

The effective number of neutrinos in non-standard scenarios

neutrino

ν

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$\nu\nu$

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ν

ν

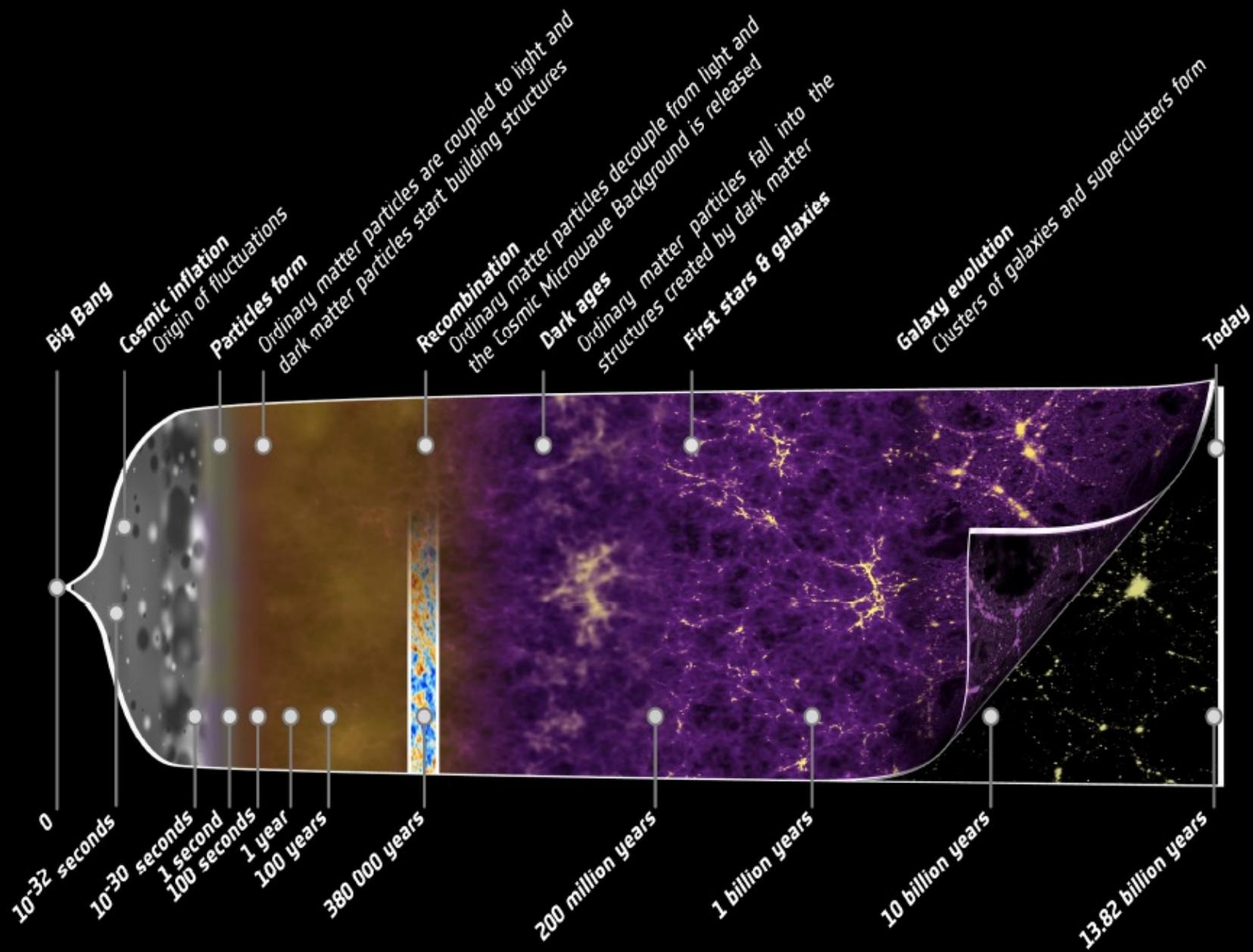
Sergio Pastor
(IFIC Valencia)



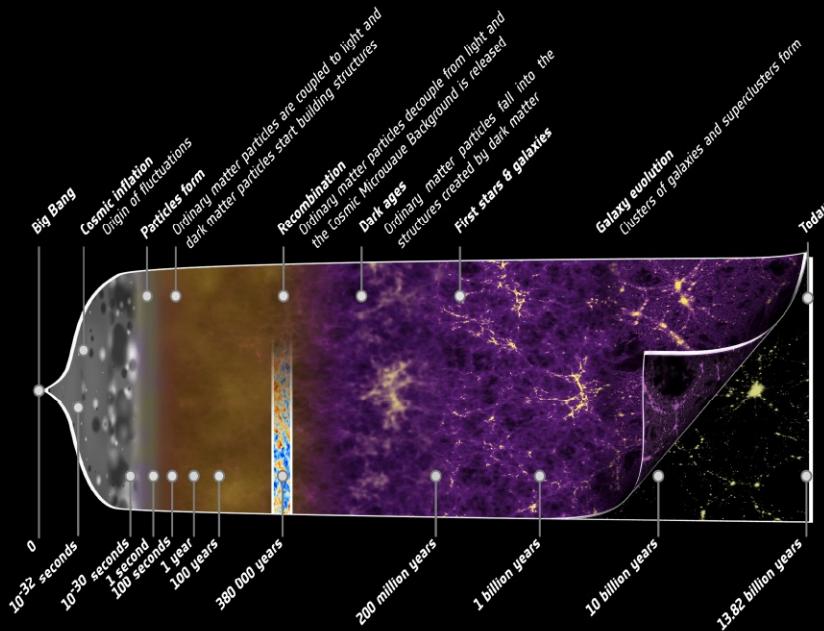
Neutrino frontiers
Focus week
GGI (2 July 2024)



Evolution of the universe



Evolution of the universe



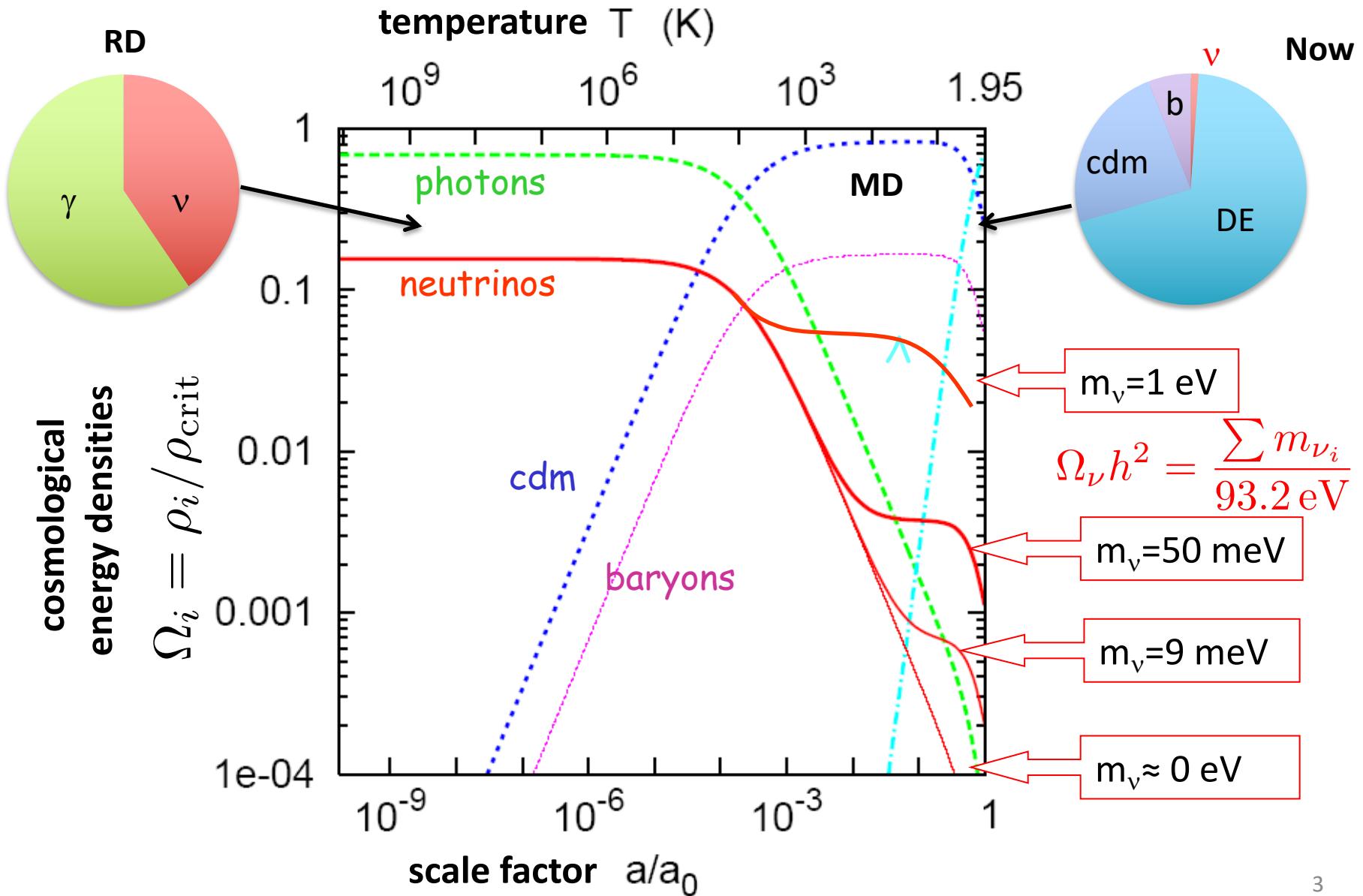
energy density: $\rho(a) = a^{-3(1+w)}$

$$\rho_R \sim a^{-4} \quad , \quad w = 1/3 \quad (\text{Radiation})$$

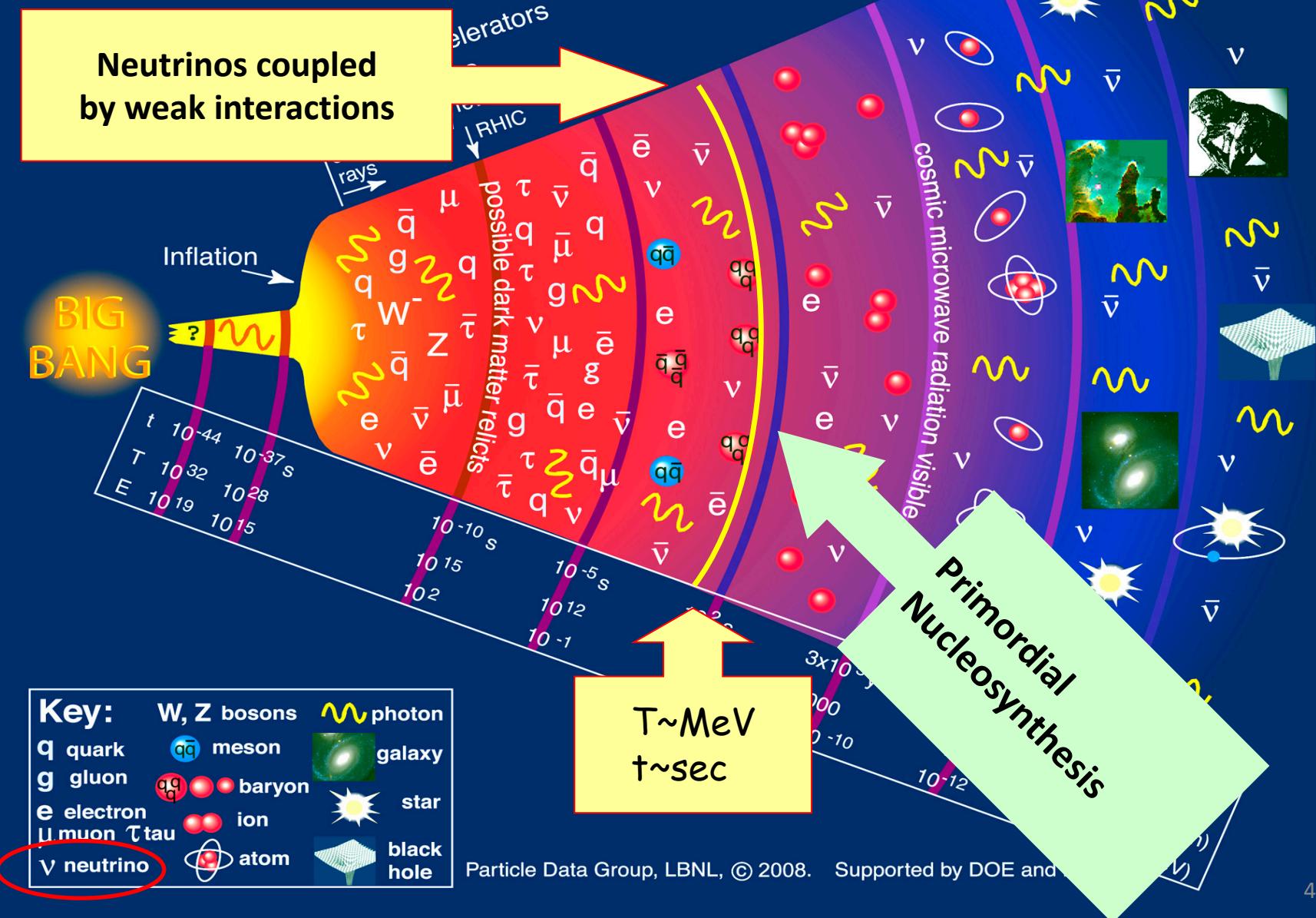
$$\rho_M \sim a^{-3} \quad , \quad w = 0 \quad (\text{Matter})$$

$$\rho_\Lambda \sim \text{const.} \quad , \quad w = -1 \quad (\text{Cosmological constant})$$

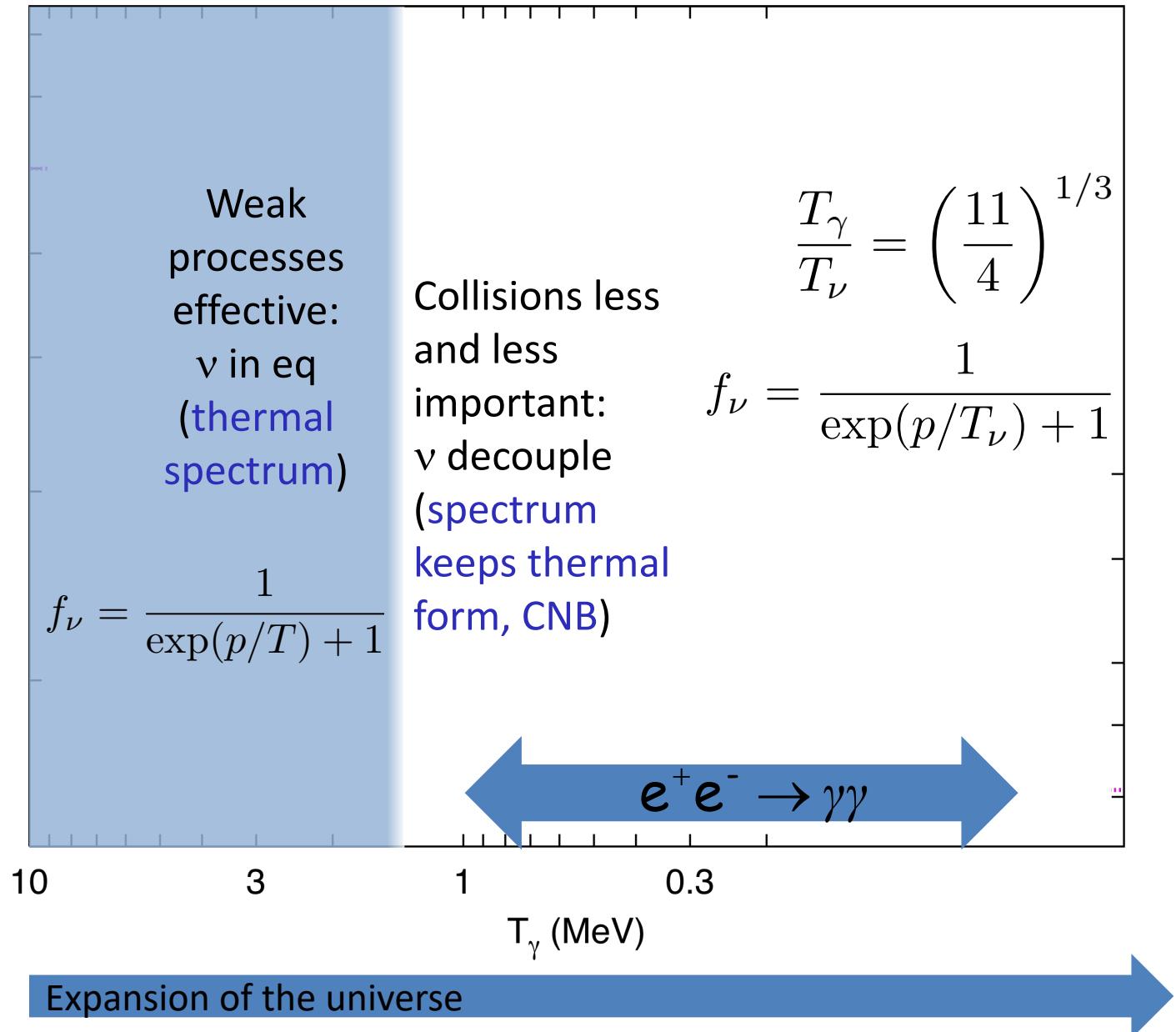
Evolution of the background densities: 1 MeV → now



History of the Universe



Neutrino decoupling and e^\pm annihilation



Relativistic particles in the universe

At $T < m_e$, the radiation content of the Universe is

$$\rho_{\text{rad}} = \rho_\gamma + \rho_\nu = \rho_\gamma \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \times 3 \right]$$

Valid for standard neutrinos in the
instantaneous decoupling approximation

Relativistic particles in the universe

At $T < m_e$, the radiation content of the Universe is

$$\rho_{\text{rad}} = \rho_\gamma + \rho_\nu + \rho_x = \rho_\gamma \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{\text{eff}} \right]$$

effective number of relativistic neutrino species
(effective number of neutrinos)

N_{eff} is a way to measure the ratio

$$\frac{\rho_\nu + \rho_x}{\rho_\gamma}$$

1960s-1970s : $N_{\text{eff}} = N_\nu$, **extra neutrinos** would enhance the cosmological expansion

>1980s: $N_{\text{eff}} = \text{additional relativistic particles}$

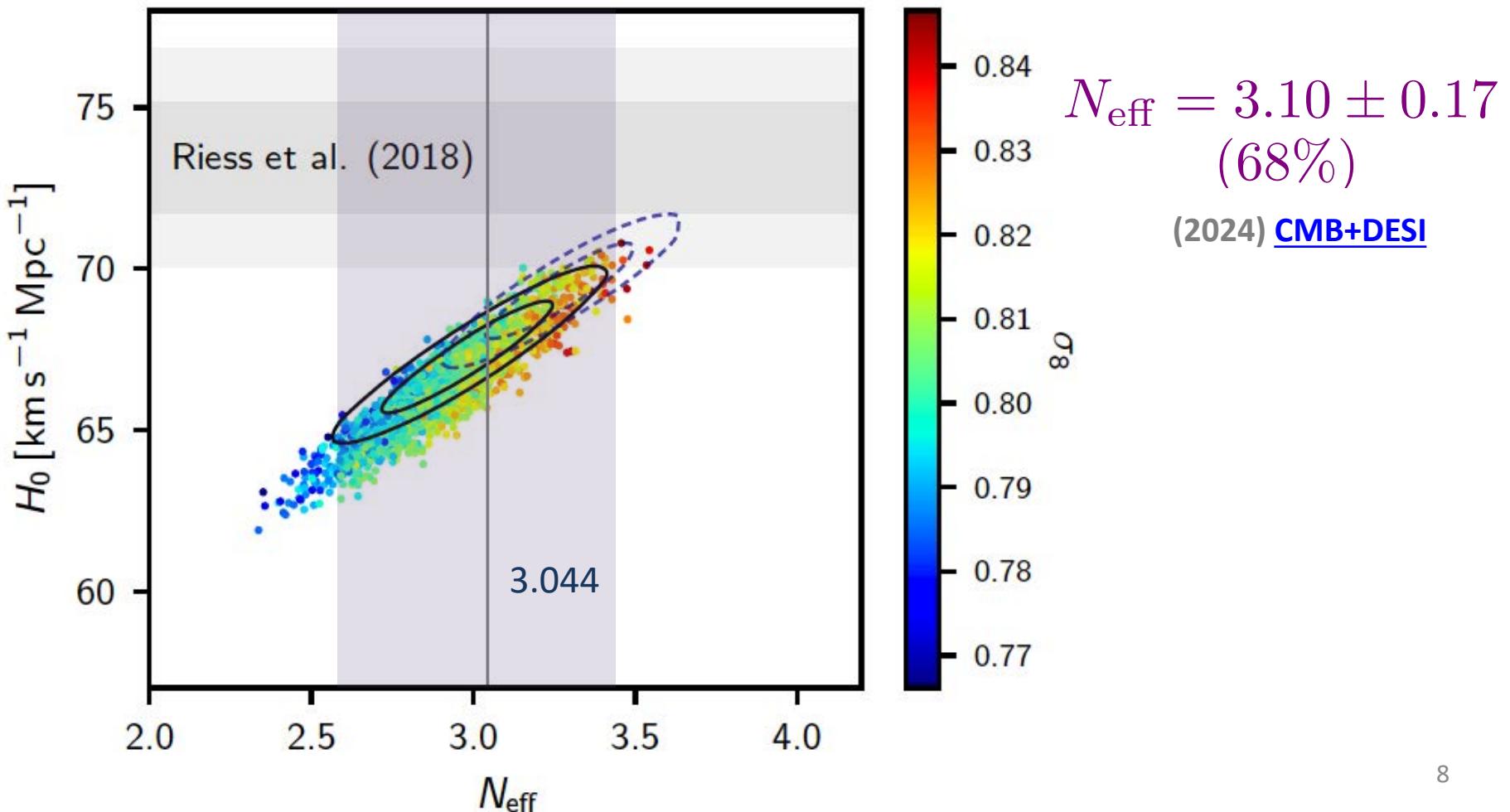
CMB anisotropies + other data

$N_{\text{eff}} \lesssim 17$ (2001) early CMB data

$N_{\text{eff}} = 4.2^{+1.2}_{-1.7}$ (2005) WMAP+...

$N_{\text{eff}} = 2.99^{+0.34}_{-0.33}$ (2018) [Planck](#)

(95%, TT,TE,EE+lowE+lensing+BAO)



Relativistic particles in the universe

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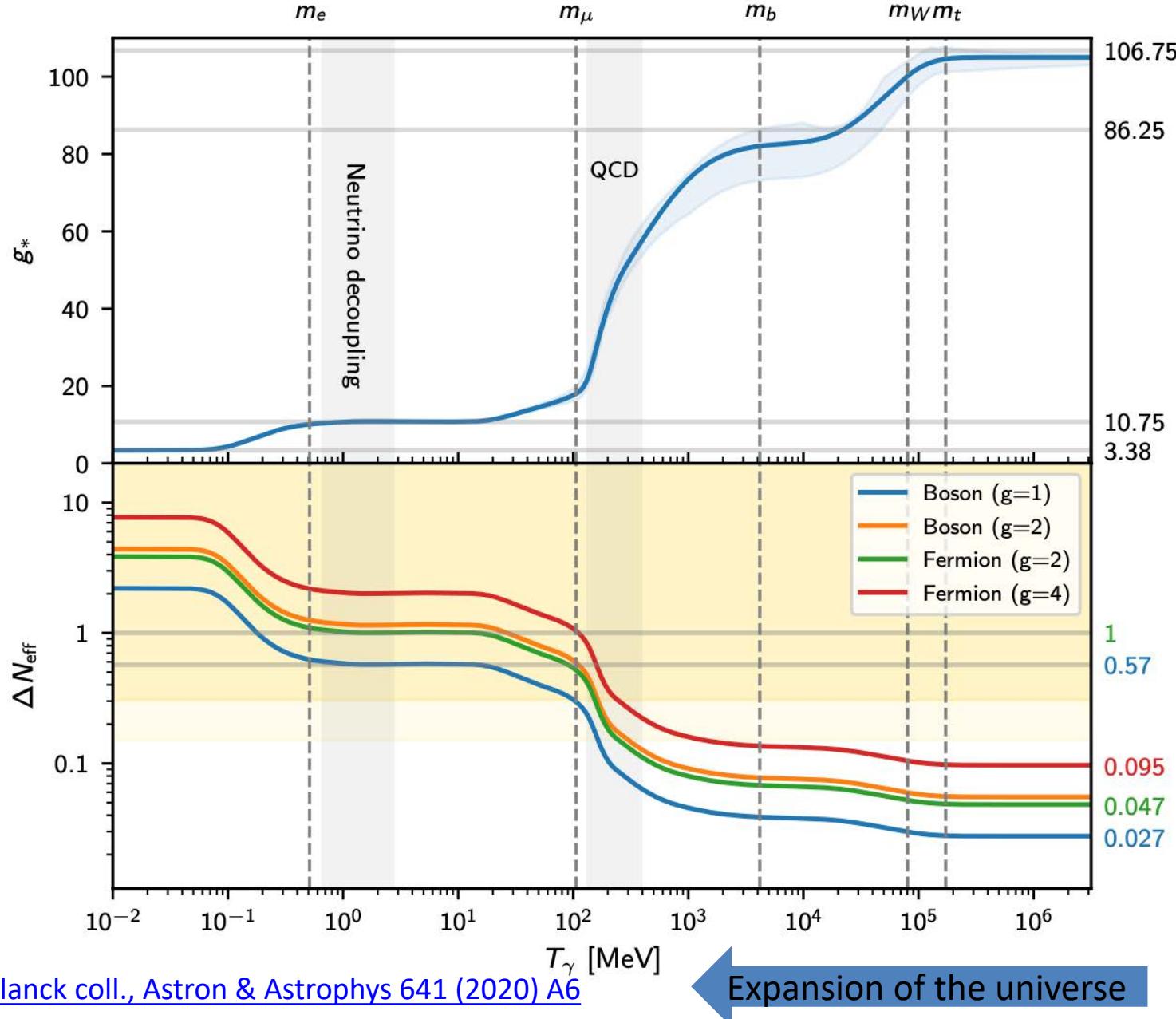
$$\rho_{\text{rad}} = \rho_\gamma + \rho_\nu + \rho_x = \rho_\gamma \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{\text{eff}} \right]$$

effective number of neutrinos

$N_{\text{eff}} \neq 3$

additional relativistic particles (scalars, pseudoscalars,
decay products of heavy particles,...)

Constraints on additional relativistic particles



Relativistic particles in the universe

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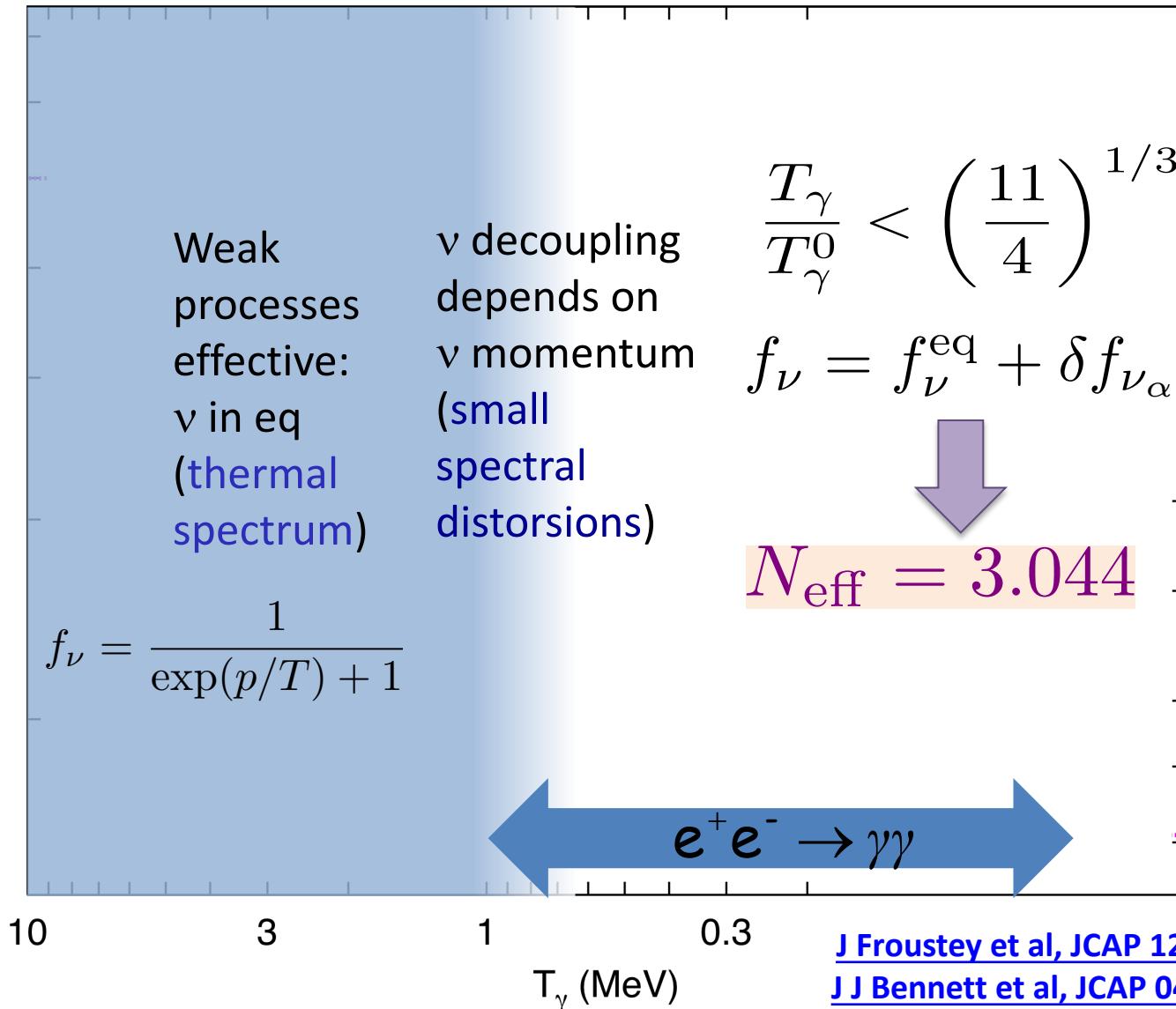
additional relativistic particles (scalars, pseudoscalars, decay products of heavy particles,...)

[J Froustey & C Pitrou, arXiv:2405.06509](#)

non-standard neutrino physics (primordial neutrino asymmetries, totally or partially thermalised **light sterile neutrinos**, **non-standard interactions with electrons**, non-unitary 3-neutrino mixing...)

[S Gariazzo et al, JCAP 03 \(2023\) 046](#)

$N_{\text{eff}} > 3$: standard case



Expansion of the universe

N_{eff} with non-standard neutrino-electron interactions

Non-standard neutrino-electron interactions

Standard and non-standard interactions (NSI) between neutrinos and electrons can be parametrised as follows:

$$\mathcal{L}_{SM} = -2\sqrt{2} G_F \left[(\bar{\nu}_e \gamma^\mu P_L e) (\bar{e} \gamma_\mu P_L \nu_e) + \sum_{X,\alpha} g_X (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\alpha) (\bar{e} \gamma_\mu P_X e) \right]$$

$$\mathcal{L}_{NSIe} = -2\sqrt{2} G_F \sum_{\alpha,\beta} \varepsilon_{\alpha\beta}^X (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta) (\bar{e} \gamma_\mu P_X e).$$

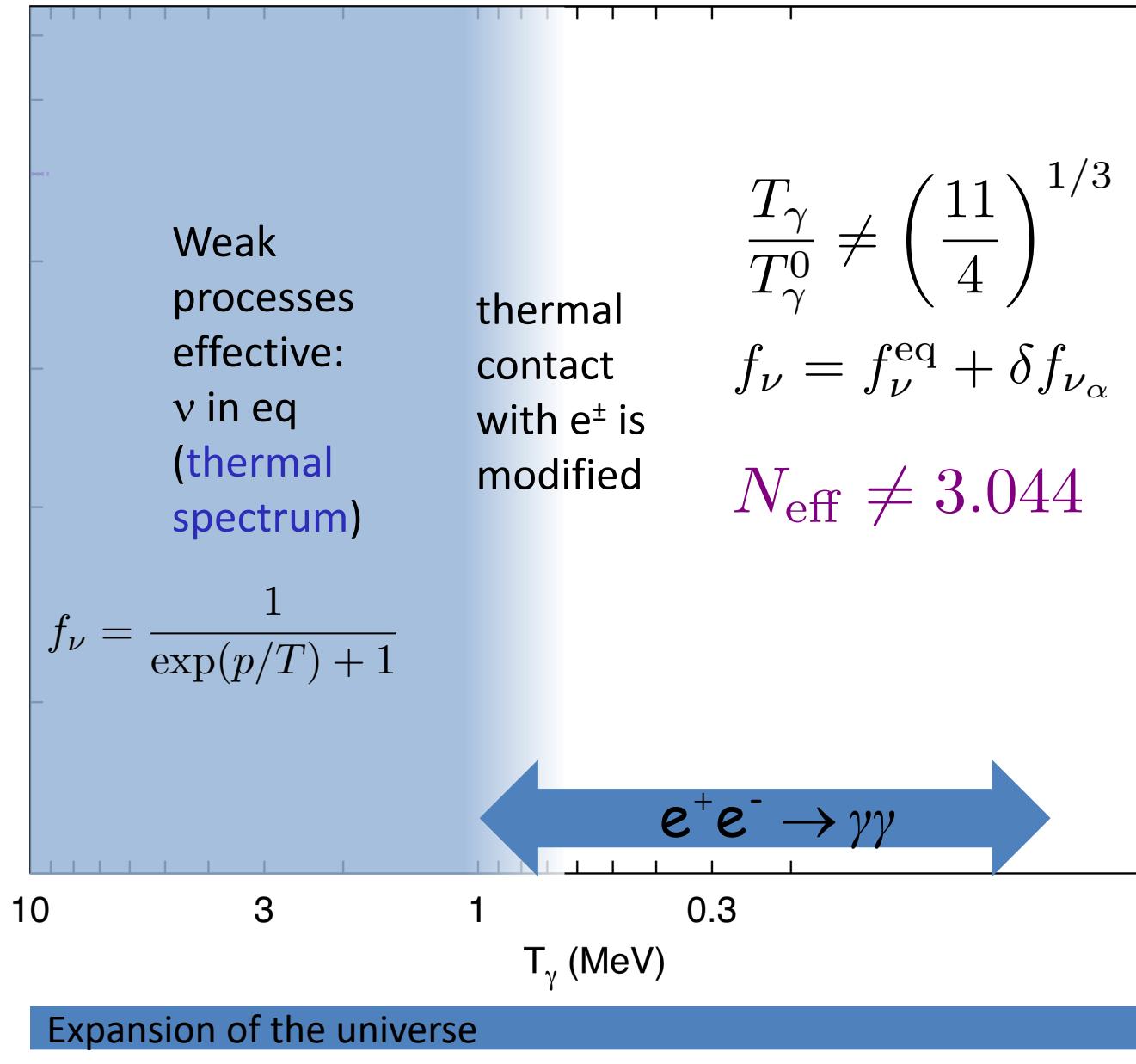
with $X \in \{L, R\}$
 $\alpha, \beta \in \{e, \mu, \tau\}$

Dimensionless coefficients $\varepsilon_{\alpha\beta}^X$ quantify the strength of the interactions with respect to the SM

$\varepsilon_{\alpha\alpha}^X$ Non-universal NSI

$\varepsilon_{\alpha\beta}^X$ (with $\alpha \neq \beta$) Flavour-changing ($\alpha \neq \beta$) NSI

N_{eff} with non-standard neutrino-electron interactions



Equations for the neutrino density matrix

**diagonal terms
(occupation numbers)**

$$\varrho_p(t) = \begin{pmatrix} \varrho_{ee} & \varrho_{e\mu} & \varrho_{e\tau} \\ \varrho_{\mu e} & \varrho_{\mu\mu} & \varrho_{\mu\tau} \\ \varrho_{\tau e} & \varrho_{\tau\mu} & \varrho_{\tau\tau} \end{pmatrix} = \begin{pmatrix} f_{\nu_e} & a_1 + ia_2 & b_1 + ib_2 \\ a_1 - ia_2 & f_{\nu_\mu} & c_1 + ic_2 \\ b_1 - ib_2 & c_1 - ic_2 & f_{\nu_\tau} \end{pmatrix}$$

off-diagonal terms

Boltzmann evolution equations (matrix form)

[G Sigl & GG Raffelt NPB 406 \(1993\) 423](#)

$$(\partial_t - H p \partial_p) \varrho_p(t) = -i \left[\left(\frac{1}{2p} \mathbb{M}_F - \frac{8\sqrt{2}G_F p}{3m_W^2} \mathbb{E} \right), \varrho_p(t) \right] + \mathcal{I} [\varrho_p(t)]$$

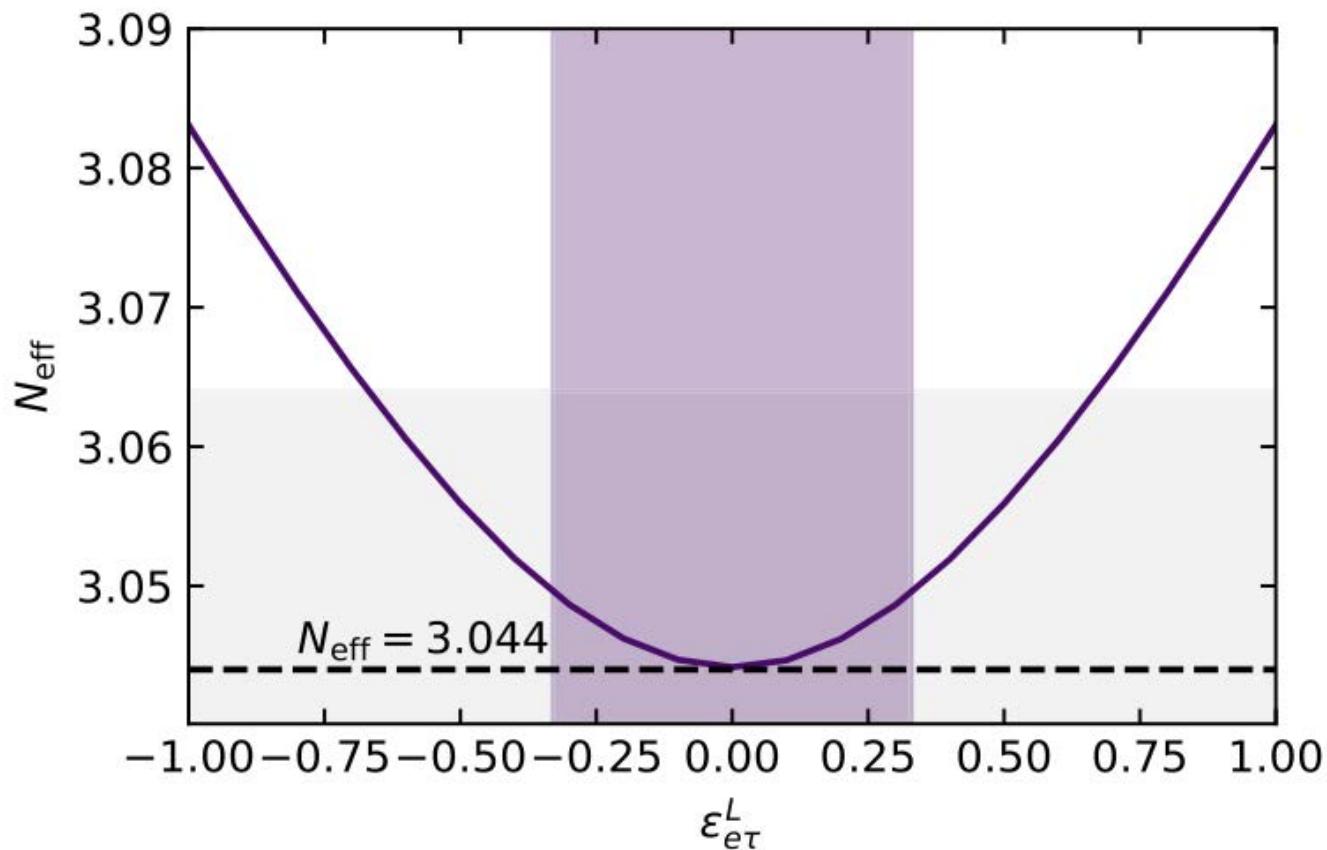
+ continuity
equation
vacuum osc.
term
matter potential
term
collision integrals
 $(\propto G_F^2)$

$$\dot{\rho} = -3H(\rho + P)$$

Code: **FORTran-Evolved Primordial
Neutrino Oscillations (FortEPiaNO)**

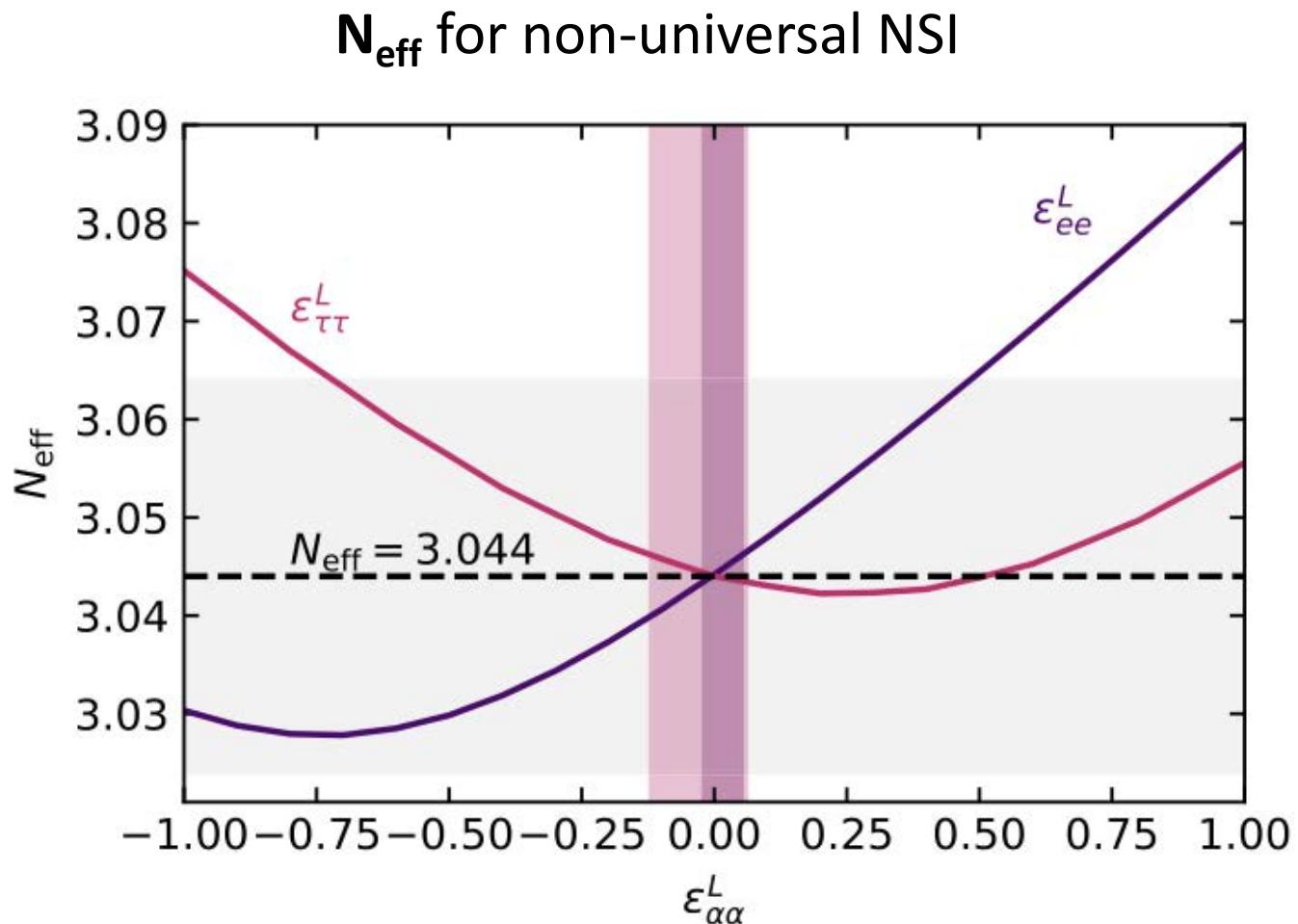
N_{eff} with only one NSI parameter

N_{eff} for flavour-changing NSI



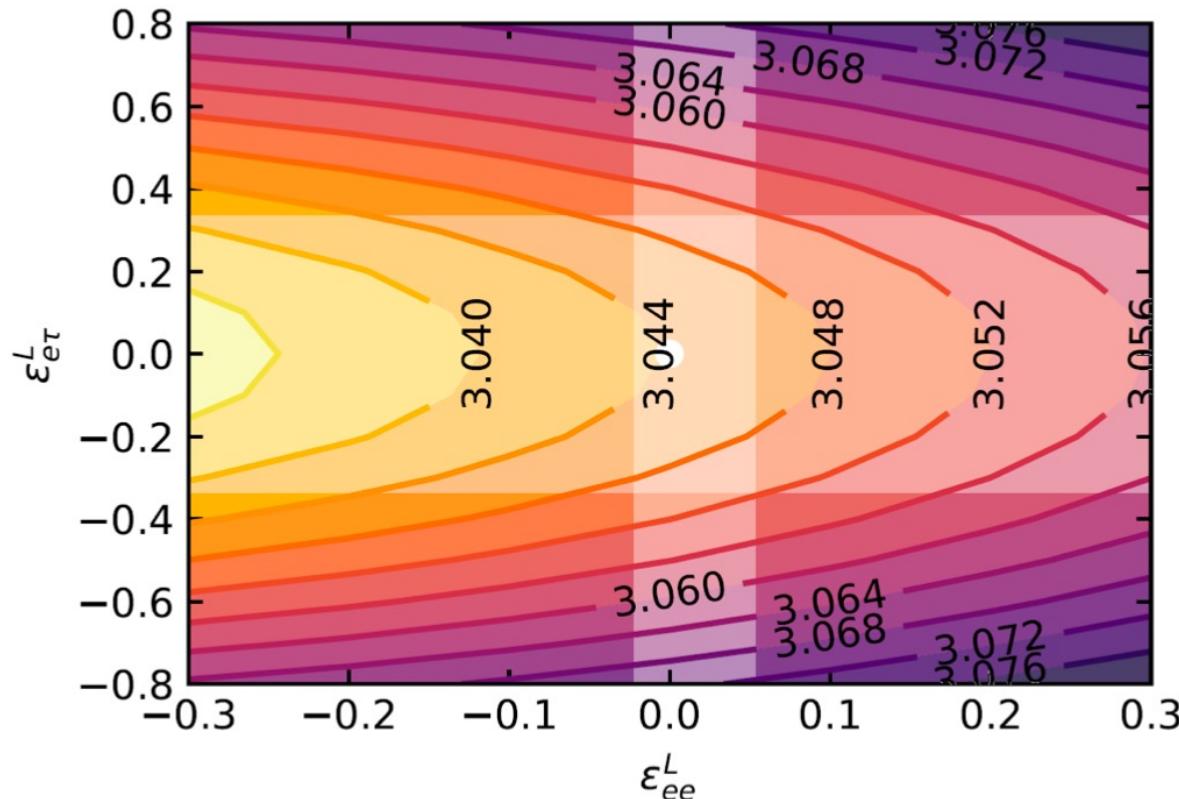
[PF de Salas et al, PLB 820 \(2021\) 136508](#)

N_{eff} with only one NSI parameter



[PF de Salas et al, PLB 820 \(2021\) 136508](#)

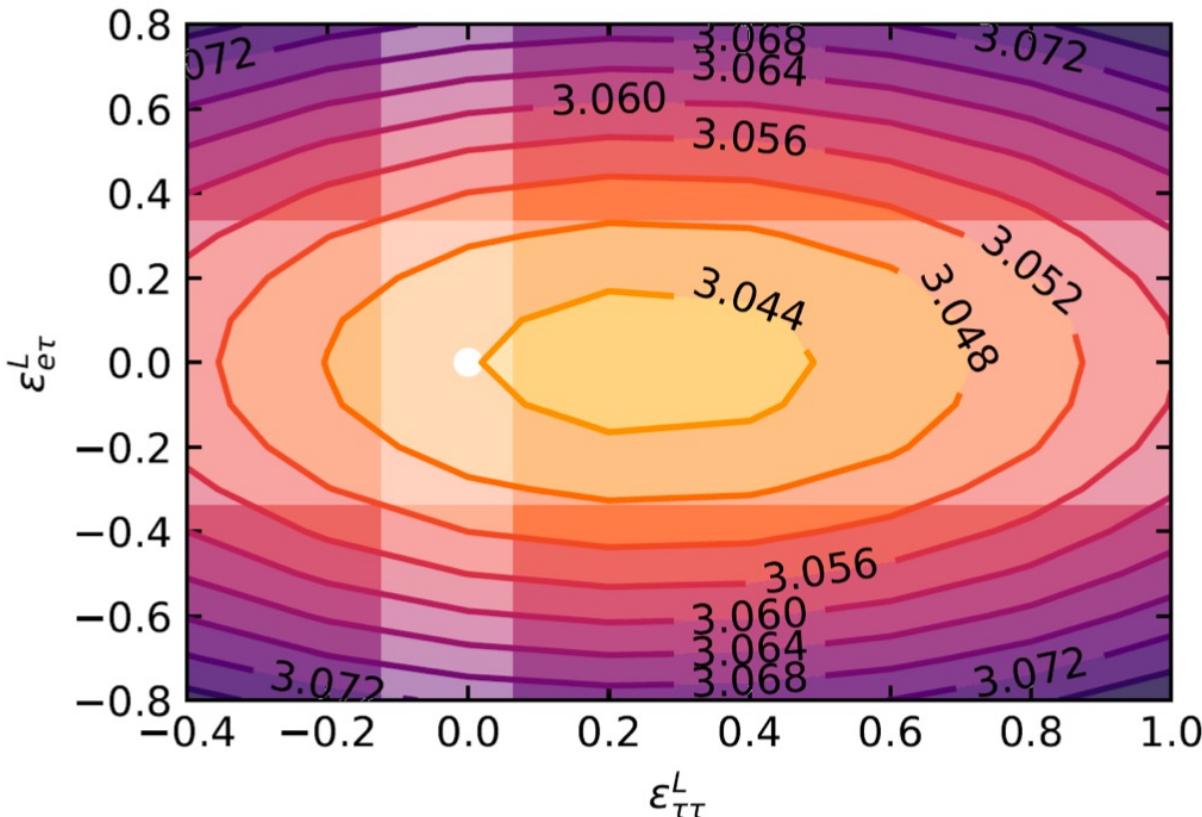
N_{eff} varying 2 NSI parameters



Future sensitivities
on N_{eff} of the order of
0.02 - 0.05

White shaded bands
correspond to terrestrial
bounds on NSI.
(One-parameter only)

N_{eff} varying 2 NSI parameters



Future sensitivities
on N_{eff} of the order of
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White shaded bands
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(One-parameter only)

N_{eff} with active-sterile neutrino oscillations

Mixing of four neutrino states?

Additional neutrino (**sterile**) states introduced in order to explain some anomalies in experimental data

4 flavour neutrinos, 4 massive neutrinos

4x4 mixing matrix
$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s_1 1} & U_{s_1 2} & U_{s_1 3} & U_{s_1 4} \end{pmatrix}$$

We consider **3 (active) + 1 (sterile)**, a perturbation of the 3-neutrino case

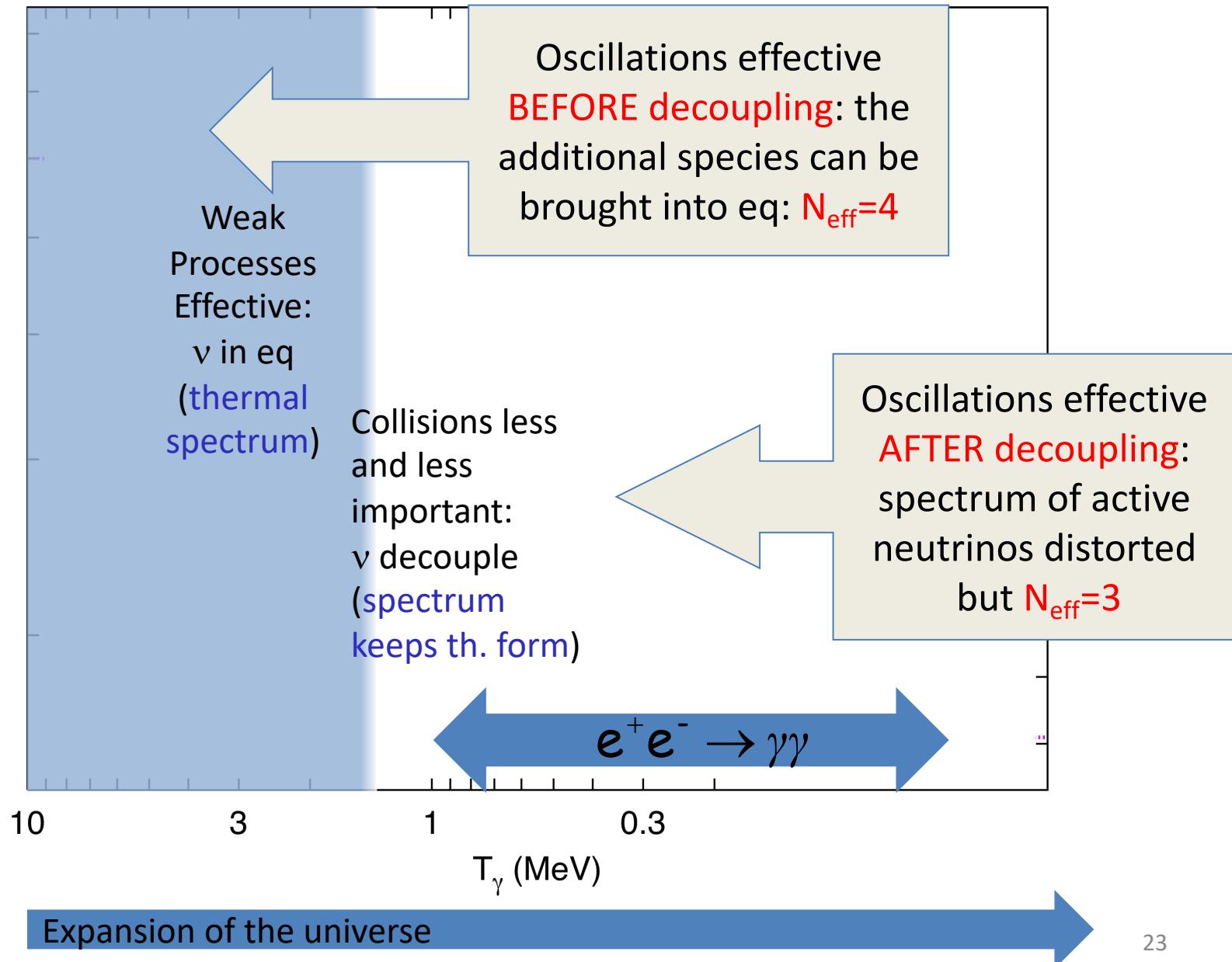
$$|U_{e4}|^2 = \sin^2 \theta_{14},$$

$$|U_{\mu 4}|^2 = \cos^2 \theta_{14} \sin^2 \theta_{24},$$

$$|U_{\tau 4}|^2 = \cos^2 \theta_{14} \cos^2 \theta_{24} \sin^2 \theta_{34},$$

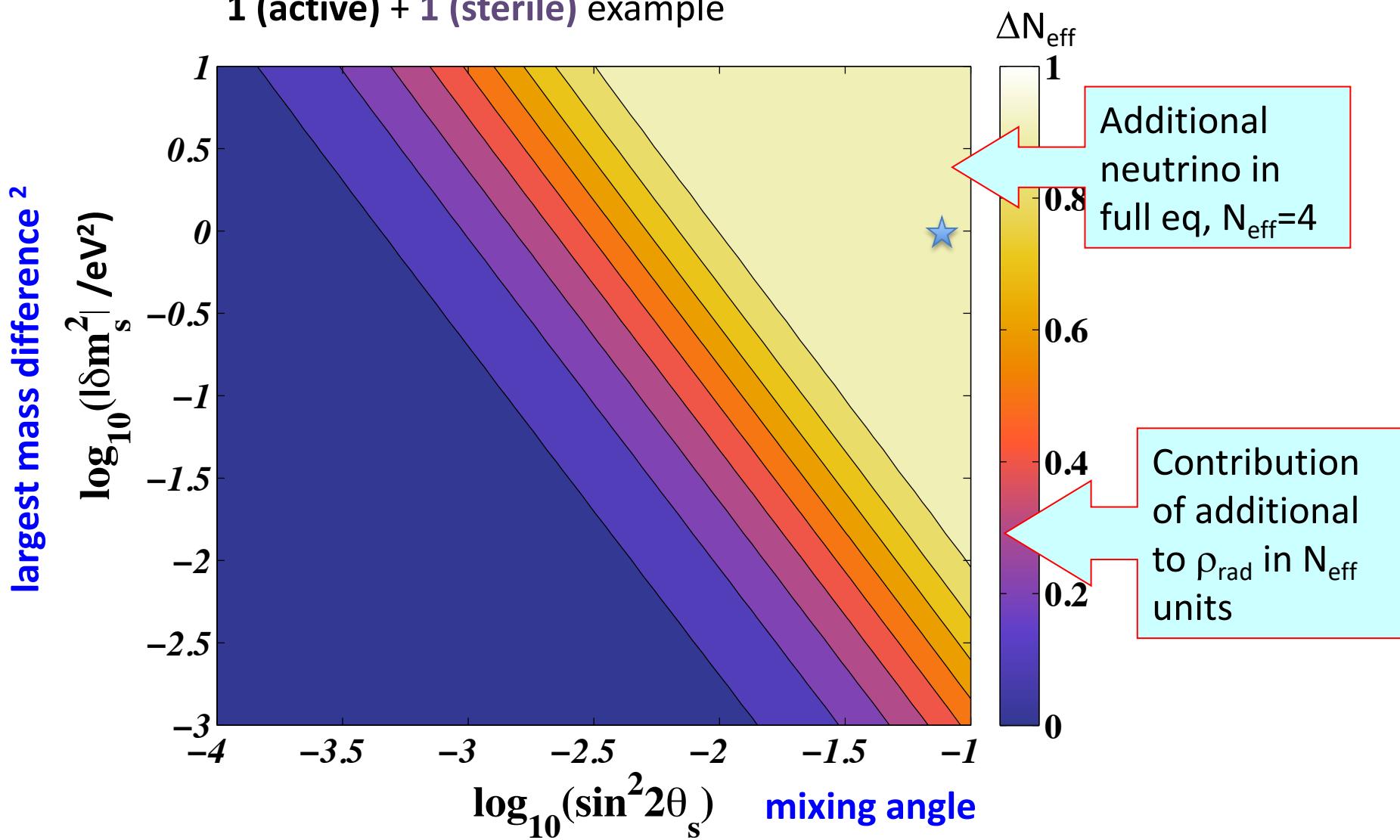
$$|U_{s_1 4}|^2 = \cos^2 \theta_{14} \cos^2 \theta_{24} \cos^2 \theta_{34}.$$

N_{eff} & active-sterile neutrino oscillations



N_{eff} & active-sterile neutrino oscillations

1 (active) + 1 (sterile) example



3+1 case: equations for the neutrino density matrix

$$\varrho(p, t) = \begin{pmatrix} \varrho_{ee} & \varrho_{e\mu} & \varrho_{e\tau} & \varrho_{es} \\ \varrho_{\mu e} & \varrho_{\mu\mu} & \varrho_{\mu\tau} & \varrho_{\mu s} \\ \varrho_{\tau e} & \varrho_{\tau\mu} & \varrho_{\tau\tau} & \varrho_{\tau s} \\ \varrho_{se} & \varrho_{s\mu} & \varrho_{s\tau} & \varrho_{ss} \end{pmatrix}$$

**diagonal terms
(occupation numbers)**
**off-diagonal
terms**

Boltzmann evolution equations (matrix form)

$$(\partial_t - H p \partial_p) \varrho_p(t) = -i \left[\left(\frac{1}{2p} \mathbb{M}_F - \frac{8\sqrt{2}G_F p}{3m_W^2} \mathbb{E} \right), \varrho_p(t) \right] + \mathcal{I}[\varrho_p(t)]$$

vacuum osc. term matter potential term collision integrals ($\propto G_F^2$)

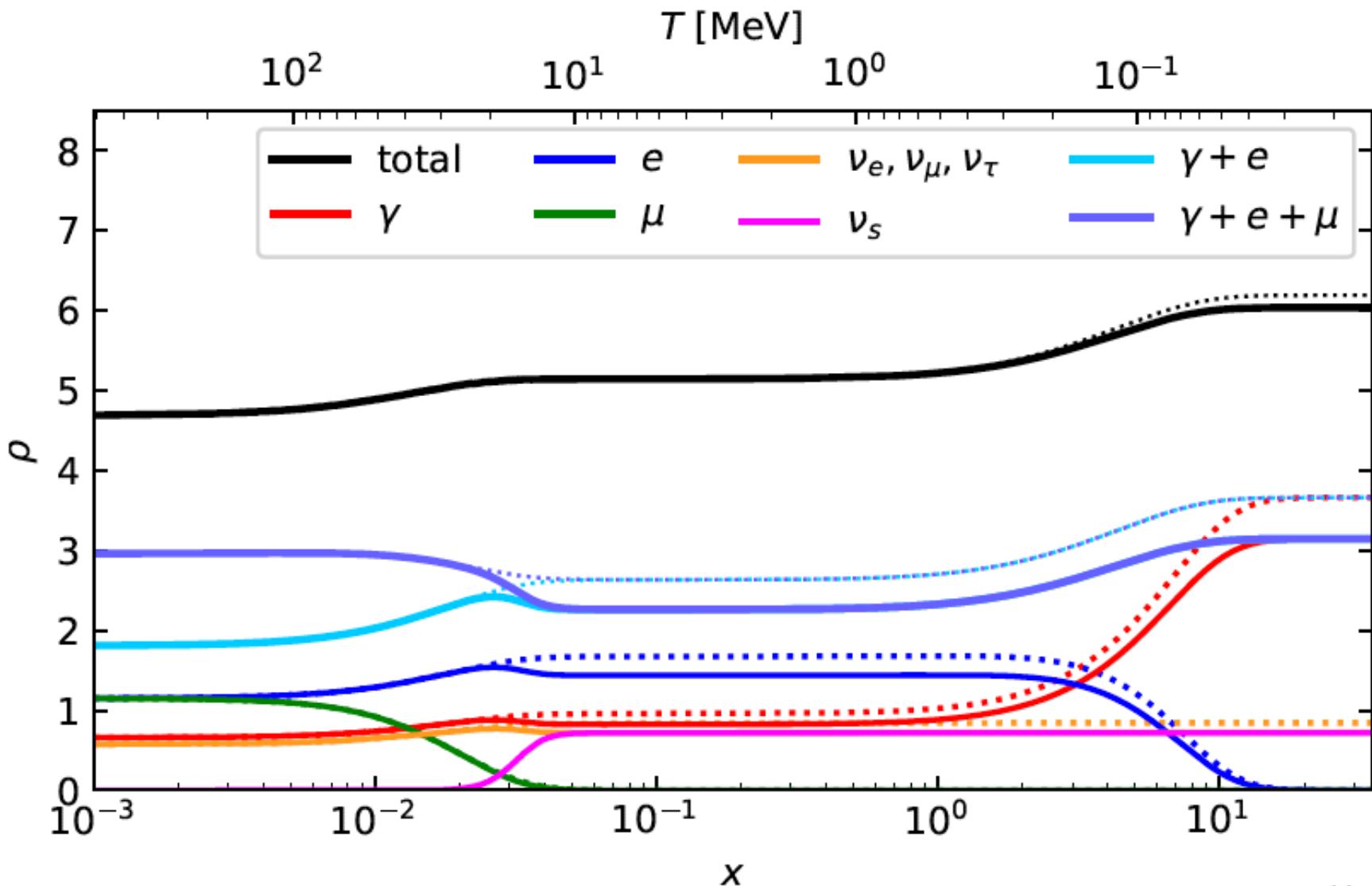
$\mathbb{M}_F = U \mathbb{M} U^\dagger$
 $\mathbb{M} = \text{diag}(m_1^2, m_2^2, m_3^2, m_4^2)$
 $U = R^{34} R^{24} R^{14} R^{23} R^{13} R^{12}$

e.g. $R^{14} = \begin{pmatrix} \cos \theta_{14} & 0 & 0 & \sin \theta_{14} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin \theta_{14} & 0 & 0 & \cos \theta_{14} \end{pmatrix}$

Code: FORTran-Evolved Primordial
Neutrino Oscillations (**FortEPiaNO**)

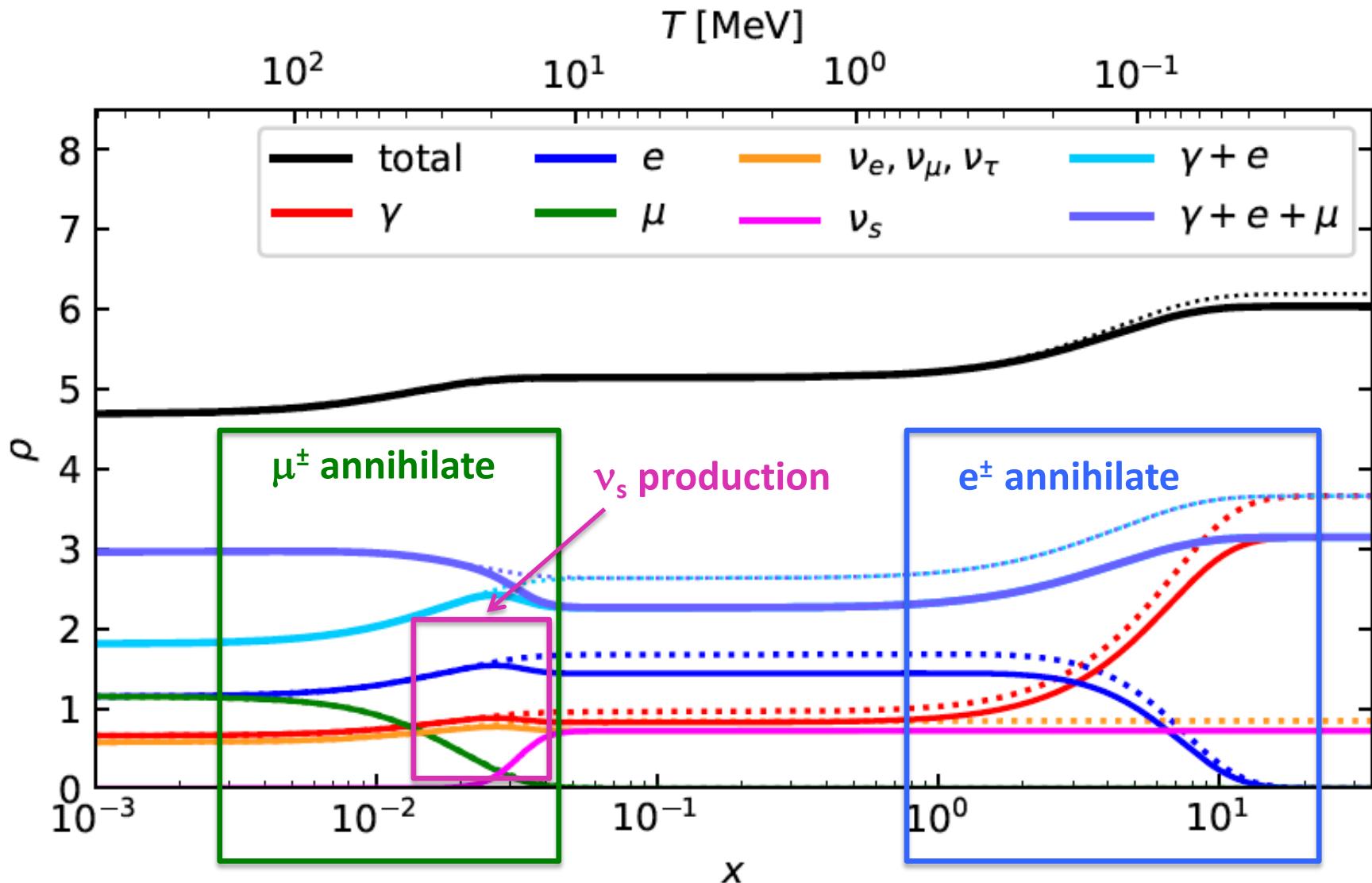
Results: evolution of energy densities (comoving)

dashed: 3ν , solid: $|U_{e4}|^2 = 10^{-2}$, $|U_{\mu 4}|^2 = |U_{\tau 4}|^2 = 0$. $\Delta m_{41}^2 = 1.29 \text{ eV}^2$

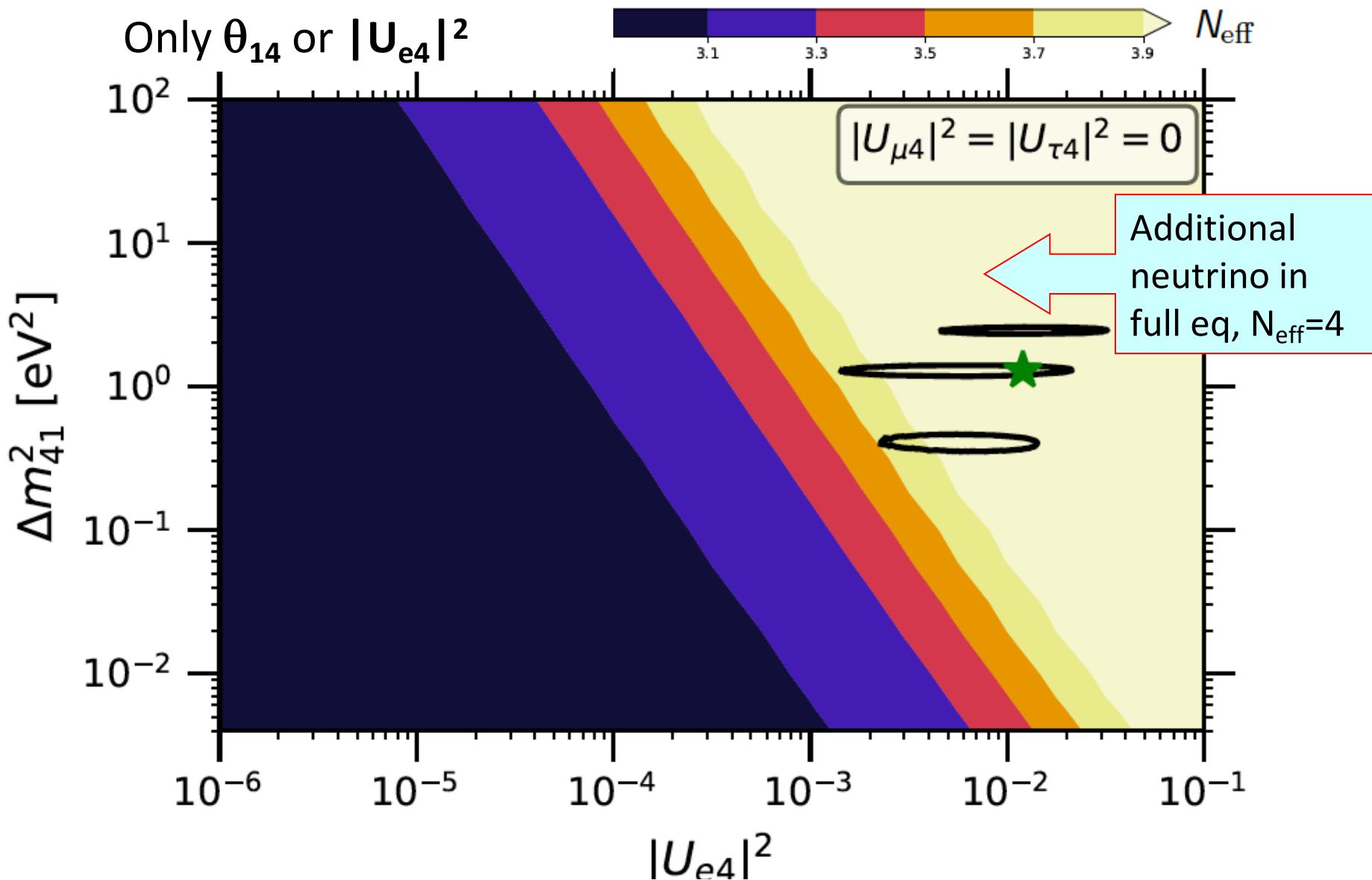


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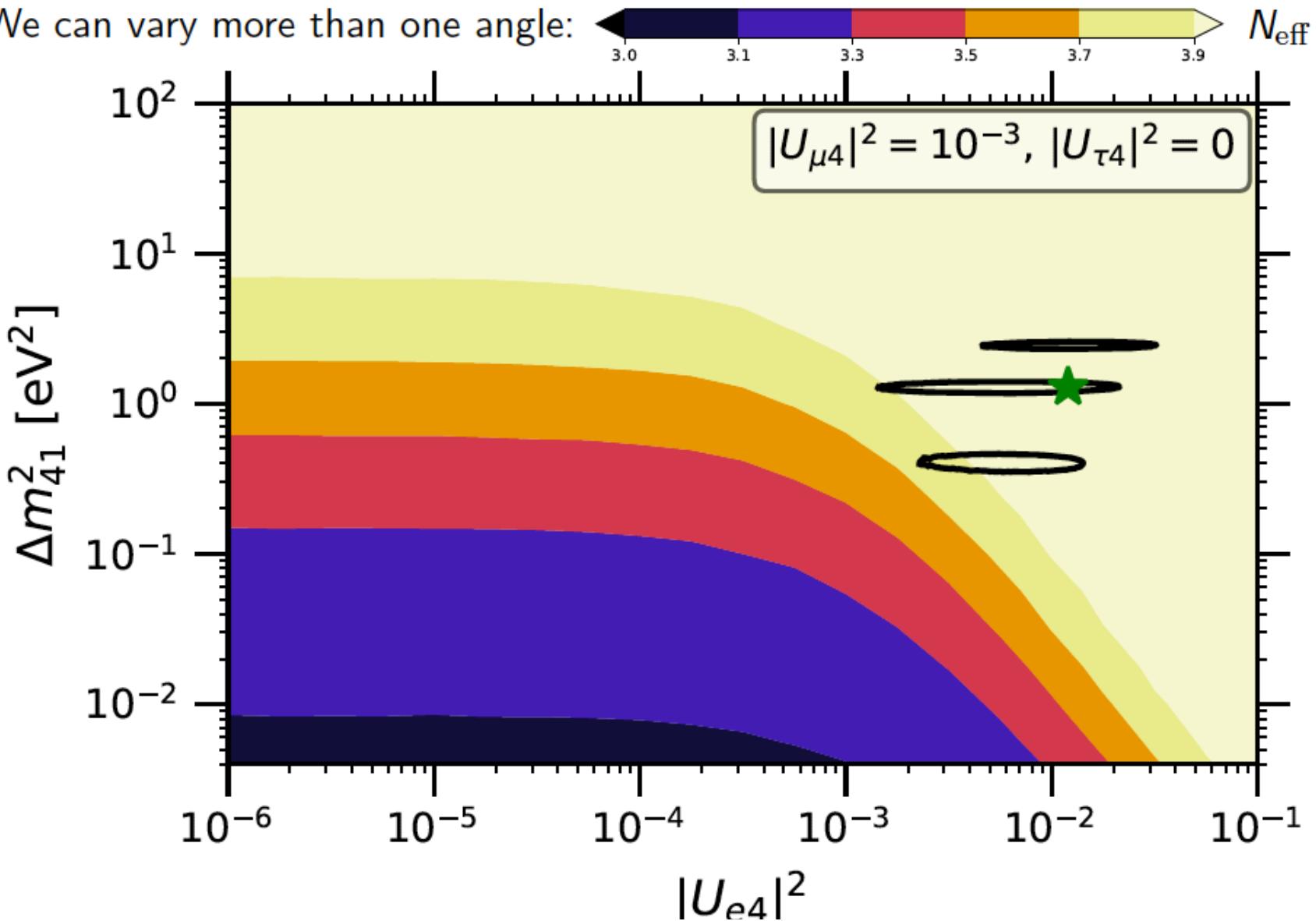


Results: final value of N_{eff} and sterile mixing parameters



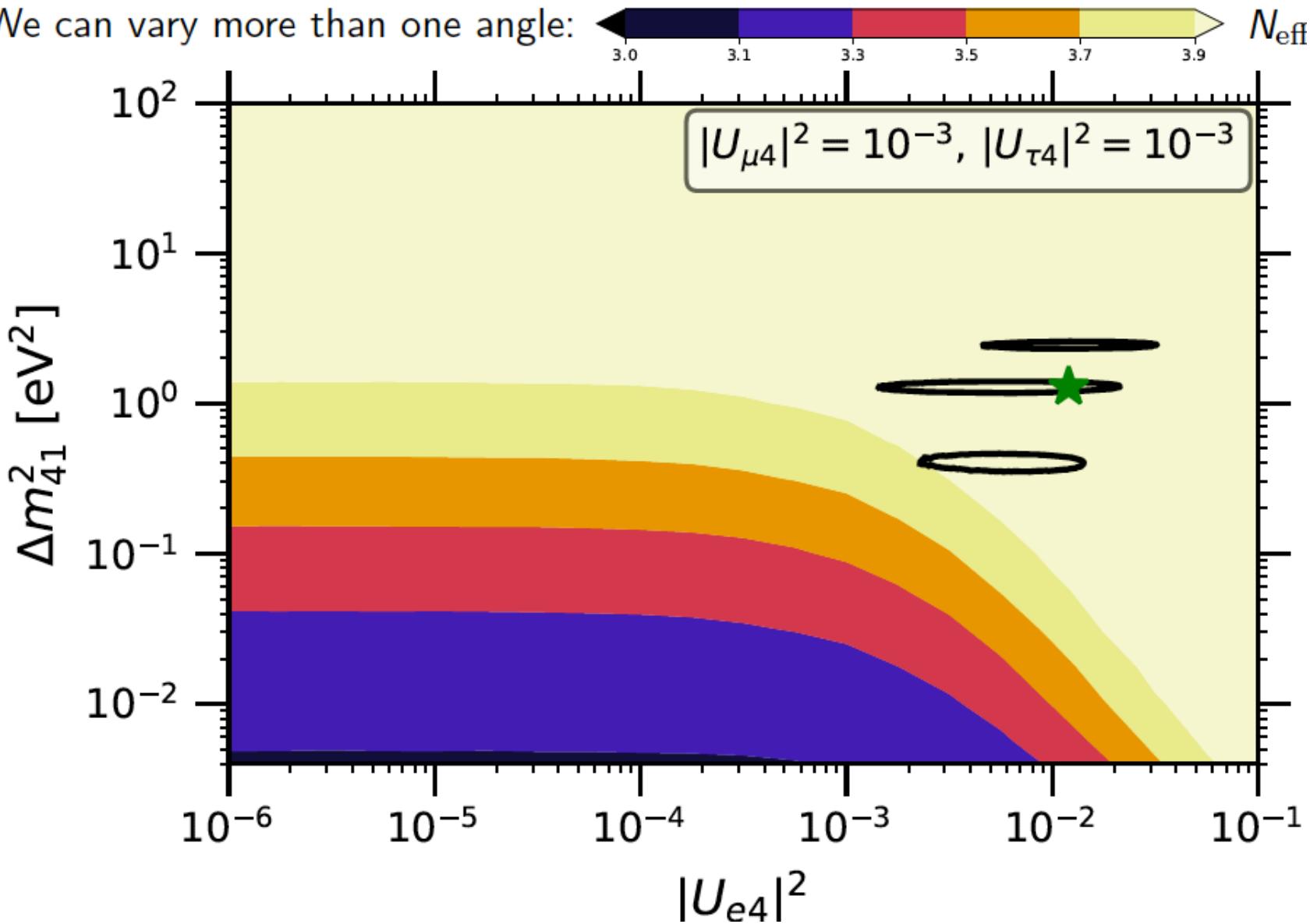
Results: final value of N_{eff} and sterile mixing parameters

We can vary more than one angle:



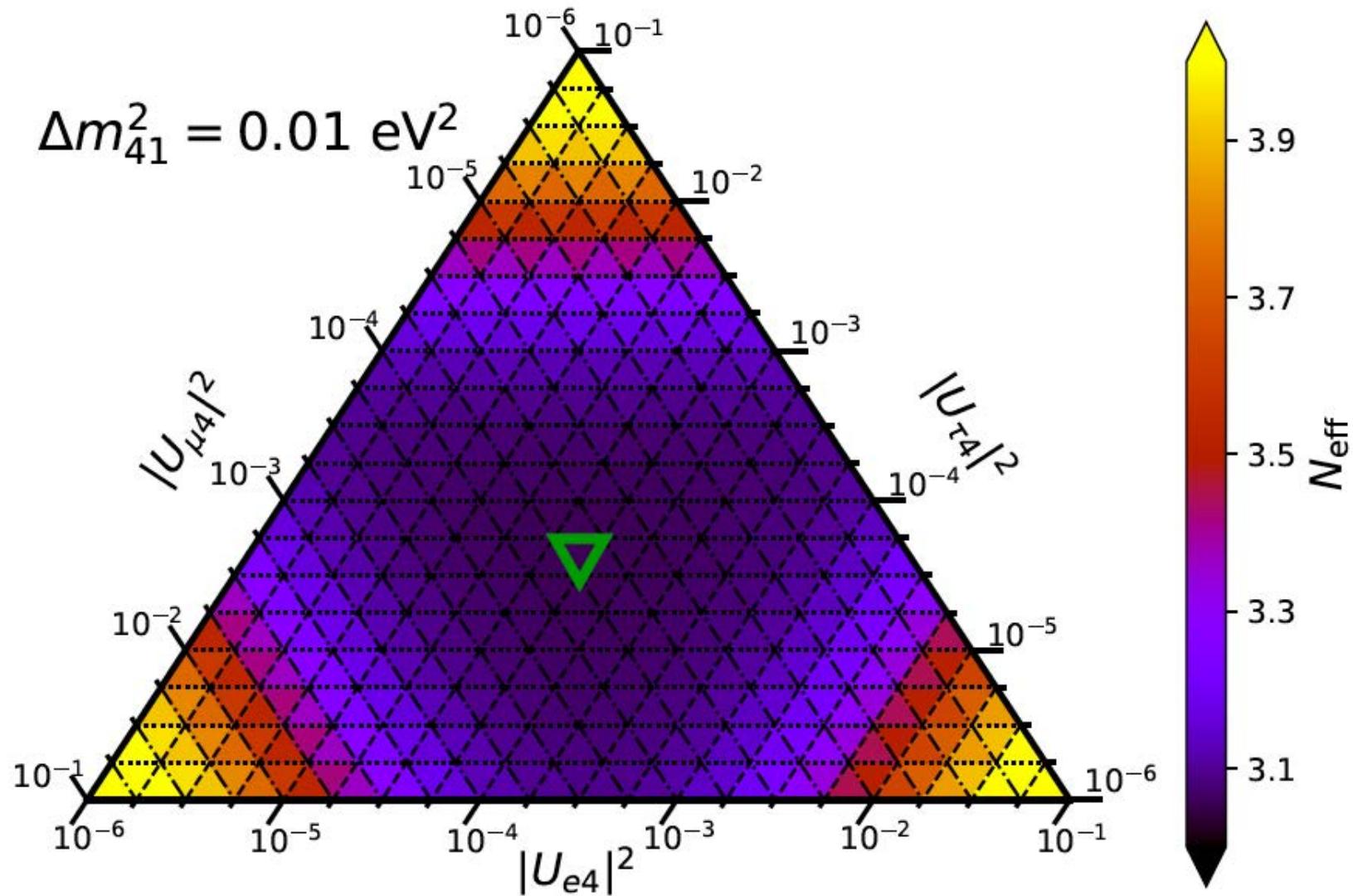
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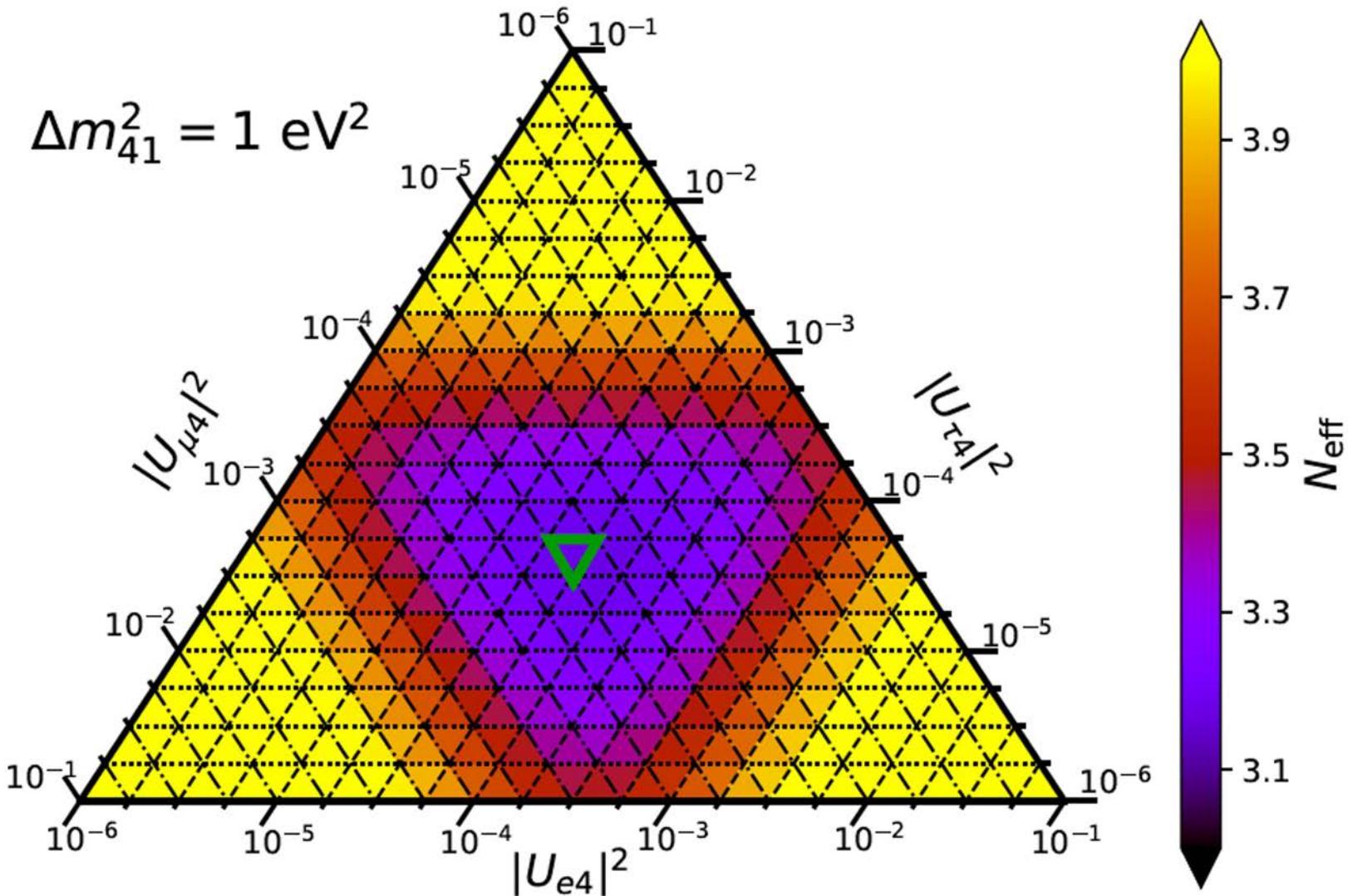
Results: final value of N_{eff} and sterile mixing parameters

Sort of ternary plot (sum of $|U_{\alpha 4}|^2$ does not add up to 1!):



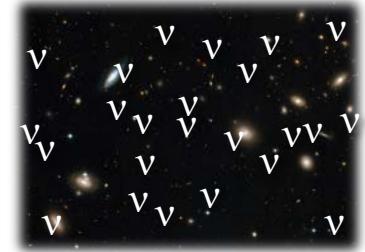
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Sort of ternary plot (sum of $|U_{\alpha 4}|^2$ does not add up to 1!):



N_{eff}

Conclusions



- ✓ N_{eff} : a parameter that quantifies the ratio of **cosmological energy densities in neutrino-like relics to photons** in the early universe
- ✓ Presence of **non-standard neutrino-electron interactions** (allowed by laboratory data) could slightly modify $N_{\text{eff}} = 3.04\text{-}3.08$
- ✓ If a **fourth (sterile) neutrino** state exists in order to explain the anomalies in oscillation measurements, it would have important **implications for the cosmological scenario ($N_{\text{eff}}, m_{\nu,\text{eff}}$)**