# Weyl anomalies in CFTs with boundaries and defects

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**Florence, 02/02/2024**

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Based on:

[Chalabi, Herzog, O'Bannon, Robinson, JS; JHEP 2022]

[Bianchi, Chalabi, Prochazka, Robinson, JS; JHEP 2021]

[Chalabi, Herzog, Ray, Robinson, JS, Stergiou; JHEP 2023]

# “Physics is like a train which does not stop at the station. One first must run fast to catch that train”

HUTP-88/A054

Applied Conformal Field Theory

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Lectures given at Les Houches summer session, June 28 – Aug. 5, 1988.

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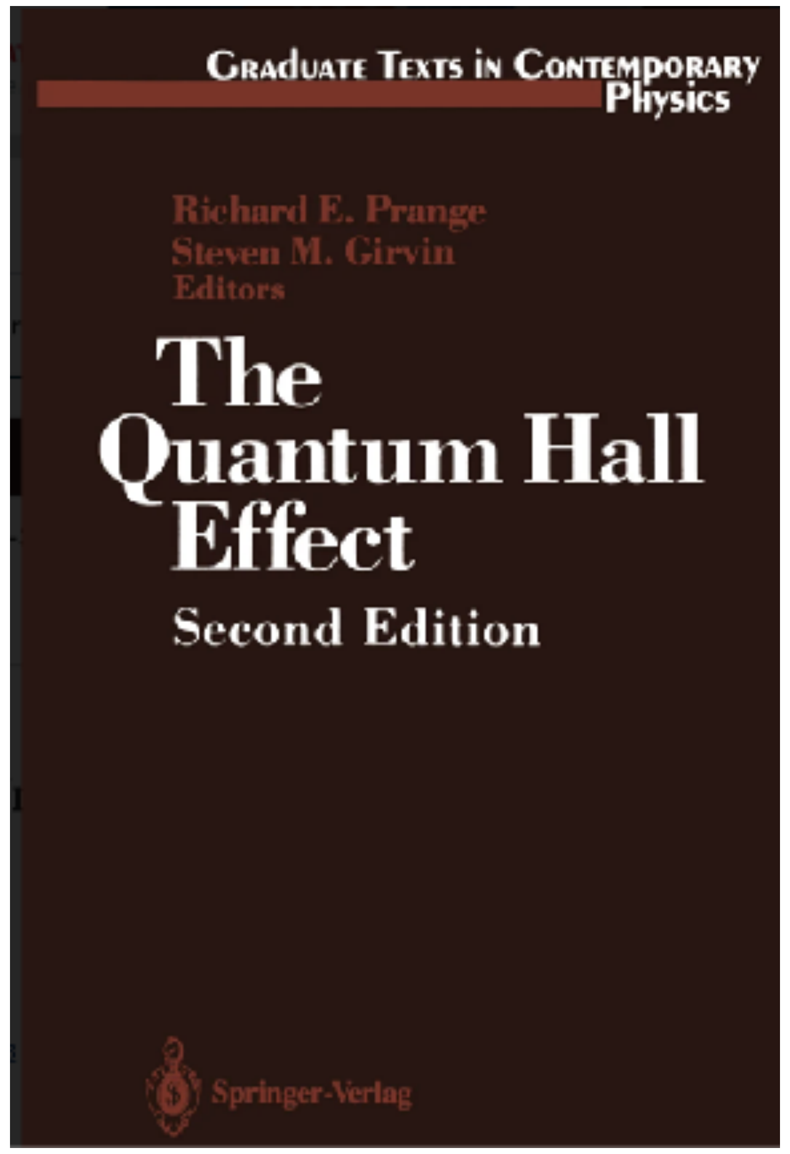


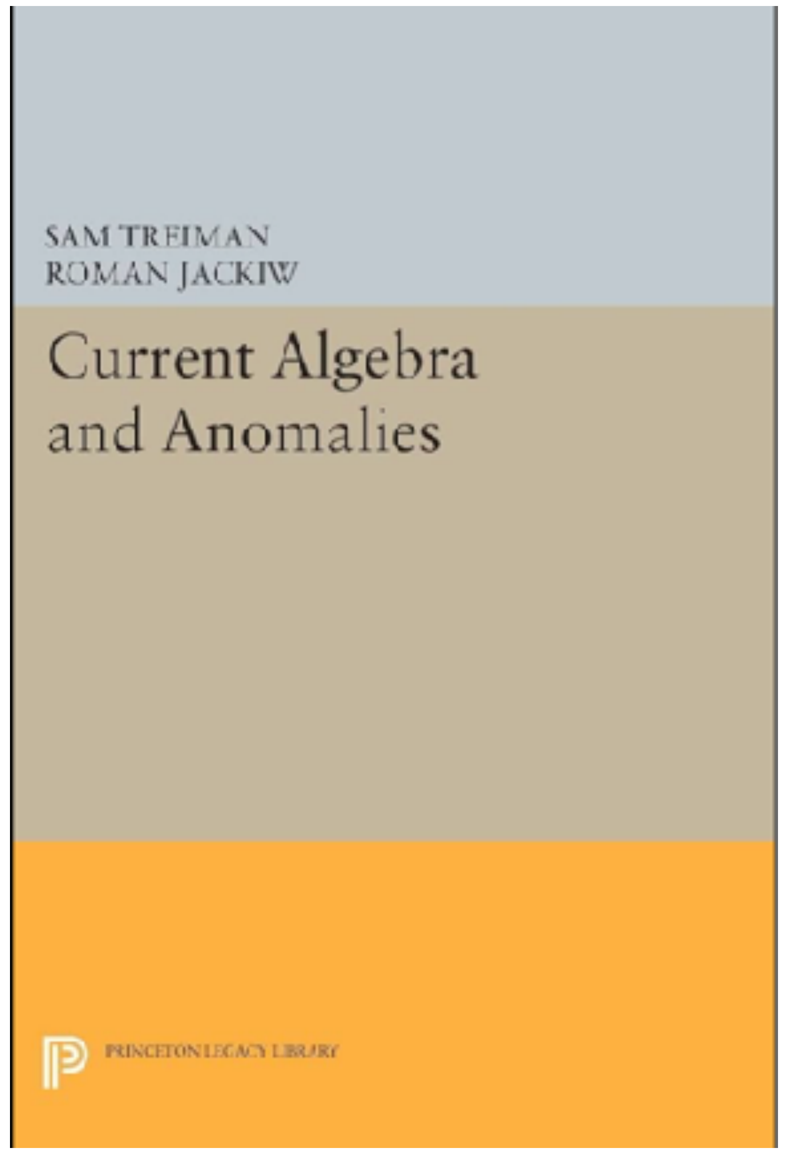


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**Topological insulators and superconductors**

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Topological insulators are new states of quantum matter which cannot be adiabatically connected to conventional insulators and semiconductors. They are characterized by a full insulating gap in the bulk and gapless edge or surface states which are protected by time-reversal symmetry. These topological materials have been theoretically predicted and experimentally observed in a variety of systems, including HgTe quantum wells, BiSb alloys, and $Bi_2Te_3$ and $Bi_2Se_3$ crystals. Theoretical models, materials properties, and experimental results on two-dimensional and three-dimensional topological insulators are reviewed, and both the topological band theory and the topological field theory are discussed. Topological superconductors have a full pairing gap in the bulk and gapless surface states consisting of Majorana fermions. The theory of topological superconductors is reviewed, in close analogy to the theory of topological insulators.

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# Defects

## Local operators are not enough

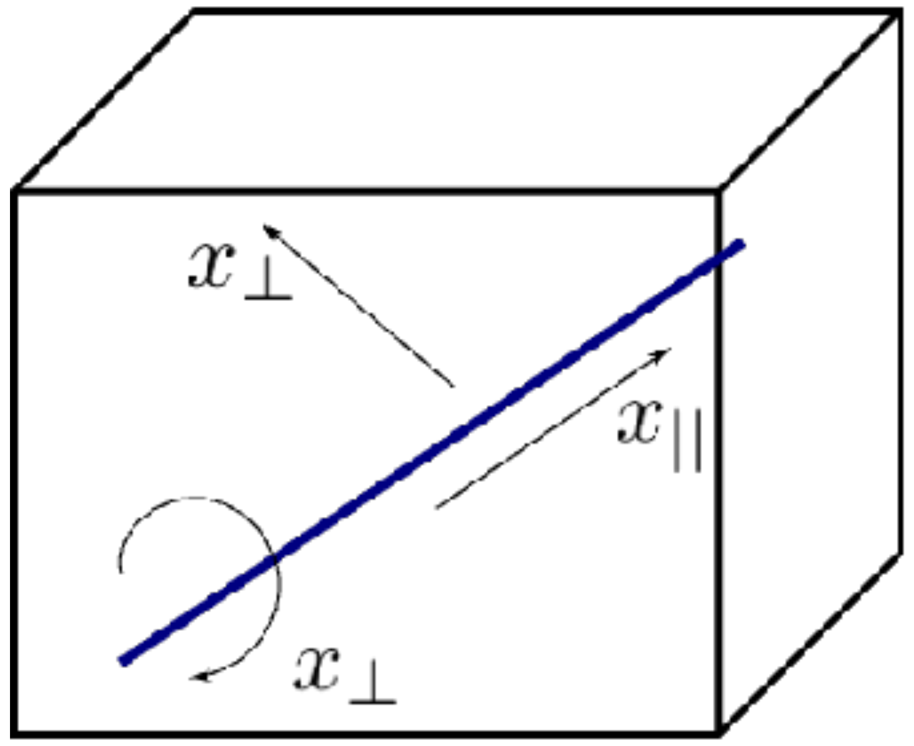
- To distinguish groups with same algebra but different global structure
- To characterise phase of matter
- Strings and Branes

## Some Examples:

- Boundaries (space-time ends on them)
- Interfaces (for instance between different phases)
- Line defects: for example Wilson lines
- Topological Defects (non-invertible symmetries)

# Defects

**Extended objects that break part of the translational symmetry**




- Localised on submanifolds of the spacetime
- p=dimension of defect space-time, q=d-p is the codimension
- The stress energy tensor is no longer conserved:

$$\partial_\mu T^{\mu\nu} = \hat{\mathcal{D}}^\nu \, \delta^q\,(\Sigma)$$

$\hat{\mathcal{D}}^\nu$ = displacement operator
protected dimension p+1

- We mainly focus on conformal field theories
- Conformal group is broken: $SO(d+1,1) \longrightarrow SO(d+1-p,1) \times SO(q)$
- New non-trivial correlators: one-point functions
- Localised contribution to anomalies: i.e. defect Weyl anomalies

# Correlators in defect CFTs

[Billò, Gonçalves, Lauria, Meineri, 2016]
[Herzog, Shrestha, 2020]

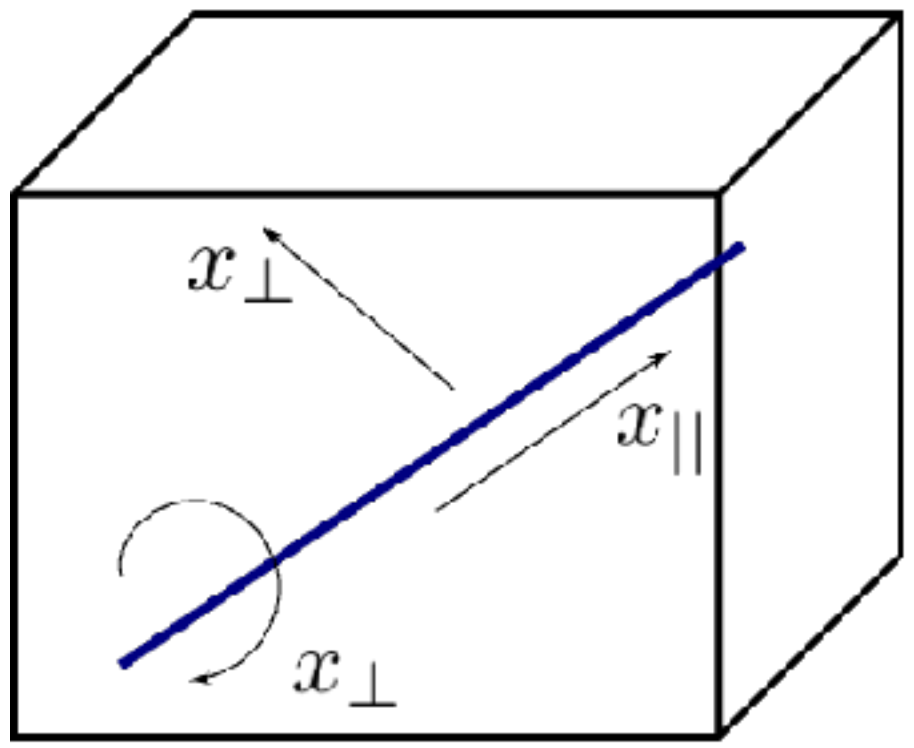
## Bulk correlators:

$$\langle \mathcal{O}(x) \rangle = \frac{A_{\mathcal{O}}}{|x|^{\Delta_{\mathcal{O}}}} \qquad \langle T^{ij} \rangle = h \frac{(d-q+1)\delta^{ij} - d\frac{x_\perp^i x_\perp^j}{|x_\perp|^2}}{|x_\perp|^d}$$

$$\langle T^{ab} \rangle = -h \frac{(q-1)\delta^{ab}}{|x_\perp|^d}$$

$i, j, k, \cdots =$ orthogonal

$a, b, c, \cdots =$ parallel




$$\langle \mathcal{O}_1(x) \mathcal{O}_2(y) \rangle = \frac{1}{|x_\perp|^{\Delta_1} |y_\perp|^{\Delta_2}} f(\xi_1, \xi_2), \qquad \xi_1 = \frac{s^2}{4|x_\perp||y_\perp|}, \ \xi_2 = \frac{x_\perp \cdot y_\perp}{|x_\perp||y_\perp|}, \ s = x - y$$

## Bulk-defect correlators:

$$\left\langle \hat{\mathcal{O}}_{\hat{\Delta}}(\mathbf{x}) \mathcal{O}_\Delta(y) \right\rangle = \frac{C_{\hat{\mathcal{O}}\mathcal{O}}}{|y_\perp|^{\Delta - \hat{\Delta}} \left(y_\perp^2 + (\mathbf{x}-\mathbf{y})^2\right)^{\hat{\Delta}}}$$

## Defect correlators:

$$\left\langle \hat{\mathcal{O}}_1(\mathbf{x}) \hat{\mathcal{O}}_2(\mathbf{y}) \right\rangle = \frac{\delta_{\Delta_{\hat{\mathcal{O}}_1}, \Delta_{\hat{\mathcal{O}}_2}}}{|\mathbf{x}-\mathbf{y}|^{2\Delta_{\hat{\mathcal{O}}_1}}} \qquad \left\langle \hat{\mathcal{D}}^i(\mathbf{y}) \hat{\mathcal{D}}^j(\mathbf{0}) \right\rangle = \frac{C_{\hat{\mathcal{D}}\hat{\mathcal{D}}}}{|\mathbf{y}|^{2p+2}} \delta^{ij}$$

$$\left\langle \hat{\mathcal{O}}_1(\mathbf{x}_1) \hat{\mathcal{O}}_2(\mathbf{x}_2) \hat{\mathcal{O}}_3(\mathbf{x}_3) \right\rangle = \frac{C_{123}}{|\mathbf{x}_{12}|^{\Delta_1+\Delta_2-\Delta_3} |\mathbf{x}_{13}|^{\Delta_1+\Delta_3-\Delta_2} |\mathbf{x}_{23}|^{\Delta_2+\Delta_3-\Delta_1}}$$

# (Bulk) Weyl Anomaly

- QFT on curved space-time
- Suppose the action invariant under Weyl rescaling (classical) $g_{\mu\nu} \longrightarrow e^{2\omega(x)} g_{\mu\nu}$

$$\delta_\omega S = -\int_{\mathcal{M}_d} d^d x \sqrt{g}\, \delta\omega\, T^\mu{}_\mu = 0 \quad \longrightarrow \quad T^\mu{}_\mu = 0$$

- Effective action is not invariant: Weyl anomaly (quantum effect) $\langle T^{\mu\nu} \rangle = -\frac{2}{\sqrt{g}} \frac{\delta W}{\delta g_{\mu\nu}}$

$$\boxed{\delta_\omega W = -\int_{\mathcal{M}_d} d^d x \sqrt{g}\, \delta\omega\, \langle T^\mu{}_\mu \rangle \neq 0} \quad \longrightarrow \quad W = \frac{a_{d-2}}{\epsilon^{d-2}} + \frac{a_{d-4}}{\epsilon^{d-4}} \cdots + \left[\int d^d x \sqrt{g}\, \langle T^\mu{}_\mu \rangle\right] \log\epsilon + \mathcal{O}(1)$$

- Local quantity of dimension $d$

$$T^\mu{}_\mu = \frac{1}{(4\pi)^{\frac{d}{2}}} \left[(-)^{\frac{d}{2}-1} a_{\mathcal{M}} E_d + \sum_n c_n I_n\right]$$

anomaly coefficients
"central charges"

curvature invariants

- Bulk anomalies absent in odd dimensions

# (Bulk) Weyl Anomaly

**2 dimensions:**

$$T^{\mu}{}_{\mu} = \frac{c}{24\pi} R$$

$c$ = Virasoro central charge

[Zamolodchikov, 1986]

$$c_{\rm UV} \geq c_{\rm IR}$$

$$\langle T(x) T(0) \rangle = \frac{c}{|x|^4}$$

**4 dimensions:**

$$T^{\mu}{}_{\mu} = \frac{1}{16\pi^2} \Big( -aE_4 + cW_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma} \Big)$$

$$a_{\rm UV} \geq a_{\rm IR}$$

[Cardy, 1988]

[Komargodski, Schwimmer, 2011]

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0) \rangle \propto \frac{c}{|x|^4} I_{\mu\nu,\rho\sigma}(x)$$

**6 dimensions:**

$$T^{\mu}{}_{\mu} = \frac{1}{(4\pi)^3} (a\, E_6 + c_1 I_1 + c_2 I_2 + c_3 I_3)$$

$$I_1 = W_{\mu\lambda\rho\nu} W^{\lambda\sigma\tau\rho} W_{\sigma}{}^{\mu\nu}{}_{\tau} \qquad I_2 = W_{\mu\nu}{}^{\lambda\rho} W_{\lambda\rho}{}^{\sigma\tau} W_{\sigma\tau}{}^{\mu\nu}$$

$$I_3 = W_{\mu\nu\lambda\rho} \left( D^2\, \delta^{\nu}{}_{\sigma} - \frac{6}{5} R\, \delta^{\nu}{}_{\sigma} + 4R^{\nu}{}_{\sigma} \right) W^{\sigma\nu\lambda\rho}$$

$a_{\rm UV} \geq a_{\rm IR}$ (conjectured)

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0) \rangle \propto \frac{c_3}{|x|^4} I_{\mu\nu,\rho\sigma}(x)$$

$$\langle TTT \rangle \propto c_1 + c_2$$

# Defect Weyl Anomaly

- In the presence of a p-dimensional defect:

$$\delta W = -\frac{1}{2}\int_{\mathcal{M}_d} d^d x\sqrt{g}\,\delta g_{\mu\nu}\,\langle T^{\mu\nu}\rangle \boxed{-\frac{1}{2}\int_{\Sigma_p} d^p y\sqrt{\bar{g}}\left(\delta g_{\mu\nu}\langle T^{\mu\nu}|_{\Sigma_p}\rangle + 2\delta X^i(y^a)\langle\mathcal{D}_i\rangle\right)}$$

- Defect-localised contribution to Weyl anomaly

$$T^\mu{}_\mu = T^\mu{}_\mu\big|_{\mathcal{M}_d} + \delta^{(q)}(x_\perp)T^\mu{}_\mu\big|_{\Sigma_p} \qquad D_\mu T^{\mu i} = \delta^{(q)}(x_\perp)\mathcal{D}^i$$

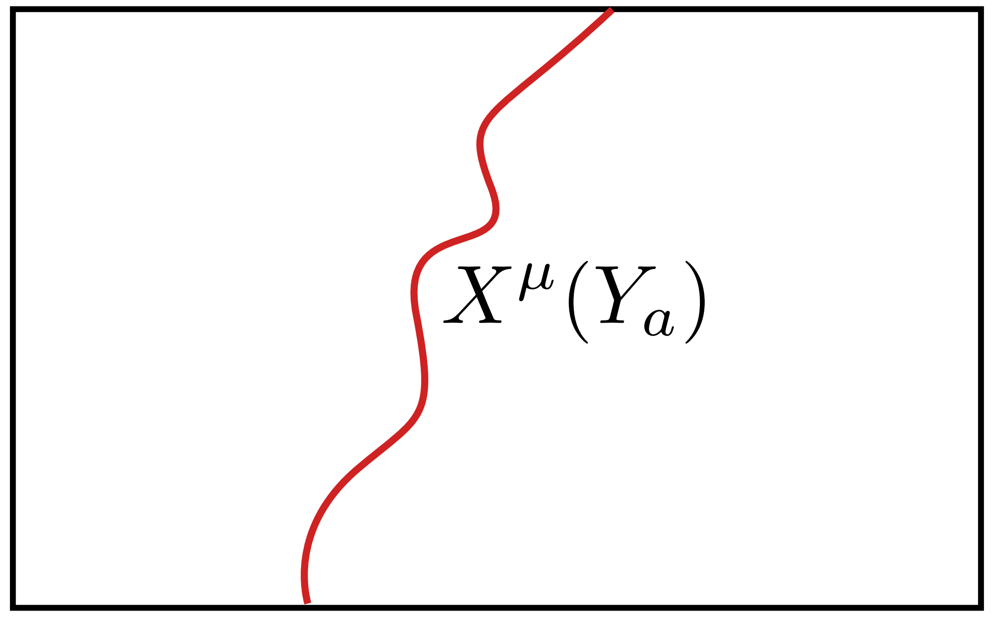
- For one-dimensional boundary is “trivial” $\quad T^\mu{}_\mu|_{\Sigma_1} = \dfrac{c}{12\pi}K$

# Defect Weyl Anomaly

$e^\mu_a = \partial_a X^\mu$ $\qquad X^\mu(Y_a)$ $\qquad$ embedding function

$\bar{g}_{ab} = e^\mu_a e^\nu_b g_{\mu\nu}$ $\qquad$ (induced metric)

$\Pi^i_{ab} = \bar{D}_a e^\mu_b$ $\qquad$ (second fundamental form)




- For two-dimensional defects [Henningson, Skenderis, 1999] [Schwimmer, Theisen, 2008]

$$T^\mu{}_\mu|_{\Sigma_2} = \frac{1}{24\pi}(b\,\bar{R} + d_1 \mathring{\Pi}^2 + d_2 W^{ab}{}_{ab})$$

$b_{\rm UV} \geq b_{\rm IR}$

[Jensen, O'Bannon, 2015]

$$\left\langle \hat{\mathcal{D}}_i(\mathbf{y})\hat{\mathcal{D}}_j(\mathbf{0})\right\rangle \propto d_1 \frac{\delta_{ij}}{|\mathbf{y}|^6} \qquad \langle T_{\mu\nu}\rangle \propto \frac{d_2}{|x_\perp|^d}$$

- For three-dimensional boundaries [Herzog, Huang, Jensen, 2015 - 2017]

$$T^\mu{}_\mu\big|_{\Sigma_3} = \frac{1}{16\pi^2}\left(a_{\mathcal{M}} E_4|_{\partial\mathcal{M}} + b_1 \mathring{K}^3 + b_2 \mathring{K}^{ab} W^c{}_{acb}\right)$$

$$E_4|_{\partial\mathcal{M}} = \delta^{abc}_{def}\left(2K^d{}_a R^{ef}{}_{bc} + \frac{8}{3}K^d{}_a K^e{}_b K^f{}_c\right)$$

$$\left\langle \hat{\mathcal{D}}\hat{\mathcal{D}}\hat{\mathcal{D}}\right\rangle \propto b_1 \qquad \left\langle \hat{\mathcal{D}}(\mathbf{y})\hat{\mathcal{D}}(\mathbf{0})\right\rangle \propto b_2 \frac{1}{|\mathbf{y}|^8}$$

# 4d Defect Weyl Anomaly

[Chalabi, Herzog, O'Bannon, Robinson, JS; JHEP 2022]

- Full Weyl anomaly for p=4 dimensional defects and codimension q>1

$$T^{\mu}{}_{\mu}|_{\Sigma_4} = \frac{1}{(4\pi)^2}\Big(\boxed{-a_\Sigma \overline{E}_4} + \boxed{d_1 \mathcal{J}_1} + \boxed{d_2 \mathcal{J}_2} + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2$$
$$+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk}$$
$$+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj}$$
$$+ d_{13} W_{ab}{}^{ab}\mathring{\Pi}^i_{cd}\mathring{\Pi}^{cd}_i + d_{14} W^a{}_{bij}\mathring{\Pi}^i{}_{ac}\mathring{\Pi}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\Pi}^i{}_{ac}\mathring{\Pi}^{jbc}$$
$$+ d_{16} W^{abcd}\mathring{\Pi}^i_{ac}\mathring{\Pi}_{ibd} + d_{17} W_a{}^{bac}\mathring{\Pi}^i_{bd}\mathring{\Pi}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\Pi}^i_{ab}\mathring{\Pi}^{jab}$$
$$+ d_{19}\mathrm{Tr}\ \mathring{\Pi}^i\mathring{\Pi}_i\mathring{\Pi}^j\mathring{\Pi}_j + d_{20}\mathrm{Tr}\ \mathring{\Pi}^i\mathring{\Pi}^j\mathring{\Pi}_i\mathring{\Pi}_j + d_{21}(\mathrm{Tr}\ \mathring{\Pi}^i\mathring{\Pi}_i)^2 + d_{22}(\mathrm{Tr}\ \mathring{\Pi}^i\mathring{\Pi}^j)(\mathrm{Tr}\ \mathring{\Pi}_i\mathring{\Pi}_j)\Big)$$

- 2 "non-trivial" invariants

$$\mathcal{J}_1 = \frac{1}{d-1}R\mathring{\Pi}^i_{ab}\mathring{\Pi}^{ab}_i - \frac{1}{d-2}N^{\mu\nu}R_{\mu\nu}\mathring{\Pi}^i_{ab}\mathring{\Pi}^{ab}_i - \frac{2}{d-2}R^a{}_b\mathring{\Pi}^i_{ac}\mathring{\Pi}^{bc}_i - \frac{1}{2}W^c{}_{acb}\Pi_i\mathring{\Pi}^{iab}$$
$$+ \frac{4}{9}W^c{}_{ica}\overline{D}^b\mathring{\Pi}^i_{ab} + \mathring{\Pi}^{iab}D_iW^c{}_{acb} - \frac{1}{2}\Pi^i\mathrm{Tr}\ \mathring{\Pi}_i\mathring{\Pi}^j\mathring{\Pi}_j + \frac{1}{16}\Pi^i\Pi_i\mathrm{Tr}\ \mathring{\Pi}^j\mathring{\Pi}_j$$
$$+ \boxed{\frac{2}{9}\overline{D}^b\mathring{\Pi}^i_{ab}\overline{D}^c\mathring{\Pi}_{ic}{}^a}$$

Related to two-point function of displacement operator

$$\mathcal{J}_2 = \frac{d-4}{d-2}W_{ab}{}^{ab}N^{\mu\nu}R_{\mu\nu} - \frac{d-4}{d-1}RW_{ab}{}^{ab} + \frac{4(d-5)}{3(d-2)}R_{ab}W_c{}^{acb}$$
$$- \frac{5(d-4)}{48}W_{ab}{}^{ab}\Pi^i\Pi_i + \frac{2(d-5)}{3}W^c{}_{ica}\overline{D}^b\mathring{\Pi}^i_{ab} + \frac{4(d+1)}{9}\mathring{\Pi}^{iab}D_iW^c{}_{acb}$$
$$- \frac{1}{3}W_{ic}{}^{ac}\overline{D}_a\Pi^i - \frac{2(d-5)}{3}\Pi^i\mathrm{Tr}\ \mathring{\Pi}_i\mathring{\Pi}^j\mathring{\Pi}_j + \frac{(d-10)}{12}\Pi^iD_iW_{ab}{}^{ab} + \boxed{\frac{1}{3}D^iD_iW_{ab}{}^{ab}},$$

Related to one-point function of the stress tensor

# Defect Weyl Anomalies and Correlators

- Insertions of the displacement operator generate perturbation of the defect's shape

- This produces a variation in the effective action (at the second order)

$$\delta_X W = -\frac{1}{2}\int_{\Sigma_p} d^p y_1\, d^p y_2\, \langle \mathcal{D}_i(\mathbf{y}_1)\mathcal{D}_j(\mathbf{y}_2)\rangle \delta X^i(\mathbf{y}_1)\delta X^j(\mathbf{y}_2) + \mathcal{O}(\delta X^3)$$

$$\left\langle \hat{\mathcal{D}}_i(\mathbf{y})\hat{\mathcal{D}}_j(0)\right\rangle = \delta_{ij}\,\frac{C_{\hat{\mathcal{D}}\hat{\mathcal{D}}}}{|\mathbf{y}|^{2p+2}}$$

- We Taylor expand and take only the term that diverges logarithmically

- From $\delta\Pi^i_{ab} = \partial_a\partial_b\delta X^i$ we find for p even (p odd no logs):

$$\boxed{\delta_X W|_{\log\varepsilon} = (-1)^{p/2+1}\frac{2^{-(p+2)}\pi^{\frac{p}{2}}}{p!\Gamma(\frac{p}{2}+2)}c_{\mathcal{D}\mathcal{D}}\int_{\Sigma_p} d^p y\,\partial^{a_1}\ldots\partial^{a_{p/2-1}}\delta\Pi^{icd}\partial_{a_1}\ldots\partial_{a_{p/2-1}}\delta\Pi^i_{cd}}$$

- We find for p=2 and p=4:

$p=2$

$$\left\langle \hat{\mathcal{D}}_i(\mathbf{y})\hat{\mathcal{D}}_j(\mathbf{0})\right\rangle = \frac{4}{3\pi^2}\frac{d_1}{|\mathbf{y}|^6}\delta_{ij}$$

$$T^\mu{}_\mu|_{\Sigma_2} \supset \frac{d_1}{24\pi}\mathring{\Pi}^2$$

$p=4$

$$\left\langle \mathcal{D}^i(\mathbf{y})\mathcal{D}^j(\mathbf{0})\right\rangle = -\frac{72}{\pi^4}\frac{d_1}{|\mathbf{y}|^{10}}\delta^{ij}$$

$$T^\mu{}_\mu|_{\Sigma_4} \supset \frac{2}{9}d_1\,\overline{D}^b\mathring{\Pi}^i_{ab}\overline{D}^c\mathring{\Pi}_{ic}{}^a$$

Unitarity

$d_1 \leq 0$

# Defect Anomalies and Correlators

- Insertions of the stress-tensor produce (bulk) metric perturbation

- This also implies variation in the anomaly contribution

$$\delta_g W = -\frac{1}{2}\int d^d x\, \langle T^{\mu\nu}\rangle\, \delta g_{\mu\nu} \quad \longrightarrow \quad \delta_g \int d^d x \sqrt{g}\, \langle T^{\mu}{}_{\mu}\rangle = -\frac{1}{2}\int d^d x\, \langle T^{\mu\nu}\rangle\, \delta g_{\mu\nu}\Big|_{\log\varepsilon}$$

- At first order only one term contributes

$$\delta_g \int_{\Sigma_4} d^4 y \sqrt{\gamma}\, \left\langle T^{\mu}{}_{\mu}|_{\Sigma_4}\right\rangle = \frac{d_2}{16\pi^2}\int_{\Sigma_4} d^4 y\, \frac{1}{3}\partial^k \partial_k \delta W^{ab}{}_{ab} \quad \longrightarrow \quad \langle T_{\mu\nu}\rangle \propto \frac{h}{|x_\perp|^d} \qquad \boxed{h = -\frac{\Gamma\left(\frac{q}{2}+1\right)}{\pi^{\frac{q}{2}+2}(q+3)} d_2}$$

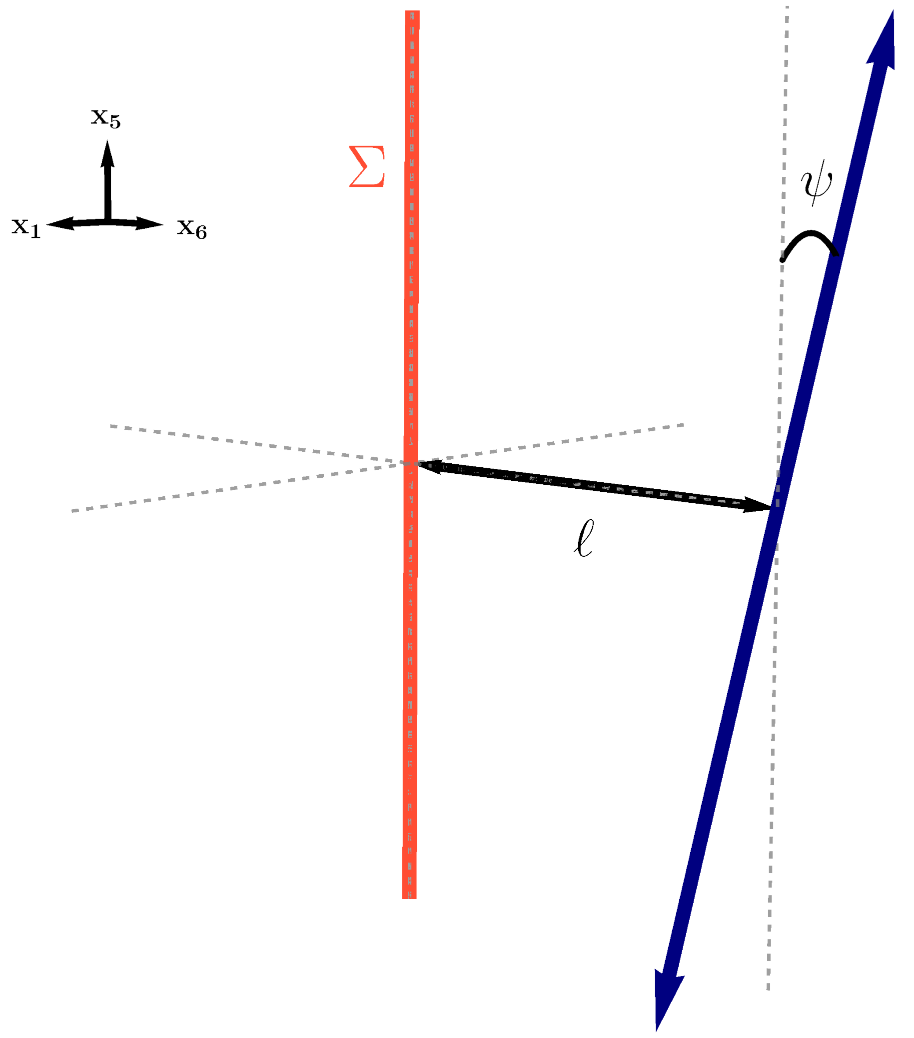
- Averaged null-energy condition $\int_{-\infty}^{\infty} du\, \langle T_{\mu\nu}\rangle\, v^\mu v^\nu \geq 0$ implies a constraint

[Jensen, O'Bannon, Robinson, Rodgers, 2018]

[Chalabi, O'Bannon, Herzog, Robinson, JS, JHEP 2022]

- Null geodesics: $t = \ell u\,, \qquad x_1 = \ell u \cos\psi\,, \qquad x_5 = \ell u \sin\psi\,, \qquad x_6 = \ell$

$$\int_{-\infty}^{\infty} du\, \langle T_{\mu\nu}\rangle\, v^\mu v^\nu = -d_2 \frac{q\,\Gamma\left(\frac{d+1}{2}\right)\Gamma\left(\frac{q}{2}\right)}{(q+3)\Gamma\left(\frac{d}{2}\right)\pi^{\frac{q+3}{2}}\ell^{d-2}}|\sin\psi| \geq 0 \quad \longrightarrow \quad \boxed{d_2 \leq 0}$$



# Defect Anomalies and Entanglement Entropy

- Contribution to EE of a spherical region with a centred defect

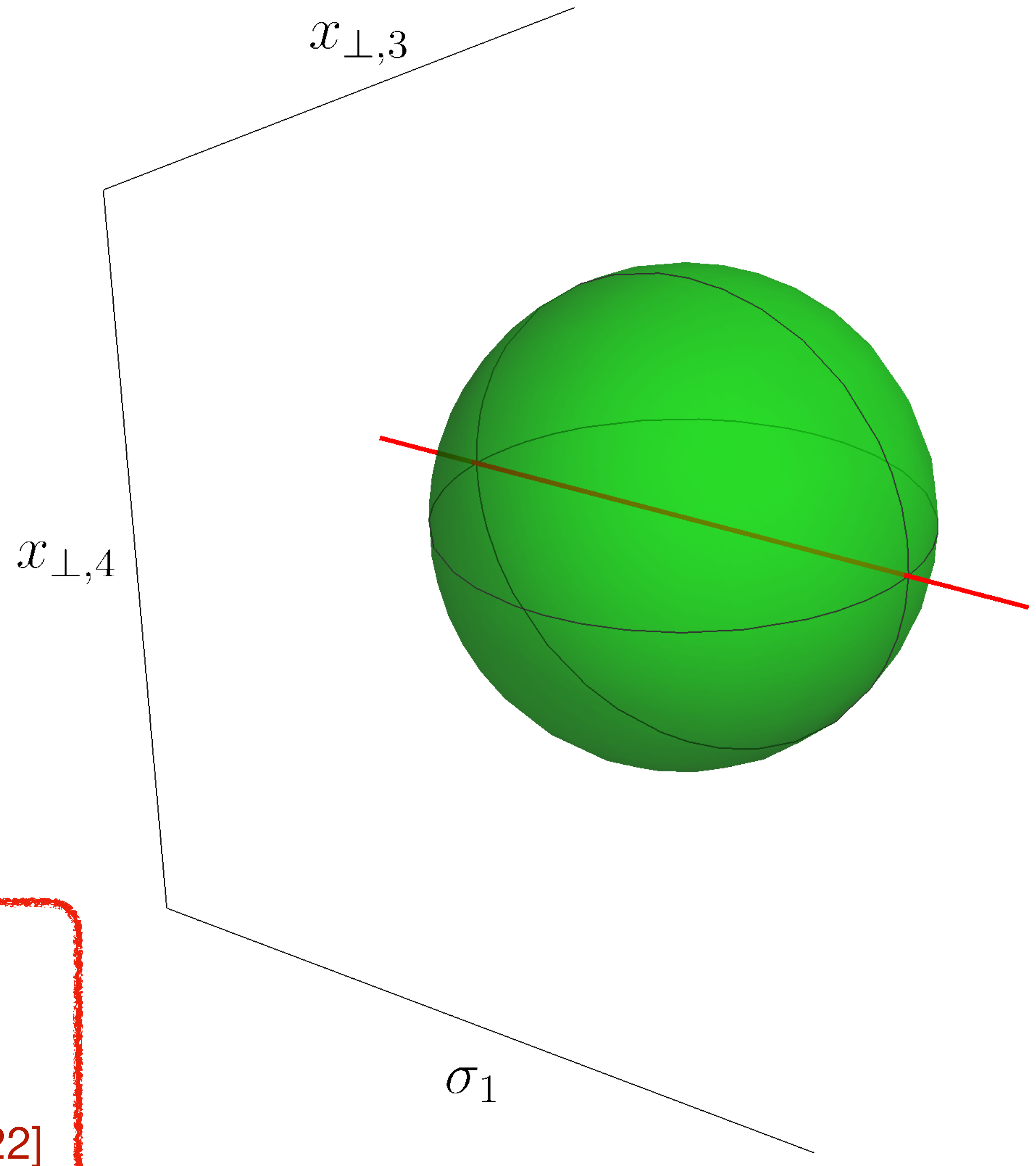
- In CFT for spherical regions we can use the CHM map

- Two contributions:

[Kobayashi, Nishioka, Sato, Watanabe, 2018]
[Jensen, O'Bannon, Robinson, Rodgers, 2018]

1) Free-energy

2) Killing Energy




$$S_{A,\Sigma_2} = \frac{1}{3}\left(b + \frac{d-3}{d-1} d_2\right) \log\left(\frac{L}{\varepsilon}\right)$$

[Chalabi, O'Bannon, Herzog, Robinson, JS; JHEP 2022]

$$S_{A,\Sigma_4} = -4\left[a_\Sigma + \frac{1}{4}\frac{(d-5)(d-4)}{d-1}\, d_2\right] \log\left(\frac{L}{\varepsilon}\right)$$

# 5-dimensional probe branes in AdS

$$I = -\frac{1}{16\pi G_N}\int dr d^d x\sqrt{g}\left(\mathcal{R}+\frac{d(d-1)}{L_{\text{AdS}}^2}\right)+T_{\text{br}}\int dr d^p y\sqrt{\bar{g}}$$

- Holographic result from the generalised Willmore Energy (probe brane) [Graham, Reichert, 2017]

- For q>1

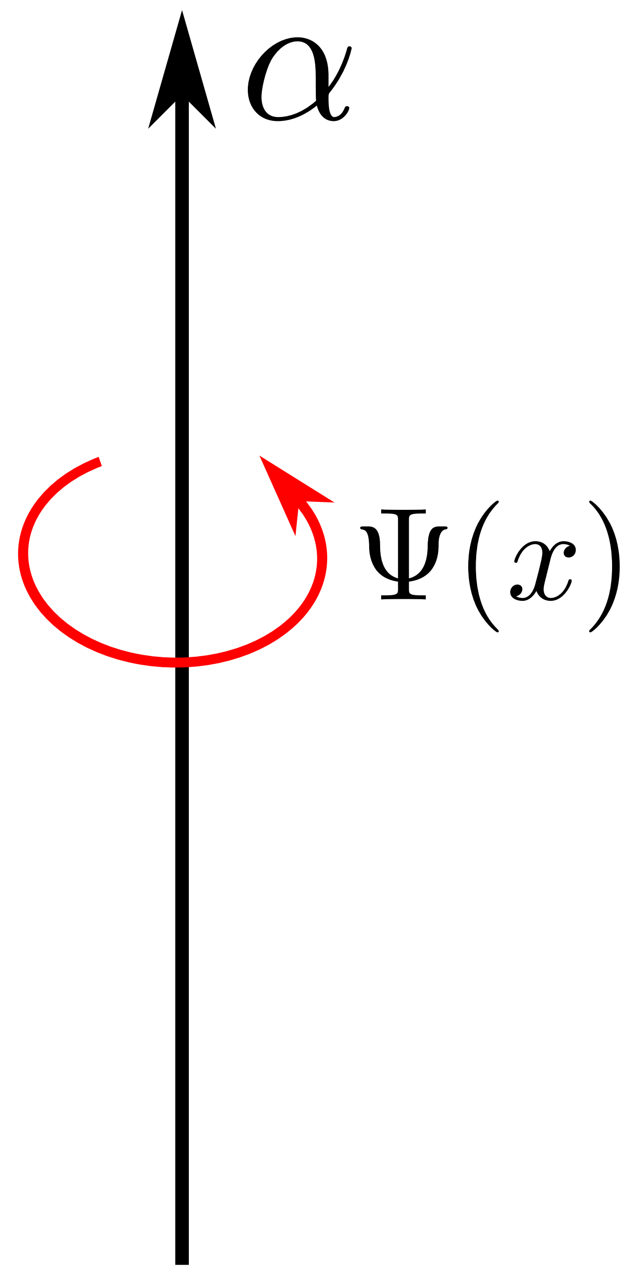
| $a_\Sigma$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ | $d_6$ | $d_7$ | $d_8$ | $d_9$ | $d_{10}$ | $d_{11}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{1}{4}$ | $-1$ | $-\frac{1}{q}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{3q}$ | $-\frac{(3q+14)}{9q}$ | $0$ | $0$ | $-\frac{2}{3q}$ | $-\frac{(q+2)}{6q}$ | $-\frac{(3q+2)}{3q}$ |

| $d_{12}$ | $d_{13}$ | $d_{14}$ | $d_{15}$ | $d_{16}$ | $d_{17}$ | $d_{18}$ | $d_{19}$ | $d_{20}$ | $d_{21}$ | $d_{22}$ | — |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $-\frac{2}{3q}$ | $-\frac{1}{2}$ | $\frac{2-q}{3q}$ | $\frac{2(q+4)}{3q}$ | $\frac{2(2q+1)}{3q}$ | $\frac{(7q+6)}{3q}$ | $-\frac{(q+6)}{3q}$ | $-1$ | $-\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{2}$ | — |

# Monodromy defects

[Bianchi, Chalabi, Prochazka, Robinson, JS; JHEP 2021]

see also [Giombi et all, 2021]




- Co-dimension 2 operator which implements a flavour symmetry rotation
- Simplest case: U(1) rotation $\boxed{\Psi(x) \to e^{-2\pi i\alpha}\Psi(x)}$
- In a Lagrangian theory it can be achieved by “gauging” the global symmetry by coupling to an external potential

$$S \longrightarrow S + \int d^d x\, J^\mu A_\mu \qquad\qquad A = \alpha\, d\theta$$

- This is a pure gauge everywhere but at the origin $\quad F_{xy} = 2\pi\,\alpha\,\delta^{(2)}(x,y)$

**Anomaly coefficients for the free scalar:**

d=4:

$$d_1 = d_2 = \frac{3}{2}\left[(1-\alpha)^2\alpha^2 + 4\xi\alpha^3\right]$$

$$b = \frac{(1-\alpha)^2\alpha^2 + 4\xi\alpha^3}{2}$$

d=6:

$$d_1 = 2d_2 = -\frac{\alpha(1-\alpha^2)(2-\alpha)}{36}\left[\alpha(1-\alpha) + \frac{6\alpha^2\xi}{2-\alpha}\right]$$

$$a_\Sigma = \frac{\alpha^2}{720}(1-\alpha)^2(3+\alpha-\alpha^2) + \frac{\alpha^3}{360}(5-3\alpha^2)\xi$$

$\alpha \in (0,1)$ monodromy parameter

$\xi = 0$ IR fixed point

$\xi = 1$ UV fixed point

# Monodromy defects

$\xi = 1$ UV fixed point $\longrightarrow$ $\left[\hat{O}_{-\alpha}(\sigma)\right] = \frac{d-2}{2} - \alpha$

$\xi = 0$ IR fixed point $\longrightarrow$ $\left[\hat{O}_{\alpha}(\sigma)\right] = \frac{d-2}{2} + \alpha$

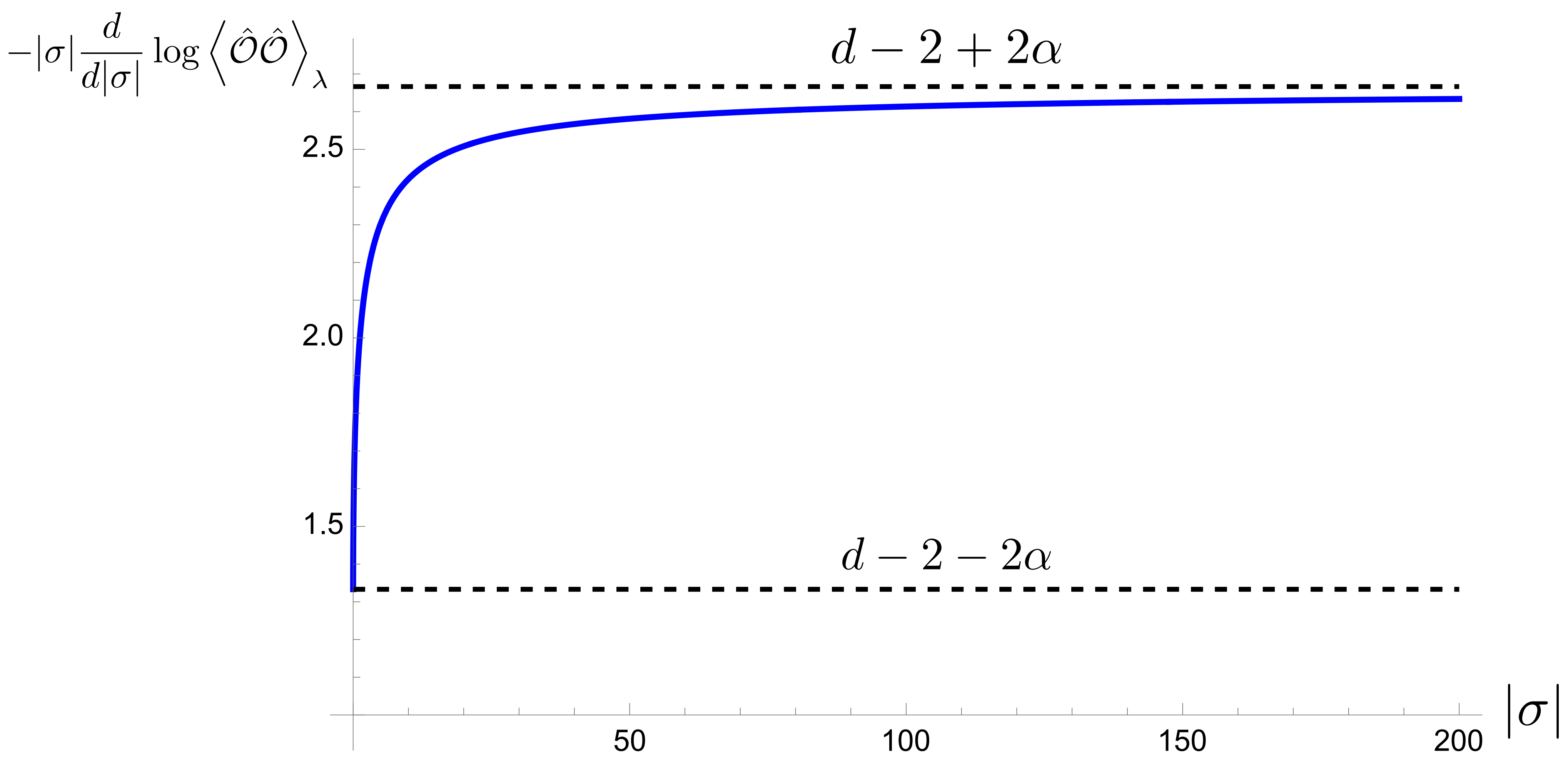
- The two choices are connected by an RG flow:

$$S_{def} = \lambda \int d^{d-2}\sigma\, \hat{O}_{-\alpha}(\sigma)\hat{O}^{\dagger}_{-\alpha}(\sigma) \qquad \lambda = \Lambda^{2\alpha}\bar{\lambda}$$

relevant deformation

$$\left\langle \hat{\mathcal{O}}_{-\alpha}(\sigma)\hat{\mathcal{O}}_{-\alpha}(0)\right\rangle_{\lambda} \longrightarrow \frac{1}{|\sigma|^{d-2-2\alpha}} \qquad \sigma \to 0$$

$$\left\langle \hat{\mathcal{O}}_{-\alpha}(\sigma)\hat{\mathcal{O}}_{-\alpha}(0)\right\rangle_{\lambda} \longrightarrow \frac{\Lambda^{-4\alpha}}{|\sigma|^{d-2+2\alpha}} \qquad |\sigma| \to \infty$$




- The Euler anomaly decreases

$$a_{\Sigma} = \frac{\alpha^2}{720}(1-\alpha)^2(3+\alpha-\alpha^2) + \frac{\alpha^3}{360}(5-3\alpha^2)\xi \longrightarrow \boxed{\begin{array}{cc} a^{(\mathrm{UV})}_{\Sigma} \geq & a^{(\mathrm{IR})}_{\Sigma} \\ \xi = 1 & \xi = 0 \end{array}}$$

defect a-theorem
[Wang, 2021]

- Analogous discussion for fermions

“Research in Physics is hard, so
you need to feel you are doing something epic”

Thank you for your attention!

and Happy Birthday Andrea!

# Higher-Derivative Theories with Boundaries

[Chalabi, Herzog, Ray, Robinson, JS, Stergiou; JHEP 2023]

- Theories with an higher number of derivatives such as:

$$S = \int d^d x \, \phi \Box^k \phi \qquad\qquad \Delta_\phi = \frac{d-2k}{2}$$

- Dimension of the fields is lower $\longrightarrow$ Many relevant boundary deformations

- Violation of the unitarity bound $\Delta \geq \dfrac{d-2}{2}$ for scalars

Boundary Primaries:

$$D\,|O_I\rangle = i\Delta\,|O_I\rangle\,,$$
$$M_{ab}\,|O_I\rangle = (M_{ab})_I{}^J\,|O_J\rangle\,, \qquad a,b = \text{parallel directions}$$
$$\boxed{K_a\,|O_I\rangle = 0}$$

- We need to compute the “single-trace” boundary primaries

$$\left|\Phi^{(k,q)}\right\rangle = \sum_{j=0}^{\lfloor \frac{q}{2} \rfloor} \alpha_j^{(k,q)} P_\parallel^{2j} P_n^{q-2j}\,|\phi\rangle \longrightarrow K_a \left|\Phi^{(k,q)}\right\rangle = 0 \qquad a = \text{parallel directions}$$

# Higher-Derivative Theories with Boundaries

- The first few boundary primaries are

$$\Phi^{(k,0)} = \phi, \qquad \Phi^{(k,1)} = \partial_n \phi, \qquad \Phi^{(k,2)} = \left(\partial_n^2 - \frac{1}{2k-3}\Box_{\parallel}\right)\phi, \qquad \Phi^{(k,3)} = \left(\partial_n^3 - \frac{3}{2k-5}\partial_n \Box_{\parallel}\right)\phi$$

- For $k=1$ only 2 primaries:

$$\delta S^{(k=1)}_{\phi,\Box} = \frac{1}{2}\int d^{d-1}x_{\parallel}(\Phi^{(1,0)}\delta\Phi^{(1,1)} - \Phi^{(1,1)}\delta\Phi^{(1,0)})$$

Neumann (N) $\Phi^{(k,1)} = \partial_n \phi = 0$

Dirichlet (D) $\Phi^{(k,0)} = \phi = 0$

- For $k=2$ we find 4 primaries:

$$\delta S^{(k=2)}_{\phi,\Box} = \frac{1}{2}\int d^{d-1}x_{\parallel}(\Phi^{(2,0)}\delta\Phi^{(2,3)} - \Phi^{(2,1)}\delta\Phi^{(2,2)} + \Phi^{(2,2)}\delta\Phi^{(2,1)} - \Phi^{(2,3)}\delta\Phi^{(2,0)})$$

- NN

$\Phi^{(2,2)} = \left(\partial_n^2 - \Box_{\parallel}\right)\phi = 0$

$\Phi^{(2,3)} = \left(\partial_n^3 + 3\partial_n\Box_{\parallel}\right)\phi = 0$

- ND

$\Phi^{(2,1)} = \partial_n \phi = 0$

$\Phi^{(2,3)} = \partial_n^3 \phi = 0$

- DN

$\Phi^{(2,0)} = \phi = 0$

$\Phi^{(2,2)} = \partial_n^2 \phi = 0$

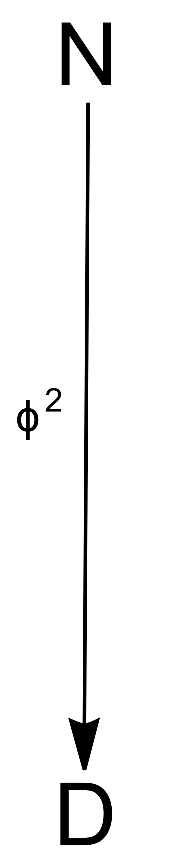
- DD

$\Phi^{(2,0)} = \phi = 0$

$\Phi^{(2,1)} = \partial_n \phi = 0$

# Boundary RG flows

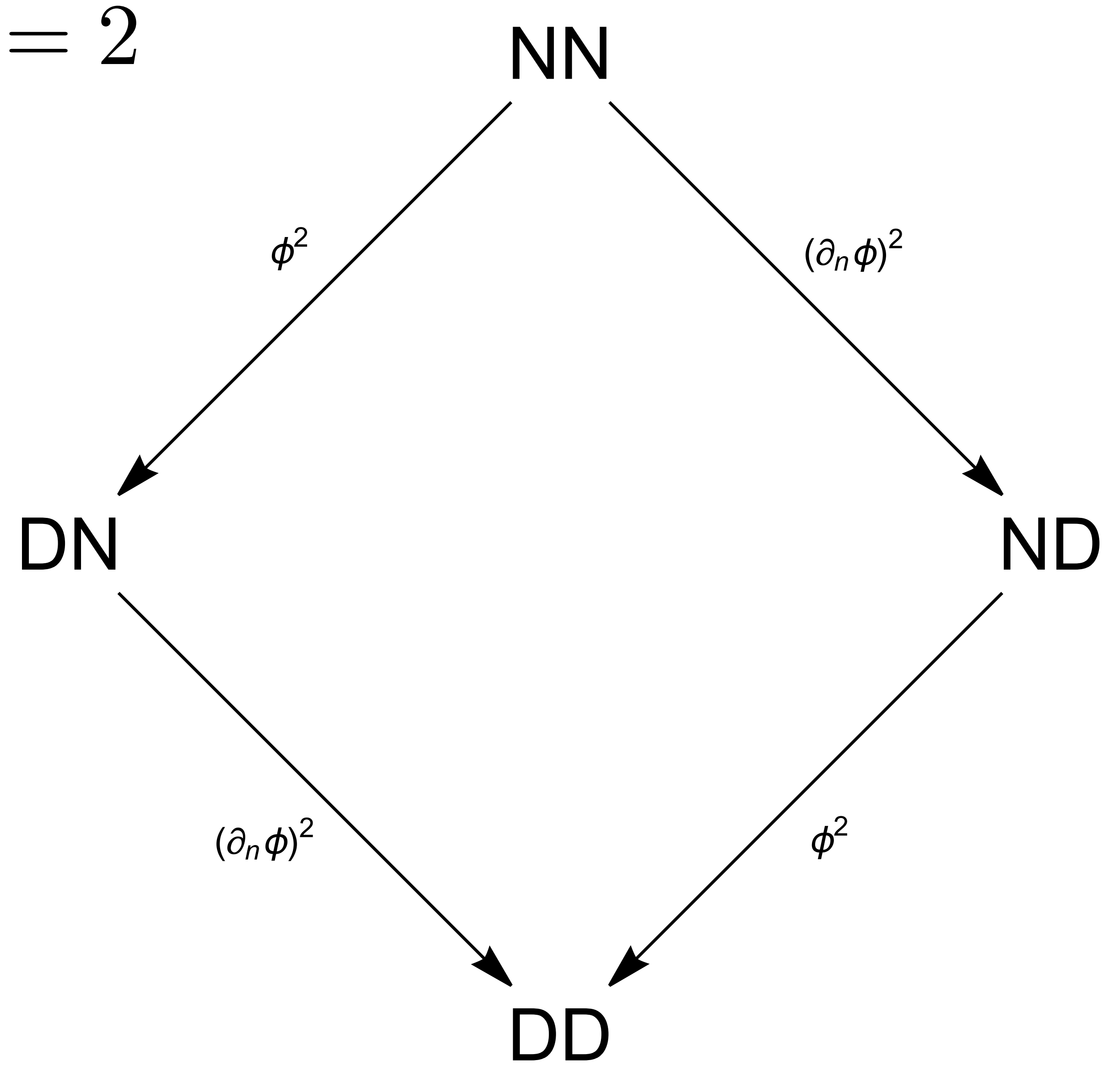
$k = 1$




- In d=5

$$\Delta a_\Sigma(N \to D) = \frac{17}{5760}$$

$k = 2$




- In d=5

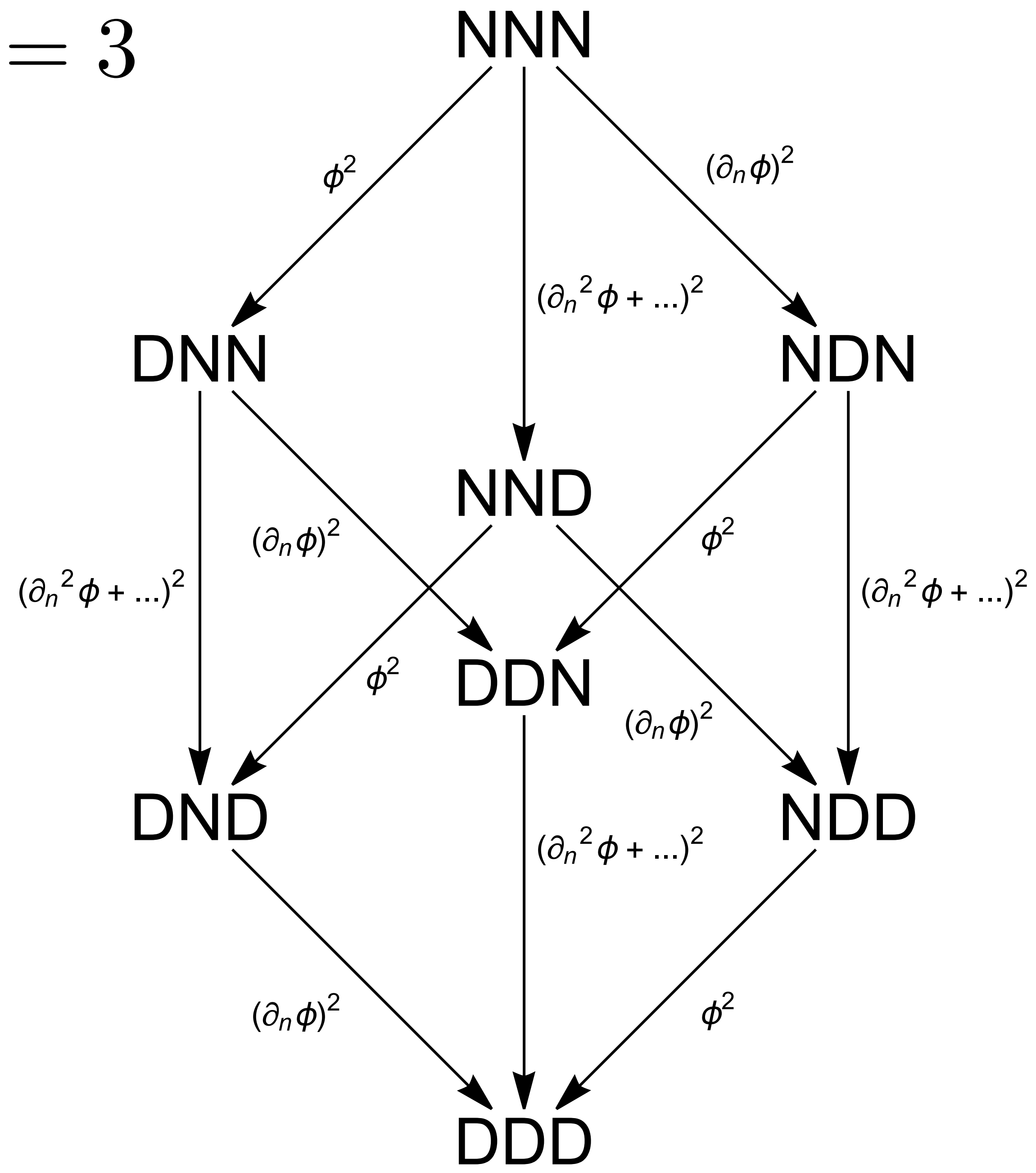
$$\Delta a_\Sigma(NN \to DN) = \frac{21}{640}$$

$$\Delta a_\Sigma(NN \to ND) = -\frac{17}{5760}$$

$$\Delta a_\Sigma(DN \to DD) = -\frac{17}{5760}$$

$$\Delta a_\Sigma(ND \to DD) = \frac{21}{640}$$

$k = 3$




- In d=5

$$\Delta a_\Sigma(\phi^2) = \frac{1375}{1152}$$

$$\Delta a_\Sigma\left((\partial_n \phi)^2\right) = \frac{21}{640}$$

$$\Delta a_\Sigma\left((\partial_n^2 \phi + \ldots)^2\right) = -\frac{17}{5760}$$

# Conclusion and Outlook

## Summary

- We discussed the defect Weyl anomalies up to 4-dimensional defects
- Some of the anomaly coefficients are related to simple correlators in flat space-time
- Examples and computation of anomaly coefficients in free and holographic theories
- Characterisation of boundary condition for free-higher derivative theories

## Outlook

- To relate additional anomaly coefficient to more complicated correlators
- Bounds on other anomaly coefficients
- Bulk theories coupled to lower dimensional one like minimal models for p=2
- Interacting higher-derivative theories with boundaries and defects?

# Algorithm for finding the Weyl Anomalies

**The algorithm (p=2, q=1 example):**

- Find a basis of terms for the anomaly $\delta_\omega W$

$$\mathcal{B}_1\,\delta\omega = \bar{R}\,\delta\omega\,, \qquad \mathcal{B}_2\,\delta\omega = R\,\delta\omega\,, \qquad \mathcal{B}_3\,\delta\omega = K^2\,\delta\omega$$

$$\tilde{\mathcal{B}}_1\,\delta\omega = \mathring{K}_{ab}\mathring{K}^{ab}\,\delta\omega$$

$$\mathcal{D}_1\,\delta\omega = K D_\perp \delta\omega\,, \qquad \mathcal{D}_2\,\delta\omega = D_\perp^2\,\delta\omega$$

$\longrightarrow$

$$\delta_\omega W = \int \sqrt{\bar{g}}\,\Big[b_1\mathcal{B}_1 + b_2\mathcal{B}_2 + b_3\mathcal{B}_3 + \\ + \tilde{b}_1\tilde{\mathcal{B}}_1 + d_1\mathcal{D}_1 + d_2\mathcal{D}_2\Big]\,\delta\omega$$

# Algorithm for finding the Weyl Anomalies

**The algorithm (p=2, q=1 example):**

- Find a basis of terms for the anomaly $\delta_\omega W$

$$\mathcal{B}_1\,\delta\omega = \bar{R}\,\delta\omega\,, \qquad \mathcal{B}_2\,\delta\omega = R\,\delta\omega\,, \qquad \mathcal{B}_3\,\delta\omega = K^2\,\delta\omega$$

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$$\mathcal{D}_1\,\delta\omega = K D_\perp \delta\omega\,, \qquad \mathcal{D}_2\,\delta\omega = D_\perp^2\,\delta\omega$$

$$\longrightarrow \quad \delta_\omega W = \int \sqrt{\bar{g}}\left[b_1\mathcal{B}_1 + b_2\mathcal{B}_2 + b_3\mathcal{B}_3 + \tilde{b}_1\tilde{\mathcal{B}}_1 + d_1\mathcal{D}_1 + d_2\mathcal{D}_2\right]\delta\omega$$

- Impose Wess-Zumino consistency $[\delta_{\omega_1}, \delta_{\omega_2}]\,W = 0$

$$(\sqrt{\bar{g}})^{-1}\delta_1(\sqrt{\bar{g}}\,\mathcal{B}_1\delta\omega_2) - (1\leftrightarrow 2) = 0$$

$$(\sqrt{\bar{g}})^{-1}\delta_1(\sqrt{\bar{g}}\,\mathcal{B}_2\delta\omega_2) - (1\leftrightarrow 2) \propto \left[\delta\omega_1 D_\perp^2\,\delta\omega_2 - (1\leftrightarrow 2)\right]$$

$$(\sqrt{\bar{g}})^{-1}\delta_1(\sqrt{\bar{g}}\,\mathcal{B}_3\delta\omega_2) - (1\leftrightarrow 2) \propto K\left[\delta\omega_1 D_\perp\delta\omega_2 - (1\leftrightarrow 2)\right]$$

$$(\sqrt{\bar{g}})^{-1}\delta_1(\sqrt{\bar{g}}\,\tilde{\mathcal{B}}_1\delta\omega_2) - (1\leftrightarrow 2) = 0$$

$$(\sqrt{\gamma})^{-1}\delta_1(\sqrt{\bar{g}}\,\mathcal{D}_1\delta\omega_2) - (1\leftrightarrow 2) = 0$$

$$(\sqrt{\bar{g}})^{-1}\delta_1(\sqrt{\bar{g}}\,\mathcal{D}_2\delta\omega_2) - (1\leftrightarrow 2) = 0$$

# Algorithm for finding the Weyl Anomalies

**The algorithm (p=2, q=1 example):**

- Find a basis of terms for the anomaly $\delta_\omega W$

$$\mathcal{B}_1\,\delta\omega = \bar{R}\,\delta\omega\,, \qquad \cancel{\mathcal{B}_2\,\delta\omega = R\,\delta\omega}\,, \qquad \cancel{\mathcal{B}_3\,\delta\omega = K^2\,\delta\omega}$$

$$\tilde{\mathcal{B}}_1\,\delta\omega = \mathring{K}_{ab}\mathring{K}^{ab}\,\delta\omega$$

$$\mathcal{D}_1\,\delta\omega = K D_\perp \delta\omega\,, \qquad \mathcal{D}_2\,\delta\omega = D_\perp^2\,\delta\omega$$

$$\longrightarrow \quad \delta_\omega W = \int \sqrt{\bar{g}}\left[b_1\mathcal{B}_1 + \cancel{b_2\mathcal{B}_2} + \cancel{b_3\mathcal{B}_3} + \tilde{b}_1\tilde{\mathcal{B}}_1 + d_1\mathcal{D}_1 + d_2\mathcal{D}_2\right]\delta\omega$$

- Impose Wess-Zumino consistency $[\delta_{\omega_1}, \delta_{\omega_2}]\,W = 0$

$$(\sqrt{\bar{g}})^{-1}\delta_1(\sqrt{\bar{g}}\,\mathcal{B}_1\delta\omega_2) - (1\leftrightarrow 2) = 0$$

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$$(\sqrt{\bar{g}})^{-1}\delta_1(\sqrt{\bar{g}}\,\mathcal{B}_3\delta\omega_2) - (1\leftrightarrow 2) \propto K\left[\delta\omega_1 D_\perp\delta\omega_2 - (1\leftrightarrow 2)\right]$$

$$(\sqrt{\bar{g}})^{-1}\delta_1(\sqrt{\bar{g}}\,\tilde{\mathcal{B}}_1\delta\omega_2) - (1\leftrightarrow 2) = 0$$

$$(\sqrt{\gamma})^{-1}\delta_1(\sqrt{\bar{g}}\,\mathcal{D}_1\delta\omega_2) - (1\leftrightarrow 2) = 0$$

$$(\sqrt{\bar{g}})^{-1}\delta_1(\sqrt{\bar{g}}\,\mathcal{D}_2\delta\omega_2) - (1\leftrightarrow 2) = 0$$

# Algorithm for finding the Weyl Anomalies

## The algorithm (p=2, q=1 example):

- Find a basis of terms for the anomaly $\delta_\omega W$

$$\mathcal{B}_1\,\delta\omega = \bar{R}\,\delta\omega\,, \qquad \cancel{\mathcal{B}_2\,\delta\omega = R\,\delta\omega}\,, \qquad \cancel{\mathcal{B}_3\,\delta\omega = K^2\,\delta\omega}$$

$$\tilde{\mathcal{B}}_1\,\delta\omega = \mathring{K}_{ab}\mathring{K}^{ab}\,\delta\omega$$

$$\mathcal{D}_1\,\delta\omega = K D_\perp \delta\omega\,, \qquad \mathcal{D}_2\,\delta\omega = D_\perp^2\,\delta\omega$$

$$\longrightarrow \quad \delta_\omega W = \int \sqrt{\bar{g}}\,\Big[b_1\mathcal{B}_1 + \cancel{b_2\mathcal{B}_2} + \cancel{b_3\mathcal{B}_3} + \tilde{b}_1\tilde{\mathcal{B}}_1 + d_1\mathcal{D}_1 + d_2\mathcal{D}_2\Big]\,\delta\omega$$

- Add all the possible counter-terms

$$W_{CT} = \sum_{i=1}^{3}\int\sqrt{\gamma}\,c_i\mathcal{B}_i \quad \longrightarrow \quad \begin{aligned} &\delta_\omega(\sqrt{\bar{g}}\,\mathcal{B}_1) = 0 \\ &\delta_\omega(\sqrt{\bar{g}}\,\mathcal{B}_2) \propto D_\perp^2\,\delta\omega \\ &\delta_\omega(\sqrt{\bar{g}}\,\mathcal{B}_3) \propto K D_\perp \delta\omega \end{aligned}$$

# Algorithm for finding the Weyl Anomalies

## The algorithm (p=2, q=1 example):

- Find a basis of terms for the anomaly $\delta_\omega W$

$$\mathcal{B}_1\,\delta\omega = \bar{R}\,\delta\omega\,, \qquad \cancel{\mathcal{B}_2\,\delta\omega = R\,\delta\omega}\,, \qquad \cancel{\mathcal{B}_3\,\delta\omega = K^2}\,\delta\omega$$

$$\tilde{\mathcal{B}}_1\,\delta\omega = \mathring{K}_{ab}\mathring{K}^{ab}\,\delta\omega$$

$$\cancel{\mathcal{D}_1\,\delta\omega = KD_\perp}\,\delta\omega\,, \qquad \cancel{\mathcal{D}_2\,\delta\omega = D_\perp^2}\,\delta\omega$$

$$\longrightarrow \quad \delta_\omega W = \int\sqrt{\bar{g}}\left[b_1\mathcal{B}_1 + \cancel{b_2\mathcal{B}_2} + \cancel{b_3\mathcal{B}_3} + \tilde{b}_1\tilde{\mathcal{B}}_1 + \cancel{d_1\mathcal{D}_1} + \cancel{d_2\mathcal{D}_2}\right]\delta\omega$$

- Add all the possible counter-terms

$$W_{CT} = \sum_{i=1}^{3}\int\sqrt{\gamma}\,c_i\mathcal{B}_i \quad \longrightarrow$$

$$\delta_\omega(\sqrt{\bar{g}}\,\mathcal{B}_1) = 0$$

$$\delta_\omega(\sqrt{\bar{g}}\,\mathcal{B}_2) \propto D_\perp^2\,\delta\omega$$

$$\delta_\omega(\sqrt{\bar{g}}\,\mathcal{B}_3) \propto KD_\perp\delta\omega$$

# Algorithm for finding the Weyl Anomalies

## The algorithm (p=2, q=1 example):

- Find a basis of terms for the anomaly $\delta_\omega W$

$$\mathcal{B}_1\, \delta\omega = \bar{R}\, \delta\omega\,, \qquad \cancel{\mathcal{B}_2\, \delta\omega = R\, \delta\omega}\,, \qquad \cancel{\mathcal{B}_3\, \delta\omega = K^2\, \delta\omega}$$

$$\tilde{\mathcal{B}}_1\, \delta\omega = \mathring{K}_{ab}\mathring{K}^{ab}\, \delta\omega$$

$$\cancel{\mathcal{D}_1\, \delta\omega = K D_\perp\, \delta\omega}\,, \qquad \cancel{\mathcal{D}_2\, \delta\omega = D_\perp^2\, \delta\omega}$$

$$\longrightarrow \quad \delta_\omega W = \int \sqrt{\bar{g}} \Big[ b_1 \mathcal{B}_1 + \cancel{b_2 \mathcal{B}_2} + \cancel{b_3 \mathcal{B}_3} + \tilde{b}_1 \tilde{\mathcal{B}}_1 + \cancel{d_1 \mathcal{D}_1} + \cancel{d_2 \mathcal{D}_2} \Big]\, \delta\omega$$

- Add all the possible counter-terms

$$W_{CT} = \sum_{i=1}^{3} \int \sqrt{\gamma}\, c_i \mathcal{B}_i \quad \longrightarrow \quad \begin{aligned} &\delta_\omega(\sqrt{\bar{g}}\, \mathcal{B}_1) = 0 \\ &\delta_\omega(\sqrt{\bar{g}}\, \mathcal{B}_2) \propto D_\perp^2\, \delta\omega \\ &\delta_\omega(\sqrt{\bar{g}}\, \mathcal{B}_3) \propto K D_\perp \delta\omega \end{aligned}$$

$$\boxed{\mathcal{A} = \int \sqrt{\gamma} \left[ -b\bar{\mathcal{R}} + \mathring{K}_{ab}\mathring{K}^{ab} \right]}$$

# Boundary Weyl Anomaly (q=1)

- We can specify our general result to q=1. We recover [Astaneh, Solodukhin, 2021]

$$T^{\mu}{}_{\mu}|_{\Sigma_4} = \frac{1}{(4\pi)^2}\Big(\boxed{-a_\Sigma \overline{E}_4} + \boxed{b_1 \mathcal{I}} + b_2(\mathrm{Tr}\ \mathring{K}^2)^2 + b_3 \mathrm{Tr}\ \mathring{K}^4 + b_4 W_{abcd}W^{abcd} + b_5 W_{anbn}W^{a}{}_{n}{}^{b}{}_{n}$$
$$+ b_6 W_{abcd}\mathring{K}^{ac}\mathring{K}^{bd} + b_7 W_{anbn}\mathring{K}^{a}{}_{c}\mathring{K}^{cb} + b_8 W_{nabc}W_{n}{}^{abc}\Big)$$

- Also in this case, the term $\mathcal{I}$ is related to the displacement two-point function

$$\mathcal{I} = -\frac{2}{3}R_{ab}\mathring{K}^{a}{}_{c}\mathring{K}^{cb} + \frac{1}{4}R\ \mathrm{Tr}\ \mathring{K}^2 - \frac{1}{3}R_{nn}\mathrm{Tr}\ \mathring{K}^2 + \frac{1}{2}W_{anbn}K\mathring{K}^{ab} + \frac{1}{16}K^2\ \mathrm{Tr}\ \mathring{K}^2$$
$$+ \mathring{K}^{ab}D_nW_{anbn} - \frac{1}{2}K\ \mathrm{Tr}\ \mathring{K}^3 + \boxed{\frac{2}{9}\overline{D}^a\mathring{K}_{ab}\overline{D}_c\mathring{K}^{bc}}$$

$\longrightarrow$ $\boxed{b_1 \leq 0}$

- Boundary anomaly coefficients for 5d probe branes

| $a_\Sigma$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ | $b_8$ |
|---|---|---|---|---|---|---|---|---|
| $\frac{1}{4}$ | $-1$ | $\frac{3}{4}$ | $-\frac{3}{2}$ | $\frac{1}{4}$ | $-1$ | $2$ | $-1$ | $-\frac{1}{2}$ |

# AdS/BCFT

[Takayanagi, 2011]
[Fujita, Takayanagi, Tonni, 2011]

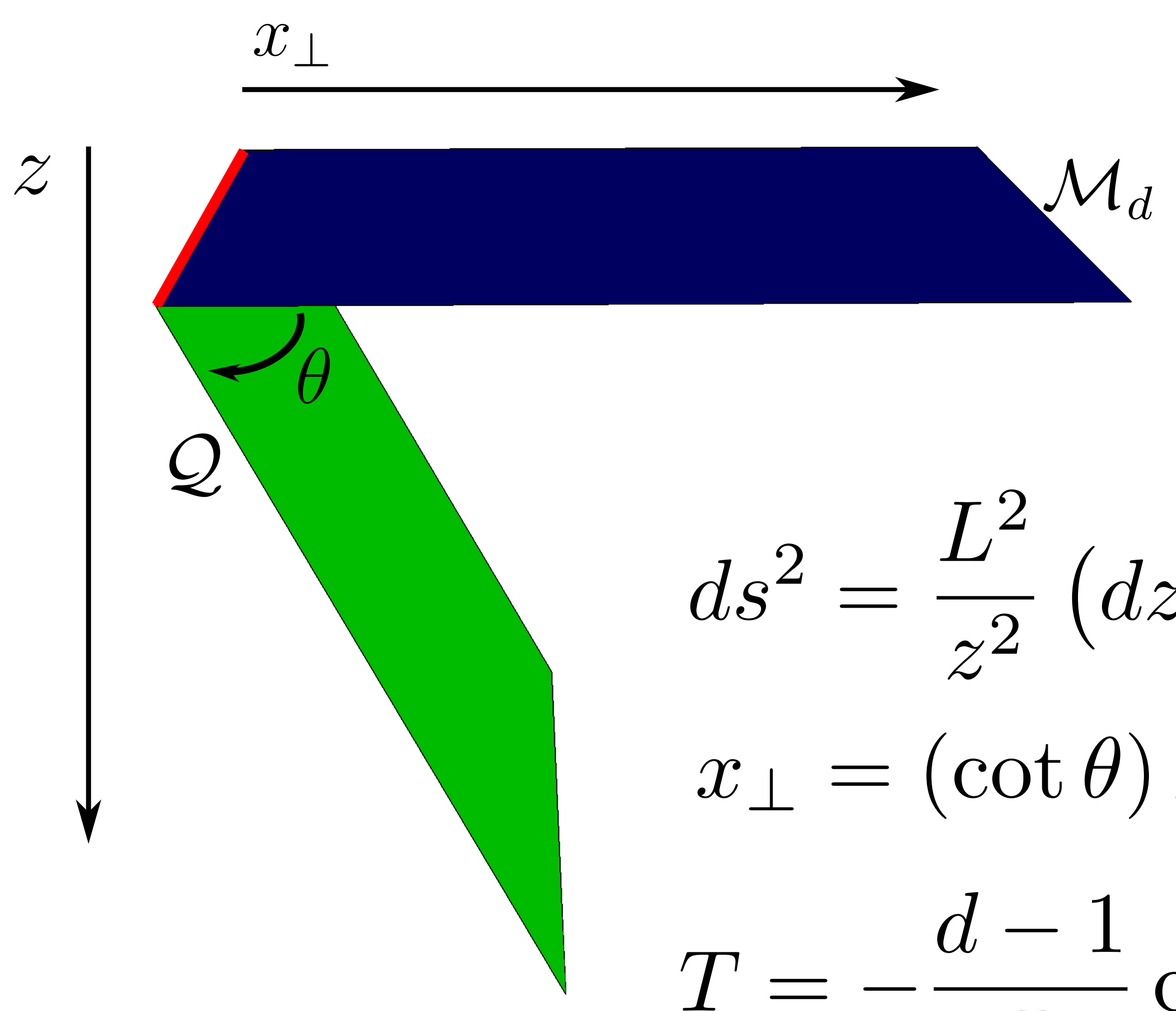
- Holographic description: boundary in the space-time

$$S_{\text{grav}} = -\frac{1}{16\pi G_N}\int_{\mathcal{N}} \sqrt{\mathfrak{g}}\,(\mathcal{R}-2\Lambda) - \frac{1}{8\pi G_N}\int_{\mathcal{Q}} \sqrt{\bar{\mathfrak{g}}}\,(\mathcal{K}-T) - \frac{1}{8\pi G_N}\int_{\mathcal{M}_d} \sqrt{\bar{\mathfrak{g}}}\,\mathcal{K}$$

Einstein equations + (Neumann) boundary condition

$$R_{MN} - \frac{1}{2}\mathcal{R}\,\mathfrak{g}_{MN} + \Lambda\mathfrak{g}_{MN} = 0 \qquad\qquad \mathcal{K}_{MN} = (\mathcal{K}-T)\,\bar{\mathfrak{g}}_{MN}$$

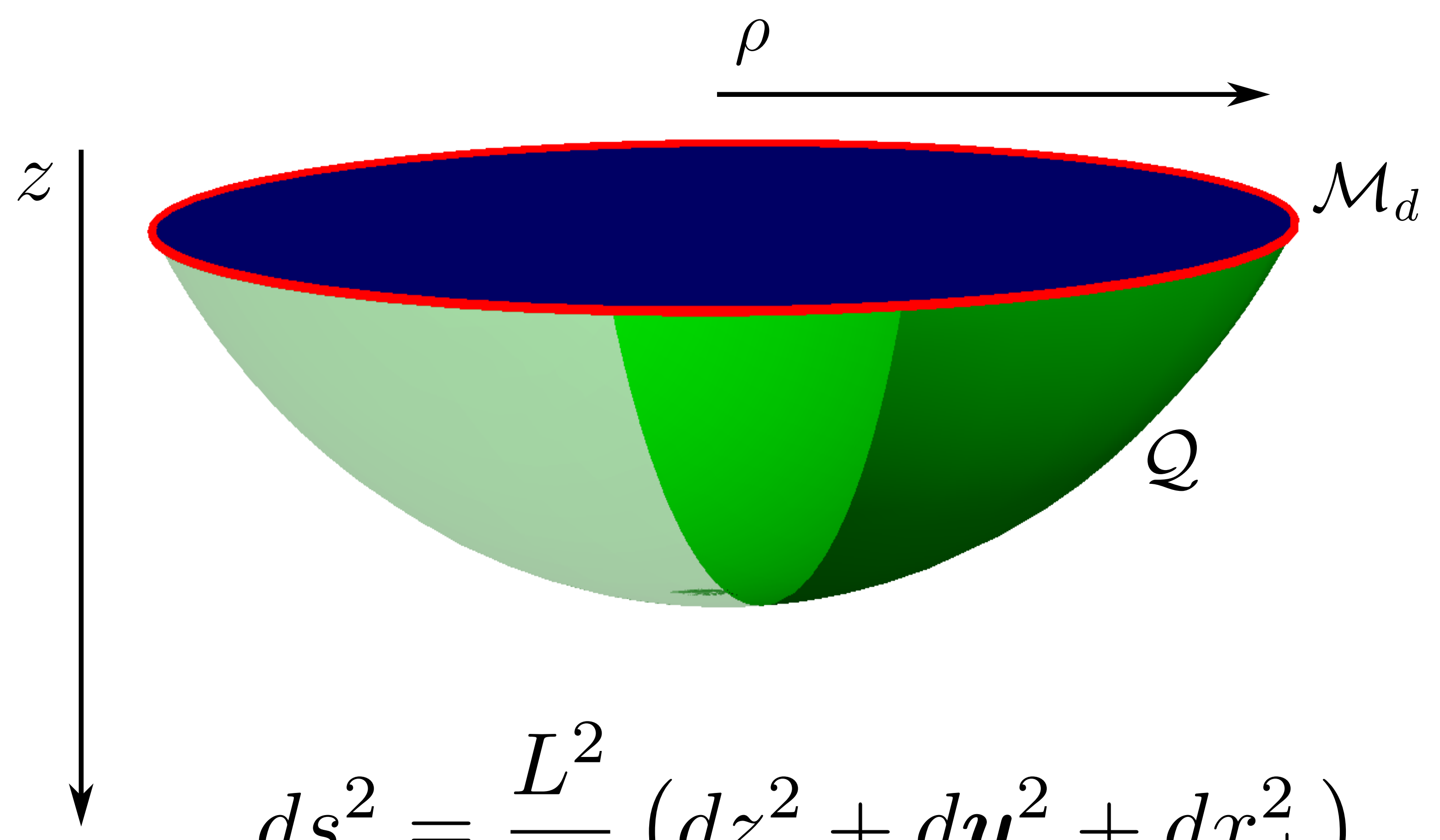
- Conformal solutions




$$ds^2 = \frac{L^2}{z^2}\left(dz^2 + d\boldsymbol{y}^2 + dx_\perp^2\right)$$

$$x_\perp = (\cot\theta)\,z$$

$$T = -\frac{d-1}{L}\cos\theta$$




$$ds^2 = \frac{L^2}{z^2}\left(dz^2 + d\boldsymbol{y}^2 + dx_\perp^2\right)$$

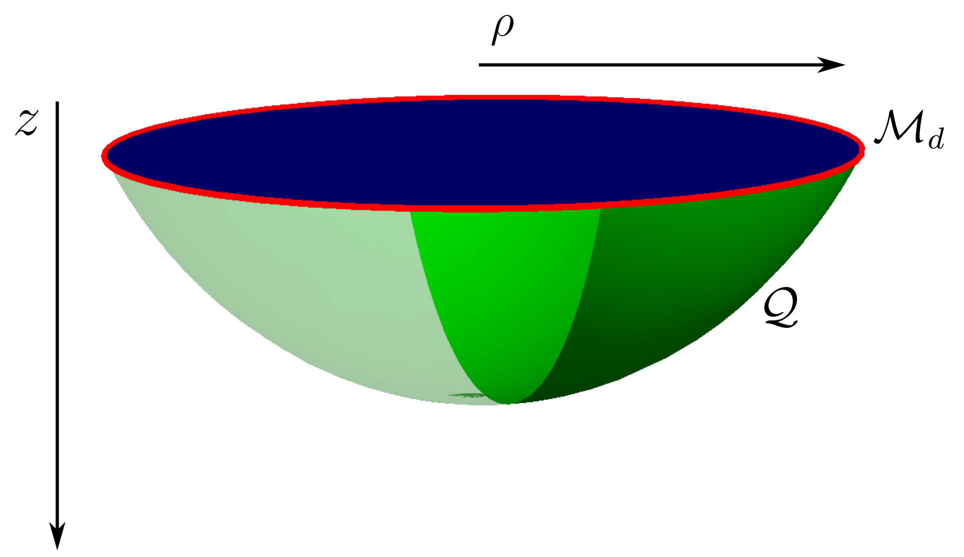
$$\rho_{\mathcal{Q}}(z) \equiv \sqrt{\frac{L^2}{\sin^2\theta} - (z + L\cot\theta)^2}$$

# AdS/BCFT

[Chalabi, Herzog, O'Bannon, Robinson, JS, 2021]

- (Half-)sphere free energy

$$S_{\rm grav} = \frac{\pi L^4}{6G_N}\left[-\frac{1+2\sin^2\theta}{\sin^2\theta}\cot\theta\log\left(\frac{L}{\epsilon}\right)+\dots\right]$$

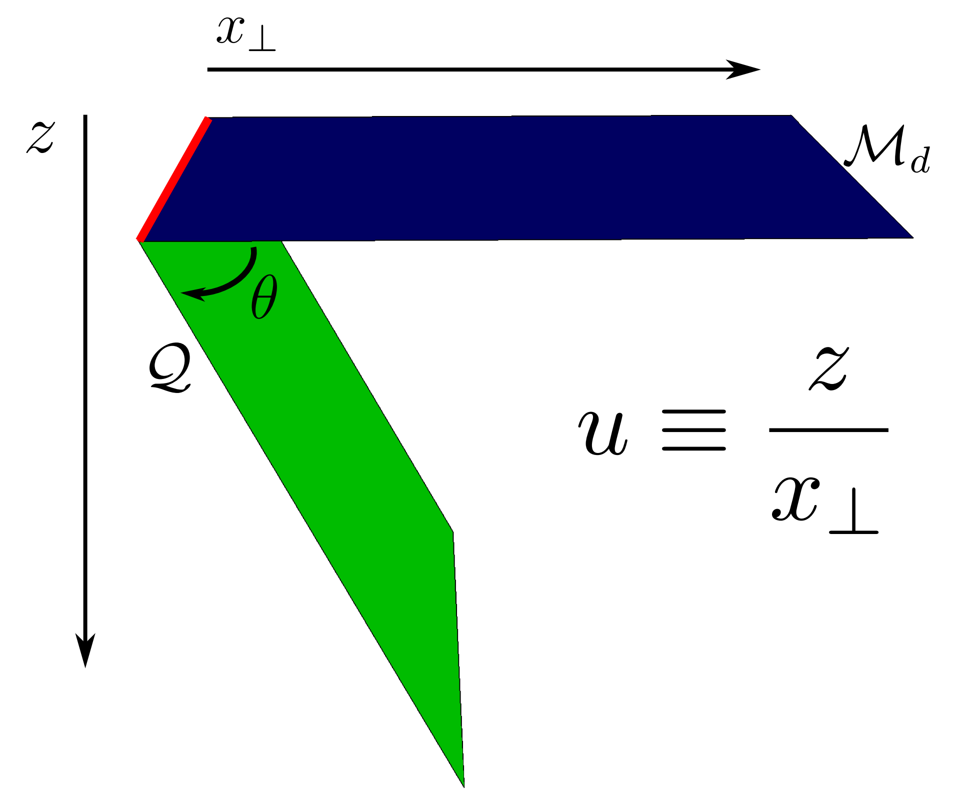



$$\boxed{a_\Sigma = -\frac{\pi L^4}{24G_N}\frac{1+2\sin^2\theta}{\sin^2\theta}\cot\theta}$$

Boundary RG flow $\theta_{\rm UV}\geq\theta_{\rm IR} \longrightarrow a_\Sigma^{(\rm UV)}\geq a_\Sigma^{(\rm IR)}$

- Metric perturbation about AdS (back-reaction)

$$ds^2 = \frac{L^2_{\rm AdS}}{z^2}\Big[dz^2+\Big(1+\epsilon^2x_\perp^2\mathcal{G}^{(2)}(u)+\epsilon^3x_\perp^3\mathcal{G}^{(3)}(u)\Big)dx_\perp^2+$$
$$+\Big(\delta_{ab}+\epsilon\,x_\perp\mathcal{F}^{(1)}_{ab}(u)+\epsilon^2\,x_\perp^2\mathcal{F}^{(2)}_{ab}(u)+\epsilon^3x_\perp^3\mathcal{F}^{(3)}_{ab}(u)\Big)dy^ady^b\Big]+\mathcal{O}(\epsilon^4)$$




$$u\equiv\frac{z}{x_\perp}$$

- Embedding for $\mathcal{Q}$

$$u_{\mathcal{Q}}(x)=\tan\theta+\epsilon B^{(1)}x_\perp+\epsilon^2B^{(2)}x_\perp^2+\epsilon^3B^{(3)}x_\perp^3+\mathcal{O}(\epsilon^4)$$

- We can solve the system of equation order by order in $\epsilon$

# AdS/BCFT

[Chalabi, Herzog, O'Bannon, Robinson, JS, 2021]

- Solution up to third order in metric perturbation
- Values of the anomaly coefficients EXACT in $\theta$

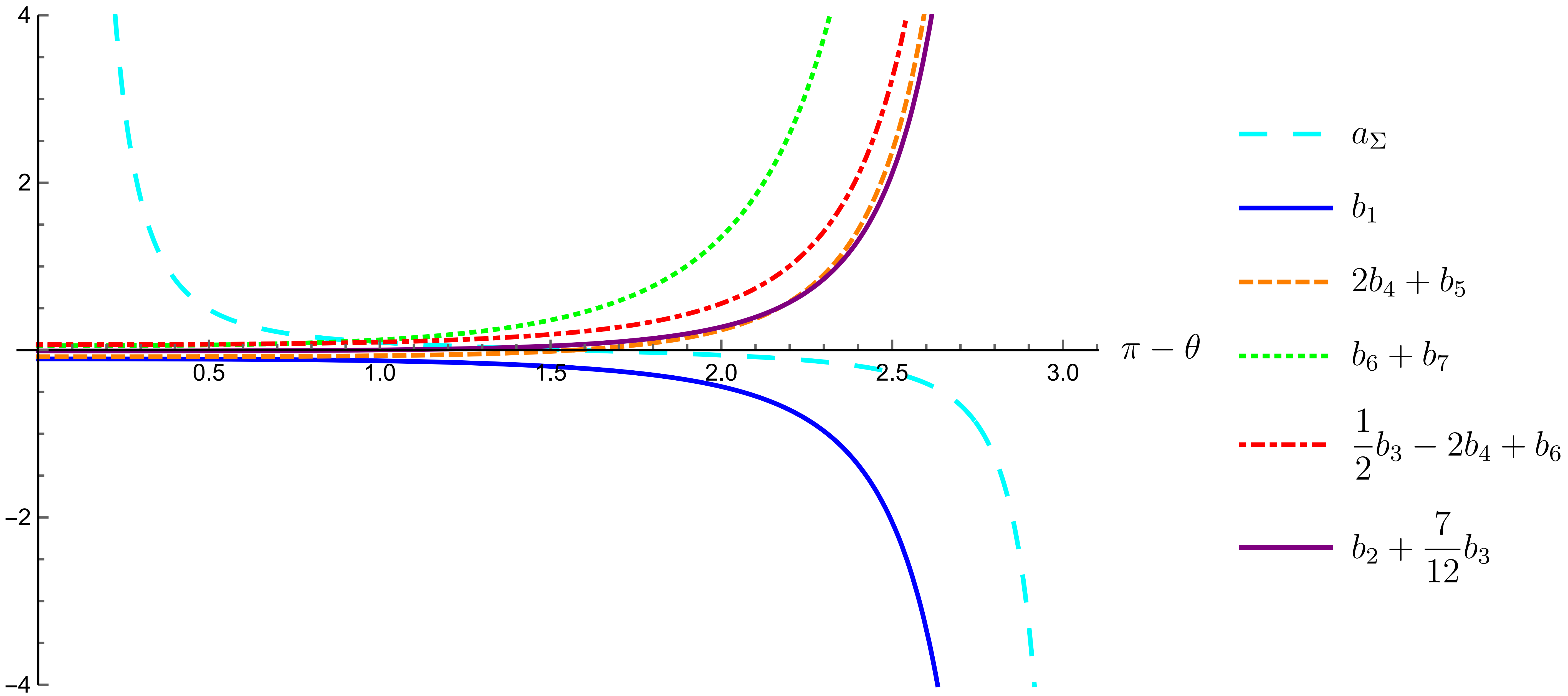
$$\boxed{b_1 = -\frac{1}{3}\frac{1}{\theta - \sin\theta\cos\theta}\frac{\pi L^4}{G_N} \qquad b_1 \leq 0}$$

$$2b_4 + b_5 = \frac{1}{4}\frac{1}{\tan\theta - \theta}\frac{\pi L^4}{G_N}$$

$$b_6 + b_7 = \frac{1}{24}\frac{13\sin\theta - 3\sin(3\theta) - 4\theta\cos\theta}{(\theta - \sin\theta\cos\theta)(\sin\theta - \theta\cos\theta)}\frac{\pi L^4}{G_N}$$

$$\frac{b_3}{2} - 2b_4 + b_6 = \frac{1}{48}\frac{13 - 3\cos(2\theta) - 10\theta\cot\theta}{(1 - \theta\cot\theta)(\theta - \sin\theta\cos\theta)}\frac{\pi L^4_{\text{AdS}}}{G_N}$$

$$\begin{aligned} b_2 + \frac{7}{12}b_3 = & \frac{1}{93312}\frac{1}{(\sin(2\theta) - 2\theta)^4(\theta\cos\theta - \sin\theta)} \times \\ & \times\Big[12\left(7963 + 16996\theta^2 - 6144\theta^4\right)\cos\theta - 24(5023 + 36\theta^2)\cos(3\theta) \\ & + 8(3719 + 4644\theta^2)\cos(5\theta) - 4699\cos(7\theta) - 57\cos(9\theta) \\ & + 24\theta(-13933 + 1488\theta^2)\sin\theta + 144\theta(600\theta^2 - 421)\sin(3\theta) \\ & + 21968\theta\sin(5\theta) - 11131\theta\sin(7\theta) + 429\theta\sin(9\theta))\Big]\frac{\pi L^4_{\text{AdS}}}{G_N} \end{aligned}$$



# Scale vs Conformal Invariance

- Example of scale (and Lorentz invariant) but not conformal boundary conditions

$$\frac{dP^a}{dt} = \int_{t,z=0} d^{d-2}x \, \left[T^{za} + \partial_t \tau^{at}\right]$$
$$\frac{dD}{dt} = \int_{t,z=0} d^{d-2}x \, \left[x_a T^{az} + \partial_t (x_a \tau^{at})\right]$$
$$\frac{dK^a}{dt} = \int_{t,z=0} d^{d-2}x \, \left[(2x^a x_b - x^2 \delta^a_b) T^{bz} + \partial_t (2x^t x_b - x^2 \delta^t_b)\tau^{bt}\right]$$

- If we take the free and not supported bc in plate theory we have to set to zero the following

$$\tilde{\Phi}^{(2,2)} \equiv (\partial_n^2 + \sigma \Box_{||})\phi, \qquad \tilde{\Phi}^{(2,3)} \equiv (\partial_n^3 + (2-\sigma)\partial_n \Box_{||})\phi$$

- This gives $T^{az} = -\partial_b \tau^{ab} \neq 0$

- Boundary scale invariance requires $\tau^a_a = \partial^a j_a$ $\longrightarrow$ $\boxed{\begin{array}{l} \sigma = -1 \\ j_a = 0 \end{array} \; \text{conformal}}$ $\quad \boxed{\begin{array}{l} \sigma \neq -1 \\ j_a \neq 0 \end{array} \; \text{non conformal}}$

- It is achieved by deforming the boundary action with $S_{\text{marg}} = \sigma \int d^{d-1}x \, \phi \, \partial^2_{||} \partial_\perp \phi$