Frontiers in Nuclear and Hadronic Physics 2025

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Centro Nazionale di Studi Avanzati dell'Istituto Nazionale di Fisica Nucleare

Arcetri, Firenze

Hadron Spectroscopy

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J. Nieves

IFIC INSTITUT DE FÍSICA CORPUSCULAR

CSIC CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

VNIVERSITAT DE VALÈNCIA

EXCELENCIA SEVERO OCHOA

- Odd parity S-wave ϕB resonances
  - ✓ Flavor SU(3) symmetry and chiral expansion
  - ✓ Chiral Unitary Approach: unitarity in coupled channels. Riemann sheets, bound, virtual and resonant states.
  - ✓ Strangeness $\mathrm{S} = -1$ sector
    - ○ $\Lambda(1405)$: double pole structure and chiral symmetry
    - ○ $\Lambda(1670)$ and $\Sigma(1620)$ bumps
  - ✓ Strangeness $\mathrm{S} = -2$ sector: $\Xi(1620)$ and $\Xi(1690)$
  - ✓ Strangeness $\mathrm{S} = -0$ sector: $\mathrm{N}(1535)$ and $\mathrm{N}(1650)$
- Odd parity D-wave ϕB resonances: $\mathrm{N}(1520), \Lambda(1520)$ and $\Xi(1820)$

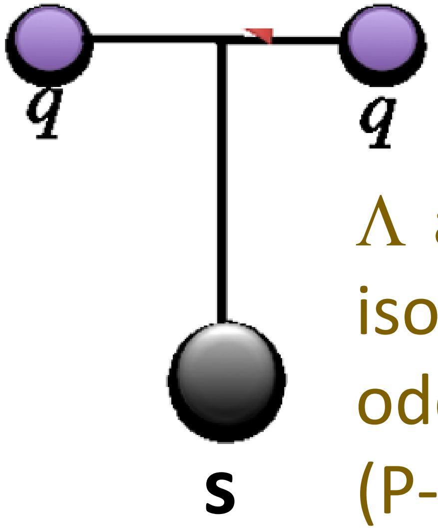
- Odd parity S-wave ϕB resonances $(J^P = 1/2^-)$

$\Lambda(1405)$ and its SU(3) symmetry partners
CUA: Chiral Unitary Approach

Λ(sqq)

			Status as seen in —		
	J^P	Overall status	$N\overline{K}$	$\Sigma\pi$	Other channels
$\Lambda(1116)$	$1/2^+$	****			$N\pi$ (weak decay)
$\Lambda(1380)$	$1/2^-$	**	**	**	
$\Lambda(1405)$	$1/2^-$	****	****	****	
$\Lambda(1520)$	$3/2^-$	****	****	****	$\Lambda\pi\pi, \Lambda\gamma, \Sigma\pi\pi$
$\Lambda(1600)$	$1/2^+$	****	***	****	$\Lambda\pi\pi, \Sigma(1385)\pi$
$\Lambda(1670)$	$1/2^-$	****	****	****	$\Lambda\eta$
$\Lambda(1690)$	$3/2^-$	****	****	***	$\Lambda\pi\pi, \Sigma(1385)\pi$

two-pole structure



Λ and Σ hyperons
isospin I=0 or I=1
odd parity ⇒ $L = 1$
(P-wave baryon states)

Σ(sqq)

			Status as seen in —			
	J^P	Overall status	$N\overline{K}$	$\Lambda\pi$	$\Sigma\pi$	Other channels
$\Sigma(1193)$	$1/2^+$	****				$N\pi$ (weak decay)
$\Sigma(1385)$	$3/2^+$	****		****	****	$\Lambda\gamma$
$\Sigma(1580)$	$3/2^-$	*	*	*	*	
$\Sigma(1620)$	$1/2^-$	*	*	*	*	
$\Sigma(1660)$	$1/2^+$	***	***	***	***	
$\Sigma(1670)$	$3/2^-$	****	****	****	****	
$\Sigma(1750)$	$1/2^-$	***	***	**	***	

scarce experimental information on the lowest-lying odd parity resonances

Ξ(ssq)

			Status as seen in —				
	J^P	Overall status	$\Xi\pi$	ΛK	ΣK	$\Xi(1530)\pi$	Other channels
$\Xi(1318)$	1/2+	****					Decays weakly
$\Xi(1530)$	3/2+	****	****				
$\Xi(1620)$		*	*				
$\Xi(1690)$		***		***	**		
$\Xi(1820)$	3/2−	***	**	***	**		
$\Xi(1950)$		***	**	**			
$\Xi(2030)$		***		**	***		

chiral symmetry gives support to their existence and fixes their spin-parity $J^P = \frac{1}{2}^-$

The Review of Particle Physics (2022)
R.L. Workman *et al.*
Prog. Theor. Exp. Phys. 2022, 083C01 (2022)

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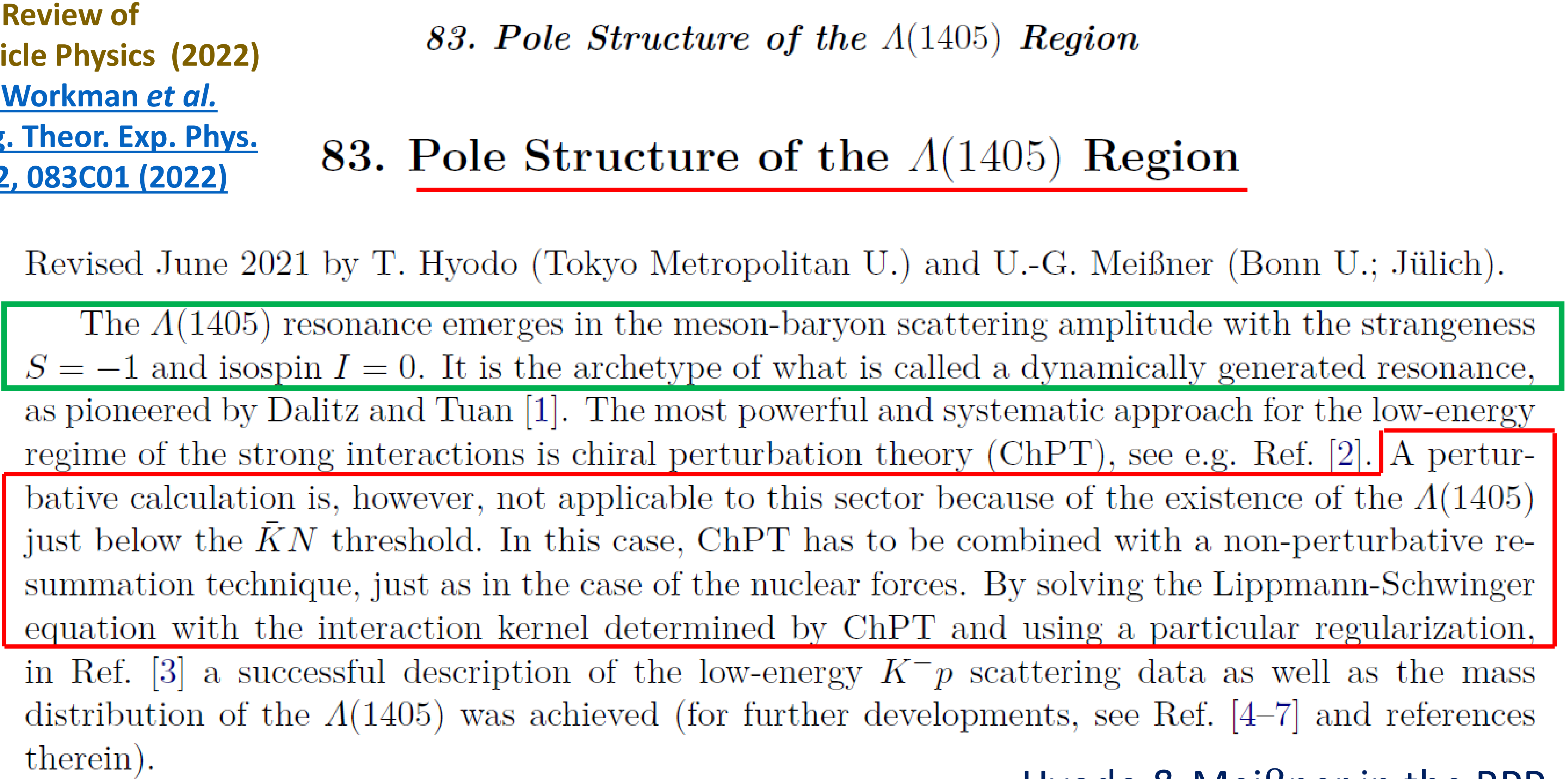


83. Pole Structure of the $\Lambda(1405)$ Region

83. Pole Structure of the $\Lambda(1405)$ Region

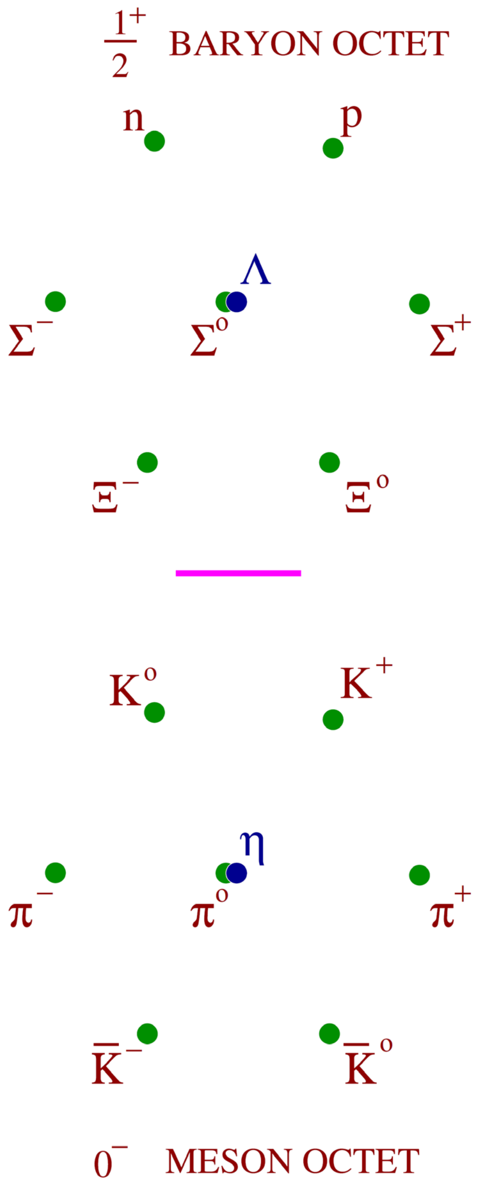
Revised June 2021 by T. Hyodo (Tokyo Metropolitan U.) and U.-G. Meißner (Bonn U.; Jülich).

The $\Lambda(1405)$ resonance emerges in the meson-baryon scattering amplitude with the strangeness $S = -1$ and isospin $I = 0$. It is the archetype of what is called a dynamically generated resonance, as pioneered by Dalitz and Tuan [1]. The most powerful and systematic approach for the low-energy regime of the strong interactions is chiral perturbation theory (ChPT), see e.g. Ref. [2]. A perturbative calculation is, however, not applicable to this sector because of the existence of the $\Lambda(1405)$ just below the $\bar{K}N$ threshold. In this case, ChPT has to be combined with a non-perturbative resummation technique, just as in the case of the nuclear forces. By solving the Lippmann-Schwinger equation with the interaction kernel determined by ChPT and using a particular regularization, in Ref. [3] a successful description of the low-energy K^-p scattering data as well as the mass distribution of the $\Lambda(1405)$ was achieved (for further developments, see Ref. [4–7] and references therein).

Hyodo & Meiβner in the RPP

SU(3) Weinberg–Tomozawa

Wigner-Eckart irreducible matrix elements



$$\underbrace{M}_{8}\;\underbrace{B}_{8}\to\underbrace{M'}_{8}\;\underbrace{B'}_{8}$$

$$\boldsymbol{\phi B \to \phi B}$$

SU(3): $8\otimes 8 = 1\oplus 8_s\oplus 8_a\oplus 10\oplus 10^*\oplus 27$, **7 independent WEIME's**, functions of s. **CS** order by order **fix them.**

At LO **SU(3) Weinberg–Tomozawa** meson-baryon lagrangian (involving the $K,\pi,\eta,\bar{K}$ meson– and the N,Σ,Λ,Ξ baryon–octets) ⇒ amplitudes depend on hadron masses and an unique parameter f

chiral symmetry

$$\mathcal{L}_1 = \mathrm{Tr}\left\{\bar{\Psi}_B\,(\mathrm{i}\not{\nabla} - M)\,\Psi_B\right\}$$

$$V^{IS}_{ab}(\sqrt{s}) = D^{IS}_{ab}\,\frac{2\sqrt{s}-M_a-M_b}{4\,f^2}$$

$$\begin{aligned}
\nabla^\mu\Psi_B &= \partial^\mu\Psi_B + [A_3^\mu,\Psi_B] \\
A_3^\mu &= \frac{1}{2}\left(u_3^\dagger\partial^\mu u_3 + u_3\partial^\mu u_3^\dagger\right) = \frac{1}{4f^2}[\Phi_3,\partial^\mu\Phi_3] \\
&+ \mathcal{O}((\Phi_3)^4) \\
U_3 = u_3^2 &= e^{\mathrm{i}\sqrt{2}\Phi_3/f}
\end{aligned}$$

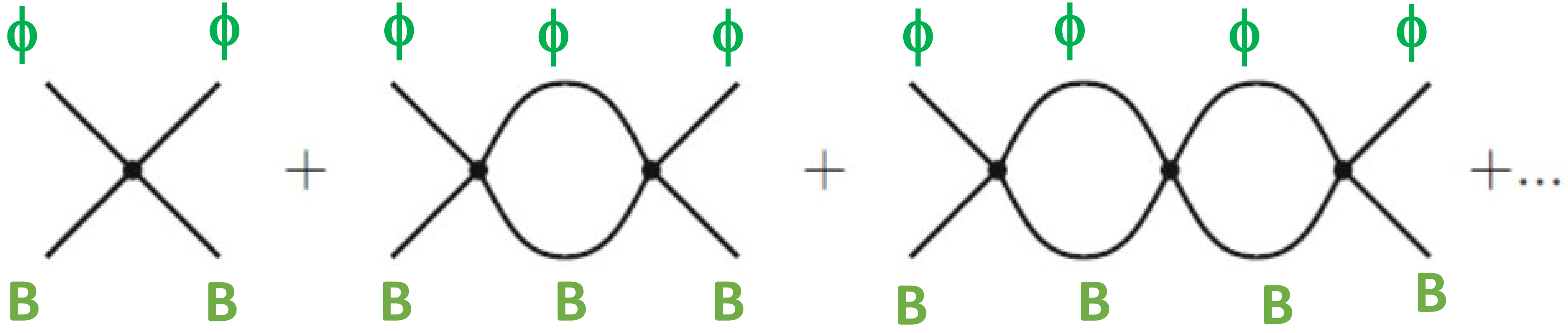
$$\Phi_3 = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$$\Psi_B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

<u>renormalization scheme</u>: consistent procedure to deal with the **ultra-Violet divergencies**: hard cutoff, subtractions, …

plus a unitarization procedure **CC BSE** + $\boxed{\text{RS}}$ ≡ **UnitarizedChPT**

<u>coupled-channels</u> Bethe Salpeter Equation + renormalization scheme



re-summation: restores elastic unitary in coupled-channels

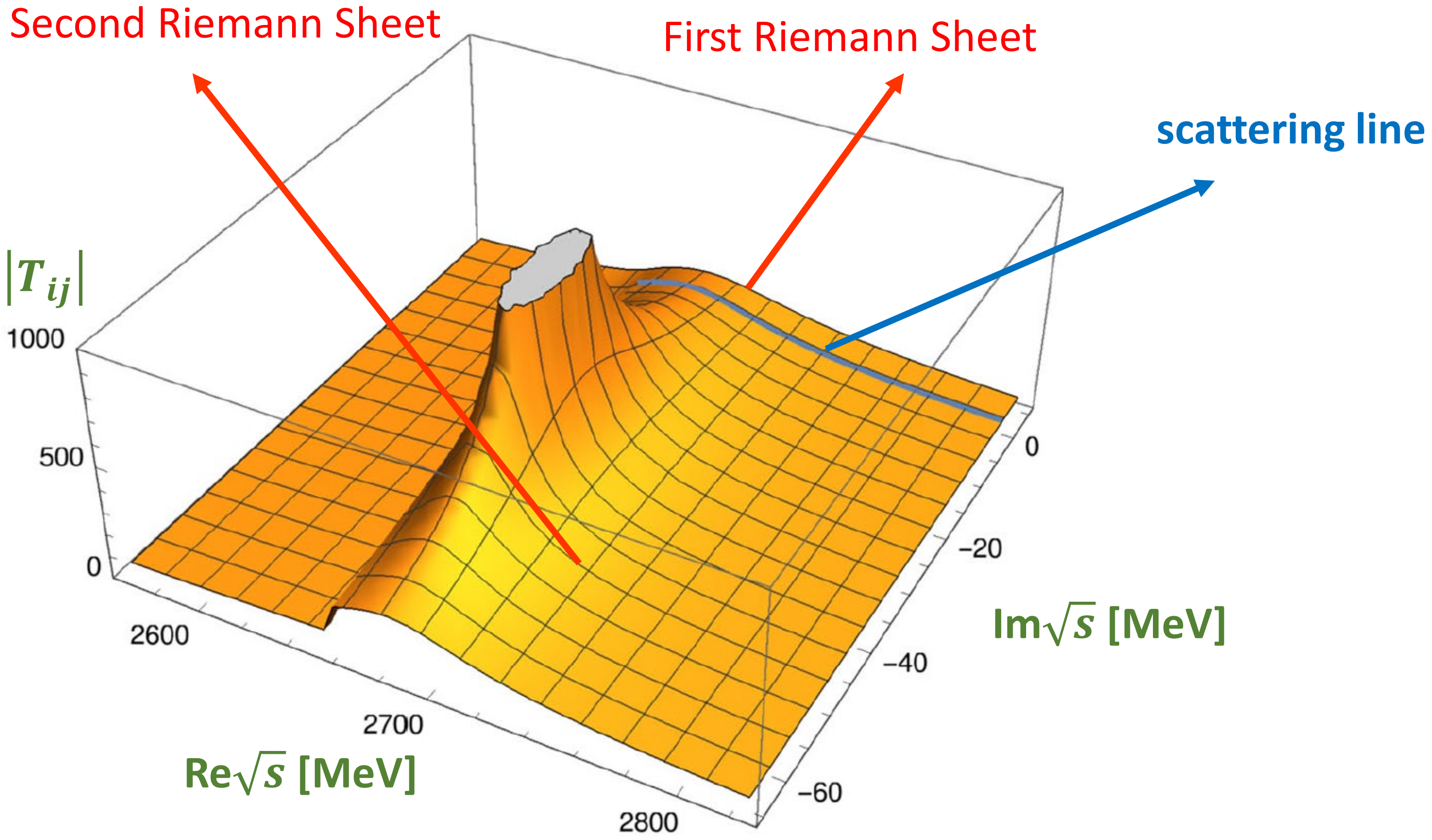
$$T^J_{IY}(\sqrt{s}) = \frac{1}{1-\underbrace{V^J_{IY}(\sqrt{s})}_{\text{WT+NLO...}}\,\underbrace{J^J_{IY}(\sqrt{s})}_{\text{loop}}}\,\overbrace{V^J_{IY}(\sqrt{s})}^{\text{WT+NLO+..}},$$

- $J=0$, S-wave $\Rightarrow$ **negative parity** $(P_\phi \times P_B = -1)$
- I (isospin), Y (hypercharge)

Looking for **poles** in the Second Riemann Sheet (SRS)

$$[T^J_{IY}]^{\text{SRS}}_{ij} = \frac{g_i g_j}{\sqrt{s}-M_R+i\Gamma_R/2} + \ldots$$

- g_i coupling of the resonance to the ϕB channel i
- pole position: $M_R - i\Gamma_R/2$ (fourth quadrant of the SRS)



[systematic approach: V^J_{IY}= <u>two particle irreducible amplitude</u> -potential-, which can be obtained in ChPT (LO+NLO+...) + perturbative corrections to the loop function]

E. Ruiz-Arriola and JN, NPA 679 (2000) 57

Renormalization

$$\boxed{T^J(s) = \frac{1}{1 - V^J(s)G^J(s)} V^J(s),}$$

$$G_i(s) = i2M_i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_i^2 + i\epsilon} \frac{1}{(P-q)^2 - M_i^2 + i\epsilon}$$

$$= \underbrace{\overline{G}_i(s)}_{\text{finite}} + \underbrace{G_i(s_{i+})}_{\text{UV divergent}} \qquad s_{i+} = (M_i + m_i)^2$$

different UV cutoffs for each meson-baryon channel

subtraction at a common scale $\mu \sim \sqrt{m^2 + M^2}$

J. Hofmann and M. Lutz, NPA763 (2005) 90

$$G_i^{\mu}(s_{i+}) = -\overline{G}_i(\mu^2)$$

common UV cutoff
$\Lambda \sim 0.5 - 1$ GeV

$$\boxed{G_i^{\Lambda}(s_{i+}) = \frac{1}{4\pi^2} \frac{M_i}{m_i + M_i} \left(m_i \ln \frac{m_i}{\boxed{\Lambda} + \sqrt{\Lambda^2 + m_i^2}} + M_i \ln \frac{M_i}{\boxed{\Lambda} + \sqrt{\Lambda^2 + M_i^2}} \right)}$$

Unitarity in coupled channels

Two particle irreducible amplitude (potential) V real (f.e. LO: WT)

$$\frac{\bar{G}(s)}{2M}=\frac{1}{(4\pi)^2}\left\{\left[\frac{M^2-m^2}{s}-\frac{M-m}{M+m}\right]\ln\frac{M}{m}+L(s)\right\}$$

PHYSICAL REVIEW D, VOLUME 64, 116008

$$L(s)\equiv L(s+\mathrm{i}\,\epsilon)=\frac{\lambda^{1/2}(s,m^2,M^2)}{s}$$

$$\times\left\{\log\left[\frac{1+\sqrt{\frac{s-s_+}{s-s_-}}}{1-\sqrt{\frac{s-s_+}{s-s_-}}}\right]-i\pi\right\},$$

$$s_-=(m-M)^2,\quad s_+=(m+M)^2,$$

$$V^{IS}_{ab}(\sqrt{s})=D^{IS}_{ab}\frac{2\sqrt{s}-M_a-M_b}{4f^2}$$

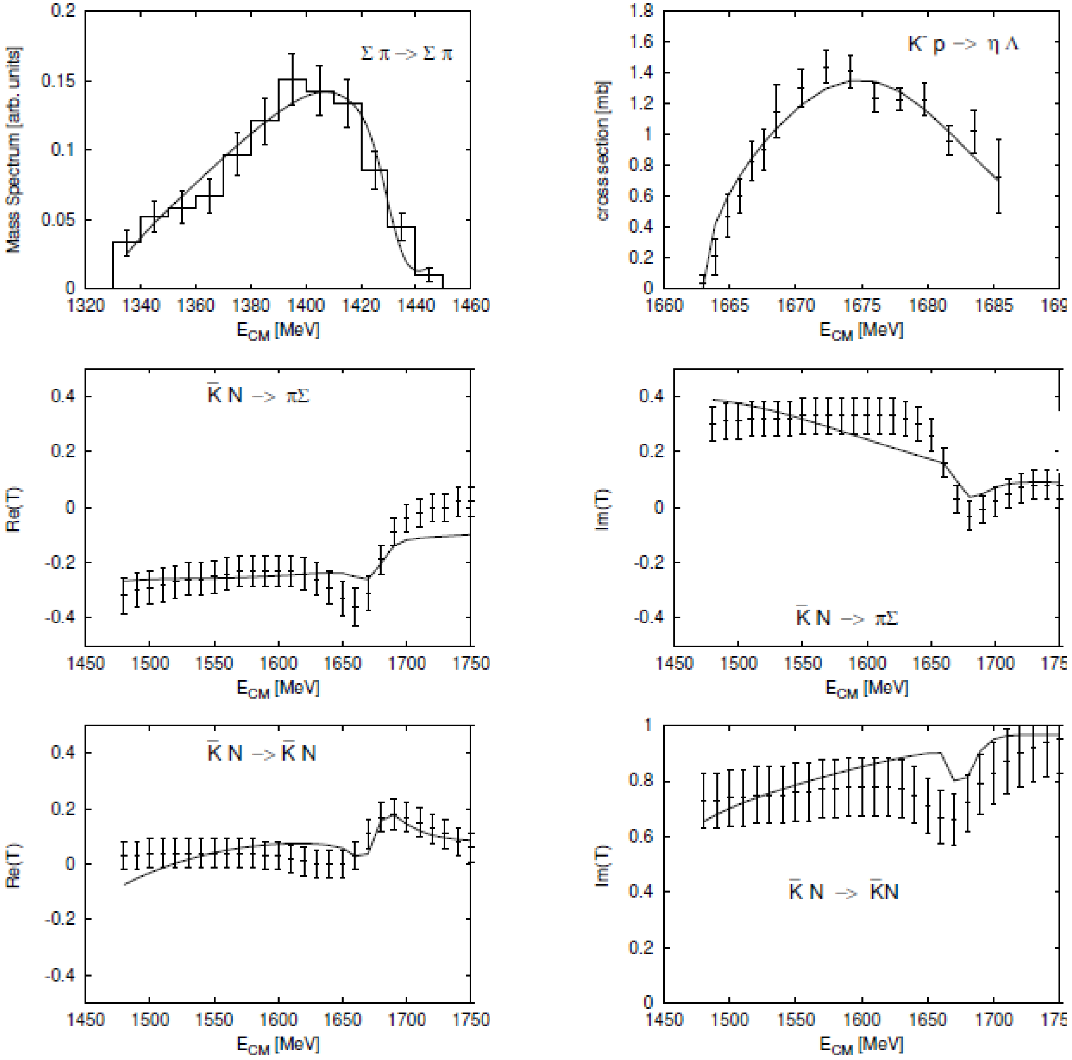
$$D^{I=0}_{S=-1}=\frac{1}{4}\begin{array}{cccccc} & \bar{K}N & \pi\Sigma & \eta\Lambda & K\Xi & \\ \left(\vphantom{\sqrt{3}}\right. & -3 & \sqrt{3/2} & -3/\sqrt{2} & 0 & \left.\vphantom{\sqrt{3}}\right)\ \bar{K}N \\ & \sqrt{3/2} & -4 & 0 & -\sqrt{3/2} & \pi\Sigma \\ & -3/\sqrt{2} & 0 & 0 & 3/\sqrt{2} & \eta\Lambda \\ & 0 & -\sqrt{3/2} & +3/\sqrt{2} & -3 & K\Xi \end{array}$$

$$2i\,\frac{\mathrm{Im}[\bar{G}(s+i\varepsilon)]}{2M}=\frac{\bar{G}(s+i\varepsilon)-\bar{G}(s-i\varepsilon)}{2M}$$

$$=-2i\,\frac{\lambda^{\frac{1}{2}}(s,M^2,m^2)}{32\pi sM}$$

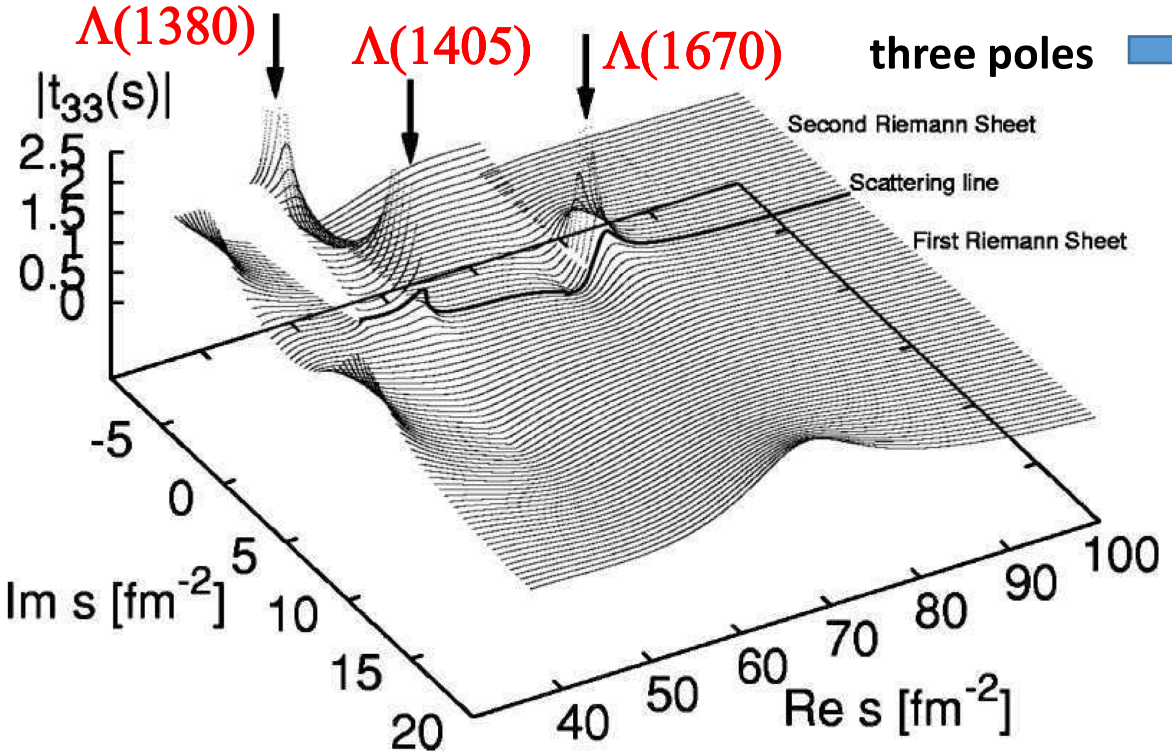
$$\mathrm{Im}[T^{-1}]=\begin{pmatrix}\mathrm{Im}[\bar{G}_{11}(s+i\varepsilon)] & 0 & 0\\ 0 & \mathrm{Im}[\bar{G}_{ii}(s+i\varepsilon)] & \vdots\\ 0 & 0 & \mathrm{Im}[\bar{G}_{nn}(s+i\varepsilon)]\end{pmatrix}$$

large number of works: Kaiser+Siegel+Weise, Oset+Ramos, Oller+Meiβner, Ikeda+Hyodo+Weise, Oller+Prades+Verbeni, Jido+Oller+Oset+Ramos+Meißner, García-Recio+Nieves+Ruiz-Arriola+Vicente-Vacas, García-Recio+Lutz+Nieves, Lutz+Kolomeitsev, Hyodo +Jido, Hyodo+Weise, Oset+Roca, Guo+Oller, Mai+Meiβner,… (**many others**)



example: García-Recio+Nieves+Ruiz-Arriola+Vicente-Vacas: PRD 67 (2003) 076009

(WT+off-shell effects: use a general RS to deal with the UV, with several counter-terms…)



three poles

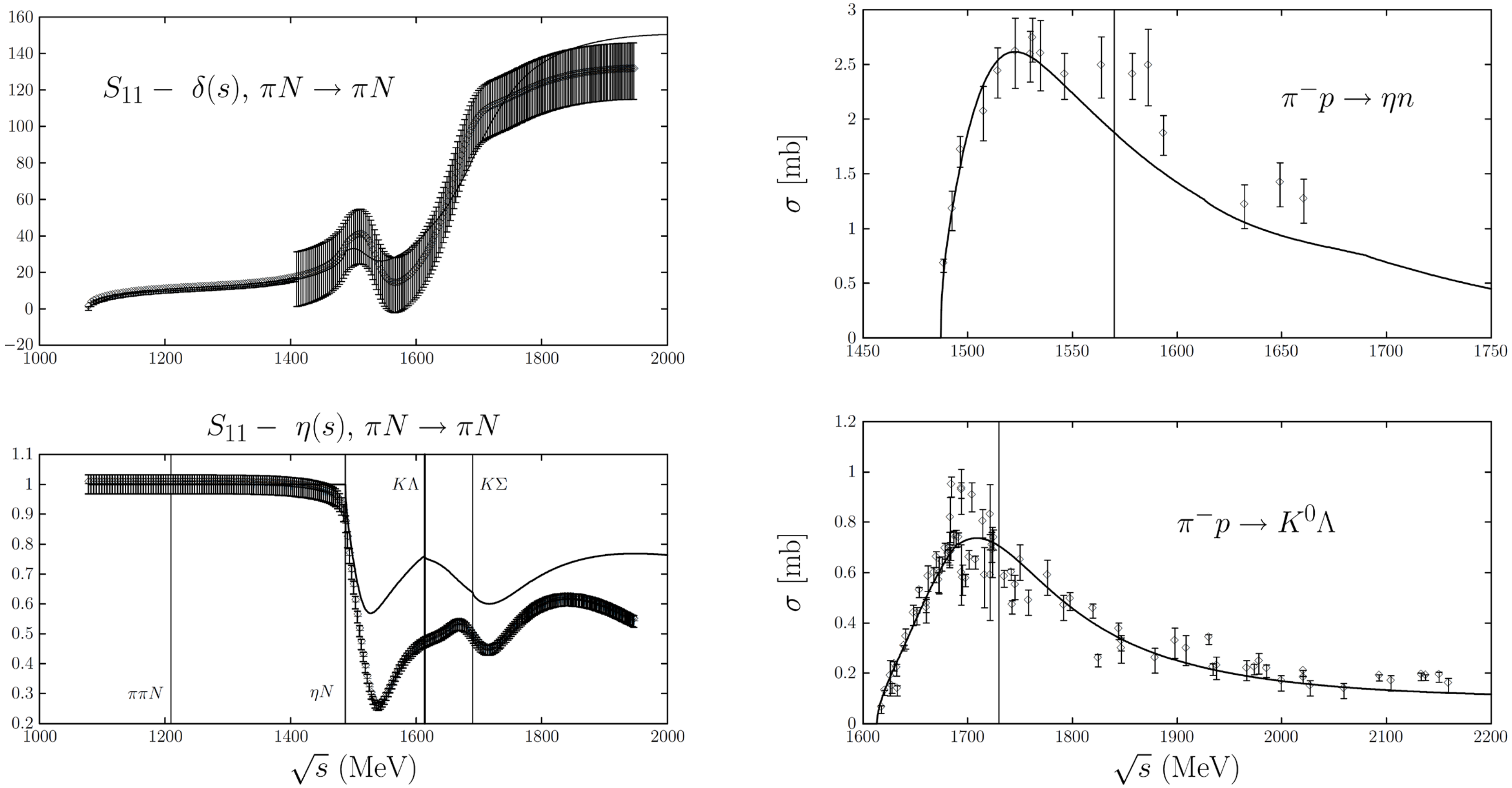
First pole: $M_R = 1368 \pm 12, \quad \Gamma_R = 250 \pm 23;$

Second pole: $M_R = 1443 \pm 3, \quad \Gamma_R = 50 \pm 7;$

double pole structure of the $\Lambda(1405)$

Third pole: $M_R = 1677.5 \pm 0.8, \quad \Gamma_R = 29.2 \pm 1.4,$ $\Lambda(1670)$

cited by the PDG in the RPP



same scheme in the strangenessless ($S=0$) sector performs also very well, and describes de S-wave resonances $N(1535)$ and $N(1650)$ $[J^P = \frac{1}{2}^-]$

Nieves+Ruiz-Arriola+Vicente-Vacas: PRD 64 (2001) 116008

Coming back to **the strange sector and the $\Lambda(1405)$:** Fitting to low-energy $K^- p$ scattering data, the $\pi\Sigma$ mass distribution of the $\Lambda(1405)$ and the precise determination of the energy shift and width of kaonic hydrogen by the SIDDHARTA collaboration

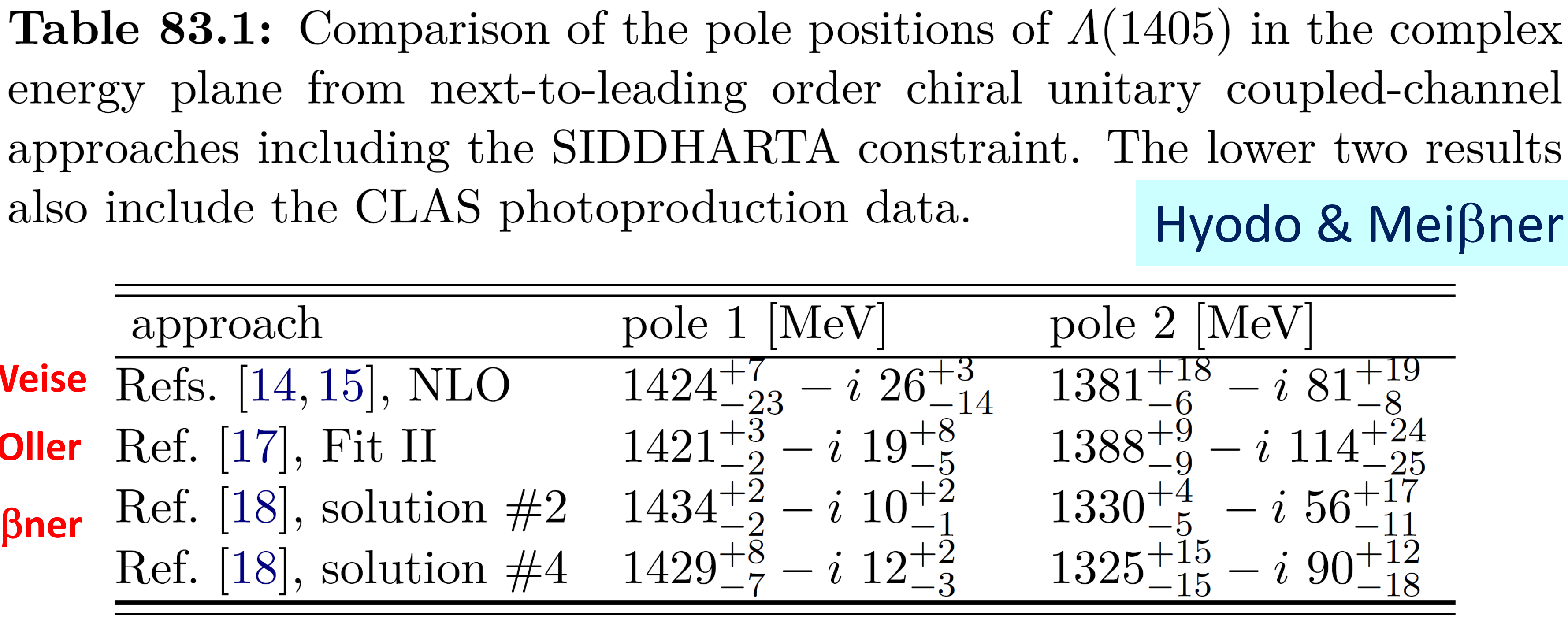


Table 83.1: Comparison of the pole positions of $\Lambda(1405)$ in the complex energy plane from next-to-leading order chiral unitary coupled-channel approaches including the SIDDHARTA constraint. The lower two results also include the CLAS photoproduction data.

Hyodo & Meiβner in the RPP

	approach	pole 1 [MeV]	pole 2 [MeV]
Ikeda+Hyodo+Weise	Refs. [14, 15], NLO	$1424^{+7}_{-23} - i\ 26^{+3}_{-14}$	$1381^{+18}_{-6} - i\ 81^{+19}_{-8}$
Guo+Oller	Ref. [17], Fit II	$1421^{+3}_{-2} - i\ 19^{+8}_{-5}$	$1388^{+9}_{-9} - i\ 114^{+24}_{-25}$
Mai+Meiβner	Ref. [18], solution #2	$1434^{+2}_{-2} - i\ 10^{+2}_{-1}$	$1330^{+4}_{-5} - i\ 56^{+17}_{-11}$
	Ref. [18], solution #4	$1429^{+8}_{-7} - i\ 12^{+2}_{-3}$	$1325^{+15}_{-15} - i\ 90^{+12}_{-18}$

- **Two poles** of the scattering amplitude in the complex energy plane **between the $\bar{K}N$ and $\pi\Sigma$ thresholds**
- The spectrum in experiments exhibits one effective resonance shape, while the existence of two poles results in the reaction-dependent line-shape
- The origin of this two-pole structure is attributed to the **two attractive channels** of the leading order interaction in the SU(3) basis (singlet and octet) and in the isospin basis (**$\bar{K}N$ and $\pi\Sigma$**)
- The leading order interaction, **Weinberg-Tomozawa term, determines de bulk of these double pole structure**

Quark mass dependence of S-wave baryon resonances

[C.García-Recio, M.F.M. Lutz and JN, PLB 582 (2004) 49]

$$D^{I=0}_{S=-1} = \frac{1}{4}\begin{array}{c} \begin{array}{cccc} \bar{K}N & \pi\Sigma & \eta\Lambda & K\Xi \end{array} \\ \left(\begin{array}{cccc} -3 & \sqrt{3/2} & -3/\sqrt{2} & 0 \\ \sqrt{3/2} & -4 & 0 & -\sqrt{3/2} \\ -3/\sqrt{2} & 0 & 0 & 3/\sqrt{2} \\ 0 & -\sqrt{3/2} & +3/\sqrt{2} & -3 \end{array}\right) \end{array} \begin{array}{c} \bar{K}N \\ \pi\Sigma \\ \eta\Lambda \\ K\Xi \end{array}$$

$$T^J_{IY}(\sqrt{s}) = \frac{1}{1 - \underbrace{V^J_{IY}(\sqrt{s})}_{\text{WT}}\,\underbrace{J^J_{IY}(\sqrt{s})}_{\text{loop}}}\,\overbrace{V^J_{IY}(\sqrt{s})}^{\boxed{\text{WT}}},$$

(only LO in ChPT)

baryon masses

$$V^{IS}_{ab}(\sqrt{s}) = D^{IS}_{ab}\,\frac{2\sqrt{s} - M_a - M_b}{4f^2}$$

pion decay constant
in the chiral limit

matrix in the coupled-channels space

$$\text{UV divergences} \to \mu^{IY}, \text{ such that } J^J_{IY}(\sqrt{s} = \mu^{IY}) = 0$$

$$\mu(I,+1) = \frac{1}{2}(m_\Lambda + m_\Sigma), \qquad \mu(I,0) = m_N$$

$$\mu(0,-1) = m_\Lambda, \qquad \mu(1,-1) = m_\Sigma,$$

$$\mu(I,-2) = m_\Xi$$

M.F.M. Lutz, E.E. Kolomeitsev, Nucl. Phys. A 700 (2002) 193

$$T(\sqrt{s} = \mu) = V(\mu), \qquad \mu = \mu(I,S)$$

coupled-channels amplitudes in the FRS and SRS depend only on the hadron masses and decay constants

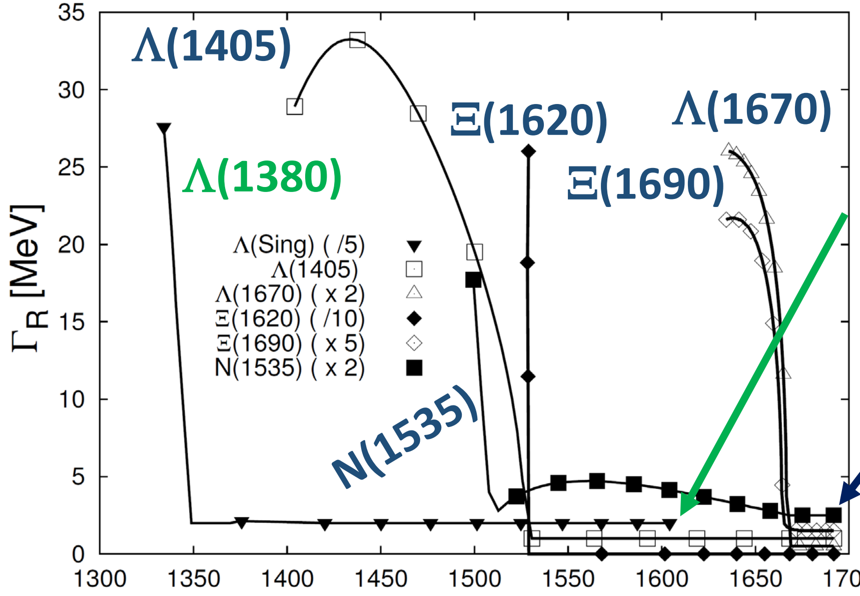
Within this scheme properties of the resonances depend only on the hadron masses

couplings!

(I,S) Resonance [MeV]	M_R [MeV] Γ_R [MeV]	$\|g_i\|^2$	ϕ_i [Rad]	BR$^{(\text{exp})}$ [%]	g_i^{b}
$(\frac{1}{2},0)$		$[\pi N]$ 0.1	1.1	45 ± 10	-0.2
N(1535) ****		$[\eta N]$ 4.7	2.7	42 ± 13	-1.6
$M = 1505 \pm 10$	1500	$[K\Lambda]$ 4.2	6.2	0	0.7
$\Gamma = 170 \pm 80$	64	$[K\Sigma]$ 11.4	6.0	0	2.5
$(0,-1)$		$[\pi\Sigma]$ 2.3	4.4	100	2.0
Λ(1405) ****		$[\bar{K}N]$ 9.3	0.3	0	2.0
$M = 1406 \pm 4$	1409	$[\eta\Lambda]$ 2.6	0.1	0	1.1
$\Gamma = 50 \pm 2$	34	$[K\Xi]$ 0.1	4.3	0	0.5
$(0,-1)$		$[\pi\Sigma]$ 0.04	1.9	40 ± 15	-1.3
Λ(1670) ****		$[\bar{K}N]$ 0.29	5.1	25 ± 5	1.2
$M = 1670 \pm 10$	1663	$[\eta\Lambda]$ 0.99	3.4	17 ± 7	-0.8
$\Gamma = 35 \pm 15$	12	$[K\Xi]$ 9.69	0.1	0	2.4
$(0,-1)$		$[\pi\Sigma]$ 8.2	5.7	100	2.4
Λ(?) ?		$[\bar{K}N]$ 5.0	2.2	0	-2.0
$M =?$	1363	$[\eta\Lambda]$ 0.5	1.6	0	-1.4
$\Gamma =?$	115	$[K\Xi]$ 0.3	5.5	0	2.0
$(1,-1)$		$[\pi\Lambda]$ 4.6	6.1	seen	0.8
Σ(1620) **		$[\pi\Sigma]$ 3.1	0.6	seen	2.1
$M \approx 1620$	1505	$[\bar{K}N]$ 12.3	3.7	22 ± 2	-2.0
$\Gamma = 87 \pm 19$	21	$[\eta\Sigma]$ 3.9	6.1	0	0.8
	see [22]	$[K\Xi]$ 0.5	3.5	0	0.1
$(\frac{1}{2},-2)$		$[\pi\Xi]$ 7.5	5.6	seen	2.6
Ξ(1620) *		$[\bar{K}\Lambda]$ 5.2	2.8	seen	-1.5
$M \approx 1620$	1565	$[\bar{K}\Sigma]$ 0.7	2.6	0	-0.8
$\Gamma = 23$	247	$[\eta\Xi]$ 0.3	4.9	0	0.3
$(\frac{1}{2},-2)$		$[\pi\Xi]$ 0.02	0.1	seen	-0.1
Ξ(1690) ***		$[\bar{K}\Lambda]$ 0.16	6.0	seen	0.9
$M = 1690 \pm 10$	1663	$[\bar{K}\Sigma]$ 5.15	3.1	seen	-2.5
$\Gamma = 10 \pm 6$	4	$[\eta\Xi]$ 2.28	3.2	0	-1.7

describes masses, widths and branching ratios of most of the components of two octets and a singlet of S-wave $J^P = \frac{1}{2}^-$ resonances:

N(1535), N(1650), Λ(1405), Λ(1670), Λ(1380), Σ(1620), Ξ(1620) and Ξ(1690)

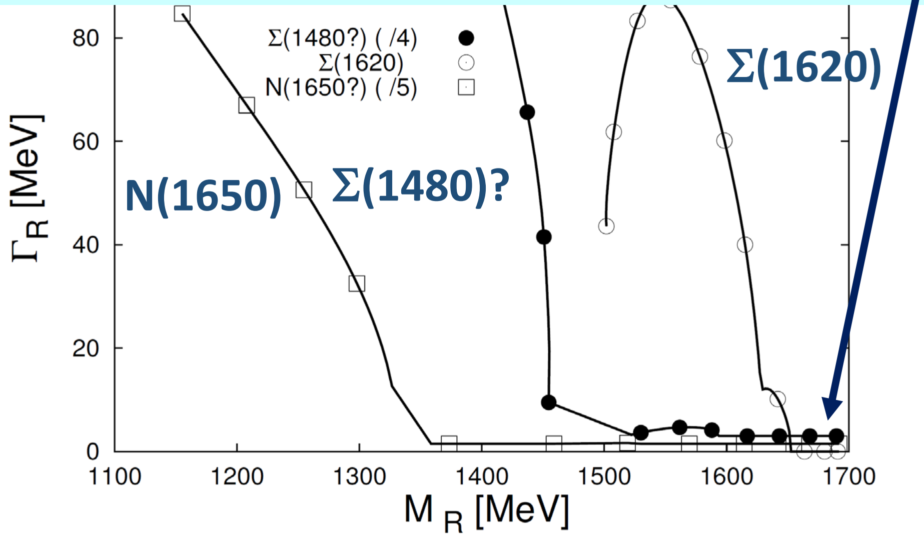


heavy SU(3) limit

i) one singlet

ii) two degenerate octets

physical world ⟸ heavy SU(3) symmetric limit

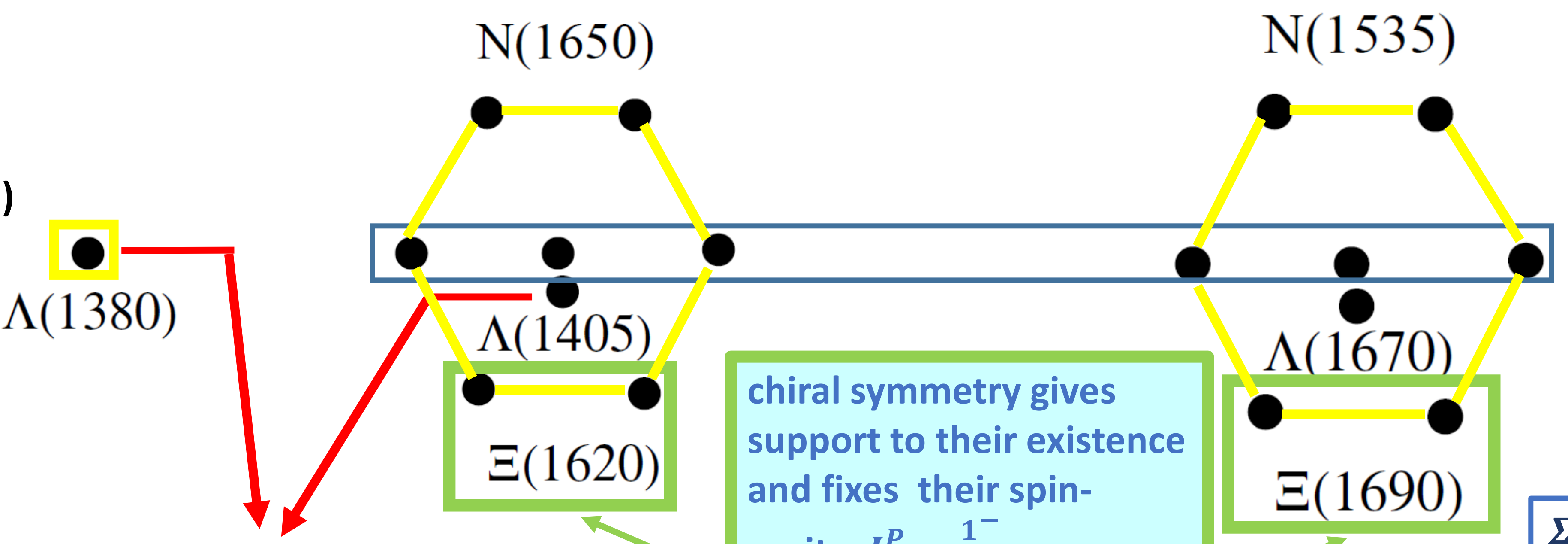


$$m_\pi^2\Big|_{\mathrm{SU(3)}} = m_\pi^2 + x(m_K^2 - m_\pi^2), \qquad x \in [0,1]$$

$J^P = \frac{1}{2}^-$

following the SU(3) trajectories: classification!

C. García-Recio, M.F.M. Lutz and JN Phys.Lett.B 582 (2004) 49.



two pole structure

chiral symmetry gives support to their existence and fixes their spin-parity $J^P = \frac{1}{2}^-$

Σ sector specially complex to analyze, and the situation is unclear

$$\underbrace{M}_{8}\ \underbrace{B}_{8} \to \underbrace{M'}_{8}\ \underbrace{B'}_{8}$$

$$\mathrm{SU}(3)\colon\ 8\otimes 8 = \boxed{1\oplus 8_s\oplus 8_a}\oplus 10\oplus 10^*\oplus 27,$$

For WT: $\lambda_{27} = 2,\ \boxed{\lambda_{8_s} = \lambda_{8_a} = -3,\ \lambda_1 = -6},\ \lambda_{10} = \lambda_{10^*} = \lambda_{8_s\to 8_a} = 0$

attractive!

$\pi\Sigma, \bar{K}N, \eta\Lambda, K\Xi$ energy levels in a **finite volume**

- ✓ Periodic boundary conditions imposes momentum quantization
- ✓ Lüscher formalism (C. Math. Phys. 105 (1986) 153 ; NPB 354 (1991) 531

infinite volume	finite volume
$\vec{q} \in \mathbb{R}^3$	$\vec{q} = \frac{2\pi}{L}\vec{n}, \quad \vec{n} \in \mathbb{Z}^3$
$\int_{\mathbb{R}^3} \frac{d^3q}{(2\pi)^3}$	$\frac{1}{L^3}\sum_{\vec{n}\in\mathbb{Z}^3}$

- ✓ In practice, changes in the T –matrix: $T(s) \longrightarrow \widetilde{T}(s,L)$ [Döring et al., EPJA47 (2011) 139]

$$\mathcal{G}_{ii}(s) \longrightarrow \tilde{\mathcal{G}}_{ii}(s,L) = \mathcal{G}_{ii}(s) + \lim_{\Lambda\to\infty}\left(\frac{1}{L^3}\sum_{\vec{n}}^{|\vec{q}|<\Lambda} I_i(\vec{q}) - \int_0^\Lambda \frac{q^2 d^3q}{(2\pi)^3} I_i(\vec{q})\right)$$

$$I_i(\vec{q}) = \frac{1}{2\omega_i(\vec{q})\omega_i'(\vec{q})}\frac{\omega_i(\vec{q})+\omega_i'(\vec{q})}{s-\left(\omega_i(\vec{q})+\omega_i'(\vec{q})\right)^2+i\epsilon}, \quad \omega_j(\vec{q}) = \sqrt{m_j^2+\vec{q}^2}, \quad \omega_j'(\vec{q}) = \sqrt{M_j^2+\vec{q}^2}$$

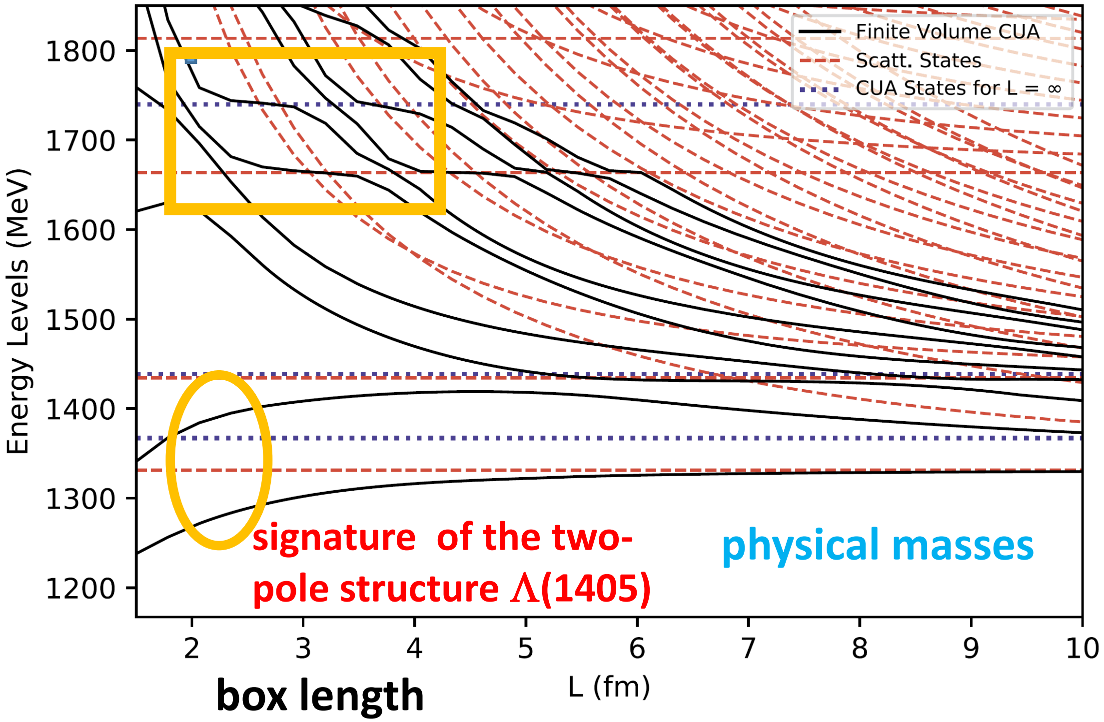
$$V(s) \longrightarrow \widetilde{V}(s,L) = V(s) \qquad T^{-1}(s) \longrightarrow \widetilde{T}^{-1}(s,L) = V^{-1}(s) - \widetilde{\mathcal{G}}(s,L)$$

Free energy levels:

$$E^i_{n,\text{free}}(L) = \omega_i\left(\frac{2\pi}{L}\vec{n}\right) + \omega_i'\left(\frac{2\pi}{L}\vec{n}\right)$$

Interacting energy levels $E_n(L)$ / $\boxed{\widetilde{T}^{-1}(E_n^2(L),L) = 0}$

[poles of the $\widetilde{T}$ matrix]



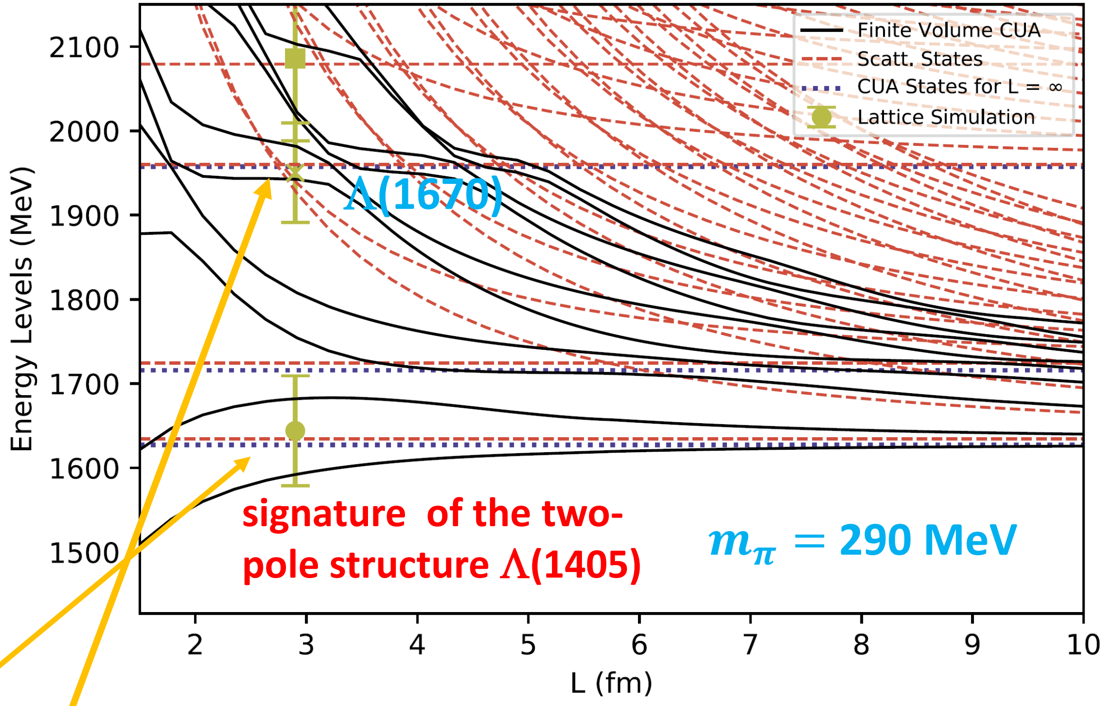
signature of the two-pole structure $\Lambda(1405)$

physical masses

box length

the lower level encodes information of the generated bound state corresponding to the lower $\Lambda(1405)$ pole at infinite box size, while the second level should be identified as a $\pi\Sigma$ scattering state.

The third and fourth levels are again scattering states that are strongly modified due to the $\bar{K}N$ interaction, while the fifth and sixth ones show two plateaus that are very close in energy. One of them is an $\eta\Lambda$ S-wave scattering state and the other one (fifth) at $L \sim 3$ fm is likely related to the **$\Lambda(1670)$** resonance, which has approached the $\eta\Lambda$ threshold from above.



$\Lambda(1670)$

signature of the two-pole structure $\Lambda(1405)$

$m_\pi = 290$ MeV

R. Pavao, P. Gubler, P. Fernandez-Soler, JN, M. Oka and T.T. Takahashi, PLB 820 (2021) 136473

The negative-parity spin-1/2 Λ baryon spectrum from lattice QCD and UChPT

- UChPT model implemented for a **finite box: energy levels**
- Comparison to LQCD data [P. Gubler, T.T. Takahashi and M. Oka, PRD94 (2016) 114518] ━🟩━ **($L\sim$ 3 fm , unquenched, only three-quark operators)**
- **LQCD is consistent with the $\Sigma\pi$ bound state influenced by the $\bar{K}N$-coupled-channels dynamics, and provides an energy level that reflects the infinite volume $\Lambda(1670)$**

Odd parity D-wave ϕB resonances: N(1520), Λ(1520) and Ξ(1820)

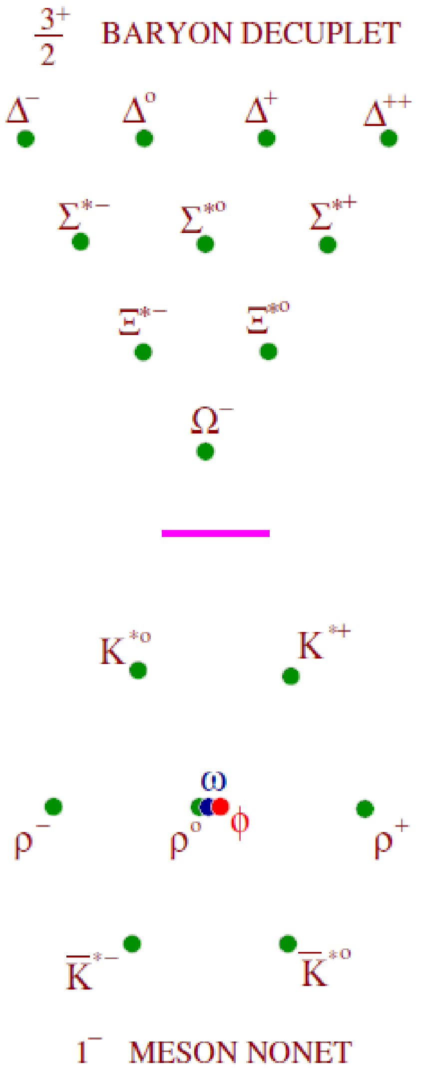
To study higher resonances, in particular those with $J^P = \frac{3}{2}^-$, or isospin-strangeness sectors where the SU(3) WT interaction is not dominant

- Consider $d-$wave or higher couplings
- Consider baryon decuplet $(\Delta, \Sigma^*, \Xi^*, \Omega)$ and vector meson nonet $(K^*, \rho, \omega, \phi, \bar{K}^*)$ effects
- Consider new meson-baryon interactions, including those involving more than two mesons (f.i. $\pi N \to \pi\pi N$)

We propose

SU(6) extension of the s-wave WT Meson-Baryon Lagrangian

WT SU(3): 8 x 10 = 8+10+27+35; λ_8= -6, λ_{10}= -3, λ_{27}= -1 , λ_{35}= 3



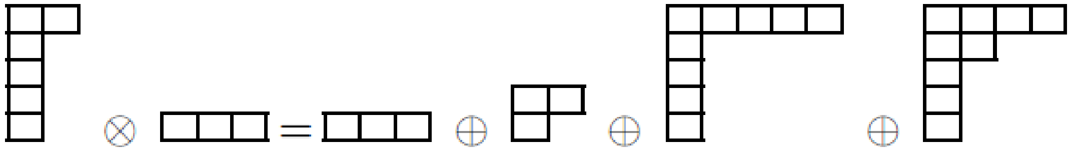
SU(6): treating the six states of a light quark (u, d or s with spin up, ↑, or down, ↓) as equivalent. For $L = 0$

$$\text{mesons}: 6 \otimes 6^* = \mathbf{35} \oplus 1 = \underbrace{8_1 \oplus 8_3 \oplus 1_3}_{\mathbf{35}} \oplus \underbrace{1_1}_{1}$$

$$\text{baryons}: 6 \otimes 6 \otimes 6 = \mathbf{56} \oplus 20 \oplus 70 \oplus 70 =$$

$$\underbrace{8_2 \oplus 10_4}_{\mathbf{56}} \oplus \underbrace{1_4 \oplus 8_2}_{20} \oplus 2 \times \left(\underbrace{10_2 \oplus 8_4 \oplus 8_2 \oplus 1_2}_{70} \right)$$

$35 \otimes 56 \equiv$



$= 56 \oplus 70 \oplus 700 \oplus 1134$, SU(6) symmetry ⇒ 4 WEIME′s

... but extending WT

$$\langle \mathcal{M}'\mathcal{B}'; JIY|V|\mathcal{M}\mathcal{B}; JIY\rangle = \sum_{\phi} V_\phi(s)\, \underbrace{\mathcal{P}^{\phi,JIY}_{\mathcal{MB},\mathcal{M}'\mathcal{B}'}}_{\text{SU(3)+SU(6)CG coeffs}}, \quad V_\phi(s) = \bar{\lambda}_\phi \frac{\sqrt{s} - M}{2\, f^2}$$

$\bar{\lambda}_{56} = -12,\ \bar{\lambda}_{70} = -18,\ \bar{\lambda}_{700} = 6,\ \bar{\lambda}_{1134} = -2,\ M = M_8 = M_{10}.$

This extension is possible because **WT Lagrangian is not just SU(3) symmetric but also chiral** $(\mathrm{SU}_L(3) \otimes \mathrm{SU}_R(3))$ **invariant,** (8 is the adjoint representation of SU(3))

$$\mathcal{L}_{\mathrm{WT}} = \mathrm{Tr}([M^\dagger, M][B^\dagger B]) = \left((M^\dagger \otimes M)_{8_a} \otimes (B^\dagger \otimes B)_8\right)_1$$

The unique SU(6) extension is then

$$\mathcal{L}_{\mathrm{WT}}^{\mathrm{SU(6)}} = \left((M^\dagger \otimes M)_{35_a} \otimes (B^\dagger \otimes B)_{35}\right)_1$$

since the 35 is the adjoint representation of SU(6).

Break SU(6) symmetry: use physical hadron masses and meson decay constants $(f_\pi, f_K, f_\eta, f_\rho, f_{K^*}, f_\omega, f_\phi)$

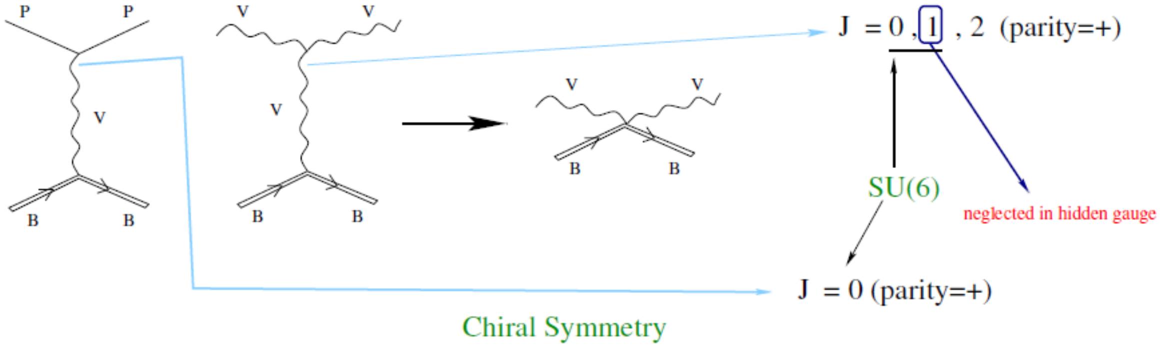
UV divergences $\rightarrow \mu^{IY}$, such that $J^J_{IY}(\sqrt{s} = \mu^{IY}) = 0$.

No free parameters !

SU(6) spin–flavor vs hidden gauge formalism for vector interactions

$$\mathcal{L}_{\mathrm{WT}}^{\mathrm{SU(6)}} = \left((M^\dagger \otimes M)_{35_a} \otimes (B^\dagger \otimes B)_{35}\right)_1, \quad \underbrace{8_1 \oplus 8_3 \oplus 1_3}_{\mathbf{35}}$$

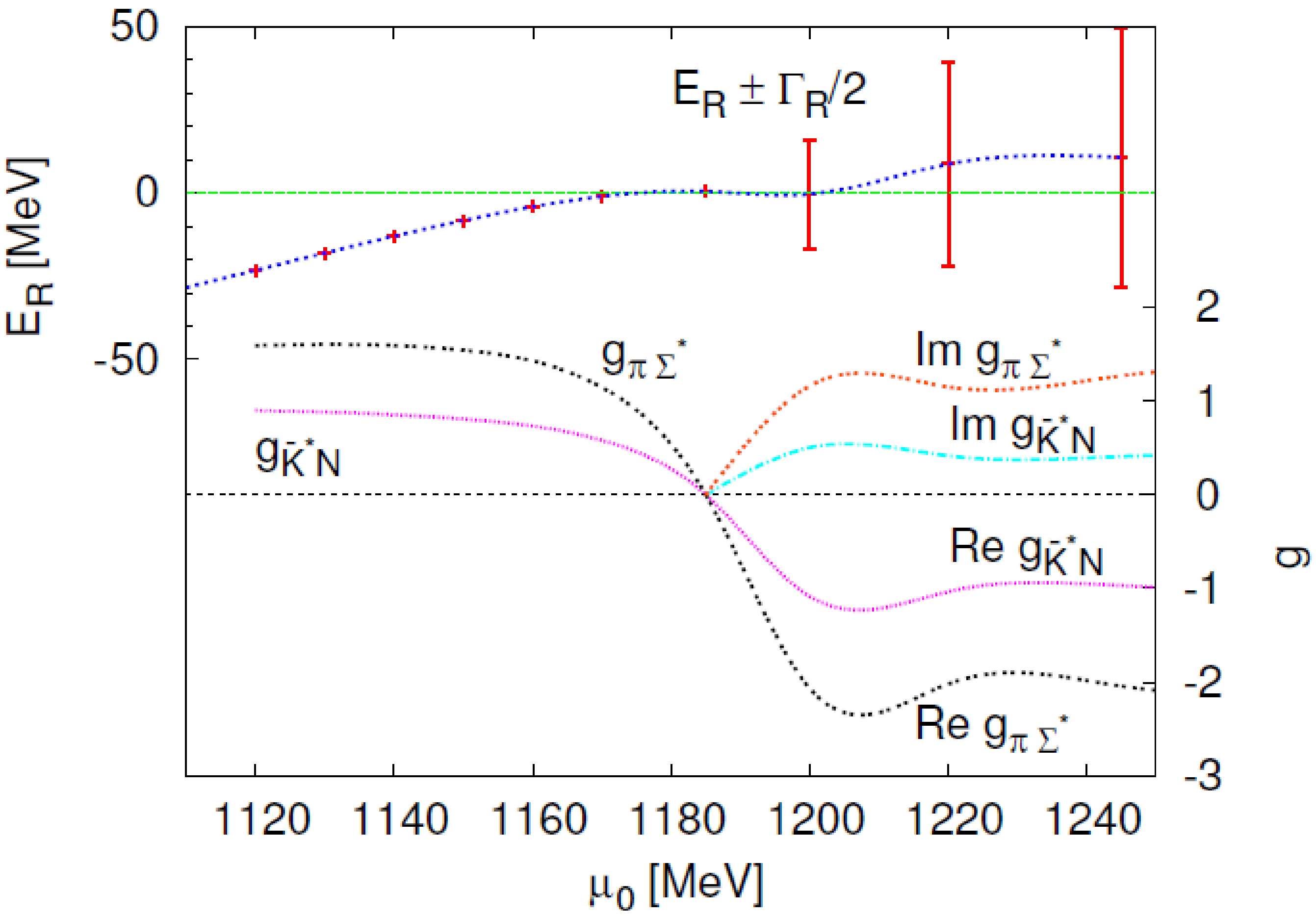
- Dynamics



- Coupled channel space, Renormalization, Symmetry breaking

We recover previous results in the $\frac{1}{2}^{-}$ sector and make new predictions

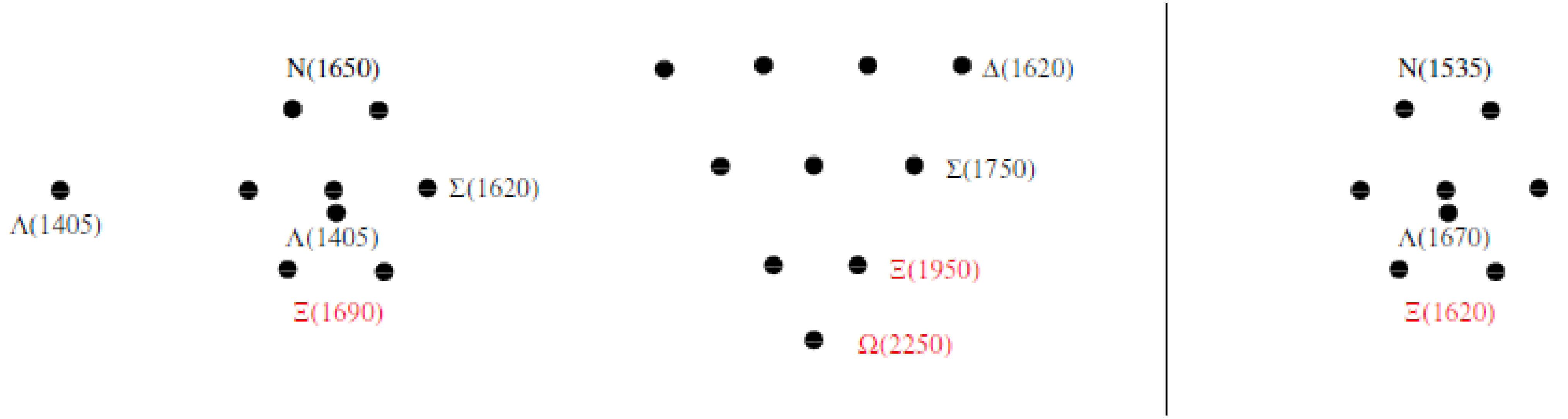
$$\pi\Sigma^*,\ \bar{K}^*N,\ \omega\Lambda,\ \rho\Sigma,\ \overbrace{K\Xi^*}^{\Lambda(1520)},\ \phi\Lambda,\ \rho\Sigma^*,\ K^*\Xi,\ K^*\Xi^*$$



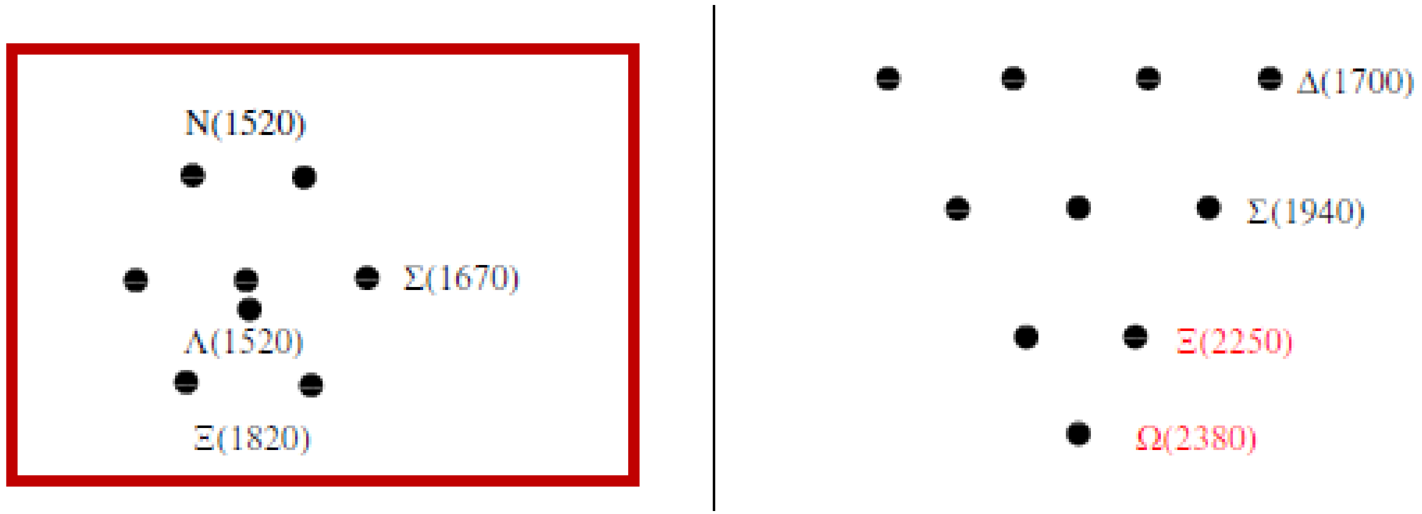
- for $J^P = \frac{3}{2}^-$ resonances (d-wave resonances), not accessible with the SU(3) WT chiral lagrangian. For instance, in the $I = 0, S = -1$ we get signals for the four star resonances $\Lambda(1520)$ and $\Lambda(1690)$, which couple to the $\Sigma^*\pi$ and $N\bar{K}^*$ channels,....
- for exotic states. Many of them fall into the **1134** representation, where the interaction is attractive $(\bar{\lambda}_{1134} = -2)$. For instance, in the $S = -4, I = J = 1/2$ sector $|\bar{K}^*\Omega\rangle = |1134; 35_2\rangle$ or for $S = +1, I = 0, J = 3/2$ $|K^*N\rangle = -|1134; 10^*_4\rangle \rightarrow$ **exotic bound states ?**

SU(6): 70 SU(6): 56

$J^P = \frac{1}{2}^-$



$J^P = \frac{3}{2}^-$



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