Duality in interacting Bose gases

Fabian Essler (Oxford)

joint work with **Bruno Bertini** (Nottingham) and **Etienne Granet** (Quantinium)

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SFT 2024 - Lectures on Statistical Field Theories

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Integrable models are typically not "adiabatically connected" to free theories:

- Hierarchy of multi-particle bound states
- weakly interacting bosons behave like fermions at low E

Is there an integrable model where elementary excitations can be understood as in Fermi liquid by simple (perturbative) dressing of elementary fermions?

Delta-function Bose gas

Lieb&Liniger '63

N-particle Hamiltonian:

$$H = \sum_{j=1}^{N} \left(-\frac{\partial^2}{\partial x_j^2} - \mu\right) + 2c \sum_{i<j} \delta(x_i - x_j)$$

2nd quantisation:

$$H = \int dx \; \Phi^\dagger(x)\left[-\partial_x^2 - \mu\right] \Phi(x) + c \int dx \; \left(\Phi^\dagger(x)\right)^2 \left(\Phi(x)\right)^2$$

$$[\Phi(x,t), \Phi^\dagger(y,t)] = \delta(x-y)$$

Consider the repulsive case c>0.

Local conservation laws:

$$[Q_n, Q_m] = 0 = [H, Q_n]$$

Smirnov, Davies & Korepin

Impenetrable bosons

"Impenetrable limit" $c \to \infty$: equivalent to free **fermions**

$$H(\infty) = -\int_0^L dx\ \Psi^\dagger(x)\partial_x^2\Psi(x) \qquad\qquad \Psi^\dagger(x) = \Phi^\dagger(x)\ e^{i\pi\int_0^x dz\ \Phi^\dagger(z)\Phi(z)}$$

$$\{\Psi(x), \Psi^\dagger(y)\} = \delta(x-y)$$

2 sectors: periodic/anti-periodic bc's for odd/even fermion #

What about $c \gg 1$?

There should be a description in terms of **weakly interacting fermions:** adiabatic continuity with $c = \infty$ (dressing elementary fermions like in Fermi liquid)

Cheon-Shigehara model

Cheon&Shigehara '98

interacting bosons $\longleftrightarrow$ interacting fermions

$$H_{\mathrm{CS}}(a,c) = \sum_{j=1}^{N} [-\frac{\partial^2}{\partial x_j^2} - \mu] + 2\sum_{i<j} \left[\frac{c}{2} - \frac{1}{a}\right] \left[\delta(x_j - x_k - a) + \delta(x_j - x_k + a)\right]$$

a is a **regulator** that needs to be taken to zero.

(a$\to$0: discontinuous wave-fns, continuous derivatives)

$$\lim_{a\to 0} \psi_{\mathrm{CS}}(x_1, \ldots, x_N) = \psi_{\mathrm{LL}}(x_1, \ldots, x_N) \prod_{j<k} \mathrm{sgn}(x_j - x_k)$$

large c: looks like

strongly interacting bosons $\longleftrightarrow$ strongly interacting fermions

No perturbative calculations possible.

Hm.

A different dual fermion theory

$$H(a,c) = \sum_{j=1}^{N} [-\frac{\partial^2}{\partial x_j^2} - \mu] + 2\sum_{i<j} V_{a,c}(x_i - x_j)$$

Regularised interaction potential: $V_{a,c}(x) = \dfrac{2\sigma''\left(\frac{x}{a}\right)}{cx + 2\sigma\left(\frac{x}{a}\right)}$

$\sigma(x)$ a smooth function s.t.

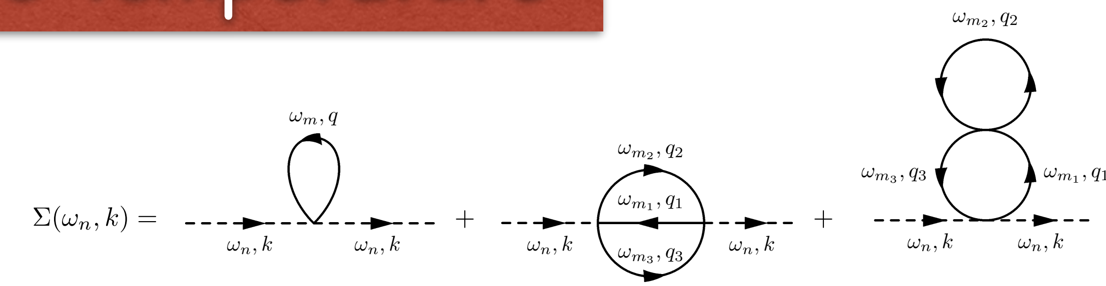
$$\sigma(-x) = -\sigma(x)\ , \qquad \sigma'(0) > 0\ , \qquad \sigma'(x \neq 0) \geq 0$$

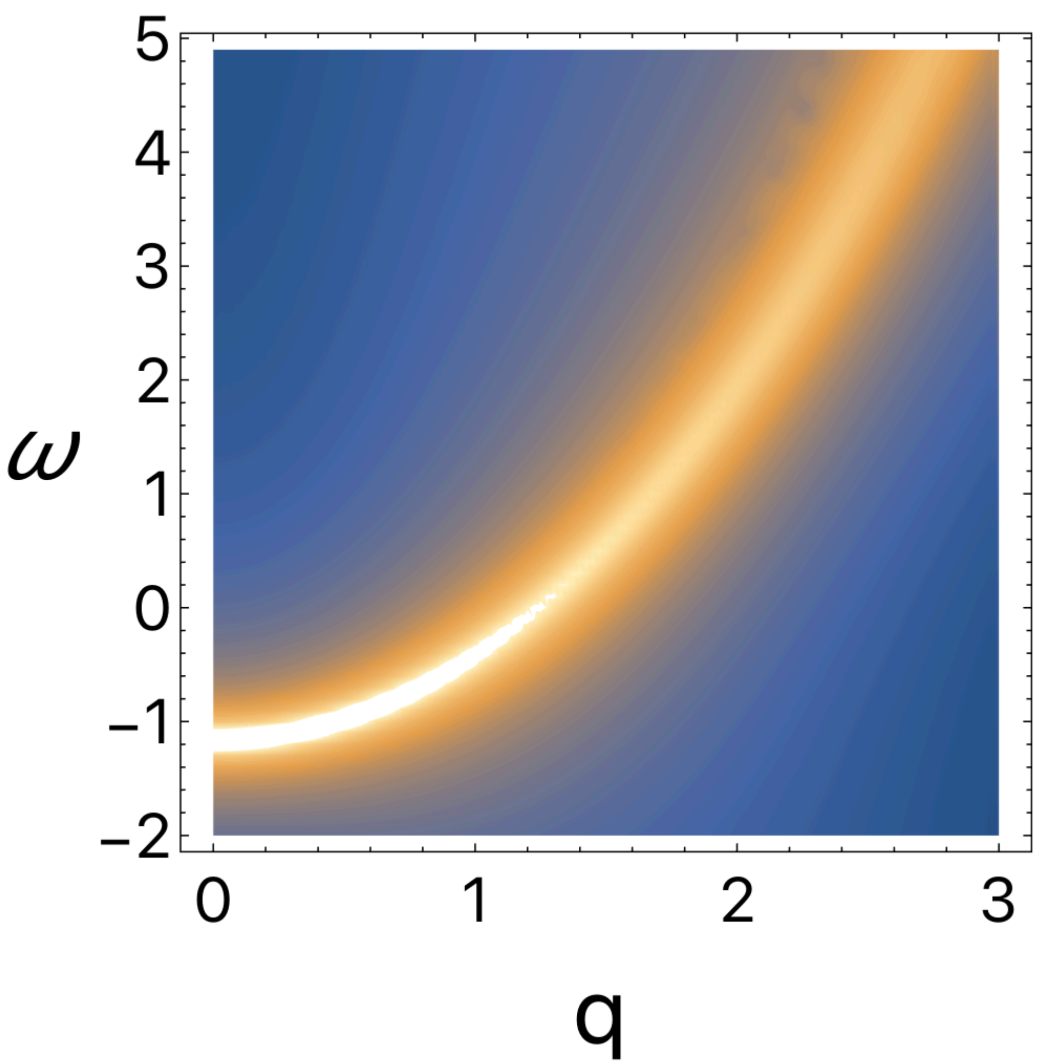
$$\lim_{x\to\infty} \sigma(x) = 1\ , \qquad \lim_{x\to\infty} x^2\sigma''(x) = 0.$$

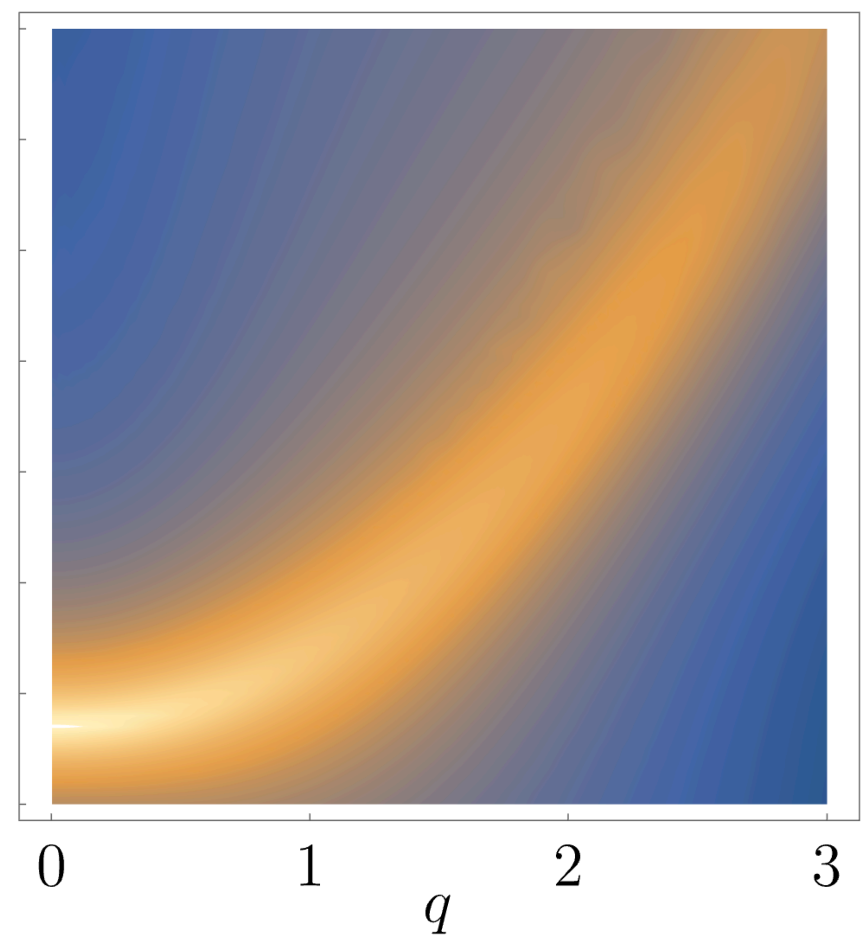
In the limit a→0 this is equivalent to $H_{CS}(a \to 0,c)$, but standard many-body perturbative calculations are possible!

Fermion spectral function at finite temperature

from fermion self-energy at order c^{-2}







$\mu = 1\ ,\ c = 4\ ,\ \beta = 0.5$

$\mu = 1\ ,\ c = 4\ ,\ \beta = 1$

Free fermion acquires finite life-time.

Conserved charges for weakly interacting fermions

Consider $$H = \sum_p \epsilon(p)\psi_p^\dagger \psi_p + \frac{\beta}{L}\sum_{\boldsymbol{p}_4} V(\boldsymbol{p}_4)\psi_{p_1}^\dagger \psi_{p_2}^\dagger \psi_{p_3}\psi_{p_4}$$

(only quartic interactions)

Try to construct conserved charges order-by-order in β

$$Q^{(n)} = \int dx Q^{(n)}(x) = \sum_p p^n \psi_p^\dagger \psi_p + \sum_{\boldsymbol{p}_4} S_1^{(n)}(\boldsymbol{p}_4)\psi_{p_1}^\dagger \psi_{p_2}^\dagger \psi_{p_3}\psi_{p_4}$$

$$+ \sum_{\boldsymbol{p}_6} S_2^{(n)}(\boldsymbol{p}_6)\psi_{p_1}^\dagger \psi_{p_2}^\dagger \psi_{p_3}^\dagger \psi_{p_4}\psi_{p_5}\psi_{p_6} + \ldots$$

where $S_k^{(n)}$ has an expansion in β starting at β^k

Can determine $S_k^{(n)}$ recursively for **any** interaction potential!

???

Charges are generally **non-local** and become quasi-local only for special **integrable** $V(\mathbf{p}_4)$!

In the spinless fermion case we find at $\mathcal{O}(\beta^2)$:

1. Cheon-Shigehara model
2. Calogero-Sutherland model.

Precisely the known integrable models!

This construction is equivalent to $\widetilde{H} = e^{-iS} H e^{iS}$, $S = \sum_{n \geq 1} \beta^n S_n$,

$$\widetilde{H} = \sum_{n \geq 1} \sum_{k_1 < \ldots < k_n} h_n(k_1, \ldots, k_n)\ \hat{n}(k_1)\hat{n}(k_2)\ldots n(k_n)$$

$$h_n(k_1, \ldots, k_n) = \begin{cases} \mathcal{O}(\beta^{n-1}) & \text{if H integrable} \\ \mathcal{O}(\beta^0) & \text{if H not integrable} \end{cases}$$

$$\widetilde{H} \prod_{j=1}^{N} c^\dagger(k_j)\,|\,0\rangle = E(\boldsymbol{k}) \prod_{j=1}^{N} c^\dagger(k_j)\,|\,0\rangle$$

Undo the unitary transformation: $A(I) = e^{iS} c^\dagger(k) e^{-iS}$, $k = \dfrac{2\pi I}{L}$

fermionic creation operators for Bethe integers!

$$H \prod_{j=1}^{N} A^\dagger(I_j)\,|\,0\rangle = E(\boldsymbol{I}) \prod_{j=1}^{N} A^\dagger(I_j)\,|\,0\rangle + \mathcal{O}(c^{-n})$$

Summary

1. Duality that maps strongly interacting Bose gas to weakly interacting fermions.
2. Perturbative analysis now feasible.
3. Generalizations to multi-species Yang-Gaudin models possible.
4. Operator equivalences ?
5. "Perturbative" construction of conserved charges to higher orders → models with interesting "pre thermal" dynamics