Boltzmann Neutrino Transport Simulation of Core-collapse Supernovae and Collective Neutrino Oscillation

Based on

- R. Akaho et al, PRD 109, 023012 (2024)
- J. Liu, R. A. et al, PRD 108, 123024 (2023)
- J. Liu, H.Nagakura, R. A. et al, arXiv:2407.10604
- R. A. et al, in prep.

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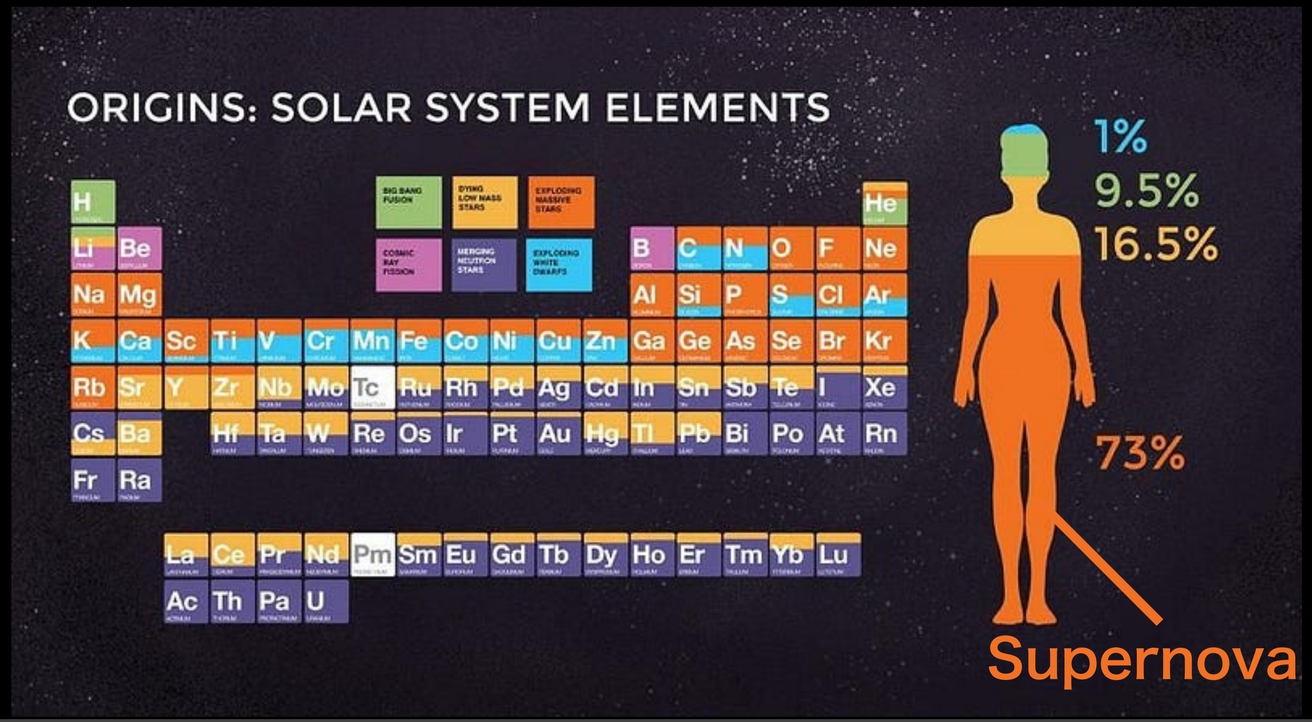
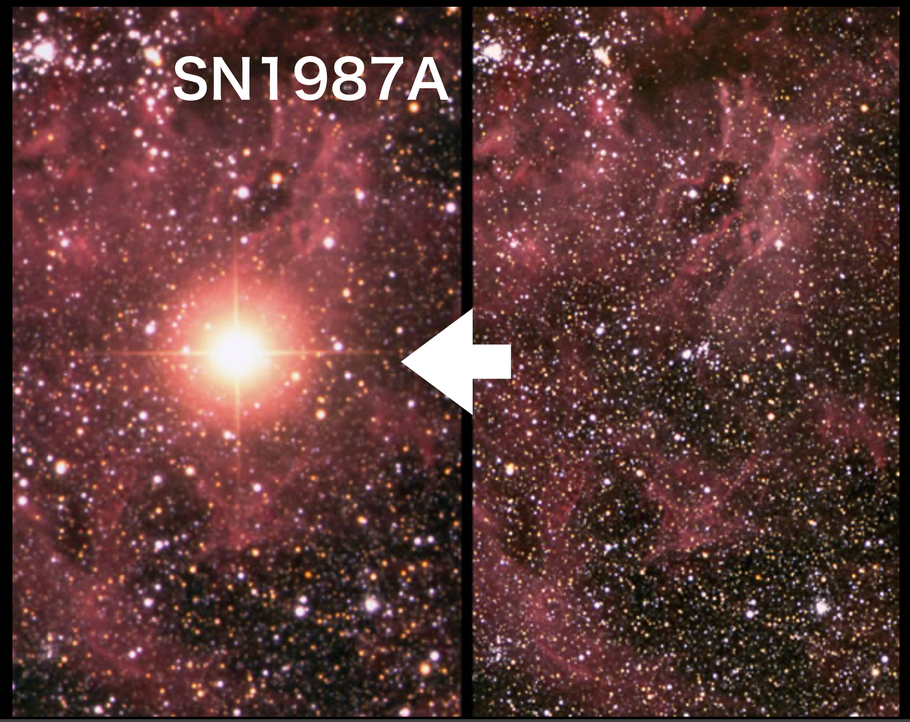
Core-collapse Supernovae (CCSNe)

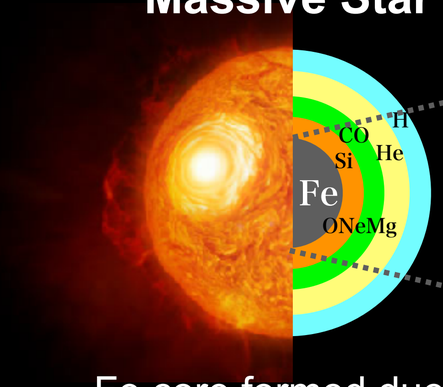
- Energetic explosion occurring at the end of stellar evolution.
- It synthesizes various elements in our universe,





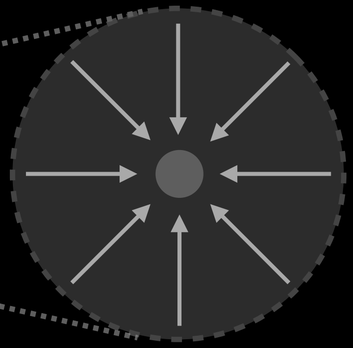
Scenario of CCSN

Massive Star



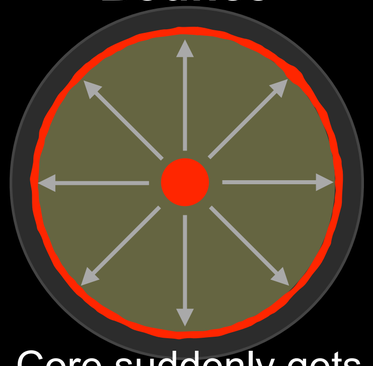
Fe core formed due to nucleosynthesis

Gravitational Collapse



When Fe core grows too massive, it collapses

Bounce



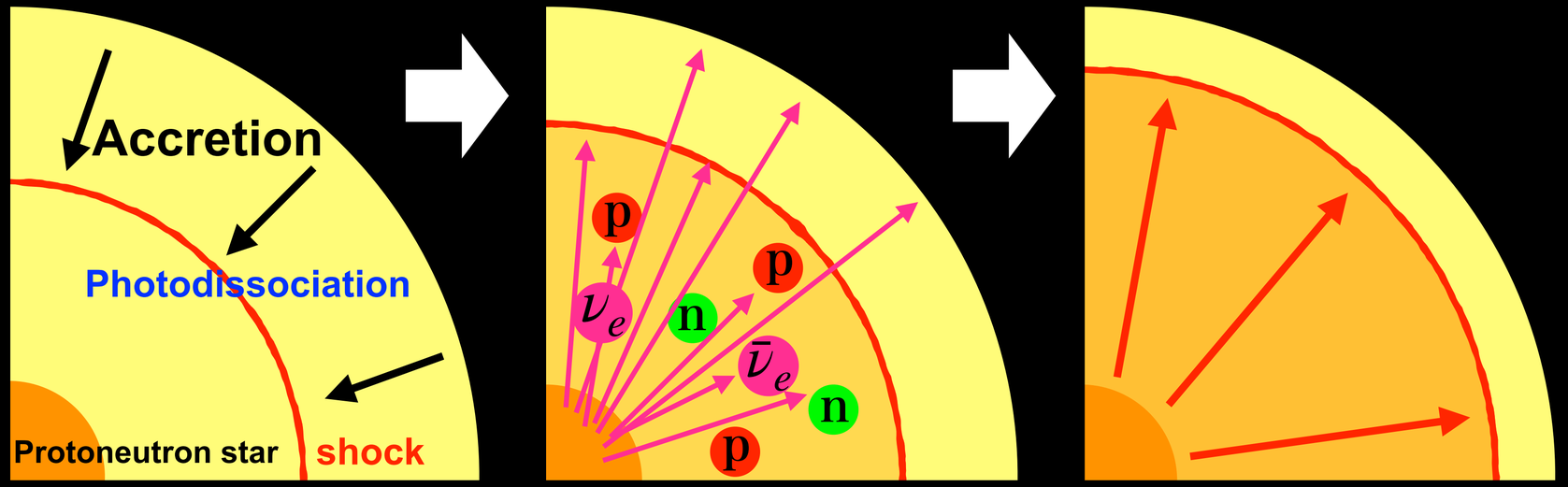
Core suddenly gets stiffened when the strong interaction take place

Explosion

Neutron star or black hole

Shock wave reach outer envelope

Neutrino heating mechanism



Shock stalls

Neutrinos heats up the matter behind the shock

Shock revival

Aim of CCSN Simulation

Reproducing Existing Observations

- Reproduce explosion energy, synthesized $^{56}\mathrm{Ni}$ mass inferred from electromagnetic observation
- Current stete-of-the-art simulation still cannot reproduce observed values

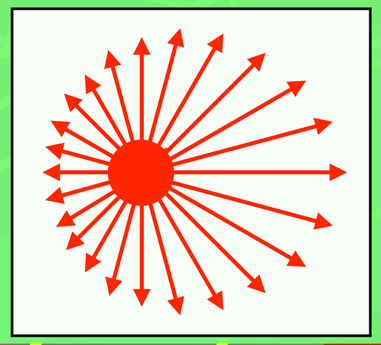
Construct Theoretical Model for Future Observations

- Accurate theoretical model should be prepared in preparation for future observations.
- Unfortunately, there are still large uncertainties remaining due to numerical methods.

Neutrinos inside CCSN

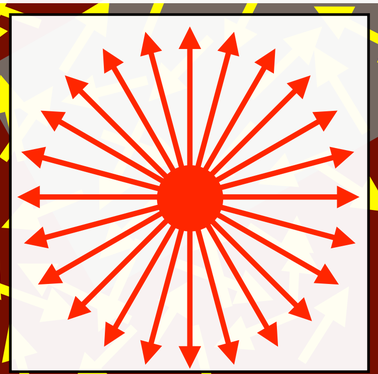
Free streaming

Intermediate: nontrivial



thermal eq. (Fermi-Dirac)

momentum: isotropic



Phase space distribution function $f(x^\mu, p^i)$

Boltzmann equation

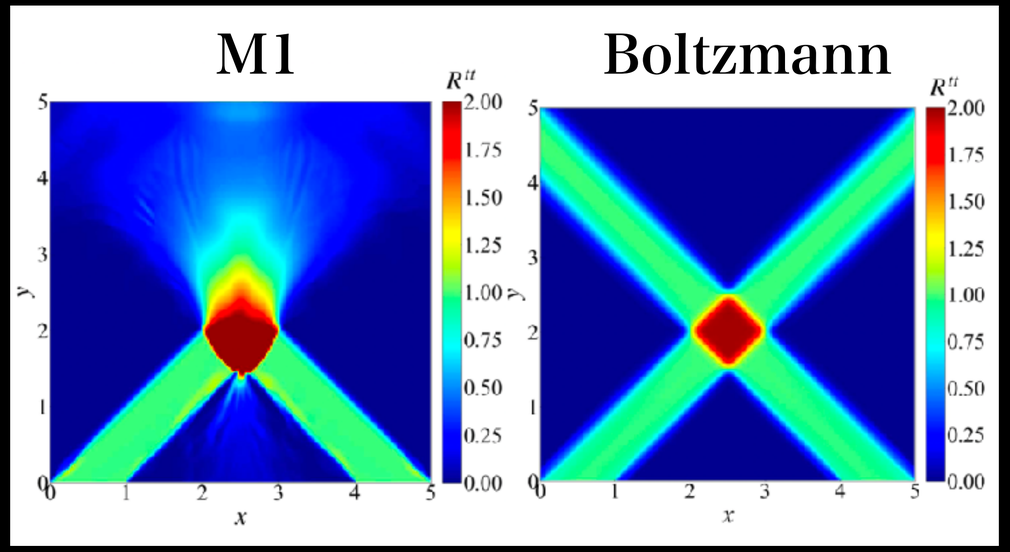
$$p^\alpha \frac{\partial f}{\partial x^\alpha} - \Gamma^i_{\alpha\beta} p^\alpha p^\beta \frac{\partial f}{\partial p^i} = \left[\frac{\delta f}{\delta t}\right]_{\text{coll}}$$

Example of Approximate Transport

Two-moment neutrino transport

Reduce momentum space information in order to save computational cost

Asahina 2020



Distribution Function

$$f(r, \theta, \phi, \epsilon, \theta_\nu, \phi_\nu)$$

Boltzmann Equation

$$\frac{\partial f}{\partial t} + p^i \frac{\partial f}{\partial x^i} + \dot{p}^i \frac{\partial f}{\partial p^i} = C$$

Multiply angular cosines in the momentum space and integrate

Moments

$$E(r, \theta, \phi, \epsilon) \qquad F^i(r, \theta, \phi, \epsilon) \qquad P^{ij}(r, \theta, \phi, \epsilon)$$

Moment Equations

$$\frac{\partial E}{\partial t} = L_1(E, F^i, P^{ij}) \qquad \frac{\partial F^i}{\partial t} = L_2(E, F^i, P^{ij})$$

Closure: $P^{ij}_{M1} = \frac{3\chi - 1}{2} P^{ij}_{thin} + \frac{3(1-\chi)}{2} P^{ij}_{thick}$

Boltzmann Neutrino Radiation Hydrodynamics Code

Boltzmann equation wrt phase space distribution function $f(t, t, \theta, \phi, \epsilon, \theta_\nu, \phi_\nu)$

$$\underbrace{\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left|_{q_i}\left[\left(e^\mu_{(0)}+\sum_{i=1}^{3} l_i e^\mu_{(i)}\right)\sqrt{-g}f\right]\right.}_{\text{Spacetime advection (propagation)}} \underbrace{-\frac{1}{\epsilon^2}\frac{\partial}{\partial\epsilon}\left(\epsilon^3 f\omega_{(0)}\right)}_{\text{Energy advection (grav. red/blue shift)}} + \underbrace{\frac{1}{\sin\theta_\nu}\frac{\partial}{\partial\theta_\nu}\left(\sin\theta_\nu f\omega_{(\theta_\nu)}\right)}_{\text{Angular advection (zenith angle } \theta_\nu)} - \underbrace{\frac{1}{\sin^2\theta_\nu}\frac{\partial}{\partial\phi_\nu}\left(f\omega_{(\phi_\nu)}\right)}_{\text{Angular advection (azimuth angle } \phi_\nu)} = \underbrace{S_{\rm rad}}_{\text{Collision } (\nu\text{-matter interaction})}$$

$$l_{(1)}=\cos\theta_\nu \quad l_{(2)}=\sin\theta_\nu\cos\phi_\nu \quad l_{(3)}=\sin\theta_\nu\sin\phi_\nu$$

$$\omega_{(0)} \equiv \epsilon^{-2} p^\mu p_\nu \nabla_\mu e^\mu_{(0)} \qquad \omega_i \equiv \epsilon^{-2} p^\mu p_\nu \nabla_\mu e^\nu_{(i)}$$

$$\omega_{(\theta_\nu)} \equiv \sum_{i=1}^{3} \omega_{(i)} \frac{\partial l_{(i)}}{\partial \theta_\nu} \qquad \omega_{(\phi_\nu)} \equiv \sum_{i=2}^{3} \omega_{(i)} \frac{\partial l_{(i)}}{\partial \phi_\nu}$$

Hydrodynamics equation

Continuity eq. $$\partial_t \rho_* + \partial_j(\rho_* v^j) = 0$$

Momentum cons. $$\partial_t S_i + \partial_j(S_i v^j + \alpha\sqrt{\gamma}P\delta_i{}^j) = -S_0\partial_i\alpha + S_j\partial_i\beta^j - \frac{1}{2}\alpha\sqrt{\gamma}S_{jk}\partial_i\gamma^{jk} - \alpha\sqrt{\gamma}G_i$$

Energy cons. $$\partial_t(S_0-\rho_*) + \partial_k((S_0-\rho_*)v^k + \sqrt{\gamma}P(v^k+\beta^k)) = \alpha\sqrt{\gamma}S^{ij}K_{ij} - S_i D^i\alpha + \alpha\sqrt{\gamma}n^\mu G_\mu$$

Lepton num. cons. $$\partial_t(\rho_* Y_e) + \partial_j(\rho_* Y_e v^j) = -\alpha\sqrt{\gamma}\Gamma$$

$$\rho_* = \alpha\sqrt{\gamma}\rho_0 u^t = \sqrt{\gamma}\rho_0\omega \qquad S_0 = \sqrt{\gamma}(\rho h w^2 - P) \qquad S_j = \rho_* h u_j \qquad S_{ij} = \rho h u_i u_j + P\gamma_{ij}$$

Boltzmann Radiation-hydro Simulation

Sumiyoshi 2012

Nagakura 2014
Nagakura 2017
Nagakura 2019

Akaho 2021
Akaho 2023

Harada in prep.

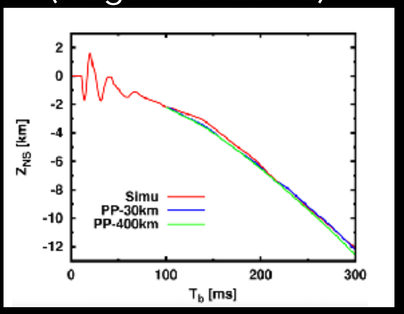
Boltzmann solver

SR Boltzmann + Newtonian hydro

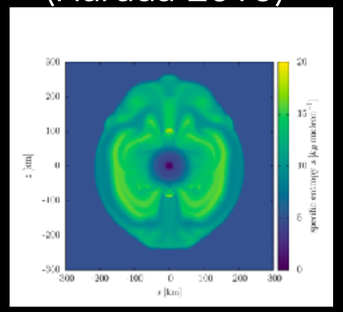
GR Boltzmann + GR hydro + 1D metric

GR Boltzmann + GR hydro + Numerical Relativity

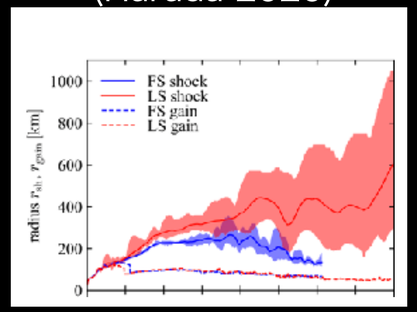
PNS kick
(Nagakura 2019)



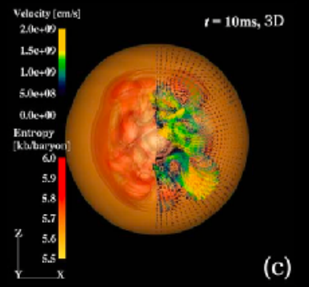
Rotating progenitor
(Harada 2019)



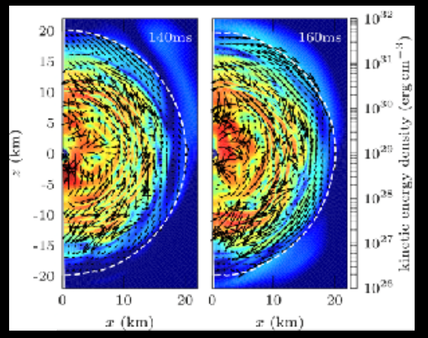
EOS dependence
(Harada 2020)



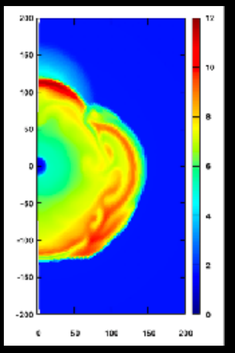
3D simulation
(Iwakami 2020)



PNS convection
(Akaho 2023)



GR CCSN simulation
(Akaho in prep.)



Basics of Neutrino Oscillation

$$\rho \equiv \begin{pmatrix} f_{\nu_e} & S_{ex} \\ S_{xe} & f_{\nu_x} \end{pmatrix}$$

Diagonal components correspond to neutrino density

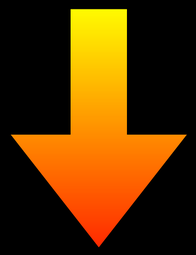
Liouville von Neumann Eq.

$$iv^\mu \partial_\mu \rho = \left[\frac{m_1^2 + m_2^2}{4E} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{m_2^2 - m_1^2}{4E} \begin{pmatrix} -\cos 2\theta_{12} & \sin 2\theta_{12} \\ \sin 2\theta_{12} & \cos 2\theta_{12} \end{pmatrix} + \sqrt{2} G_F n_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \rho \right]$$

Vacuum (mass difference leads to oscillation)

Matter (only diagonal)

neutrino oscillation has been neglected for CCSN core



Dilute matter ($\rho \ll 10^5$g/cc): vacuum oscillation

Vacuum potential = matter potential: MSW resonance

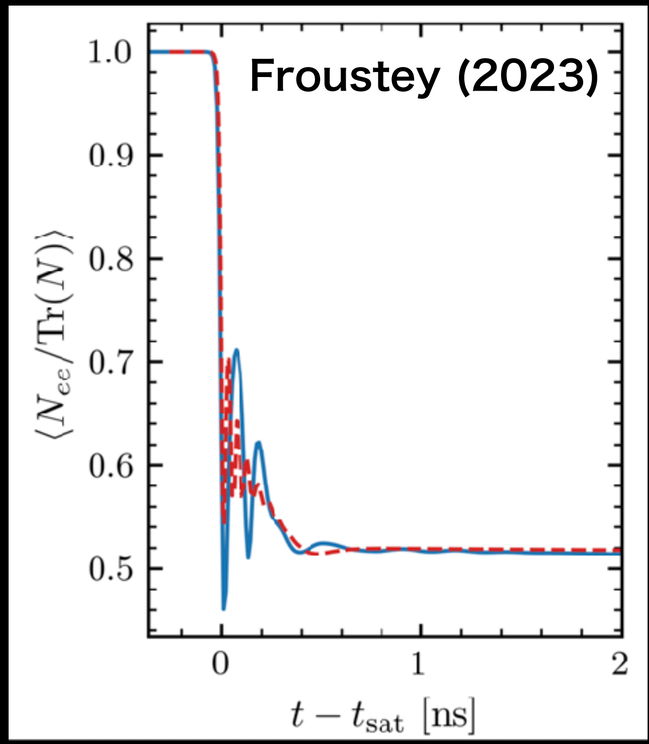
High density matter ($\rho \gg 10^5$g/cc): matter suppression (no oscillation)

Collective Neutrino Oscillation

$$iv^\mu \partial_\mu \rho = \left[H_{\rm vac} + H_{\rm mat} + \sqrt{2} G_F \nu_\mu \int dP' \rho(x, P') v'^\mu, \rho \right]$$

neutrino self-interaction

- Inside CCSNe or neutron star mergers, "collective oscillation" may occur due to high neutrino density
- It can dramatically change neutrino distribution, which may affect CCSN dynamics/signals, and also BNSMs.



Timescale

Collective		Weak interaction		Dynamical
$t_{\nu\nu} \sim \dfrac{1}{cG_F n_\nu} \sim 10^{-9}\,\mathrm{sec}$	$<$	$t_{\rm weak} \sim \dfrac{m_u}{cG_F E_\nu^2 \rho} \sim 10^{-8}\,\mathrm{sec}$	$\ll$	$t_{\rm dyn} \sim 1/\sqrt{G\rho} \sim 10^{-2}\,\mathrm{sec}$

Collective Neutrino Flavor Instabilities

$$i v^{\mu} \partial_{\mu} \rho = \left[H_{\mathrm{vacuum}} + H_{\mathrm{matter}} + H_{\nu\nu}, \rho \right] + iC$$

- **Slow instability**
  Energy crossing

- **Fast instability**
  Angular crossing in momentum space
  Requires multi-angle neutrino transport

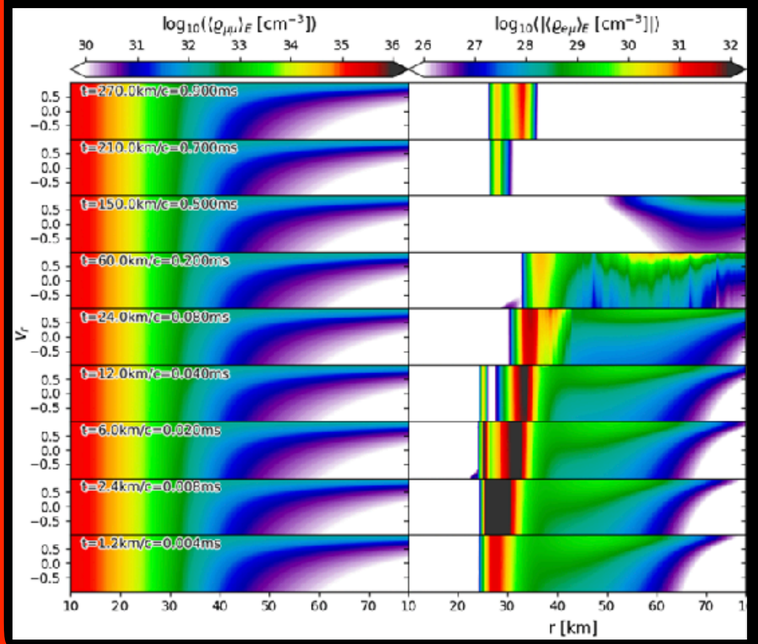
- **Collisional instability**
  Disparity in collision rates

Global Simulation of Collective Oscillation

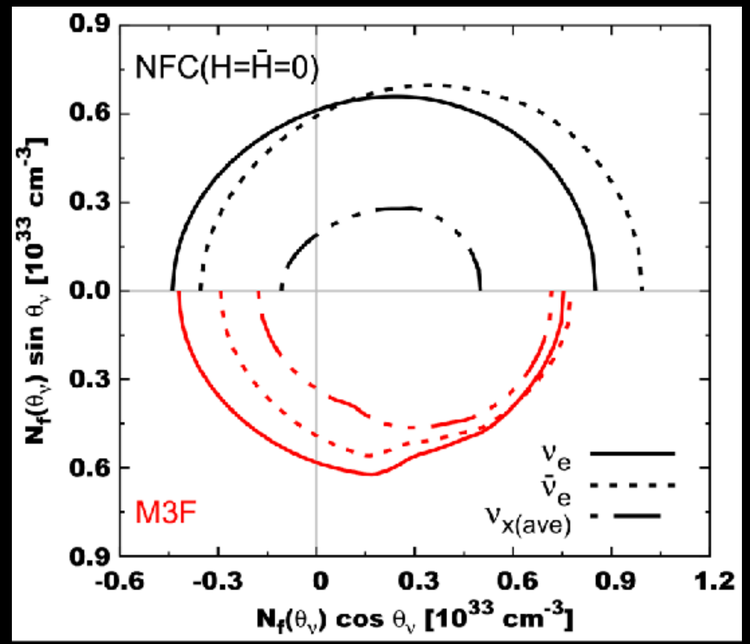
- Ideally, full quantum equation should be solved for the neutrino transport.
- However, it is too computationally expensive (**typical wavekength $\sim$ cm !**)

CCSNe

Xiong 2023

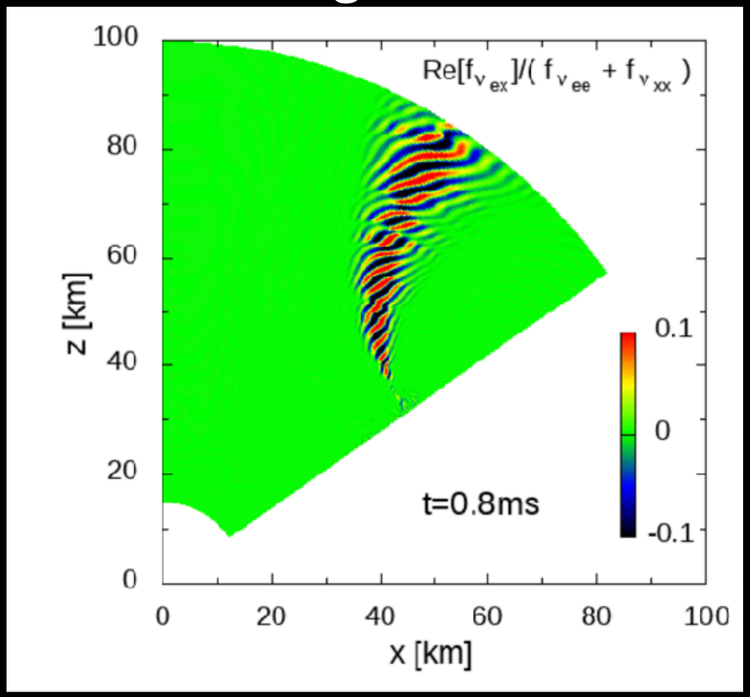


Nagakura 2023



BNMSs

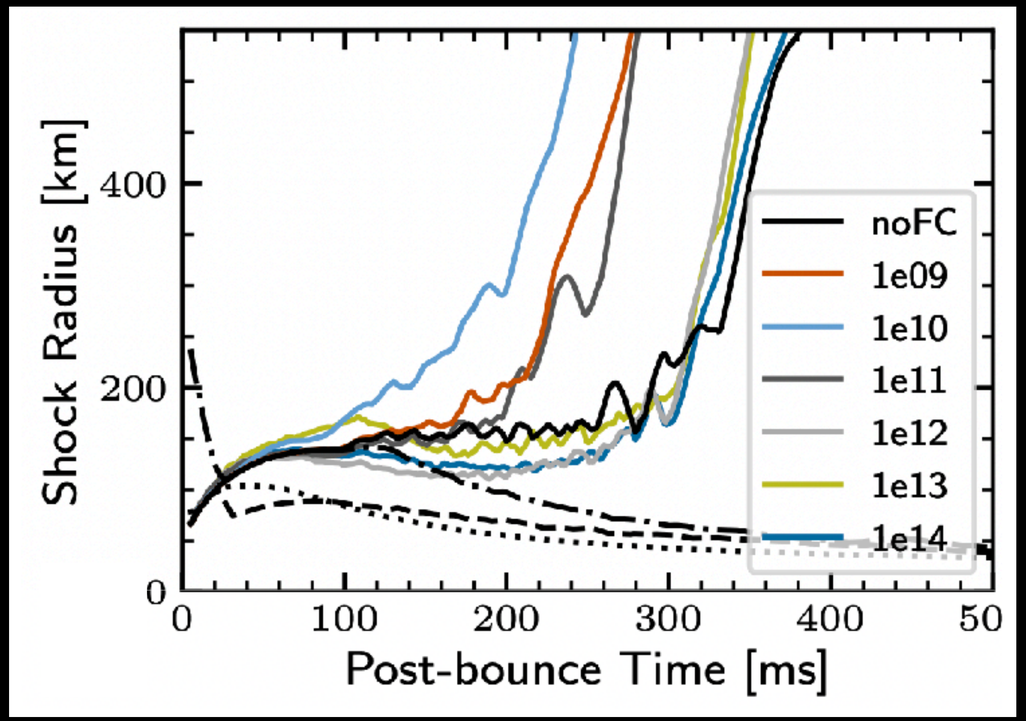
Nagakura 2023



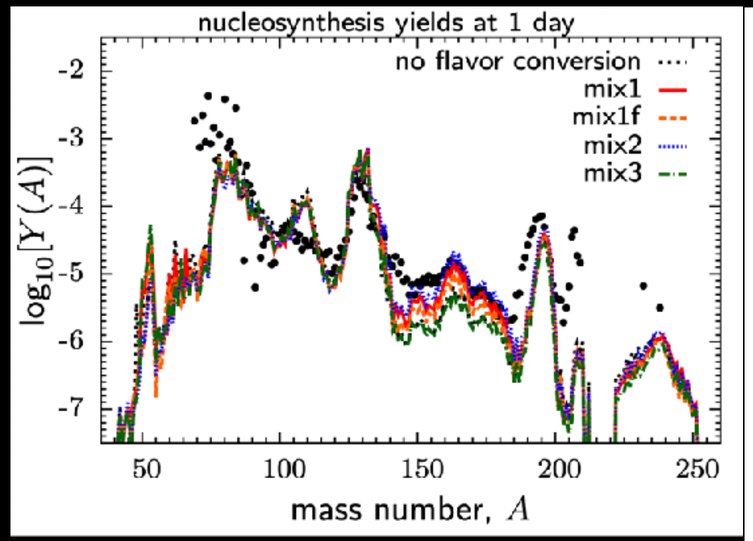
Classical Simulation + Mixing

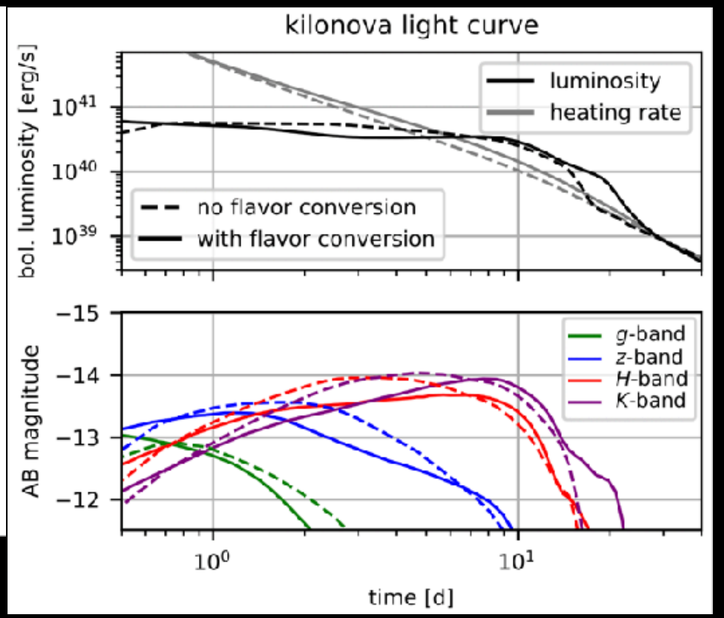
- There are several attempts to include the effects of flavor conversion to the classical simulations.
- However, (1)detecting the flavor instability and (2)imposing neutrino distribution after the flavor conversion is tighly nontrivial problem.

CCSN simulation with flavor mixing
Ehring 2023



BH accretion disk simulation with flavor mixing
Just 2022





Linear Stability Analysis

In this study, linear stability analysis (LSA) is performed to judge the occurrence of collective oscillation

$$\rho \equiv \begin{pmatrix} f_{\nu_e} & S_{ex} \\ S_{xe} & f_{\nu_x} \end{pmatrix}$$

$$iv^\mu \partial_\mu \rho = \left[\sqrt{2} G_F v^\mu \int dP' \rho(x, P') v'_\mu , \rho \right] + \frac{i}{2} \{ \mathrm{diag}(\Gamma_{\nu_e}(x,P), \Gamma_{\nu_x}(x,P)), \rho_{\mathrm{eq}} - \rho \}$$

Assuming $f \gg S$, the off-diagonal component becomes

$$v^\mu (i\partial_\mu - \Lambda_{0e\mu} + \Lambda_{0x\mu}) S_{ex} + i\Gamma_{ex} S_{ex} = 0$$

$$\Lambda^\mu_{0\alpha} \equiv \int dP f_{\nu_\alpha}(x,P) v^\mu$$

Plane-wave ansatz $S_{ex}(x,P) = \tilde{S}_{ex}(x,k) e^{ikx}$

$$\Pi^{\mu\nu}_{ex}(k) a_\nu(k) = 0$$

$$\Pi^{\mu\nu}_{ex}(k) = \eta^{\mu\nu} + \sqrt{2} G_F \int dP \frac{(f_{\nu_e} - f_{\nu_x}) v^\mu v^\nu}{v^\lambda (k_\lambda - \Lambda_{0e\lambda} + \Lambda_{0x\lambda}) + i\Gamma_{ex}}$$

$$a^\mu(k) \equiv \sqrt{2} G_F \int \mathrm{d}P \tilde{S}_{ex}(k,P) v^\mu$$

$$\Pi^{\mu\nu}_{ex}(k)a_\nu(k) = 0$$

For the existence of nontrivial solution of a, $\det \Pi_{ex}(k) = 0$ **Dispersion relation**

If ω has **negative imaginary part**, off-diagonal component **grows exponentially** with time.

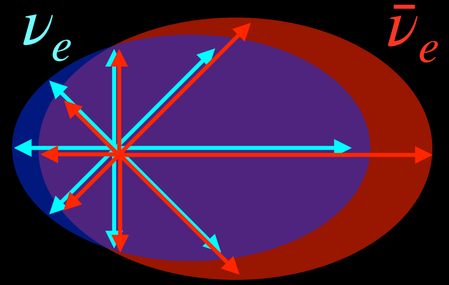
In this study, two limits are considered to analytically estimate the growth rate.

Neglect collision terms

Neglect angular distribution

Fast Flavor Instability (FFI)

- Equivalent to the angular crossing (Morinaga 2021) (**requires Boltzmann neutrino transport!**)
- Growth rate proportional to ν density



Collisional Flavor Instability (CFI)

- Caused by the existence of collision terms
- Growth rate proportional to the square-root of neutrino density.

(Most of previous studies only consider FFI)

Growth Rate (FFI)

For a two-beam model, the growth rate is given as

$$\sigma = (1 - v_{\nu_e} \cdot v_{\bar{\nu}_e}) \, \mathrm{Re}\sqrt{-F_{\nu_e} F_{\bar{\nu}_e}}$$

with the collimated beams

$$4\pi F_{\bar{\nu}_e} \delta(v - v_{\bar{\nu}_e}) \qquad 4\pi F_{\nu_e} \delta(v - v_{\nu_e})$$

Above suggests following formula to estimate the growth rate for arbitrary distribution

$$\sigma \equiv \sqrt{-\left(\int_{\Delta G>0} \frac{d\Omega}{4\pi} \Delta G\right)\left(\int_{\Delta G<0} \frac{d\Omega}{4\pi} \Delta G\right)}$$

$$\Delta G = \frac{\sqrt{2} G_F}{2\pi^2} \int \left(f_{\nu_e} - f_{\bar{\nu}_e}\right) \nu^2 d\nu$$

Growth Rate (CFI)

$$\Pi^{\mu\nu}_{ex}(k) = \eta^{\mu\nu} + \sqrt{2}G_F \int_{-\infty}^{\infty} \frac{E^2 dE}{2\pi^2} \int \frac{d\Omega_p}{4\pi} \frac{f_{\nu_e}(E,v) - f_{\nu_x}(E,v)}{\omega - v^\mu k_\mu + i\Gamma_{ex}(E)}$$

$$\begin{pmatrix} 1 & \cos\theta_\nu & \sin\theta_\nu\cos\phi_\nu & \sin\theta_\nu\sin\phi_\nu \\ \cos\theta_\nu & \cos^2\theta_\nu & \sin\theta_\nu\cos\theta_\nu\cos\phi_\nu & \sin\theta_\nu\cos\theta_\nu\sin\phi_\nu \\ \sin\theta_\nu\cos\phi_\nu & \sin\theta_\nu\cos\theta_\nu\cos\phi_\nu & \sin^2\theta_\nu\cos^2\phi_\nu & \sin^2\theta_\nu\cos\phi_\nu\cos\phi_\nu \\ \sin\theta_\nu\sin\phi_\nu & \sin\theta_\nu\cos\theta_\nu\sin\phi_\nu & \sin^2\theta_\nu\sin\phi_\nu\cos\phi_\nu & \sin^2\theta_\nu\sin^2\phi_\nu \end{pmatrix}$$

nonzero after integration

Angular integration is simplified and

$$I \equiv \sqrt{2}G_F \int_{-\infty}^{\infty} \frac{E^2 \mathrm{d}E}{2\pi^2} \frac{f_{\nu_e}(E) - f_{\nu_x}(E)}{\omega + i\Gamma_{ex}(E)} = -1,3$$

$$G \equiv \frac{n_{\nu_e} + n_{\bar{\nu}_e} - n_{\nu_x} - n_{\bar{\nu}_x}}{2}$$

$$A \equiv \frac{n_{\nu_e} - n_{\bar{\nu}_e} - n_{\nu_x} + n_{\bar{\nu}_x}}{2}$$

By imposing a monochromatic distribution,

$$\sigma \equiv \mathrm{Im}\omega$$

Isotropy-preserving branch ($I = -1$)

$$\omega_{\pm}^{\mathrm{pres}} = -A - i\gamma \pm \sqrt{A^2 - \alpha^2 + 2iG\alpha},$$

Isotropy-breaking branch ($I = 3$)

$$\omega_{\pm}^{\mathrm{break}} = -\frac{A}{3} - i\gamma \pm \sqrt{(\frac{A}{3})^2 - \alpha^2 - \frac{2}{3}iG\alpha},$$

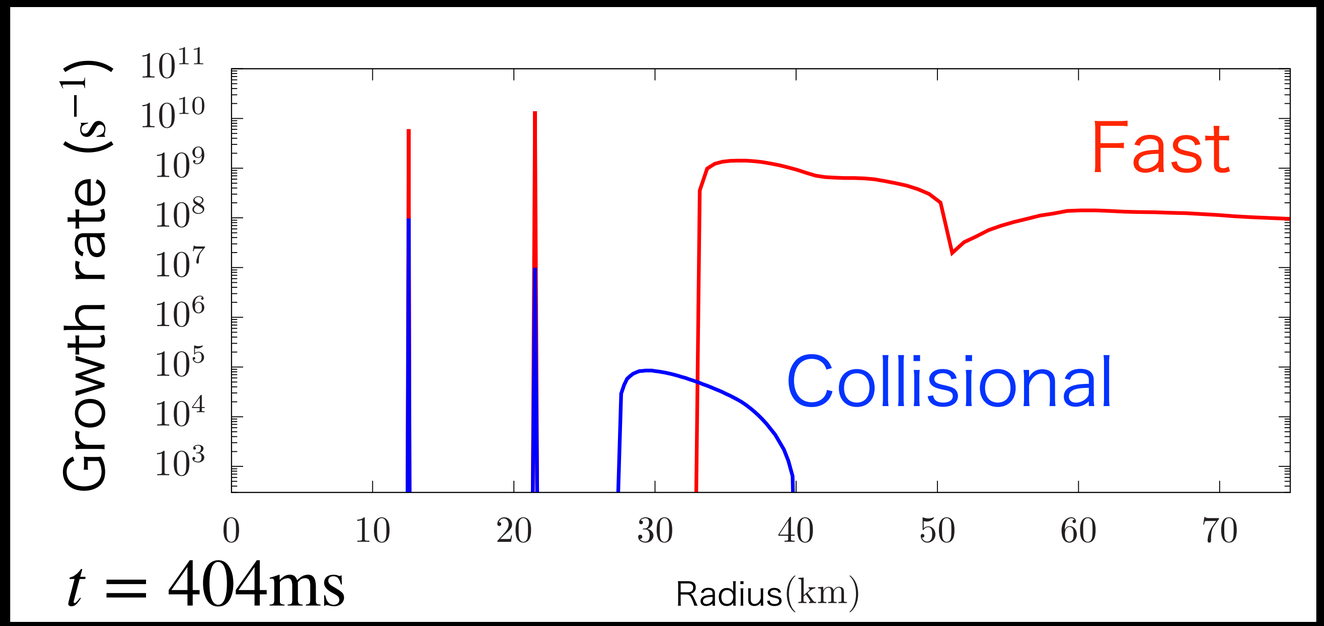
$$\gamma \equiv \frac{\Gamma_{\nu_e} + \Gamma_{\bar{\nu}_e} + \Gamma_{\nu_x} + \Gamma_{\bar{\nu}_x}}{2}$$

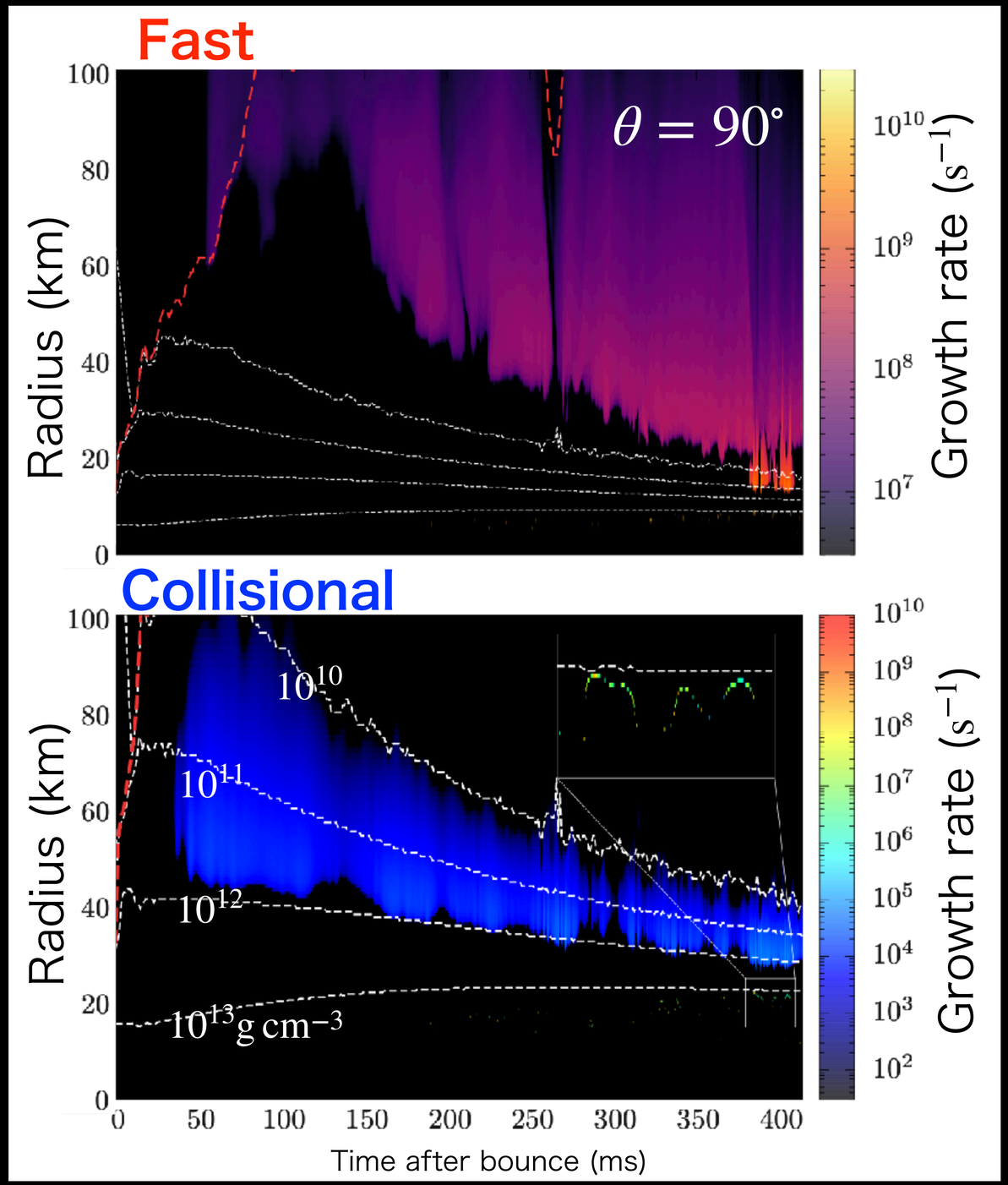
$$\alpha \equiv \frac{\Gamma_{\nu_e} - \Gamma_{\bar{\nu}_e} + \Gamma_{\nu_x} - \Gamma_{\bar{\nu}_x}}{2}$$

$$\Gamma_i \equiv \sqrt{2}G_F \int \frac{E^2 dE}{2\pi^2} \Gamma(E) f(E)$$

Post-process Analysis

- Stability analysis is performed for2D CCSN simulation with Boltzmann neutrino transport
- $11.2M_{\odot}$ progenitor, Lattimer-Swesty EOS
- Both CFI and FFI are found, and if both of them occur, FFI dominates

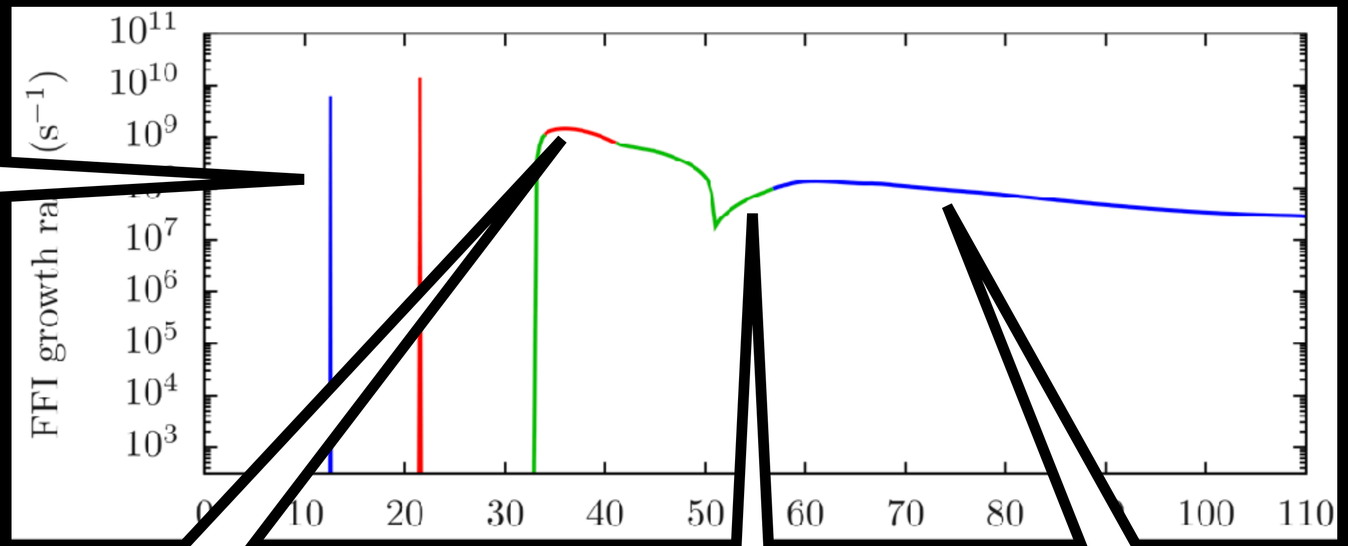


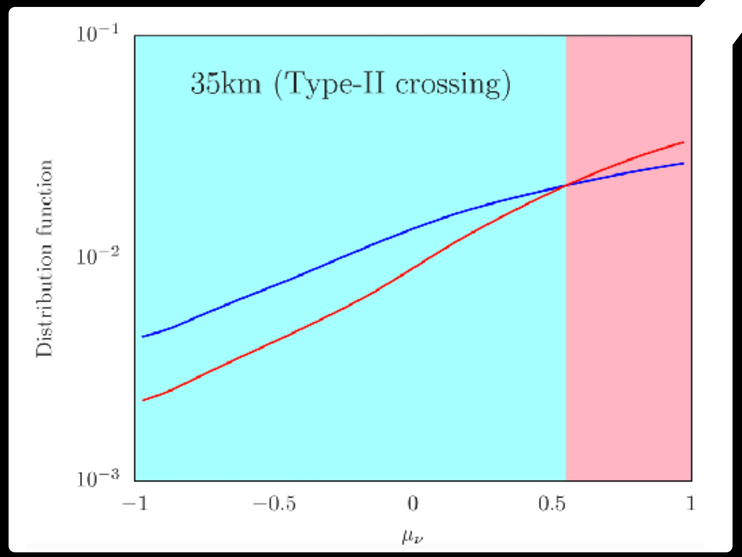


Properties of FFI

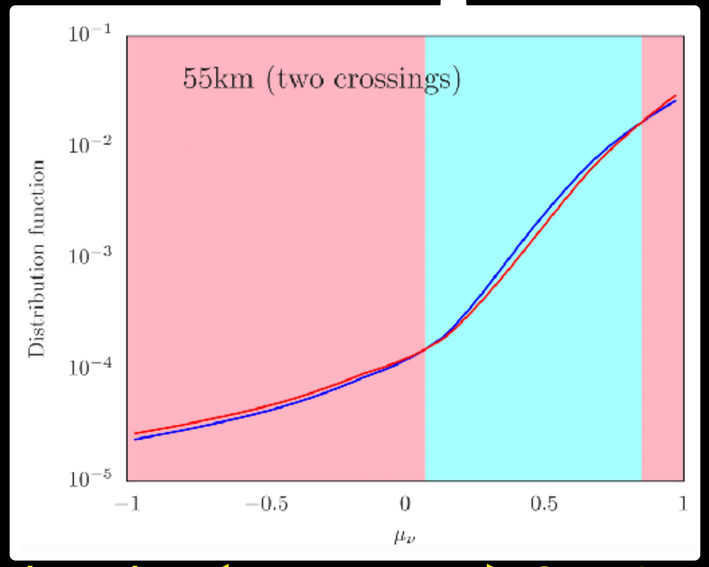
Neutrino chemical potential $\mu = 0$

$$n_{\nu_e} \sim n_{\bar{\nu}_e}$$

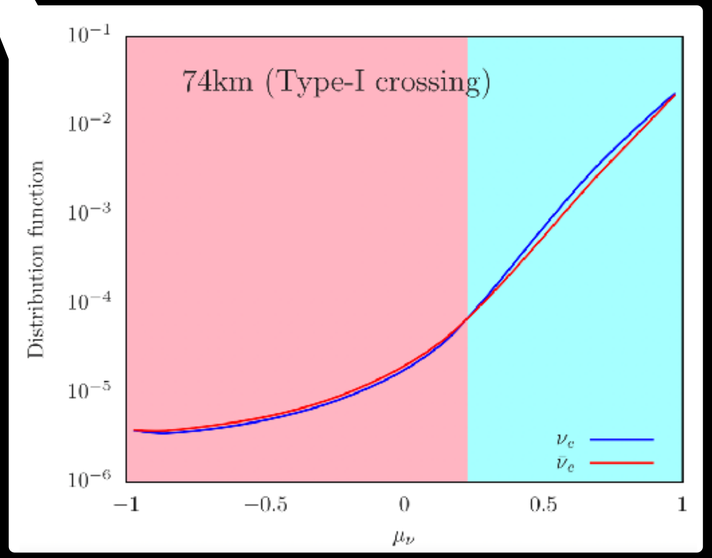




Incoming ⟷ Outgoing



Incoming ⟷ Outgoing

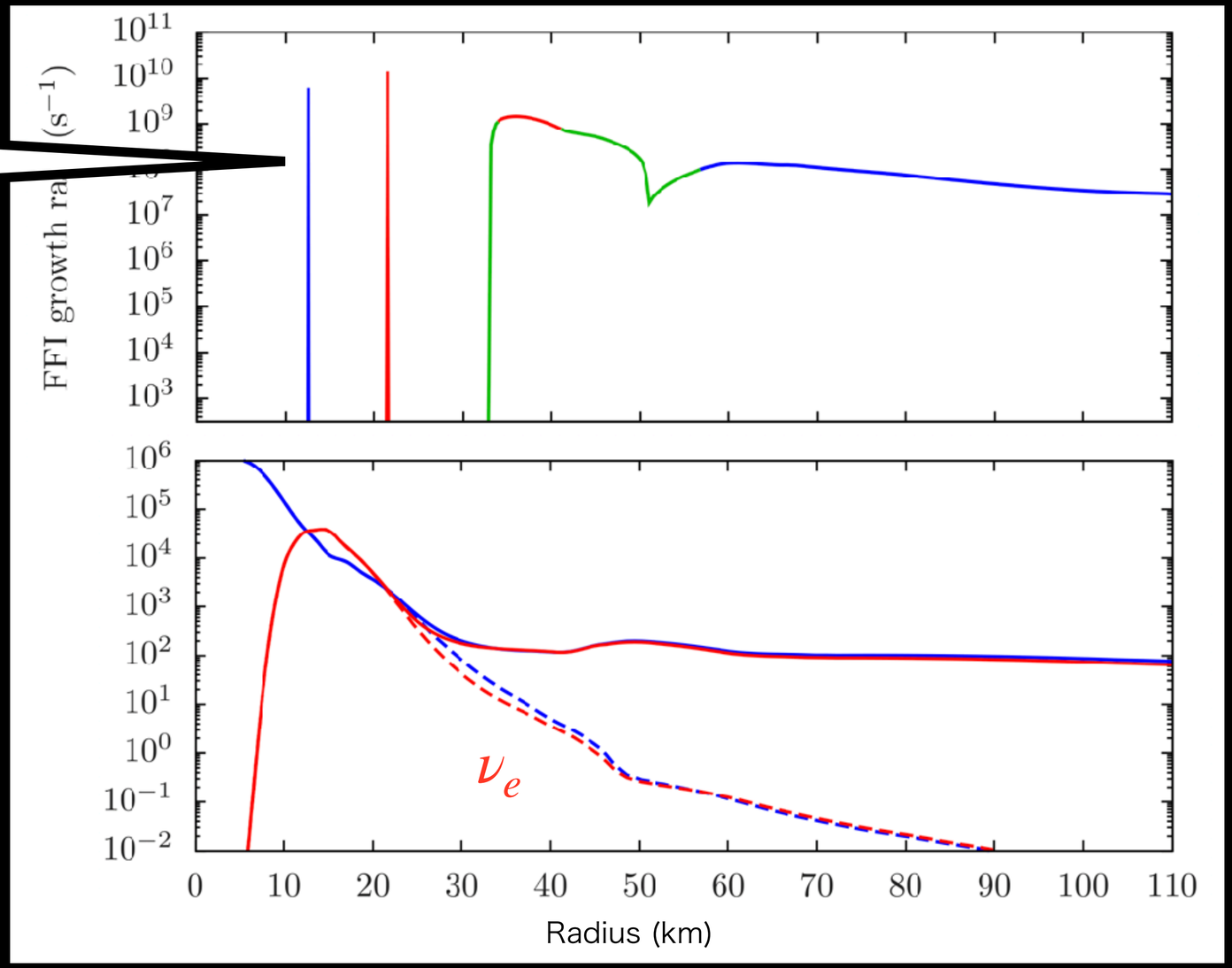


Incoming ⟷ Outgoing

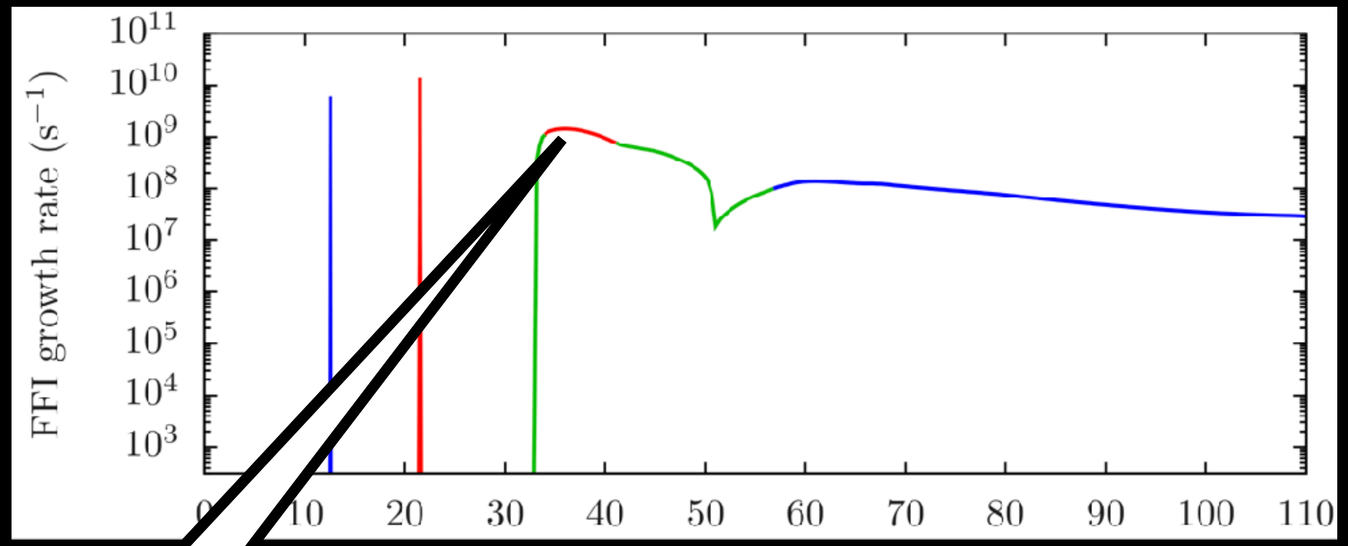
Properties of FFI

Neutrino chemical potential $\mu = 0$

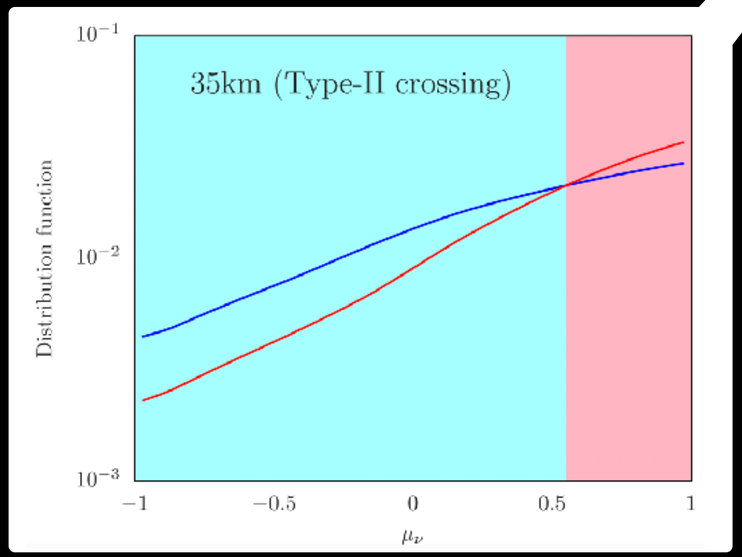
$$n_{\nu_e} \sim n_{\bar{\nu}_e}$$



Properties of FFI

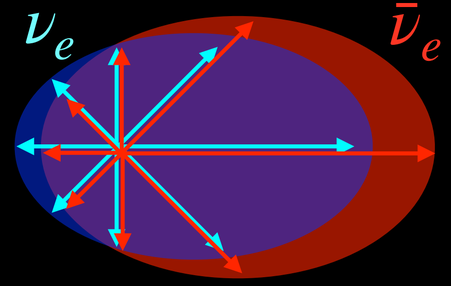


Angular distributions



Incoming ⟷ Outgoing

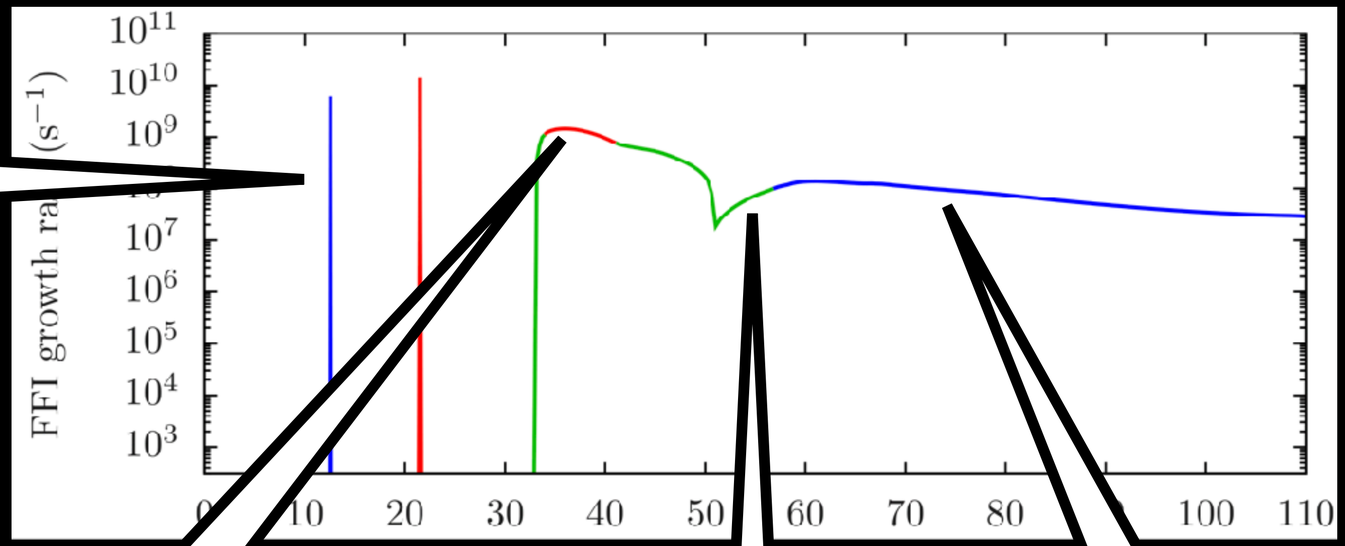
In between $\bar{\nu}_e$ sphere (inside) and ν_e sphere (outside), there is a region where $\bar{\nu}_e$ becomes more forward-peaked than ν_e



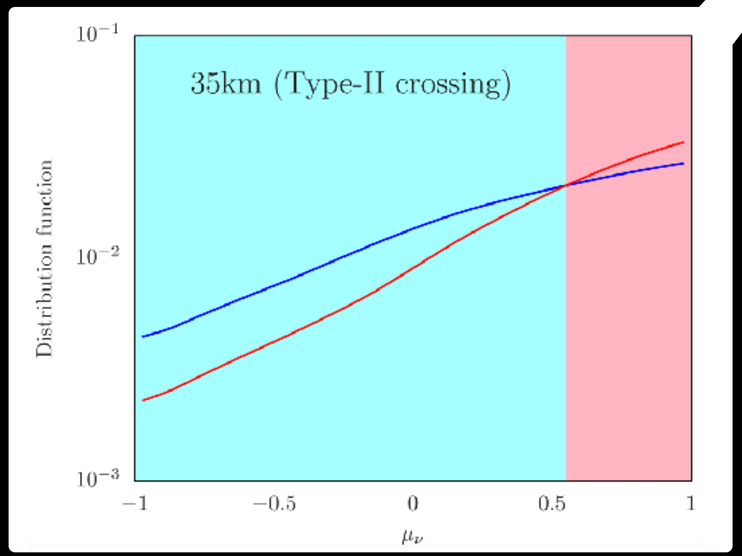
Properties of FFI

Neutrino chemical potential $\mu = 0$

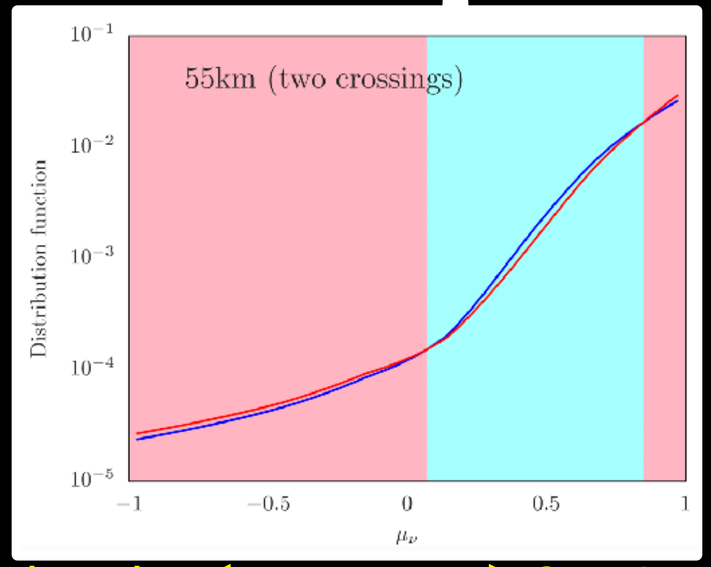
$$n_{\nu_e} \sim n_{\bar{\nu}_e}$$



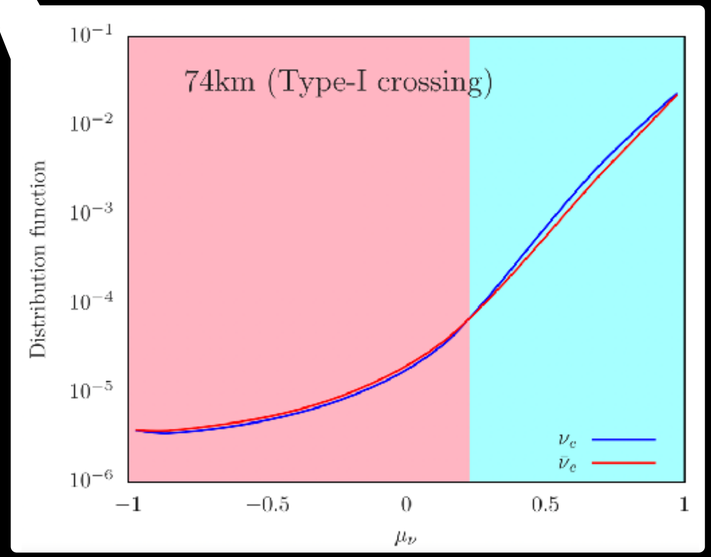
Angular distributions



Incoming ⟷ Outgoing



Incoming ⟷ Outgoing

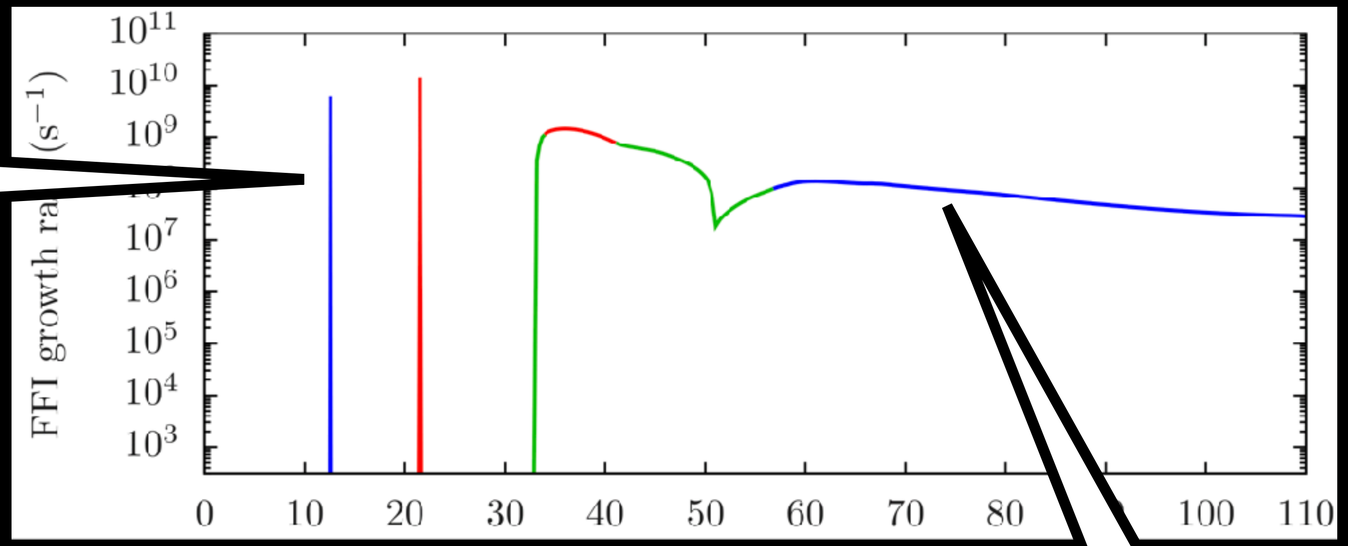


Incoming ⟷ Outgoing

Properties of FFI

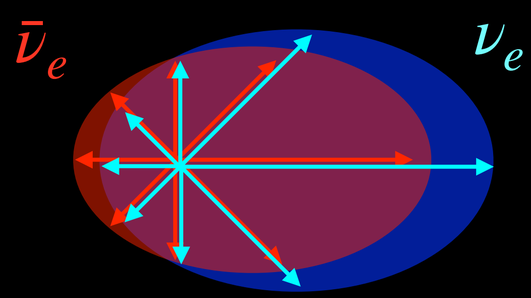
Neutrino chemical potential $\mu = 0$

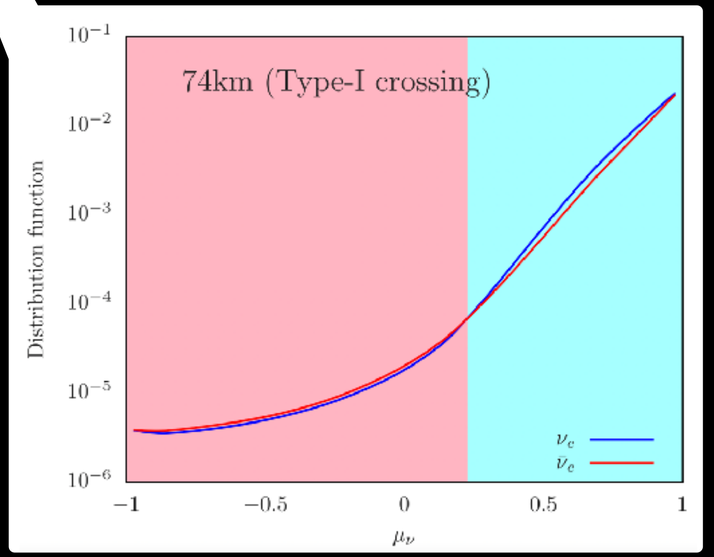
$$n_{\nu_e} \sim n_{\bar{\nu}_e}$$



Angular distributions

Since $\bar{\nu}_e$ has higher mean energy, its back-scattering by nucleon dominates over ν_e



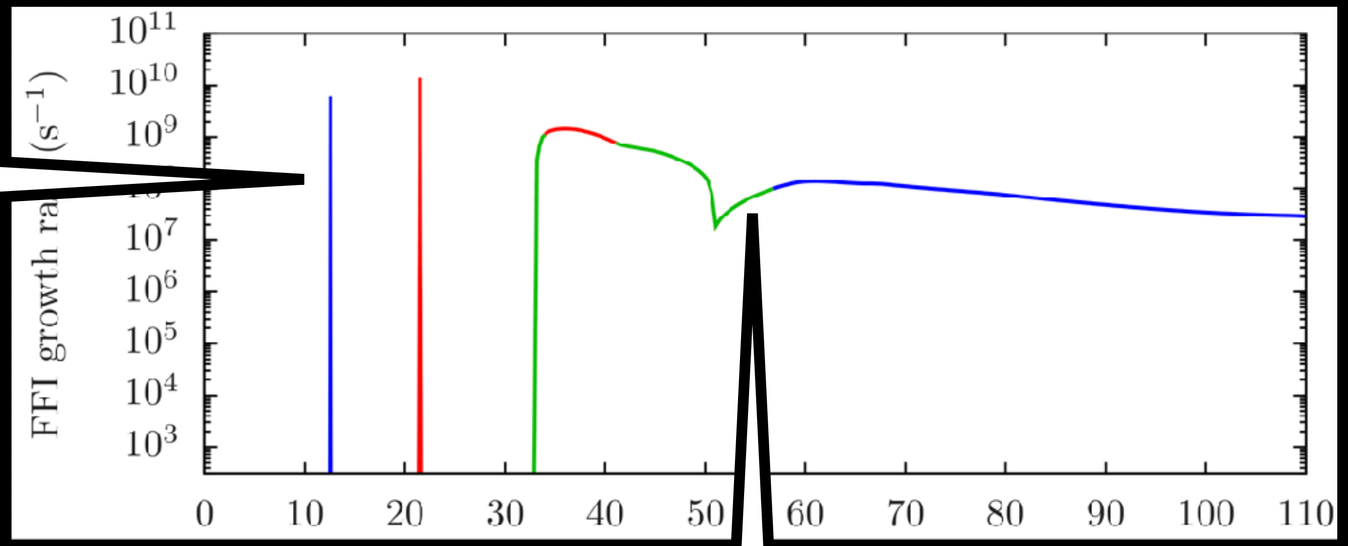


Incoming ⟷ Outgoing

Properties of FFI

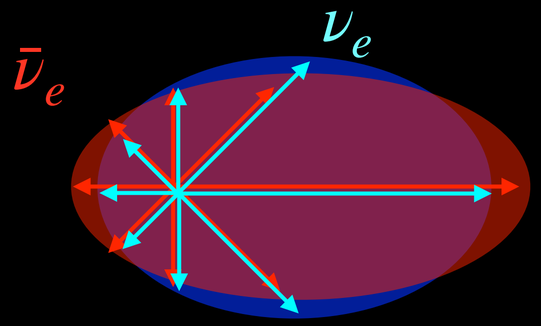
Neutrino chemical potential $\mu = 0$

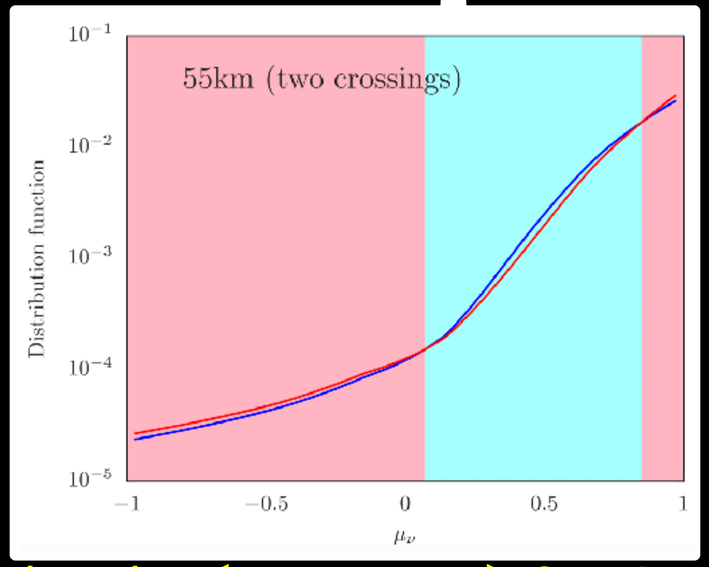
$$n_{\nu_e} \sim n_{\bar{\nu}_e}$$



Angular distributions

In the intermediate region, both mechanisms works and multiple crossing may appear





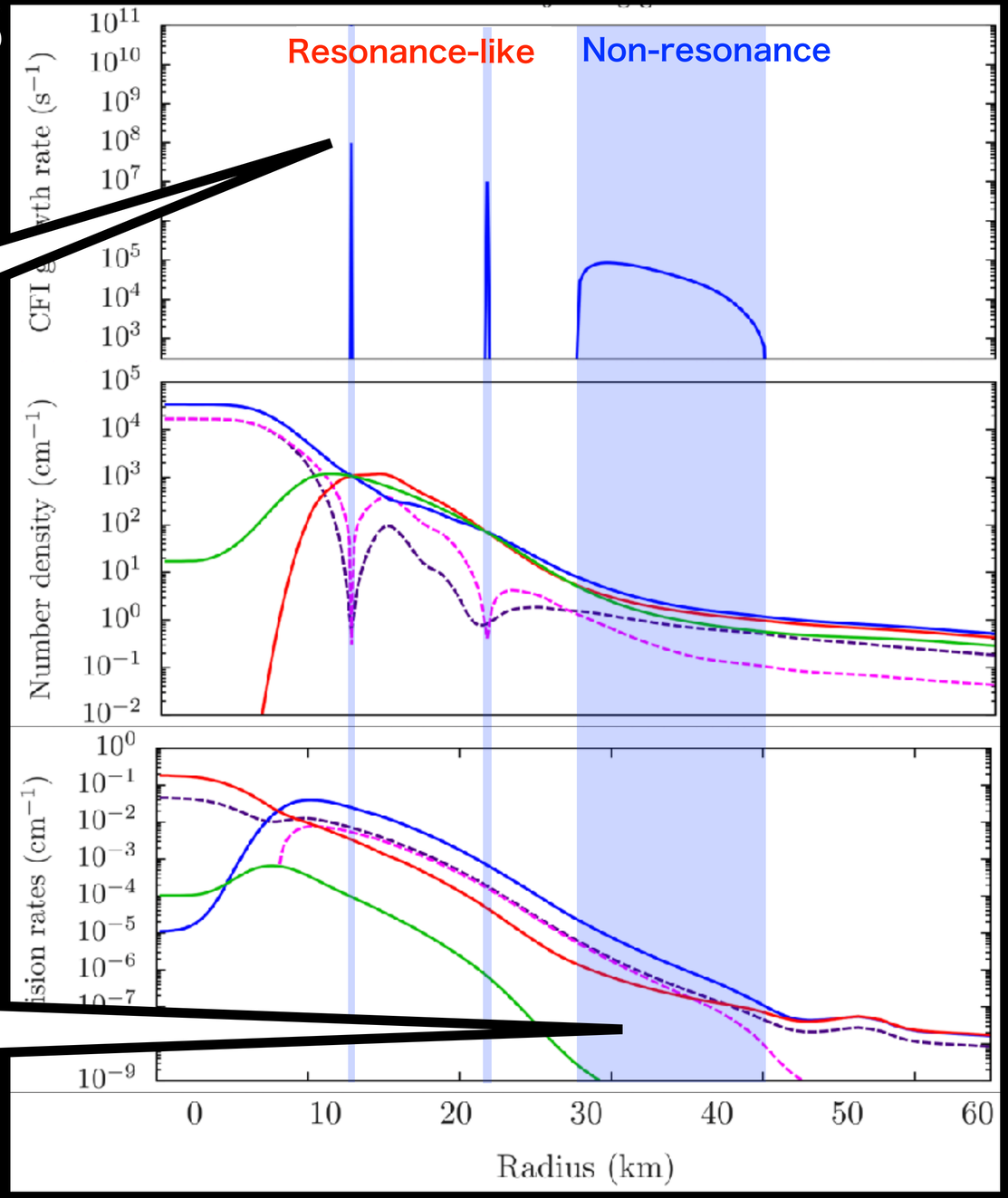
Incoming ⟷ Outgoing

Properties of CFI

In the similar way as FFI, $\mu = 0$ is desirable for CFI

This region did not appear in previous 1D study (Liu, RA+ 2023)

- $n_{\nu_e} > n_{\nu_x}$ & $n_{\bar{\nu}_e} > n_{\nu_x}$ because only ν_x is decoupled
- $\Gamma_e \sim \bar{\Gamma}_e$ due to low-Y_e environment



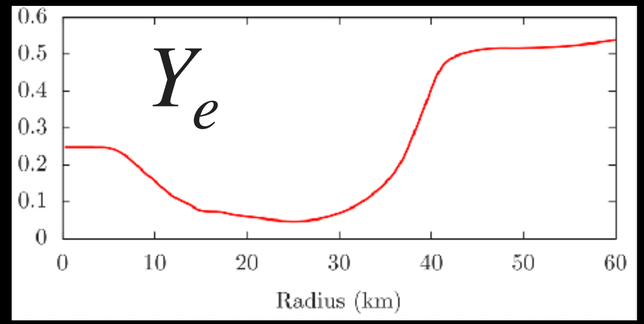
Non-resonance CFI (usual)

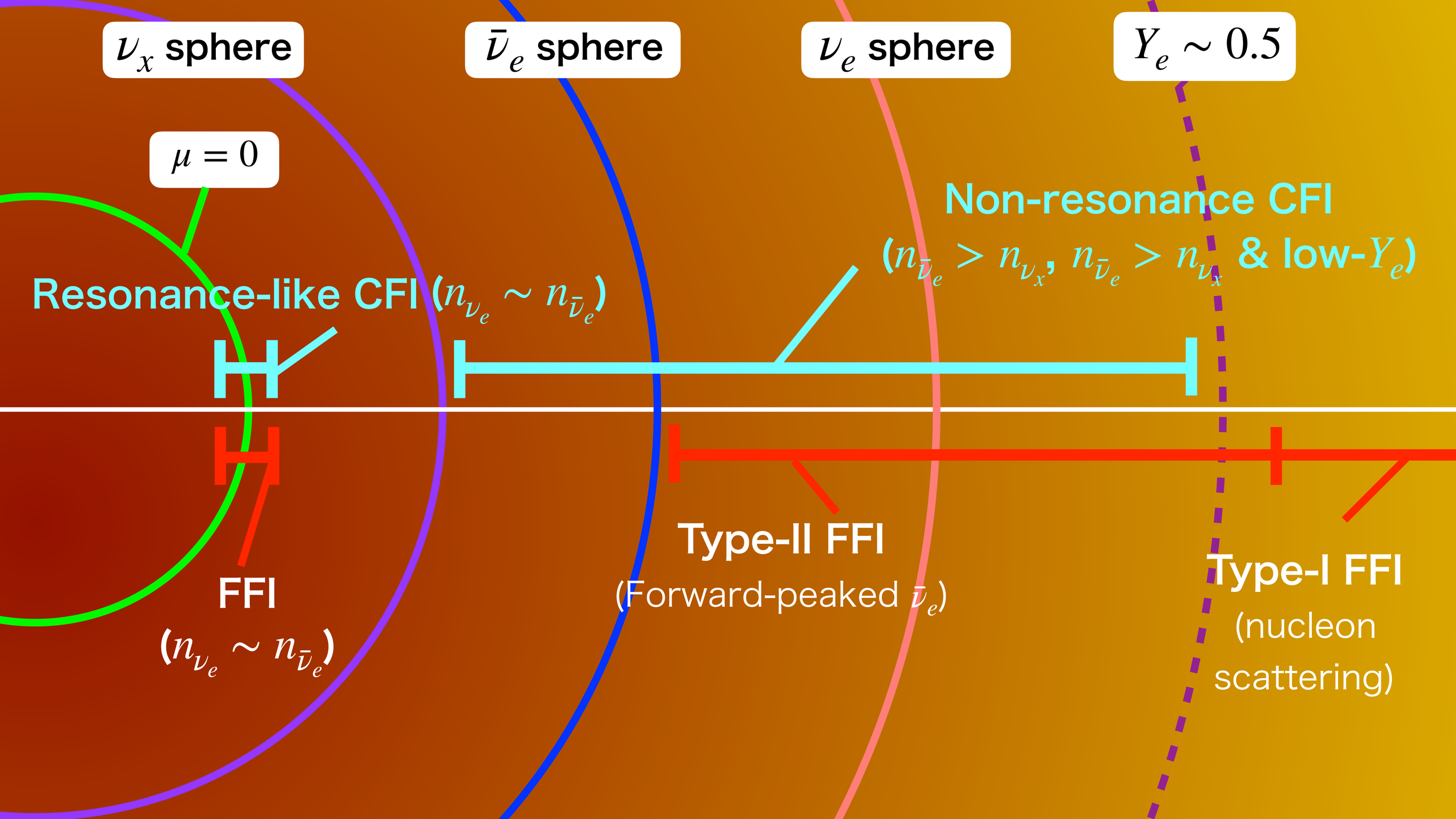
$$\sigma = \begin{cases} -\gamma + \dfrac{|G\alpha|}{|A|}, & (A^2 \gg |G\alpha|), \\ -\gamma + \sqrt{|G\alpha|}, & (A^2 \ll |G\alpha|) \end{cases}$$

Resonance-like CFI (Only when $A \sim 0$)

$$A = \frac{n_{\nu_e} - n_{\bar{\nu}_e}}{2} \qquad G = \frac{n_{\nu_e} + n_{\bar{\nu}_e} - 2n_{\nu_x}}{2}$$

$$\gamma = \frac{\Gamma_e + \bar{\Gamma}_e + 2\Gamma_x}{2} \qquad \alpha = \frac{\Gamma_e - \bar{\Gamma}_e}{2}$$




ν_x sphere
$\bar{\nu}_e$ sphere
ν_e sphere
$Y_e \sim 0.5$
$\mu = 0$
Resonance-like CFI ($n_{\nu_e} \sim n_{\bar{\nu}_e}$)
Non-resonance CFI
($n_{\bar{\nu}_e} > n_{\nu_x}$, $n_{\bar{\nu}_e} > n_{\nu_x}$ & low-Y_e)
FFI
($n_{\nu_e} \sim n_{\bar{\nu}_e}$)
Type-II FFI
(Forward-peaked $\bar{\nu}_e$)
Type-I FFI
(nucleon
scattering)

Effect of the Stellar Rotation

- Due to the centrifugal force, low-Y_e matter expands to the equatorial direction, which results in the enhancement of both FFI and CFI.

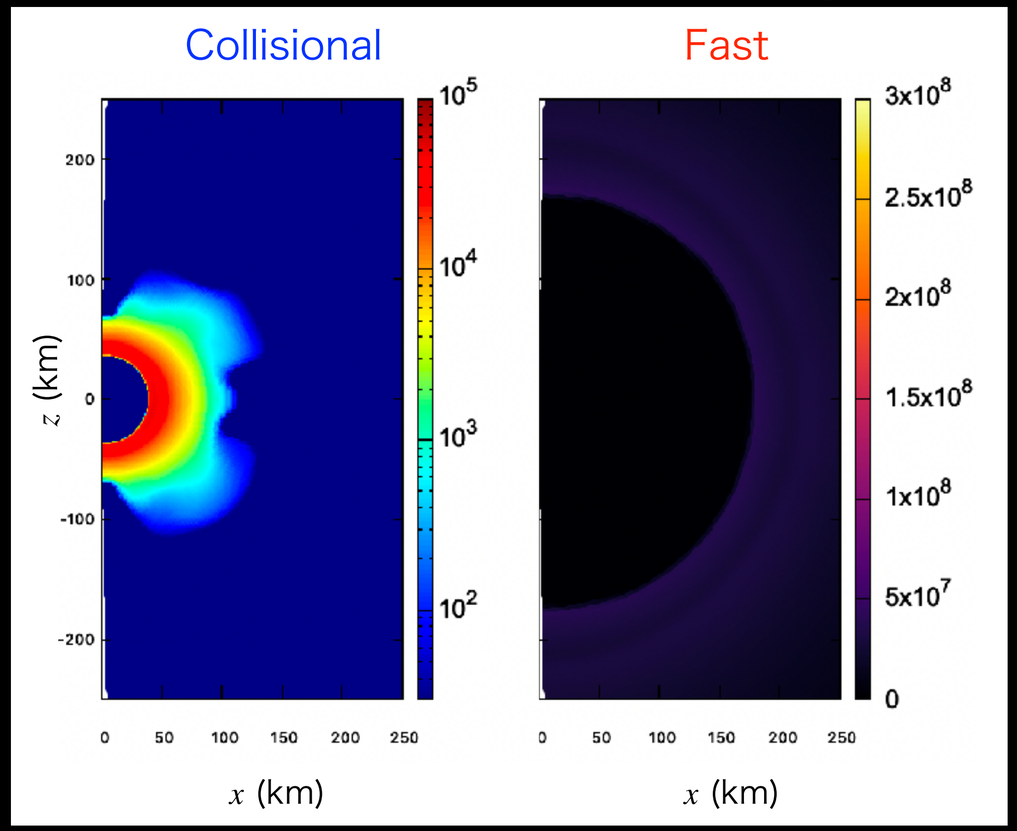
Furusawa-Togashi EOS

$15M_\odot$ progenitor

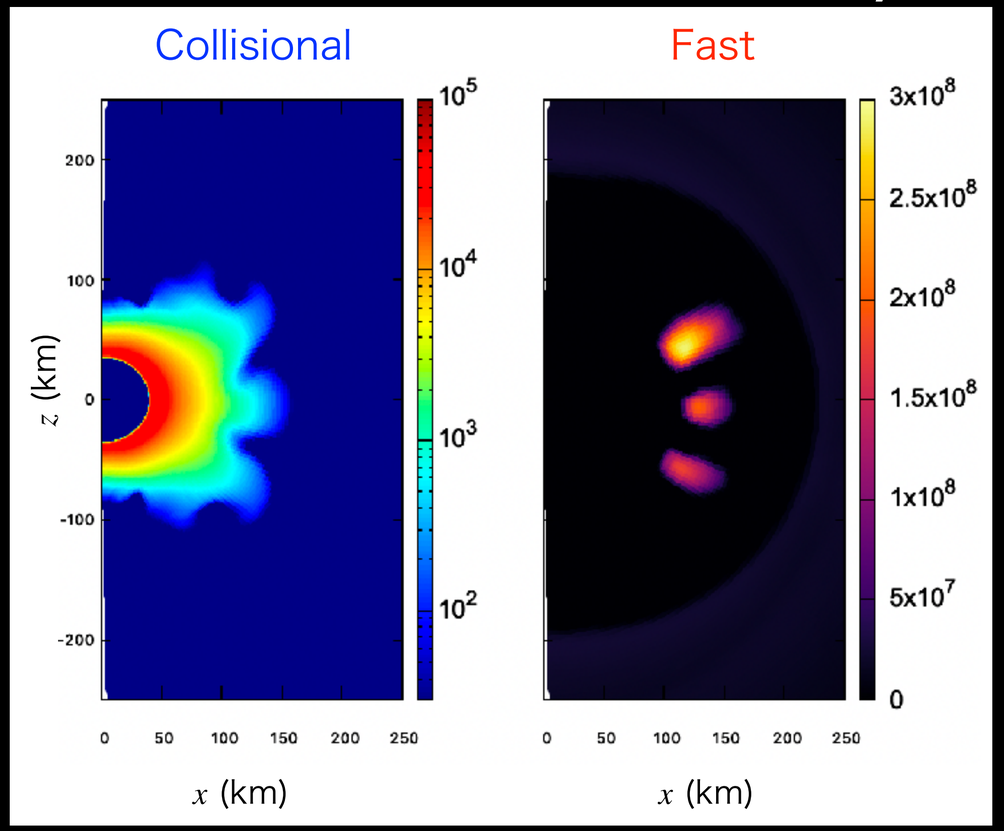
Angular momentum distribution:

$$\frac{2,4\,\mathrm{rad/s}}{1+(r/10^3\,\mathrm{km})^2}$$

2rad/s

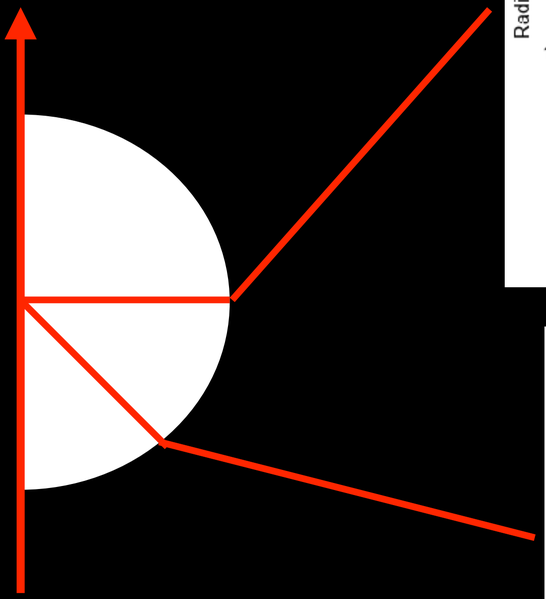


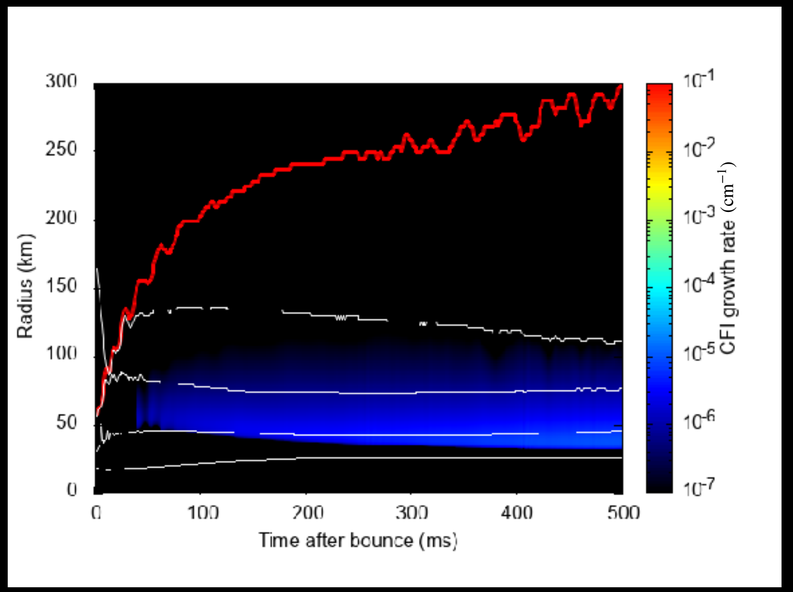
4rad/s

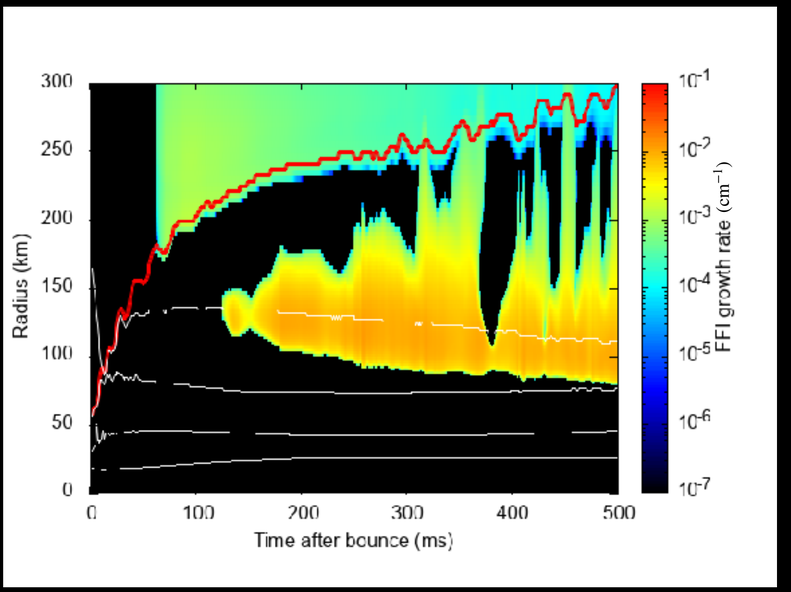


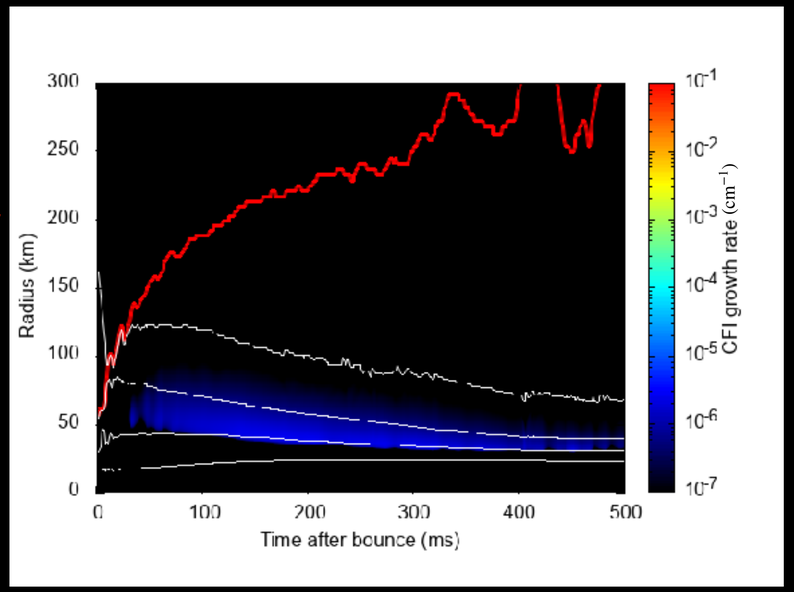
Effect of the Stellar Rotation

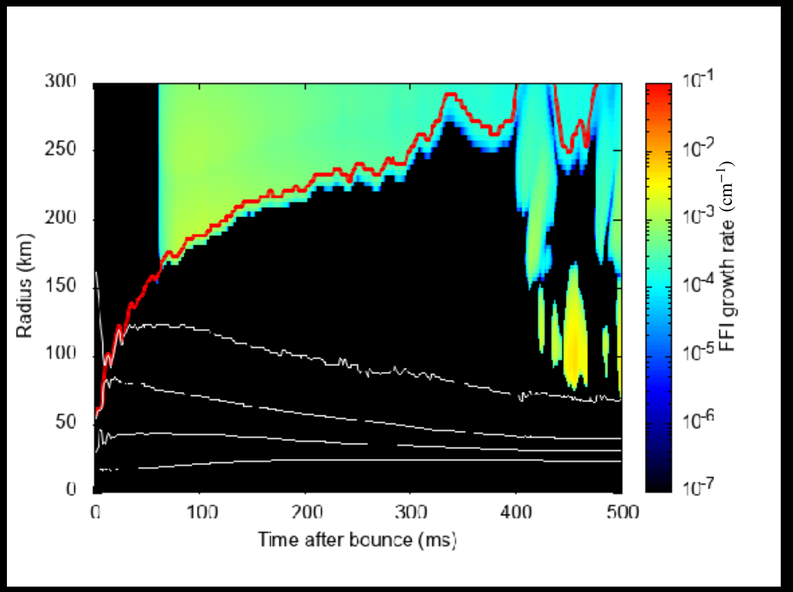
Rotation axis





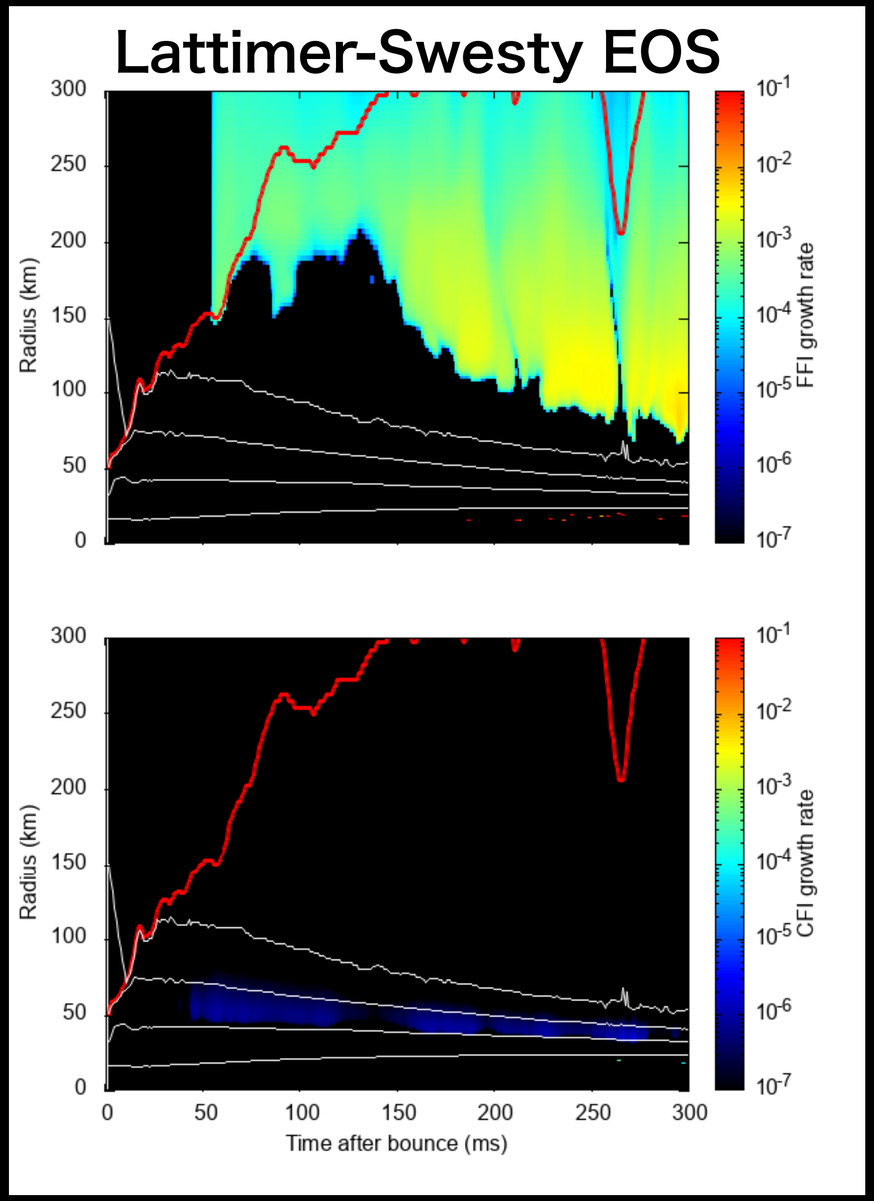


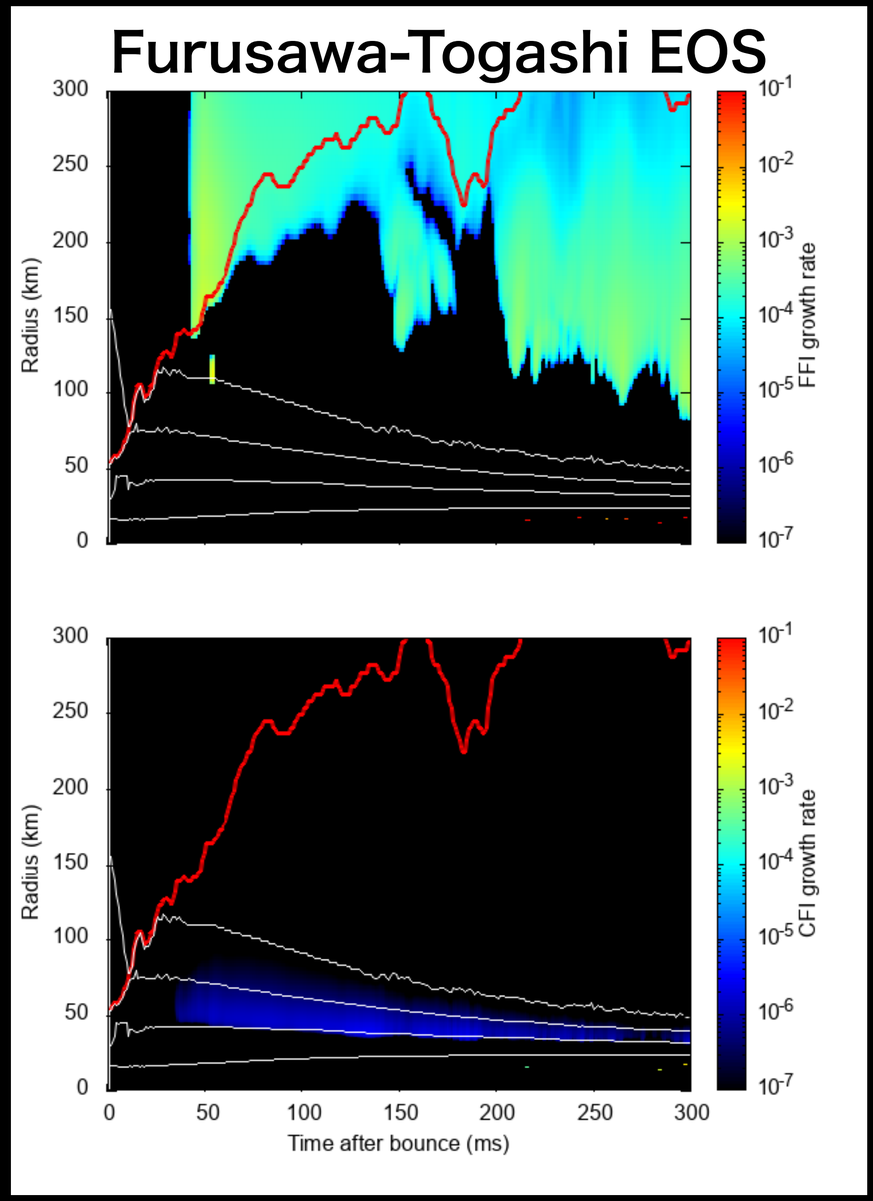




EOS Dependence

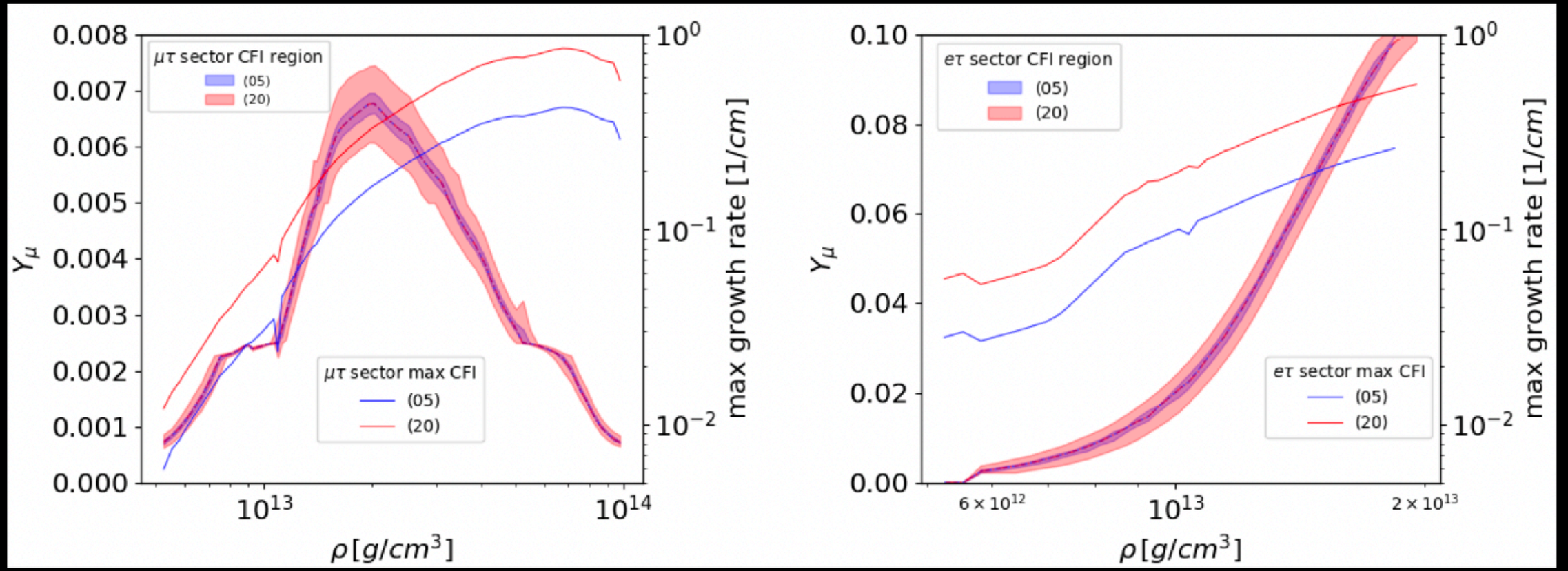
$11.2M_{\odot}$ progenitor





Effects of Muon on CFI (arXiv:2407.10604)

In CCSNe, muons can also appear in CCSNe (Boling 2017)

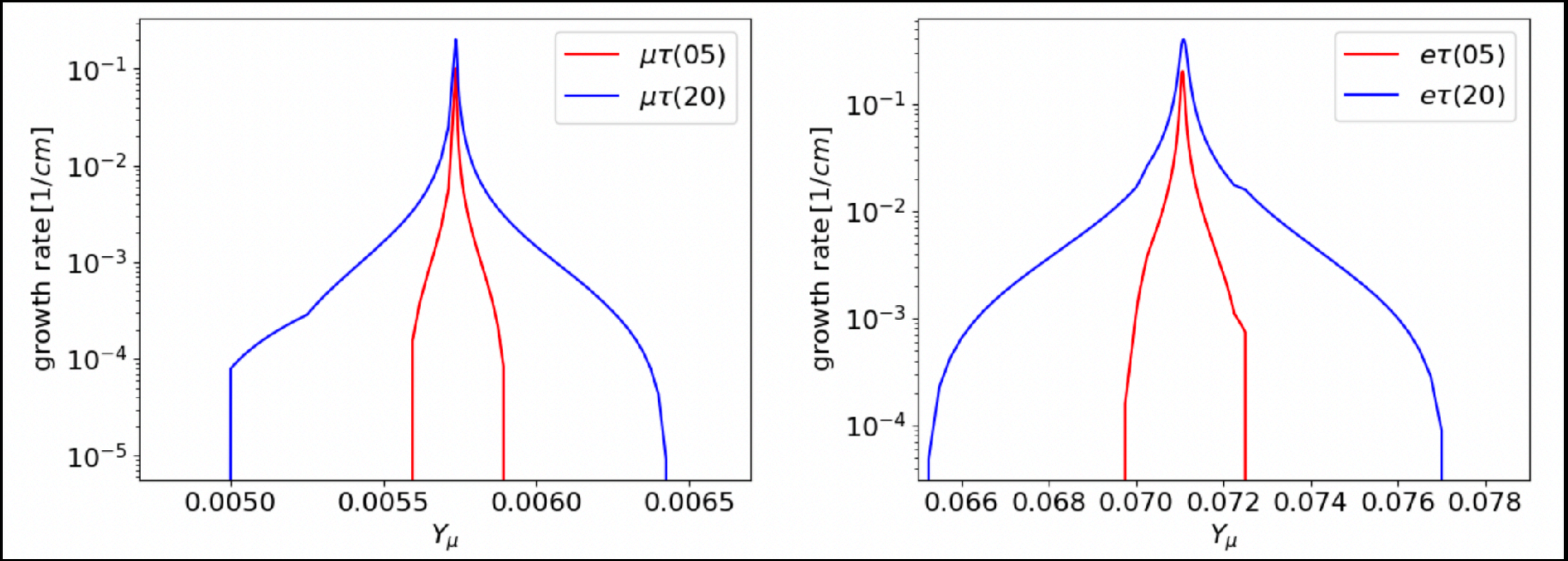


$$\sigma = \begin{cases} -\gamma + \dfrac{|G\alpha|}{|A|}, & (A^2 \gg |G\alpha|), \\ -\gamma + \sqrt{|G\alpha|}, & (A^2 \ll |G\alpha|) \end{cases}$$

$$G \equiv \frac{n_{\nu_i} + n_{\bar{\nu}_i} - n_{\nu_j} - n_{\bar{\nu}_j}}{2} \qquad \alpha \equiv \frac{\Gamma_{\nu_i} - \Gamma_{\bar{\nu}_i} + \Gamma_{\nu_j} - \Gamma_{\bar{\nu}_j}}{2}$$

$$A \equiv \frac{n_{\nu_i} - n_{\bar{\nu}_i} - n_{\nu_j} + n_{\bar{\nu}_j}}{2} \qquad \gamma \equiv \frac{\Gamma_{\nu_i} + \Gamma_{\bar{\nu}_i} + \Gamma_{\nu_j} + \Gamma_{\bar{\nu}_j}}{2}$$

Effects of Muon on CFI



Discussion

- FFI & CFI commonly occurs in CCSNe
- FFI growth rate is orders of magnitude larger than CFI if they occur in the same place. However, CFI occurs at inner radii (upstream), which may modify the appearance of FFI at larger radii.

Future Tasks

- Classical simulation + back-reaction of flavor conversion (such as Ehring+2023)