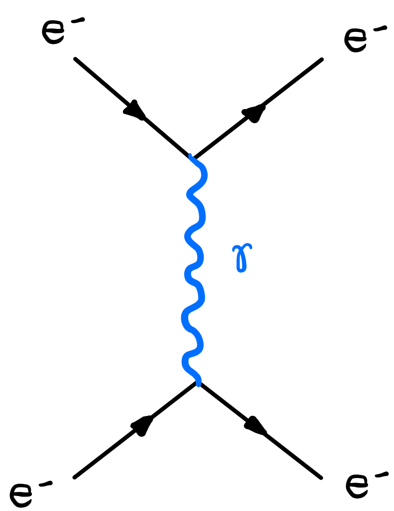
Non-linear Dynamical Models in Complex Systems: Fermionic vs Bosonic

Premio "Milla Baldo Ceolin" 2023



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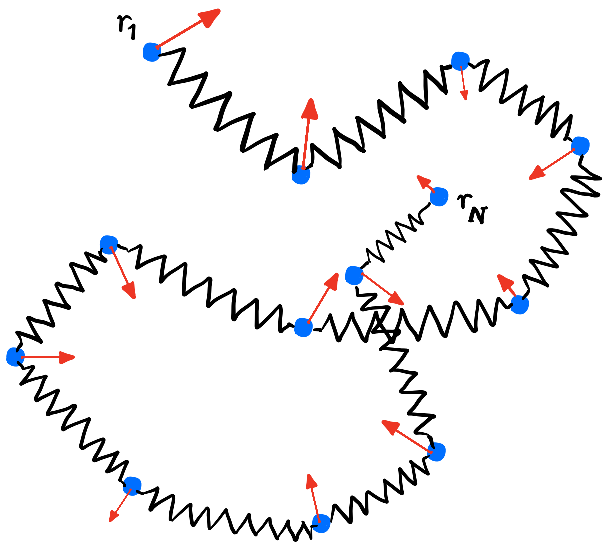
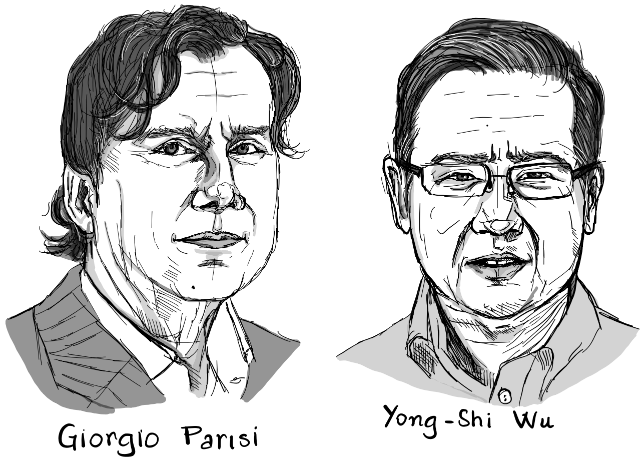
08.10.2024



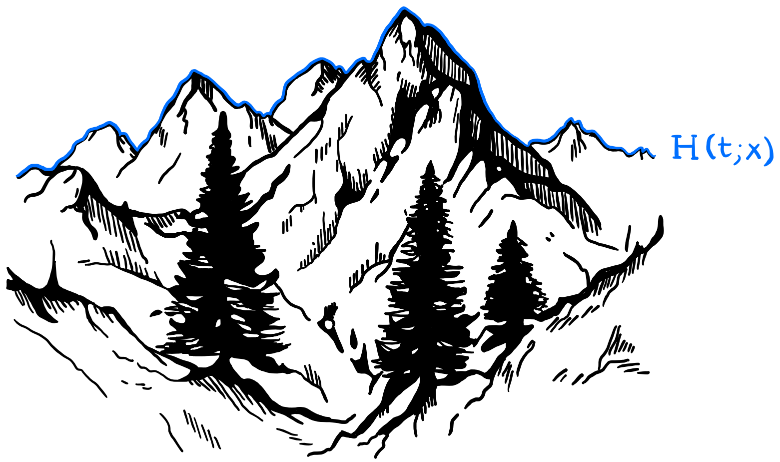
State of the art

Why Stochastic PDEs?

- Quantum Field Theory:
  Stochastic quantization[1]



- Complex systems:
  KPZ equation[2]



[1] G. Parisi and Y. S. Wu, *S.S.* (1981), **24**: 483

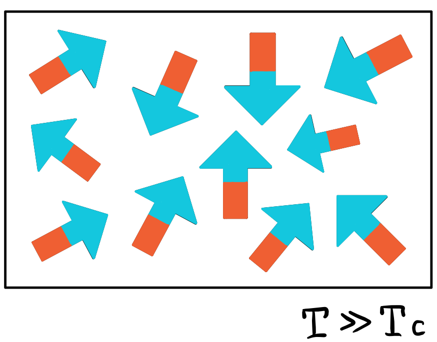
[2] M. Kardar, G. Parisi and Y.C. Zhang, *P.R.L.*, **56**, 9, 889-892

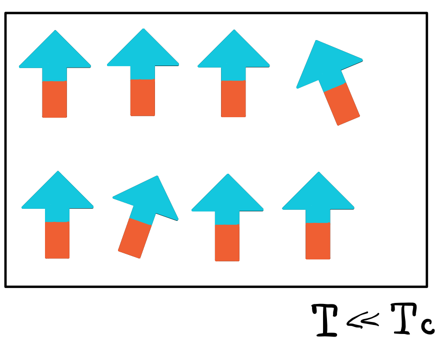
Aim of MSc thesis

Study of stochastic non-linear Fermionic PDEs of Dirac type

Thirring model

$$\not{D}^{\rho}_{\rho'}\psi^{\rho'} + \lambda g(x)(\bar{\psi}\gamma^{\mu}\psi)(\gamma_{\mu})^{\rho'}_{\rho}\psi^{\rho'} = \xi^{\rho}$$

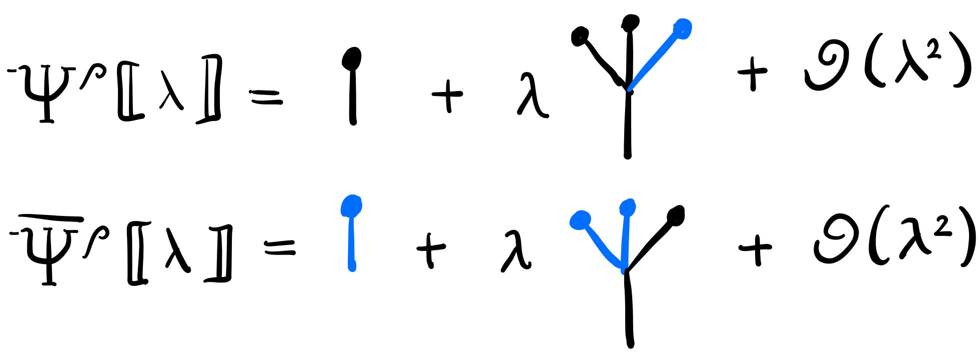




Results

- Explicit computation of **expectation values** and **correlation functions**

- Solution expressed in terms of a perturbative series.



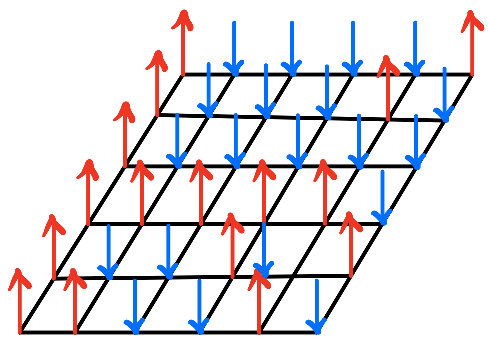
PhD project

- **Bosonization**: *Thirring* $\Leftrightarrow$ *Sine-Gordon* models[1].
  Applications in QFT and condensed matter physics.

[1] S. Coleman, *P.R.D.* **11**, 2088 (1975).
[2] D. J. Gross and A. Neveu, *P.R.D.* (1974), 10 (10): 3235–3253.
[3] A. Bonicelli, C. Dappiaggi and P. Rinaldi *CMP*, (2024)

PhD project

- **Bosonization**: *Thirring* $\Leftrightarrow$ *Sine-Gordon* models[1]. Applications in QFT and condensed matter physics.
- Analysis of the ***Gross-Neveu model*** in $2D$ with N Dirac spinors[2] $\rightarrow$ the random bond *Ising model*



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- **Convergence** of the pertubative series in the Fermionic scenario[3]

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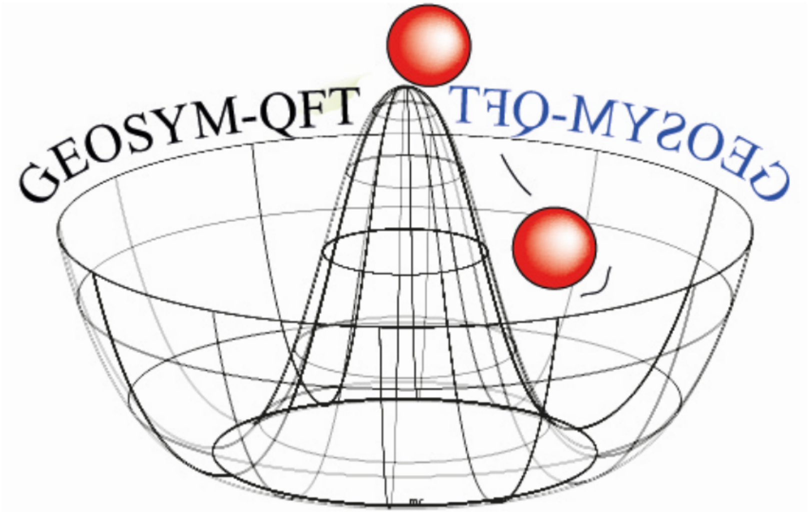
Connection with INFN activities

* Stochastic quantization ⇒ GEOSYM_QFT



* Statistical physics ⇒ Low-dimensional systems
* Complex systems

Two-point correlation function[4] up to order $O(\lambda^2)$

$$\mathbb{E}(\hat{\psi}[|\lambda|] \otimes \hat{\psi}[|\lambda|])(f_1 \otimes f_2) = \omega_2(f_1 \otimes f_2) =$$
$$= Q(f_1 \otimes f_2) - 3\lambda Q \cdot (1 \otimes (P_\chi \circledast C))(f_1 \otimes f_2 + f_2 \otimes f_1) + O(\lambda^2)$$

A similar result can be devised in the Fermionic scenario.

Numerical simulations can be used to explicitly calculate the integrals appearing in formulae of this type.

[4] C. Dappiaggi, N. Drago, P. Rinaldi and L. Zambotti, *C.G.M.* **24** (2022)