

A photograph of the Leaning Tower of Pisa and the adjacent Pisa Cathedral (Duomo di Pisa) at sunset. The sun is low on the horizon, creating a bright lens flare effect that highlights the tower's tilt. The sky is a mix of blue and warm orange and yellow hues. In the foreground, there is a well-maintained green lawn where several people are walking or sitting.

A QI approach to detect QPTs in Central Spin Models

Miriam Patricolo

Supervisor: Professor Ettore Vicari



Università di Pisa

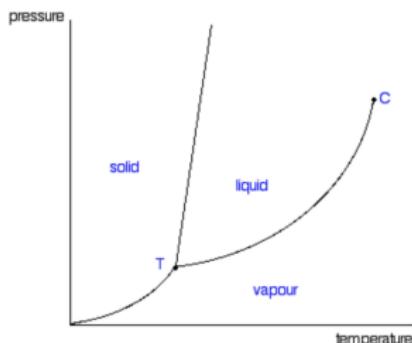
First order

Phase Transitions

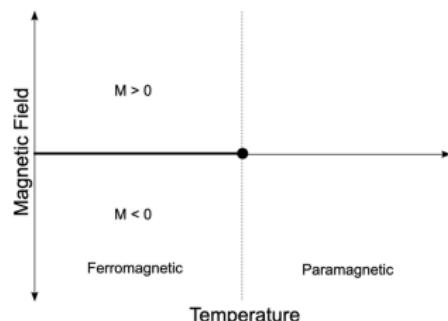
Continuos



Universal critical behaviors shared by
a large class of models



Examples of
CPT



- ★ Power laws ruled by *critical exponents*

	Exponent	Definition	Conditions
Specific heat	α	$C \propto t ^{-\alpha}$	$t \rightarrow 0, h = 0$
Order parameter	β	$m \propto (-t)^\beta$	$t \rightarrow 0^-, h = 0$
Susceptibility	γ	$\chi \propto t^{-\gamma} $	$t \rightarrow 0, h = 0$
Critical isotherm	δ	$B \propto m ^\delta sign(m)$	$h \rightarrow 0, t = 0$
Correlation length	ν	$\xi \propto t ^{-\nu}$	$t \rightarrow 0, h = 0$
Correlation function	η	$G(r) \propto r ^{-d+2-\eta}$	$t = 0, h = 0$

- ★ Spontaneous Symmetry Breaking

- ★ Existence of an order parameter

- ★ Universality

* Power laws ruled by

critical exponents

Renormalization group theory

* Spontaneous Symmetry Breaking

RG flow in Hamiltonian space

Critical behavior \longleftrightarrow fixed points

* Existence of an order parameter

Derivation of critical exponents

* Universality

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Critical isotherm	δ	$B \propto m ^\delta \text{sign}(m)$	$h \rightarrow 0, t = 0$
Correlation length	ν	$\xi \propto t ^{-\nu}$	$t \rightarrow 0, h = 0$
Friction	η	$G(r) \propto r ^{-d+2-\eta}$	$t = 0, h = 0$

You're welcome!





d -dimensional quantum theories



$d + 1$ -dimensional classical theories



d-dimensional quantum theories

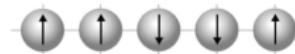


d + 1-dimensional classical theories



Paradigmatic model: **The quantum Ising chain**

$$H_{IS} = - \sum_j \sigma_j^x \sigma_{j+1}^x - g \sum_j \sigma_j^z - h \sum_j \sigma_j^x$$



The critical point $g_c = 1$ divides the ordered from the disordered phase

Continuous QT at $g = g_c$

$$\xi \sim |g - g_c|^{-\nu}; \Delta \sim \xi^{-z}, z = 1$$

FOQTs line driven by h , when $g < g_c$

$$\lim_{h \rightarrow 0^\pm} M = \pm m_0$$

- We apply methods from the RG theory to a new theoretical laboratory (the Central Spin Model)
- We introduce tools to characterize QTs



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Spotlight singularities at QPTs

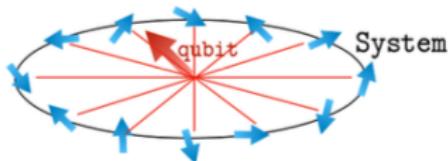
Quantum information
theory tools

- Hilbert basis: $\{|0\rangle, |1\rangle\} \rightarrow a|0\rangle + b|1\rangle, \quad |a|^2 + |b|^2 = 1$
- Performing a measure inevitably disturbs the state
- Composite systems: $|\Psi\rangle_A \otimes |\Psi\rangle_B \in \mathcal{H}_A \otimes \mathcal{H}_B,$

$$\begin{cases} \{|0\rangle_A, |1\rangle_A\} \\ \{|0\rangle_B, |1\rangle_B\} \end{cases} \Rightarrow |\Psi\rangle_{AB} = a(|0\rangle_A \otimes |0\rangle_B) + b(|1\rangle_A \otimes |1\rangle_B)$$

- $|\Psi\rangle_{AB} = \sum_{i,\mu} a_{i\mu} |i\rangle_A \otimes |\mu\rangle_B; \quad \rho_A = Tr_B [|\Psi\rangle\langle\Psi|]$

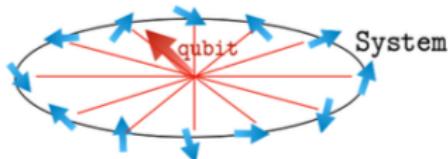
Central Spin Model



Qubit system $Q \rightarrow H_q = -s\Sigma^z$

Many-body system $S \rightarrow H_S = H_{ls} = -\sum_j \sigma_j^x \sigma_{j+1}^x - g \sum_j \sigma_j^z - h \sum_j \sigma_j^x$

Central Spin Model



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- How to formulate scaling ansatz in the RG framework?
- Can we use the **qubit alone** to achieve information about the many-body system behavior?

Interaction term	Qubit term	
$H_{qS,1} = -w\Sigma^z \sum_j \sigma_j^z$	$H_q = -s\Sigma^z$	Commutative
$H_{qS,2} = -v\Sigma^x \sum_j \sigma_j^x$	$H_q = -s\Sigma^z$	Non-commutative
$H_{qS,3} = -u\Sigma^x \sum_j \sigma_j^x$	$H_q = -s\Sigma^z$	Commutative

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$$H_2 = - \sum_j \sigma_j^x \sigma_{j+1}^x - g \sum_j \sigma_j^z - s \Sigma^z - v \Sigma^x \sum_j \sigma_j^x$$

FSS AT CQT OF THE ISING RING

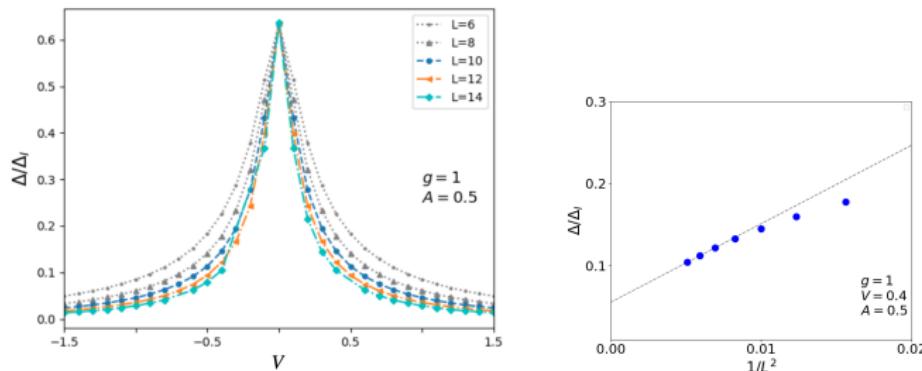
$$W = (g - g_c)L^{y_r}, \quad y_r = 1$$

$$A = \frac{s}{L^{-z}} = sL^z, \quad z = 1$$

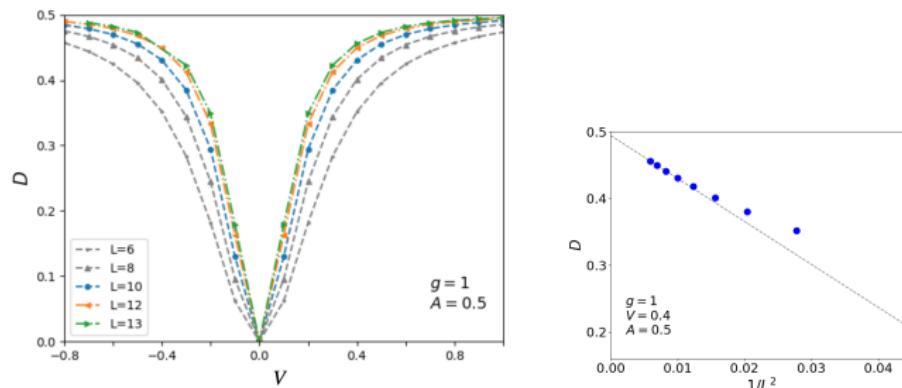
$$V = \frac{vk_h}{h} = \frac{vL^{y_h} h}{h} = vL^{15/8} \Rightarrow y_v = 15/8.$$

FSS ansatz for the energy gap:

$$\Delta(L, g, s, v) \approx L^{-z} \varepsilon(W, A, V) \Rightarrow \frac{\Delta(L, g=1, s, v)}{\Delta_L} \approx \varepsilon_c(A, V)$$



While the decoherence is expected to obey: $D(L, g, s, v) \approx D(W, A, k_v)$.



Phase diagram for finite s and v

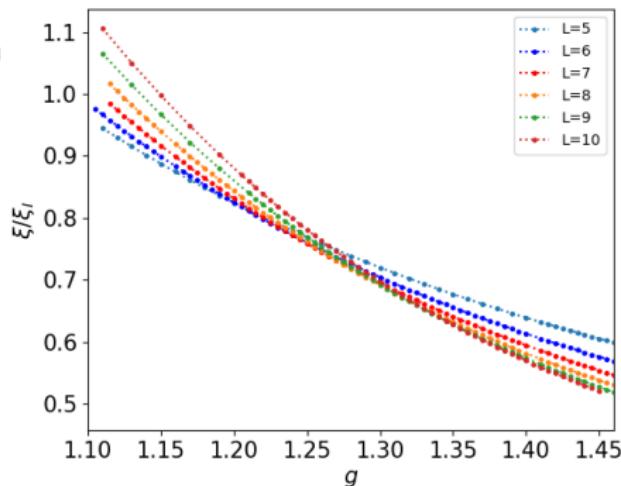
A global \mathbf{Z}_2 symmetry is preserved → Is still present an Ising-like transition?

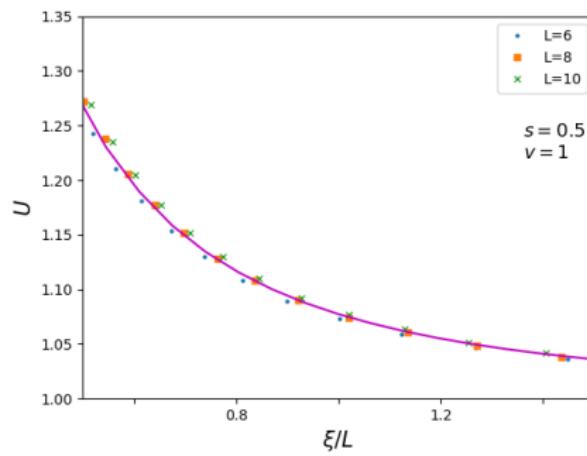
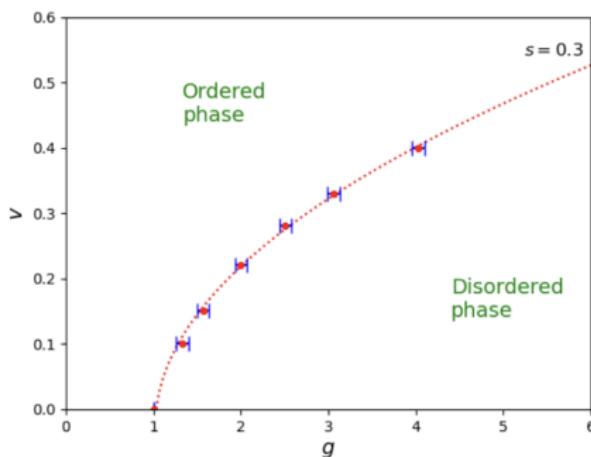
$$\mathbf{Z}_2 : P = \prod_j \sigma_j^z \quad \tilde{P} = \Sigma^z \quad [H, P\tilde{P}] = 0$$

- * The **crossing method** gives us an estimation of the shifted critical point



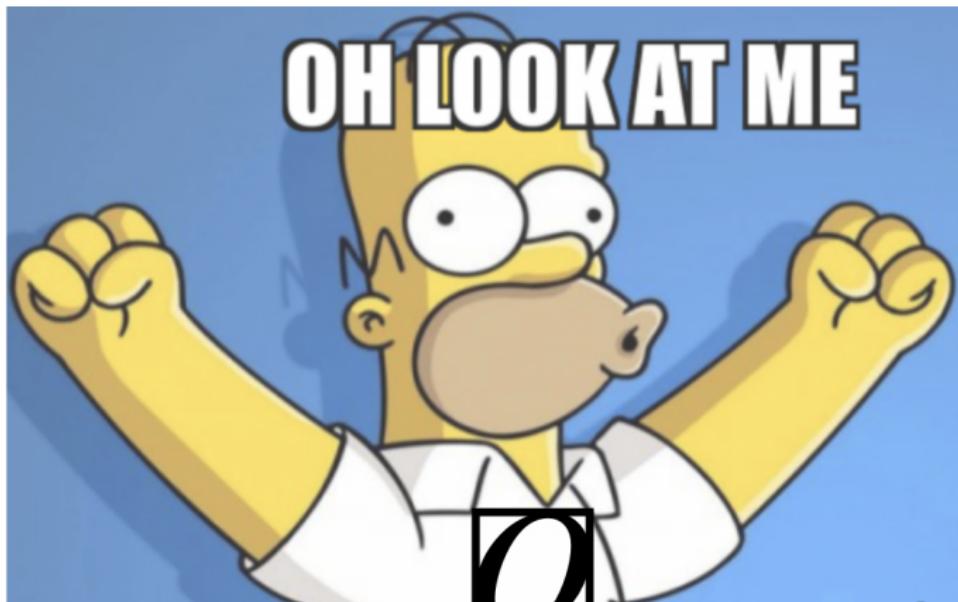
We are able to build
the rising phase diagram!



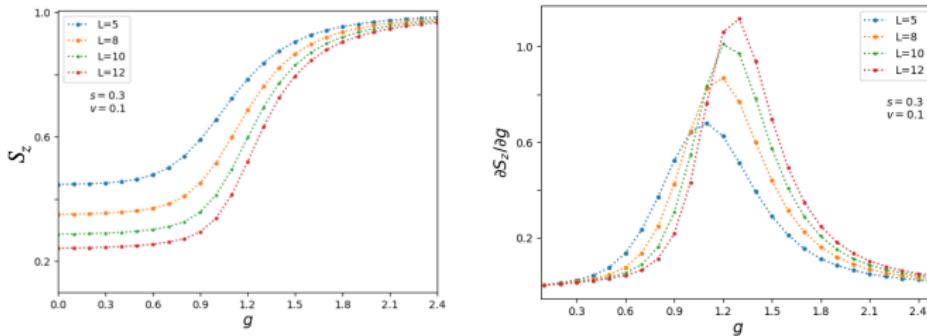


Ising universality class

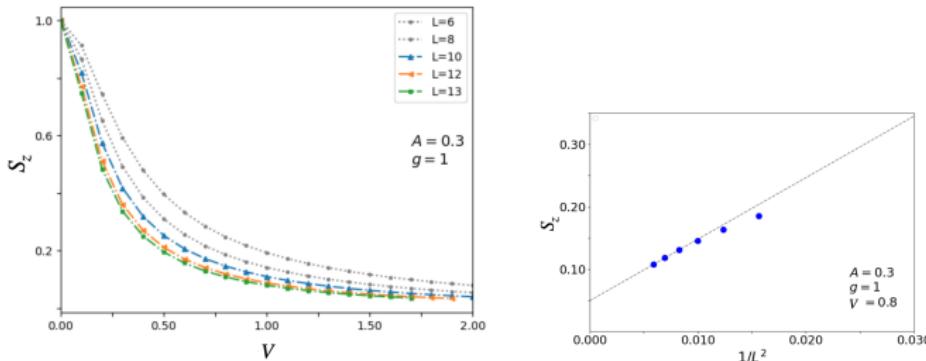
The qubit perspective



$$S_x \equiv \langle \Sigma^x \rangle, \quad S_y \equiv \langle \Sigma^y \rangle, \quad S_z \equiv \langle \Sigma^z \rangle$$



We formulate appropriate ansatz to be verified: $S_z(L, g, s, v) \approx S_z(W, A, V)$





Welcome to my current world

Phase diagram of the central spin model:

$$\begin{array}{c} \text{300} \\ \text{r}_N \\ \text{8} \\ \text{0} \\ \text{0} \\ \text{r}_{\text{c}} \\ \text{Fermi liquid} \\ \text{d-SG} \\ \text{Charge order} \\ \text{Spin order} \\ \text{T}_{\text{SDW}} \\ \text{T}_{\text{AF}} \\ \text{Pseudogap} \\ \text{T}_{\text{C}, \text{onset}} \\ \text{T}_{\text{SC}, \text{onset}} \\ \text{T}_{\text{CDW}} \\ \text{T}_{\text{c}} \\ \rho_{\text{min}} \\ \rho_{\text{c1}} \\ 0.1 \\ \rho_{\text{c2}} \\ 0.2 \\ \rho_{\text{c3}} \\ \rho_{\text{max}} \end{array}$$

Self-energy:

Functional RG (fRG):

$$S = - \int_0^\beta d\tau d\tau' \sum_{\mathbf{k}} \bar{c}_{\sigma \mathbf{k}}(\tau) \mathcal{G}^{-1}(\mathbf{k}, \tau - \tau') c_{\sigma \mathbf{k}}(\tau') + S_{\text{int}}$$

introduce cutoff in Gaussian part of the action $\mathcal{G} \rightarrow \mathcal{G}^\Lambda$

treat degrees of freedom in a scale ordered way