Local-equilibrium transport of oscillating neutrinos

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LA-UR-24-26818

Overview

Flavor mixing in neutron star mergers & core-collapse supernovae is estimated to have significant effects, but the quantum kinetic equation (QKE) is computationally intractable.

$$\underbrace{i\left(\partial_t + \hat{\mathbf{p}} \cdot \partial_{\mathbf{r}}\right)\rho}_{\text{Particle advection}} = \underbrace{[H, \rho]}_{\text{Flavor mixing}} + \underbrace{iC}_{\text{Collisions}}$$

This talk:
A coarse-grained transport theory based on local mixing equilibrium.

I. Foundations

Studies have questioned the accuracy of quantum kinetics by comparing many-body and mean-field evolution.

$$H = \sum_{\mathbf{p}} \omega_{\mathbf{p}} \mathbf{B} \cdot \hat{\mathbf{J}}_{\mathbf{p}} + \underbrace{\mu \sum_{\mathbf{p},\mathbf{q}} (1 - \cos\theta_{\mathbf{pq}})\, \hat{\mathbf{J}}_{\mathbf{q}} \cdot \hat{\mathbf{J}}_{\mathbf{p}}}_{\text{Pairing interaction}}$$

where each $\mathbf{J}$ is a flavor isospin operator.

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The many-body Hamiltonian used in this literature *requantizes the forward-scattering Hamiltonian*—which fundamentally alters the physics.

Johns, 2305.04916 (*Neutrino many-body correlations*)

Calculations have recently been done using the correct many-body Hamiltonian:

Neutrino many-body flavor evolution: the full Hamiltonian

Vincenzo Cirigliano,[1] Srimoyee Sen,[2] and Yukari Yamauchi[1]

[1] *Institute for Nuclear Theory, University of Washington, Seattle, WA 98195, USA*
[2] *Department of Physics and Astronomy, Iowa State University, Ames, IA, 50011*
(Dated: April 26, 2024)

We study neutrino flavor evolution in the quantum many-body approach using the full neutrino-neutrino Hamiltonian, including the usually neglected terms that mediate non-forward scattering processes. Working in the occupation number representation with plane waves as single-particle states, we explore the time evolution of simple initial states with up to $N = 10$ neutrinos. We discuss the time evolution of the Loschmidt echo, one body flavor and kinetic observables, and the one-body entanglement entropy. For the small systems considered, we observe 'thermalization' of both flavor and momentum degrees of freedom on comparable time scales, with results converging towards expectation values computed within a microcanonical ensemble. We also observe that the inclusion of non-forward processes generates a faster flavor evolution compared to the one induced by the truncated (forward) Hamiltonian.

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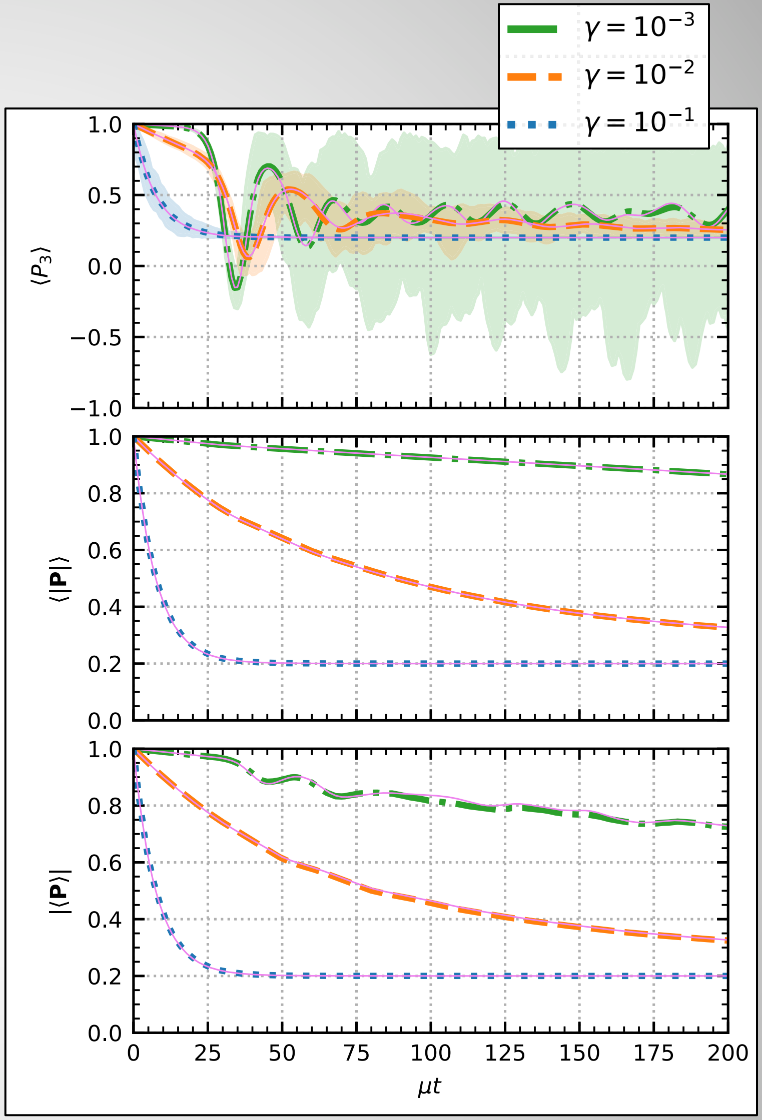
Coherent forward scattering (G_F vs. $G_F{}^2$) is a kinetic phenomenon. Most neutrino many-body models aren't actually expected to be described by kinetics.

Johns, 2305.04916 (*Neutrino many-body correlations*)
Also see Shalgar & Tamborra, 2304.13050

We devised the **once-in-a-lifetime encounter (OILE) model** to illustrate the emergence of coherent flavor evolution in many-body systems with brief interaction times.

Kost, **Johns**, & Duan, 2402.05022
(*Once-in-a-lifetime encounter models for neutrino media*)

Compare with Friedland & Lunardini 2003



II. Paradigms

1. **Direct numerical simulation**

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2. **Asymptotic-state subgrid modeling**

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2. **Asymptotic-state subgrid modeling**

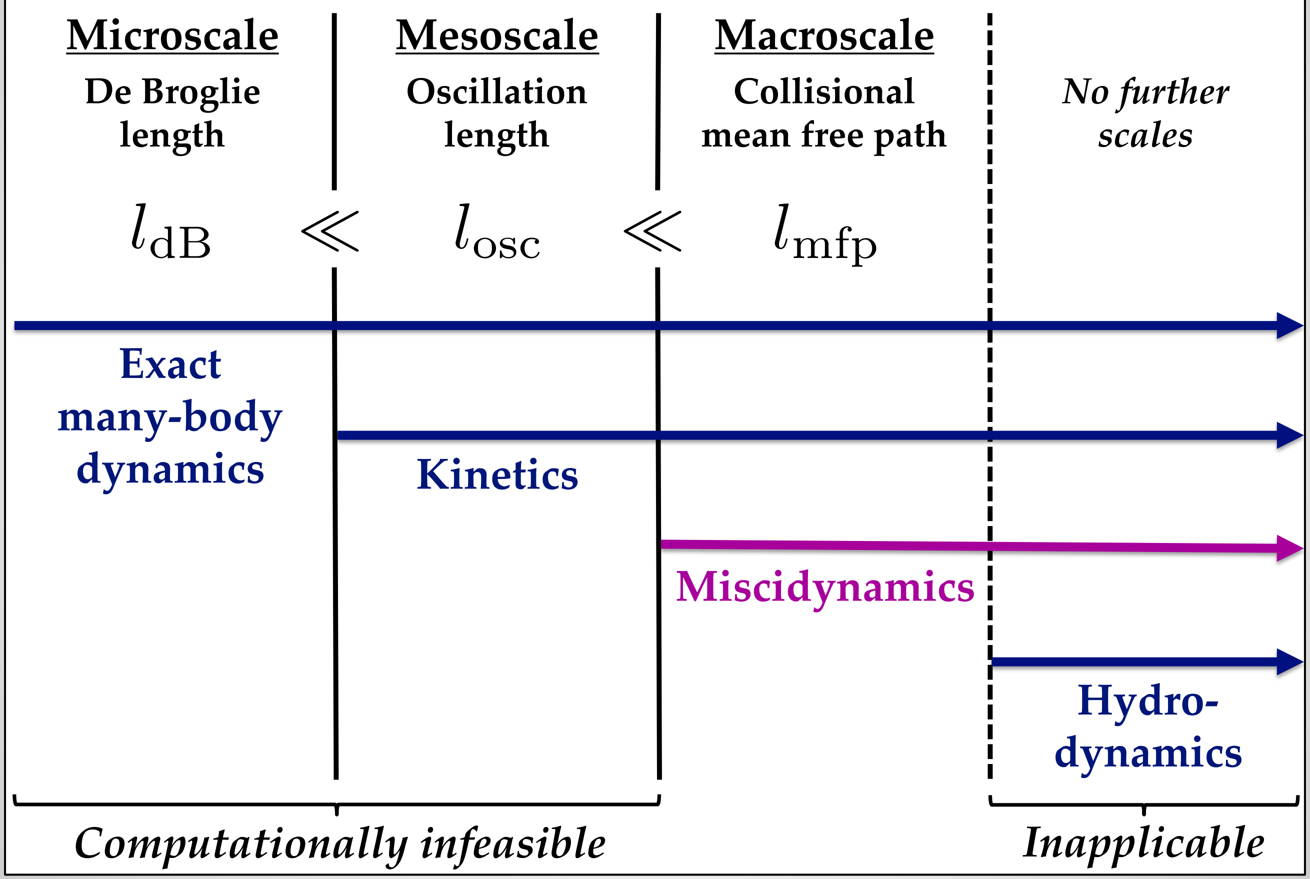
3. **Miscidynamics** (*i.e.*, transport near local mixing equilibrium)

$$i\left(\partial_t + \hat{\mathbf{p}} \cdot \partial_{\mathbf{r}}\right)\rho = [H, \rho] + iC$$

Vanishes in *mixing* eq

Vanishes in *collisional* eq

Length scales, coarse-grainings, & transport theories



Johns, 2306.14982 (*Thermodynamics of oscillating neutrinos*)

Asymptotic-state subgrid models are not self-consistent: If fast instabilities are instantaneous, then angular crossings can't form in the first place.

Johns, 2401.15247 (*Subgrid modeling of neutrino oscillations in astrophysics*)

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with $\gamma \to \infty$

Nagakura, **Johns**, & Zaizen, 2312.16285
(*BGK subgrid model for neutrino quantum kinetics*)

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with $\left[H_{\boldsymbol{p}}^{\text{eq}}, \rho_{\boldsymbol{p}}^{\text{eq}} \right] = 0$

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Essential qualities of miscidynamics:
- Astrophysical driving
- Self-consistent equilibrium

Johns, 2401.15247 (*Subgrid modeling of neutrino oscillations in astrophysics*)

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Why would there be rapid relaxation or uncorrelated fluctuations? We'll explore **ergodicity** and **thermodynamics** as possible explanations.

Miscidynamics generalizes adiabatic quantum evolution.

MSW
adiabaticity

Nonlinearity
(self-interactions)

Raffelt-Smirnov
adiabaticity
Raffelt & Smirnov 2007

Inhomogeneity,
advection,
& collisions

Miscidynamic
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In particular, we can have adiabatic *incoherent* evolution. Control parameters include flavor polarizations $|\boldsymbol{P}|$ as well as couplings λ and μ.

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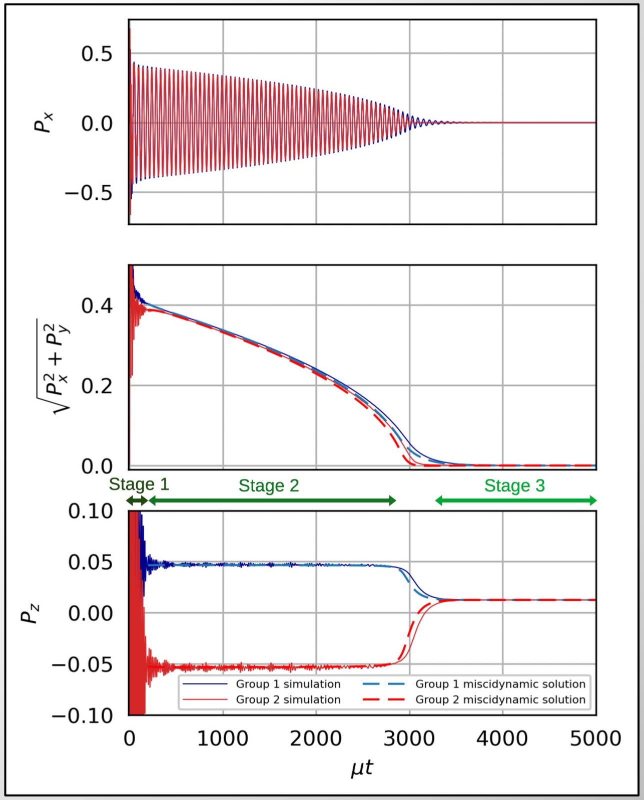
Nontrivial relaxation in OILE model



Kost, **Johns**, & Duan, in preparation

Current status of miscidynamics

- The **adiabatic theory** (what I'm presenting) is largely complete, *i.e.*, simulation-ready.
- The **nonadiabatic theory** (with finite and/or correlated deviations from mixing eq) is still being developed.
- The nonadiabatic theory is definitely needed for some models, but it's unclear whether it's needed for astrophysics. There may be **adiabatic adjustment**.

The adiabatic hypothesis

Johns, 2306.14982 (*Thermodynamics of oscillating neutrinos*)
Johns, 2401.15247 (*Subgrid modeling of neutrino oscillations in astrophysics*)

III. Flavor ergodicity

Ergodic hypothesis

Neutrinos uniformly explore all accessible parts of flavor space.

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Lichtenberg & Lieberman

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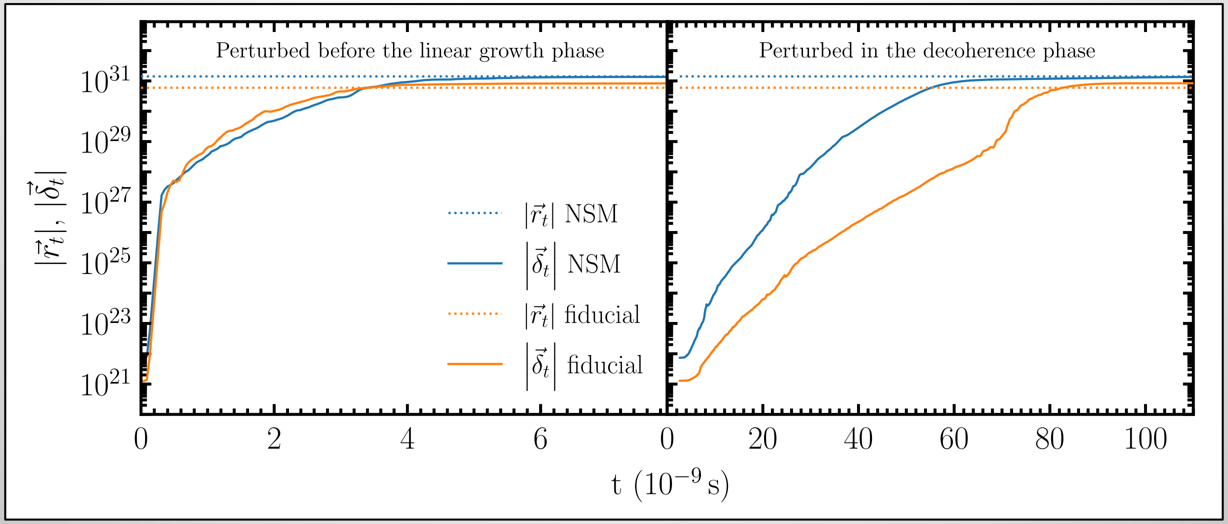
Lichtenberg & Lieberman

Numerical evidence of chaotic flavor evolution



Urquilla & Richers, 2401.01936

If ergodicity is assumed, the iff relationship between **fast instabilities & angular crossings** becomes easy to prove.

Johns, 2402.08896 (*Ergodicity demystifies fast neutrino flavor instability*)
Proofs without ergodicity: Morinaga 2022 and Fiorillo & Raffelt 2024

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Johns, 2402.08896 (*Ergodicity demystifies fast neutrino flavor instability*)
Proofs without ergodicity: Morinaga 2022 and Fiorillo & Raffelt 2024

The insufficiency of angular crossings for instability of the **fast flavor pendulum** is also illuminated by ergodicity: the infinite number of invariants restricts the ergodic subspace.

Johns, Nagakura, Fuller, & Burrows, 1910.05682
(*Neutrino oscillations in supernovae: Angular moments and fast instabilities*)

IV. Neutrino quantum thermodynamics

Guided by ergodicity, equate coarse-grained averages with **ensemble expectation values**:

$$\langle \rho \rangle \cong \rho^{\mathrm{eq}}$$

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ρ^{eq} maximizes entropy S subject to constraints. Assuming there are no fine-grained spatial correlations,

$$S = -V \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \mathrm{Tr}\left[\rho_{\boldsymbol{p}} \log \rho_{\boldsymbol{p}} + (1-\rho_{\boldsymbol{p}}) \log(1-\rho_{\boldsymbol{p}})\right]$$

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$\Big\downarrow$ Fix total energy & neutrino number at each $\boldsymbol{p}$

$$\rho^{\mathrm{eq}}_{\boldsymbol{p}} = \frac{1}{e^{\beta(H^{\mathrm{eq}}_{\boldsymbol{p}} - \mu_{\boldsymbol{p}})} + 1}$$

Johns, 2306.14982 (*Thermodynamics of oscillating neutrinos*)

Pathway #3 to the miscidynamic equation goes through neutrino quantum thermodynamics:

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Negligible thermal fluctuations in the thermodynamic limit

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Negligible thermal fluctuations in the thermodynamic limit

Adiabaticity ($\Delta S = 0$) makes S-maximization unnecessary:

1. Evolve $|\boldsymbol{P}|$ under collisions & advection.
2. Obtain new $\boldsymbol{P}^{\text{eq}}$ using $|\boldsymbol{P}|$ from step 1.

Johns, 2306.14982 (*Thermodynamics of oscillating neutrinos*)

V. Collisional flavor instabilities

The asymmetry between neutrino & antineutrino interaction rates can cause collisional flavor instability.

Johns, 2104.11369 (*Collisional flavor instabilities of supernova neutrinos*)

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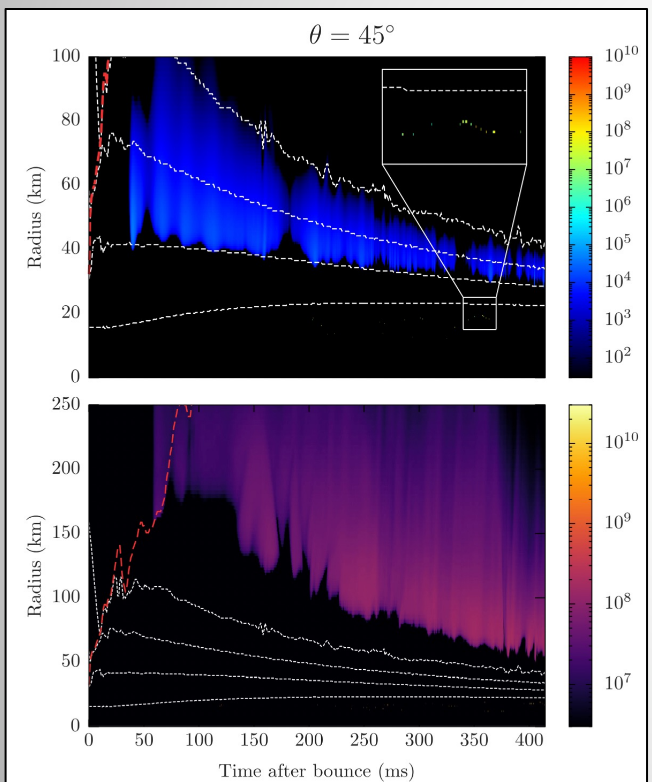
2D SN simulation



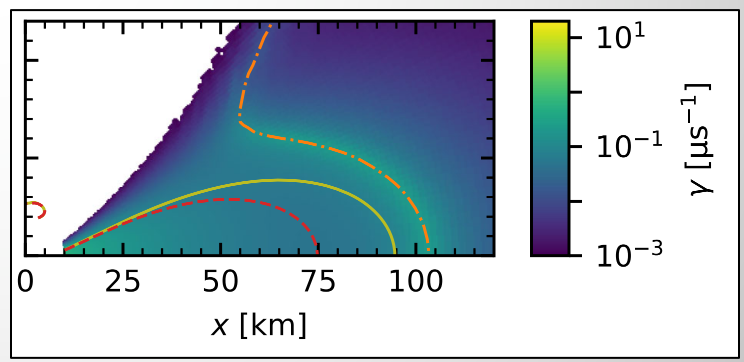
CFI growth rate (s^{-1})

FFI growth rate (s^{-1})

Akaho, Liu, Nagakura, Zaizen, & Yamada 2024

Evidence points to CFI being prevalent in SNe & mergers, with growth rates from Γ up to $\sqrt{\Gamma\mu}$.

BH accretion disk (t = 50 ms)



CFI growth rate

Xiong, **Johns**, Wu, & Duan, 2212.03750
(*Collisional flavor instability in dense neutrino gases*)

Estimating CFI effects is uniquely challenging because of kilometer-scale growth.

Johns & Nagakura, 2206.09225 [collision-related modeling issues]
(*Self-consistency in models of neutrino scattering and fast flavor conversion*)
Xiong, Wu, Martinez-Pinedo, Fischer, George, Lin, & **Johns,** 2210.08254 [a first attempt at estimates]
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Analytic insights from idealized models:

***Slow* flavor pendulum** Gravitational field due to mass splitting

Hannestad, Raffelt, Sigl, & Wong 2006 [SNe]
Johns & Fuller, 1709.00518 [early universe]
(*Strange mechanics of the neutrino flavor pendulum*)

***Fast* flavor pendulum** Gravitational field due to spectral asymmetry

Johns, Nagakura, Fuller, & Burrows, 1910.05682
(*Neutrino oscillations in supernovae: Angular moments and fast instabilities*)
Padilla-Gay, Tamborra, & Raffelt 2022
Fiorillo & Raffelt 2023

***Collisional* flavor pendulum** Adiabatic spin reversal due to collisional asymmetry

Johns & Rodriguez, 2312.10340
(*Collisional flavor pendula and neutrino quantum thermodynamics*)

Miscidynamic adiabaticity

Summary

I. Foundations

- No evidence that QKE is inadequate.
- New type of many-body model: OILE.

II. Paradigms

- Asymptotic-state models aren't self-consistent.
- Miscidynamics = local-equilibrium transport.

III. Flavor ergodicity

- A hypothetical but potentially useful concept (*e.g.*, understanding instability).

IV. Neutrino quantum thermodynamics

- Sheds new light on neutrino oscillations.

V. Collisional flavor instabilities

- Challenging even to estimate their effects.