EM}: l^{\pm}(k) + N(p) &\rightarrow l^{\pm}(k') + N(p') \\
\text{CC}: \nu(k) + n(p) &\rightarrow l^{-}(k') + p(p') \\
\bar{\nu}(k) + p(p) &\rightarrow l^{+}(k') + n(p') \\
\text{NC}: \nu(k) + N(p) &\rightarrow \nu(k') + N(p') \\
\bar{\nu}(k) + N(p) &\rightarrow \bar{\nu}(k') + N(p')
\end{aligned}$$



- Cross section:

$$\frac{d\sigma}{dk'_0 d\Omega(\vec{k}')} = \frac{G_F^2}{(2\pi)^2} \frac{|\vec{k}'|}{k_0} L_{\mu\nu} W^{\mu\nu}$$

For EM: $L_{\mu\nu} \rightarrow L_{\mu\nu}^{(\text{sym})}$ $\quad \frac{G_F^2}{(2\pi)^2} \rightarrow \frac{\alpha^2}{q^4}$

$$W^{\mu\nu} = \frac{1}{2M} \int \frac{d^3 p'}{2E'} \delta^4(k' + p' - k - p) H^{\nu\mu}$$

$$H^{\alpha\beta} = \text{Tr}\left[(\not{p} + M)\, \gamma^0\, (\Gamma^{\alpha})^{\dagger}\, \gamma^0\, (\not{p}' + M)\, \Gamma^{\beta} \right]$$

$$\langle N' | J^{\mu} | N \rangle = \bar{u}(p') \Gamma^{\mu} u(p) = \mathcal{V}^{\mu} - \mathcal{A}^{\mu}$$

Electroweak nucleon current

$$\langle N'|\, J^\mu \,|N\rangle = \bar{u}(p')\Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

$$\mathcal{V}^\mu \;=\; \bar{u}(p')\left[\gamma^\mu F_1 + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2\right]u(p)$$

$$\mathcal{A}^\mu \;=\; \bar{u}(p')\left[\gamma^\mu\gamma_5 F_A + \frac{q^\mu}{M}\gamma_5 F_P\right]u(p)$$

- $F_1 \leftarrow$ Dirac form factor
- $F_2 \leftarrow$ Pauli form factor
- $F_A \leftarrow$ axial form factor
- $F_P \leftarrow$ pseudoscalar form factor

- The most general structure is reduced using Dirac algebra and motion eq.
- T-invariance $\Rightarrow F_i = F_i^*$
- $G = Ce^{i\pi\frac{\tau_2}{2}}$ parity transformations
- $F_i = F_i\,(q^2) \Leftrightarrow 2\; p\cdot q + q^2 = 0$

Electroweak nucleon current

$$\langle N'|\, J^\mu \,|N\rangle = \bar{u}(p')\Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

- $\Gamma^\mu \leftarrow$ 4-vector constructed from:

$$(1)\ p^\mu\ ,\ p'^\mu$$

$$(2)\ \epsilon_{\alpha\beta\mu\nu}\ ,\ g^{\mu\nu}$$

$$(3)\ \left\{\gamma_\mu\ ,\ \gamma_5\ ,\ \gamma_\mu\gamma_5\ ,\ \sigma_{\mu\nu} = \tfrac{i}{2}\left[\gamma_\mu, \gamma_\nu\right]\right\}$$

Any other combination of γ matrices can be reduced to (3).

For example:

$$\gamma_\mu\gamma_\nu = g_{\mu\nu} - i\sigma_{\mu\nu}$$

$$\gamma^\alpha\sigma^{\mu\nu} = i\left(g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\mu}\right)\gamma_\beta + \epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta$$

Electroweak nucleon current

$$\langle N'|\, J^\mu \,|N\rangle = \bar{u}(p')\Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

- $\Gamma^\mu \leftarrow$ 4-vector constructed from:

$$(1)\ p^\mu\ ,\ p'^\mu$$

$$(2)\ \epsilon_{\alpha\beta\mu\nu}\ ,\ g^{\mu\nu}$$

$$(3)\ \left\{\gamma_\mu\ ,\ \gamma_5\ ,\ \gamma_\mu\gamma_5\ ,\ \sigma_{\mu\nu} = \tfrac{i}{2}\left[\gamma_\mu, \gamma_\nu\right]\right\}$$

Using:

- Dirac algebra
- Dirac equation: $(\not{p} - M)u(p) = \bar{u}(p')(\not{p}' - M) = 0$

one finds:

$$\Gamma^\mu = \gamma^\mu F_1 + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2 + \frac{q^\mu}{M}F_S - \gamma^\mu\gamma_5 F_A - \frac{i}{2M}\sigma^{\mu\nu}q_\nu\gamma_5 F_T - \frac{q^\mu}{M}\gamma_5 F_P$$

Electroweak nucleon current

$$\langle N'|\, J^\mu \,|N\rangle = \bar{u}(p')\Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

$$\Gamma^\mu = \gamma^\mu F_1 + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2 + \frac{q^\mu}{M}F_S - \gamma^\mu\gamma_5 F_A - \frac{i}{2M}\sigma^{\mu\nu}q_\nu\gamma_5 F_T - \frac{q^\mu}{M}\gamma_5 F_P$$

- Time reversal (T) transformation:

$$T\left(\bar{u}\Gamma^\mu u\right)T^\dagger = \sum_i F_i^* \bar{u}(\mathcal{O}_i)_\mu u \qquad\qquad T l_\mu T^\dagger = l^\mu$$

the interaction is proportional to:

$$\begin{aligned} l_\mu \bar{u}\Gamma^\mu u &= \sum_i F_i l_\mu \bar{u}\mathcal{O}_i^\mu u \\ T\left(l_\mu \bar{u}\Gamma^\mu u\right)T^\dagger &= \sum_i F_i^* l_\mu \bar{u}\mathcal{O}_i^\mu u \end{aligned}$$

- T-inv ⇒ $F_i = F^*_i$

Electroweak nucleon current

$$\langle N'|\, J^\mu \,|N\rangle = \bar{u}(p')\Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

$$\Gamma^\mu = \gamma^\mu F_1 + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2 + \frac{q^\mu}{M}F_S - \gamma^\mu\gamma_5 F_A - \frac{i}{2M}\sigma^{\mu\nu}q_\nu\gamma_5 F_T - \frac{q^\mu}{M}\gamma_5 F_P$$

- Parity (P) transformation:

$$P\bar{u}(p_0', \vec{p}\,')\Gamma^\mu(q_0, \vec{q})u(p_0, \vec{p})P^\dagger = \bar{u}(p_0', -\vec{p}\,')\gamma_0\Gamma^\mu(q_0, -\vec{q})\gamma_0 u(p_0, -\vec{p})$$

$\gamma^\mu \,,\, \sigma^{\mu\nu}q_\nu \,,\, q^\mu$ ← transform as vectors

$\gamma^\mu\gamma_5 \,,\, \sigma^{\mu\nu}\gamma_5 q_\nu \,,\, q^\mu\gamma_5$ ← transform as axial-vectors

Electroweak nucleon current

$$\langle N'|\, J^\mu \,|N\rangle = \bar{u}(p')\Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

$$\mathcal{V}^\mu = \bar{u}(p')\left[\gamma^\mu F_1 + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2 + \frac{q^\mu}{M}F_S\right]u(p)$$

$$\mathcal{A}^\mu = \bar{u}(p')\left[\gamma^\mu\gamma_5 F_A + \frac{i}{2M}\sigma^{\mu\nu}q_\nu\gamma_5 F_T + \frac{q^\mu}{M}\gamma_5 F_P\right]u(p)$$

- G-parity (G) transformation: $G = Ce^{i\pi\frac{\tau_2}{2}}$
  - Isospin rotation: $e^{i\pi\frac{\tau_2}{2}}\begin{pmatrix} q_u \\ q_d \end{pmatrix} = i\tau_2\begin{pmatrix} q_u \\ q_d \end{pmatrix} = \begin{pmatrix} q_d \\ -q_u \end{pmatrix}$

$$G\mathcal{V}^\mu G^\dagger = \mathcal{V}^\mu$$ ← except for the F$_S$ term

$$G\mathcal{A}^\mu G^\dagger = -\mathcal{A}^\mu$$ ← except for the F$_T$ term

  - $\Rightarrow F_S = F_T = 0 \Leftrightarrow$ absence of 2nd class currents

Electroweak nucleon current

- Vector and EM form factors:

$V_a^\alpha = \mathcal{V}^\alpha \frac{\tau_a}{2}$ ← isovector current $V_Y^\alpha = \mathcal{V}_Y^\alpha I$ ← hypercharge (isoscalar) current

$$\langle p| V_{\rm EM}^\alpha |p\rangle = \langle p| V_3^\alpha + \frac{1}{2} V_Y^\alpha |p\rangle = \frac{\mathcal{V}^\alpha + \mathcal{V}_Y^\alpha}{2} \equiv \mathcal{V}_p^\alpha$$

$$\langle n| V_{\rm EM}^\alpha |n\rangle = \langle n| V_3^\alpha + \frac{1}{2} V_Y^\alpha |n\rangle = \frac{-\mathcal{V}^\alpha + \mathcal{V}_Y^\alpha}{2} \equiv \mathcal{V}_n^\alpha$$

Then: $\langle p| V_{\rm CC}^\alpha |n\rangle = \langle p| V_1^\alpha + i V_2^\alpha |n\rangle = \mathcal{V}^\alpha = \mathcal{V}_p^\alpha - \mathcal{V}_n^\alpha$

$$\begin{aligned} \langle p| V_{\rm NC}^\alpha |p\rangle &= \langle p| (1 - 2\sin^2\theta_W) V_3^\alpha - \sin^2\theta_W V_Y^\alpha |p\rangle \\ &= \left(\frac{1}{2} - \sin^2\theta_W\right) \mathcal{V}^\alpha - \sin^2\theta_W \mathcal{V}_Y^\alpha \\ &= \left(\frac{1}{2} - 2\sin^2\theta_W\right) \mathcal{V}_p^\alpha - \frac{1}{2} \mathcal{V}_n^\alpha \end{aligned}$$

- Vector CC and NC form factors can be expressed in terms of EM ones

Electron scattering on the nucleon

$$\mathrm{EM}: l^{\pm}(k) + N(p) \rightarrow l^{\pm}(k') + N(p')$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} = \frac{\alpha^2}{4k_0^2 \sin^4\frac{\theta}{2}} \frac{k_0'}{k_0} \cos^2\frac{\theta}{2}$$

← Scattering on a point-like spinless target in Lab

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \left[1 - \frac{q^2}{2M^2}\tan^2\frac{\theta}{2}\right]$$

← Scattering on a point-like spin 1/2 target in Lab

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \left[\frac{G_E^2 - \frac{q^2}{4M^2}G_M^2}{1 - \frac{q^2}{4M^2}} - \frac{q^2}{2M^2}G_M^2\tan^2\frac{\theta}{2}\right]$$

$$q^2 = -4k_0k_0'\sin^2\frac{\theta}{2}$$

- Rosenbluth separation ⇒ Sachs f.f. G_E , G_M

$$G_E = F_1 + \frac{q^2}{4m_N^2}F_2$$

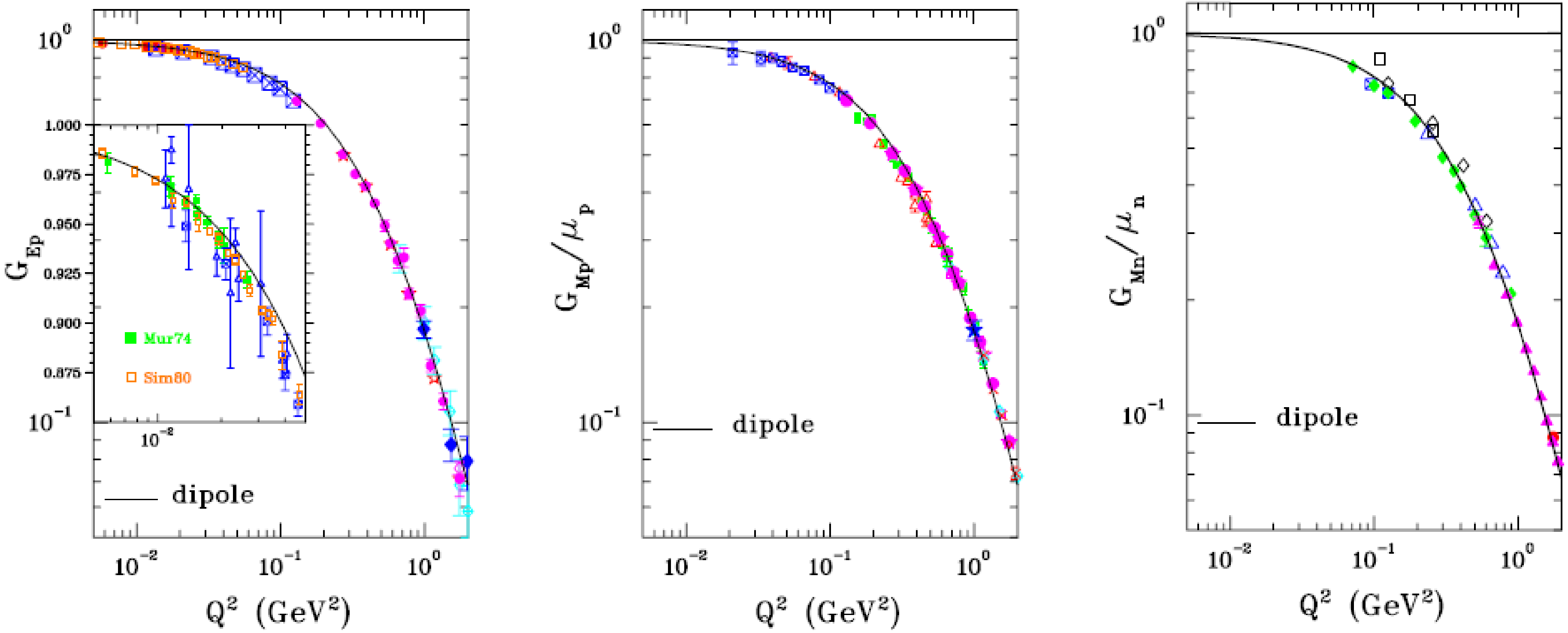
$$G_M = F_1 + F_2$$

Nucleon EM form factors

- $Q^2 \lesssim 1$ GeV^2

$$G_E^p(q^2) = G_D(q^2)\,,\; G_M^p = \mu_p G_D(q^2)\,,\; G_M^p = \mu_p G_D(q^2)\,,\; G_D = \left(1 - \frac{q^2}{M_V^2}\right)^{-2}$$

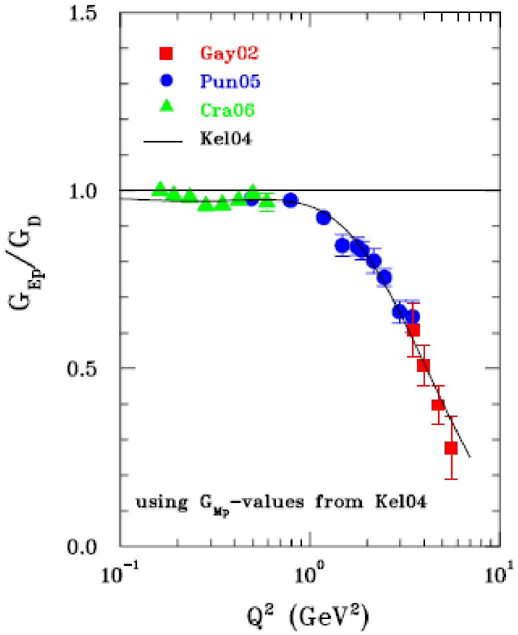
$$\mu_p = 2.793\,,\; \mu_n = -1.913\,,\; M_V^2 = 0.71 \text{ GeV}^2$$

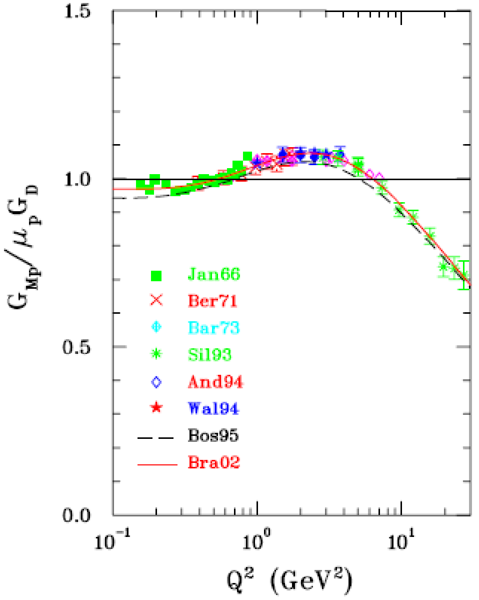


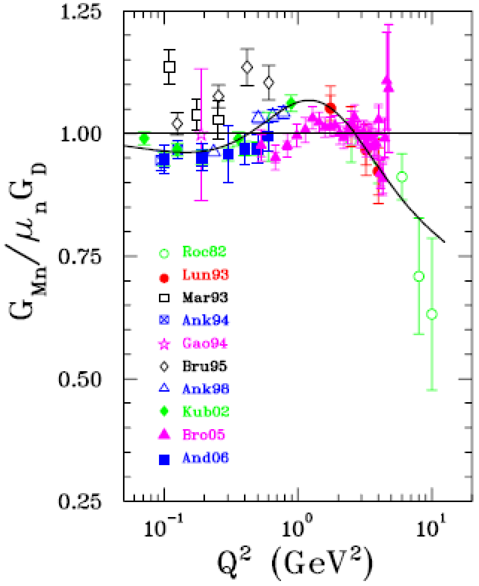
Perdrisat et al., Prog.Part.Nucl.Phys. 59 (2007) 694-764

Electron scattering on the nucleon

- More precision, particularly at high Q^2, with polarization techniques (Jlab)
  - Polarization transfer: $\vec{e} + p \rightarrow e + \vec{p}$
  - Double polarization: $\vec{e} + \vec{p} \rightarrow e + p$
- $Q^2 \gtrsim 1$ GeV^2





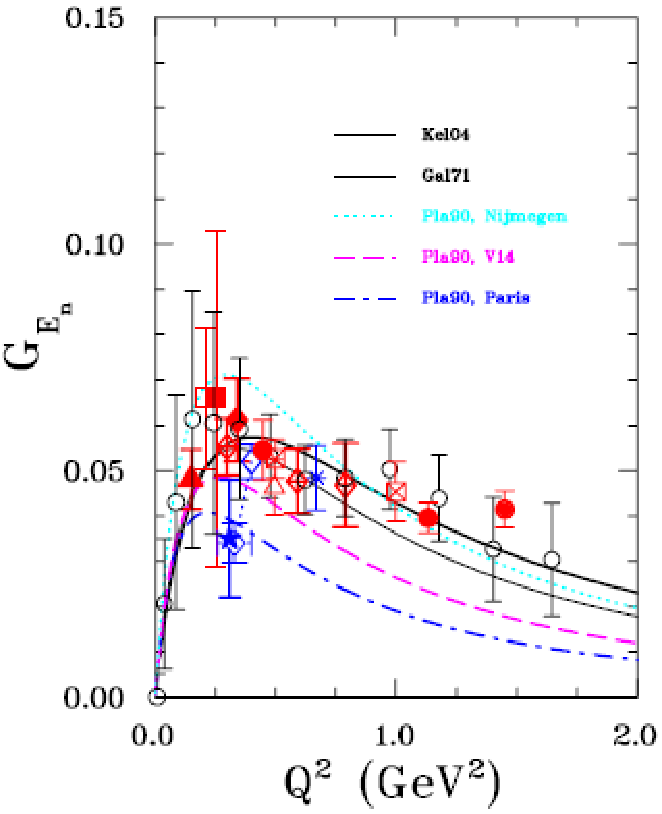


Perdrisat et al., Prog.Part.Nucl.Phys. 59 (2007) 694-764

Nucleon EM form factors

- Neutron electric form factor

$$G_E^n(0) = 0$$



Perdrisat et al., Prog.Part.Nucl.Phys. 59 (2007) 694-764

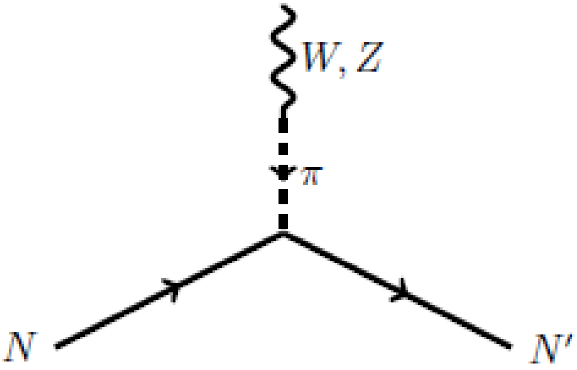
Electroweak nucleon current

- Consequences of PCAC and pion-pole dominance

$$\langle n| (-i)\mathcal{V}^\mu_\pi |p\rangle = \langle n| (-i)\mathcal{L}_{N\pi\pi} \left|p\pi^-\right\rangle (-i)D_\pi(q)(-i) \left\langle\pi^-\right| \mathcal{A}^\mu_- |0\rangle$$

$$\mathcal{L}_{NN\pi} = -\frac{g_{NN\pi}}{2f_\pi}\bar{N}\gamma_\mu\gamma_5(\partial^\mu\vec{\pi})\vec{\tau}N \qquad D_\pi = \frac{1}{q^2-m_\pi^2} \qquad \left\langle\pi^-\right| A^\mu_- |0\rangle = -\sqrt{2}f_\pi i q^\mu$$

$$\langle n| \mathcal{V}^\mu_\pi |p\rangle \quad = \quad -g_{NN\pi}F_{NN\pi}(q^2)\frac{1}{q^2-m_\pi^2}\bar{u}\not{q}\gamma_5 q^\mu u \qquad F_{NN\pi}(0) = 1$$



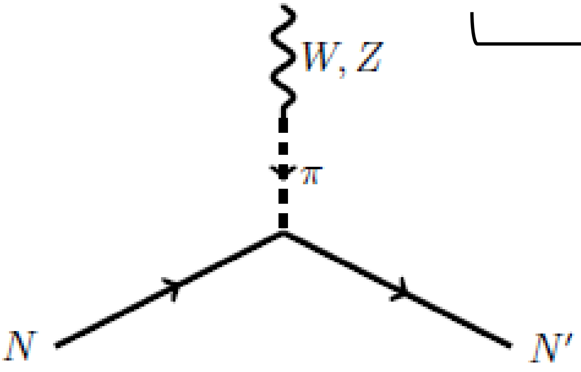
Electroweak nucleon current

- Consequences of PCAC and pion-pole dominance

$$\langle n|\,(-i)\mathcal{V}^{\mu}_{\pi}\,|p\rangle = \langle n|\,(-i)\mathcal{L}_{N\pi\pi}\,\left|p\pi^{-}\right\rangle(-i)D_{\pi}(q)(-i)\left\langle\pi^{-}\right|\mathcal{A}^{\mu}_{-}\,|0\rangle$$

$$\mathcal{L}_{NN\pi} = -\frac{g_{NN\pi}}{2f_{\pi}}\bar{N}\gamma_{\mu}\gamma_{5}(\partial^{\mu}\vec{\pi})\vec{\tau}N \qquad D_{\pi} = \frac{1}{q^{2}-m_{\pi}^{2}} \qquad \left\langle\pi^{-}\right|A^{\mu}_{-}\,|0\rangle = -\sqrt{2}f_{\pi}iq^{\mu}$$

$$\begin{aligned}\langle n|\,\mathcal{V}^{\mu}_{\pi}\,|p\rangle &= -g_{NN\pi}F_{NN\pi}(q^{2})\frac{1}{q^{2}-m_{\pi}^{2}}\bar{u}\not{q}\gamma_{5}q^{\mu}u \qquad F_{NN\pi}(0)=1\\ &= \underbrace{-g_{NN\pi}F_{NN\pi}(q^{2})\frac{2M}{q^{2}-m_{\pi}^{2}}}_{=\frac{F_{P}}{M}}\bar{u}\gamma_{5}q^{\mu}u\end{aligned}$$



$$\mathcal{A}^{\mu} = \bar{u}(p')\left[\gamma^{\mu}\gamma_{5}F_{A} + \frac{q^{\mu}}{M}\gamma_{5}F_{P}\right]u(p)$$

Electroweak nucleon current

- Consequences of PCAC and pion pole dominance

$$\begin{aligned} \langle n | \mathcal{V}_\pi^\mu | p \rangle &= -g_{NN\pi} F_{NN\pi}(q^2) \frac{1}{q^2 - m_\pi^2} \bar{u} \not{q} \gamma_5 q^\mu u \\ &= \underbrace{-g_{NN\pi} F_{NN\pi}(q^2) \frac{2M}{q^2 - m_\pi^2}}_{= \frac{F_P}{M}} \bar{u} \gamma_5 q^\mu u \end{aligned}$$

PCAC: $\langle n | q_\mu \mathcal{A}^\mu | p \rangle = 0 \qquad m_\pi \to 0$

$$\bar{u} \left[\not{q} \gamma_5 F_A - g_{NN\pi} F_{NN\pi}(q^2) \frac{2M}{q^2 - m_\pi^2} q^2 \gamma_5 \right] u = 0 \qquad m_\pi \to 0$$

$$F_A(q^2) = g_{NN\pi} F_{N\pi\pi}(q^2)$$

$F_A(0) \equiv g_A = g_{NN\pi}$ ← Goldberger-Treiman relation

Electroweak nucleon current

- Consequences of PCAC and pion pole dominance

$$\mathcal{A}^\mu = \bar{u}(p')\left[\gamma^\mu\gamma_5 F_A(q^2) + \frac{q^\mu}{M}\gamma_5 F_A(q^2)\frac{2M}{Q^2+m_\pi^2}\right]u(p)$$

- F_P has a small impact on CCQE cross sections (except for ν_τ !)
  - appear in terms proportional to $(m_l/M)^4$
- F_P does not contribute to NCE cross sections
- F_p is studied in muon capture $\mu^- + p \to \nu_\mu + n$
  - $F_P(0)/g_A$ consistent with the PCAC+pion-pole
- F_p pion-pole enhancement at low q^2

Nucleon axial form factor

- What is known:
  - $F_A(0) = g_A \leftarrow \beta$ decay
  - $F_A(\infty) \sim Q^{-4} \leftarrow$ QCD

$$F_A(Q^2) = g_A - \frac{1}{6}\langle r_A^2 \rangle Q^2 + \ldots$$

- Main source of information: bubble chamber (ANL, BNL, FNAL,CERN) data

- Dipole ansatz: Bodek et al., EPJC 53 (2008)

$$F_A(Q^2) = g_A \left(1 + \frac{Q^2}{M_A^2}\right)^{-2} \qquad \langle r_A^2 \rangle = \frac{12}{{M_A}^2}$$

  - $<r_A^2> = 0.453(12)$ fm^2
- z-expansion: Meyer et al., PRD 93 (2016)
  - $<r_A^2> = 0.46(22)$ fm^2
- Neural networks + Bayesian statistics: LAR, Graczyk, Saúl-Sala, PRC 99 (2019)
  - $<r_A^2> = 0.471(15)$ fm^2 ← ANL only so far

- All methods obtain similar $F_A(Q^2)$...
- ... but with different errors

Lattice QCD in a nutshell

- Green functions:

$$\mathcal{G}(x_1,\ldots,x_n)=\frac{1}{Z}\int[d\phi]\phi(x_1)\cdots\phi(x_n)e^{iS(\phi)}$$

- Euclidean time: $x_4 = i\,x_0\;,\;S=\;i\,S_E$

$$\mathcal{G}(x_1,\ldots,x_n)=\frac{1}{Z_E}\int[d\phi]\phi(x_1)\cdots\phi(x_n)e^{-S_E(\phi)}$$

- Regularization: finite lattice spacing a
- Finite lattice volume V
- Unphysical (large) fermion masses
- For QCD (and nucleons)
  - 2-point functions: $\Rightarrow m_N(a,V,m_q)\rightarrow\;m_N(0,\infty,m_q(\mathrm{phys}))$

$$\mathcal{G}(x)=\frac{1}{Z_E}\int[dUd\bar{\psi}\psi]N(x)N^\dagger(0)e^{-S_E}=<0|N(x)N^\dagger(0)|0>$$

- $N(x)$ interpolating field: 3-quark system with nucleon quantum numbers
- $U=1-A_\mu(x)a\hat{e}^\mu+\ldots$ lattice gauge fields

Lattice QCD in a nutshell

- Green functions:

$$\mathcal{G}(x_1,\ldots,x_n) = \frac{1}{Z}\int [d\phi]\phi(x_1)\cdots\phi(x_n)e^{iS(\phi)}$$

- Euclidean time: $x_4 = i\,x_0\;,\; S = \; i\,S_E$

$$\mathcal{G}(x_1,\ldots,x_n) = \frac{1}{Z_E}\int [d\phi]\phi(x_1)\cdots\phi(x_n)e^{-S_E(\phi)}$$

- Regularization: finite lattice spacing a
- Finite lattice volume V
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  - 2-point functions: $\Rightarrow m_N(a,V,m_q) \to \; m_N(0,\infty,m_q(\text{phys}))$

$$\mathcal{G}(x) = \frac{1}{Z_E}\int [dU d\bar{\psi}\psi] N(x) N^\dagger(0) e^{-S_E} =< 0|N(x)N^\dagger(0)|0 >$$

$$\mathcal{G}(x) = \sum_n < 0|N(x)|n >< n|N^\dagger(0)|0 > e^{-E_n x_4} =< 0|N(x)N^\dagger(0)|0 >$$

Lattice QCD in a nutshell

- Green functions:

$$\mathcal{G}(x_1,\ldots,x_n) = \frac{1}{Z}\int [d\phi]\phi(x_1)\cdots\phi(x_n)e^{iS(\phi)}$$

- Euclidean time: $x_4 = i\,x_0$, $S = \; i\,S_E$

$$\mathcal{G}(x_1,\ldots,x_n) = \frac{1}{Z_E}\int [d\phi]\phi(x_1)\cdots\phi(x_n)e^{-S_E(\phi)}$$

- Regularization: finite lattice spacing a
- Finite lattice volume V
- Unphysical (large) fermion masses
- For QCD (and nucleons)
  - 3-point functions: $\Rightarrow F_A(a, V, m_q) \to \; F_A(0, \infty, m_q(\mathrm{phys}))$

$$\mathcal{G}^\mu(x,y) = \frac{1}{Z_E}\int [dU d\bar{\psi}\psi] N(x)A^\mu(y)N^\dagger(0)e^{-S_E} =< 0|N(x)A^\mu(y)N^\dagger(0)|0 >$$

  - $A^\mu(y) = \bar{\psi}(y)\gamma^\mu\gamma_5\psi(y)$ axial current

Lattice QCD in a nutshell

- Green functions:

$$\mathcal{G}(x_1,\ldots,x_n) = \frac{1}{Z}\int [d\phi]\phi(x_1)\cdots\phi(x_n)e^{iS(\phi)}$$

- Euclidean time: $x_4 = i\,x_0\,,\; S = \; i\,S_E$

$$\mathcal{G}(x_1,\ldots,x_n) = \frac{1}{Z_E}\int [d\phi]\phi(x_1)\cdots\phi(x_n)e^{-S_E(\phi)}$$

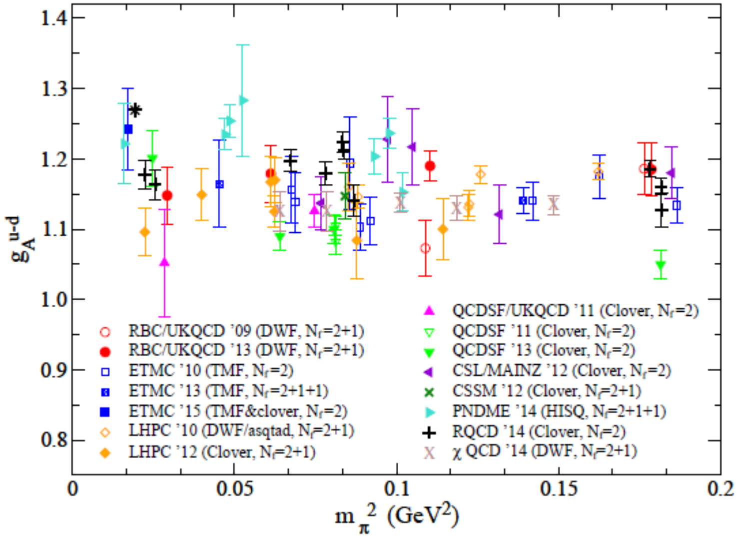
- Regularization: finite lattice spacing a
- Finite lattice volume V
- Unphysical (large) fermion masses
- For QCD (and nucleons)
  - 3-point functions: $\Rightarrow F_A(a,V,m_q) \rightarrow \; F_A(0,\infty,m_q(\text{phys}))$

$$\mathcal{G}^\mu(x,y) = \frac{1}{Z_E}\int [dUd\bar{\psi}\psi]N(x)A^\mu(y)N^\dagger(0)e^{-S_E} =< 0|N(x)A^\mu(y)N^\dagger(0)|0 >$$

$$= \sum_{n,m} < 0|N(x)|n >< n|A^\mu(y)|m >< m|N^\dagger(0)|0 > e^{-E_n(x_4-y_4)-E_m y_4} =< 0|N(x)A^\mu(y)N^\dagger(0)|0 >$$

F_A in LQCD

- g_A : lower than exp. values were once obtained



Constantinou, PoS CD15 (2015) 009

$g_A = 1.2754(13)_{exp}(2)_{RC}$

M. Gorchtein and C.-Y. Seng, JHEP 53 (2021)

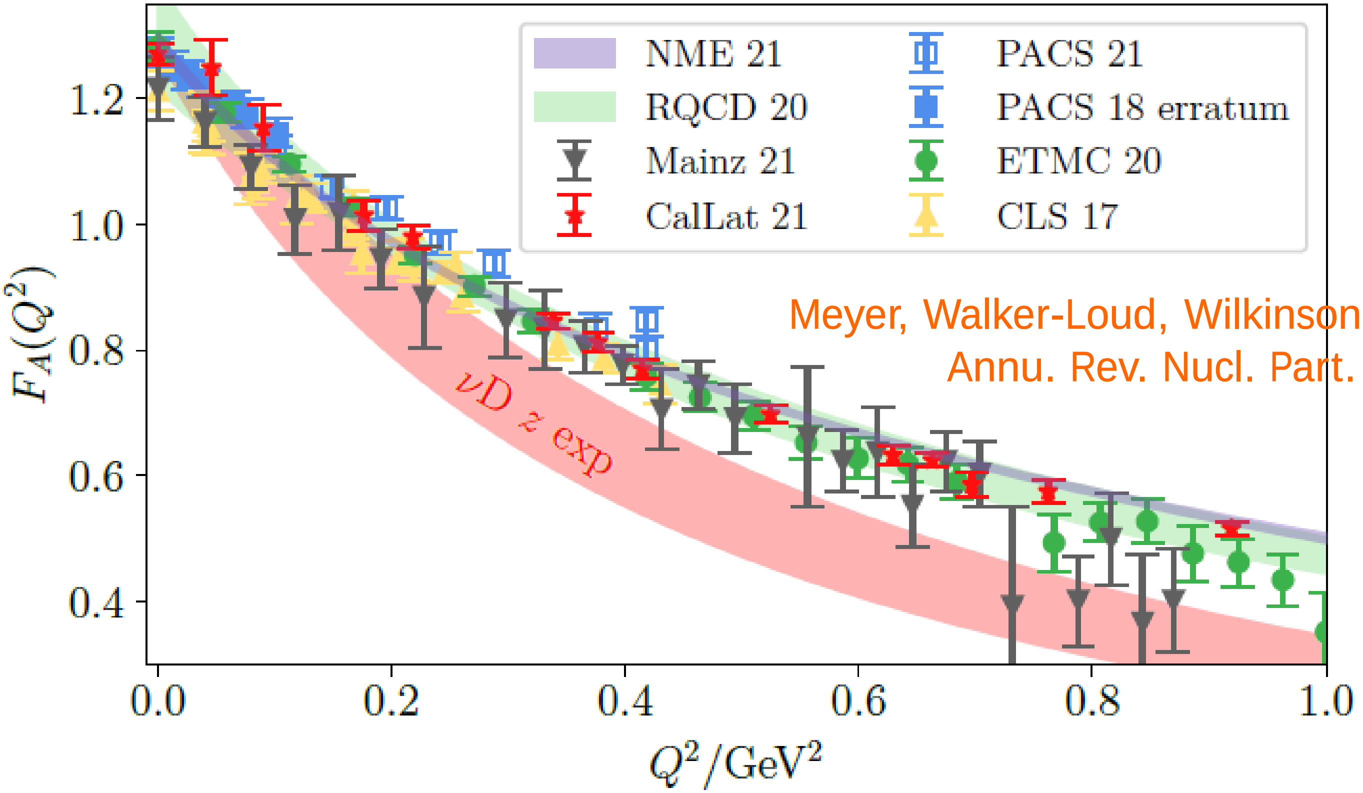
- Progress (for both g_A and F_A)
  - improved algorithms for a careful treatement of excited states
  - low pion masses

FLAG 2021
Flavour Lattice Averaging Group

$g_A = 1.246(28)$

Alexandrou et al., PRD 96 (2017); PRD103 (2021)
Capitani et al., Int. J. Mod. Phys. A 34 (2019)
Gupta et al., PRD 96 (2017); Park et al., PRD 105 (2022)
Chang et al., Nature 558 (2018)
Bali et al., JHEP 05 (2020)
Shintani, PRD 99; PRD 102(erratum) (2020)

F_A: Exp. vs LQCD



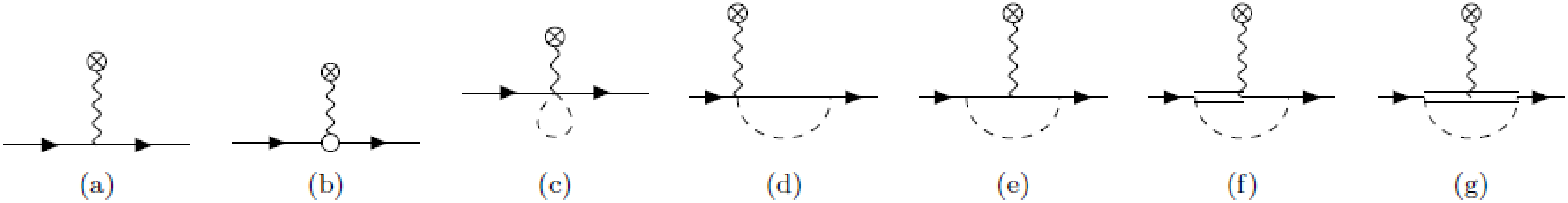
Meyer, Walker-Loud, Wilkinson,
Annu. Rev. Nucl. Part. Sci. 2022. 72:1–30

F_A in ChPT

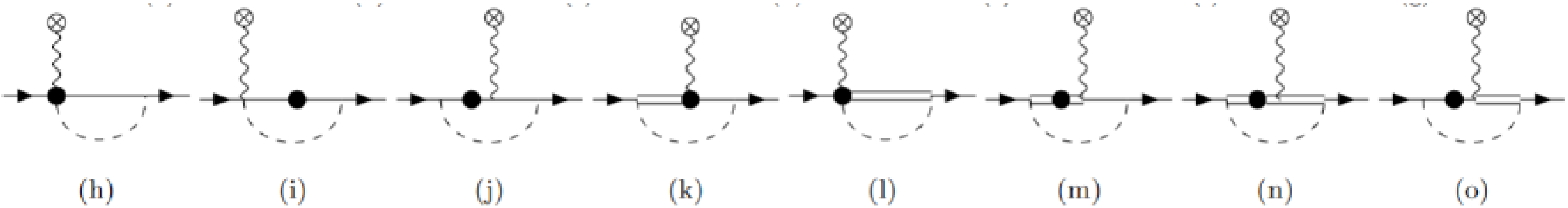
$$F_A(Q^2, M_\pi^2) = g + 4d_{16} M_\pi^2 + d_{22} Q^2 + F_A^{(\mathrm{loops})} + F_A^{(wf)}$$

- Up to O(p^3) Yao, LAR, Vicente Vacas, PRD 96 (2017)

$\delta = m_\Delta - m_N \sim \mathcal{O}(p)$



- Up to O(p^4) Alvarado, LAR, PRD 105 (2022); to be published



- Differences between O(p^3) and O(p^4) provide a measure of the systematic error arising from the truncation of the perturbative expansion.

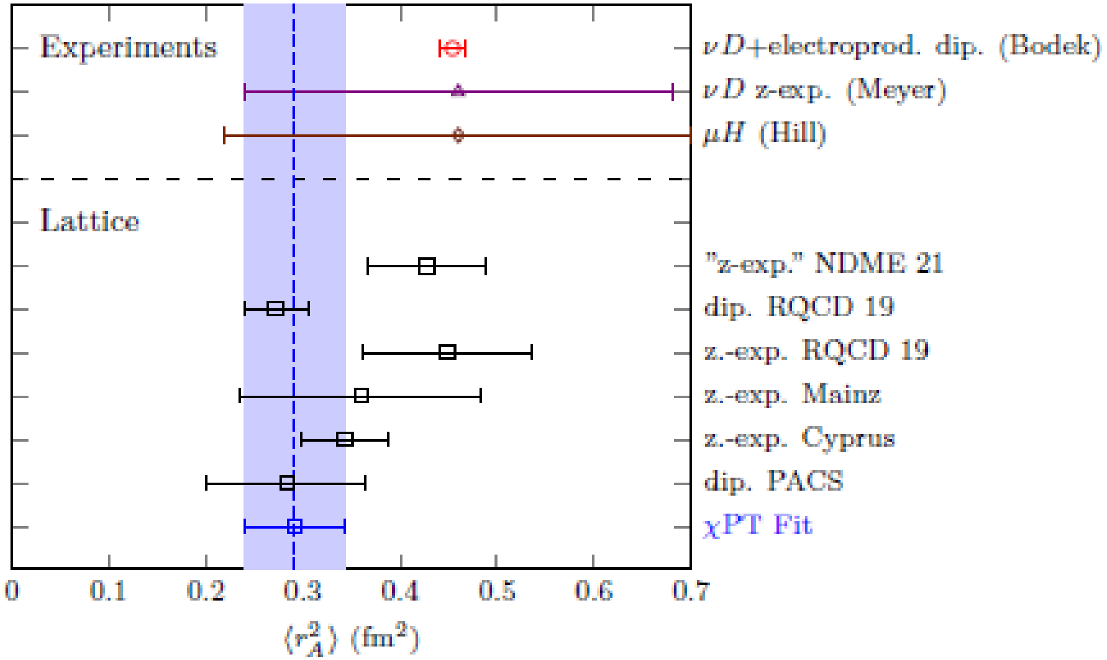
$M_\pi{}^2 = (m_u + m_d)< \bar{u}u + \bar{d}d >/(2f_\pi{}^2) + \ldots$; $< \bar{u}u + \bar{d}d > \leftarrow$ condensate

F_A in ChPT & LQCD

- ChPT analysis: $Q^2 < 0.36$ GeV^2, $M_\pi < 400$ MeV , $M_\pi L > 3.5$
  - Model-independent extrapolations to the physical M_π

$$F_A(Q^2, M_\pi^2) = g + 4d_{16}M_\pi^2 + d_{22}Q^2 + F_A^{(\mathrm{loops})} + F_A^{(wf)}$$

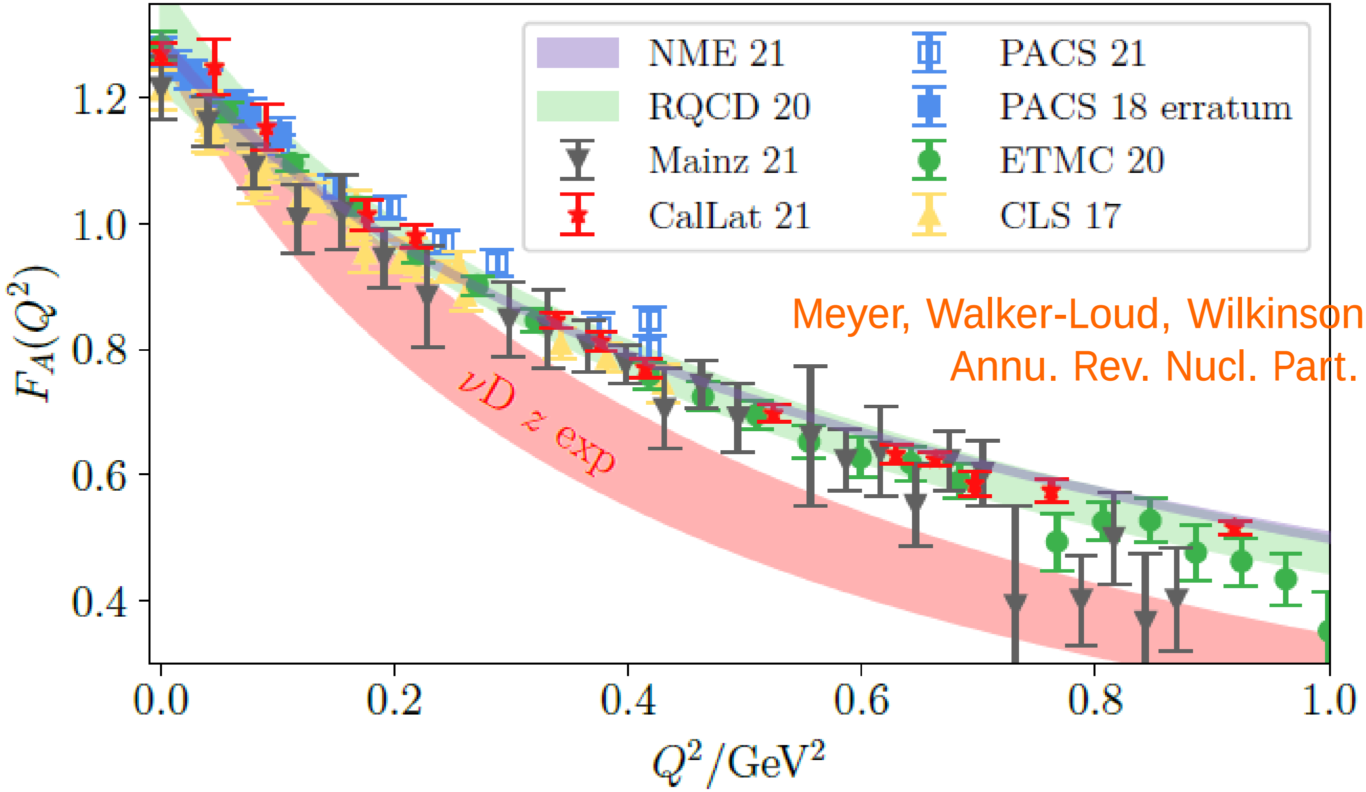
$$F_A(q^2) = g_A \left[1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \mathcal{O}(q^4)\right]$$



$\langle r_A^2\rangle = 0.291(52)$ fm^2 ⇔ $M_A = 1.27(11)$ GeV   F. Alvarado, LAR

in tension with empirical determinations

F_A: Exp. vs LQCD

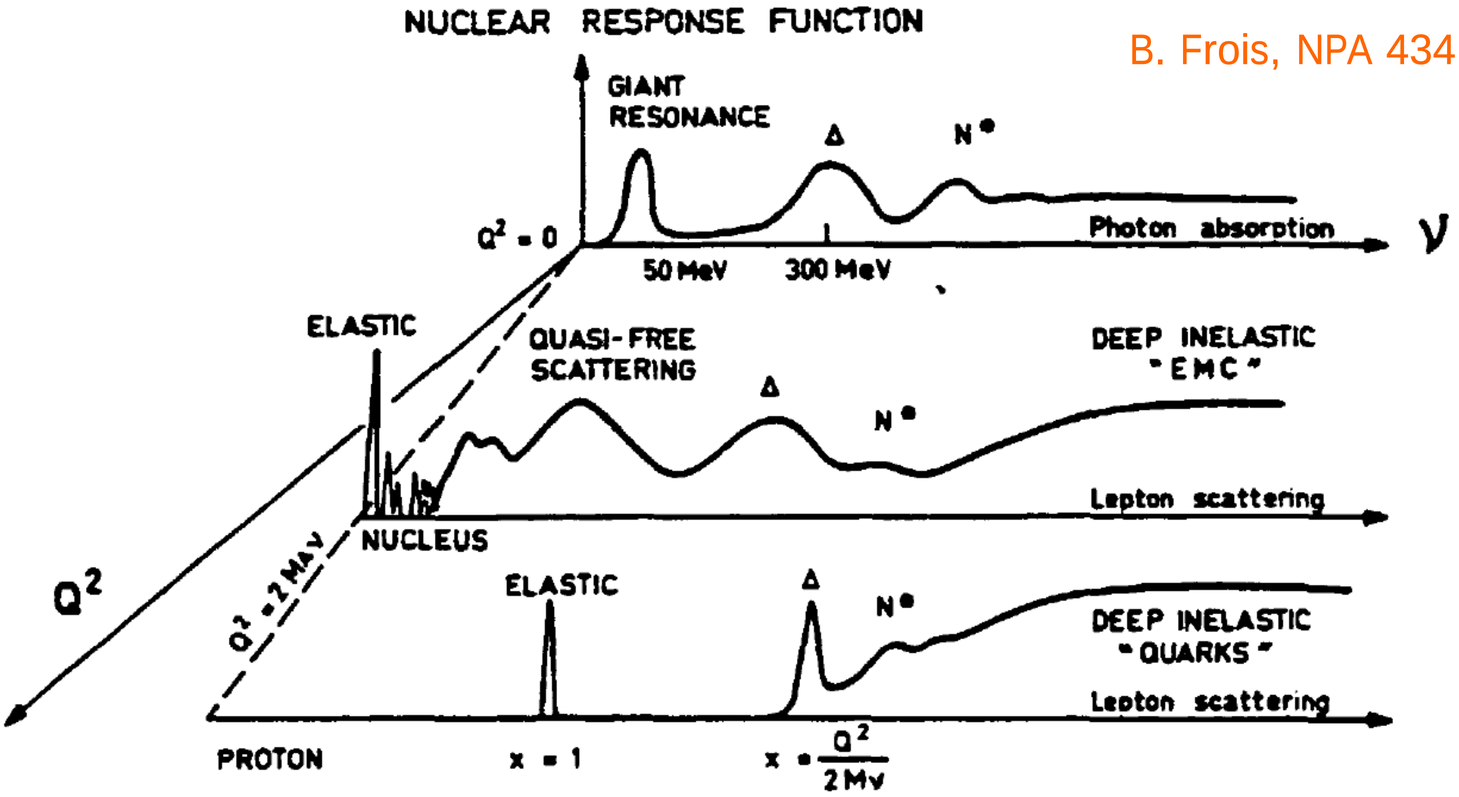


Meyer, Walker-Loud, Wilkinson, Annu. Rev. Nucl. Part. Sci. 2022. 72:1–30

- How reliable are old bubble chamber experiments?
  - new measurements on H and D are needed.
- Do LQCD present results still hide uncontrolled systematics?

Neutrino interactions on nuclei

- Multiscale (even at a given E_ν), multi-nucleon problem



B. Frois, NPA 434 (1985)

- Shell structure, collective excitations, QE peak, ...
- initial state description: non-relativistic)
- final state interactions: (relativistic) NN, πN, ...

The nucleon-nucleon interaction

- Constrained by
  - Deuteron properties
  - NN scattering data
- At low energies ⇒ non-relativistic potential $V = V(\vec{r}_i, \vec{p}_i, \vec{\sigma}_i, \vec{\tau}_i)\,, \quad i = 1, 2$
- Symmetries:
  - Translational invariance: $V(\vec{r}_i, \ldots) = V(\vec{r} = \vec{r}_1 - \vec{r}_2, \ldots)$
  - Galilean invariance: $V(\vec{p}_i, \ldots) = V(\vec{p} = \vec{p}_1 - \vec{p}_2, \ldots)$
  - Parity invariance: $V(\vec{r}, \vec{p}, \vec{\sigma}_i, \vec{\tau}_i) = V(-\vec{r}, -\vec{p}, \vec{\sigma}_i, \vec{\tau}_i)$
  - Time reversal invariance: $V(\vec{r}, \vec{p}, \vec{\sigma}_i, \vec{\tau}_i) = V(\vec{r}, -\vec{p}, -\vec{\sigma}_i, \vec{\tau}_i)$
  - Isospin invariance: $V = V_0 + V_\tau(\vec{\tau}_1 \cdot \vec{\tau}_2)$

The nucleon-nucleon interaction

- Important terms:
  - Local central potential

$$V_C = V_0(r) + V_\sigma(r)(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_\tau(r)(\vec{\tau}_1 \cdot \vec{\tau}_2) + V_{\sigma\tau}(r)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2)$$

  - Tensor force
    - non central
    - explains the deuteron electric quadrupole moment

$$V_T = \left[V_{T_0}(r) + V_{T_\tau}(\vec{\tau}_1 \cdot \vec{\tau}_2)\right] S_{12}$$

$$S_{12} = \frac{(\vec{r} \cdot \vec{\sigma}_1)(\vec{r} \cdot \vec{\sigma}_2)}{r^2} - \frac{1}{3}(\vec{\sigma}_1 \cdot \vec{\sigma}_1)$$

  - Spin-orbit force
    - most relevant nonlocal term
    - revealed in NN scattering through polarization observables
    - needed to obtain magic nuclei

$$V_{LS} = V_{LS}(r)\vec{L} \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)\,, \;\; \vec{L} = (\vec{r} \times \vec{p})$$

The nucleon-nucleon interaction

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  - Local central potential

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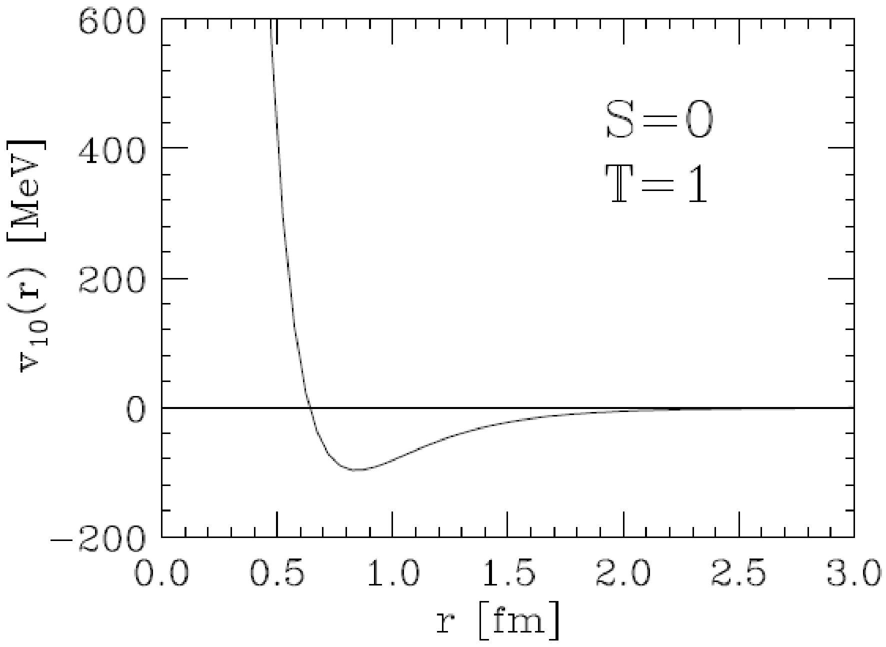
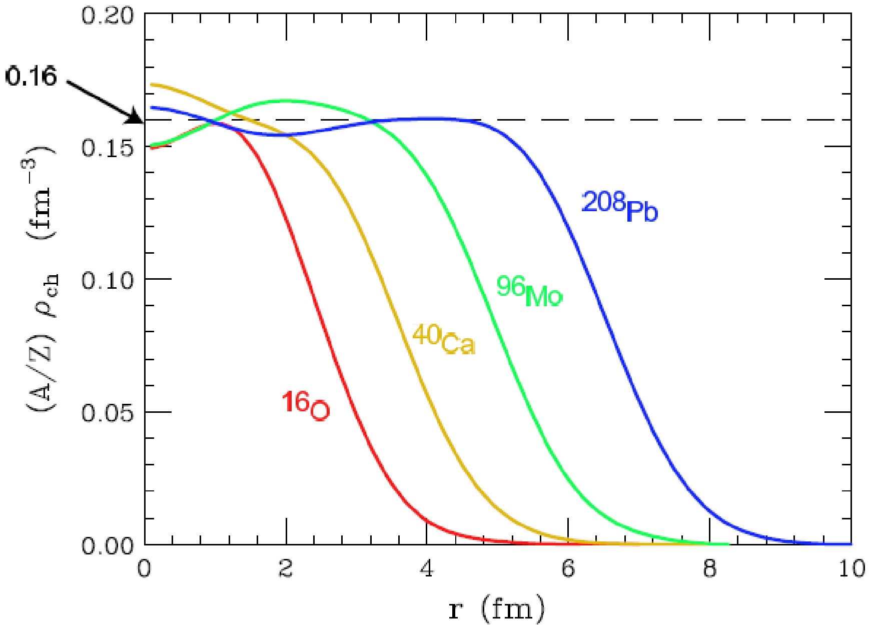
M. Göppert Mayer

$$V_{LS} = V_{LS}(r)\vec{L}\cdot(\vec{\sigma}_1 + \vec{\sigma}_2)\,,\;\; \vec{L} = (\vec{r}\times\vec{p})$$

The nucleon-nucleon interaction

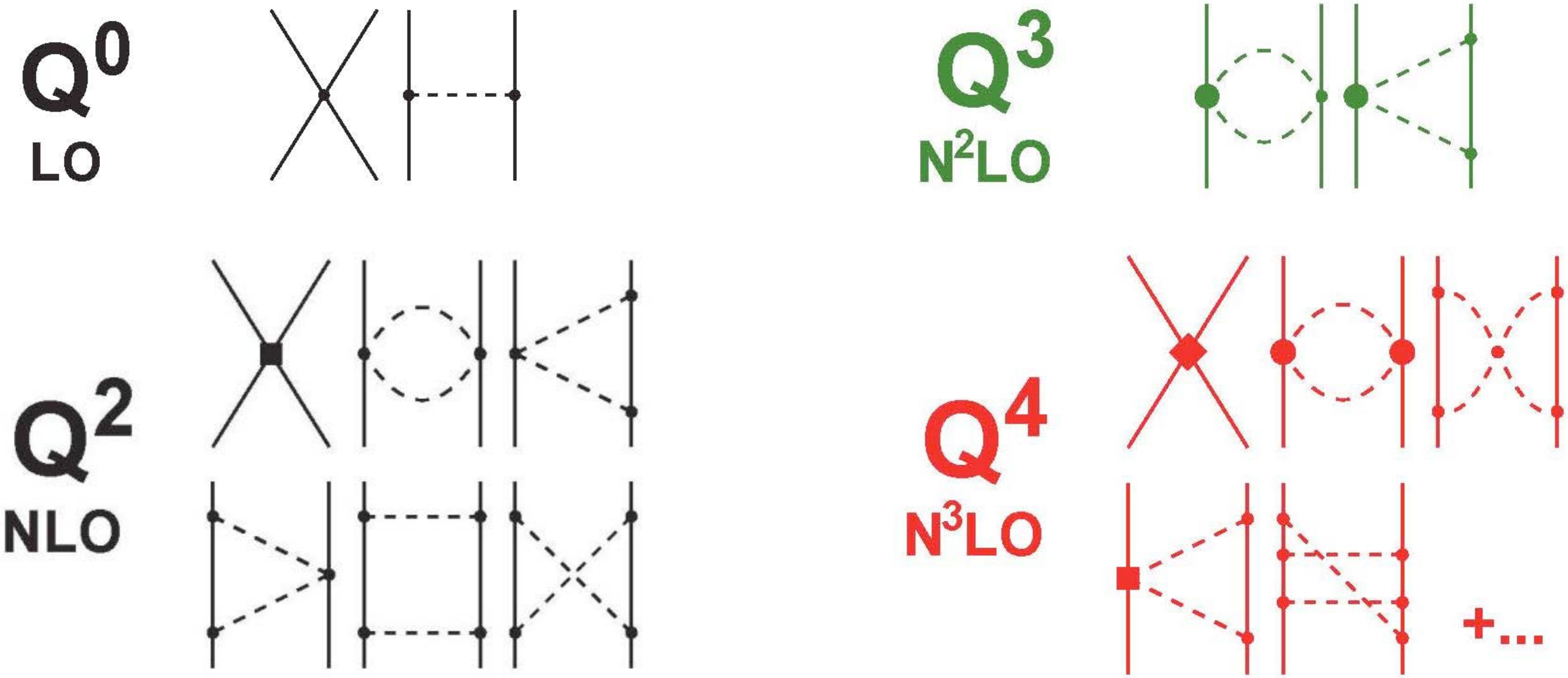
- Short range
- Attraction at intermediate r
- Strong repulsion at r < 0.5 fm
  - Consistent with saturation





The nucleon-nucleon interaction

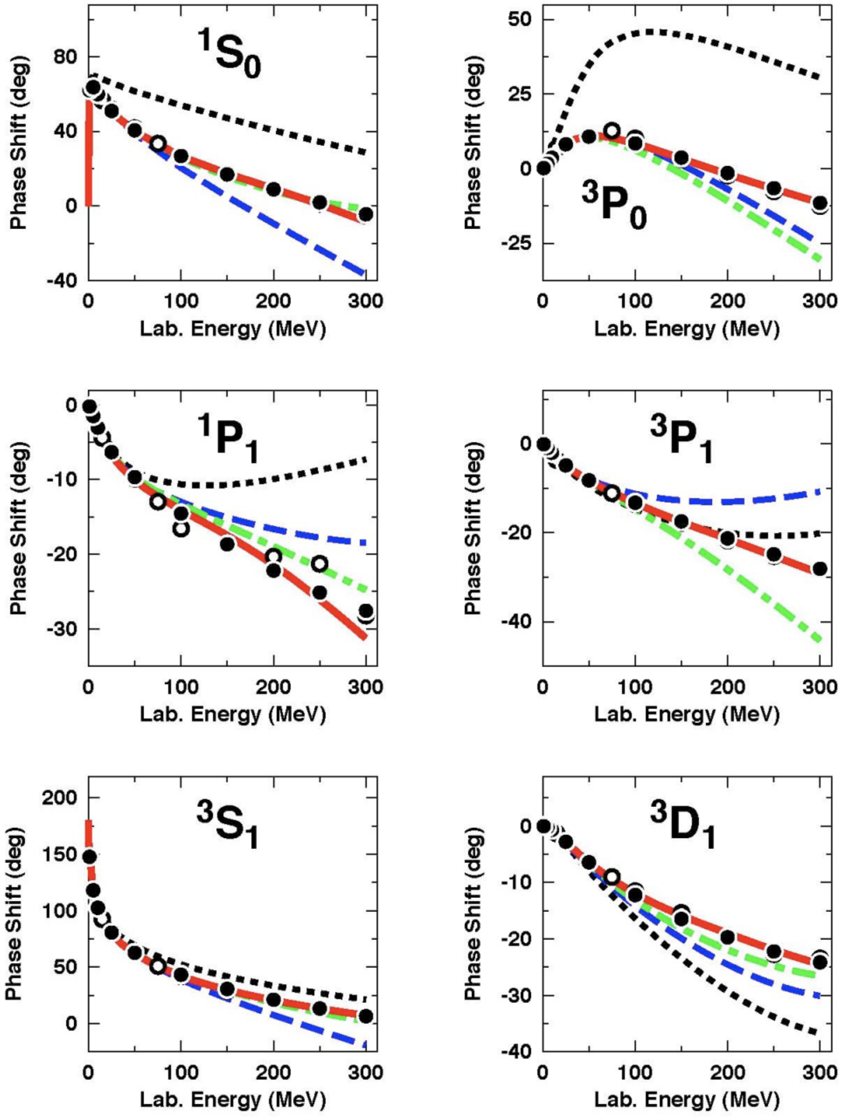
- Effective Field Theory of NN (and NNN) interactions
  - Obeys the symmetries of QCD
  - Systematic expansion in powers of (small) momenta
  - In terms of π, N and contact interactions (LECs fitted to experiment)



R. Machleidt , NTSE 2013

The nucleon-nucleon interaction

- Effective Field Theory of NN (and NNN) interactions



(small) momenta

actions (LECs fitted to experiment)

R. Machleidt , NTSE 2013

Nuclear Many-Body Theory

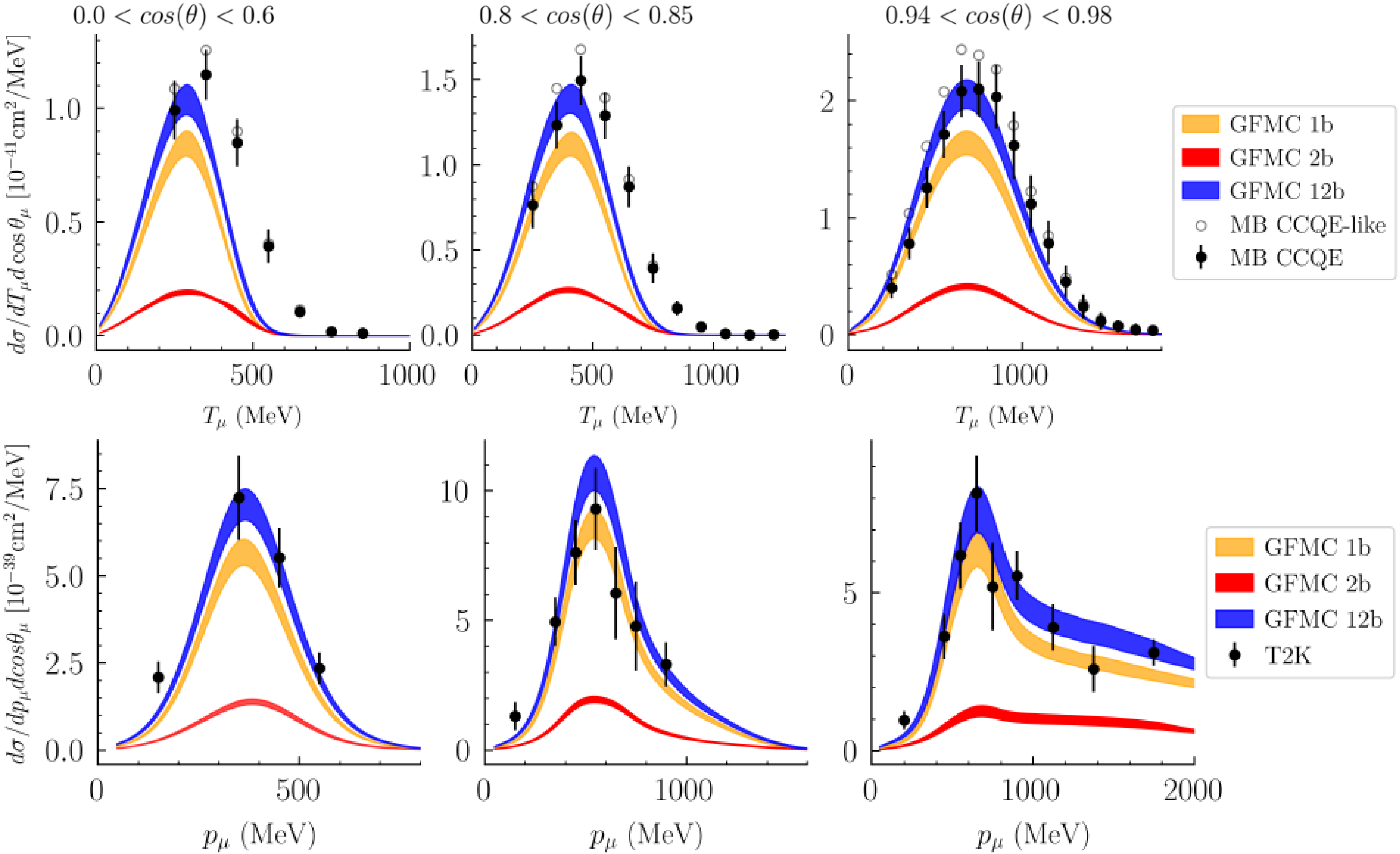
- Hamiltonian:

$$H = \sum_{i}^{A} T_i + \sum_{j>i}^{A} V_{ij} + \sum_{k>j>i}^{A} V_{ijk}$$

- Ab initio methods:
  - Properties of H fixed at A $\leq$ 3
  - Computationally demanding: light nuclei < ^{12}C
  - Green's function MC
    - Nuclear response function in Euclidean time
    - DOF: π, N but no Δ(1232)
    - Non-relativistic framework
    - Lovato et al.: semi-inclusive ν-nucleus scattering in the QE region

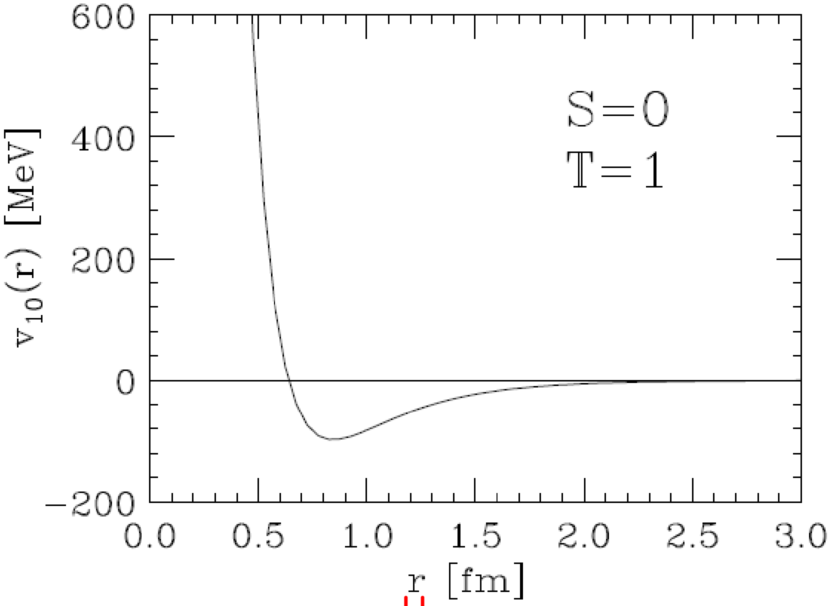
Ab initio

- Green's function MC   Simons et al., 2210.02455



The nucleon-nucleon interaction

- Saturation density ρ_0 = 0.16 fm^{-3} $\Rightarrow$ $r_{12} \sim 2$ fm
- At $r_{12} \sim 2$ fm, the NN interaction is “weak” !



$\Downarrow$

- Nucleons in nuclei follow single particle orbits in a mean-field potential created by all nucleons

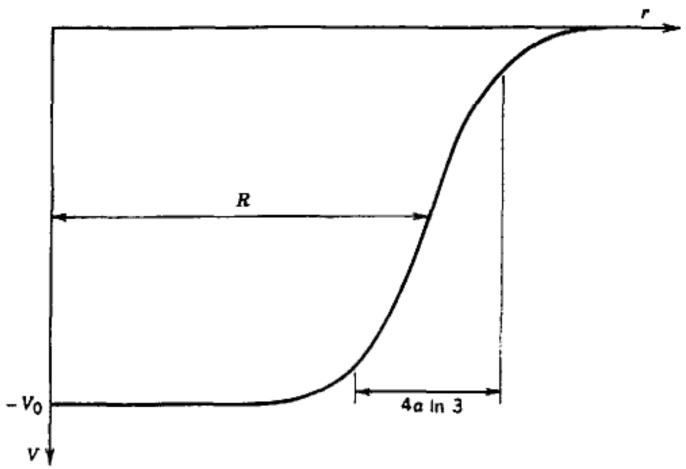
$$\sum_{j>i}^{A} V_{ij} + \sum_{k>j>i}^{A} V_{ijk} \approx \sum_{i}^{A} \tilde{V}(i)$$

$\leftarrow$ Hartree-Fock approximation

Independent particle models

- Shell Model
- Schrödinger eq. in the mean field potential
- Woods-Saxon potential

$$V(r) = -\frac{V_0}{1+\exp\left[(r-R)/a\right]}$$



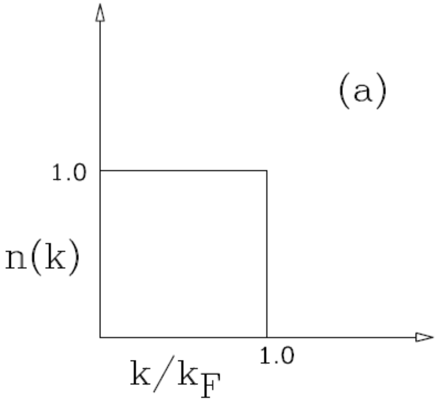
- Spin-Orbit potential: magic numbers
- Explains spin and parity of many nuclei
- Fair description of magnetic dipole and electric quadrupole moments
- Can be extended to deformed nuclei
- Relativistic extensions have been developed

Independent particle models

- Fermi gas model (of nuclear matter)

$$H = \sum_i^A T_i = \sum_i^A \frac{\vec{p}_i^{\,2}}{2M} \qquad \text{or} \qquad H_{\rm rel} = \sum_i^A T_i = \sum_i^A \sqrt{\vec{p}_i^{\,2} + M^2}$$

- Free fermions (nucleons) in a box of volume V ($\to \infty$) at T=0
- Number of occupied states:



$$N = 2V \int \frac{d^3p}{(2\pi)^3} n(|\vec{p}|) \qquad n(p) = \theta(p - p_F)$$

n(p) ← occupation number

P_F ← Fermi momentum

$$\rho = \frac{N}{V} = \frac{1}{3\pi^2} p_F^3$$ for protons or neutrons separately

$$\rho = \frac{N}{V} = \frac{2}{3\pi^2} p_F^3$$ for isospin symmetric nuclear matter

$\rho = \rho_0 = 0.16$ fm^{-3} $\Rightarrow$ $p_F = 263$ MeV

Independent particle models

- Fermi gas model for nuclei
  - Global FG:
    - p_F = const. for a given nucleus
    - Fit parameter in (e,e') scattering
  - Local FG:

$$\rho_{p,n} = \rho_{p,n}(r) \qquad p_F^{p,n} = p_F^{p,n}(r) = \sqrt[3]{\frac{3}{2}\pi^2 \rho_{p,n}(r)}$$

ρ_p(r) $\leftarrow$ from experiment

ρ_n(r) $\leftarrow$ from realistic calculations of the ground state

  - space-momentum correlations absent in the Global FG

Nucleon propagator in the medium

- Green's function:

$$iG(x,x') = \frac{\langle \phi_0 \left| T\left[\psi(x)\psi^\dagger(x')\right] \right| \phi_0 \rangle}{\langle \phi_0 | \phi_0 \rangle}$$

- $\phi_0 \leftarrow$ ground state of the system: $H|\phi_0\rangle = E|\phi_0\rangle$

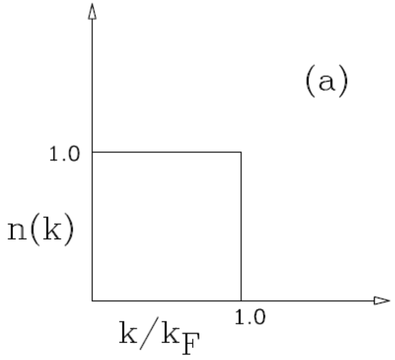
- Free nucleon propagator in the medium
- ϕ_0 : system of non-interacting nucleons $\Leftrightarrow$ Fermi gas

$$\begin{aligned} D(p) &= (\not{p} + M) G_0(p) \\ G_0(p) &= \frac{1}{p^2 - M^2 + i\epsilon} + 2\pi i \delta(p^2 - M^2)\theta(p^0) n(\vec{p}) \\ &= \frac{n(\vec{p})\theta(p^0)}{p^2 - M^2 - i\epsilon} + \frac{1 - n(\vec{p})\theta(p^0)}{p^2 - M^2 + i\epsilon} \\ &= \frac{1}{p^0 + E_p - i\epsilon} \left[\frac{n(\vec{p})}{p^0 - E_p - i\epsilon} + \frac{1 - n(\vec{p})}{p^0 - E_p + i\epsilon} \right] \end{aligned}$$

hole

particle

$$n(p) = \theta(p_F - p)$$



$$E_p = \sqrt{\vec{p}^2 + M^2}$$

Spectral functions

- Full nucleon propagator in the medium
- Lehmann/spectral representation:

$$
\begin{aligned}
D(p) &= (\not{p} + M)G(p) \\
G(p) &= \frac{1}{p^0 + E_p - i\epsilon}\left[\int_{-\infty}^{\mu} \frac{\mathcal{A}_h(\omega, \vec{p})}{p^0 - \omega - i\epsilon} d\omega + \int_{\mu}^{\infty} \frac{\mathcal{A}_p(\omega, \vec{p})}{p^0 - \omega + i\epsilon} d\omega\right]
\end{aligned}
$$

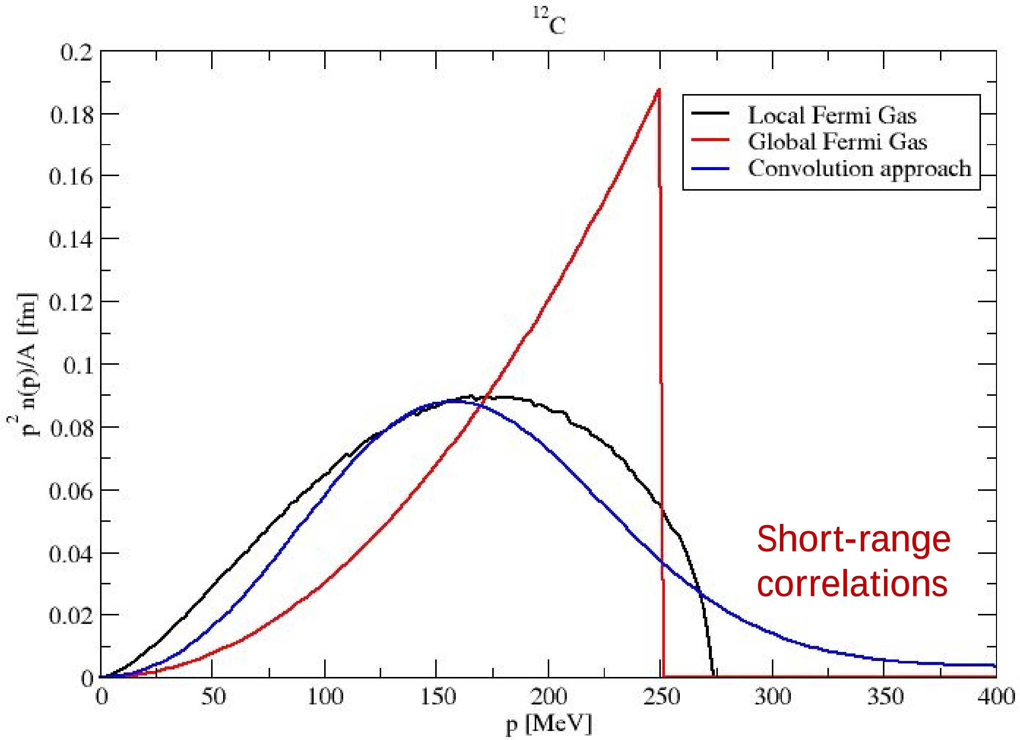
$$\mu^2 = \vec{p}_F^{\,2} + M^2 + \mathrm{Re}\Sigma(\mu, p_F)$$

$$\mathcal{A}_{p,h}(p) = \mp\frac{1}{\pi}\frac{\mathrm{Im}\Sigma(p)}{[p^2 - M^2 - \mathrm{Re}\Sigma(p)]^2 + [\mathrm{Im}\Sigma(p)]^2}$$

- The hole (particle) spectral function $A_{h(p)}(p^0,\mathbf{p})$ represents the probability of removing (adding) a nucleon of momentum $|\mathbf{p}|$ changing the energy of the system by p^0
- Occupation number: $n(\vec{p}) = \int dp_0 (2p_0) \mathcal{A}_h(p^0, \vec{p})$

Spectral functions

-

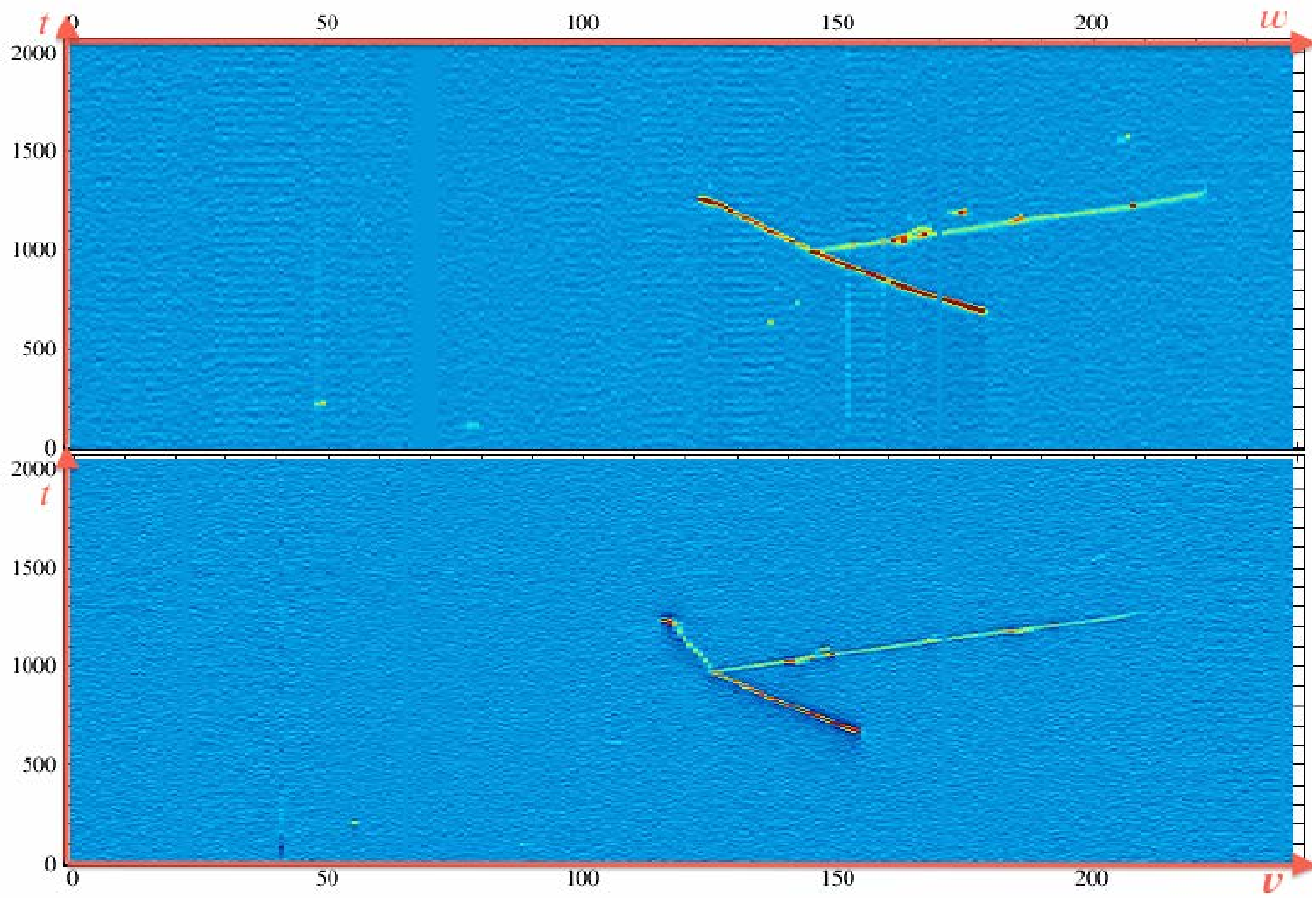


Short-range correlations

- Occupation number: $n(\vec{p}) = \int dp_0 (2p_0) \mathcal{A}_h(p^0, \vec{p})$

Short range correlations

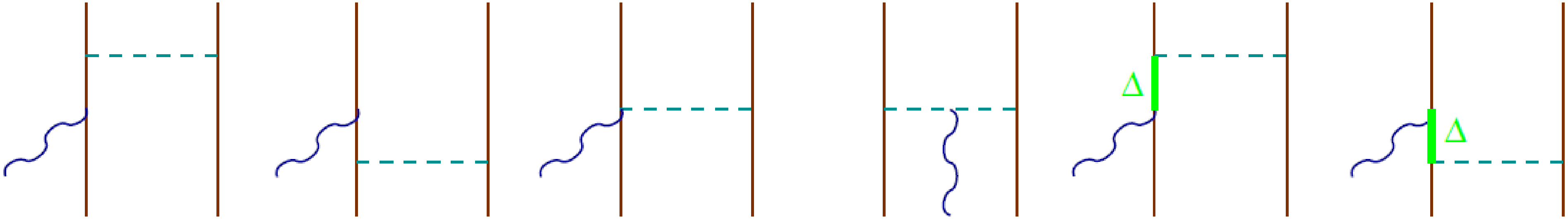
$$\nu_\mu \ (np)_{\text{correl}} \to \mu^- \ p(\vec{p})\, p(-\vec{p})$$



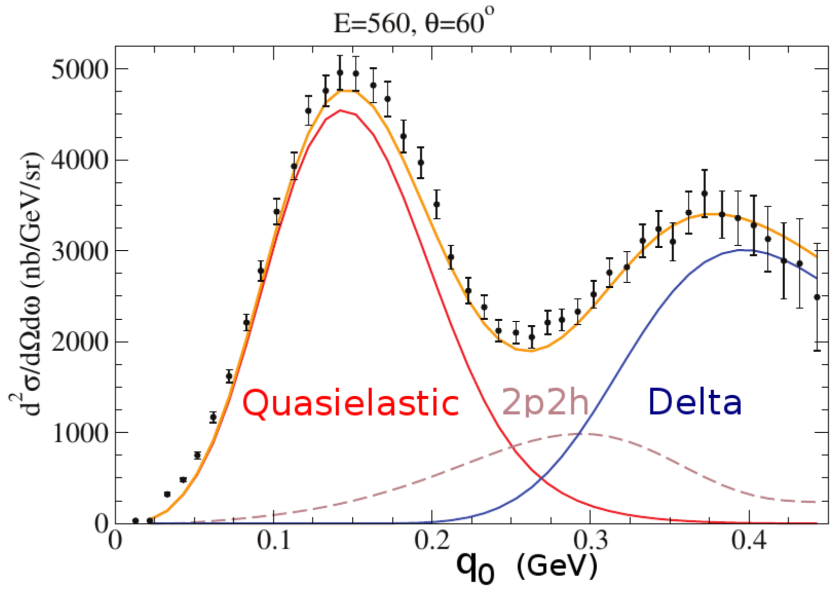
Hammer event at ArgoNeut

Two-nucleon currents

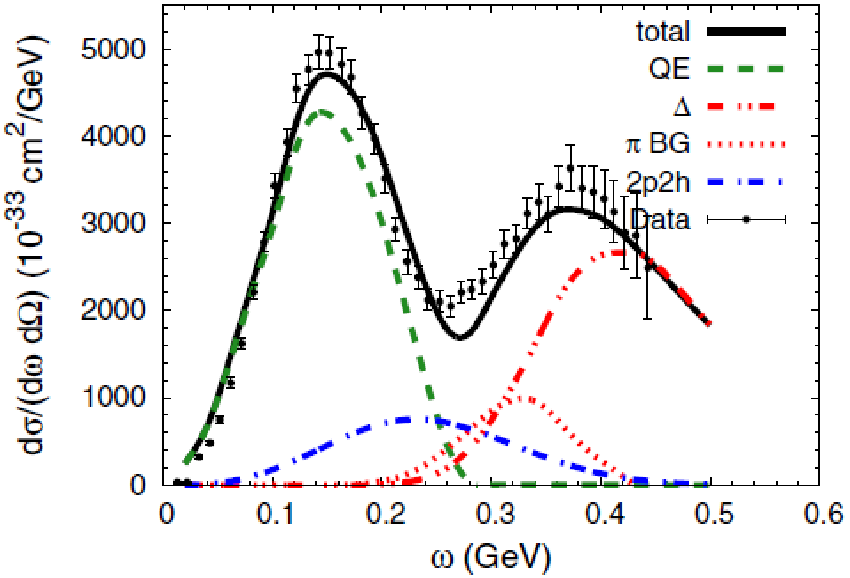
- 2-nucleon EW currents exist



- Sizable contribution can be inferred from A(e,e')X



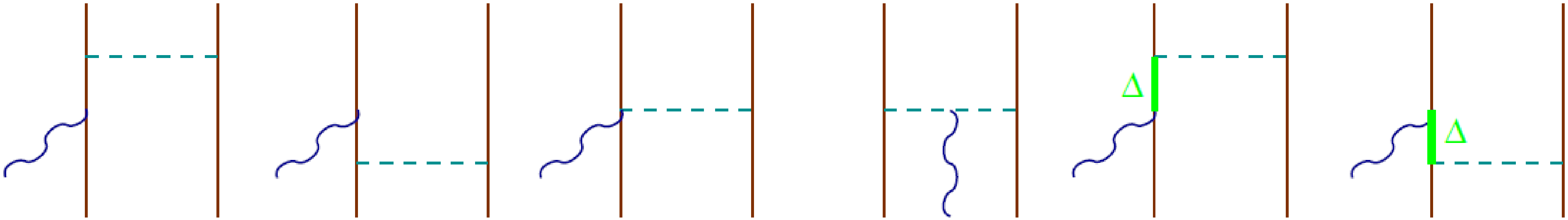
Megias et al., PRD 94 (2016)



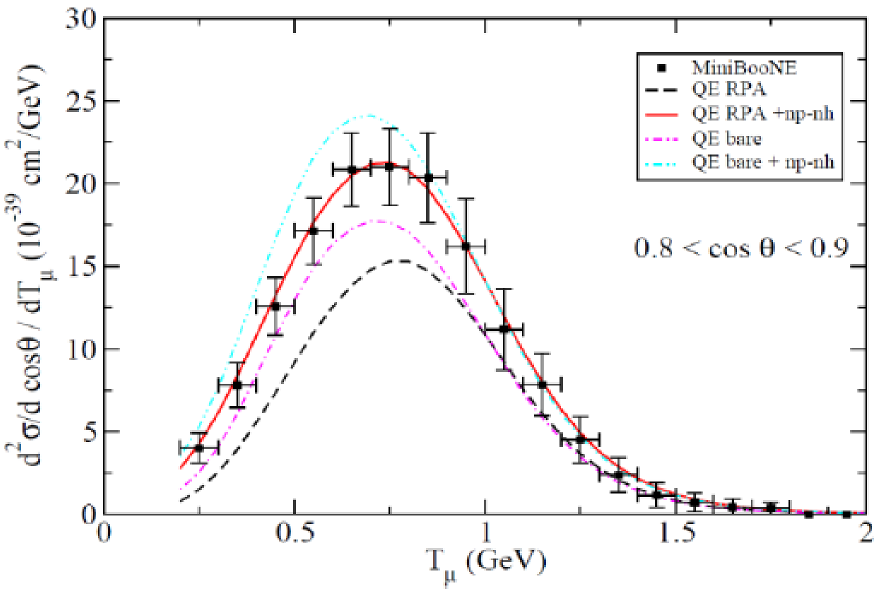
Gallsmeiter et al., PRD 94 (2016)

Two-nucleon currents

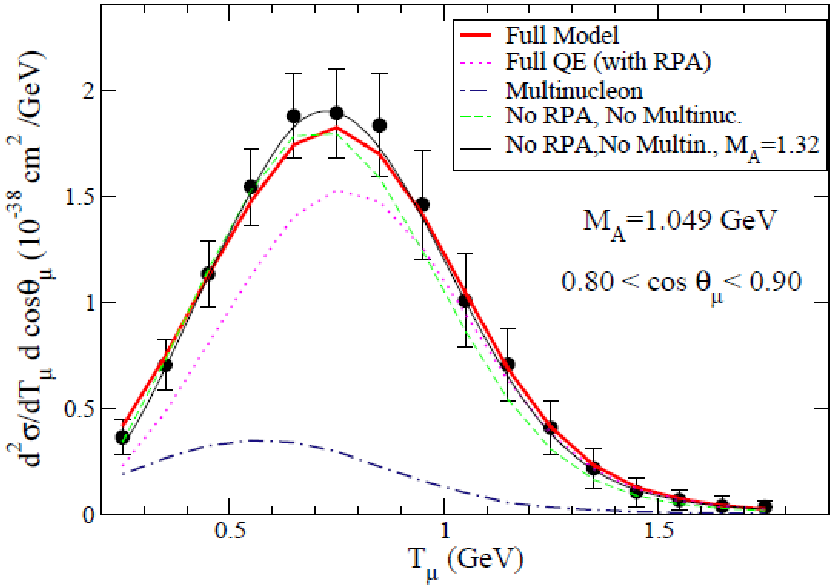
- 2-nucleon EW currents exist



- needed to explain MiniBooNE & T2K 0π data



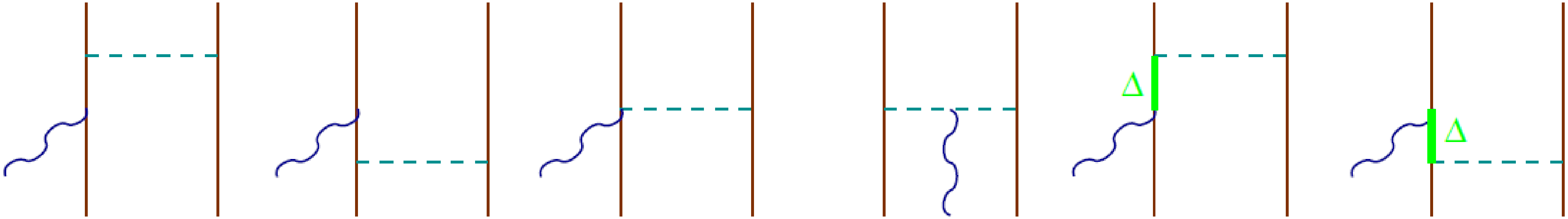
Martini et al.



Nieves et al.

Two-nucleon currents

- 2-nucleon EW currents exist



- source of bias in E_ν kinematic reconstruction @ T2K

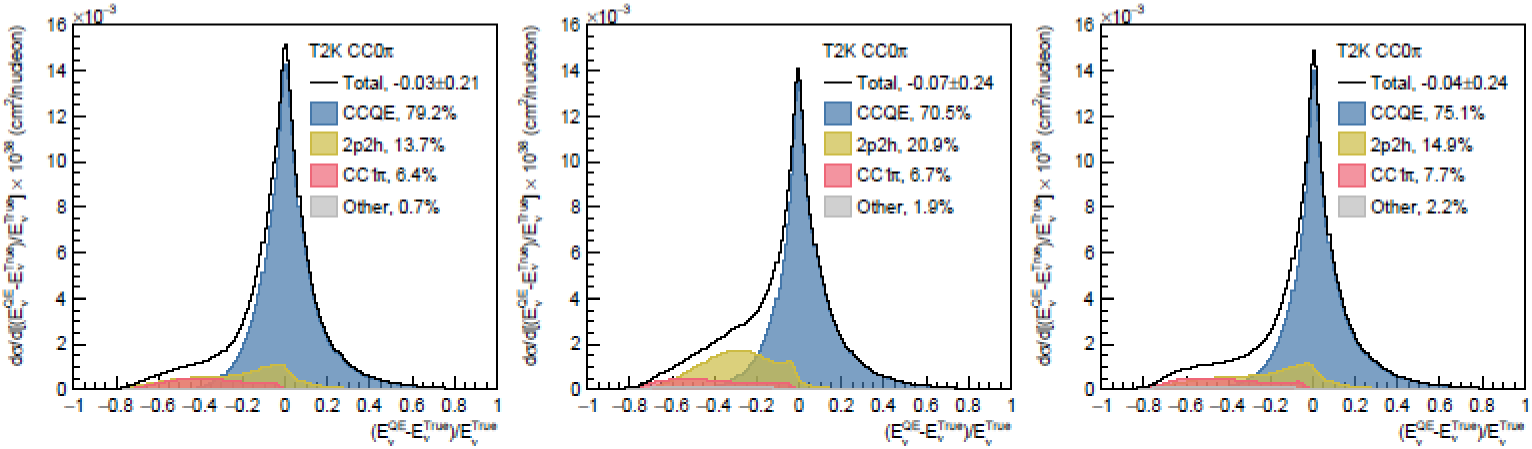
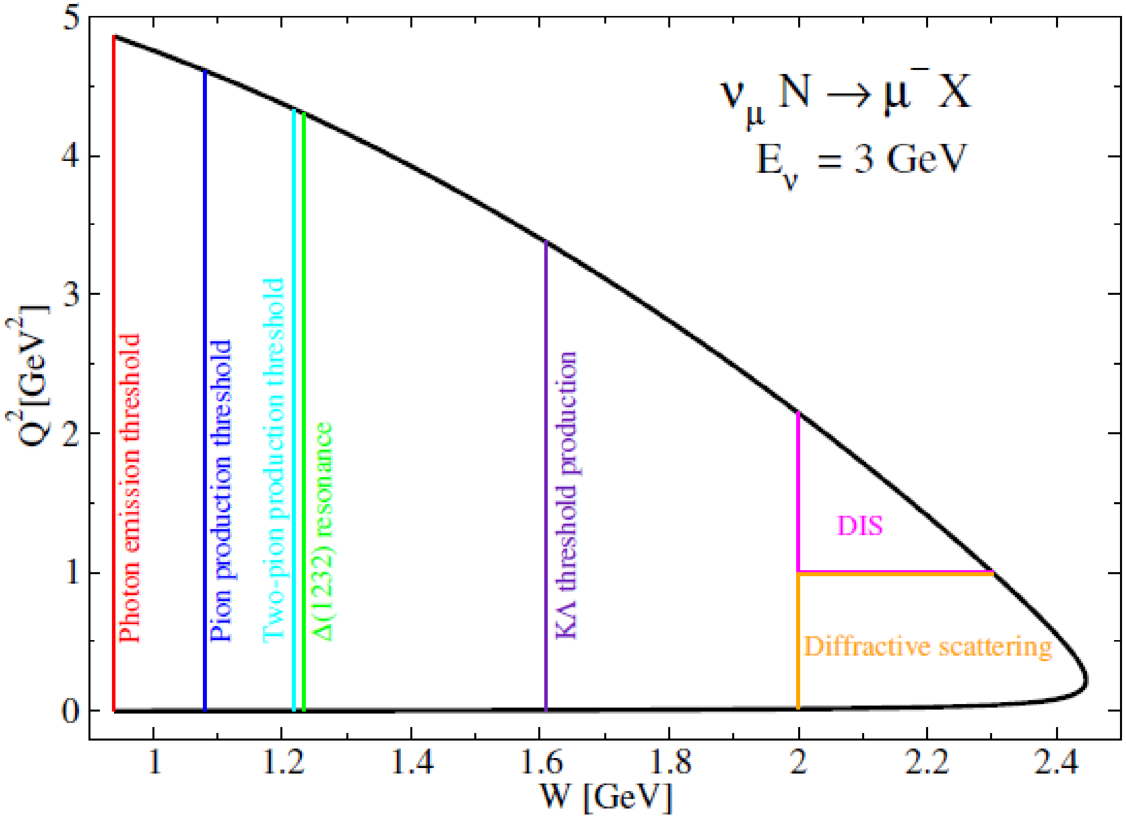
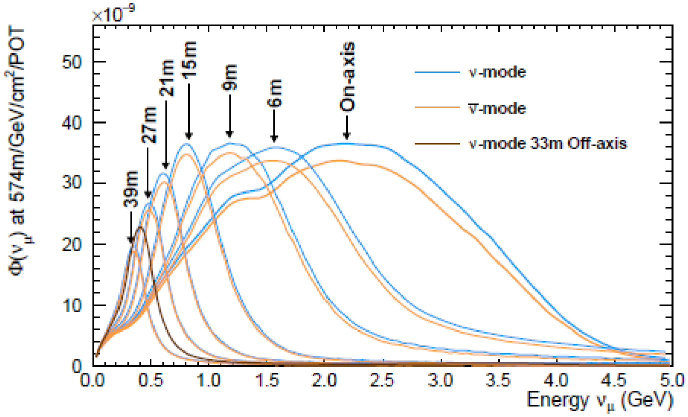


FIG. 7: Biases in a T2K-like kinematic CCQE energy estimator away from the true neutrino energy, using NuWro 19.01 (left), `GENIE G18_02a` (middle), and `NEUT v5.4.0` (right). The prediction for `GENIE G18_10a`, with a 2p2h calculation by Nieves et al., closely matches that of NEUT.

C. Wret

Inelastic scattering



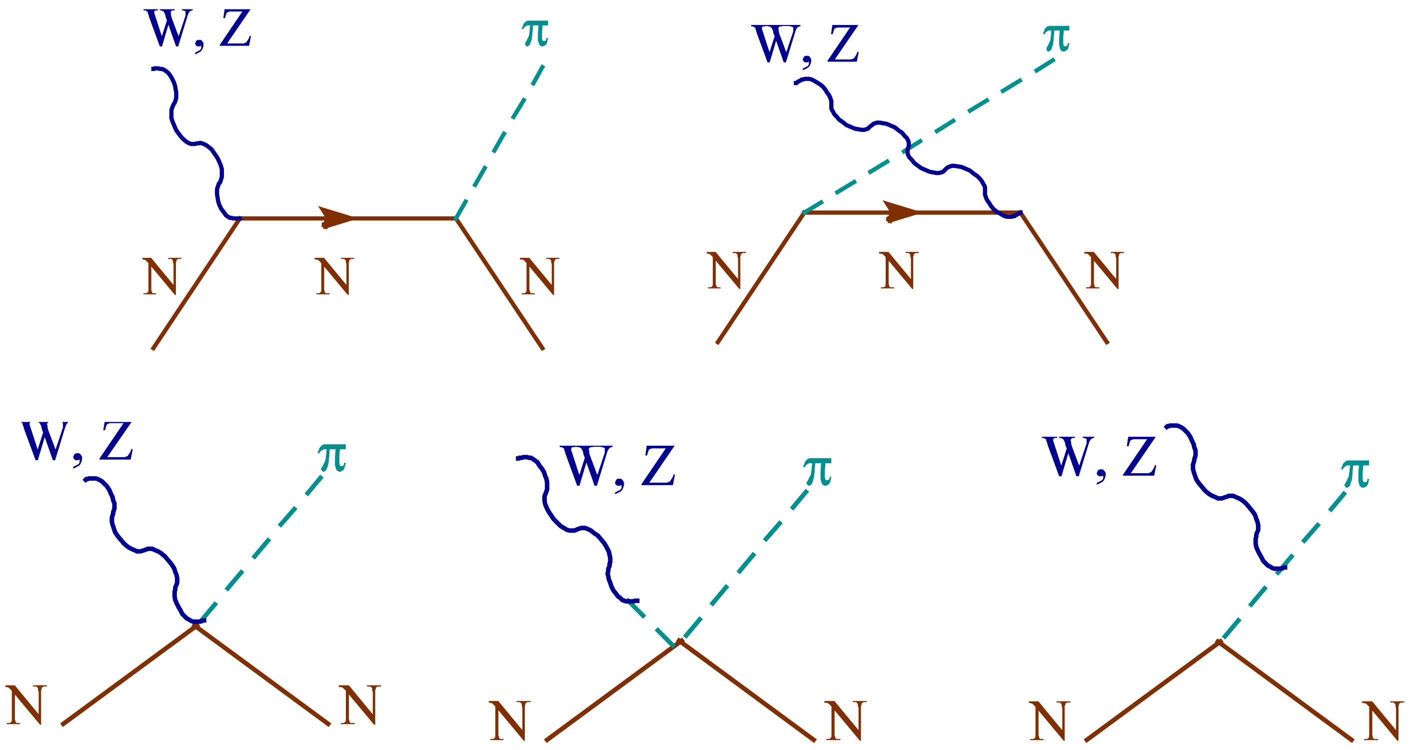


DUNE flux @ ND, 2002.03005

1π production on the nucleon

$$\nu_l \, N \to l \, \pi \, N'$$

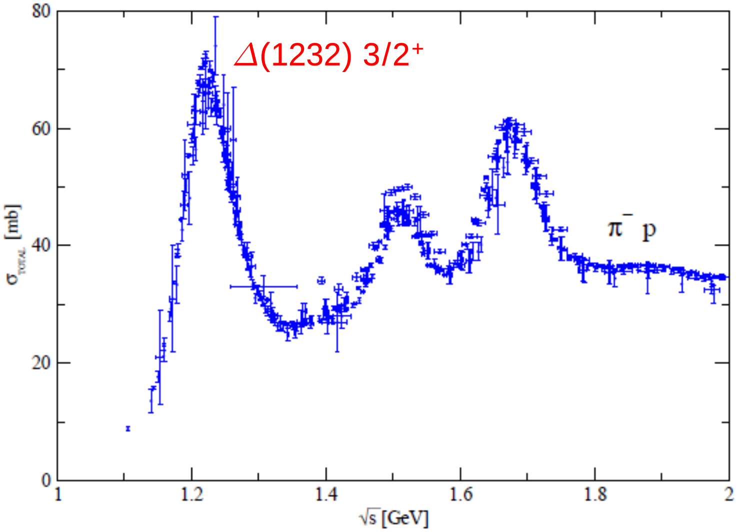
- Threshold behavior dictated by chiral symmetry: Hernandez et al., PRD76 (2007)



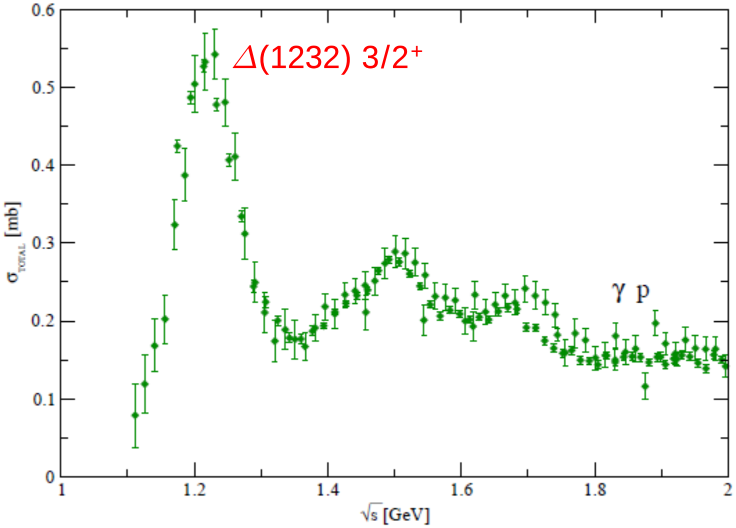
- Can be systematically improved using ChPT Yao et al., PRD 98 (2018)

Baryon resonances

- Nucleons are extended objects ⇒ excitation spectrum



$$\pi\, N \to R \to \pi N,\ \pi\pi N,\ \eta N,\ \Lambda K\ \ldots$$

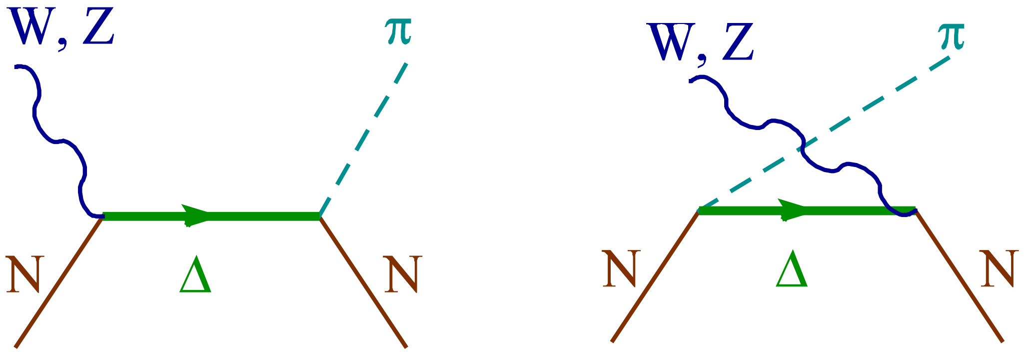


$$\gamma\, N \to R \to \pi N,\ \pi\pi N,\ \eta N,\ \Lambda K\ \ldots$$

From PDG database

1π production on the nucleon

- Δ (1232) excitation:



- N-Δ transition current:

$$J^\mu = \bar{\psi}_\mu \left[\left(\frac{C_3^V}{M}(g^{\beta\mu}\not{q} - q^\beta\gamma^\mu) + \frac{C_4^V}{M^2}(g^{\beta\mu} q\cdot p' - q^\beta p'^\mu) + \frac{C_5^V}{M^2}(g^{\beta\mu} q\cdot p - q^\beta p^\mu) \right)\gamma_5 \right.$$
$$\left. + \frac{C_3^A}{M}(g^{\beta\mu}\not{q} - q^\beta\gamma^\mu) + \frac{C_4^A}{M^2}(g^{\beta\mu} q\cdot p' - q^\beta p'^\mu) + C_5^A g^{\beta\mu} + \frac{C_6^A}{M^2} q^\beta q^\mu \right] u$$

Weak Resonance excitation

- $\Delta(1232)$ $J^P=3/2^+$

$$J_\alpha = \bar{u}^\mu(p')\left[\left(\frac{C_3^V}{M_N}(g_{\alpha\mu}\not{q} - q_\alpha\gamma_\mu) + \frac{C_4^V}{M_N^2}(g_{\alpha\mu}q\cdot p' - q_\alpha p'_\mu) + \frac{C_5^V}{M_N^2}(g_{\alpha\mu}q\cdot p - q_\alpha p_\mu)\right)\gamma_5\right.$$
$$\left. + \frac{C_3^A}{M_N}(g_{\alpha\mu}\not{q} - q_\alpha\gamma_\mu) + \frac{C_4^A}{M_N^2}(g_{\alpha\mu}q\cdot p' - q_\beta p'_\mu) + C_5^A g_{\alpha\mu} + \frac{C_6^A}{M_N^2}q_\alpha q_\mu\right]u(p)$$

C^V_{3-5} , C^A_{3-6} ← N-Δ transition form factors

- Rarita-Schwinger fields: spin 3/2

$$u_\mu(p, s_\Delta) = \sum_{\lambda,s}\left(1\lambda\frac{1}{2}s\middle|\frac{3}{2}s_\Delta\right)\epsilon_\mu(p,\lambda)u(p,s)$$

- Eq. of motion: $(\not{p} - M_\Delta)\,u_\mu = 0$

- with constrains: $\gamma^\mu u_\mu = p^\mu u_\mu = 0$

Weak Resonance excitation

- $\Delta(1232)$ J^P=3/2$^+$

$$J_\alpha = \bar{u}^\mu(p') \left[\left(\frac{C_3^V}{M_N}(g_{\alpha\mu}\not{q} - q_\alpha\gamma_\mu) + \frac{C_4^V}{M_N^2}(g_{\alpha\mu}q\cdot p' - q_\alpha p'_\mu) + \frac{C_5^V}{M_N^2}(g_{\alpha\mu}q\cdot p - q_\alpha p_\mu) \right)\gamma_5 \right.$$
$$\left. + \frac{C_3^A}{M_N}(g_{\alpha\mu}\not{q} - q_\alpha\gamma_\mu) + \frac{C_4^A}{M_N^2}(g_{\alpha\mu}q\cdot p' - q_\beta p'_\mu) + C_5^A g_{\alpha\mu} + \frac{C_6^A}{M_N^2}q_\alpha q_\mu \right] u(p)$$

C_{3-5}^V , C_{3-6}^A ← N-Δ transition form factors

- Isospin symmetry ⇒ vector CC and NC form factors
  - can be expressed in terms of EM ones
  - extracted from data on π photo- and electro-production

Weak Resonance excitation

- *Δ*(1232) $J^P=3/2^+$

$$J_\alpha = \bar{u}^\mu(p') \left[\left(\frac{C_3^V}{M_N}(g_{\alpha\mu}\not{q} - q_\alpha\gamma_\mu) + \frac{C_4^V}{M_N^2}(g_{\alpha\mu} q\cdot p' - q_\alpha p'_\mu) + \frac{C_5^V}{M_N^2}(g_{\alpha\mu} q\cdot p - q_\alpha p_\mu)\right)\gamma_5 \right.$$
$$\left. + \frac{C_3^A}{M_N}(g_{\alpha\mu}\not{q} - q_\alpha\gamma_\mu) + \frac{C_4^A}{M_N^2}(g_{\alpha\mu} q\cdot p' - q_\beta p'_\mu) + C_5^A g_{\alpha\mu} + \frac{C_6^A}{M_N^2} q_\alpha q_\mu \right] u(p)$$

C^V_{3-5} , C^A_{3-6} ← N-*Δ* transition form factors

- Axial transition form factors
  - Poorly known (if at all...)
  - π-pole dominance + PCAC $\Rightarrow$ $C_6^A = -\frac{M_N^2}{q^2 - m_\pi^2} C_5^A$
  - off diagonal Goldberger-Treiman relation $\Rightarrow$ $C_5^A(0) = \sqrt{\frac{2}{3}} g_{\Delta N\pi}$

$$g_{\Delta N\pi} \Leftrightarrow \Gamma(\Delta \to N\pi)$$

  - Deviations from GTR arise from chiral symmetry breaking
    - expected only at the few % level

Weak Resonance excitation

- Δ(1232) $J^P=3/2^+$

$$J_\alpha = \bar{u}^\mu(p') \left[\left(\frac{C_3^V}{M_N}(g_{\alpha\mu}\not{q} - q_\alpha\gamma_\mu) + \frac{C_4^V}{M_N^2}(g_{\alpha\mu}q\cdot p' - q_\alpha p'_\mu) + \frac{C_5^V}{M_N^2}(g_{\alpha\mu}q\cdot p - q_\alpha p_\mu) \right)\gamma_5 \right.$$
$$\left. + \frac{C_3^A}{M_N}(g_{\alpha\mu}\not{q} - q_\alpha\gamma_\mu) + \frac{C_4^A}{M_N^2}(g_{\alpha\mu}q\cdot p' - q_\beta p'_\mu) + C_5^A g_{\alpha\mu} + \frac{C_6^A}{M_N^2}q_\alpha q_\mu \right] u(p)$$

C^V_{3-5} , C^A_{3-6} ← N-Δ transition form factors

- Axial transition form factors
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  - off diagonal Goldberger-Treiman relation ⇒ $C_5^A(0) = \sqrt{\dfrac{2}{3}} g_{\Delta N\pi}$
  - From ANL and BNL data on $\nu_\mu \, d \rightarrow \mu^- \, \pi^+ \, p \, n$

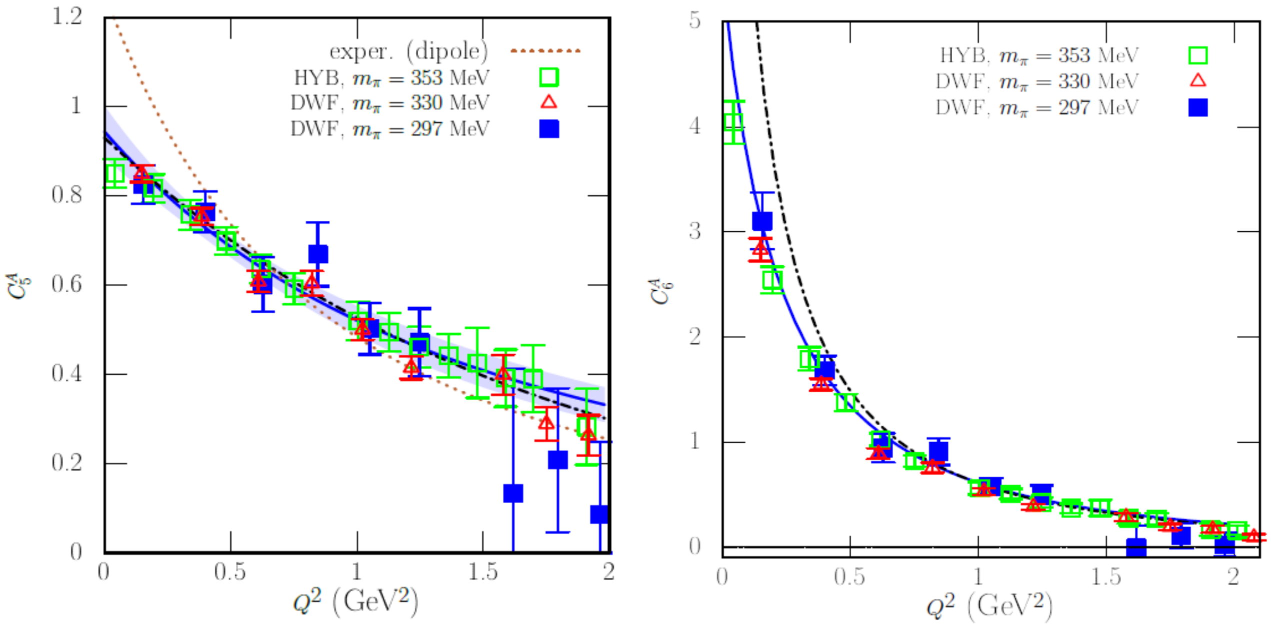
$$C_5^A = C_5^A(0)\left(1 + \frac{Q^2}{M_{A\Delta}^2}\right)^{-2}$$

$M_{A\,\Delta}$ = 0.95 ± 0.06 GeV

LAR, Hernandez, Nieves, Vicente Vacas, PRD93(2016)

LQCD & meson production

- Early N-Δ(1232) axial FF with heavy m_q Alexandrou et al.,PRD83 (2011)

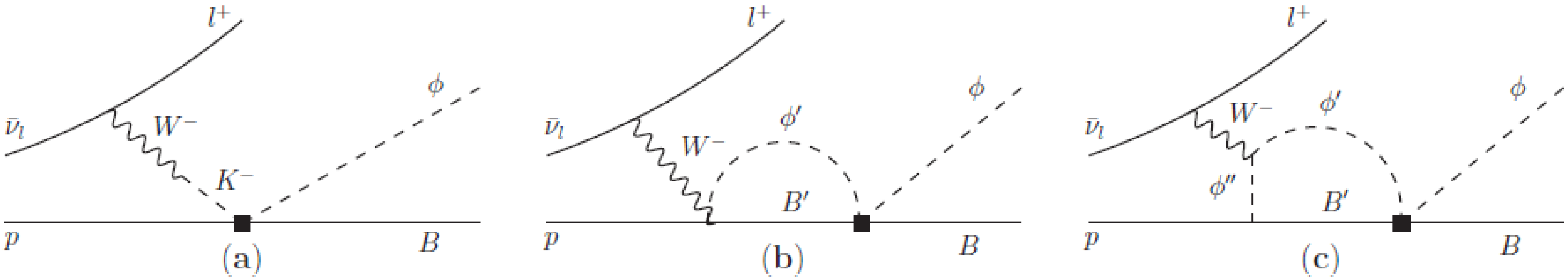


- Calculations of N-Δ,N-N* transition FF should become available in the next 5-10 years LAR et al., Snowmass 2021, 2203.09030
- Control systematic uncertainties is challenging

Weak meson production

- Beyond the $\Delta(1232)$ region:
  - relevant for DUNE $\nu_l \, N \to l \, N' \, \pi$
  - several exclusive channels: $\nu_l \, N \to l \, N' \, \pi\pi \, , \, \eta$

    $\nu_l \, N \to l \, \Lambda(\Sigma) \, K$
  - Dynamical models:
    - Unitarization (in coupled channels)



    - Vector current can be constrained with $\gamma \, N \to N \, \pi$, $e \, N \to e' \, N \, \pi$
    - Axial current at $q^2 \to 0$ can be constrained with $\pi \, N \to N \pi$

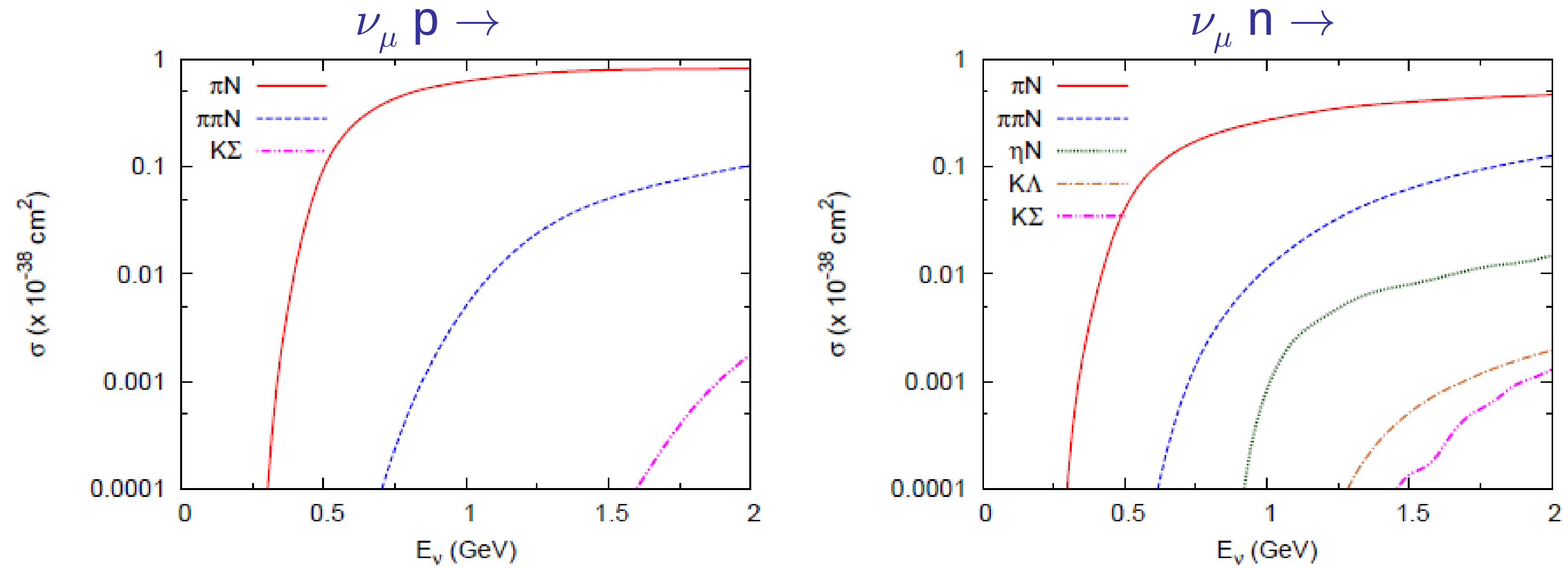
$$\left.\frac{d\sigma_{CC\pi}}{dE_l \, d\Omega_l}\right|_{q^2=0} = \frac{G_F^2 V_{ud}^2}{2\pi^2} \frac{2 f_\pi^2}{\pi} \frac{E_l^2}{E_\nu - E_l} \sigma_{\pi N}$$

Weak meson production

- Beyond the $\Delta(1232)$ region:
  - relevant for DUNE
  - several exclusive channels:

$$\nu_l \, N \to l \, N' \, \pi$$
$$\nu_l \, N \to l \, N' \, \pi\pi \, , \, \eta$$
$$\nu_l \, N \to l \, \Lambda(\Sigma) \, K$$

  - Dynamical models:
    - E.g.: DCC Model Nakamura et al., PRD92 (2015)



- Very limited information about the axial current at $q^2 \neq 0$
- Direct or indirect measurement on H and/or D needed

1π production on nuclei

- Incoherent 1π production in nuclei

$$\nu_l\, A \rightarrow l\, \pi\, X$$

  - Modification of the $\Delta(1232)$ properties in the medium

$$D_\Delta \Rightarrow \tilde{D}_\Delta(r) = \frac{1}{(W+M_\Delta)(W-M_\Delta-\mathrm{Re}\Sigma_\Delta(\rho)+i\tilde{\Gamma}_\Delta/2-i\mathrm{Im}\Sigma_\Delta(\rho))}$$

$\tilde{\Gamma}_\Delta \leftarrow$ Free width $\Delta \rightarrow N\,\pi$ modified by Pauli blocking

$$\mathrm{Re}\Sigma_\Delta(\rho) \approx 40\mathrm{MeV}\frac{\rho}{\rho_0}$$

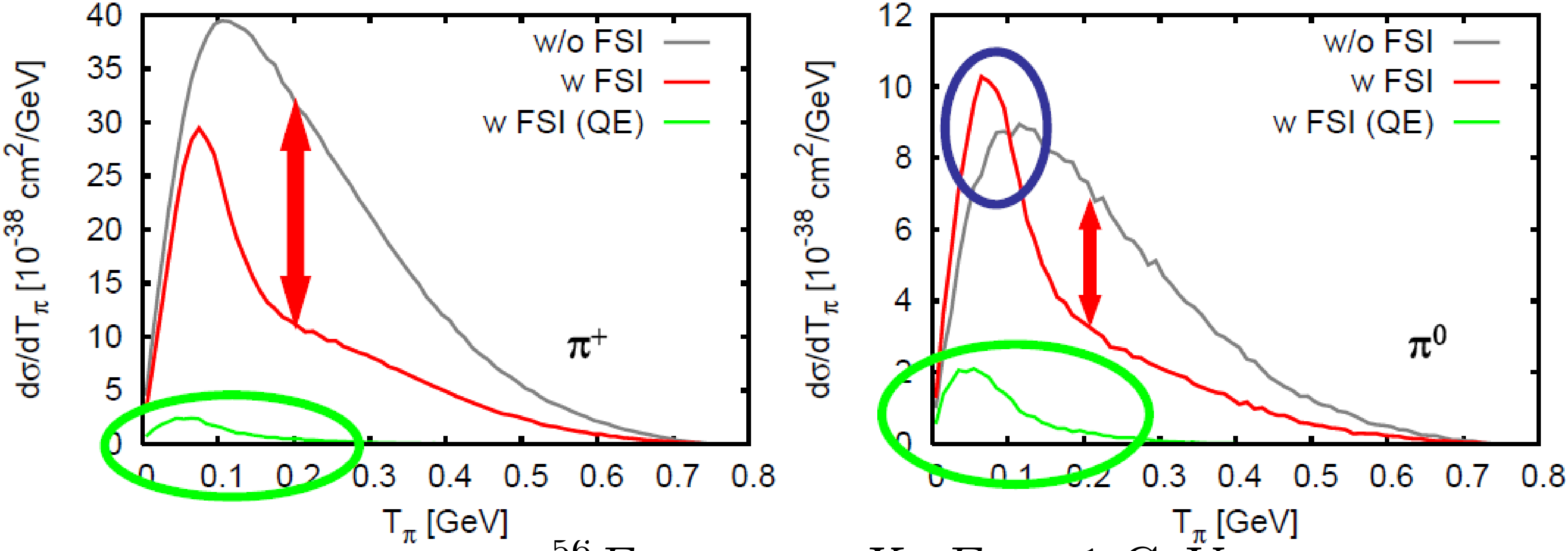
$\mathrm{Im}\Sigma_\Delta(\rho) \leftarrow$

- $\Delta\, N \rightarrow N\, N$
- $\Delta\, N \rightarrow N\, N\, \pi$
- $\Delta\, N\, N \rightarrow N\, N\, N$

  - π propagation (scattering, charge exchange), absorption (FSI)
    - semiclassical cascade, transport models

1π production on nuclei

- GiBUU Leitner, LAR, Mosel, PRC 73 (2006)
  - Effects of FSI on pion kinetic energy spectra
    - strong absorption in Δ region
    - side-feeding from dominant π^+ into π^o channel
    - secondary pions through FSI of initial QE protons



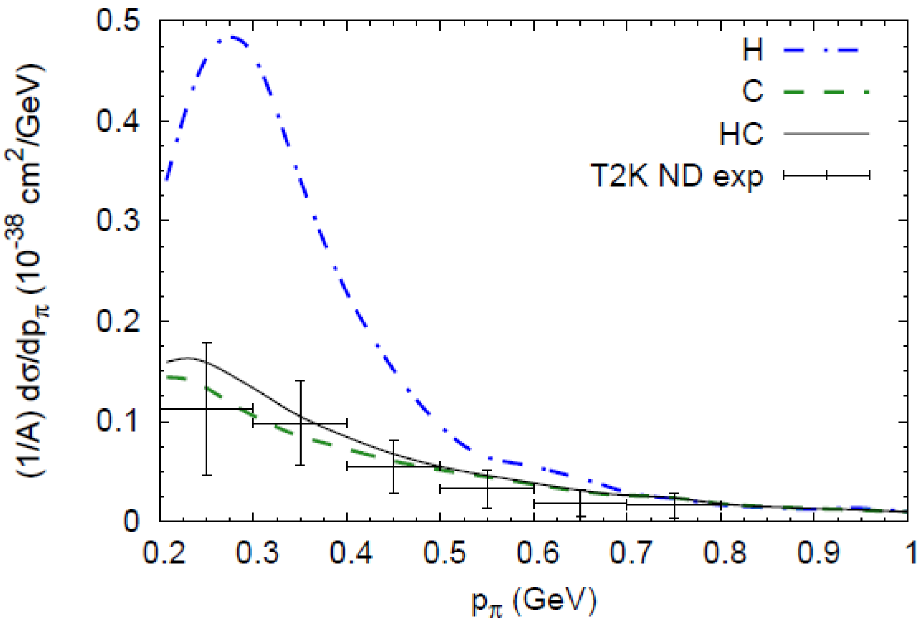
$$\nu_\mu + {}^{56}Fe \rightarrow \mu^- \, \pi \, X \quad E_\nu = 1 \text{ GeV}$$

π production on ^{12}C

Comparison to T2K:

Mosel, Gallmeister, PRC99 (2019)

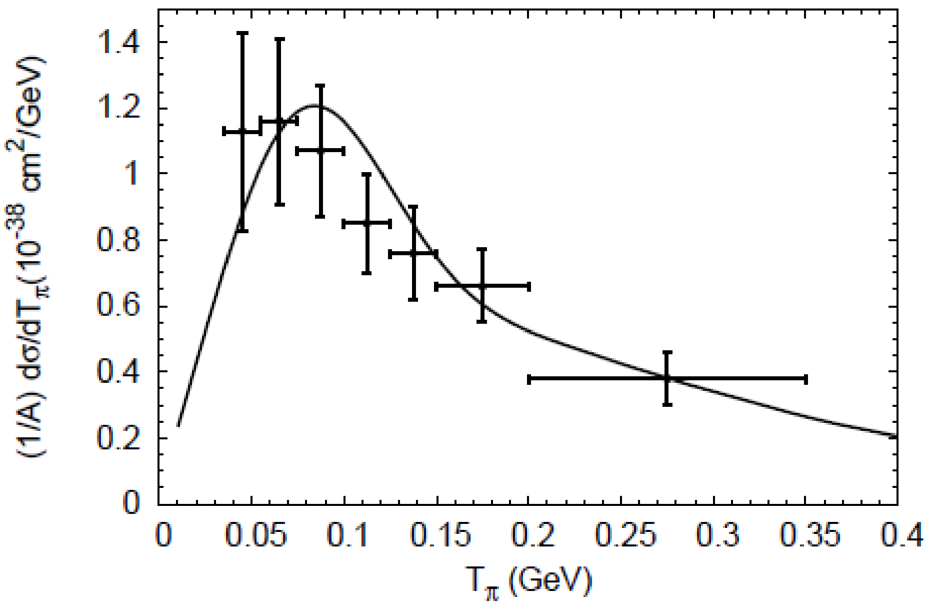
$CC\pi^{\pm}$ data: R. Castillo, PhD Thesis (2015)



Comparison to MINERvA:

Mosel, Gallmeister, PRC96 (2017)

$CC\pi^{\pm}$ data: Eberly et al., PRD 92 (2015)



Outlook

- Neutrino interactions with matter
  - are present in many interesting and relevant physical processes.
  - are critical for the success of the neutrino oscillation program in the precision era.
  - are determined by fundamental properties of hadrons and nuclei.

- Ongoing progress in neutrino-interaction theory
  - Lattice QCD
  - Effective Field Theory
  - Phenomenological models
  - Monte Carlo simulations

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