Combinatorial Quantum Gravity
q-it from bits?

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40 Years of Conformal Field Theory
Conference in honour of Andrea Cappelli
Florence February 2024

The problem of quantum gravity

GR: space-time = dynamical manifold governed by Ricci curvature R

$$S_{\mathrm{EH}} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R - 2\Lambda)$$

Quantization:

- Time concepts in QM and GR incompatible (see e.g. Kuchar)
- GR perturbatively non-renormalizable, problems at short scales

Many approaches, make sense of $\int_{\mathrm{manifolds}} e^{-S_{\mathrm{EH}}^{\mathrm{Euclidean}}}$

J. A. Wheeler: both GR and QM emerge from more fundamental theory involving only binary d.o.f : **"it from bit"** hypothesis

Today: space-time from entanglement **Here**: stick to original proposal preeminence of QM

It from qubit

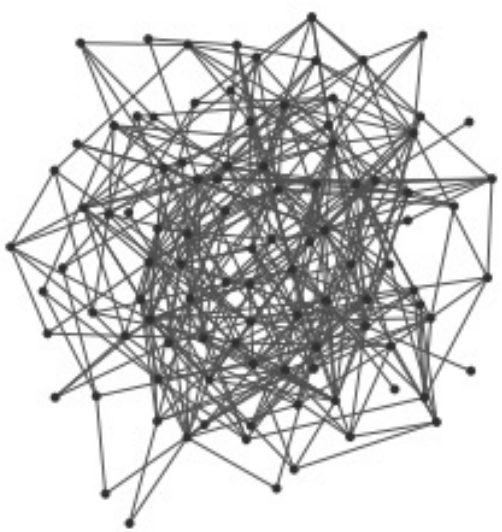
Q-it from bit

Bits = edges of random graph
Combinatorial quantum gravity

Last 15 years → discrete geometry on metric spaces
Purely combinatorial formulation of Einstein's idea

C. A. T. , JHEP 1709 (2017) 45

Random graphs



Note: graph can be viewed as entanglement pattern if desired

Combinatorial quantum gravity

$$Z = \sum_{\text{random graphs}} e^{-\frac{H}{g}}$$

$$H = -\sum_i \kappa(i)$$

$$\kappa(i) = \sum_{j\sim i} \kappa(i,j)$$

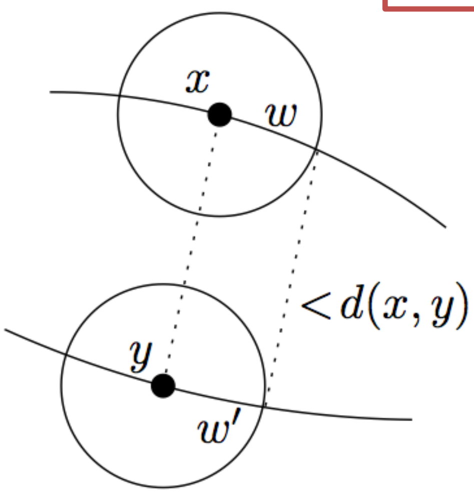
$\varkappa$ (i,j) : **Ollivier curvature**

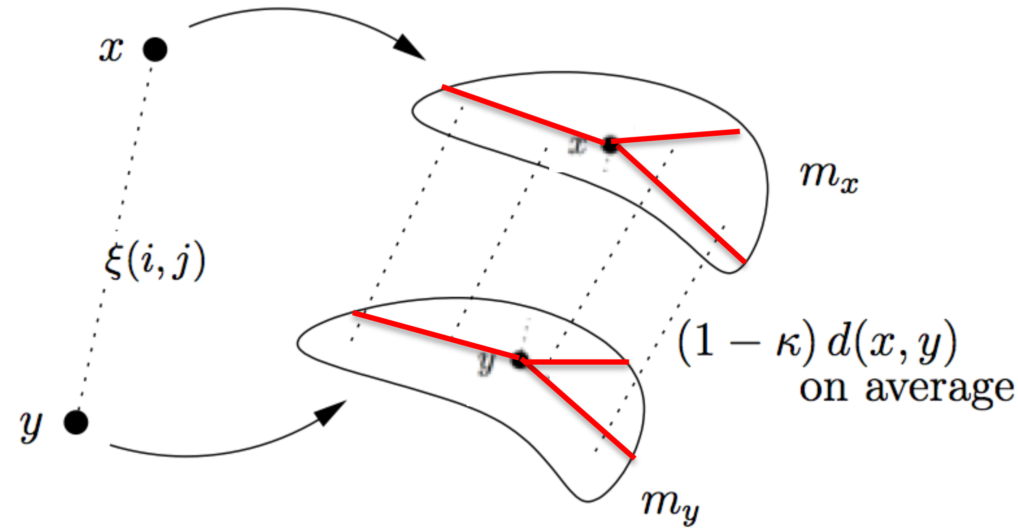
Y. Ollivier, *Comptes Rendus Math*. **345** (2007) 643.

Combinatorial Ricci curvature (Ollivier)

$$\boxed{\kappa(x,y) := 1 - \frac{W_1(m_x, m_y)}{d(x,y)}}$$

x, y vertices
d(x,y)= graph dist.
m = unif. prob. distr. on unit ball





Wasserstein distance

$$W(\mu_1, \mu_2) = \inf \sum_{i,j} \xi(i,j) d(i,j)$$

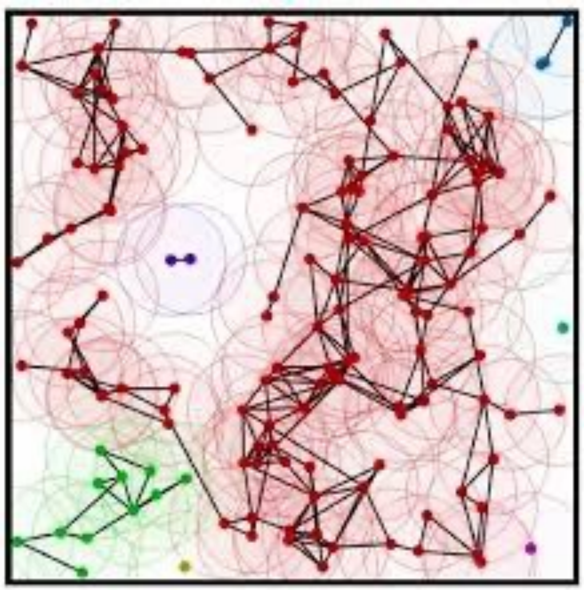
Transference plan $\xi(i,j)$

$$\sum_j \xi(i,j) = \mu_1(i)\ , \qquad \sum_i \xi(i,j) = \mu_2(j)$$

Convergence to Ricci

Combinatorics ➜ geometry requires introducing a length scale for the edges of the random graph

Random geometric graphs



Sprinkle D-dim. Riemann manifold M with N points by a Poisson process, then connect points with $d < \varepsilon_N$

If edges weighted by manifold distances, then

$$\frac{\kappa(x,y)}{\epsilon_{\mathrm{N}}^2} \to \frac{\mathrm{Ric}(\mathrm{v},\mathrm{v})}{2(\mathrm{D}+2)}$$

P. Van Horn, W. Cunnigham, G. Lippner, C.A. T. & D. Krioukov, arXiv:2008.01209

C.Kelly, C.A.T. & F. Biancalana, Phys. Rev. D105, 124002 (2022)

The graph ensemble

- **Random 2d-regular graphs →** d=manifold dimension
- **Incompressible graphs**: hard-core condition, short loops can share at most one edge

$$\kappa(uv) = \frac{\triangle_{uv}}{d} - \left[1 - \frac{2 + \triangle_{uv} + \square_{uv}}{d}\right]_+ - \left[1 - \frac{2 + \triangle_{uv} + \square_{uv} + \pentagon_{uv}}{d}\right]_+$$

Continuous phase transition: **randomness to geometry**

↓

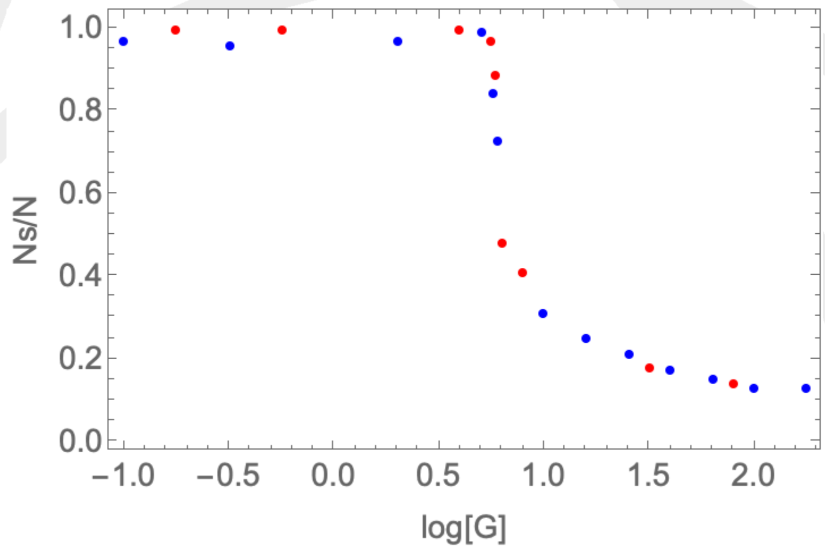
Condensation of 4-cycles (square loops) on the graph

↓

Emergence of a scale → Planck length

C. A. T. , JHEP 1709 (2017) 45, C. Kelly, C. A. T. & F. Biancalana, Class. Quantum Grav. 36, 125012 (2019);
Class. Quantum Grav. 38, 075008 (2019);

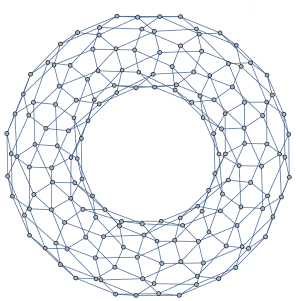
d=2, surfaces from random graphs



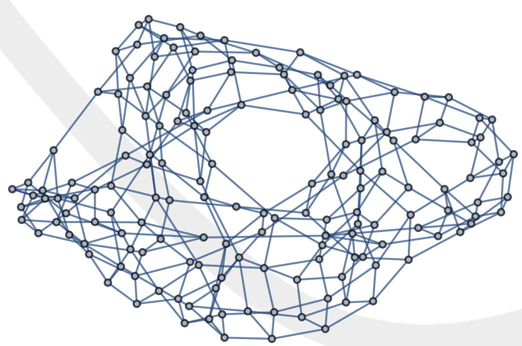
Geometric phase

Random phase

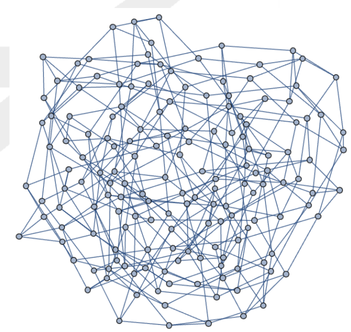
Genus lowering transition



$g=0$



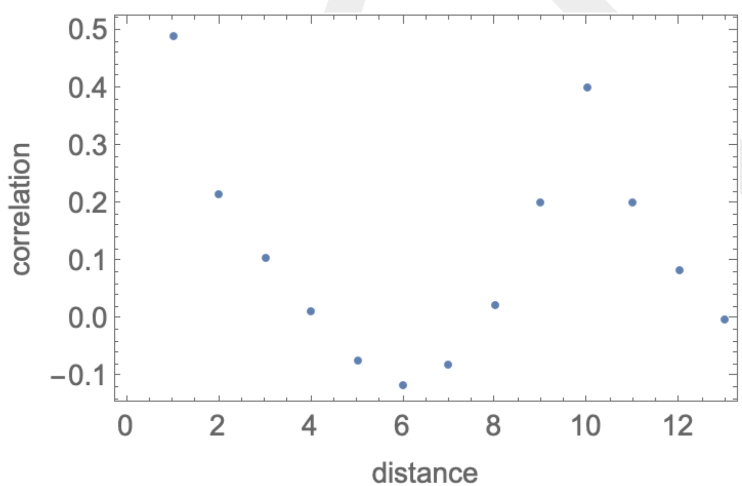
$g \ll g_{cr}$

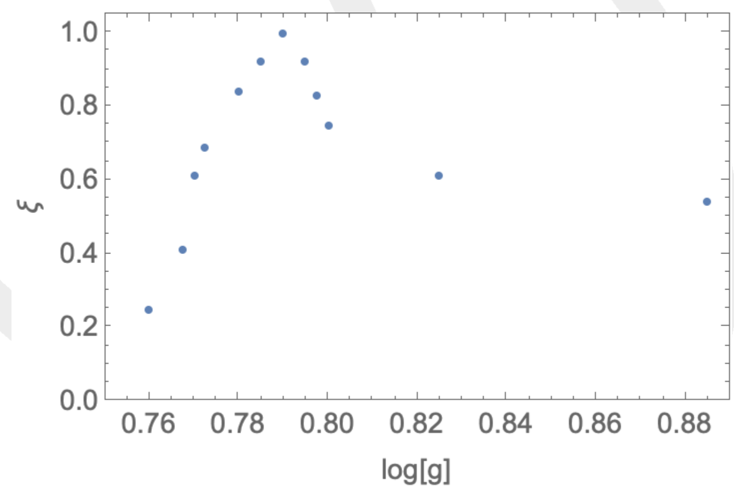


$g \gg g_{cr}$

Emergent Planck length

Correlation functions **$G(d) = \langle N_S(v_i)\, N_S(v_j) \rangle_C$** between numbers of squares **$N_S(v)$** at vertices v_i and v_j at graph distance d



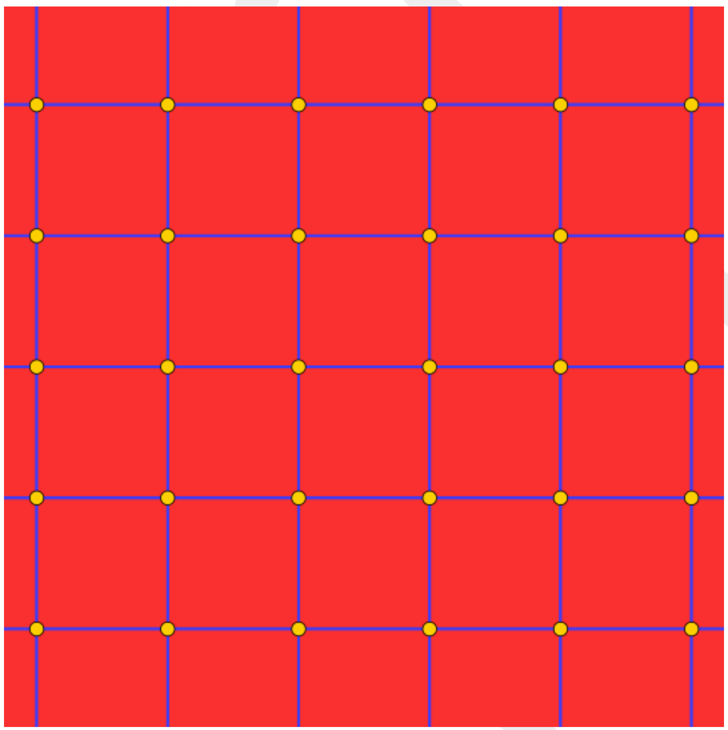


$g < g_{cr}$: random bubbles of scale $N_r = O(N^r)$, diameter $\rightarrow \log(N_r)$

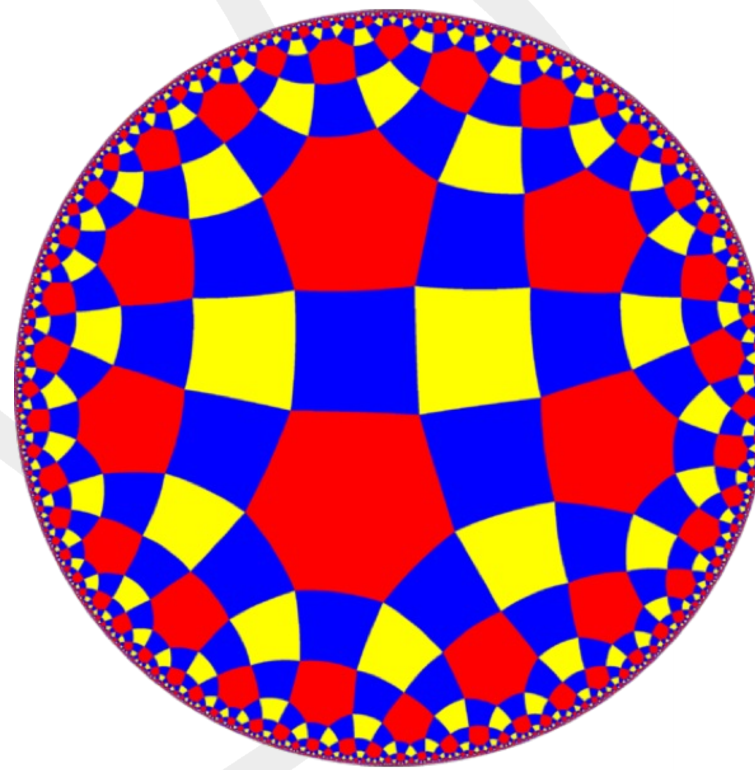
Length ℓ for edges, $\ell \rightarrow 0$, $N \rightarrow \infty$, bubble size $\ell_P = r(g)\, \ell \log(N)$ fixed

Infinite geometric surfaces with bubbles of randomness of scale ℓ_P

Condensation of loops = birth of manifolds

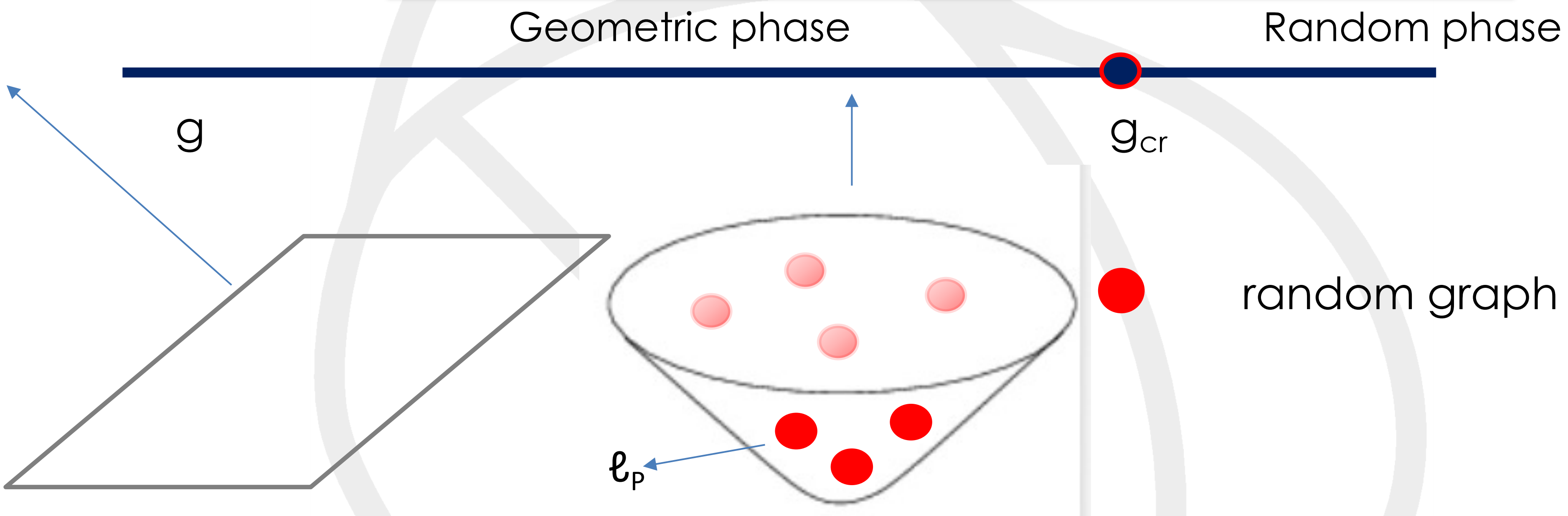


$g = 0$, 4 squares per vertex
flat plane



$0 < g < g_{cr}$, 3 squares per vertex
hyperbolic plane, negative curvature

Emergence of hyperbolic universe with defects



Hyperbolic space: **negative curvature**

= “randomness” bubbles of scale ℓ_P
= excitation of energy given by [–(excess negative curvature)]

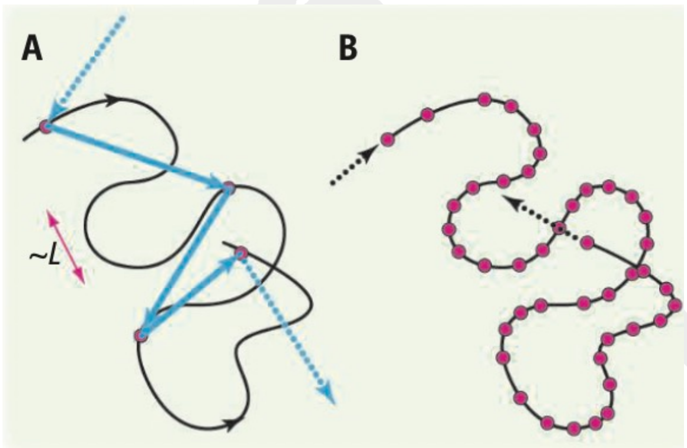
C. A. T. JHEP 04 (2022) 019

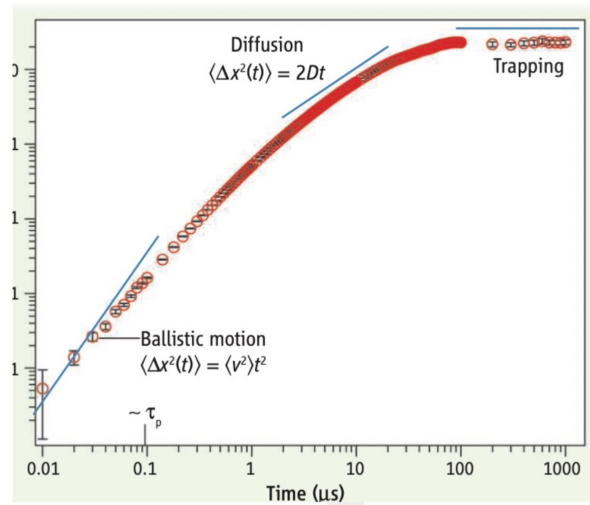
Fluctuations (from underlying graph): need **universal time t (before/after)**

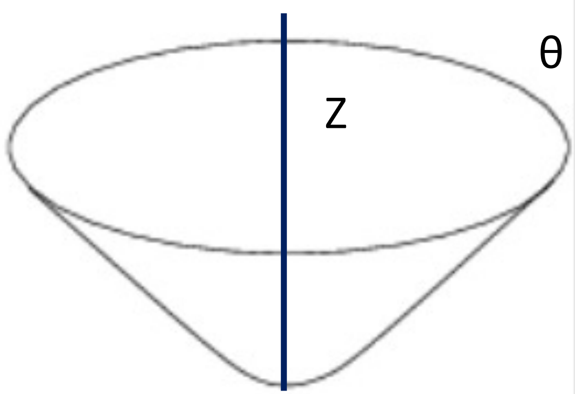
diffuse on the hyperbolic space

Ballistic diffusion on hyperbolic space

Simplicity ➜ diffusion of point excitations (non-relativistic)







$$\mathbf{x} = \begin{pmatrix} \frac{1}{H}\sinh Hz\ \cos\theta \\ \frac{1}{H}\sinh Hz\ \sin\theta \\ \frac{1}{H}\cosh Hz \end{pmatrix}$$

Curvature : $R = -H^2$

Emergence of coordinate time
Effective de Sitter space-time

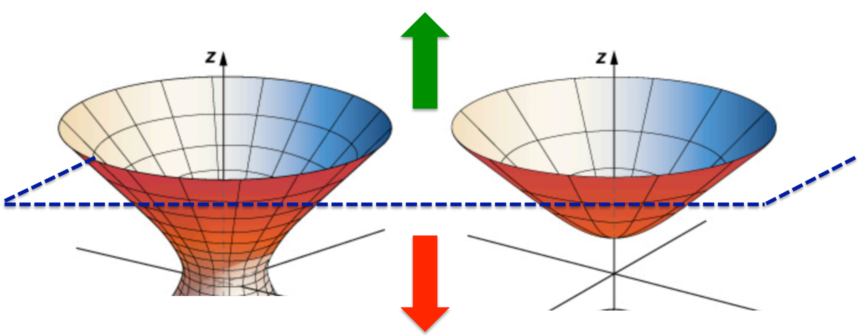
radial $$z_t = \frac{H}{2} t$$ + corr. + sub-dominant random component

angular $$\left(\frac{\partial^2}{\partial z^2} + H \frac{\partial}{\partial z} - 4H^2 e^{-2Hz} \frac{\partial^2}{\partial \theta^2} \right) u(z, \theta) = 0$$ + corr. + random

Wave equation on **effective de Sitter space,** $R_{eff} = + H^2$

Lorentzian ≈ Riemannian to leading order

Lorentzian space-time good description



Fundamental Riemannian manifold

● = matter particles
E ∝ R, trace of Einstein eqs.

Matter and space made of same stuff (random/geometric)

C. A. T. JHEP 03 (2023) 186

Random component
Quantum mechanics in 3D?

- Ballistic diffusion too fast to see spectral dimensions (spectral gap)
- Sub-dominant random component = infinite Brownian loop

J.-P. Anker et al. *Rev. Mat. Iberoamericana* **18** (2002) 41, E.B. Davies and N. Mandouvalis, *Proc. Lond. Math. Soc.* **3-57** (1988) 182.

Riemannian $$K(t,\rho) \asymp t^{-\frac{3}{2}} \mathrm{e}^{-\frac{\rho^2}{4t}}$$

Effective Lorentzian $$K_{\mathrm{eff}}(\tilde{t},\rho) \asymp \left(\frac{m}{i\hbar\tilde{t}}\right)^{\frac{3}{2}} \mathrm{e}^{-\frac{m\rho^2}{2i\hbar\tilde{t}}}$$

Hyperbolic distance inherited in 3D

Local limit theorem in negative curvature
F. Ledrappier and S. Lim, arXiv:1503.04156.

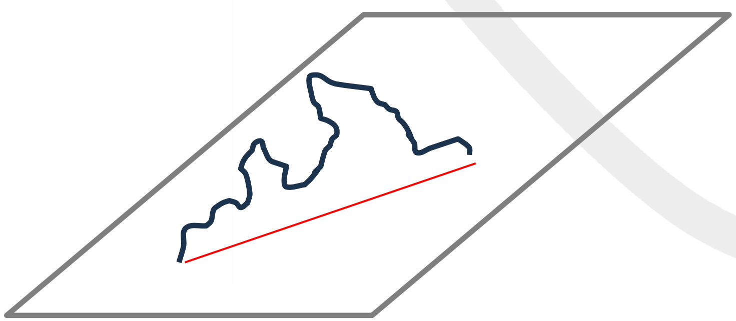
Spectral dimension **always 3** independent of top. dimension

Holographic quantum mechanics in 3+1 dimensions
Holographic (3+1)-dim. metric Minkowski space with gravity interactions

Conclusions

Purely combinatorial formulation of GR

- Space, time and matter emerge from graphs (only combinatorics)
- Space-time and matter made of same stuff: two different phases
- D=2: random surface with two scales, UV Planck scale and IR radius of curvature.
- $\ell < \ell_P$: $d_{haus} = \infty$, matter particles
- $\ell_P < \ell < \ell_{curv}$: $d_{haus} = d_{spectral} = 2$, diffusion as dynamics
- $\ell_{curv} < \ell$: $d_{spectral} = 3$, Minkowski space + quantum mechanics as effective dynamics
- Curvature effects in 3D (GR) inherited from 2D holographic screen



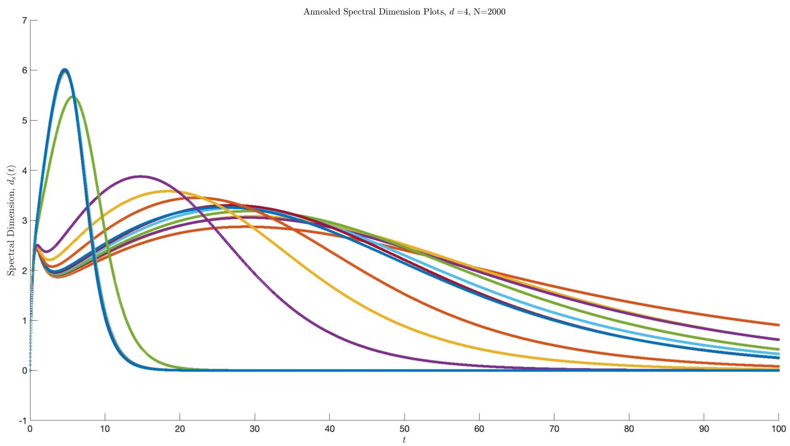
Maybe it is QM that uses one more dimension??

C. A. T. Universe 9, 499 (2023)

Swiss Scientific
TECHNOLOGIES

Happy birthday Andrea

Spectral curves
Hausdorff dimension



$$K(t) = \mathrm{tr}\, \mathrm{e}^{\Delta t}$$

$$d_\mathrm{s}(t) = -2\frac{d\,\ln K(t)}{d\,\ln t}$$