Frontiers in Nuclear and Hadronic Physics 2025

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Centro Nazionale di Studi Avanzati dell'Istituto Nazionale di Fisica Nucleare

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Hadron Spectroscopy

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EXCELENCIA SEVERO OCHOA

- QCD Lagrangian
  - ✓ Color degrees of freedom. Confinement, asymptotic freedom. Quark model and non qqq and $q\bar{q}$ structures
  - ✓ Local gauge group transformations
  - ✓ Symmetries and conserved currents

- Chiral symmetry
  - ✓ Wigner-Weyl & Nambu-Goldstone realizations of chiral symmetry
  - ✓ Linear and non-linear σ −models: renormalization
  - ✓ Weinberg-Tomozawa interaction

- Odd parity S-wave ϕB resonances
  - ✓ Flavor SU(3) symmetry and chiral expansion
  - ✓ Chiral Unitary Approach: unitarity in coupled channels. Riemann sheets, bound, virtual and resonant states.
  - ✓ Strangeness $\mathrm{S}=-1$ sector
    - o $\Lambda(1405)$: double pole structure and chiral symmetry
    - o $\Lambda(1670)$ and $\Sigma(1620)$ bumps
  - ✓ Strangeness $\mathrm{S}=-2$ sector: $\Xi(1620)$ and $\Xi(1690)$
  - ✓ Strangeness $\mathrm{S}=-0$ sector: $\mathrm{N}(1535)$ and $\mathrm{N}(1650)$
- Odd parity D-wave ϕB resonances: $\mathrm{N}(1520), \Lambda(1520)$ and $\Xi(1820)$

- Lowest lying odd parity resonances in the charm sector: $SU(6)_{lsf} \times$ HQSS model
  - ✓ $\Lambda_c(2595)$ and $\Lambda_c(2625)$
  - ✓ Ω_c (LHCb) and Ξ_c excited states
  - ✓ Extension to the bottom sector
- Lowest lying $\left(\frac{1}{2}\right)^-$ and $\left(\frac{3}{2}\right)^-$ Λ_Q resonances: from the strange to the bottom sectors
  - ✓ Interplay between chiral meson-baryon and CQM degrees of freedom and the role played by the renormalization scheme
  - ✓ Molecular content, HQSS and thresholds: $\Lambda_b(5912)/\Lambda_c(5920)$, $\Lambda_c(2595)/\Lambda_c(2625)$ and $\Lambda(1405)/\Lambda(1520)$
  - ✓ Higher resonances: $\Lambda_b(6070)$ and $\Lambda_c(2765)$; molecules versus CQM 2S states

Exercises for Lectures on Hadron Spectroscopy

1. Derive the equations of motion of QCD
2. Obtain the SU(3) light-flavor vector and axial currents and their commutation relations
3. Use the Jacobi identity to constrain the non-linear σ −model for pions
4. Study the SU(3) limit of the odd parity S-wave ϕB resonance-spectrum obtained from the chiral unitarity approach (CUA) with the Weinberg-Tomozawa interaction
5. Discuss the extension of the CUA to finite volume: energy-levels
6. $K^{-}p$ correlation function from high multiplicity collisions and chiral SU(3) dynamics

Some references:

- Taizo Muta, *Foundations of quantum chromodynamics: An Introduction to perturbative methods in gauge theories* (World Scientific Lecture Notes In Physics, vol. 5, 1987).
- Stefan Scherer and Matthias R. Schindler, *A Chiral perturbation theory primer* [hep-ph/0505265 & Lect. Notes Phys. 830 (2012) pp.1-338]
- Pedro Pascual and Rolf Tarrach, *QCD: renormalization for the practitioner,* [Lect. Notes Phys. 194 (1984) pp. 1-277]
- Feng-Kun, Guo and Christoph Hanhart, Ulf-G. Meiβner, *Hadronic Molecules,* [*Rev. Mod. Phys.* 90 (2018) 015004, *Rev. Mod. Phys.* 94 (2022) 029901 (erratum)]
- Juan Nieves, Albert Feijoo, Miguel Albaladejo, Meng-Lin Du, *Lowest lying* $\left(\frac{1}{2}\right)^{-}$ *and* $\left(\frac{3}{2}\right)^{-}$ Λ_Q *resonances: from the strange to the bottom sectors* [Prog. Part. Nucl. Phys. 137 (2024) 104118 **and references therein**]

QCD Lagrangian

$$\mathcal{L}_{QCD} = \frac{1}{2g^2}\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \frac{i}{2}\,\overline{q^A}\gamma_\mu D^\mu q^A - \frac{i}{2}\,\overline{D^\mu q^A}\gamma_\mu q^A - m_A \bar{q}^A q^A$$

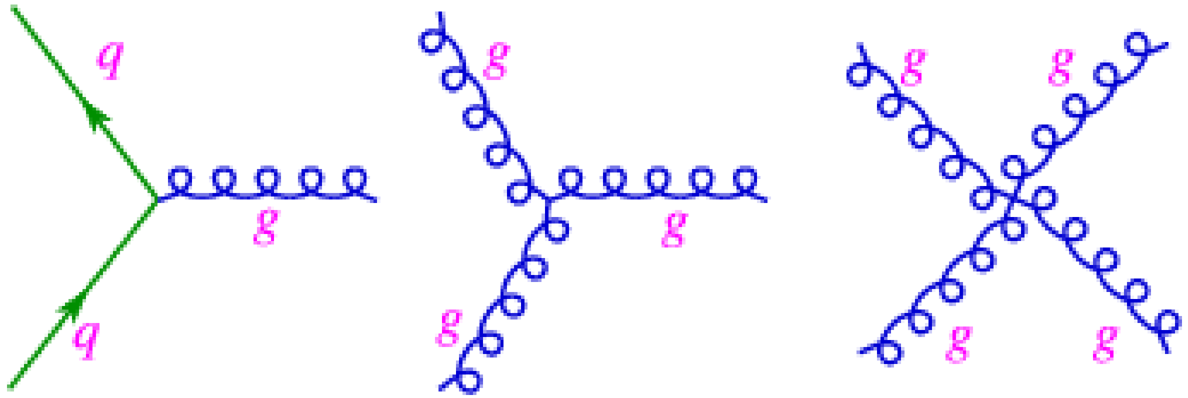
$A = 1, \ldots, N_F$ (flavor index)

$$D^\mu_{\alpha\beta} = \delta_{\alpha\beta}\partial^\mu - \underbrace{igT^a_{\alpha\beta}B^\mu_a(x)}_{B^\mu_{\alpha\beta}(\mathrm{x})}$$

$T^a_{\alpha\beta} = \frac{\lambda^a_{\alpha\beta}}{2}$, $\lambda^{a=1,\ldots,N_C^2-1}$ (Gell-Mann matrices)

α, β: color indices; $[T_a, T_b] = if_{abc}T_c$

$q^A_\alpha(x)$ quark field: flavor and color



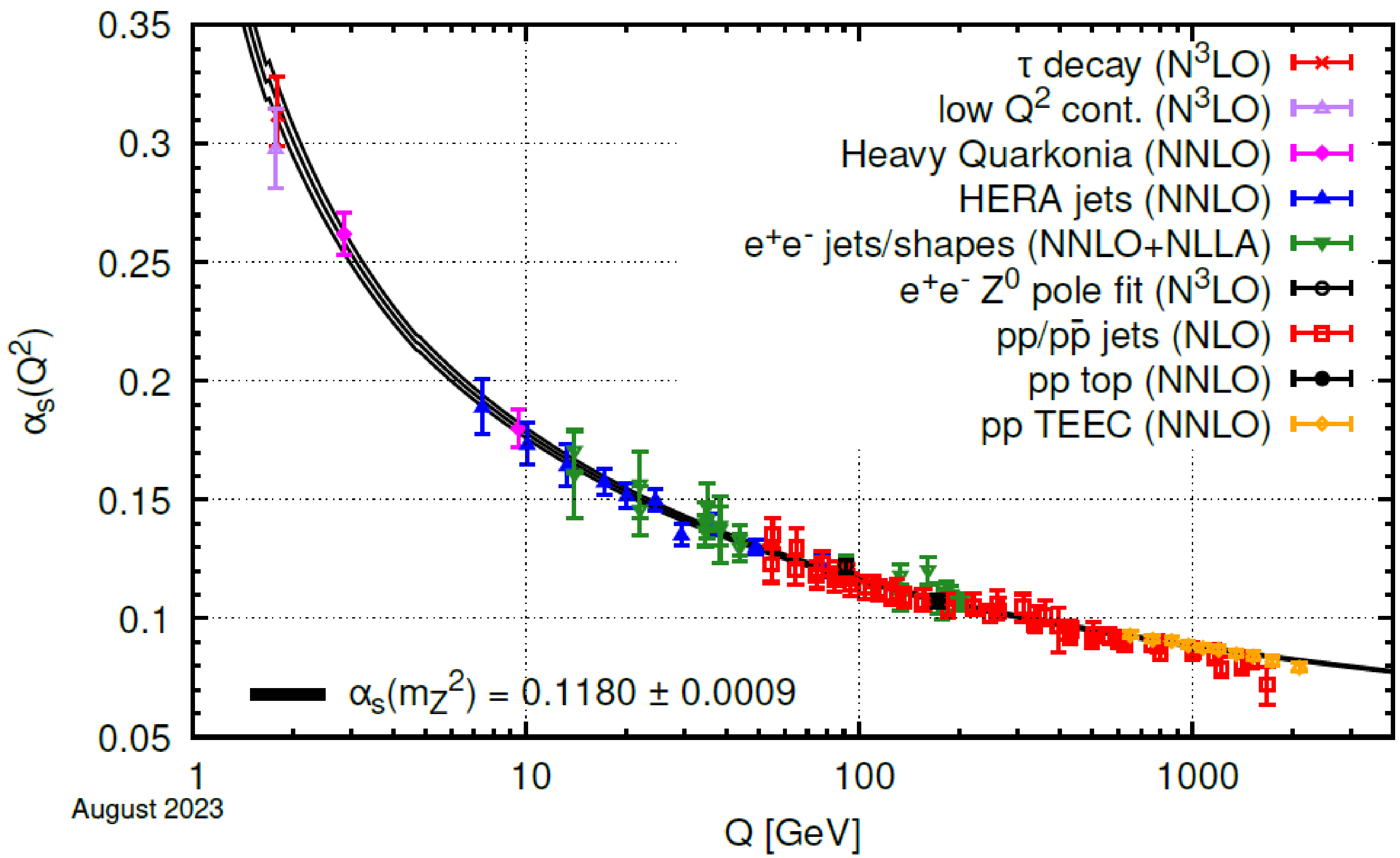
$$F^{\mu\nu}(x) = -[D^\mu, D^\nu]$$
$$= \partial^\mu B^\nu - \partial^\nu B^\mu - [B^\mu, B^\nu]$$

$G_{\alpha\beta}(x) = \left[e^{-igT_a\theta_a(x)}\right]_{\alpha\beta}$ **(local gauge transformation !)**

$$D^\mu \longrightarrow GD^\mu G^{-1},\ F^{\mu\nu} \longrightarrow GF^{\mu\nu}G^{-1}$$
$$q^A \longrightarrow Gq^A$$
$$B^\mu \longrightarrow GB^\mu G^{-1} + (\partial^\mu G)G^{-1}$$

$\mathcal{L}_{QCD} \rightarrow \mathcal{L}_{QCD}$

invariant!



Confinement and asymptotic freedom.

The Review of Particle Physics (2024)
S. Navas *et al.* (Particle Data Group),
Phys. Rev. D **110**, 030001 (2024)

$$\left(i\gamma_\mu D^\mu - m_A\right)q^A = 0$$

$$\left[D^\mu, F_{\mu\nu}\right] = -ig^2 T_a \sum_A \bar{q}^A\, T_a \gamma_\nu q^A$$

Classical equations of motion

Symmetries and conserved currents

Symmetry $\mathbf{U}$ $\{\tau_a\}$ ⟺ **Conserved charges** Q_a

$$\mathcal{L}[\phi] = \mathcal{L}[\phi'],\ \phi' = U[\tau_a\theta_a]\phi = \phi + \delta_a\phi\ \theta_a \Rightarrow j_a^\mu(x) = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta_a\phi$$

Noether Theorem: $\partial_\mu j_a^\mu = 0;\ Q_a = \int d^3x\, j_a^0(t,\vec{x});\quad \frac{dQ_a}{dt} = 0$

- **Global $\mathrm{U_B}(1)$:** $q' = e^{-i\theta}q$ **(same phase for all flavors and colors)**

  $J^\mu = \sum_{A,\alpha}\bar{q}_\alpha^A\gamma^\mu q_\alpha^A,\quad \partial_\mu J^\mu = 0,\ \mathrm{B} = \int d^3x\, J^0(t,\vec{x})$ **= baryon number**

- **Global** $\mathrm{U}(1)\times\mathrm{U}(1)\times\overbrace{\dots}^{N_F}\times\mathrm{U}(1)$: $q'^A = e^{-i\theta_A}q$ **(phase depend on flavor)**

  $J_A^\mu = \sum_\alpha \bar{q}_\alpha^A\gamma^\mu q_\alpha^A,\quad \partial_\mu J_A^\mu = 0,\ N_A = \int d^3x\, J_A^0(t,\vec{x})$ **= flavor number**

  (gluons do not change flavor)

- **Global SU(2) rotation for two flavors $m_A = m_B$**

$$q'^A_\alpha = \left[e^{-i\vec{\sigma}\vec{\theta}/2}\right]^A_B q^B_\alpha$$ (isospin symmetry for *u* and *d* quarks)

If $m_A = m\ \forall\ A \longrightarrow \mathrm{SU}(N_F)$ (e.g. SU(3) symmetry for *u*,*d* and *s*)

- **Let us suppose $m_A = 0\ \forall\ A \Rightarrow \mathrm{SU_L}(N_F) \times \mathrm{SU_R}(N_F)$ rotation**

$$q'_\alpha = e^{-i\theta_A T^A} q_\alpha \quad \text{(vector)}$$

$$q'_\alpha = e^{-i\theta_A T^A \gamma_5} q_\alpha \quad \text{(axial)}$$

currents:

$$V^A_\mu = \sum_\alpha \sum_{YZ} \bar{q}^Y_\alpha \gamma_\mu (T^A)_{YZ} q^Z_\alpha, \quad \partial_\mu V^{\mu A} = i \sum_\alpha \sum_{YZ} (m_Y - m_Z) \bar{q}^Y_\alpha (T^A)_{YZ} q^Z_\alpha$$

$$A^A_\mu = \sum_\alpha \sum_{YZ} \bar{q}^Y_\alpha \gamma_\mu \gamma_5 (T^A)_{YZ} q^Z_\alpha, \quad \partial_\mu A^{\mu A} = i \sum_\alpha \sum_{YZ} (m_Y + m_Z) \bar{q}^Y_\alpha (T^A)_{YZ} \gamma_5 q^Z_\alpha$$

$$Q^A(t)=\int d^3x\, V_A^0(t,\vec{x}\,),\ Q_5^A(t)=\int d^3x\, A_A^0(t,\vec{x}\,)$$ & $$\left\{q_i(x),q_j^\dagger(y)\right\}_{x^0=y^0}=\delta_{ij}\,\delta^4(x-y)$$

$$[Q^A(t),Q^B(t)]=if_{ABC}Q^c(t)$$

$$\left[Q^A(t),Q_5^B(t)\right]=if_{ABC}Q_5^C(t)$$

$$\left[Q_5^A(t),Q_5^B(t)\right]=if_{ABC}Q^c(t)$$

$$\left[Q_L^A(t),Q_L^B(t)\right]=if_{ABC}Q_L^C(t)$$

$$\left[Q_R^A(t),Q_R^B(t)\right]=if_{ABC}Q_R^C(t)$$

$$\left[Q_L^A(t),Q_R^B(t)\right]=0$$

$$q_{R,L}=\frac{1\pm\gamma_5}{2}q,\qquad q=q_L+q_R$$

$$q_{(L,R)}\to q'_{(L,R)}=e^{-i\theta^A_{(L,R)}T^A}\,q_{(L,R)}$$

$$L_\mu^A=\frac{V_\mu^A-A_\mu^A}{2},\quad R_\mu^A=\frac{V_\mu^A+A_\mu^A}{2}$$

$$\langle 0|[Q_5^A,P^B]|0\rangle=-\frac{1}{2}\langle 0|\bar{q}\{\lambda^A,\lambda^B\}q|0\rangle=-\frac{2}{3}\langle 0|\bar{q}q|0\rangle$$

$$P^B=\bar{q}\gamma_5\frac{\lambda^B}{2}\,q$$

Current Algebra ('60)

Symmetry realizations

Symmetry $U\{\tau_a\}$ $\Longleftrightarrow$ Conserved charges $\boldsymbol{Q_a}$

Wigner-Weyl

$$Q_a|0>=0$$

- Exact symmetry
- Degenerate multiplets
- Linear representation

Nambu-Goldstone

$$Q_a|0>\neq\ 0$$

- Spontaneously Broken Symmetry
- Massless Goldstone Bosons
- Non-Linear representation

Chiral symmetry

$(m_q = 0)$ Chiral limit

$$\mathcal{L}_{QCD}^{0} = \frac{1}{2g^2}\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \bar{q}_L i\gamma_\mu D^\mu q_L + \bar{q}_R i\gamma_\mu D^\mu q_R \qquad q^T \equiv (u, d, s)$$

- $\mathcal{L}_{QCD}^{0}$ **invariant under** $\mathbf{G \equiv SU(3)_L \otimes SU(3)_R}$**:**

$$\bar{\mathbf{q}}_L \to g_L\, \bar{\mathbf{q}}_L \quad ; \quad \bar{\mathbf{q}}_R \to g_R\, \bar{\mathbf{q}}_R \quad ; \quad (g_L, g_R) \in \mathbf{G}$$

- **Only** $\mathbf{SU(3)_V}$ **in the hadronic spectrum:** $(\pi, K, \eta)_{0^-}\; ;\; (\rho, K^*, \omega)_{1^-}\; ;\; \cdots$

$$M_{0^-} < M_{0^+} \quad ; \quad M_{1^-} < M_{1^+}$$

- **The** $\mathbf{0^-}$ **octet is nearly massless:** $\mathbf{m_\pi \approx 0}$

- **The vacuum is not invariant (SSB):** $\langle 0 | (\bar{\mathbf{q}}_L \mathbf{q}_R + \bar{\mathbf{q}}_R \mathbf{q}_L) | 0 \rangle \neq 0$

8 Massless 0^- Goldstone Bosons

$\mathcal{Q}|0> \neq 0$

Goldstone Theorem

$$\mathcal{Q} = \int d^3x\, j^0(x) \quad ; \quad \partial_\mu j^\mu = 0 \quad ; \quad \exists\, \mathcal{O} : \; v(t) \equiv \langle 0|\, [\mathcal{Q}(t), \mathcal{O}]\, |0\rangle \neq 0$$

$$\Downarrow$$

$$\exists\, |n\rangle : \; \langle 0|\mathcal{O}|n\rangle\, \langle n|j^0|0\rangle \neq 0 \quad ; \quad E_n\, \delta^{(3)}(\vec{p}_n) = 0 \quad ; \quad M_n = 0$$

Proof: $j^0(x) = \mathrm{e}^{iP\cdot x}\, j^0(0)\, \mathrm{e}^{-iP\cdot x} \quad ; \quad \sum_n |n\rangle\langle n| = 1$

$$v(t) = \sum_n \int d^3x\, \{\langle 0|j^0(x)|n\rangle\langle n|\mathcal{O}|0\rangle - \langle 0|\mathcal{O}|n\rangle\langle n|j^0(x)|0\rangle\}$$

$$= \sum_n \int d^3x\, \{\mathrm{e}^{-ip_n\cdot x}\, \langle 0|j^0(0)|n\rangle\langle n|\mathcal{O}|0\rangle - \mathrm{e}^{ip_n\cdot x}\, \langle 0|\mathcal{O}|n\rangle\langle n|j^0(0)|0\rangle\}$$

$$= (2\pi)^3 \sum_n \delta^{(3)}(\vec{p}_n)\, \{\mathrm{e}^{-iE_n t}\, \langle 0|j^0(0)|n\rangle\langle n|\mathcal{O}|0\rangle - \mathrm{e}^{iE_n t}\, \langle 0|\mathcal{O}|n\rangle\langle n|j^0(0)|0\rangle\} \neq 0$$

$$\frac{d}{dt} v(t) = 0 = -i\,(2\pi)^3 \sum_n \delta^{(3)}(\vec{p}_n)\, E_n\, \{\, \mathrm{e}^{-iE_n t}\, \langle 0|j^0(0)|n\rangle\langle n|\mathcal{O}|0\rangle + \mathrm{e}^{iE_n t}\, \langle 0|\mathcal{O}|n\rangle\langle n|j^0(0)|0\rangle\}$$

Courtesy A.Pich
EFT, IFIC 2015

Pions: $\mathbf{SU(2) \times SU(2)}$ linear $\boldsymbol{\sigma}$ −model

$$[Q^A(t), Q^B(t)] = i\epsilon_{ABC} Q^c(t)$$

$$[Q^A(t), Q_5^B(t)] = i\epsilon_{ABC} Q_5^C(t)$$

$$[Q_5^A(t), Q_5^B(t)] = i\epsilon_{ABC} Q^c(t)$$

$$[Q^i, \pi^j] = i\epsilon_{ijk}\pi^k$$

$$\boldsymbol{[Q_5^j, \pi^k] \equiv -i\sigma\delta_{jk}}$$

$$[Q^j, \sigma] = 0$$

$$[Q_5^j, \sigma] = i\pi^j$$

$$[Q^j, \sigma^2 + \vec{\pi}^2] = 0$$

$$[Q_5^j, \sigma^2 + \vec{\pi}^2] = 0$$

$$[Q^j, \mathcal{L}] = [Q_5^j, \mathcal{L}] = 0$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{\mu^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 + \cdots$$

$$\vec{V}_\mu = \vec{\pi} \times \partial_\mu\vec{\pi}, \ \vec{A}_\mu = \sigma\partial_\mu\vec{\pi} - (\partial_\mu\sigma)\,\vec{\pi}, \ \partial_\mu\vec{V}^\mu = \partial_\mu\vec{A}^\mu = 0$$

- $Q^i|0> = Q_5^i|0> = 0$ (Wigner-Weyl realization)
- π and σ have the same mass
- $f_\pi = 0$ (pion and sigma fields annihilate the vacuum.

$$<0|A_j^\mu(0)|\pi^k(p)> = 0 = -if_\pi\delta_{jk}p_\mu$$

Pions: $SU(2) \times SU(2)$ linear σ –model

$$\Sigma(x) = \sigma(x) + i\vec{\tau}\,\vec{\pi}(x) = \begin{pmatrix} \sigma + i\pi^0 & i\sqrt{2}\pi^+ \\ i\sqrt{2}\pi^- & \sigma - i\pi^0 \end{pmatrix}$$

$$\mathcal{L} = \frac{1}{4}\,\mathrm{Tr}\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) - \frac{\mu^2}{4}\,\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right) - \frac{\lambda}{16}\,\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right)^2 + \cdots$$

- **vector** $\begin{cases} \pi'^j = e^{i\vec{Q}.\vec{\theta}}\pi^j\, e^{-i\vec{Q}.\vec{\theta}} = \pi^j - \theta_k \epsilon_{kjm}\pi^m + \mathcal{O}(\theta^2) \\ \sigma' = e^{i\vec{Q}.\vec{\theta}}\sigma\, e^{-i\vec{Q}.\vec{\theta}} = \sigma \end{cases}$ $\quad \boldsymbol{\Sigma' = g_V \Sigma g_V^\dagger,}$ $\quad \boldsymbol{g_V = e^{-i\vec{\tau}.\vec{\theta}/2} \in SU(2)_V}$

- **axial** $\begin{cases} \pi'^j = e^{i\vec{Q}_5.\vec{\theta}}\pi^j e^{-i\vec{Q}_5.\vec{\theta}} = \pi^j + \sigma\pi^j + \mathcal{O}(\theta^2) \\ \sigma' = e^{i\vec{Q}_5.\vec{\theta}}\sigma e^{-i\vec{Q}_5.\vec{\theta}} = \sigma - \vec{\theta}\vec{\pi} + \mathcal{O}(\theta^2) \end{cases}$ $\quad \boldsymbol{\Sigma' = g_A^\dagger \Sigma g_A^\dagger,}$ $\quad \boldsymbol{g_A = e^{-i\vec{\tau}.\vec{\theta}/2} \in SU(2)_A}$

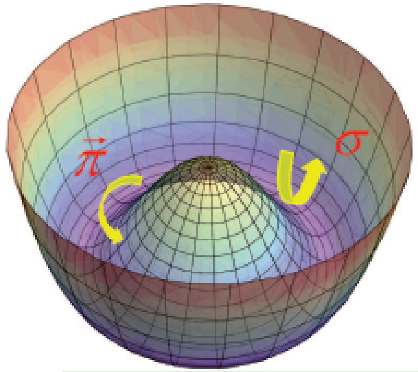
$O(4) \sim SU(2)_L \otimes SU(2)_R$ Symmetry: $\Sigma \to g_R \Sigma g_L^\dagger;\quad g_{L,R} \in SU(2)_{L,R}$

Sigma Model

$\Phi^T \equiv (\sigma, \vec{\pi})$

Courtesy A.Pich
EFT, IFIC 2015

$$\mathcal{L}_\sigma = \frac{1}{2}\,\partial_\mu \Phi^T \partial^\mu \Phi \;-\; \frac{\lambda}{4}\,\left(\Phi^T\Phi - v^2\right)^2$$



Spontaneous Symmetry Breaking

Global Symmetry: $O(4) \sim SU(2) \otimes SU(2)$

- $v^2 < 0$: $\quad m_\Phi^2 = -\lambda\, v^2$
- $v^2 > 0$: $\quad \langle 0|\sigma|0\rangle = v \quad , \quad \langle 0|\vec{\pi}|0\rangle = 0$

$$Q^i|0> = 0$$

$$\left\langle 0\middle|\left[Q_5^j, \pi^k\right]\middle|0\right\rangle = -i\delta_{jk}\langle 0|\sigma|0\rangle \neq 0$$

$$Q_5^i|0> \neq 0$$

SSB: $O(4) \to O(3)$ $\quad \left[\frac{4\times 3}{2} - \frac{3\times 2}{2} \;=\; 3 \text{ broken generators}\right]$

$$\mathcal{L}_\sigma = \frac{1}{2}\left\{\partial_\mu\hat{\sigma}\,\partial^\mu\hat{\sigma} + \partial_\mu\vec{\pi}\,\partial^\mu\vec{\pi} - M^2\hat{\sigma}^2\right\} - \frac{M^2}{2v}\,\hat{\sigma}\left(\hat{\sigma}^2+\vec{\pi}^2\right) - \frac{M^2}{8v^2}\left(\hat{\sigma}^2+\vec{\pi}^2\right)^2$$

$f_\pi = \text{v} \qquad \hat{\sigma} \equiv \sigma - v \quad ; \quad M^2 = 2\,\lambda\, v^2$

$$\vec{A}_\mu = (\hat{\sigma} + f_\pi)\partial_\mu\vec{\pi} - (\partial_\mu\hat{\sigma})\,\vec{\pi}$$

$$\partial_\mu \vec{V}^\mu = 0; \;\; \partial_\mu \vec{A}^\mu = 0$$

3 Massless Goldstone Bosons

Pions: $\mathrm{SU}(2) \times \mathrm{SU}(2)$ non-linear σ −model

$$[Q^A(t), Q^B(t)] = i\epsilon_{ABC} Q^c(t)$$

$$[Q^A(t), Q_5^B(t)] = i\epsilon_{ABC} Q_5^C(t)$$

$$[Q_5^A(t), Q_5^B(t)] = i\epsilon_{ABC} Q^c(t)$$

$$[Q^i, \pi^j] = i\epsilon_{ijk}\pi^k$$

$$\left[Q_5^j, \pi^k\right] \equiv -i f_{jk}(\vec{\pi}) = -i\{\delta_{jk} f(\vec{\pi}^2) + \pi_j \pi_k g(\vec{\pi}^2)\}$$

$$g(\vec{\pi}^2) = \frac{1 + 2f(\vec{\pi}^2) f'(\vec{\pi}^2)}{f(\vec{\pi}^2) - 2f'(\vec{\pi}^2)\vec{\pi}^2}$$

usual choice

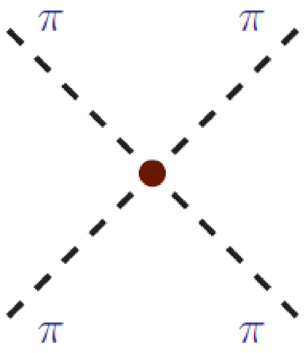
$$f(\vec{\pi}^2) = \sqrt{f_\pi^2 - \vec{\pi}^2} \Rightarrow g(\vec{\pi}^2) = 0$$

redefinition of the pion field: $\vec{\pi} \longrightarrow \vec{\phi}$ and introduce

$$U = \frac{\Sigma}{\sqrt{2}} = \frac{f_\pi}{\sqrt{2}} \xi^2 = \frac{f_\pi}{\sqrt{2}} e^{i\vec{\tau}\vec{\phi}/f_\pi}, \quad \mathrm{U} \rightarrow g_R U g_L^\dagger \Rightarrow U' = \begin{cases} g_V U g_V^\dagger & \text{vector} \\ g_A^\dagger U g_A^\dagger & \text{axial} \end{cases}$$

$$\mathcal{L} = \frac{1}{2}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) = \frac{1}{2}\partial_\mu \vec{\phi}\,\partial^\mu \vec{\phi} - \frac{1}{6f_\pi^2}\left(\vec{\phi}^2 \partial_\mu \vec{\phi}\,\partial^\mu \vec{\phi} - \left(\vec{\phi}\,\partial_\mu \vec{\phi}\right)\left(\vec{\phi}\,\partial^\mu \vec{\phi}\right)\right) + \mathcal{O}\left(\frac{\phi^6}{f_\pi^4}\right)$$

Chiral Symmetry Determines the Interaction:



$$T\left(\pi^+\pi^0 \to \pi^+\pi^0\right) = \frac{t}{f_\pi^2}, \qquad t = (p'_+ - p_+)^2$$

Weinberg

Non-Linear Lagrangian: $2\pi \to 2\pi, 4\pi, \cdots$ related

$$\mathcal{L} = \frac{1}{2}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{f_\pi}{2\sqrt{2}}\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right), \qquad \chi = 2B_0\begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$

Explicit Symmetry Breaking. Isospin limit $m_u = m_d \Rightarrow m_\pi^2 = 2B_0 m$

Pions and nucleons: $\mathbf{SU(2)\times SU(2)}$ non-linear $\boldsymbol{\sigma}$ −model

ψ: isospin doublet $\quad \psi \to \psi' = \begin{cases} e^{-i\vec{\tau}\vec{\theta}/2}\psi & \text{vector} \\ e^{-i\gamma_5\vec{\tau}\vec{\theta}/2}\psi & \text{axial} \end{cases}$

$$\tilde{\Sigma} = \frac{\Sigma+\Sigma^\dagger}{2} + \gamma_5\frac{\Sigma-\Sigma^\dagger}{2} = f_\pi\, e^{i\gamma_5\vec{\tau}\vec{\phi}/f_\pi} = f_\pi V^2,\; V = e^{i\gamma_5\vec{\tau}\vec{\phi}/2f_\pi} = \frac{\xi+\xi^\dagger}{2} + \gamma_5\frac{\xi-\xi^\dagger}{2}$$

$$\mathcal{L} = \frac{1}{2}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \bar{\psi} i\gamma_\mu\partial^\mu\psi - g\bar{\psi}\tilde{\Sigma}\psi$$ chiral invariant

Unitary transformation: $\psi_\omega = V\psi$

$$\mathcal{L} = \frac{1}{2}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \bar{\psi}_\omega V^\dagger\left(i\gamma_\mu\partial^\mu - \overbrace{g f_\pi}^{M_N} V^2\right)V^\dagger\psi_\omega$$

$$= \frac{1}{2}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \bar{\psi}_\omega i\gamma_\mu\left[\partial^\mu + \frac{1}{2}\left(\xi\partial^\mu\xi^\dagger + \xi^\dagger\partial^\mu\xi\right)\right]\psi_\omega - M_N\bar{\psi}_\omega\psi_\omega$$ → invariant

$$+\, \bar{\psi}_\omega\gamma_\mu\gamma_5\frac{i}{2}\left(\xi\partial^\mu\xi^\dagger - \xi^\dagger\partial^\mu\xi\right)\psi_\omega$$ → invariant

$$\mathcal{L} = \frac{1}{2}\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - \frac{1}{6f_\pi^2}\left(\vec{\phi}^2\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - \left(\vec{\phi}\partial_\mu\vec{\phi}\right)\left(\vec{\phi}\partial^\mu\vec{\phi}\right)\right) + \bar{\psi}_\omega[i\gamma_\mu\partial^\mu - M_N]\psi_\omega$$

$$- \frac{1}{4f_\pi^2}\bar{\psi}_\omega\gamma_\mu\vec{\tau}\left(\vec{\phi}\times\partial^\mu\vec{\phi}\right)\psi_\omega + \frac{\mathbf{g_A}}{2f_\pi}\bar{\psi}_\omega\gamma_\mu\gamma_5\vec{\tau}\partial^\mu\vec{\phi}\psi_\omega + \mathcal{O}\left(\frac{1}{f_\pi^4}\right)$$

Weinberg-Tomozawa Goldberger-Treiman $\mathbf{g_A} \sim \mathbf{1.26}$

$$T(N\to N\pi^i) = -ig_{\pi NN}\,\bar{u}(p')\gamma_5\tau^i u(p)\,, \qquad g_{\pi NN} = \frac{g_A M_N}{f} = 12.8$$

Generalization to $\mathrm{SU(3)_L} \otimes \mathrm{SU(3)_R}$

$$B(x) \equiv \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix},$$

$$\mathbf{\Phi} \equiv \frac{\vec{\lambda}}{\sqrt{2}}\,\vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

$$\mathcal{L}_1 = \mathrm{Tr}\{\bar{B}(i\not{\nabla} - M_B)B\} + \tfrac{1}{2}\mathcal{D}\,\mathrm{Tr}\{\bar{B}\gamma^\mu\gamma_5\{u_\mu, B\}\} + \tfrac{1}{2}\mathcal{F}\,\mathrm{Tr}\{\bar{B}\gamma^\mu\gamma_5[u_\mu, B]\}.$$

$$\nabla_\mu B = \partial_\mu B + \tfrac{1}{2}[u^\dagger\partial_\mu u + u\partial_\mu u^\dagger, B],$$

$$U = u^2 = e^{i\sqrt{2}\Phi/f}, \quad u_\mu = iu^\dagger\partial_\mu U u^\dagger$$

$$\mathcal{L}_{MB\to MB} = \frac{i}{4f^2}\mathrm{Tr}\{\bar{B}\gamma^\mu[[\Phi, \partial_\mu\Phi], B]\}.$$

Weinberg-Tomozawa

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