CP violation in light neutrino oscillations and heavy neutrino decays
—— a general and explicit connection ——

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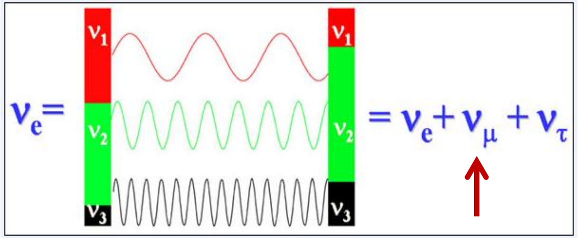
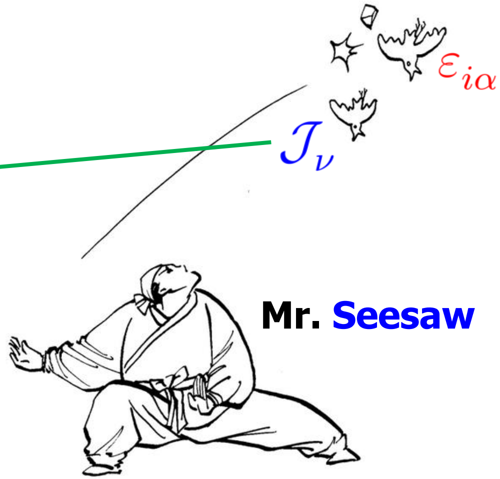
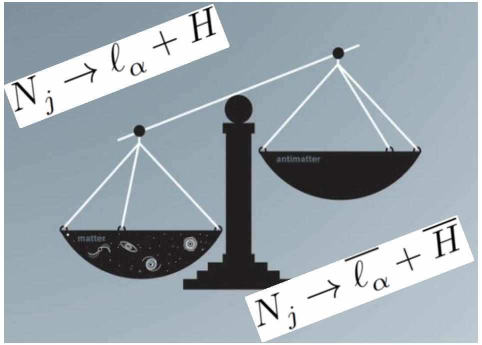
CP violation





cosmic CP violation



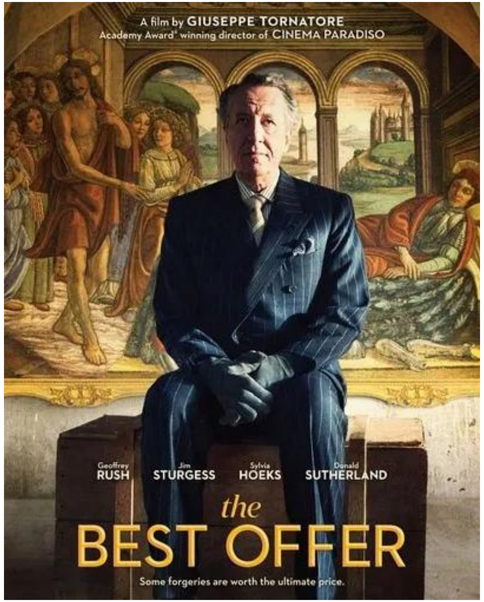
ZZX, 2406.01142

Neutrino Frontiers at the Galileo Galilei Institute —— Focus Week 1—5 July 2024, Arcetri, Firenze

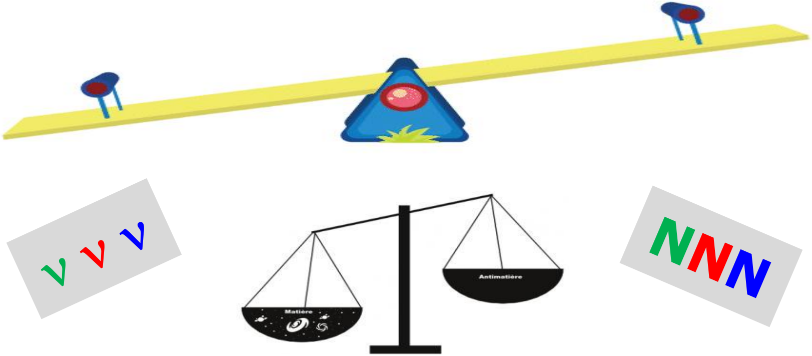
Killing two birds with one stone?

- There are many **mechanisms**/**models** on the origin of ***tiny neutrino masses*** **on the market** (see, e.g., ZZX, Phys. Rept. 854 (2020) 1—147)

- Which do you like to buy? Waiting for the best offer?



The best offer is the canonical seesaw



- But in 1996 Wilfried Buchmüller and Michael Plümacher made a very strong statement: "There is NO direct connection between the CP violation and generation mixing at high and low energies" (hep-ph/9608308). *Is this conclusion really valid?*

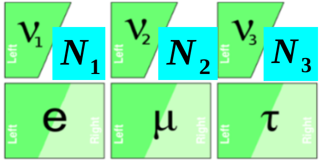
OUTLINE

- Why is seesaw most convincing
- The flavor structure of seesaw
- The J-invariant in ν-oscillations
- CP violation in heavy *N*–decays

A way out of the SM —— seesaw?

- **Fundamentals of the electroweak SM structure → reasons for zero ν-mass:**
  - **The Lorentz invariance**
  - **Local $SU(2)_L \times U(1)_Y$ gauge symmetries**
  - **The Higgs mechanism**
  - **Renormalizability (no d ≥ 5 operators)**

Plus *economical* particle content:
- **No right-handed neutrino fields**
- **Only one Higgs doublet**



$$m_1 = m_2 = m_3 = 0$$

Go beyond the d=4 operators

SMEFT

The unique d=5 operator for ν-mass and Majorana nature

$$\mathcal{O}_{\rm W} = \frac{\overline{\ell_{\rm L}}\widetilde{H}\widetilde{H}^T\ell_{\rm L}^c}{\Lambda}$$

Steven Weinberg 1979

R-handed neutrinos + Majorana character

- **The most convincing *UV* mechanism = canonical seesaw**

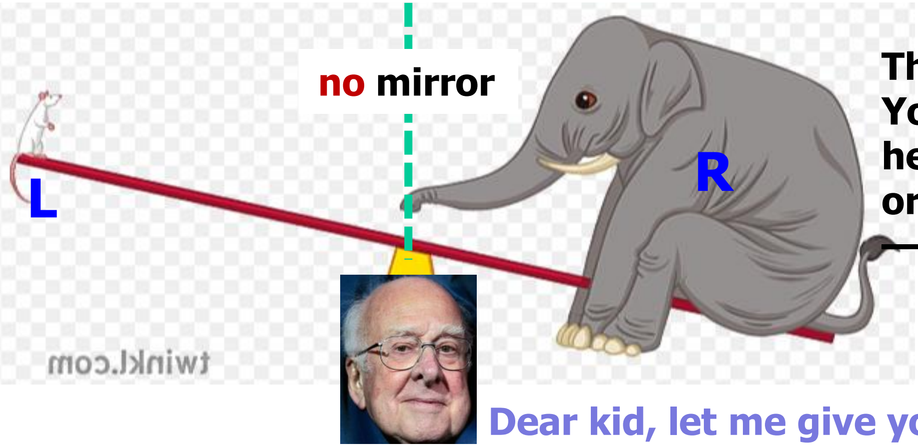
fully consistent with the SMEFT spirit, most natural/economical extension of the SM. *Bonus: Leptogenesis, SO(10) GUT-friendly…*

Integrate out the heavy dof

Three key issues of the seesaw

- **Right-handed neutrino fields are not the mirror counterparts of the left-handed ones**

It is said that I was weightless at birth, and it was you who fed me up a bit.



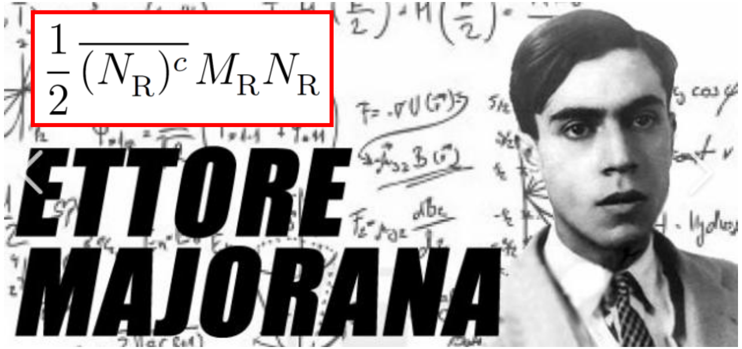
That is true, my love! You may call me heavy neutrinos or heavy neutral leptons —— both sound strange!

Dear kid, let me give you mass on behalf of God

- **Yukawa interactions —— the Higgs fields play a crucial role, as they do in generating masses for the charged fermions in the SM.**

- **The Majorana nature of massive neutrinos:**
  N **and** N^{c} **may have *self-interactions*, respecting all the fundamental symmetries of the SM.**

Gell-Mann's totalitarian principle (1956)
Everything not forbidden is compulsory!



How seesaw works?

The seesaw mechanism formally works far above the Fermi scale, before SSB (ZZX, 2301.10461):

$$-\mathcal{L}_{\rm lepton} = \overline{\ell_{\rm L}} Y_l H l_{\rm R} + \overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

$$= \overline{l_{\rm L}} Y_l l_{\rm R} \phi^0 + \frac{1}{2}\overline{\begin{bmatrix}\nu_{\rm L} & (N_{\rm R})^c\end{bmatrix}} \underline{\begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R}\end{pmatrix}} \begin{bmatrix}(\nu_{\rm L})^c \\ N_{\rm R}\end{bmatrix} + \overline{\nu_{\rm L}} Y_l l_{\rm R} \phi^+ - \overline{l_{\rm L}} Y_\nu N_{\rm R} \phi^- + {\rm h.c.}$$

The basis transformation related to the origin of active Majorana neutrino masses even before SSB:

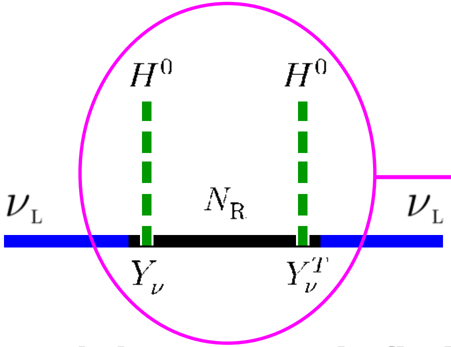
$$\mathbb{U}^\dagger \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R}\end{pmatrix} \mathbb{U}^* = \begin{pmatrix} D_\nu & \mathbf{0} \\ \mathbf{0} & D_N \end{pmatrix}$$

working masses:
$$\begin{cases} D_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\} & \textbf{light} \\ D_N \equiv {\rm Diag}\{M_4, M_5, M_6\} & \textbf{heavy} \end{cases}$$

SSB $\Downarrow$

$$\begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix}$$

6×6 mass matrix



Integrating out the heavy degrees of freedom:

$$\boxed{-\mathcal{L}_{\rm mass} = \frac{1}{2}\overline{\nu_{\rm L}} M_\nu \nu_{\rm L}^c + {\rm h.c.}} \qquad M_\nu \simeq -Y_\nu \frac{\langle H\rangle^2}{M_{\rm R}} Y_\nu^T$$

Consistent with the dim-5 Weinberg operator!

If you can untie Weinberg's knot, you will find new heavy Majorana neutrinos at a superhigh scale.

OUTLINE

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- The J-invariant in ν-oscillations
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A way out of the unknown flavor structure

A ***block parametrization*** of **active-sterile flavor mixing in the seesaw framework:**

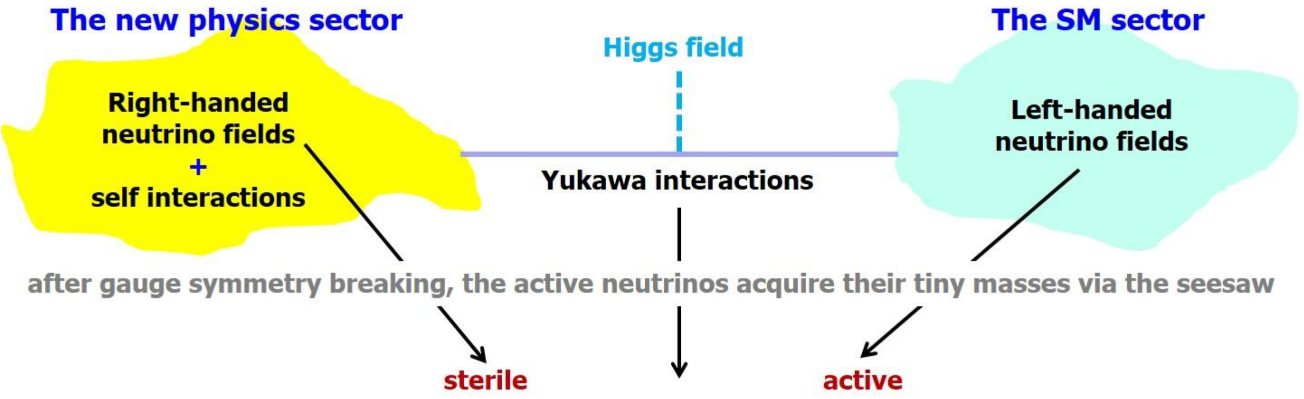
- ♦ **reflects salient features of the seesaw dynamics**
- ♦ **offers *generic* + *explicit* expressions of observables using the Euler-like angles and phases (ZZX, 1110.0083)**

The weak charged-current interactions of leptons:

$U = AU_0$: **the PMNS matrix;**
R **: an analogue for heavy.**

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{\begin{pmatrix} e & \mu & \tau \end{pmatrix}_{\rm L}} \gamma^\mu \left[U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_4 \\ N_5 \\ N_6 \end{pmatrix}_{\rm L} \right] W^-_\mu + {\rm h.c.}$$

oscillations ⟵ light **heavy ⟶ leptogenesis**



Block parametrization

$$\mathbb{U} = \begin{pmatrix} I & 0 \\ 0 & U_0' \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} U_0 & 0 \\ 0 & I \end{pmatrix}$$

interplay

3 angles + 3 phases $\Rightarrow O_{56}O_{46}O_{45}$

$O_{23}O_{13}O_{12}$ **3 angles + 3 phases**

$$O_{36}O_{26}O_{16}O_{35}O_{25}O_{15}O_{34}O_{24}O_{14}$$

9 mixing angles + 9 CP-violating phases

seesaw + unitarity:
$$\begin{cases} UD_\nu U^T + RD_N R^T = \mathbf{0} \\ UU^\dagger + RR^\dagger = I \end{cases}$$

Original vs derivational seesaw parameters

$$U_0 = \begin{pmatrix} c_{12}c_{13} & \hat{s}^*_{12}c_{13} & \hat{s}^*_{13} \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{12}c_{23} - \hat{s}^*_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{13}\hat{s}^*_{23} \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}^*_{12}\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix}$$

derivational parameters!

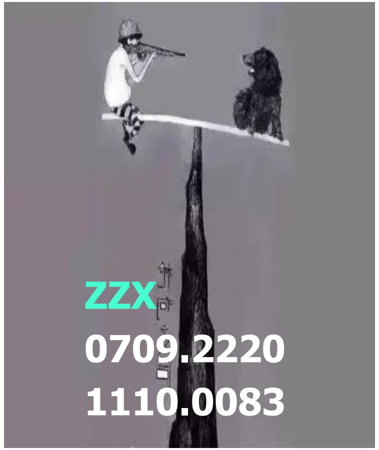
$$c_{ij} \equiv \cos\theta_{ij}$$
$$\hat{s}_{ij} \equiv e^{i\delta_{ij}} \sin\theta_{ij} \ \ (\text{for } 1 \leq i < j \leq 6)$$

$$A = \begin{pmatrix} c_{14}c_{15}c_{16} & 0 & 0 \\ \begin{array}{l} -c_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - c_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} \\ -\hat{s}_{14}\hat{s}^*_{24}c_{25}c_{26} \end{array} & c_{24}c_{25}c_{26} & 0 \\ \begin{array}{l} -c_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + c_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -c_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} + \hat{s}_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} \\ +\hat{s}_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} - \hat{s}_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & \begin{array}{l} -c_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} - c_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} \\ -\hat{s}_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & c_{34}c_{35}c_{36} \end{pmatrix}$$

$$R = \begin{pmatrix} \hat{s}^*_{14}c_{15}c_{16} & \hat{s}^*_{15}c_{16} & \hat{s}^*_{16} \\ \begin{array}{l} -\hat{s}^*_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} \\ +c_{14}\hat{s}^*_{24}c_{25}c_{26} \end{array} & -\hat{s}^*_{15}\hat{s}_{16}\hat{s}^*_{26} + c_{15}\hat{s}^*_{25}c_{26} & c_{16}\hat{s}^*_{26} \\ \begin{array}{l} -\hat{s}^*_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -\hat{s}^*_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} - c_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -c_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} + c_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & \begin{array}{l} -\hat{s}^*_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} - c_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ +c_{15}c_{25}\hat{s}^*_{35}c_{36} \end{array} & c_{16}c_{26}\hat{s}^*_{36} \end{pmatrix}$$

The original seesaw parameters in A+R:

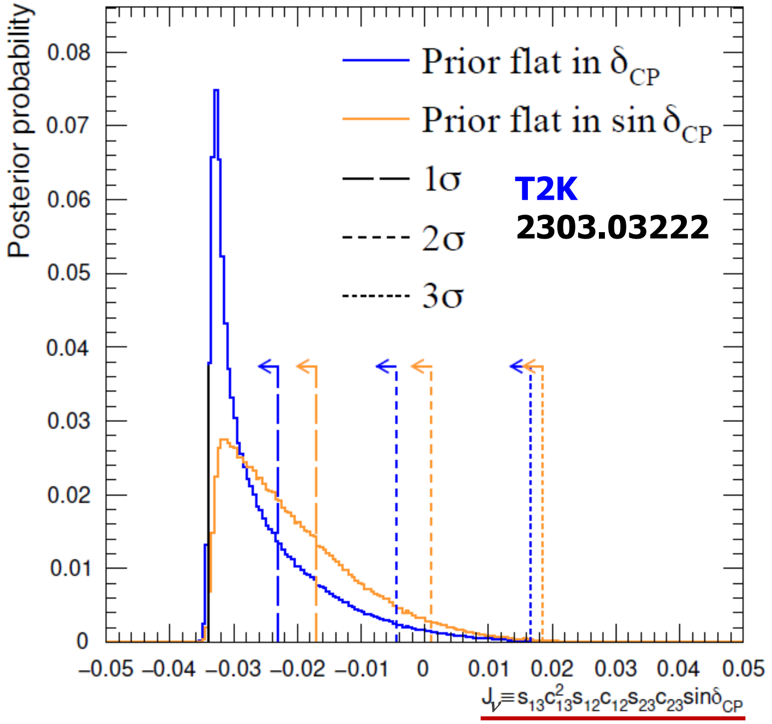
9 angles + 6 phases



You may calculate everything that can in principle be measured, in terms of 18 seesaw parameters.

Two kinds of CP violation

♦ **3-flavor ν-oscillations are established and a 2σ hint for CPV is achieved.**



$$P(\nu_\mu \to \nu_e) = -4\sum_{i<j}\left(\mathcal{R}_{ij}\sin^2\frac{\Delta_{ji}L}{4E}\right) - 8\mathcal{J}_\nu \prod_{i<j}\sin\frac{\Delta_{ji}L}{4E}$$

connection

?

seesaw

+

leptogenesis

THE FIRST

general

+

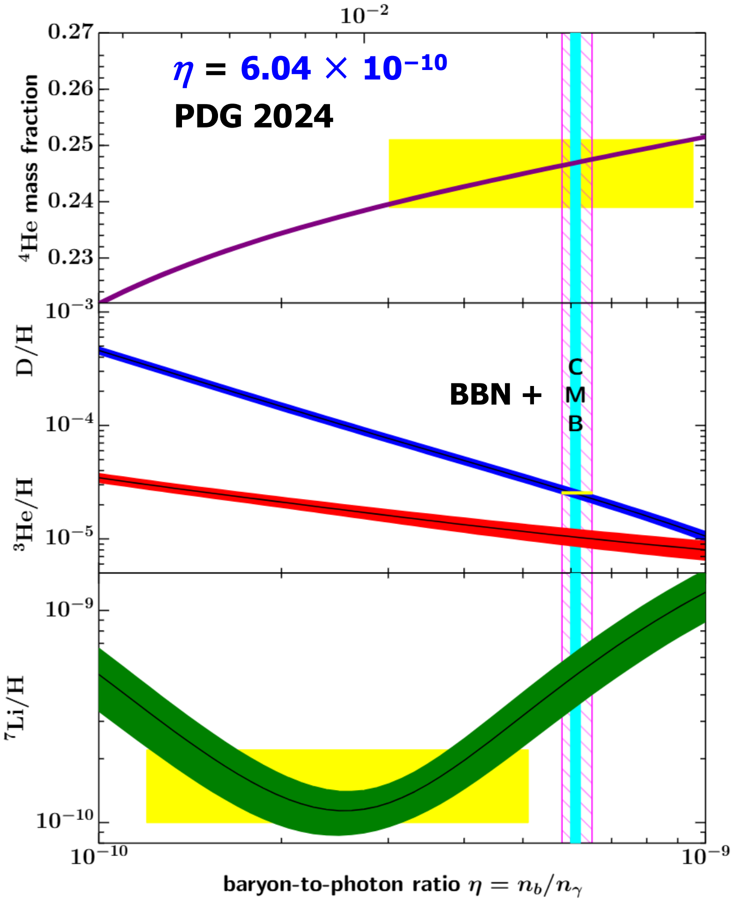
explicit

connection

↓

ZZX, 2406.01142

♦ **Cosmic CPV is already established.**



OUTLINE

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A bridge between light and heavy

- **The exact seesaw formula —— a bridge between the original and derivational flavor parameters:**

$$\boxed{U D_\nu U^T + R D_N R^T = \mathbf{0}} \longrightarrow \boxed{M_\nu \equiv U_0 D_\nu U_0^T = \left(\mathrm{i} A^{-1} R\right) D_N \left(\mathrm{i} A^{-1} R\right)^T}$$

Degrees of freedom (mass + mixing angle + CPV phase): 3 + 3 + 3 (derivational) ⟵ 3 + 9 + 6 (original)

- **The Jarlskog invariant of CP violation in ν-oscillations:**

$$\boxed{\mathcal{J}_\nu \equiv \mathrm{Im}\left[\left(U_0\right)_{\alpha i}\left(U_0\right)_{\beta i'}\left(U_0\right)^*_{\alpha i'}\left(U_0\right)^*_{\beta i}\right]}$$

(α, β) run cyclically over (e, μ, τ), (i, i') run cyclically over $(1, 2, 3)$

$$\left\{\begin{array}{l} D_\nu \equiv \mathrm{Diag}\{m_1, m_2, m_3\} \\ D_N \equiv \mathrm{Diag}\{M_4, M_5, M_6\} \\ \Delta_{ii'} \equiv m_i^2 - m_{i'}^2 \end{array}\right.$$

- **On the one hand, we use the light degrees of freedom to get the relation (ZZX, 2306.02362)**

$$\mathcal{J}_\nu = \frac{\mathrm{Im}\left[\left(M_\nu M_\nu^\dagger\right)_{e\mu}\left(M_\nu M_\nu^\dagger\right)_{\mu\tau}\left(M_\nu M_\nu^\dagger\right)_{\tau e}\right]}{\Delta_{21}\Delta_{31}\Delta_{32}}$$

⟵ **already measured**

- **On the other hand, we use the original seesaw-related parameters to calculate the same quantity *in the leading order approximation* of $A^{-1}R$, because the non-unitarity of U characterized by $R \neq 0$ has been well constrained by precision measurements (M. Blennow et al, 2306.01040)**

$$A^{-1} R \simeq \begin{pmatrix} \hat{s}^*_{14} & \hat{s}^*_{15} & \hat{s}^*_{16} \\ \hat{s}^*_{24} & \hat{s}^*_{25} & \hat{s}^*_{26} \\ \hat{s}^*_{34} & \hat{s}^*_{35} & \hat{s}^*_{36} \end{pmatrix}$$

Is this approximation really safe?

- **Of course, one may use the non-unitary PMNS matrix *U* = *AU*$_0$ to define the more general *Jarlskog* invariants to describe CP violation in neutrino oscillations. But one can show that their leading terms are the same, coming from the unitarity limit (ZZX, 1110.0083):**

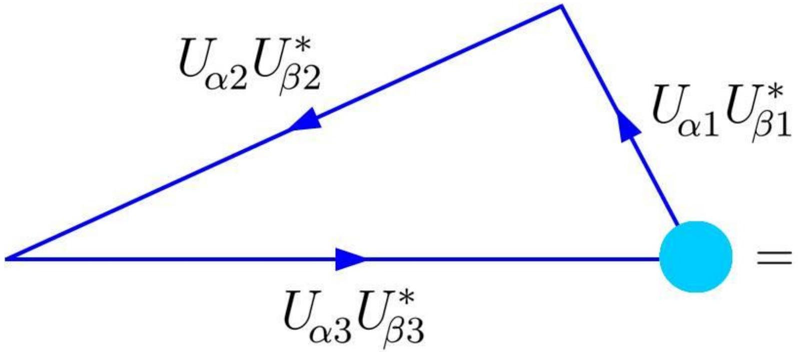
$$\boxed{\mathcal{J}^{ii'}_{\alpha\beta} = \mathcal{J}_\nu + \text{corrections}}$$

$\leq$ **1%** $\qquad$ < **0.01%**

$$\boxed{\mathcal{J}^{ii'}_{\alpha\beta} \equiv \text{Im}\left(U_{\alpha i} U_{\beta i'} U^*_{\alpha i'} U^*_{\beta i}\right)}$$

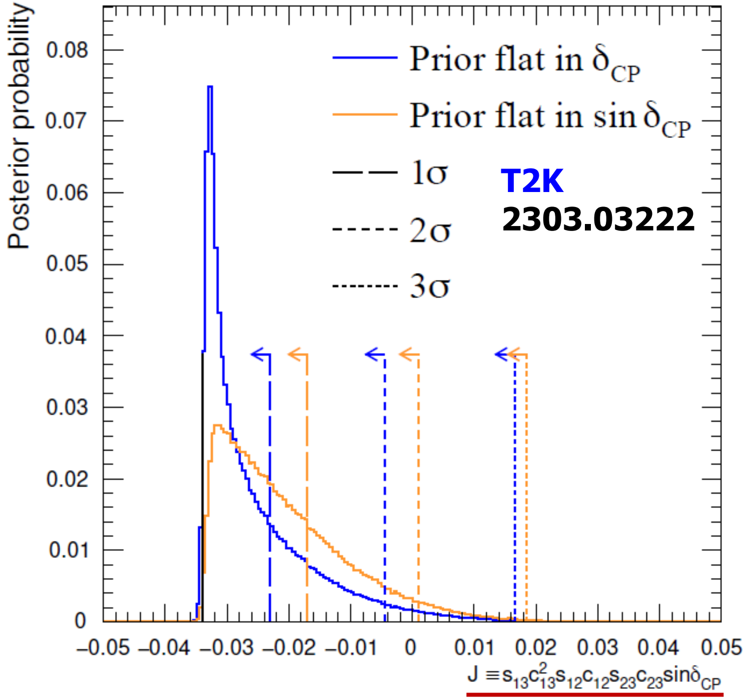
(α, β) run cyclically over (e, μ, τ)
(i, i') run cyclically over $(1, 2, 3)$

- **Yes, absolutely safe, at least by 2044!**



$$= \sum_{j=4}^{6} \left(R_{\alpha j} R^*_{\beta j}\right)$$

$$\boxed{\begin{aligned} \alpha &= e \\ \beta &= \mu \end{aligned}}$$



How many terms to be calculated?

♦ Let us classify the analytical results in terms of the products of heavy Majorana neutrino masses.

$$I_{jk} \equiv \sum_{i=1}^{3} \hat{s}^*_{ij} \hat{s}_{ik} = I^*_{kj} \; , \; (j,k = 4,5,6)$$

$$\mathrm{Im}\left[\left(M_\nu M_\nu^\dagger\right)_{e\mu} \left(M_\nu M_\nu^\dagger\right)_{\mu\tau} \left(M_\nu M_\nu^\dagger\right)_{\tau e}\right]$$

$$\left(M_\nu M_\nu^\dagger\right)_{e\mu} = \sum_{j=4}^{6} \sum_{k=4}^{6} M_j M_k I_{jk} \hat{s}^*_{1j} \hat{s}_{2k}$$

$$\left(M_\nu M_\nu^\dagger\right)_{\mu\tau} = \sum_{j=4}^{6} \sum_{k=4}^{6} M_j M_k I_{jk} \hat{s}^*_{2j} \hat{s}_{3k}$$

$$\left(M_\nu M_\nu^\dagger\right)_{\tau e} = \sum_{j=4}^{6} \sum_{k=4}^{6} M_j M_k I_{jk} \hat{s}^*_{3j} \hat{s}_{1k}$$

<u>CPV from 2-family or (and) 3-family interferences</u>

Term 6 0 0: $M_4^6, \; M_5^6, \; M_6^6$ ×

Term 5 1 0: $M_4^5 M_5, \; M_4^5 M_6, \; M_5^5 M_4, \; M_5^5 M_6, \; M_6^5 M_4, \; M_6^5 M_5$ ×

Term 4 2 0: $M_4^4 M_5^2, \; M_4^4 M_6^2, \; M_5^4 M_4^2, \; M_5^4 M_6^2, \; M_6^4 M_4^2, \; M_6^4 M_5^2$ √

Term 3 3 0: $M_4^3 M_5^3, \; M_4^3 M_6^3, \; M_5^3 M_6^3$ √

Term 4 1 1: $M_4^4 M_5 M_6, \; M_5^4 M_4 M_6, \; M_6^4 M_4 M_5$ √

Term 3 2 1: $M_4^3 M_5^2 M_6, \; M_4^3 M_6^2 M_5, \; M_5^3 M_4^2 M_6, \; M_5^3 M_6^2 M_4, \; M_6^3 M_4^2 M_5, \; M_6^3 M_5^2 M_4$ √

Term 2 2 2: $M_4^2 M_5^2 M_6^2$ √

♦ There are totally 6 independent original CP-violating phases in the canonical seesaw mechanism, measuring the *inter-family interference effects* in all processes of heavy and light Majorana neutrinos.

$$\begin{cases} \alpha_i \equiv \delta_{i4} - \delta_{i5} \\ \beta_i \equiv \delta_{i5} - \delta_{i6} \\ \gamma_i \equiv \delta_{i6} - \delta_{i4} \end{cases}$$

$$\alpha_i + \beta_i + \gamma_i = 0 \; (\mathrm{for} \; i = 1, 2, 3)$$

So we arrive at ... (1)

- The **general** and **explicit** expression of the **Jarlskog invariant** in the seesaw mechanism is a linear combination of the above **5** terms:

$$\mathcal{J}_\nu = \frac{T_{\mathbf{33}} + T_{\mathbf{42}} + T_{\mathbf{411}} + T_{\mathbf{321}} + T_{\mathbf{222}}}{\Delta_{21}\Delta_{31}\Delta_{32}}$$

- The two ***2-family interference*** terms are:

$$T_{\mathbf{33}} = \underline{M_4^3 M_5^3} \left(I_{44}I_{55} - |I_{45}|^2\right) \left[\sum_{i=1}^{3} s_{i4}s_{i5}s_{i'4}s_{i'5}\left(s_{i4}^2 s_{i''5}^2 + s_{i''4}^2 s_{i'5}^2 - s_{i'4}^2 s_{i''5}^2 - s_{i''4}^2 s_{i5}^2\right) \underline{\sin\left(\alpha_i + \alpha_{i'}\right)} - \sum_{i=1}^{3} s_{i4}^2 s_{i5}^2 \left(s_{i'4}^2 s_{i''5}^2 - s_{i''4}^2 s_{i'5}^2\right) \underline{\sin 2\alpha_i}\right]$$
$$+ \,\mathrm{term}\{4 \to 5,\ 5 \to 6;\ \alpha_i \to \beta_i\} + \mathrm{term}\{4 \to 6,\ 5 \to 4;\ \alpha_i \to \gamma_i\}$$

i, i' and i'' run cyclically over 1, 2 and 3

$$T_{\mathbf{42}} = \underline{M_4^2 M_5^2} \left(I_{44}I_{55} - |I_{45}|^2\right) \left[\sum_{i=1}^{3} s_{i4}s_{i5}s_{i'4}s_{i'5}\left(\underline{M_4^2} I_{44} s_{i''4}^2 - \underline{M_5^2} I_{55} s_{i''5}^2\right) \underline{\sin\left(\alpha_i - \alpha_{i'}\right)}\right]$$
$$+ \,\mathrm{term}\{4 \to 5,\ 5 \to 6;\ \alpha_i \to \beta_i\} + \mathrm{term}\{4 \to 6,\ 5 \to 4;\ \alpha_i \to \gamma_i\}$$

Switching off the 3rd heavy neutrino species "6", we can immediately arrive at the results in the *minimal seesaw* case (ZZX, 2306.02362): ➡ **9 combinations of 3 original CPV phases**

So we arrive at … (2)

- The simplest ***3-family interference*** term is obtained as follows:

$$T_{\mathbf{411}} = \underline{M_4^4 M_5 M_6} I_{44} s_{14} s_{24} s_{34} \left[- s_{14} s_{24} s_{34} \sum_{i=1}^{3} s_{i5}^2 \left[s_{i'6}^2 \underline{\sin 2\left(\alpha_i + \gamma_{i'}\right)} - s_{i''6}^2 \underline{\sin 2\left(\alpha_i + \gamma_{i''}\right)} \right] \right.$$

$$+ \sum_{i=1}^{3} s_{i4} \left(s_{i'4}^2 - s_{i''4}^2 \right) \left[s_{i5}^2 s_{i'6} s_{i''6} \underline{\sin\left(2\alpha_i + \gamma_{i'} + \gamma_{i''}\right)} - s_{i6}^2 s_{i'5} s_{i''5} \underline{\sin\left(\alpha_{i'} + \alpha_{i''} + 2\gamma_i\right)} \right]$$

$$+ \sum_{i=1}^{3} s_{i4} s_{i'5} s_{i'6} \left(s_{i4}^2 + s_{i''4}^2 \right) \left[s_{i'5} s_{i''6} \underline{\sin\left(\alpha_{i'} - \beta_{i'} + \gamma_{i''}\right)} - s_{i'6} s_{i''5} \underline{\sin\left(\alpha_{i''} - \beta_{i'} + \gamma_{i'}\right)} \right]$$

$$+ \sum_{i=1}^{3} s_{i4} s_{i''5} s_{i''6} \left(s_{i4}^2 + s_{i'4}^2 \right) \left[s_{i'5} s_{i''6} \underline{\sin\left(\alpha_{i'} - \beta_{i''} + \gamma_{i''}\right)} - s_{i'6} s_{i''5} \underline{\sin\left(\alpha_{i''} - \beta_{i''} + \gamma_{i'}\right)} \right]$$

$$\left. + \sum_{i=1}^{3} 2 s_{i4} s_{i5} s_{i6} \left(s_{i'4}^2 + s_{i''4}^2 \right) \left[s_{i'5} s_{i''6} \underline{\sin\left(\alpha_{i'} - \beta_i + \gamma_{i''}\right)} - s_{i'6} s_{i''5} \underline{\sin\left(\alpha_{i''} - \beta_i + \gamma_{i'}\right)} \right] \right]$$

$$+ \mathrm{term}\left\{ (4,5,6) \to (5,4,6)\,;\ (\alpha_i,\ \beta_i,\ \gamma_i) \to -(\alpha_i,\ \gamma_i,\ \beta_i) \right\}$$

$$+ \mathrm{term}\left\{ (4,5,6) \to (6,5,4)\,;\ (\alpha_i,\ \beta_i,\ \gamma_i) \to -(\beta_i,\ \alpha_i,\ \gamma_i) \right\}$$

The terms *T_{321}* + *T_{222}* are very complicated and can be found in ZZX, 2406.01142.

Counting the phase combinations (1)

♦ **After a very tedious survey of all terms of the Jarlskog invariant, we find 240 linear combinations of the 6 original seesaw phase parameters —— 72 of them:**

$$\sin\alpha_1,\quad \sin\alpha_2,\quad \sin\alpha_3;\quad \sin\beta_1,\quad \sin\beta_2,\quad \sin\beta_3;\quad \sin\gamma_1,\quad \sin\gamma_2,\quad \sin\gamma_3;$$

$$\sin 2\alpha_1,\quad \sin 2\alpha_2,\quad \sin 2\alpha_3;\quad \sin 2\beta_1,\quad \sin 2\beta_2,\quad \sin 2\beta_3;\quad \sin 2\gamma_1,\quad \sin 2\gamma_2,\quad \sin 2\gamma_3;$$

$$\begin{array}{llllll}
\sin(\alpha_1+\alpha_2), & \sin(\alpha_2+\alpha_3), & \sin(\alpha_3+\alpha_1); & \sin(\beta_1+\beta_2), & \sin(\beta_2+\beta_3), & \sin(\beta_3+\beta_1); \\
\sin(\gamma_1+\gamma_2), & \sin(\gamma_2+\gamma_3), & \sin(\gamma_3+\gamma_1); & & & \\
\sin(\alpha_1-\alpha_2), & \sin(\alpha_2-\alpha_3), & \sin(\alpha_3-\alpha_1); & \sin(\beta_1-\beta_2), & \sin(\beta_2-\beta_3), & \sin(\beta_3-\beta_1); \\
\sin(\gamma_1-\gamma_2), & \sin(\gamma_2-\gamma_3), & \sin(\gamma_3-\gamma_1); & & &
\end{array}$$

$$\begin{array}{llllll}
\sin(\alpha_1+\beta_2), & \sin(\alpha_1+\beta_3), & \sin(\alpha_2+\beta_1), & \sin(\alpha_2+\beta_3), & \sin(\alpha_3+\beta_1), & \sin(\alpha_3+\beta_2); \\
\sin(\alpha_1+\gamma_2), & \sin(\alpha_1+\gamma_3), & \sin(\alpha_2+\gamma_1), & \sin(\alpha_2+\gamma_3), & \sin(\alpha_3+\gamma_1), & \sin(\alpha_3+\gamma_2); \\
\sin(\beta_1+\gamma_2), & \sin(\beta_1+\gamma_3), & \sin(\beta_2+\gamma_1), & \sin(\beta_2+\gamma_3), & \sin(\beta_3+\gamma_1), & \sin(\beta_3+\gamma_2);
\end{array}$$

$$\begin{array}{llllll}
\sin 2(\alpha_1+\beta_2), & \sin 2(\alpha_1+\beta_3), & \sin 2(\alpha_2+\beta_1), & \sin 2(\alpha_2+\beta_3), & \sin 2(\alpha_3+\beta_1), & \sin 2(\alpha_3+\beta_2); \\
\sin 2(\alpha_1+\gamma_2), & \sin 2(\alpha_1+\gamma_3), & \sin 2(\alpha_2+\gamma_1), & \sin 2(\alpha_2+\gamma_3), & \sin 2(\alpha_3+\gamma_1), & \sin 2(\alpha_3+\gamma_2); \\
\sin 2(\beta_1+\gamma_2), & \sin 2(\beta_1+\gamma_3), & \sin 2(\beta_2+\gamma_1), & \sin 2(\beta_2+\gamma_3), & \sin 2(\beta_3+\gamma_1), & \sin 2(\beta_3+\gamma_2);
\end{array}$$

Counting the phase combinations (2)

- **Of 240 linear combinations of the 6 original seesaw phase parameters —— 60 of them:**

$$\sin(2\alpha_1-\beta_2)\,,\quad \sin(2\alpha_1-\beta_3)\,,\quad \sin(2\alpha_2-\beta_1)\,,\quad \sin(2\alpha_2-\beta_3)\,,\quad \sin(2\alpha_3-\beta_1)\,,\quad \sin(2\alpha_3-\beta_2)\,;$$
$$\sin(2\alpha_1-\gamma_2)\,,\quad \sin(2\alpha_1-\gamma_3)\,,\quad \sin(2\alpha_2-\gamma_1)\,,\quad \sin(2\alpha_2-\gamma_3)\,,\quad \sin(2\alpha_3-\gamma_1)\,,\quad \sin(2\alpha_3-\gamma_2)\,;$$
$$\sin(2\beta_1-\alpha_2)\,,\quad \sin(2\beta_1-\alpha_3)\,,\quad \sin(2\beta_2-\alpha_1)\,,\quad \sin(2\beta_2-\alpha_3)\,,\quad \sin(2\beta_3-\alpha_1)\,,\quad \sin(2\beta_3-\alpha_2)\,;$$
$$\sin(2\beta_1-\gamma_2)\,,\quad \sin(2\beta_1-\gamma_3)\,,\quad \sin(2\beta_2-\gamma_1)\,,\quad \sin(2\beta_2-\gamma_3)\,,\quad \sin(2\beta_3-\gamma_1)\,,\quad \sin(2\beta_3-\gamma_2)\,;$$
$$\sin(2\gamma_1-\alpha_2)\,,\quad \sin(2\gamma_1-\alpha_3)\,,\quad \sin(2\gamma_2-\alpha_1)\,,\quad \sin(2\gamma_2-\alpha_3)\,,\quad \sin(2\gamma_3-\alpha_1)\,,\quad \sin(2\gamma_3-\alpha_2)\,;$$
$$\sin(2\gamma_1-\beta_2)\,,\quad \sin(2\gamma_1-\beta_3)\,,\quad \sin(2\gamma_2-\beta_1)\,,\quad \sin(2\gamma_2-\beta_3)\,,\quad \sin(2\gamma_3-\beta_1)\,,\quad \sin(2\gamma_3-\beta_2)\,;$$

$$\sin(\alpha_1+\alpha_2+2\beta_3)\,,\quad \sin(\alpha_1+\alpha_2+2\gamma_3)\,,\quad \sin(\alpha_1+\alpha_3+2\beta_2)\,,\quad \sin(\alpha_1+\alpha_3+2\gamma_2)\,,$$
$$\sin(\alpha_2+\alpha_3+2\beta_1)\,,\quad \sin(\alpha_2+\alpha_3+2\gamma_1)\,;\quad \sin(\beta_1+\beta_2+2\alpha_3)\,,\quad \sin(\beta_1+\beta_2+2\gamma_3)\,,$$
$$\sin(\beta_1+\beta_3+2\alpha_2)\,,\quad \sin(\beta_1+\beta_3+2\gamma_2)\,,\quad \sin(\beta_2+\beta_3+2\alpha_1)\,,\quad \sin(\beta_2+\beta_3+2\gamma_1)\,;$$
$$\sin(\gamma_1+\gamma_2+2\alpha_3)\,,\quad \sin(\gamma_1+\gamma_2+2\beta_3)\,,\quad \sin(\gamma_1+\gamma_3+2\alpha_2)\,,\quad \sin(\gamma_1+\gamma_3+2\beta_2)\,,$$
$$\sin(\gamma_2+\gamma_3+2\alpha_1)\,,\quad \sin(\gamma_2+\gamma_3+2\beta_1)\,;$$

$$\sin(\alpha_1+\beta_2+\gamma_3)\,,\quad \sin(\alpha_1+\beta_3+\gamma_2)\,,\quad \sin(\alpha_2+\beta_1+\gamma_3)\,,\quad \sin(\alpha_2+\beta_3+\gamma_1)\,,$$
$$\sin(\alpha_3+\beta_1+\gamma_2)\,,\quad \sin(\alpha_3+\beta_2+\gamma_1)\,.$$

Counting the phase combinations (3)

- Of **240** linear combinations of the **6** **original seesaw phase parameters** —— **54** of them:

$$
\begin{array}{llll}
\sin\left(\alpha_1+\alpha_2-\beta_1\right), & \sin\left(\alpha_1+\alpha_2-\beta_2\right), & \sin\left(\alpha_1+\alpha_2-\beta_3\right), & \sin\left(\alpha_1+\alpha_3-\beta_1\right), \\
\sin\left(\alpha_1+\alpha_3-\beta_2\right), & \sin\left(\alpha_1+\alpha_3-\beta_3\right), & \sin\left(\alpha_2+\alpha_3-\beta_1\right), & \sin\left(\alpha_2+\alpha_3-\beta_2\right), \\
\sin\left(\alpha_2+\alpha_3-\beta_3\right); & \sin\left(\alpha_1+\alpha_2-\gamma_1\right), & \sin\left(\alpha_1+\alpha_2-\gamma_2\right), & \sin\left(\alpha_1+\alpha_2-\gamma_3\right), \\
\sin\left(\alpha_1+\alpha_3-\gamma_1\right), & \sin\left(\alpha_1+\alpha_3-\gamma_2\right), & \sin\left(\alpha_1+\alpha_3-\gamma_3\right), & \sin\left(\alpha_2+\alpha_3-\gamma_1\right), \\
\sin\left(\alpha_2+\alpha_3-\gamma_2\right), & \sin\left(\alpha_2+\alpha_3-\gamma_3\right); & \sin\left(\beta_1+\beta_2-\alpha_1\right), & \sin\left(\beta_1+\beta_2-\alpha_2\right), \\
\sin\left(\beta_1+\beta_2-\alpha_3\right), & \sin\left(\beta_1+\beta_3-\alpha_1\right), & \sin\left(\beta_1+\beta_3-\alpha_2\right), & \sin\left(\beta_1+\beta_3-\alpha_3\right), \\
\sin\left(\beta_2+\beta_3-\alpha_1\right), & \sin\left(\beta_2+\beta_3-\alpha_2\right), & \sin\left(\beta_2+\beta_3-\alpha_3\right); & \sin\left(\beta_1+\beta_2-\gamma_1\right), \\
\sin\left(\beta_1+\beta_2-\gamma_2\right), & \sin\left(\beta_1+\beta_2-\gamma_3\right), & \sin\left(\beta_1+\beta_3-\gamma_1\right), & \sin\left(\beta_1+\beta_3-\gamma_2\right), \\
\sin\left(\beta_1+\beta_3-\gamma_3\right), & \sin\left(\beta_2+\beta_3-\gamma_1\right), & \sin\left(\beta_2+\beta_3-\gamma_2\right), & \sin\left(\beta_2+\beta_3-\gamma_3\right); \\
\sin\left(\gamma_1+\gamma_2-\alpha_1\right), & \sin\left(\gamma_1+\gamma_2-\alpha_2\right), & \sin\left(\gamma_1+\gamma_2-\alpha_3\right), & \sin\left(\gamma_1+\gamma_3-\alpha_1\right), \\
\sin\left(\gamma_1+\gamma_3-\alpha_2\right), & \sin\left(\gamma_1+\gamma_3-\alpha_3\right), & \sin\left(\gamma_2+\gamma_3-\alpha_1\right), & \sin\left(\gamma_2+\gamma_3-\alpha_2\right), \\
\sin\left(\gamma_2+\gamma_3-\alpha_3\right); & \sin\left(\gamma_1+\gamma_2-\beta_1\right), & \sin\left(\gamma_1+\gamma_2-\beta_2\right), & \sin\left(\gamma_1+\gamma_2-\beta_3\right), \\
\sin\left(\gamma_1+\gamma_3-\beta_1\right), & \sin\left(\gamma_1+\gamma_3-\beta_2\right), & \sin\left(\gamma_1+\gamma_3-\beta_3\right), & \sin\left(\gamma_2+\gamma_3-\beta_1\right), \\
\sin\left(\gamma_2+\gamma_3-\beta_2\right), & \sin\left(\gamma_2+\gamma_3-\beta_3\right); & &
\end{array}
$$

Counting the phase combinations (4)

- **Of 240 linear combinations of the 6 original seesaw phase parameters —— 54 of them:**

$$\sin(\alpha_1+\beta_2-\gamma_1)\,,\quad \sin(\alpha_1+\beta_2-\gamma_2)\,,\quad \sin(\alpha_1+\beta_2-\gamma_3)\,,\quad \sin(\alpha_1+\beta_3-\gamma_1)\,,$$
$$\sin(\alpha_1+\beta_3-\gamma_2)\,,\quad \sin(\alpha_1+\beta_3-\gamma_3)\,,\quad \sin(\alpha_2+\beta_1-\gamma_1)\,,\quad \sin(\alpha_2+\beta_1-\gamma_2)\,,$$
$$\sin(\alpha_2+\beta_1-\gamma_3)\,,\quad \sin(\alpha_2+\beta_3-\gamma_1)\,,\quad \sin(\alpha_2+\beta_3-\gamma_2)\,,\quad \sin(\alpha_2+\beta_3-\gamma_3)\,,$$
$$\sin(\alpha_3+\beta_1-\gamma_1)\,,\quad \sin(\alpha_3+\beta_1-\gamma_2)\,,\quad \sin(\alpha_3+\beta_1-\gamma_3)\,,\quad \sin(\alpha_3+\beta_2-\gamma_1)\,,$$
$$\sin(\alpha_3+\beta_2-\gamma_2)\,,\quad \sin(\alpha_3+\beta_2-\gamma_3)\,;\quad \sin(\alpha_1-\beta_1+\gamma_2)\,,\quad \sin(\alpha_1-\beta_1+\gamma_3)\,,$$
$$\sin(\alpha_1-\beta_2+\gamma_2)\,,\quad \sin(\alpha_1-\beta_2+\gamma_3)\,,\quad \sin(\alpha_1-\beta_3+\gamma_2)\,,\quad \sin(\alpha_1-\beta_3+\gamma_3)\,,$$
$$\sin(\alpha_2-\beta_1+\gamma_1)\,,\quad \sin(\alpha_2-\beta_1+\gamma_3)\,,\quad \sin(\alpha_2-\beta_2+\gamma_1)\,,\quad \sin(\alpha_2-\beta_2+\gamma_3)\,,$$
$$\sin(\alpha_2-\beta_3+\gamma_1)\,,\quad \sin(\alpha_2-\beta_3+\gamma_3)\,,\quad \sin(\alpha_3-\beta_1+\gamma_1)\,,\quad \sin(\alpha_3-\beta_1+\gamma_2)\,,$$
$$\sin(\alpha_3-\beta_2+\gamma_1)\,,\quad \sin(\alpha_3-\beta_2+\gamma_2)\,,\quad \sin(\alpha_3-\beta_3+\gamma_1)\,,\quad \sin(\alpha_3-\beta_3+\gamma_2)\,;$$
$$\sin(\alpha_1-\beta_1-\gamma_2)\,,\quad \sin(\alpha_1-\beta_1-\gamma_3)\,,\quad \sin(\alpha_1-\beta_2-\gamma_1)\,,\quad \sin(\alpha_1-\beta_2-\gamma_3)\,,$$
$$\sin(\alpha_1-\beta_3-\gamma_1)\,,\quad \sin(\alpha_1-\beta_3-\gamma_2)\,,\quad \sin(\alpha_2-\beta_1-\gamma_2)\,,\quad \sin(\alpha_2-\beta_1-\gamma_3)\,,$$
$$\sin(\alpha_2-\beta_2-\gamma_1)\,,\quad \sin(\alpha_2-\beta_2-\gamma_3)\,,\quad \sin(\alpha_2-\beta_3-\gamma_1)\,,\quad \sin(\alpha_2-\beta_3-\gamma_2)\,,$$
$$\sin(\alpha_3-\beta_1-\gamma_2)\,,\quad \sin(\alpha_3-\beta_1-\gamma_3)\,,\quad \sin(\alpha_3-\beta_2-\gamma_1)\,,\quad \sin(\alpha_3-\beta_2-\gamma_3)\,,$$
$$\sin(\alpha_3-\beta_3-\gamma_1)\,,\quad \sin(\alpha_3-\beta_3-\gamma_2)\,.$$

OUTLINE

- Why is seesaw most convincing
- The flavor structure of seesaw
- The J-invariant in ν-oscillations
- CP violation in heavy *N*–decays

CPV in heavy Majorana neutrino decays

- **The flavor-dependent CP-violating asymmetries in LNV decays of heavy Majorana neutrinos:**

$$\varepsilon_{j\alpha} \equiv \frac{\Gamma(N_j \to \ell_\alpha + H) - \Gamma(N_j \to \overline{\ell_\alpha} + \overline{H})}{\sum_\alpha \left[\Gamma(N_j \to \ell_\alpha + H) + \Gamma(N_j \to \overline{\ell_\alpha} + \overline{H})\right]}$$

$$\simeq \frac{1}{8\pi\langle H\rangle^2 \sum_\beta \left|R_{\beta j}\right|^2} \sum_{k=4}^{6} \left\{ M_k^2 \,\mathrm{Im}\left[\left(R^*_{\alpha j} R_{\alpha k}\right) \sum_\beta \left[\left(R^*_{\beta j} R_{\beta k}\right) \xi(x_{kj}) + \left(R_{\beta j} R^*_{\beta k}\right) \zeta(x_{kj}) \right] \right] \right\}$$

- **Baryogenesis via leptogenesis in the early Universe:**

A net lepton number asymmetry

$$\boxed{Y_{\rm L} \equiv \frac{n_{\rm L} - n_{\overline{\rm L}}}{s} = \frac{1}{g_*} \sum_{j,\alpha} \kappa_{j\alpha} \varepsilon_{j\alpha}}$$

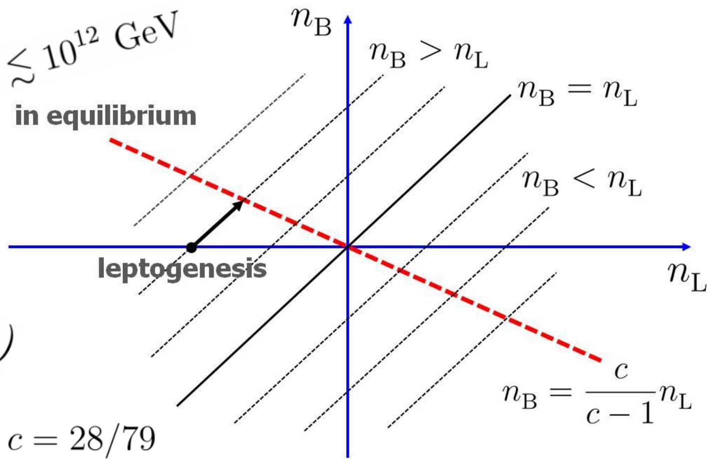
sphaleron-induced (B-L)-conserving process in thermal equilibrium

$$\boxed{Y_{\rm B} \equiv \frac{n_{\rm B} - n_{\overline{\rm B}}}{s} = -c Y_{\rm L}}$$

A net baryon number asymmetry

$$\Gamma(N_j \to \ell_\alpha + H) < \mathcal{H}(T = M_j)$$

$$10^2 \ {\rm GeV} \lesssim T \lesssim 10^{12} \ {\rm GeV}$$



How many phase combinations?

- A **flavor-dependent** CP-violating asymmetry of the first (j = 4) heavy Majorana neutrino decays:

$$\varepsilon_{4e} = \frac{1}{8\pi\langle H\rangle^2 I_{44}}\left\{M_5^2 s_{14}s_{15}\left[\xi(x_{54})\sum_{i=1}^{3} s_{i4}s_{i5}\,\underline{\sin(\alpha_1+\alpha_i)} + \zeta(x_{54})\left[s_{24}s_{25}\,\underline{\sin(\alpha_1-\alpha_2)} + s_{34}s_{35}\,\underline{\sin(\alpha_1-\alpha_3)}\right]\right] - M_6^2 s_{14}s_{16}\left[\xi(x_{64})\sum_{i=1}^{3} s_{i4}s_{i6}\,\underline{\sin(\gamma_1+\gamma_i)} + \zeta(x_{64})\left[s_{24}s_{26}\,\underline{\sin(\gamma_1-\gamma_2)} + s_{34}s_{36}\,\underline{\sin(\gamma_1-\gamma_3)}\right]\right]\right\}$$

The formulas for "5" and "6" can be similarly written out.

$$\varepsilon_{4\mu} = \frac{1}{8\pi\langle H\rangle^2 I_{44}}\left\{M_5^2 s_{24}s_{25}\left[\xi(x_{54})\sum_{i=1}^{3} s_{i4}s_{i5}\,\underline{\sin(\alpha_2+\alpha_i)} + \zeta(x_{54})\left[s_{14}s_{15}\,\underline{\sin(\alpha_2-\alpha_1)} + s_{34}s_{35}\,\underline{\sin(\alpha_2-\alpha_3)}\right]\right] - M_6^2 s_{24}s_{26}\left[\xi(x_{64})\sum_{i=1}^{3} s_{i4}s_{i6}\,\underline{\sin(\gamma_2+\gamma_i)} + \zeta(x_{64})\left[s_{14}s_{16}\,\underline{\sin(\gamma_2-\gamma_1)} + s_{34}s_{36}\,\underline{\sin(\gamma_2-\gamma_3)}\right]\right]\right\}$$

The formulas for "5" and "6" can be similarly written out.

How many phase combinations?

- A **flavor-dependent** CP-violating asymmetry of the first ($j = 4$) heavy Majorana neutrino decays:

$$\varepsilon_{4\tau} = \frac{1}{8\pi\langle H\rangle^2 I_{44}} \left\{ M_5^2 s_{34} s_{35} \left[\xi(x_{54}) \sum_{i=1}^{3} s_{i4} s_{i5} \underline{\sin(\alpha_3 + \alpha_i)} + \zeta(x_{54}) \left[s_{14} s_{15} \underline{\sin(\alpha_3 - \alpha_1)} + s_{24} s_{25} \underline{\sin(\alpha_3 - \alpha_2)} \right] \right] - M_6^2 s_{34} s_{36} \left[\xi(x_{64}) \sum_{i=1}^{3} s_{i4} s_{i6} \underline{\sin(\gamma_3 + \gamma_i)} + \zeta(x_{64}) \left[s_{14} s_{16} \underline{\sin(\gamma_3 - \gamma_1)} + s_{24} s_{26} \underline{\sin(\gamma_3 - \gamma_2)} \right] \right] \right\}$$

The formulas for "5" and "6" can be similarly written out.

- The **flavor-independent** CP-violating asymmetry $\underline{\varepsilon_4 \equiv \varepsilon_{4e} + \varepsilon_{4\mu} + \varepsilon_{4\tau}}$, for example:

$$\varepsilon_4 = \frac{1}{8\pi\langle H\rangle^2 I_{44}} \left[\sum_{i=1}^{3} \sum_{i'=1}^{3} s_{i4} s_{i'4} \left[M_5^2 \xi(x_{54}) s_{i5} s_{i'5} \underline{\sin(\alpha_i + \alpha_{i'})} - M_6^2 \xi(x_{64}) s_{i6} s_{i'6} \underline{\sin(\gamma_i + \gamma_{i'})} \right] \right]$$

- Totally **27** linear combinations of the **6** **original seesaw phase parameters** in CP violation of three heavy Majorana neutrino decays ($i, i' = 1, 2, 3$):

$$\sin(\alpha_i \pm \alpha_{i'}) \; , \; \sin(\beta_i \pm \beta_{i'}) \; , \; \sin(\gamma_i \pm \gamma_{i'})$$

The connection can be more direct!

- The analytical results obtained above imply that the *CP-violating asymmetries* can be expressed as a linear combinations of the sines of the 6 original seesaw phase parameters:

$$\boxed{\begin{aligned} \varepsilon_{j\alpha} &= \sum_{i=1}^{3}\left(C'_{\alpha i}\sin\alpha_i + C'_{\beta i}\sin\beta_i\right) \\ \varepsilon_j &= \sum_{i=1}^{3}\left(C''_{\alpha i}\sin\alpha_i + C''_{\beta i}\sin\beta_i\right) \end{aligned}}$$

It's then straightforward to extract the coefficients from the formulas of CP violating asymmetries.

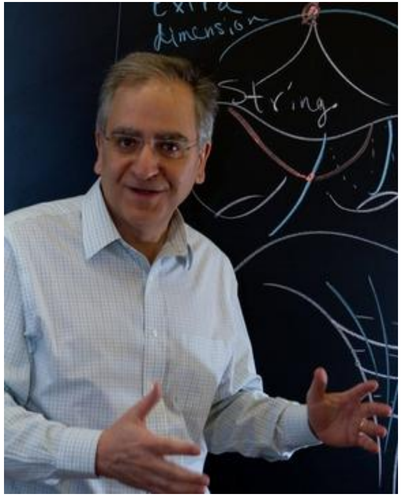
- In comparison, the achieved result of the Jarlskog invariant implies that it can also be expressed as a linear combinations of the sines of the 6 original seesaw phase parameters:

$$\boxed{\mathcal{J}_\nu = \sum_{i=1}^{3}\left(C_{\alpha i}\sin\alpha_i + C_{\beta i}\sin\beta_i\right)}$$

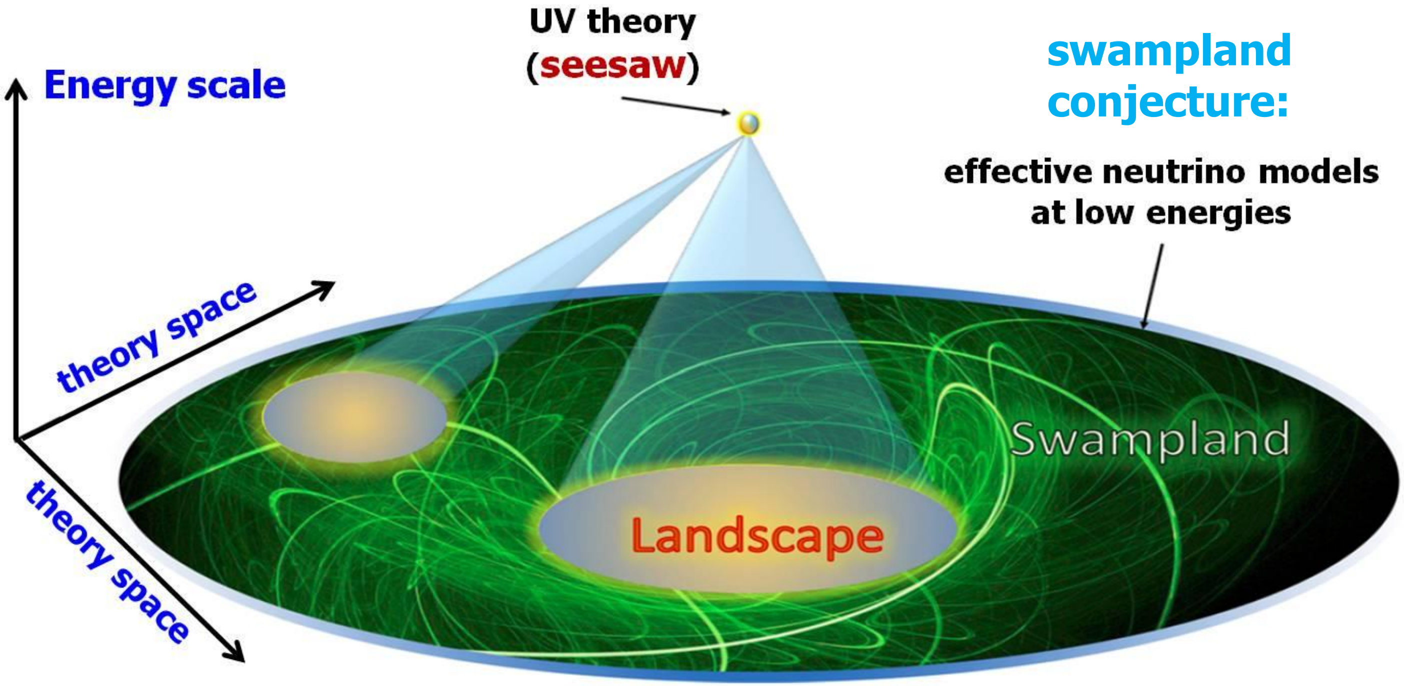
It is straightforward to extract the lengthy coefficients from the T_{33} + T_{42} + T_{411} + T_{321} + T_{222} terms, but the expressions are so complicated that they cannot be presented here.

Concluding remark (1)

♦ Extending the SM framework in a way that is as natural and economical as possible, we have argued that the canonical seesaw mechanism is most convincing to give mass to the active neutrinos. It is fully consistent with the spirit of Weinberg's EFT and thus should be located in the landscape of neutrino physics.



Cumrun Vafa 2005



Concluding remark (2)

♦ The **new era** of precision measurements, as characterized by **JUNO**, **DUNE** and **T2HK**, is coming. It is **high time** to experimentally test the **canonical seesaw** in a **systematical** and **model-independent** way at low energies.

♦ This becomes possible, with the help of a **complete Euler-like block** parametrization of the seesaw flavor structure, since it makes ***analytical*** calculations of all observables possible. The present talk give a ***PoC*** example by clarifying the **Buchmüller-Plümacher** claim. **For the first time**, we have shown that a **direct**, **explicit** and **model-independent** connection exists between CP violation at **high** and **low** energy scales.

♦ **A take-home message:** to really test the seesaw, you should calculate everything by using the **original seesaw parameters** instead of the **derivational ones** or a **mixture**.

♦ We are trying to calculate all the light degrees of freedom along this line of thought.

GRAZIE