# Degenerate Oscillation in Neutron Star


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Fu, **SFG**, Guo & Wang [arXiv:2405.08591]



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# Outline

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- Degenerate Oscillation
- Neutron-Antineutron Oscillation in NS
- Neutron Star Cooling & GUT

# Neutrino Oscillation

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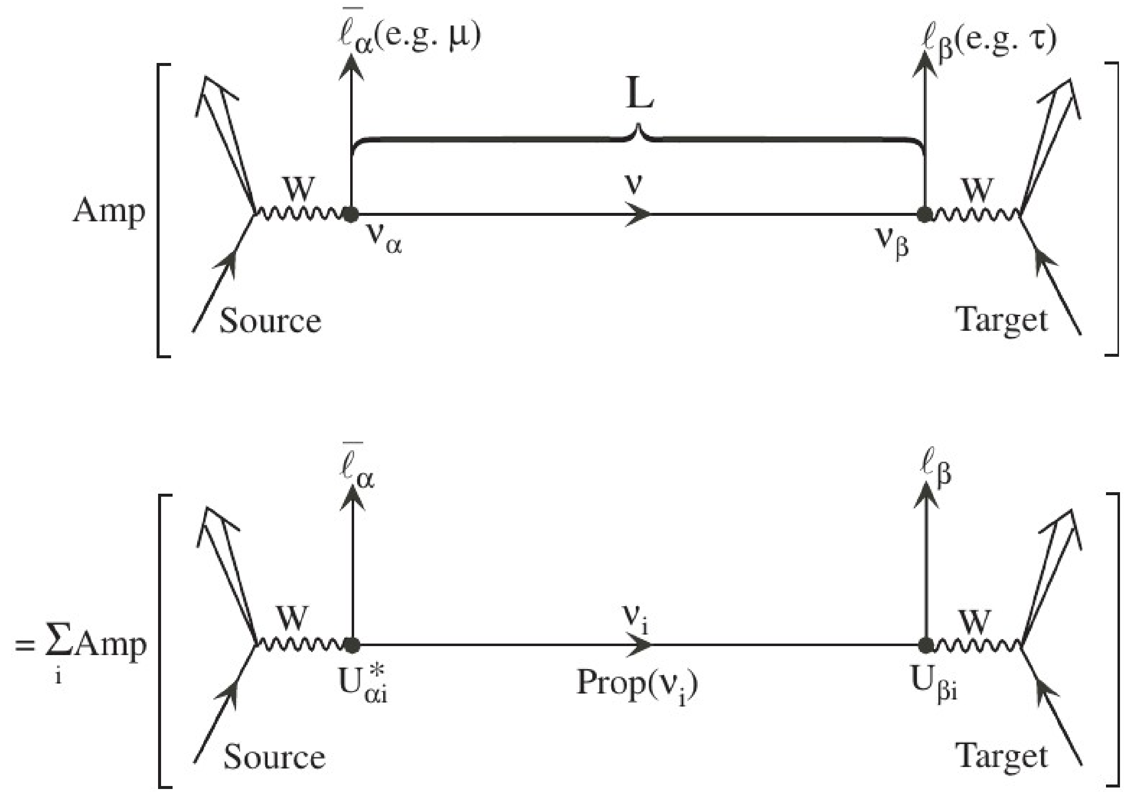



[Kayser, https://arxiv.org/abs/hep-ph/0506165]

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i \rightarrow \boxed{\sum_i U_{\alpha i} e^{i(E_i t - \vec{P}_i \cdot \vec{x})} \nu_i} = \boxed{\sum_i U_{\alpha i} P_i U^\dagger_{\beta i} \nu_\beta} \equiv \sum_\beta A_{\alpha\beta} \nu_\beta$$

# ν Oscillation in Matter



PHYSICAL REVIEW D VOLUME 17, NUMBER 9 1 MAY 1978

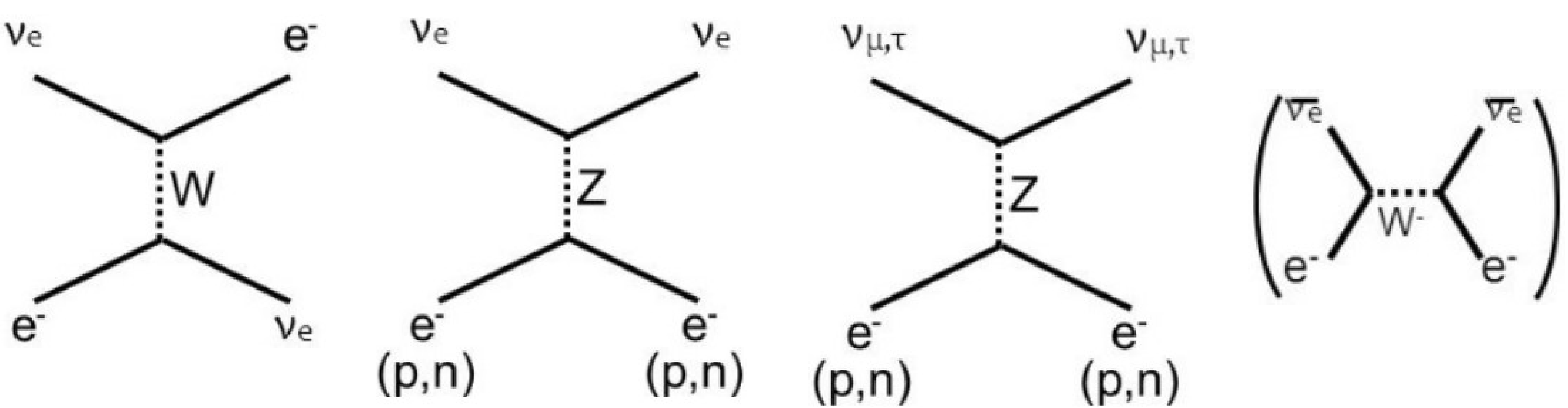
## Neutrino oscillations in matter

L. Wolfenstein
*Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213*

(Received 6 October 1977; revised manuscript received 5 December 1977)

The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.

$$\mathcal{H} = \frac{MM^\dagger}{2E_\nu} \pm V$$



# Collective Neutrino Oscillation

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$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x}\psi = \mathsf{H}\psi$$

$$\mathsf{H} = \frac{\Delta m^2}{2E}\mathsf{B} + \lambda\mathsf{L} + \mathsf{H}_{\nu\nu}$$

$$\mathsf{B} = \mathsf{U}\left(\frac{1}{2}\mathrm{diag}[-1,\, 1]\right)\mathsf{U}^\dagger = \frac{1}{2}\begin{bmatrix} -\cos 2\theta_\mathrm{v} & \sin 2\theta_\mathrm{v} \\ \sin 2\theta_\mathrm{v} & \cos 2\theta_\mathrm{v} \end{bmatrix}$$

$$= \frac{\Delta m^2}{2E}\mathsf{B} + \sqrt{2}G_\mathrm{F} n_e \mathsf{L} + \sqrt{2}G_\mathrm{F}\int \mathrm{d}^3\mathbf{p}'(1 - \hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')(\rho_{\mathbf{p}'} - \bar{\rho}_{\mathbf{p}'})$$

$$[\rho_{\mathbf{p}'}(t,\mathbf{x})]_{\alpha\beta} = \sum_{\nu'} n_{\nu',\mathbf{p}'}(t,\mathbf{x})\langle\nu_\alpha|\psi_{\nu',\mathbf{p}'}(t,\mathbf{x})\rangle\langle\psi_{\nu',\mathbf{p}'}(t,\mathbf{x})|\nu_\beta\rangle,$$

$$[\bar{\rho}_{\mathbf{p}'}(t,\mathbf{x})]_{\beta\alpha} = \sum_{\bar{\nu}'} n_{\bar{\nu}',\mathbf{p}'}(t,\mathbf{x})\langle\bar{\nu}_\alpha|\psi_{\bar{\nu}',\mathbf{p}'}(t,\mathbf{x})\rangle\langle\psi_{\bar{\nu}',\mathbf{p}'}(t,\mathbf{x})|\bar{\nu}_\beta\rangle,$$

Duan, Fuller & Qian [arXiv:1001.2799]

# Degeneracy with Boltzmann Eq.

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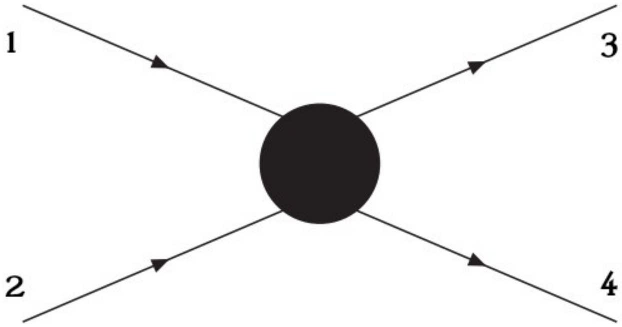
**Boltzmann Equation:**

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} + \frac{\partial f}{\partial \mathbf{p}}\frac{d\mathbf{p}}{dt} = \mathbb{C}[f]$$




$$\mathbb{C}[f] = \int \frac{d^3p_1}{(2\pi)^3 2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} \int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4)$$

$$|\mathcal{M}|^2 \{f_1 f_2 [1 \pm f_3][1 \pm f_4] - f_3 f_4 [1 \pm f_1][1 \pm f_2]\}$$

**Degeneracy is described by phase space factors:** $1 \pm f$

# Description with 2nd Quantization

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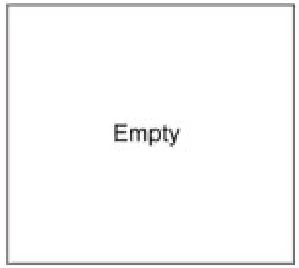
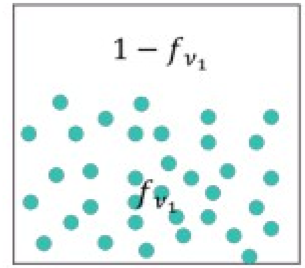



vs




$$|N\rangle \equiv \frac{(a^\dagger)^N}{\sqrt{N!}}|0\rangle$$

$$a^\dagger|N\rangle = \sqrt{N+1}|N+1\rangle$$

$$a|N\rangle = \sqrt{N}|N-1\rangle$$

$$a^\dagger a|N\rangle = N|N\rangle$$

$$aa^\dagger|N\rangle = (1 \pm N)|N\rangle$$

$$|\Omega\rangle \qquad f(\mathbf{x},\mathbf{p})$$

$$N \equiv \int f(\mathbf{x},\mathbf{p}) d^3\mathbf{x} d^3\mathbf{p}$$

$$n_\mathbf{p} \equiv \int f(\mathbf{x},\mathbf{p}) d^3\mathbf{x}$$

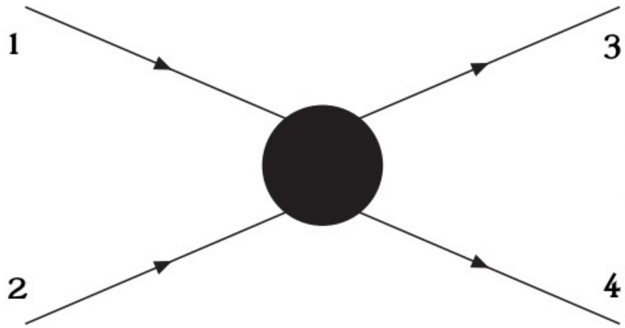
$$a^\dagger_\mathbf{p} \qquad a_\mathbf{p} \qquad \hat{n}_\mathbf{p} = a^\dagger_\mathbf{p} a_\mathbf{p}$$

$$a^\dagger_\mathbf{p} a_\mathbf{p}|\Omega\rangle = \int d^3\mathbf{x} f(\mathbf{x},\mathbf{p})|\Omega\rangle$$

$$a_\mathbf{p} a^\dagger_\mathbf{p}|\Omega\rangle = \int d^3\mathbf{x}[1 \pm \mathbf{x} f(\mathbf{x},\mathbf{p})]|\Omega\rangle$$

# Degeneracy with External State






$$|\mathcal{M}|^2\{f_1 f_2[1 \pm f_3][1 \pm f_4] - f_3 f_4[1 \pm f_1][1 \pm f_2]\}$$

**Initial State:**

$$\mathcal{M} \sim \langle \Omega + f | \cdots a | i + \Omega \rangle$$

$$|\mathcal{M}|^2 \sim \mathcal{M}^* \mathcal{M}$$

$$= \langle i + \Omega | \cdots a^\dagger a | i + \Omega \rangle$$

$$\Downarrow$$

$$f$$

**Final State:**

$$\mathcal{M} \sim \langle \Omega + f | a^\dagger \cdots | i + \Omega \rangle$$

$$|\mathcal{M}|^2 \sim \mathcal{M}^* \mathcal{M}$$

$$= \langle i + \Omega | a a^\dagger \cdots | i + \Omega \rangle$$

$$\Downarrow$$

$$1 \pm f$$

# Degenerate Oscillation

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## In QFT, fermion mixing is described as

$$\psi_i \equiv \sum_\alpha U^*_{\alpha i}\psi_\alpha \qquad \psi_i \sim a u e^{-ip_i\cdot x} + b^\dagger v e^{ip_i\cdot x}$$

**SFG**, Chui-Fan Kong, Pedro Pasquini [2310.04077]

## Fermion oscillation needs to involve 3 parts:

$$\mathcal{M}_{\beta\alpha} \equiv \mathcal{M}_d \left[\sum_i U_{\beta i}U^*_{\alpha i} e^{ip_i\cdot(x-y)} \langle\Omega|a_{\mathbf{p}i}a^\dagger_{\mathbf{p}i}|\Omega\rangle\right]\mathcal{M}_p$$

**Spinors u & v combined into $M_p$ & $M_d$**

$$1 - f_i$$

**Pauli blocking factor already appears in amplitude!**

# Degenerate Oscillation in QFT

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$$\mathcal{M}_{\beta\alpha} = \mathcal{M}_d \left[ \sum_i U_{\beta i} U^*_{\alpha i} e^{i p_i \cdot (x-y)} (1 - f_i) \right] \mathcal{M}_p$$

## Events detected w/o oscillation

$$N_\alpha(x = y) \propto \sum_\beta \left| \sum_i U_{\beta i} U^*_{\alpha i} (1 - f_i) \right|^2$$

## Events detected after oscillation

$$N_{\alpha\to\beta}(x - y) \propto \left| \sum_i U_{\beta i} U^*_{\alpha i} e^{i p_i \cdot (x-y)} (1 - f_i) \right|^2$$

## Fraction of events in β flavor

$$P_{\alpha\beta}(x - y) \equiv N_{\alpha\to\beta}(x - y) / N_\alpha(x = y)$$

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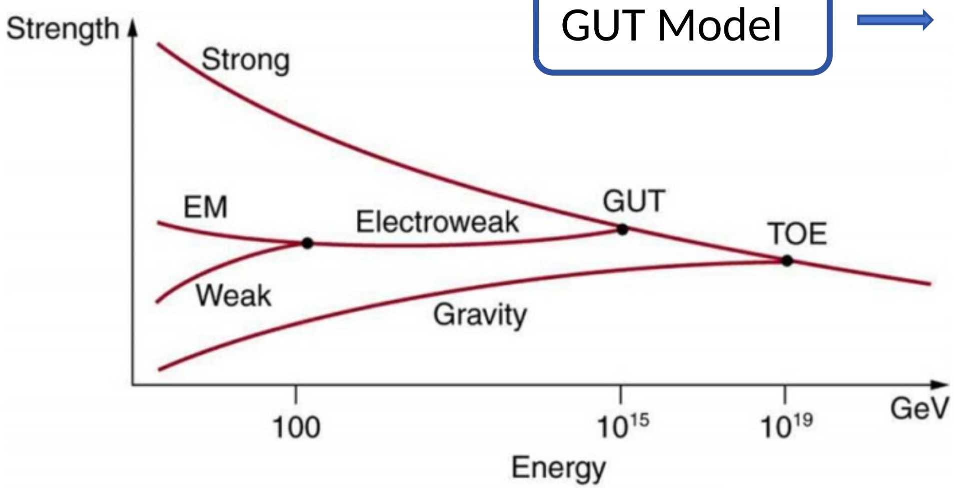
- **Neutron-Antineutron Oscillation in NS**
- **Neutron Star Cooling & GUT**

# GUT & Baryon Number Violation






GUT Model → Baryon Number Violation → { Proton Decay ; Neutron-Antineutron Oscillation }

Table 1

| GUT model | Is $N-\bar{N}$ observable? | Implications |
|---|---|---|
| (NON SUSY) | | |
| $SU(5)$ | No | $\Delta(B-L)=0$ |
| $SU(2)_L \times SU(2)_R \times SU(4)_c$ | Yes | $M_c \simeq 10^5$ GeV |
| Minimal SO(10) | No | |
| $E_6$ | No | |
| (SUSY GUT) | | |
| $[SU(3)]^3$ | Yes | Induced breaking of R-parity |
| $SO(10)$ | No | |

**Table Caption:** This table summarizes the observability of neutron-anti-neutron oscillation in various GUT models.

# Degenerate n-nbar Oscillation



**Being neutral, neutron can have Majorana mass term:**

$$\mathrm{H} \approx \begin{pmatrix} H_{11} & \delta m \\ \delta m & H_{22} \end{pmatrix}$$

## Mixing between neutron & antineutron

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix} = \mathrm{U} \begin{pmatrix} n \\ \bar{n} \end{pmatrix} \qquad \mathrm{U} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad \tan 2\theta = \frac{2\delta m}{H_{22} - H_{11}}$$

## Neutron-antineutron oscillation!

$$P_{n\bar{n}} = \frac{c^2 s^2 (1-f_1)^2 + c^2 s^2 (1-f_2)^2}{c^2 (1-f_1)^2 + s^2 (1-f_2)^2}$$

$$- \frac{2(1-f_1)(1-f_2) c^2 s^2 \cos(\Delta E t)}{c^2 (1-f_1)^2 + s^2 (1-f_2)^2} \qquad \Delta E \equiv \sqrt{(H_{11} - H_{22})^2 + 4\delta m^2}$$

# Degenerate n-nbar Oscillation

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$$P_{n\bar{n}} = \frac{c^2 s^2 (1-f_1)^2 + c^2 s^2 (1-f_2)^2}{c^2 (1-f_1)^2 + s^2 (1-f_2)^2}$$

$$- \frac{2(1-f_1)(1-f_2) c^2 s^2 \cos(\Delta E t)}{c^2 (1-f_1)^2 + s^2 (1-f_2)^2}$$

**reduces to the usual one with** $f_i \to 0$

$$P_{n\bar{n}} = 4 c^2 s^2 \sin^2 \left( \frac{\Delta E t}{2} \right)$$

**in a dense neutron environment** $f_1 \to 1, f_2 \to 0$

$1 - f_1 \ll s \ll 1$ $\quad$ $P_{n\bar{n}} \to c^2 \sim 1$ $\quad$ X

$s \ll 1 - f_1 \ll 1$ $\quad$ $P_{n\bar{n}} \to \dfrac{s^2}{(1-f_1)^2} \ll 1$ $\quad$ ✔

# Zero Distance Limit

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$$P_{n\bar{n}} \approx \frac{s^2}{(1-f_1)^2} \qquad P_{nn} \approx c^2$$

**The degeneracy effect already appears at zero distance!**

**Standing fraction of antineutron in neutron star.**

$$f_1(\mathbf{p}) = \frac{1}{e^{\frac{\varepsilon_n(\mathbf{p})-\mu}{T}}+1} \approx f_n$$

$$R(\mathbf{p}) \equiv \frac{f_{\bar{n}}}{f_n} = \frac{P_{n\bar{n}}}{P_{nn}} \approx \frac{\tan^2\theta}{[1-f_1(\mathbf{p})]^2} \qquad f_{\bar{n}} \approx \frac{s^2}{[1-f_1(\mathbf{p})]^2} f_1$$

**Note that only neutron is in thermal equilibrium!**

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# Neutron Degeneracy in NS

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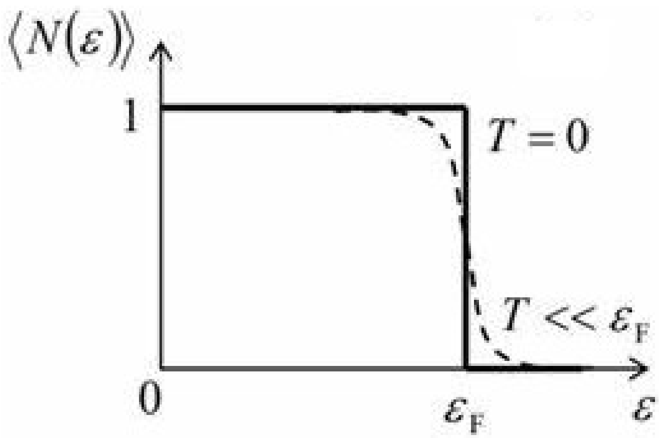
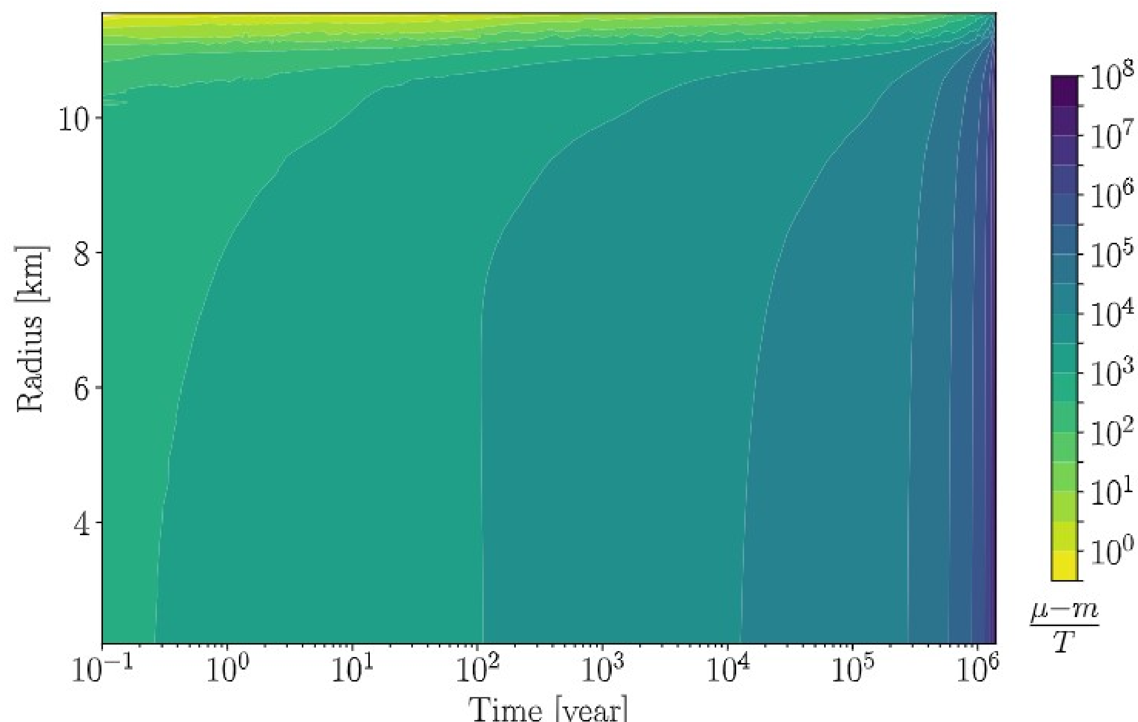
$$f_1(\mathbf{p}) = \frac{1}{e^{\frac{\varepsilon_n(\mathbf{p})-\mu}{T}}+1} \qquad \varepsilon_n(\mathbf{p}) = \sqrt{\mathbf{p}^2+m^2}$$

**Degenerate fermi gas:**

$$k_F = (3\pi^2 n_n)^{1/3} \qquad \mu = \sqrt{k_F^2+m^2}$$




Fu, **SFG**, Guo & Wang [arXiv:2405.08591]

# Antineutron Distribution

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$$f_{\bar{n}} \approx \frac{s^2}{[1 - f_1(p)]^2} f_1$$

$$n_{\bar{n}} = \frac{m^2 T}{2\pi^2} e^{\frac{\mu}{T}} \left[ 2K_2\left(\frac{m}{T}\right) + e^{\frac{\mu}{T}} K_2\left(\frac{2m}{T}\right) \right] s^2$$

**Huge enhancement**

$$\frac{\mu - m}{T} \sim \mathcal{O}(10^4) \quad \Rightarrow \quad e^{\frac{\mu - m}{T}} \sim 10^{\mathcal{O}(1000)}$$

# Antineutron Annihilation



**During a single annihilation, nucleon number reduces by 2**

$$\mathrm{d}N = -2\langle \sigma v \rangle n_n n_{\bar{n}} \mathrm{d}t \mathrm{d}V$$

**Replacing the antineutron number density** $n_{\bar{n}} = R n_n$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\frac{2R}{1+R} n_n \langle \sigma v \rangle N \equiv -\Gamma N$$

$$n_n = 10^{38}\,\mathrm{cm}^{-3}$$

$$\langle \sigma v \rangle \approx 10^{-15}\,\mathrm{cm}^3/\mathrm{s}$$

⇒ $$\Gamma \approx 2R \times 10^{23}\,\mathrm{s}^{-1}$$

**NS lifetime > $10^6$ yrs** ⇒ $$R \approx \frac{s^2}{[1 - f_1(\mathbf{p})]^2} \lesssim 10^{-37}$$

# NS Cooling Data

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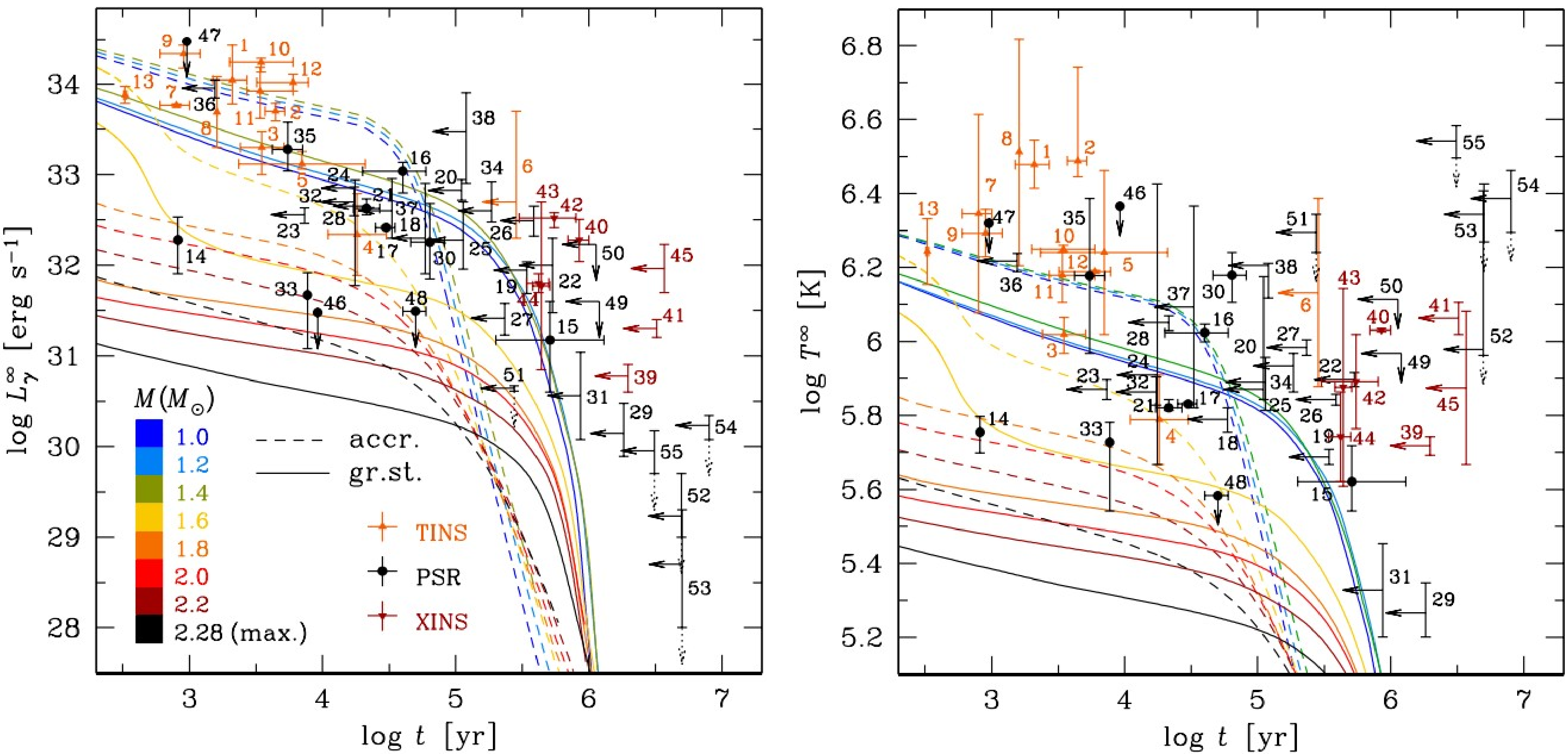
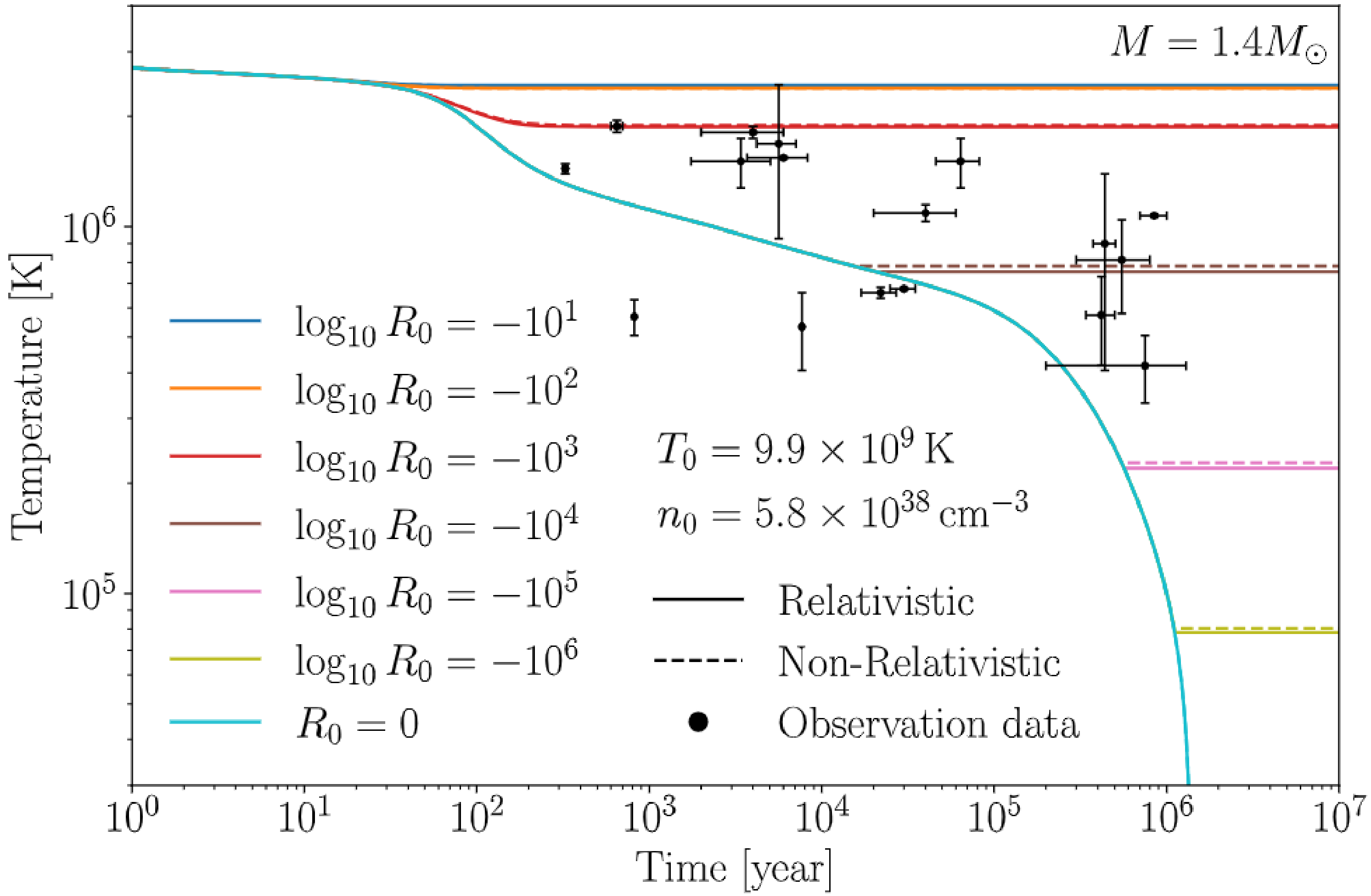



Potekhin, Zyuzin, Yakovlev, Beznogov & Shibanov
MNRAS 496, 5052-5071 (2020) [arXiv:2006.15004]

# NS Cooling




$$R \approx \left(\frac{T}{T_0}\right)^{\frac{3}{2}} e^{\frac{2(\mu - m)}{T} - \frac{2(\mu_0 - m)}{T_0}} R_0 \lesssim 10^{-43}$$

# NS Cooling

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$$R \approx \left(\frac{T}{T_0}\right)^{\frac{3}{2}} e^{\frac{2(\mu - m)}{T} - \frac{2(\mu_0 - m)}{T_0}} R_0 \lesssim 10^{-43}$$

$$\log_{10} R_0 \lesssim -\mathcal{O}(10^4) \qquad R_0 \equiv R(T_0, n_0)$$

$$T_0 \equiv 9.9 \times 10^9 \mathrm{K}$$

$$s^2 \lesssim 10^{-\mathcal{O}(10^4)}$$

$$n_0 \equiv 5.8 \times 10^{38} \mathrm{cm}^{-3}$$

**Heating power**

$$2Rn_n^2 \langle \sigma v \rangle V m \quad \sim 10^{24} \mathrm{W}$$

**Cooling power**

$$4\pi R^2 \sigma T^4 \sim 10^{21}\,\mathrm{W}$$

# Constraints on GUT

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$$\mathrm{H} \approx \begin{pmatrix} H_{11} & \delta m \\ \delta m & H_{22} \end{pmatrix} \qquad s \sim \frac{\delta m}{\Delta H}$$

$$\delta m \sim \frac{\Lambda_{\mathrm{QCD}}^6}{M_X^5} \qquad \Lambda_{\mathrm{QCD}} \sim 180\,\mathrm{MeV}$$

$$\Delta H = \mathcal{O}(\mathrm{MeV}) \sim \mathcal{O}(\mathrm{GeV})$$

$$s^2 \lesssim 10^{-\mathcal{O}(10^4)} \quad \Rightarrow \quad M_X > 10^{\mathcal{O}(1000)}\mathrm{GeV}$$

**which is far beyond the Planck scale!**

# Summary

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- **Degenerate Oscillation**

  1. Consistent picture of degeneracy in external & intermediate state

- **Neutron-Antineutron Oscillation in NS**

  1. Concrete realization with n-nbar oscillation
  2. Standing fraction of antineutron

- **Neutron Star Cooling & GUT**

  1. Degeneracy enhancement
  2. Very strong constraint

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# Thank You

# Superfluidity in NS