Theory of fast neutrino flavor evolution

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Niels Bohr Institute, Copenhagen

Based on works with G. Raffelt, G. Sigl

GGI Neutrino Frontiers

KØBENHAVNS UNIVERSITET
UNIVERSITY OF COPENHAGEN

VILLUM FONDEN

Collective flavor conversions

$$\nu_\mu \longrightarrow \nu_e$$

$$\bar{\nu}_\mu \longrightarrow \bar{\nu}_e$$

Interaction eigenstates

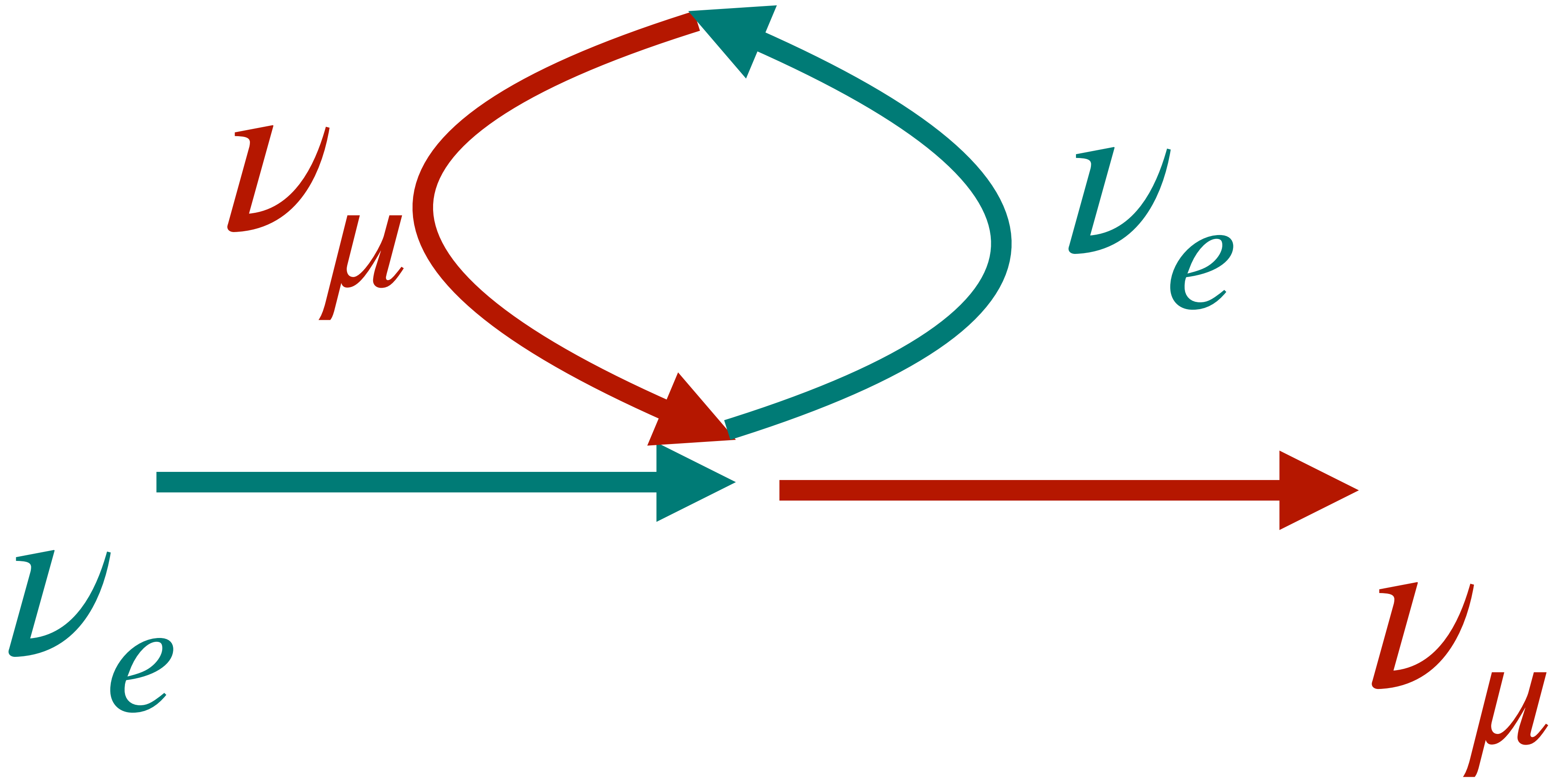
$\neq$

Mass eigenstates

Does not require other neutrinos

Collective flavor conversions

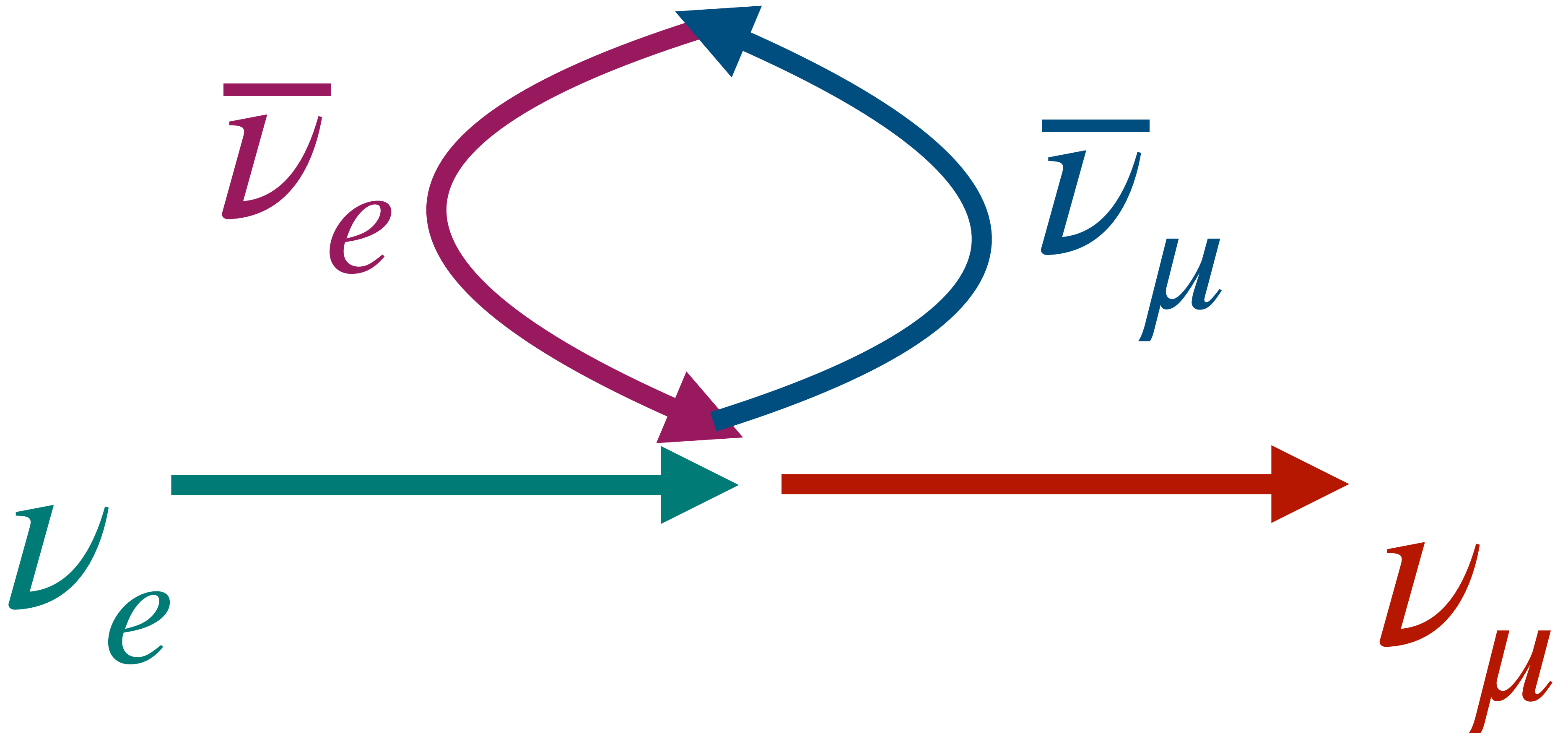
$\alpha_e | \nu_e > + \alpha_\mu | \nu_\mu >$



Refractive flavor exchange among different energies and directions

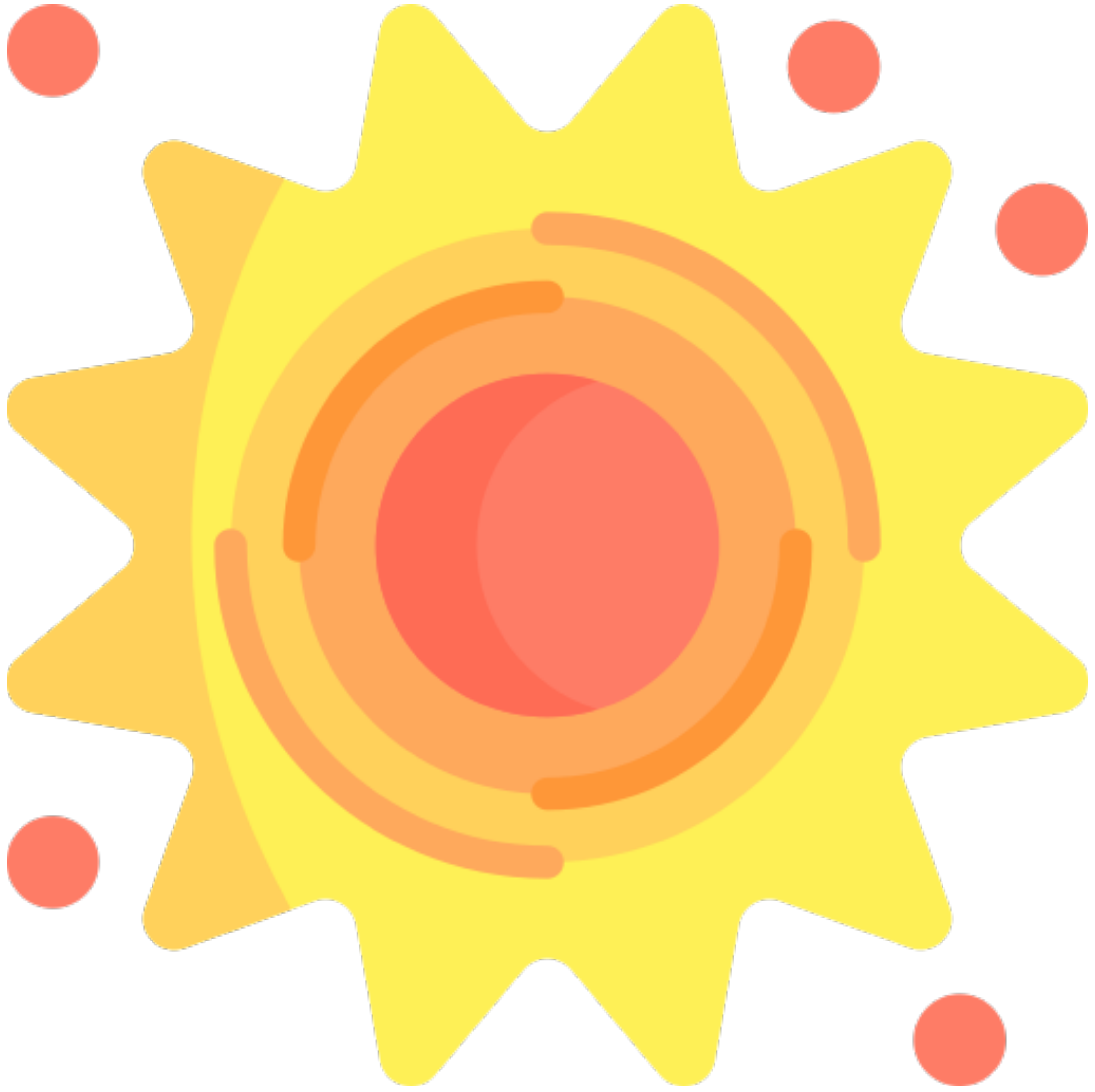
Non linear!

$\alpha_e | \bar{\nu}_e > + \alpha_\mu | \bar{\nu}_\mu >$



Quantum superposition neutrinos infect other neutrinos!

Does it matter?



High densities in supernovae (SNe) and neutron star mergers (NSMs)

Does it happen?

Most likely **yes!**

Abbar et al., 1812.06883; Li et al., 2103.02616; Abbar et al., 1911.01983; Nagakura et al., 1910.04288; Abbar et al., 2012.06594; Nagakura et al., 2108.07281; Wu et al., 1701.06580; …

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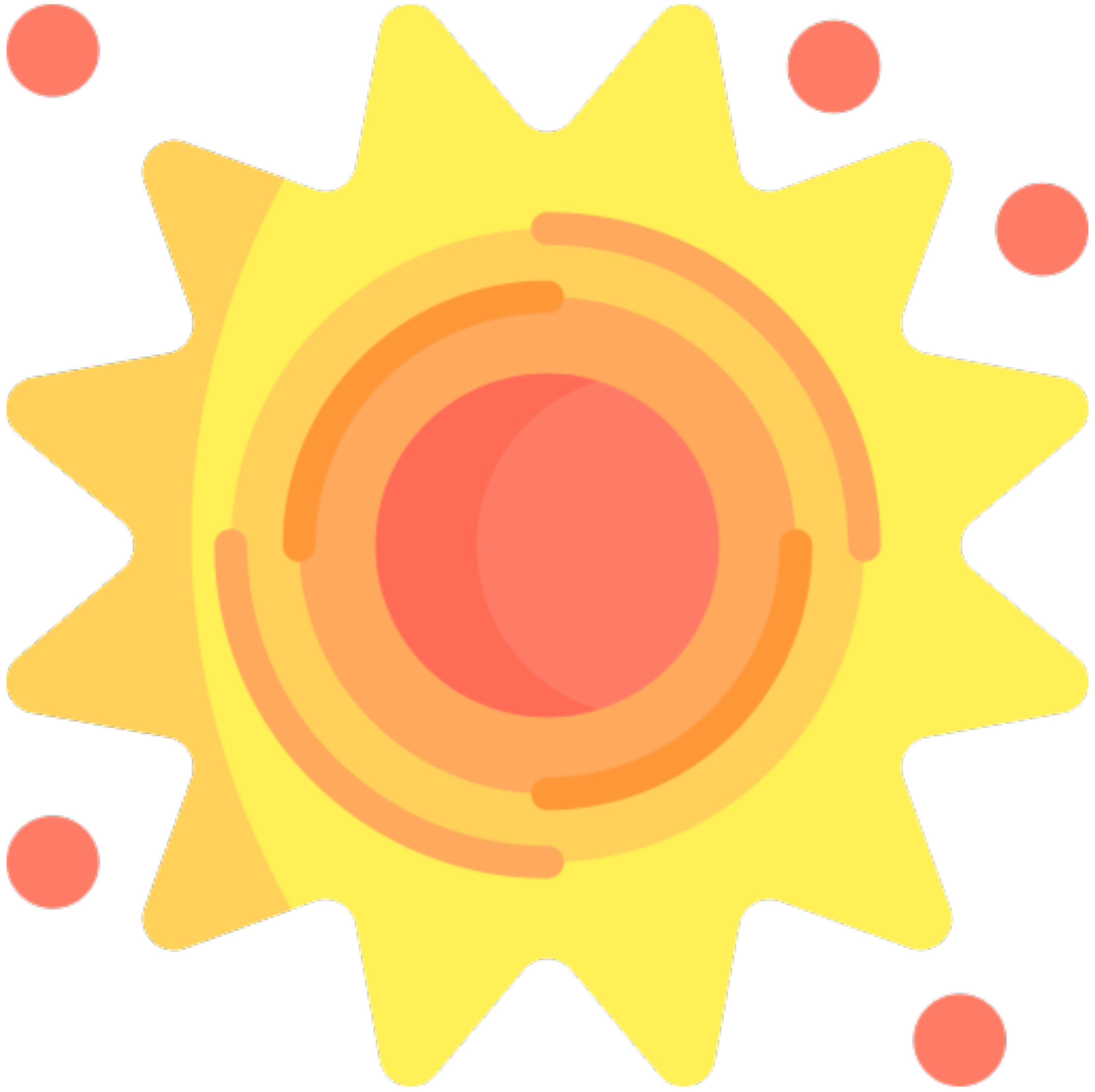

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Does it affect neutrino observations?

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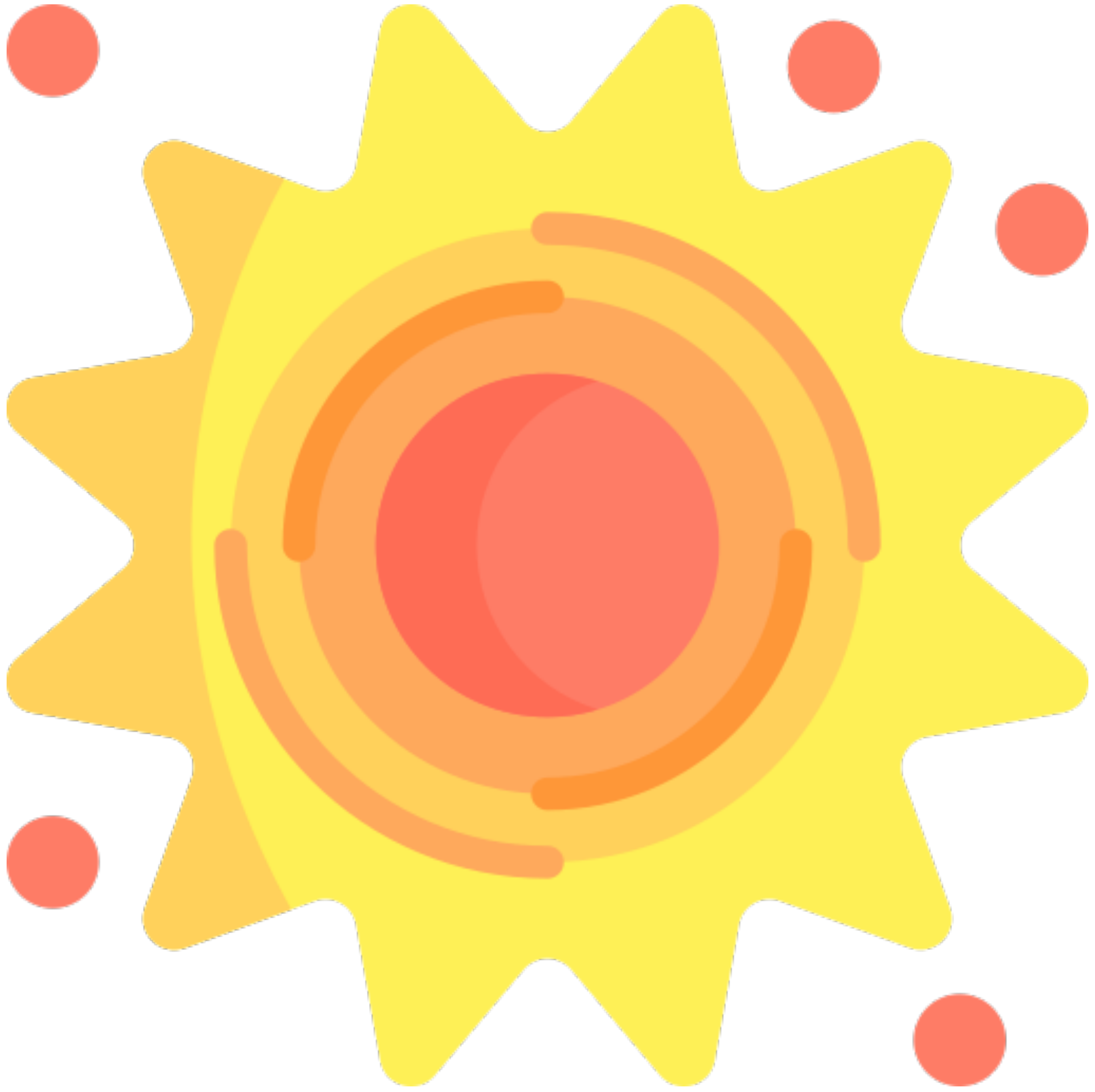
Does it matter?



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Does it affect neutrino observations?

Most likely **yes!**

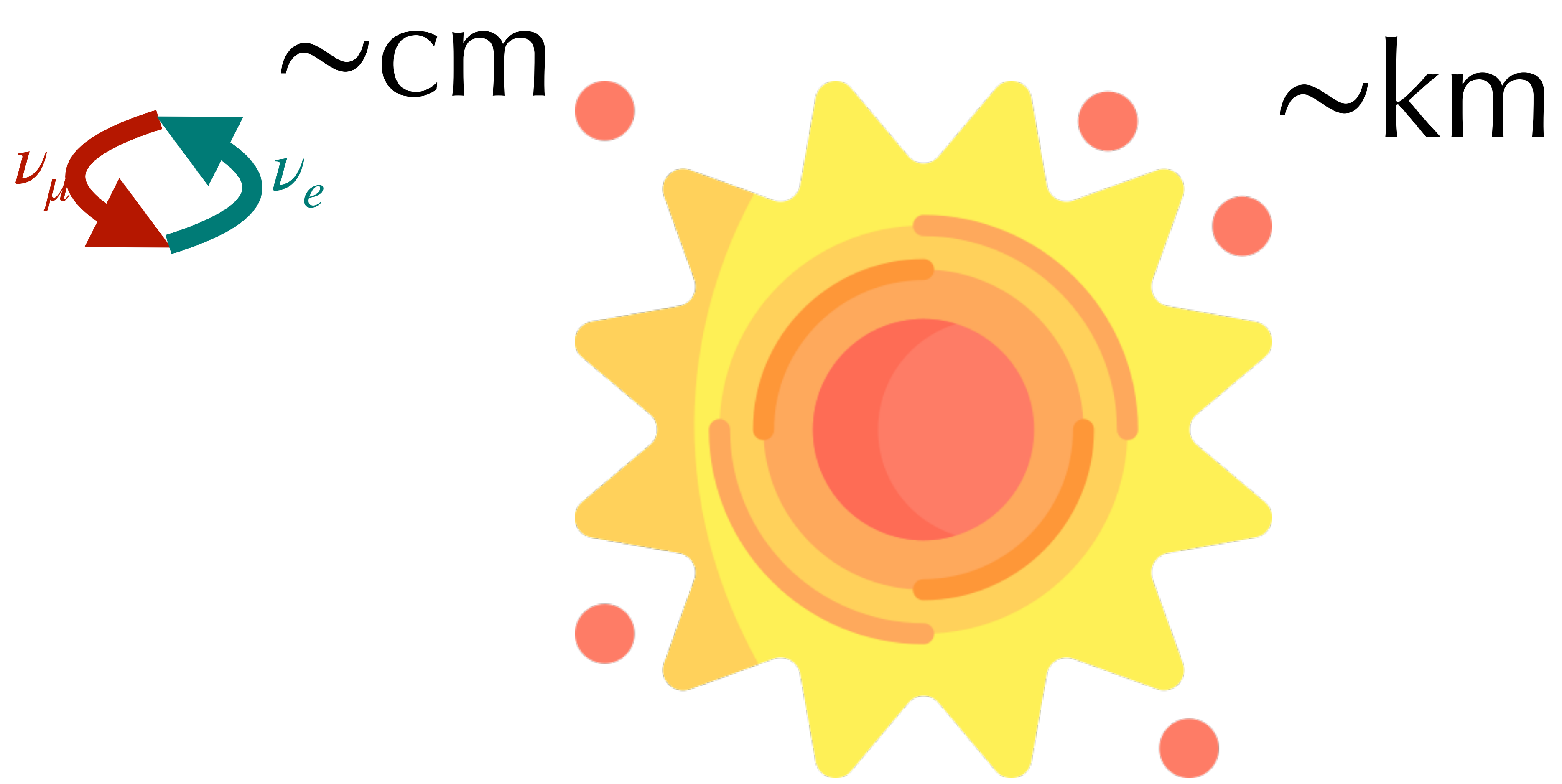
Does it affect SN evolution?

Likely **yes!**

Ehring et al., 2301.11938, 2305.11207

Theoretical interest

(One of the) most exotic many-body systems (driven by **weak interactions!**)



Intrinsically **multi-scale** problem (challenging!)

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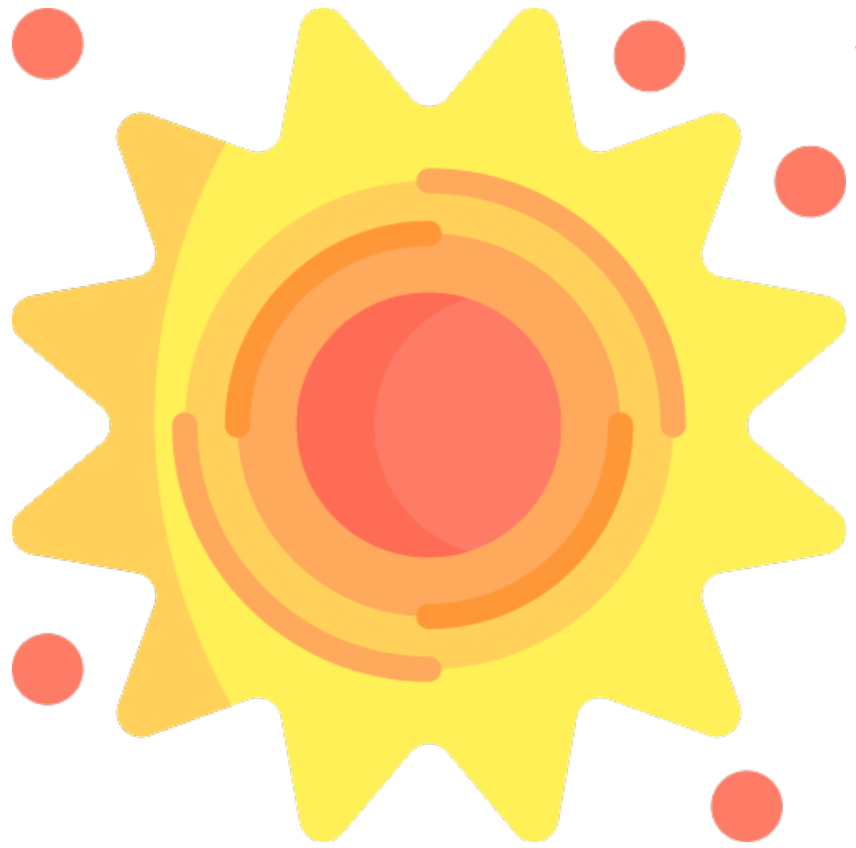
(One of the) most exotic many-body systems (driven by **weak interactions!**)

~cm



~km

Intrinsically **multi-scale** problem (challenging!)

Turbulence

Convection

MHD turbulence

Fast flavor conversions

Theoretical interest

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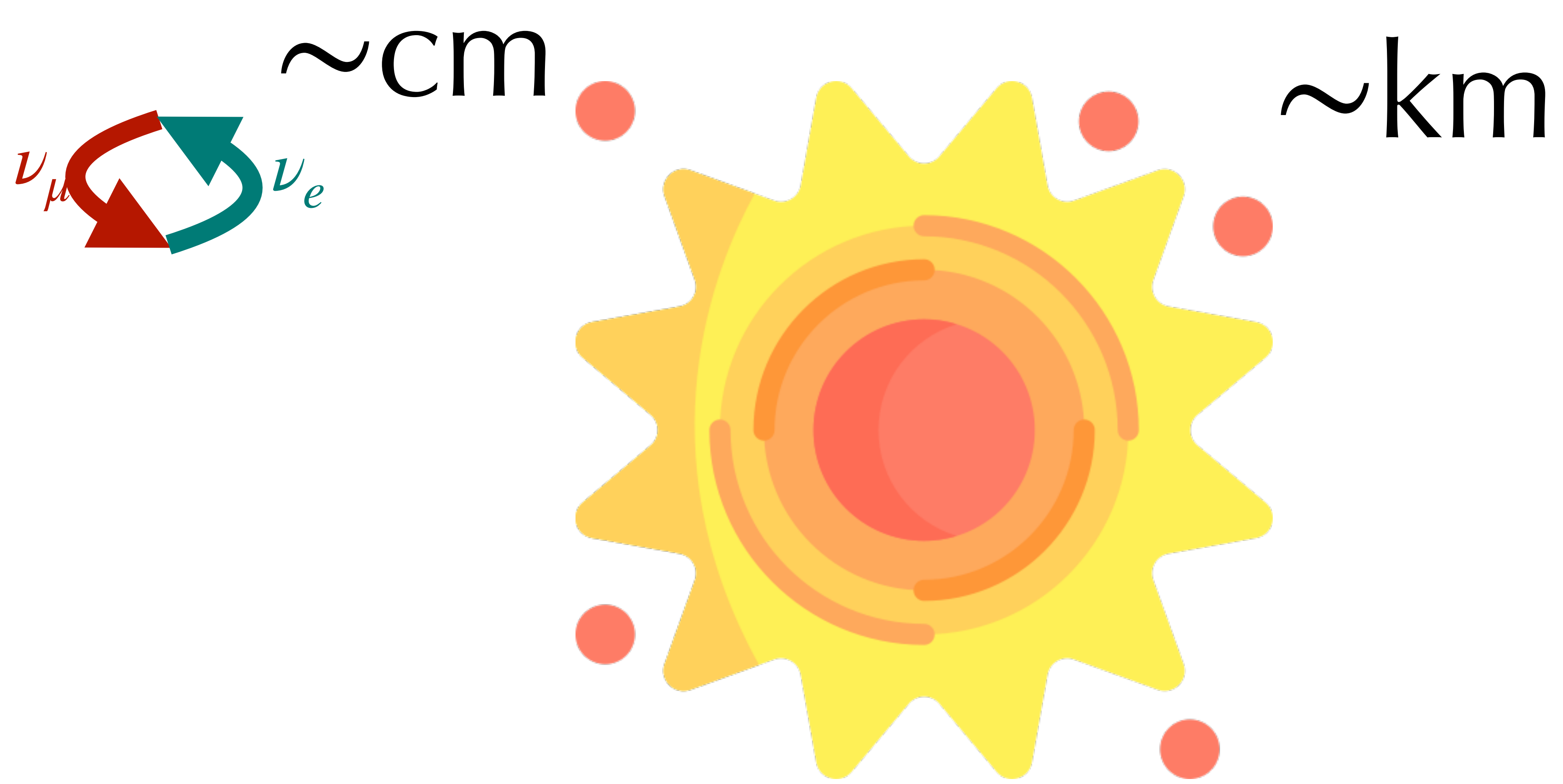

Intrinsically **multi-scale** problem (challenging!)

Turbulence	Kolmogorov-Obukhov
Convection	Mixing length
MHD turbulence	Kraichnan, Goldreich-Sridhar, …
Fast flavor conversions	???

Outline

- Quantum kinetic equations
- **Stable** systems: the meaning of ELN crossings
  - Flavor waves
  - Landau damping — the plasma analogy
- **Unstable** systems
  - Growth of flavor waves
  - Quasi-linear saturation of instability

Quantum kinetic equations

$$\alpha_e|\nu_e> + \alpha_\mu|\nu_\mu>$$

$$|\alpha_e|^2$$

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{e\mu} \\ \rho_{\mu e} & \rho_{\mu\mu} \end{pmatrix}$$

$$\alpha_\mu^* \alpha_e$$

$$|\alpha_\mu|^2$$

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Dolgov, Sov. J. Nucl. Phys.,1981

Rudzsky, Astrophys. Space Sci., 1990,

Sigl, Raffelt, Nucl. Phys. B, 1993

$$\partial_t \rho$$

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$$\partial_t \rho \qquad + v \partial_r \rho$$

Advection

Quantum kinetic equations

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Dolgov, Sov. J. Nucl. Phys.,1981

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$$\partial_t \rho \qquad + v\partial_r \rho \qquad = -i[\mathcal{H}, \rho]$$

Advection

Interaction

$|\overrightarrow{P}_v|$ conserved!

$$\mathcal{H} \propto \sqrt{2} G_F \sum_{\prime} \rho'$$

Quantum kinetic equations

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{e\mu} \\ \rho_{\mu e} & \rho_{\mu\mu} \end{pmatrix}$$

Instability driven by **advection** and **interaction** (similar to **plasma waves**)

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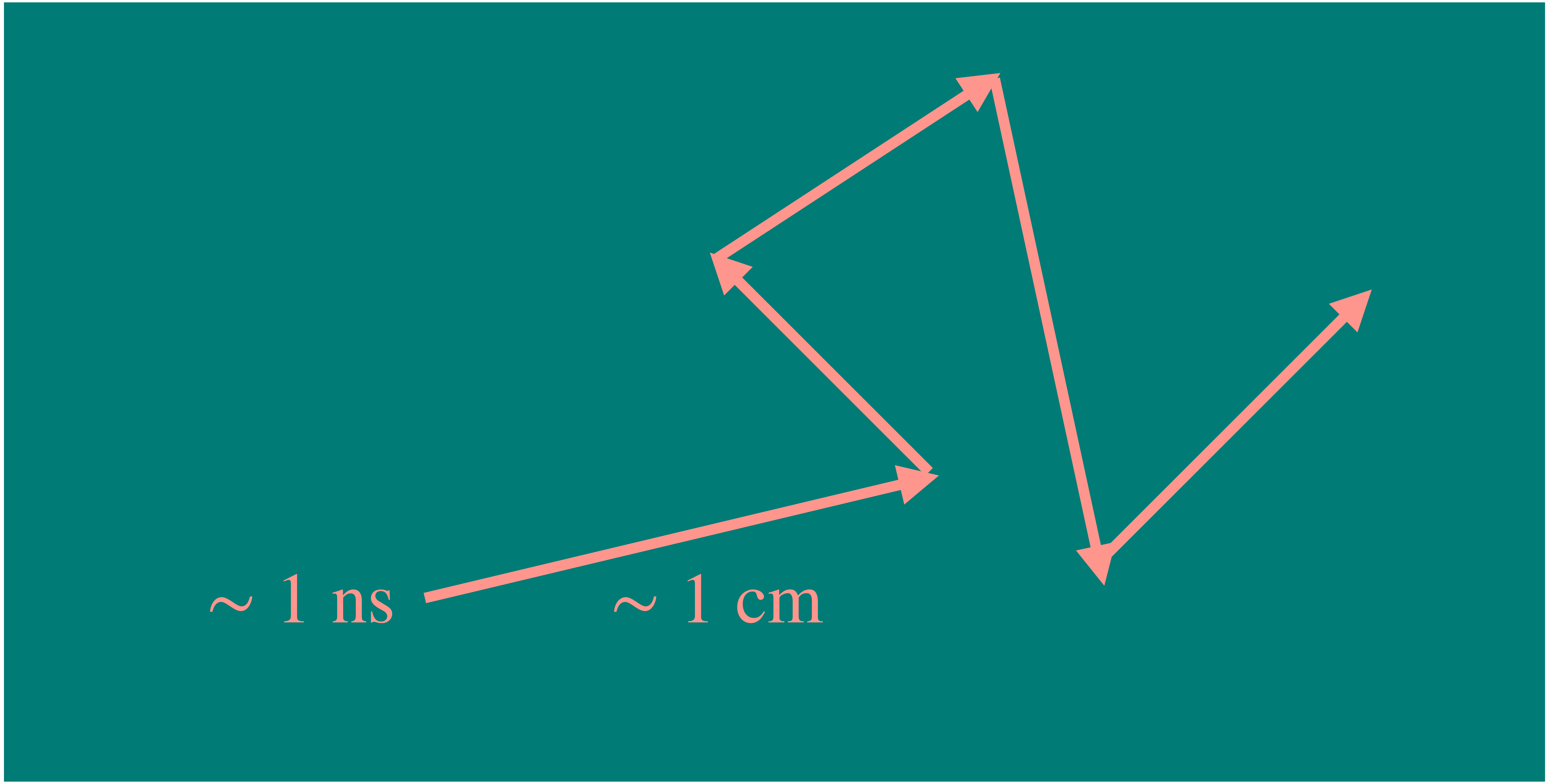
~ 1 ns → $\mathscr{H} \propto \sqrt{2} G_F \sum_{'} \rho'$

Quantum kinetic equations



Spontaneous breaking of homogeneity!

Theory of fast neutrino flavor evolution

*Based on **DF**, Raffelt, 2406.06708*

Conservation laws

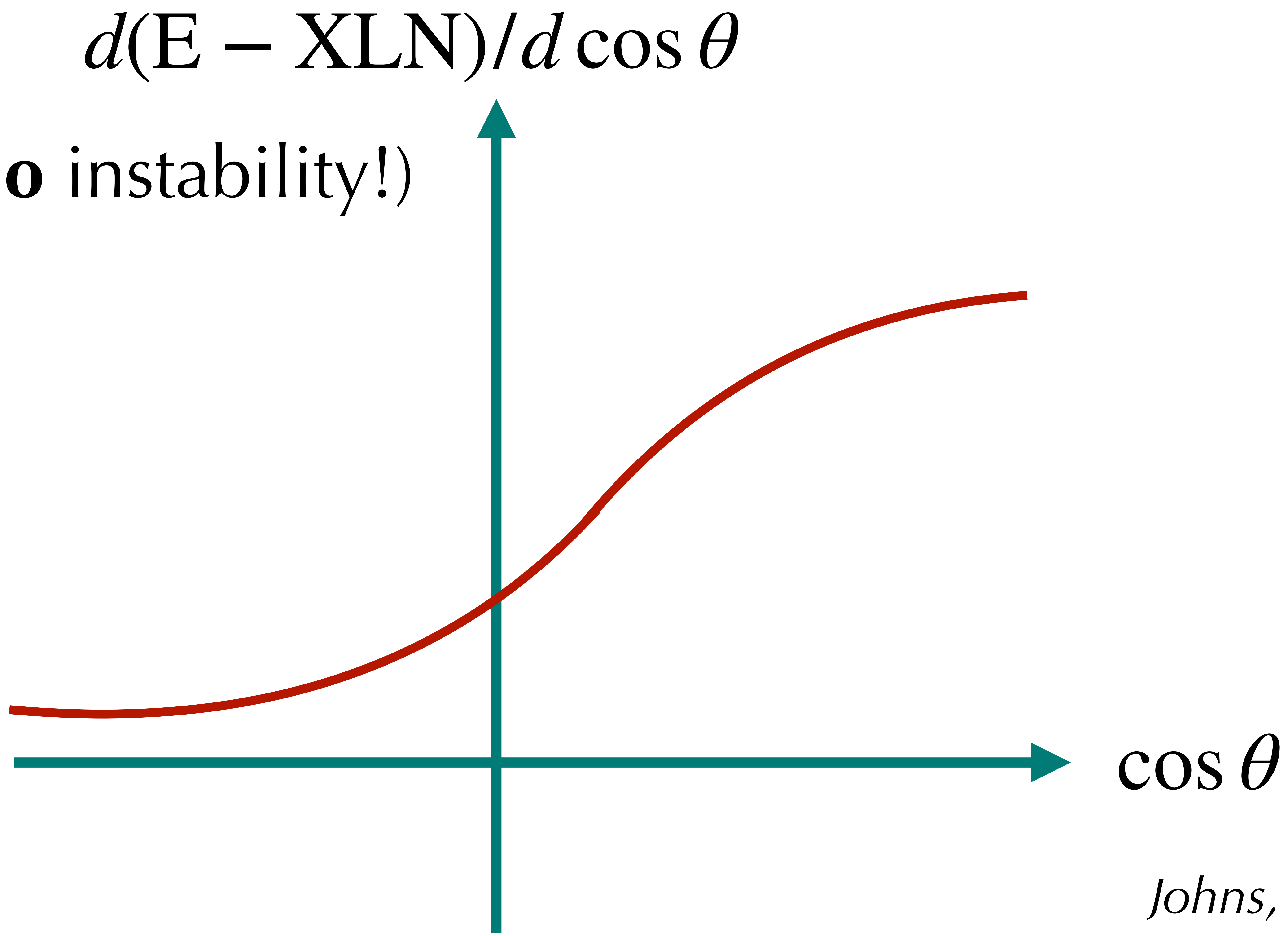
$$(n_{\nu_e} - n_{\bar{\nu}_e}) - (n_{\nu_\mu} - n_{\bar{\nu}_\mu})$$

conserved!

E-XLN conservation

Conservation laws

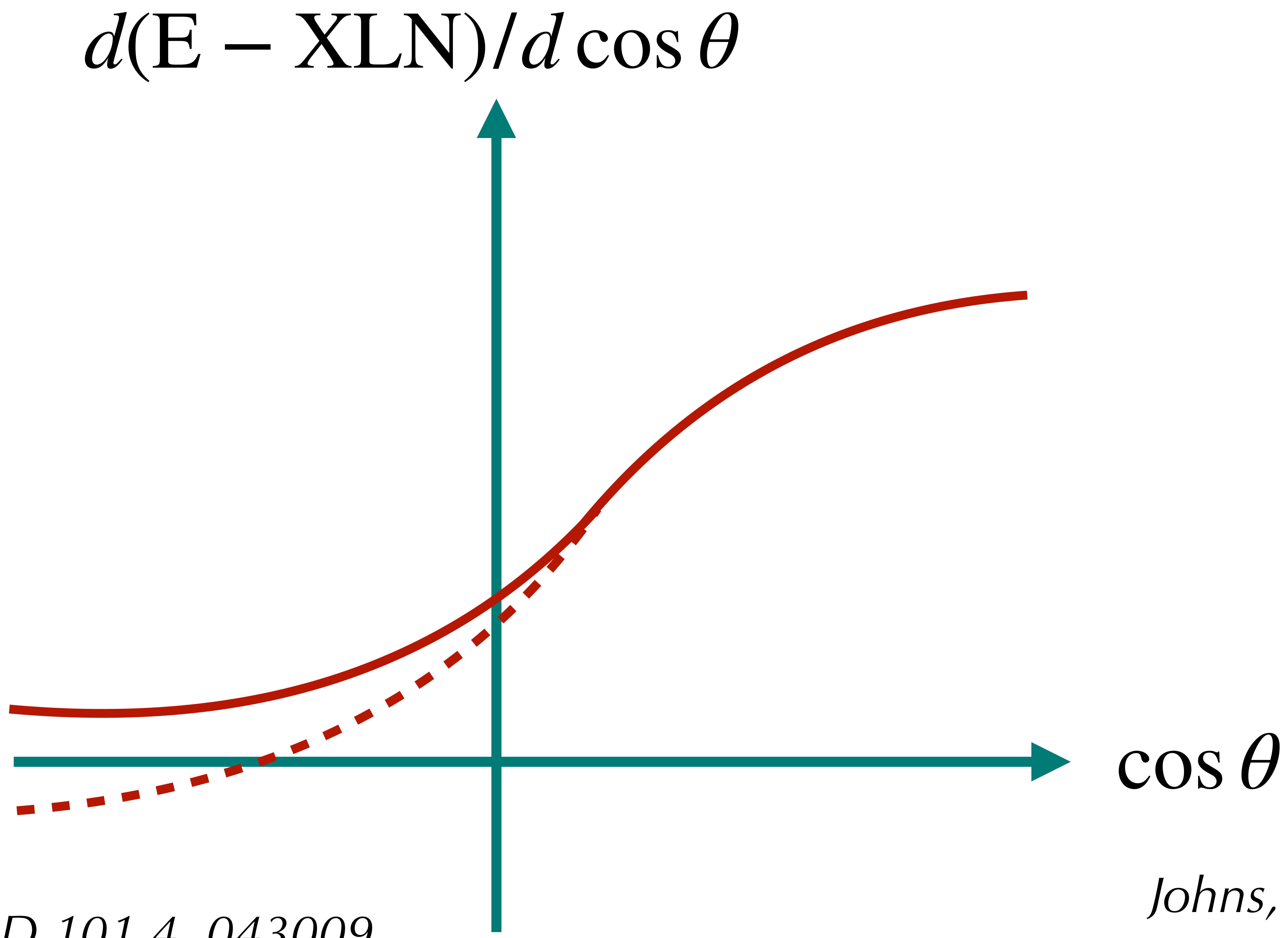
Nothing can move (**no** instability!)



Johns, 2402.08896

***DF**, Raffelt, 2406.06708*

Conservation laws



Things can move

Instability? Only if no more conservation laws!

Simple counterexample: homogeneous system (infinite conservation laws)

Johns et al., PRD 101 4, 043009

***DF**, Raffelt, PRD 107 4, 043024; PRD 107 12, 123024*

Johns, 2402.08896

***DF**, Raffelt, 2406.06708*

Stable systems

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{e\mu} \\ \rho_{\mu e} & \rho_{\mu\mu} \end{pmatrix}$$

$$\rho_{e\mu} \ll \rho_{ee}, \rho_{\mu\mu}$$

Neutrinos are mostly in flavor eigenstates!

$\rho_{e\mu}$

Flavor waves

Stable systems

Stable systems

Stable systems

If $u \neq c \cos\theta$ no net effect!

Stable systems

If $u = c\cos\theta$ resonance!

Stable systems

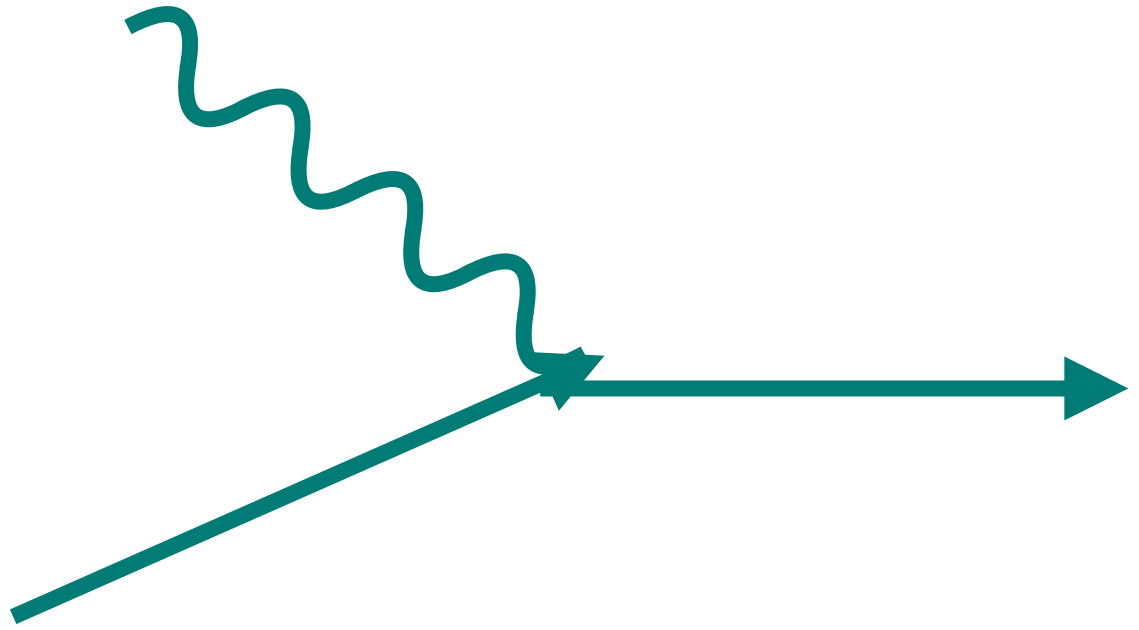
Flavor waves can only be damped ⟶ **Landau damping!**

Resonant neutrinos move in phase with the wave

Stable systems

Flavor waves can only be damped → **Landau damping!**

Resonant neutrinos move in phase with the wave

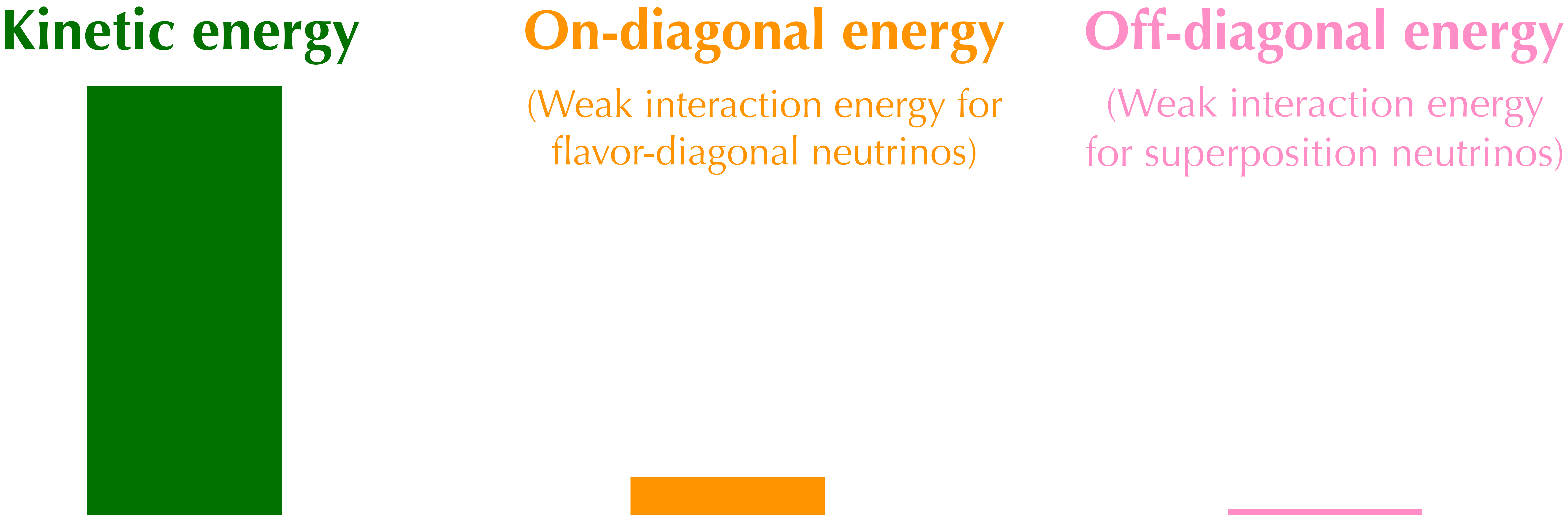


Cherenkov absorption causes damping

Stable systems

Flavor waves can only be damped → **Landau damping!**

Resonant neutrinos move in phase with the wave



DF, *Raffelt, Sigl, PRL 133 2, 021002*

Stable systems

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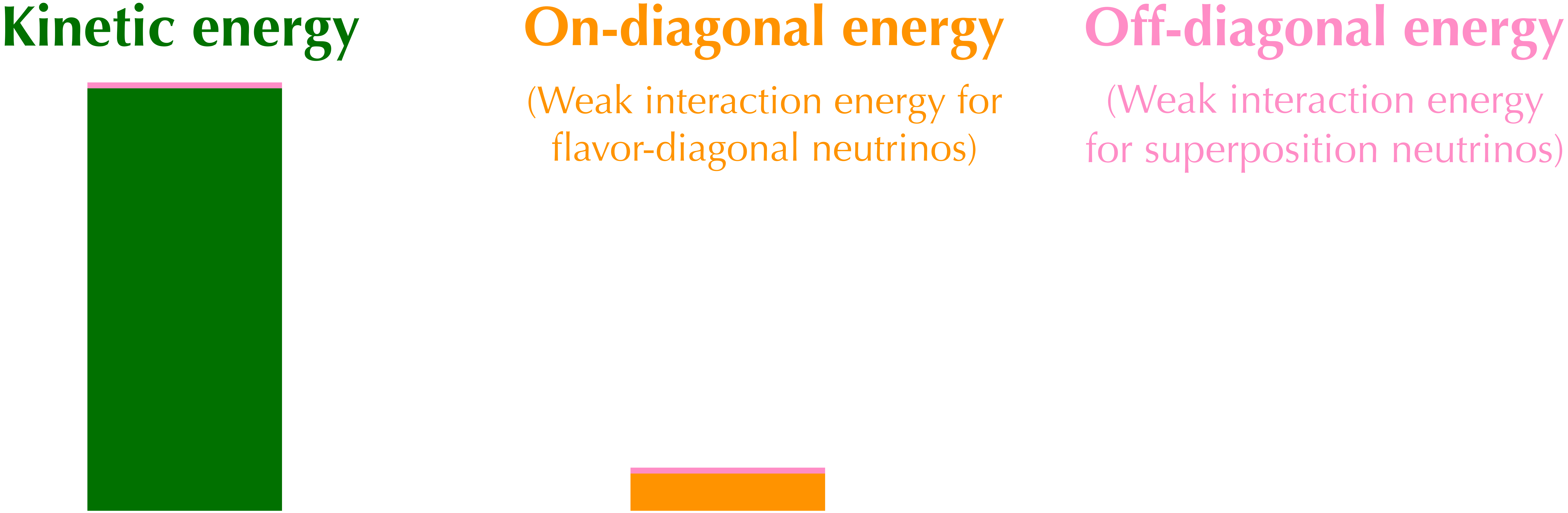
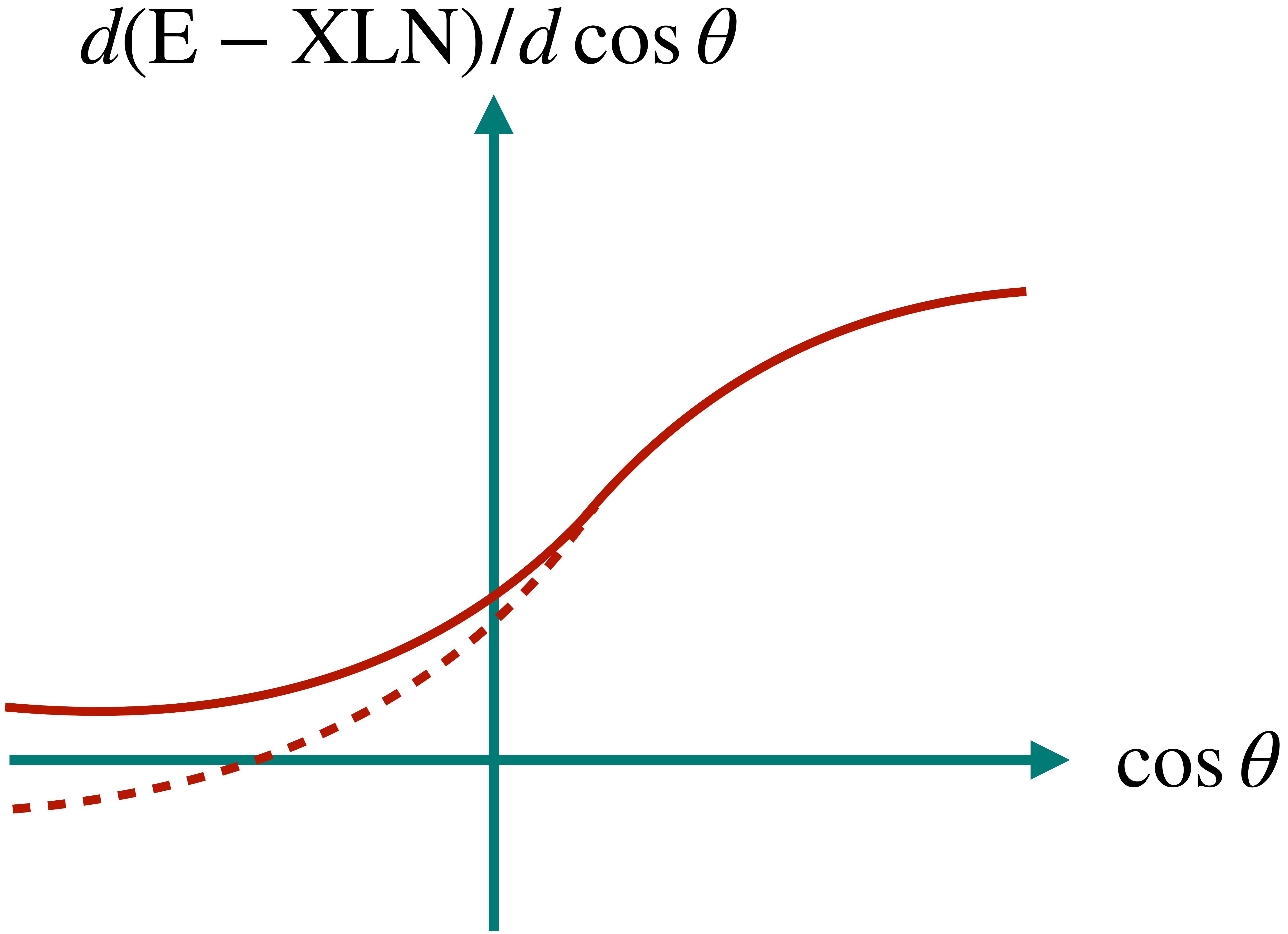
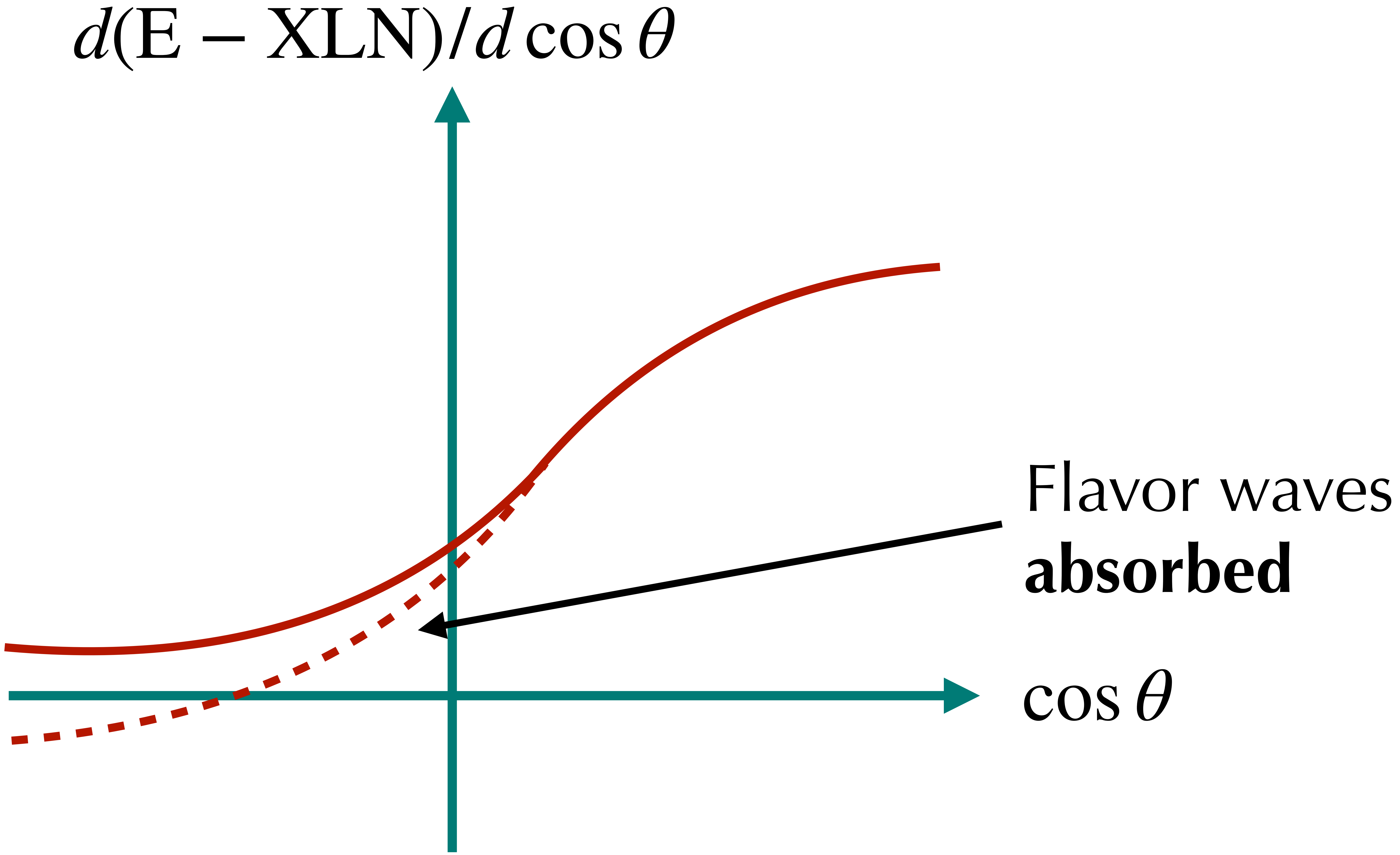


***DF**, Raffelt, Sigl, PRL 133 2, 021002*

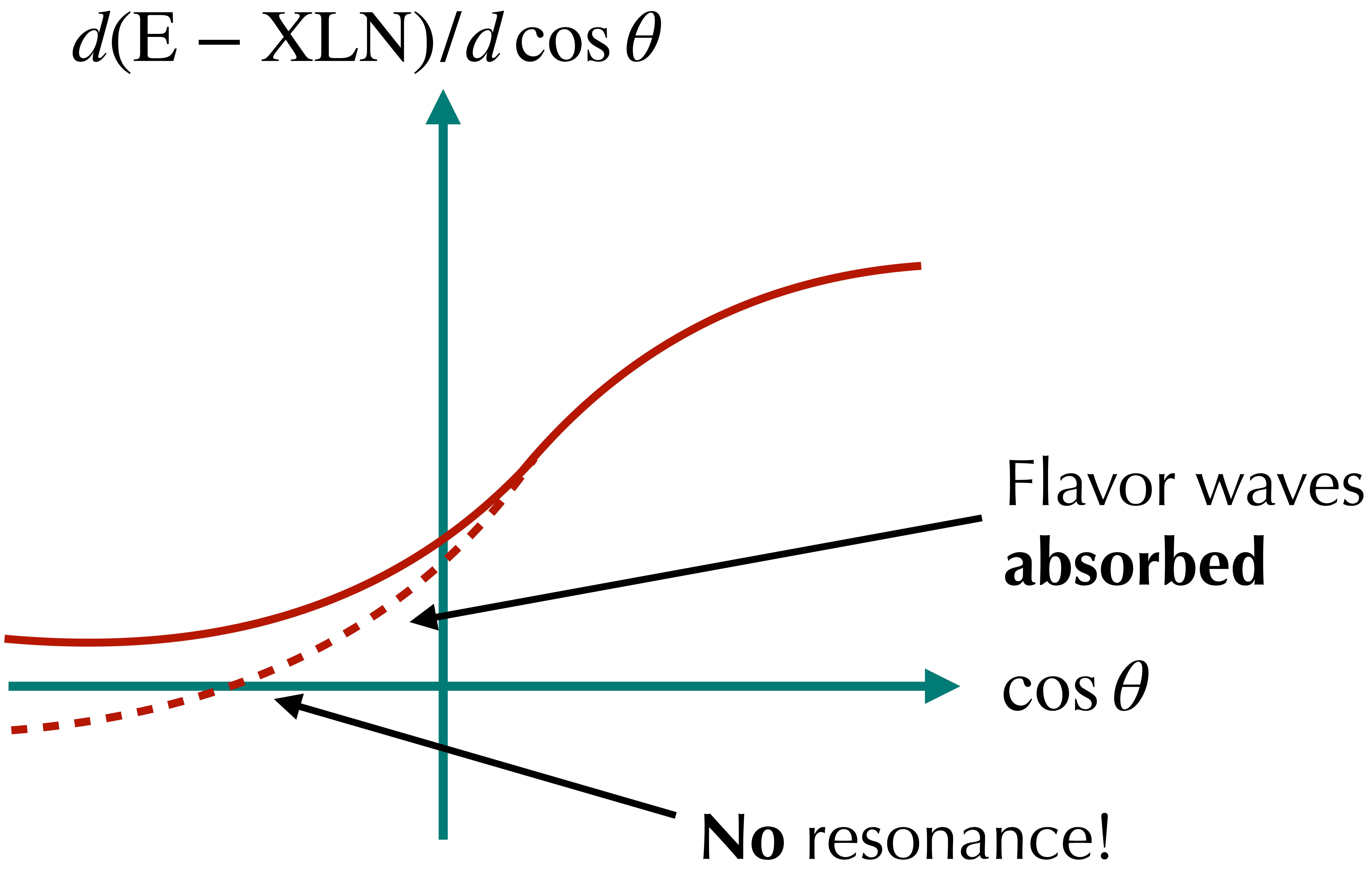
E-XLN crossing



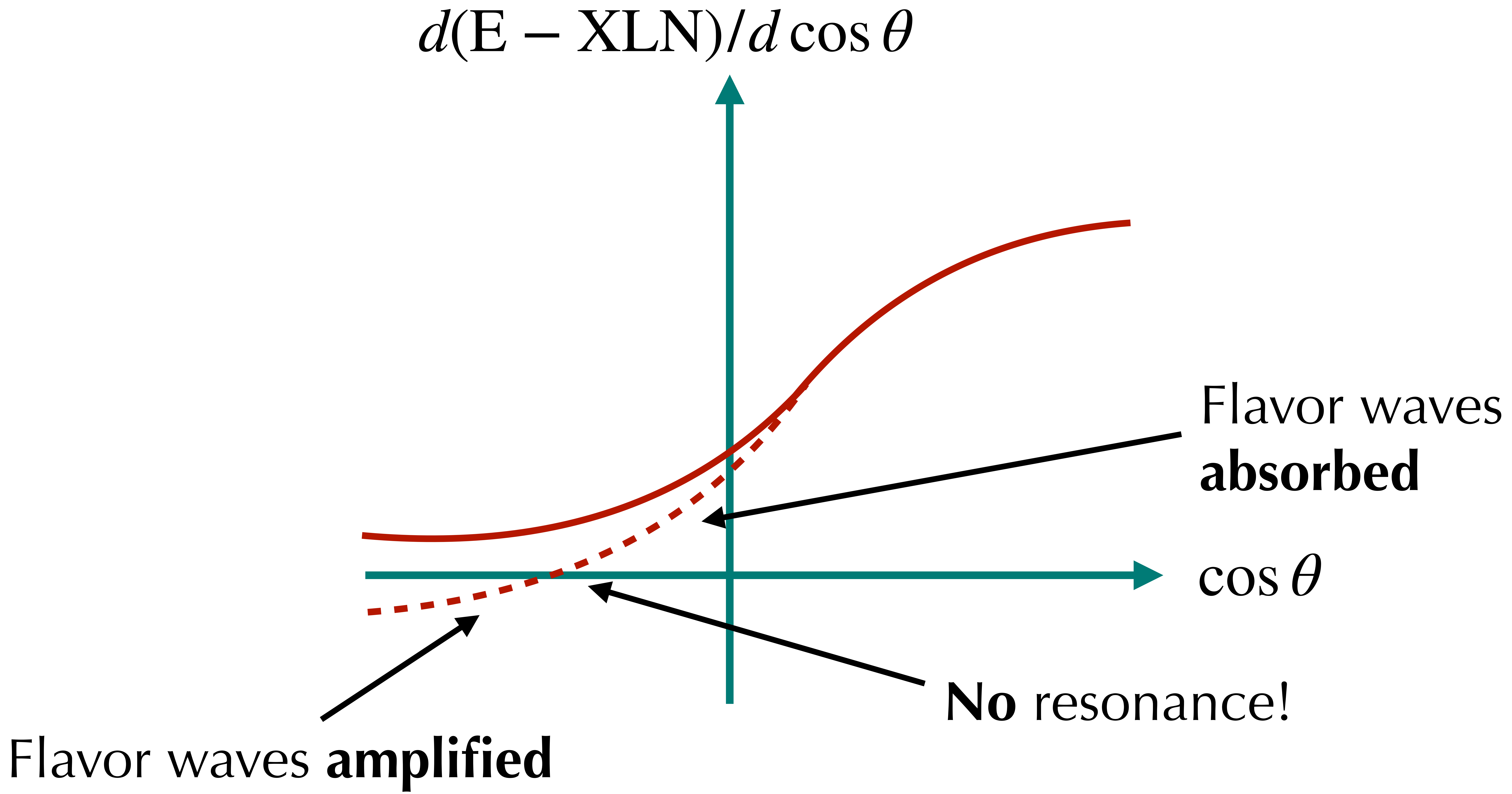
E-XLN crossing



E-XLN crossing



E-XLN crossing

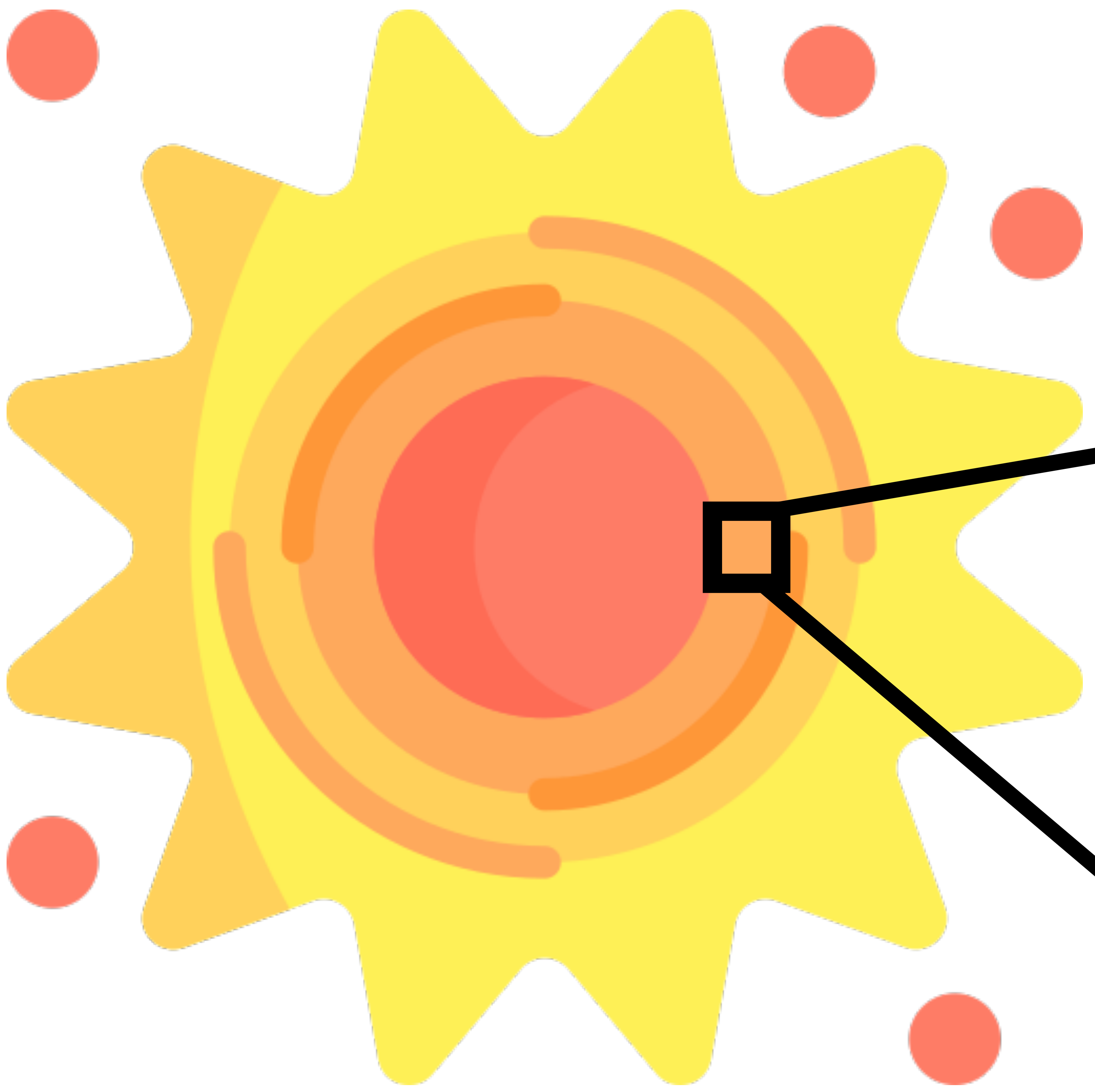


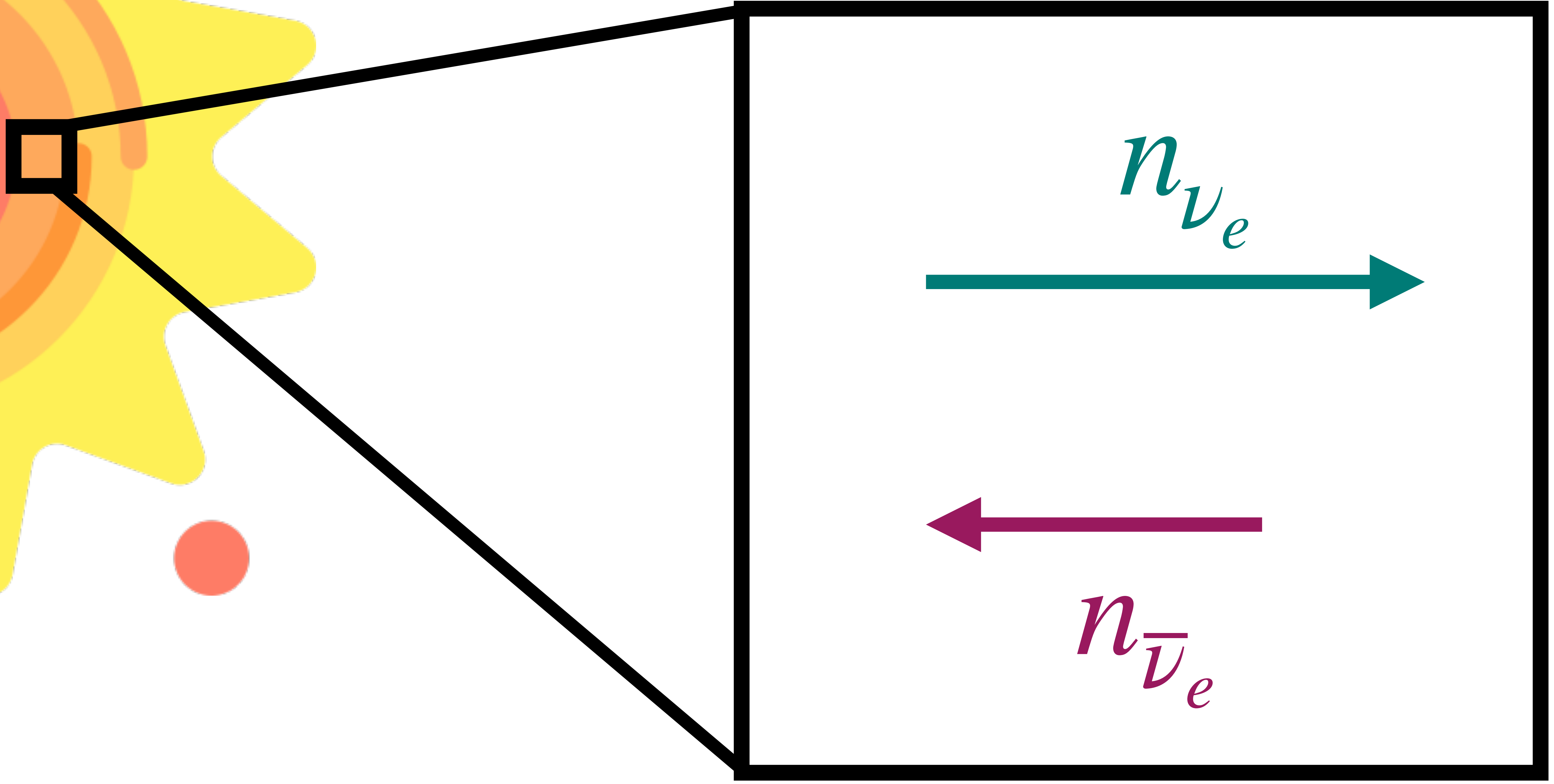
Relaxation of instability

Relaxation of instability

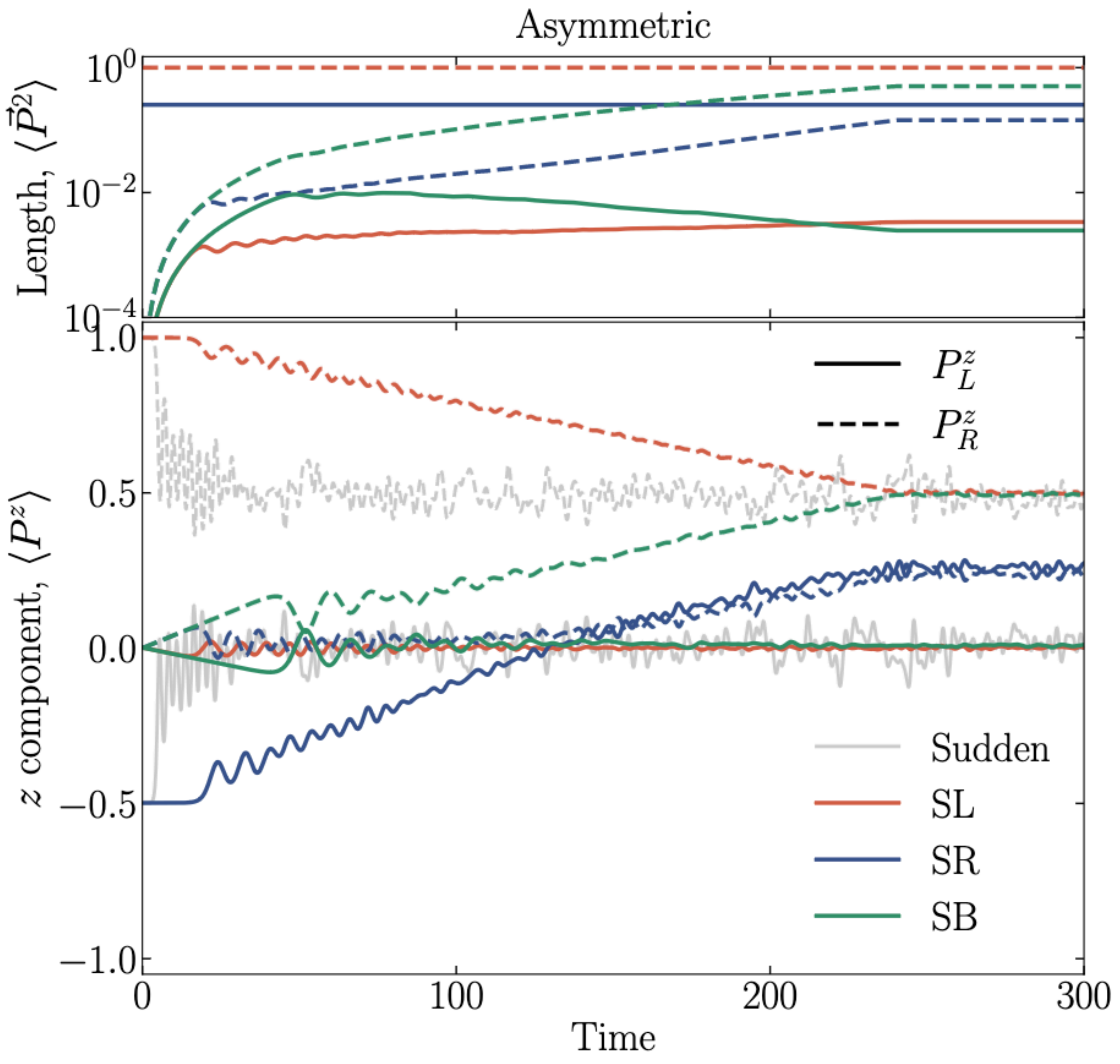
$$\mu \simeq \sqrt{2} G_F n \simeq \mathrm{cm}^{-1}$$

As simple as possible, but no simpler!

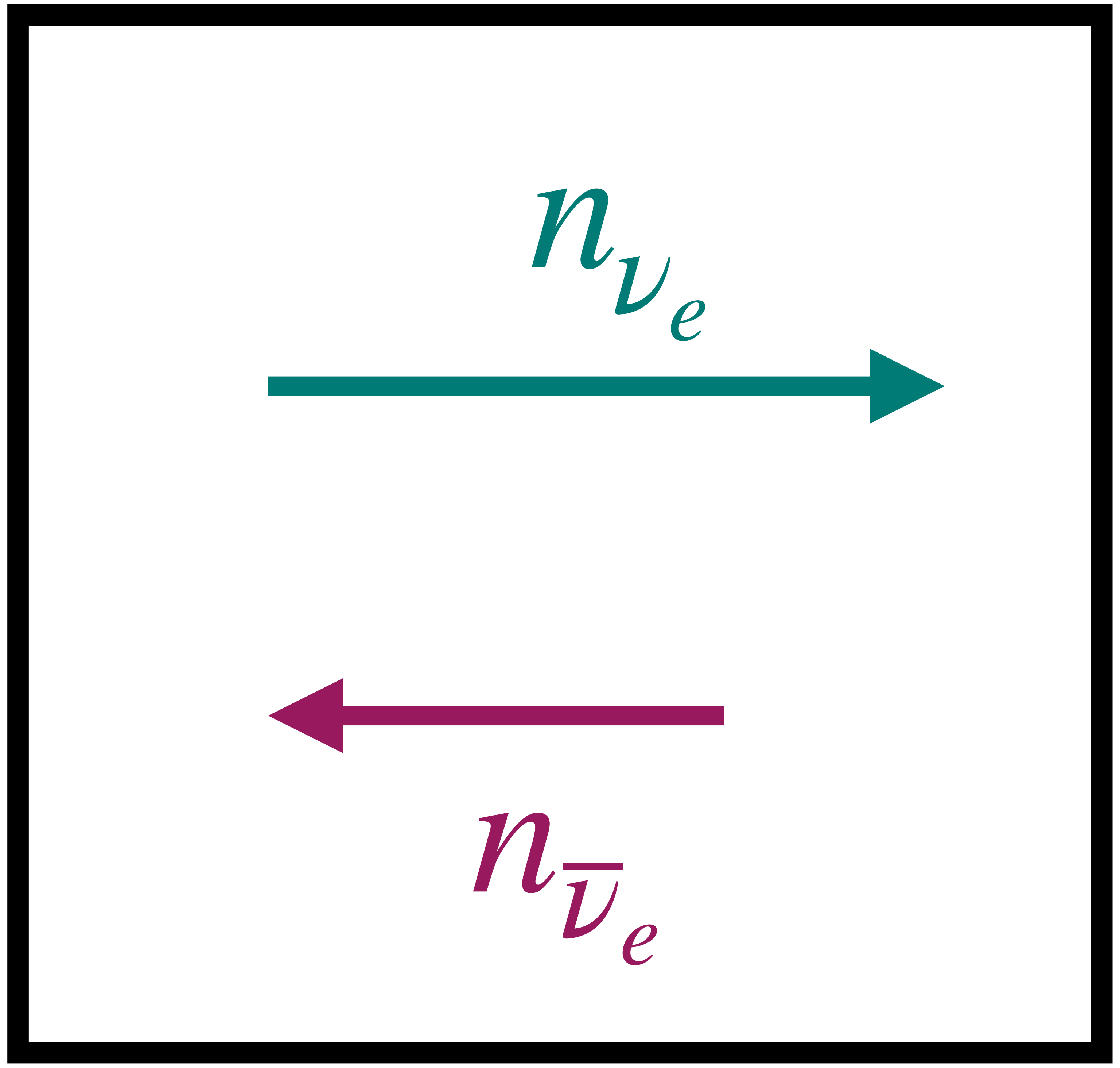




Relaxation of instability

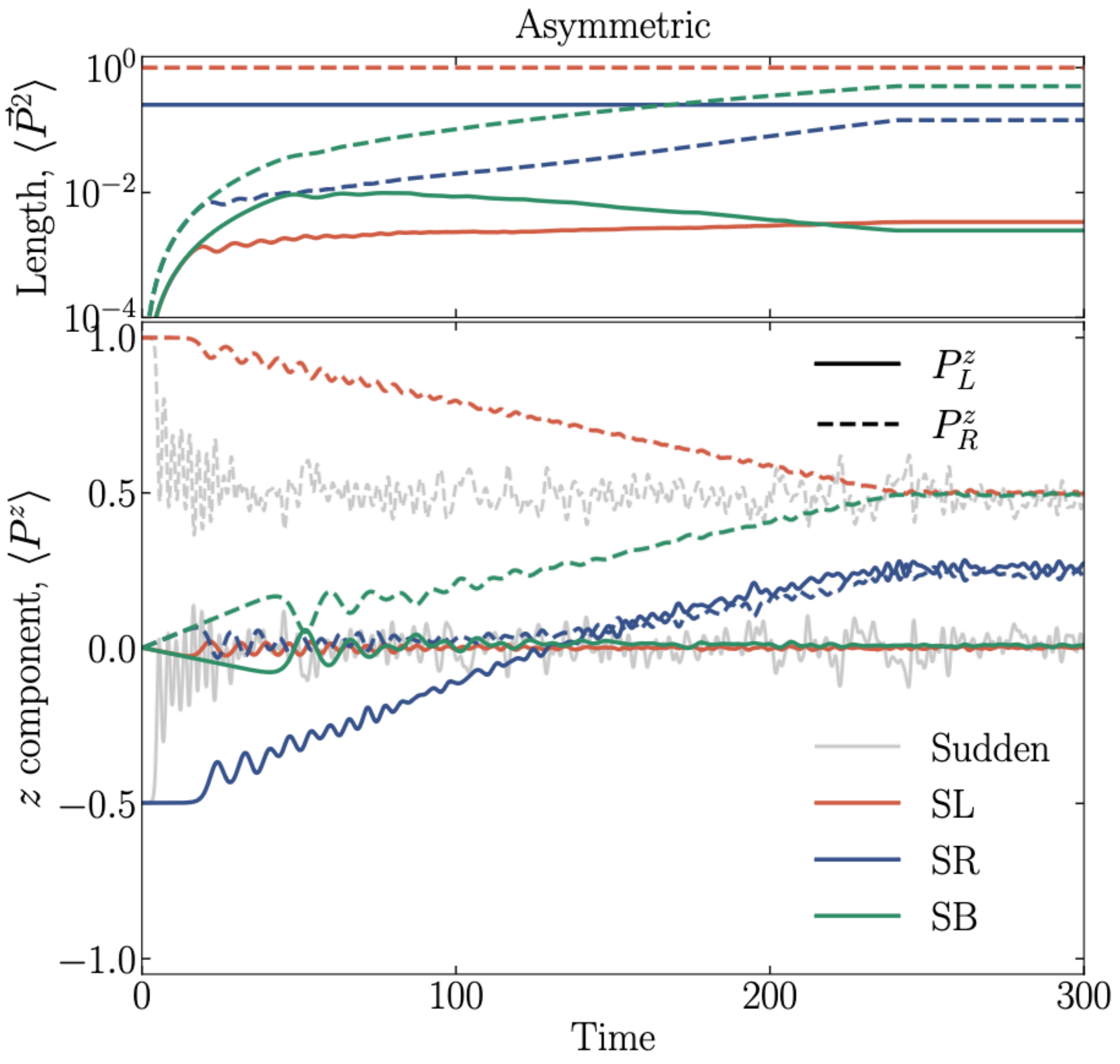


Space-time fluctuating, but average leads to **removal of angular crossing**



***DF**, Raffelt, 2403.12189*

Relaxation of instability



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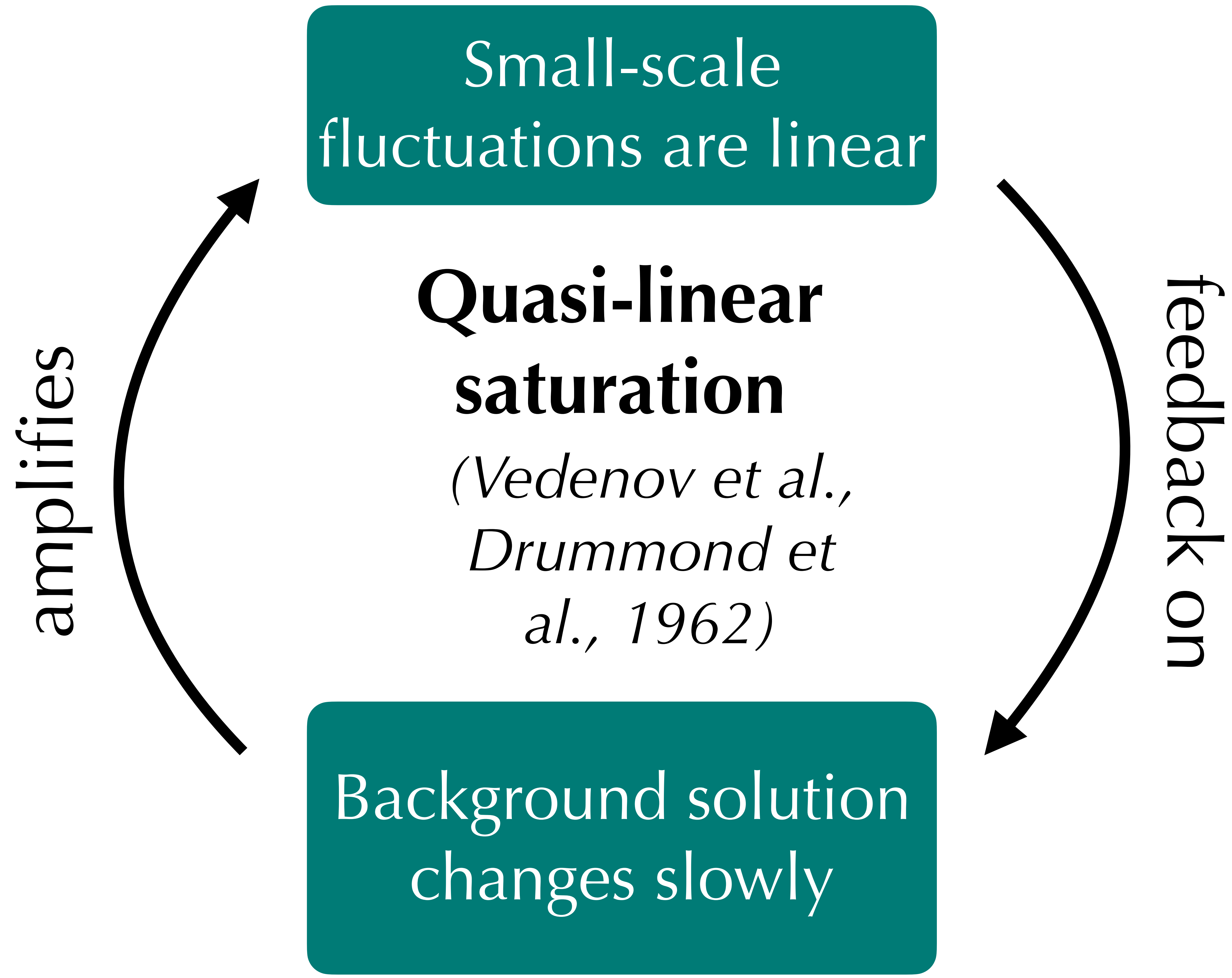

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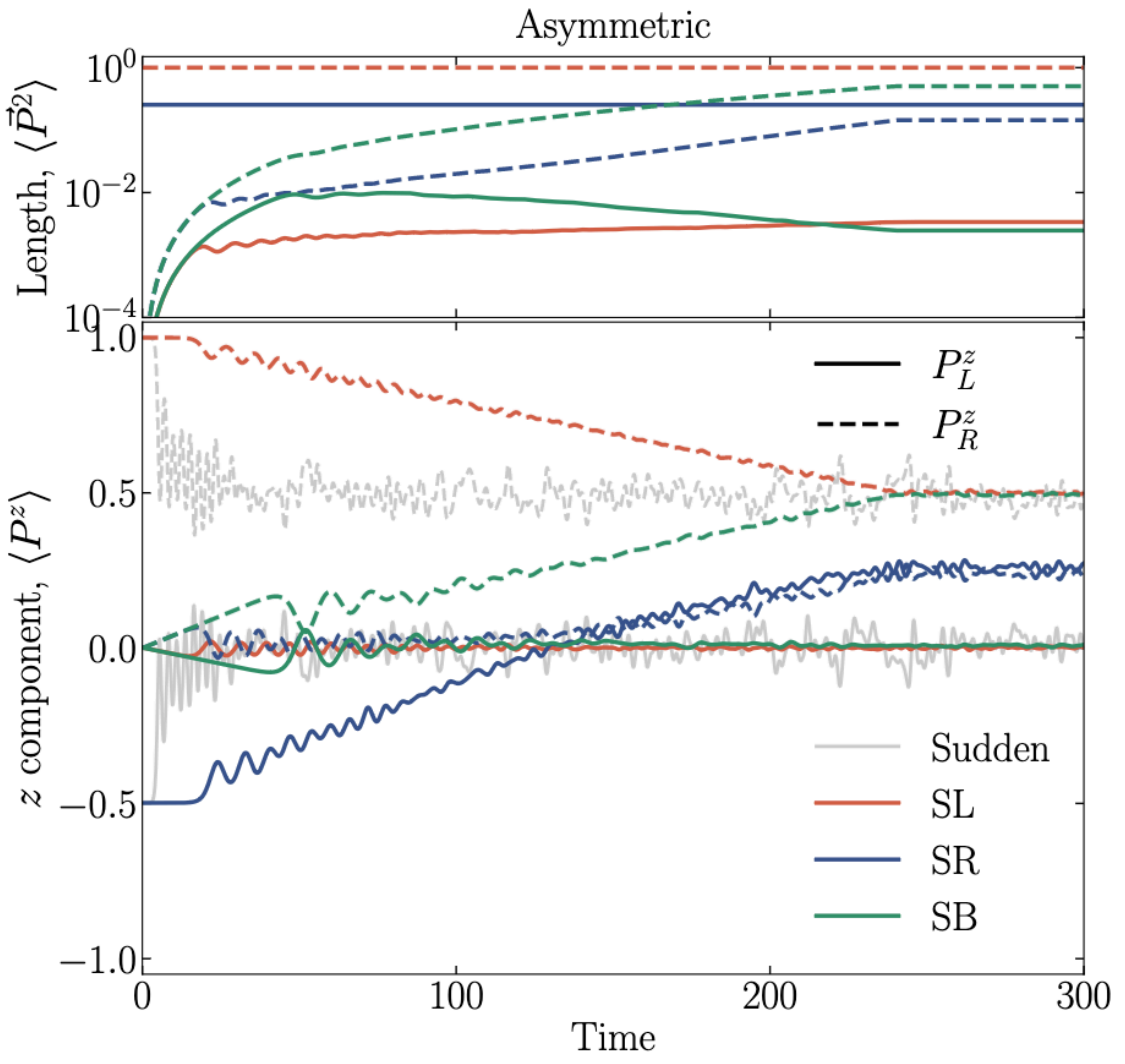
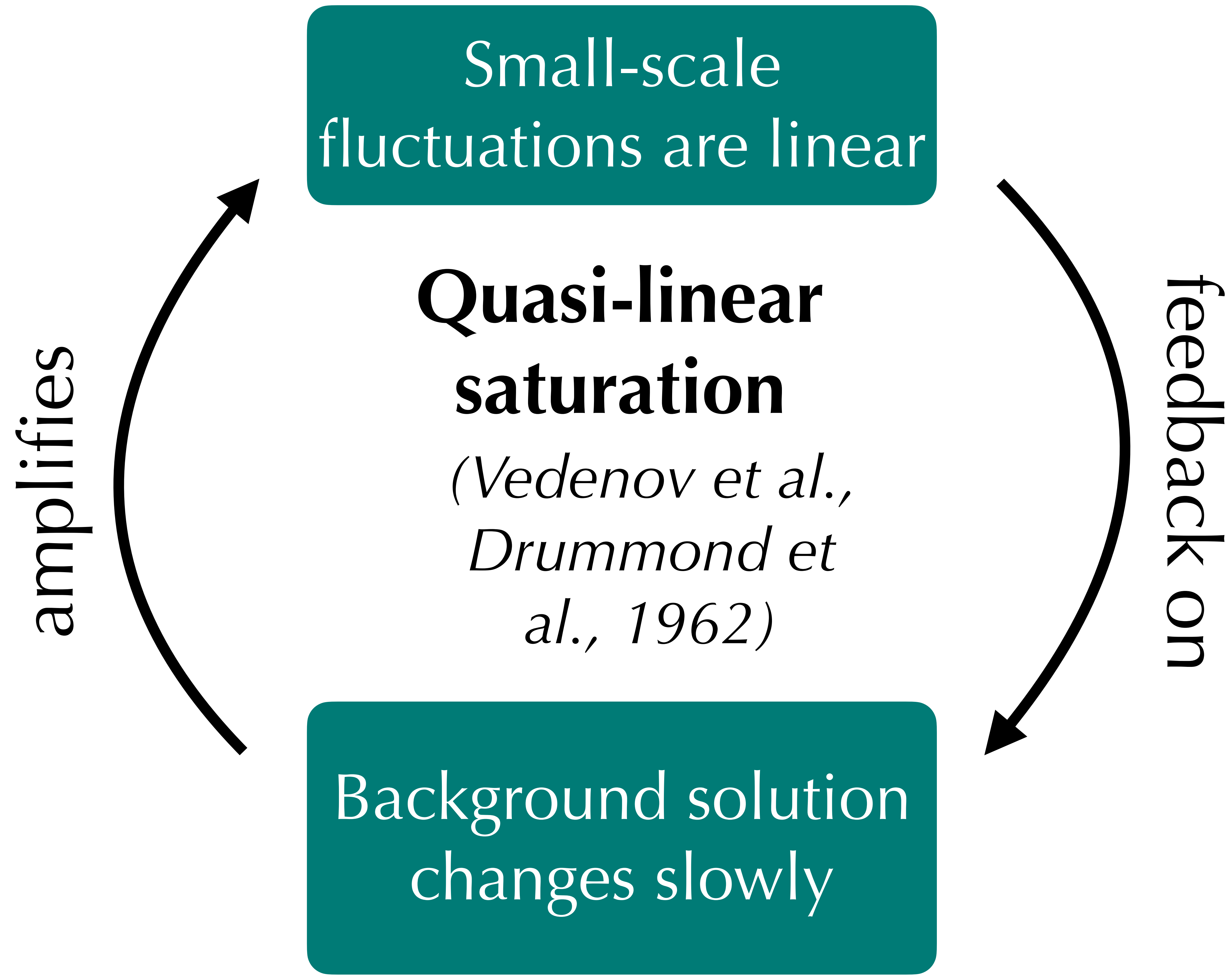


System sticks to the closest stable state (which may depend on history!)



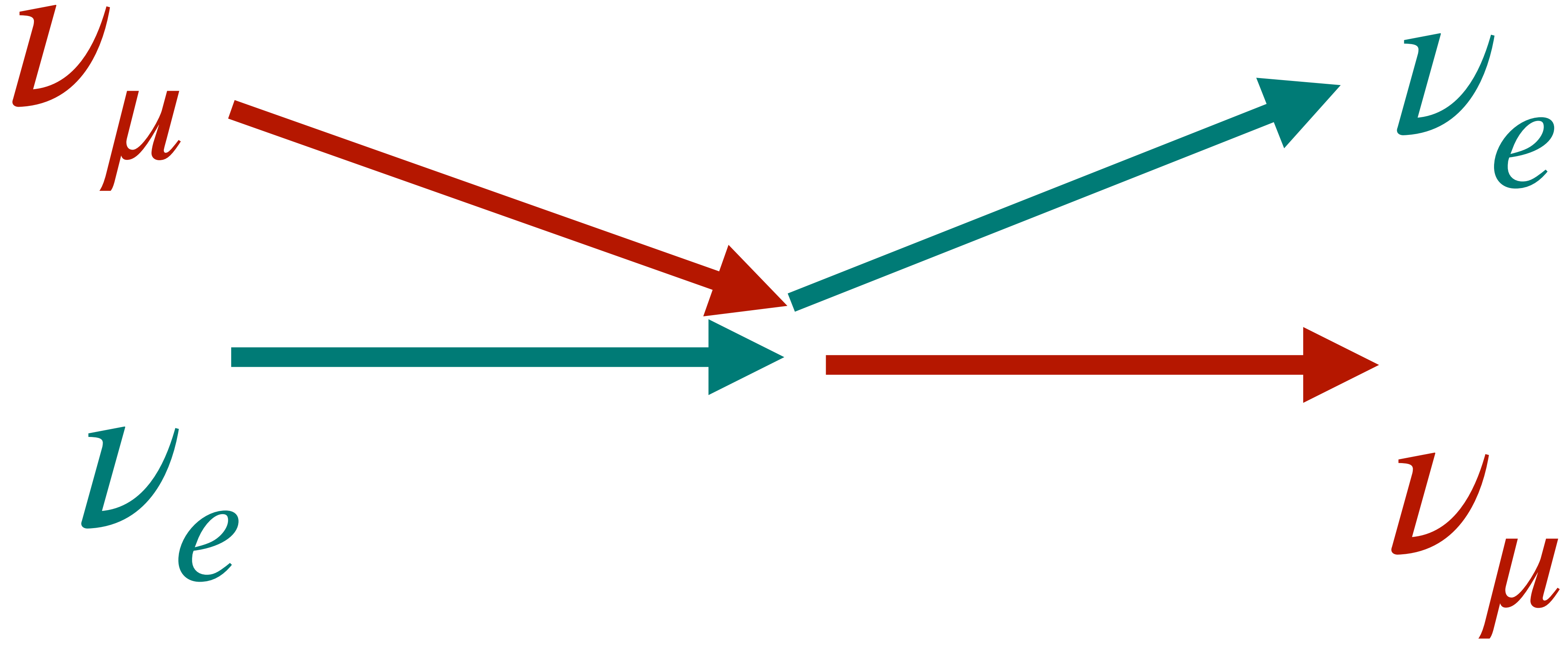
***DF**, Raffelt, 2403.12189*

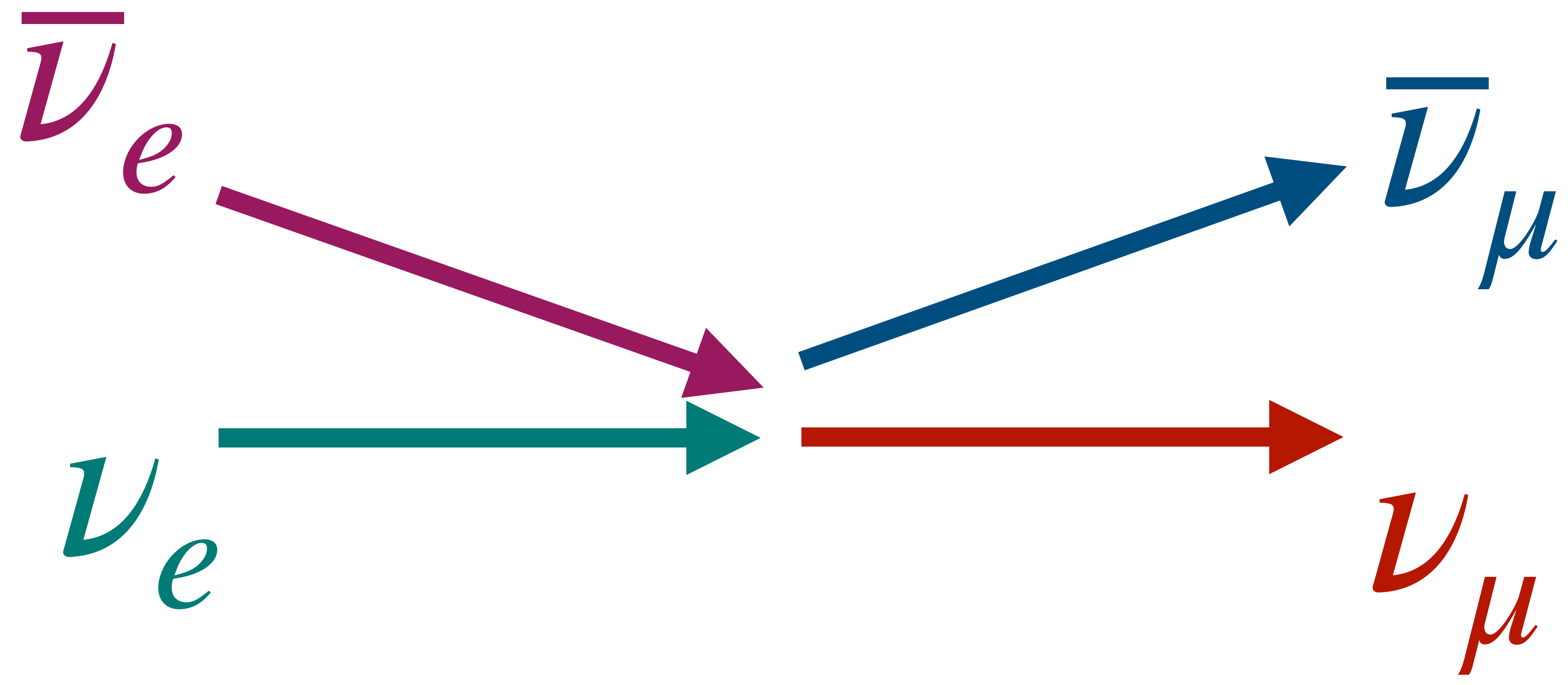
Conclusions

- Framework to intuitively understand flavor instabilities
- **Conservation laws** can protect from instability
- Instability = **resonant** emission of flavor waves from **flipped neutrinos**
- **Saturation of instability** tends to the closest stable configuration (predicted by **quasi-linear** framework)

Thank you!

Collective flavor conversions





Refractive flavor exchange among different energies and directions

Non linear!

Quantum kinetic equations

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{e\mu} \\ \rho_{\mu e} & \rho_{\mu\mu} \end{pmatrix}$$

Density matrix

$$P^z = \rho_{ee} - \rho_{\mu\mu}$$

Nearly ν_e

$$P^x = \mathrm{Re}(\rho_{e\mu})$$

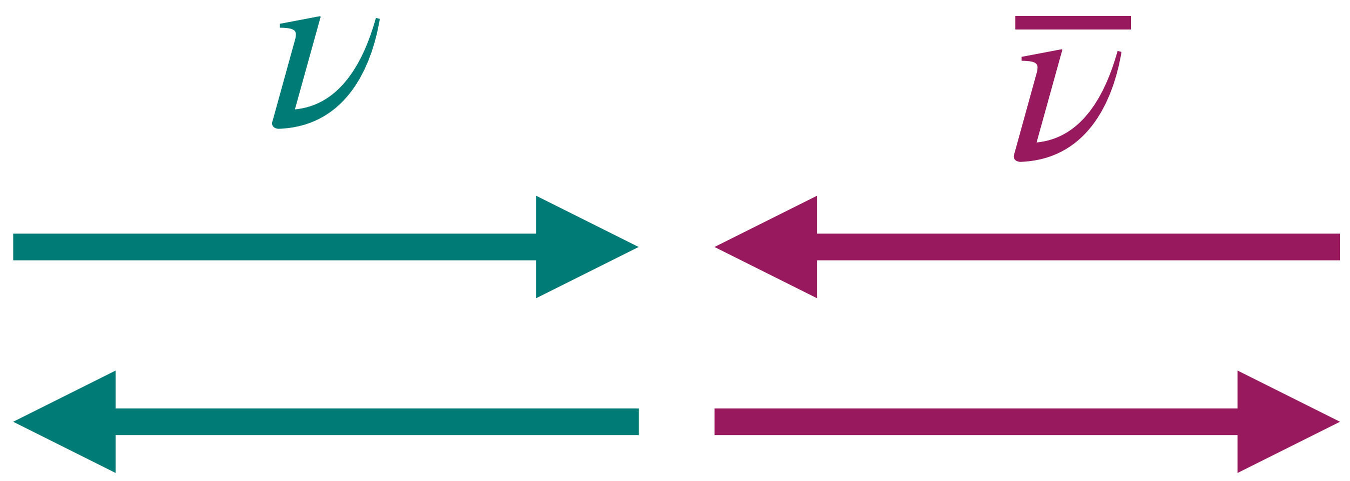
$$P^y = -\,\mathrm{Im}(\rho_{e\mu})$$

$$P^z = \bar{\rho}_{\mu\mu} - \bar{\rho}_{ee}$$

Nearly $\bar{\nu}_\mu$

A concrete example

Can conversions happen without flavor?



Neutrino-antineutrino collective oscillations?

Proposed in Sawyer, PRD 2023

A concrete example

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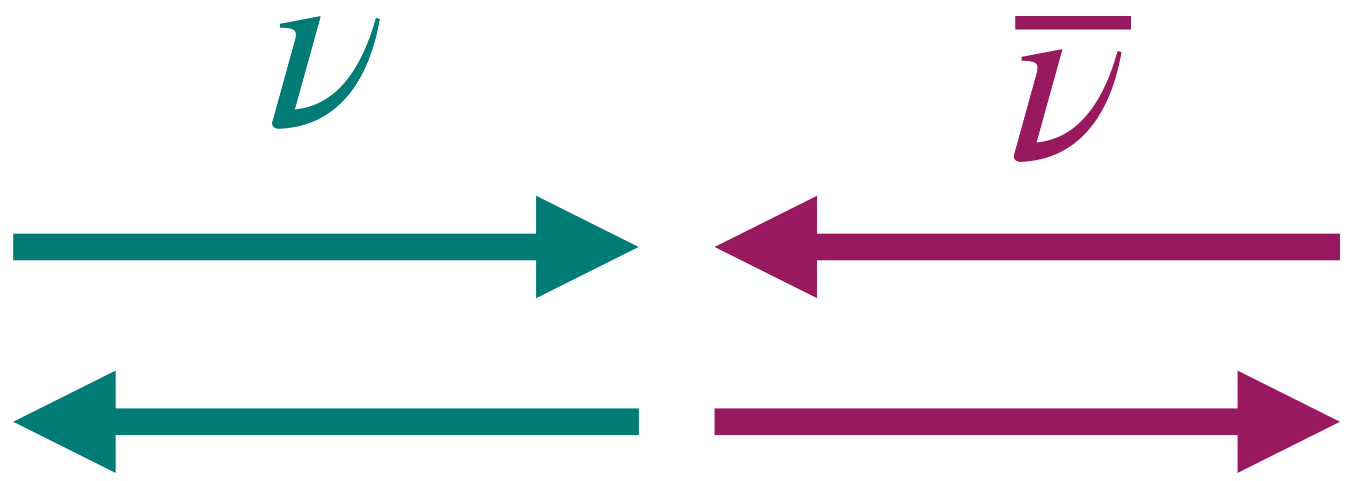


Neutrino-antineutrino collective oscillations?

Proposed in Sawyer, PRD 2023

Helicity violation!

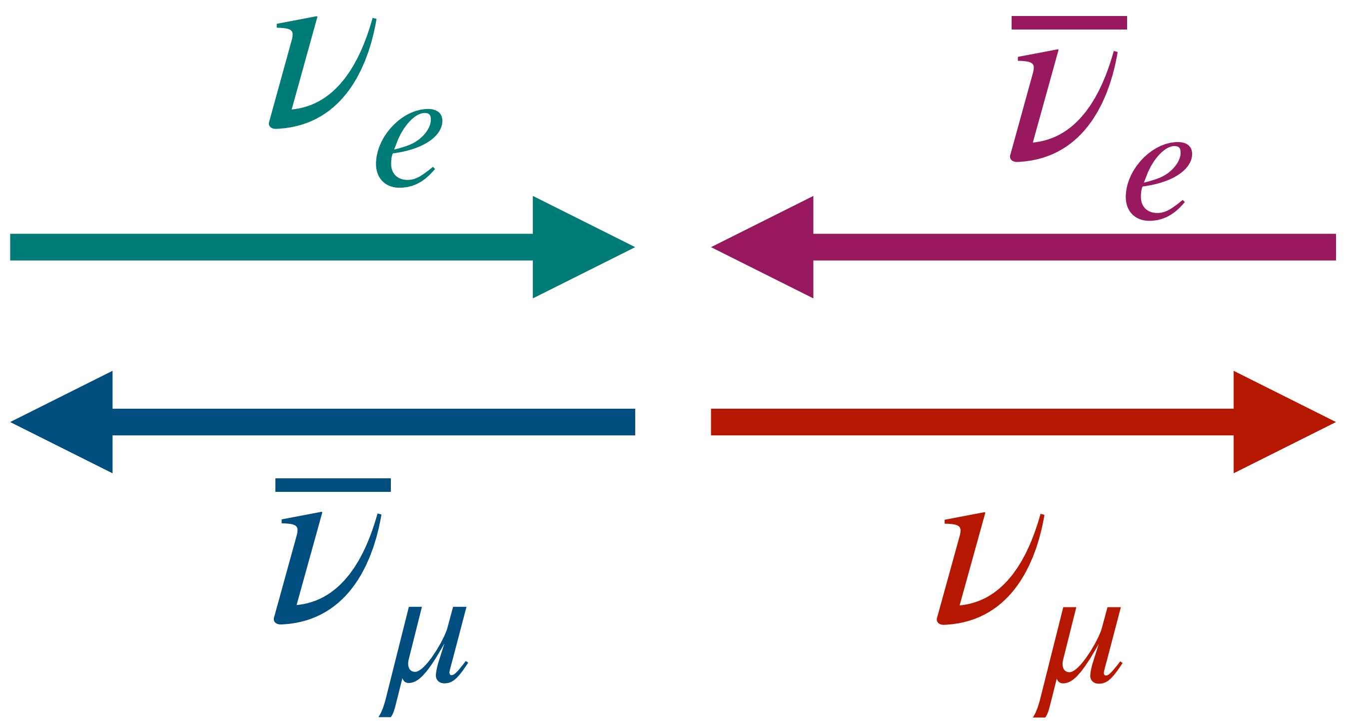
$\overline{\nu}\nu$ conversions can be neglected, but only by previously unnoticed argument!

__DF__, Raffelt, Sigl, 2401.02478

A concrete example

Exponential growth of off-diagonal components

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{e\mu} \\ \rho_{\mu e} & \rho_{\mu\mu} \end{pmatrix}$$



Can we predict the final state of the system?

Are there **conserved quantities**?

Conserved quantities

$$\sum \rho = \begin{pmatrix} \rho_{ee} + \bar{\rho}_{\mu\mu} & \rho_{e\mu} + \bar{\rho}_{\mu e} \\ \rho_{\mu e} + \bar{\rho}_{e\mu} & \rho_{\mu\mu} + \bar{\rho}_{ee} \end{pmatrix}$$

Total lepton number

Conserved quantities

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Total lepton number

Homogeneous systems

Infinite conservation laws (**Gaudin invariants**) ***DF**, Raffelt, 2301.09650*

Broken for inhomogeneous, except special solutions (flavor solitons) ***DF**, Raffelt, 2303.12143*

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Inhomogeneities grow!

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Inhomogeneities grow!

Energy must be conserved (right?)

Energy in collective oscillations

$$E = \mathrm{K} \quad + \quad U$$

$\sim 10\ \mathrm{MeV}$ $\qquad\qquad$ $\sim \mathrm{cm}^{-1} \sim 10^{-1}\ \mathrm{meV}$

Energy in collective oscillations

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- Standard quantum kinetic equations
- Neutrino motion decoupled from collective conversions ($dK/dt = 0$)

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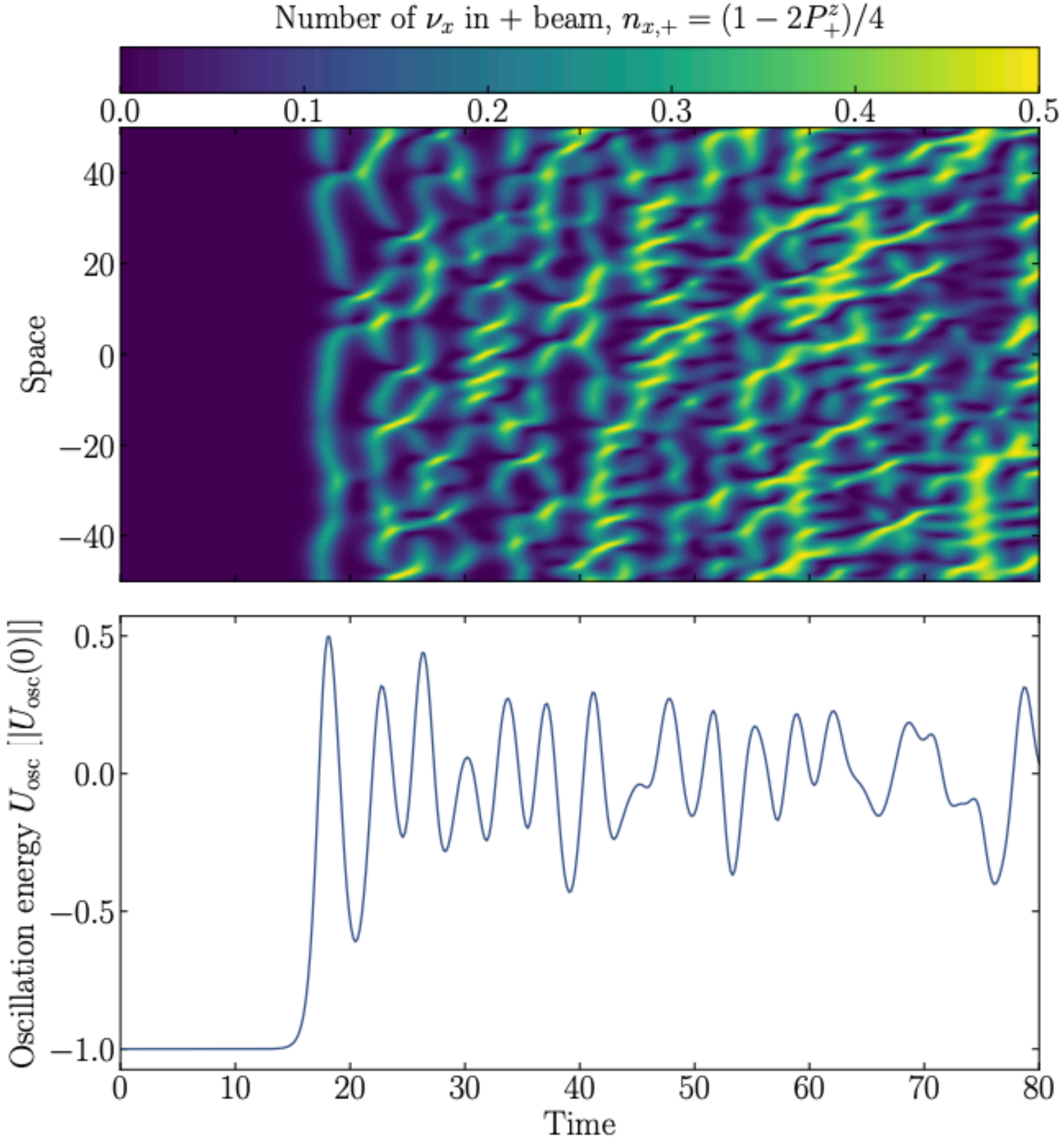
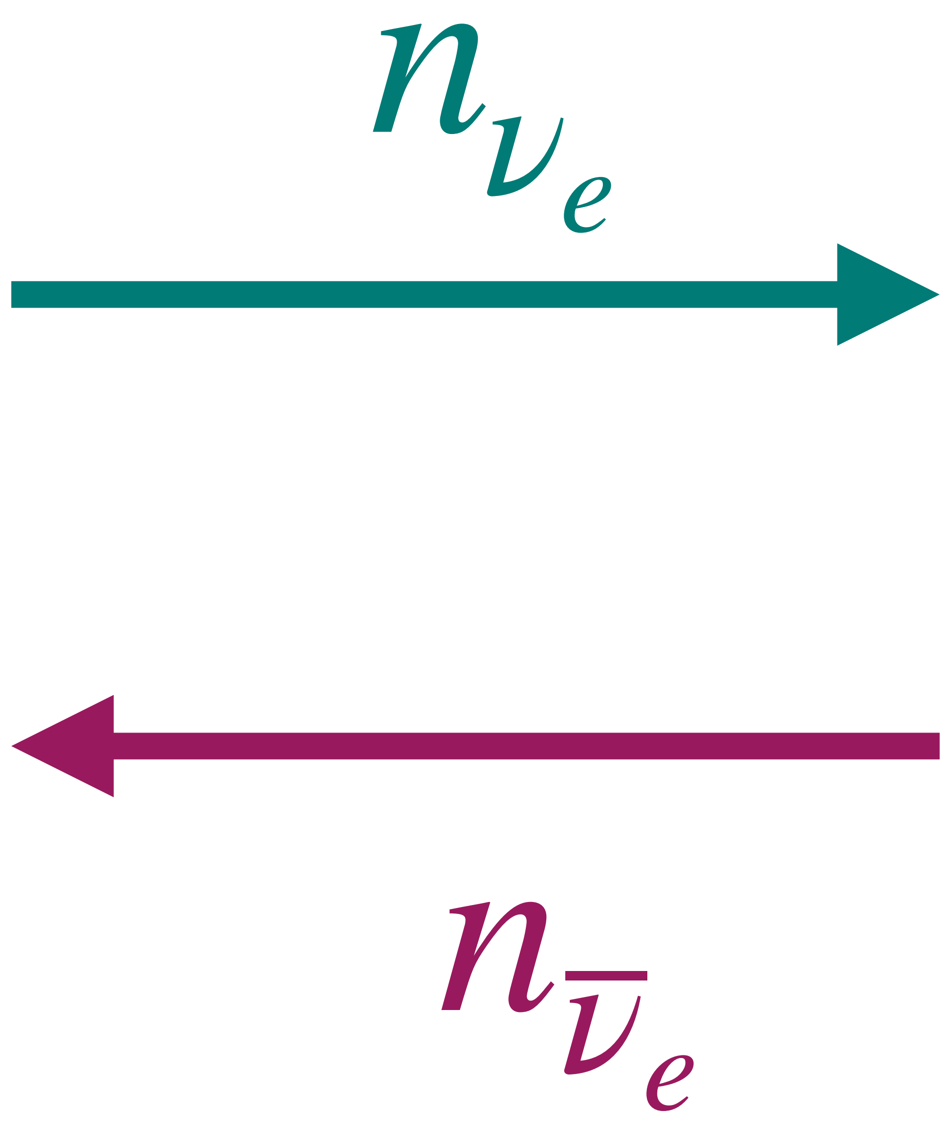
$$\frac{dU}{dt} \neq 0!$$

Energy in collective oscillations





Initially

Average U was initially -1, finally oscillates around 0

Maximal energy violation!

***DF**, Raffelt, Sigl, 2401.05278*

Energy in collective oscillations

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$$\frac{dU}{dt} \neq 0!$$

Energy in collective oscillations

Gradients in flavor composition → Force → Neutrinos accelerated (or slowed) by inhomogeneous flavor conversions!

Energy in collective oscillations

$$E = \quad K \quad + \quad U$$

~ 10 MeV $\qquad \sim \mathrm{cm}^{-1} \sim 10^{-1}$ meV

Interaction energy is not conserved!

Quasi-linear relaxation

$$\rho = \langle \rho \rangle + \delta\rho$$

Slowly-varying background

Rapidly-varying fluctuation

$$\partial_t \delta\rho + \vec{v} \cdot \vec{\nabla} \delta\rho = -i[\langle H \rangle, \delta\rho] - i[\delta H, \langle \rho \rangle]$$

Fluctuations are treated **linearly**

$$\partial_t \rho = -i\langle [\delta H, \delta\rho] \rangle$$

Fluctuations **non-linearly** feedback and lead to **background relaxation**