Alison Warman

# Superconformal anomalies from superconformal Chern-Simons polynomials

Based on [C.Imbimbo, D.Rovere, A.Warman, *JHEP* 05 (2024) 277]

**Women in Theoretical Physics**
**Premio Nazionale "Milla Baldo Ceolin" 2023**

Oct 8, 2024

## Symmetries and anomalies

- An ordinary continuous global symmetry of an action implies, at the classical level, conservation of the corresponding Noether current, e.g. in massless QED

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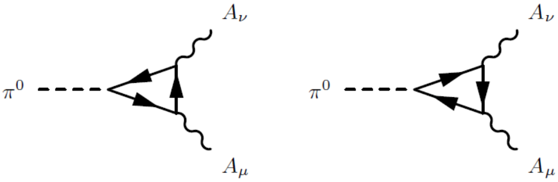
$$\partial_\mu j^\mu = 0 \quad , \quad \partial_\mu j^\mu_A = 0$$

- However, quantum effects can break a symmetry $\Rightarrow$ there is an anomaly

$$\partial_\mu j^\mu_A = \mathcal{A} \neq 0$$



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- SCFTs are quantum field theories with:
- conformal symmetry (invariance under scale transformations), that has many applications both in condensed matter (e.g. phase transitions) and in high energy physics (e.g. the AdS/CFT correspondence);
- supersymmetry, which exchanges bosons and fermions. It is useful for studying models of strongly coupled QFTs and provides candidate particles for Beyond Standard Model physics

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- To determine Yang-Mills anomalies, Stora and Zumino defined

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- Anomalies (in $d = 4$) are computed from the exact relations

$$\begin{cases} P_6 = \delta Q_5 \\ P_6 = 0 \end{cases} \quad \Rightarrow \quad \delta Q_5 = 0$$

where $P_6$ is an invariant polynomial cubic in $\boldsymbol{F}$ and
$Q_5$ is the associated Chern-Simons class

$$\text{e.g.} \quad P_6 = \mathrm{tr}_R(\boldsymbol{F}^3) \,, \quad Q_5 = \mathrm{tr}_R\left(\boldsymbol{A}\boldsymbol{F}^2 - \tfrac{1}{2}\boldsymbol{A}^3\boldsymbol{F} + \tfrac{1}{10}\boldsymbol{A}^5\right)$$

## Original results

- We generalized the Stora-Zumino framework to $d = 4$ , $\mathcal{N} = 1$ SCFTs and showed that (certain) superconformal anomalies can be obtained from superconformal Chern-Simons polynomials

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- This revealed the topological nature of these conformal and supersymmetry anomalies, providing a non-perturbative formulation which enabled us to calculate them exactly
- A possible future direction is to provide a holographic (AdS/CFT) interpretation of our results

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- include contributions from the full superconformal algebra where $[\cdot, \cdot]$ is the super-Lie bracket
- and extra terms coming from local supersymmetry $\Rightarrow$ possible obstruction to the anomaly mechanism

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- Details in [C.Imbimbo, D.Rovere, A.Warman, *JHEP* 05 (2024) 277]

# Current research: Non-Invertible Symmetries

- I am currently at the University of Oxford supervised by Prof. Sakura Schäfer-Nameki, supported by the UKRI Frontier Research Grant, underwriting the ERC Advanced Grant "Generalized Symmetries in Quantum Field Theory and Quantum Gravity"

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- And have many future projects planned!

# The Galileo Galilei Institute For Theoretical Physics

Centro Nazionale di Studi Avanzati dell'Istituto Nazionale di Fisica Nucleare

M I L L A B A L D O C E O L I N A W A R D

# Thank you!

Istituto Nazionale di Fisica Nucleare