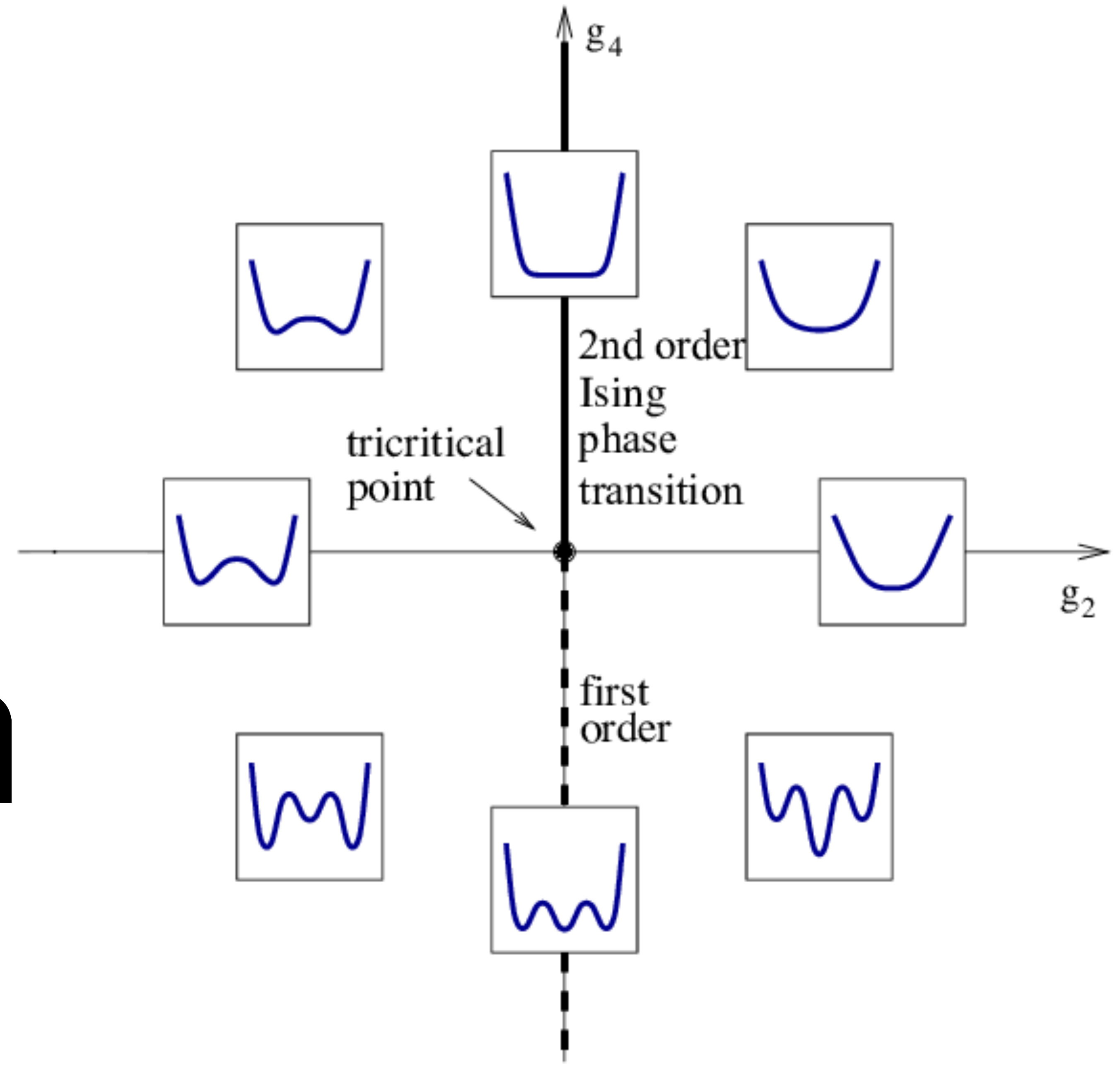

Tri-Critical Ising Phase Transition in a Ladder of JJa.

Lorenzo Maffi[1], Matteo Rizzi[2], Niklas Tausendpfund[2], Michele Burrello[1]

[1]*Niels Bohr Institute,* University of Copenhagen
[2]*Institute for Theoretical Physics,* Cologne

Center for Quantum Devices

VILLUM FONDEN

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Two must-have papers

Tools

Pleasure

Two must-have papers

Tools

Applied Conformal Field Theory

Paul Ginsparg†

Lyman Laboratory of Physics
Harvard University
Cambridge, MA 02138

Lectures given at Les Houches summer session, June 28 – Aug. 5, 1988.

To appear in Les Houches, Session XLIX, 1988, *Champs, Cordes et Phénomènes Critiques/ Fields, Strings and Critical Phenomena*, ed. by E. Brézin and J. Zinn-Justin, ©Elsevier Science Publishers B.V. (1989).

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Superconductivity for Particular Theorists*)

Steven WEINBERG

Theory Group, Department of Physics, University of Texas, Austin, TX 78712

(Received December 10, 1985)

No one did more than Nambu to bring the idea of spontaneously broken symmetries to the attention of elementary particle physicists. And, as he acknowledged in his ground-breaking 1960 article "Axial Current Conservation in Weak Interactions", Nambu was guided in this work by an analogy with the theory of superconductivity, to which Nambu himself had made important contributions. It therefore seems appropriate to honor Nambu on his birthday with a little pedagogical essay on superconductivity, whose inspiration comes from experience with broken symmetries in particle theory. I doubt if anything in this article will be new to the experts, least of all to Nambu, but perhaps it may help others, who like myself are more at home at high energy than at low temperature, to appreciate the lessons of superconductivity.

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Quantum simulation of the Tricritical Ising model in tunable Josephson junction ladders

[ArXiv 2310.18300]

Niklas Tausendpfund
(Cologne)

Matteo Rizzi
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Michele Burrello
(NBI, Copenhagen)

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- Tricritical Ising universality class… Why is it interesting?
- Models and experiment proposals
- Hybrid Josephson junctions

Our model

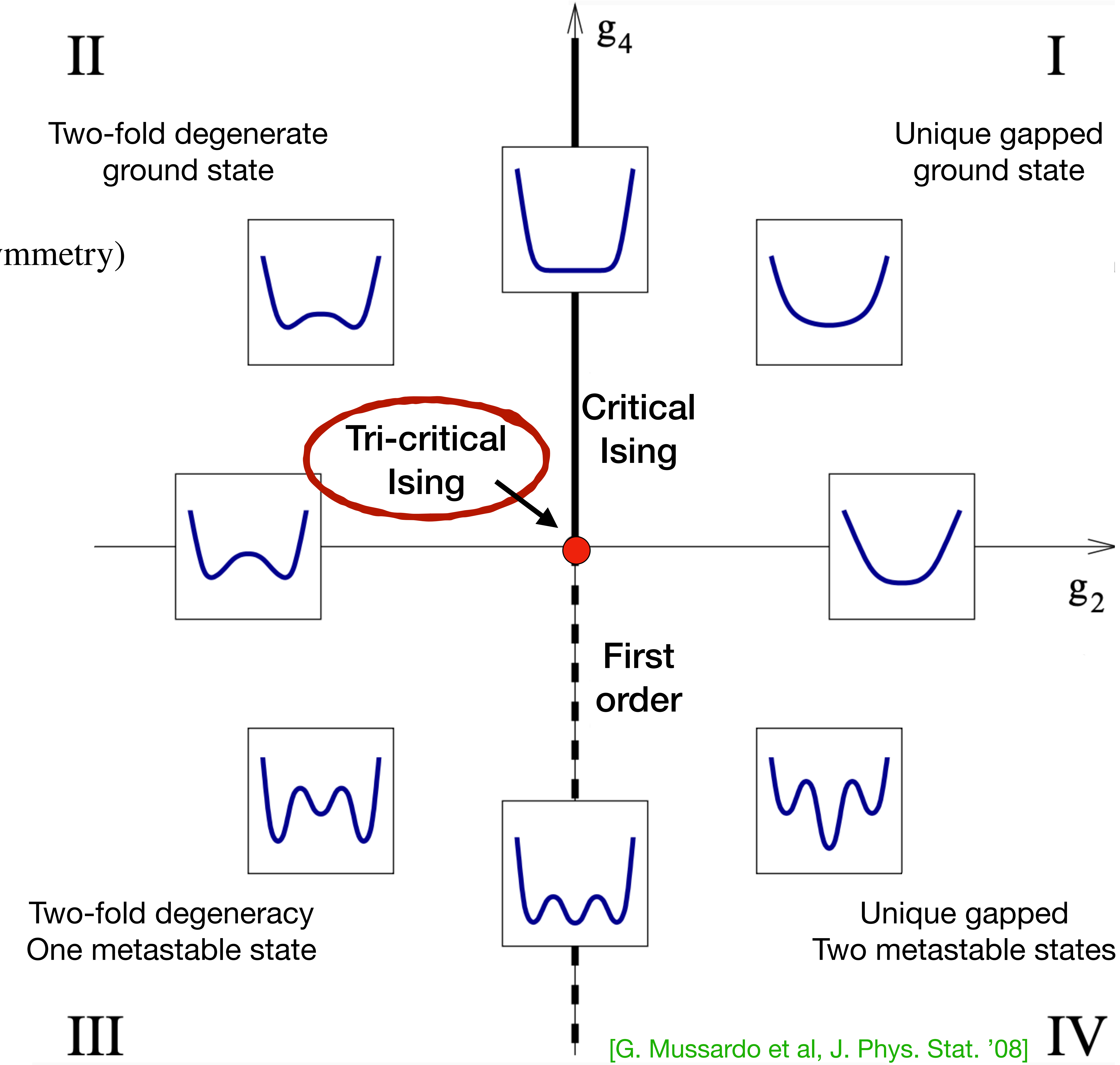
- Single triple junction element
- 1D ladder
- Signatures and observables

Tri-Critical Ising

- Landau-Ginzburg effective description:

$$V(\varphi) = g_2\varphi^2 + g_4\varphi^4 + \varphi^6 \qquad (\varphi \to -\varphi, \quad \mathbb{Z}_2 - \text{symmetry})$$

- Merging of three different phases



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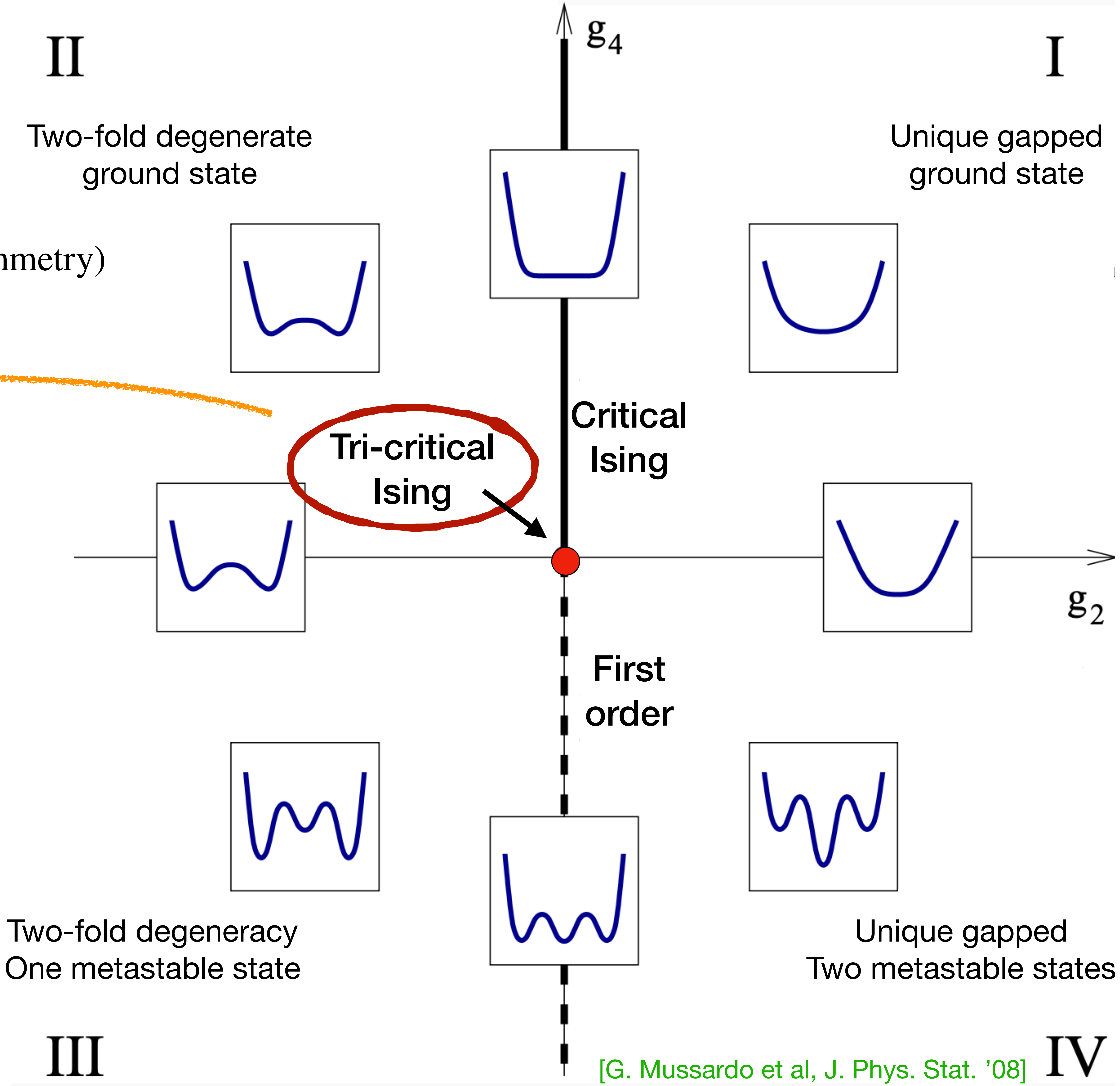
CFT

- 4 relevant operators: $\{\sigma, \sigma', \epsilon, \tau\}$



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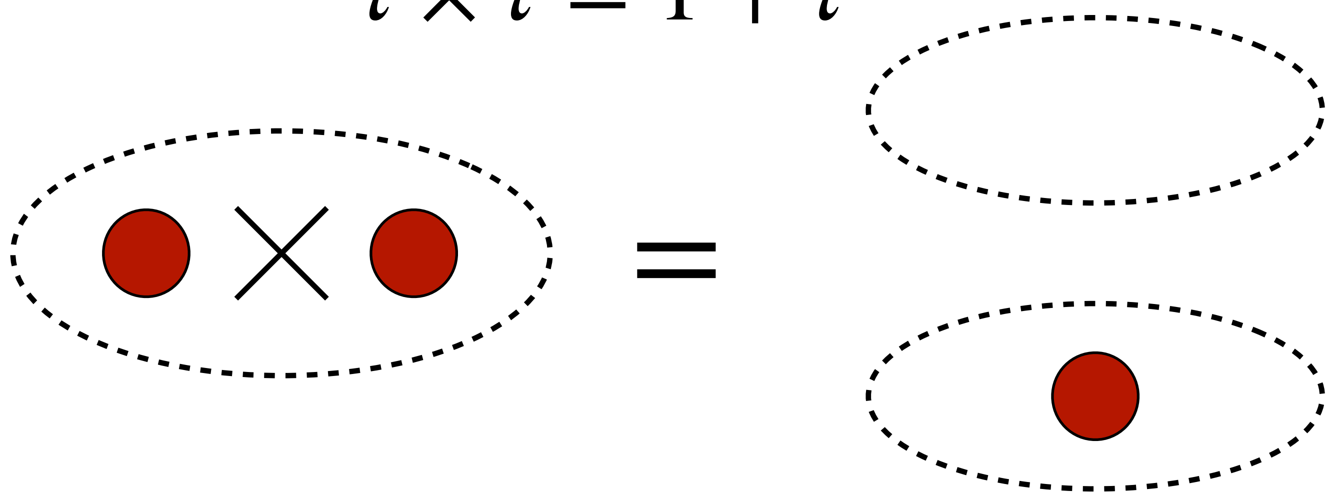
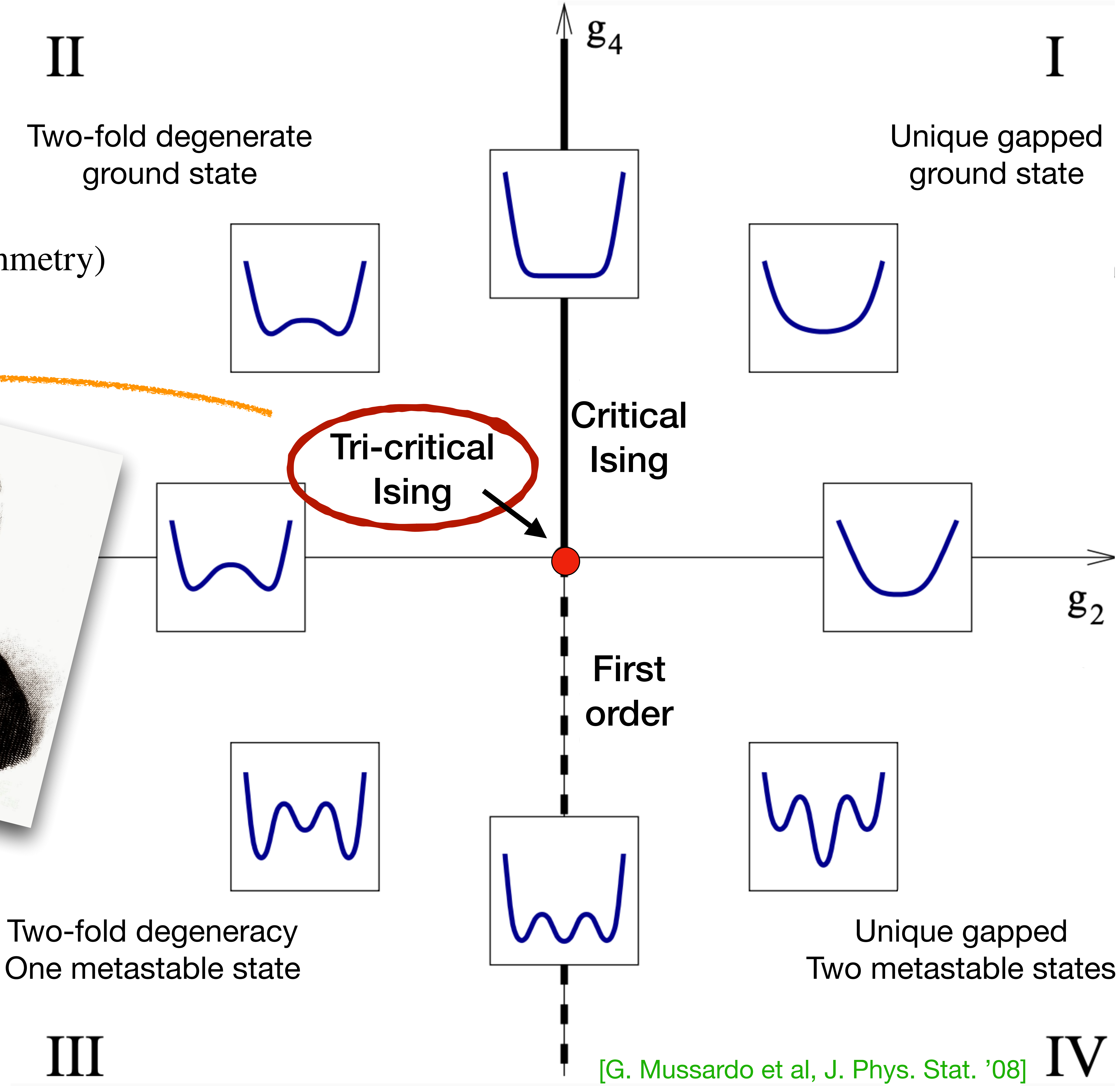
$\tau \times \tau = 1 + \tau$





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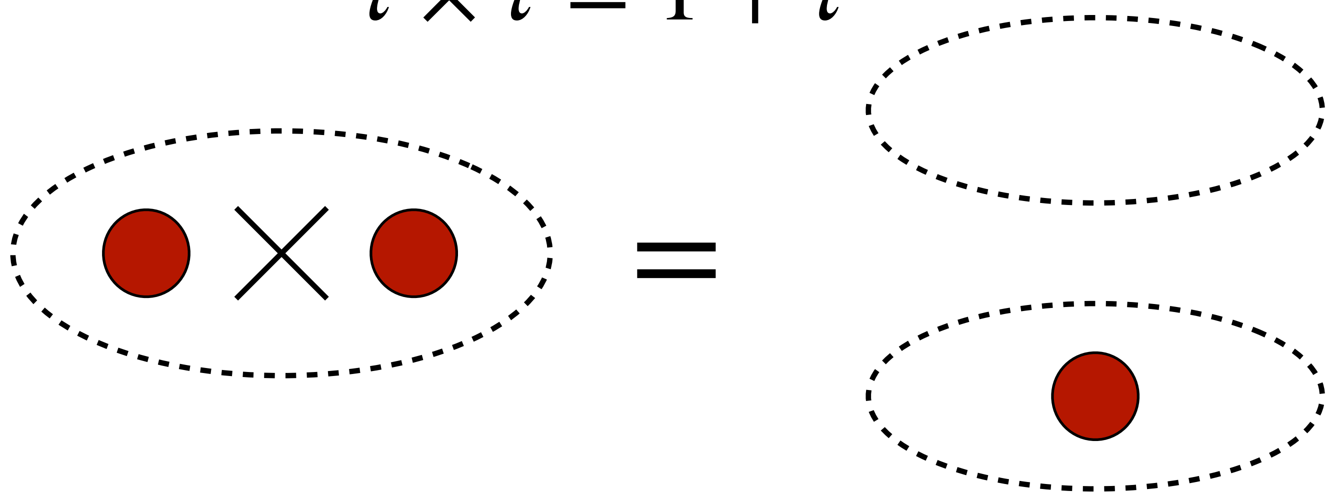

- Power law decay:

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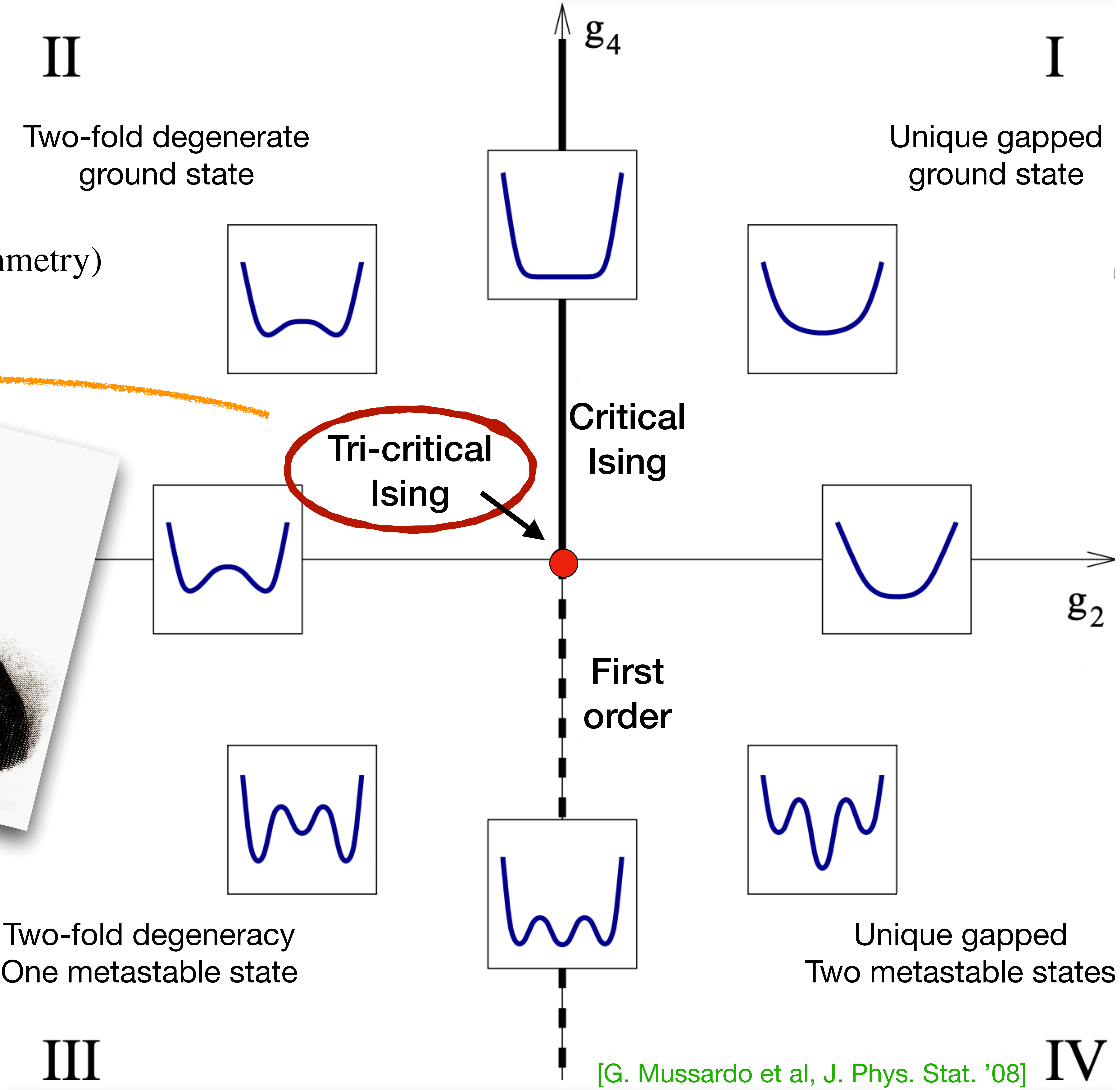

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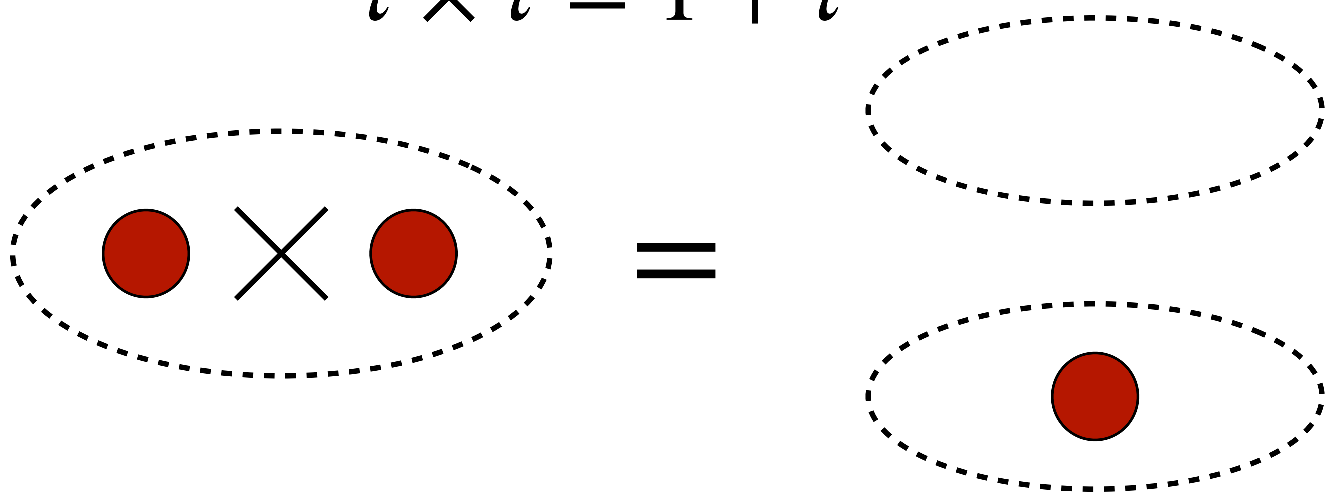

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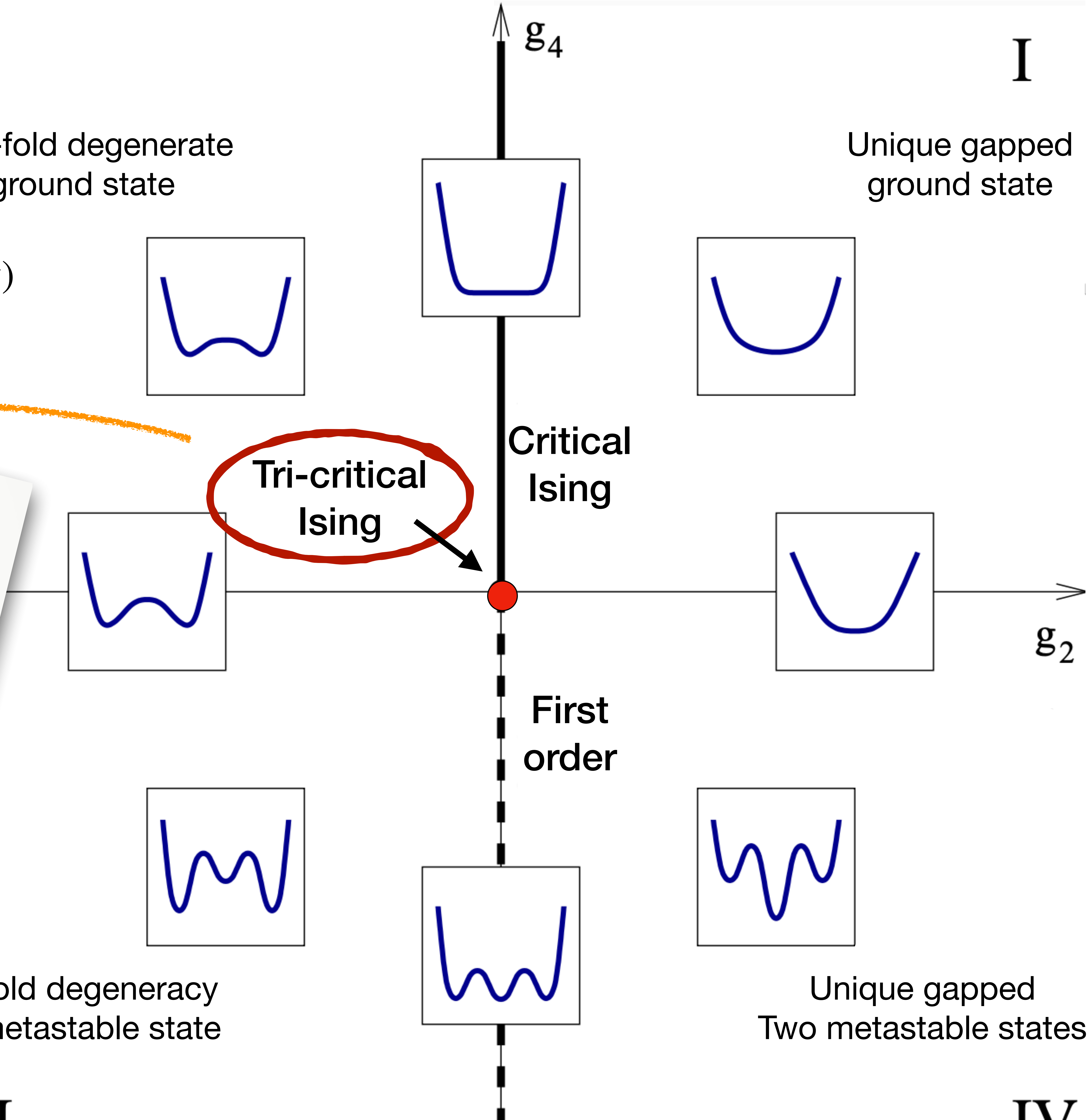
- Emergence of Supersymmetry



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Spin models & other realisations

No 1D Quantum physical realization (related proposals)

- Spin 1 quantum Blume-Capel model
- Strong interacting Rydberg atoms
- Majorana fermions on a lattice
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Quantum field theory

- **Multi-frequency Sine-Gordon model**

[Mussardo, Delfino '97]

$$S = S_{c=1}[\varphi] + \\ + \int dxdt \left(\mu_1 \cos\varphi + \mu_2 \cos(2\varphi) + \mu_3 \cos(3\varphi) \right)$$

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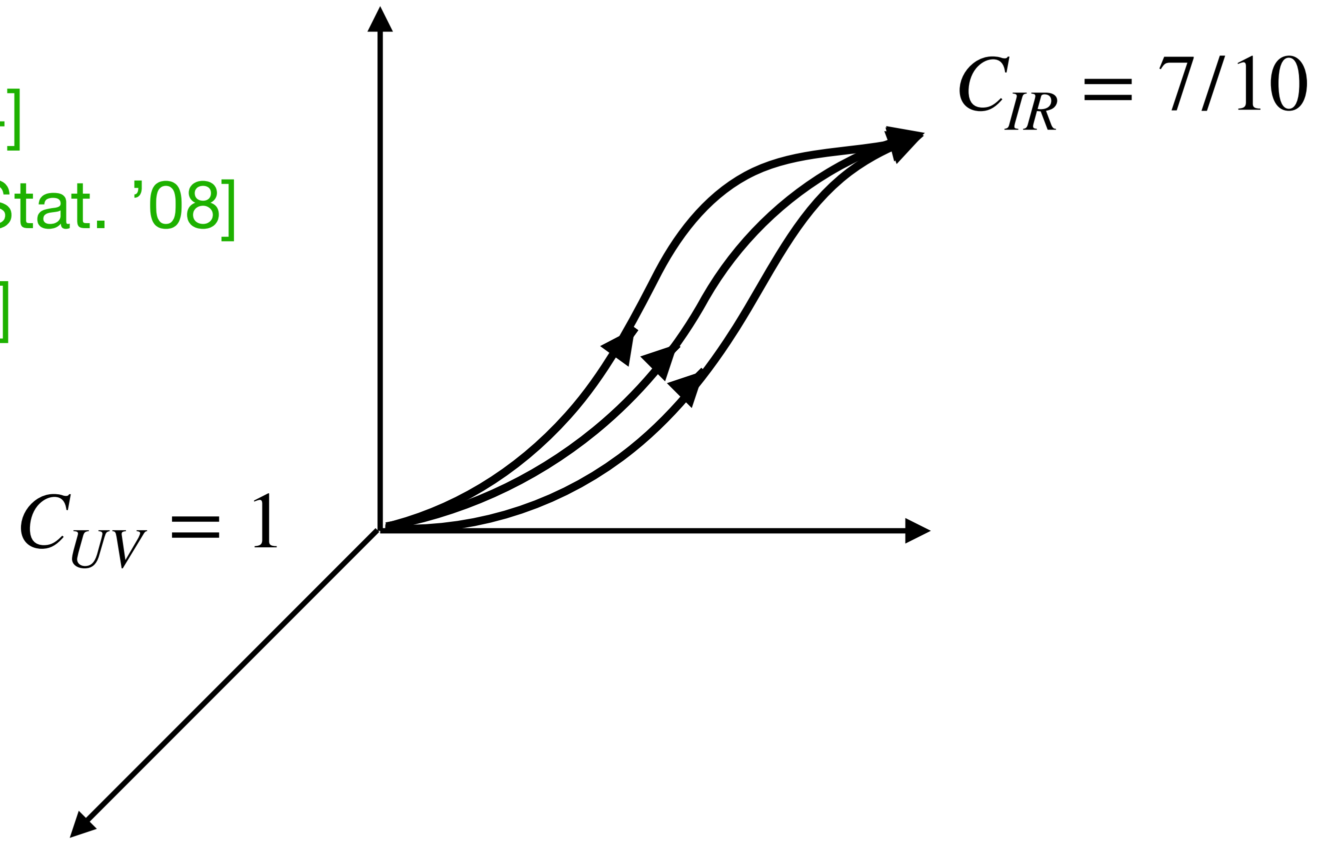

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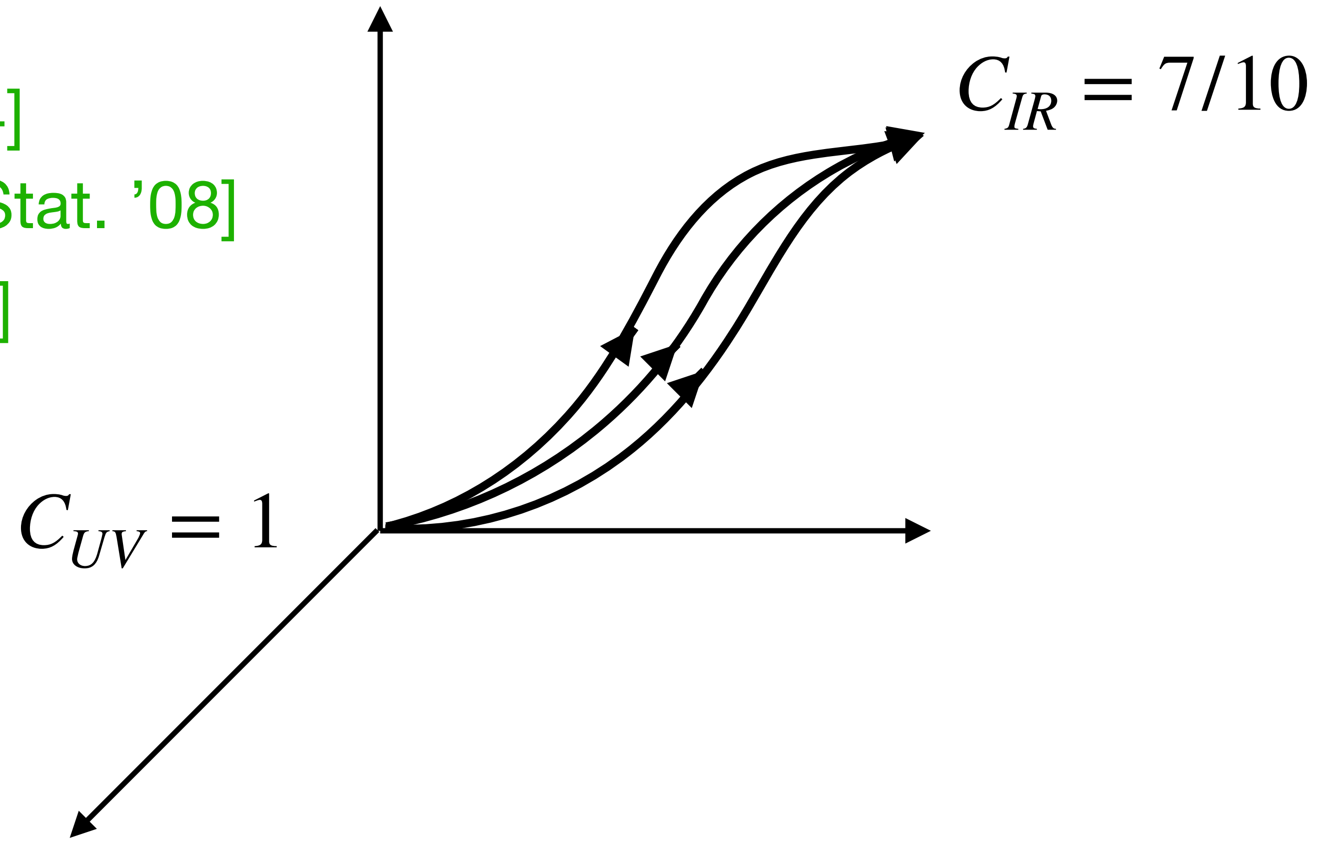

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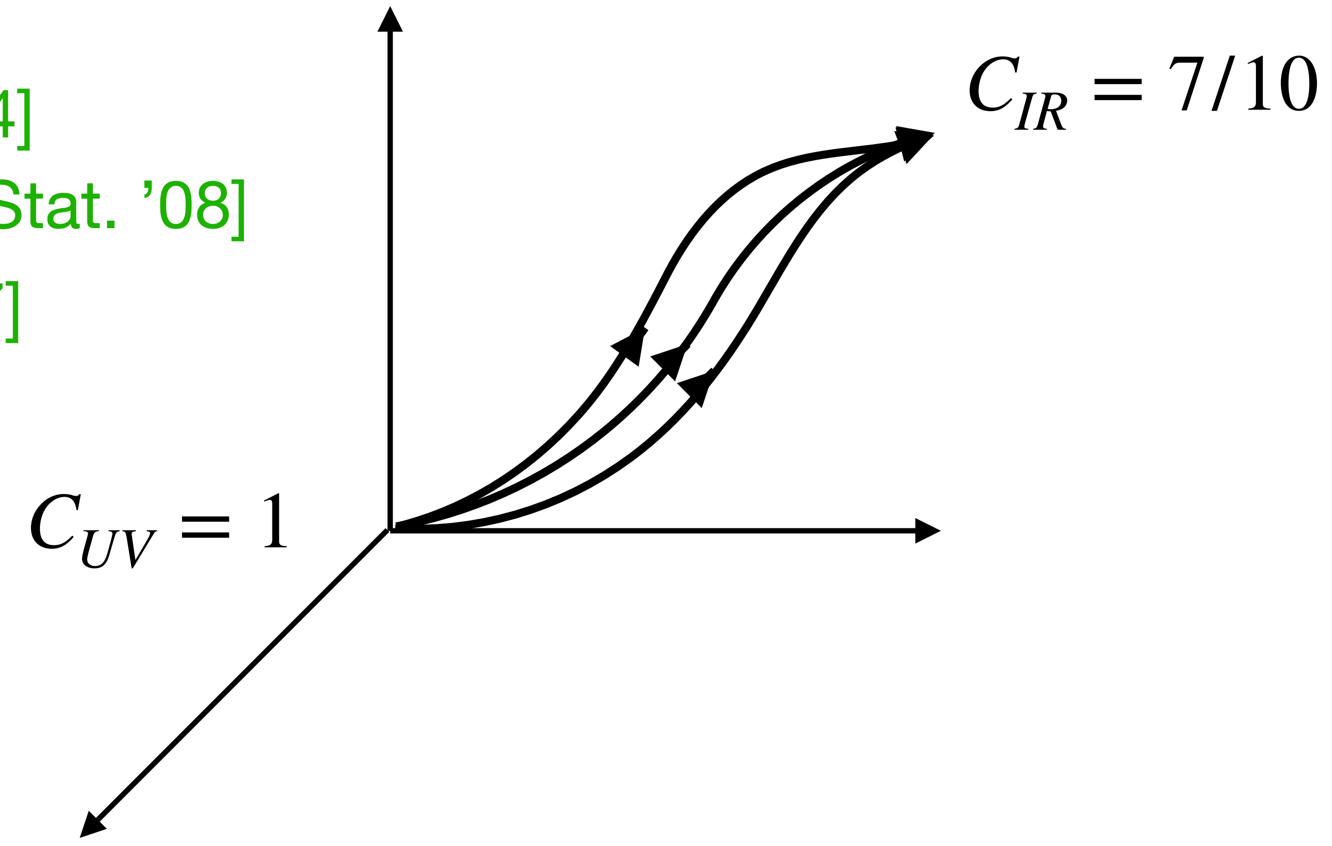

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The quantum sine-Gordon model with quantum circuits

Ananda Roy [a,*], Dirk Schuricht [b], Johannes Hauschild [c], Frank Pollmann [a,d], Hubert Saleur [e]

[a] Department of Physics, T42, Technische Universität München, 85748 Garching, Germany
[b] Institute for Theoretical Physics, Center for Extreme Matter and Emergent Phenomena, Utrecht University, Princetonplein 5, 3584 CE Utrecht, The Netherlands
[c] Department of Physics, University of California, Berkeley, CA 94720, USA
[d] Munich Center for Quantum Science and Technology (MCQST), 80799 Munich, Germany
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Received 24 July 2020; received in revised form 3 May 2021; accepted 13 May 2021
Available online 25 May 2021
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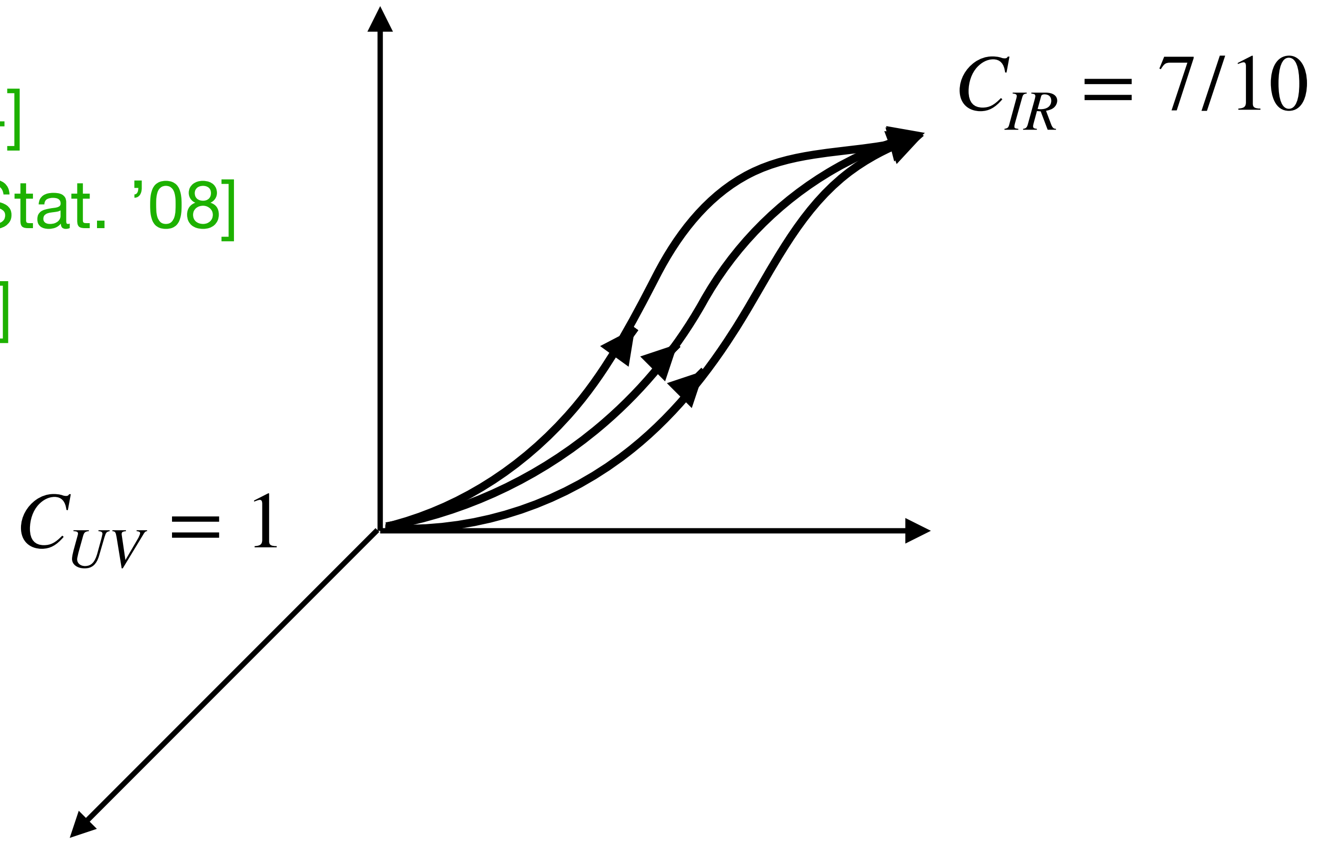

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Quantum Electronic Circuits for Multicritical Ising Models

[PRB '24]

Ananda Roy*
Department of Physics and Astronomy, Rutgers University, Piscataway, NJ 08854-8019 USA

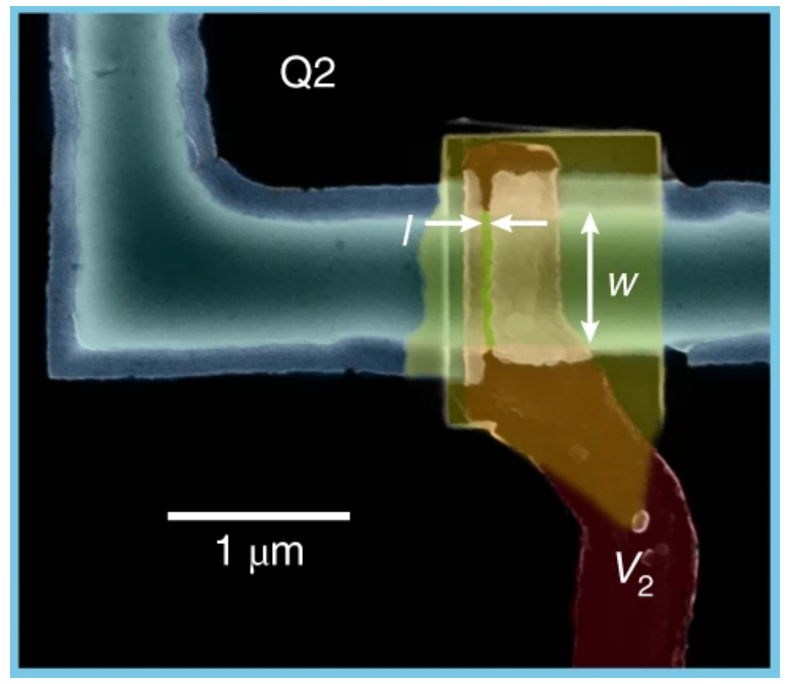
Multicritical Ising models and their perturbations are paradigmatic models of statistical mechanics. In two space-time dimensions, these models provide a fertile testbed for investigation of numerous non-perturbative problems in strongly-interacting quantum field theories. In this work, analog superconducting quantum electronic circuit simulators are described for the realization of these multicritical Ising models. The latter arise as perturbations of the quantum sine-Gordon model with p-fold degenerate minima, $p = 2, 3, 4, \ldots$. The corresponding quantum circuits are constructed with Josephson junctions with $\cos(n\phi + \delta_n)$ potential with $1 \leq n \leq p$ and $\delta_n \in [-\pi, \pi]$. The sim-

Hybrid Josephson Junctions

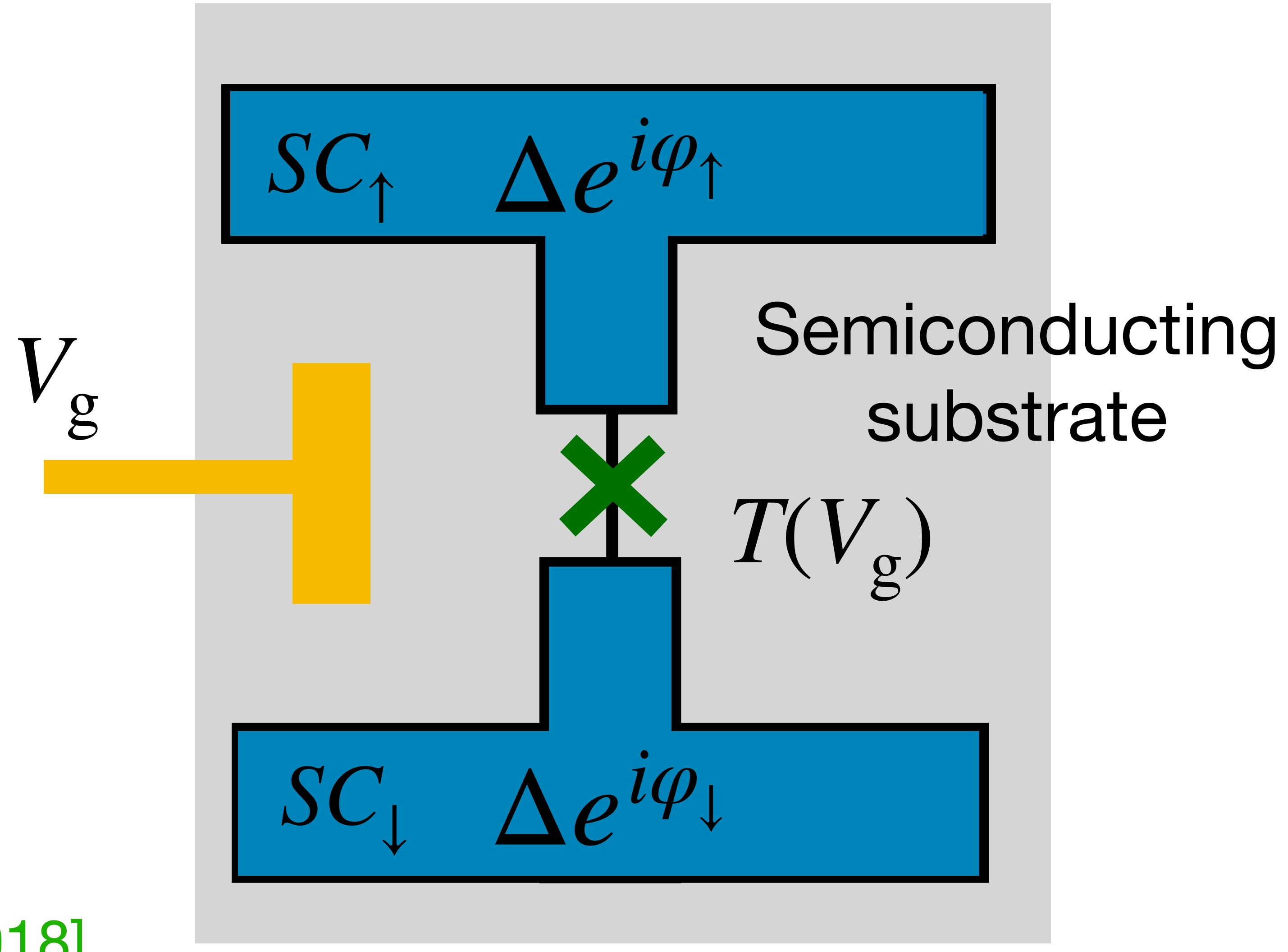
- Local degrees of freedom: Cooper pairs on the SC islands

$$\left[\hat{N}_j, e^{i\hat{\varphi}_{j'}}\right] = -\delta_{j,j'}\, e^{i\hat{\varphi}_j}$$

$$\left(\varphi = \varphi_\uparrow - \varphi_\downarrow\right)$$



[C. Marcus et al. Nat. Phys. 2018]



- Josephson effect: Cooper pair tunneling through the normal region

$$V(\varphi) = -E_J\cos(\varphi) = -\frac{E_J}{2}\left(e^{i\varphi_\uparrow}e^{-i\varphi_\downarrow} + \mathrm{H.c}\right)$$

- The semiconductor sustains also multiple Cooper pairs coherent tunneling processes

$$V(\varphi) = -\Delta\sqrt{1 - T\sin^2(\varphi/2)} \qquad T = T(V_g) \in [0,1]$$

[Benakker PRL 1991]

- The Josephson potential contains higher harmonics

$$V(\varphi) \simeq -\Delta\left[\left(\frac{T}{4} + \frac{T^2}{16}\right)\cos\varphi + \frac{T^2}{64}\cos 2\varphi + \ldots\right]$$

- We target the local potential

$$V(\varphi) = \mu_1\cos\varphi + \mu_2\cos(2\varphi) + \mu_3\cos(3\varphi)$$

One JJ is not enough…

Outline

Motivations

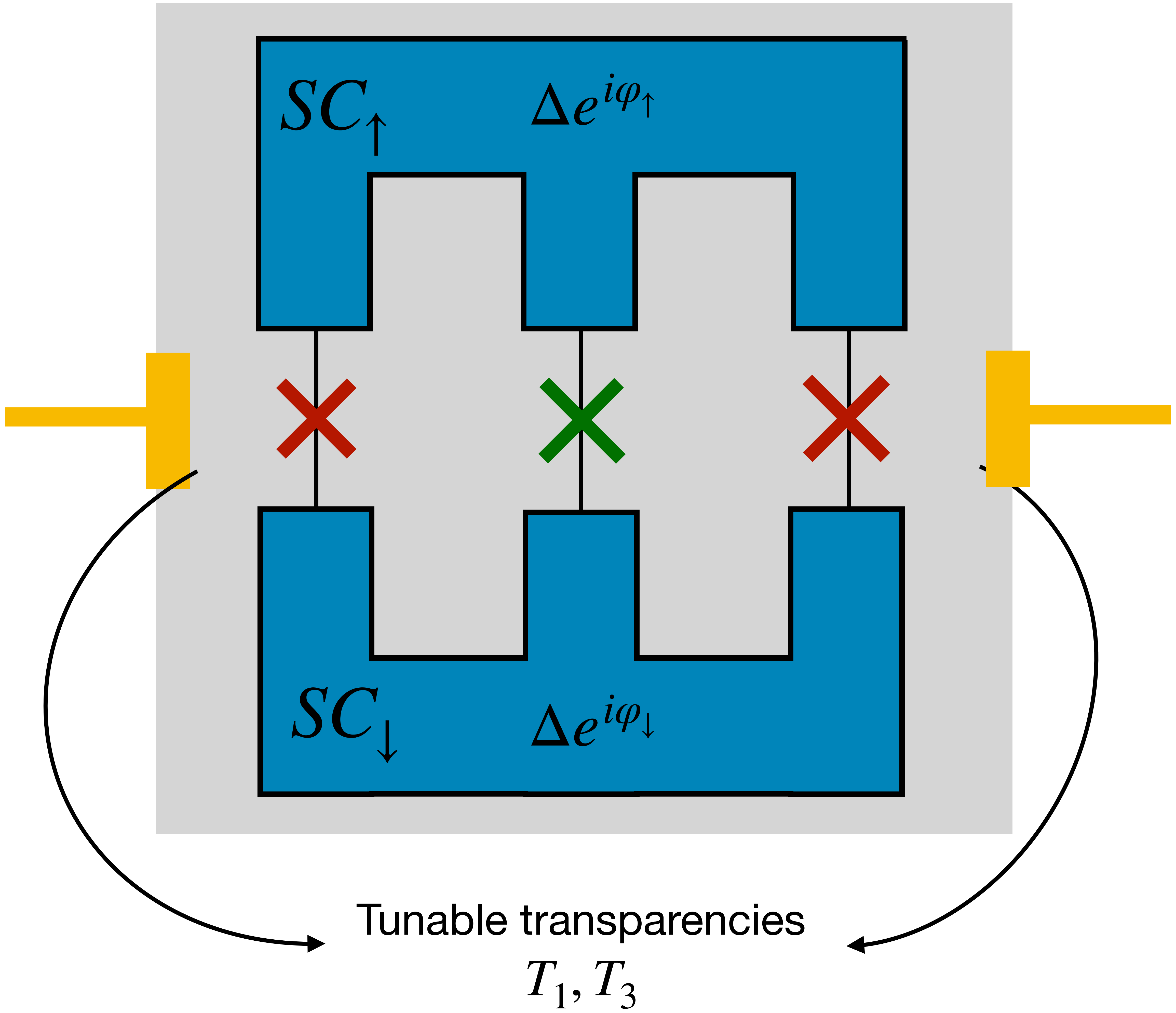
- Tricritical Ising universality class…What is it? Why is it interesting?
- Models and experiment proposals
- Hybrid Josephson junctions

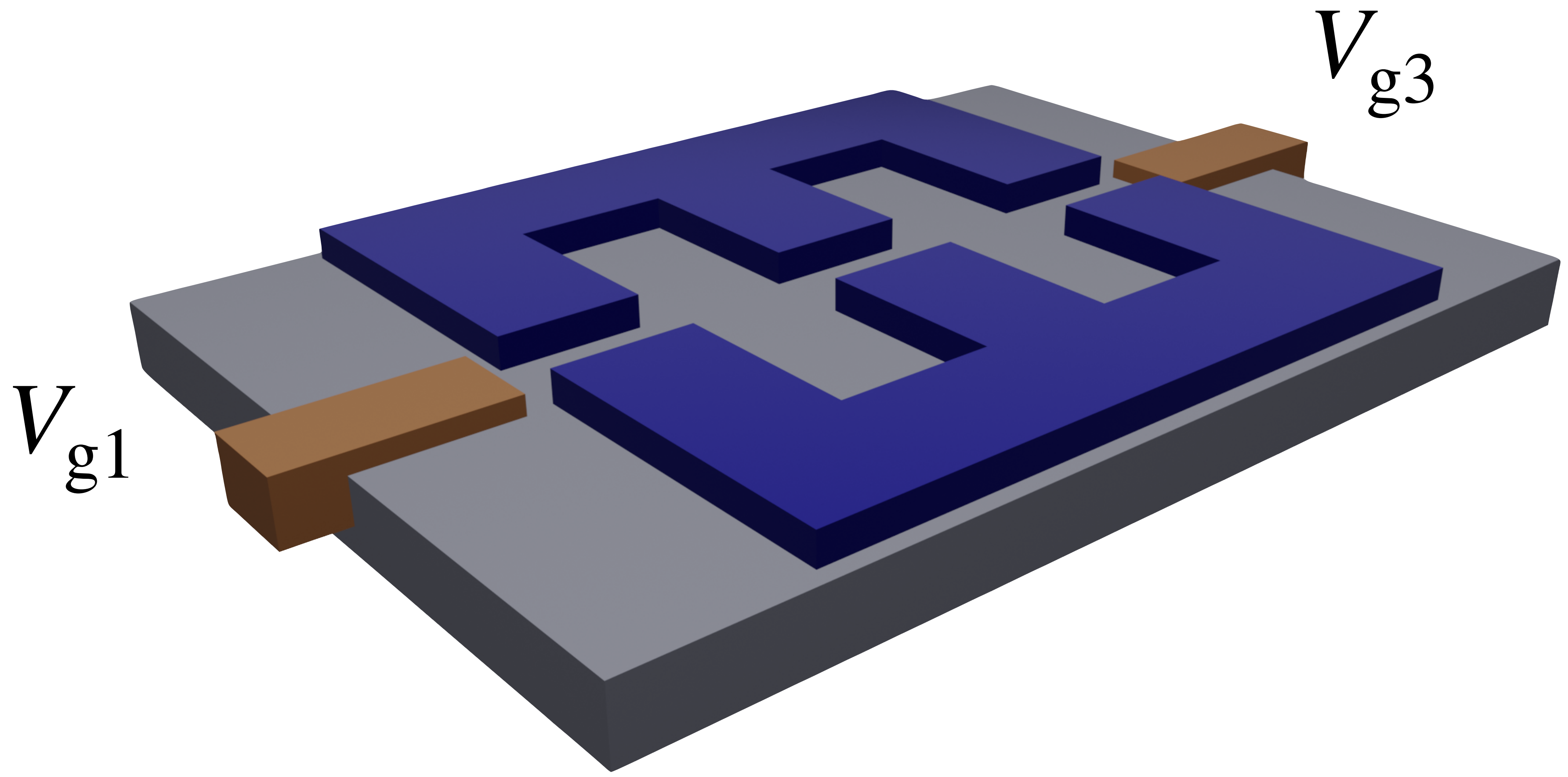
Our model

- Single triple junction element
- 1D ladder
- Signatures and observables

Local potential: Triple JJs element

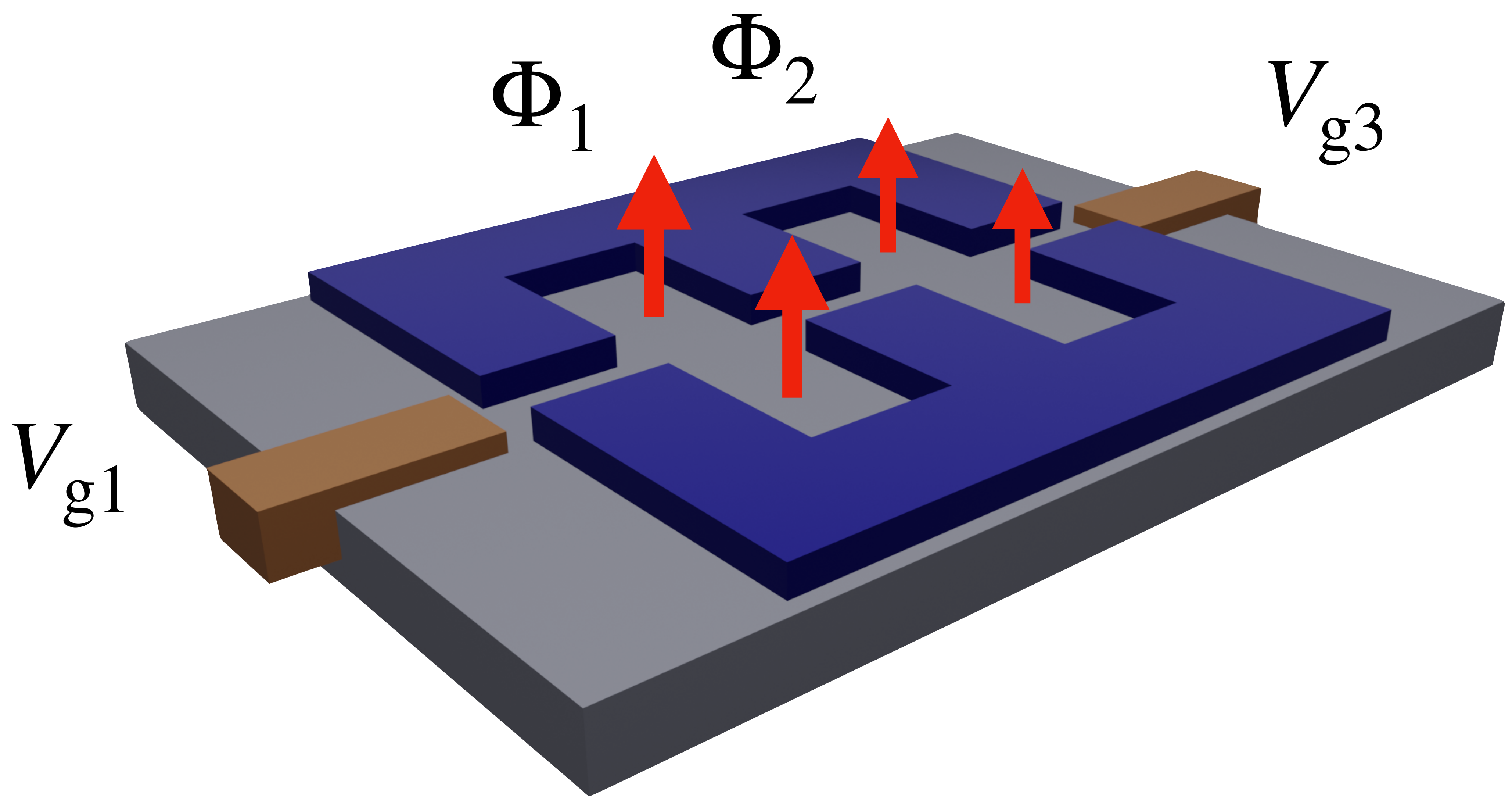
- Two E-shaped SC islands: 3 junctions T_1, T_2, T_3

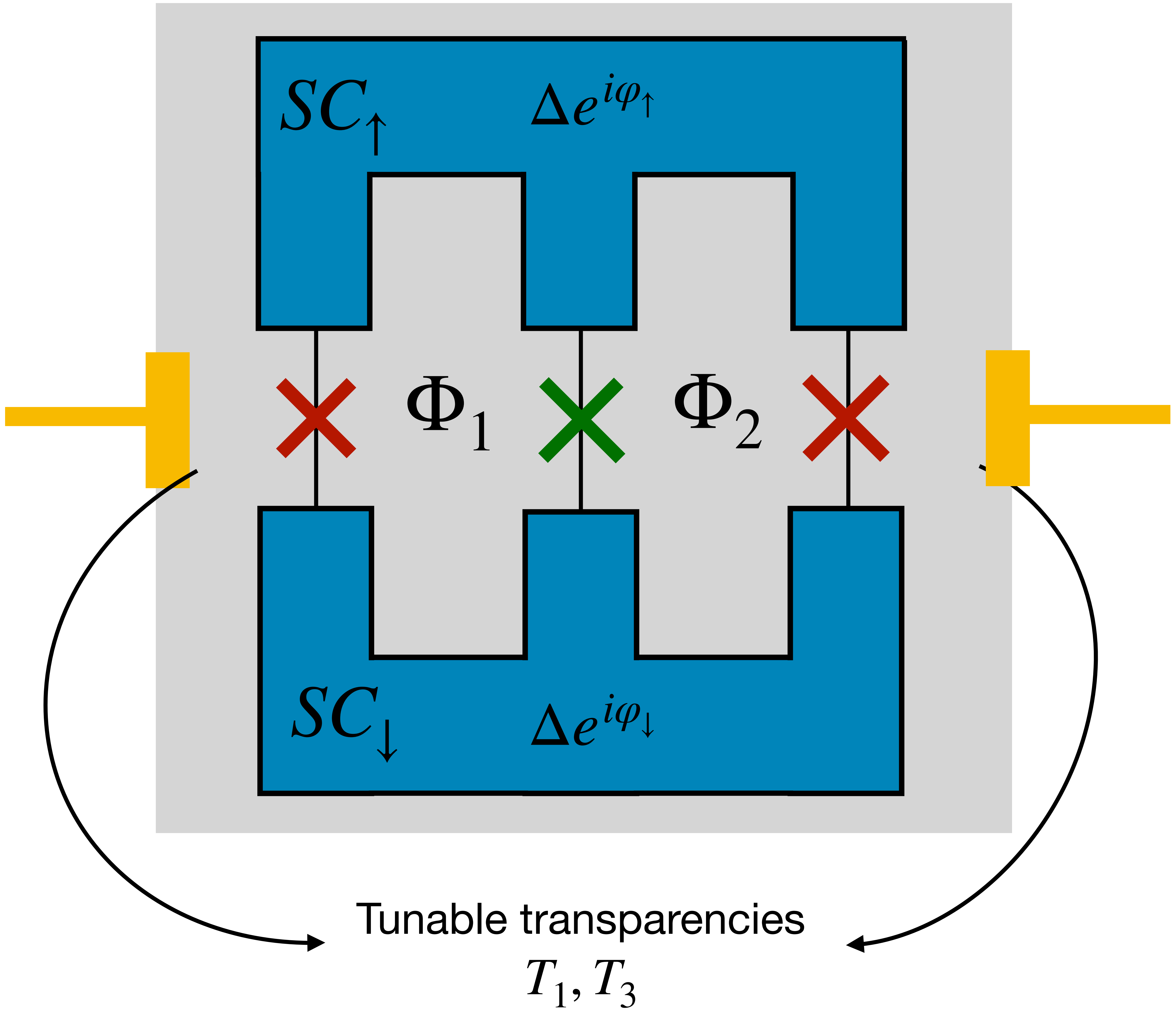




Local potential: Triple JJs element

- Two E-shaped SC islands: 3 junctions T_1, T_2, T_3
- 2 Loops: 2 magnetic fluxes Φ_1 and Φ_2





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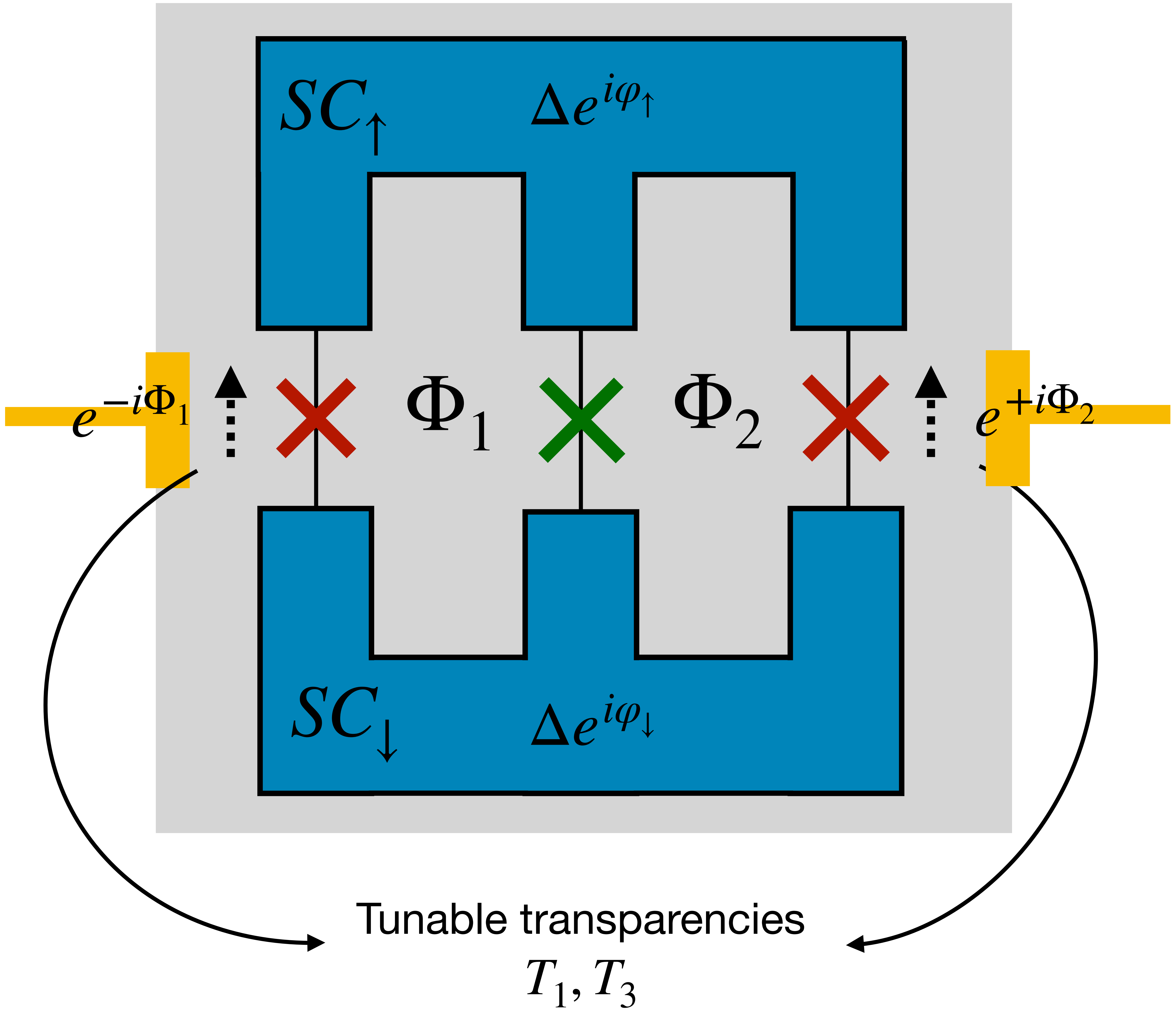

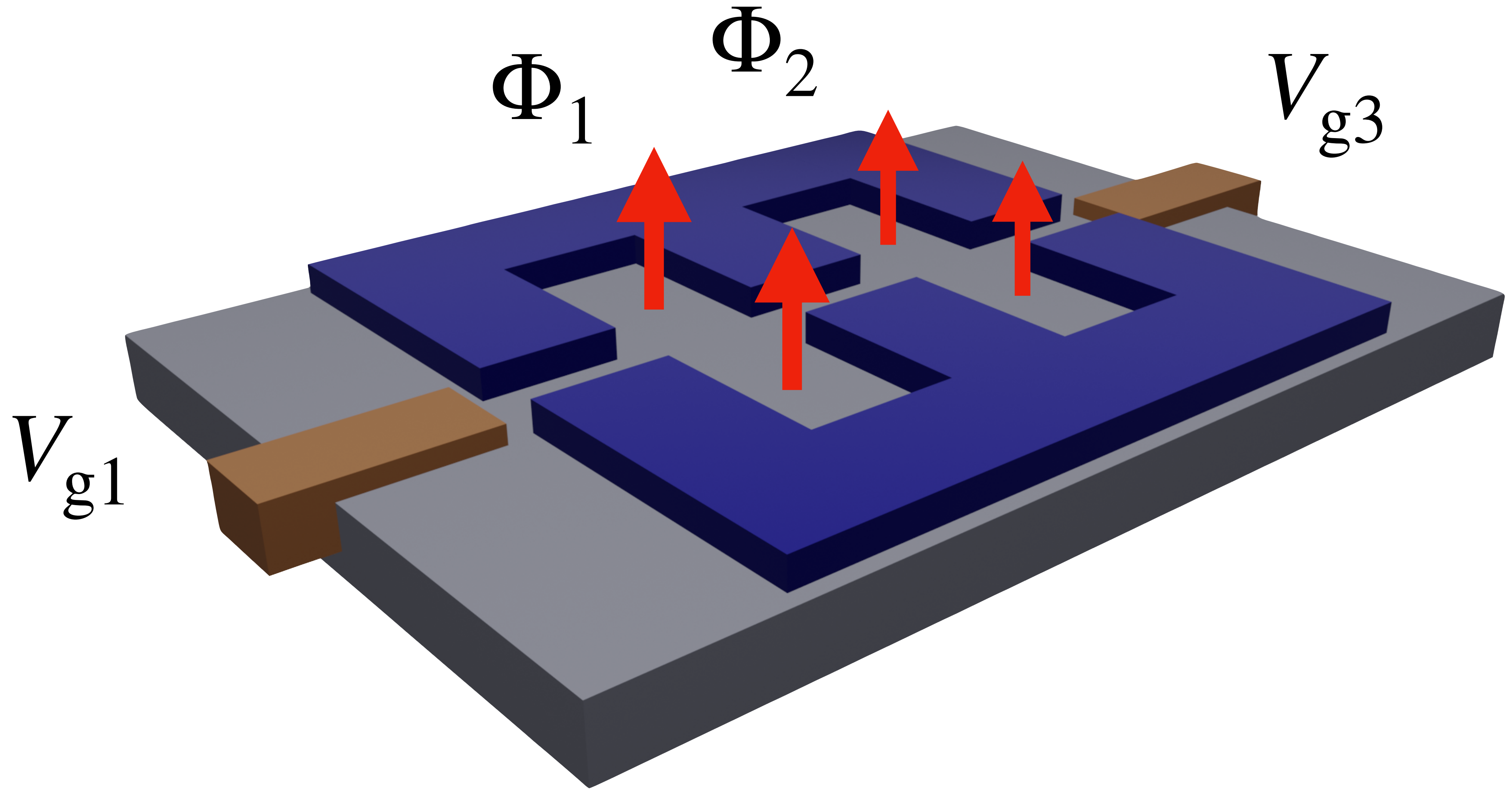

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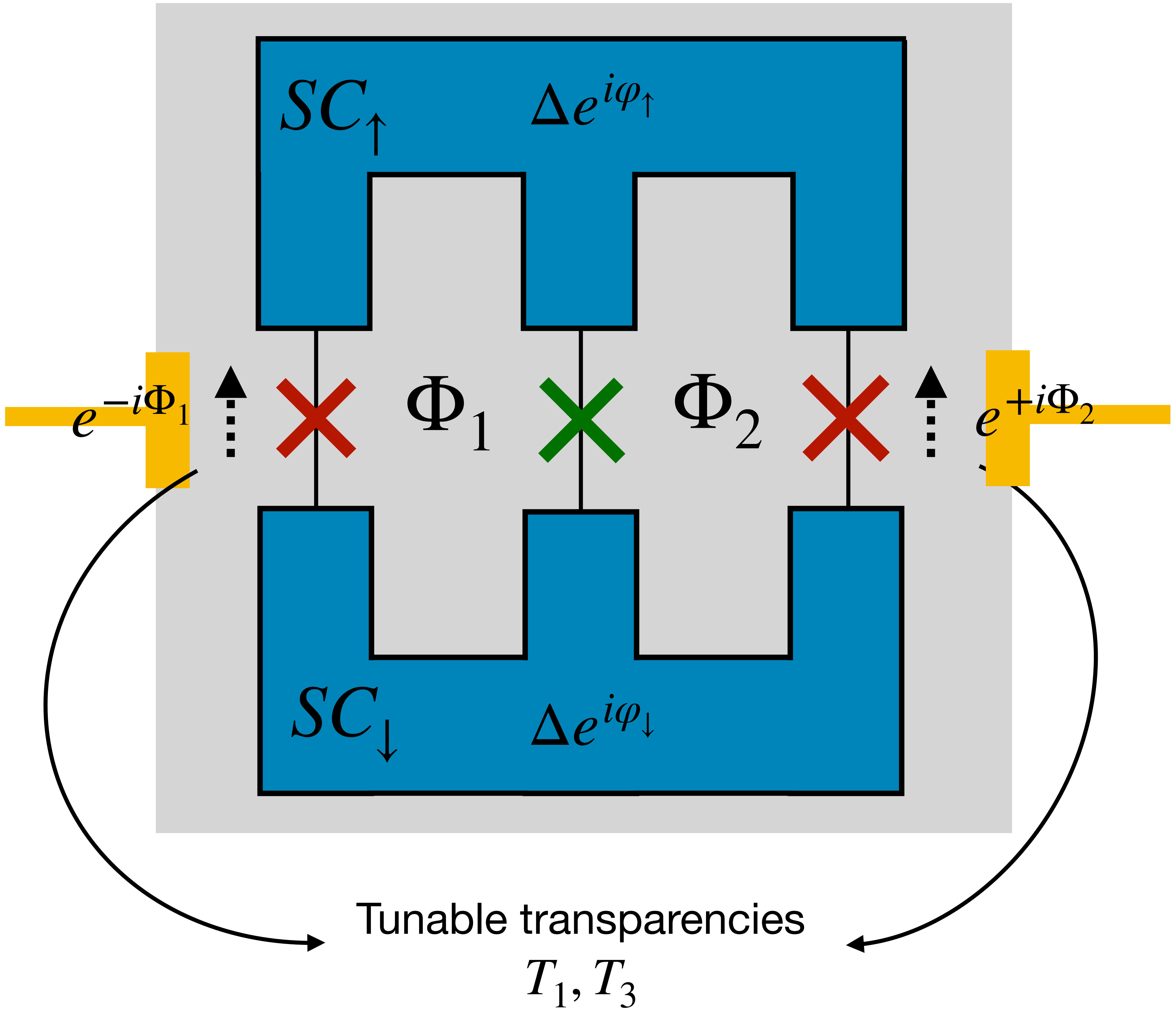

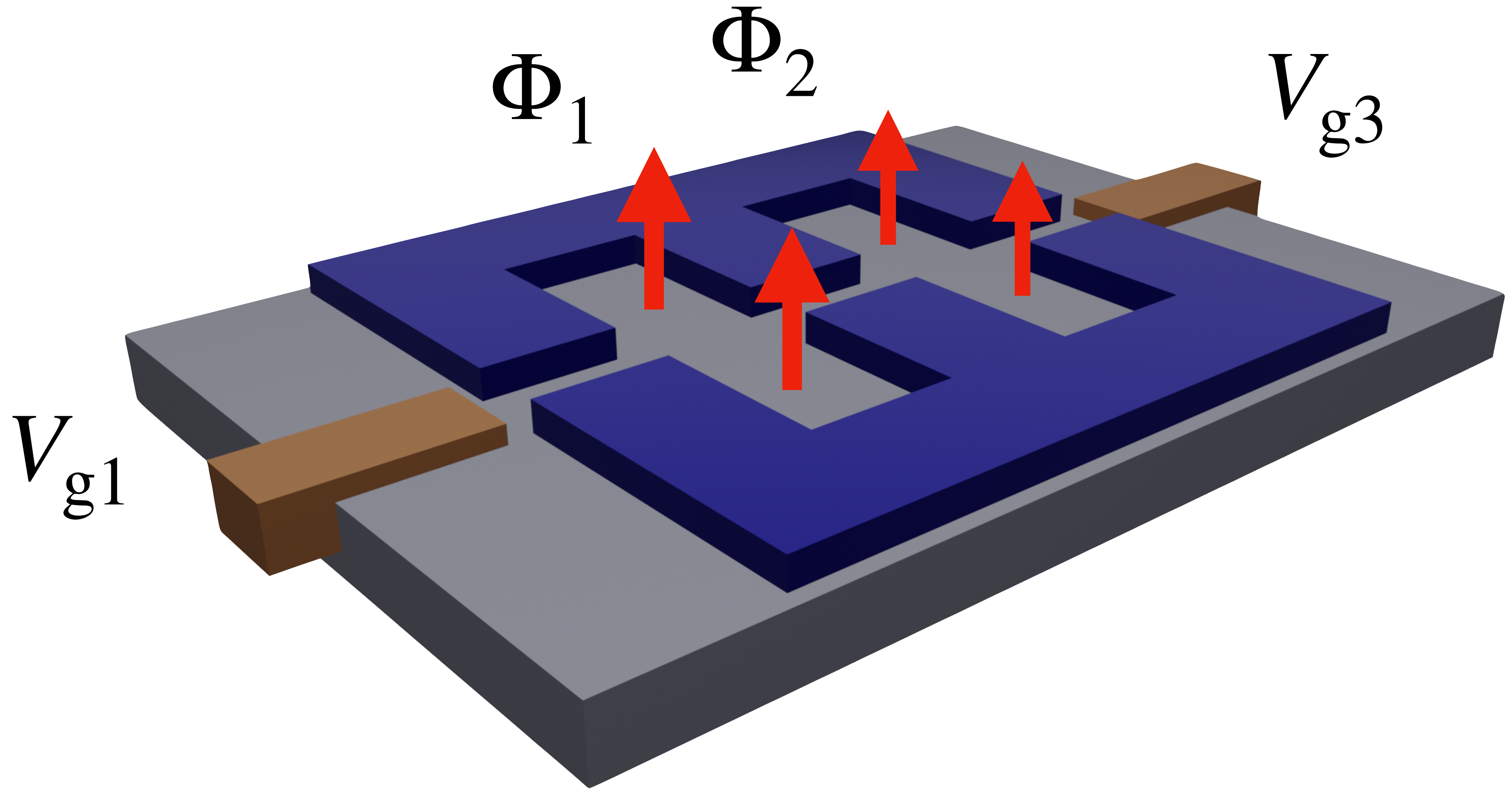

- Local potential $(\varphi = \varphi_\uparrow - \varphi_\downarrow)$

$$V_{\text{loc}}(\varphi) = V^{(1)}(\varphi - \Phi_1) + V^{(2)}(\varphi) + V^{(3)}(\varphi + \Phi_2)$$

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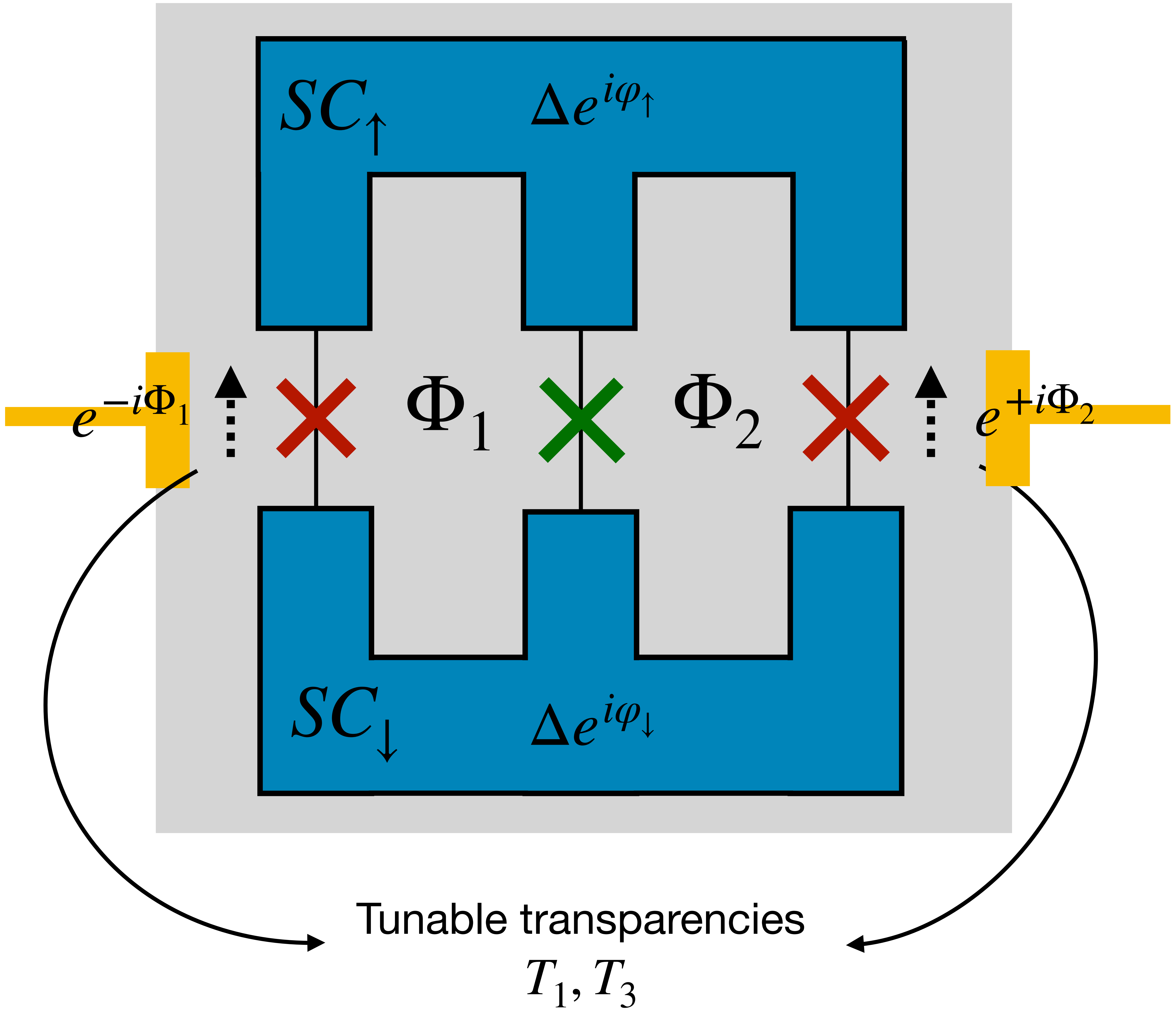

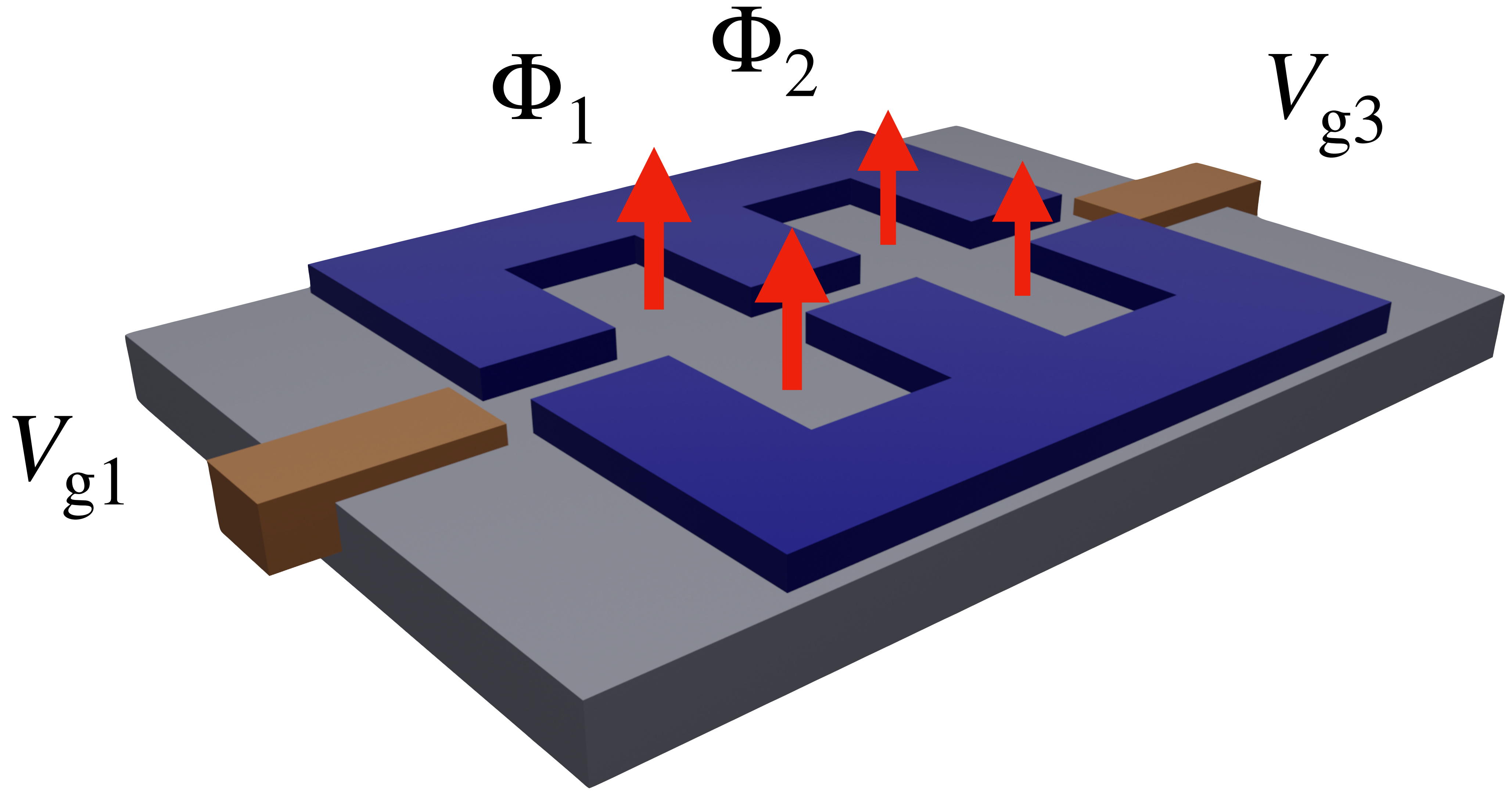

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$\varphi \to -\varphi$ ↓ $\mathbb{Z}_2$ – symmetry

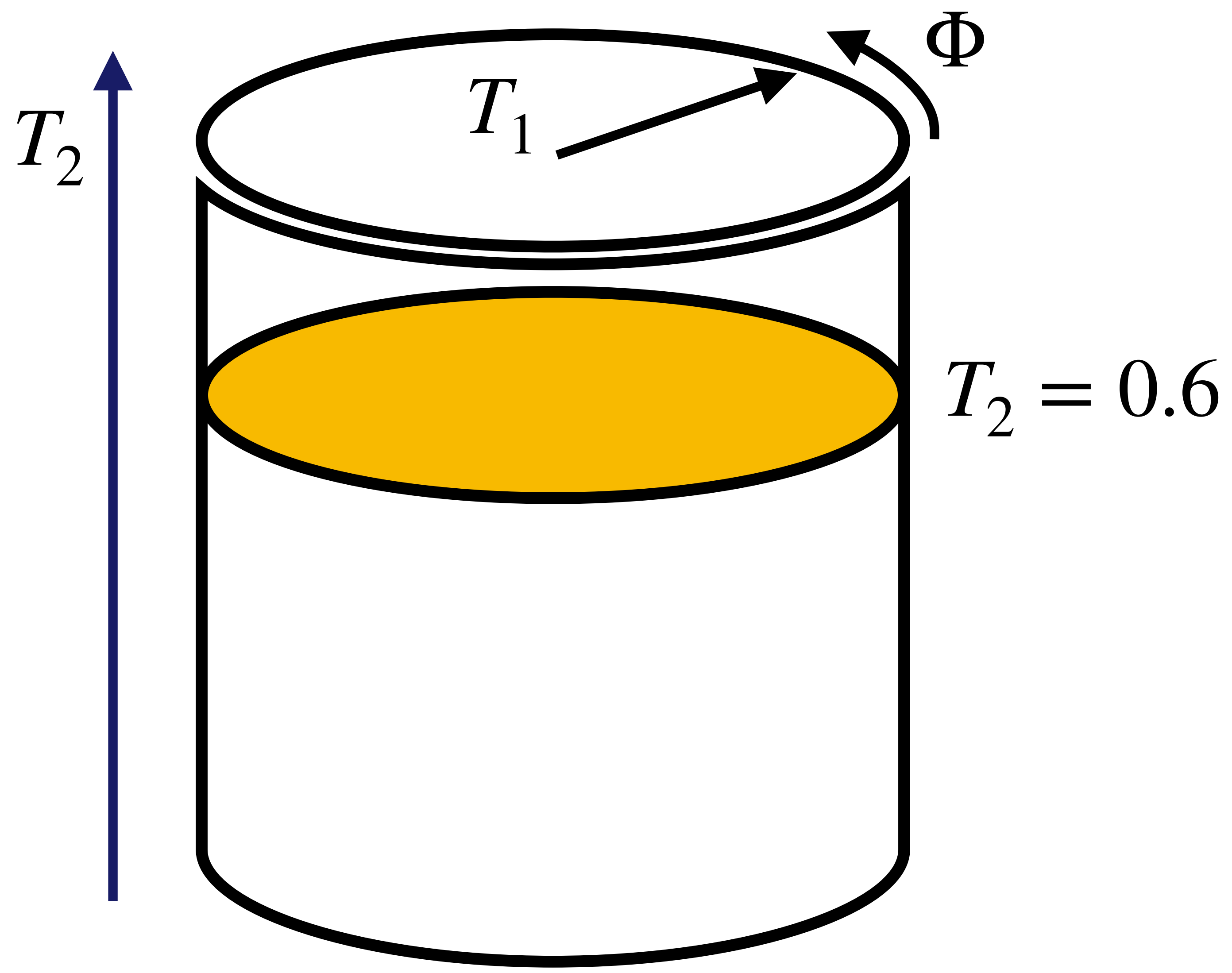
3 Parameters:

- Transparencies $\left\{T_1 = T_3, T_2\right\}$
- Flux $\left\{\Phi_1 = \Phi_2 \equiv \Phi\right\}$

$$V_{\rm loc}(\varphi) = \mu_1 \cos\varphi + \mu_2 \cos 2\varphi + \mu_3 \cos 3\varphi + \ldots$$

Semiclassical analysis

- Three-dimensional parameter space



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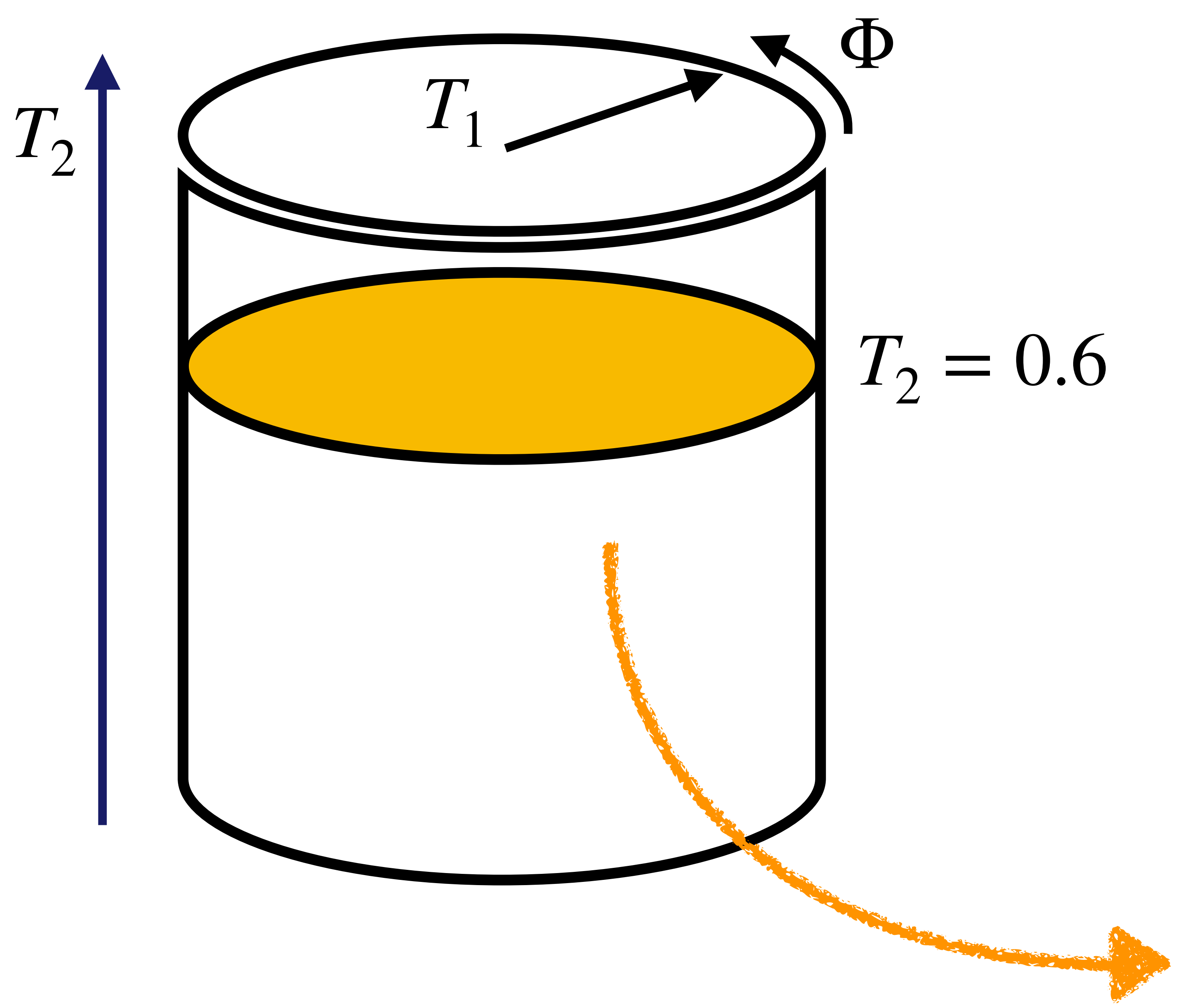

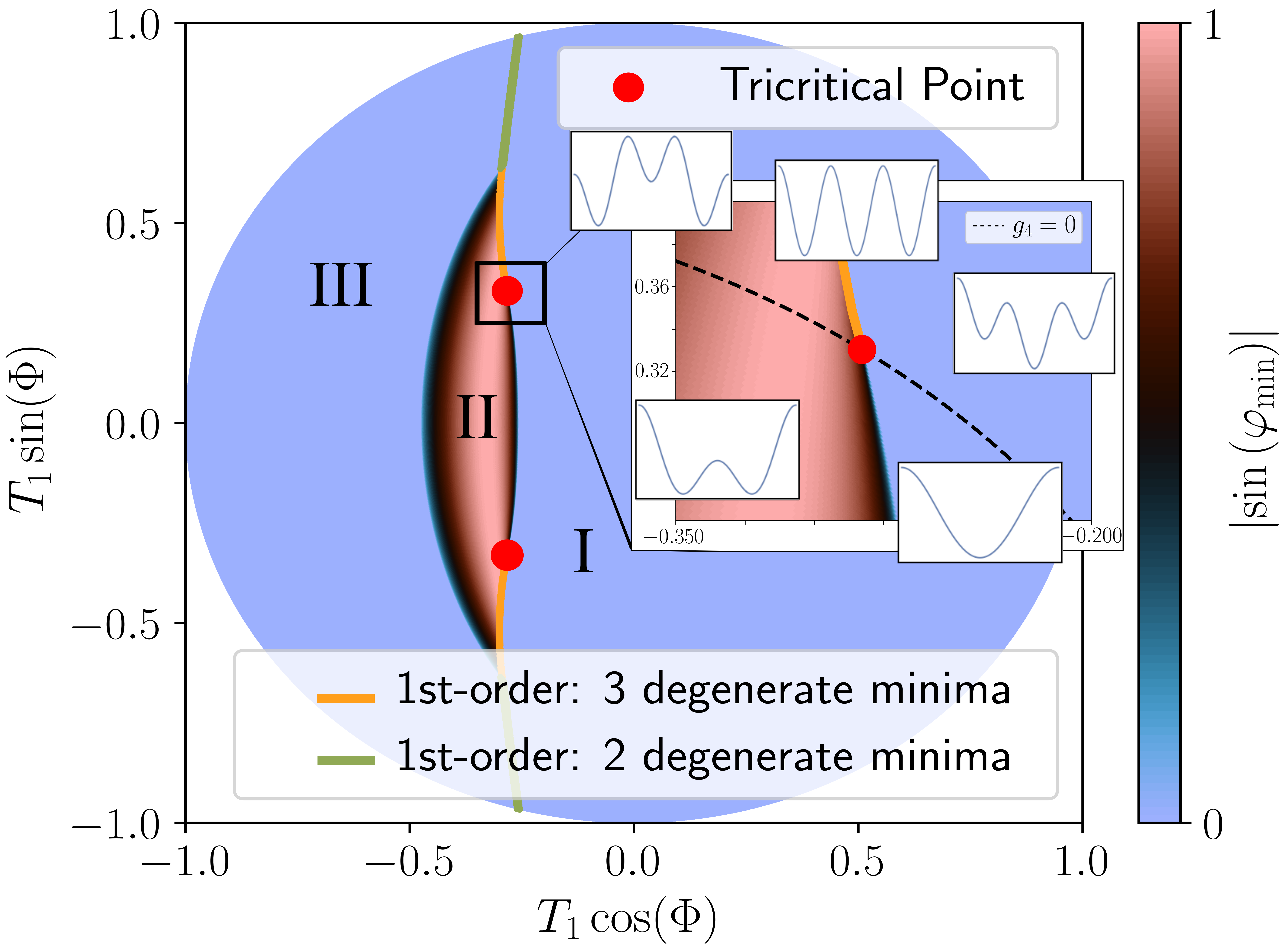

- Order parameter: $\left|\sin\left(\varphi_{\min}\right)\right|$

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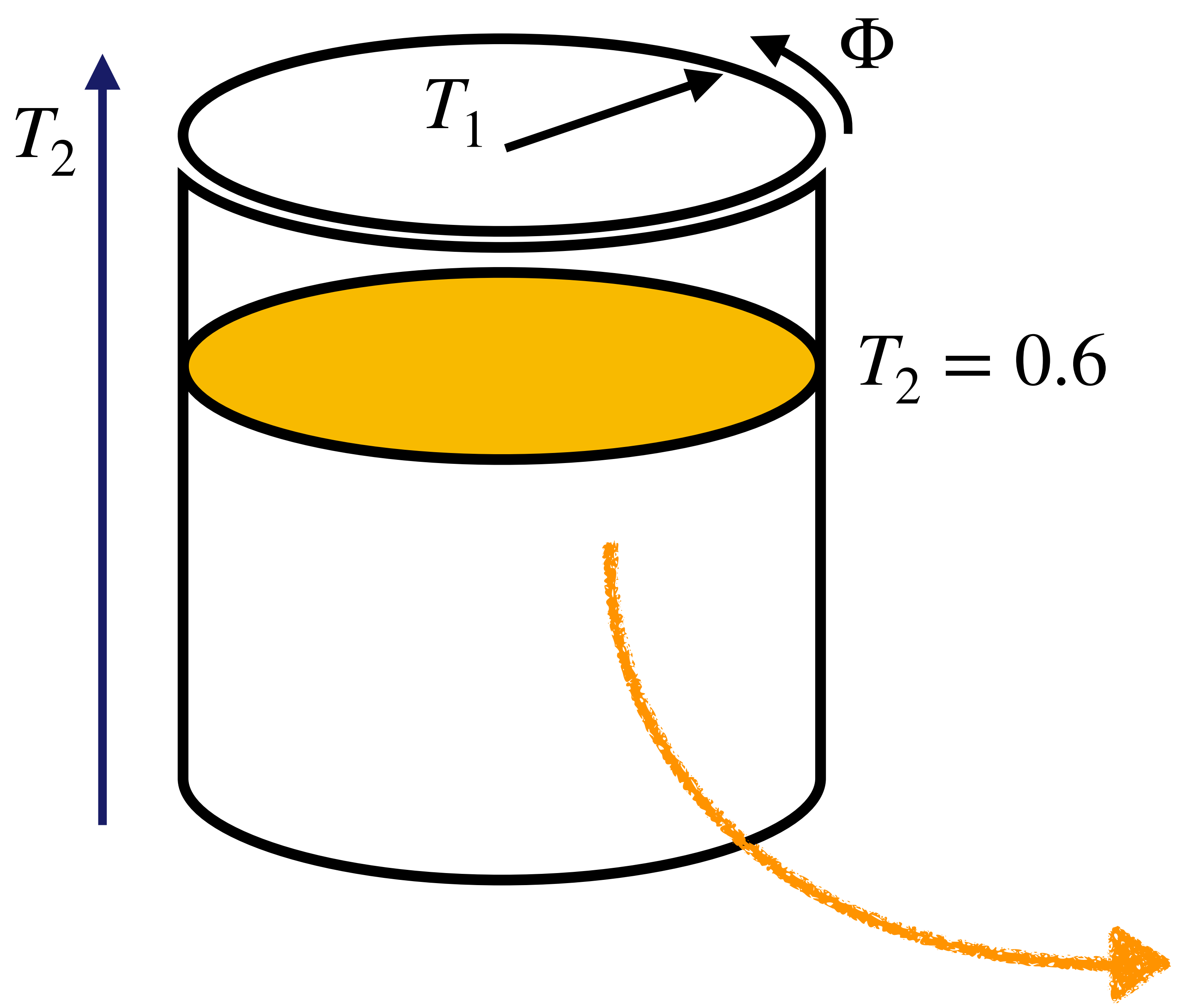

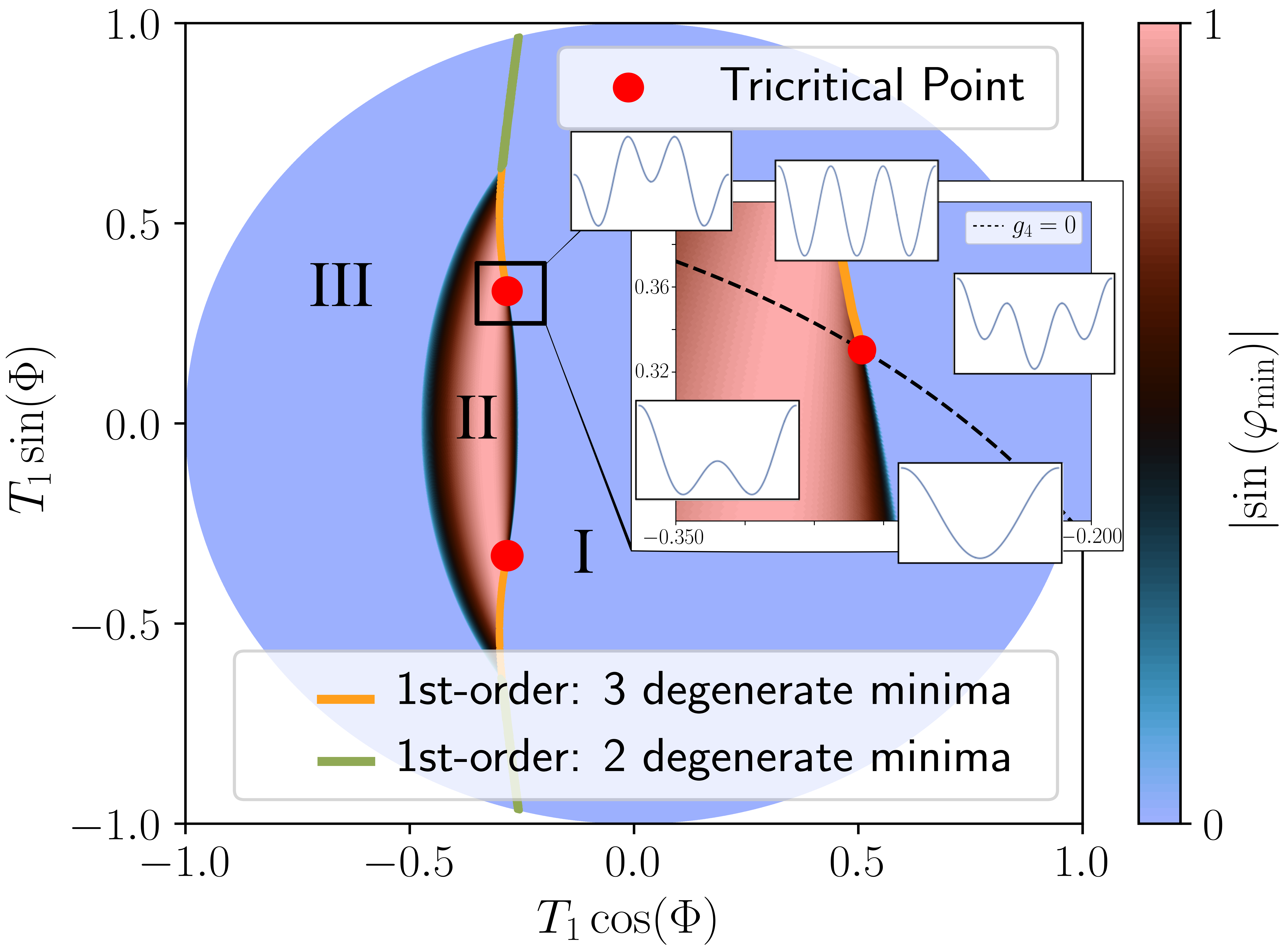

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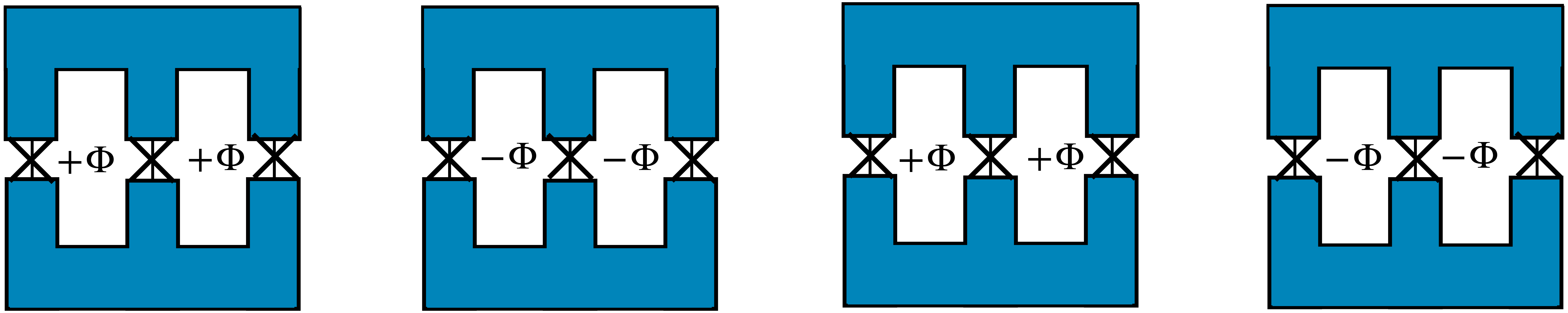
We need to promote the phase difference to a field $\varphi \to \varphi(x)$

Ladder construction

$$\left[\hat{N}_{j,x}\,,\,e^{i\,\hat{\varphi}_{j',x'}}\right] = -\,\delta_{j,j'}\,\delta_{x,x'}\,e^{i\,\hat{\varphi}_{j,x}} \qquad j = \uparrow\,,\downarrow$$

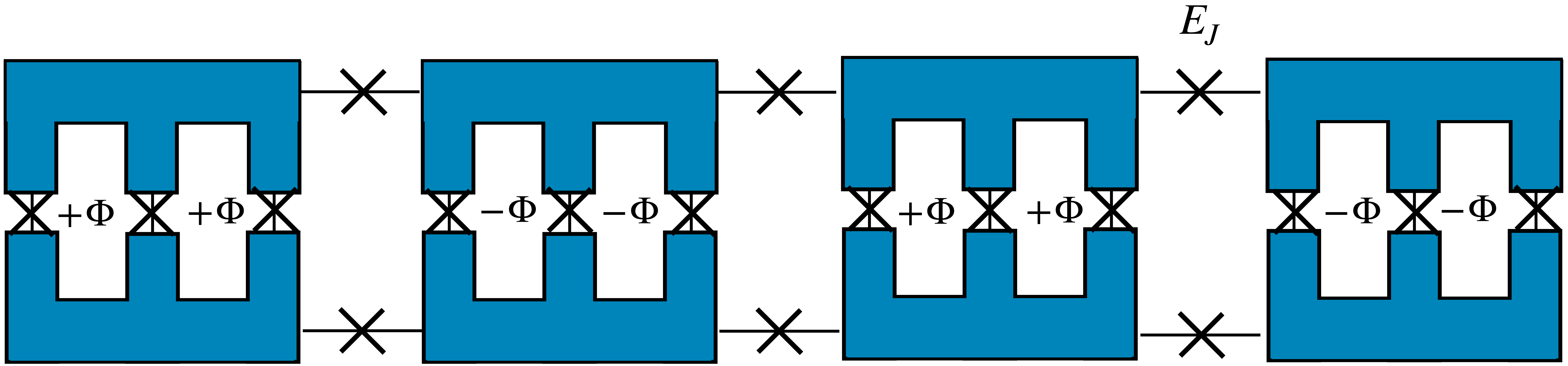


$$H = \sum_{x=0}^{L} V_{\mathrm{loc}}\left(\varphi_{\uparrow,x} - \varphi_{\downarrow,x}\right)$$

Ladder construction

$$\left[\hat{N}_{j,x}, e^{i\hat{\varphi}_{j',x'}}\right] = -\delta_{j,j'}\delta_{x,x'}\, e^{i\hat{\varphi}_{j,x}} \qquad j = \uparrow, \downarrow$$

- Standard Josephson junction along the leg

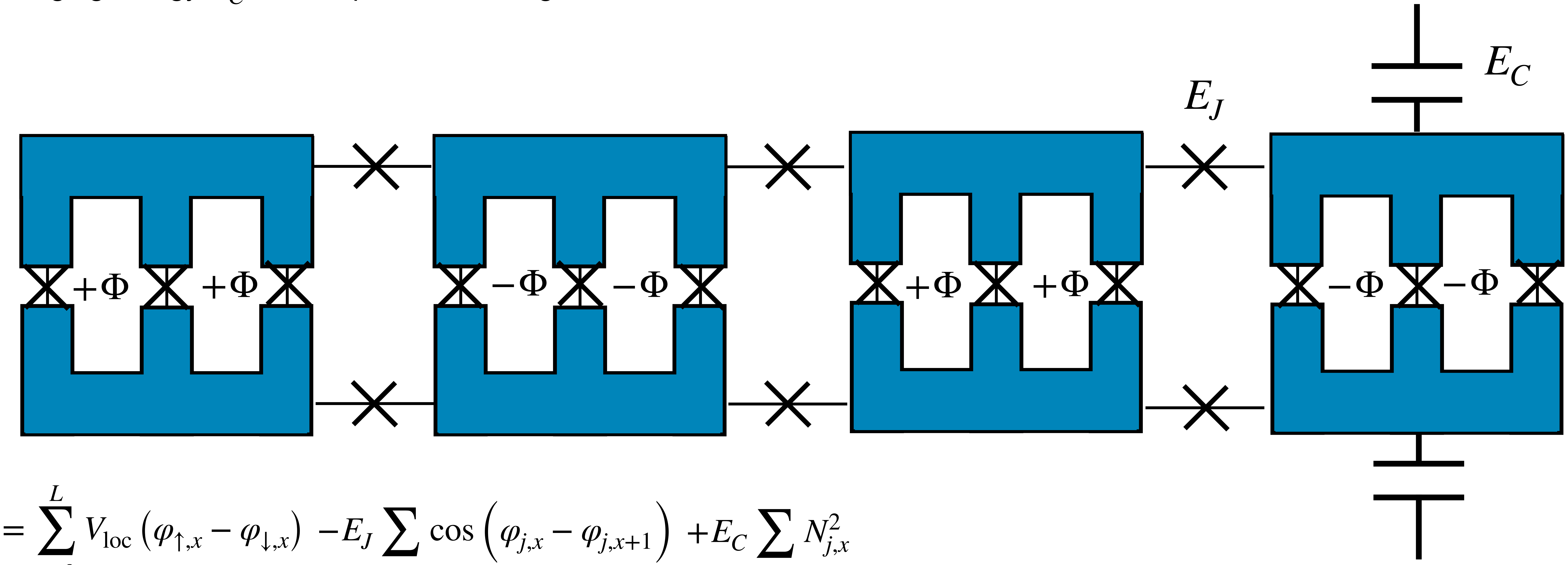


$$H = \sum_{x=0}^{L} V_{\text{loc}}\left(\varphi_{\uparrow,x} - \varphi_{\downarrow,x}\right) - E_J \sum_{j=\uparrow\downarrow} \cos\left(\varphi_{j,x} - \varphi_{j,x+1}\right)$$

Ladder construction

$$\left[\hat{N}_{j,x}, e^{i\hat{\varphi}_{j',x'}}\right] = -\delta_{j,j'}\delta_{x,x'}\,e^{i\hat{\varphi}_{j,x}} \qquad j = \uparrow, \downarrow$$

- Standard Josephson junction along the leg
- Charging energy E_C with respect to a background

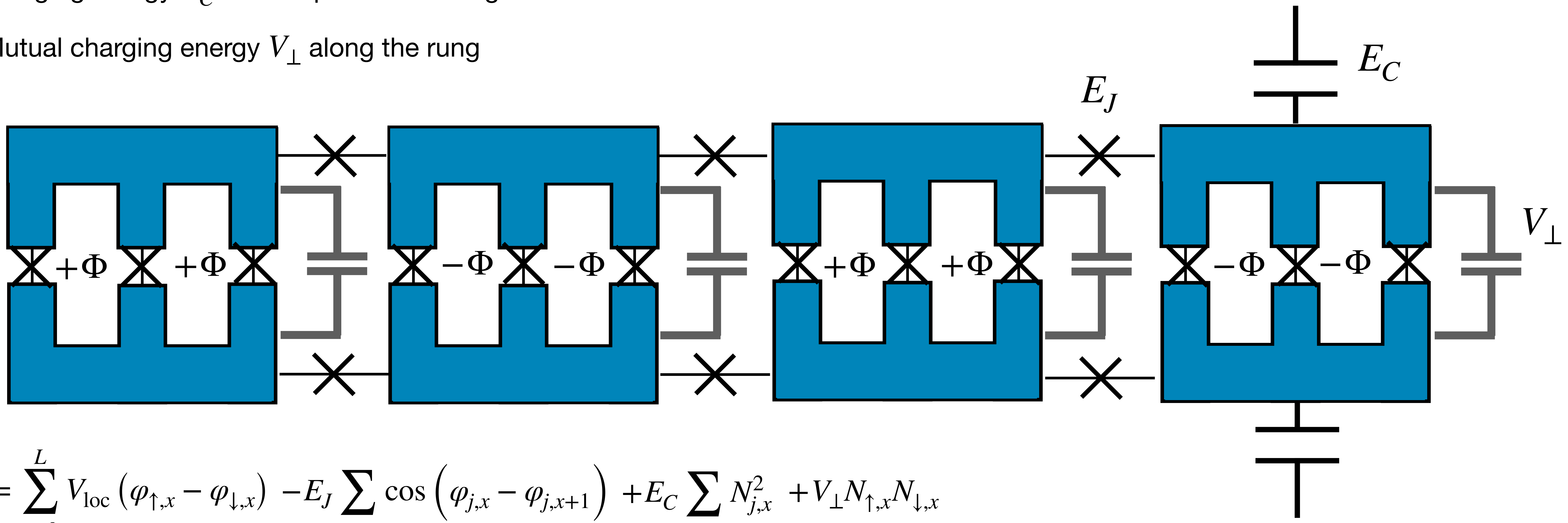


$$H = \sum_{x=0}^{L} V_{\text{loc}}\left(\varphi_{\uparrow,x} - \varphi_{\downarrow,x}\right) - E_J \sum_{j=\uparrow\downarrow} \cos\left(\varphi_{j,x} - \varphi_{j,x+1}\right) + E_C \sum_{j=\uparrow\downarrow} N_{j,x}^2$$

Ladder construction

$$\left[\hat{N}_{j,x}, e^{i\hat{\varphi}_{j',x'}}\right] = -\delta_{j,j'}\delta_{x,x'}e^{i\hat{\varphi}_{j,x}} \qquad j = \uparrow, \downarrow$$

- Standard Josephson junction along the leg
- Charging energy E_C with respect to a background
- Mutual charging energy $V_\perp$ along the rung



$$H = \sum_{x=0}^{L} V_{\rm loc}\left(\varphi_{\uparrow,x} - \varphi_{\downarrow,x}\right) - E_J \sum_{j=\uparrow\downarrow} \cos\left(\varphi_{j,x} - \varphi_{j,x+1}\right) + E_C \sum_{j=\uparrow\downarrow} N_{j,x}^2 + V_\perp N_{\uparrow,x} N_{\downarrow,x}$$

Bosonization - harmonic approximation $(E_J \gg E_C, V_\perp, V_{\text{loc}})$

$$\left[N_{j,x}, e^{i\varphi_{j',x'}}\right] = -\delta_{j,j'}\delta_{x,x'}e^{i\varphi_{j,x}}$$

$$H = \sum_{x=0}^{L} V_{\text{loc}}\left(\varphi_{\uparrow,x} - \varphi_{\downarrow,x}\right)$$

$$\frac{\left(\varphi_{\uparrow,x} + \varphi_{\downarrow,x}\right)}{\sqrt{2}} \to \varphi_c(x)$$

$$\frac{\left(\varphi_{\uparrow,x} - \varphi_{\downarrow,x}\right)}{\sqrt{2}} \to \varphi_s(x)$$

$$\sum_{x=0}^{L} \to \frac{1}{a}\int_{x=0}^{L} dx$$

$$N_{j,x} \to -a\frac{\partial_x \theta_{j,x}}{\pi}$$

2 Luttinger Liquids: charge/spin sector $\varphi_{c/s}(x)$

$$H = \sum_{q\in\{c,s\}} u_q \int \frac{dx}{2\pi}\left[K_q\left(\partial_x\varphi_q\right)^2 + \frac{1}{K_q}\left(\partial_x\theta_q\right)^2\right] + $$
$$+\int dx\left[\mu_1\cos\left(\sqrt{2}\varphi_s\right) + \mu_2\cos\left(2\sqrt{2}\varphi_s\right) + \mu_3\cos\left(3\sqrt{2}\varphi_s\right)\right].$$

Effective potential

- Luttinger parameters

$$K_{c/s} = \pi\sqrt{\frac{E_J}{(2E_c \pm V_\perp)}}$$

$$u_{c/s} = a\sqrt{E_J\left(2E_c \pm V_\perp\right)}$$

- The first three harmonics are relevant for $K_s > 9/4$.
- The other harmonics are less relevant. Moreover $\left|\mu_{n\geq4}/\mu_{n\leq3}\right| \lesssim 10^{-2}$.

Three-frequency SG model In the 'spin' sector

Bosonization - harmonic approximation

$(E_J \gg E_C, V_\perp, V_{\text{loc}})$

$$\left[N_{j,x}, e^{i\varphi_{j',x'}}\right] = -\delta_{j,j'}\delta_{x,x'}e^{i\varphi_{j,x}}$$

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$$\frac{(\varphi_{\uparrow,x} + \varphi_{\downarrow,x})}{\sqrt{2}} \to \varphi_c(x)$$

$$\frac{(\varphi_{\uparrow,x} - \varphi_{\downarrow,x})}{\sqrt{2}} \to \varphi_s(x)$$

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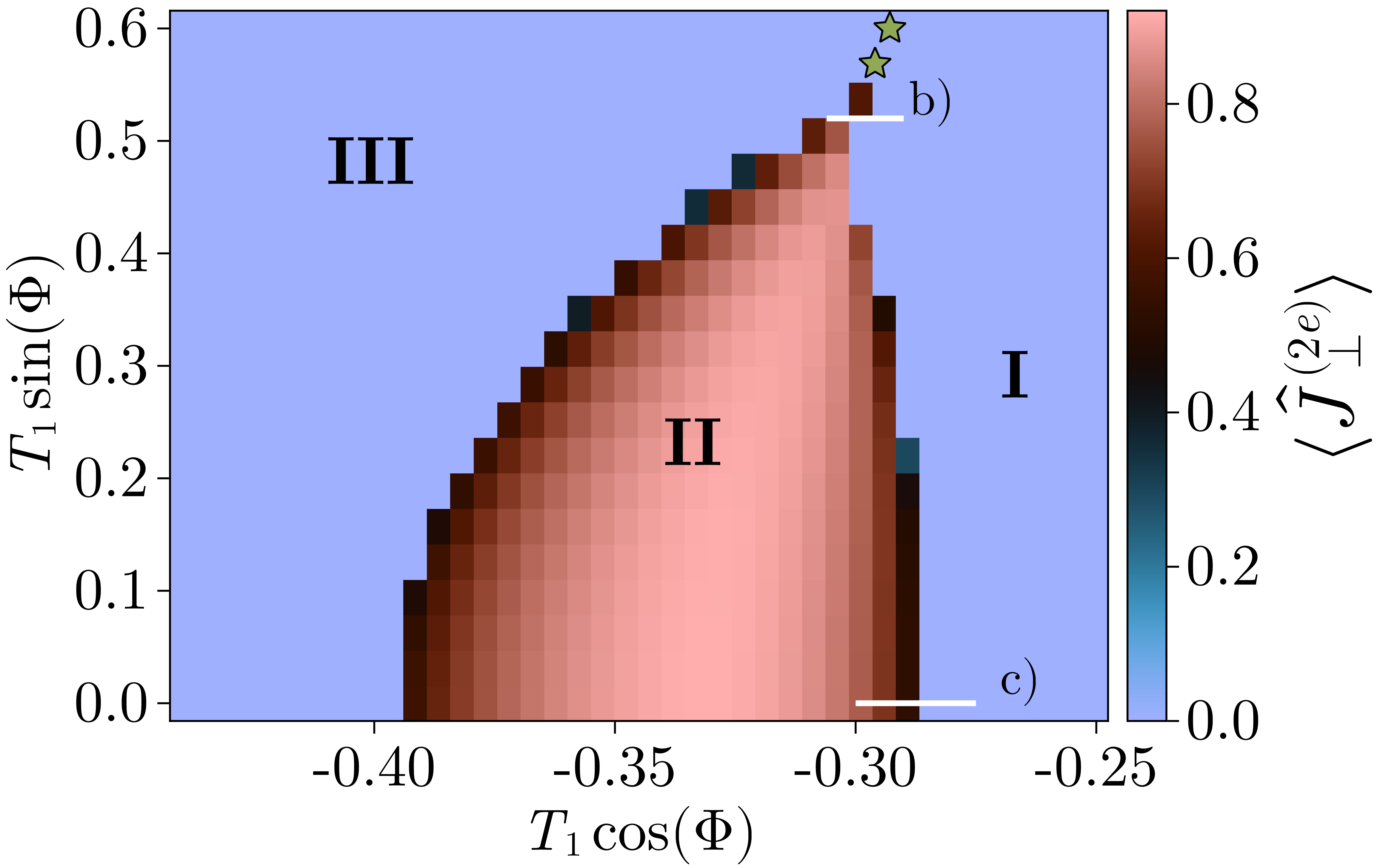
Three-frequency SG model In the 'spin' sector

Phase-diagram from Tensor Networks (VUMPS)

(Truncation local Hilbert space: $\left|N_{j,x}\right| \leq 8$, Bond-dimension $D = 600$)



- Order parameter:

$$\hat{J}_\perp^{(2e)}(x) = \sin\left(\sqrt{2}\,\hat{\varphi}_s(x)\right)$$

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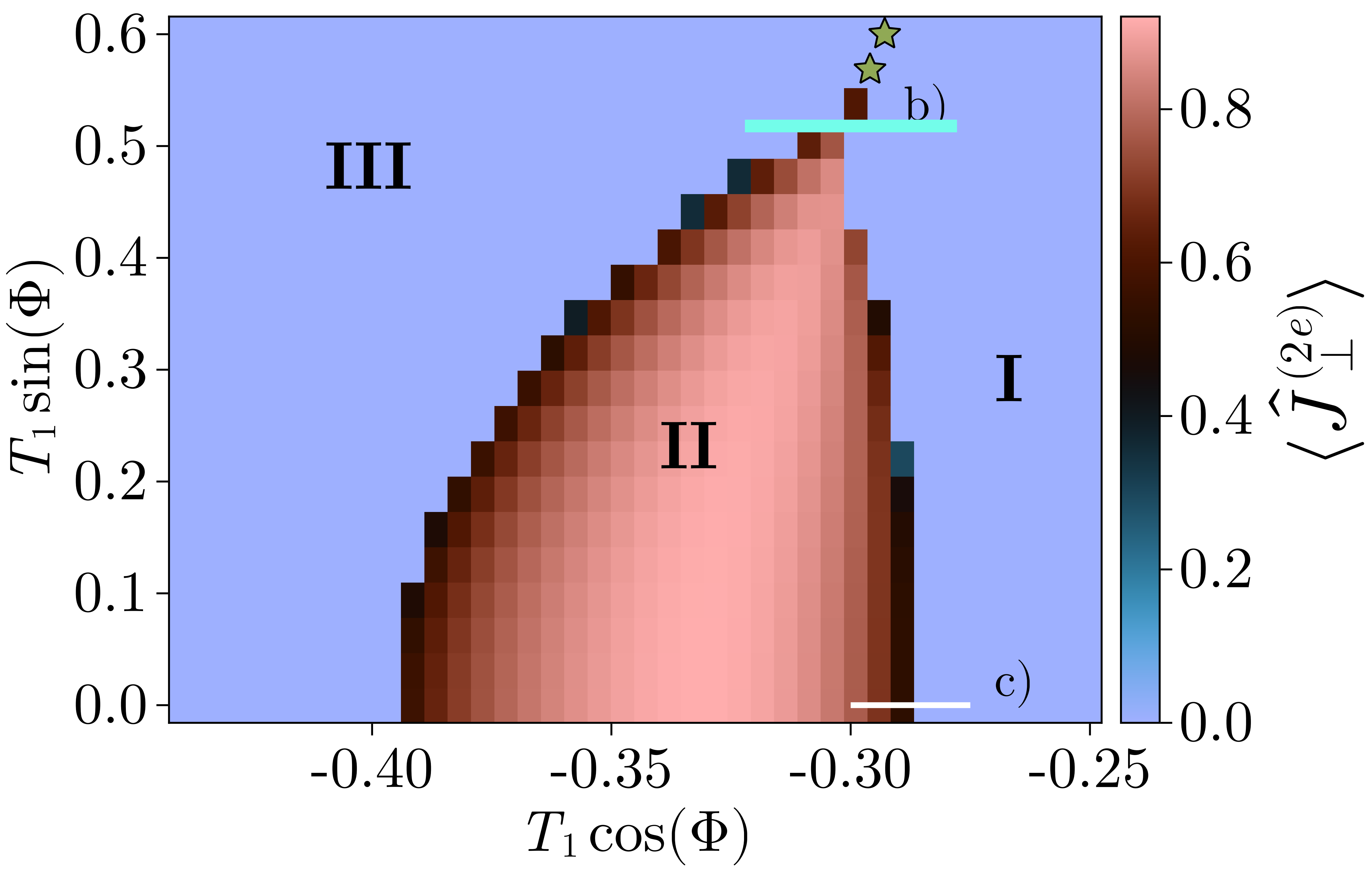

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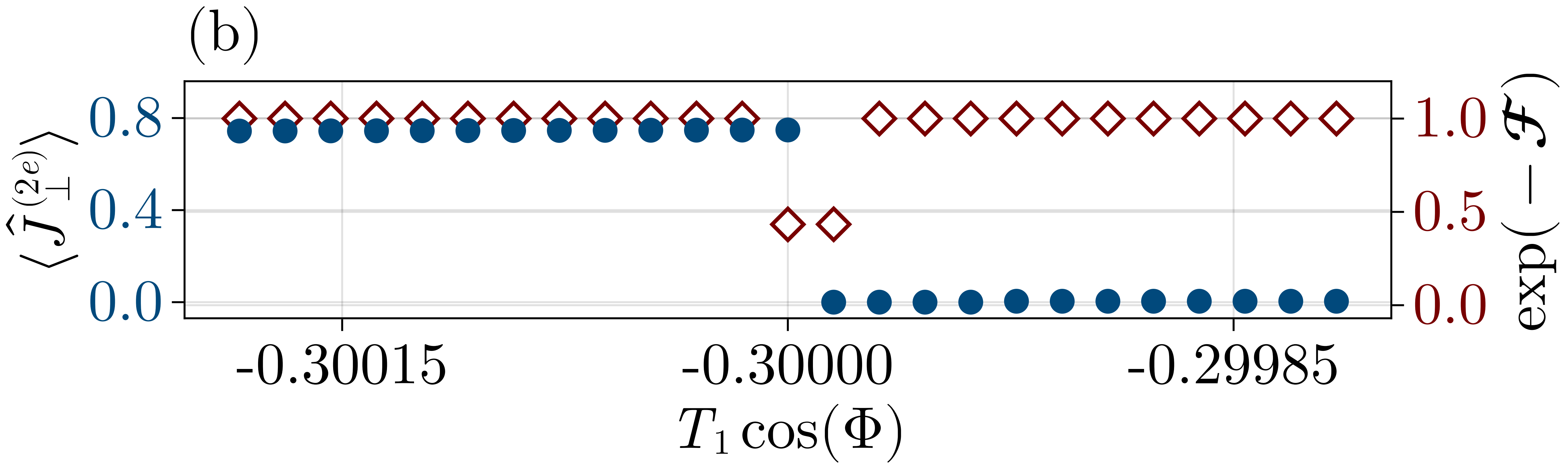
- **First order transition between I and II**



- Fidelity per lattice sites (VUMPS $\Longrightarrow N \to \infty$)

$$e^{-\mathcal{F}} = \lim_{N\to\infty} \left(\langle \psi\left(\mathbf{X} - \delta\right) | \psi\left(\mathbf{X} + \delta\right)\rangle\right)^{1/N}$$

[Rams et al PRL '11]

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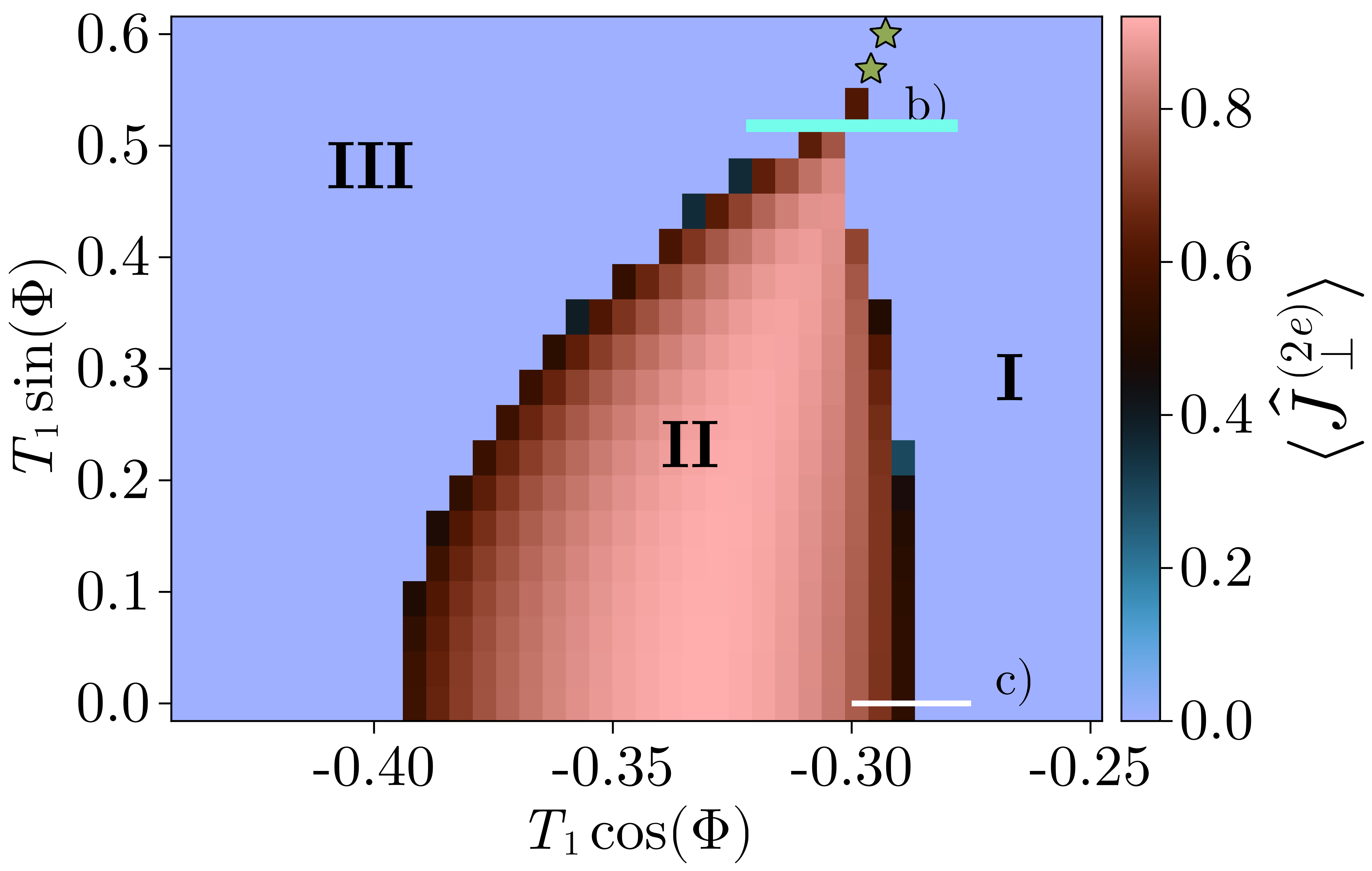

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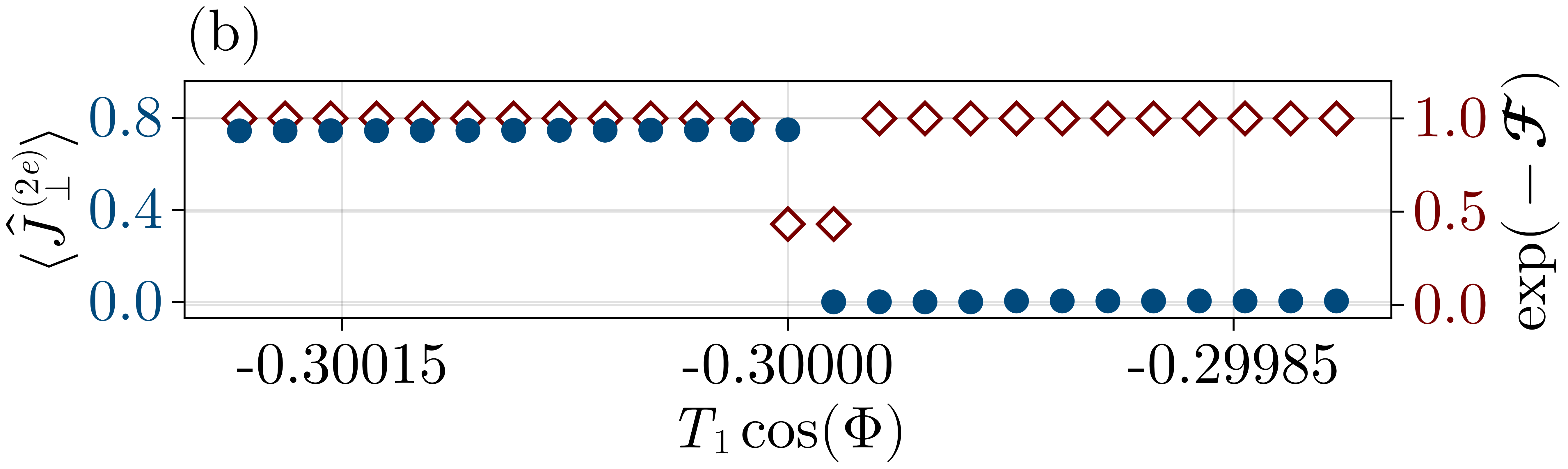

- Fidelity per lattice sites (VUMPS $\implies N \to \infty$)

$$e^{-\mathcal{F}} = \lim_{N\to\infty} \left(\langle \psi\left(\mathbf{X} - \delta\right) | \psi\left(\mathbf{X} + \delta\right)\rangle\right)^{1/N}$$

[Rams et al PRL ’11]

The wavefunction is discontinuous

Phase-diagram from Tensor Networks (VUMPS)

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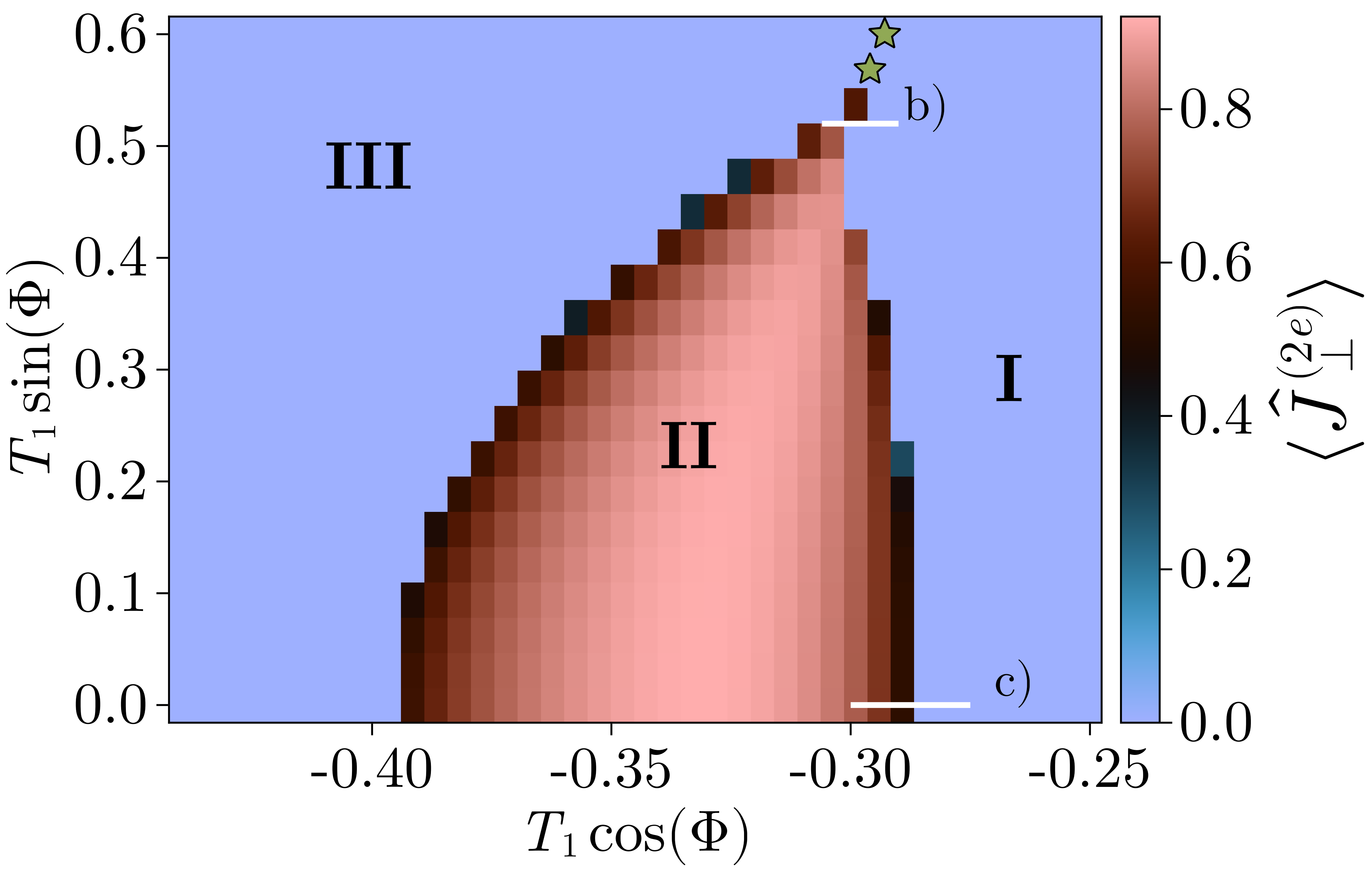

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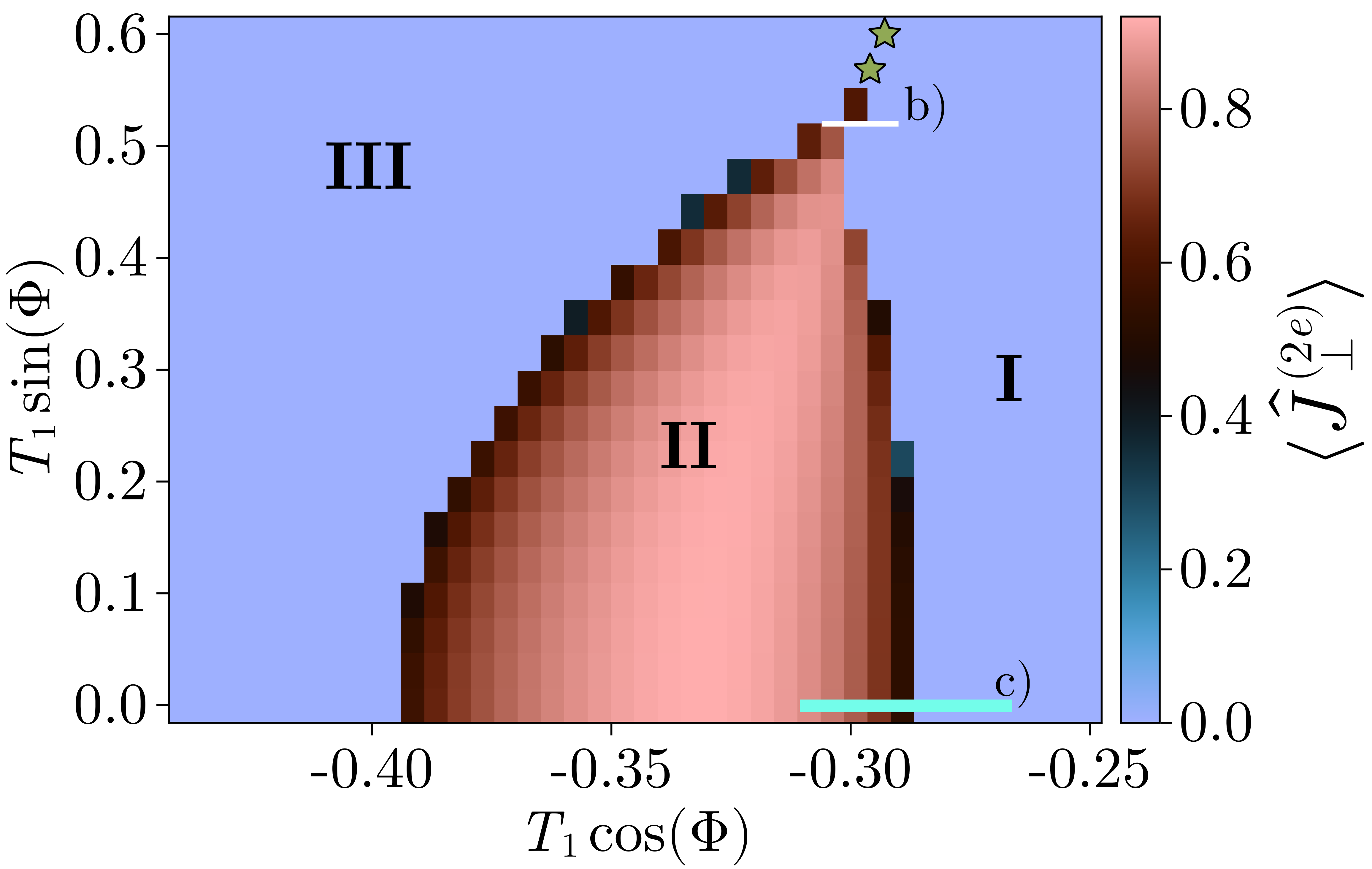
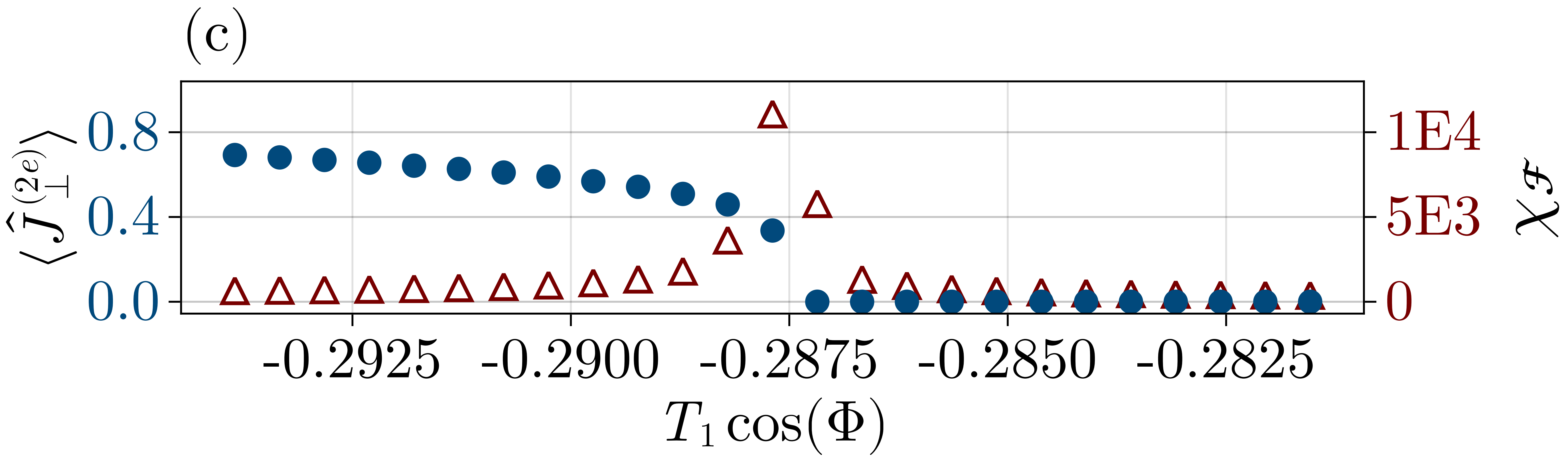


- Order parameter:

$$\hat{J}_\perp^{(2e)}(x) = \sin\left(\sqrt{2}\,\hat{\varphi}_s(x)\right)$$

- **Second order transition between I and II**



- Fidelity susceptibility (VUMPS $\Longrightarrow N \to \infty$)

$$\chi_F = -\lim_{N\to\infty} \frac{1}{N\delta^2} \left(\langle \psi\,(\mathbf{X}-\delta)\,|\,\psi\,(\mathbf{X}+\delta)\rangle\right)$$

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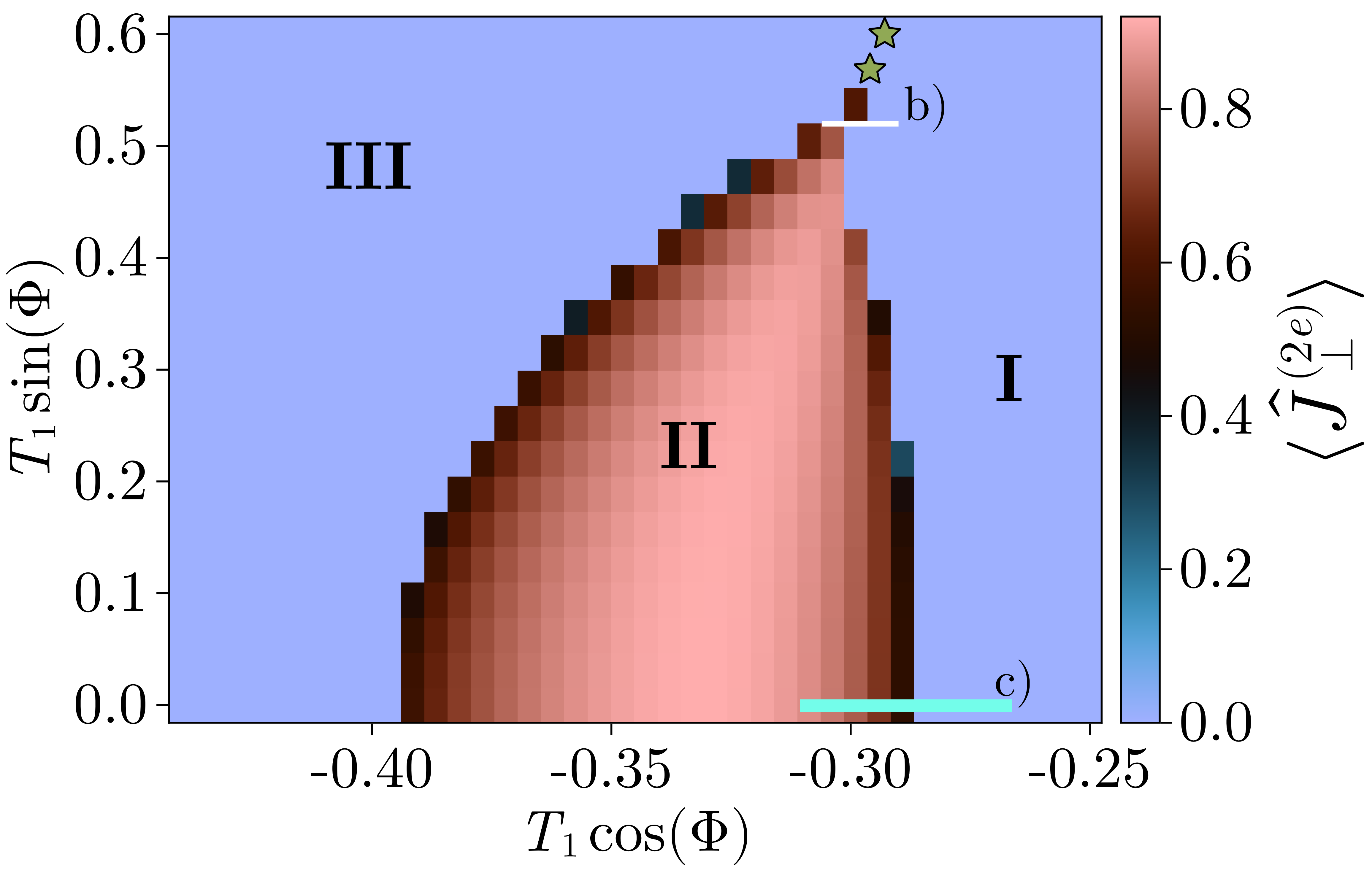

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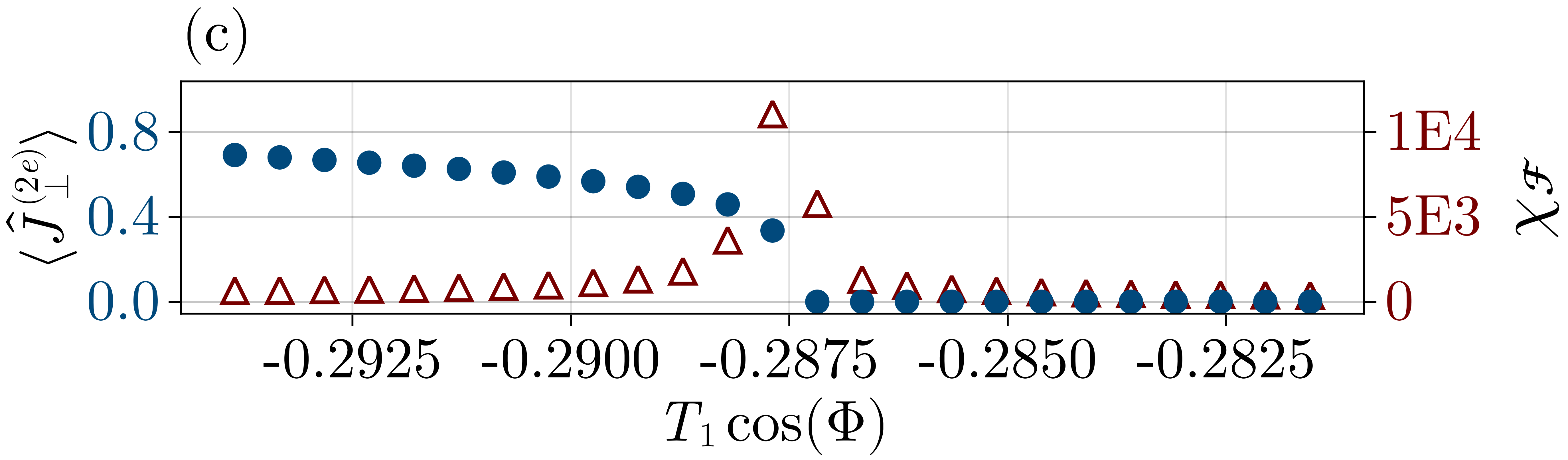

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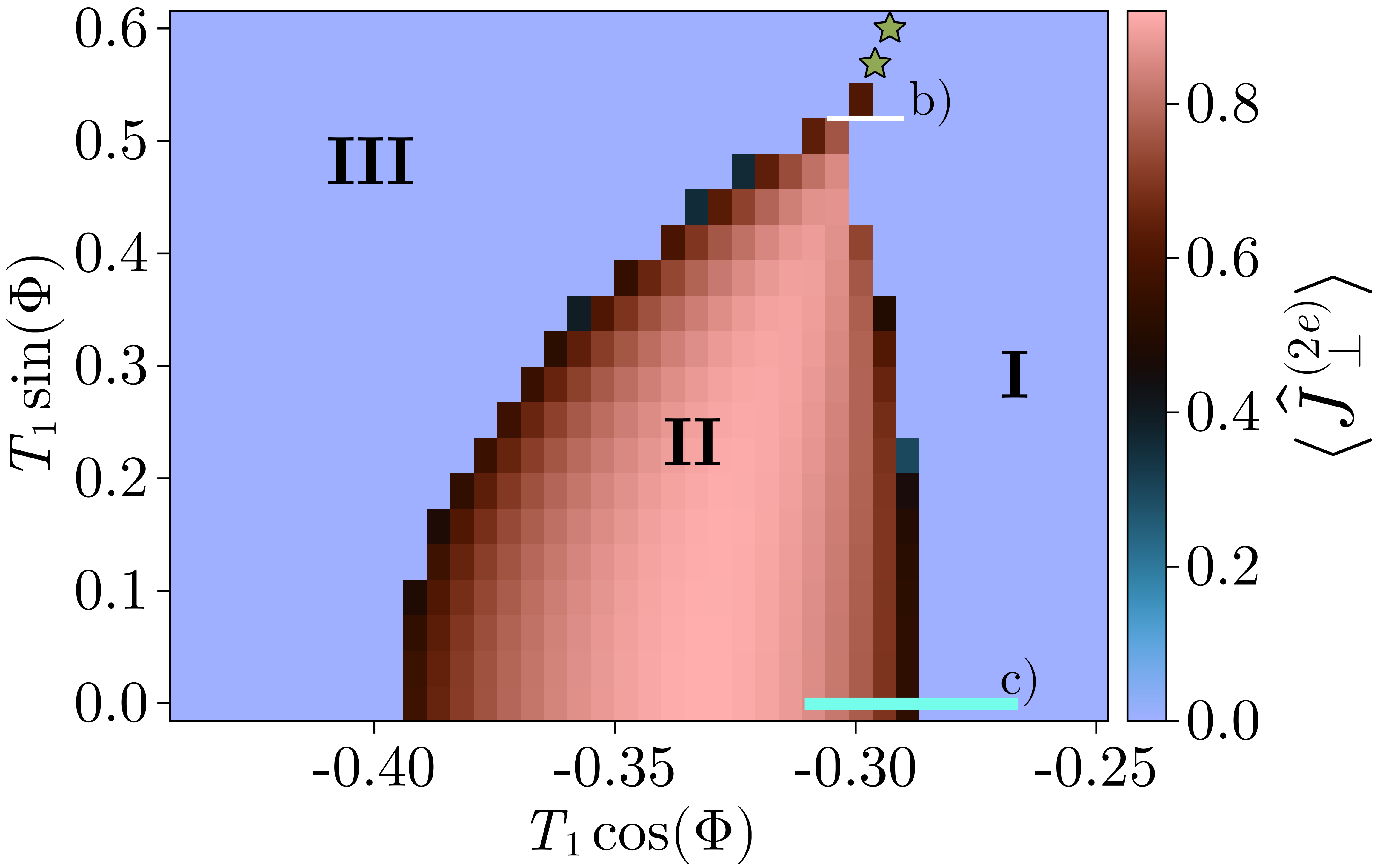
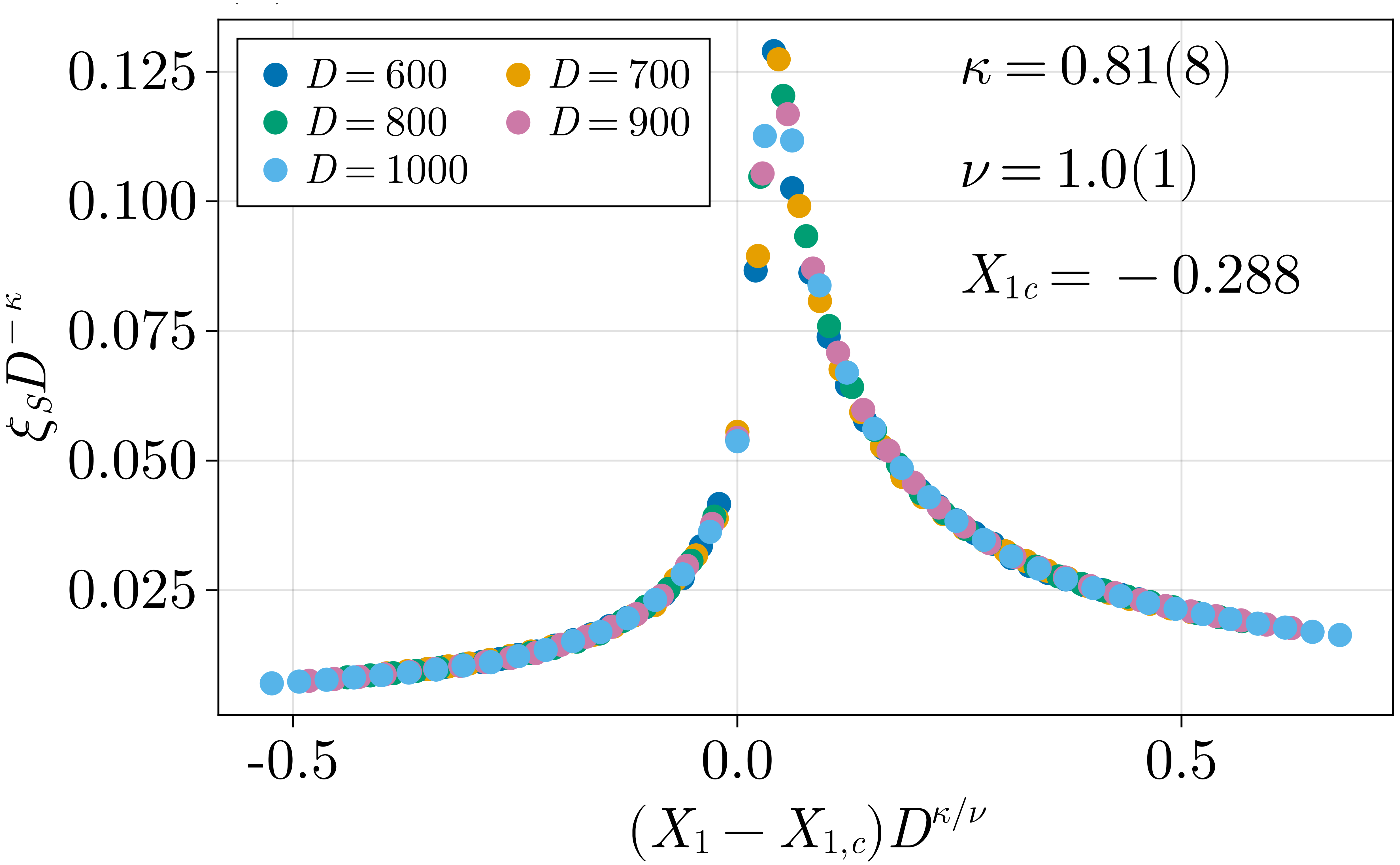
Continuous phase transition

Phase-diagram from Tensor Networks (VUMPS)

(Truncation local Hilbert space: $\left|N_{j,x}\right| \leq 8$, Bond-dimension $D \leq 1000$)



- Ising transition between I and II



- Order parameter:

$$\hat{J}_\perp^{(2e)}(x) = \sin\left(\sqrt{2}\,\hat{\varphi}_s(x)\right)$$

- Finite-entanglement scaling of the spin correlation length ξ_s [Tagliacozzo et al PRB '08]
- At the critical point $\xi_s \propto D^\kappa$ [Pollmann et al PRL '09]

Ising phase transition $\nu_{\mathrm{IS}} = 1$

Phase-diagram from Tensor Networks (VUMPS)

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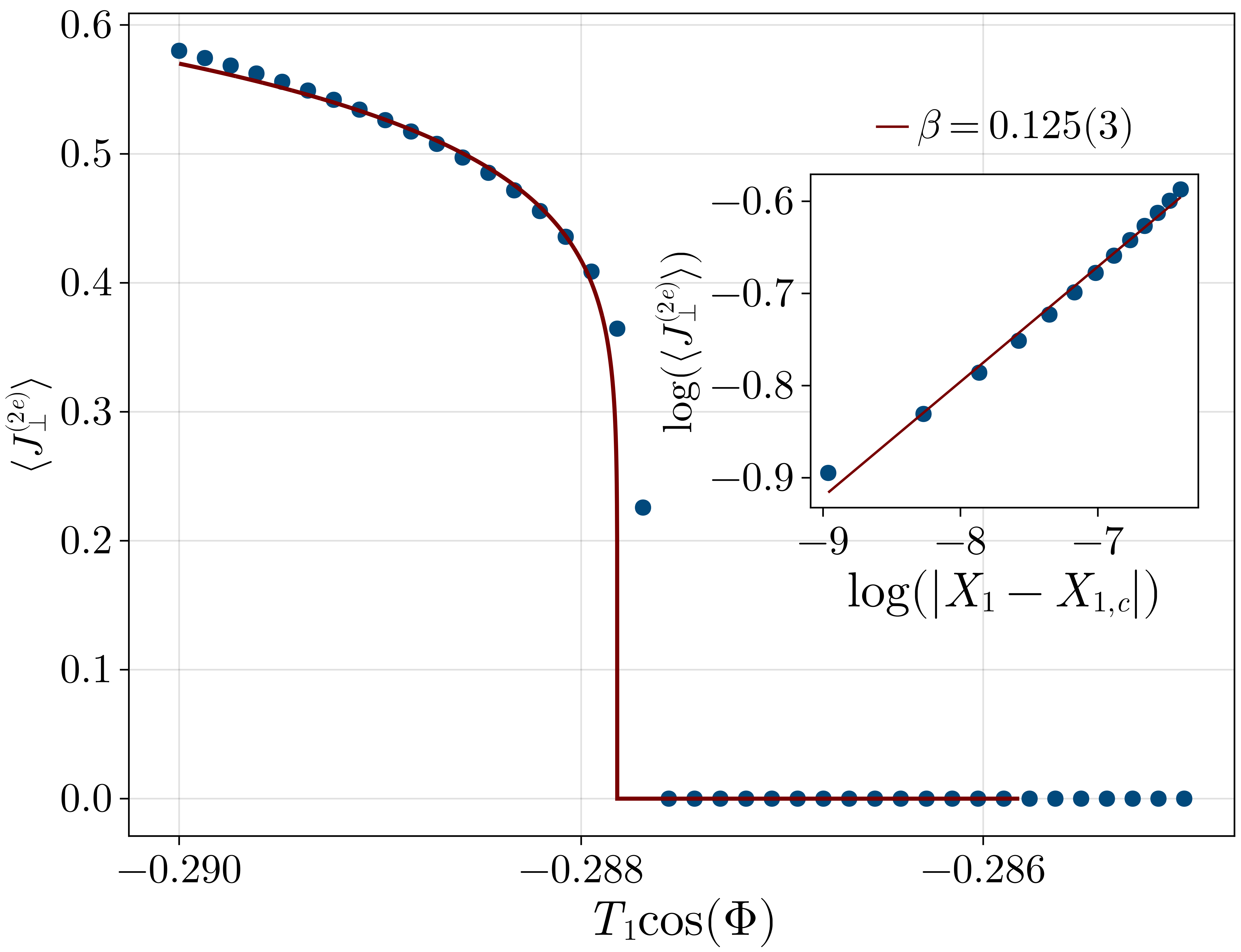
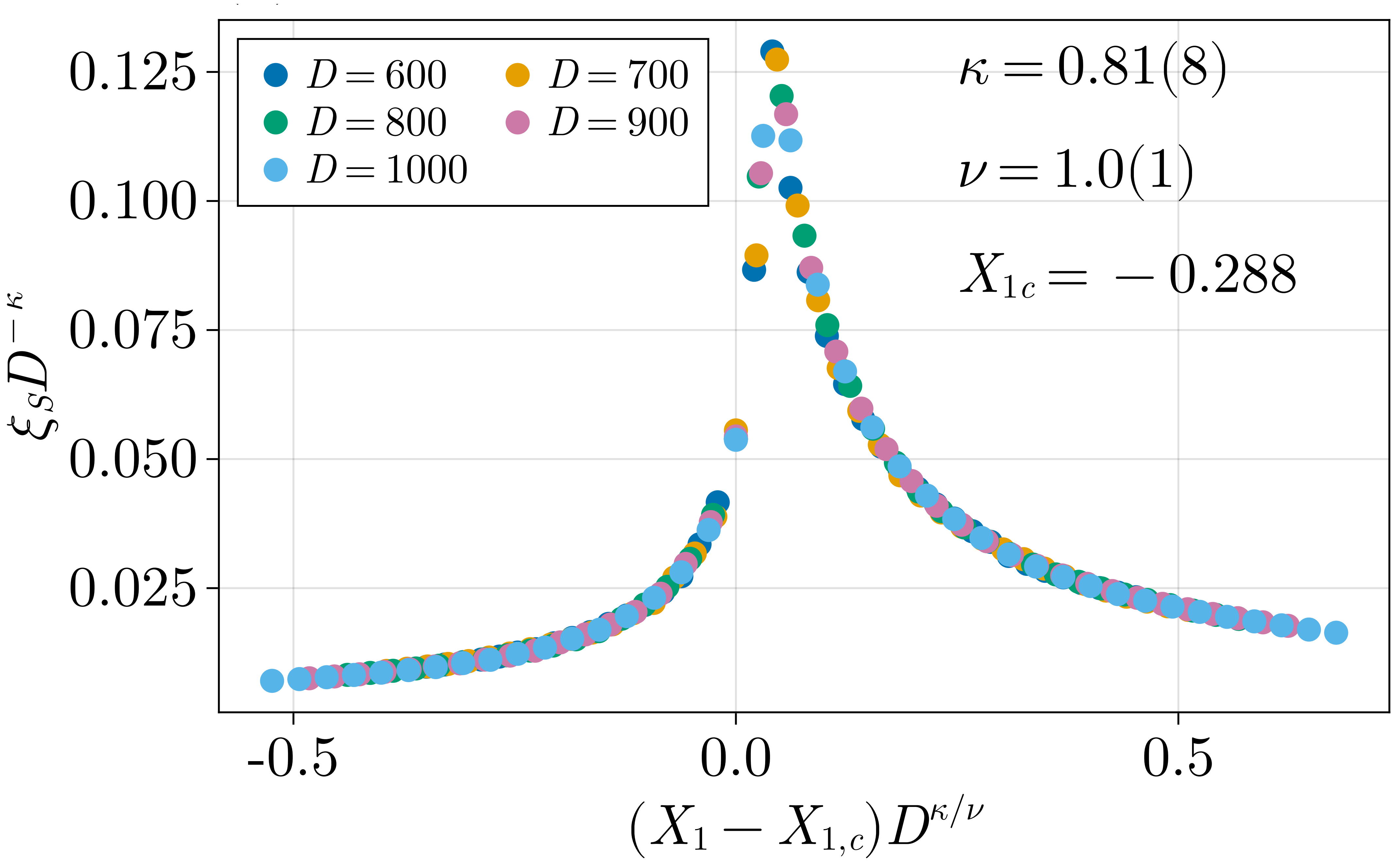


- Order parameter: $\langle \hat{J}_{\perp}^{(2e)} \rangle \propto \left| X_1 - X_{1c} \right|^{\beta}$

Ising phase transition $\beta_{\rm IS} = 1/8$

- **Ising transition between I and II**



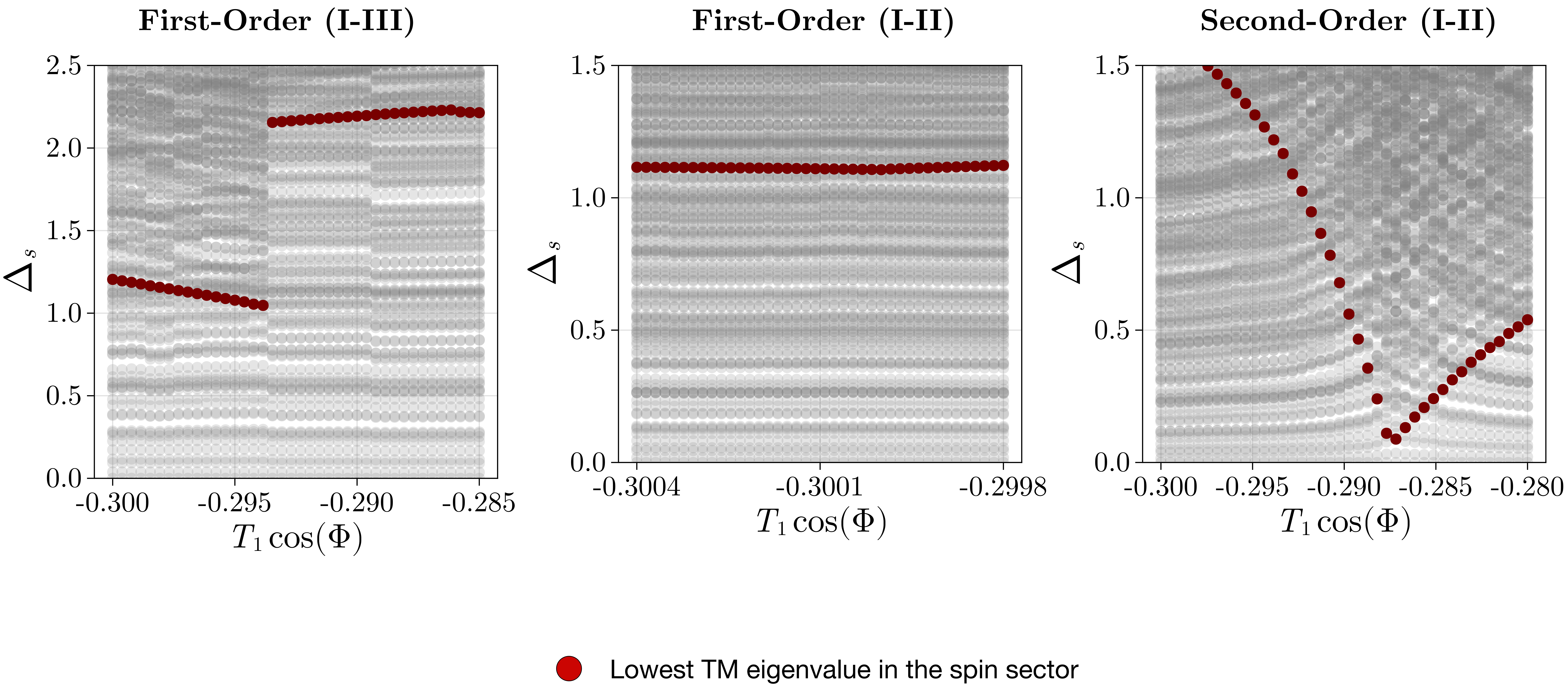
- Finite-entanglement scaling of the spin correlation length ξ_s

[Tagliacozzo et al PRB '08]

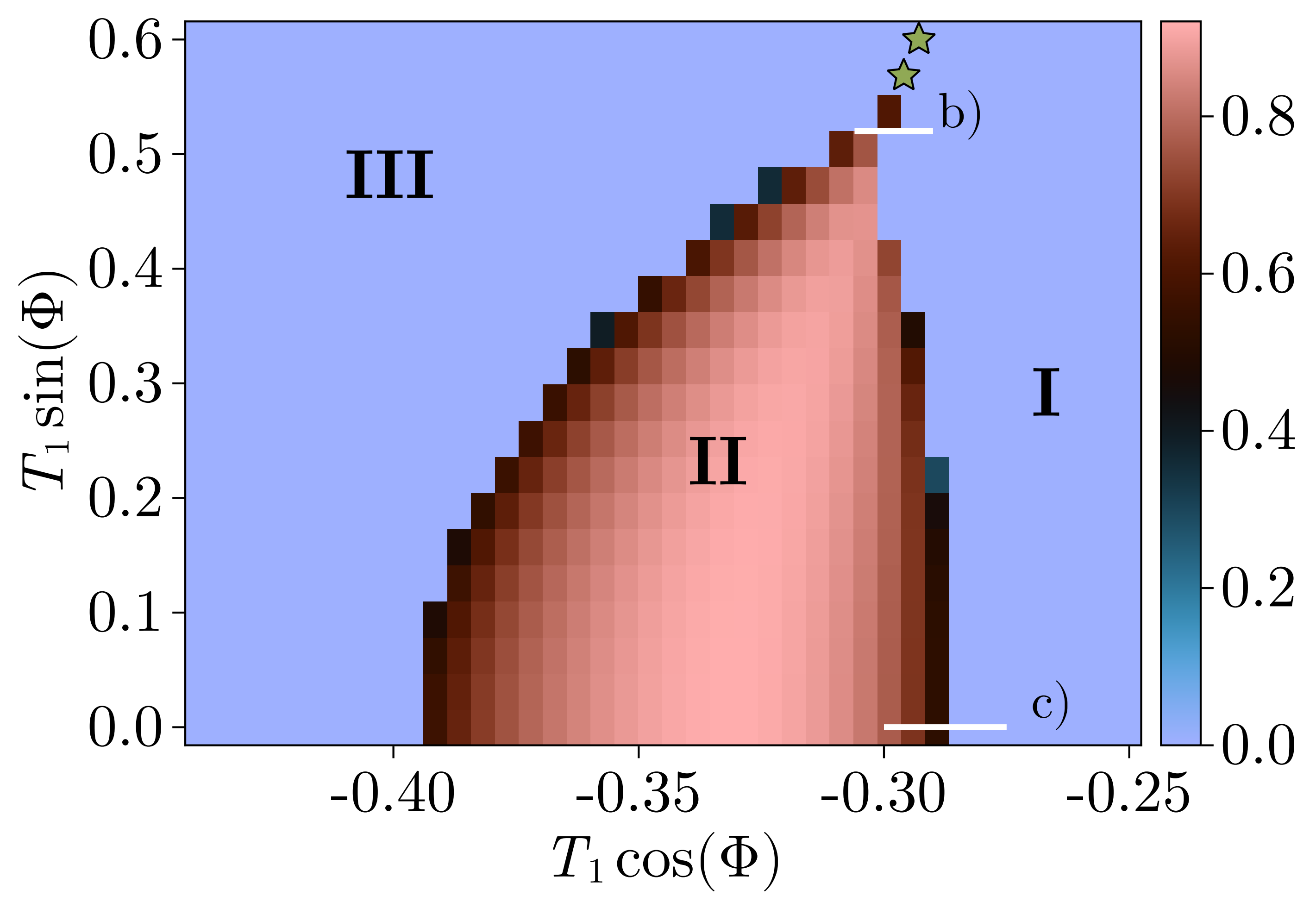
[Pollmann et al PRL '09]

Ising phase transition $\nu_{\rm IS} = 1$

Transfer Matrix spectrum

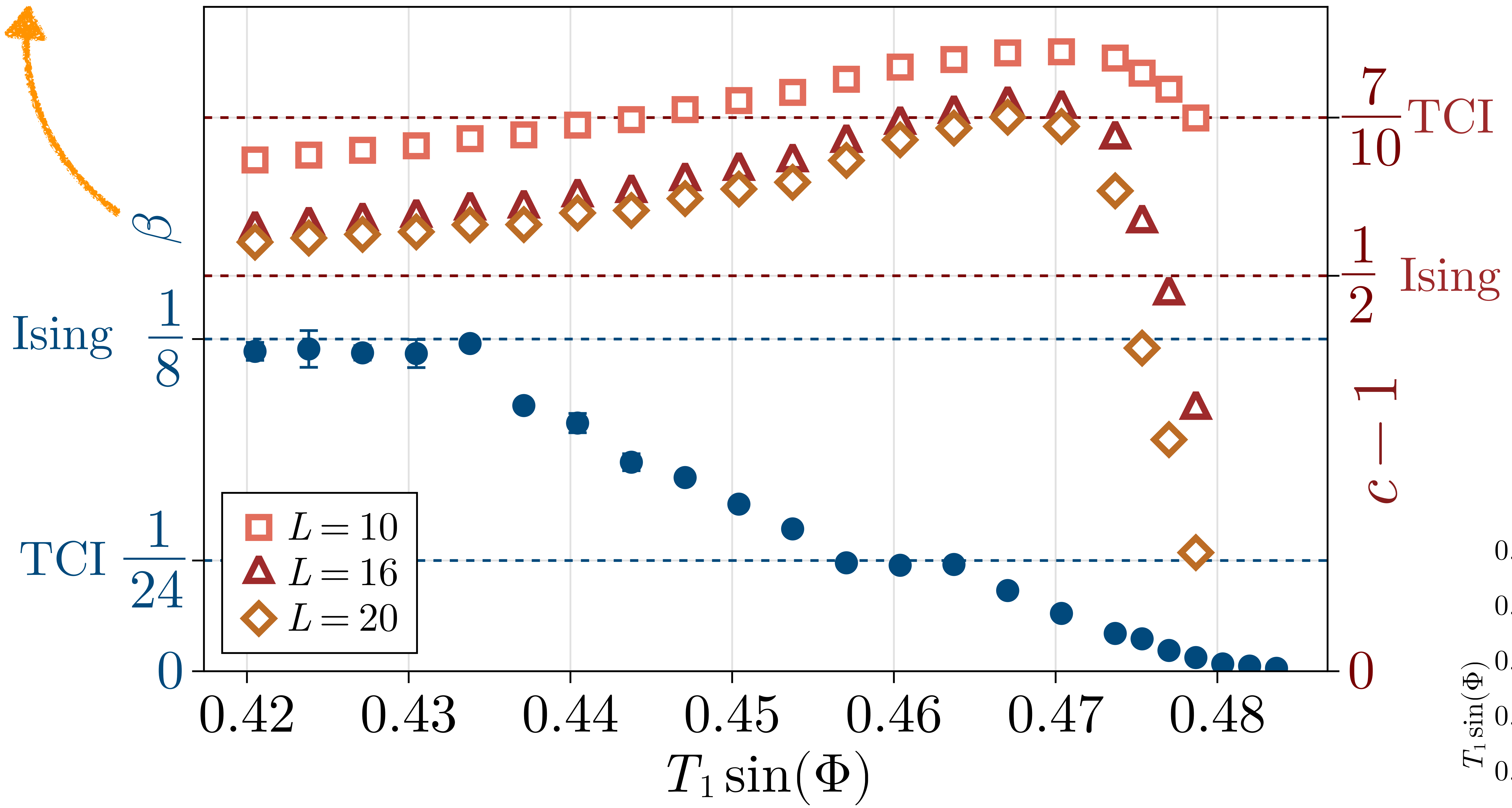


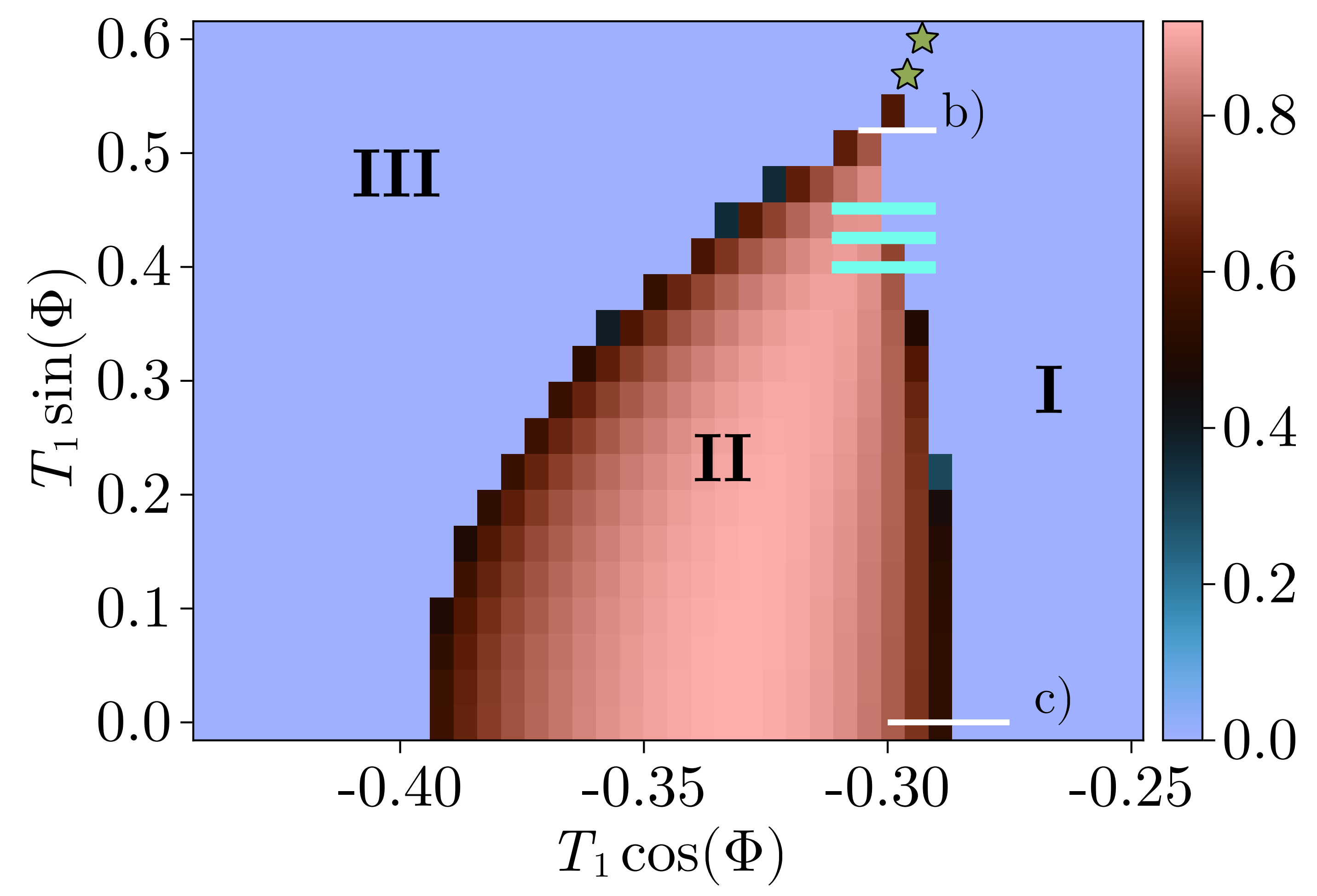
Tricritical Ising point



Tricritical Ising point

VUMPS simulation $D = 600$



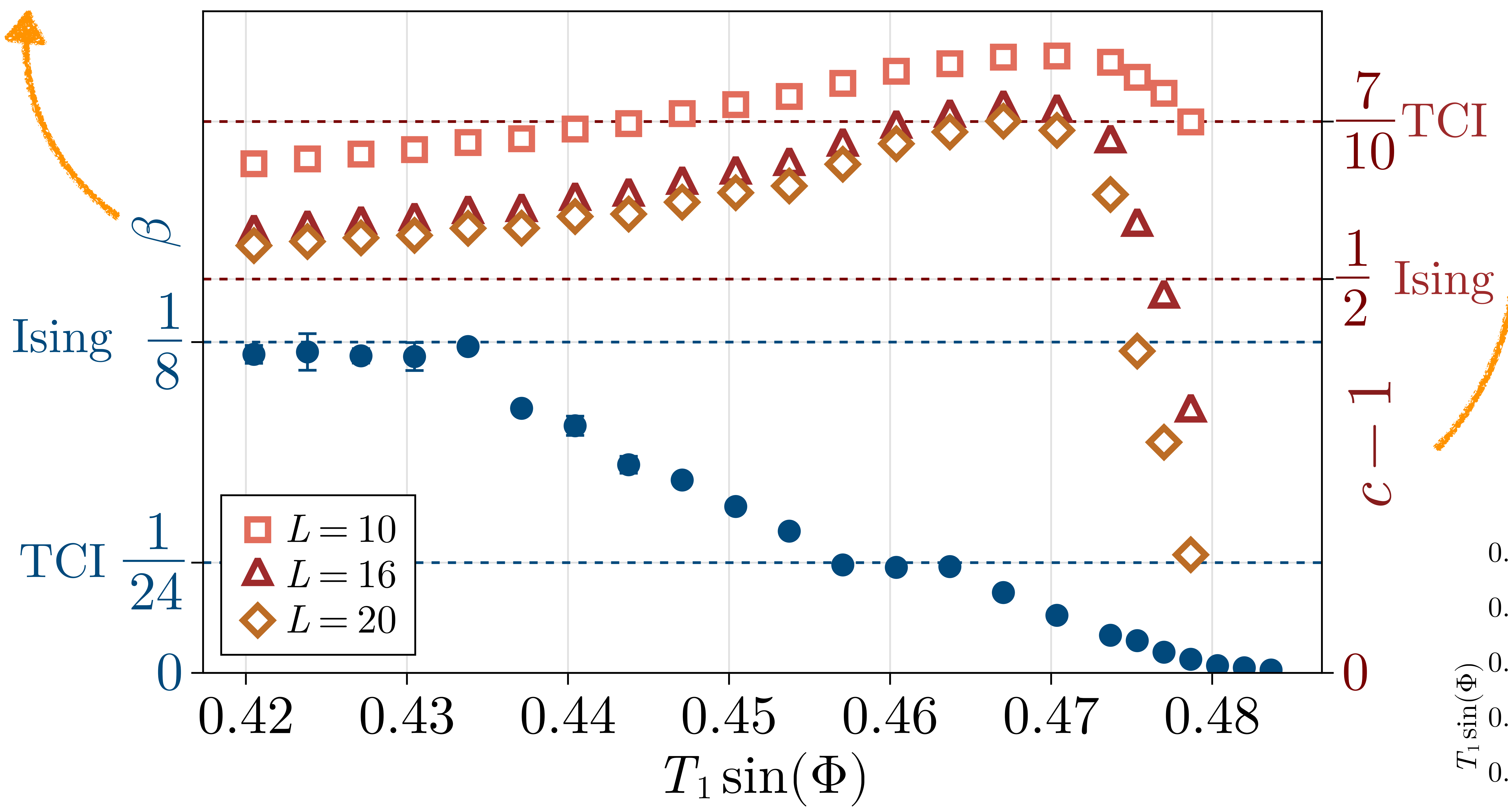


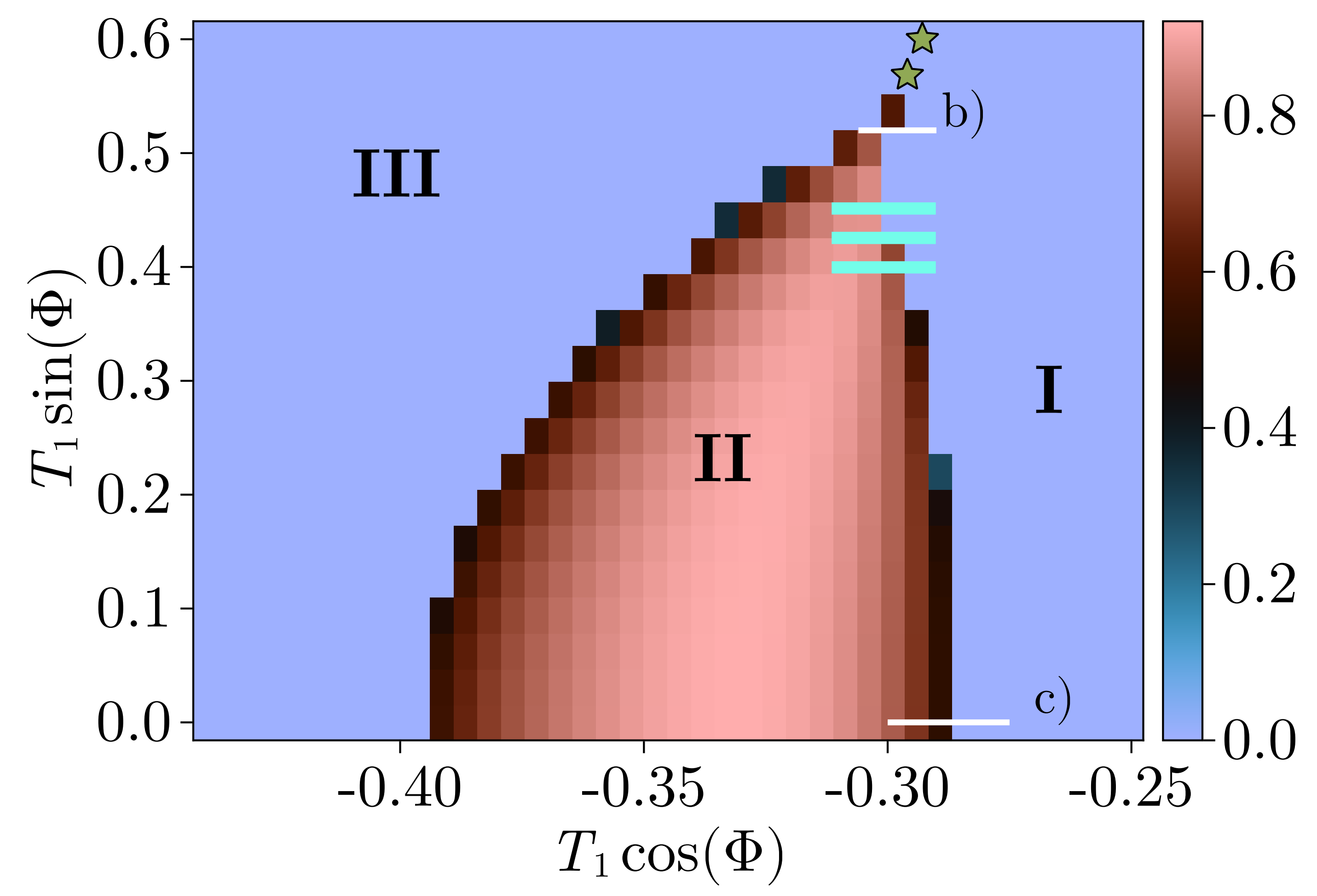
Tricritical Ising point

VUMPS simulation $D = 600$

DMRG finite size simulation

(Entanglement entropy)





Summary

- Hybrid Josephson junctions can be electrically tuned
- This opens the path for solid-state analog quantum simulations
- Ladder models offer the possibility of engineering CFTs
- Ladder geometry and quantum field theory limit
- We can hope to achieve a tricritical Ising point
- The central charge (heat transport!) can be used to distinguish them

Niklas Tausendpfund
(Cologne)

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Michele Burrello
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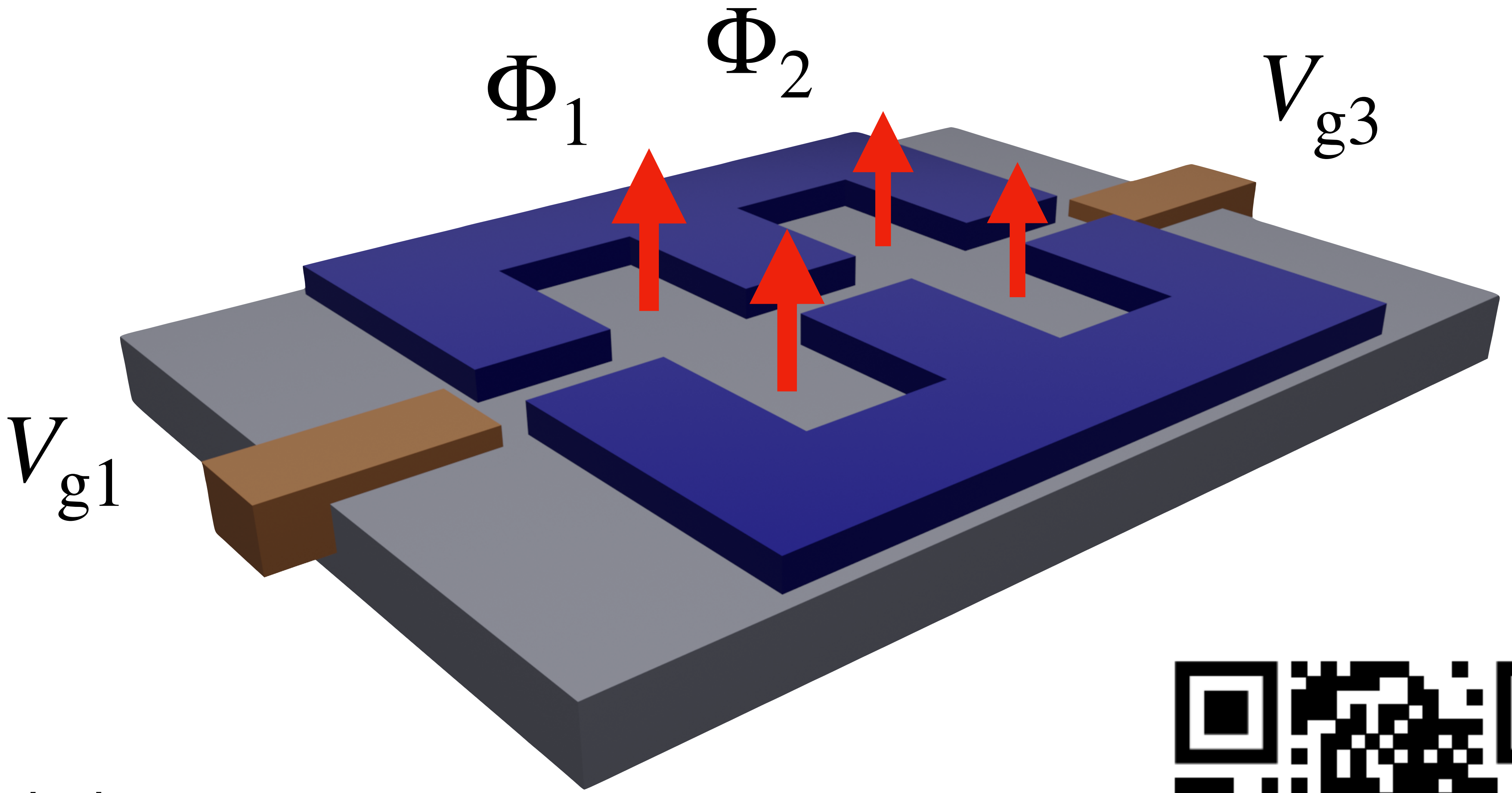

Perspectives

- Signature of Fibonacci operator τ
- Away from criticality: can we measure masses with spectroscopy techniques
- Scale up to 2D keeping a massless theory on the edge $\longmapsto$ Topological phases in the bulk With Fibonacci anyons

[Franz et al PRB ’20]

[ArXiv 2310.18300]

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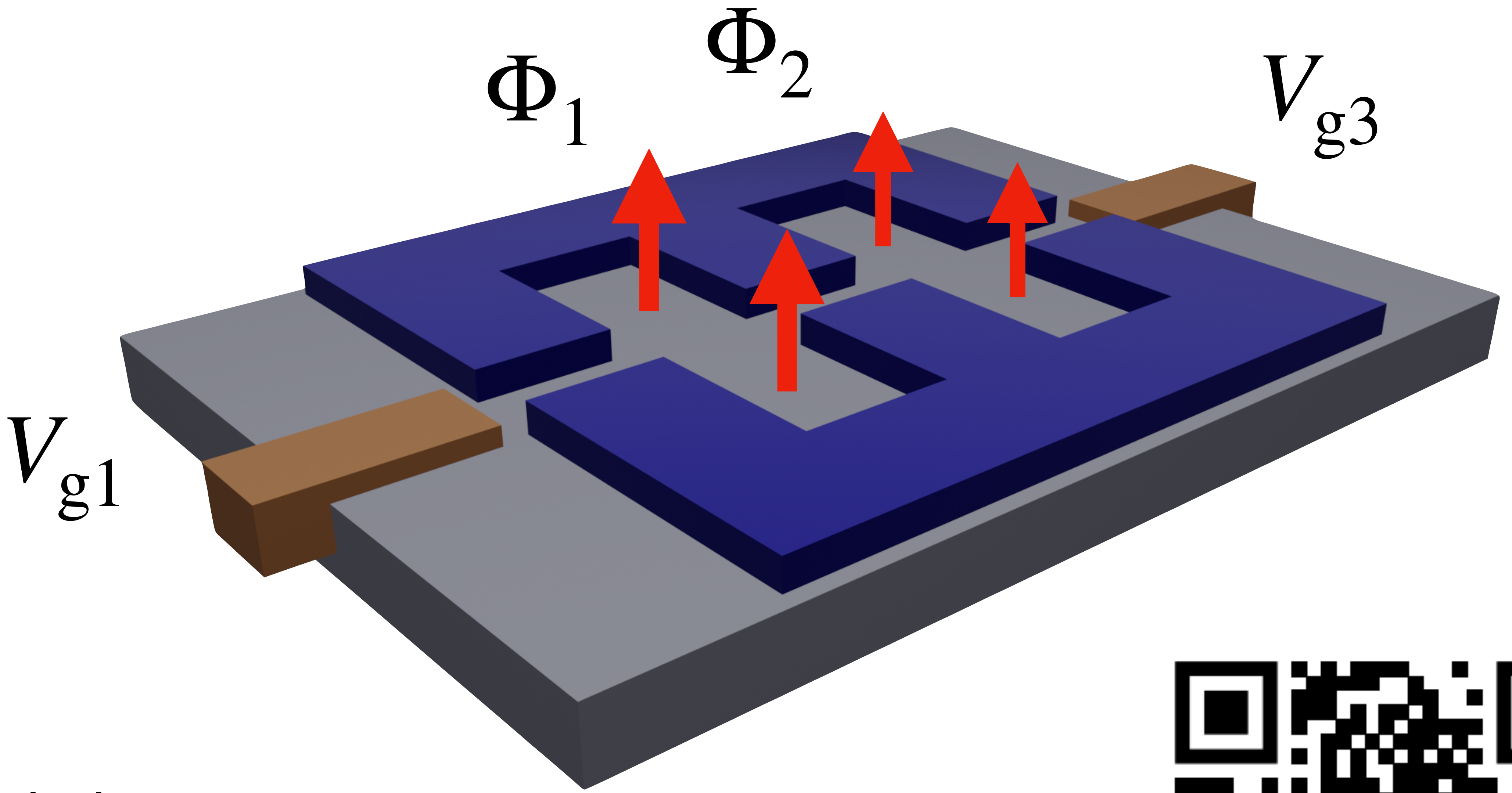

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Thank you all!

[ArXiv 2310.18300]

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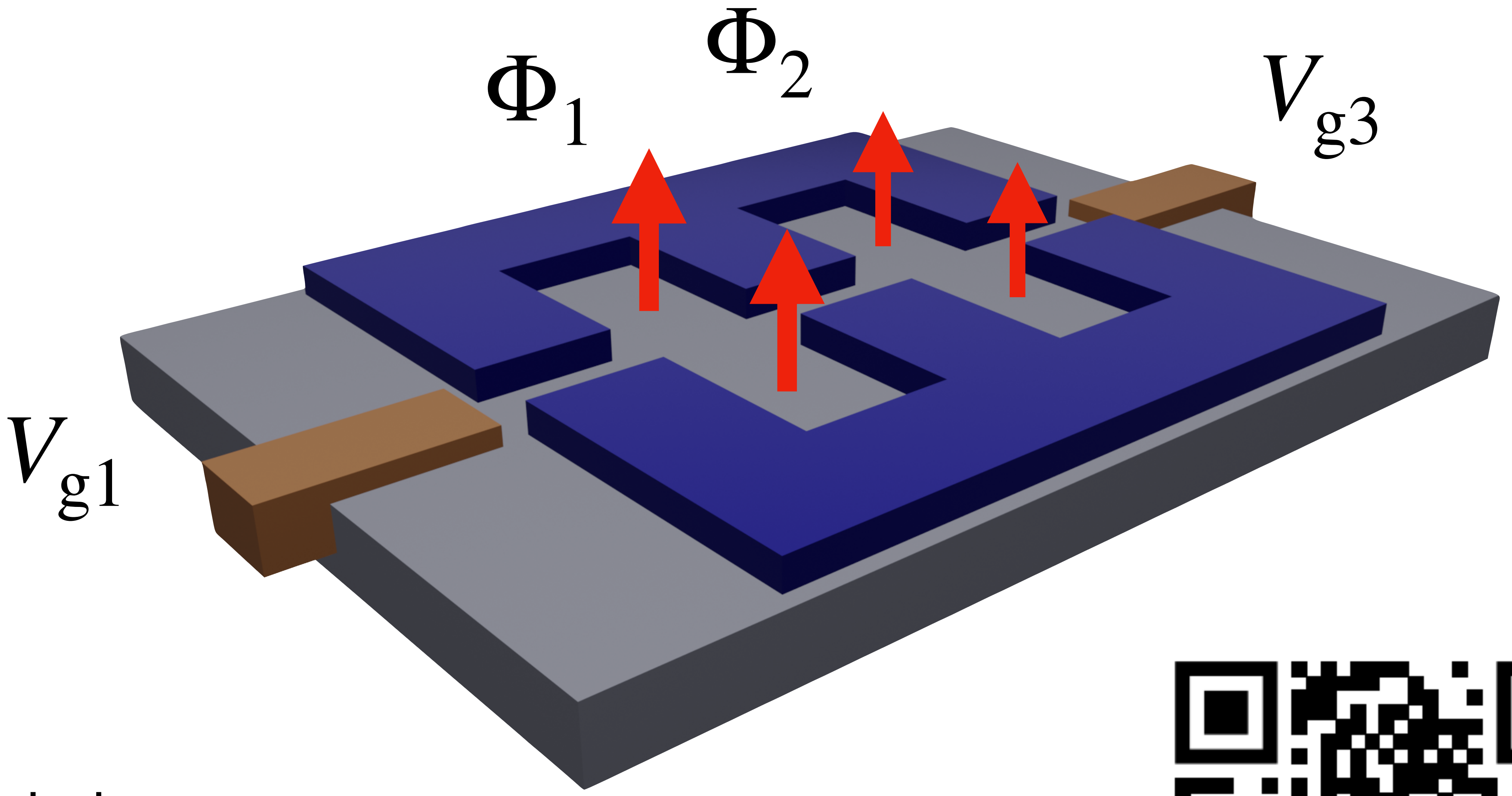

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[ArXiv 2310.18300]

Thank you all!

...and happy birthday Andrea!!

Spin models & other realisations

No 1D Quantum physical realization (related proposals)

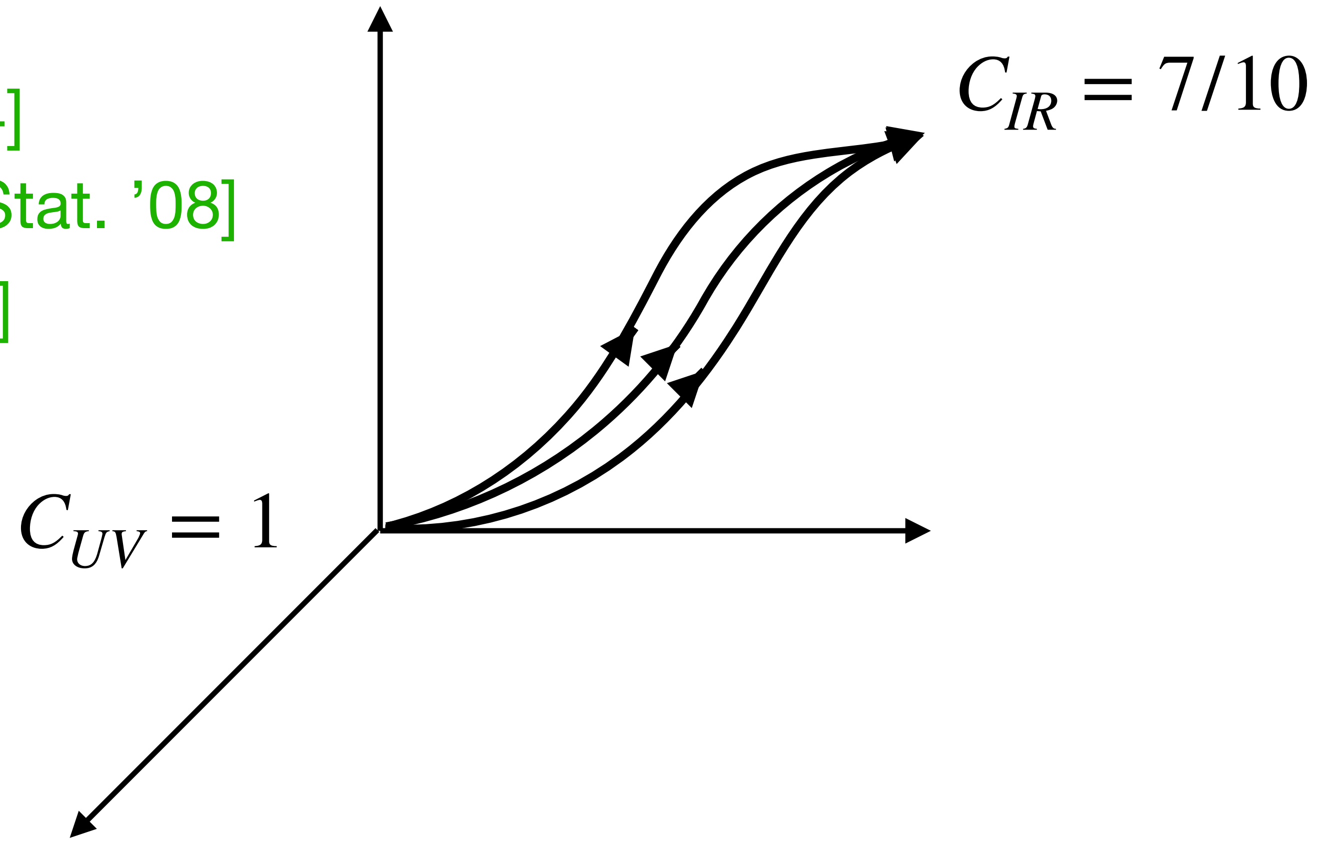
- Spin 1 quantum Blume-Capel model [Slagle et al PRB '21]
- Strong interacting Rydberg atoms [Oreg et al PRL '19]
- Majorana fermions on a lattice [Franz et al PRB '16]
- Staggered fermionic chains [Essler al PRB '17]

Quantum field theory

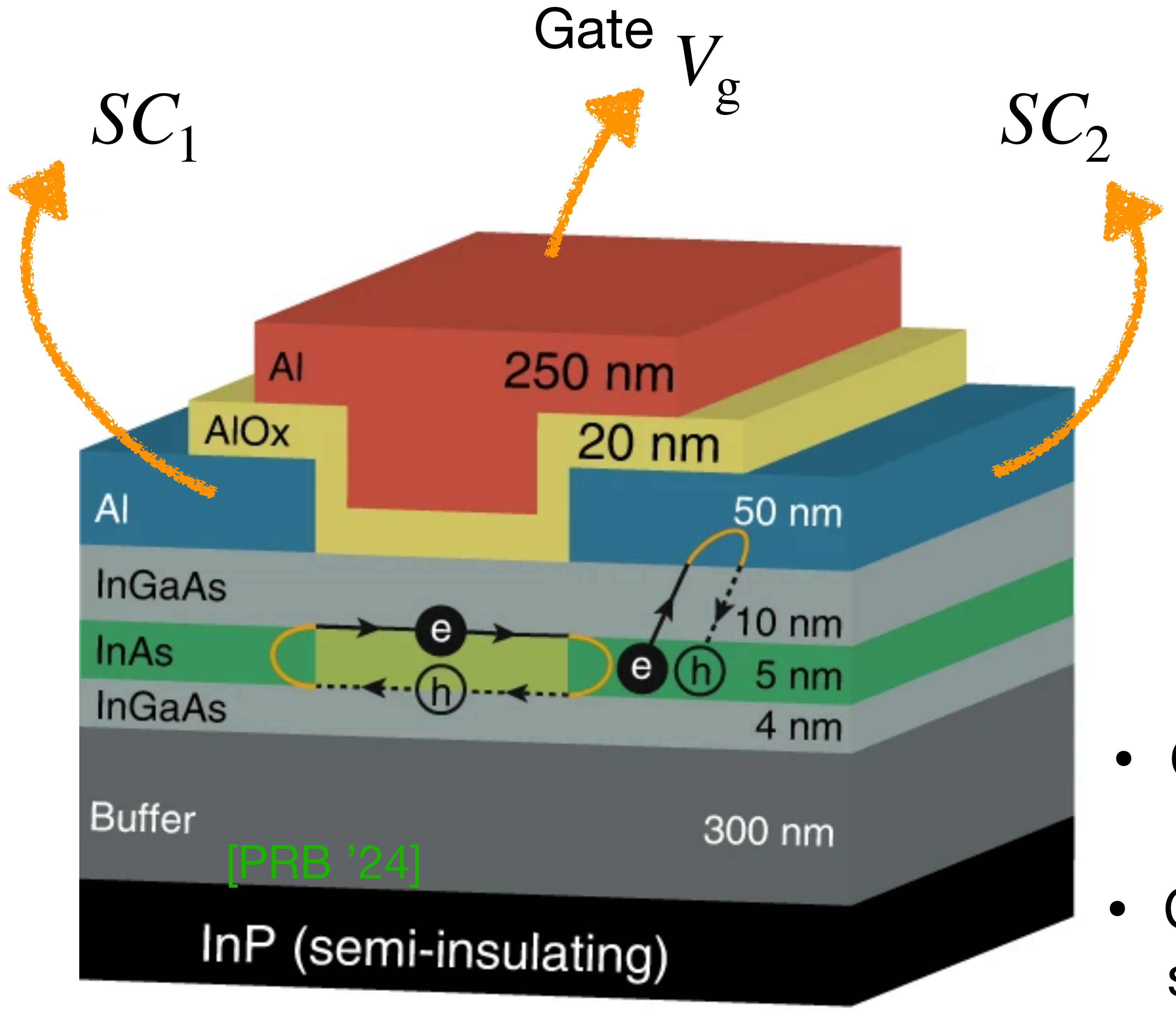
- **Multi-frequency Sine-Gordon model** [Mussardo, Delfino '97]

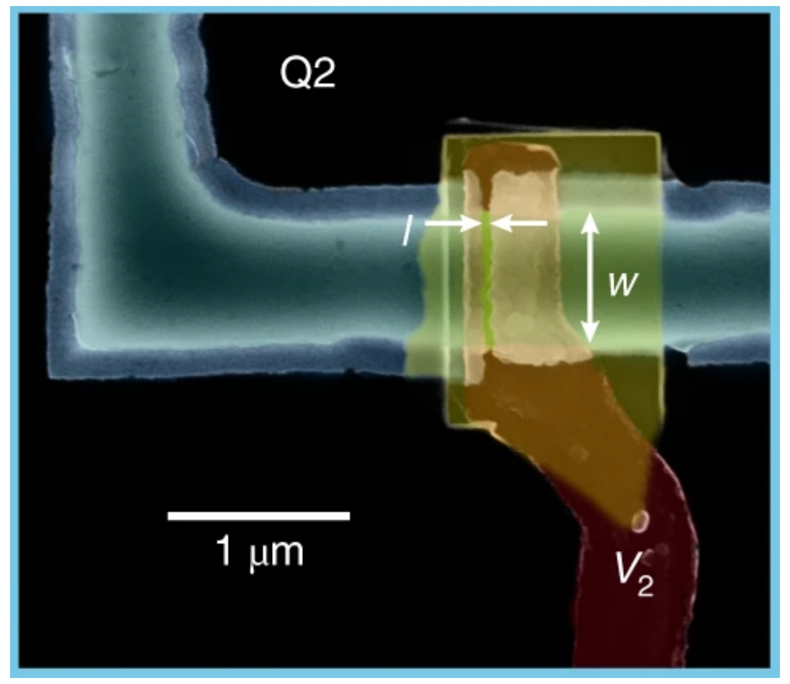
$$S = S_{c=1}[\varphi] + $$
$$+ \int dxdt \left(\mu_1 \cos\varphi + \mu_2 \cos(2\varphi) + \mu_3 \cos(3\varphi) \right)$$

[Toth, J.Phys. A '04]
[Mussardo et al. J. Phys. Stat. '08]
[Essler et al. PRB '17]



- Advances hybrid Superconductor/Semiconductor Junctions

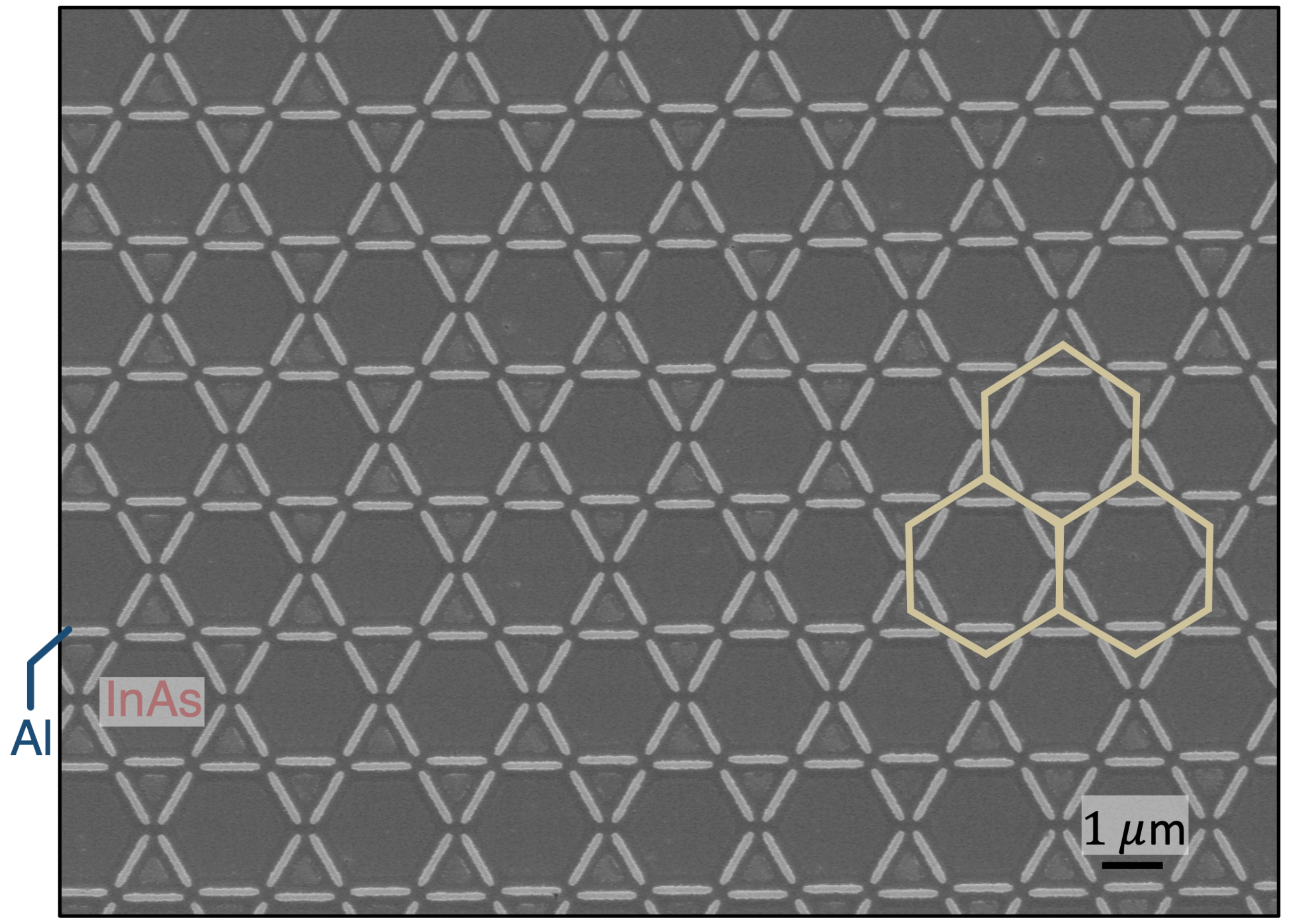




[C. Marcus et al. Nat. Phys. 2018]

- Coherence from superconductor
- Control via electrostatic gate from semiconductor
- Scalability and designing capability from lithography

[Marcus's Lab in Copenhagen]



Charge basis

Model $\left[\hat{N}_{j,x}, e^{i\hat{\varphi}_{j',x'}}\right] = -\delta_{j,j'}\delta_{x,x'}e^{i\hat{\varphi}_{j,x}}$

$$V_{\text{loc}} = \sum_{x=0}^{L} \mu_1 \cos\left(\hat{\varphi}_{\uparrow,x} - \hat{\varphi}_{\downarrow,x}\right) + \mu_2 \cos\left(2\left(\hat{\varphi}_{\uparrow,x} - \hat{\varphi}_{\downarrow,x}\right)\right) + \mu_3 \cos\left(3\left(\hat{\varphi}_{\uparrow,x} - \hat{\varphi}_{\downarrow,x}\right)\right)$$

$$\hat{H}_{\text{J}} = -E_J \sum_{j=\uparrow\downarrow} \sum_{x=0}^{L-1} \cos\left(\hat{\varphi}_{j,x} - \hat{\varphi}_{j,x+1}\right)$$

$$\hat{H}_{\text{C}} = E_C \sum_{j=\uparrow\downarrow} \sum_{x=0}^{L} \left(\hat{N}_{j,x} - n_g\right)^2$$

$$\hat{H}_{\perp} = V_{\perp} \sum_{x=0}^{L} \hat{N}_{\uparrow,x}\hat{N}_{\downarrow,x}$$

We can add a rung-charging interaction!

$$K_s \simeq \pi \sqrt{\frac{E_J}{2\left(E_c - V_{\perp}\right)}} > \frac{9}{4}$$

Discrete charge-basis

$$\left\{\tilde{n}_{j,x} = N_{j,x} - n_g\right\}$$

$E_c \gg E_J$

We can restrict the local Hilbert space

$$\left|\tilde{n}\right| < N_{\max}$$

$$\left(\dim\left(\mathscr{H}_{j,x}\right) = 2N_{\max} + 1\right)$$

We can define the local (non-unitary) operator

Es. $N_{\max} = 2$

$$e^{-i\hat{\varphi}_{j,x}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}_{j,x} \equiv \hat{\Sigma}^{\dagger}_{j,x}$$

$$e^{i\hat{\varphi}_{j,x}} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{j,x} \equiv \hat{\Sigma}_{j,x}$$

Diagonal charging terms

$$\hat{H} = \sum_{x=0}^{L} E_C \sum_{j=\uparrow\downarrow} \left(\tilde{n}_{j,x}\right)^2 + V_{\perp}\tilde{n}_{\uparrow,x}\tilde{n}_{\downarrow,x} + n_g V_{\perp}\left(\tilde{n}_{\uparrow,x} + \tilde{n}_{\downarrow,x}\right) +$$

$$-E_J/2 \sum_{j=\uparrow\downarrow} \left(\Sigma^{+}_{j,x}\Sigma^{-}_{j,x+1} + \Sigma^{-}_{j,x}\Sigma^{+}_{j,x+1}\right) +$$

Hopping term

$$+\frac{\mu_1}{2}\left(\Sigma^{-}_{\uparrow,x}\Sigma^{+}_{\downarrow,x} + \Sigma^{+}_{\uparrow,x}\Sigma^{-}_{\downarrow,x}\right) +$$

$$+\frac{\mu_2}{2}\left(\left(\Sigma^{-}_{\uparrow,x}\right)^2\left(\Sigma^{+}_{\downarrow,x}\right)^2 + \left(\Sigma^{+}_{\uparrow,x}\right)^2\left(\Sigma^{-}_{\downarrow,x}\right)^2\right) +$$

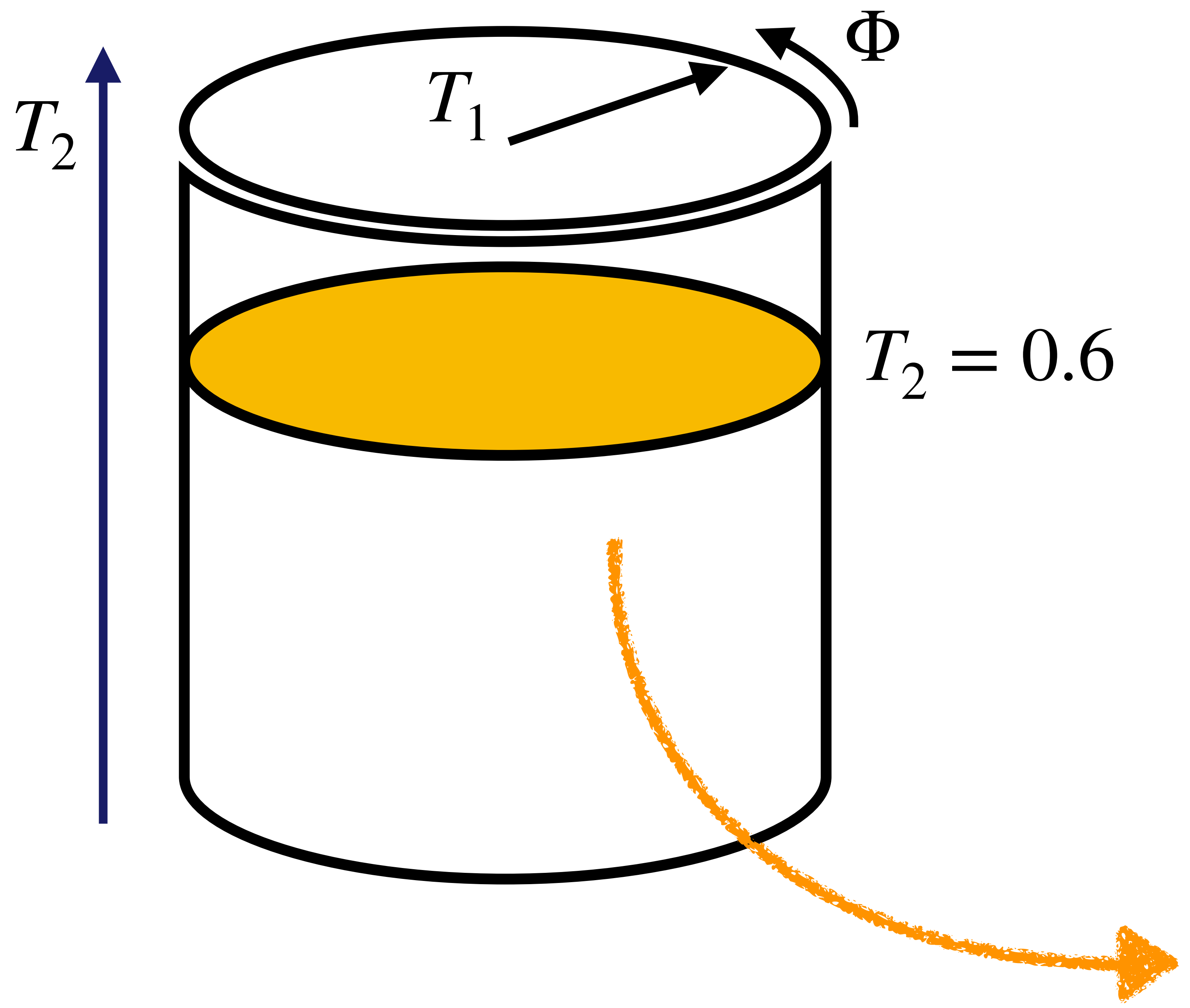
$$+\frac{\mu_3}{2}\left(\left(\Sigma^{-}_{\uparrow,x}\right)^3\left(\Sigma^{+}_{\downarrow,x}\right)^3 + \left(\Sigma^{+}_{\uparrow,x}\right)^3\left(\Sigma^{-}_{\downarrow,x}\right)^3\right)$$

Interaction

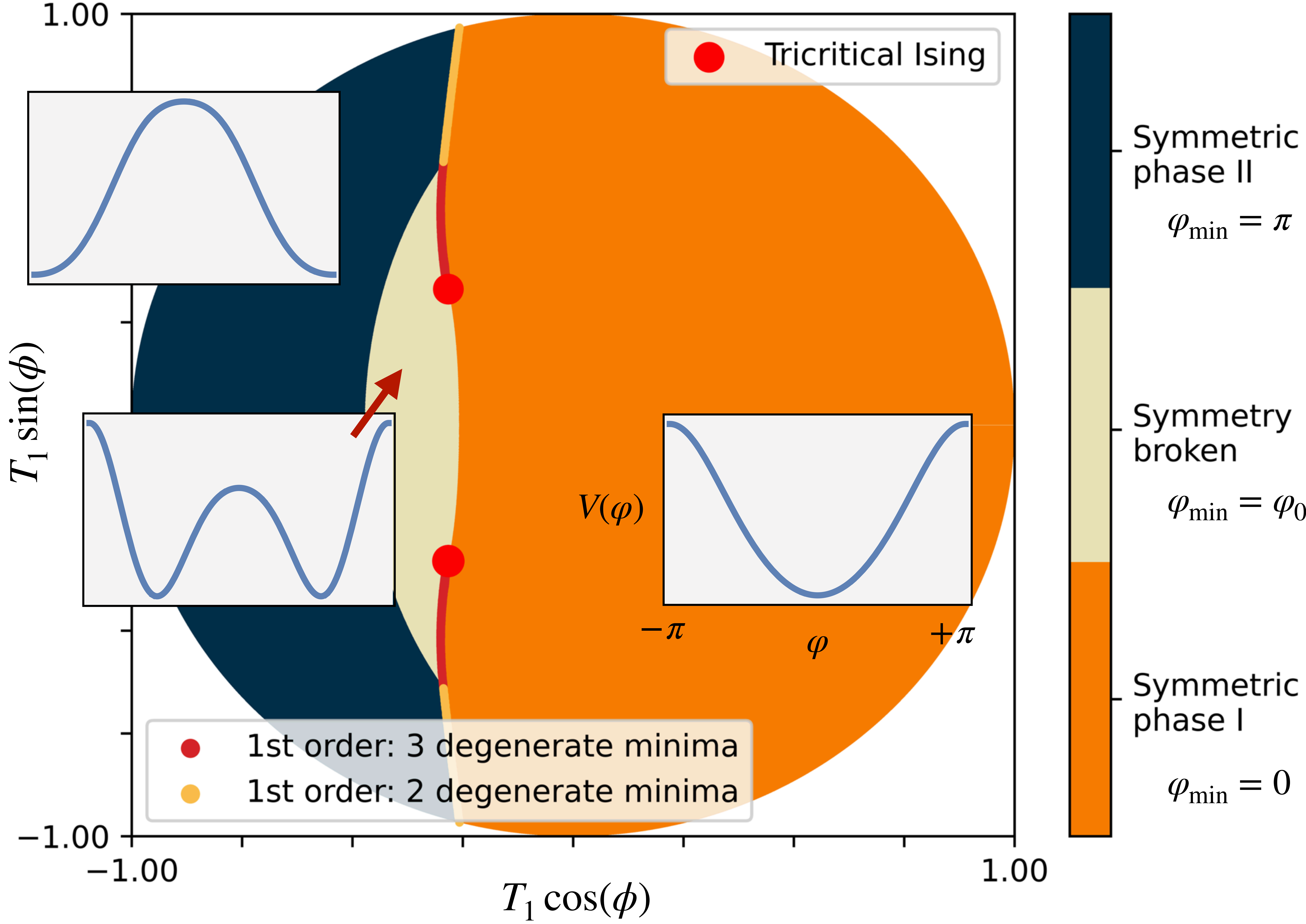
Staggered fluxes

Semiclassical analysis

- Three-dimensional parameter space

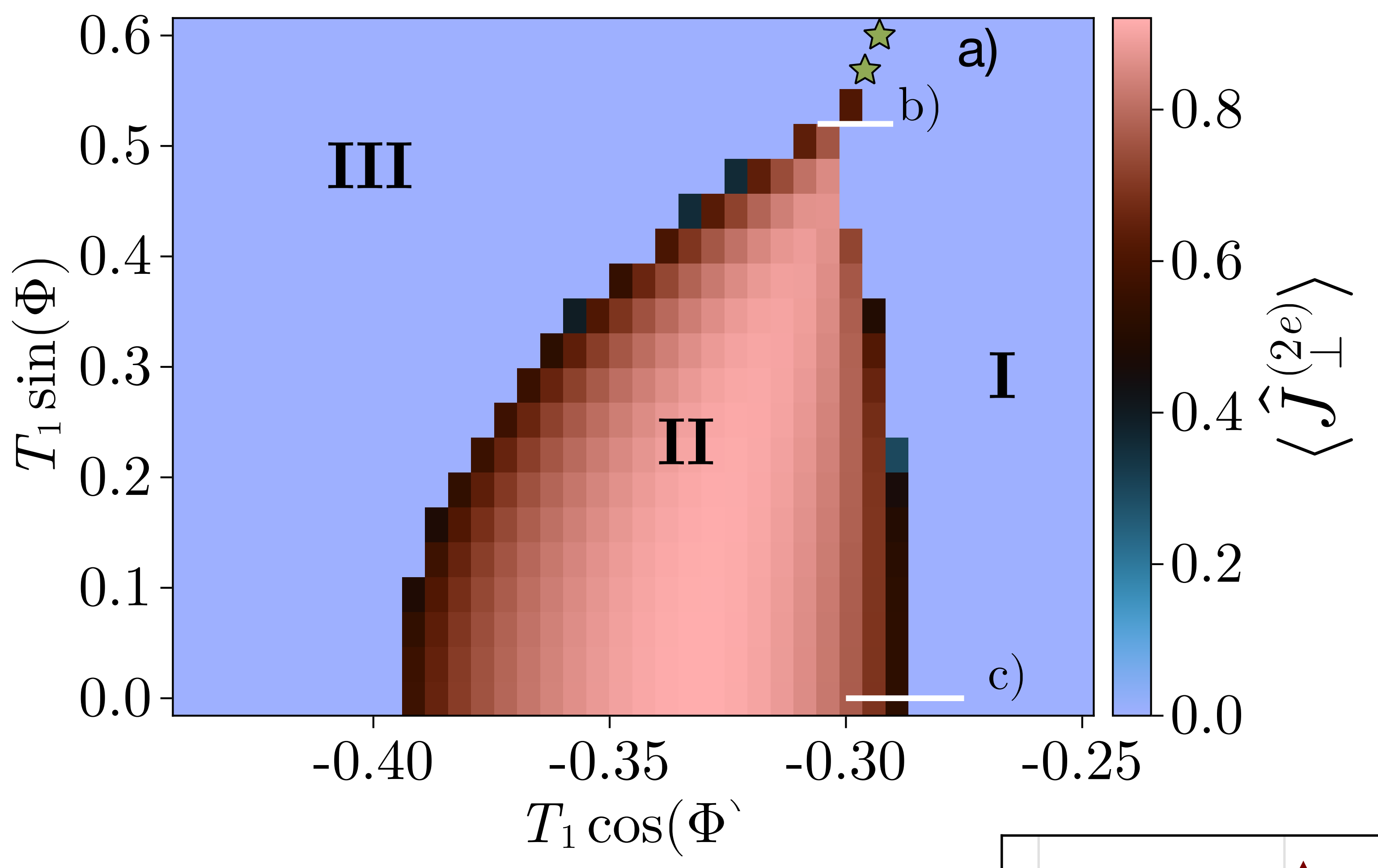


- Landscape of minima

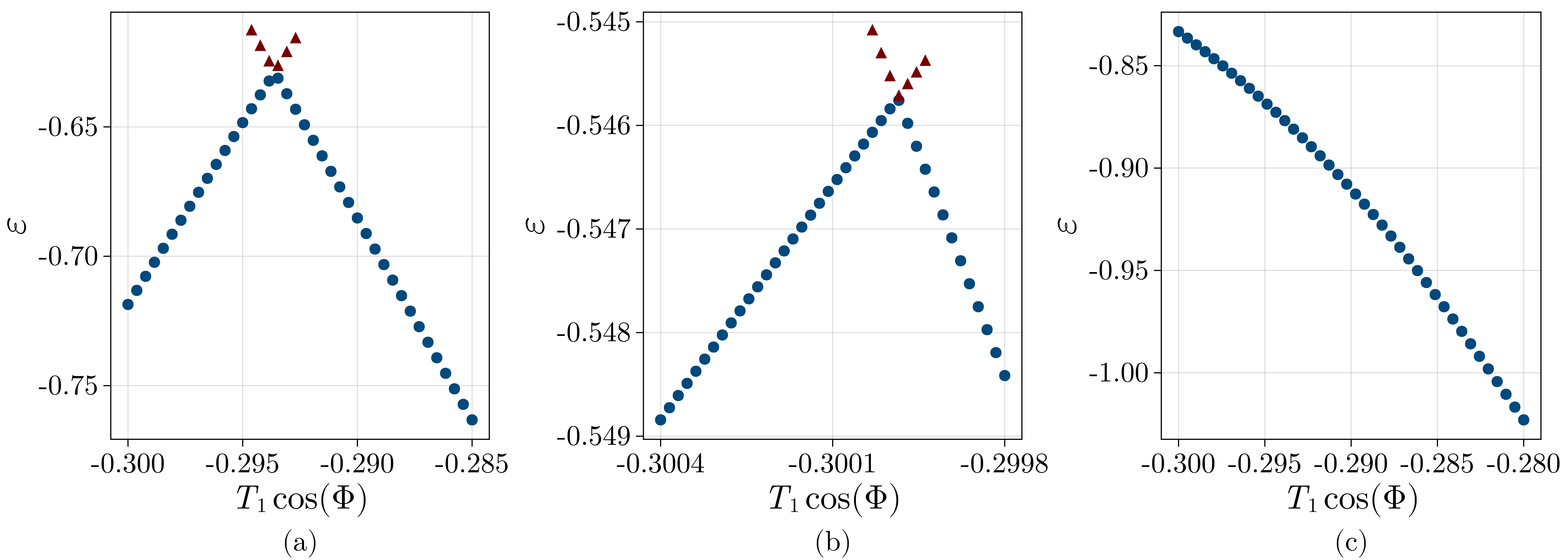


Phase-diagram from Tensor Networks (VUMPS)

(Truncation local Hilbert space: $\left|N_{j,x}\right| \leq 8$, Bond-dimension $D = 600$)

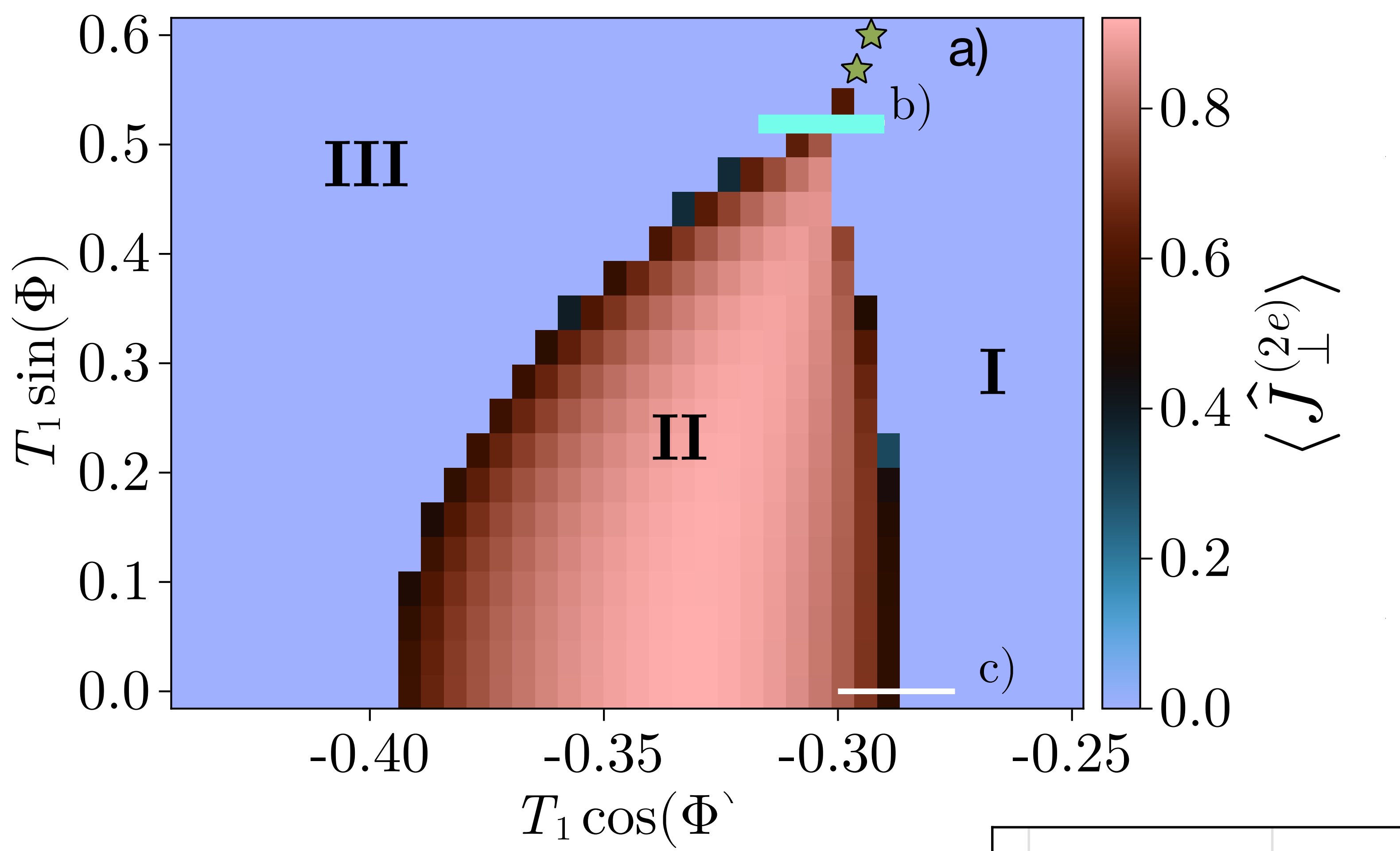


- Hysteresis at the first order transitions

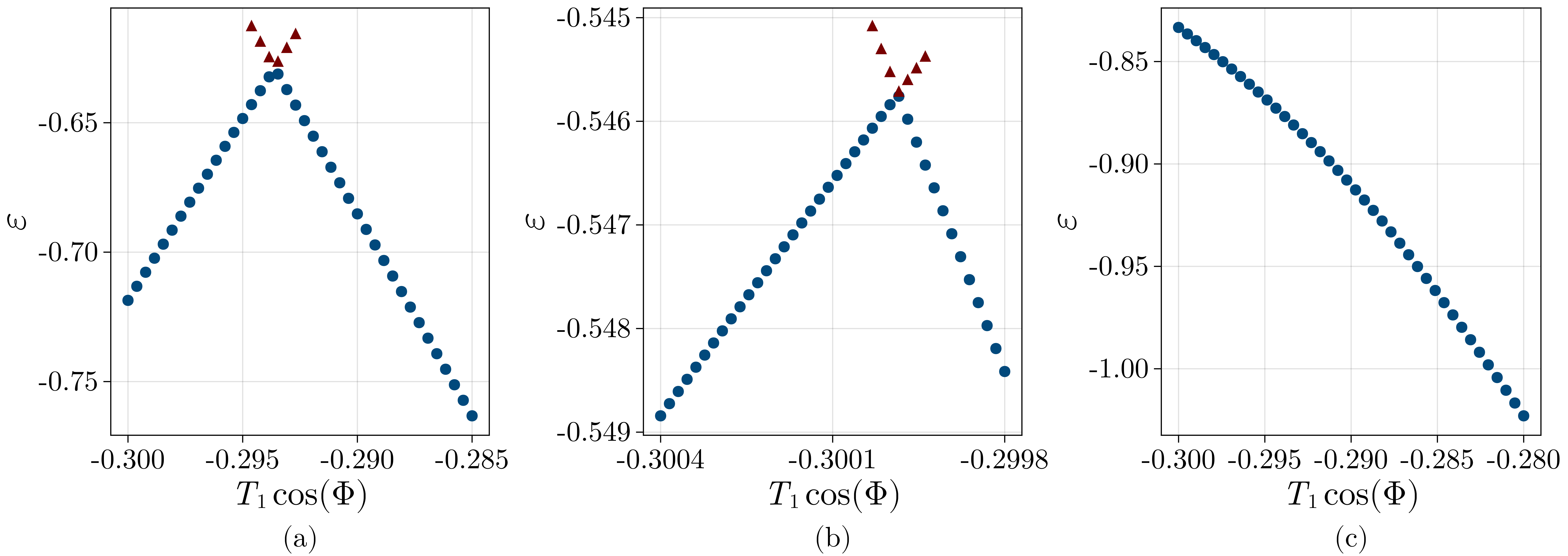


Phase-diagram from Tensor Networks (VUMPS)

(Truncation local Hilbert space: $\left|N_{j,x}\right| \leq 8$, Bond-dimension $D = 600$)



- Hysteresis at the first order transitions



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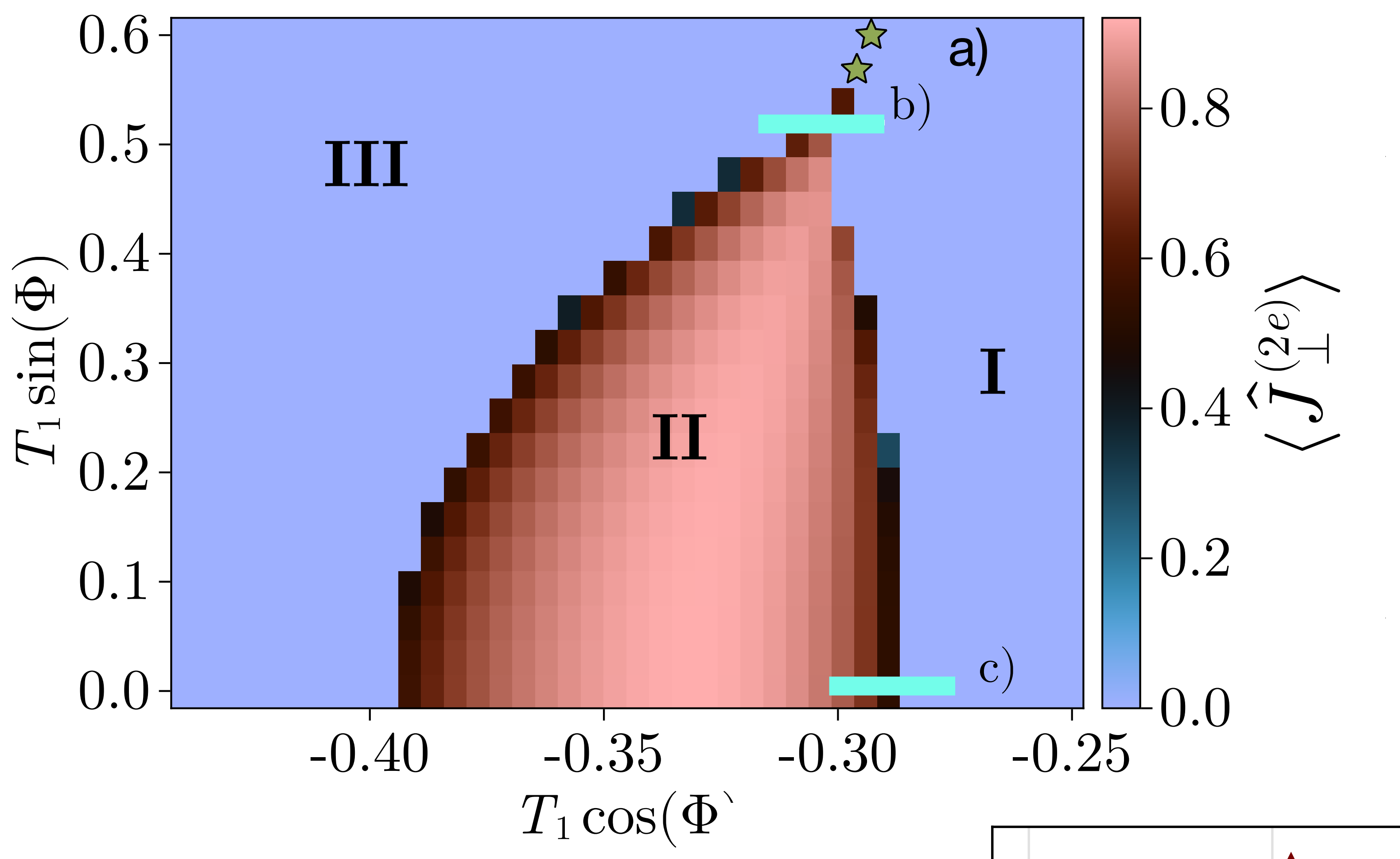
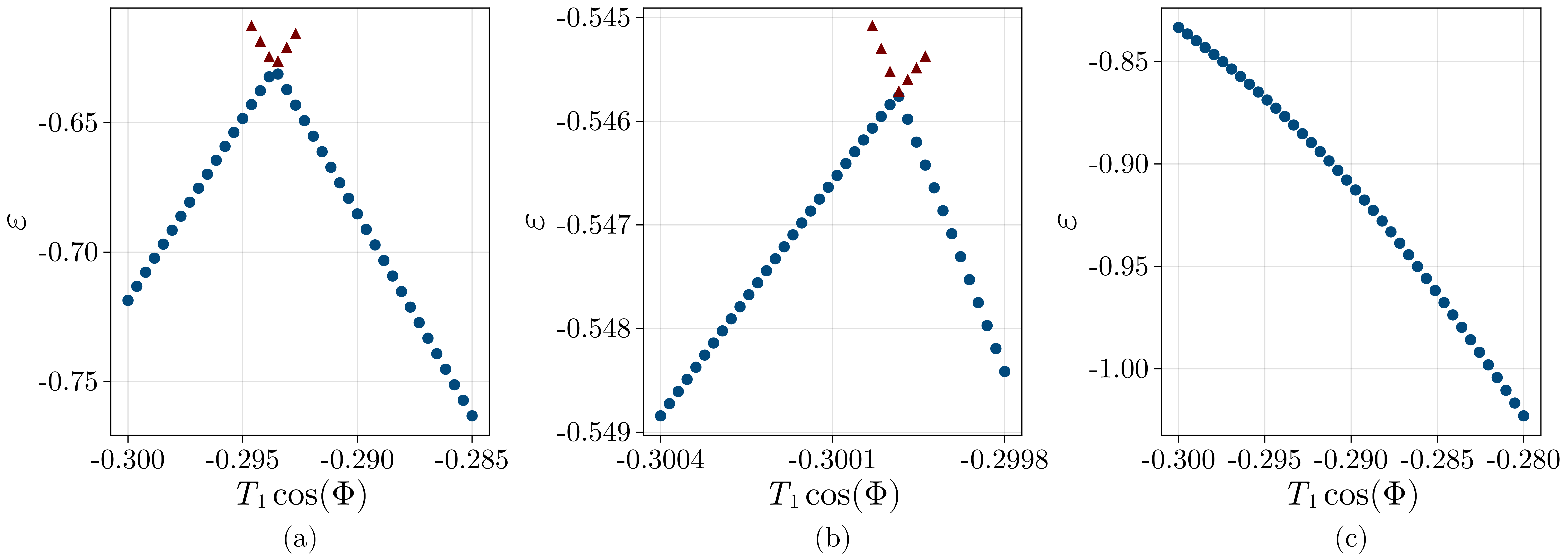
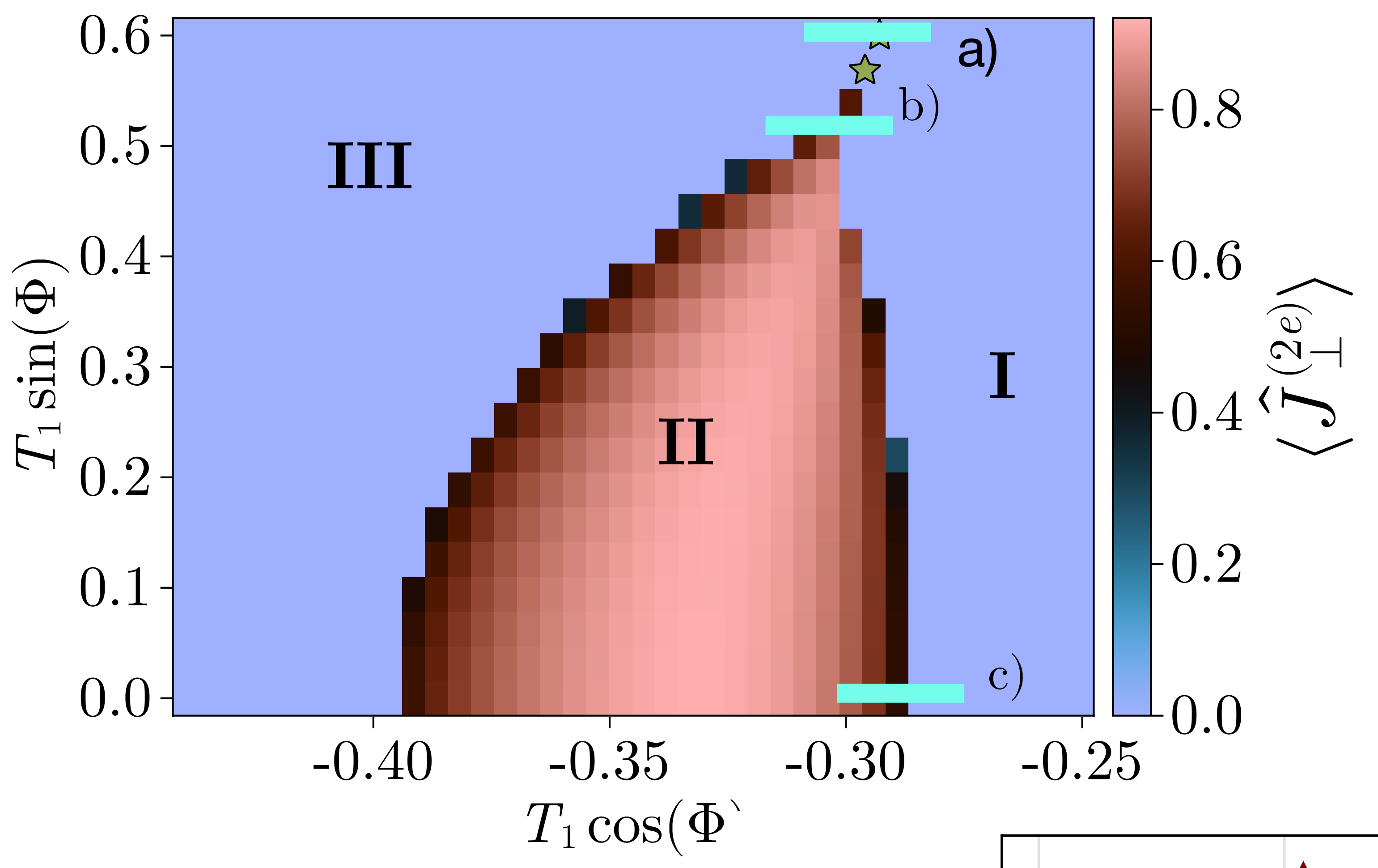


- Hysteresis at the first order transitions

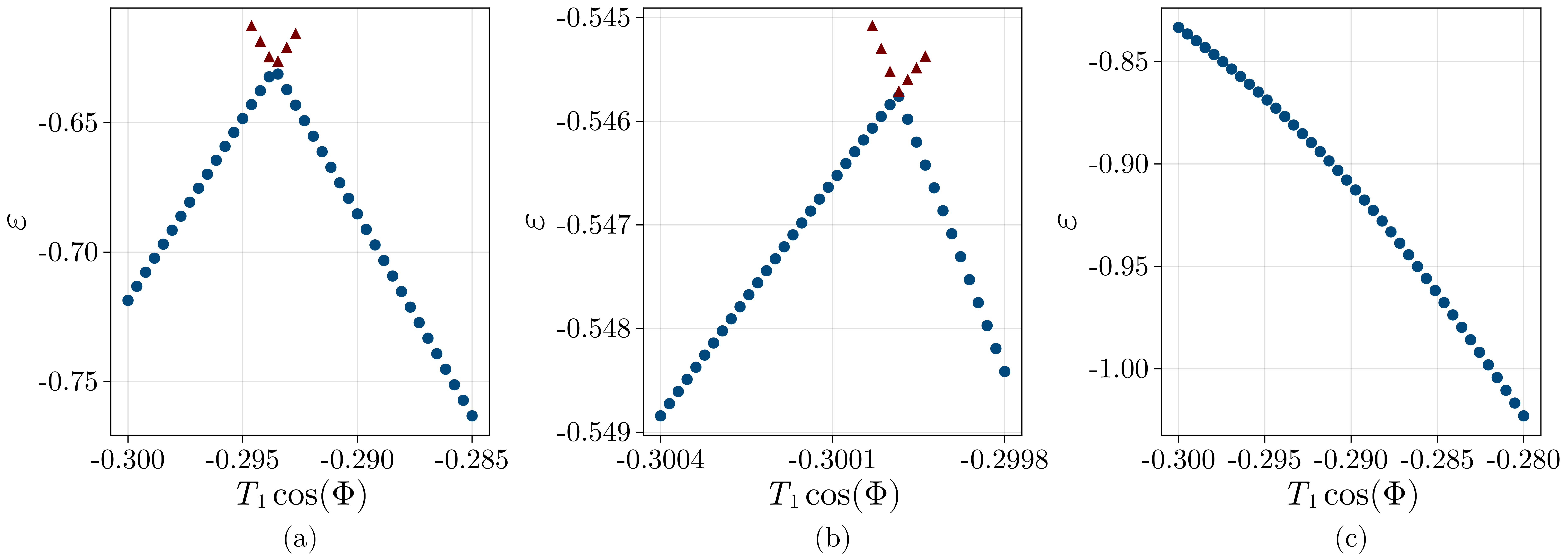


Phase-diagram from Tensor Networks (VUMPS)

(Truncation local Hilbert space: $\left|N_{j,x}\right| \leq 8$, Bond-dimension $D = 600$)

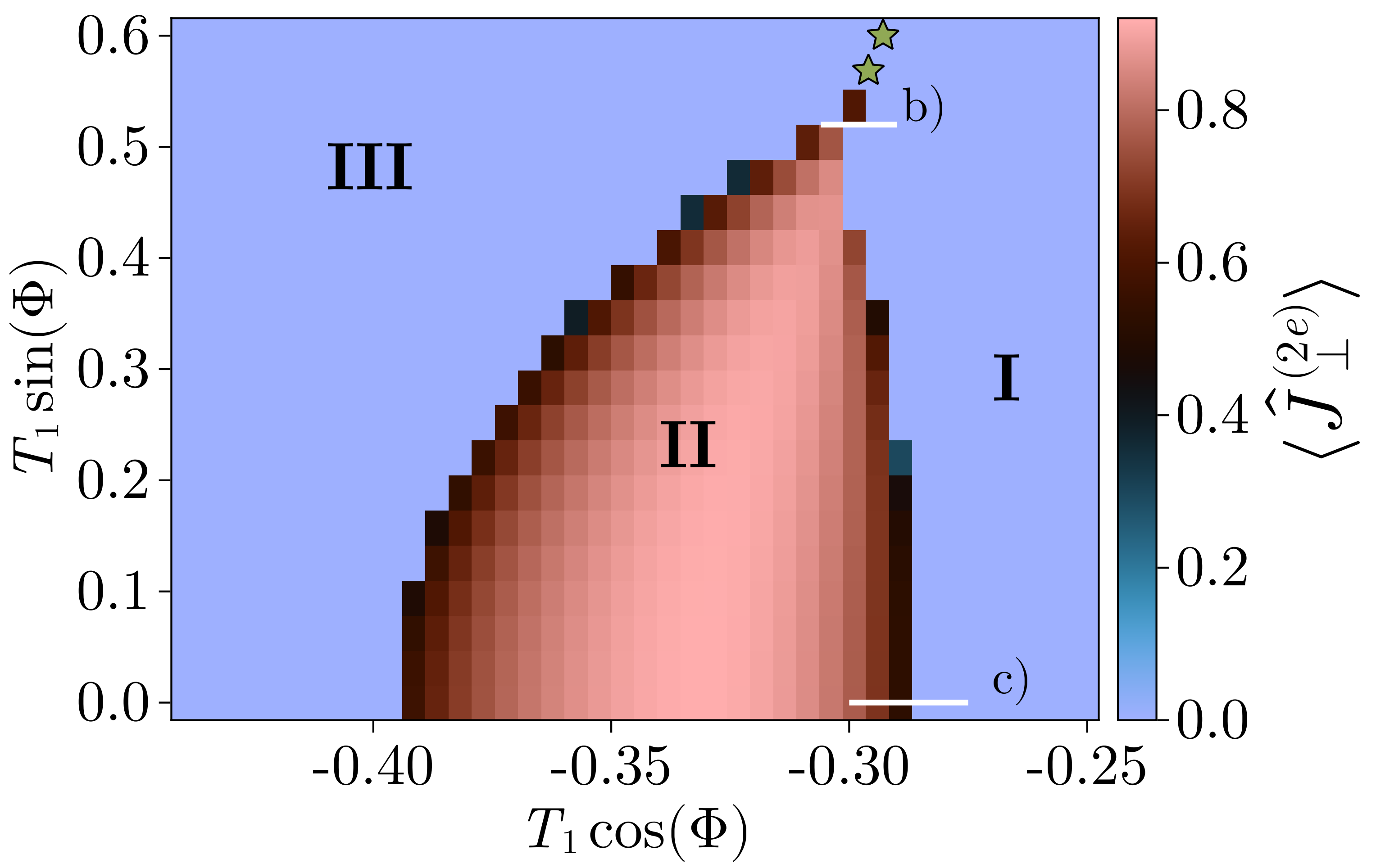


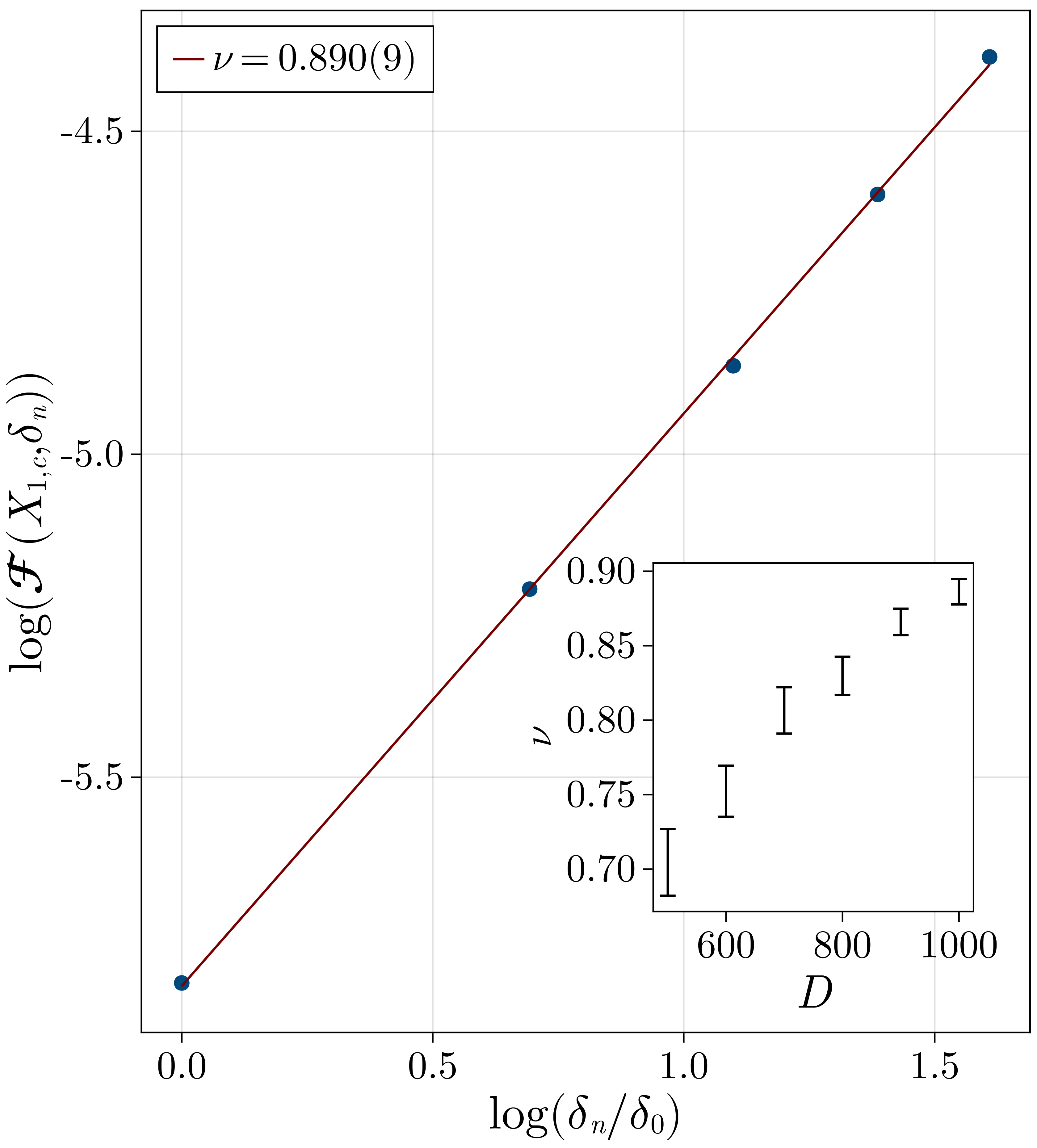
- Hysteresis at the first order transitions



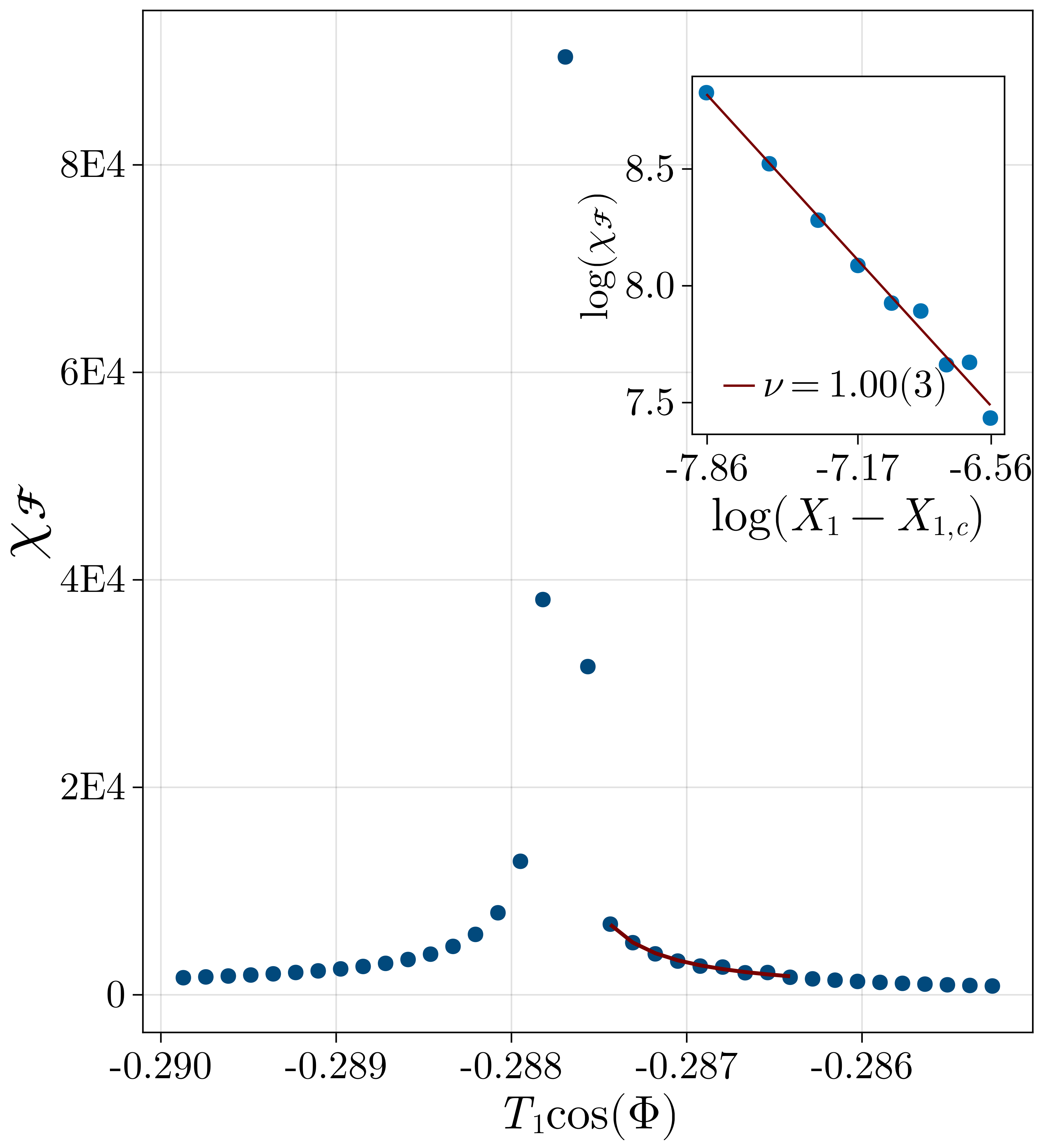
Phase-diagram from Tensor Networks (VUMPS)

(Truncation local Hilbert space: $|N_{j,x}| \leq 8$, Bond-dimension $D = 600$)





(a)



(b)

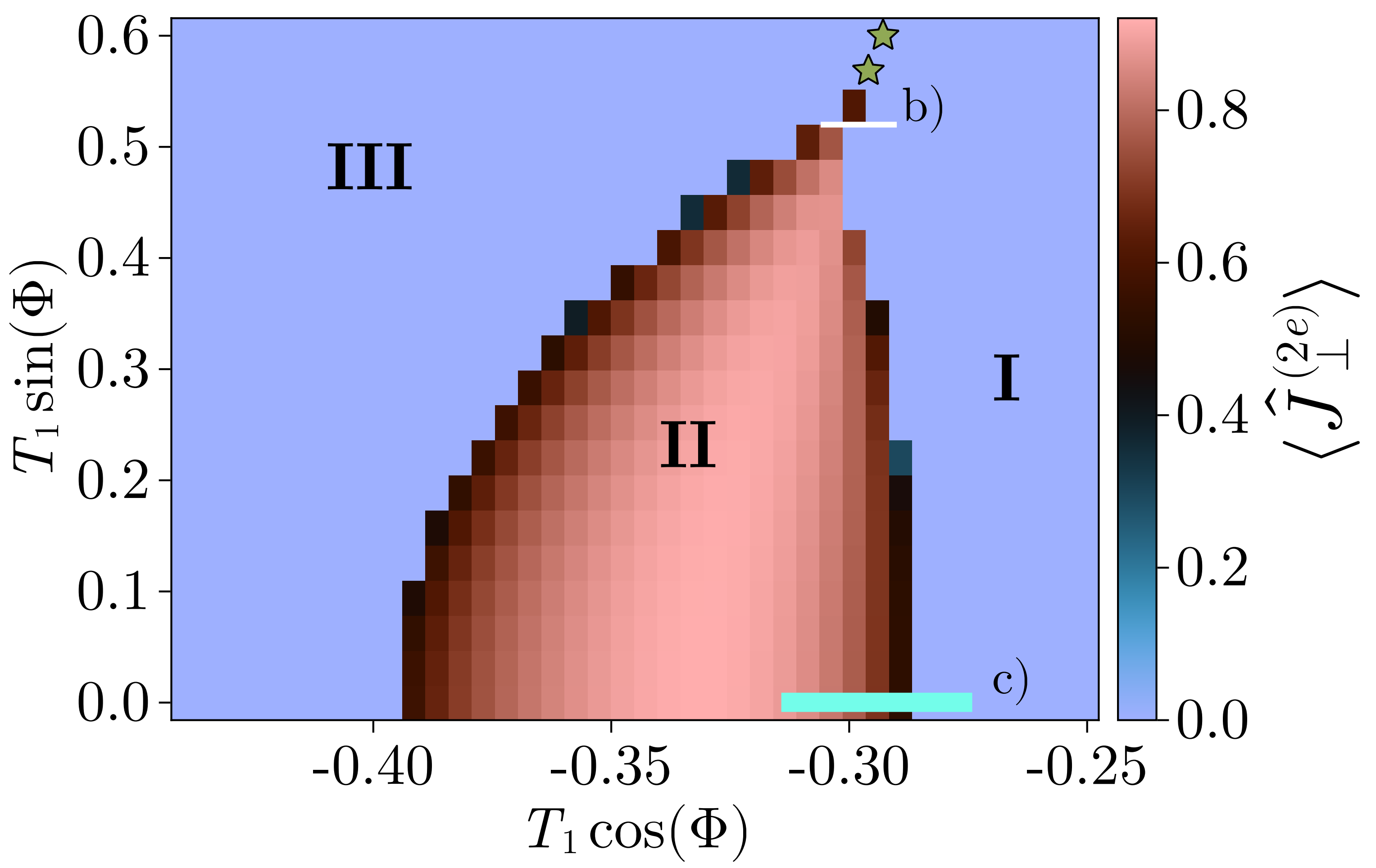
$$\mathcal{F}(X_{1c}, \delta) \sim |\delta|^{D\nu}$$

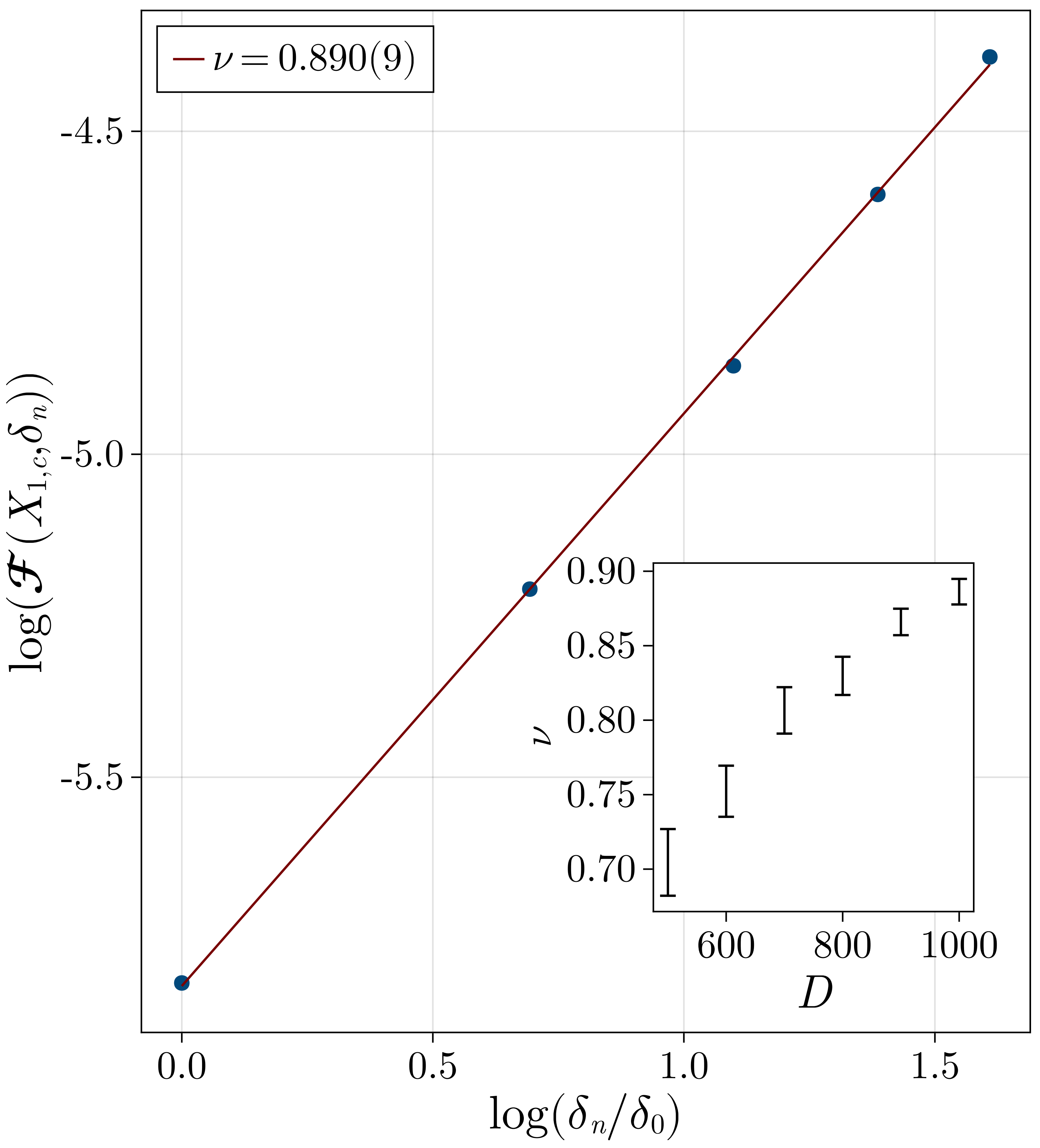
$$\chi_F(X_1, \delta) \sim |X_1 - X_{1c}|^{D\nu - 2}$$

[Rams et al PRL ’11]

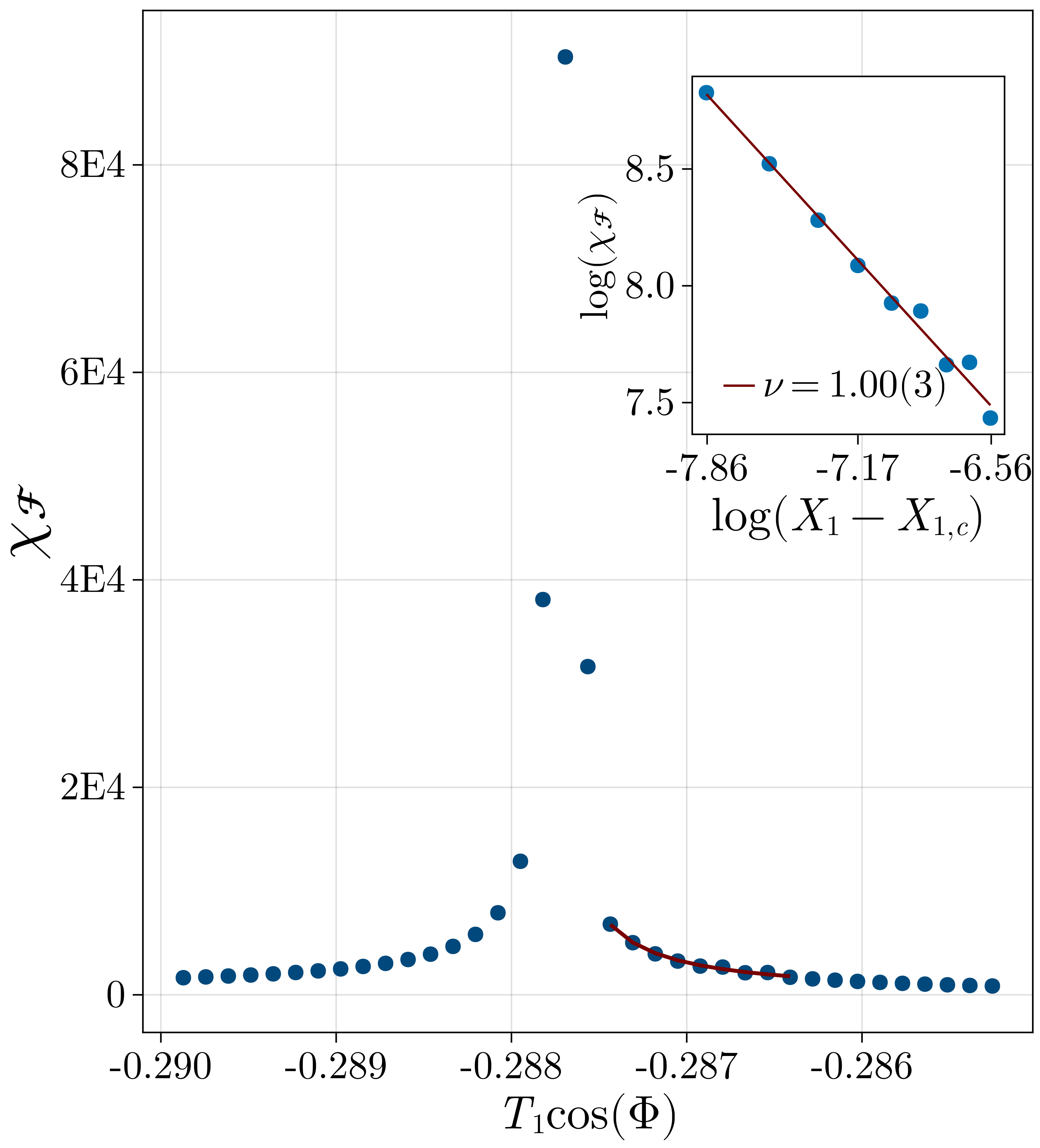
Phase-diagram from Tensor Networks (VUMPS)

(Truncation local Hilbert space: $\left| N_{j,x} \right| \leq 8$, Bond-dimension $D = 600$)





(a)



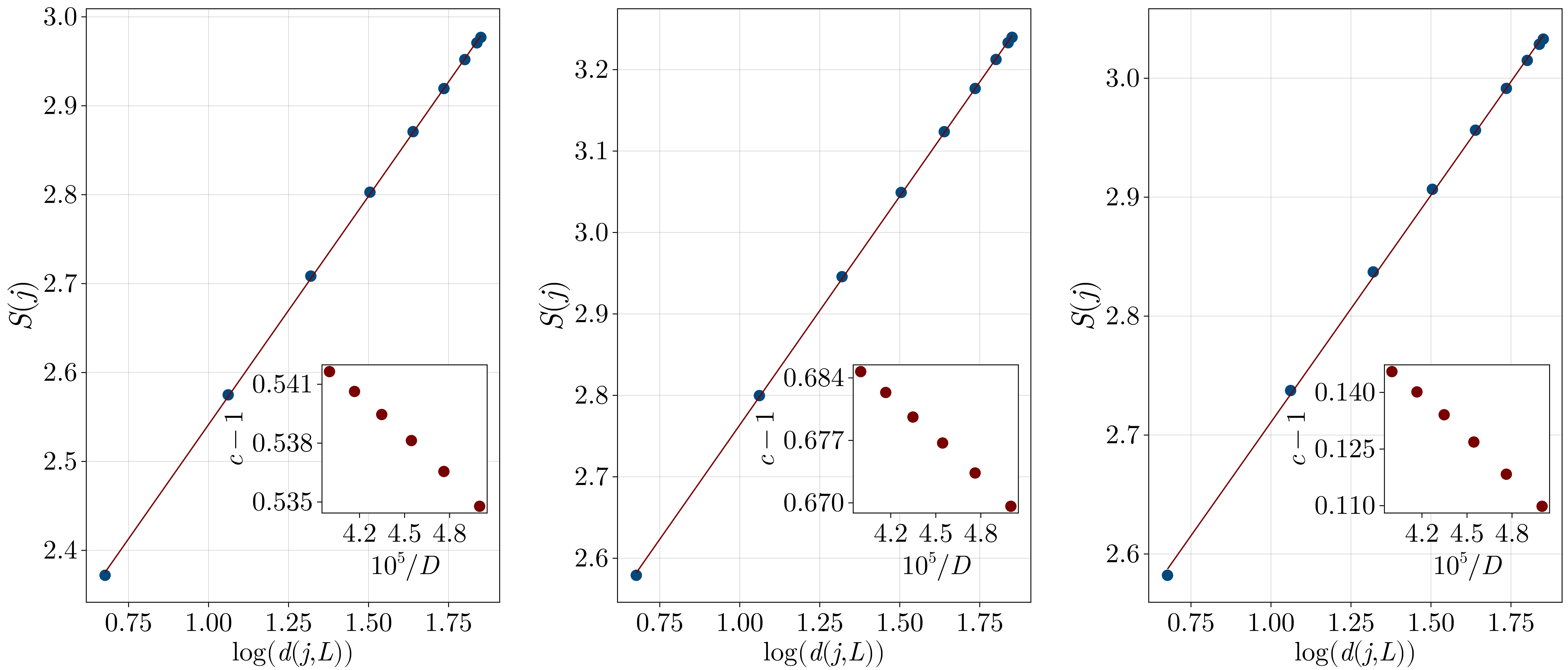
(b)

$$\mathcal{F}\left(X_{1c}, \delta\right) \sim |\delta|^{D\nu}$$

$$\chi_F\left(X_1, \delta\right) \sim |X_1 - X_{1c}|^{D\nu - 2}$$

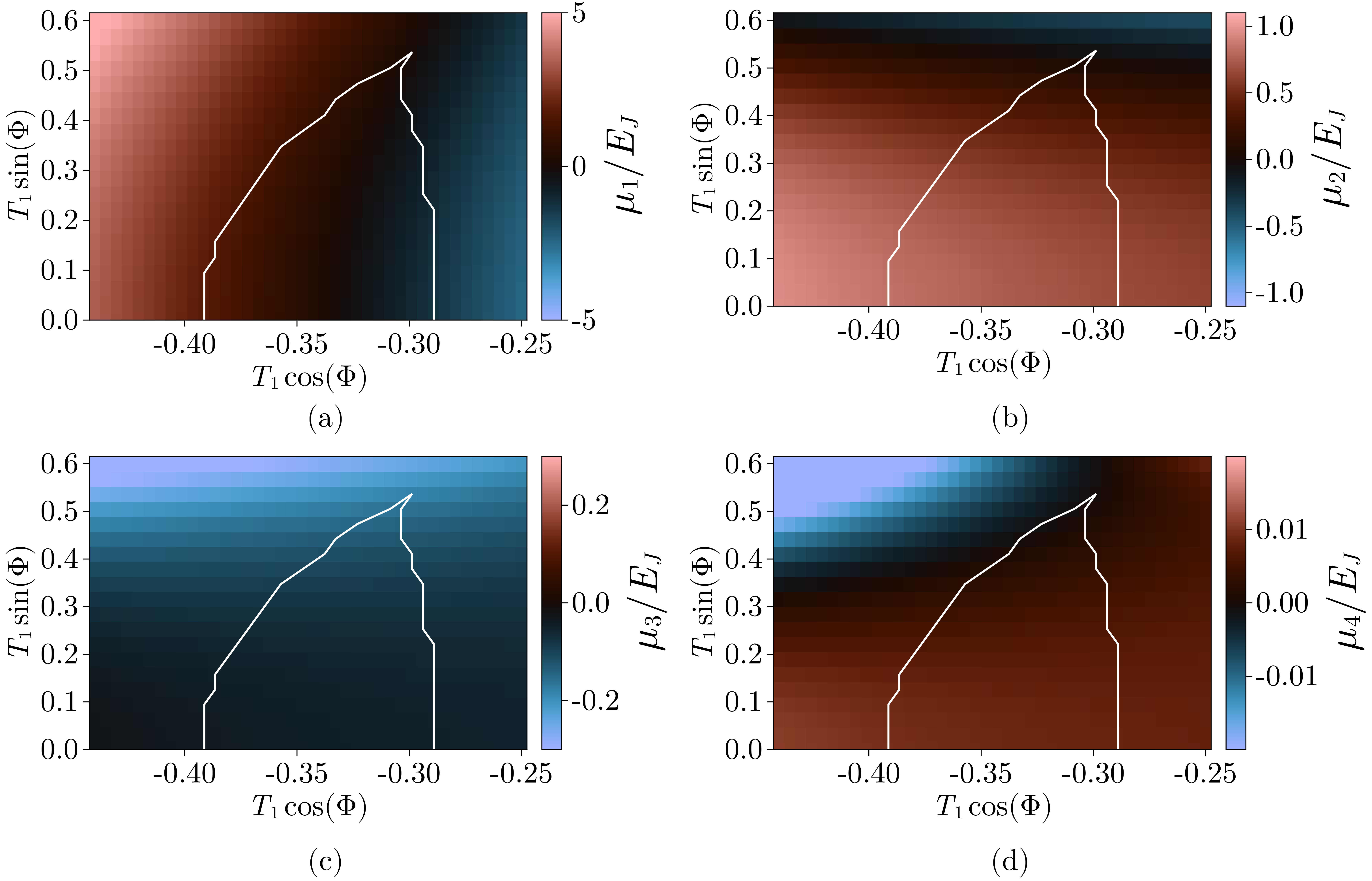
[Rams et al PRL '11]

Central charge and entanglement entropy



(a) (b) (c)

Fourier components of the local potential



(a) (b) (c) (d)