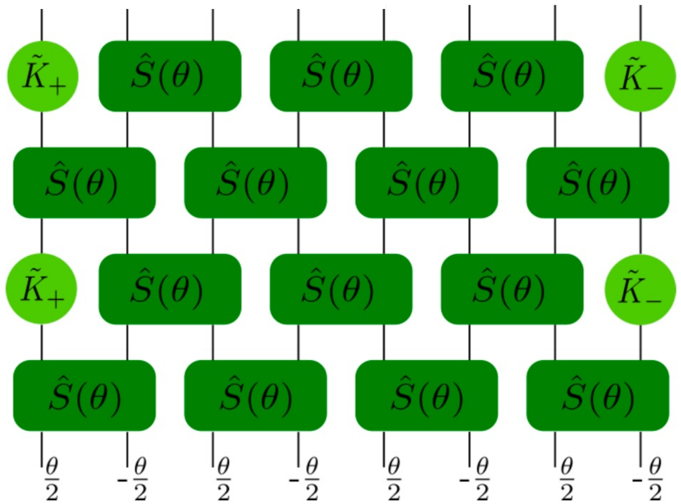
Brick Wall Quantum Circuits with Global Fermionic Symmetry

40 Years CFT



Forty Years of CFT & Cappelli Fest - GGI– 2 Feb 2024

Universiteit van Amsterdam

Kareljan Schoutens

Institute for Theoretical Physics

QuSoft

Research Center for Quantum Software

with

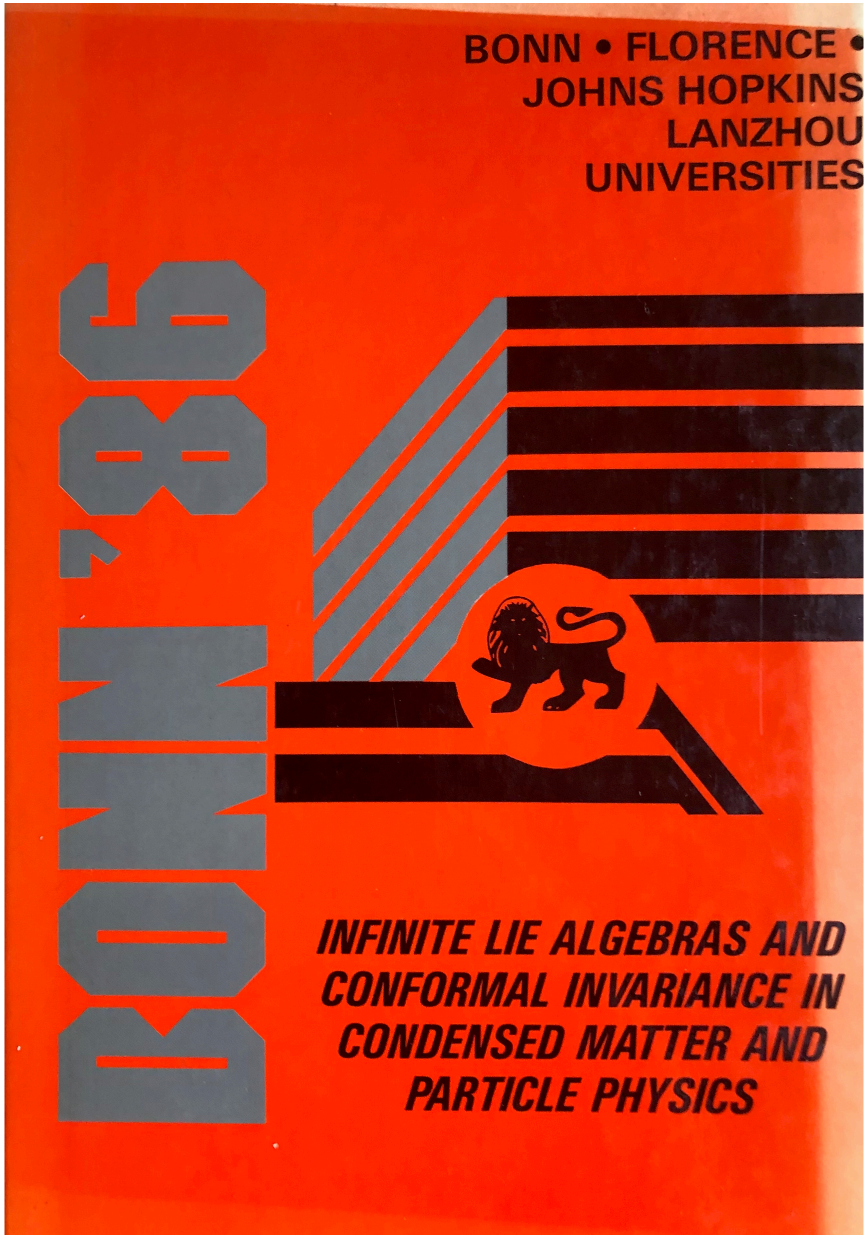
Pietro Richelli

Alberto Zorzato

Quantum Delta NL

Year 3 - 1986

Johns Hopkins Workshop in Bonn, with Jean-Bernard explaining the CIZ Classification of Modular Invariant Partition Functions



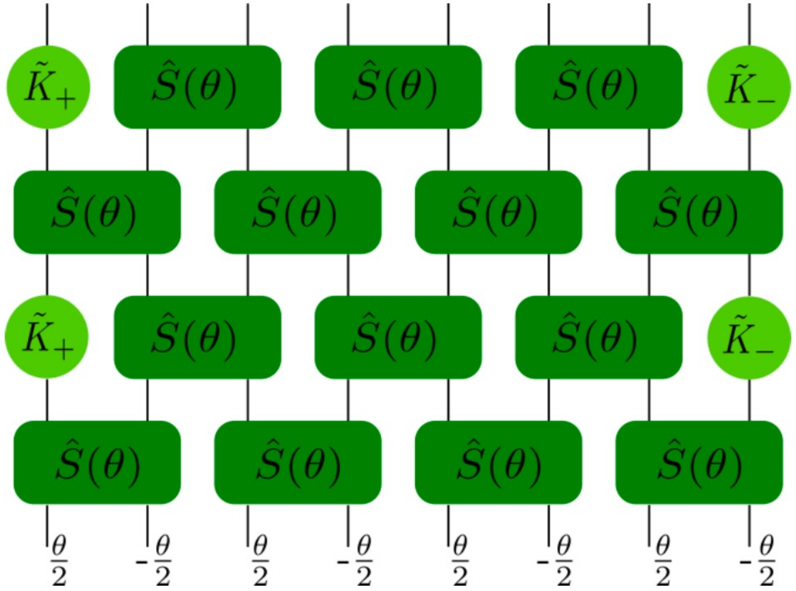
Year 18 - 2001

Amsterdam Summer Workshop
on Flux, Charge, Topology and Statistics, 2001

Year 25 - 2008

Conference on Low-dimensional Quantum Field Theories and Applications, GGI 2008 (??)

Year 40 - 2023

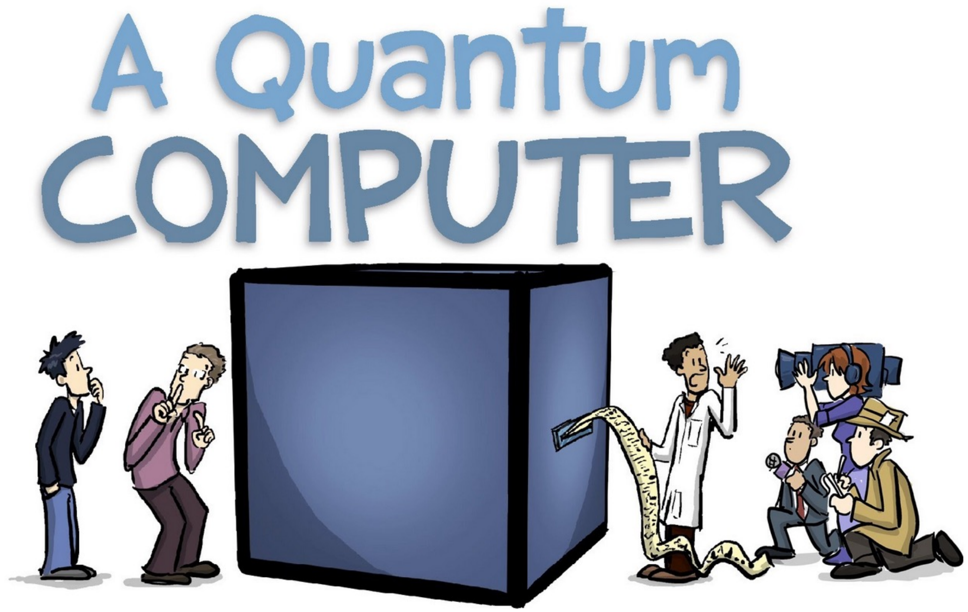


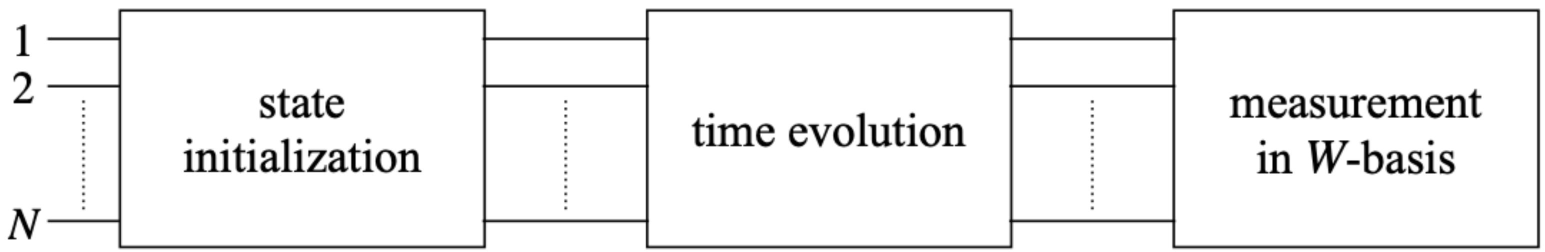
with

P. Richelli, A. Zorzato

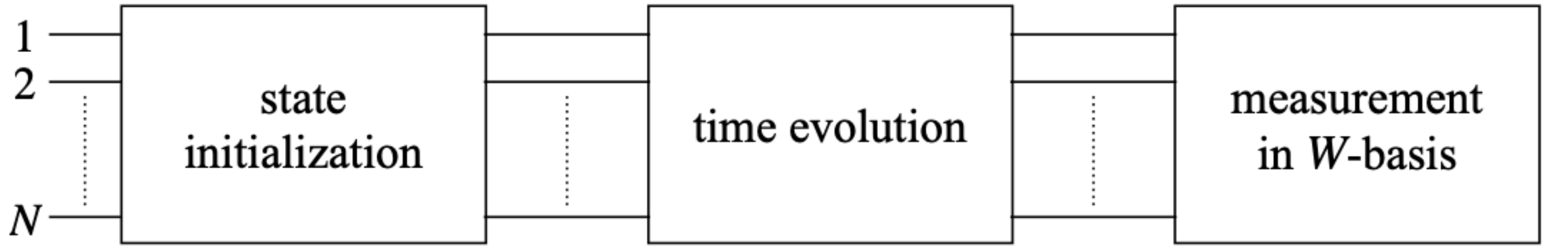
Study of class of **Brick Wall quantum circuits** that are integrable, `free fermionic' and possess a global fermionic symmetry and that derive from factorizable particle scattering in perturbed superconformal field theory

Context: quantum dynamics on a quantum computer

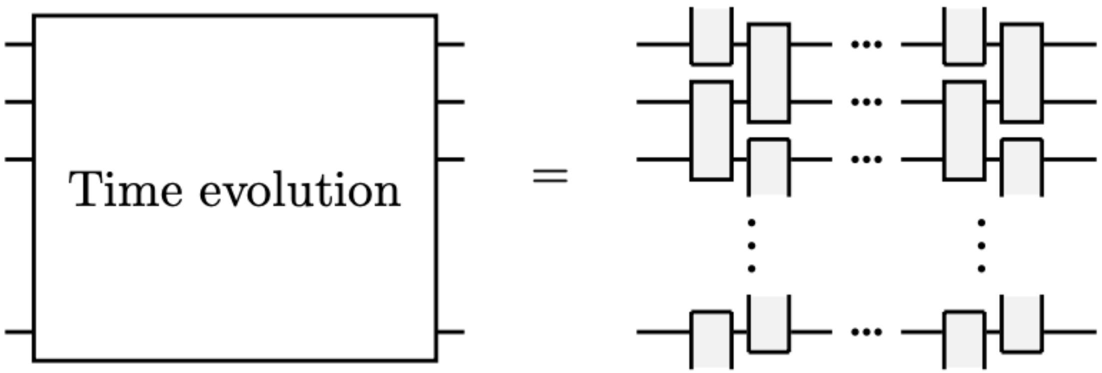




Context: quantum dynamics



Time evolution implemented using a **brick wall** type quantum circuit:



figures from arXiv:2208.00576

Context: quantum dynamics

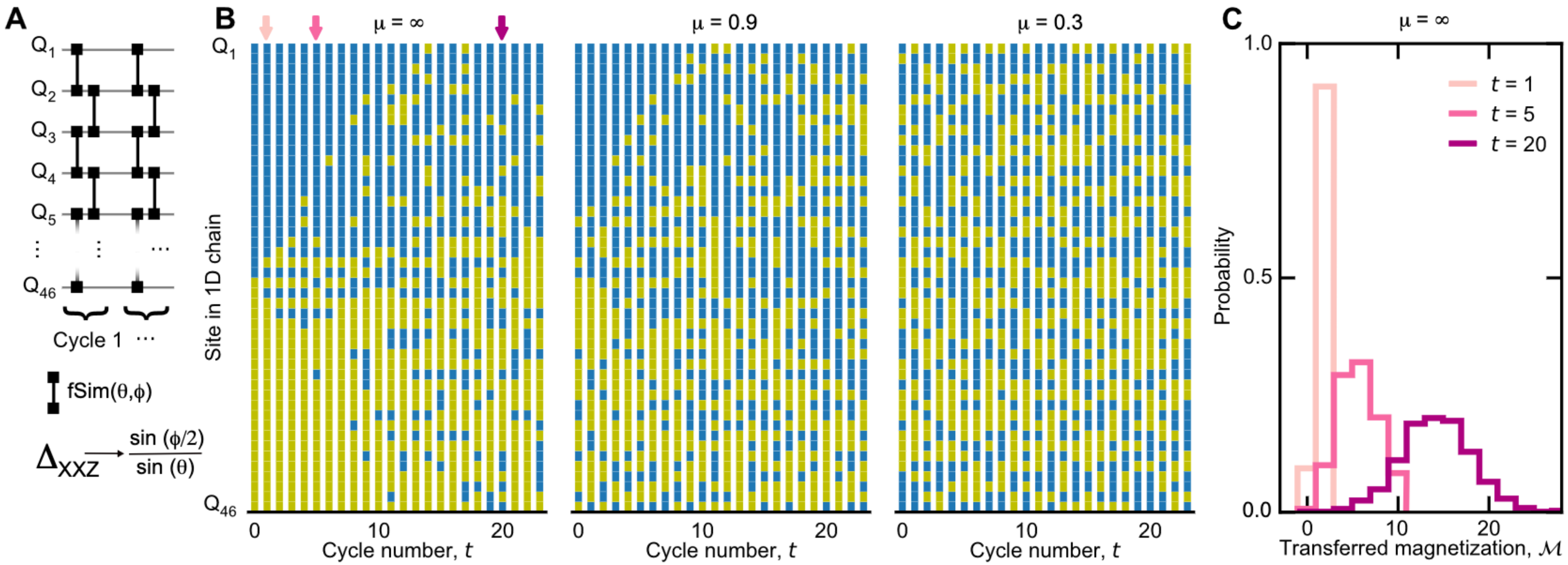
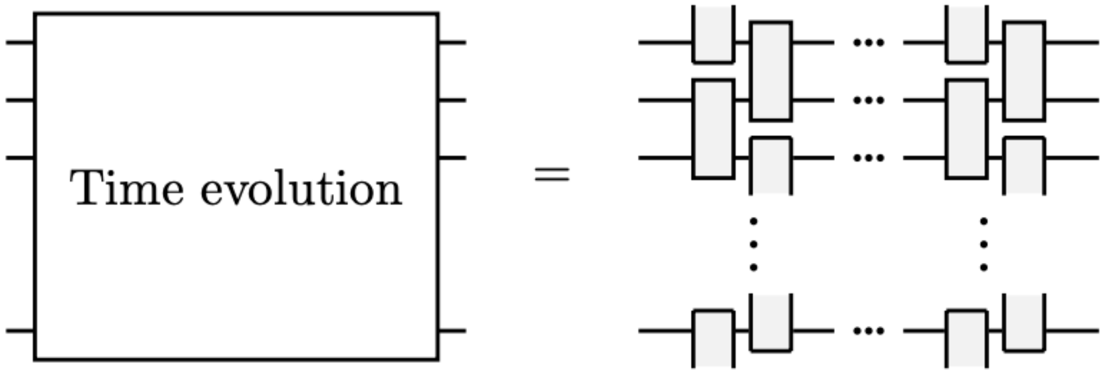


FIG. 1. **Domain wall relaxation in the Heisenberg XXZ spin chain**. (**A**) Schematic of the unitary gate sequence used in this work, where fSim gates are applied in a Floquet scheme on a 1D chain of $N_Q = 46$ qubits. (**B**) Relaxation dynamics as a function of site and cycle number for $\mu = \infty, 0.9$, and 0.3 for initially prepared domain-wall states with $2\langle S^z \rangle = \pm \tanh \mu$. (**C**) Histogram showing the probability distribution of transferred magnetization after $t = 1$, 5 and 20 cycles (arrows in **B**) for $\mu = \infty$.

Google Quantum AI and collaborators, arXiv:2306.09333

Integrable quantum circuits



If the 2-qubit `brick' satisfies a Yang-Baxter Equation (YBE) the Floquet dynamics given by the quantum circuit is integrable

Gritsev, Polkolnikov, 2017

Vanicat, Zadnik, Prosen, 2018

Miao, Gritsev, Kurlov, 2022

Maruyoshi et al, 2022

...

Integrable quantum circuits – fundamental symmetries

Reading qubit states as

$|0\rangle \rightarrow |\uparrow\rangle, \quad |1\rangle \rightarrow |\downarrow\rangle$

natural symmetry is *SU(2)*, as in the XXX chain
or *U(1)* as in an XXZ chain

Reading qubit states as

$|0\rangle \rightarrow |b\rangle, \quad |1\rangle \rightarrow |f\rangle$

natural symmetry is fermionic

- do we have a solution of YBE with fermionic symmetry?

S-matrices in integrable 1+1D qft

Early 1990's: study of factorizable S-matrices in integrable massive 1+1D qft arising from a relevant perturbation of a CFT.

Year 6 - 1989

Example: E_8 structured scattering theory for Ising CFT with magnetic perturbation [Zamolodchikov, 1989]

Clue: for integrable perturbation, 2-body S-matrices satisfy a YBE → provide `integrable' 2-qubit gate

S-matrices in integrable 1+1D qft

Factorizable S-matrices with fermionic symmetry arise for perturbations of Superconformal Field Theories that preserve integrability and supersymmetry

Year 3 - 1986

Volume 185, number 1,2 PHYSICS LETTERS B 12 February 1987

MODULAR INVARIANT PARTITION FUNCTIONS OF SUPERCONFORMAL THEORIES

A. CAPPELLI [1]

Service de Physique Théorique, CEN-Saclay, 91191 Gif-sur-Yvette Cedex, France

Received 29 October 1986

The modular invariant partition functions of two-dimensional minimal superconformal theories are obtained by extending a systematic method developed for conformal theories. They are classified in three infinite series and a few exceptional cases and labelled by simply laced Lie algebras.

Year 7 - 1990

Nuclear Physics B344 (1990) 665–695
North-Holland

SUPERSYMMETRY AND FACTORIZABLE SCATTERING

Kareljan SCHOUTENS*

Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, NY 11794-3840, USA

Received 20 February 1990

We analyze supersymmetric particle theories in $1+1$ dimensions that exhibit factorizable scattering. We propose the general form $\hat{S} = \hat{S}_{\mathrm{BF}} \hat{S}_{\mathrm{B}}$ for the S-matrix, where $\hat{S}_{\mathrm{B}}$ is a purely bosonic S-matrix and $\hat{S}_{\mathrm{BF}}$ describes the mixing of bosonic and fermionic particles. We derive a general expression for $\hat{S}_{\mathrm{BF}}$.

Susy in integrable 1+1D qft

Found in my 1990 paper:

$$\check{\mathbf{S}}(\alpha, \gamma, \theta)$$

- most general supersymmetric 2-body S-matrices satisfying (graded) YBE, with

  $\boldsymbol{\alpha}$ – coupling strength

  $\boldsymbol{\gamma}$ – log of particle mass ratio

  $\boldsymbol{\theta}$ – difference of particle rapidities

Susy in 1+1D integrable qft

- $\check{\mathbf{S}}(\alpha,\gamma,\theta)$ matched with specific perturbed SCFT, later confirmed by TBA analysis

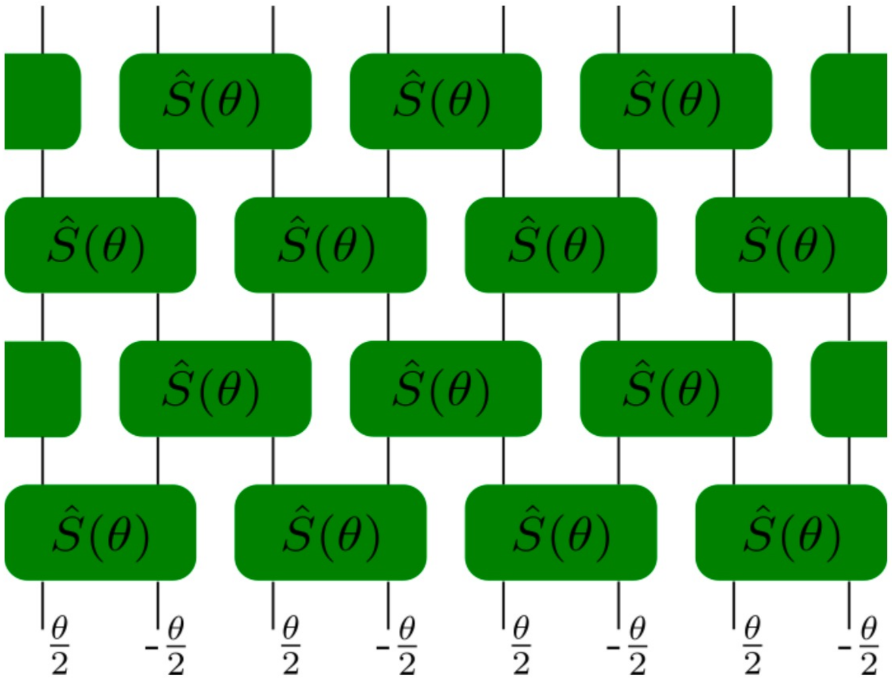
M. Moriconi, KjS - 1995

- $\check{\mathbf{S}}(\alpha,\gamma,\theta)$ satisfies `free fermion' (or `matchgate') property, as a consequence we can write

$$\check{\mathbf{S}}_{i,i+1}(\alpha,\gamma,\theta) = \exp[i\mathbf{E}_{i,i+1}].$$

Brick Wall quantum circuit

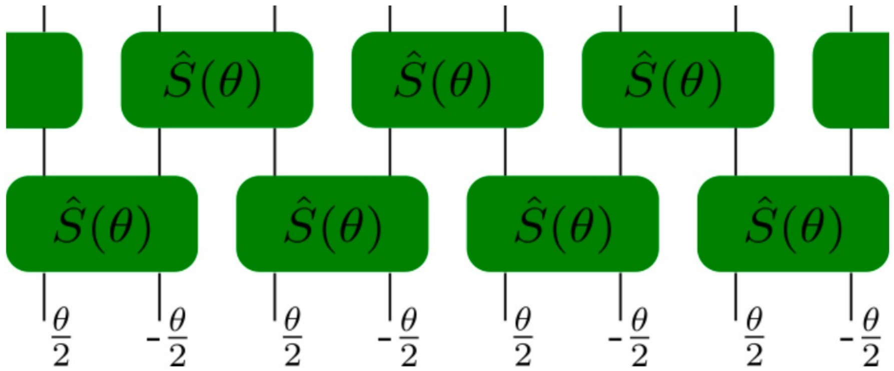
- period n=2
- masses m_1, m_2, ...
- rapidities $\theta/2$, - $\theta/2$, ...
- PBC



Note that gate connecting sites *1*, *L* has string of *Z*-operators to enforce graded tensor product

Essler, Korepin, 1992

Brick Wall quantum circuits



$$\mathbf{U_F}(\theta) = \exp[i\mathbf{E}(\theta)] \overset{\theta \to 0}{=} \mathbf{U_F}(0) + i\mathbf{U_F}(0)\mathbf{H}_\gamma(\theta) + o(\theta^2)\,,$$

Both $\mathbf{E}(\boldsymbol{\theta})$ and $\mathbf{H}_\gamma(\boldsymbol{\theta})$ have free fermionic form,
we obtained closed-form (but involved) expressions for their spectral structure

Spectral structure of $\mathbf{H}_\gamma$

- For $\boldsymbol{\gamma}$ =0 (equal masses) $\mathbf{H}_\gamma$ takes the form of a Kitaev chain Hamiltonian at criticality
- 1-particle dispersion relations for general $\boldsymbol{\alpha}$, $\boldsymbol{\gamma}$

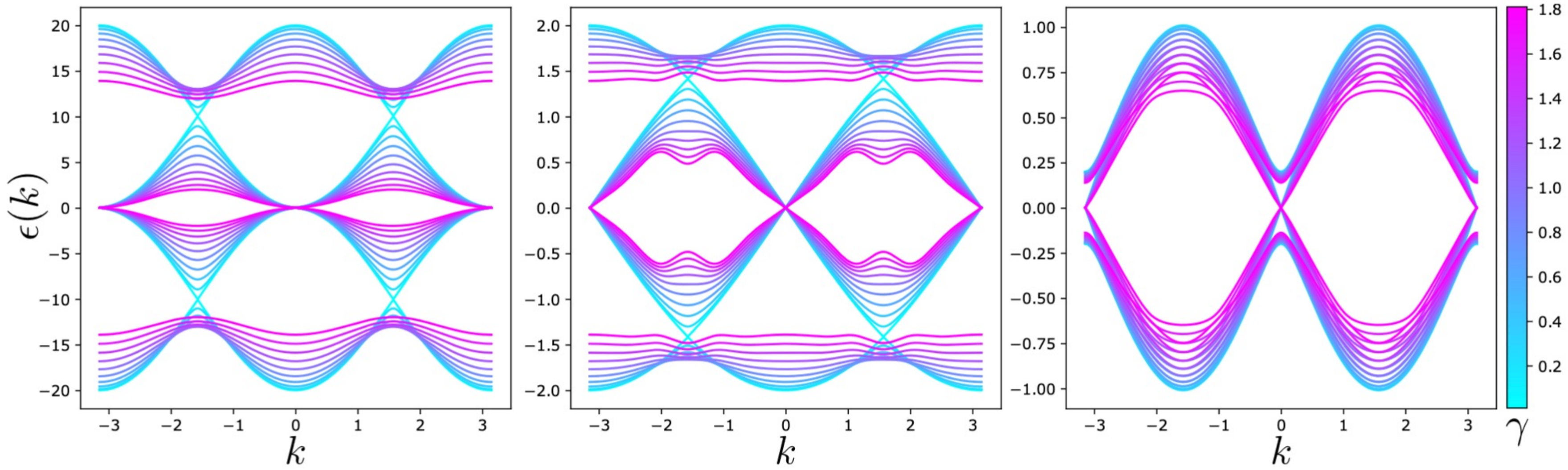


Figure 2: Dispersions for $\mathbf{H}_\gamma$. From the left: $\alpha = 0.1, 1, 10$

Spectral structure of $\mathbf{H}_\gamma$

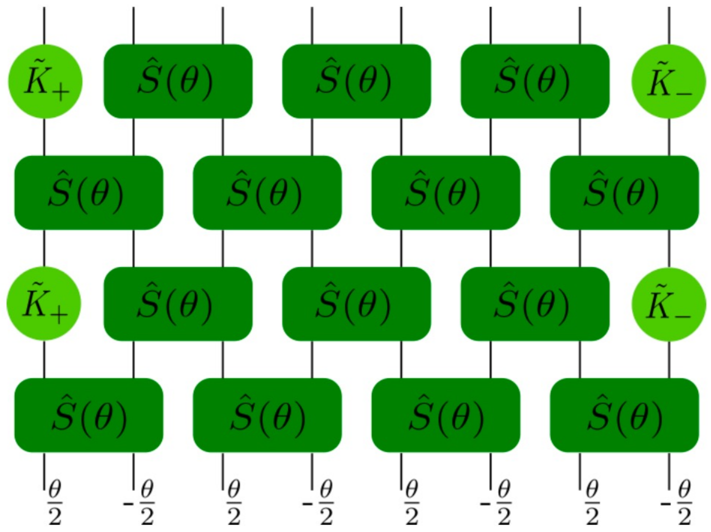
- $\mathbf{H}_\gamma$ found to be critical for all $\boldsymbol{\alpha}$, $\boldsymbol{\gamma}$; criticality protected by global fermionic symmetry

- Breaking the fermionic symmetry opens a gap a leads to topological phases (class BDI), similar to those in the Kitaev and SSH models

Brick wall circuits - OBC

- Open BC require boundary terms that respect the fermionic symmetry

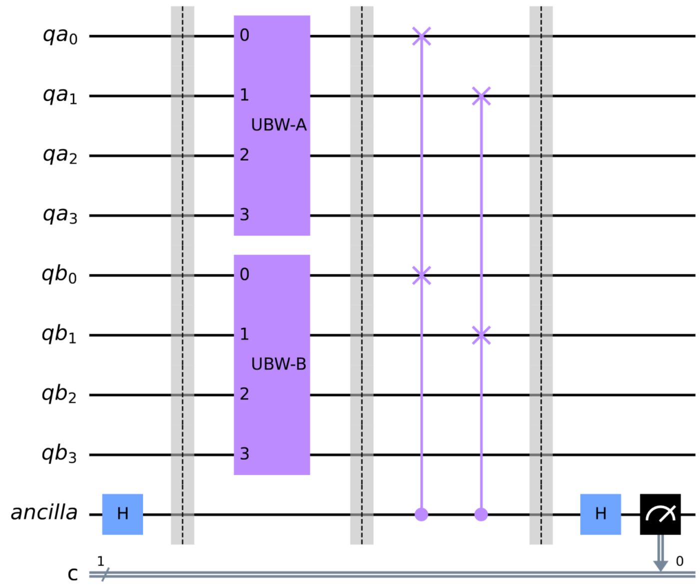
M. Moriconi, KjS - 1996

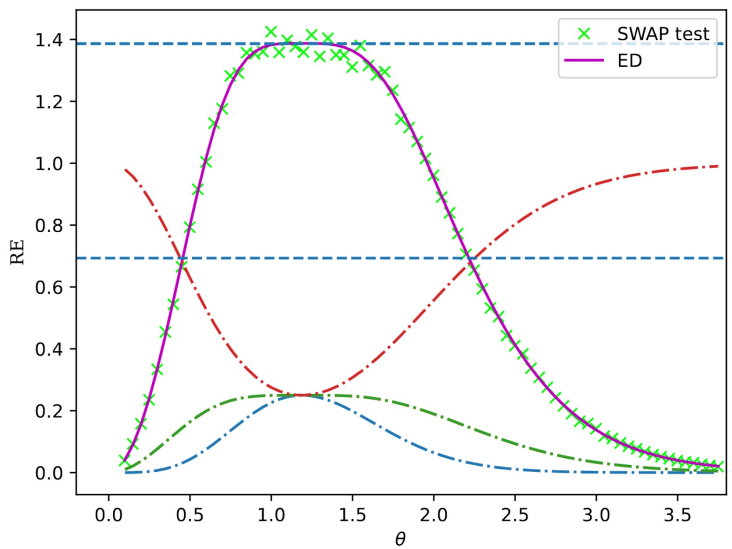


- For unequal masses, OBC circuit needs *2L* layers
- OBC avoids need for global parity operators

Quantum (quench) dynamics

- Highly structured circuits offer large degree of analytical control to study quench dynamics, build-up of Entanglement Entropy, etc
- Straightforward to implement on quantum hardware





Build-up of Rényi Entropy (RE) for L=4 OBC circuit on |0000>, $\boldsymbol{\alpha}$=1.2, $\boldsymbol{\gamma}$=20

65 Years Andrea

Congratulations!