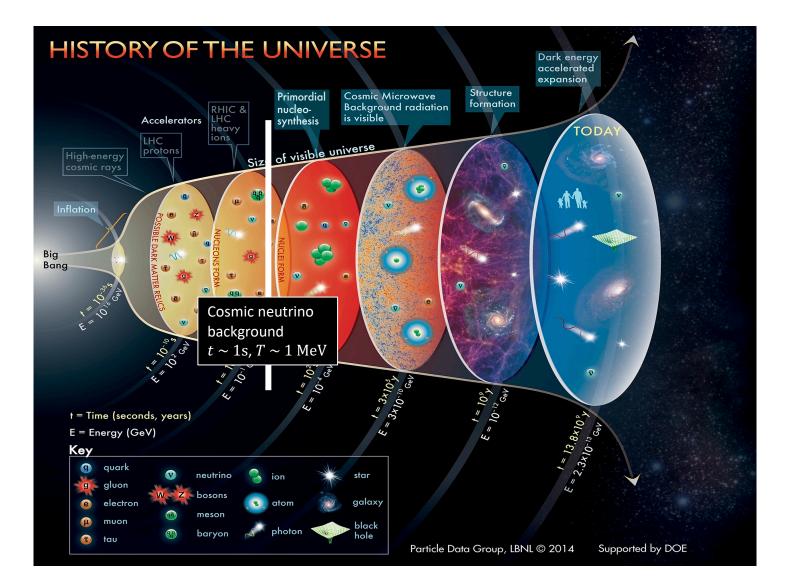
Constraining the neutrino lifetime with precision cosmology

Barenboim, Chen, Hannestad, Oldengott, Tram & Y³W, *JCAP* 03 (2021) 087 [arXiv:2011.01502 [astro-ph.CO]] Chen, Oldengott, Pierobon & Y³W, *EPJC* 82 (2022) 7, 640 [arXiv:2203.09075 [hep-ph]]

Yvonne Y. Y. Wong, UNSW Sydney

Neutrino Frontiers Focus Week, GGI Florence, July 1 - 5, 2024

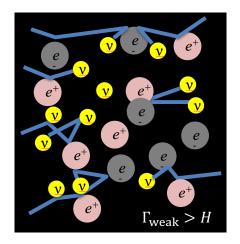


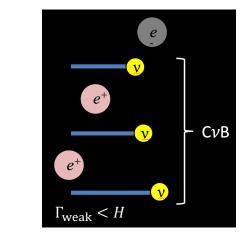
Formation of the $C\nu B...$

Expansion rate: $H \sim M_{\rm pl}^{-2} T^2$

Interaction rate: $\Gamma_{\text{weak}} \sim G_F^2 T^5$

The CvB is formed when neutrinos decouple from the cosmic plasma.

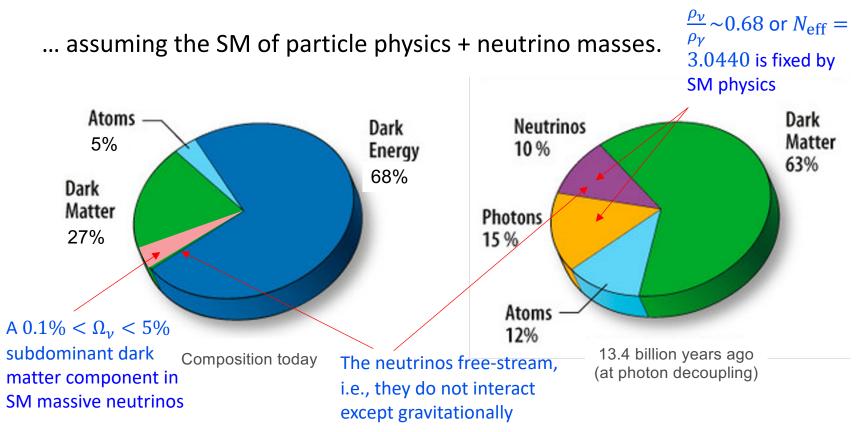




Neutrinos "free-stream" to infinity.

Above $T \sim 1$ MeV, even the Weak Interaction occurs efficiently enough to allow neutrinos to scatter off e^+e^- and other neutrinos, and attain thermodynamic equilibrium. **Below** $T \sim 1$ MeV, expansion dilutes plasma, and reduces interaction rate: the universe becomes transparent to neutrinos.

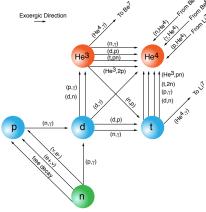
Three key predictions of the $C\nu B...$



Testing CvB predictions against observations...

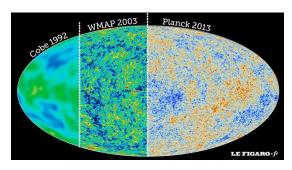
We cannot (yet) detect the $C\nu B$ in the lab. But we can look its imprints on cosmological observables to see if they are consistent with expectations

Light element abundances



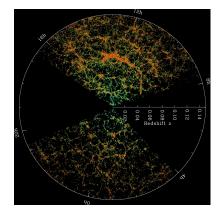
Properties of the $C\nu B$ N_{eff} (expansion rate) probed:

CMB anisotropies



 $N_{\rm eff}$ (expansion rate) $\sum m_{\nu}$ (perturbation growth) Interactions (relativistic free-streaming) Lifetime (relativistic free-streaming)

Large-scale matter distribution



 $\sum m_{
u}$ (perturbation growth)

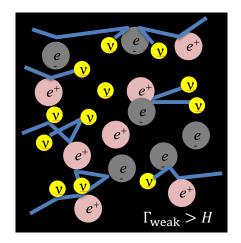
Testing relativistic neutrino freestreaming...

Formation of the $C\nu B...$

Expansion rate: $H \sim M_{\rm pl}^{-2} T^2$

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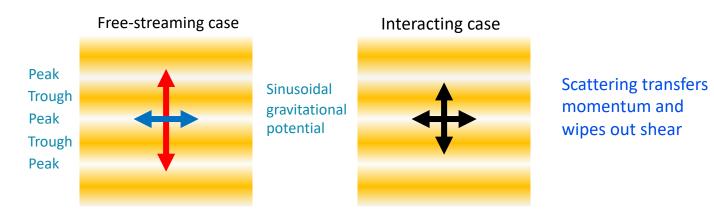
 e^+ v e^+ v e^+ v $r_{weak} < H$ v v $r_{weak} < H$

Above $T \sim 1$ MeV, even the Weak Interaction occurs efficiently enough to allow neutrinos to scatter off e^+e^- and other neutrinos, and attain thermodynamic equilibrium. **Below** $T \sim 1$ MeV, expansion dilutes plasma, and reduces interaction rate: the universe becomes transparent to neutrinos.

Free-streaming in inhomogeneities...

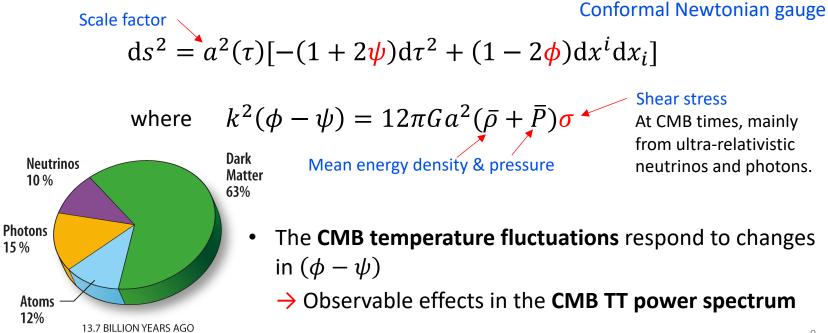
Standard Model neutrinos free-stream after decoupling.

- Relativistic free-streaming in a spatially inhomogeneous background induces shear stress (or momentum anisotropy) in the neutrino fluid.
- Conversely, interactions transfer momentum and, if sufficiently efficient, can wipe to out shear stress.

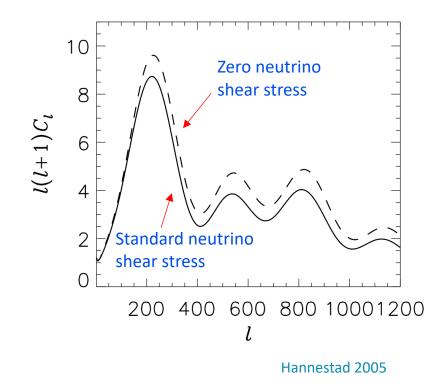


Why is this interesting for the CMB?

Neutrino shear stress (or lack thereof) leaves distinct imprints on the spacetime metric perturbations at CMB formation times.



Neutrino shear & the CMB TT spectrum...

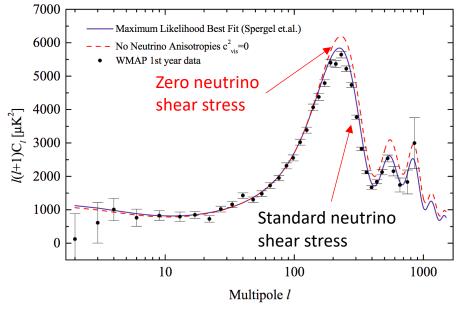


Removing neutrino shear stress enhances power at multipoles $\ell \gtrsim 200$.

- Effect is mildly degenerate with the primordial fluctuation amplitude and spectral tilt.
- But even with WMAP-1st year data, it was already possible to exclude zero neutrino shear stress at $\gtrsim 2\sigma$.

Neutrino shear & the CMB TT spectrum...

Melchiorri & Trotta 2005



Removing neutrino shear stress enhances power at multipoles $\ell \gtrsim 200$.

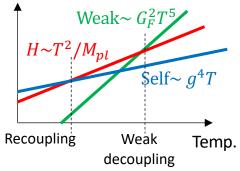
- Effect is mildly degenerate with the primordial fluctuation amplitude and spectral tilt.
- But even with WMAP-1st year data, it was already possible to exclude zero neutrino shear stress at $\gtrsim 2\sigma$.

A more modern take...

Recent analyses usually split the discussion of relativistic free-streaming constraints on neutrino self-interactions into two limiting behaviours:

• Decoupling scenario:

- Realised, by, e.g., 2-to-2 contact interaction
- Delay neutrino decoupling to CMB temperatures ($T \sim 0.2 1 \text{ eV}$) i.e., neutrinos enter the CMB epoch with no anisotropic stress.
- → How late into the CMB epoch can data tolerate no anisotropic stress?
- Recoupling scenario:
 - Realised by, e.g., 2-to-2 scattering with light or massless mediator, relativistic 2-to-1 decay.
 - → How early in the CMB epoch can neutrinos begin to lose anisotropic stress?



Recoupling from relativistic invisible neutrino decay ...

Invisible neutrino decay...

Invisible here means the decay products do **not** include a photon.

• SM 1 \rightarrow 3 decay: $v_j \rightarrow v_i v_k \overline{v}_k$, but the rate is suppressed by m_v^6 .

→ For sub-eV neutrino masses, the neutrino lifetime would be $> 10^{10}$ longer than the present age of the universe, i.e., not very interesting. Bahcall, Cabibbo & Yahil 1972

• Beyond SM: generically one could consider

SM neutrinos
$$\nu_H \rightarrow \nu_l + \phi$$
 Some almost massless boson (scalar, pseudo-scalar, vector)

- More freedom with the coupling strength and hence lifetime.
- Predicted by a many extensions to the SM (mostly linked to neutrino mass generation or dark matter). Gelmini & Roncadelli 1981; Chikashige, Mohapatra & Peccei 1981; Schechter & Valle 1982; Dror 2020; Ekhterachian, Hook, Kumar & Tsai 2021; etc.

Isotropisation timescale...

Given the decay process, the **key to using relativistic free-streaming requirements** to constrain invisible neutrino decay is knowing the rate at which neutrino shear stress is lost due to the interaction.

→ What is the **isotropisation timescale** given a specific interaction?

Tracking neutrino perturbations...

The standard approach is to use the **relativistic Boltzmann equation** to describe the neutrino phase space distribution $f_i(x^{\mu}, P^i)$.

Liouville operator
$$P^{\mu} \frac{\partial f_i}{\partial x^{\mu}} - \Gamma^{\nu}_{\rho\sigma} P^{\rho} P^{\sigma} \frac{\partial f_i}{\partial P^{\nu}} = 0$$

Gravitational effects

Integrate in momentum:

 $\ell = 0 \rightarrow$ density and pressure

- perturbations
- $\ell = 1 \rightarrow$ velocity perturbations $\ell \ge 2 \rightarrow$ anisotropies

• Linearise and go to Fourier space $x^i \leftrightarrow k^i$

• Split into $f_i(x^{\mu}, P^i) = \bar{f}_i(x^0, |P^i|) + F_i(x^{\mu}, P^i)$

• **Decompose** $F_i(x^o, k^i, P^i)$ into a Legendre series in $k \cdot P$.

Ma & Bertschinger 1995

Adding a short-range particle interaction...

To describe a **short-range interaction**, add a **collision integral** to the RHS of the relativistic Boltzmann equation for $f_i(x^{\mu}, P^i)$.

Liouville operator
$$P^{\mu} \frac{\partial f_i}{\partial x^{\mu}} - \Gamma^{\nu}_{\rho\sigma} P^{\rho} P^{\sigma} \frac{\partial f_i}{\partial P^{\nu}} = C[f]$$
 Collision integral

Gravitational effects

Integrate in momentum:

 $\ell \geq 2 \rightarrow$ anisotropies

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 $\ell = 1 \rightarrow$ velocity perturbations

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- **Decompose** $F_i(x^o, k^i, P^i)$ into a Legendre series in $k \cdot P$.

Ma & Bertschinger 1995

Collision integral and the isotropisation rate...

Given an interaction Lagrangian, the collision integral for $f_i(x^{\mu}, P^i)$ is

$$C[f] = \frac{1}{2} \left(\prod_{j=1}^{N} \int g_{j} \frac{\mathrm{d}^{3} \mathbf{n}_{j}}{(2\pi)^{3} 2E_{j}(\mathbf{n}_{j})} \right) \left(\prod_{k=1}^{M} \int g_{k} \frac{\mathrm{d}^{3} \mathbf{n}_{k}}{(2\pi)^{3} 2E_{k}(\mathbf{n}_{k})} \right)$$

$$\times (2\pi)^{4} \delta_{D}^{(4)} \left(p + \sum_{j=1}^{N} n_{j} - \sum_{k=1}^{M} n_{k}' \right) |\mathcal{M}_{i+j_{1}+\dots+j_{N}\leftrightarrow k_{1}+\dots+k_{M}}|^{2}$$

$$\times [f_{k_{1}}\cdots f_{k_{N}}(1\pm f_{i})(1\pm f_{j_{1}})\cdots (1\pm f_{j_{N}}) - f_{i}f_{j_{1}}\cdots f_{j_{N}}(1\pm f_{k_{1}})\cdots (1\pm f_{k_{M}})$$

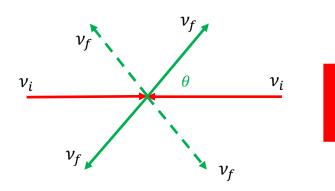
• To compute the isotropisation rate, follow the previous procedure of linearisation and decomposition into a Legendre series.

 \rightarrow The damping rate of the quadrupole ($\ell = 2$) moment represents the lowest-order isotropisation rate of the neutrino ensemble.

Tedious stuff, but this is really the only correct way to calculate these things, else you can get it very wrong... However, the result can usually be understood in simple terms. \rightarrow **Next slide**

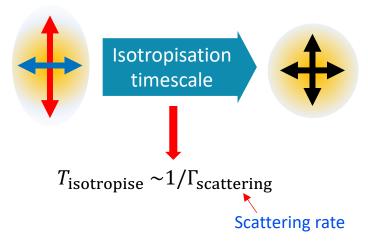
Warm-up: Isotropisation from self-interaction...

Consider a 2 \rightarrow 2 scattering event $v_i + v_i \rightarrow v_f + v_f$.



• The probability of v_f emitted at any angle θ is the same for all $\theta \in [0, \pi]$.

→ Particles in two head-on ν_i beams need only scatter once to transfer their momenta equally in all directions.

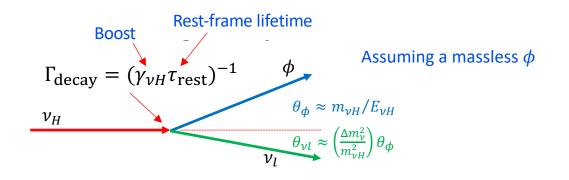


That was easy.... Now let's try relativistic $1 \rightarrow 2$ decay+inverse...

Isotropisation from relativistic $1 \rightarrow 2$ decay...

How long does it take $\nu_H \rightarrow \nu_l + \phi$ and its inverse process to wipe out momentum anisotropies? (Hint: it's **not** the lifetime of ν_H .)

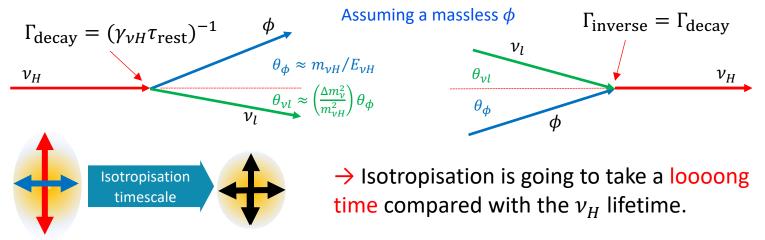
• In relativistic decay, the decay products are **beamed**.



Isotropisation from relativistic $1 \rightarrow 2$ decay...

How long does it take $\nu_H \rightarrow \nu_l + \phi$ and its inverse process to wipe out momentum anisotropies? (Hint: it's **not** the lifetime of ν_H .)

- In relativistic decay, the decay products are **beamed**.
- Inverse decay also only happens when the daughter particles meet **strict momentum/angular requirements**.



How long?

Part 1

Two works in the 2000s that considered how long it would take relativistic $1 \rightarrow 2$ decay and inverse decay to isotropise a neutrino ensemble.



Neither work actually ٠ calculated it... But this is the isotropisation timescale they (sort of^{*}) used:

$$T \sim (\theta_{\nu l} \theta_{\phi})^{-1} \gamma_{\nu H} \tau_{\text{rest}}$$

Their argument is as follows.

Phys. Rev. D 72, 103514 - Published 14 November 2005

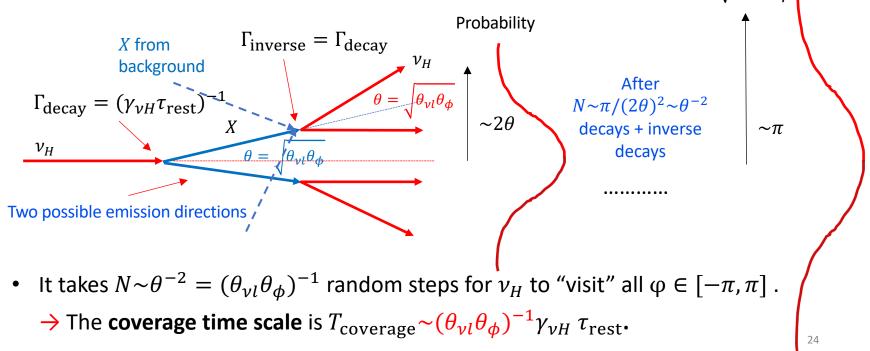
* Sort of, because both works assumed two massless daughters.

How long?

Part 1

Let's look at what happens to v_H after one decay and inverse decay.

• For simplicity, let's say $\nu_H \to XX$, and we track one X emitted at $\theta = \sqrt{\theta_{\nu l} \theta_{\phi}}$.



How long?



Z. Chacko, Lawrence J. Hall, Takemichi Okui, and Steven J. Olive Phys. Rev. D 70, 085008 – Published 12 October 2004

PHYSICAL REVIEW D covering particles, fields, gravitation, and cosmology									
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Constraining invisible neutrino decays with the cosmic microwave background

Steen Hannestad and Georg G. Raffelt Phys. Rev. D **72**, 103514 – Published 14 November 2005

Part 1

• Taking $T_{coverage}$ to be the isotropisation timescale and assuming massless decay products, the free-streaming bound on the v_H rest-frame lifetime was found to be:

$$au_{\mathrm{rest}} \gtrsim 10^9 \left(\frac{m_{\nu H}}{0.05 \,\mathrm{eV}}\right)^3 \mathrm{s}$$

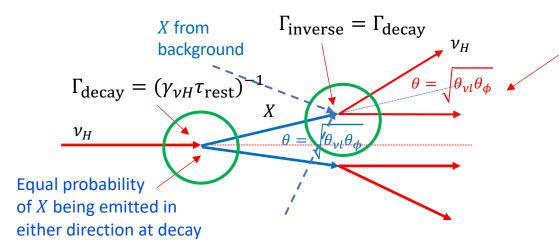
Hannestad & Raffelt 2005

Many updates to the number since (e.g., WMAP to Planck), but no one really questioned the modelling behind this bound in the next 15 years...

Barenboim, Chen, Hannestad, Oldengott, Tram & Y³W 2021 Chen, Oldengott, Pierobon & Y³W 2022

Actually, $T_{coverage}$ is only the first half of the story!

• It is NOT the isotropisation time scale and here's the reason.

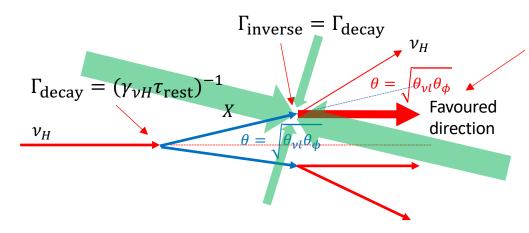


Emission direction of v_H at inverse decay depends on the momentum anisotropy of the background X that recombines with the emitted X.

Barenboim, Chen, Hannestad, Oldengott, Tram & Y³W 2021 Chen, Oldengott, Pierobon & Y³W 2022

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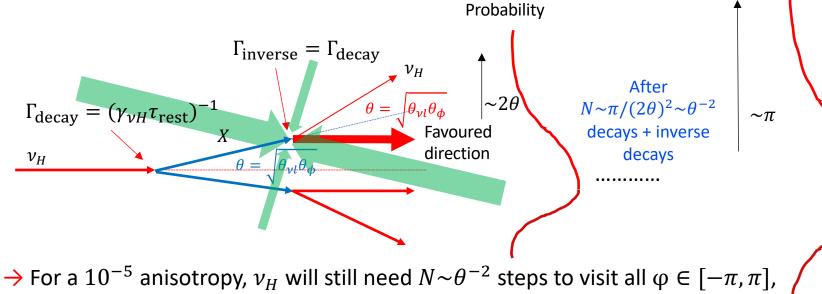


Emission direction of v_H at inverse decay depends on the momentum anisotropy of the background X that recombines with the emitted X. \rightarrow Random walk of v_H in θ space is biased towards the anisotropy of X.

Barenboim, Chen, Hannestad, Oldengott, Tram & Y³W 2021 Chen, Oldengott, Pierobon & Y³W 2022

Actually, $T_{coverage}$ is only the first half of the story!

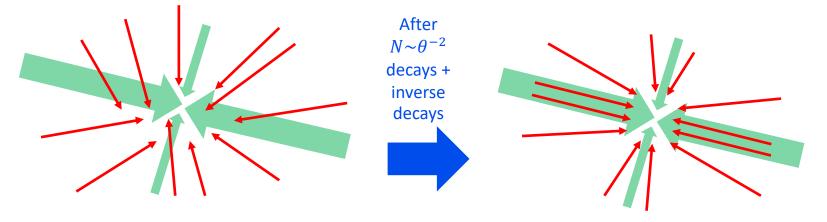
It is NOT the isotropisation time scale and here's the reason.



but there will be a higher concentration of steps in the anisotropy's direction.

That was for just one particle v_H .

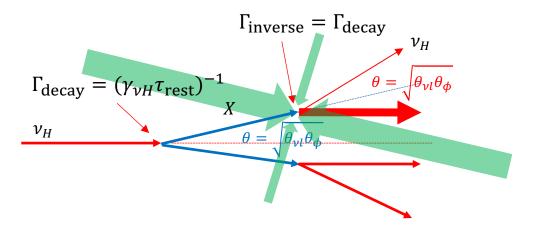
• Suppose now we have a whole ensemble of v_H s random-walking in the same anisotropic background.



 Thus, after T_{coverage}, the v_H ensemble will not become isotropic, but will end up almost as anisotropic as the background...

Almost as anisotropic (or how long part 2)...

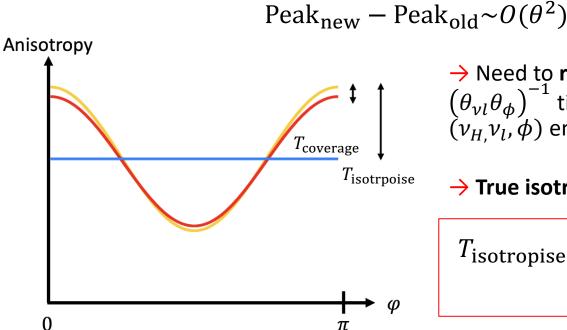
After one coverage time, the anisotropy of v_H will be smeared over $\sim \theta = \sqrt{\theta_{\nu l} \theta_{\phi}}$ relative to the anisotropy of *X*, because v_H is **always emitted at an angle** $\pm \theta$ relative to *X* in an inverse decay.



 \rightarrow Even though total isotropisation of v_H is not possible after one coverage time, a small amount of anisotropy is inevitably lost as a result.

Almost as anisotropic (or how long part 2)...

Smearing over $\sim \theta$ reduces the peak anisotropy after one coverage time by an amount:



→ Need to **repeat** coverage $M \sim \theta^{-2} = (\theta_{\nu l} \theta_{\phi})^{-1}$ times to completely rid the (ν_{H}, ν_{l}, ϕ) ensemble of anisotropy.

→ True isotropisation time scale:

$$T_{\text{isotropise}} \sim \left(\theta_{\phi} \theta_{\nu l}\right)^{-1} T_{\text{coverage}} \\ \sim \left(\theta_{\phi} \theta_{\nu l}\right)^{-2} \gamma_{\nu H} \tau_{\text{rest}}$$

OK, that was hand-waving. But...

The isotropisation rate is calculable...

Given an interaction Lagrangian, the collision integral for $f_i(x^{\mu}, P^i)$ is

$$C[f] = \frac{1}{2} \left(\prod_{j=1}^{N} \int g_{j} \frac{\mathrm{d}^{3} \mathbf{n}_{j}}{(2\pi)^{3} 2E_{j}(\mathbf{n}_{j})} \right) \left(\prod_{k=1}^{M} \int g_{k} \frac{\mathrm{d}^{3} \mathbf{n}_{k}}{(2\pi)^{3} 2E_{k}(\mathbf{n}_{k})} \right)$$
$$\times (2\pi)^{4} \delta_{D}^{(4)} \left(p + \sum_{j=1}^{N} n_{j} - \sum_{k=1}^{M} n_{k}' \right) |\mathcal{M}_{i+j_{1}+\dots+j_{N}\leftrightarrow k_{1}+\dots+k_{M}}|^{2}$$
$$\times [f_{k_{1}}\cdots f_{k_{N}}(1\pm f_{i})(1\pm f_{j_{1}})\cdots(1\pm f_{j_{N}}) - f_{i}f_{j_{1}}\cdots f_{j_{N}}(1\pm f_{k_{1}})\cdots(1\pm f_{k_{M}})$$

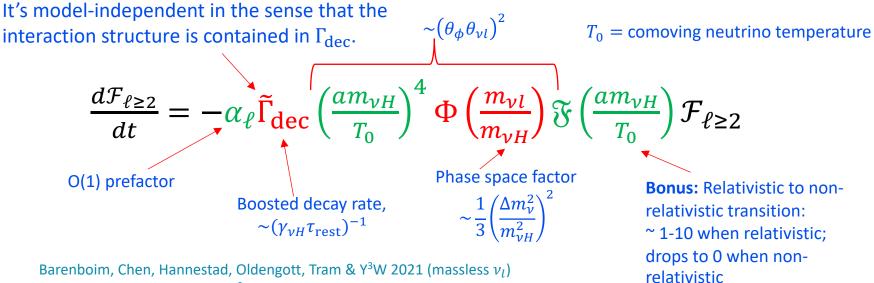
• To compute the isotropisation rate, follow the previous procedure of linearisation and decomposition into a Legendre series.

 \rightarrow The damping rate of the quadrupole ($\ell = 2$) moment represents the lowest-order isotropisation rate of the neutrino ensemble.

In fact, we calculated the rate loooong before we understood what was going on physically...

Massless ϕ ν_H The isotropisation rate is calculable... ν_l

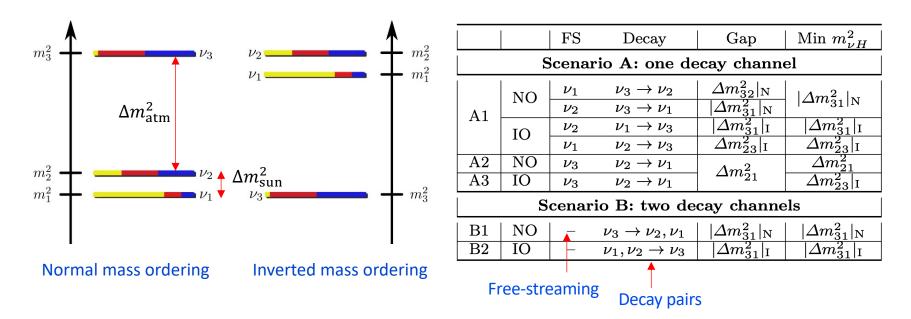
With some reasonable approximations (e.g., separation of scales), we have calculated the damping rate of the ℓ th neutrino kinetic moment from relativistic $v_H \rightarrow v_l + \phi$ and its inverse process:



Barenboim, Chen, Hannestad, Oldengott, Tram & Y³W 2021 (massless v_1) Chen, Oldengott, Pierobon & Y³W 2022 (massive v_l + full Boltzmann hierarchy) Revised constraints on the neutrino lifetime...

Decay scenarios...

Global neutrino oscillation data currently point to two possible orderings of neutrino masses \rightarrow several possible decay/free-streaming patterns.



Decay scenarios...

These scenarios look very different from one another...

• Phenomenologically, however, there are only two independent parameters.

 $d\mathcal{F}_{\ell\geq 2}$ $\alpha_{\ell} a^{6} Y \mathfrak{F}(aX) \mathcal{F}_{\ell>2}$ $d\tau$

"Mass" of decaying neutrino

$$X = 298 \left(\frac{m_{\nu H}}{0.05 eV}\right)$$

Effective isotropisation rate

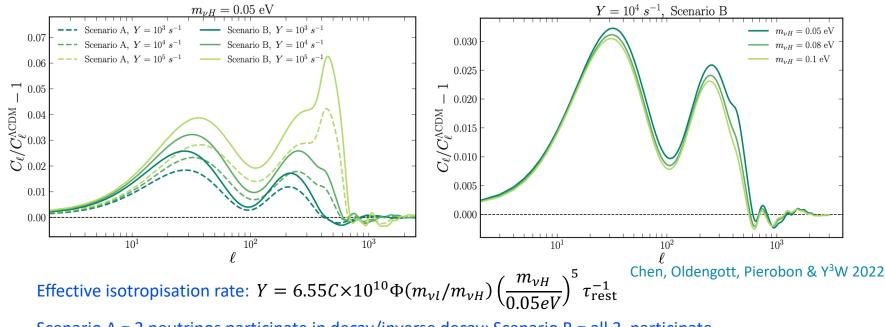
$$Y = 6.55C \times 10^{10} \Phi \left(\frac{m_{\nu_l}}{m_{\nu H}}\right) \left(\frac{m_{\nu H}}{0.05eV}\right)^5 \tau_{\text{rest}}^{-1}$$

Mass gap

Relativistic to NR transition									
			FS	Decay	Gap	$ \operatorname{Min} m_{\nu H}^2 $			
⁶ Y $\mathfrak{F}(aX)\mathcal{F}_{\ell\geq 2}$	Scenario A: one decay channel								
$\int \mathcal{Y}(\alpha \Lambda) \mathcal{J}_{\ell \geq 2}$	A1	NO	ν_1	$ u_3 ightarrow u_2$	$egin{array}{ c c c c c c c } arDelta m_{32}^2 _{ m N} & arphi arDelta m_{31}^2 _{ m N} \end{array}$	$ arDelta m_{31}^2 _{ m N}$			
			ν_2	$\nu_3 \rightarrow \nu_1$					
ine		ΙΟ	ν_2	$ u_1 \rightarrow \nu_3 $	$ \Delta m_{31}^2 _{\rm I}$	$ \Delta m^2_{31} _{\mathrm{I}}$			
ino			ν_1	$ u_2 ightarrow u_3$	$ \Delta m^2_{23} _{ m I}$	$\Delta m^2_{23} _{ m I}$			
	A2	NO	ν_3	$ u_2 ightarrow u_1$	Δm^2_{21}	Δm^2_{21}			
	A3	IO	ν_3	$ u_2 ightarrow u_1$	Δm_{21}	$\Delta m^2_{23} _{ m I}$			
	Scenario B: two decay channels								
ate	B1	NO	_	$ u_3 ightarrow u_2, u_1$	$ \Delta m^2_{31} _{ m N}$	$ \Delta m^2_{31} _{ m N}$			
$(m_{u}) (m_{u})^{5}$	B2	IO	—	$ u_1, \nu_2 ightarrow u_3$	$ \Delta m^2_{31} _{\mathrm{I}}$	$\frac{ \Delta m_{31}^2 _{\mathrm{I}}}{ \Delta m_{31}^2 _{\mathrm{I}}}$			
$\left(\frac{m_{\nu_l}}{m_{\nu H}}\right) \left(\frac{m_{\nu H}}{0.05 eV}\right)^5 \tau_{\rm rest}^{-1}$	Free-streaming Decay pairs								
Lifetime									

Signatures in the CMB TT power spectrum...

Fractional deviations in the CMB TT power spectrum from Λ CDM for various the effective isotropisation rate Y and v_H masses.

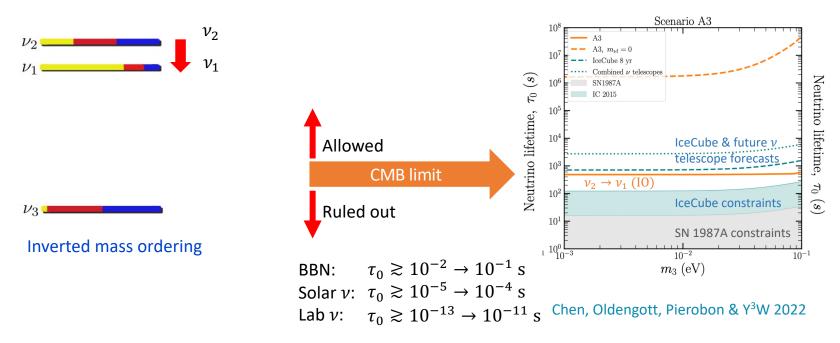


CMB lower bounds on the neutrino lifetime...

We derive constraints on Y at a set of fixed X using Planck 2018 TTTEEE+ low+lensing, and translate the constraints to a revised lower bound on the neutrino lifetime:

CMB lower bounds on the neutrino lifetime...

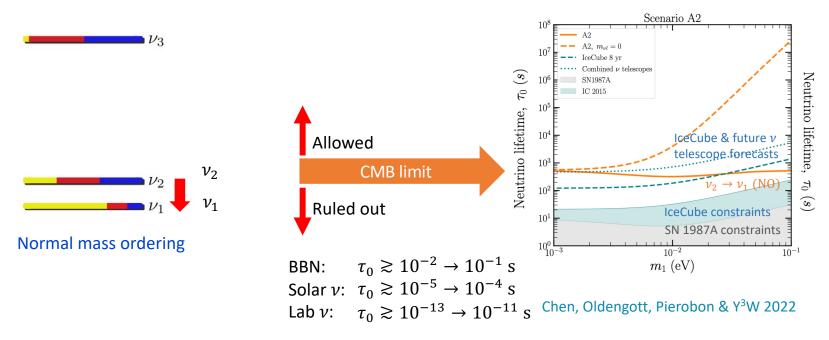
... currently the best limits on invisible neutrino decay $v_H \rightarrow v_l + \phi$.



* IceCube constraints & forecasts from Song et al. 2021

CMB lower bounds on the neutrino lifetime...

... currently the best limits on invisible neutrino decay $v_H \rightarrow v_l + \phi$.



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Summary...

- We can use **precision cosmological observables** to constrain nonstandard neutrino properties like **relativistic invisible neutrino decay**.
- But **mapping the decay rate** to the **isotropisation rate** that ultimately changes the CMB observable can be a tricky task.
- We have calculated the isotropisation rate from first-principles and relaxed the CMB constraint on the neutrino lifetime by several orders of magnitude relative to old works using an incorrect rate.
 - Barenboim et al. 2021: massless daughters; 3 orders of magnitude bound relaxation at $m_{\nu} = 0.05$ eV.
 - Chen et al. 2022: massive daughters + full Boltzmann hierarchy + "handwaving" explanation; up to another 5 orders of magnitude relaxation.