Neutrino flavor transformation in dense environments

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Neutrino Frontiers — *Galileo Galilei Institute*

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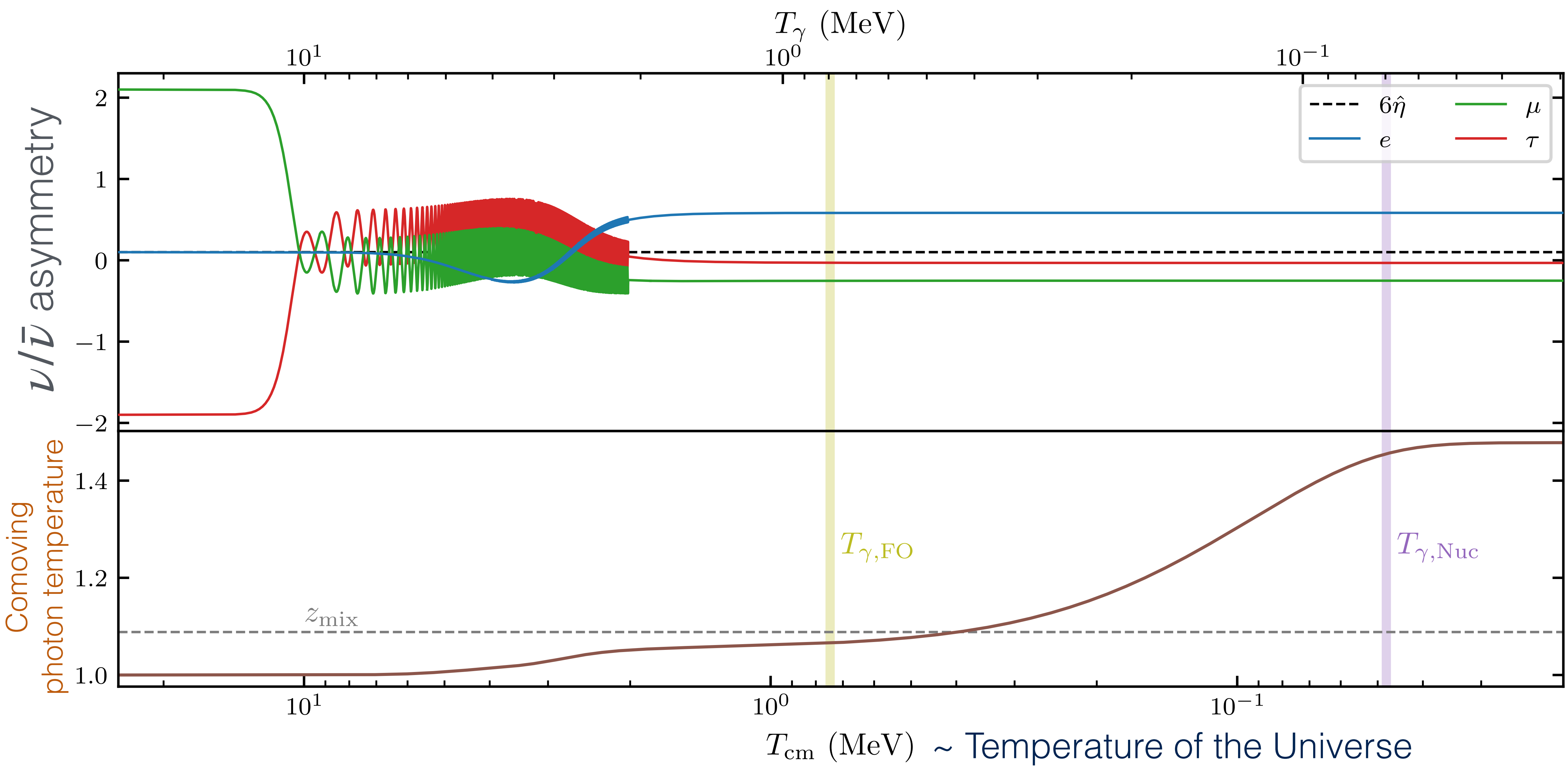
Side note • Constraints on primordial asymmetries

J. Froustey, C. Pitrou [2405.06509]

- Neutrino evolution code in the early Universe with non-zero asymmetries

$$i\left(\frac{\partial \varrho}{\partial t} - Hp\frac{\partial \varrho}{\partial p}\right) = [\mathcal{H}_{\rm vac} + \mathcal{H}_{\rm mat} + \mathcal{H}_{\rm self}, \varrho] + i\,\mathcal{C}$$



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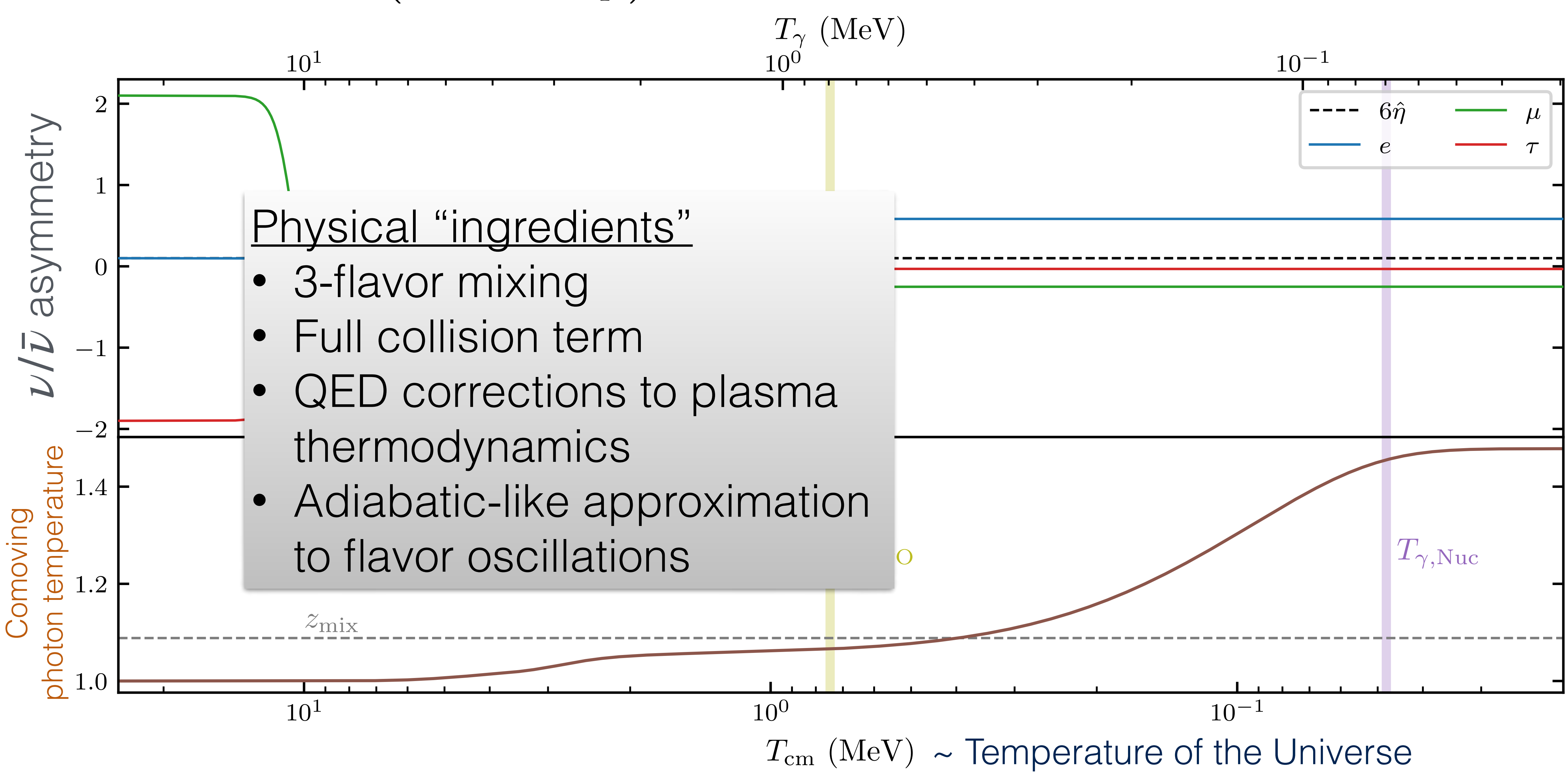

Physical "ingredients"

- 3-flavor mixing
- Full collision term
- QED corrections to plasma thermodynamics
- Adiabatic-like approximation to flavor oscillations

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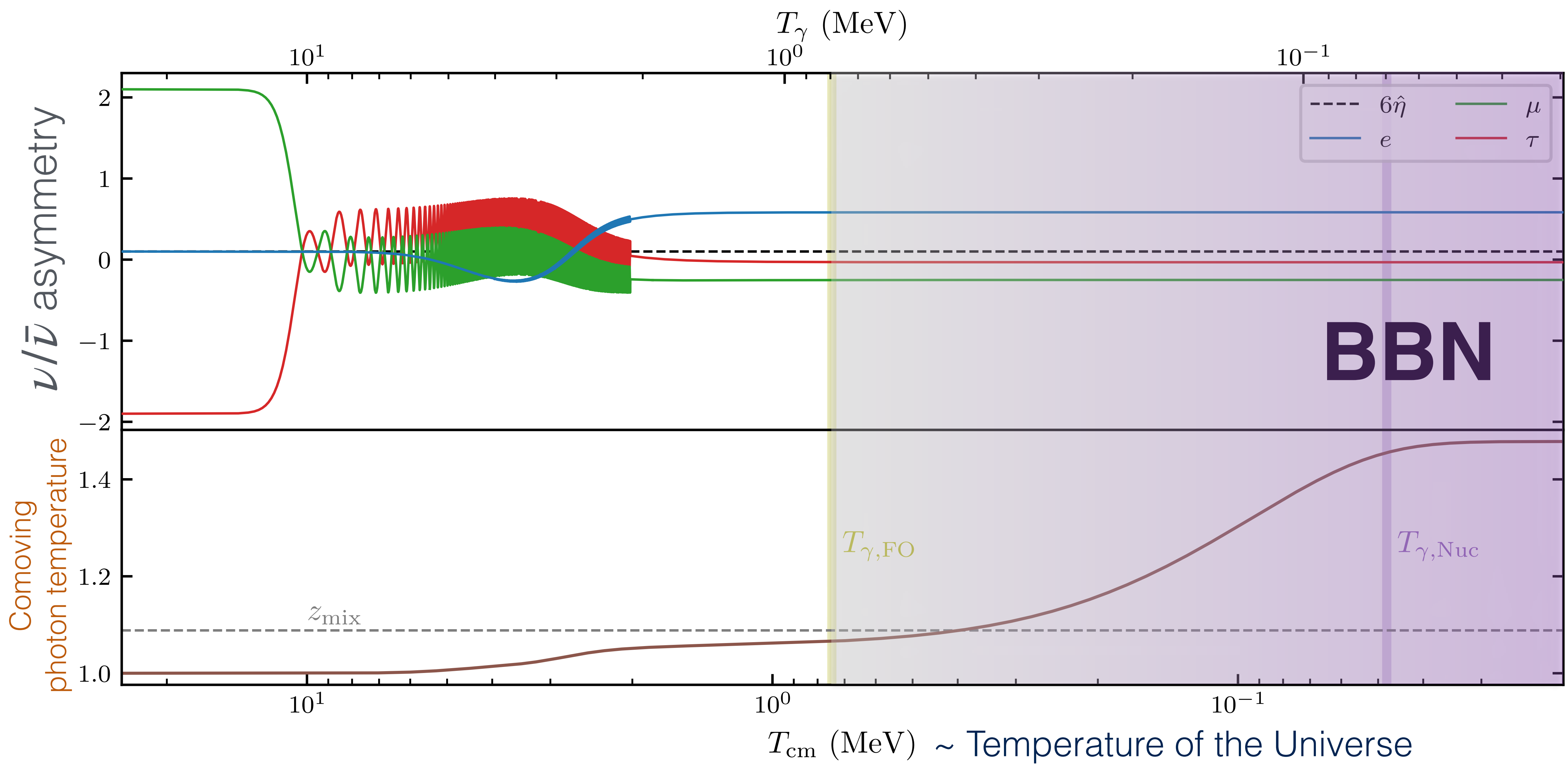
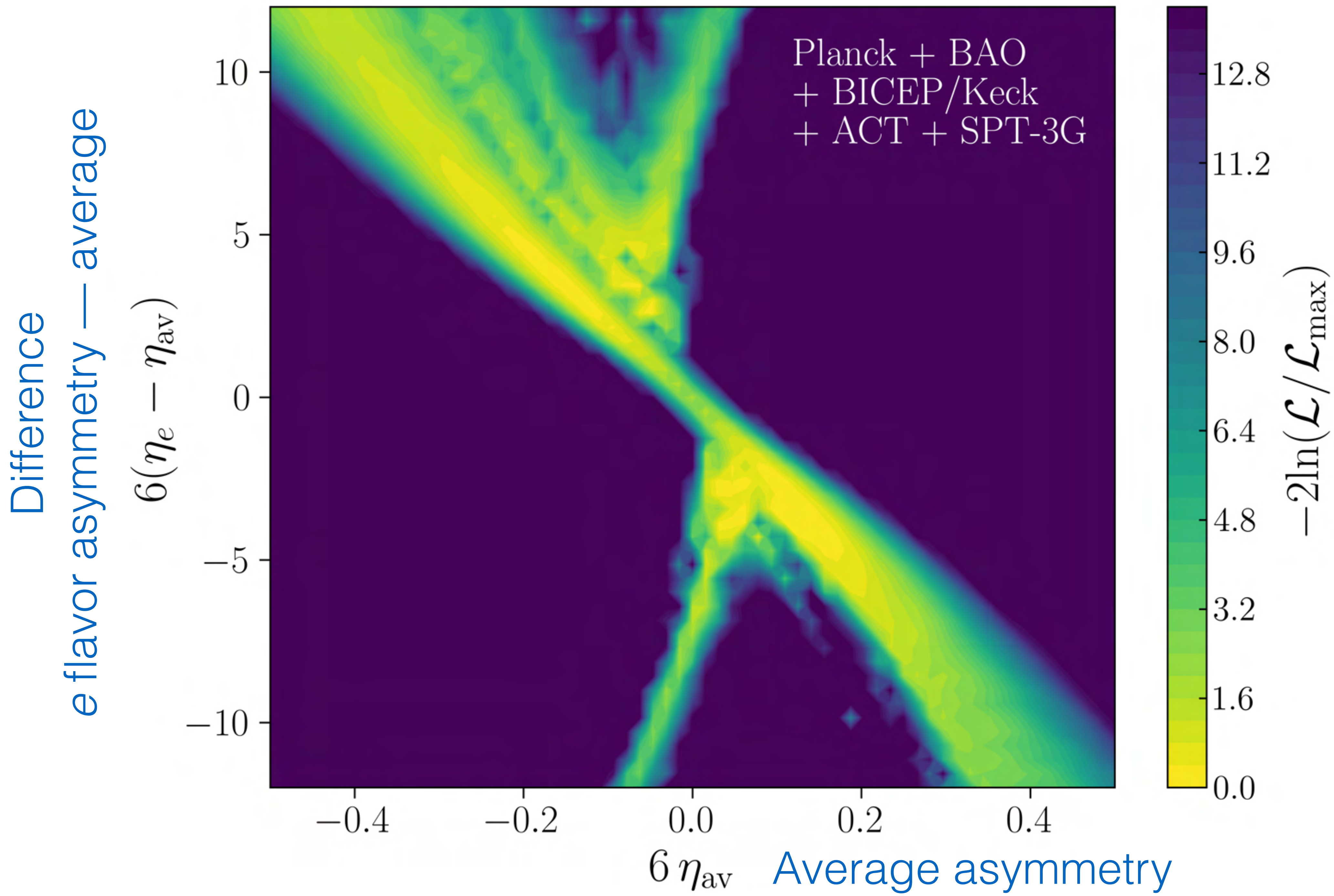


Side note • Constraints on primordial asymmetries

J. Froustey, C. Pitrou [2405.06509]

- Output used in a BBN code (**PRIMAT**): likelihood based on CMB + primordial abundance measurements.



From the early Universe to neutron star mergers

- The early Universe is fantastic: **homogeneous, isotropic**
- It is a dense environment $\Longrightarrow$ interesting neutrino physics
- Astrophysical environments (supernovae, neutron star mergers): many more intricate phenomena! *(Therefore, less detailed physics)*
- In particular, neutrino/antineutrino asymmetries + anisotropies lead to new flavor conversion mechanisms.

How do neutrinos change flavor in dense astrophysical environments?

↓

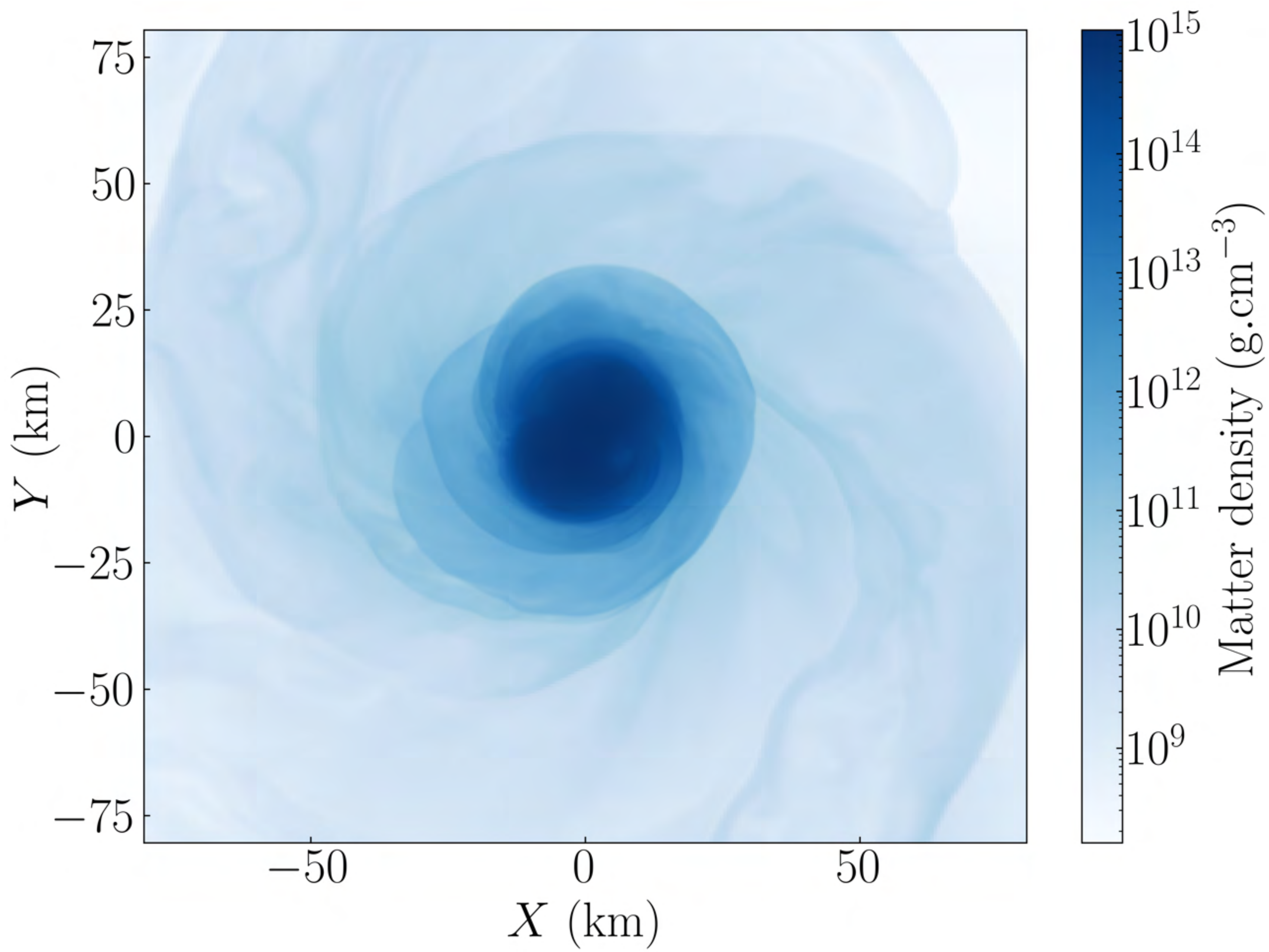
Neutrino flavor instabilities in binary neutron star mergers

Based on:

[2207.00214] E. Grohs et al., *Neutrino Fast Flavor Instability in three dimensions for a Neutron Star Merger*, Phys. Lett. B 846 (2023)
[2309.00972] E. Grohs et al., *Two-Moment Neutrino Flavor Transformation with applications to the Fast Flavor Instability in Neutron Star Mergers,* ApJ 963 (2024)
[2311.11968] J. Froustey et al., *Neutrino fast flavor oscillations with moments: linear stability analysis and application to neutron star mergers*, Phys. Rev. D 109 (2024)

NSM simulation snapshot (equatorial slice)

- Merger of $1.3\,M_\odot$ and $1.4\,M_\odot$ neutron stars, general relativistic radiation hydrodynamics M1 simulation (*F. Foucart et al.*).
- Snapshot taken 3 ms post-merger



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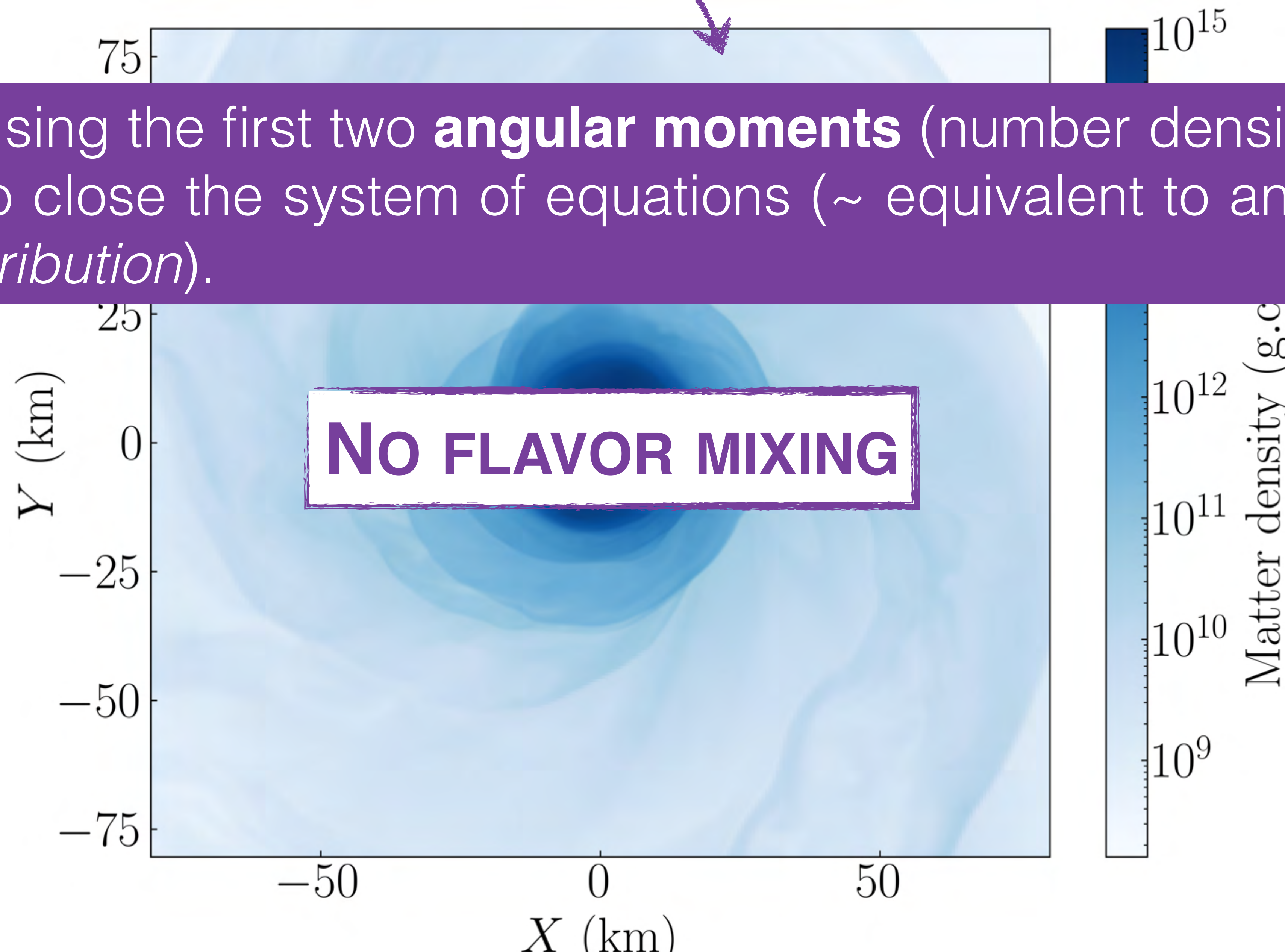


Simulation using the first two **angular moments** (number density, flux) and a **closure** to close the system of equations (~ equivalent to an *underlying angular distribution*).

NO FLAVOR MIXING

Flavor instabilities

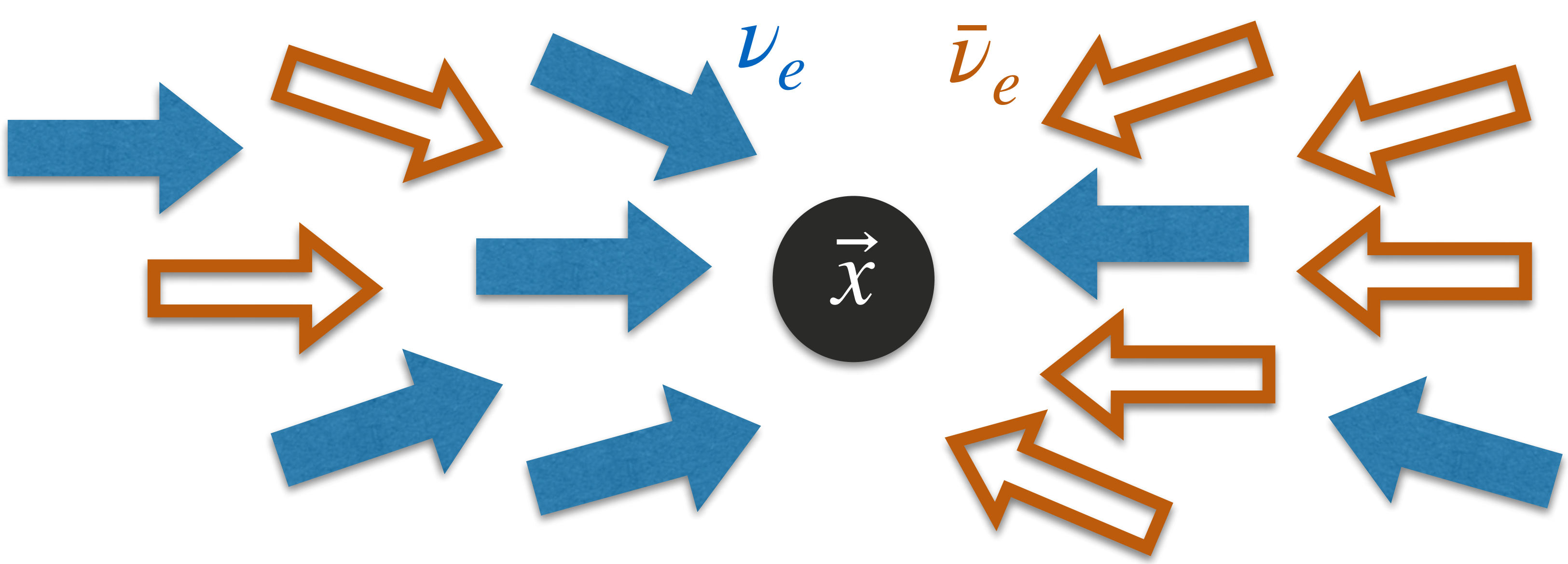
- Dense astrophysical environments: many flavor transformation mechanisms, among which:

See review by *M. C. Volpe*, [2301.11814]

- **Fast flavor instability (FFI)**

R. F. Sawyer, [0503013]

related to an *angular crossing* between neutrino and antineutrino distributions



$$f_{\nu_e}(\vec{x}, p, \theta_1) - f_{\bar{\nu}_e}(\vec{x}, p, \theta_1) > 0$$

and

$$f_{\nu_e}(\vec{x}, p, \theta_2) - f_{\bar{\nu}_e}(\vec{x}, p, \theta_2) < 0$$

- **Collisional flavor instability (CFI)**

L. Johns, [2104.11369]

related to *different collision rates* between neutrinos and antineutrinos

Introducing the QKEs

- In order to describe the evolution of a statistical ensemble of neutrinos: combination of kinetic theory and quantum mechanics.

↓ *Boltzmann equation* ↓ *Flavor mixing*

- Generalization of distribution functions: (1-body reduced) **“density matrix”**

$$\begin{pmatrix} f_{\nu_e} & \\ & f_{\nu_x} \end{pmatrix} \longrightarrow \begin{pmatrix} \varrho_{ee} & \varrho_{ex} \\ \varrho_{xe} & \varrho_{xx} \end{pmatrix}$$

- Quantum Kinetic Equations:

$$\boxed{i\left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}\right)\varrho_{\alpha\beta} = [\mathcal{H}_{\rm self}, \varrho]_{\alpha\beta} + i\, C_{\alpha\beta}}$$

$$\text{with} \quad \mathcal{H}_{\rm self} = \frac{\sqrt{2}G_F}{(2\pi)^3}\int \mathrm{d}^3\vec{q}\,(1-\cos\theta)\left[\varrho(t,\vec{x},\vec{q}) - \bar{\varrho}(t,\vec{x},\vec{q})\right]$$

Multi-angle analysis

- In order to quickly and systematically study the occurrence of flavor instabilities: **linear stability analysis** (LSA).

- To deal with (anti)neutrino angular distributions: “multi-angle” LSA.

- Discretization of the density matrix with azimuthal angle $\mu_n \equiv \cos\theta_n$

$$\varrho(\mu_n) = \begin{pmatrix} \varrho_{ee,n} & a_{ex,n} e^{-\mathrm{i}(\Omega t - \vec{k}\cdot\vec{r})} \\ a_{xe,n} e^{-\mathrm{i}(\Omega t - \vec{k}\cdot\vec{r})} & \varrho_{xx,n} \end{pmatrix}$$

$\mathrm{Im}(\Omega) > 0$

$=$ unstable!

Classical distributions from the NSM simulation

Multi-angle analysis · Classical distributions

- How to obtain angular distributions from a moment calculation?

Maximum entropy closure $\Longleftrightarrow$ $$\psi_{aa}(\theta) = \frac{N_{aa}}{4\pi}\frac{Z_{aa}}{\sinh(Z_{aa})}e^{Z_{aa}\cos(\theta-\theta_{\vec{F}_{aa}})}$$

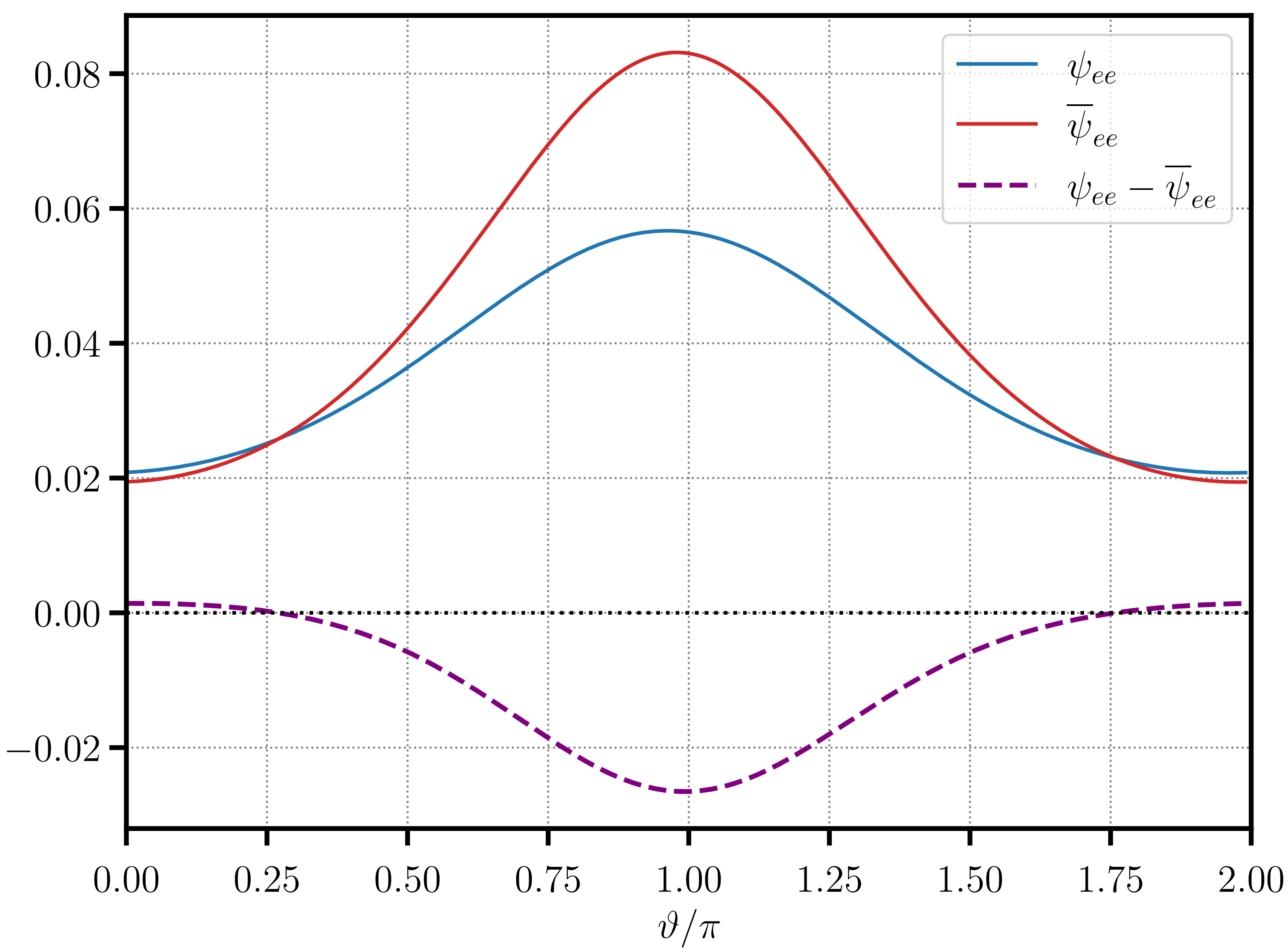
$$\frac{|\vec{F}_{aa}|}{N_{aa}} = \frac{1}{\tanh(Z_{aa})} - \frac{1}{Z_{aa}}$$

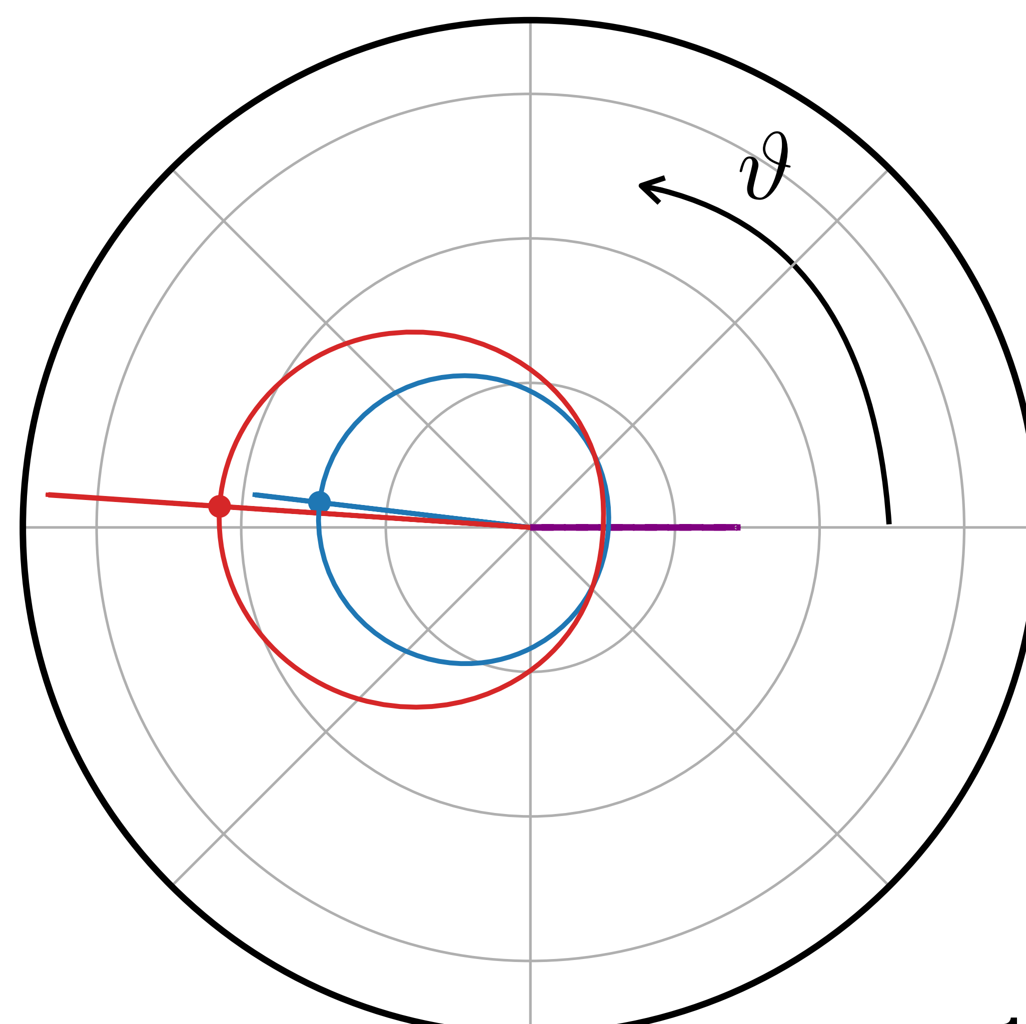
Multi-angle analysis · Classical distributions

- How to obtain angular distributions from a moment calculation?

Maximum entropy closure $\Longleftrightarrow$ $\psi_{aa}(\theta) = \dfrac{N_{aa}}{4\pi}\dfrac{Z_{aa}}{\sinh(Z_{aa})} e^{Z_{aa}\cos(\theta - \theta_{\vec{F}_{aa}})}$

Example of NSM point:





Multi-angle analysis

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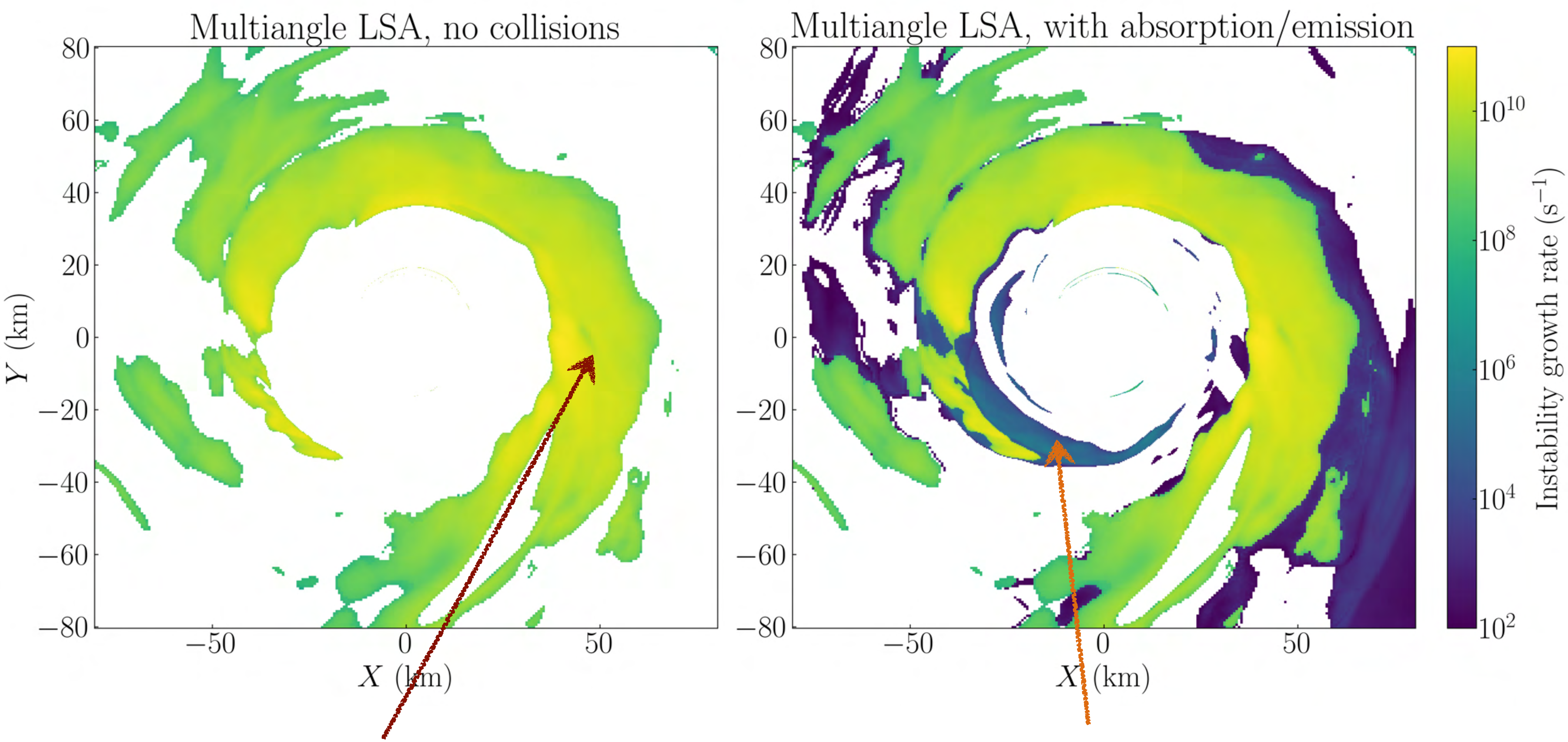
- Linearization of the QKEs $\longrightarrow$ Eigenvalues $\Omega(\vec{k})$

$\longrightarrow$ Fastest growing mode

Instability growth rate $\quad \max_{\vec{k}} \left\{ \mathrm{Im}[\Omega(\vec{k})] \right\} \equiv \mathrm{Im}(\Omega)_{\max}$

- Need to diagonalize a $(2 \times N_{\mu_n})^2$ matrix

Linear stability analysis — Results



FFI

Very large growth rate $\sim 10^{10}\,s^{-1}$

CFI

Growth rate $\sim \Gamma \sim 10^{6}\,s^{-1}$

“Moment” Quantum Kinetic Equations

- Angular moments of the density matrix:

$$\begin{matrix}\textbf{Number density}\\ \textbf{Flux}\\ \textbf{Pressure tensor}\end{matrix}\quad \begin{bmatrix} N \\ F^i \\ P^{ij} \end{bmatrix} = \int \mathrm{d}p \frac{p^2}{(2\pi)^3} \int \mathrm{d}\boldsymbol{\Omega} \begin{bmatrix} 1 \\ p^i/p \\ p^i p^j/p^2 \end{bmatrix} \varrho(t, \vec{x}, \vec{p})$$

- QKEs for moments (simplifying assumption: mono-energetic p):

$$i\left(\frac{\partial N}{\partial t} + \frac{\partial F^j}{\partial x^j}\right) = \sqrt{2}G_F\left[N - \overline{N}, N\right] - \sqrt{2}G_F\left[(F - \overline{F})_j, F^j\right] + i\,C_N$$

$$i\left(\frac{\partial F^i}{\partial t} + \frac{\partial P^{ij}}{\partial x^j}\right) = \sqrt{2}G_F\left[N - \overline{N}, F^i\right] - \sqrt{2}G_F\left[(F - \overline{F})_j, P^{ij}\right] + i\,C_F$$

"Moment" Quantum Kinetic Equations

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Pressure tensor

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Closure $P^{ij}_{\alpha\beta}\left(N_{\alpha\beta}, F^k_{\alpha\beta}\right)$

Linear stability analysis

$$N = \begin{pmatrix} N_{ee} & A_{ex} e^{-\mathrm{i}(\Omega t - \vec{k}\cdot\vec{r})} \\ A_{xe} e^{-\mathrm{i}(\Omega t - \vec{k}\cdot\vec{r})} & N_{xx} \end{pmatrix}$$

Classical moments = output from the NSM simulation

$$F^j = \begin{pmatrix} F^j_{ee} & B^j_{ex} e^{-\mathrm{i}(\Omega t - \vec{k}\cdot\vec{r})} \\ B^j_{xe} e^{-\mathrm{i}(\Omega t - \vec{k}\cdot\vec{r})} & F^j_{xx} \end{pmatrix}$$

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- Classical maximum entropy **closure**:

$$P^{\mathrm{zz}}_{ee} = \chi\left(\frac{|\vec{F}_{ee}|}{N_{ee}}\right) N_{ee} \; , \quad P^{\mathrm{zz}}_{xx} = \chi\left(\frac{|\vec{F}_{xx}|}{N_{xx}}\right) N_{xx}$$

$$\chi(\hat{f}) \equiv \frac{1}{3} + \frac{2\hat{f}^2}{15}\left(3 - \hat{f} + 3\hat{f}^2\right)$$

Linear stability analysis

$$N = \begin{pmatrix} N_{ee} & A_{ex} e^{-\mathrm{i}(\Omega t - \vec{k}\cdot\vec{r})} \\ A_{xe} e^{-\mathrm{i}(\Omega t - \vec{k}\cdot\vec{r})} & N_{xx} \end{pmatrix}$$

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- "Quantum-extended" maximum entropy **closure**:

$$P^{\mathrm{zz}}_{ee} = \chi\left(\frac{|\vec{F}_{ee}|}{N_{ee}}\right) N_{ee}$$

$$P^{\mathrm{zz}}_{xx} = \chi\left(\frac{|\vec{F}_{xx}|}{N_{xx}}\right) N_{xx}$$

$$P^{\mathrm{zz}}_{ex} = \chi\left(\frac{|\vec{F}_{ee} + \vec{F}_{xx}|}{N_{ee} + N_{xx}}\right) N_{ex}$$

Linear stability analysis

- Linearly expand the QKEs to get the system of equations:

$$\boxed{S_{\vec{k}} \cdot Q + \Omega\, \mathbb{I} \cdot Q = 0}$$

"Stability matrix"
[8 × 8 matrix]

$$Q = \begin{pmatrix} A_{ex} \\ B^{\mathrm{x}}_{ex} \\ B^{\mathrm{y}}_{ex} \\ B^{\mathrm{z}}_{ex} \\ \bar{A}_{ex} \\ \bar{B}^{\mathrm{x}}_{ex} \\ \bar{B}^{\mathrm{y}}_{ex} \\ \bar{B}^{\mathrm{z}}_{ex} \end{pmatrix}$$

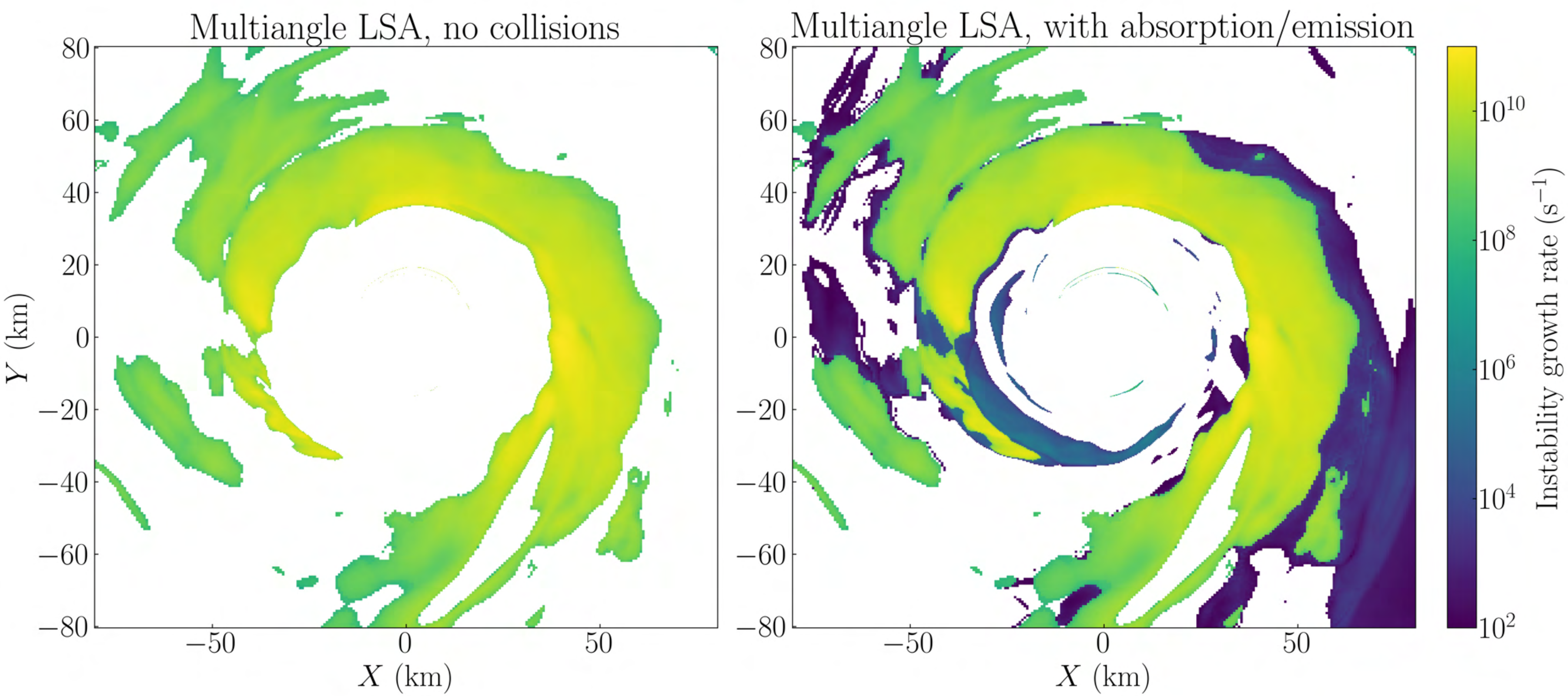
- Non-zero solution only if:

$$\det\left(S_{\vec{k}} + \Omega\, \mathbb{I}\right) = 0 \implies \Omega(\vec{k})$$

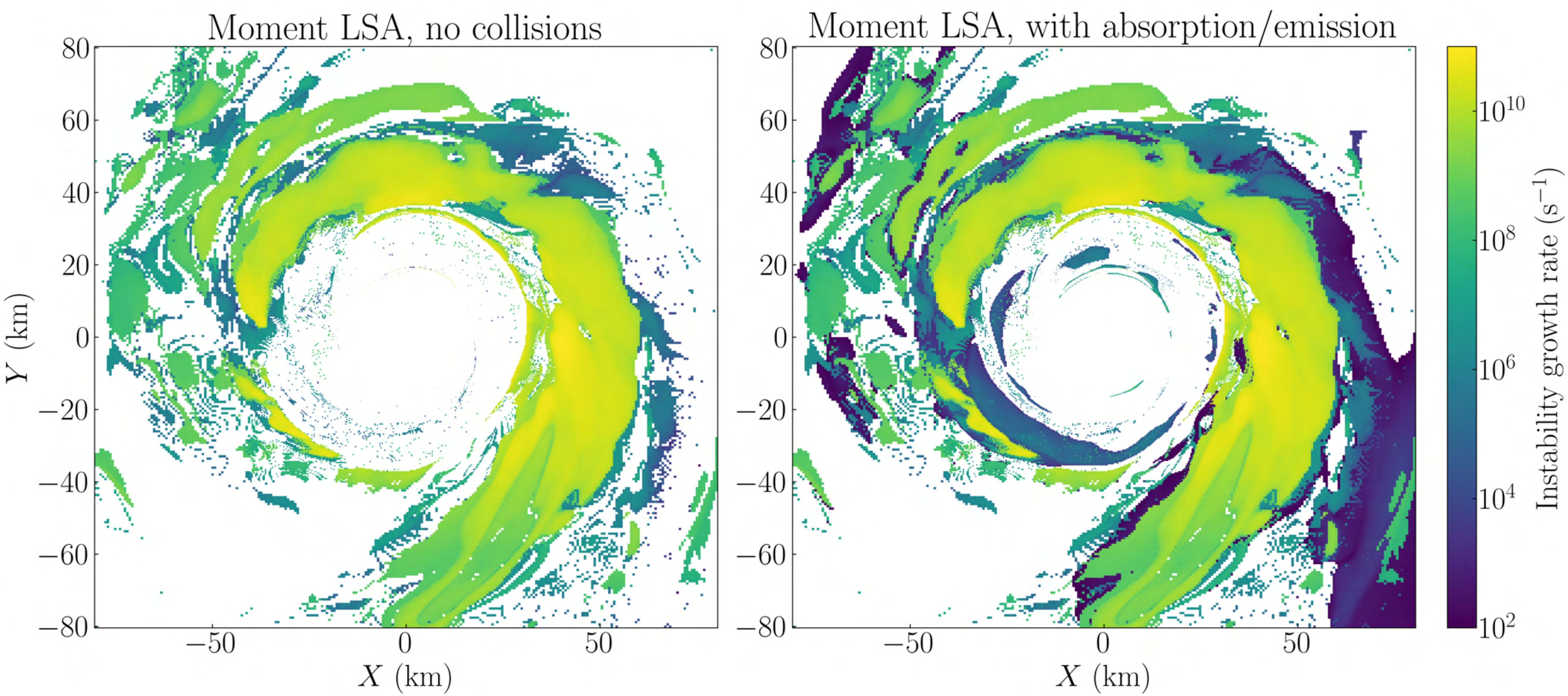
- Fastest growing mode:

$$\max_{\vec{k}} \left\{ \mathrm{Im}[\Omega(\vec{k})] \right\} \equiv \mathrm{Im}(\Omega)_{\max}$$ **Instability growth rate**

Results — Multi-angle

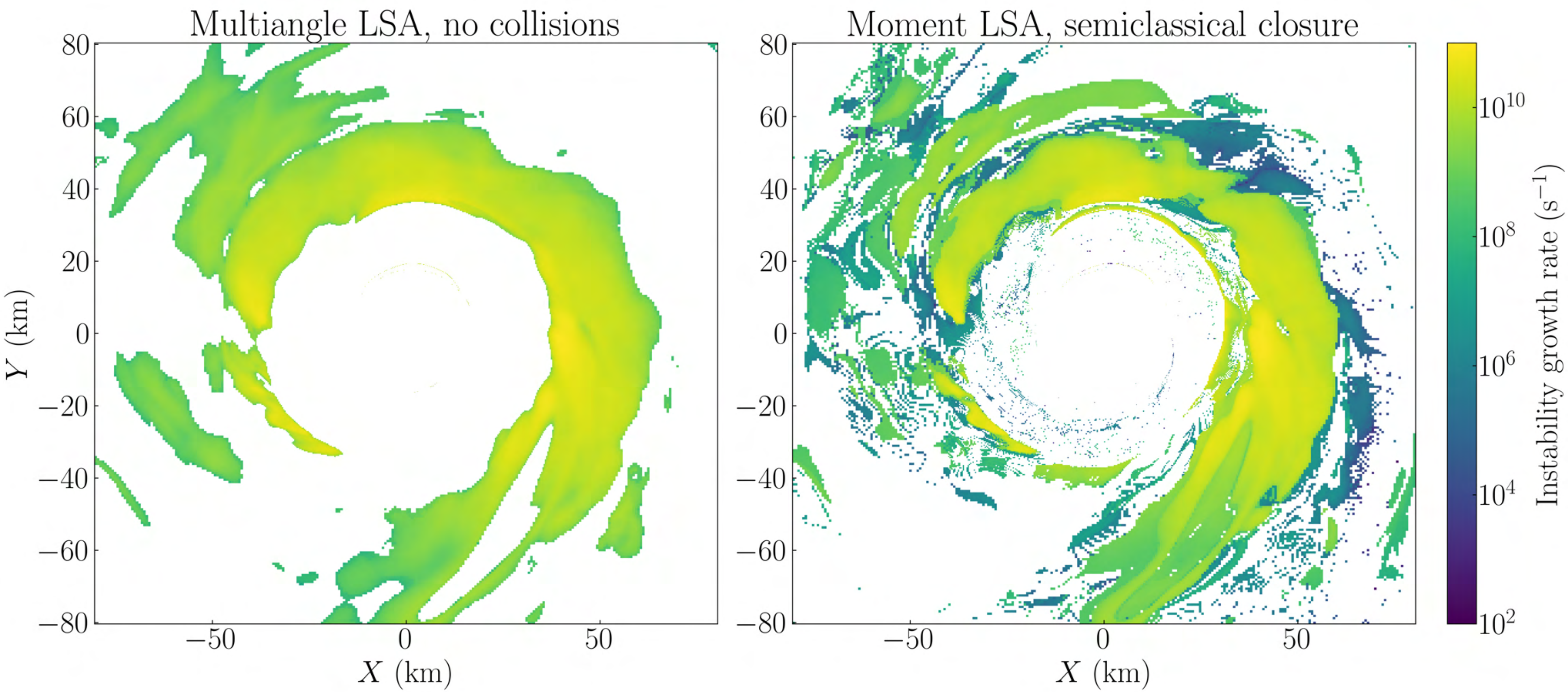


Results — Moments



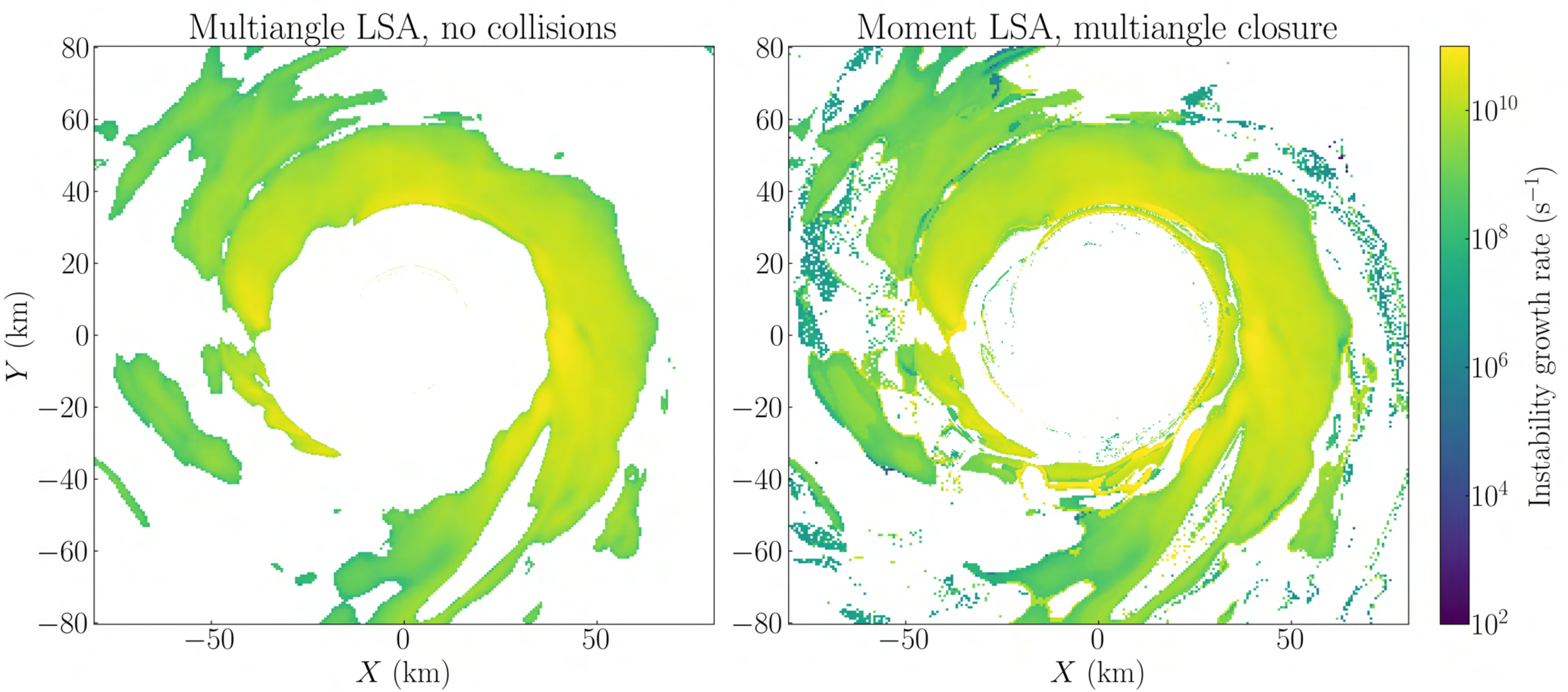
- Good identification of main instability regions, but artificial additional unstable regions (limitations of the closure);
- Additional effect of collisional instabilities well-reproduced

Role of the closure for Moment-LSA



$$P_{ex}^{zz} = \chi\left(\frac{|\vec{F}_{ee} + \vec{F}_{xx}|}{N_{ee} + N_{xx}}\right) N_{ex}$$

Role of the closure for Moment-LSA



$$P^{zz}_{ex} = \kappa^{\text{multi}-\text{LSA}}_{ex}(\vec{k}_{\max}) e^{i\phi^{\text{multi}-\text{LSA}}_{ex}(\vec{k}_{\max})} N_{ex}$$

Conclusion

J. Froustey et al., *PRD* 109, 043046 (2024) [2311.11968]
E. Grohs et al., *ApJ* 963, 11 (2024) [2309.00972]

Moments can describe flavor instabilities.

- Linear stability analysis provides a powerful, “cheap” framework to provide guidance for calculations.
- We confirm the “ubiquity” of FFI in neutron star mergers.
- Regions of collisional instabilities, but impact unclear.
- Shortcomings of the method:
    - Dependence on the choice of closure
    - Parametrization of closure and exploration of possibilities ongoing
- Additional step towards the goal of including flavor transformation in large-scale simulations.