Anomalous fluid dynamics as bosonization

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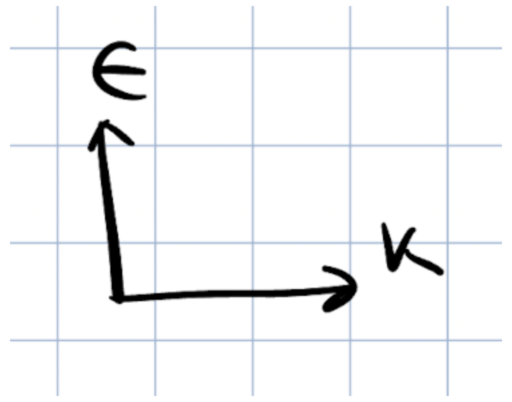
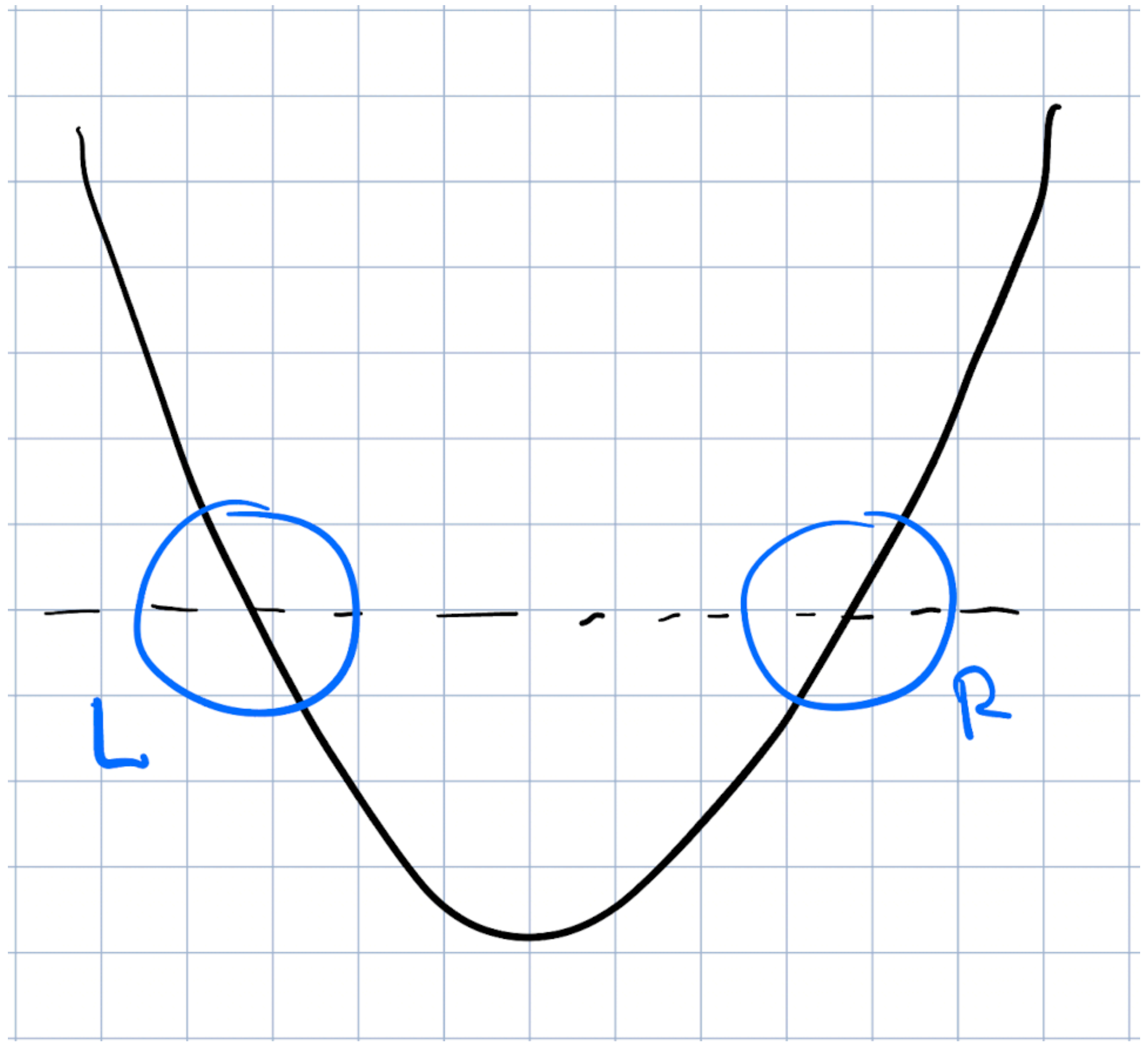
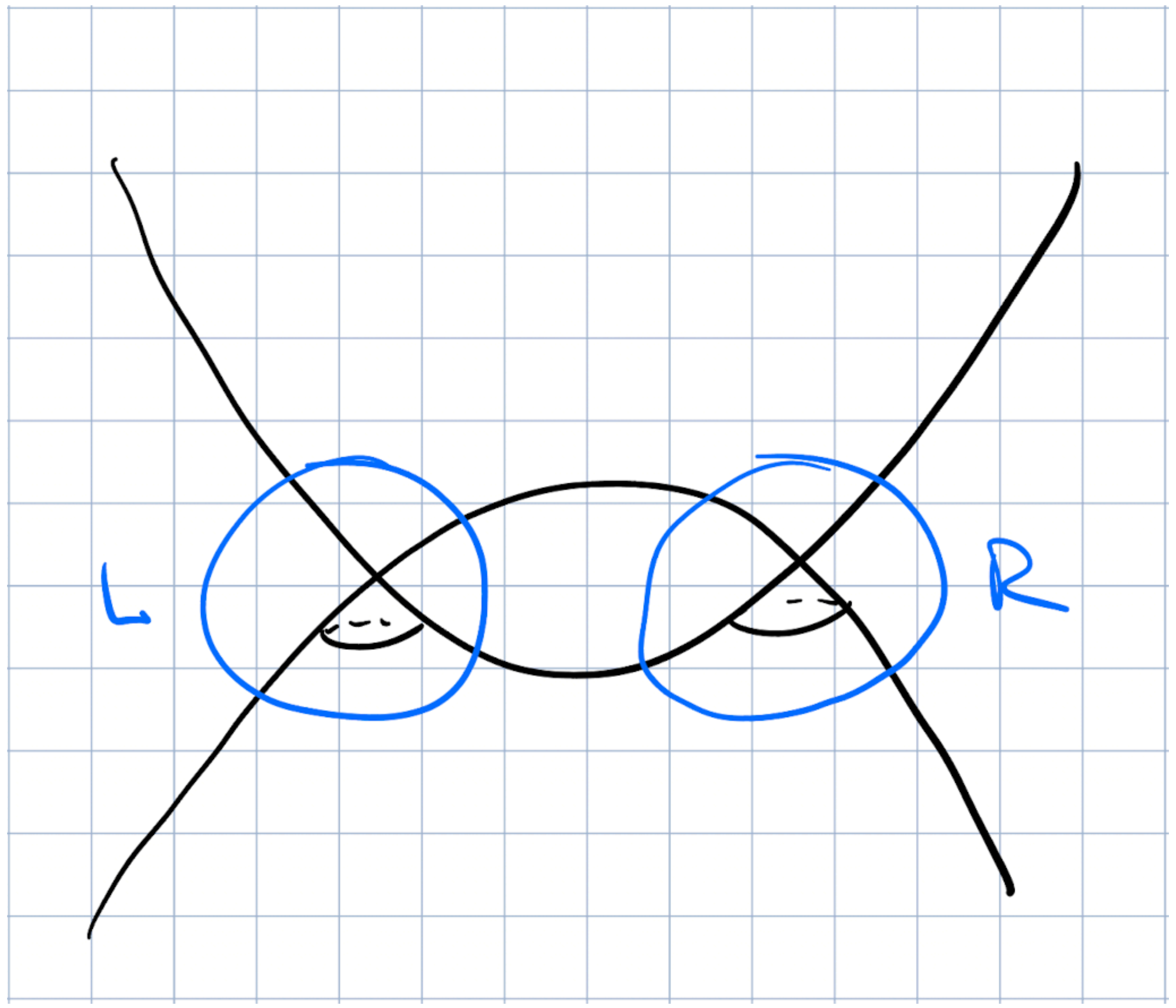
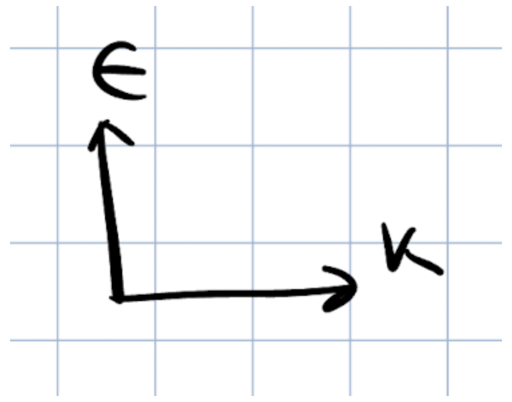
Fourty Years of Conformal Field Theory, GGI, Florence

Chiral anomaly in pictures

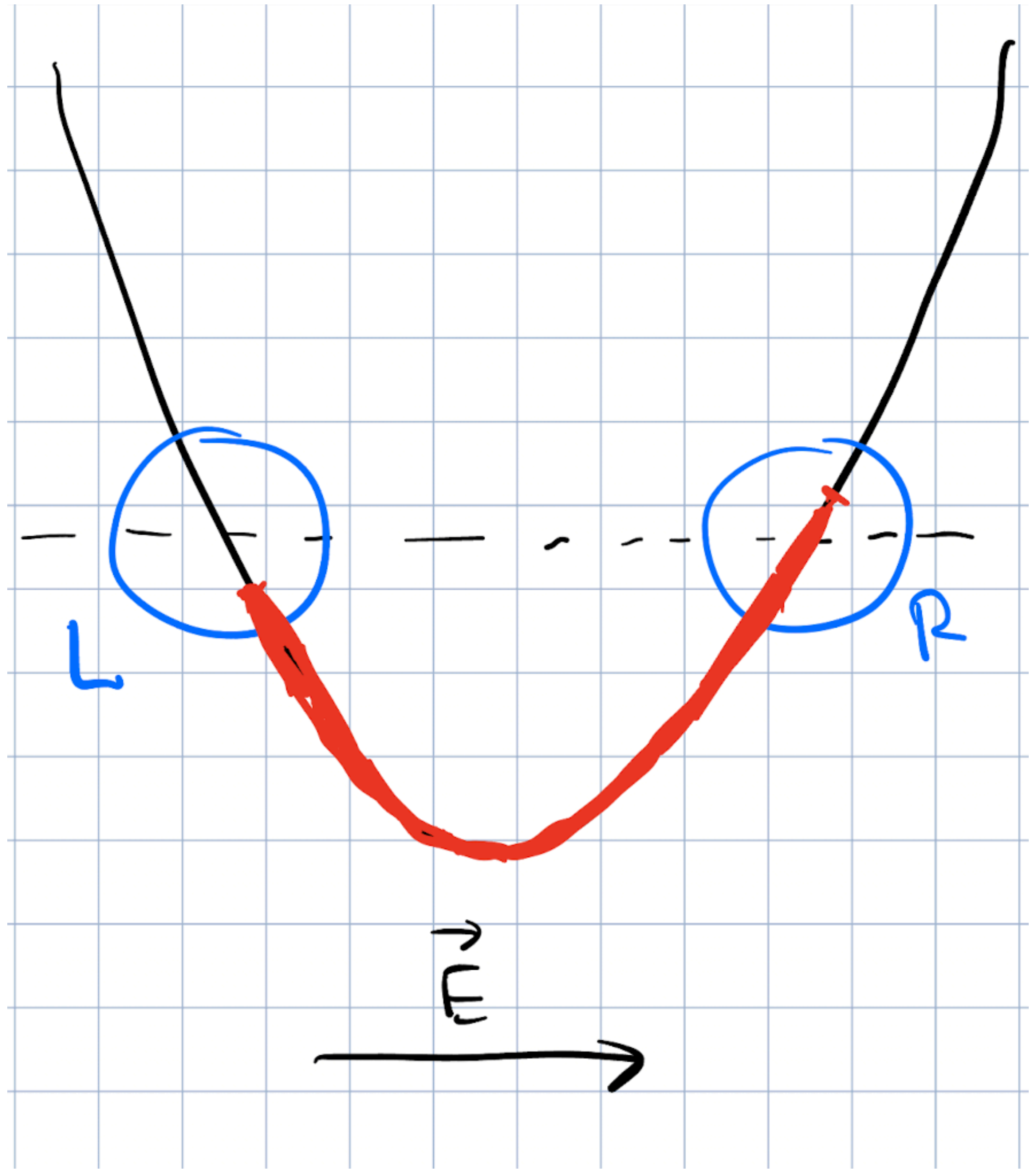


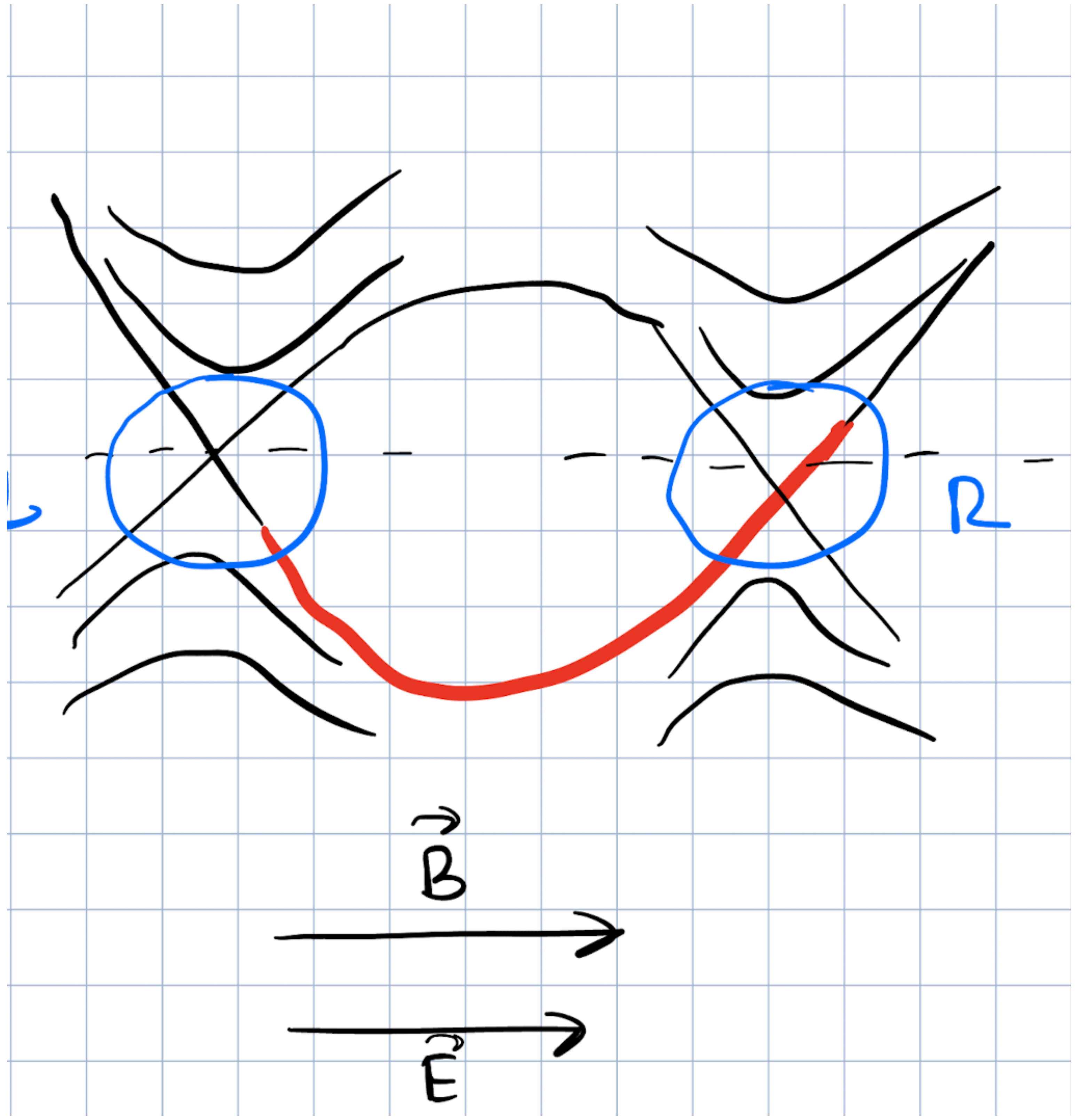






Chiral anomaly in pictures





$$\frac{d}{dt}(n_R - n_L) \sim E$$

$$\partial_\mu j^\mu_5 \sim E$$

$$\frac{d}{dt}(n_R - n_L) \sim \vec{E} \cdot \vec{B}$$

$$\partial_\mu j^\mu_5 \sim \vec{E} \cdot \vec{B}$$

1+1 charged boson

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2$$

$j^\mu = \partial^\mu \phi$ Noether current $\partial_\mu j^\mu = 0$

$\tilde{j}^\mu = \epsilon^{\mu\nu}\partial_\nu \phi$ Topological current $\partial_\mu \tilde{j}^\mu = 0$

1+1 charged boson

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2$$

$j = \partial\phi$ Noether current $\partial j = 0$

$\tilde{j} = [d\phi]$ Topological current $\partial\tilde{j} = 0$

Gauging 1+1 charged boson

$$\mathcal{L} = \frac{1}{2}(\partial\phi - A)^2$$

$j = \partial\phi - A$ Noether current $\partial j = 0$

$\tilde{j} = [d\phi]$ Topological current $\partial\tilde{j} = 0$

Not gauge-invariant!

Gauging 1+1 charged boson

$$\mathcal{L} = \frac{1}{2}(\partial\phi - A)^2$$

$j = \partial\phi - A$	Noether current	$\partial j = 0$
$\tilde{j} = [d\phi - A]$	Topological current	$\partial\tilde{j} = -[dA]$

Gauge-invariant!

Not conserved

Gauging 1+1 charged boson

$$\mathcal{L} = \frac{1}{2}(\partial\phi - A)^2 + \tilde{A} \wedge (d\phi - A)$$

$j = \partial\phi - A$ Noether current $\partial j = [d\tilde{A}]$

$\tilde{j} = [d\phi - A]$ Topological current $\partial\tilde{j} = -[dA]$

Both covariant currents are not conserved! Mixed vector-axial anomaly.

Anomaly Inflow

$$\mathcal{L} = \frac{1}{2}(\partial\phi - A)^2 + \tilde{A} \wedge (d\phi - A) - \int_{M_3} \tilde{A} \wedge dA$$

$j = \partial\phi - A$ Noether current $\partial j = [d\tilde{A}]$

$\tilde{j} = [d\phi - A]$ Topological current $\partial\tilde{j} = -[dA]$

Both covariant currents are not conserved! Mixed vector-axial anomaly.

Anomaly Inflow

$$\mathcal{L} = P(\partial\phi - A) + \tilde{A} \wedge (d\phi - A) - \int_{M_3} \tilde{A} \wedge dA$$

Pressure, equation of state

$j = \partial\phi - A$ Noether current $\partial j = [d\tilde{A}]$

$\tilde{j} = [d\phi - A]$ Topological current $\partial\tilde{j} = -[dA]$

Both covariant currents are not conserved! Mixed vector-axial anomaly.

Euler fluid and helicity conservation in 3d

$$v_t + (v\nabla)v = -\nabla\mu$$

$$\rho_t + \nabla(\rho v) = 0$$

$$dP = \mu d\rho$$

barotropic equation of state

Consequence of equations: conservation of helicity $$\partial_t \int v(\nabla \times v) = 0$$

$$\partial_t(v\omega) + \nabla(\ldots) = 0$$

additional conservation law!

$\omega = \nabla \times v$ - vorticity

$h = v \cdot \omega$ - helicity

Euler fluid and helicity conservation in 3d

$$v_t + (v\nabla)v = -\nabla\mu$$

$$dP = \mu d\rho$$

barotropic equation of state

$$\rho_t + \nabla(\rho v) = 0$$

$$\partial_t(v\omega) + \nabla(\ldots) = 0$$

Is there a mixed anomaly between two conservation laws?

Euler fluid and helicity conservation in 3d

$$v_t + (v\nabla)v = -\nabla\mu + E + v \times B$$

$$\rho_t + \nabla(\rho v) = 0$$

$$\partial_t\Big((v + A) \cdot (\omega + B)\Big) + \nabla(\ldots) = 0$$

Not gauge-invariant!

Euler fluid and helicity conservation in 3d

$$v_t + (v\nabla)v = -\nabla\mu + E + v \times B$$

$$\rho_t + \nabla(\rho v) = 0$$

$$\partial_t\Big(v \cdot (\omega + 2B)\Big) + \nabla(\ldots) = 2E \cdot B$$

Gauge-invariant!

Not conserved

Anomaly for axial current in presence of vector gauge field!

Covariant notations

$$\pi_0 - A_0 = -\mu - \frac{v^2}{2}$$

$$\pi_i - A_i = v_i$$

- covariant notations

$$d\pi \wedge d\pi = 0$$

covariant form of
helicity conservation

$$\partial_\mu \left(\epsilon^{\mu\nu\lambda\rho} \pi_\nu \partial_\lambda \pi_\rho \right) = 0$$

for A=0

$$\partial_t (v\omega) + \nabla(\ldots) = 0$$

Not gauge-invariant!

Covariant notations

$$\pi_0 - A_0 = -\mu - \frac{v^2}{2}$$

$$\pi_i - A_i = v_i$$

- covariant notations

$$d\pi \wedge d\pi = 0$$

covariant form of helicity conservation

$$v \cdot (\omega + 2B)$$

$$2E \cdot B$$

$$d\Big((\pi - A) \wedge d(\pi + A)\Big) = \cancel{d\pi \wedge d\pi} - dA \wedge dA = -dA \wedge dA$$

Gauge-invariant!

Not conserved

$$\partial \tilde{j} = 2E \cdot B$$

Actions in 1d and 3d

$$\mathcal{L} = P(\pi - A) + \tilde{A} \wedge (\pi - A) - \int_{M_3} \tilde{A} \wedge dA$$

$$d\pi = 0 \qquad \pi = d\phi$$

$$\partial j = [d\tilde{A}] \qquad \partial \tilde{j} = -[dA]$$

$$\mathcal{L} = P(\pi - A) + \tilde{A} \wedge (\pi - A) \wedge d(\pi + A) - \int_{M_5} \tilde{A} \wedge dA \wedge dA$$

$$d\pi \wedge d\pi = 0 \qquad \pi = d\phi + \alpha d\beta$$

$$\partial j = -2\left[dA \wedge d\tilde{A}\right] \qquad \partial \tilde{j} = -\left[dA \wedge dA\right]$$

Even more general!

$$\mathcal{L} = P(\pi - A) + \tilde{A} \wedge (\pi - A) \wedge d(\pi + A) - \int_{M_5} \tilde{A} \wedge dA \wedge dA$$
$$+ \psi \Big[d\pi \wedge d\pi + \alpha d\tilde{A} \wedge d\tilde{A} \Big] - \int_{M_5} \alpha \tilde{A} \wedge d\tilde{A} \wedge d\tilde{A}$$

$$\partial j = -2 \Big[dA \wedge d\tilde{A} \Big]$$
$$\partial \tilde{j} = - \Big[dA \wedge dA \Big] - 3\alpha \Big[d\tilde{A} \wedge d\tilde{A} \Big]$$

Most general axial-vector mixed anomaly as in fermionic theories

Conclusions

- A variational principle constructed for a fluid with mixed axial-vector anomaly
- The anomaly inflow picture for 3+1 fluid is analogous to 1+1 bosonization picture
- The most general mixed anomaly requires two 5d Chern-Simons terms
- Mixed axial-gravitational anomalies can be obtained similarly
- There are multiple versions of anomalous fluid dynamics to be explored…

Buon Compleanno, Andrea!

Grazie per aver condiviso la tua passione
per la fisica, la musica e molte altre cose!