Exercises for Lectures on Exotic Hadrons

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1. Try to understand this statement as a consequence of heavy quark spin symmetry. Hint: in e^+e^- collisions, the leading production mechanism of heavy meson pairs is from the vector current $\bar{Q}\gamma^\mu Q$ which couples to the virtual photon, *i.e.*, $e^+e^- \to \gamma^* \to \bar{Q}Q$ with the $Q\bar{Q}$ pair in an S-wave.

2. Why is the isospin of the negative C-parity $\pi^+\pi^-$ system equal to 1?

3. The $X(3872)$ was found in the $J/\psi\pi^+\pi^-$ final state. It is expected to have a bottom partner X_b in various models. Suppose the X_b has a mass tens of MeV below the $B\bar{B}^*$ threshold. Are the $\Upsilon(1S,2S,3S)\pi^+\pi^-$ final states suitable for searching for the X_b? Why?

4. Show that the combinations of the LO contact terms in the S-wave interaction matrix elements for $D^{(*)}\bar{D}^{(*)}$ scattering are as follows:

$$\begin{aligned}
\begin{pmatrix} D\bar{D} \\ D^*\bar{D}^* \end{pmatrix}: &\quad V^{(0^{++})} = \begin{pmatrix} C_{IA} & \sqrt{3}C_{IB} \\ \sqrt{3}C_{IB} & C_{IA}-2C_{IB} \end{pmatrix}, \\
\begin{pmatrix} D\bar{D}^* \\ D^*\bar{D}^* \end{pmatrix}: &\quad V^{(1^{+-})} = \begin{pmatrix} C_{IA}-C_{IB} & 2C_{IB} \\ 2C_{IB} & C_{IA}-C_{IB} \end{pmatrix}, \\
D\bar{D}^*: &\quad V^{(1^{++})} = C_{IA}+C_{IB}, \\
D^*\bar{D}^*: &\quad V^{(2^{++})} = C_{IA}+C_{IB},
\end{aligned}$$

where $C_{IA} = \frac{1}{4}(3F_{I1}+F_{I0})$, $C_{IB} = \frac{1}{4}(F_{I1}-F_{I0})$. Here, F_{Is_L} are defined as $F_{I0} = \left\langle \frac{1}{2},\frac{1}{2},0 \middle| \hat{\mathcal{H}} \middle| \frac{1}{2},\frac{1}{2},0 \right\rangle_I$, $F_{I1} = \left\langle \frac{1}{2},\frac{1}{2},1 \middle| \hat{\mathcal{H}} \middle| \frac{1}{2},\frac{1}{2},1 \right\rangle_I$ in terms of $\left\langle s_{\ell 1}, s_{\ell 2}, s_L \middle| \hat{\mathcal{H}} \middle| s'_{\ell 1}, s'_{\ell 2}, s_L \right\rangle_I$ with $s_{\ell i}$ the angular momentum of the light (anti)quark in meson-i and s_L the total spin of the light quark-antiquark pair.

5. Show that the Lagrangian for the $D^{(*)}\bar{D}^{(*)}$ interaction in S-wave

$$\begin{aligned}\mathcal{L}_{4H} =& -\frac{1}{4}\,\mathrm{Tr}\left[H^{a\dagger}H_b\right]\mathrm{Tr}\left[\bar{H}^c\bar{H}_d^\dagger\right]\left(F_A\delta_a^b\delta_c^d + F_A^\lambda\vec{\lambda}_a^b\cdot\vec{\lambda}_c^d\right)\\ &+\frac{1}{4}\,\mathrm{Tr}\left[H^{a\dagger}H_b\sigma^m\right]\mathrm{Tr}\left[\bar{H}^c\bar{H}_d^\dagger\sigma^m\right]\left(F_B\delta_a^b\delta_c^d + F_B^\lambda\vec{\lambda}_a^b\cdot\vec{\lambda}_c^d\right)\end{aligned}$$

can also be written as single trace terms.