

Few words about Andrea

40 years of CFT

Jacopo Viti (INFN, Florence)

GGI, February 2024

1. Entanglement in CFTs

- Struggling with Andrea in 2007 : “**Entanglement entropies** in two-dimensional **CFTs**”

- Entropies for free fermions at finite temperature
- **Full solution:** Azeynagi, Nishioka, Takayanagi [PRD 2007]
- **Excited states:** Alcaraz, Berganca, Sierra [PRL 2011]

UNIVERSITÀ DEGLI STUDI DI FIRENZE
Facoltà di Scienze Matematiche Fisiche e Naturali



Tesi di Laurea Specialistica in Fisica

**ENTROPIA DI ENTANGLEMENT IN
TEORIE INVARIANTI CONFORMI
BIDIMENSIONALI**

Candidato: **Jacopo Viti**

Relatore: **Dott. Andrea Cappelli**

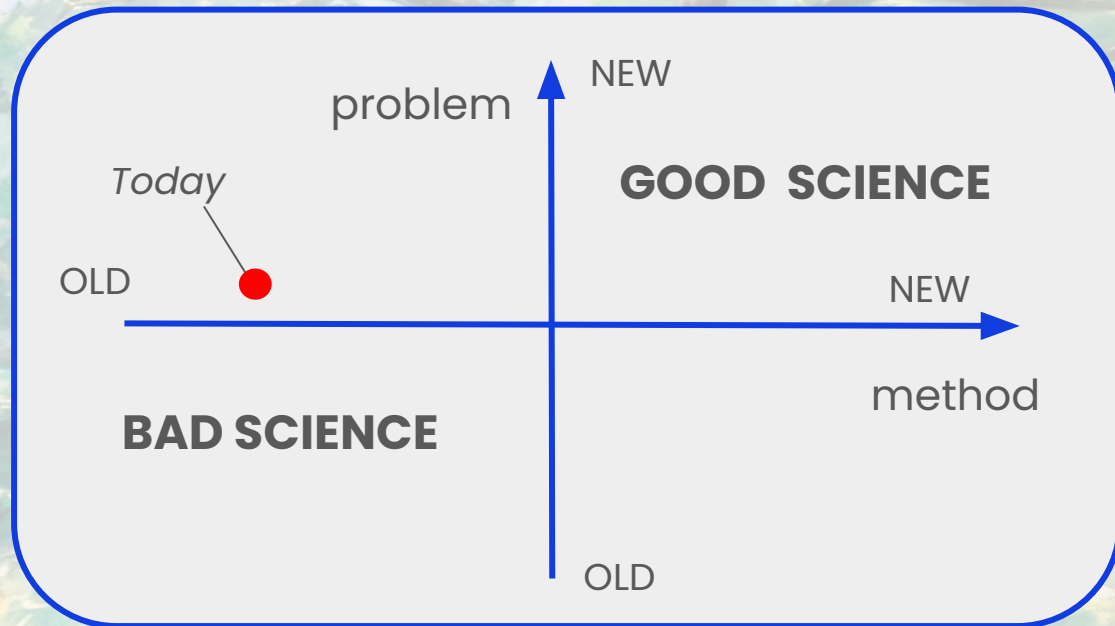
Anno Accademico 2006/07

2. Capparelli last theorem (in his own words)

“It is likely that if you write a paper with Andrea, it will be your **last good paper**”

3. Cappelli taxonomy for research

- **Classification** of modular invariant CFT p.f. [Cappelli, Itzykson, Zuber NPB 1986]



**Cappelli
complementarity
principle (in his own
words):**

"If you want to have a good approximation of reality, negate everything Andrea told you."

Tracy-Widom distribution in the six-vertex model with DWBC

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1. The six vertex model

Basics

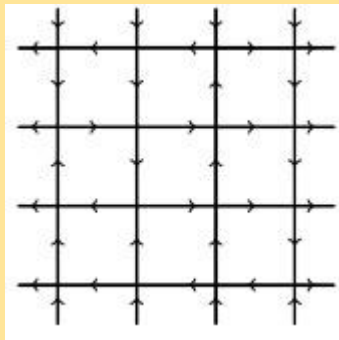
- Residual entropy of **ice** [Pauling '35]

- **Vertex weights**

$$W(a_i) = a, W(b_i) = b, W(c_i) = c$$

- Exact **free energy** on a square lattice with pbc [Lieb '67]
- Different kind of bcs (free and fixed)

Six possible vertex (v) configurations



$$Z = \sum_{\text{conf}} \prod_v W(v)$$

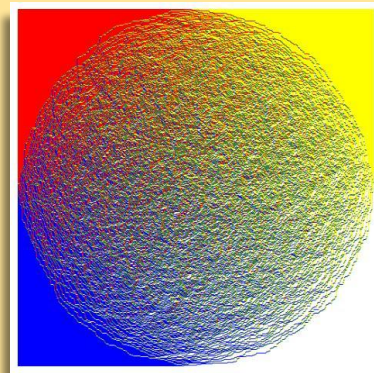
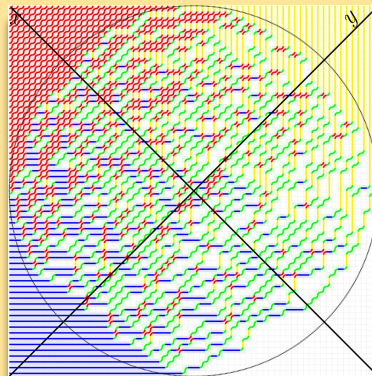
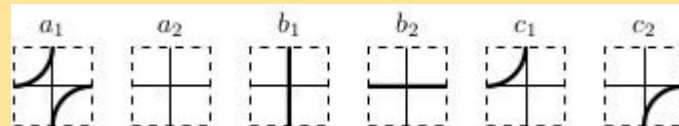
2. Domain wall boundary conditions

Domain wall bcs [Korepin '82]

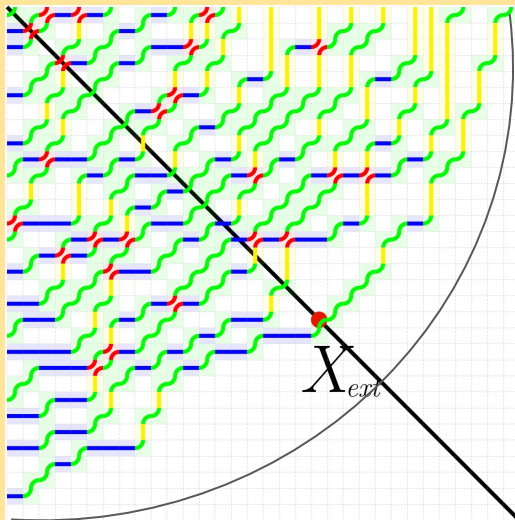
- Paths come in from the top and go out on the left
- Vertices fluctuate within **deterministic curves** [Jokush, Propp and Shor '98] - [Colomo and Pronko 2010] for $N \rightarrow \infty$
- Define for later purposes

$$\Delta = \frac{a^2 + b^2 - c^2}{2ab}$$

Non-intersecting path interpretation



3. Boundary fluctuations



Theorem [Johansson 2005]

Consider the first occurrence of a vertex different from the one on the frozen corner.

For $\Delta = 0$ and N large enough :

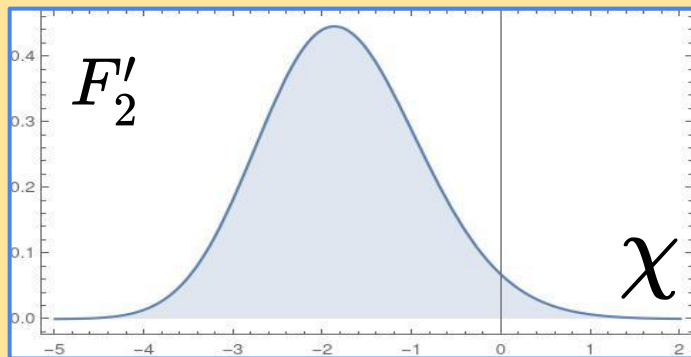
$$X_{ext} = \Lambda N + \Gamma N^{1/3} \chi; \quad \text{Prob}(\chi) = F'_2(\chi)$$

- **Purpose:** Test numerically universality of the **Tracy-Widom** distribution at $\Delta \neq 0$

4. Tracy-Widom distribution

- Fluctuations of the **largest eigenvalue** of an $N \times N$ Hermitian random matrix

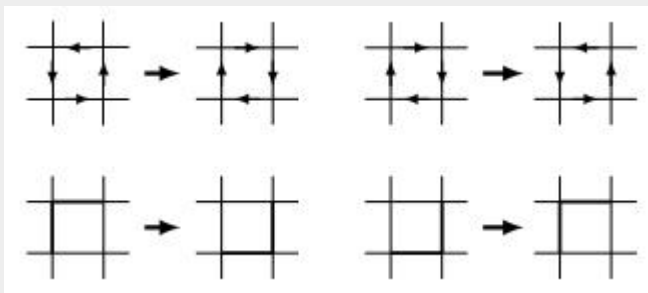
$$\text{Prob} \left(aN^{\frac{1}{6}} (\lambda_{\max} - \sqrt{2N}) < \chi \right) = F_2(\chi)$$



- Vicious Random walkers, random permutations, KPZ equation (**Universality**, see [Deift 2006], also in Quanta magazine 2014)

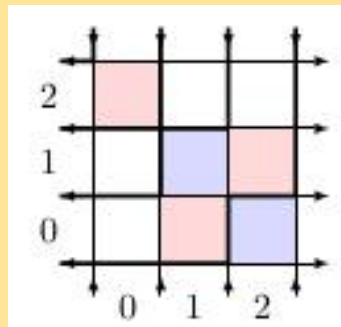
5. Monte Carlo algorithm

- **Local** Glauber dynamics



- Flip a vertex with probability

$$P(v) = \frac{\prod_{v' \in \text{plaquette}(v)} W(v')}{R}$$



- **Algorithm** proposed by [Allison, Reshetikhin 2006]
- Rejections for $\Delta < -1$
- Density profiles, several bcs. [Lyberg, Korepin and V. 2016, 2018].
- **GPU** implementation [Keating, Sridhar 2018]

6. Direct analysis

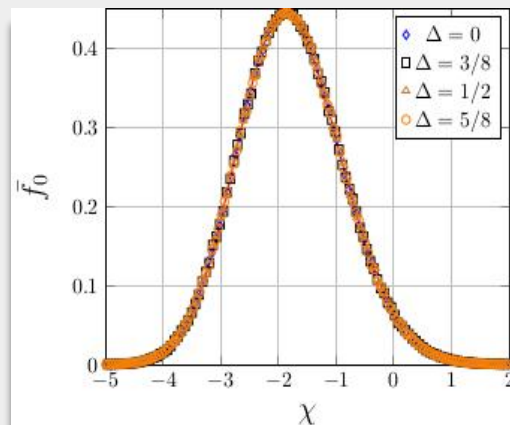
[with I. Lyberg, V. Korepin, 2023]

- At finite N construct the histogram:

$$f_N \left(\frac{X_{ext} - \Lambda_N}{\Gamma_N} \right)$$

- Fit to the TW distribution and determine:

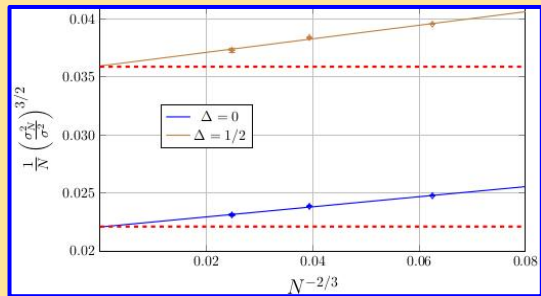
$$\Lambda_N, \Gamma_N$$



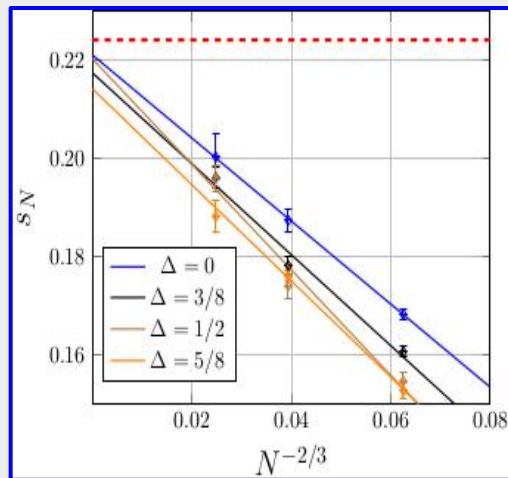
The fitting procedure does not distinguish TW from a Gaussian

7. Central moments

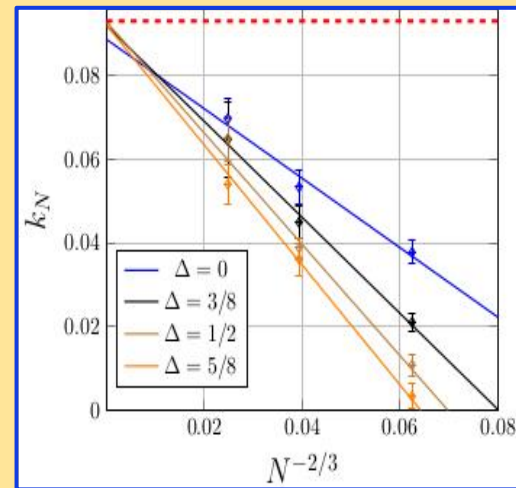
[with I. Lyberg V. Korepin 2023.]



- From the **variance** we can extract Γ^3
- See: [Allegra, Dubail, Stephan, V; 2016]; [Spohn, Praehofer; 2023]



- **Skewness**
(third moment)



- **Excess kurtosis**
(fourth moment)

8. Conclusions, recap

- In the **six vertex model** with DWbc the case $\Delta = 0$ maps to free fermions. Obtaining results away from this point is hard.
- Provided **numerical evidence** of the existence of Tracy–Widom scaling for fluctuations of the arctic curves for $\Delta \neq 0$ (Universality)
- **Analytical approaches** (?)