Solar models, solar neutrinos and helioseismology

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(*) partly based on work done in collaboration with **Aldo Serenelli (ICE and IEEC, BCN)**

The Standard Solar Model (SSM)

Our comprehension of the Sun is based on the **Standard Solar Model (SSM).**
This implies:

✓ Stellar structure equations;
(α = mixing length)

✓ Chemical evolution paradigm:
ZAMS homogenous model (Y_{ini} , Z_{ini})
Nuclear reactions + elemental diffusion

✓ Knowledge of the properties of solar plasma
(i.e. opacity, equation of state, nuc. cross sections);

No free parameters
The unknown quantities
- α, Y_{ini} , Z_{ini} ,
are fixed by requiring that the present Sun (t_{sun}=4.57 Gyr) reproduces its observational properties
- R_{sun} , L_{sun} , $(Z/X)_{Surf}$

The Standard Solar Model (SSM)

The predictions of SSMs can be **falsified** by other observations. e.g.:

- *Solar neutrinos:*

Hydrogen fusion in the solar core produce a huge amount of neutrinos that can be measured in suitable detectors (Davis 1964, Bahcall 1964)

$4H + 2e^- \rightarrow {}^4He + 2\nu_e + \text{energy}$

Solar Neutrino Problem
Nuclear energy generation (cross sections, etc.)

- *Helioseismology:*

Solar oscillations originally discovered by Leighton at al. 1962 and interpreted as standing acoustic waves

Elemental Diffusion
Opacity, EoS, …

Constant improvement in SSM constitutive physics was triggered during last decades by solar neutrino and helioseismic data.

Helioseismology

The Sun is a non radial oscillator. The observed oscillation frequencies can be used to determine the properties of the Sun. Linearizing around a known solar model:

$$\frac{\delta\nu_{nl}}{\nu_{nl}} = \int_0^R dr\ K^{nl}_{u,Y}(r)\,\frac{\delta u}{u}(r) + \int_0^R dr\ K^{nl}_{Y,u}(r)\,\delta Y + \frac{F(\nu_{nl})}{\nu_{nl}}$$

squared isothermal sound speed

Related to temperature stratification in the sun

surface helium abundance

See Basu & Antia 07 for a review

Impressive agreement with SSM predictions …

Surface helium abundance

$$Y_b = 0.2485 \pm 0.0035$$

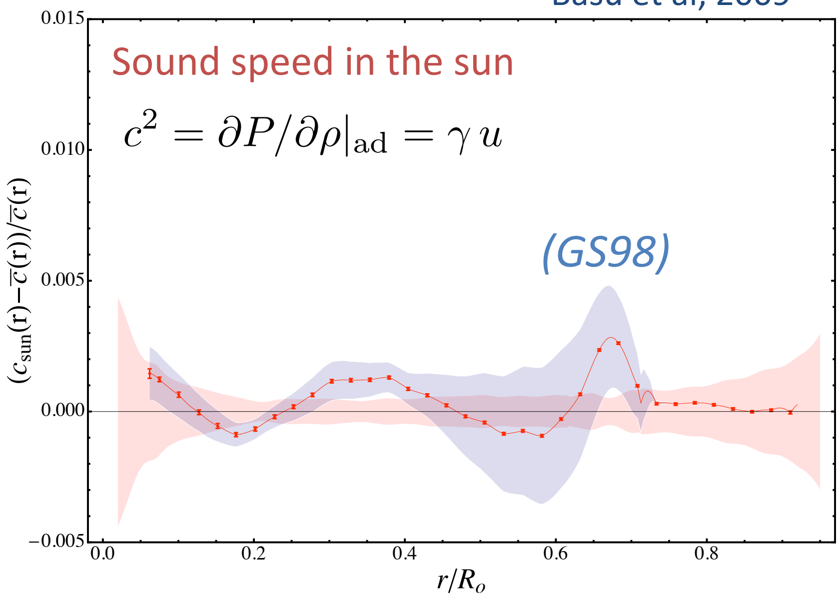
$$Y_b = 0.243 \quad \textit{(GS98)}$$

Inner radius of the solar convective envelope

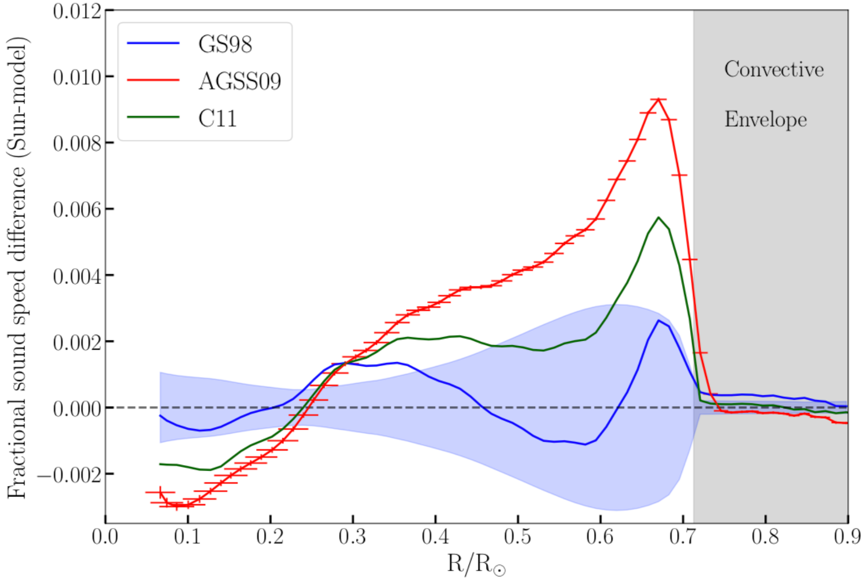
$$R_b/R_\odot = 0.713 \pm 0.001$$

$$R_b/R_\odot = 0.712 \quad \textit{(GS98)}$$

Basu et al, 2009



… till few years ago



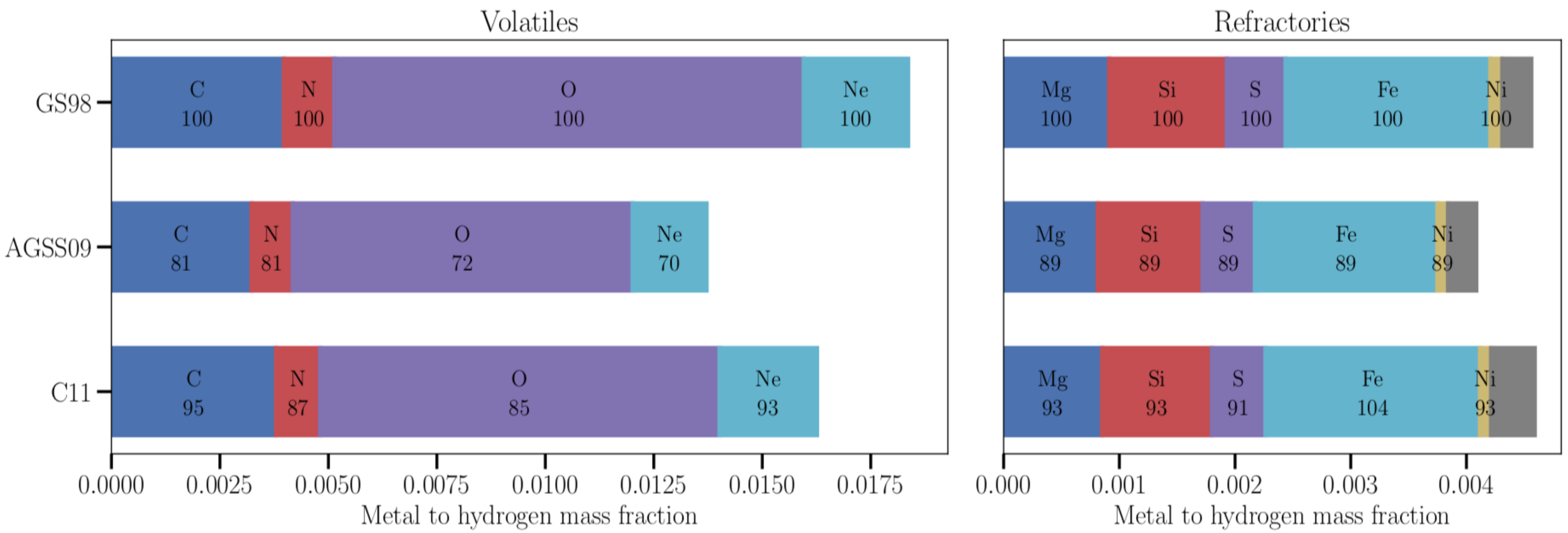
Downward revision of solar surface abundances

Solar surface composition is a fundamental input for SSMs → determined with spectroscopic techniques (3D models of solar atmosphere, NLTE corrections, …)

1998
1D - LTE

2009
3D – LTE/NLTE

2011
3D – LTE/NLTE



Orebi Gann et al. 2021/2022

GS98: Grevesse & Sauval 1998 – AGSS09: Asplund, Grevesse et al. 2009 – C11: Caffau et al. 2011

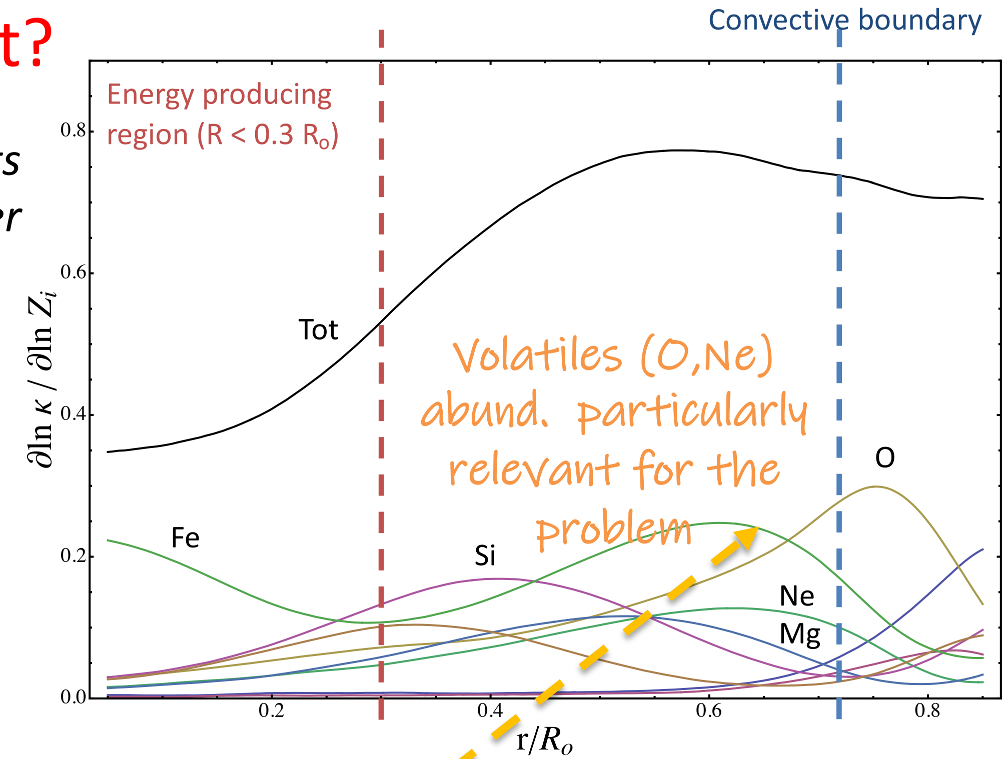
Why metals are so important?

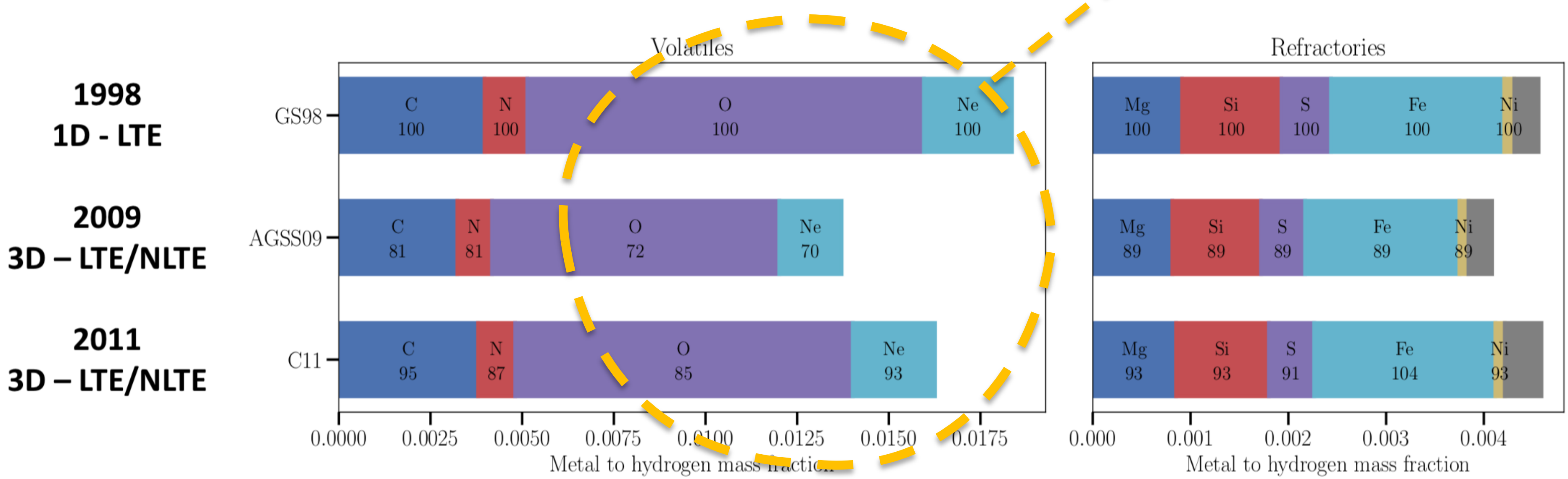
A change of the solar composition affects the efficiency of radiative energy transfer in the core of the Sun

Composition opacity change:

$$\delta\kappa(r) = \sum_j \frac{\partial \ln \kappa(r)}{\partial \ln Z_j}\, \delta z_j$$

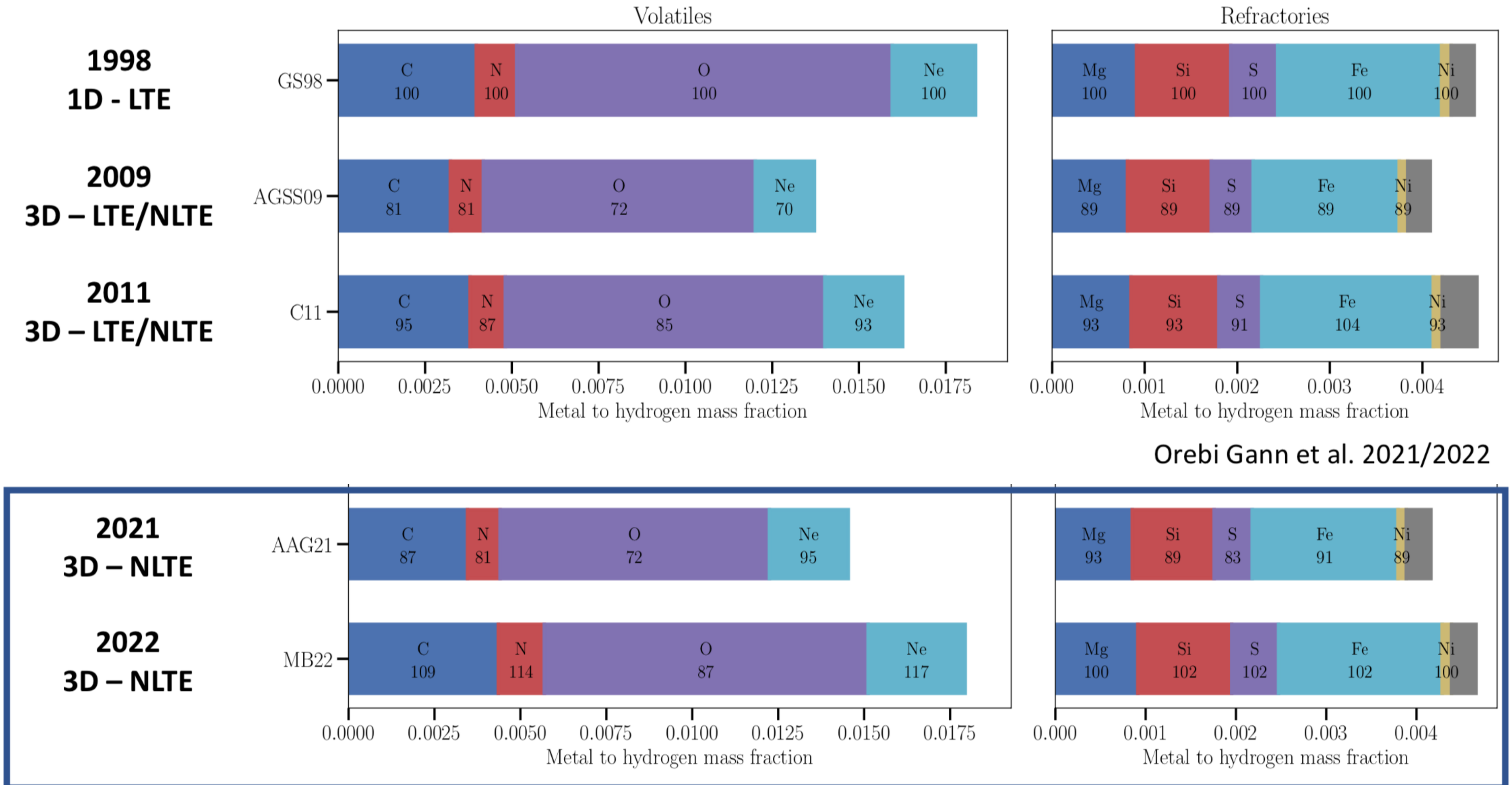
Different temperature stratification





Orebi Gann et al. 2021/2022

Updates in solar abundances



Orebi Gann et al. 2021/2022

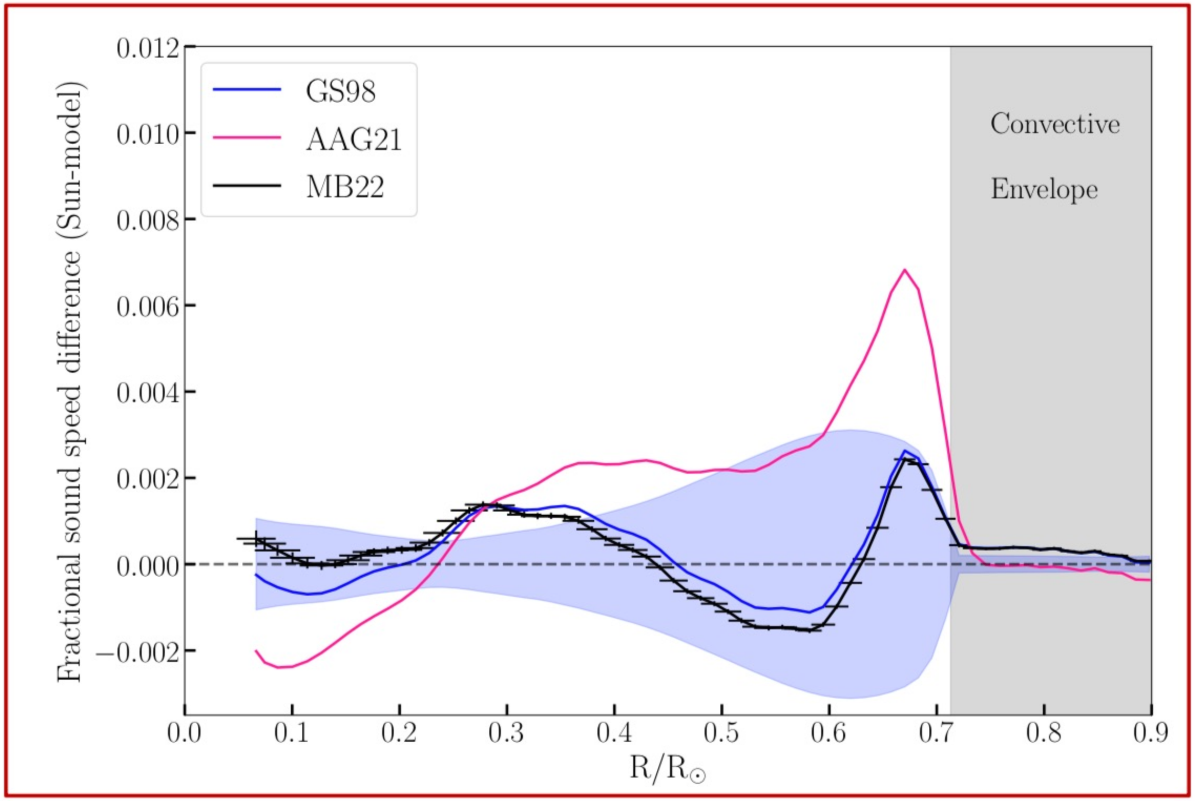
AAG21: Asplund, Amarsi & Grevesse 2021 – MB22: Magg, Bergemann et al. 2022

Solar composition “dichotomy” still persists but now based on 3D NLTE abundances

Helioseismic results

Model	$R_{CZ}/R_{\odot}$	Y_S
MB22-phot	0.7123	0.2439
MB22-met	0.7120	0.2442
AAG21	0.7197	0.2343
AGSS09-met	0.7231	0.2316
GS98	0.7122	0.2425
C11	0.7162	0.2366

Situation in 2022



Helioseismic determinations

$$R_b/R_{\odot} = 0.713 \pm 0.001$$

$$Y_b = 0.2485 \pm 0.0035$$

Magg et al. 2022

HZ surface composition provide a better description of helioseismic data

Can we conclude that LZ abundances are wrong?

The interpretation is complicated by the **opacity-composition degeneracy**.

$$\delta\kappa(r) = \delta\kappa_{\rm I}(r) + \sum_j \frac{\partial \ln \kappa(r)}{\partial \ln Z_j}\delta z_j$$

Intrinsic opacity change (e.g. opacity table "errors")

Composition opacity change

Q. Is opacity of the solar plasma sufficiently well calculated?

Note that the **shape** of **opacity variation** may be **as important as** its overall **magnitude**

$$\delta Q = \int dx\, K_Q(x)\, \delta\kappa(x)$$

Fractional variation of observable quantity Q

Fractional variation of opacity at a given point x

Q = generic observable (e.g. surf. helium, conv. radius, sound speed, ν fluxes)

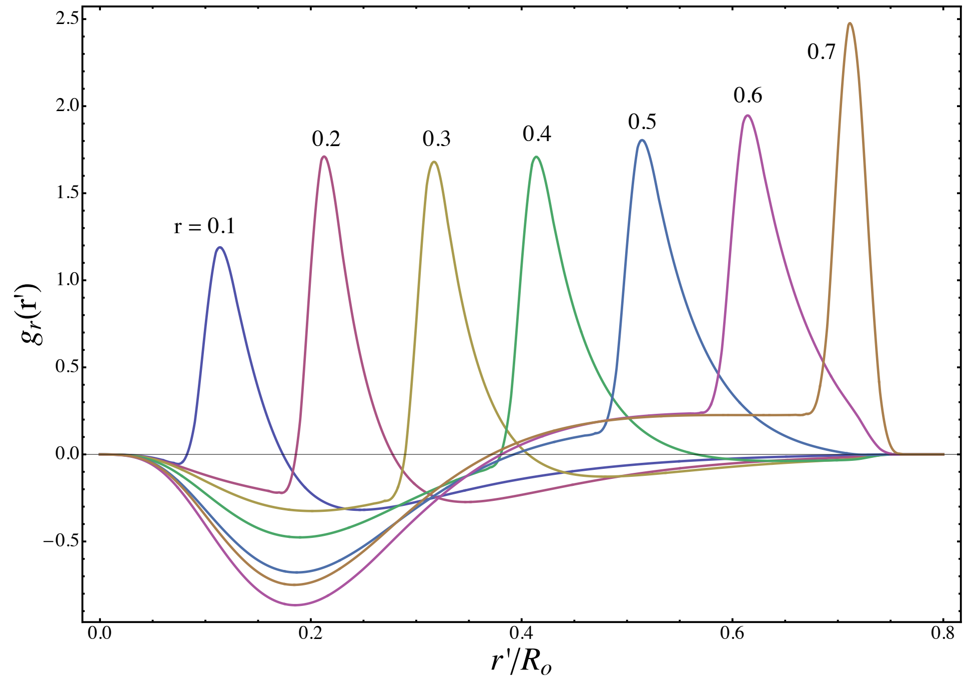
$x = \frac{r}{R_o}, \ln\left(\frac{T}{T_c}\right), \ldots$

$K_Q(x) =$ Opacity Kernel

Tripathy & Christensen-Dalsgaard, 1998 (static or w/o diffusion)
Villante et al. 2010 (Linear Solar Models)
Villante, 2010 (LSM, diffusion)
Vinyoles et al, 2017 (Full evolut. Models)

The sound speed kernels

$$\delta u(r) = \int dr' \; K_u(r, r') \; \delta\kappa(r')$$



The kernels are not positive definite → compensating effects can occur …

$$\delta u_0(r) = \int dr' K_u(r, r') \simeq 0$$

The sound speed is *insensitive to a global rescaling of opacity*

$$\frac{GMm_u}{R} \sim \frac{k_B T}{\mu} = \frac{P}{\rho} = u$$

Tripathy & Christensen-Dalsgaard, 1998 (static or w/o diffusion)
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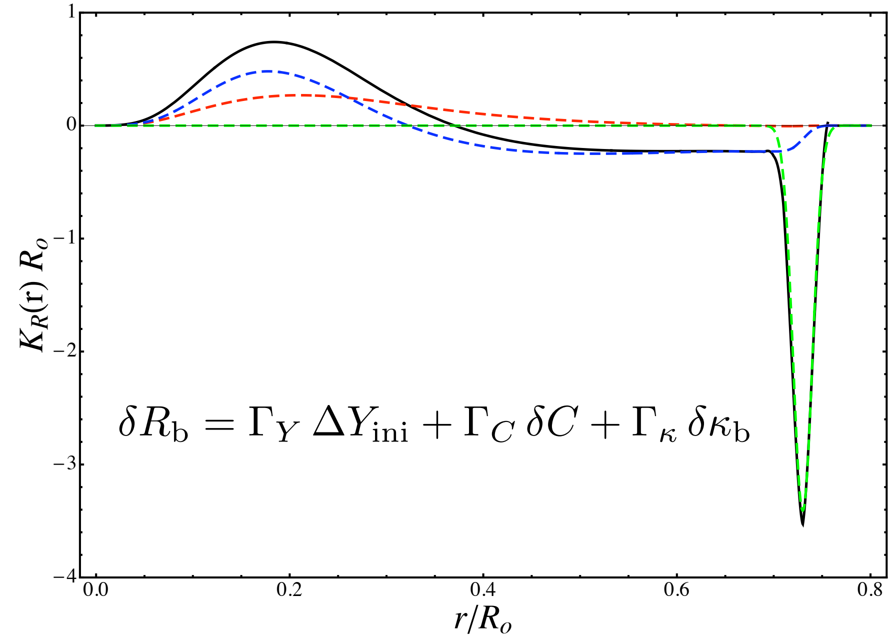
The convective radius and the surface helium abundance

Convective radius:

$$\delta R_{\rm b} = \int dr\; K_{\rm R}(r)\; \delta\kappa(r)$$

$$\begin{aligned} \delta R_{\rm b} &= 0.12\; A_{\rm in} - 0.14\; A_{\rm out} \\ &\simeq 0.13\; (A_{\rm in} - \; A_{\rm out}) \end{aligned}$$

$$\delta R_{\rm b} = -0.02\; A_0 - 0.10\; A_1$$

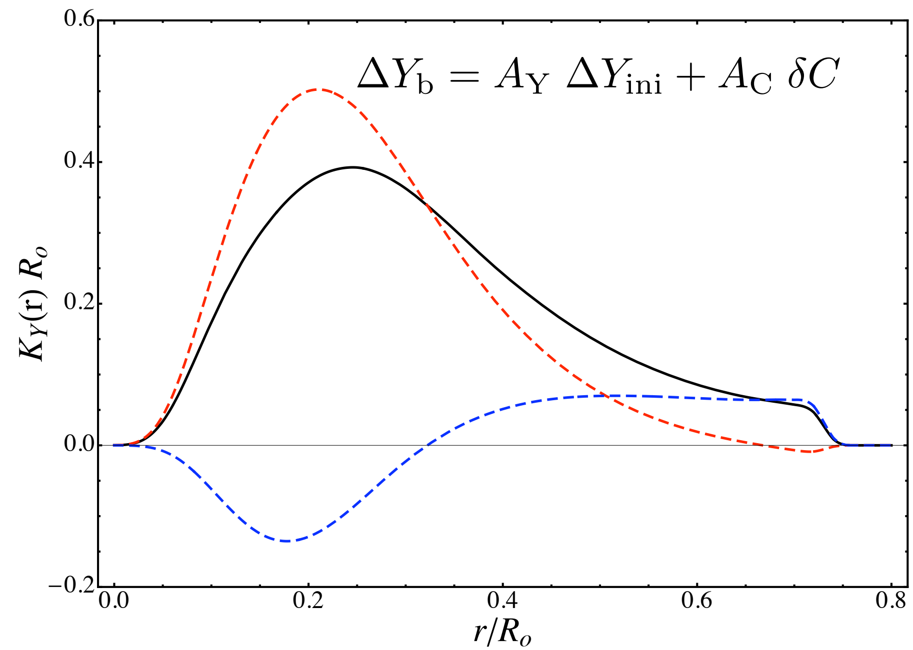


Surface helium:

$$\Delta Y_{\rm b} = \int dr\; K_{\rm Y}(r)\; \delta\kappa(r)$$

$$\begin{aligned} \Delta Y_{\rm b} &= 0.073\; A_{\rm in} + 0.069\; A_{\rm out} \\ &\simeq 0.07\; (A_{\rm in} + A_{\rm out}) \end{aligned}$$

$$\Delta Y_{\rm b} = 0.142\; A_0 + 0.062\; A_1$$



To reproduce helioseismic results:

$A_{\rm in} = 0.07 \pm 0.04 \qquad A_{\rm out} = 0.21 \pm 0.04$

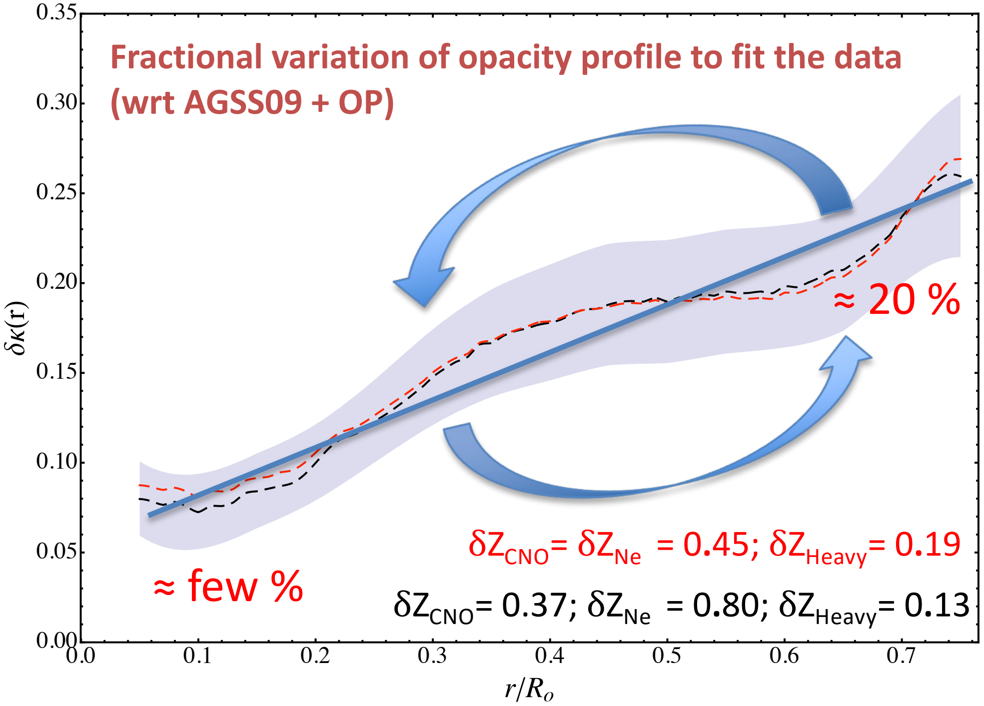
The solar opacity profile

F.L. Villante and B. Ricci - **Astrophys.J.714:944-959,2010**
F.L. Villante – **Astrophys.J.724:98-110,2010**
F.L. Villante, A. Serenelli et al., **Astrophys.J. 787 (2014) 13**

The **“optimal” composition/opacity profile** of the Sun can be determined from obs. data

Note that:

- The sound speed and the convective radius determine the tilt of $\delta\kappa(r)$ (but not the scale)
- The surface helium and the neutrino fluxes determine the scale for $\delta\kappa(r)$



The **opacity at the bottom of the convective envelope** can be directly inferred from **helioseismic observables**:

$$\delta\kappa_b = C_Y\,\Delta Y_b + C_R\,\delta R_b + C_\rho\,\delta\rho_b = 0.24 \pm 0.03$$

(wrt AGS05 + OP)

$$\begin{cases} C_Y = 6.27 \\ C_R = -11.71 \\ C_\rho = -1.58 \end{cases}$$

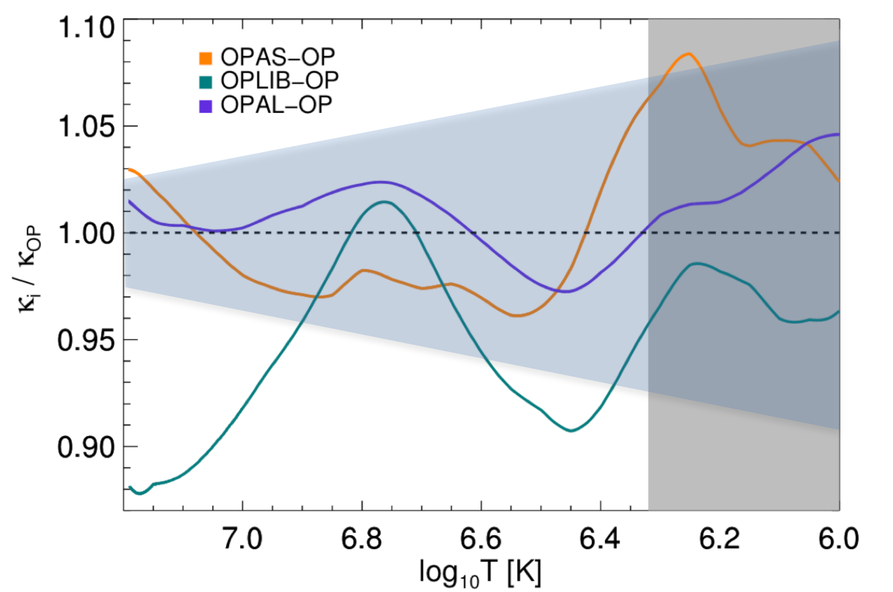
F.L. Villante – **Astrophys.J.724:98-110,2010**

Paramaterizing uncertainty in opacity calculations …

Opacity uncertainty in B16-SSMs is parameterized as:

$$\delta\kappa(T) = \kappa_a + (\kappa_b/\Delta)\ \ln(T/T_C)$$

κ_a, κ_b = random variables
(means equal to 0 and variances σ_a= 0.02 and σ_b= 0.067)



This prescription is motivated by:

- Opacity calculations more accurate at the solar core (~2%) than at the base of the convective envelope (~7%);

- It avoids underestimating the opacity error contribution to sound speed and convective radius (sensitive to tilt and not to scale of opacity)

… but **it still remains** a very simplified description of the real situation

Neutrinos

Hydrogen Burning: PP chain and CNO cycle

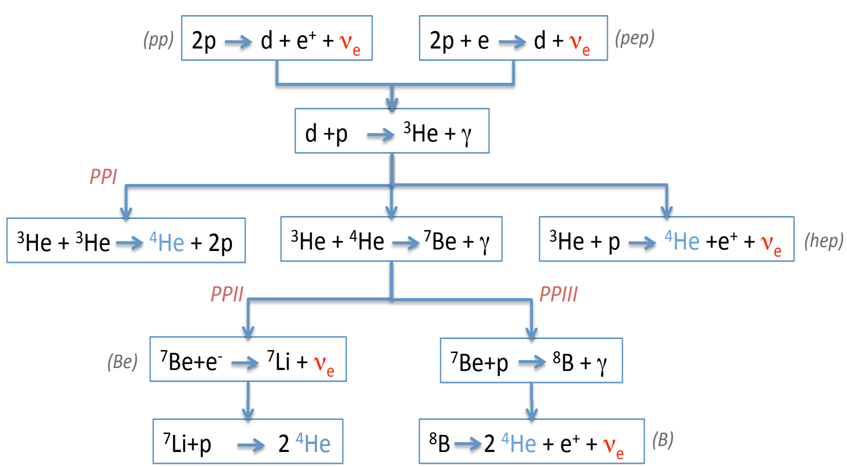
The Sun is powered by nuclear reactions that transform H into ^4He:

$$4H + 2e^- \rightarrow {}^4He + 2\nu_e + \text{energy}$$

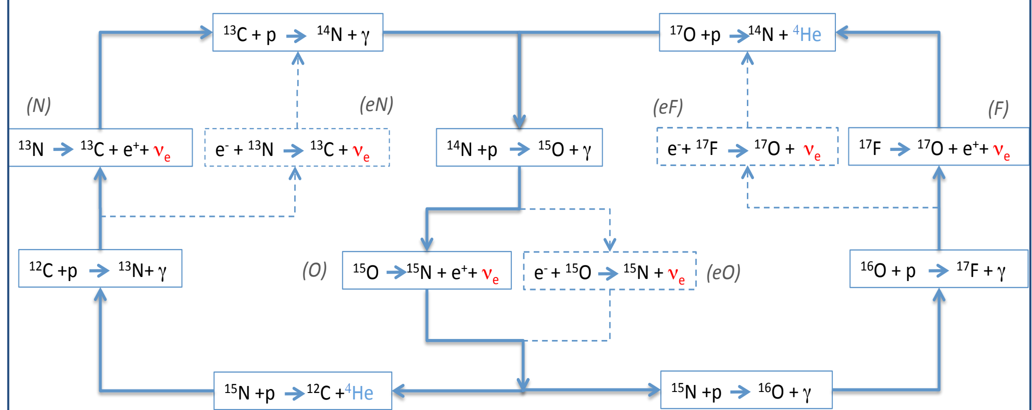
Q = 26,7 MeV (globally)

Free stream – 8 minutes to reach the earth
Direct information on the energy producing region.

The PP-chain



The CN-NO (bi-)cycle

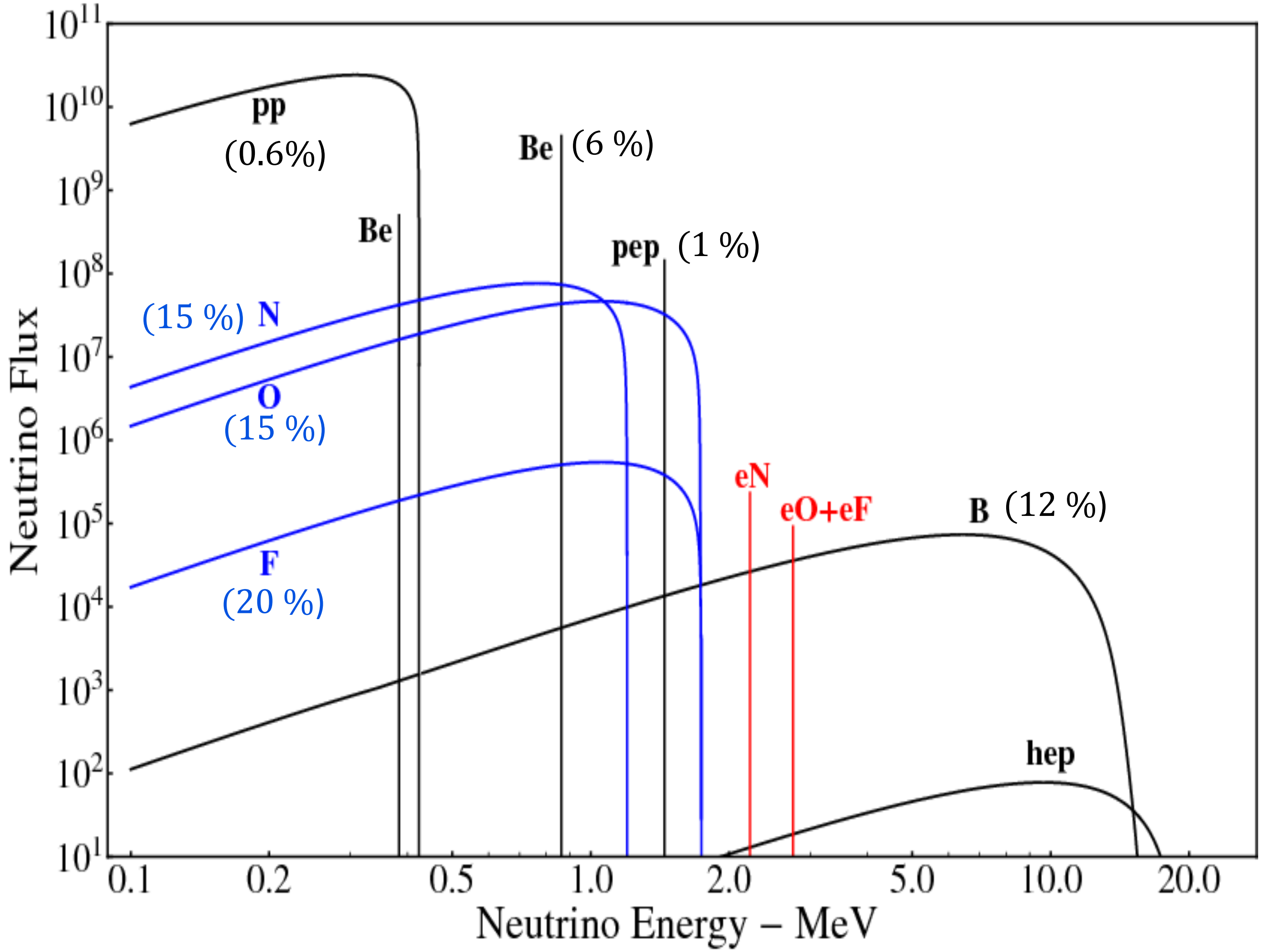


The pp chain is responsible for about 99% of the total energy (and neutrino) production.

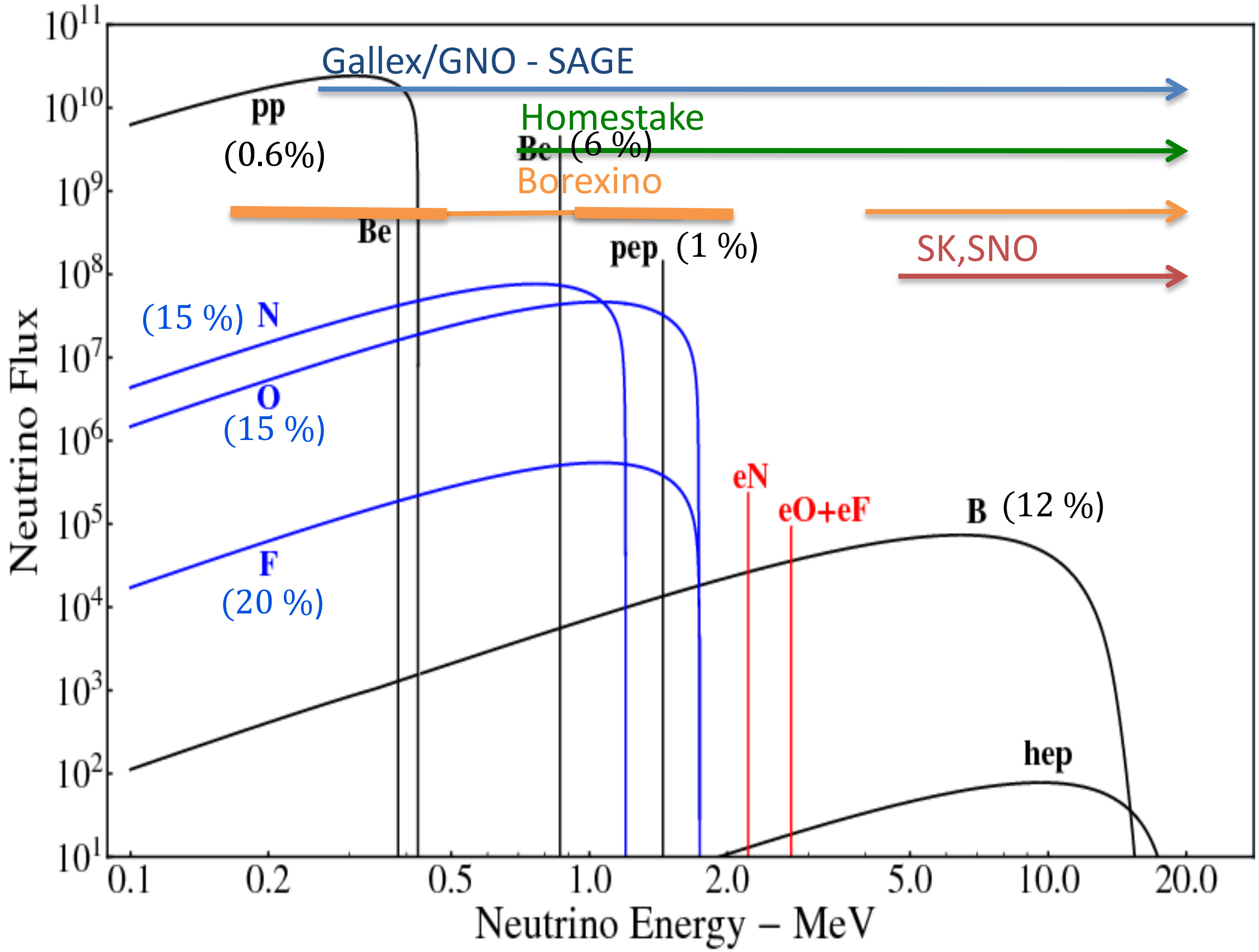
C, N and O nuclei are used as catalysts for hydrogen fusion.

CNO (bi-)cycle is responsible for about 1% of the total neutrino (and energy) budget. Important for more advanced evolutionary stages

The solar neutrino spectrum



The solar neutrino spectrum



The different comp. of the solar neutrinos flux have been **directly** determined with accuracy level:

pp: ~ 10%
pep: ~ 10%
^{7}Be: ~ 3 %
^{8}B: ~ 2 %
CNO: ~ 20%

Recent Milestones from **Borexino**:

- ^{7}Be (and ^{8}B) neutrino direct detection [PRL 2008]
- pp (and pep) neutrinos direct detection [Nature 2014, 2018]
- CNO neutrinos signal identification [Nature 2020, PRL 2022, arXiv: 2307.14636]

Status of direct determination of solar neutrino fluxes after Borexino

[Gonzales-Garcia et al, JHEP 2024]

Implementing the **solar luminosity constraint**:

$$f_{pp} = 0.9969^{+0.0041}_{-0.0039}\,[^{+0.0095}_{-0.0092}]\,, \qquad \Phi_{pp} = 5.941^{+0.024}_{-0.023}\,[^{+0.057}_{-0.055}] \times 10^{10}\ \mathrm{cm^{-2}\ s^{-1}}\,,$$

$$f_{^7\mathrm{Be}} = 1.019^{+0.020}_{-0.017}\,[^{+0.047}_{-0.041}]\,, \qquad \Phi_{^7\mathrm{Be}} = 4.93^{+0.10}_{-0.08}\,[^{+0.23}_{-0.20}] \times 10^{9}\ \mathrm{cm^{-2}\ s^{-1}}\,,$$

$$f_{pep} = 1.000^{+0.016}_{-0.018}\,[^{+0.041}_{-0.042}]\,, \qquad \Phi_{pep} = 1.421^{+0.023}_{-0.026}\,[^{+0.058}_{-0.060}] \times 10^{8}\ \mathrm{cm^{-2}\ s^{-1}}\,,$$

$$f_{^{13}\mathrm{N}} = 1.25^{+0.17}_{-0.14}\,[^{+0.47}_{-0.40}]\,, \qquad \Phi_{^{13}\mathrm{N}} = 3.48^{+0.47}_{-0.40}\,[^{+1.30}_{-1.10}] \times 10^{8}\ \mathrm{cm^{-2}\ s^{-1}}\,,$$

$$f_{^{15}\mathrm{O}} = 1.22^{+0.17}_{-0.14}\,[^{+0.46}_{-0.39}] \qquad \Phi_{^{15}\mathrm{O}} = 2.53^{+0.34}_{-0.29}\,[^{+0.94}_{-0.80}] \times 10^{8}\ \mathrm{cm^{-2}\ s^{-1}}\,,$$

$$f_{^{17}\mathrm{F}} = 1.03^{+0.20}_{-0.20}\,[^{+0.47}_{-0.48}]\,, \qquad \Phi_{^{17}\mathrm{F}} = 5.51^{+0.75}_{-0.63}\,[^{+2.06}_{-1.75}] \times 10^{7}\ \mathrm{cm^{-2}\ s^{-1}}\,,$$

$$f_{^8\mathrm{B}} = 1.036^{+0.020}_{-0.020}\,[^{+0.047}_{-0.048}]\,, \qquad \Phi_{^8\mathrm{B}} = 5.20^{+0.10}_{-0.10}\,[^{+0.24}_{-0.24}] \times 10^{6}\ \mathrm{cm^{-2}\ s^{-1}}\,,$$

$$f_{hep} = 3.8^{+1.1}_{-1.2}\,[^{+2.7}_{-2.7}]\,, \qquad \Phi_{hep} = 3.0^{+0.9}_{-1.0}\,[^{+2.2}_{-2.1}] \times 10^{4}\ \mathrm{cm^{-2}\ s^{-1}}\,.$$

Not implementing the **solar luminosity constraint**:

$$f_{pp} = 1.038^{+0.076}_{-0.066}\,[^{+0.18}_{-0.16}]\,, \qquad \Phi_{pp} = 6.19^{+0.45}_{-0.39}\,[^{+1.1}_{-1.0}] \times 10^{10}\ \mathrm{cm^{-2}\ s^{-1}}\,,$$

$$f_{^7\mathrm{Be}} = 1.022^{+0.022}_{-0.018}\,[^{+0.051}_{-0.042}]\,, \qquad \Phi_{^7\mathrm{Be}} = 4.95^{+0.11}_{-0.089}\,[^{+0.25}_{-0.22}] \times 10^{9}\ \mathrm{cm^{-2}\ s^{-1}}\,,$$

$$f_{pep} = 1.039^{+0.082}_{-0.065}\,[^{+0.19}_{-0.16}]\,, \qquad \Phi_{pep} = 1.48^{+0.11}_{-0.09}\,[^{+0.26}_{-0.22}] \times 10^{8}\ \mathrm{cm^{-2}\ s^{-1}}\,,$$

$$f_{^{13}\mathrm{N}} = 1.16^{+0.19}_{-0.19}\,[^{+0.50}_{-0.45}]\,, \qquad \Phi_{^{13}\mathrm{N}} = 3.32^{+0.53}_{-0.54}\,[^{+1.40}_{-1.24}] \times 10^{8}\ \mathrm{cm^{-2}\ s^{-1}}\,,$$

$$f_{^{15}\mathrm{O}} = 1.16^{+0.19}_{-0.19}\,[^{+0.49}_{-0.44}] \qquad \Phi_{^{15}\mathrm{O}} = 2.41^{+0.38}_{-0.39}\,[^{+1.02}_{-0.90}] \times 10^{8}\ \mathrm{cm^{-2}\ s^{-1}}\,,$$

$$f_{^{17}\mathrm{F}} = 1.01^{+0.16}_{-0.16}\,[^{+0.45}_{-0.38}]\,, \qquad \Phi_{^{17}\mathrm{F}} = 5.25^{+0.84}_{-0.85}\,[^{+2.21}_{-1.97}] \times 10^{6}\ \mathrm{cm^{-2}\ s^{-1}}\,,$$

$$f_{^8\mathrm{B}} = 1.034^{+0.020}_{-0.021}\,[^{+0.052}_{-0.051}]\,, \qquad \Phi_{^8\mathrm{B}} = 5.192^{+0.10}_{-0.11}\,[^{+0.26}_{-0.26}] \times 10^{6}\ \mathrm{cm^{-2}\ s^{-1}}\,,$$

$$f_{hep} = 3.6^{+1.2}_{-1.1}\,[^{+3.0}_{-2.6}]\,, \qquad \Phi_{hep} = 2.9^{+1.0}_{-0.9}\,[^{+2.4}_{-2.1}] \times 10^{4}\ \mathrm{cm^{-2}\ s^{-1}}\,.$$

$$\frac{L_\odot(\text{neutrino-inferred})}{L_\odot} = 1.038^{+0.069}_{-0.060}\,[^{+0.17}_{-0.15}]\,.$$

The importance of measuring pp-neutrinos

Assuming that the Sun is **stable:**

$$L_{\odot} + L_{\nu} \; (+\, L_x) = \varepsilon_n + \underbrace{\varepsilon_g}$$

Gravothermal energy prod. $O(10^{-4}\, L_{\odot})$

where:

$L_{\odot}$ **Radiative Luminosity**

$$L_{\nu} = 4\pi D^2 \sum_i \langle E_{\nu} \rangle_i \, \Phi_i$$

Neutrino Luminosity

$(+\, L_x)$ **Additional (exotic) energy losses**

$$\varepsilon_n = 4\pi D^2 \sum_i \frac{Q}{2} \Phi_i$$

Energy released by nuclear reactions (Q=27.3 MeV)
up to $O(10^{-3}\, L_{\odot})$ corrections due to incomplete pp-chain and CNO-cycle
[Vescovi et al., 2021]

The importance of measuring pp-neutrinos

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$$L_\odot + L_\nu \; (+\, L_x) = \varepsilon_n + \underbrace{\varepsilon_g}$$

Gravothermal energy prod. O(10^{-4} $L_\odot$)

where:

$L_\odot$ — **Radiative Luminosity**

$$L_\nu = 4\pi D^2 \sum_i \langle E_\nu \rangle_i \, \Phi_i$$ — **Neutrino Luminosity**

$(+\, L_x)$ — **Additional (exotic) energy losses**

$$\varepsilon_n = 4\pi D^2 \sum_i \frac{Q}{2} \Phi_i$$

Energy released by nuclear reactions (Q=27.3 MeV)
up to O(10^{-3} $L_\odot$) corrections due to incomplete pp-chain and CNO-cycle
[Vescovi et al., 2021]

$$\underbrace{L_\odot} \;\; (+L_x) = 4\pi D^2 \sum_i \left(\frac{Q}{2} - \langle E_\nu \rangle_i\right) \underbrace{\Phi_i}$$

Radiative luminosity
(Heat diff. time ≈ 10^5 year)

Neutrino fluxes
t_ν = 8 min

pp-neutrinos direct detection allows us to test:

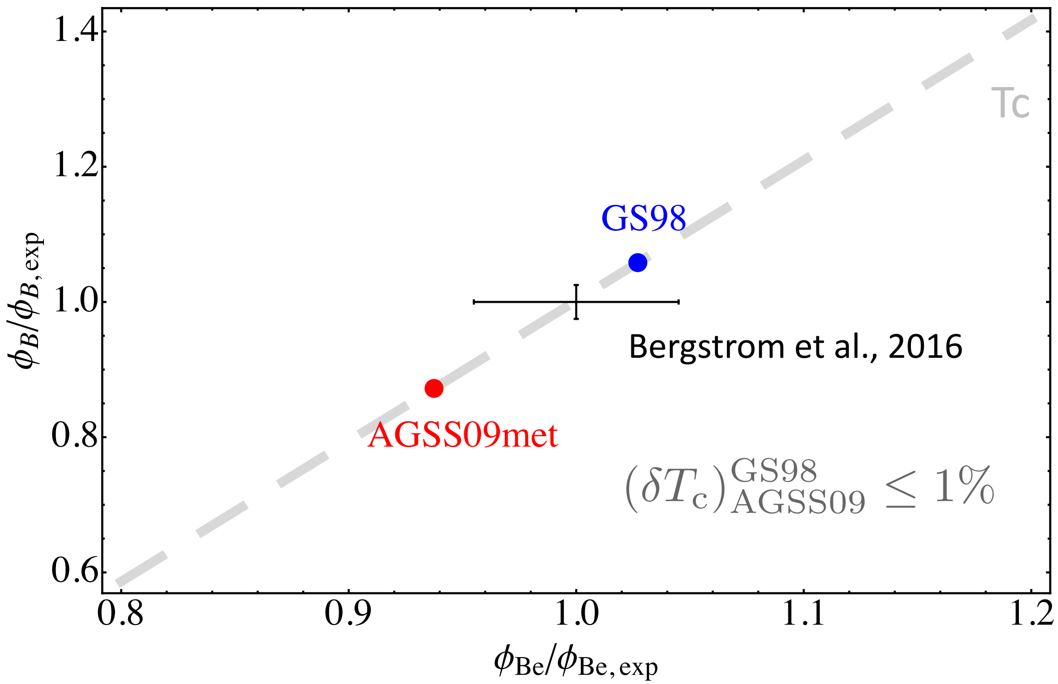
- Solar stability
- Global energy balance of the Sun
- Additional energy losses/sources

$$\frac{L_\odot(\text{neutrino-inferred})}{L_\odot} = 1.038^{+0.069}_{-0.060} \, \left[^{+0.17}_{-0.15}\right].$$

Gonzales-Garcia et al, JHEP 2024

The ^{7}Be and ^{8}B neutrino fluxes

N.Vinyoles et al. ApJ 2017 [arXiv:1611.09867v1]



^{7}Be and ^{8}B neutrinos depend on the core temperature T_c and on the cross sections that control the branching of different pp-chain terminations

$$\beta_{\mathrm{Be}} = \gamma_{34} + (\gamma_{11} - \gamma_{33})/2 \sim 11$$
$$\beta_{\mathrm{B}} = \beta_{\mathrm{Be}} + \gamma_{17} + 1/2 \simeq 24$$

$$\delta\Phi(^7\mathrm{Be}) = \delta S_{34} + \frac{1}{2}\,(\delta S_{11} - \delta S_{33}) + \beta_{\mathrm{Be}}\,\delta T_{\mathrm{c}}$$

$$\delta\Phi(^8\mathrm{B}) = (\delta S_{17} - \delta S_{e7}) + \delta S_{34} + \frac{1}{2}\,(\delta S_{11} - \delta S_{33}) + \beta_{\mathrm{B}}\,\delta T_{\mathrm{c}}$$

N.B. The core temperature is a function of surface composition and enviromental parameters

$$\delta T_{\mathrm{c}} = f\,(\delta X_{\mathrm{i}}, \delta(\mathrm{opa}), \delta(\mathrm{diffu}), \ldots)$$

The ^{7}Be and ^{8}B neutrino fluxes

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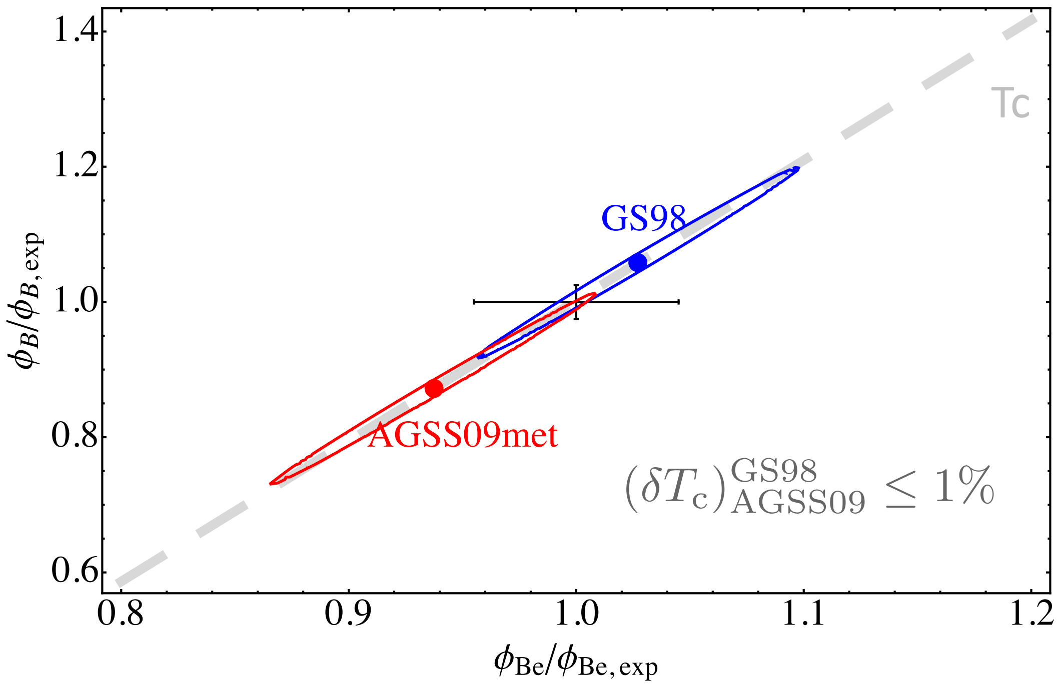
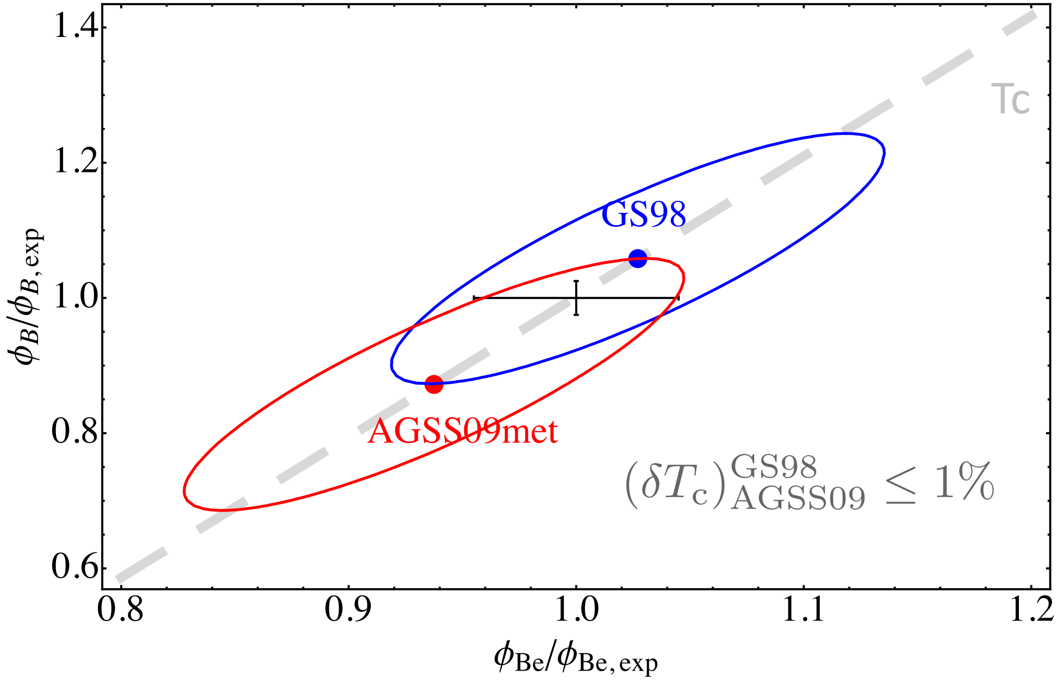


Theoretical uncertainties dominate the error budget. These are due to:

- Surface composition
- Environmental parameters: opacity (few %), diffusion coeff. (15%), etc
- Nuclear cross section: S_{17}(4.7%), S_{33}(5.2%), S_{34}(5.4%) dominant error sources

The ^{7}Be and ^{8}B neutrino fluxes

N.Vinyoles et al. ApJ 2017 [arXiv:1611.09867v1]



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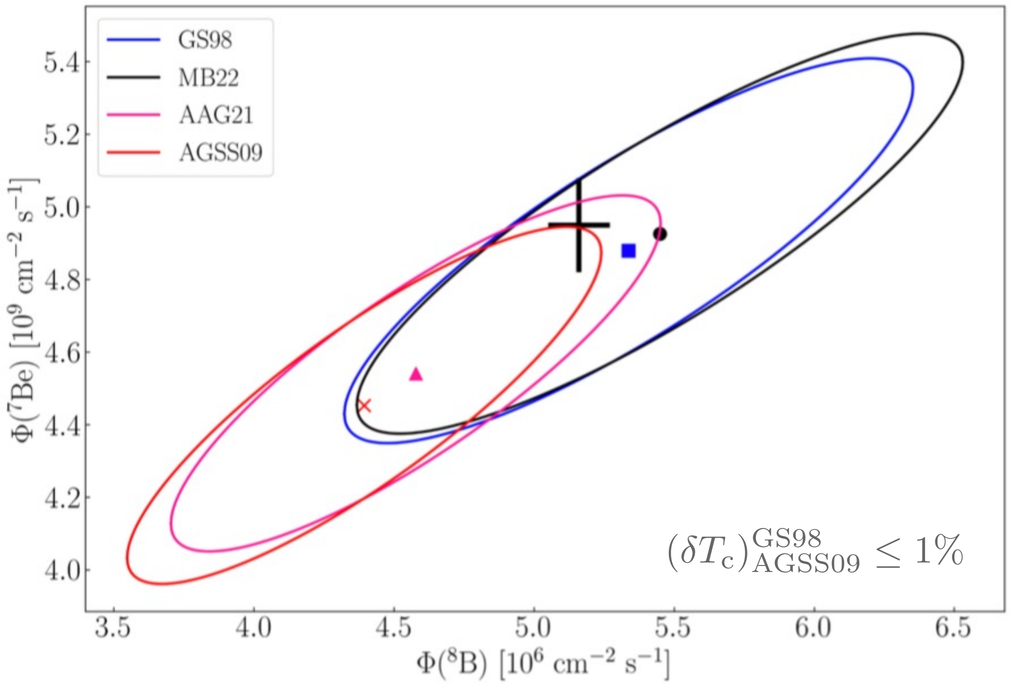
The ^{7}Be and ^{8}B neutrino fluxes

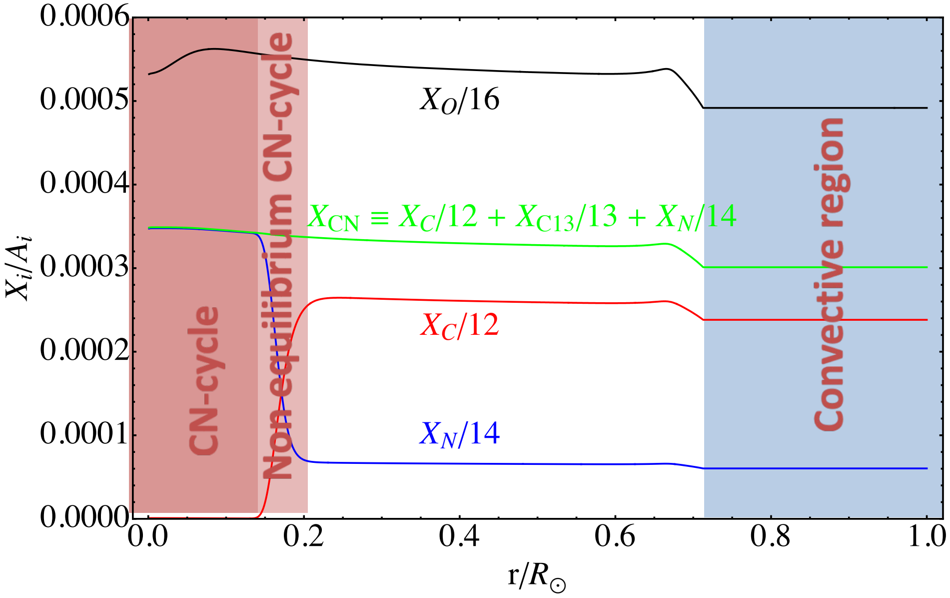


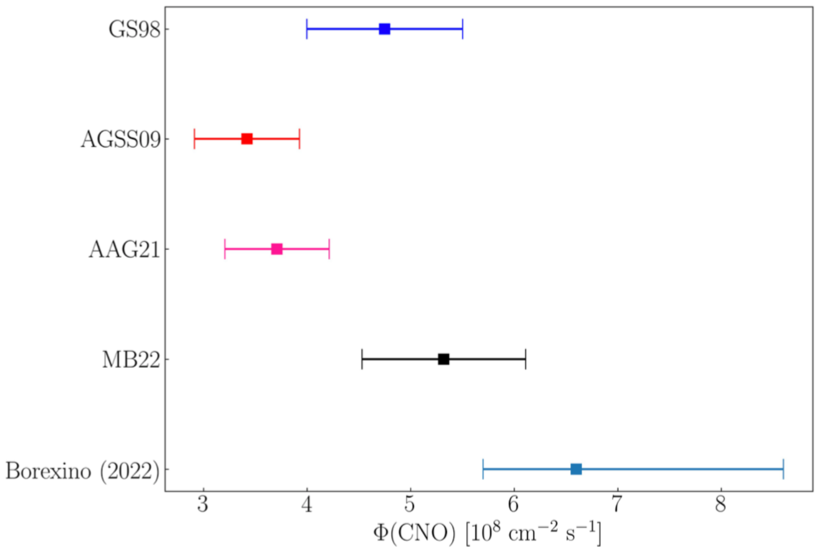
At the moment, **^{7}Be and ^{8}B neutrinos:**

- constrain the core temperature at < 1% level
- do not determine the core composition with suff. accuracy

N.B. ^{7}Be and ^{8}B neutrinos alone do not break composition-opacity degeneracy

CNO neutrino fluxes





- CNO neutrino fluxes also directly depend on the carbon+nitrogen in the core of the Sun (X_{CN})

Assuming equal C and N fractional variations (i.e. $\delta X_{\rm N}^{\rm core} = \delta X_{\rm C}^{\rm core} \equiv \delta X_{\rm CN}^{\rm core}$):

$$\delta\Phi(^{15}{\rm O}) = \delta X_{\rm CN}^{\rm core} + \beta_{\rm O}\,\delta T_{\rm c} + \delta S_{114}$$

$$\delta\Phi(^{13}{\rm N}) = \delta X_{\rm CN}^{\rm core} + \beta_{\rm N}\,\delta T_{\rm c} + f\,\delta S_{114}$$

$$\beta_{\rm O} = 20$$

$$\beta_{\rm N} = f\,\beta_{\rm O} = 15$$

$$f \simeq 0.7$$

Removing composition-opacity degeneracy

The combined measurement of pp-chain and CNO-cycle neutrinos can be used to directly **infer the solar core composition.** *Indeed:*

- The (strong) dependence on T_c (and opacity) can be eliminated by using ^{8}**B-neutrinos as solar thermometer;**
- The additional dependence of CNO-neutrinos on X_{CN} can be used to infer core composition

In practical terms, one can form a weighted ratio of e.g. ^{8}B and ^{15}O neutrino fluxes that is:

- Essentially independent on environmental parameters (including opacity);
- **Directly proportional to Carbon+Nitrogen abundance in the solar core**

Serenelli et al., PRD 2013
See also (application to BX obs. rate):
Agostini et al, EPJ 2021
Villante & Serenelli, Frontiers 2021

$$\delta\Phi(^{15}\mathrm{O}) - x\,\delta\Phi(^{8}\mathrm{B}) \simeq \boxed{\delta X_{\mathrm{CN}}^{\mathrm{core}}} + \delta S_{114} - x\left(\delta S_{17} - \delta S_{e7} + \delta S_{34} + \frac{\delta S_{11}}{2} - \frac{\delta S_{33}}{2}\right)$$

$$x = \frac{\beta_{\mathrm{O}}}{\beta_{\mathrm{B}}} \sim 0.8$$

Solar neutrino fluxes - opacity kernels

F.L. Villante – **Astrophys.J.724:98-110,2010**

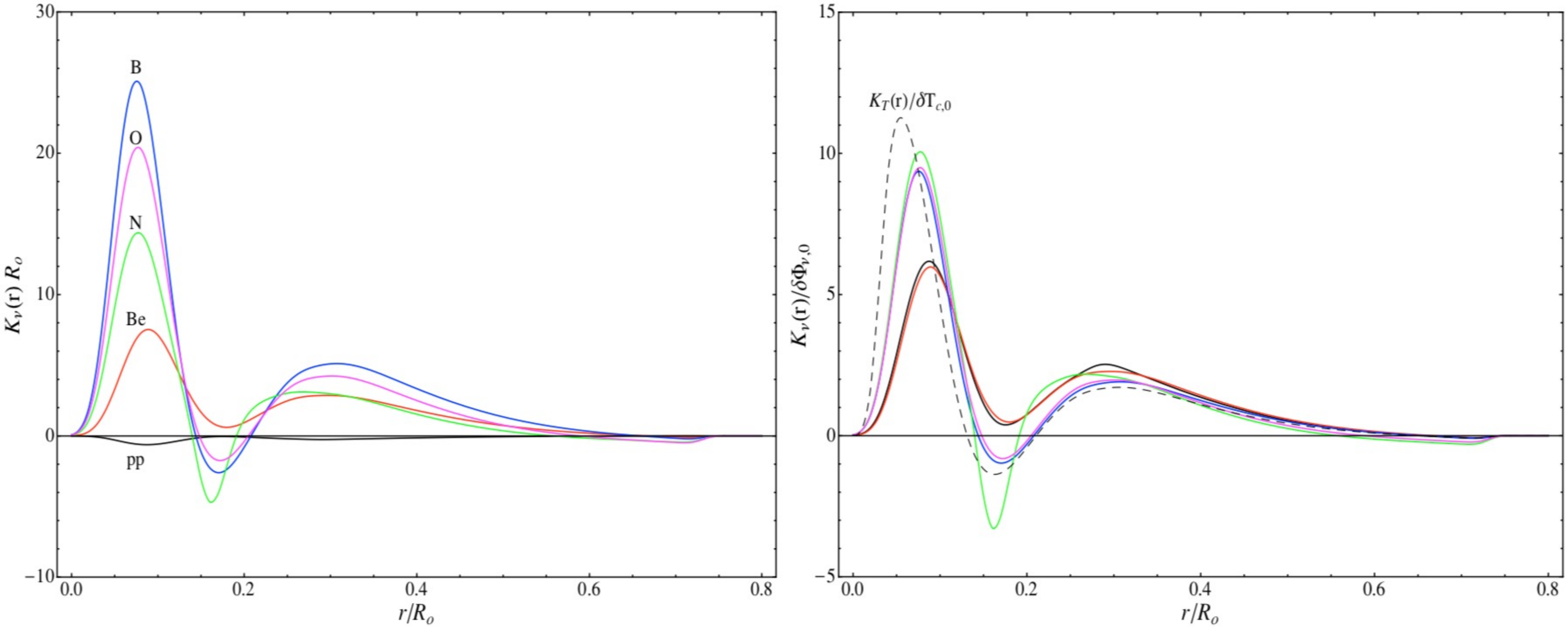


Figure 6. Left panel: the solar neutrino kernels $K_\nu(r)$ defined in Equation (32). Right panel: the solid lines are the normalized solar neutrino kernels $K_\nu(r)/\delta\Phi_{\nu,0}$. The dashed line shows the normalized kernel $K_T(r)/\delta T_{c,0}$ defined in Equation (36), that describes the response of the solar central temperature to localized opacity modifications.

Probing solar composition with neutrinos

By considering

$$\frac{R^{Bx}_{CNO}}{R^{SSM}_{CNO}} = \frac{R^{Bx}_{^{15}O}}{R^{SSM}_{^{15}O}} = \frac{\Phi^{Bx}_{^{15}O}}{\Phi^{SSM}_{^{15}O}} = 1.35^{+0.24}_{-0.16}$$

Borexino CNO neutrino signal (scaled to GS98 prediction)
[Borexino: PRL 2022, arXiv: 2307.14636]

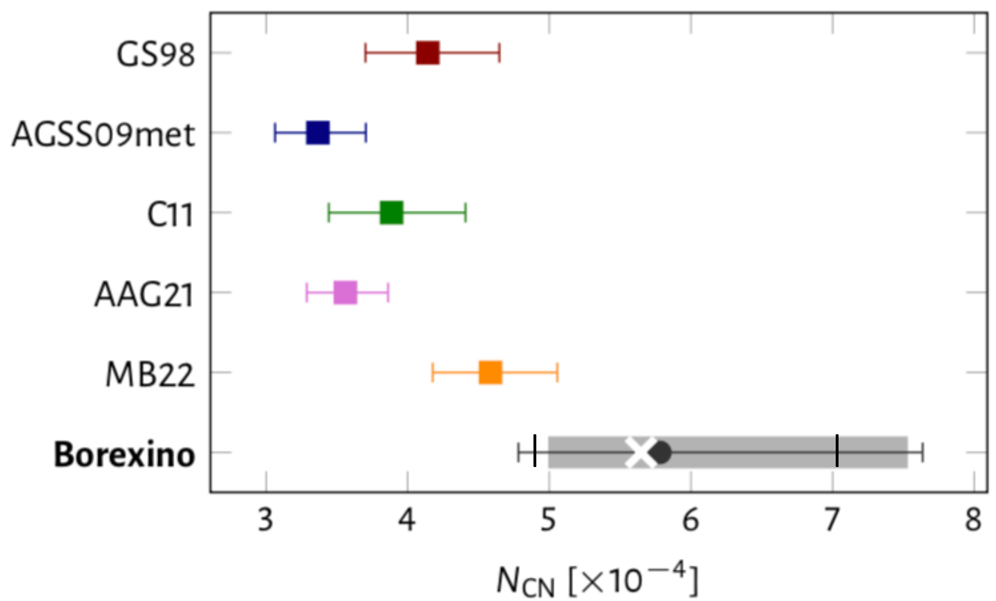
$$\frac{\Phi^{global}_{^8B}}{\Phi^{SSM}_{^8B}} = 0.96 \pm 0.027$$

^8B flux determined from global analysis (scaled to GS98 prediction)

One obtains:

$$\frac{(N_C + N_N)/N_H}{[(N_C + N_N)/N_H]^{SSM}} = 1.35 \times (0.96)^{-0.769} \times$$
$$\times \left[1 \pm \left({}^{+0.18}_{-0.12}(CNO) \pm 0.097(nucl) \pm 0.023(^8B) \pm 0.005(env) \pm 0.027(diff) \pm 0.022(O/N)\right)\right]$$

Note: reduced error wrt Borexino, PRL 2022



N.B.
This determination is robust wrt to environmental parameters variations (including opacity).

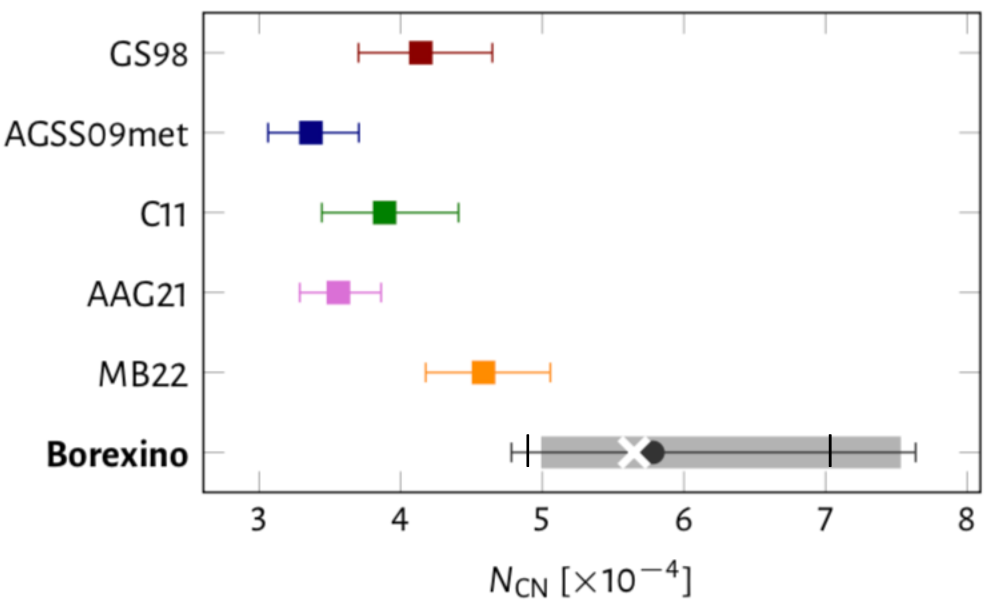
Only limited by nuclear reaction uncertainties:

$S_{114} \rightarrow 7.6\%$
$S_{17} \rightarrow 3.5\%$
$S_{34} \rightarrow 3.4\%$

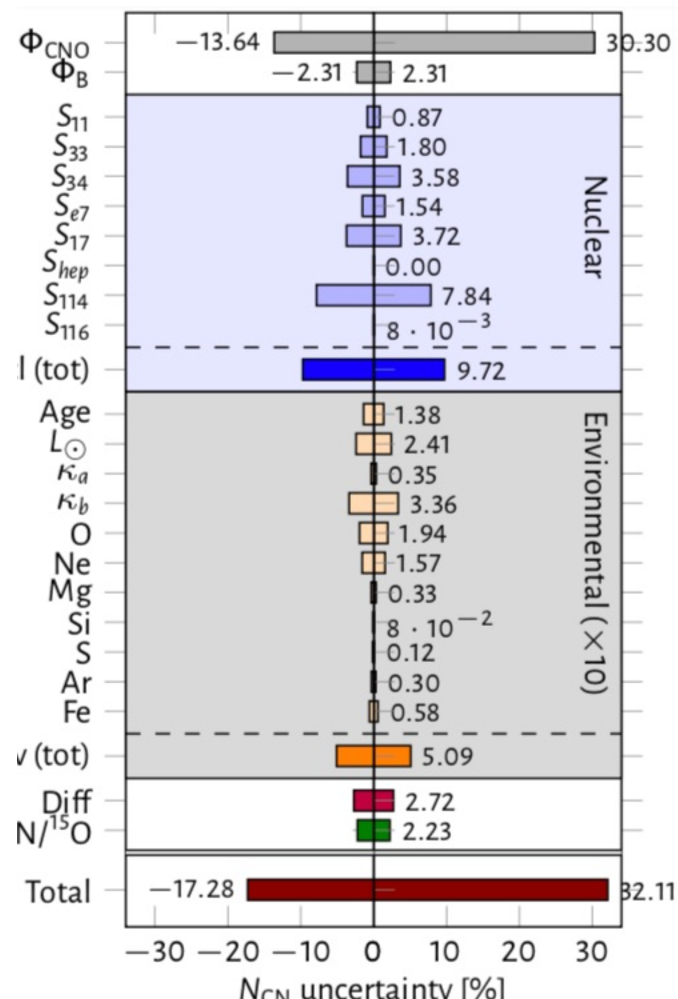
Probing solar composition with neutrinos

$$\frac{(N_C + N_N)/N_H}{[(N_C + N_N)/N_H]^{SSM}} = 1.35 \times (0.96)^{-0.769} \times$$

$$\times \left[1 \pm \left({}^{+0.18}_{-0.12}(\mathrm{CNO}) \pm 0.097(\mathrm{nucl}) \pm 0.023(^8\mathrm{B}) \pm 0.005(\mathrm{env}) \pm 0.027(\mathrm{diff}) \pm 0.022(\mathrm{O/N})\right)\right]$$



Error contributions

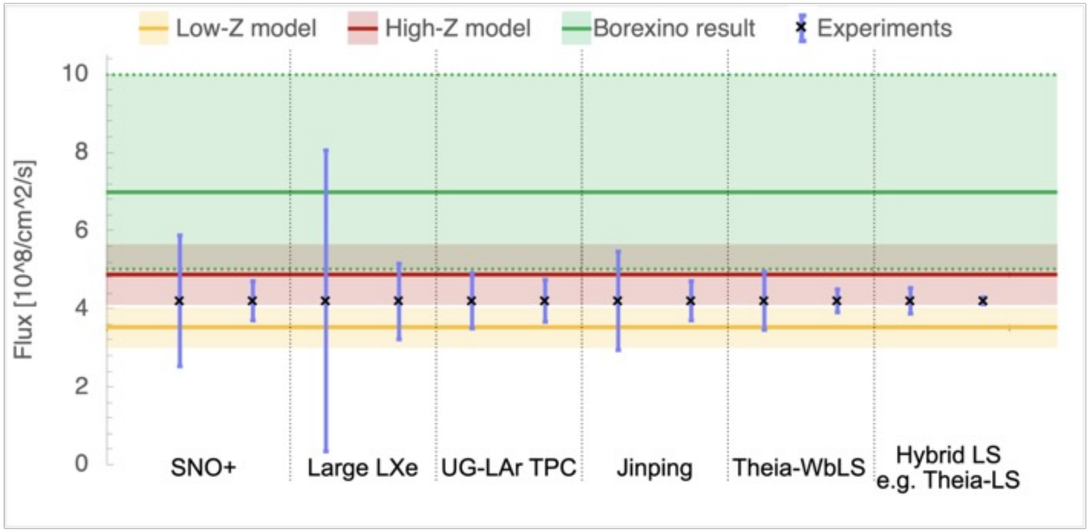


Future perspectives

Borexino has opened the way to CNO neutrino detection

Improvements on the experimental side will be provided in the future by planned detectors, e.g.:
- SNO+
- JUNO
- Jinping
- Hyper-Kamiokande
- THEIA
- DUNE
- Dark Matter experiments

....



ARNP – Orebi Gann et al. in press

Note that: some minor components (hep and ecCNO) of the solar neutrino flux are still undetected

- ecCNO neutrinos: A challenge for gigantic ultra-pure LS detectors (Villante, PLB 2015)
  Expt. requirements: *as clean (and deep) as Borexino; as large as JUNO*

Conclusions

- Solar neutrino physics entered the precision era.
- Borexino has opened the way to CNO neutrino detection
- Some unsolved puzzles could be addressed → (Present and future) CNO neutrino measurements, combined with precise determinations of ^{8}B and ^{7}Be fluxes, can shed light on the solar abundance problem
- To exploit the full potential of future measurements → improvements in the SSM constitutive physics are needed [nuclear cross sections and radiative opacities]

Solar Fusion Cross Sections III (INT-22-82W)
July 2022, UC Berkeley, Berkeley, CA, USA
e-Print: 2405.06470 [astro-ph.SR]

Thank you

Standard Solar Models

Stellar structure equations are solved, starting from a ZAMS model to present solar age (we neglect rotation, magnetic fields, etc.):

$$
\begin{aligned}
\frac{\partial m}{\partial r} &= 4\pi r^2 \rho \\
\frac{\partial P}{\partial r} &= -\frac{G_{\rm N} m}{r^2}\rho \\
P &= P(\rho, T, X_i) \\
\frac{\partial l}{\partial r} &= 4\pi r^2 \rho\, \epsilon(\rho, T, X_i) \\
\frac{\partial T}{\partial r} &= -\frac{G_{\rm N} m T \rho}{r^2 P}\nabla
\end{aligned}
$$

$$
\nabla = {\rm Min}(\nabla_{\rm rad}, \nabla_{\rm ad}) \rightarrow \begin{cases} \nabla_{\rm rad} = \dfrac{3}{16\pi a c\, G_{\rm N}} \dfrac{\kappa(\rho, T, X_i)\, l\, P}{m\, T^4} \\ \nabla_{\rm ad} = (d \ln T / d \ln P)_{\rm s} \simeq 0.4 \end{cases}
$$

Chemical evolution driven by nuclear reaction, diffusion and gravitational settling, convection

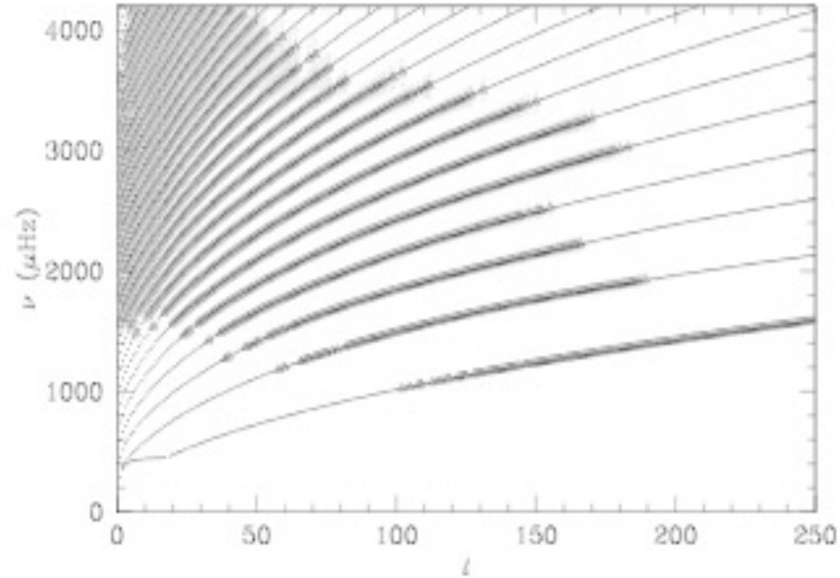
Standard input physics for equation of states, nuclear reaction rates, opacity, etc.

Free-parameters (mixing length, Y_{ini}, Z_{ini}) adjusted to match the observed properties of the Sun (radius, luminosity, Z/X).

Note that equations are non-linear → Iterative method to determine mixing length, Y_{ini}, Z_{ini}

The solar abundance problem

Oscillation frequencies of the sun



360 days of observation of the MDI instrument (errors multiplied by 5000)

The **downward revision** of heavy elements photospheric abundances leads to SSMs which **do not correctly reproduce helioseismic observables**

The Sun is a non radial oscillator. The observed oscillation frequencies can be used to determine the properties of the Sun. Linearizing around a known solar model:

$$\frac{\delta\nu_{nl}}{\nu_{nl}} = \int_0^R dr\; K^{nl}_{u,Y}(r)\,\frac{\delta u}{u}(r) + \int_0^R dr\; K^{nl}_{Y,u}(r)\,\delta Y + \frac{F(\nu_{nl})}{\nu_{nl}}$$

squared isothermal sound speed

Related to temperature stratification in the sun

surface helium abundance

See Basu & Antia 07 for a review

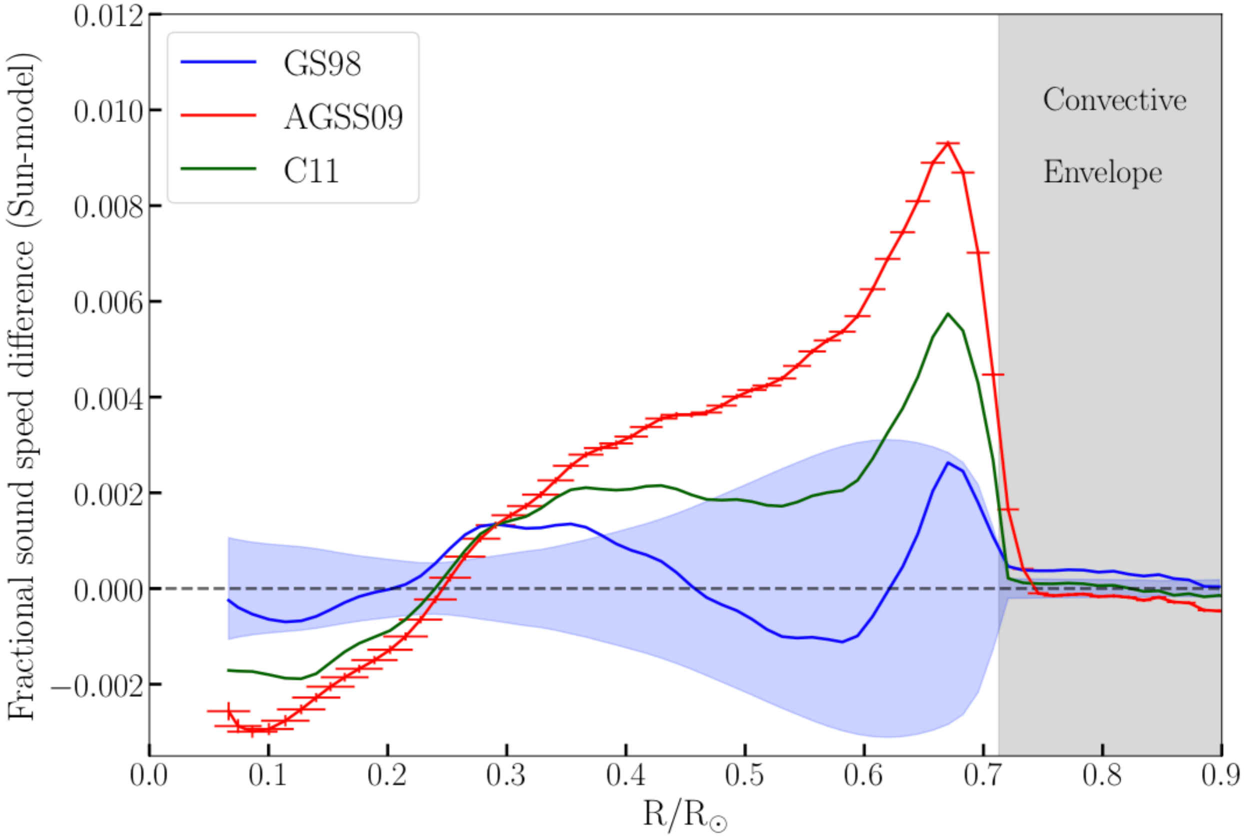
The solar abundance problem

Model	$R_{CZ}/R_\odot$	Y_S
MB22-phot	0.7123	0.2439
MB22-met	0.7120	0.2442
AAG21	0.7197	0.2343
AGSS09-met	0.7231	0.2316
GS98	0.7122	0.2425
C11	0.7162	0.2366

Helioseismic determinations

$$R_b/R_\odot = 0.713 \pm 0.001$$

$$Y_b = 0.2485 \pm 0.0035$$

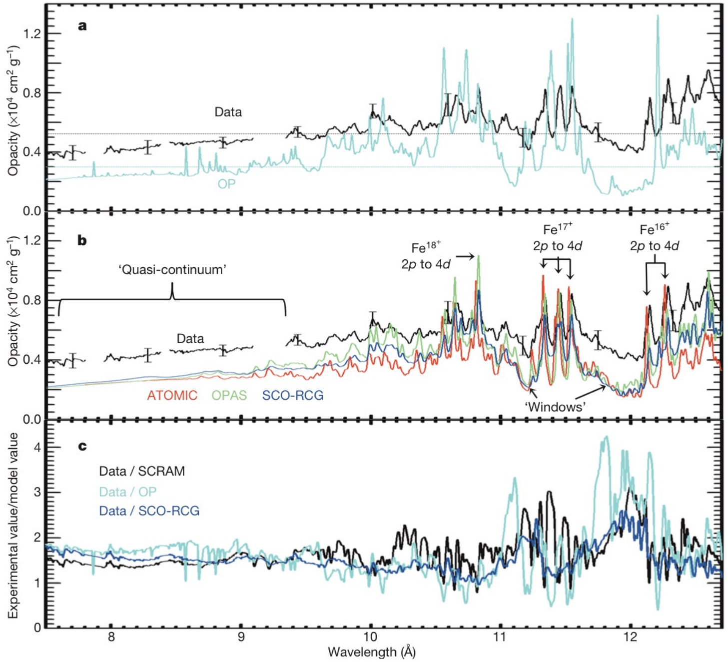


Magg et al. 2022

Wrong opacity?

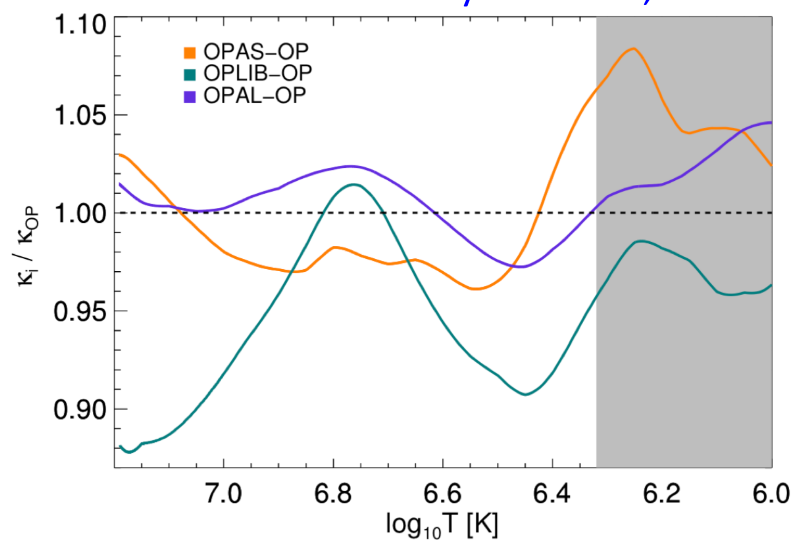
- Opacity is being measured at stellar interiors conditions (Bailey et al., Nature 2015);
- Monochromatic opacity is higher than expected for iron (up to a factor 2);
- Total opacity (integrated over the wavelength and summed over the composition) is increased by about 7%

Bailey et al., Nature 2015



- Different opacity tables may differ "locally" by a large amount (up to 10%) and with a complicated pattern

Vinyoles et al., 2017



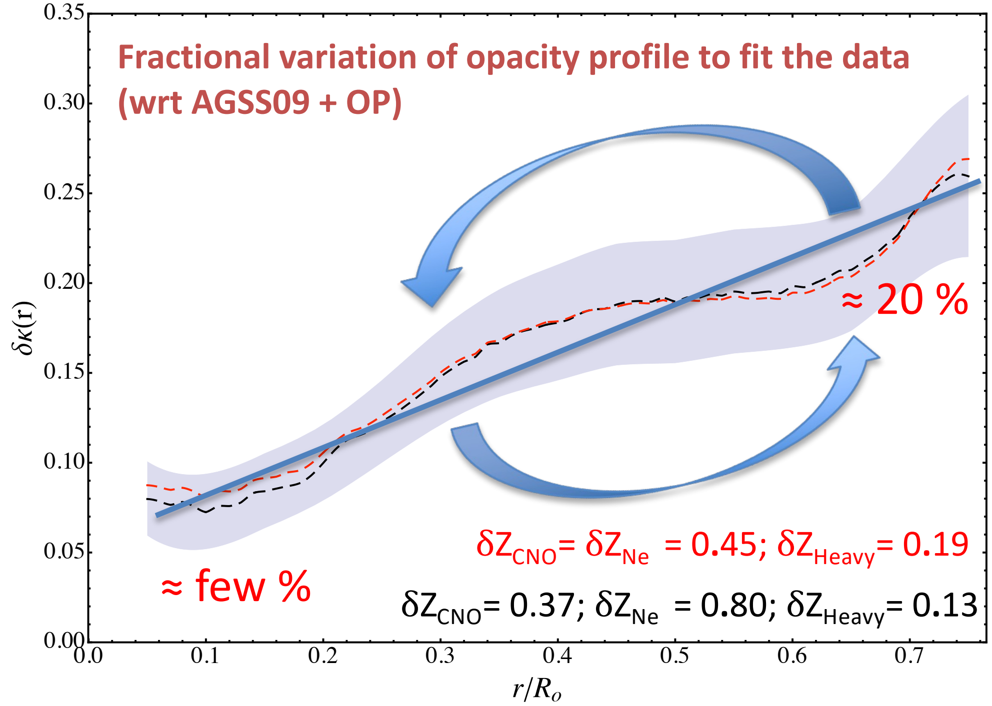
The solar opacity profile

F.L. Villante and B. Ricci - ***Astrophys.J.714:944-959,2010***
F.L. Villante – **Astrophys.J.724:98-110,2010**
F.L. Villante, A. Serenelli et al., ***Astrophys.J. 787 (2014) 13***

The **"optimal" opacity profile** of the Sun can be determined from obs. data

Note that:

- The sound speed and the convective radius determine the tilt of $\delta\kappa(r)$ (but not the scale)
- The surface helium and the neutrino fluxes determine the scale for $\delta\kappa(r)$



The interpretation is however complicated by the **opacity-composition degeneracy**. *Which fraction of the required* $\delta\kappa(r)$ *has to be ascribed to intrinsic* ($\delta\kappa_I(r)$) *and/or composition opacity changes?*

$$\delta\kappa(r) = \delta\kappa_{\rm I}(r) + \sum_j \frac{\partial \ln \kappa(r)}{\partial \ln Z_j}\delta z_j$$

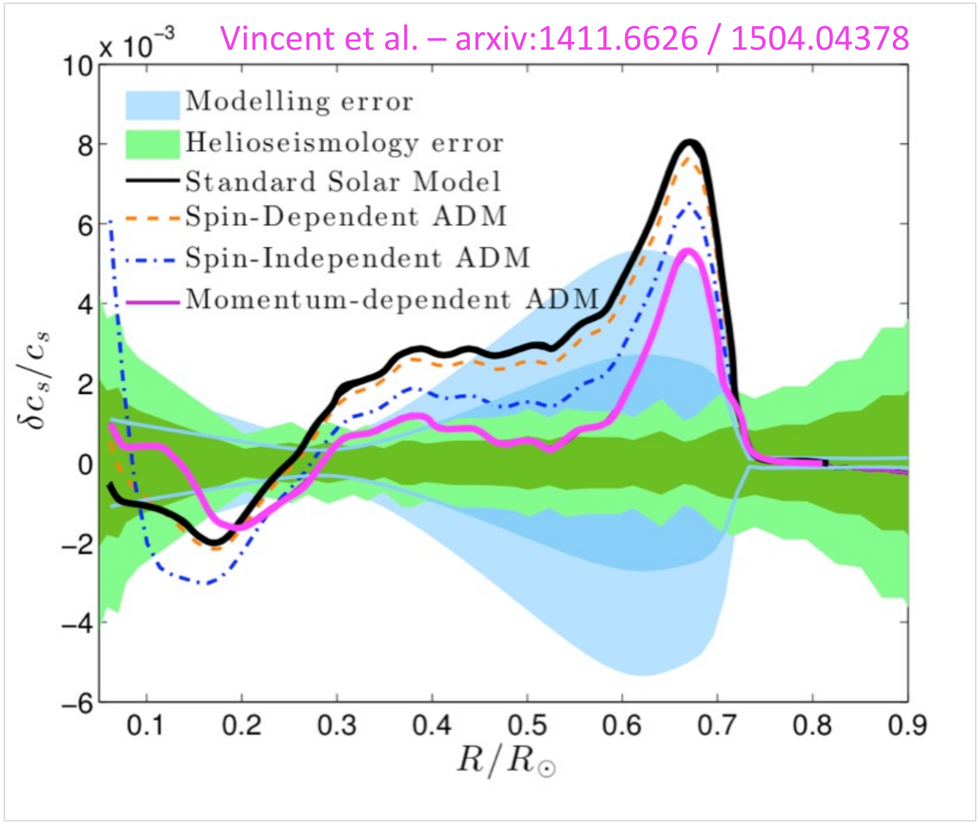
Opacity table "errors"
Non standard effects (WIMPs in solar core)
...

different admixtures $\{\delta z_i\}$ can do equally well the job

Asymmetric DM

DM accumulation in the solar core:

→ Additional energy transport;
→ **Reduction** of the "effective opacity";
→ Modification of temperature profile;



Agreement with helioseismic data can be improved. However:

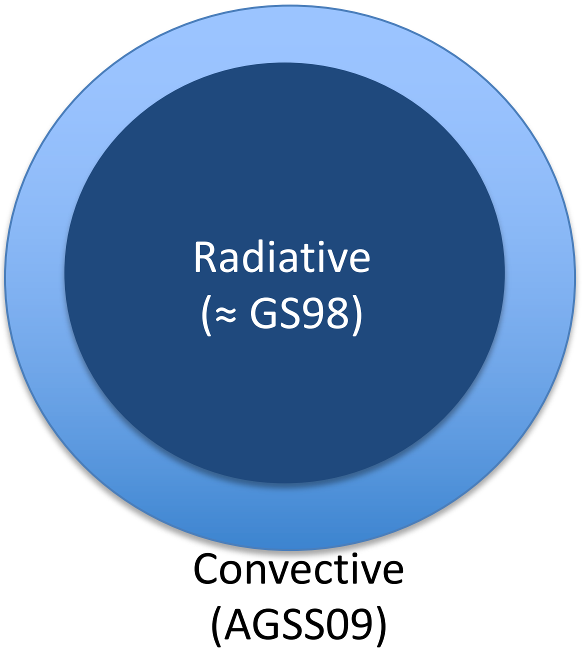
→ DM accumulation do not provide the optimal opacity profile;
→ Potential tension with neutrino fluxes and surface helium;
→ Caveat: DM evaporation not accounted for (relevant for few GeV masses)

$$\sigma = \sigma_0 \left(\frac{q}{q_0}\right)^2 \qquad \begin{cases} m_\chi = 3\ \mathrm{GeV} \\ \sigma_0 = 10^{-37}\ \mathrm{cm}^2 \\ q_0 = 40\ \mathrm{MeV} \end{cases}$$

Wrong chemical evolution?

Helioseismic observables and neutrino fluxes are sensitive to **the metallicity of the radiative interior of the Sun.**

The observations determine **the chemical composition of the convective envelope** (2-3% of the solar mass).



Difference between AGSS09 and GS98 correspond to $\approx 40 M_{\oplus}$ of metal, when integrated over the Sun's convective zone.

Could this difference be accounted in non standard chemical evolution scenarios (e.g. by accretion of material with non standard composition)?

See A. Serenelli et al. – ApJ 2011

This is a well posed and extremely important question but …

… no satisfactory solutions have been proposed up to now, in my opinion

Hydrogen Burning: PP chain and CNO cycle

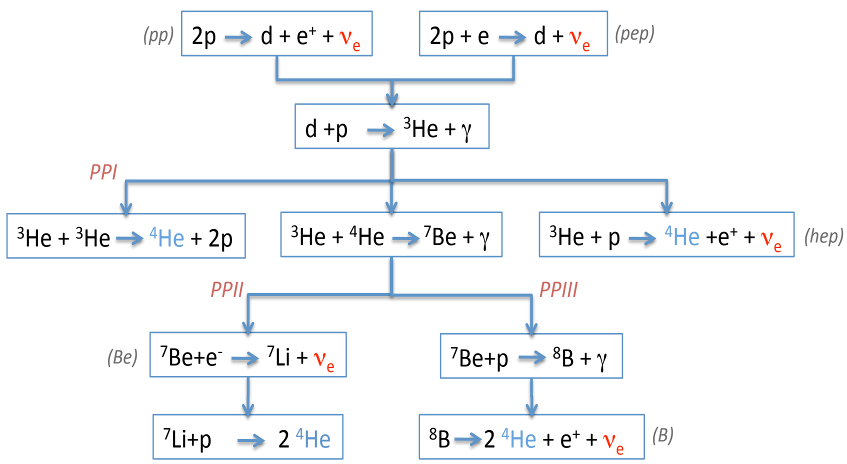
The Sun is powered by nuclear reactions that transform H into ^{4}He:

$$4H + 2e^- \rightarrow {}^4He + 2\nu_e + \text{energy}$$

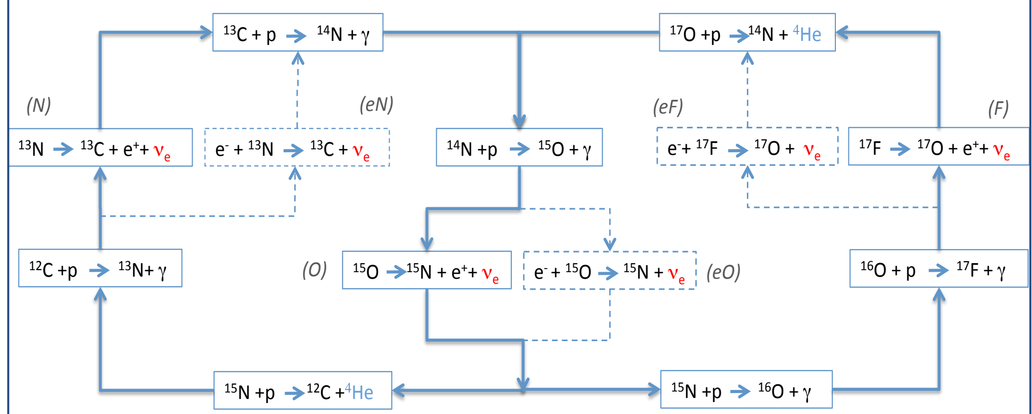
→ Q = 26,7 MeV (globally)

Free stream – 8 minutes to reach the earth
Direct information on the energy producing region.

The PP-chain



The CN-NO (bi-)cycle



The pp chain is responsible for about 99% of the total energy (and neutrino) production.

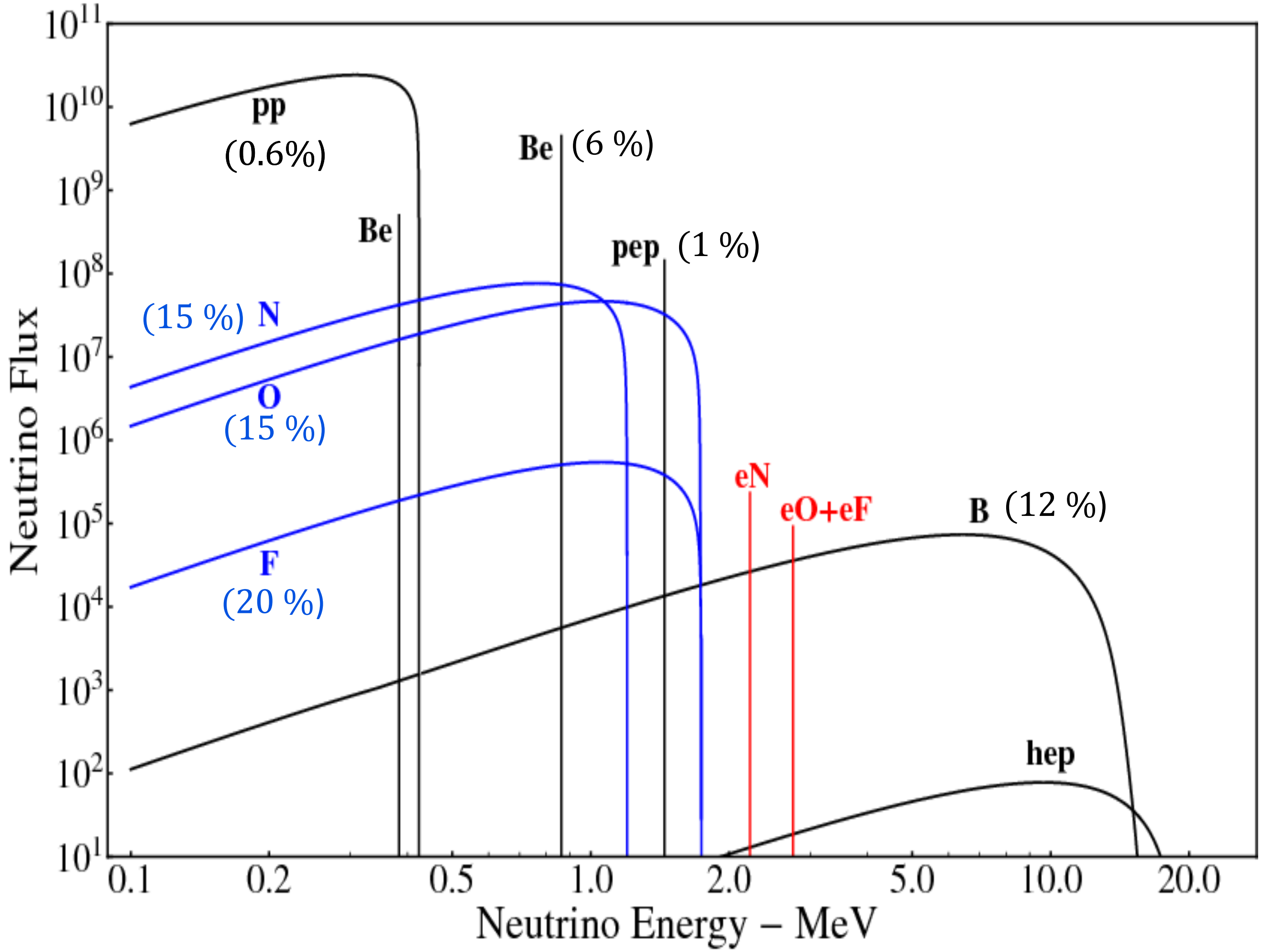
C, N and O nuclei are used as catalysts for hydrogen fusion.

CNO (bi-)cycle is responsible for about 1% of the total neutrino (and energy) budget. Important for more advanced evolutionary stages

The solar neutrino spectrum

The Sun is powered by nuclear reactions that transform H into ^4He:

$4H + 2e^- \rightarrow {}^4He + 2\nu_e$ + energy



Status of direct determination of solar neutrino fluxes after Borexino

[Gonzales-Garcia et al, JHEP 2024]

Implementing the **solar luminosity constraint**:

$$f_{\rm pp} = 0.9969^{+0.0041}_{-0.0039}\,[^{+0.0095}_{-0.0092}]\,, \qquad \Phi_{\rm pp} = 5.941^{+0.024}_{-0.023}\,[^{+0.057}_{-0.055}] \times 10^{10}\ {\rm cm^{-2}\ s^{-1}}\,,$$

$$f_{^7{\rm Be}} = 1.019^{+0.020}_{-0.017}\,[^{+0.047}_{-0.041}]\,, \qquad \Phi_{^7{\rm Be}} = 4.93^{+0.10}_{-0.08}\,[^{+0.23}_{-0.20}] \times 10^{9}\ {\rm cm^{-2}\ s^{-1}}\,,$$

$$f_{\rm pep} = 1.000^{+0.016}_{-0.018}\,[^{+0.041}_{-0.042}]\,, \qquad \Phi_{\rm pep} = 1.421^{+0.023}_{-0.026}\,[^{+0.058}_{-0.060}] \times 10^{8}\ {\rm cm^{-2}\ s^{-1}}\,,$$

$$f_{^{13}{\rm N}} = 1.25^{+0.17}_{-0.14}\,[^{+0.47}_{-0.40}]\,, \qquad \Phi_{^{13}{\rm N}} = 3.48^{+0.47}_{-0.40}\,[^{+1.30}_{-1.10}] \times 10^{8}\ {\rm cm^{-2}\ s^{-1}}\,,$$

$$f_{^{15}{\rm O}} = 1.22^{+0.17}_{-0.14}\,[^{+0.46}_{-0.39}] \qquad \Phi_{^{15}{\rm O}} = 2.53^{+0.34}_{-0.29}\,[^{+0.94}_{-0.80}] \times 10^{8}\ {\rm cm^{-2}\ s^{-1}}\,,$$

$$f_{^{17}{\rm F}} = 1.03^{+0.20}_{-0.20}\,[^{+0.47}_{-0.48}]\,, \qquad \Phi_{^{17}{\rm F}} = 5.51^{+0.75}_{-0.63}\,[^{+2.06}_{-1.75}] \times 10^{7}\ {\rm cm^{-2}\ s^{-1}}\,,$$

$$f_{^8{\rm B}} = 1.036^{+0.020}_{-0.020}\,[^{+0.047}_{-0.048}]\,, \qquad \Phi_{^8{\rm B}} = 5.20^{+0.10}_{-0.10}\,[^{+0.24}_{-0.24}] \times 10^{6}\ {\rm cm^{-2}\ s^{-1}}\,,$$

$$f_{\rm hep} = 3.8^{+1.1}_{-1.2}\,[^{+2.7}_{-2.7}]\,, \qquad \Phi_{\rm hep} = 3.0^{+0.9}_{-1.0}\,[^{+2.2}_{-2.1}] \times 10^{4}\ {\rm cm^{-2}\ s^{-1}}\,.$$

Not implementing the **solar luminosity constraint**:

$$f_{\rm pp} = 1.038^{+0.076}_{-0.066}\,[^{+0.18}_{-0.16}]\,, \qquad \Phi_{\rm pp} = 6.19^{+0.45}_{-0.39}\,[^{+1.1}_{-1.0}] \times 10^{10}\ {\rm cm^{-2}\ s^{-1}}\,,$$

$$f_{^7{\rm Be}} = 1.022^{+0.022}_{-0.018}\,[^{+0.051}_{-0.042}]\,, \qquad \Phi_{^7{\rm Be}} = 4.95^{+0.11}_{-0.089}\,[^{+0.25}_{-0.22}] \times 10^{9}\ {\rm cm^{-2}\ s^{-1}}\,,$$

$$f_{\rm pep} = 1.039^{+0.082}_{-0.065}\,[^{+0.19}_{-0.16}]\,, \qquad \Phi_{\rm pep} = 1.48^{+0.11}_{-0.09}\,[^{+0.26}_{-0.22}] \times 10^{8}\ {\rm cm^{-2}\ s^{-1}}\,,$$

$$f_{^{13}{\rm N}} = 1.16^{+0.19}_{-0.19}\,[^{+0.50}_{-0.45}]\,, \qquad \Phi_{^{13}{\rm N}} = 3.32^{+0.53}_{-0.54}\,[^{+1.40}_{-1.24}] \times 10^{8}\ {\rm cm^{-2}\ s^{-1}}\,,$$

$$f_{^{15}{\rm O}} = 1.16^{+0.19}_{-0.19}\,[^{+0.49}_{-0.44}] \qquad \Phi_{^{15}{\rm O}} = 2.41^{+0.38}_{-0.39}\,[^{+1.02}_{-0.90}] \times 10^{8}\ {\rm cm^{-2}\ s^{-1}}\,,$$

$$f_{^{17}{\rm F}} = 1.01^{+0.16}_{-0.16}\,[^{+0.45}_{-0.38}]\,, \qquad \Phi_{^{17}{\rm F}} = 5.25^{+0.84}_{-0.85}\,[^{+2.21}_{-1.97}] \times 10^{6}\ {\rm cm^{-2}\ s^{-1}}\,,$$

$$f_{^8{\rm B}} = 1.034^{+0.020}_{-0.021}\,[^{+0.052}_{-0.051}]\,, \qquad \Phi_{^8{\rm B}} = 5.192^{+0.10}_{-0.11}\,[^{+0.26}_{-0.26}] \times 10^{6}\ {\rm cm^{-2}\ s^{-1}}\,,$$

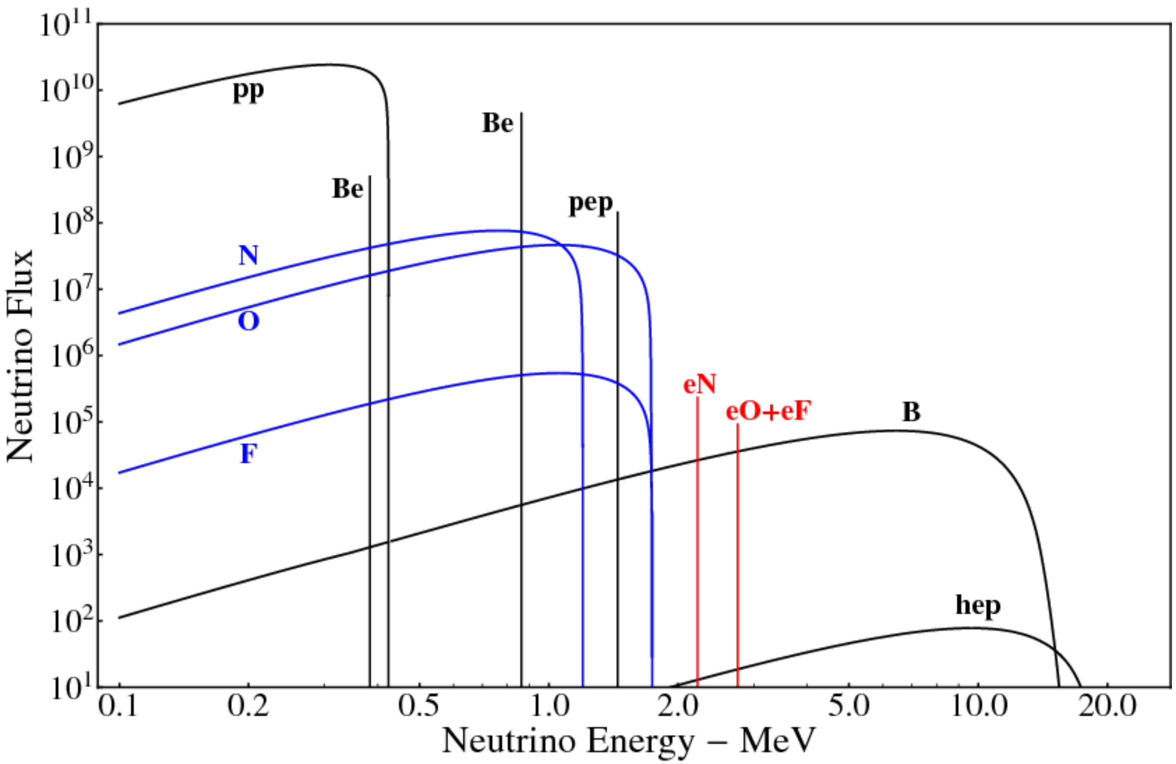
$$f_{\rm hep} = 3.6^{+1.2}_{-1.1}\,[^{+3.0}_{-2.6}]\,, \qquad \Phi_{\rm hep} = 2.9^{+1.0}_{-0.9}\,[^{+2.4}_{-2.1}] \times 10^{4}\ {\rm cm^{-2}\ s^{-1}}\,.$$

$$\frac{L_\odot(\text{neutrino-inferred})}{L_\odot} = 1.038^{+0.069}_{-0.060}\,[^{+0.17}_{-0.15}]\,.$$

ecCNO neutrinos

In the CN-NO cycle, besides the conventional CNO neutrinos (blue lines), monochromatic ecCNO neutrinos (red lines) are also produced by **electron capture** reactions:

$$^{13}\text{N} + e^- \rightarrow {}^{13}\text{C} + \nu_e \qquad E_\nu = 2.220 \text{ MeV}$$
$$^{15}\text{O} + e^- \rightarrow {}^{15}\text{N} + \nu_e \qquad E_\nu = 2.754 \text{ MeV}$$
$$^{17}\text{F} + e^- \rightarrow {}^{17}\text{O} + \nu_e \qquad E_\nu = 2.761 \text{ MeV}$$



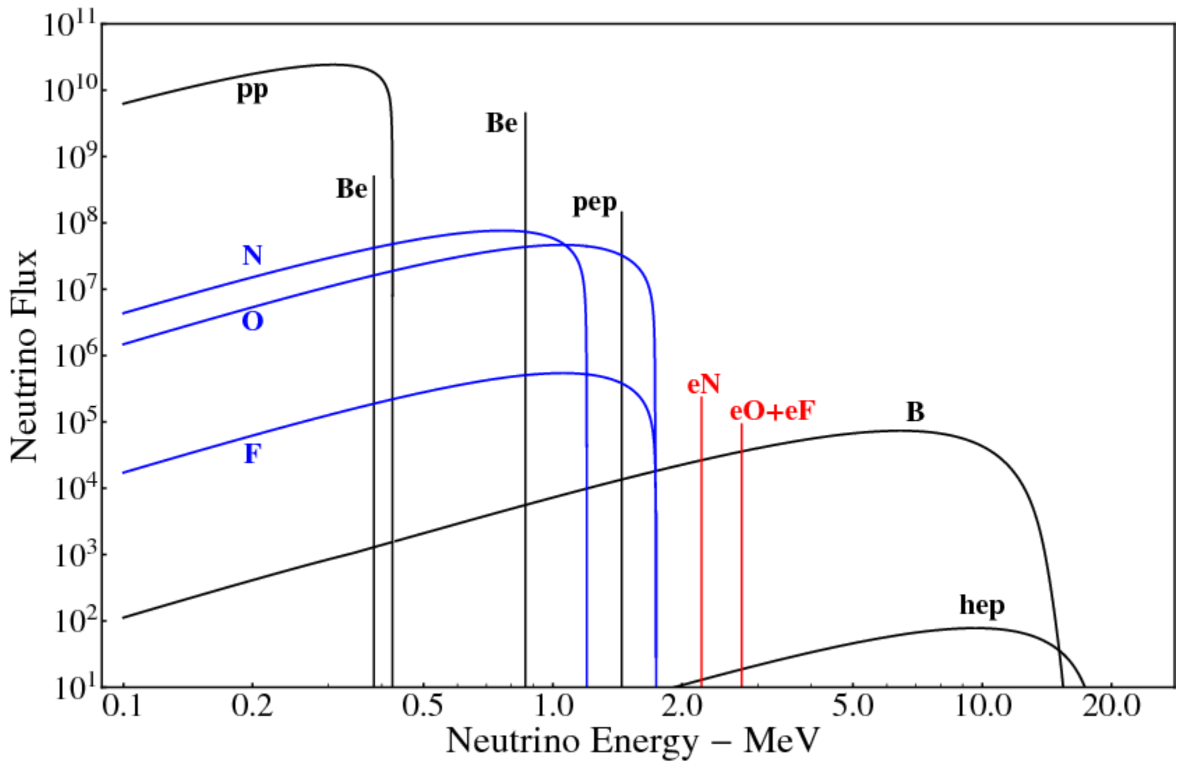
F.L. Villante, PLB 742 (2015) 279-284
L.C. Stonehill et al, PRC 69, 015801 (2004)
J.N. Bahcall, PRD 41, 2964 (1990).

ecCNO neutrinos

The ecCNO fluxes are extremely low: $\Phi_{ecCNO} \approx (1/20)\ \Phi_B$. Detection is extremely difficult but could be rewarding. Indeed:

- ecCNO neutrinos are sensitive to the **metallic content of the solar core** (same infos as CNO neutrinos);

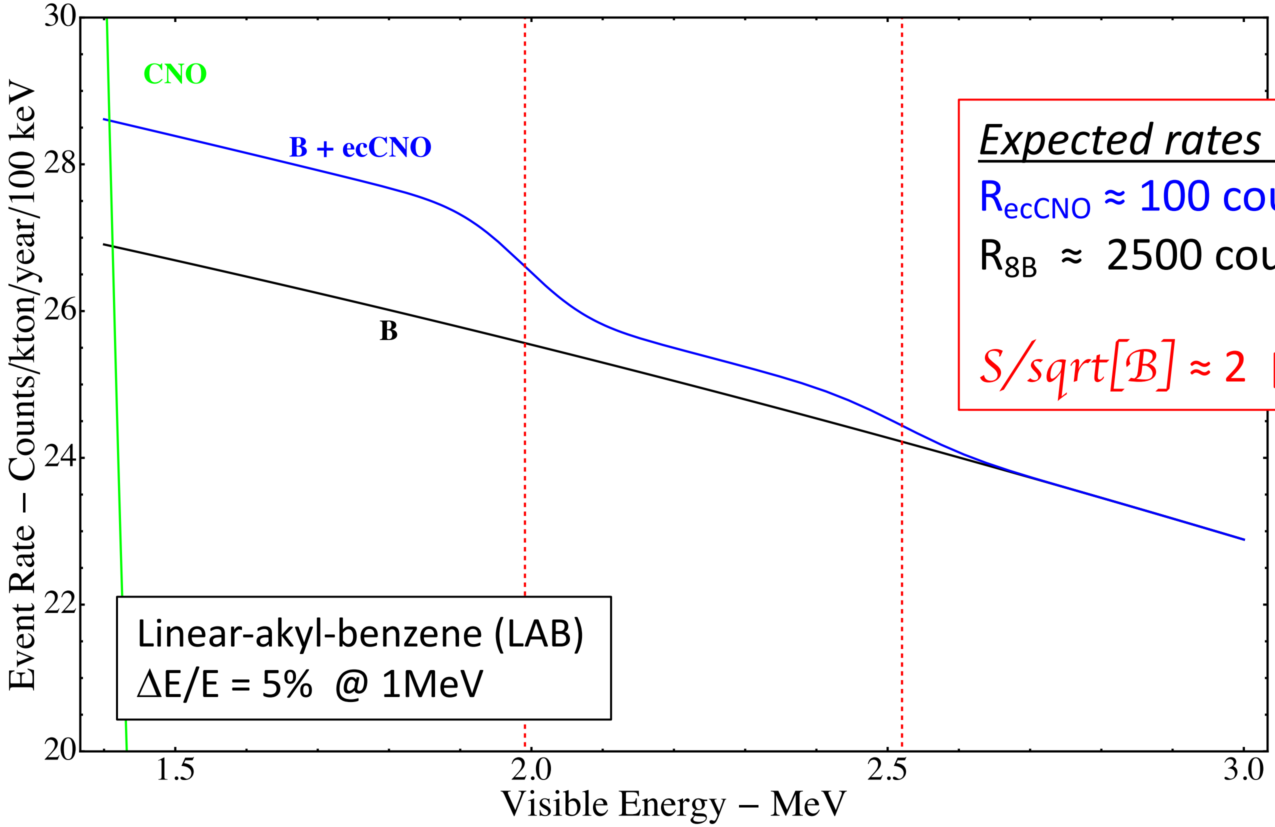
- Being monochromatic, they probe the solar neutrino **survival probability** at specific energies ($E_\nu \cong 2.5$ MeV) exactly **in the transition region**.



F.L. Villante, PLB 742 (2015) 279-284
L.C. Stonehill et al, PRC 69, 015801 (2004)
J.N. Bahcall, PRD 41, 2964 (1990).

Expected rates in Liquid Scintillators

- ν − e elastic scattering of ecCNO neutrinos produces Compton shoulders (smeared by energy resolution) at 2.0 and 2.5 MeV;
- ecCNO neutrino signal has to be extracted statistically from the (irreducible) ^{8}B neutrino background.



Expected rates [1.5 MeV, 2.5 MeV]

R_{ecCNO} ≈ 100 counts/10 kton/year

R_{8B} ≈ 2500 counts/10kton/year

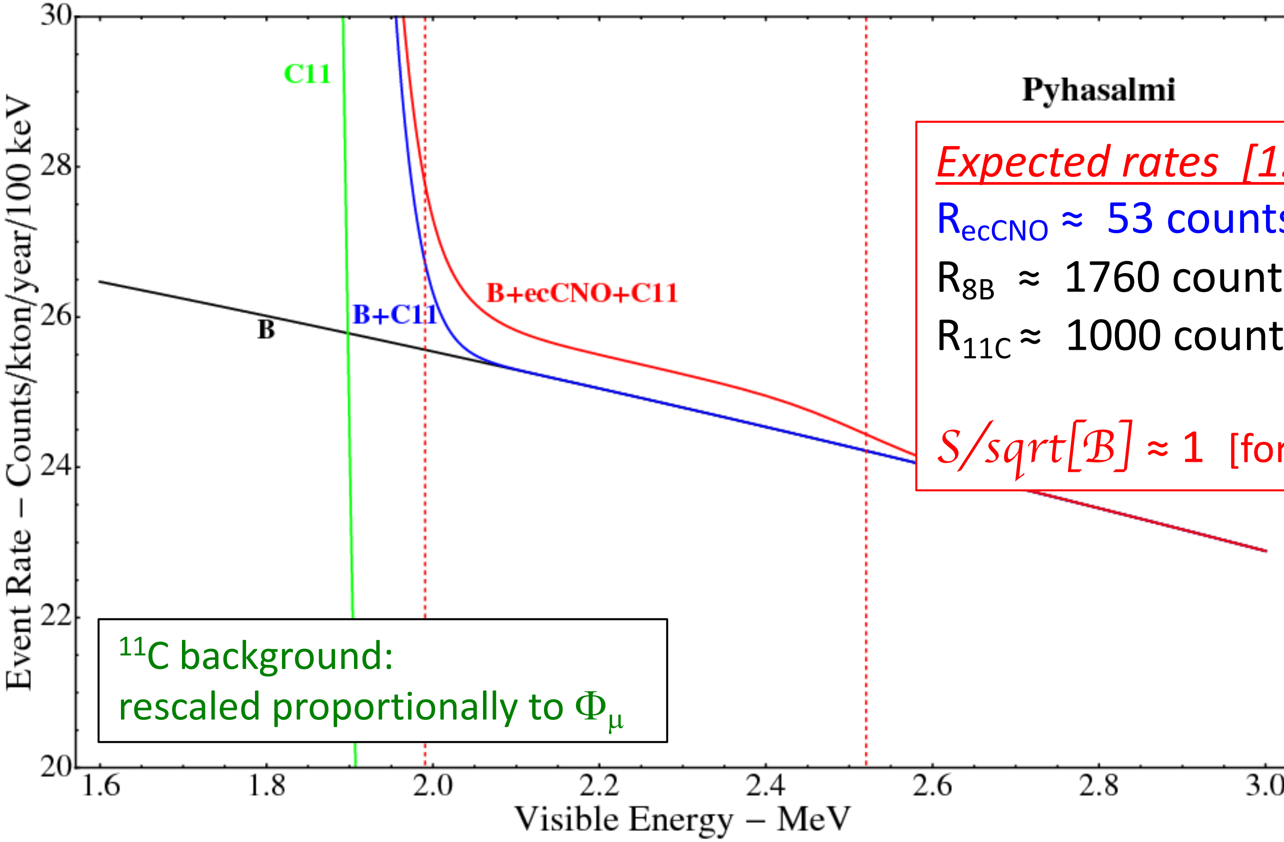
$S/sqrt[B]$ ≈ 2 [for 10kton × year exposure]

F.L. Villante, Phys.Lett. B742 (2015) 279-284

Expected rates in Liquid Scintillators

Additional background sources:

- **Intrinsic:** negligible/tagged (with Borexino Phase-I radio-purity levels);
- **External:** reduced by self-shielding (Fid. mass reduced from 50 to ≈20 kton in LENA);
- **Cosmogenic:** ^{11}C overlap with the observation window.



Expected rates [1.8 MeV, 2.5 MeV]

$R_{ecCNO} \approx$ 53 counts/10 kton/year

$R_{8B} \approx$ 1760 counts/10kton/year

$R_{11C} \approx$ 1000 counts/10kton/year

$S/sqrt[\mathcal{B}] \approx 1$ [for 10kton × year exposure]

Signal comparable to stat. fluctuations for exposures 10 kton × year or larger.

100 counts / year above 1.8 MeV in 20 kton detector → 3σ detection in 5 year in LENA

F.L. Villante, Phys.Lett. B742 (2015) 279-284

Removing composition-opacity degeneracy

The combined measurement of pp-chain and CNO-cycle neutrinos can be used to directly **infer the solar core composition.** *Indeed:*

- The (strong) dependence on T_c (and opacity) can be eliminated by using ^{8}**B-neutrinos as solar thermometer;**
- The additional dependence of CNO-neutrinos on X_{CN} can be used to infer core composition

In practical terms, one can form a weighted ratio of e.g. ^{8}B and ^{15}O neutrino fluxes that is:

- Essentially independent on environmental parameters (including opacity);
- **Directly proportional to Carbon+Nitrogen abundance in the solar core**

Serenelli et al., PRD 2013
See also (application to BX obs. rate):
Agostini et al, EPJ 2021
Villante & Serenelli, Frontiers 2021

$$\varphi_{^{15}\mathrm{O}}/\varphi_{^{8}\mathrm{B}}^{0.769} = x_{\mathrm{C}}^{0.802}\, x_{\mathrm{N}}^{0.204}\, x_{D}^{0.181}$$

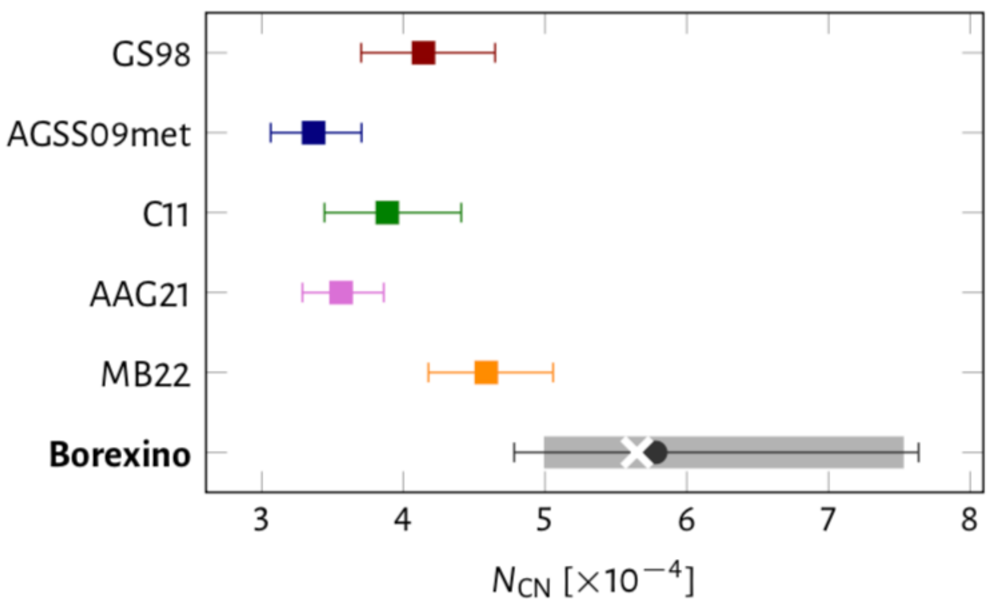
$$\times \left[x_{S_{11}}^{-0.866}\, x_{S_{33}}^{0.345}\, x_{S_{34}}^{-0.689}\, x_{S_{e7}}^{0.769}\, x_{S_{17}}^{-0.791}\, x_{S_{hep}}^{0.000}\, x_{S_{114}}^{1.046}\, x_{S_{116}}^{0.001}\right] \quad \text{(nucl)}$$

$$\times \left[x_{\mathrm{Age}}^{0.313}\, x_{L_{\odot}}^{0.602}\, x_{\kappa_a}^{0.018}\, x_{\kappa_b}^{-0.050}\right] \quad \text{(solar)}$$

$$\times \left[x_{\mathrm{O}}^{0.006}\, x_{\mathrm{Ne}}^{-0.003}\, x_{\mathrm{Mg}}^{-0.003}\, x_{\mathrm{Si}}^{0.001}\, x_{\mathrm{S}}^{0.001}\, x_{\mathrm{Ar}}^{0.001}\, x_{\mathrm{Fe}}^{0.005}\right] \quad \text{(met)}$$

Probing solar composition with neutrinos

$$\frac{(N_C + N_N)/N_H}{[(N_C + N_N)/N_H]^{SSM}} = 1.35 \times (0.96)^{-0.769} \times$$
$$\times \left[1 \pm \left(^{+0.303}_{-0.136}(\mathrm{CNO}) \pm 0.097(\mathrm{nucl}) \pm 0.023(^{8}\mathrm{B}) \pm 0.005(\mathrm{env}) \pm 0.027(\mathrm{diff}) \pm 0.022(\mathrm{O/N})\right)\right]$$



Error contributions