Spin-Flavor Precession Phase Effects in Supernova

Yamaç Pehlivan

MSGSÜ Physics Department

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Neutrino Magnetic Moment

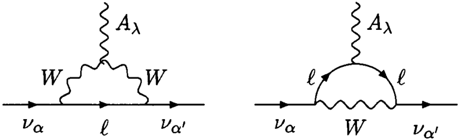
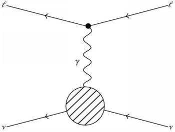


image credit: Mohapatra, 2004.

- From minimally extended standard model:

$$\mu_{\nu_\alpha} = 3.2 \times 10^{-19} \mu_B \left(\frac{m_{\nu_\alpha}}{eV} \right)$$



- Earth based limits from enhancement of $e - \overset{(-)}{\nu}_e$ scattering:
  - GEMMA collaboration (2013): $\mu_{\bar{\nu}_e} < 2.9 \times 10^{-11} \mu_B$
  - Giunti & Ternes (2023): $\mu_{\nu_e} < 1.3 \times 10^{-11} \mu_B$

- Astrophysics (cooling of Red giants through plasmon decay):
  - Raffelt (1990), Arceo-Diaz *et al* (2015): $\mu_{\bar{\nu}_e} < 2 \times 10^{-12} \mu_B$

Majorana neutrinos, two flavors

- Majorana neutrinos, two flavor picture
  - Negative helicity $\longrightarrow \nu_e$ and ν_x
  - Positive helicity $\longrightarrow \bar{\nu}_e$ and $\bar{\nu}_x$

- μB mixes *neutrino* and *antineutrino* degrees of freedom.

- Flavor diagonal component of μ identically vanish.

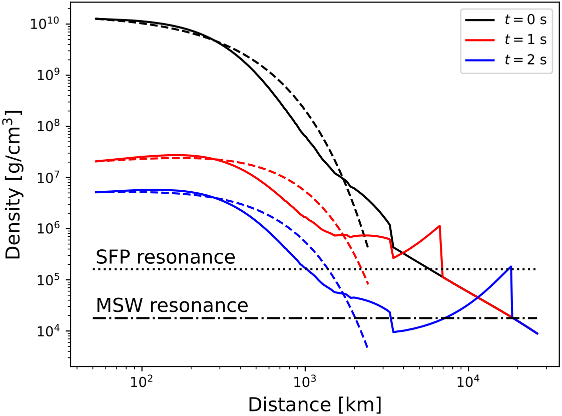
$$H_\mu = \mu B \left(|\nu_e\rangle \langle \bar{\nu}_x| + |\bar{\nu}_x\rangle \langle \nu_e| - |\nu_x\rangle \langle \bar{\nu}_e| - |\bar{\nu}_e\rangle \langle \nu_x| \right).$$

- B is the component perpendicular to neutrino momentum.

- Density matrix: $\rho = \left(\begin{array}{cc|cc} \rho_{ee} & \rho_{ex} & \rho_{e\bar{e}} & \rho_{e\bar{x}} \\ \rho_{xe} & \rho_{xx} & \rho_{x\bar{e}} & \rho_{x\bar{x}} \\ \hline \rho_{\bar{e}e} & \rho_{\bar{e}x} & \rho_{\bar{e}\bar{e}} & \rho_{\bar{e}\bar{x}} \\ \rho_{\bar{x}e} & \rho_{\bar{x}x} & \rho_{\bar{x}\bar{e}} & \rho_{\bar{x}\bar{x}} \end{array}\right)$

- Hamiltonian: $H(r) = \left(\begin{array}{cc|cc} \multicolumn{2}{c|}{H_{\nu\leftrightarrow\nu}(r)} & 0 & \mu B(r) \\ & & -\mu B(r) & 0 \\ \hline 0 & -\mu B(r) & \multicolumn{2}{c}{H_{\bar{\nu}\leftrightarrow\bar{\nu}}(r)} \\ \mu B(r) & 0 & & \end{array}\right)_{\text{flavor}}$

Supernova Model

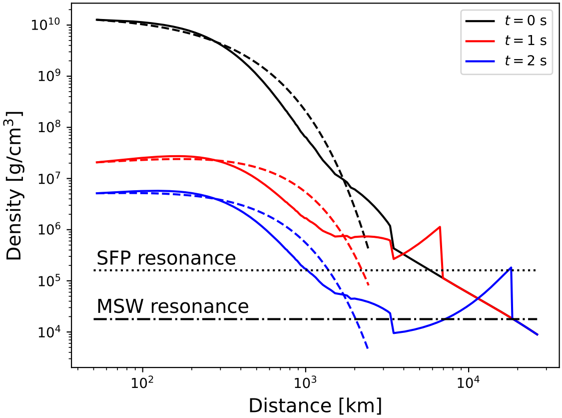


SFP resonance: $$\frac{\delta m^2}{2E_\nu}\cos 2\theta = \frac{\sqrt{2}G_F n(r)}{m_n}(1-2Y_e)$$

MSW resonance: $$\frac{\delta m^2}{2E_\nu}\cos 2\theta = \frac{\sqrt{2}G_F n(r)}{m_n}Y_e$$

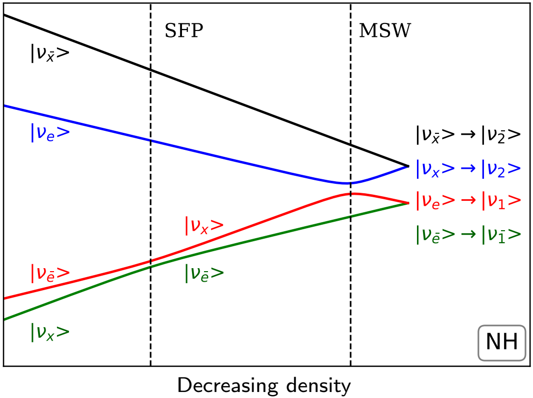
$Y_e = 0.45$ SFP resonance happens before the MSW resonance.

Supernova Model



- Solid lines:
  - $6M_\odot$ helium core presupernova model of Nomoto *et al* (1987)
  - Parametric shock *a la* Fogli *et al* (2003).
- Dotted lines: Best exponential fit to $n(r) = n_0 e^{-r/r_{\mathbf{mat}}}$
- Magnetic field profile: $B(r) = 10^{15}\, G \left(\frac{50\text{km}}{r}\right)^2$

SFP and MSW resonances



$$\tan 2\theta_B(r) = \frac{2\mu B(r)}{\frac{\delta m^2}{2E}\cos\theta - \frac{\sqrt{2}G_F n(r)}{m_n}(1-2Y_e)}$$

$$|r_3\rangle = \cos\theta_B(r)\,|\nu_x\rangle + \sin\theta_B(r)\,|\bar{\nu}_e\rangle$$

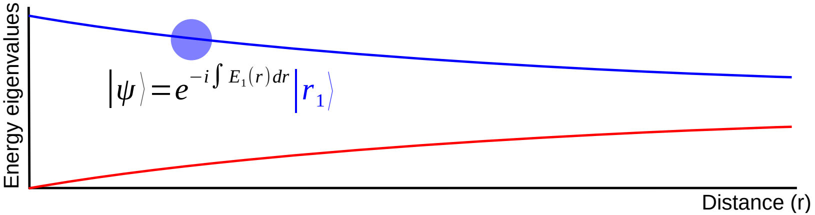
$$|r_4\rangle = -\sin\theta_B(r)\,|\nu_x\rangle + \cos\theta_B(r)\,|\bar{\nu}_e\rangle$$

$$\tan 2\theta_M(r) = \frac{\frac{\delta m^2}{2E_\nu}\sin 2\theta}{\frac{\delta m^2}{2E_\nu}\cos 2\theta - \frac{\sqrt{2}G_F n(r)}{m_n}Y_e}$$

$$|r_2\rangle = \cos\theta_M(r)\,|\nu_e\rangle + \sin\theta_M(r)\,|\nu_x\rangle$$

$$|r_3\rangle = -\sin\theta_M(r)\,|\nu_e\rangle + \cos\theta_M(r)\,|\nu_x\rangle$$

Adiabatic Evolution



- Adiabatic evolution is completely determined by the energy spectrum.

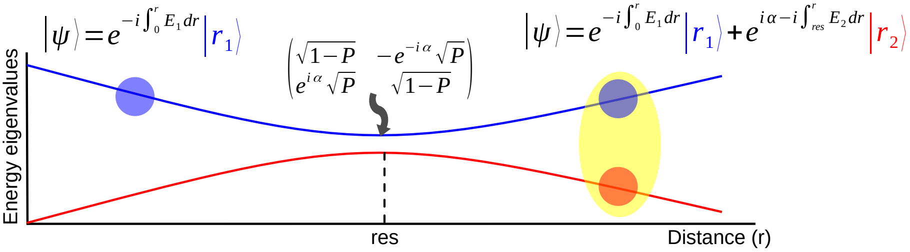
$$H(r)\,|r_1\rangle = E_1(r)\,|r_1\rangle \qquad H(r)\,|r_2\rangle = E_2(r)\,|r_2\rangle$$

- If a neutrino starts in an energy eigenstate, it stays in the same eigenstate

- Density operator in energy eigenbasis:

$$\rho(r) = |\psi\rangle\langle\psi| = |r_1\rangle\langle r_1| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_E$$

Partial violation of adiabaticity



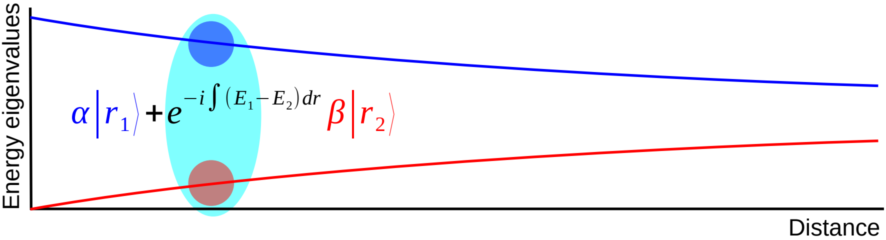
- A partial violation of adiabaticity
  - P = Landau-Zener jumping probability
  - α = Stoke's phase

- Density operator in energy eigenbasis:

$$\rho(r) = \begin{pmatrix} 1-P & \text{phases} \\ \text{phases} & P \end{pmatrix}_E \xrightarrow[\text{decoherence}]{r \longrightarrow \infty} \begin{pmatrix} 1-P & 0 \\ 0 & P \end{pmatrix}_{\text{mass}}$$

- Off-diagonal terms $\sim e^{-(r/r_{\text{coh}})^2}$ with $r_{\text{coh}} \sim 10^6$ km.

Phase Effect



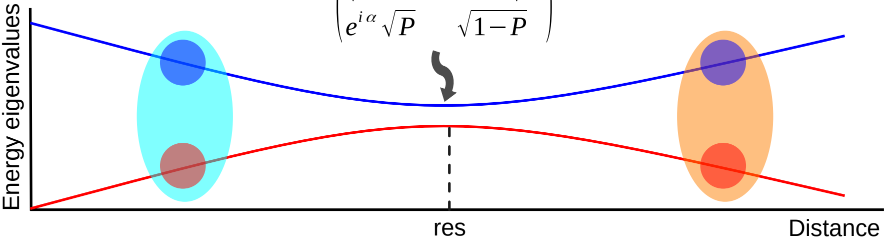
- The neutrino may already be in a superposition of two energy eigenstates before the resonance

- A relative phase is acquired by the components

$$\rho(r) = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* e^{-i\int(E_1-E_2)dr} \\ \alpha^*\beta e^{i\int(E_1-E_2)dr} & |\beta|^2 \end{pmatrix}_E$$

Phase Effect

$$\begin{pmatrix} \sqrt{1-P} & -e^{-i\alpha}\sqrt{P} \\ e^{i\alpha}\sqrt{P} & \sqrt{1-P} \end{pmatrix}$$



$$\rho(r) = \begin{pmatrix} |\alpha|^2(1-P)+|\beta|^2P+\sqrt{P(1-P)}\,\mathrm{Re}(\alpha\beta^* e^{i\phi}) & \dots \\ \dots & |\beta|^2(1-P)+|\alpha|^2P-\sqrt{P(1-P)}\,\mathrm{Re}(\alpha\beta^* e^{i\phi}) \end{pmatrix}_E$$

In the diagonal we have the phase

$$\phi = \alpha - \int_0^{\text{res}} (E_1(r) - E_2(r))dr$$

This phase depends very sensitively on neutrino energy and on external conditions.

$-1 \leq e^{-i\phi} \leq 1 \implies$ uncertainty in survival probabilities after decoherence.

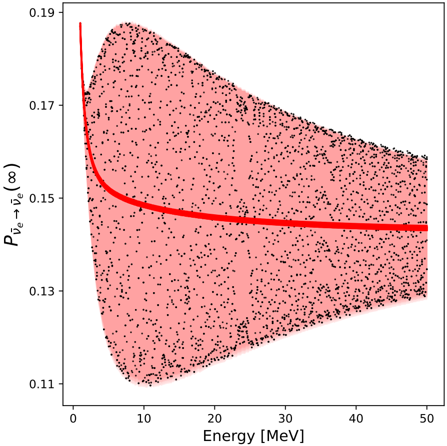
Appearance of phase effect in SFP ($\theta = 0$)

$$\rho(\infty) = \begin{pmatrix} |\alpha|^2(1-P)+|\beta|^2 P \pm \sqrt{P(1-P)}\,\mathrm{Re}(\alpha\beta^*) & 0 \\ 0 & |\beta|^2(1-P)+|\alpha|^2 P \mp \sqrt{P(1-P)}\,\mathrm{Re}(\alpha\beta^*) \end{pmatrix}_{\text{mass}}$$

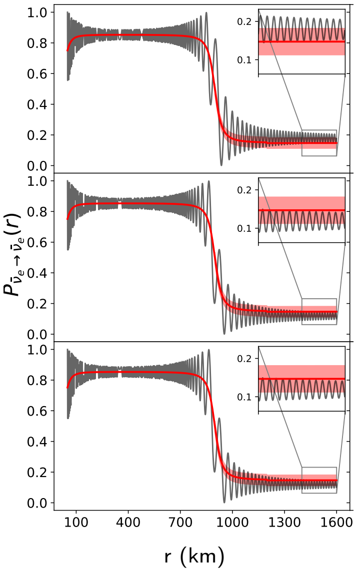
Survival probability at Earth:

$P_{\nu_\alpha \to \nu_\alpha}(\infty) =$ Classical probability $\pm$ phase effect

- $t = 5$s post-bounce.
- Red and $\pm$: Analytically expected
- Black dots: Numerical results with slightly varied ($\sim$ %0.1) conditions
- Detecting many neutrinos
  $\longrightarrow$ averaging
  $\longrightarrow$ ignore the phase effect



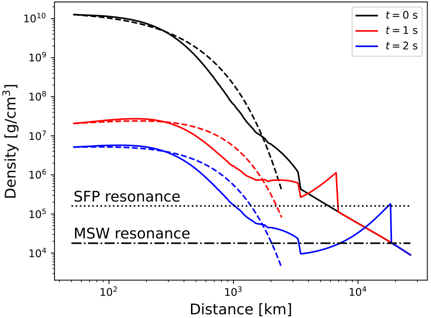
SFP phase effect appears with only one partially adiabatic SFP resonance



- Neutrinos are already born into superpositions of energy eigenstates if μB is large.

$$H(r) = \left(\begin{array}{cc|cc} \multicolumn{2}{c|}{H_{\nu\leftrightarrow\nu}(r)} & 0 & \mu B(r) \\ & & -\mu B(r) & 0 \\ \hline 0 & -\mu B(r) & \multicolumn{2}{c}{H_{\bar{\nu}\leftrightarrow\bar{\nu}}(r)} \\ \mu B(r) & 0 & & \end{array} \right)_{\text{flavor}}$$

- Phase effects associated with MSW resonances need two partially adiabatic MSW resonances.

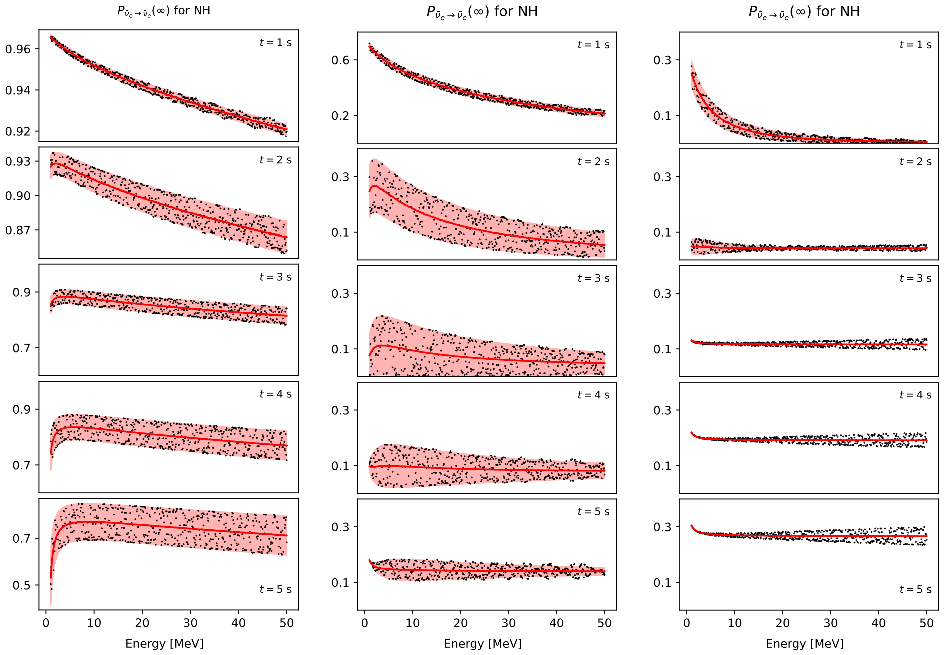


MSW and SFP resonances ($\theta \neq 0$)

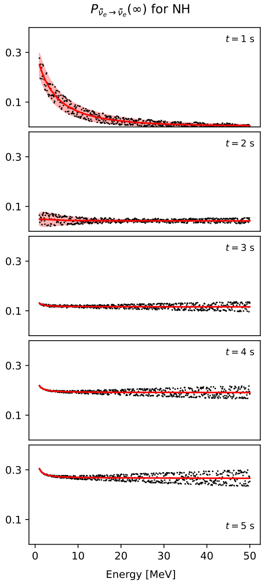
P_B = LZ jumping probability for SFP resonance

P_M = LZ jumping probability for MSW resonance

$$\rho(\infty) = \begin{pmatrix} \text{Classical} \\ \pm\text{ phase effects} & 0 & 0 & 0 \\ 0 & \begin{array}{l}[(1-P_M)\rho_{22}(0)+P_M((1-P_B)\rho_{33}(0)+P_B\rho_{44}(0))] \\ \pm 2\sqrt{(1-P_B)P_B}\,|\rho_{34}(0)|P_M \\ \pm 2\sqrt{(1-P_M)P_M}[\sqrt{P_B}\,|\rho_{24}(0)|+\sqrt{1-P_B}\,|\rho_{23}(0)|]\end{array} & 0 & 0 \\ 0 & 0 & \begin{array}{l}\text{Classical} \\ \pm\text{ phase effects}\end{array} & 0 \\ 0 & 0 & 0 & \begin{array}{l}\text{Classical} \\ \pm\text{ phase effects}\end{array} \end{pmatrix}_{\text{mass}}$$

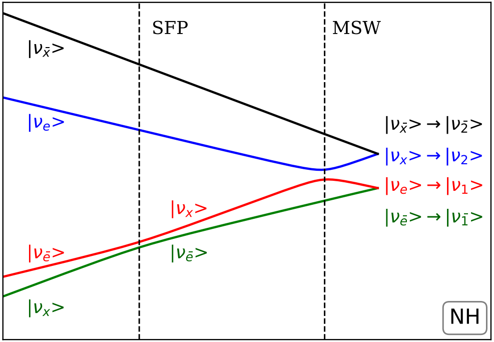


$\mu = 1 \times 10^{-16} \mu_B$ $\mu = 5 \times 10^{-16} \mu_B$ $\mu = 1 \times 10^{-15} \mu_B$



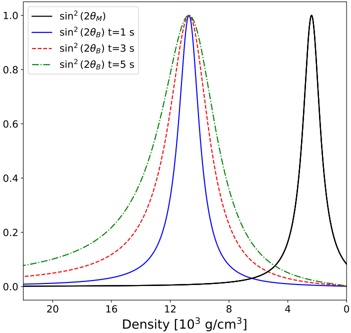
$\mu = 10 \times 10^{-16} \mu_B$

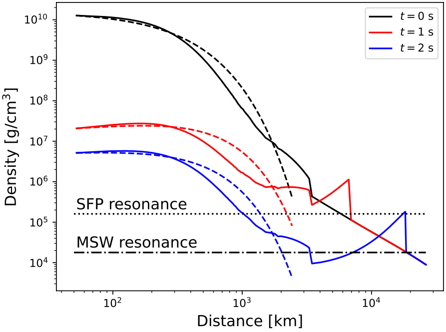
- For large μB, SFP resonance becomes adiabatic. Phase effect should be lost.
- But SFP broadens and overlaps with MSW resonance.
- Turns into a three level QM problem.



Decreasing density

MSW resonance is universal, SFP resonance is not



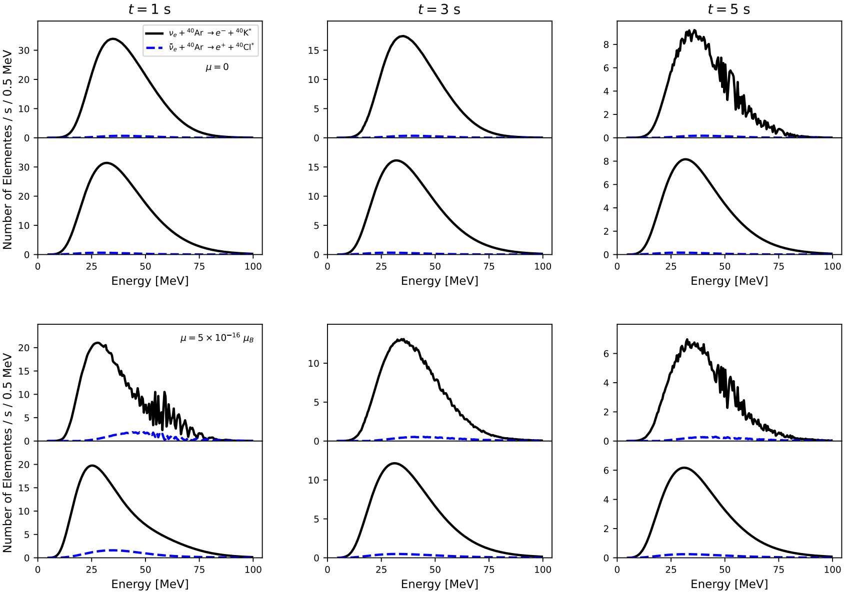


$$\sin^2 2\theta_M = \frac{\left(\frac{\delta m^2}{2E_\nu}\sin 2\theta\right)^2}{\left(\frac{\delta m^2}{2E_\nu}\sin 2\theta\right)^2+\left(\frac{\delta m^2}{2E_\nu}\cos 2\theta - \frac{\sqrt{2}G_F n}{m_n}Y_e\right)^2}$$

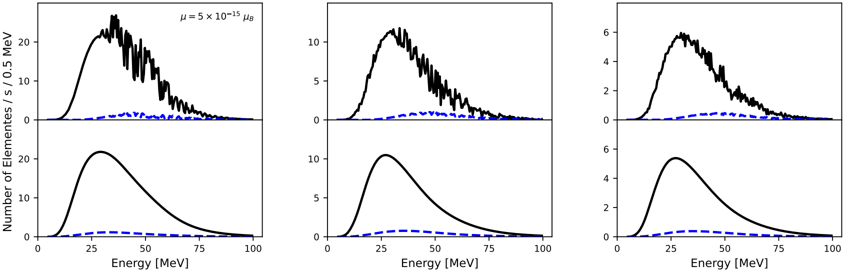
$$\sin^2 2\theta_B = \frac{(2\mu B)^2}{(2\mu B)^2+\left(\frac{\delta m^2}{2E}\cos\theta - \frac{\sqrt{2}G_F n}{m_n}(1-2Y_e)\right)^2}$$

- At larger post-bounce times both resonances move inward.
- MSW resonance width is unaffected (universal)
- SFP resonance becomes wider with stronger magnetic field

Observability of phase effects: DUNE event rates



Observability of phase effects: DUNE event rates



Conclusions:

- SFP phase effects appear earlier than MSW phase effects.
- May appear even when all resonances are seemingly "adibatic."
- Investigation of coupling effects $\Longrightarrow$ observability?
- Changing electron fraction?
- Coupling with collective oscillations?