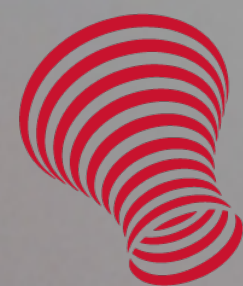


# WOMEN IN THEORETICAL PHYSICS

## “MILLA BALDO CEOLIN” NATIONAL PRIZE 2023



Istituto Nazionale di Fisica Nucleare  
Sezione di Perugia



**strong.**  
niels bohr  
institute

*Marta Cocco | University of Perugia & Niels Bohr Institute*  
*October 8, 2024*





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UNIVERSITÀ DEGLI STUDI  
DI PERUGIA

**Università degli Studi di Perugia**

DIPARTIMENTO DI FISICA E GEOLOGIA

Corso di Laurea Magistrale in Fisica

TESI DI LAUREA:

# **Tidal deformations of a slowly spinning binary system**

Candidato:  
**Marta Cocco**

Relatore:  
**Prof.ssa Marta Orselli**



Istituto Nazionale di Fisica Nucleare  
Sezione di Perugia



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niels bohr  
institute

Anno Accademico 2022/2023

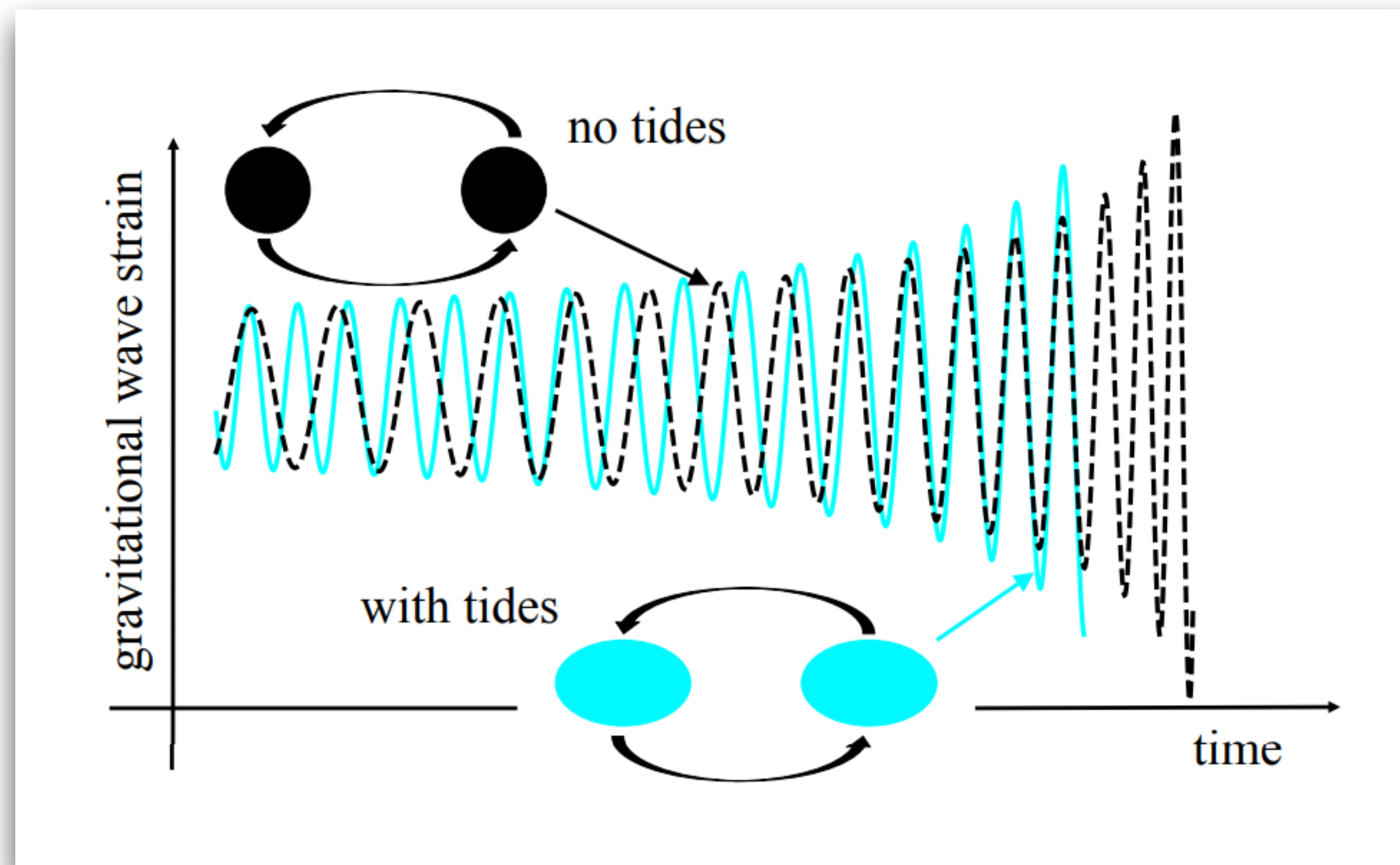
**ICAL PHYSICS**  
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# Tidal forces

Tidal effects can have an impact on gravitational waves emitted by binary systems

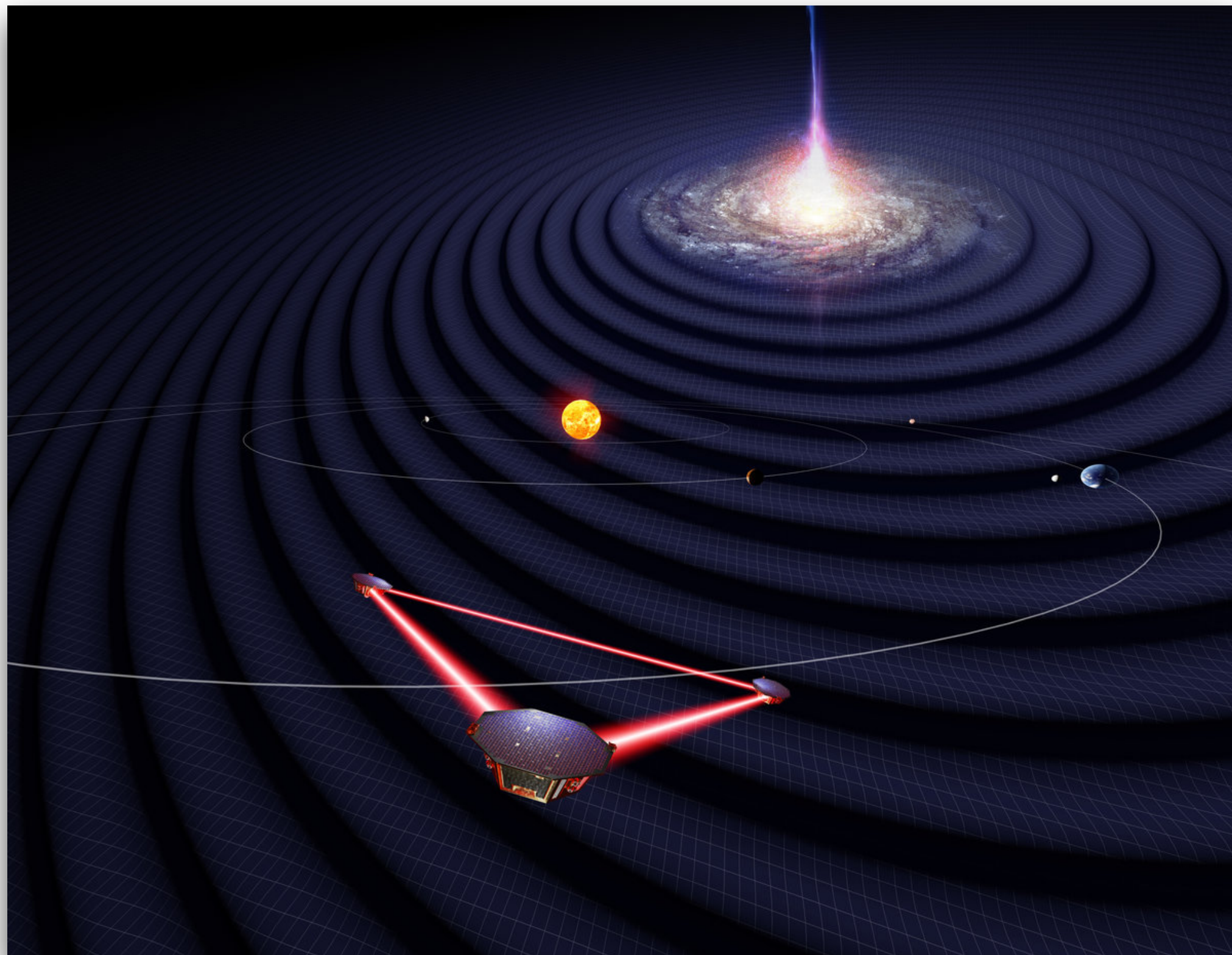
[T. Hinderer (2008); E. E. Flanagan and T. Hinderer (2008)]



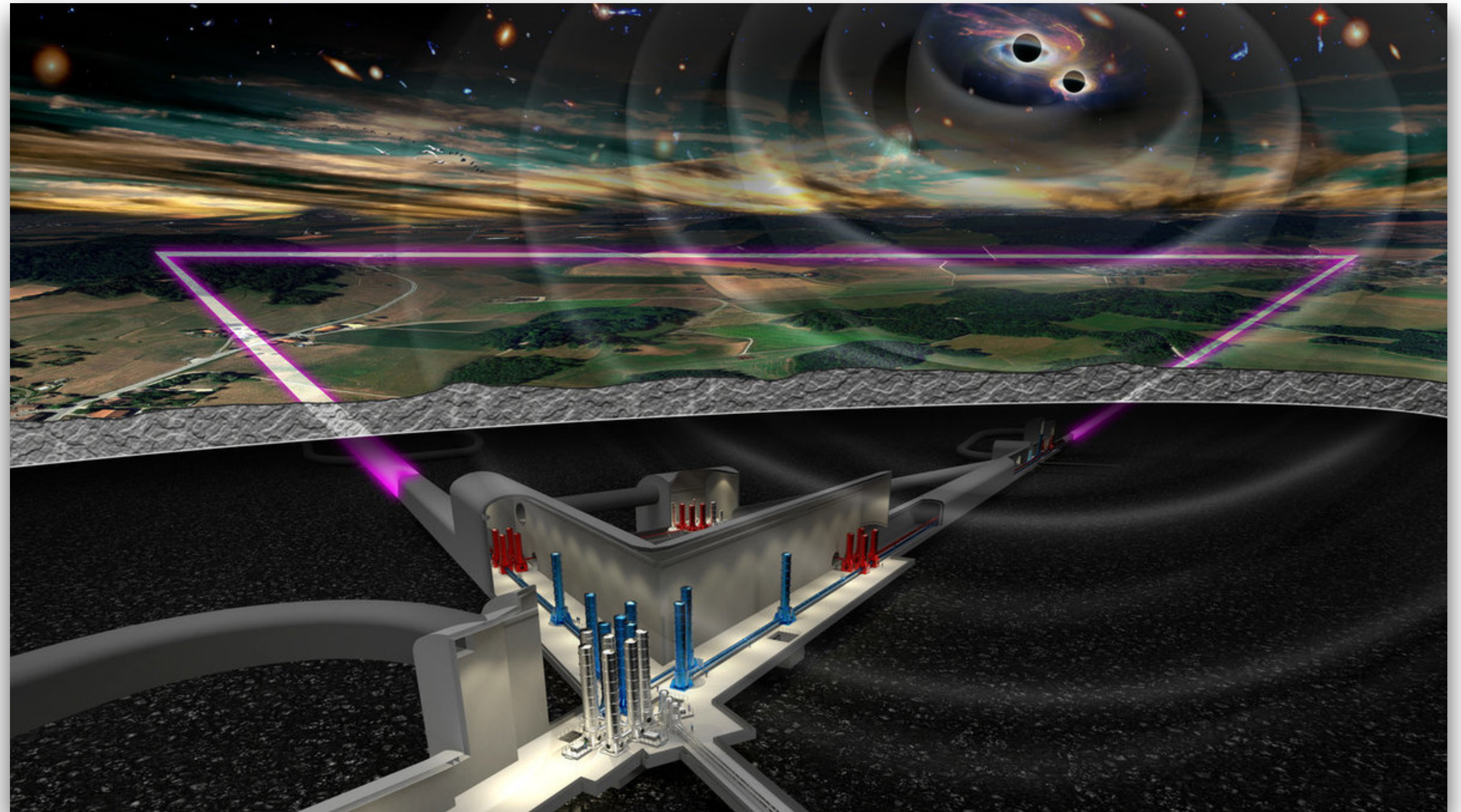
[N. Yunes, M. Coleman Miller, K. Yagi (2022)]



# Tidal forces



LISA [Max Planck Institute]

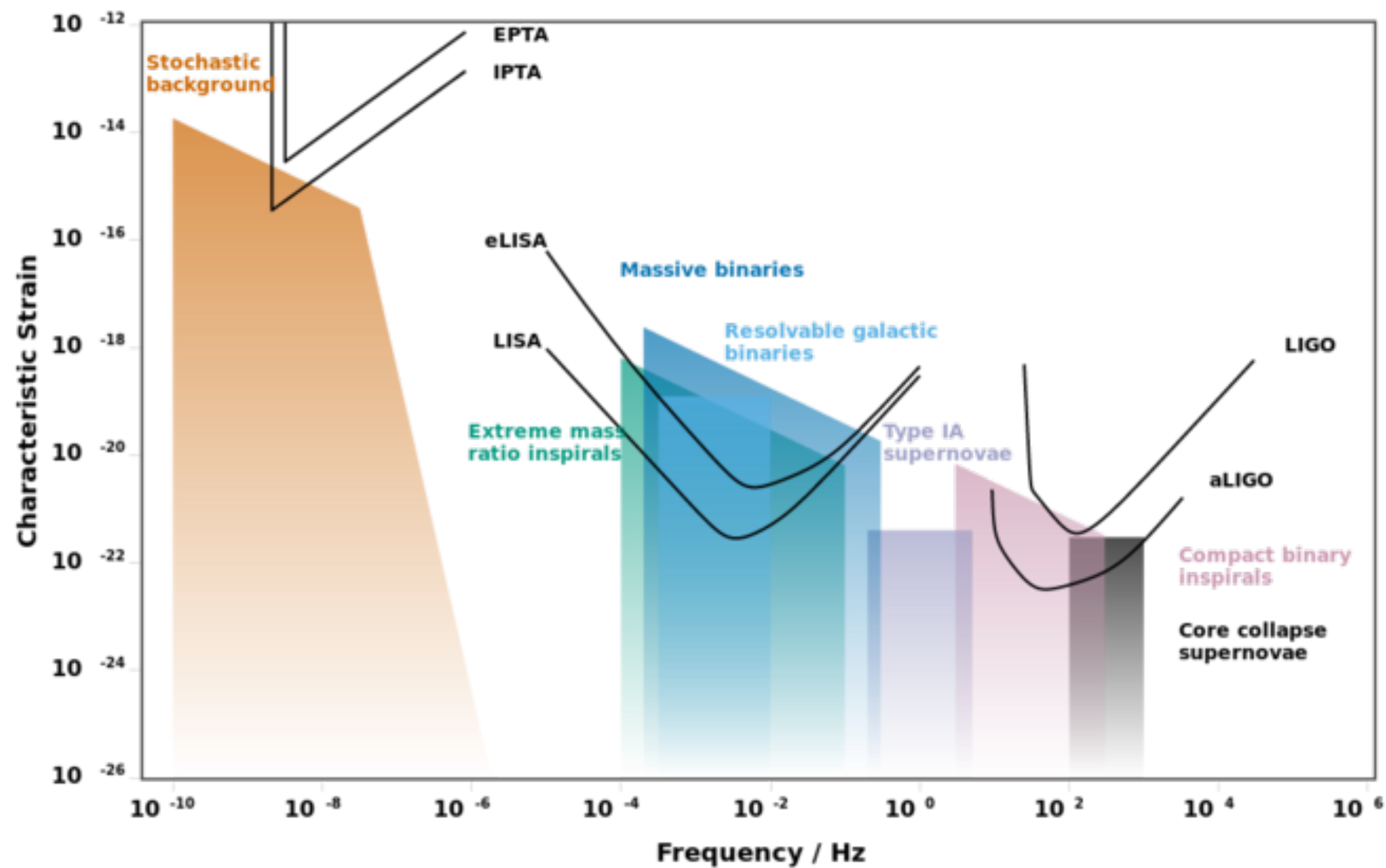


Einstein Telescope [Max Planck Institute]

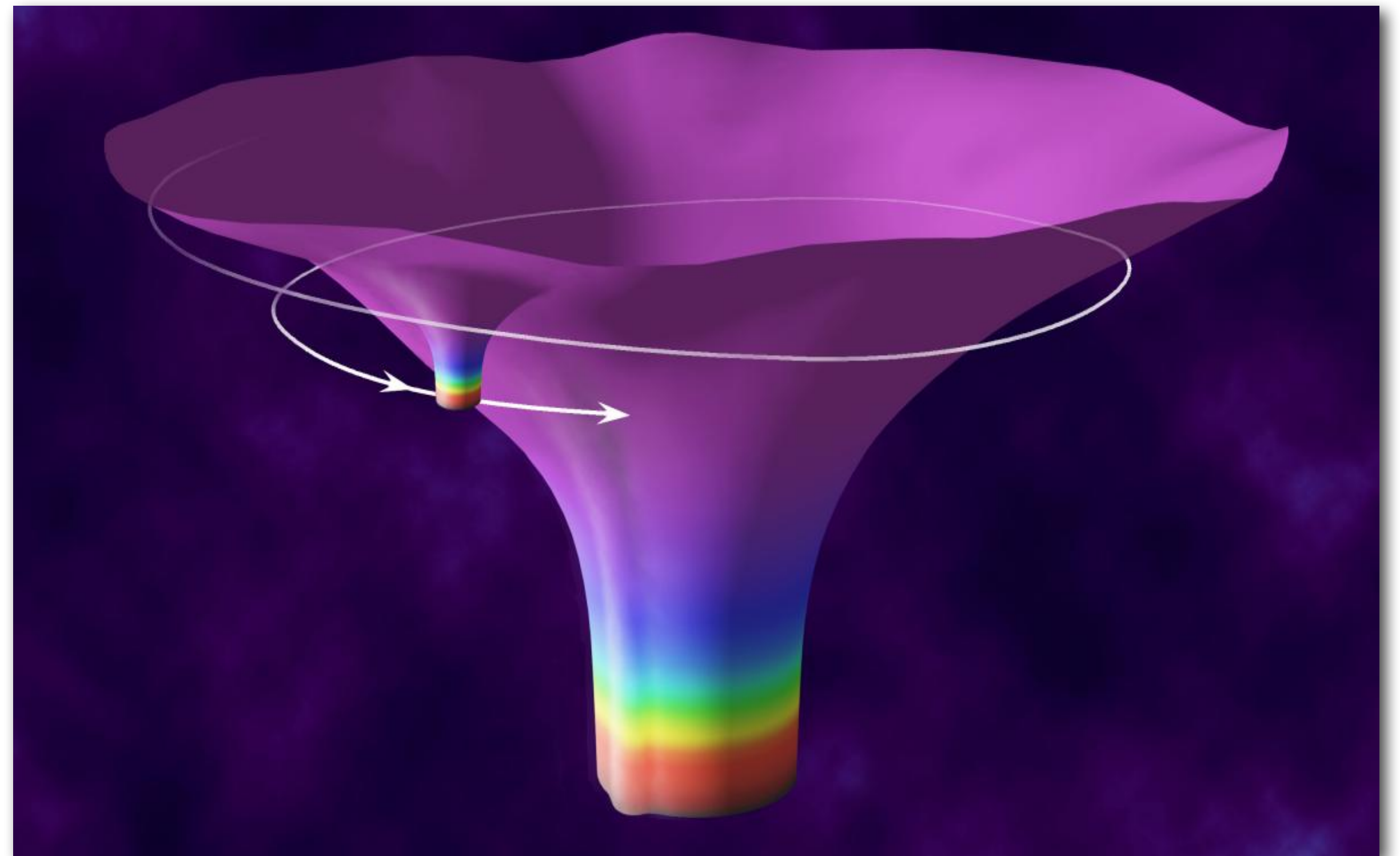
The upcoming generation of gravitational waves detectors (e.g. LISA, ET) will provide an improved sensitivity and a wider frequency range



# Tidal forces



[C. Moore, R. Cole, C. Berry (2013)]



Extreme mass ratio inspiral [Wikipedia]

LISA will detect **EMRIs**

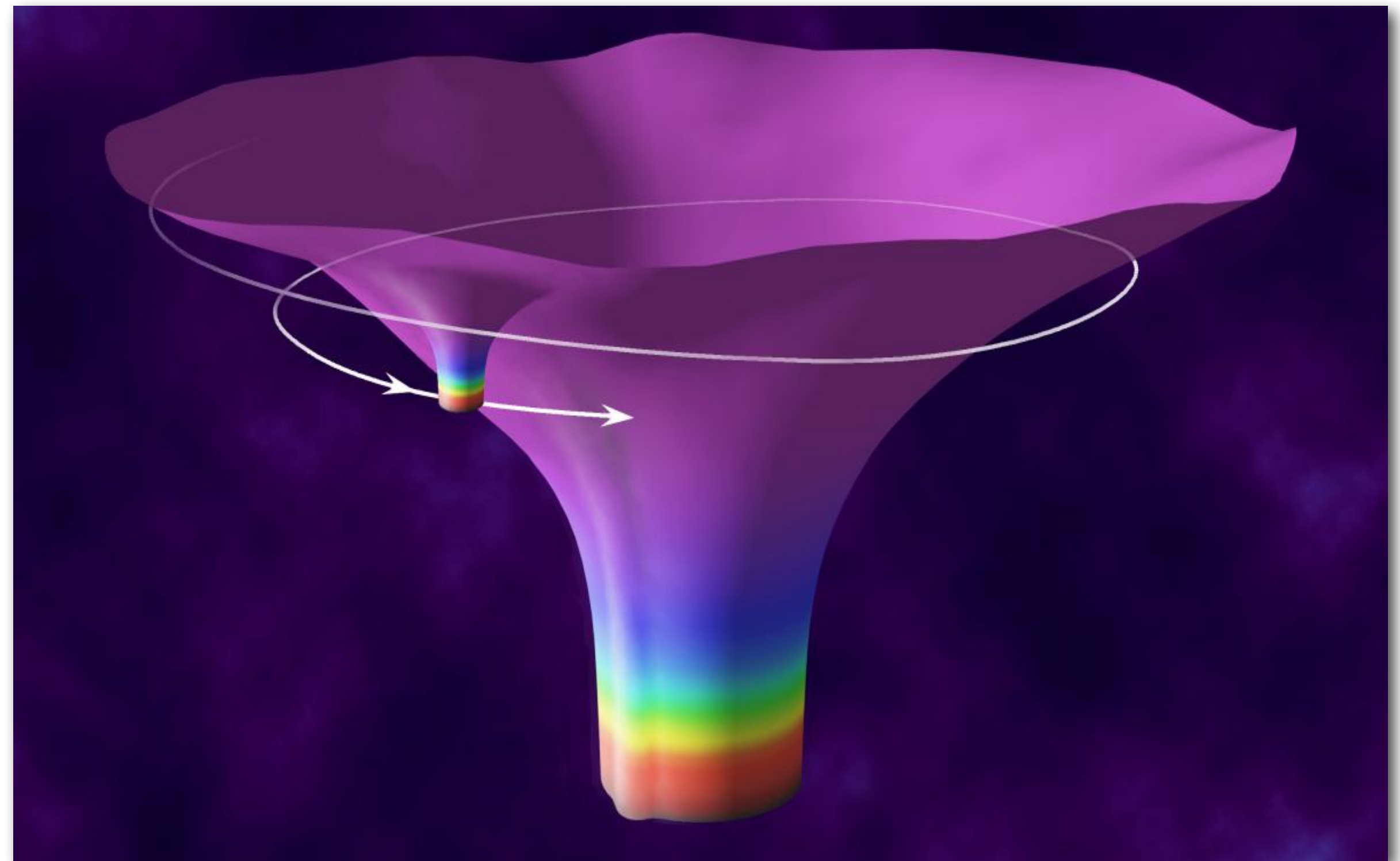
EMRIs are a probe of the surrounding environment: accretion disks, dark matter, supermassive black hole



# Tidal forces

In my Master's thesis, I studied:

- the effects of tidal forces
- exerted by a third supermassive black hole
- on the EMR binary system's dynamics

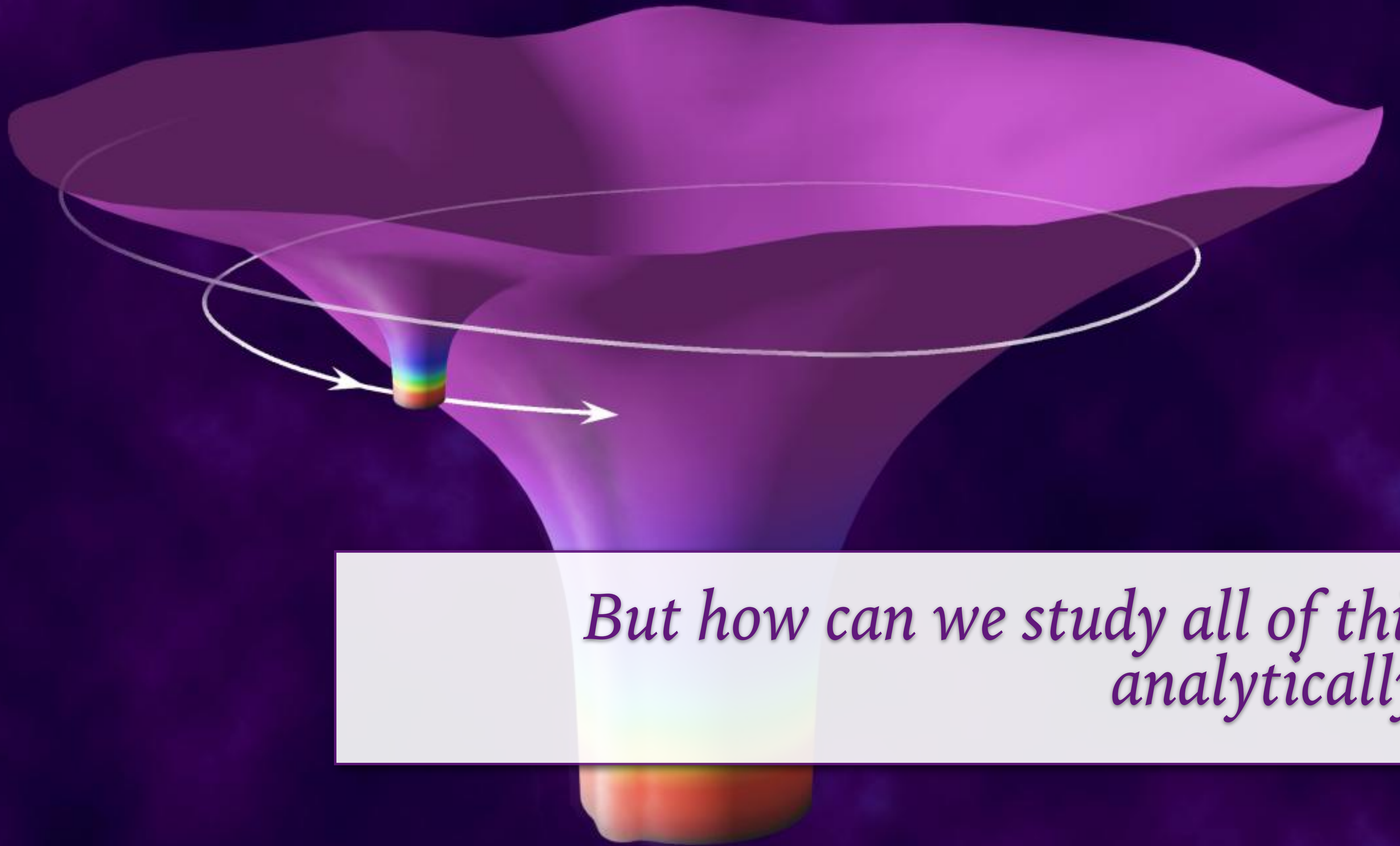


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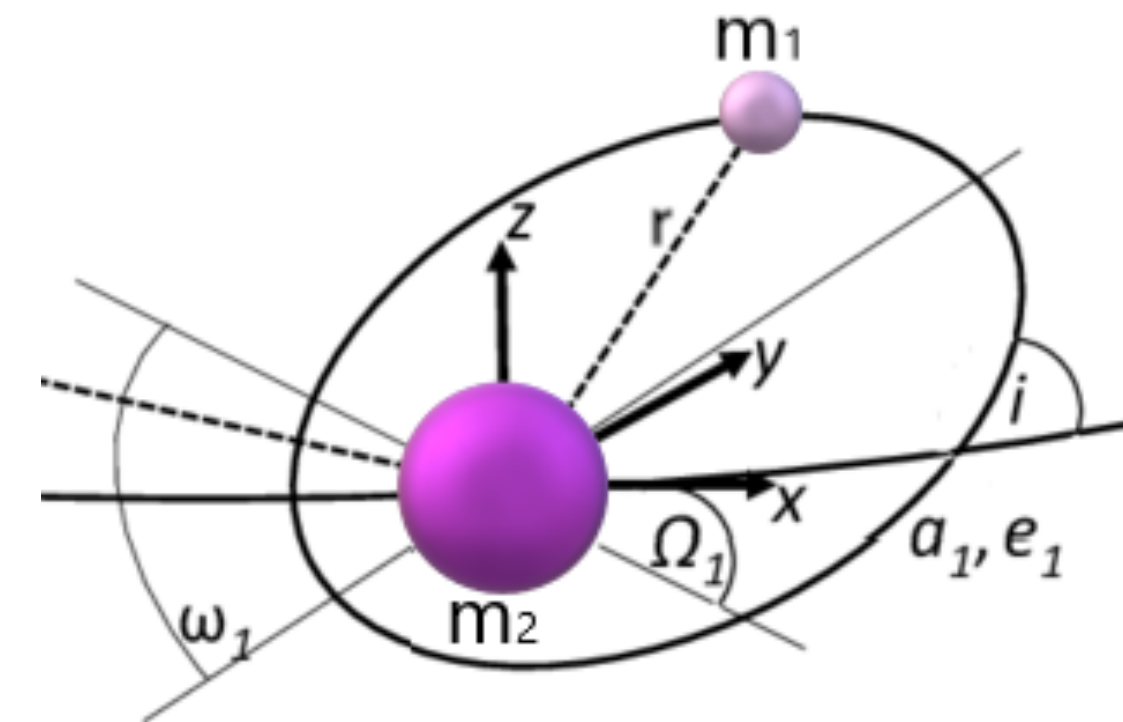
*But how can we study all of this,  
analytically?*



# Tidal effects on an EMR system

Hierarchical condition

$$\frac{m_1 \ll m_2}{\text{EMR}}$$



[B. Deme (2021)]



# Tidal effects on an EMR system

Hierarchical condition

$$m_1 \ll m_2 \ll m_3$$

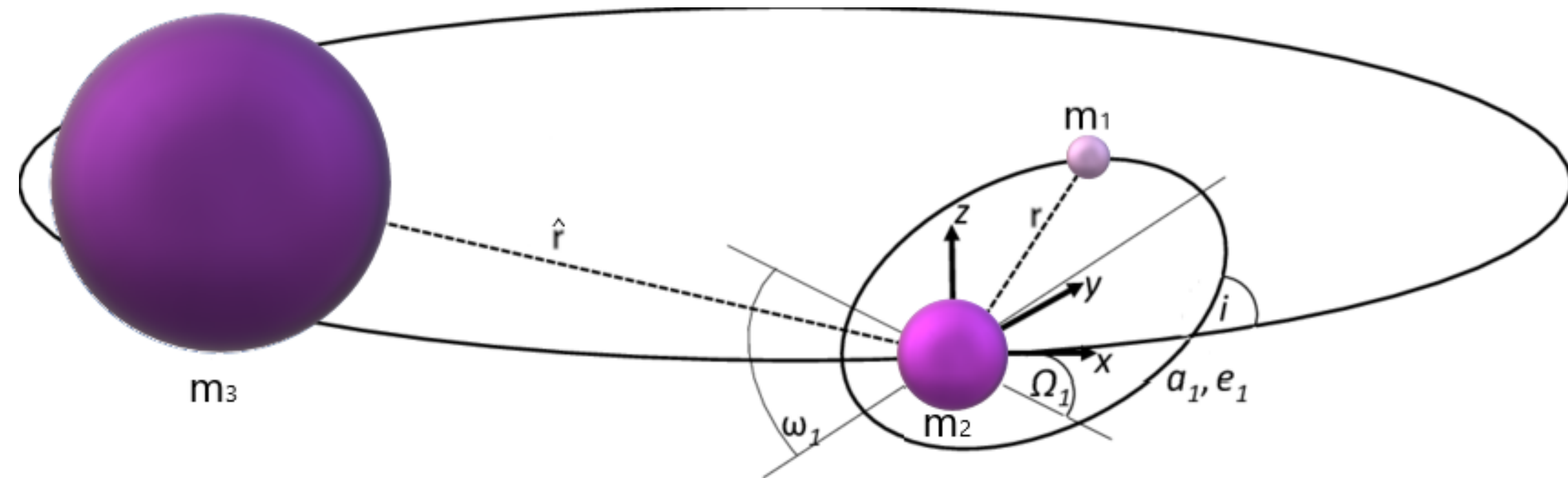
EMR



Small-tide approximation

- The presence of the external black hole is written through **tidal forces**, small corrections to spacetime of the inner binary

- Quadrupole tidal moments  $\sim \mathcal{O} \left( \frac{m_2}{\mathcal{R}} \right)^2$



[B. Deme (2021)]



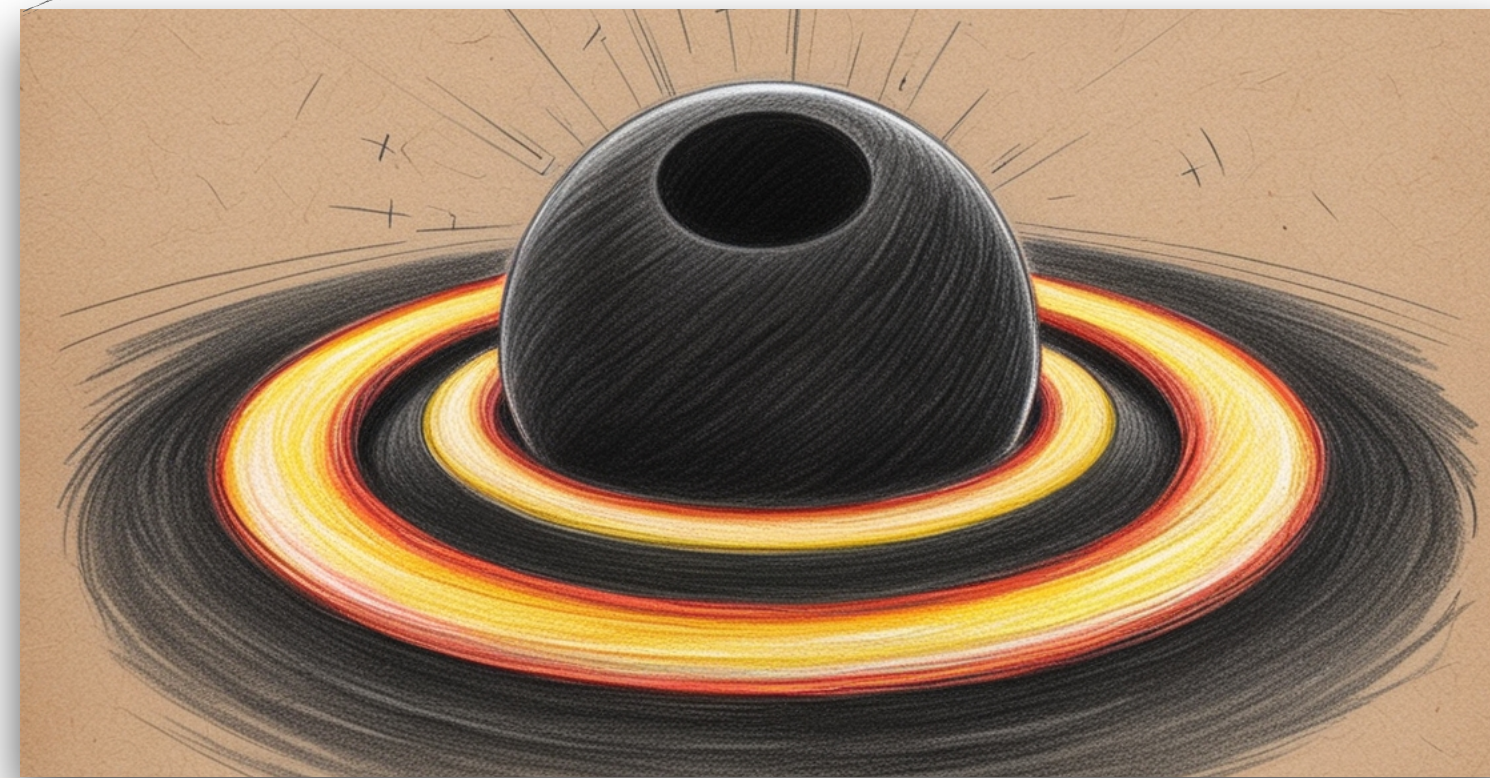
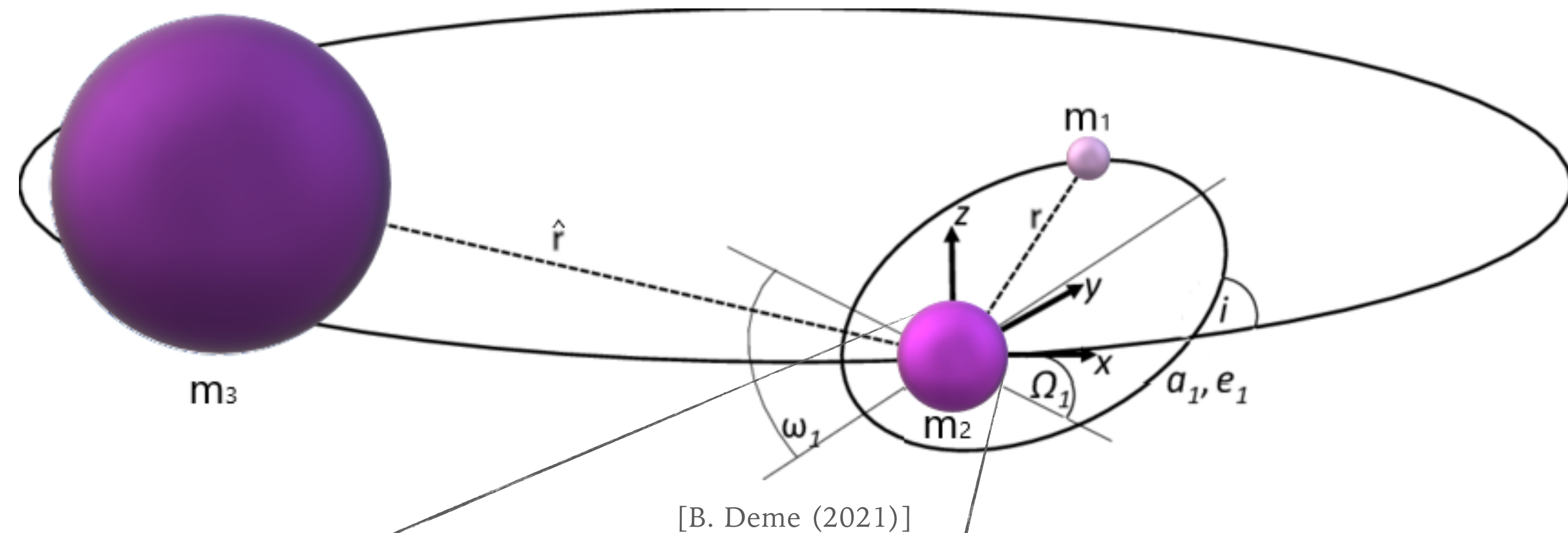
# Tidal effects on an EMR system

- Previous analysis:  $m_2$  was non-rotating black hole

[F. Camilloni, G. Grignani, T. Harmark, R. Oliveri, M. Orselli and D. Pica (2023)]

- Now:  $m_2$  is slowly rotating

- **Coupling terms** between the black hole's spin  $\chi_2$  and the quadrupole tidal moments





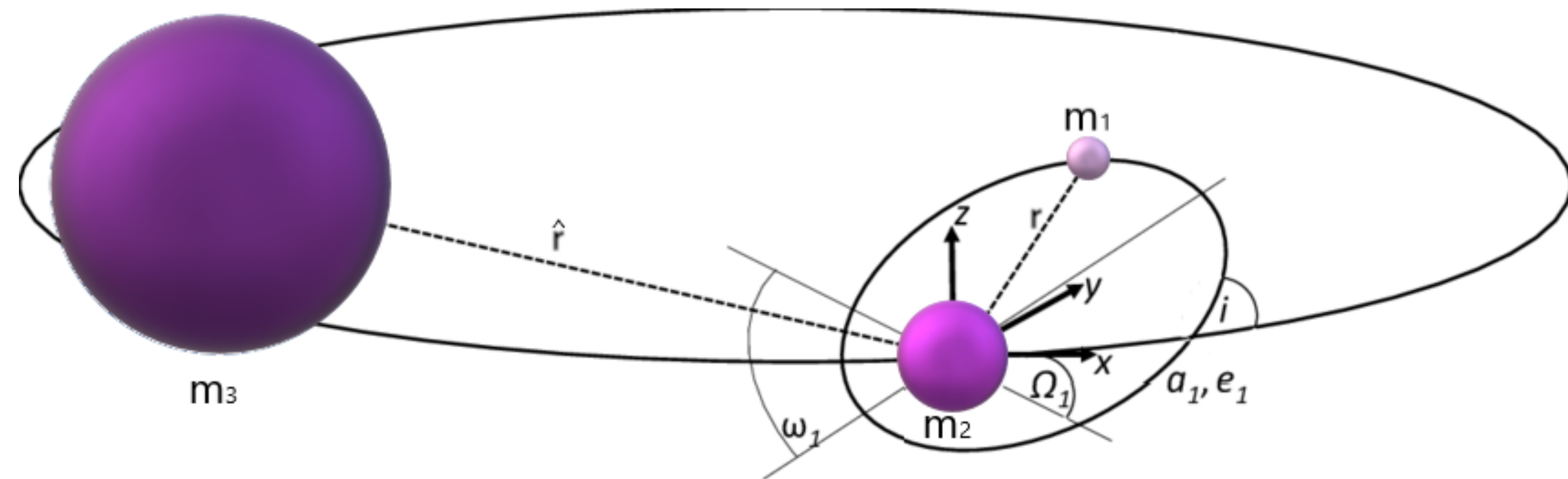
# Tidal effects on an EMR system

The particle  $m_1$  moves in a metric  $g_{\mu\nu}$   
[E. Poisson (2015)] :

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

non-rotating black hole

- black hole's ( $m_2$ ) slow rotation  $\propto \chi_2 \ll 1$
- presence of the external black hole  $m_3 \propto \mathcal{E}_{ab}, \mathcal{B}_{ab}$
- coupling between the spin and tidal moments  $\propto \chi_2 \mathcal{E}_{ab}, \chi_2 \mathcal{B}_{ab}$



[B. Deme (2021)]



# Tidal effects on an EMR system

- We can study the dynamics of the test particle  $m_1$  in the EMR system, in the metric:

$$g_{\mu\nu} = \bar{g}_{\mu\nu}$$

Radius  $r$

Energy  $E$

Angular momentum  $L$


$$\mathcal{A} = \mathcal{A}_0$$

- the **ISCO**, which stands for the Innermost Stable Circular Orbit (massive particles)
- the **photon sphere** (massless particles)



# Tidal effects on an EMR system


- We can study the dynamics of the test particle  $m_1$  in the EMR system, in the metric:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}^{(rot)}$$

Radius  $r$

Energy  $E$

Angular momentum  $L$


$$\mathcal{A} = \mathcal{A}_0 + \chi_2 \mathcal{A}_{rot}$$

- the **ISCO**, which stands for the Innermost Stable Circular Orbit (massive particles)
- the **photon sphere** (massless particles)



# Tidal effects on an EMR system

- We can study the dynamics of the test particle  $m_1$  in the EMR system, in the metric:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}^{(rot)} + h_{\mu\nu}^{(tidal)}$$

Radius  $r$

Energy  $E$

Angular momentum  $L$


$$\mathcal{A} = \mathcal{A}_0 + \chi_2 \mathcal{A}_{rot} + \eta \mathcal{A}_{tidal}$$

- the **ISCO**, which stands for the Innermost Stable Circular Orbit (massive particles)
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# Tidal effects on an EMR system

- We can study the dynamics of the test particle  $m_1$  in the EMR system, in the metric:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}^{(rot)} + h_{\mu\nu}^{(tidal)} + h_{\mu\nu}^{(coupling)}$$

Radius  $r$

Energy  $E$

Angular momentum  $L$


$$\mathcal{A} = \mathcal{A}_0 + \chi_2 \mathcal{A}_{rot} + \eta \mathcal{A}_{tidal} + (\chi_2 \eta + \eta_1) \mathcal{A}_{coupling}$$

- the **ISCO**, which stands for the Innermost Stable Circular Orbit (massive particles)
- the **photon sphere** (massless particles)



# Tidal effects on an EMR system

$$r_{ISCO} = 6m_2 - 4\chi_2\sqrt{\frac{2}{3}}m_2\cos\beta + \eta\left(3072m_2 - \chi_2\frac{27320}{3}\sqrt{\frac{2}{3}}m_2\cos\beta\right) + \eta_1(\chi_2)\frac{16}{3}\sqrt{\frac{2}{3}}m_2$$

If we place the inner binary in an ISCO around a Kerr BH of mass  $m_3$ :

$$\eta = \frac{m_2^2 m_3}{2 (\hat{r}_{ISCO}^\sigma)^3} \left[ 1 - \frac{1}{2}(1 + 4 \sin^2 \gamma) \sin^2 \beta \right]$$

$$\eta_1 = \chi_2 \frac{m_2^2 m_3}{(\hat{r}_{ISCO}^\sigma)^3} \cos \beta$$



# Tidal effects on an EMR system

$$r_{ISCO} = 6m_2 - 4\chi_2\sqrt{\frac{2}{3}}m_2\cos\beta + \eta\left(3072m_2 - \cancel{\chi_2\frac{27320}{3}\sqrt{\frac{2}{3}}m_2\cos\beta}\right) + \eta_1(\chi_2)\cancel{\frac{16}{3}\sqrt{\frac{2}{3}}m_2}$$

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In the non-rotating case,

$$\eta_1 = \cancel{\chi_2 \frac{m_2^2 m_3}{(\hat{r}_{ISCO}^\sigma)^3} \cos \beta}$$



# Tidal effects on an EMR system

$$r_{ISCO} = 6m_2 - 4\chi_2\sqrt{\frac{2}{3}}m_2\cos\beta + \eta\left(3072m_2 - \chi_2\frac{27320}{3}\sqrt{\frac{2}{3}}m_2\cos\beta\right) + \eta_1(\chi_2)\frac{16}{3}\sqrt{\frac{2}{3}}m_2$$

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$$\eta_1 = \chi_2 \frac{m_2^2 m_3}{(\hat{r}_{ISCO}^\sigma)^3} \cos\beta \rightarrow 0$$

In the non-rotating case, there exist orbital configurations in which the tidal effects vanish

# Tidal effects on an EMR system

$$r_{ISCO} = 6m_2 - 4\chi_2\sqrt{\frac{2}{3}}m_2\cos\beta + \eta\left(3072m_2 - \chi_2\frac{27320}{3}\sqrt{\frac{2}{3}}m_2\cos\beta\right) + \eta_1(\chi_2)\frac{16}{3}\sqrt{\frac{2}{3}}m_2 \quad !!!$$

If we place the inner binary in an ISCO around a Kerr BH of mass  $m_3$ :

$$\eta = \frac{m_2^2 m_3}{2(\hat{r}_{ISCO}^\sigma)^3} \left[ 1 - \frac{1}{2}(1 + 4\sin^2\gamma)\sin^2\beta \right] = 0 \rightarrow \sin^2\beta^*(\gamma) = \frac{2}{1 + 4\sin^2\gamma}$$

$$\eta_1 = \chi_2 \frac{m_2^2 m_3}{(\hat{r}_{ISCO}^\sigma)^3} \cos\beta \quad !!!$$

In the rotating case, there exist orbital configurations in which the tidal effects vanish, but we have a residual tidal effect thanks to the spin coupling!





*Does this project have a future?*

*My academic life, now*

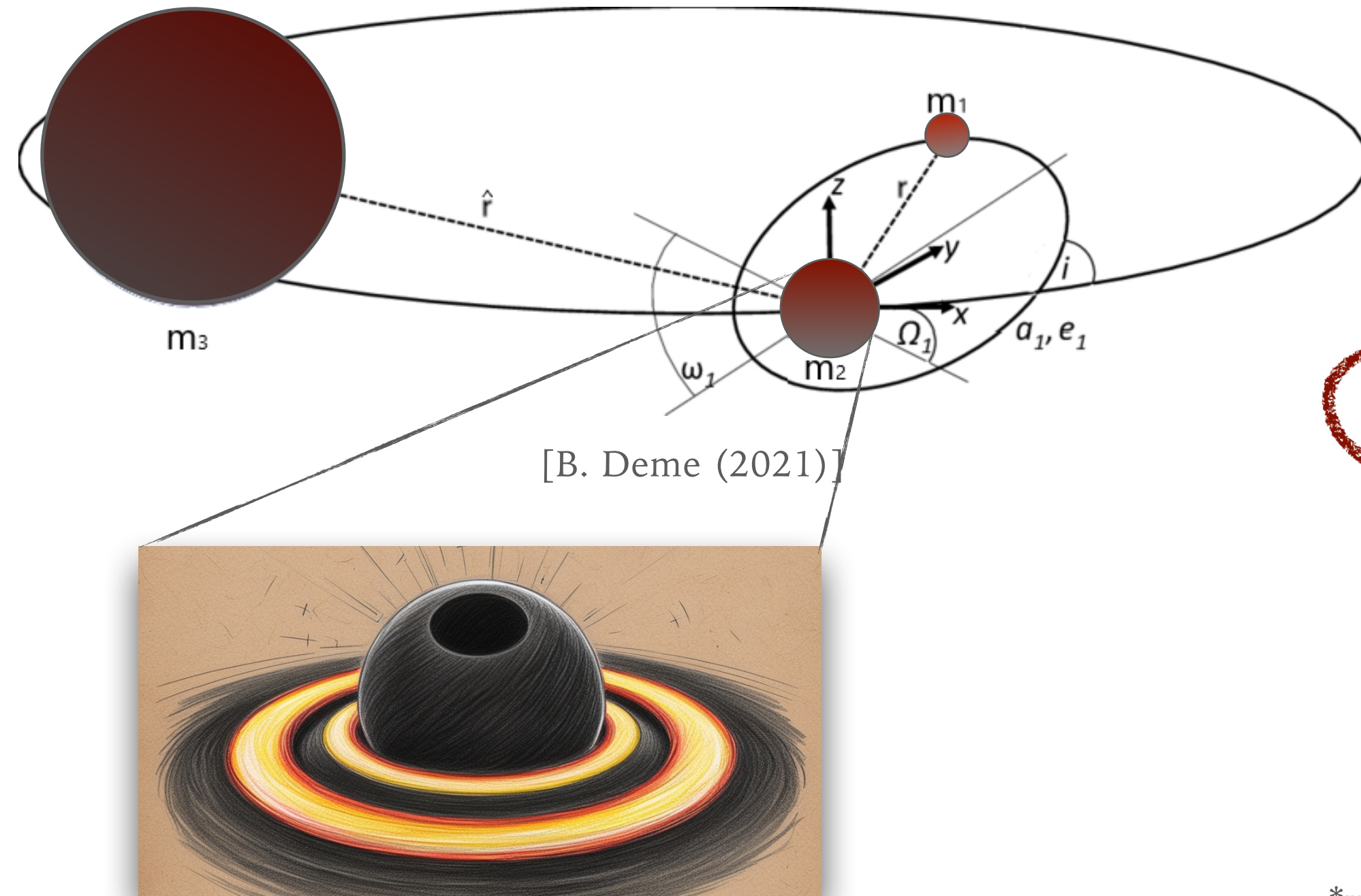


# PhD Program:

**START:**  
**1/11/2023 (UNIPG)**  
**1/12/2023 (NBI)**

**END (planned):**  
**11/2026**

## Tidal deformations in hierarchical 3-body systems



*Strong field spin couplings in hierarchical triples\**

Same scenario of my Master's thesis, but now the perturbed black hole  $m_2$  is ~~slowly~~ **ROTATING!**

Linear perturbations to the Kerr metric:

Teukolsky equation + metric reconstruction techniques

\*work in progress | M. Cocco, G. Grignani, T. Harmark, M. van de Meent, M. Orselli, D. Pereñiguez



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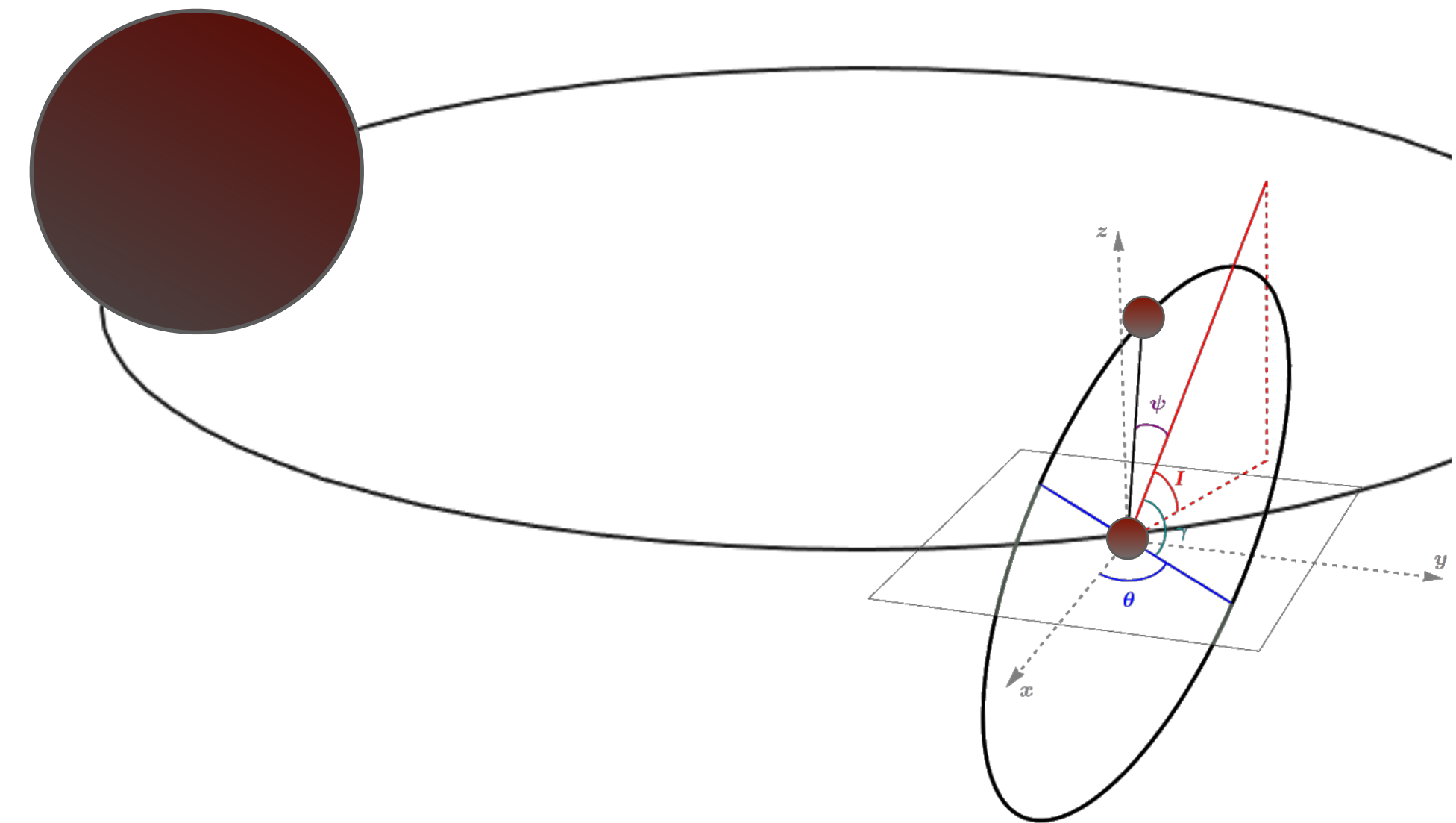
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## Tidal deformations in hierarchical 3-body systems

*Precession resonance in a 3-body system in a Strong Gravity regime\**

A BHB with COMPARABLE masses perturbed by a black hole



\*in preparation | M. Cocco, G. Grignani, T. Harmark, M. Orselli, D. Pica



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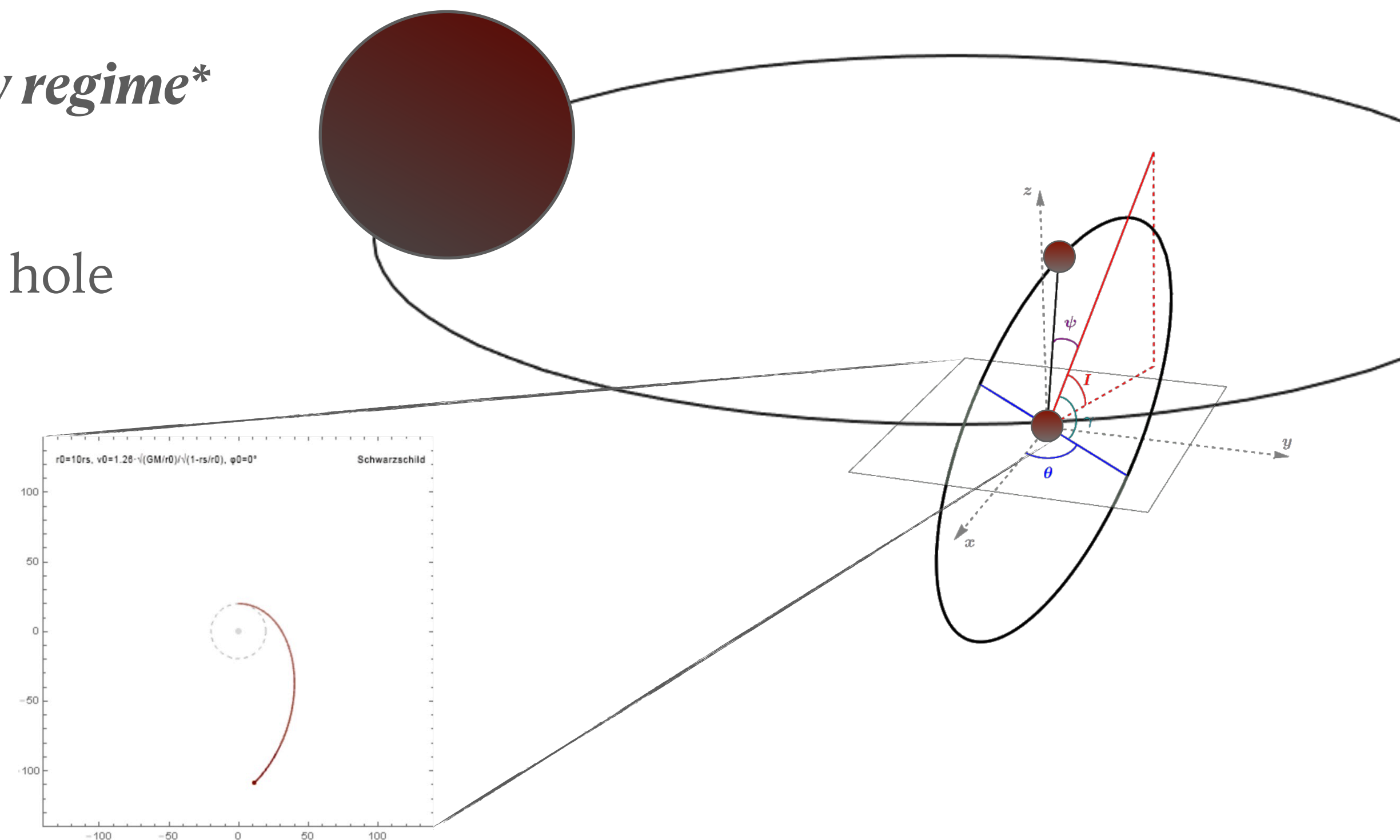
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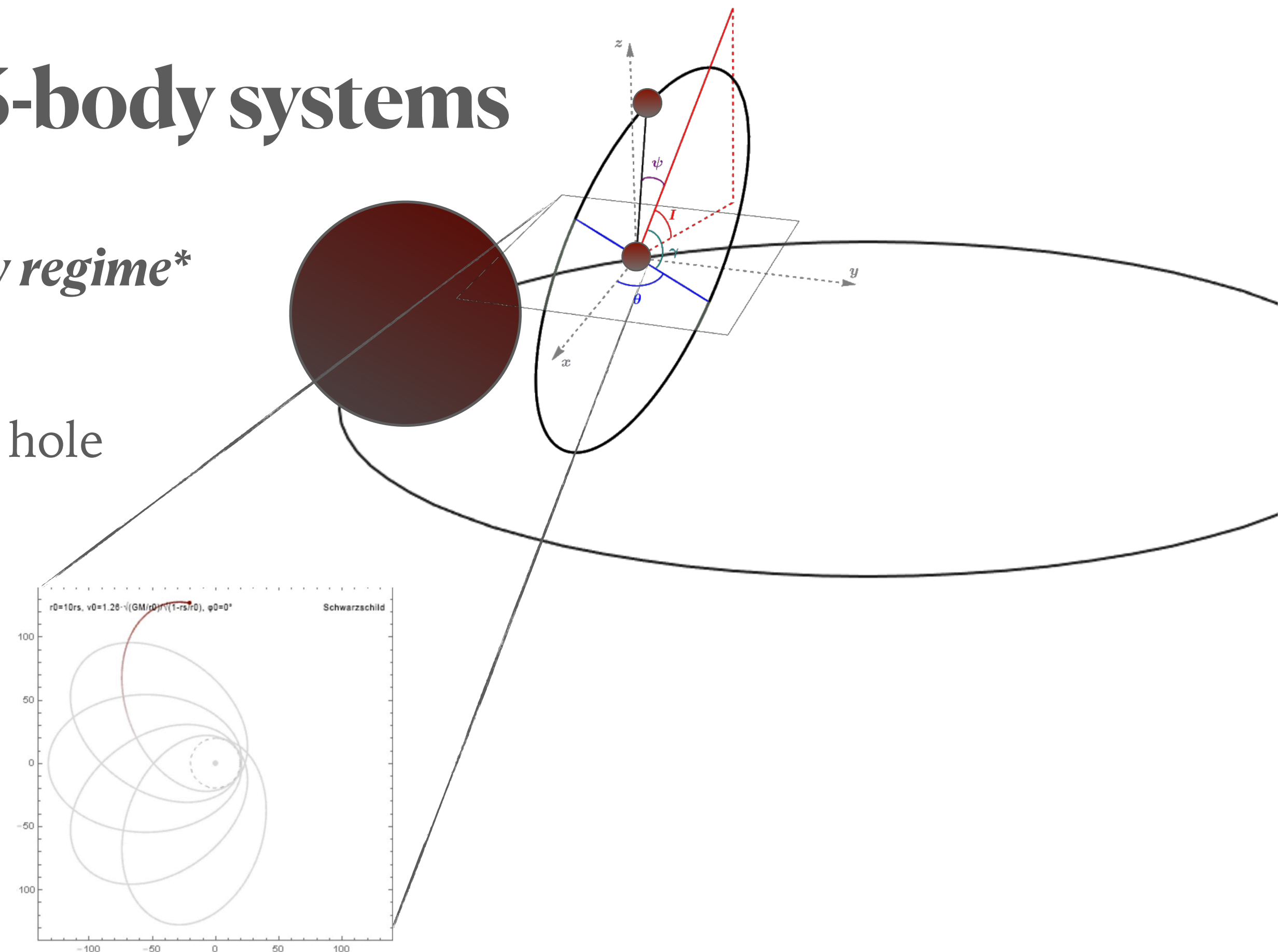
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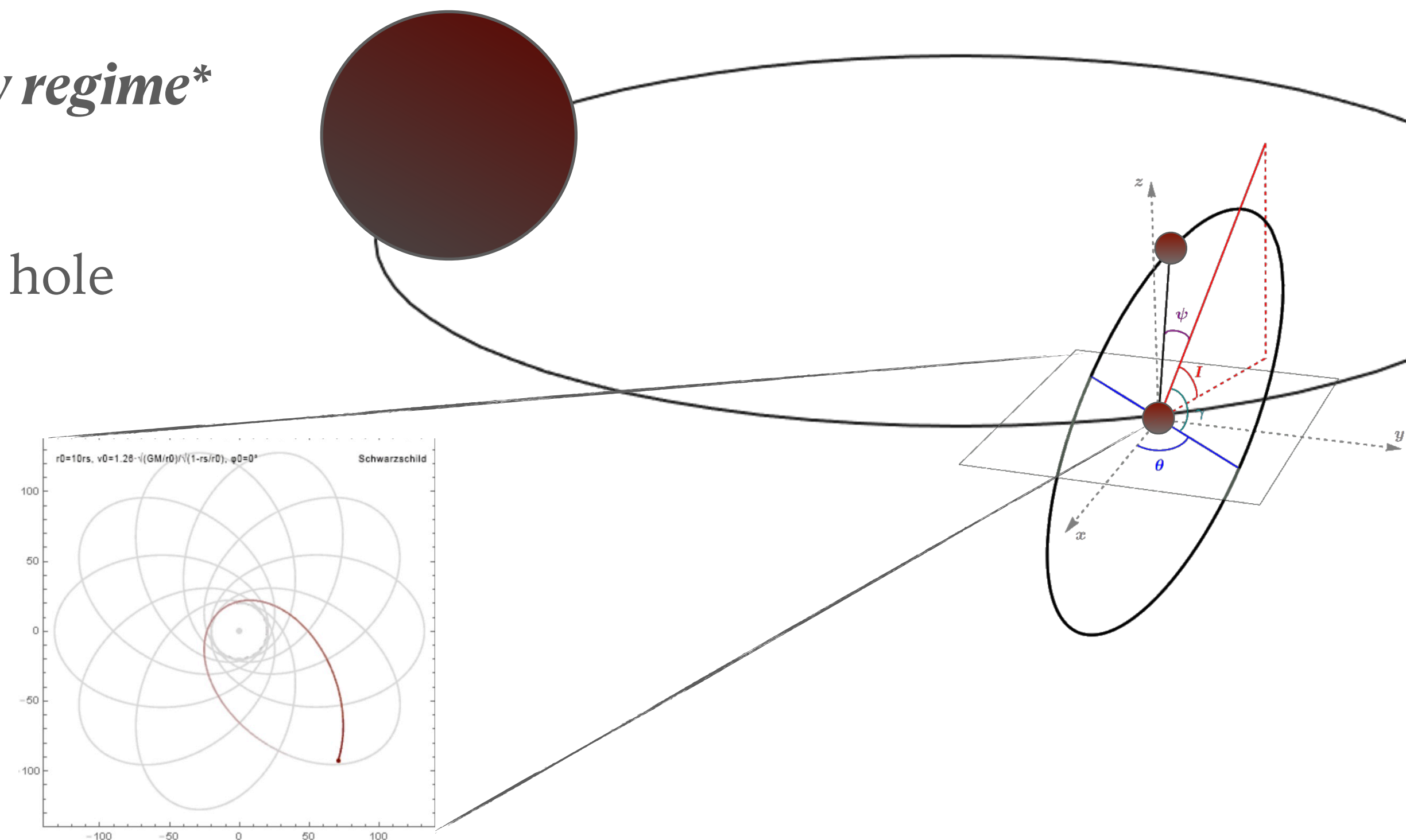
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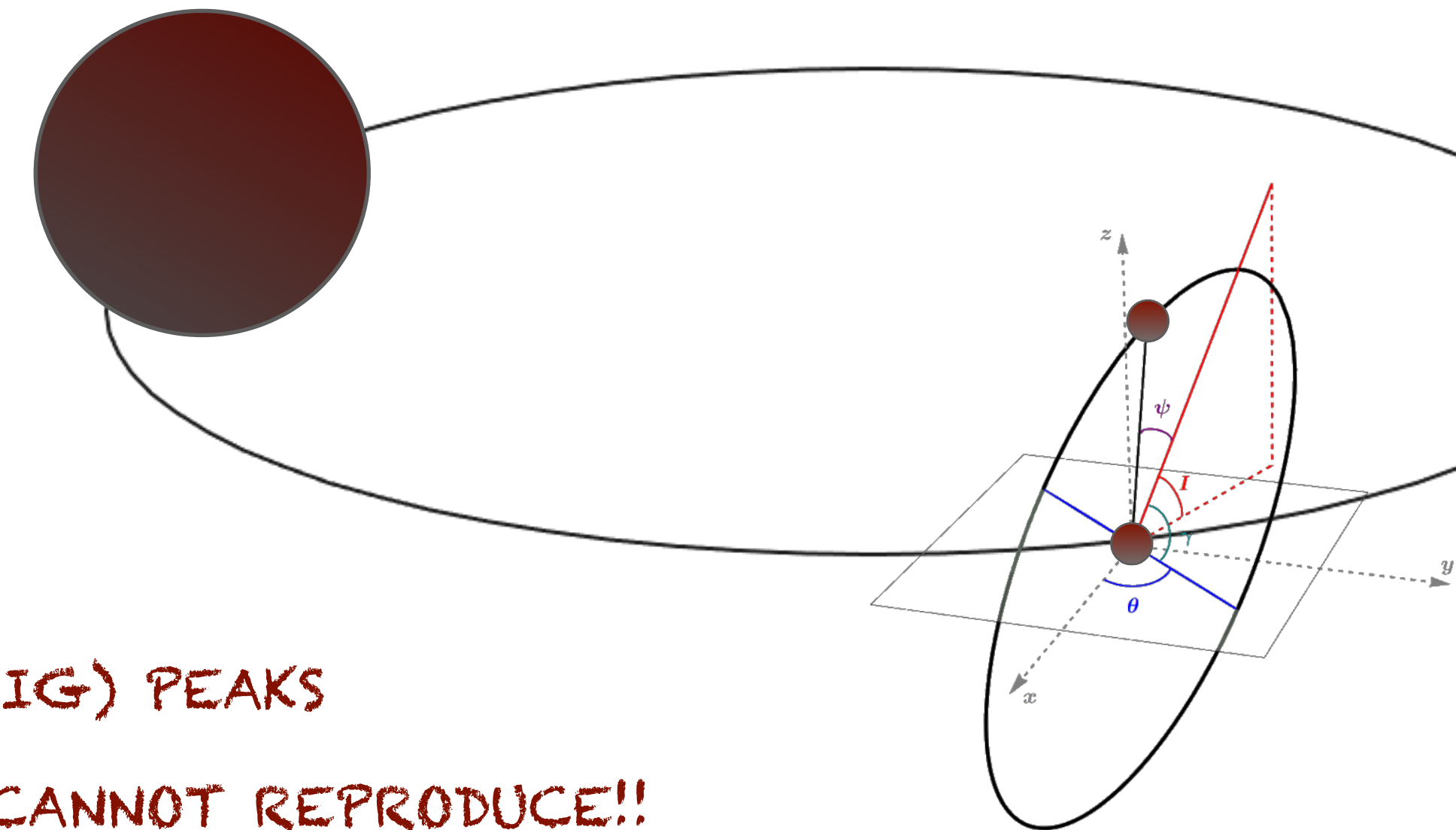
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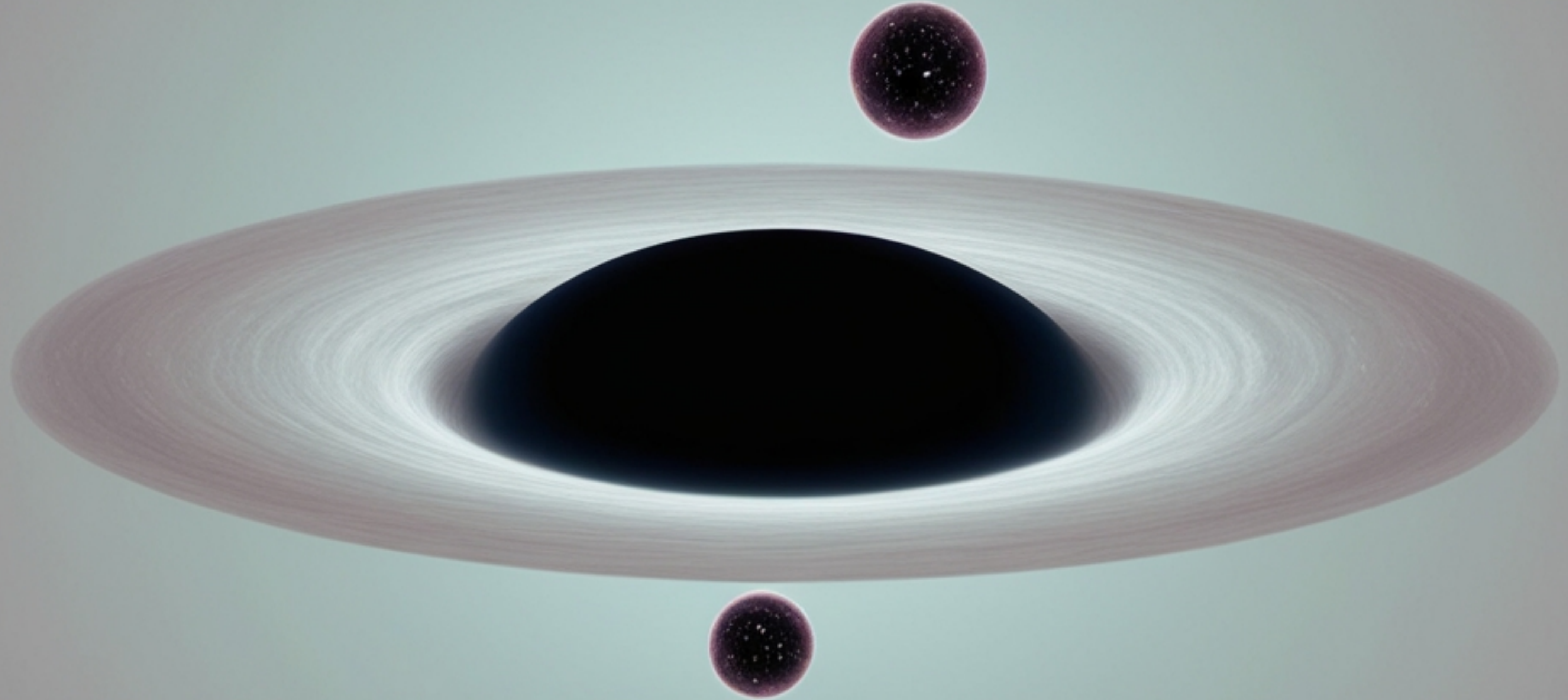
WITH GR WE SEE (BIG) PEAKS

THAT A CLASSICAL APPROACH CANNOT REPRODUCE!!



\*in preparation | M. Cocco, G. Grignani, T. Harmark, M. Orselli, D. Pica





THANK YOU!

