

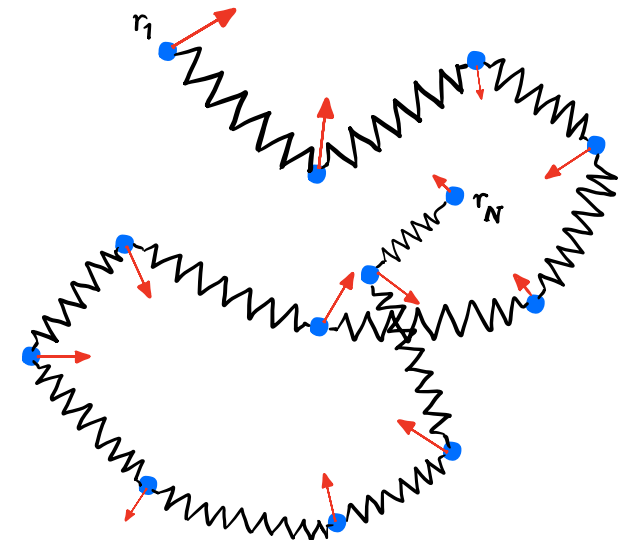
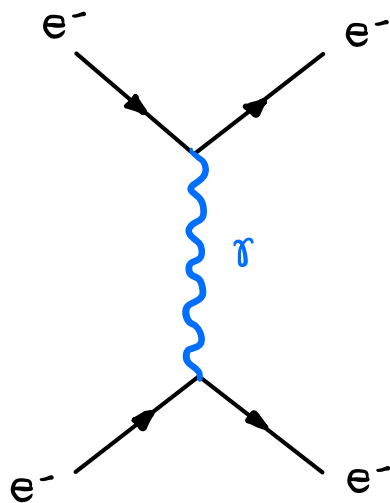
Non-linear Dynamical Models in Complex Systems: Fermionic vs Bosonic

Premio "Milla Baldo Ceolin" 2023

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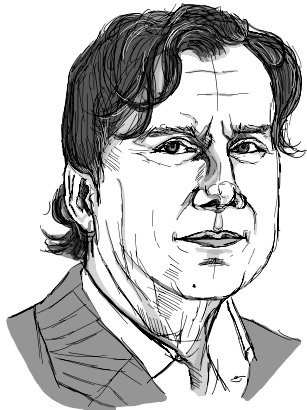
08.10.2024



State of the art

Why **Stochastic PDEs**?

✿ Quantum Field Theory:
Stochastic quantization¹

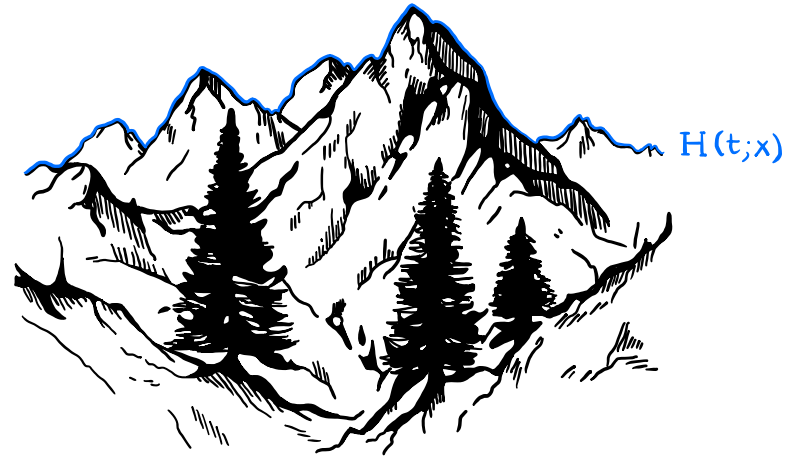


Giorgio Parisi



Yong-Shi Wu

✿ Complex systems:
KPZ equation²



¹G. Parisi and Y. S. Wu, *S.S.* (1981), **24**: 483

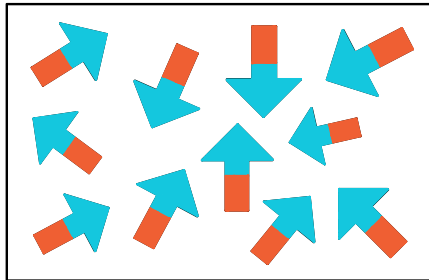
²M. Kardar, G. Parisi and Y.C. Zhang, *P.R.L.*, **56**, 9, 889-892

Aim of MSc thesis

Study of stochastic non-linear Fermionic PDEs of Dirac type

Thirring model

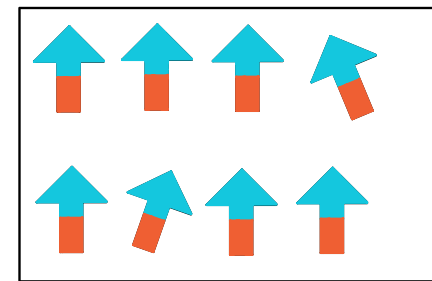
$$\not{D}_{\rho'}^{\rho} \psi^{\rho'} + \lambda g(x) (\bar{\psi} \gamma^{\mu} \psi) (\gamma_{\mu})_{\rho}^{\rho'} \psi^{\rho'} = \xi^{\rho}$$



$T \gg T_c$



Walter Thirring



$T \ll T_c$

Results

- Explicit computation of **expectation values** and **correlation functions**
- Solution expressed in terms of a **perturbative series**.

$$\begin{aligned}\langle \Psi^{\mathcal{A}}[\lambda] \rangle &= \text{diagram 1} + \lambda \text{diagram 2} + \mathcal{O}(\lambda^2) \\ \langle \overline{\Psi}^{\mathcal{A}}[\lambda] \rangle &= \text{diagram 3} + \lambda \text{diagram 4} + \mathcal{O}(\lambda^2)\end{aligned}$$

The diagrams are Feynman-like graphs. Diagram 1 is a single vertical line with a black dot at the top. Diagram 2 is a vertical line with a black dot at the top, branching into two lines: one black and one blue, both ending in dots. Diagram 3 is a single vertical line with a blue dot at the top. Diagram 4 is a vertical line with a blue dot at the top, branching into two lines: one blue and one black, both ending in dots.

PhD project

✱ **Bosonization:** *Thirring* $\Leftrightarrow$ *Sine-Gordon* models¹.
Applications in **QFT** and **condensed matter physics**.

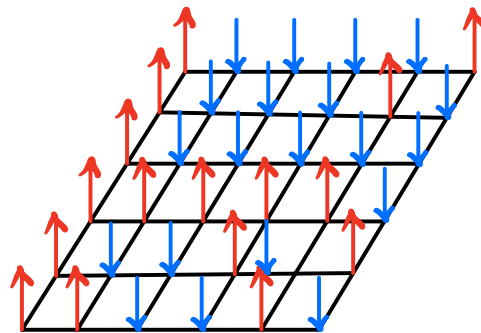
¹S. Coleman, *P.R.D.* **11**, 2088 (1975).

²D. J. Gross and A. Neveu, *P.R.D.* (1974), 10 (10): 3235–3253.

³A. Bonicelli, C. Dappiaggi and P. Rinaldi *CMP*, (2024)           

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- ✱ **Bosonization:** *Thirring* $\Leftrightarrow$ *Sine-Gordon* models¹.
Applications in **QFT** and **condensed matter physics**.
- ✱ Analysis of the ***Gross-Neveu model*** in $2D$ with N Dirac spinors² $\rightarrow$ the **random bond Ising model**



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- ✱ **Convergence** of the perturbative series in the Fermionic scenario³

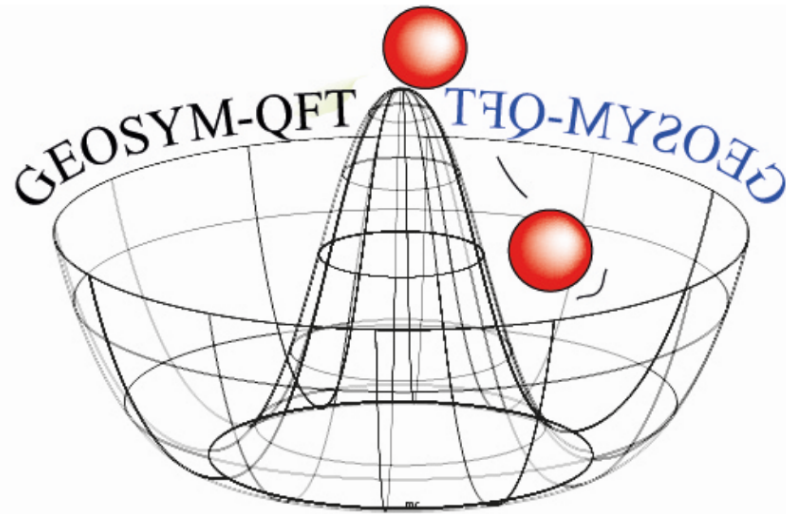
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Connection with INFN activities

* Stochastic quantization $\Rightarrow$ GEOSYM_QFT



* Statistical physics $\Rightarrow$ Low-dimensional systems

* Complex systems

Backup slide: An example of computation performed

Two-point correlation function⁴ up to order $O(\lambda^2)$

$$\begin{aligned}\mathbb{E}(\hat{\psi}[|\lambda|] \otimes \hat{\psi}[|\lambda|])(f_1 \otimes f_2) &= \omega_2(f_1 \otimes f_2) = \\ &= Q(f_1 \otimes f_2) - 3\lambda Q \cdot (1 \otimes (P_\chi * C))(f_1 \otimes f_2 + f_2 \otimes f_1) + O(\lambda^2)\end{aligned}$$

A similar result can be devised in the Fermionic scenario.

Numerical simulations can be used to explicitly calculate the integrals appearing in formulae of this type.

⁴C. Dappiaggi, N. Drago, P. Rinaldi and L. Zambotti, *C.G.M.* **24** (2022)