Towards the discovery of the Diffuse Supernova Neutrino Background

Neutrino Frontiers - GGI

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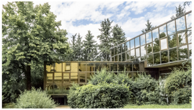
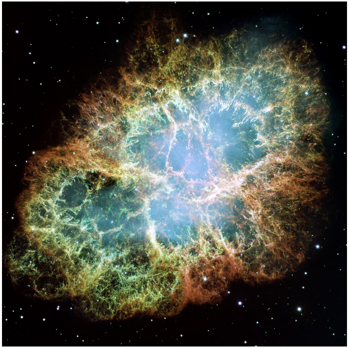


Outline

1. Supernova neutrinos & the Diffuse Supernova Neutrino Background (DSNB)
2. DSNB detection
3. Current status of DSNB searches and next-generation experiments
4. Latest results for Super-Kamiokande with gadolinium

Supernovae physics

- Core-collapse supernovae are among the most cataclysmic phenomena in the Universe and essential elements of the dynamics of the cosmos
- Underlying mechanism still poorly understood and requires knowledge of the core of the collapsing star
- 10^{58} neutrinos emitted in a burst ($\sim$ 99% of gravitational binding energy) $\Rightarrow$ carry unique and unambiguous information about this core



Crab Nebula
six-light-year-wide expanding
remnant of SN1054

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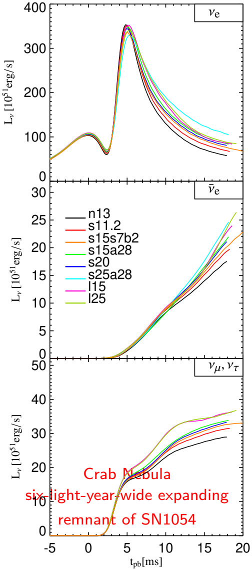
Phys. Rev. D 71, 063003 (2005)



Crab Nebula
six-light-year-wide expanding
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Progenitor star between 11.2 and 25 $M_\odot$

Supernova physics (cont'd)

- So far, only SN1987a in LMC has been detected by neutrino experiments, all current detectors ready to detect one nearby
- If burst happens in the galatic center $\Rightarrow \sim 8000$ neutrinos in Super-Kamiokande... but only few times per century in our galaxy

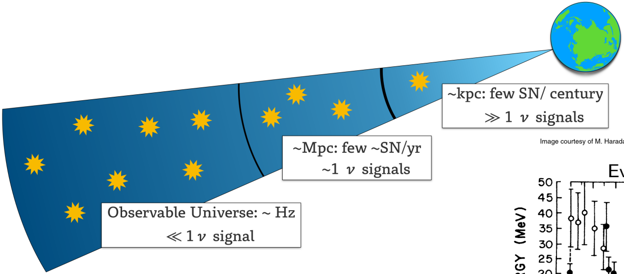
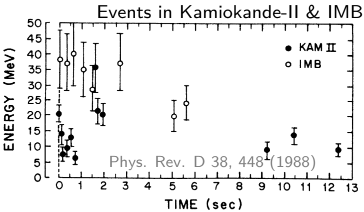


Image courtesy of M. Harada

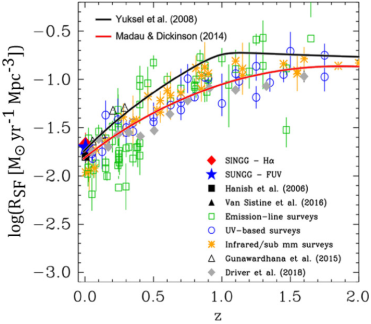


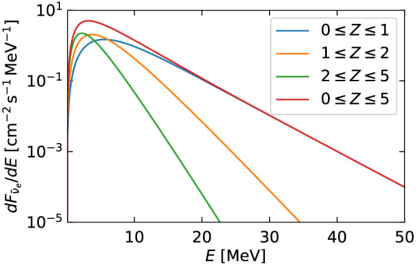
$\Rightarrow$ quest for the Diffuse Supernova Neutrino Background (DSNB)

The Diffuse Supernova Neutrino Background

- Composed by neutrinos of all past SN of all flavors whose energies have been redshifted when propagating to the Earth ⇒ information not only on the SN neutrino emission process but also star formation and Universe expansion history

$$\frac{d\phi_{\mathrm{DSNB}}}{dE} = \iint R_{SN}(z, M) \left[\frac{dF\left(E\left(1+z\right), M\right)}{dM} \right] \left| c\frac{dt}{dz} \right| dzdM$$





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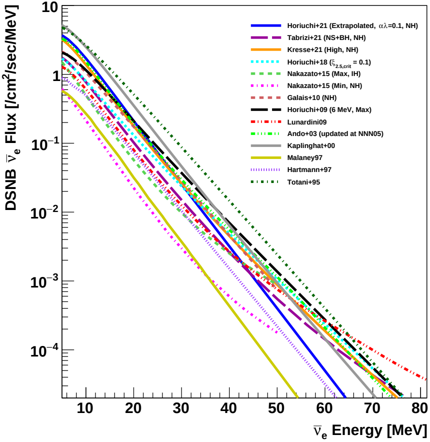
- Normalisation mostly determined by SN rate, related to cosmic star formation rate

- Shape depends on many parameters :
  - fraction of BH-forming SN,
  - effective neutrino energies (core temperature),
  - the expansion of the Universe
  - neutrino oscillation inside the star (dense matter),
  - neutrino mass-hierarchy,
  - neutrino decay and other NSI

- Rich physics program depending on the precision on the DSNB spectrum measurement

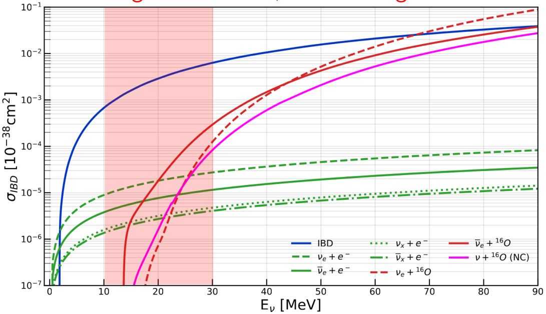
Phys. Rev. D 104 (2021) 12, 122002

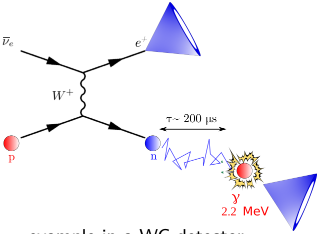


DSNB detection

- Usual detection channel : inverse β-decay (IBD) $\bar{\nu}_e + p \rightarrow e^+ + n$
- Searched at $\mathcal{O}(10)$ MeV, bounded by reactor + spallation background at lower energy and atmospheric neutrinos at higher energies
- In order to disentangle signal from backgrounds, neutron detection in coincidence with the positron is mandatory

region of interest, IBD has largest cross-section





example in a WC detector

Highlight of recent search and prospects

- w/ current detectors
  - Liquid scintillator (LS) :
    KamLAND (1 kt),
    Borexino ($\mathcal{O}$(100) t)
  - Water Cherenkov (WC) :
    Super-Kamiokande (22.5 kt),
    SNO (0.7 kt)
  - Gd-loaded WC detectors : SK-Gd
    $\Rightarrow$ **SK-VI** discussed later
- Next generation experiments for DSNB detection
  - WC : Hyper-Kamiokande
  - LS : JUNO

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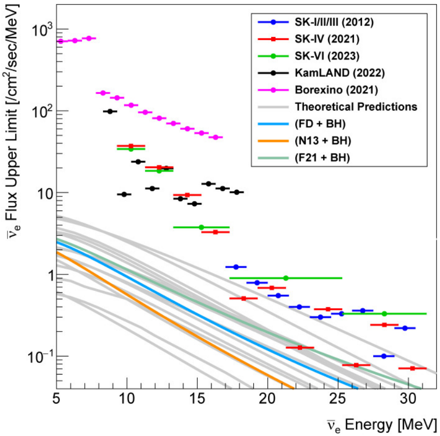

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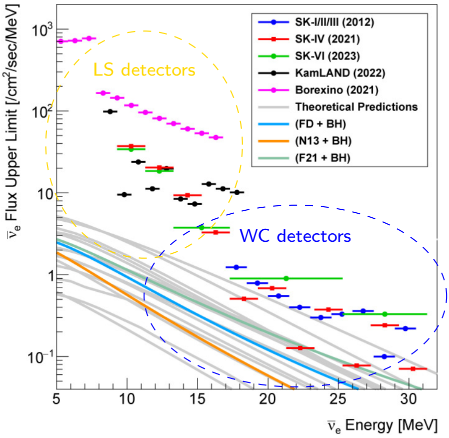
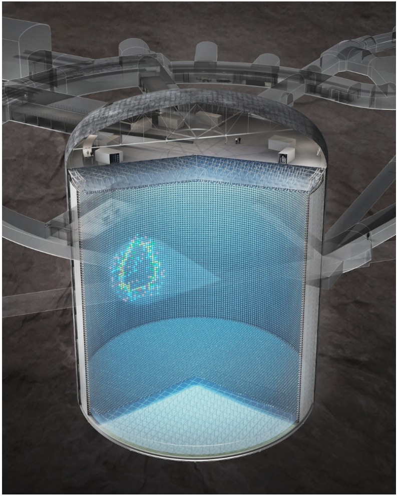
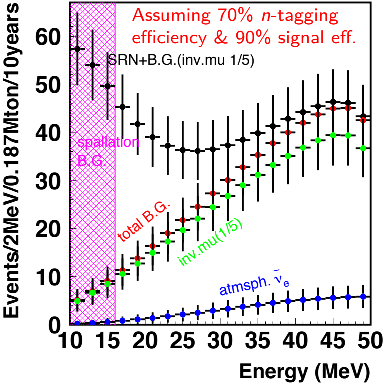




- 258 kton WC detector @ Kamioka, Japan
- Will start data taking in 2027
- Expected $> 4\sigma$ in 10 years due to its large volume and upgraded phototubes ($\times 2$ eff.)



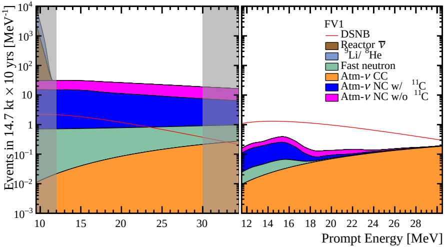
arXiv:1805.04163 [physics.ins-det]

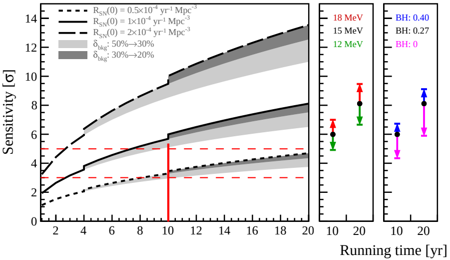
Future experiments - Hyper-Kamiokande (cont'd)

Cavern excavation on June 11, 2024

- 20 kton LS detector @ Jianmen, China ⇒ construction finished in 2024
- Assuming 80% signal efficiency and effective background reduction using muon veto and PSD method ⇒ S/B ∼ 3.5
- 5σ within 10 years for optimistic prediction

Pinched Fermi-Dirac





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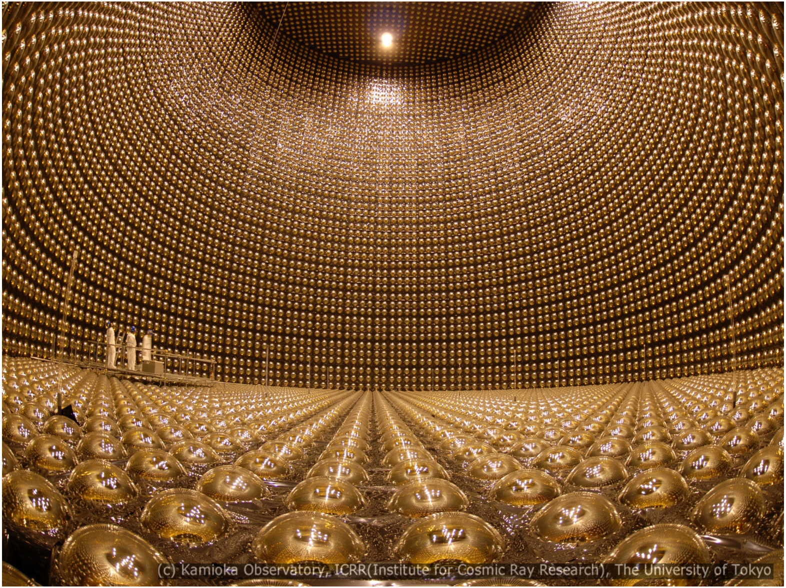
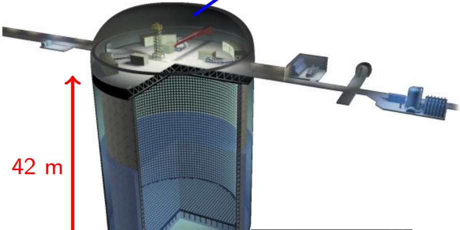
Latest Super-Kamiokande results with Gd

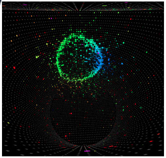


The Super-Kamiokande experiment

- 50 kton water Cherenkov detector (currently doped with Gd)
- Located in Kamioka, Japan, under Mt. Ikenoyama : 1 km rock overburden (2.7 km water equivalent)
- Optically divided into an inner detector (ID) with a fiducial volume of 22.5 kton and an outer detector (OD), instrumented with
  - ID : $\sim$ 11000 inward facing large 20"-PMTs, 40% photo-coverage
  - OD : 1885 8"-PMTs primarily used as veto

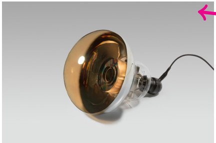
Running for more than 25 years and still has a lot to teach !

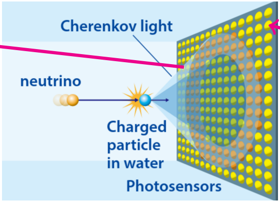


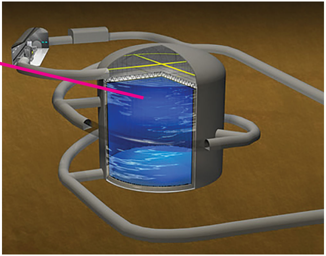


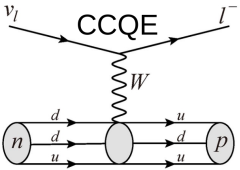
Neutrino detection with water Cherenkov detectors

PhotoMultiplier Tube (PMT)



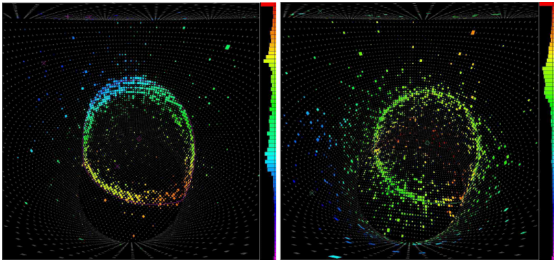






Muon

Electron



courtesy of SK collaboration

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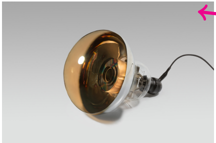
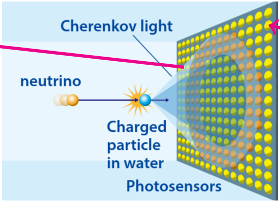
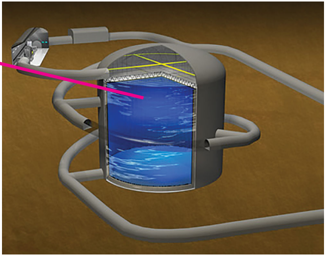
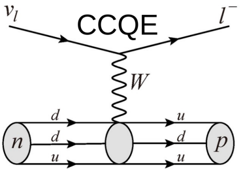
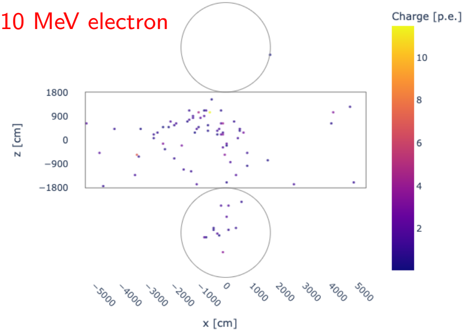










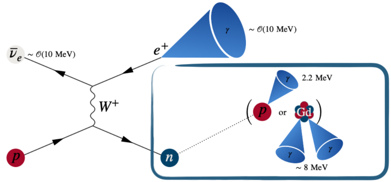
Detector phases

- SK experiment has collected data during 7 phases

Phase	Period	Event
SK-I	1996.4 to 2001.7	Start of the experiment
SK-II	2002.10 to 2005.10	20% photo-coverage after accident
SK-III	2006.7 to 2008.8	Full photo-coverage (40%) restored
SK-IV	2008.9 to 2018.5	Upgraded electronics
SK-V	2019.1 to 2020.8	Detector upgraded for Gd-loading
SK-VI	2020.8 to 2022.6	0.01% Gd-doping
SK-VII	since 2022.6	0.03% Gd-doping

- Highly versatile multi-purpose experiment in the MeV - TeV range : solar & atmospheric neutrinos, supernovae neutrinos, DSNB, neutrino astrophysics, proton-decay, dark matter, beam neutrino (T2K)
- Regarding the search for the DNSB, neutron tagging is only possible since SK-IV period with its upgraded electronics

The SK-Gd upgrade



Neutron capture

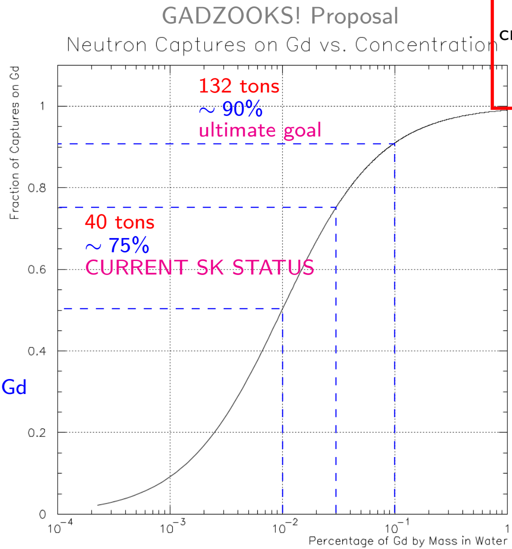
Motivations

- Improve neutron detection at SK
- SK-IV: neutron tagging possible but inefficient
- Dissolve Gd sulfate in water to enhance neutron signal : very high neutron capture cross-section + 8 MeV photon cascade

Upgrade process

- 2002: first proof of concept
- 2009: small scale prototype detector started (EGADS)
- 2018: SK detector refurbishment (SK-V)
- 2020: SK detector w/ 0.01% Gd (SK-VI)
- 2022: SK detector w/ 0.03% Gd (SK-VII, currently running)
- SK detector w/ 0.1% Gd?

Gd thermal neutron cross section : 49.7 kb ($\sigma_H = 0.33$ b)
8 MeV γ cascade



13.2 tons of $Gd_2(SO_4)_3 8H_2O$
in 50 ktons water
~ 50% capture on Gd
August 2020

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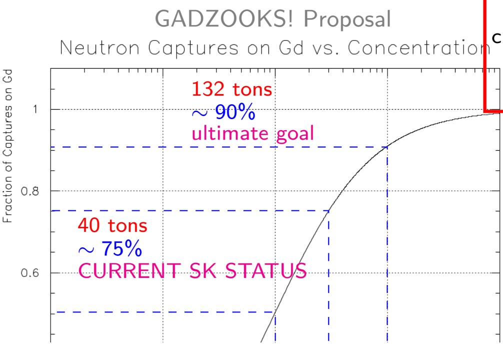

13.2 tons of $Gd_2(SO_4)_38H_2O$ in 50 ktons water

$\sim$ 50% capture on Gd

August 2020

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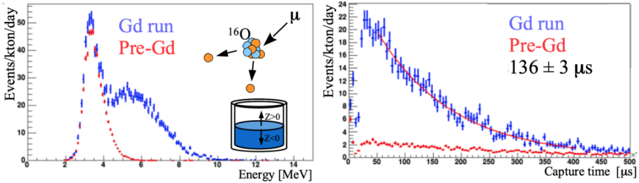
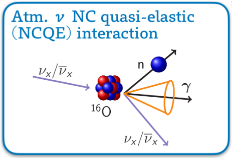
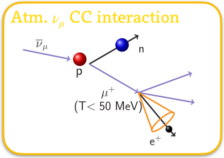


Figure 5: Reconstructed energy for spallation neutrons candidates for runs with and without Gd in the lower region of the detector (left) and capture time of the neutron candidates (right).

1. **Atmospheric neutrinos**
   - NCQE interactions (inducing nuclear γ ray) is the main atm. ν background $\lesssim$ 20 MeV
   - at higher energies $\gtrsim$ 30 MeV CCQE interactions and π production dominate, mostly Michel-e^- from invisible μ/π decay (up $\sim$ 50 MeV)
   - ν_e/ν_μ CC can be largely reduced using neutron tagging
   - $\bar{\nu}_e$ CCQE are irreducible $\Rightarrow$ sets the analysis upper threshold

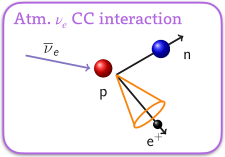


2. **Cosmic ray muon spallation**
   - 2700 mwe $\Rightarrow$ $\sim$ 2 Hz of muon in SK inducing spallation of ^{16}O
   - misidentification of decays of produced radioactive isotopes as IBD
   - below 20 MeV spallation events are 10^6 times higher than DSNB events
   - drastically reduced by neutron tagging apart from βn emitters and accidental coincidence



3. **Reactor neutrinos**
   - Irreducible, overwhelm DSNB events $\lesssim$ 8 MeV $\Rightarrow$ sets the analysis lower threshold

4. **Solar neutrinos**
   - ν_e from the *pp* chain (^{8}B & *hep*) up to $\sim$ 20 MeV
   - easily removed using neutron tagging and directional cuts using θ_{Sun}



Super-Kamiokande DSNB analysis - Reduction steps

Data Quality

Remove poorly reconstructed events,
Rn events, events with OD activity

↓

Spallation Background

Remove long-lived, high-energy,
spallation isotopes (^{8}Li, ^{8}B, ^{9}Li, ^{9}C)
remove events near "neutron clouds"

↓

Atmospheric Background

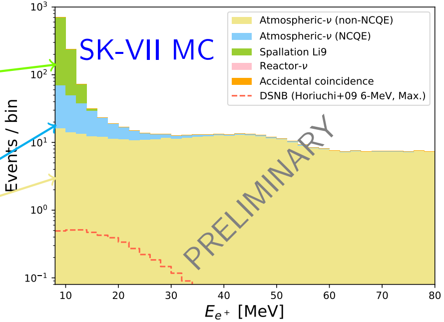
Remove multi-cone prompt events
remove μ/π events

↓

Neutron Tagging

[2 different ML algorithms BDT/NN]

no reduction applied



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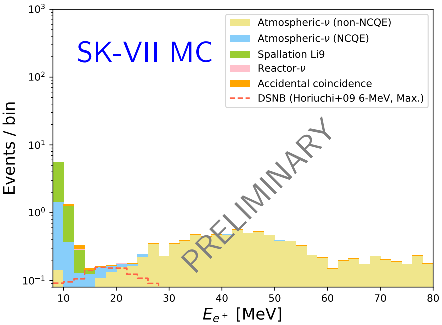

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Neutron Tagging

[2 different ML algorithms BDT/NN]

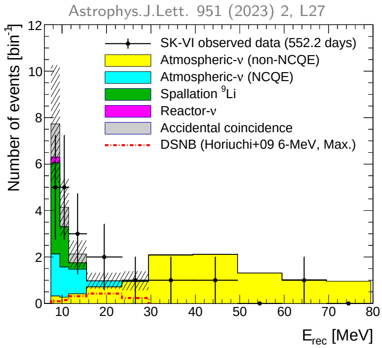
→ **Cut Optimization and Systematics Evaluation**

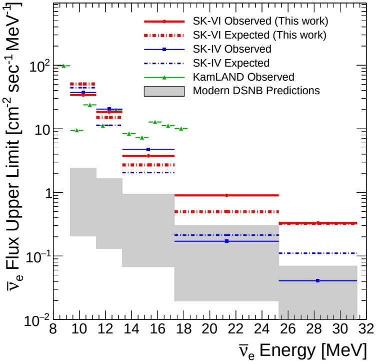
→ **DSNB model-independent (binned analysis)**

→ **DSNB model-dependent (spectral analysis)**

The very first results with Gadolinium (0.01%)

- Quick analysis for proving power of neutron tagging with Gd
- 552.2 days with 0.01% Gd concentration (SK-VI)
- Cut-based neutron tagging $\Rightarrow$ 36% n-tag efficiency
- Requires $N_n = 1$ to select IBD candidates, dominant background is NCQE





- Close to 2970 days of pure water (SK-IV) $\rightarrow$ SK-Gd era is moving us along quickly!

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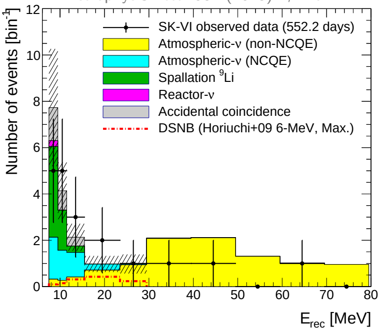

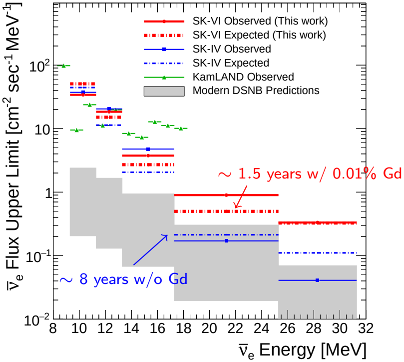

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Analysis improvement

- Additional 404 days with 0.03% Gd (SK-VII)

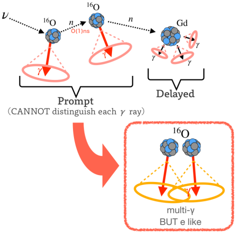
  ⇒ 956 days of SK-Gd data

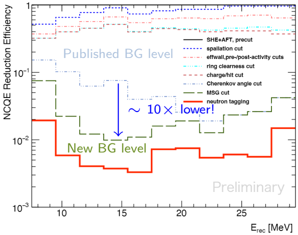
- New reduction for NCQE events that targets multi-cone events

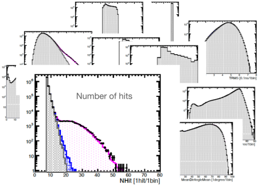
  ⇒ remove ∼ 90% of NCQE

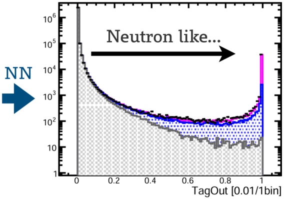
- Two machine learning based neutron tagging methods :
  - Boosted Decision Tree (already used in SK-IV)
  - Neural Network (new)

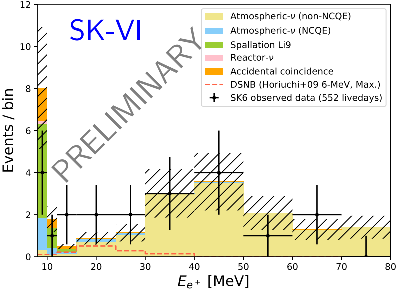
  ⇒ > 60% n-tag efficiency in SK-VII

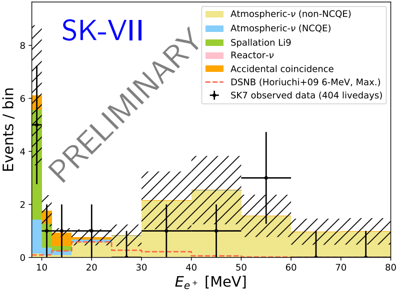


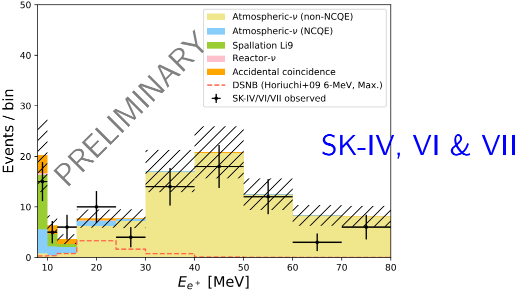




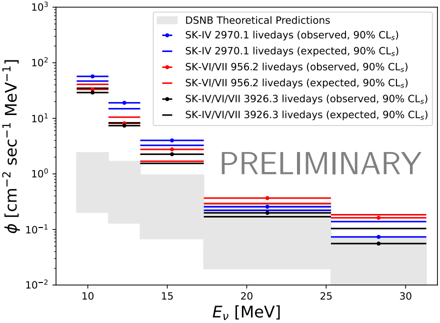



SK-VI
PRELIMINARY
Atmospheric-ν (non-NCQE)
Atmospheric-ν (NCQE)
Spallation Li9
Reactor-ν
Accidental coincidence
DSNB (Horiuchi+09 6-MeV, Max.)
SK6 observed data (552 livedays)
Events / bin
E_{e^+} [MeV]
0
2
4
6
8
10
12
10
20
30
40
50
60
70
80


SK-VII
PRELIMINARY
Atmospheric-ν (non-NCQE)
Atmospheric-ν (NCQE)
Spallation Li9
Reactor-ν
Accidental coincidence
DSNB (Horiuchi+09 6-MeV, Max.)
SK7 observed data (404 livedays)
Events / bin
E_{e^+} [MeV]
0
2
4
6
8
10
20
30
40
50
60
70
80


SK-IV, VI & VII
PRELIMINARY
Atmospheric-ν (non-NCQE)
Atmospheric-ν (NCQE)
Spallation Li9
Reactor-ν
Accidental coincidence
DSNB (Horiuchi+09 6-MeV, Max.)
SK-IV/VI/VII observed
Events / bin
E_{e^+} [MeV]
0
10
20
30
40
50
10
20
30
40
50
60
70
80

Binned analysis - Upper limit on DSNB flux



956 days of SK-Gd + 2970 days
Model independent analysis
Only use $N_n = 1$ events

Differential upper limit for
$9.3 < E_\nu < 31.3$ MeV
World stringent sensitivity

- $CLs \equiv p_{S+BG}/(1 - p_{BG})$ upper limits are conservative by construction

$$\phi_{90} = \frac{N_{90}}{\sigma_{\mathrm{IBD}}\, T\, \epsilon_{\mathrm{IBD}}\, N_p\, \Delta E}$$

- Limits are highly similar in the various energy bins for BDT / NN analysis
- Differences are driven either by neutron tagging technique and/or the evaluation of accidental coincidences
- This result takes into account the correlation between SK phases due to the same backgrounds and their predicted levels per bin
- In the near future, we expect appreciable sensitivity to a wide variety of models...

Spectral fitting analysis - Fit procedure

- We fit the number of observed events $(N_s, \vec{N}_b)$ that maximizes the following extended likelihood :

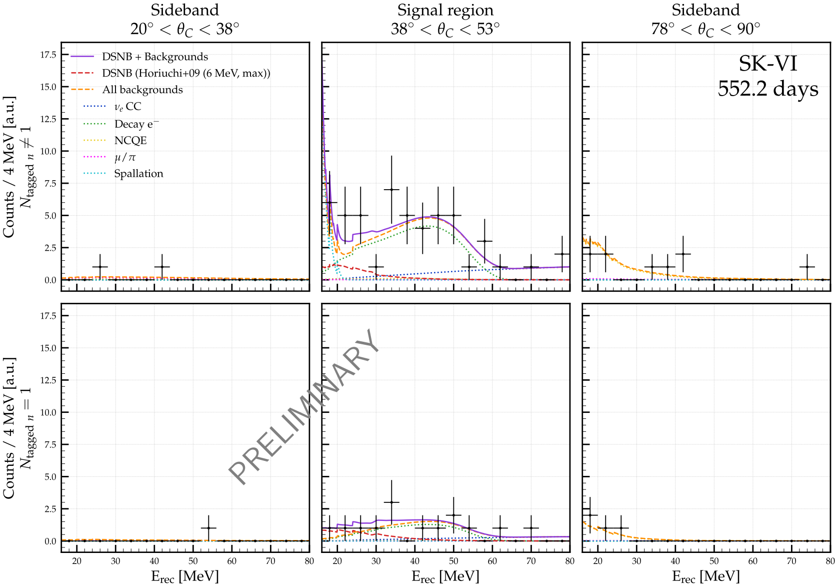
$$\mathcal{L}\left(\text{data}\,|\,N_s, \vec{N}_b, \vec{\varepsilon}\right) = \mathcal{L}(\vec{\varepsilon_0}|\vec{\varepsilon}) \times e^{-\sum_{j\in s+b} N_j} \times \prod_{i=1}^{N_{\text{data}}} \sum_{j\in s+b} N_j \cdot \text{PDF}_j\left(E^i, \theta_C^i, N^i_{\text{tagged}\,n}\,|\,\vec{\varepsilon}\right)$$

with 3 regions in θ_C and 2 regions in $N_{\text{tagged}\,n}$

- Systematics encoded as shape nuisance parameters $(\vec{\varepsilon})$:
  - $\varepsilon_{\text{CCQE}}$: ν_e CC spectrum in the signal region
  - $\varepsilon_{\text{spall}}$: spallation PDF in the signal region
  - $\varepsilon_{\text{NCQE}}$: relative normalization between signal / high-θ_C region
  - $\varepsilon_{\text{ntag}}$: relative normalization of spectra between the 2 n-tag regions

- Statistical approach chosen : frequentist inference

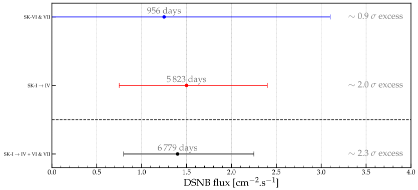
Spectral fitting analysis - SK-VI best fit

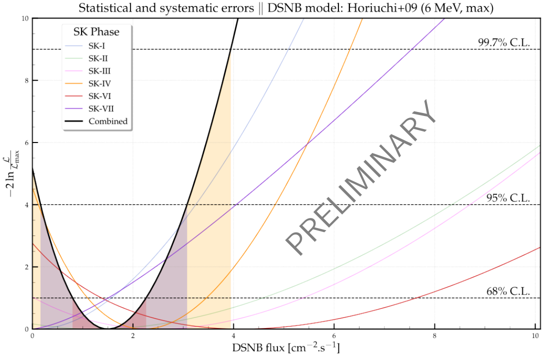


Spectral fitting analysis - Fit results (all phases + BDT)

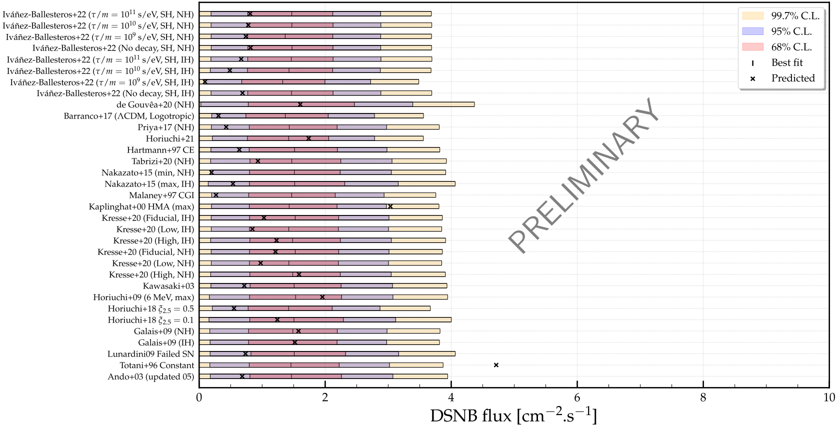
Combined Results

- DSNB (Horiuchi+09)
- Best fit rate : 2.9 events.year^{-1}
- 90% C.L. upper limit : 5.0 events.year^{-1}
- Best fit flux (> 17.3 MeV): 1.4 cm^{-2}.year^{-1}
- 90% C.L. upper limit : 2.5 cm^{-2}.year^{-1}
- Combined (stat. + syst.) $\sim$ 2.3 σ excess





Spectral fitting analysis - Confidence levels (SK I to IV, SK-VI to VII)



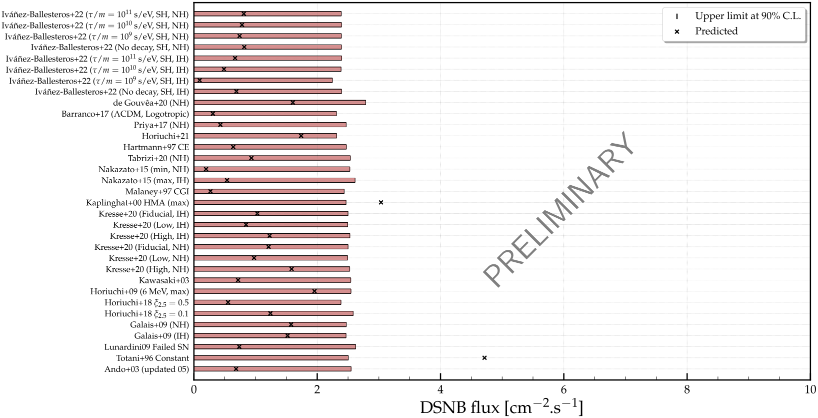
Currently dominated by statistical error,
model-dependent analysis is (almost) model-independent

Summary and perspectives

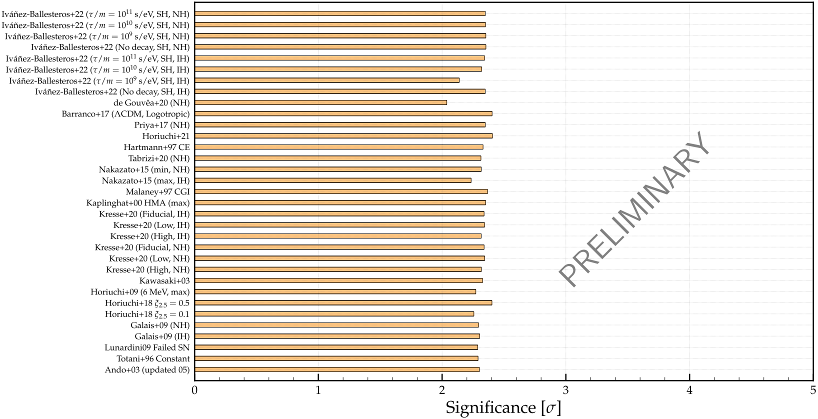
- DSNB flux carries valuable information about not only the supernova neutrino flux, but also the history of star formation, cosmic-expansion and neutrino properties
- Worldwide neutrino detectors sensitive to $\mathcal{O}(10)$ MeV running and foreseen to observe the DSNB
- Super-Kamiokande experiment published first results for the SK-Gd era in 2023
- Latest updates of DSNB search in SK-Gd presented @ Neutrino 2024 conference (and today !)
- No significant DSNB signal, however, some excess appears to be visible in the signal region : 2.3σ from the best-fit
- New work started to lower the analysis threshold, requires a consequent work on the spallation PDF
- Looking forward to discovery of DSNB in the next decade !!

BACK-UP SLIDES

Spectral fitting analysis - Upper limits



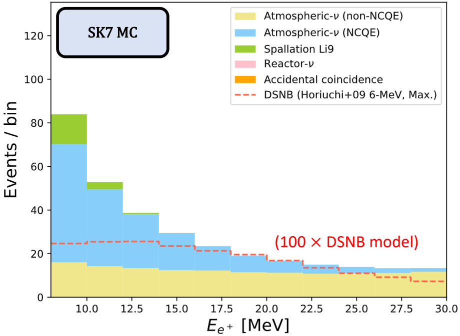
Spectral fitting analysis - Excess significance



Super-K DSNB analysis: Reduction steps

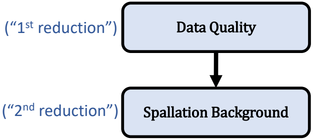
("1st reduction") **Data Quality**

- Only keep events that are **well reconstructed** by BONSAI (bsgood>0.5).
- Event **reconstruction becomes difficult near the walls** he detector (dwall>200cm).
- **Radon contamination** is concentrated near the detector walls (dwall+effwall).
- Focus on **events without OD activity** that would be associated with cosmic ray muons (trigger requirements).

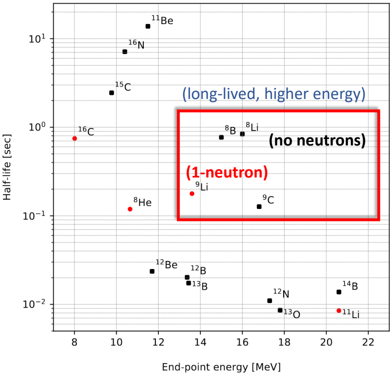


Reduction steps

Super-K DSNB analysis: Reduction steps



- Spallation **isotopes with longer half-lives are more difficult to reject** since it is harder to associate them to a parent muon.
- Isotopes without neutrons can be **largely rejected when tagging neutron** captures.
- **Li-9** produces a neutron in an IBD-like signal.



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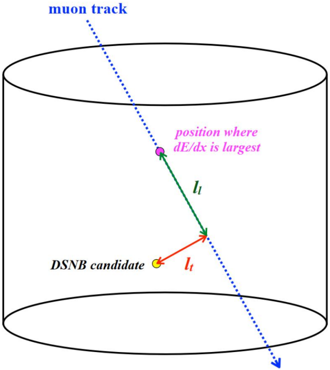
Super-K DSNB analysis: Reduction steps

("1st reduction") **Data Quality**

("2nd reduction") **Spallation Background**

- Remove **events within 1 ms** a muon passing through the tank (time cut).
- Remove **events within 4 m** to a low-energy event (multiple-spallation).
- Remove events identified within/close to a "**neutron cloud**" created by passing muons.
- Apply a set of energy-dependent **box cuts** and **spallation likelihood cuts**.



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Super-K DSNB analysis: Reduction steps

("1st reduction") **Data Quality**

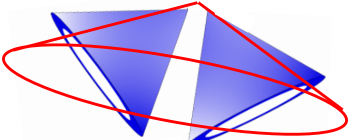
↓

("2nd reduction") **Spallation Background**

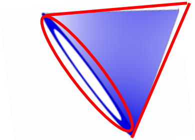
↓

("3rd reduction") **Atmospheric Background**

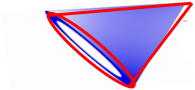
- Atmospheric neutrino interactions can produce **multi-cone prompt events**.
- Heavy charged leptons near the Cherenkov threshold have **small opening angles**.
- θ_c: Opening angle assuming one cone.



(multi-cone reconstructed as single-cone)



(IBD-like single-cone $\theta_c \approx 42°$)



(μ/π-like single cone near threshold)

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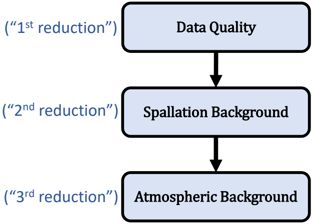
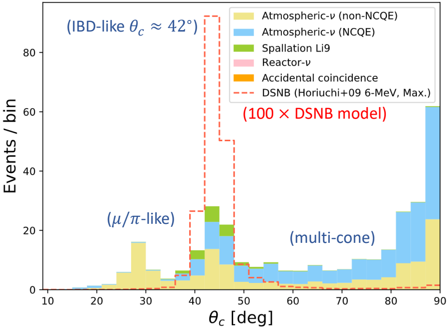
Super-K DSNB analysis: Reduction steps





- Atmospheric neutrino interactions can produce **multi-cone prompt events**.
- Heavy charged leptons near the Cherenkov threshold have **small opening angles**.
- θ_c: Opening angle assuming one cone.

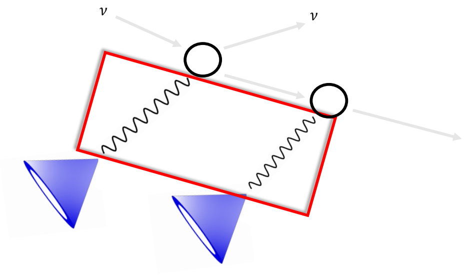
Super-K DSNB analysis: Reduction steps

("1st reduction") **Data Quality**

("2nd reduction") **Spallation Background**

("3rd reduction") **Atmospheric Background**

- Atmospheric neutrino interactions can produce **multi-cone prompt events**.



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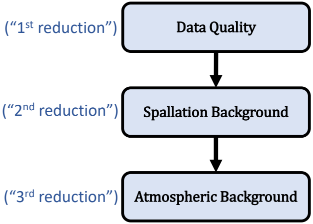
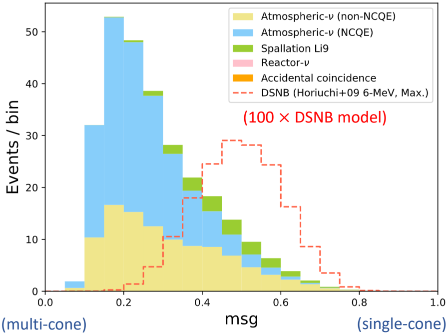
10

Reduction steps

Super-K DSNB analysis: Reduction steps



- Atmospheric neutrino interactions can produce **multi-cone prompt events**.
- Multiple scattering goodness (MSG): a repurposed variable capable of **distinguishing between likely single- and multi-cone events**.



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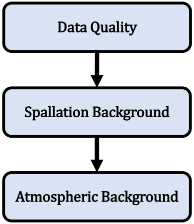
Reduction steps

Super-K DSNB analysis: Reduction steps

(“1st reduction”)

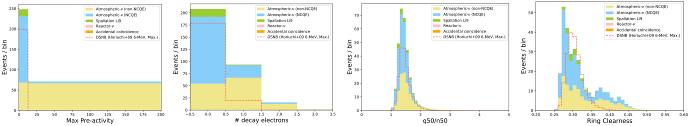
(“2nd reduction”)

(“3rd reduction”)



- IBD events will have **low PMT activity before the main prompt peak**, unlike some double-peak atmospheric neutrino backgrounds (maxpre).
- **μ/π decays** can be tagged as “**decay electrons**” (nmue).
- Heavy charged leptons will deposit **more charge-per-PMT** than IBD at higher energies (q50/n50).
- These particles will also create **clearer Cherenkov rings** than lighter $e^{\pm}$ (L_{clear}).

(see backup)



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Reduction steps

Super-K DSNB analysis: Reduction steps

("1st reduction") **Data Quality**

↓

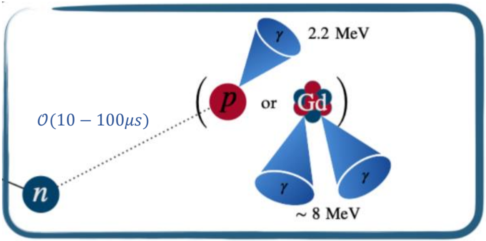
("2nd reduction") **Spallation Background**

↓

("3rd reduction") **Atmospheric Background**

↓

("4th reduction") **Neutron Tagging**



- The **Boosted Decision Tree (BDT)** neutron tagging tool was used in the 2021 DSNB paper and was updated for SK-Gd.
- The **Neural Network (NN)** tool has been introduced as a new technique in SK-Gd for the DSNB analysis.
- These two techniques are used in SK-Gd analysis.

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Super-K DSNB analysis: Reduction steps

("1st reduction") **Data Quality**

↓

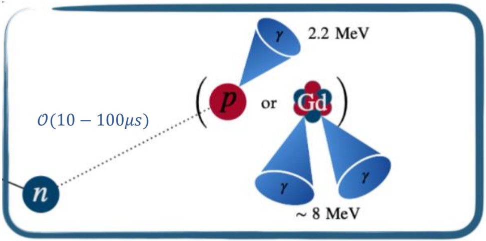
("2nd reduction") **Spallation Background**

↓

("3rd reduction") **Atmospheric Background**

↓

("4th reduction") **Neutron Tagging**



- **Both** the BDT and the NN **achieve less than 0.1% mistag rates** for neutron tagging **efficiencies above 60%** in SK7.
- Note: The **exact conditions/inputs** that bring the BDT/NN to these final performances are **not the same**.

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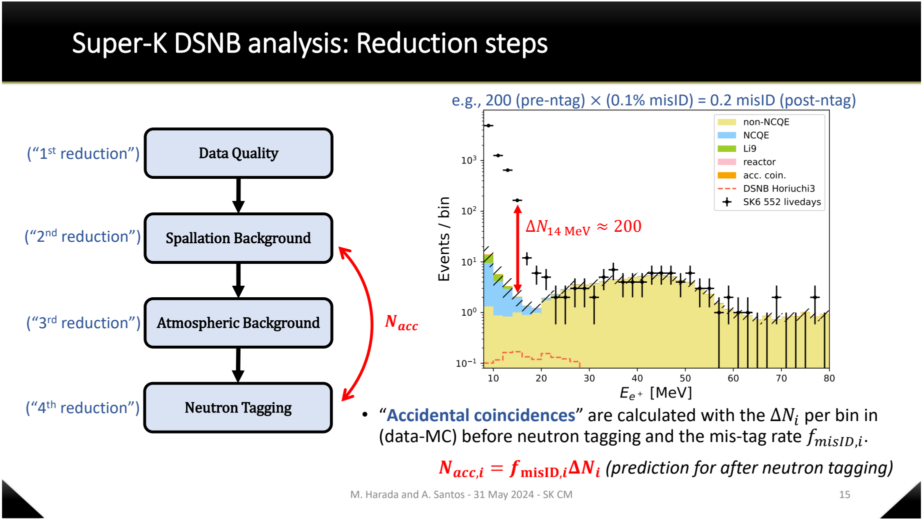
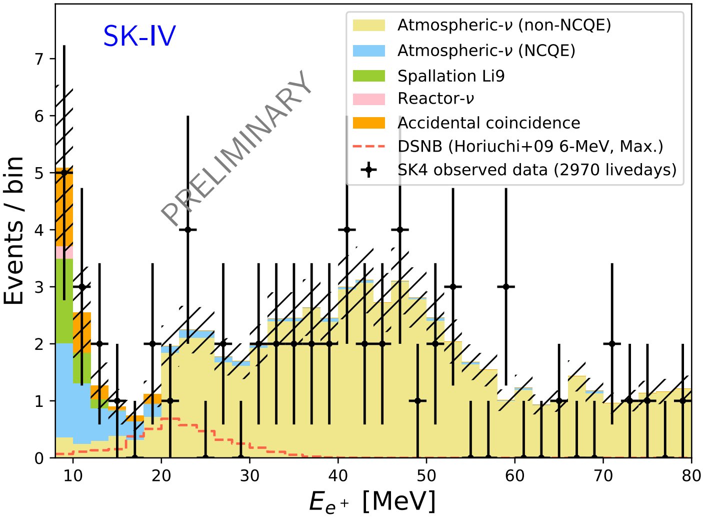
Reduction steps



Binned analysis - SK-IV energy spectrum



Comments on the likelihood

Construction of the likelihood :

$$\mathcal{L}\left(\text{data}\,|\,N_s,\vec{N}_b,\vec{\varepsilon}\right)=\prod_{i=1}^{N_{\text{data}}} PDF_{S+B}\left(E^i,\theta_C^i,N_n^i\,|\,\vec{\varepsilon}\right)$$

where

$$PDF_{S+B}\left(E^i,\theta_C^i,N_n^i\,|\,\vec{\varepsilon}\right)=\sum_{j\,\in\,S+B} f_j\cdot PDF_j\left(E^i,\theta_C^i,N_n^i\,|\,\vec{\varepsilon}\right)\quad\text{with}\quad\sum_j f_j=1\quad\text{(by definition)}$$

The maximization of this likelihood does not allow for determining the absolute normalisation. This can be overcome by using the extended likelihood (for N_{data} events, coming from Poisson distributions with mean N_j). We add following term in the likelihood

$$\frac{\left(\sum_j N_j\right)^{N_{\text{data}}} e^{-\sum_j N_j}}{N_{\text{data}}!}$$

We obtain

$$\mathcal{L}\left(\text{data}\,|\,N_s,\vec{N}_b,\vec{\varepsilon}\right)=\frac{\left(\sum_j N_j\right)^{N_{\text{data}}} e^{-\sum_j N_j}}{N_{\text{data}}!}\prod_{i=1}^{N_{\text{data}}}\sum_{j\,\in\,S+B} f_j\cdot PDF_j\left(E^i,\theta_C^i,N_n^i\,|\,\vec{\varepsilon}\right)$$

Disregarding the constant term $N_{\text{data}}!$ which has no influence in the maximization and moving the term $\left(\sum_j N_j\right)^{N_{\text{data}}}$ within the product, we obtain :

$$\mathcal{L}\left(\text{data}\,|\,N_s,\vec{N}_b,\vec{\varepsilon}\right)=e^{-\sum_j N_j}\prod_{i=1}^{N_{\text{data}}}\left(\sum_j N_j\right)\sum_{j\,\in\,S+B} f_j\cdot PDF_j\left(E^i,\theta_C^i,N_n^i\,|\,\vec{\varepsilon}\right)$$

By definition

$$f_j=\frac{N_j}{\sum_j N_j}\quad\Leftrightarrow\quad N_j=\left(\sum_j N_j\right)f_j$$

and therefore

$$\mathcal{L}\left(\text{data}\,|\,N_s,\vec{N}_b,\vec{\varepsilon}\right)=e^{-\sum_j N_j}\prod_{i=1}^{N_{\text{data}}}\sum_{j\,\in\,S+B} N_j\cdot PDF_j\left(E^i,\theta_C^i,N_n^i\,|\,\vec{\varepsilon}\right)$$

The last term is a penalty term in the likelihood to take into account our prior knowledge on the systematic parameters $\vec{\varepsilon}$. The final likelihood is :

$$\mathcal{L}\left(\text{data}\,|\,N_s,\vec{N}_b,\vec{\varepsilon}\right)=\mathcal{L}\left(\vec{\varepsilon}_0\,|\,\vec{\varepsilon}\right)e^{-\sum_j N_j}\prod_{i=1}^{N_{\text{data}}}\sum_{j\,\in\,S+B} N_j\cdot PDF_j\left(E^i,\theta_C^i,N_n^i\,|\,\vec{\varepsilon}\right)$$