Majorana Phase in two flavor neutrino oscillation with neutrino decay

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Based on : ***Phys.Rev.D* 107 (2023) 1, 013002**
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Neutrino Frontiers, Florence

Jun 25 - Jun 28, 2024

Indian Institute of Technology Bombay

The Galileo Galilei Institute For Theoretical Physics

Oscillations in Vacuum

- In general, neutrino mass eigenstates ν_i mix via a unitary matrix to introduce flavor states ν_α of neutrinos

$$\boxed{\nu_\alpha = U\ \nu_i = O\ U_{ph}\ \nu_i}$$

where $\nu_\alpha = (\nu_e \quad \nu_\mu)^T$ and $\nu_i = (\nu_1 \quad \nu_2)^T$, and

$$O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad U_{ph} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

- Hamiltonian that governs oscillations in vacuum,

$$\mathcal{H} = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} \qquad a_1 = m_1^2/2E \text{ and } a_2 = m_2^2/2E$$

$$= \frac{(a_1 + a_2)}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{(a_2 - a_1)}{2} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Oscillations in Vacuum

$$i\frac{d}{dt}\nu_i(t) = \left[\frac{(a_1 + a_2)}{2}\sigma_0 - \frac{(a_2 - a_1)}{2}\sigma_z\right]\nu_i(t)$$

Flavor basis

$$i\frac{d}{dt}\nu_\alpha(t) = \left[\frac{(a_1 + a_2)}{2}\sigma_0 - \frac{(a_2 - a_1)}{2}OU_{ph}\sigma_z U_{ph}^\dagger O^T\right]\nu_\alpha$$

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Phase ϕ disappears from the evolution equation

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{(a_2-a_1)t}{2}\right) \equiv P_{e\mu}^{\text{vac}}$$

Motivation

➡ Oscillation probabilities depend on Majorana phases for **neutrino decoherence**, with an off-diagonal term in decoherence matrix.

F. Benatti et al. Phys.Rev.D 64(2001),085015
A. capolupo et al. Phys. Lett. B 7929(2019) 298-303

➡ Towards distinguishing Dirac from Majorana with **Gravitational waves**.

Stephen S. King et al. 2306.05389

Our Goal

What are the other possibilities under which the Majorana phase appear in neutrino oscillation probabilities and what are the outcomes due this new effect??

Our Proposal

- The decay-Hamiltonian $\mathcal{H} = M - i\Gamma/2,$

$$\mathcal{M} = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}, \quad \Gamma/2 = \begin{pmatrix} b_1 & \frac{1}{2}\eta e^{i\xi} \\ \frac{1}{2}\eta e^{-i\xi} & b_2 \end{pmatrix}$$

- For a system of two particles that can oscillate into each other, these matrices can have off diagonal terms, as in the case of neutral meson system.

A. de Gouvea et al. Phys. Lett. B 742 (2015) 1407.6631

A. Dighe et al. Phys. Rev. Lett. **129**, 011802

The decay eigenstates are not aligned with the mass eigenstates.

Evolution Equation

- Evolution equation in mass basis

$$i\frac{d}{dt}\nu_i(t) \;=\; \left[\frac{(a_1+a_2)}{2}\sigma_0 - \frac{(a_2-a_1)}{2}\sigma_z \;-\; \frac{i}{2}\Big((b_1+b_2)\sigma_0 + \vec{\sigma}.\vec{\Gamma}\Big)\right]\;\nu_i(t)$$

- In flavor basis

$$\begin{aligned} i\frac{d}{dt}\nu_\alpha(t) \;&=\; \Bigg[\frac{(a_1+a_2)}{2}\sigma_0 - \frac{(a_2-a_1)}{2}O\sigma_z O^T \\ &\quad - \frac{i}{2}(b_1+b_2)\sigma_0 - \frac{i}{2}O\underline{U_{ph}(\vec{\sigma}.\vec{\Gamma})U^\dagger_{ph}}O^T\Bigg]\,\nu_\alpha(t) \end{aligned}$$

$$\text{where }\; \vec{\Gamma} \;=\; [\eta\cos\xi, -\eta\sin\xi, -(b_2 - b_1)]$$

Evolution Equation

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$$\left. -\;\frac{i}{2}(b_1+b_2)\sigma_0 - \frac{i}{2}O\underline{U_{ph}(\vec{\sigma}.\vec{\Gamma})U_{ph}^{\dagger}}O^T\right]\nu_\alpha(t)$$

σ_x and σ_y do not commute with U_{ph}

$$\sin\xi, -(b_2 - b_1)]$$

Probability calculation

- The time evolution operator for neutrinos in the mass eigenbasis is $\mathcal{U} = e^{-i\mathcal{H}t}$, can be expanded in the basis spanned by σ_0 and Pauli matrices

This expansion is parameterized by a complex four-vector

$$n_\mu = Tr[(-i\mathcal{H}t).\sigma_\mu]/2$$

$$\boxed{\mathcal{U} = e^{n_0}\left[\cosh n\ \sigma_0 + \frac{\vec{n}.\vec{\sigma}}{n}\sinh n\right]}$$

In flavor basis

$$\mathcal{U}_f = U\mathcal{U}U^{-1} \qquad P_{\alpha\beta} = |(\mathcal{U}_f)_{\alpha\beta}|^2$$

Nielsen, M., Chuang, I. (2010). Quantum Computation and Quantum Information

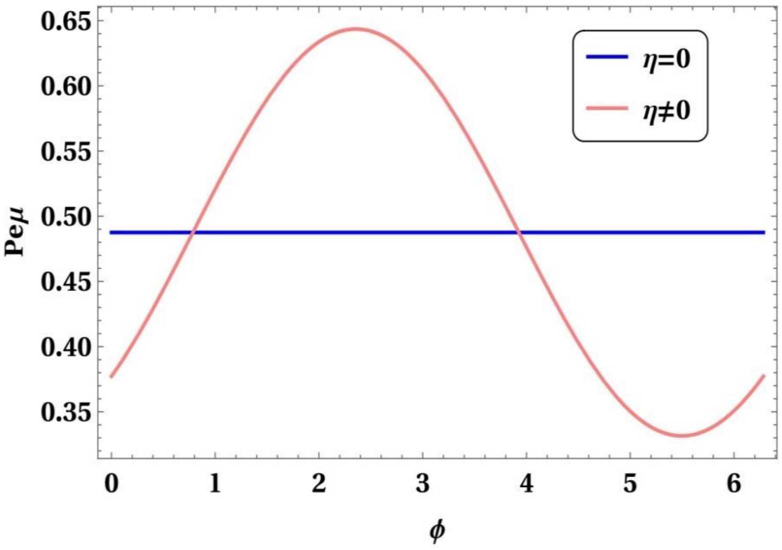
Outcome-1

In the limit

$$b_1 = b = b_2 \text{ and } \eta \ll |a_2 - a_1|$$

$$P_{e\mu} = e^{-2bt}\left(P_{e\mu}^{\text{vac}} + 2\eta \sin(\xi - \phi)\mathcal{B}\right)$$

$$\mathcal{B} = \frac{\sin(2\theta)\sin^2\left[\frac{1}{2}t(a_2 - a_1)\right]}{(a_2 - a_1)}$$



Outcome-2

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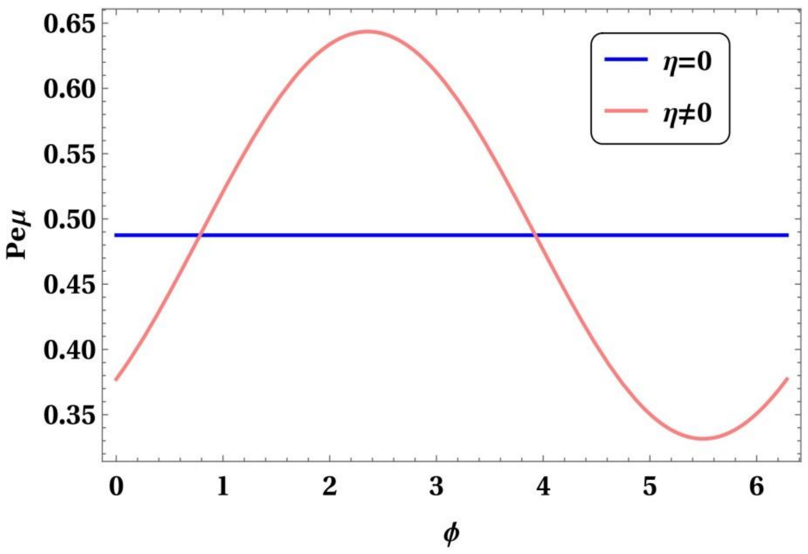

CPT Conservation $\bar{M} = M \quad \text{and} \quad \bar{\Gamma} = \Gamma^*$

$$\phi \to -\phi \qquad \xi \to -\xi$$

$$P_{\bar{e}\bar{\mu}} = e^{-2bt}\left(P^{\text{vac}}_{\bar{e}\bar{\mu}} - 2\eta\sin(\xi - \phi)\mathcal{B}\right)$$

CP Violation $P_{\bar{e}\bar{\mu}} \neq P_{e\mu}$

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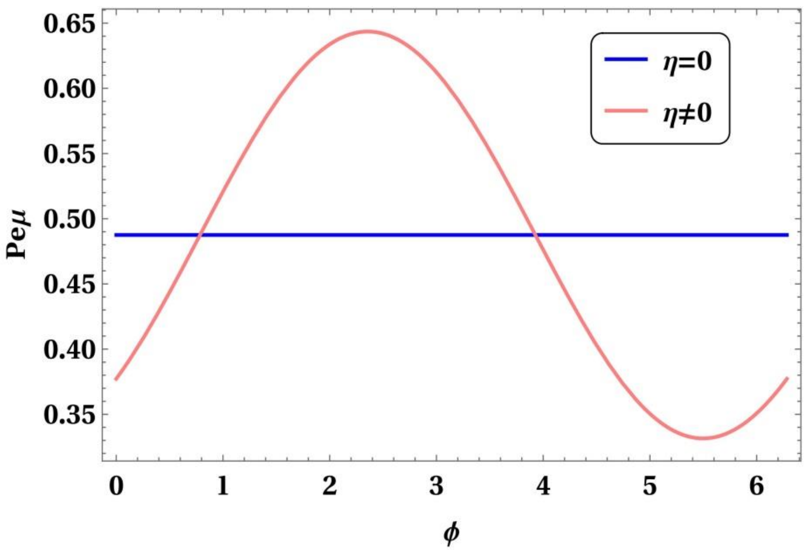

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CP Violation $P_{\bar{e}\bar{\mu}} \neq P_{e\mu}$

- Due to Majorana phase ϕ
- Due to decay phase ξ
- Due to both phases

Outcome-3

$$P_{ee} = e^{-2bt}\left(P_{ee}^{\text{vac}} - \eta\cos(\xi-\phi)\mathcal{A}\right)$$

$$P_{e\mu} = e^{-2bt}\left(P_{e\mu}^{\text{vac}} + 2\eta\sin(\xi-\phi)\mathcal{B}\right)$$

$$\mathcal{A} = \frac{\sin(2\theta)\sin\left[(a_2-a_1)\,t\right]}{(a_2-a_1)}$$

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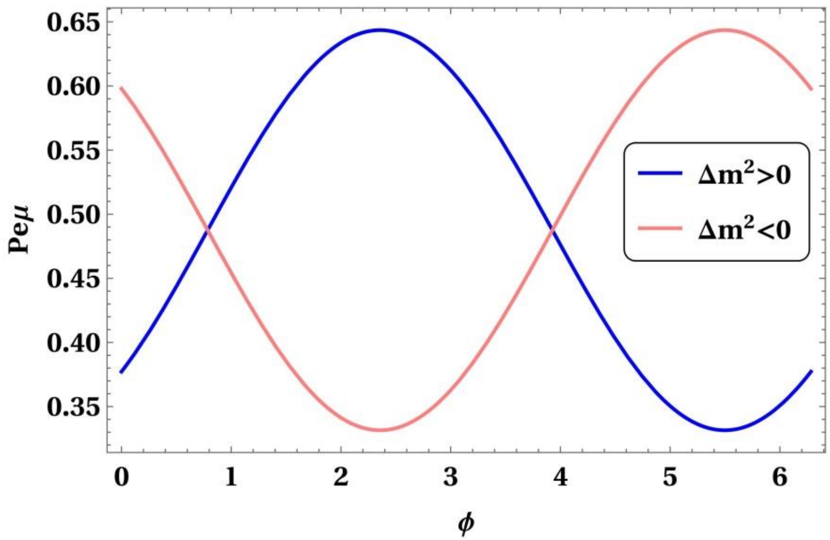

Sensitive to Mass Hierarchy

Do we ever observe this effect ??

- Supernova 1987A, $\tau_\nu \geq 5.7 \times 10^5 \text{ s } (m_\nu/\text{eV})$

  $\Gamma_\nu \equiv b \approx 10^{-21}$ for m_ν =1eV.

J.A.Frieman et al. Phys.Lett.B 200(1988) 115-121

The new effects considered in this work are of order

$$\eta/(a_2 - a_1) = \eta E/\Delta m^2$$

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⟹ Effects are of **order 10%** for $\Delta m^2 \approx 10^{-4}\,\text{eV}^2$ if ***E*=10^7 GeV**

★ **Ultra high energy neutrinos** from astrophysical sources provide a platform to study the effect.

Thank you for your attention!

Backup slides

At any time t, the state of the system $|\psi(t)\rangle$ can be written as a linear combination of the light neutrino eigenstates $|\nu_{0i}\rangle$ and the $|\phi_{0k}\rangle$ as

$$|\psi(t)\rangle = \sum_i c_i(t)|\nu_{0i}\rangle + \sum_k C_k(t)|\phi_{0k}\rangle,$$

$$P_{\nu\to\nu} = \sum_i |c_i(t)|^2 = 1 - \sum_k |C_k(t)|^2 \leq 1,$$

Using Weisskopf-Wigner approximation (WW)

$$\begin{aligned} M_{ij} &= (E_i - \bar{E})\delta_{ij} + \langle \nu_{0i}|\mathscr{H}'|\nu_{0j}\rangle \\ &\quad - \sum_k \frac{\langle \nu_{0i}|\mathscr{H}'|\phi_{0k}\rangle\langle \phi_{0k}|\mathscr{H}'|\nu_{0j}\rangle}{E(k) - \bar{E}}, \end{aligned}$$

$$\Gamma_{ij} = \pi \sum_k \langle \nu_{0i}|\mathscr{H}'|\phi_{0k}\rangle\langle \phi_{0k}|\mathscr{H}'|\nu_{0j}\rangle \delta(E(k) - \bar{E}).$$