A QI approach to detect QPTs in Central Spin Models

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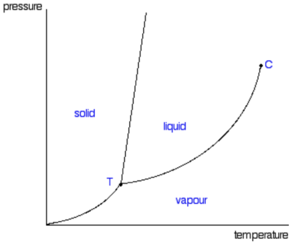
First order

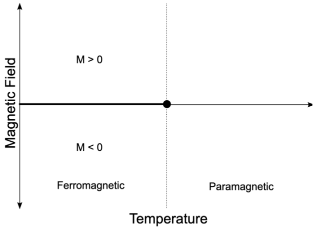
Phase Transitions

Continuos

Universal critical behaviors shared by a large class of models



Examples of CPT



	Exponent	Definition	Conditions
Specific heat	α	$C \propto \|t\|^{-\alpha}$	$t \to 0,\ h=0$
Order parameter	β	$m \propto (-t)^{\beta}$	$t \to 0^-,\ h=0$
Susceptibility	γ	$\chi \propto \|t^{-\gamma}\|$	$t \to 0,\ h=0$
Critical isotherm	δ	$B \propto \|m\|^{\delta} sign(m)$	$h \to 0,\ t=0$
Correlation length	ν	$\xi \propto \|t\|^{-\nu}$	$t \to 0,\ h=0$
Correlation function	η	$G(r) \propto \|r\|^{-d+2-\eta}$	$t=0, h=0$

- ⋆ Power laws ruled by *critical exponents*
- ⋆ Spontaneous Symmetry Breaking
- ⋆ Existence of an order parameter
- ⋆ Universality

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⋆ Power laws ruled by critical exponents

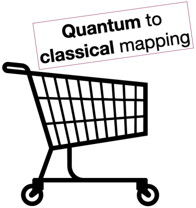
Renormalization group thory

⋆Spontaneous Symmetry Breaking

- ☑ RG flow in Hamiltonian space
- ☑ Critical behavior ⟷ fixed points
- ☑ Derivation of critical exponents

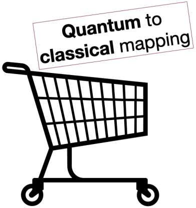
⋆Existence of an order parameter

⋆Universality



d-dimensional quantum theories

$d+1$-dimensional classical theories

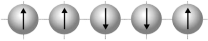


d-dimensional quantum theories

$d+1$-dimensional classical theories

Paradigmatic model: **The quantum Ising chain**

$$H_{IS} = -\sum_j \sigma_j^x \sigma_{j+1}^x - g\sum_j \sigma_j^z - h\sum_j \sigma_j^x$$



The critical point $gc = 1$ divides the ordered from the disordere phase

Continous QT at $g = g_c$

$$\xi \sim |g - g_c|^{-\nu}; \Delta \sim \xi^{-z}, z = 1$$

FOQTs line driven by h, when $g < g_c$

$$\lim_{h\to 0^{\pm}} M = \pm m_0$$

- We apply methods from the RG theory to a new theoretical laboratory (the Central Spin Model)
- We introduce tools to characterize QTs

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Spotlight singularities at QPTs

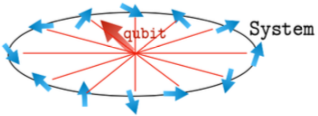
Quantum information theory tools

- Hilbert basis: $\{|0\rangle, |1\rangle\} \rightarrow a|0\rangle + b|1\rangle, \quad |a|^2 + |b|^2 = 1$
- Performing a measure inevitably disturbs the state
- Composite systems: $|\Psi\rangle_A \otimes |\Psi\rangle_B \in \mathcal{H}_A \otimes \mathcal{H}_B$,

$$\begin{cases} \{|0\rangle_A, |1\rangle_A\} \\ \{|0\rangle_B, |1\rangle_B\} \end{cases} \Rightarrow |\Psi\rangle_{AB} = a(|0\rangle_A \otimes |0\rangle_B) + b(|1\rangle_A \otimes |1\rangle_B)$$

- $|\Psi\rangle_{AB} = \sum_{i,\mu} a_{i\mu} |i\rangle_A \otimes |\mu\rangle_B; \quad \rho_A = Tr_B\left[|\Psi\rangle\langle\Psi|\right]$

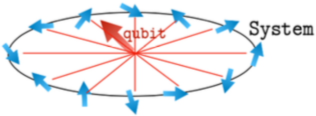
Central Spin Model



Qubit system Q $\rightarrow H_q = -s\Sigma^z$

Many-body system $S \rightarrow H_S = H_{Is} = -\sum_j \sigma_j^x \sigma_{j+1}^x - g\sum_j \sigma_j^z - h\sum_j \sigma_j^x$

Central Spin Model



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- How to formulate scaling ansatz in the RG framework?
- Can we use the **qubit alone** to achieve information about the many-body system behavior?

Interaction term	Qubit term	
$H_{qS,1} = -w\Sigma^z \sum_j \sigma_j^z$	$H_q = -s\Sigma^z$	Commutative
$H_{qS,2} = -v\Sigma^x \sum_j \sigma_j^x$	$H_q = -s\Sigma^z$	Non-commutative
$H_{qS,3} = -u\Sigma^x \sum_j \sigma_j^x$	$H_q = -s\Sigma^z$	Commutative

- ◇ The qubit decouples from the Ising ring
- ◇ The qubit scale energy s will be treated as a relevant perturbation

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$$H_2 = -\sum_j \sigma_j^x \sigma_{j+1}^x - g \sum_j \sigma_j^z - s\Sigma^z - v\Sigma^x \sum_j \sigma_j^x$$

FSS at CQT of the Ising ring

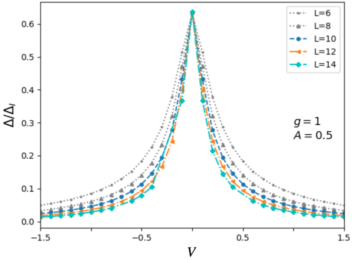
$$W = (g - g_c)L^{y_r}, \qquad y_r = 1$$

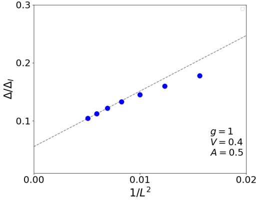
$$A = \frac{s}{L^{-z}} = sL^z, \qquad z = 1$$

$$V = \frac{vk_h}{h} = \frac{vL^{y_h}h}{h} = vL^{15/8} \Rightarrow y_v = 15/8.$$

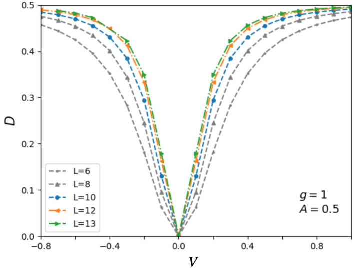
FSS ansatz for the energy gap:

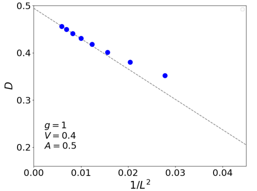
$$\Delta(L, g, s, v) \approx L^{-z}\varepsilon(W, A, V) \Rightarrow \frac{\Delta(L,g=1,s,v)}{\Delta_I} \approx \varepsilon_c(A, V)$$





While the decoherence is expected to obey: $D(L, g, s, v) \approx \mathcal{D}(W, A, k_v)$.





Phase diagram for finite s and v

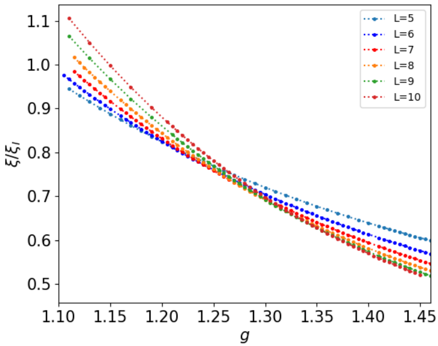
A global $\mathbf{Z}_2$ symmetry is preserved$\rightarrow$ Is still present an Ising-like transition?

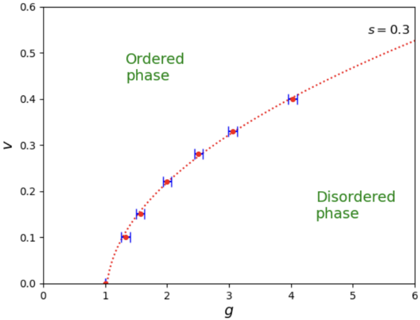
$$\mathbf{Z}_2 : P = \prod_j \sigma_j^z \qquad \tilde{P} = \Sigma^z \qquad \left[H, P\tilde{P}\right] = 0$$

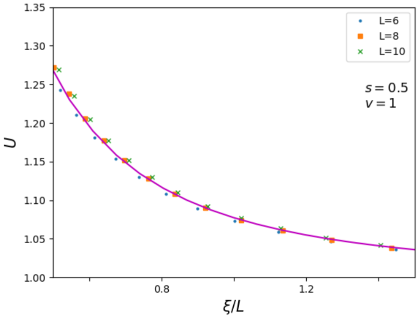
⋆ The **crossing method** gives us an estimation of the shifted critical point

⇓

We are able to build the rising phase diagram!

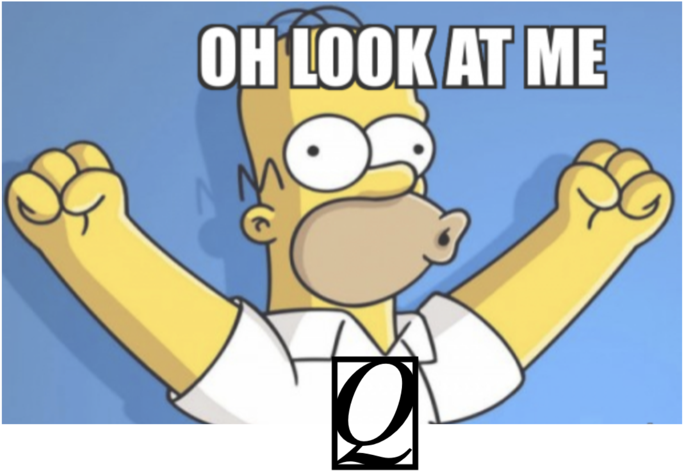




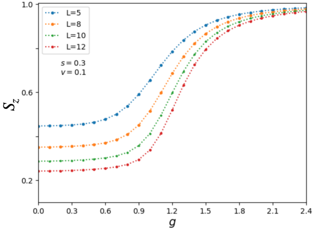


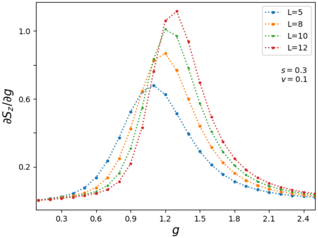
Ising universality class

The qubit perspective

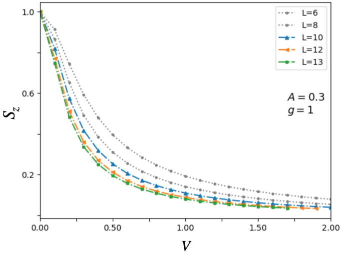


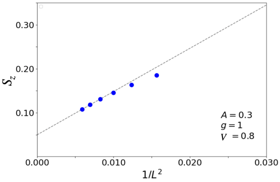
$$S_x \equiv \langle \Sigma^x \rangle, \quad S_y \equiv \langle \Sigma^y \rangle, \quad S_z \equiv \langle \Sigma^z \rangle$$





We formulate appropriate ansatz to be verified: $S_z(L, g, s, v) \approx \mathcal{S}_z(W, A, V)$





Welcome to my current world