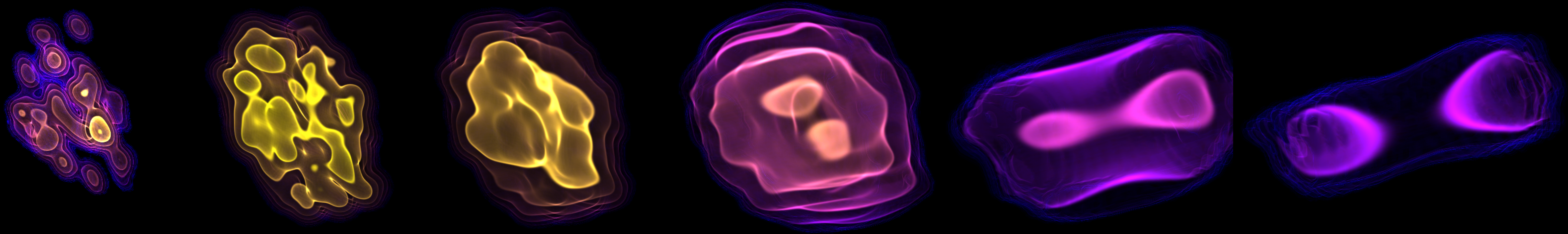
APPLICATIONS OF CAUSALITY CONDITIONS IN HEAVY-ION PHENOMENOLOGY

CHUN SHEN

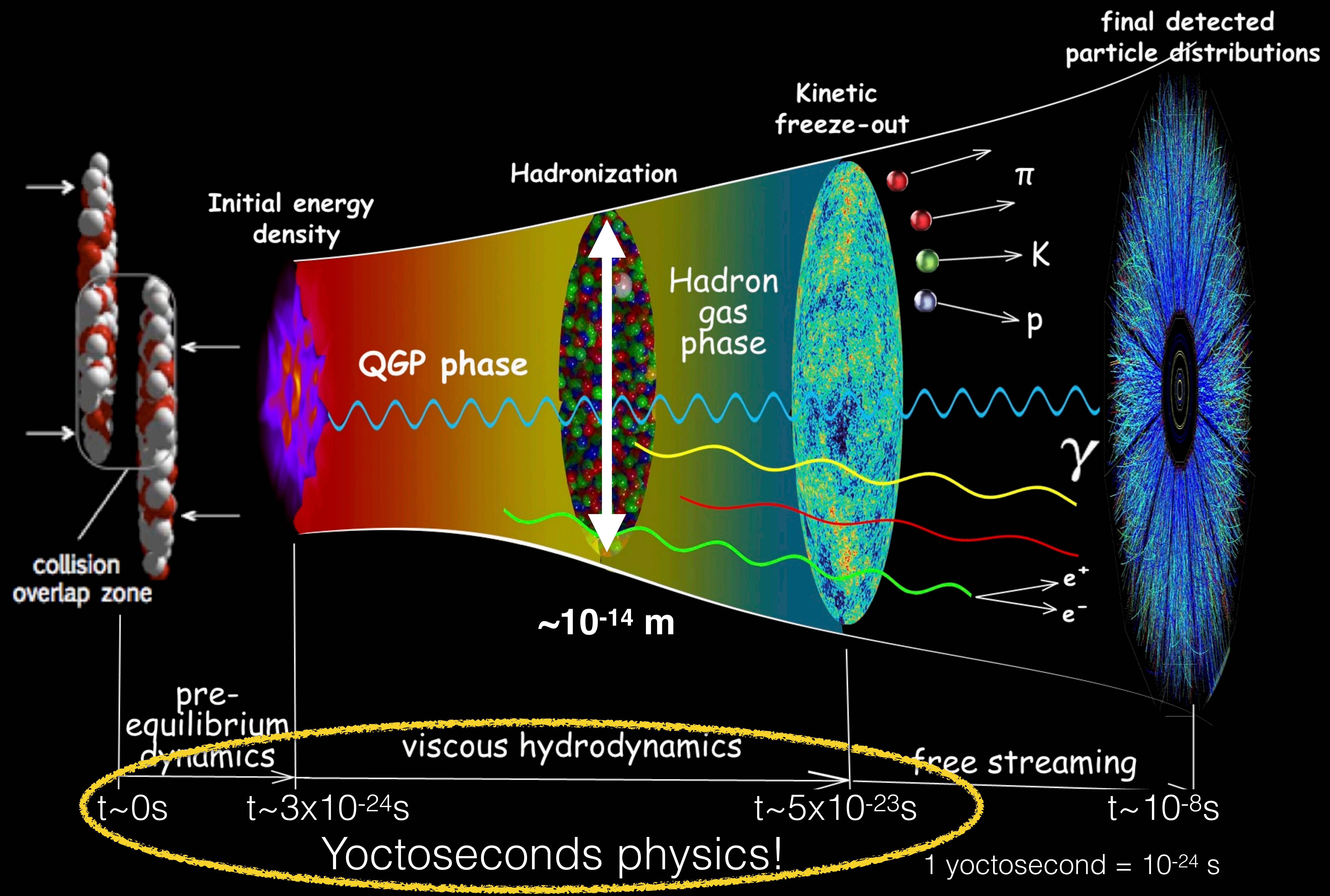


In collaboration with Cheng Chiu, Gabriel Denicol, and Matt Luzum

C. Chiu and C. Shen, Phys. Rev. C 103, 064901 (2021)
C. Chiu, G. Denicol, M. Luzum, and C. Shen, arXiv:2504.xxxxx

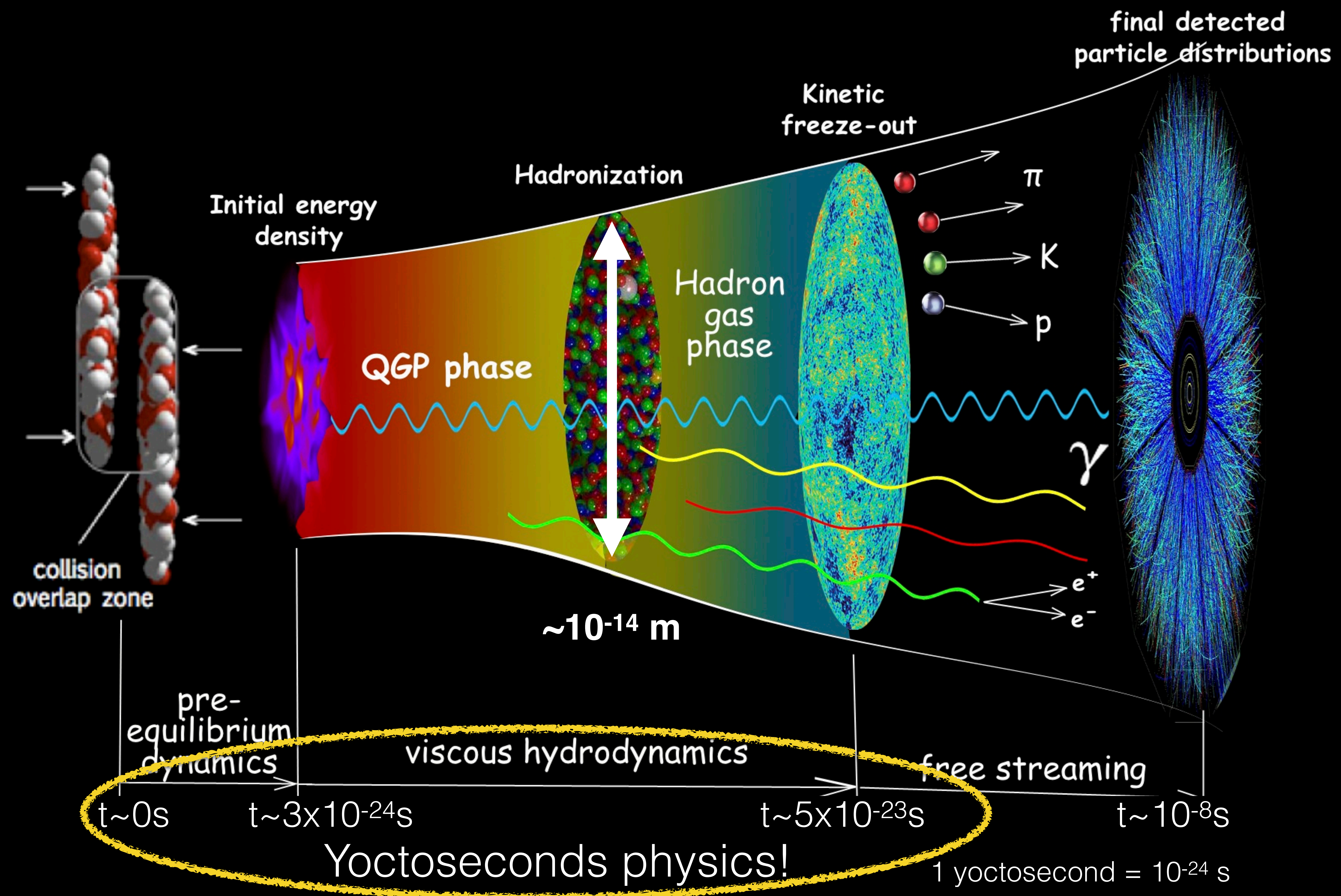
April 14, 2025
Foundations and Applications of Relativistic Hydrodynamics

NUCLEAR MATTER UNDER EXTREME CONDITIONS



Heavy-ion collisions are tiny and have ultra-fast dynamics

NUCLEAR MATTER UNDER EXTREME CONDITIONS

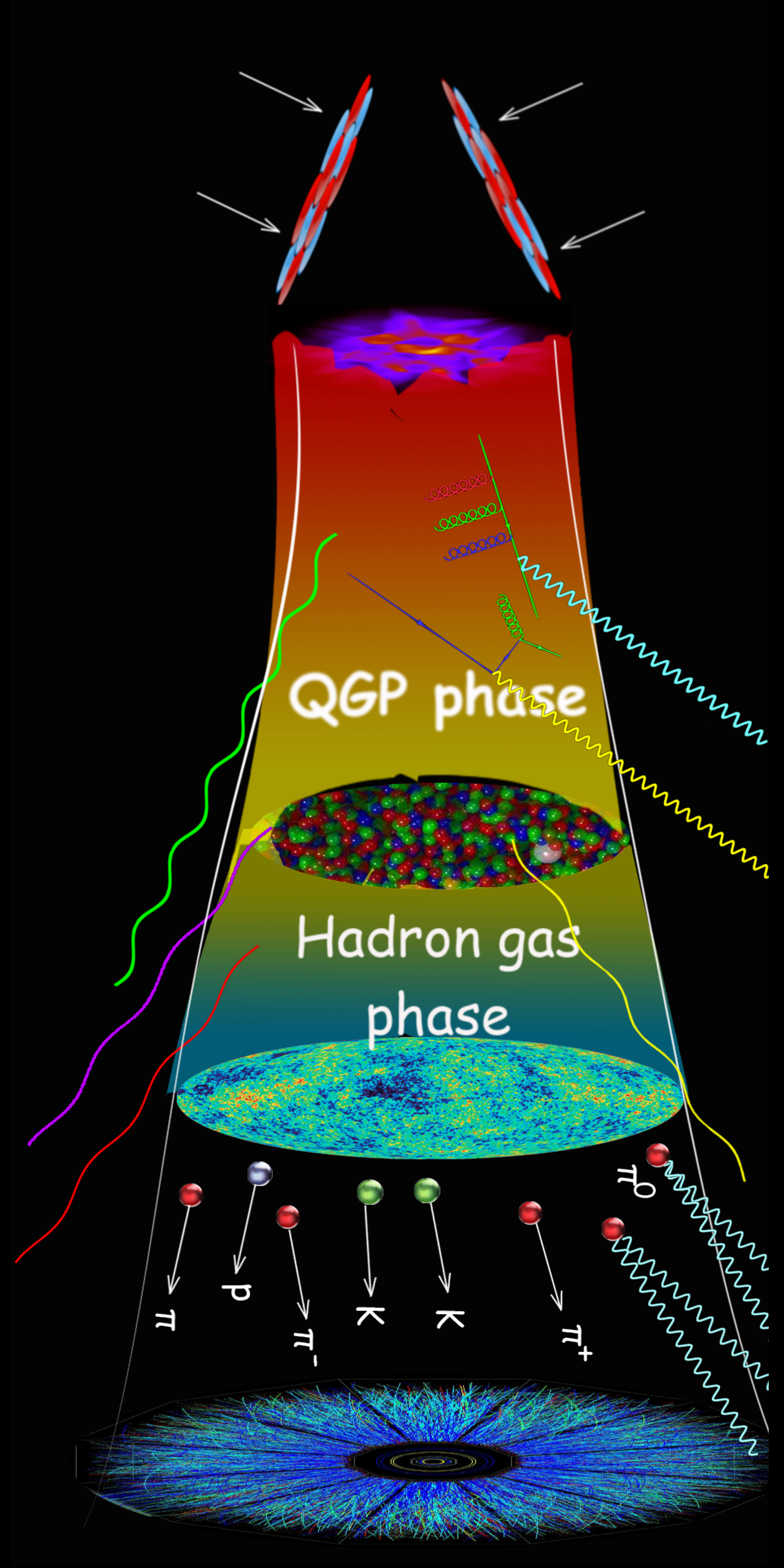


Heavy-ion collisions are tiny and have ultra-fast dynamics

A variety of particles are emitted from the collisions

Multi-messenger nature of heavy-ion physics

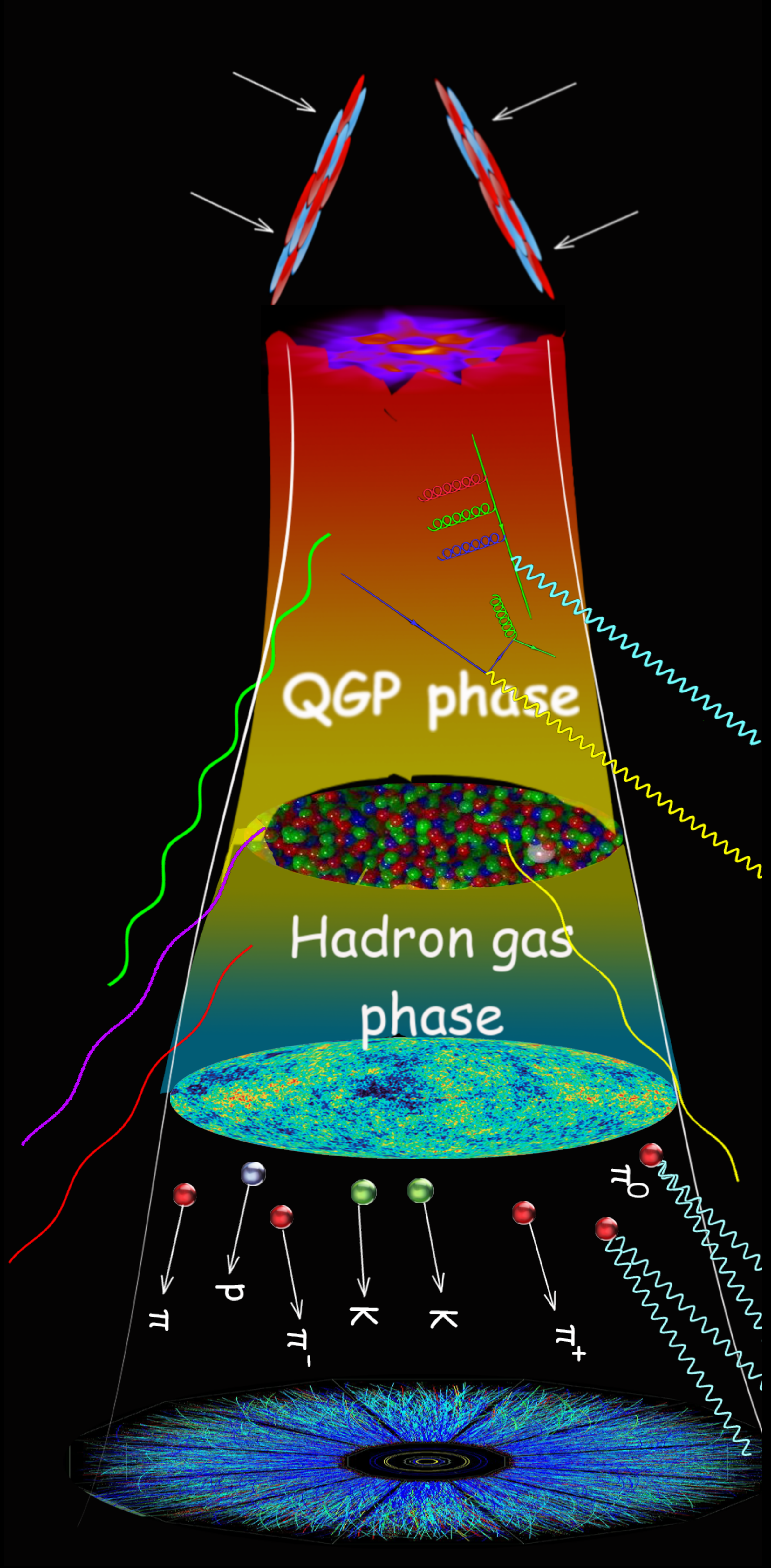
DEFINING THE QUARK-GLUON PLASMA



Which **properties of hot QCD matter** can we determine from relativistic heavy ion data (LHC, RHIC, and future FAIR/NICA/JPAC)?

Equation of State $T^{\mu\nu} \Longleftrightarrow e, P, s$ $c_s^2 = \partial P / \partial e\|_{s/n}$	Spectra, collective flow, femtoscopy
Shear and bulk viscosities $\eta/s(T, \mu_B)$, $\zeta/s(T, \mu_B)$	Anisotropic flow v_n Flow correlations
Charge diffusion D_B, D_Q, D_S	Balance functions
Electromagnetic emissivity	Photons and dileptons
Energy-momentum transport $\hat{q}, \hat{e}, \hat{e}_2, \ldots$	Jets and heavy-quarks

DEFINING THE QUARK-GLUON PLASMA



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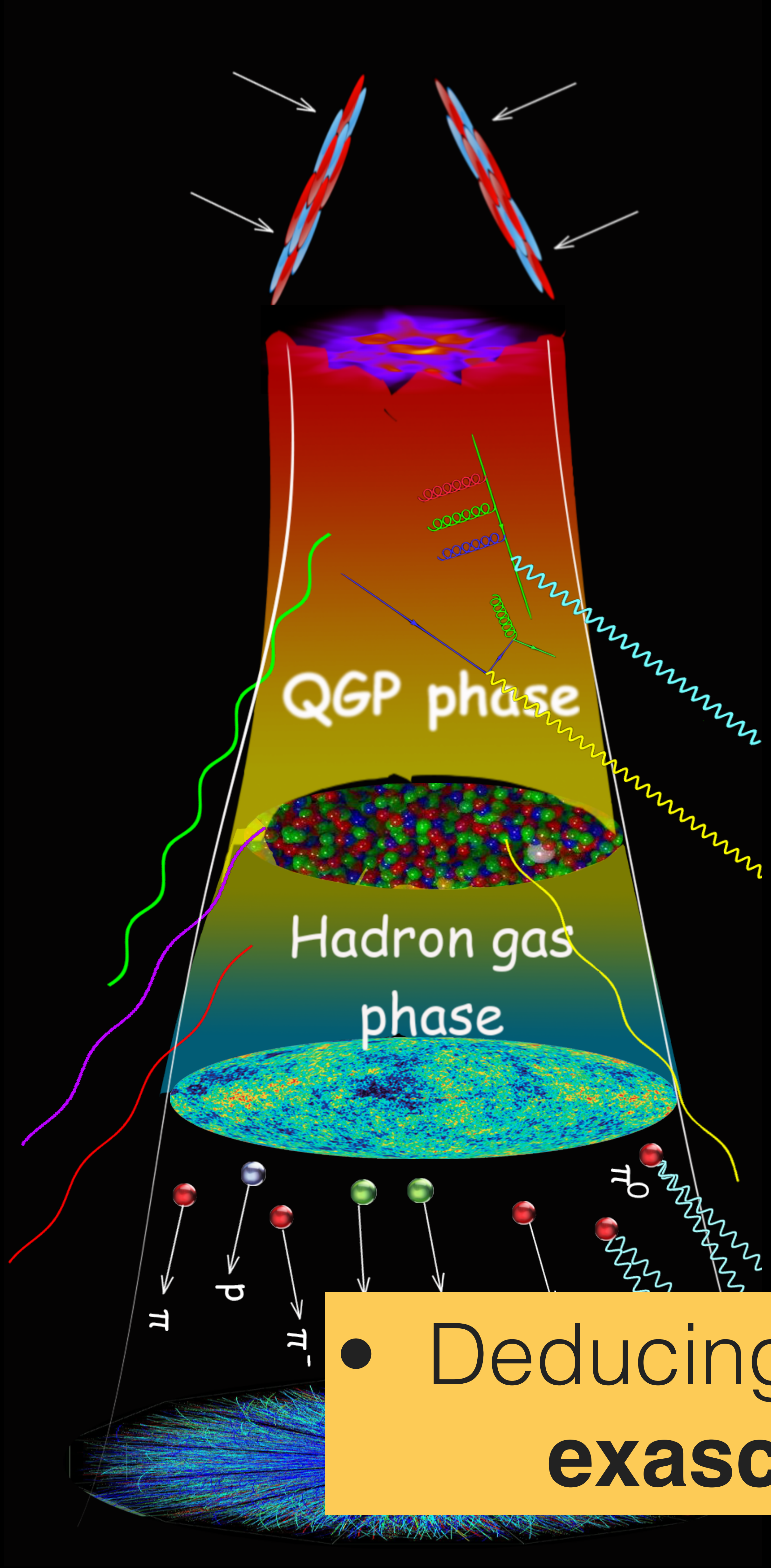

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Charge diffusion D_B, D_Q, D_S

Electromagnetic emissivity

Spectra, collective flow, femtoscopy

Anisotropic flow v_n

Flow correlations

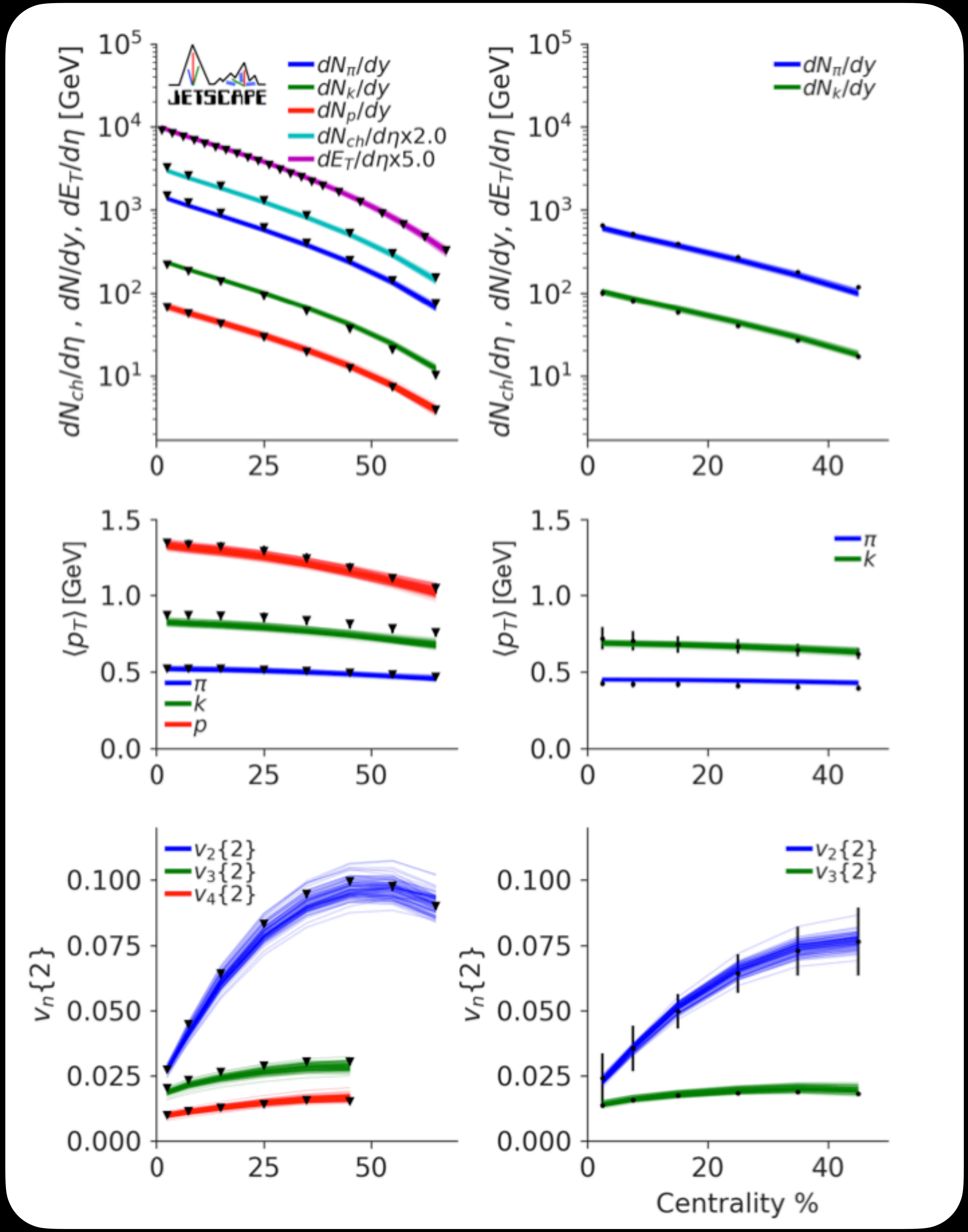
Balance functions

Photons and dileptons

quarks

- Deducing the QGP properties from experimental data requires **exascale computing** with advanced statistical methods

GLOBAL BAYESIAN CONSTRAINTS ON QGP VISCOSITY



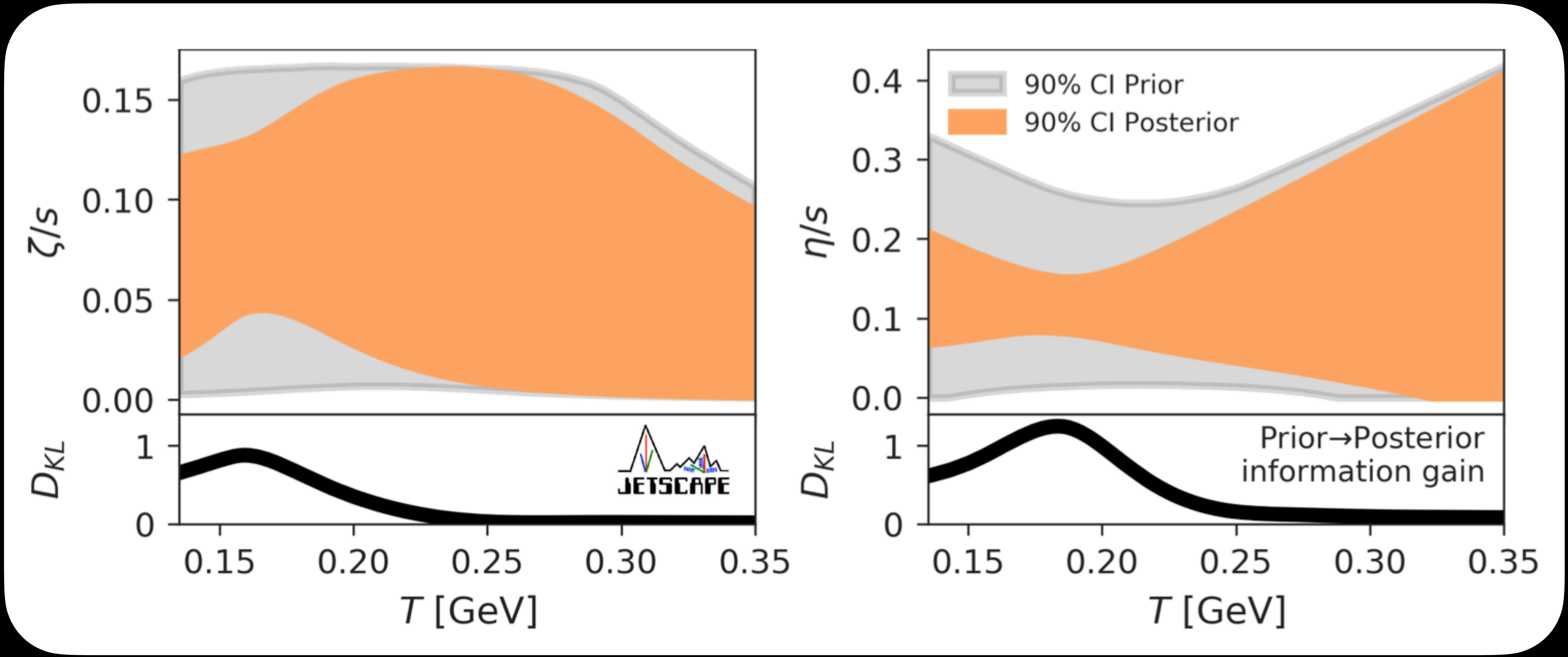
S. Pratt, E. Sangaline, P. Sorensen and H. Wang, Phys. Rev. Lett. 114, 202301 (2015)

J. E. Bernhard, J. S. Moreland, S. A. Bass, J. Liu and U. Heinz, Phys. Rev. C94, 024907 (2016)

J. E. Bernhard, J. S. Moreland and S. A. Bass, Nature Phys. 15, 1113-1117 (2019)

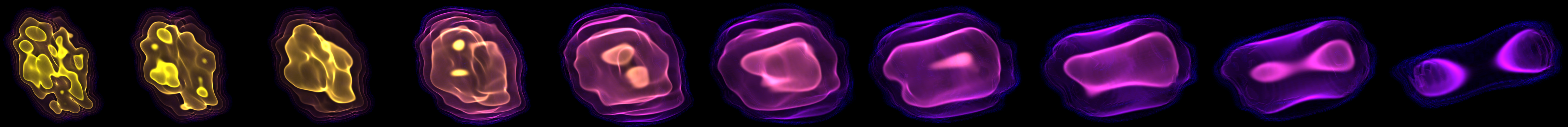
G. Nijs, W. Van Der Schee, U. Gursoy and R. Snellings,
Phys. Rev. Lett. 126, 202301 (2021) & Phys. Rev. C103, 054909 (2021)

D. Everett *et al.* [JETSCAPE],
Phys. Rev. Lett.126, 242301 (2021) & Phys. Rev. C103, 054904 (2021)



- Precision hadronic measurements can systematically constrain the QGP viscosity

THE MULTI-STAGE THEORETICAL FRAMEWORK



$T^{\mu\nu}$ 0^+ 0.5-1 → τ (fm)

Initial State +
Pre-equilibrium dynamics

Hydrodynamics

Hadronic Transport

$$T^{\mu\nu}_{\rm pre.\,eq} = T^{\mu\nu}_{\rm hydro}$$

+ Landau Matching
with lattice EoS

$$T^{\mu\nu}_{\rm hydro} = T^{\mu\nu}_{\rm particles}$$

Cooper-Frye
particlization

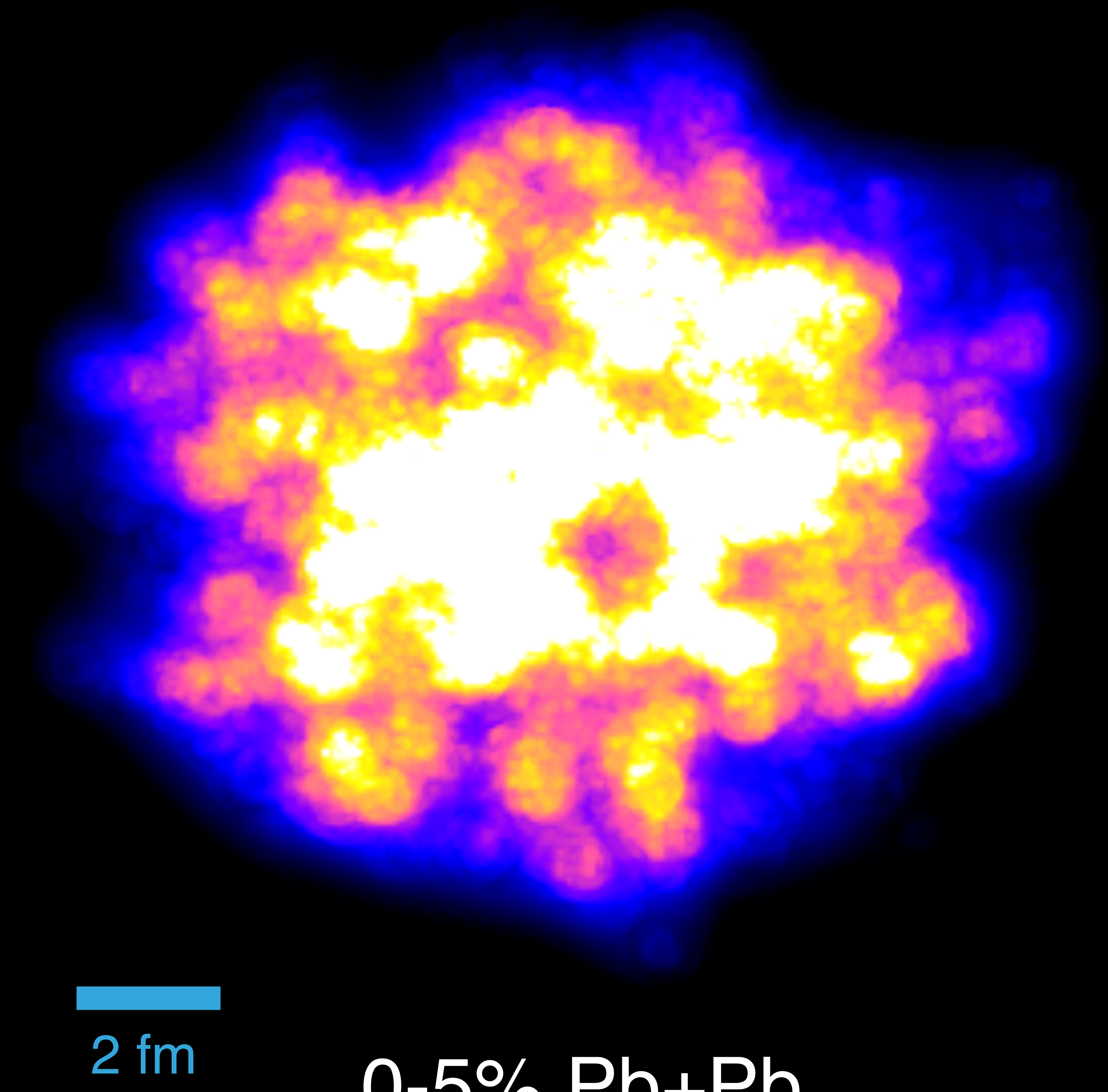
- Continuously connect the system's energy-momentum tensor $T^{\mu\nu}$ between different stages

PUSHING HYDRODYNAMICS TO ITS LIMIT

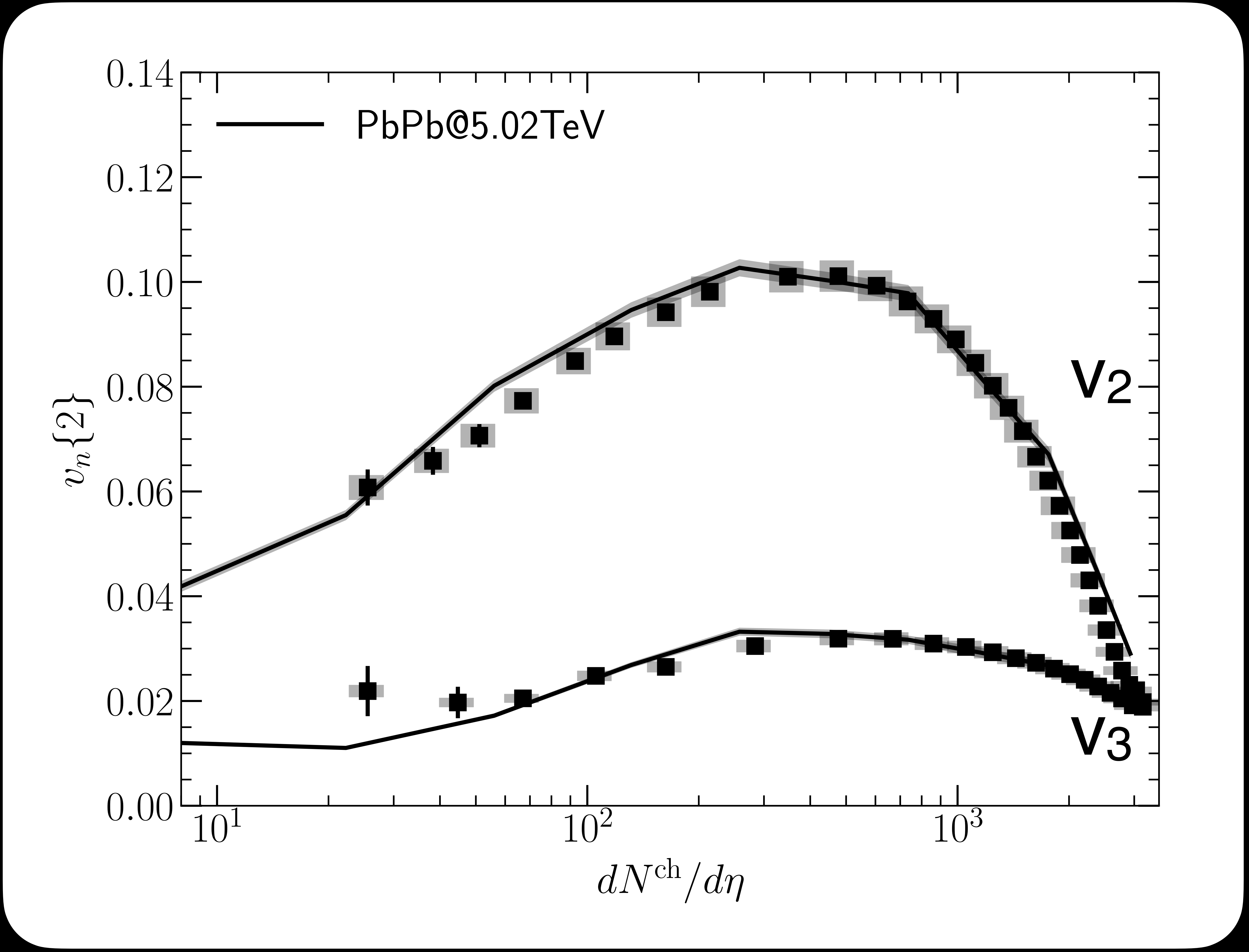
A UNIVERSAL DESCRIPTION OF FLOW IN ALL SYSTEMS

ALICE Collaboration, Phys. Rev. Lett. 123, 142301 (2019)

B. Schenke, C. Shen and P. Tribedy, Phys. Rev. C **102**, 044905 (2020)



0-5% Pb+Pb



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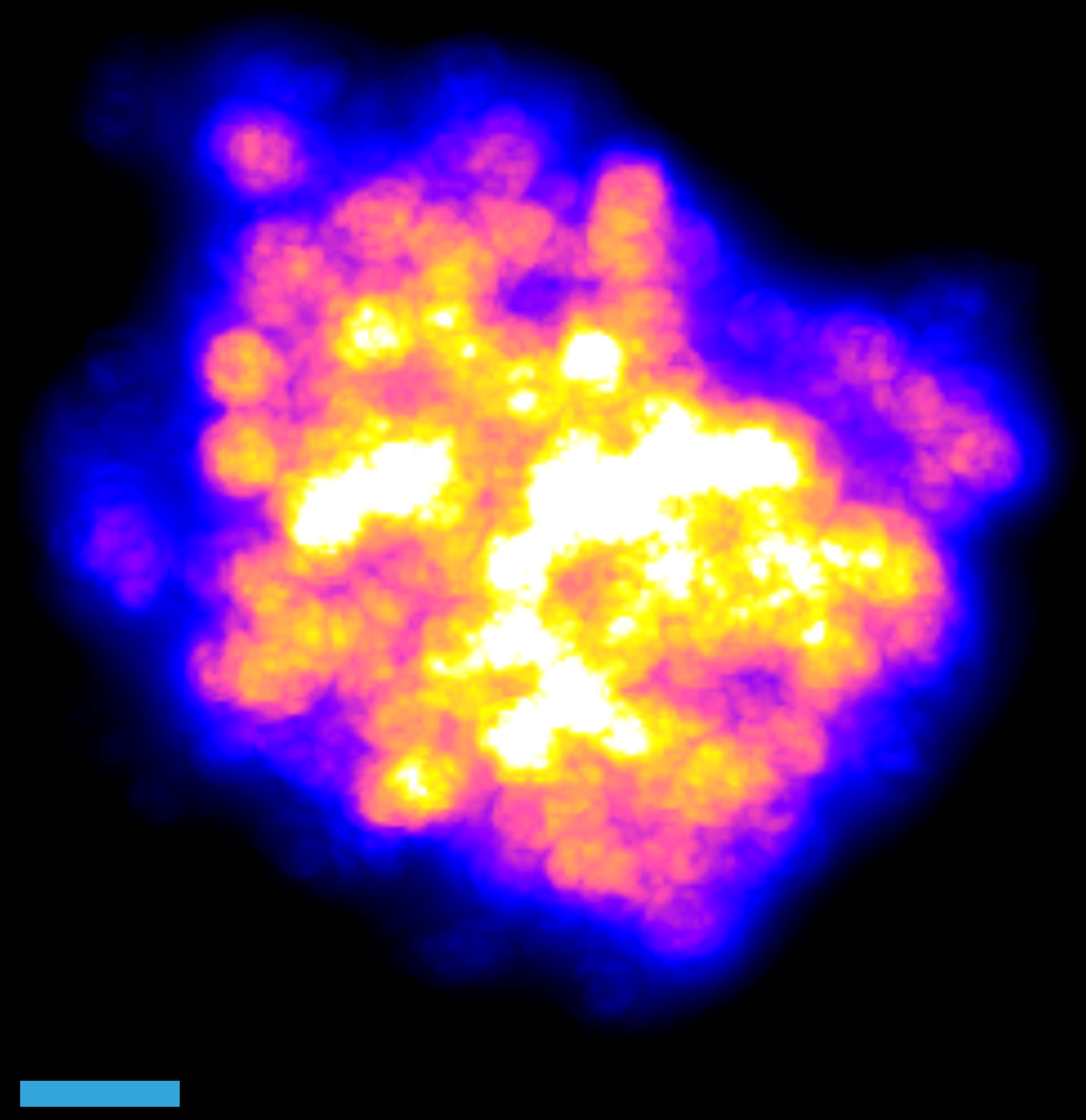
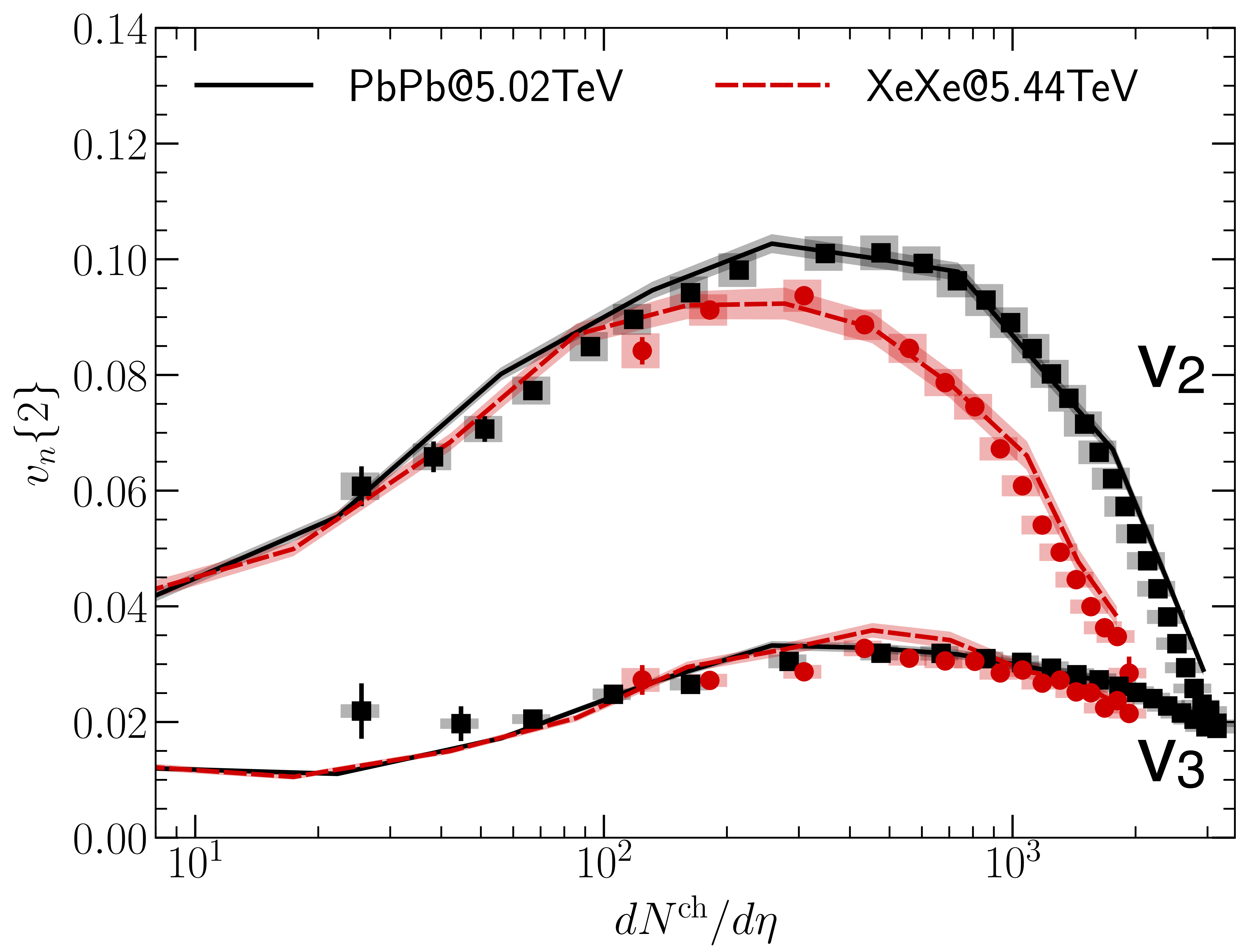


0-5% Xe+Xe



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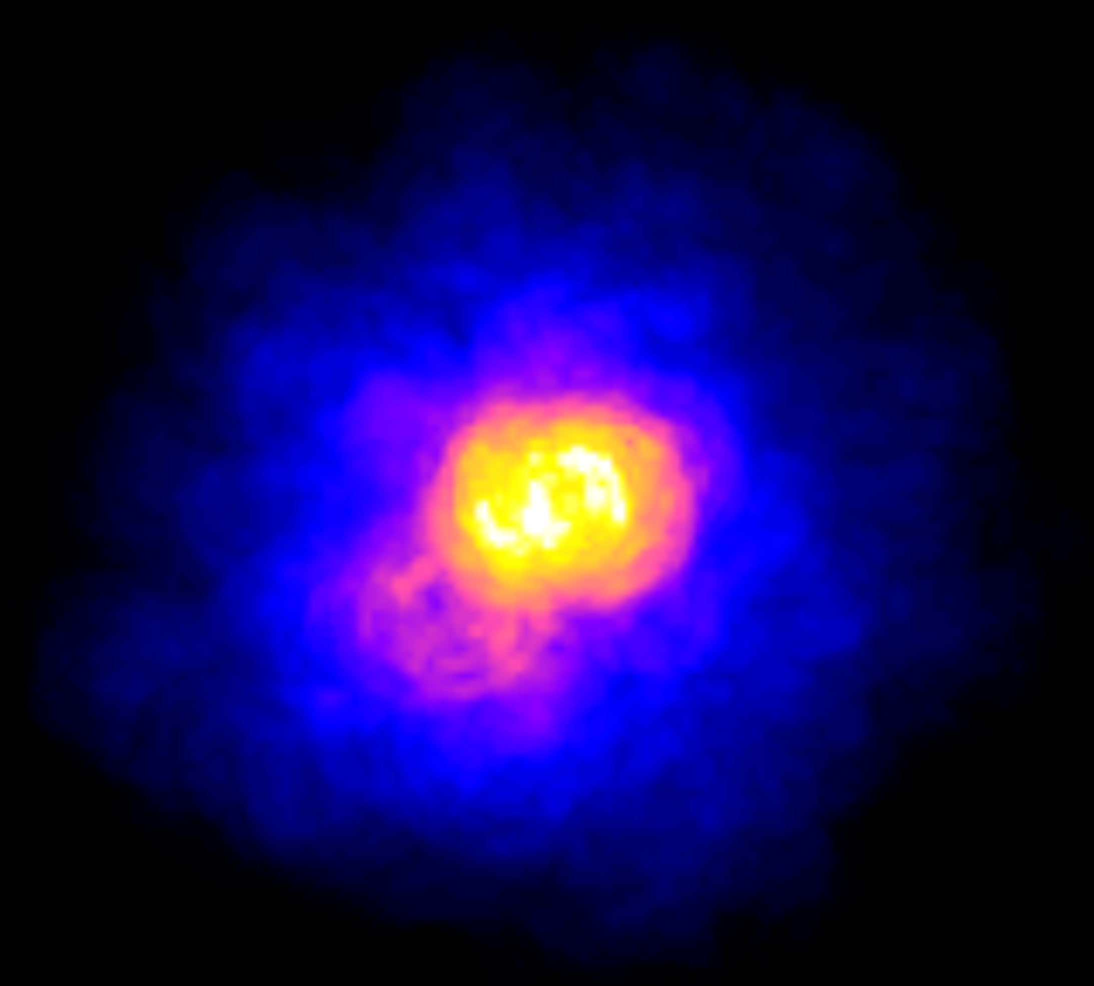
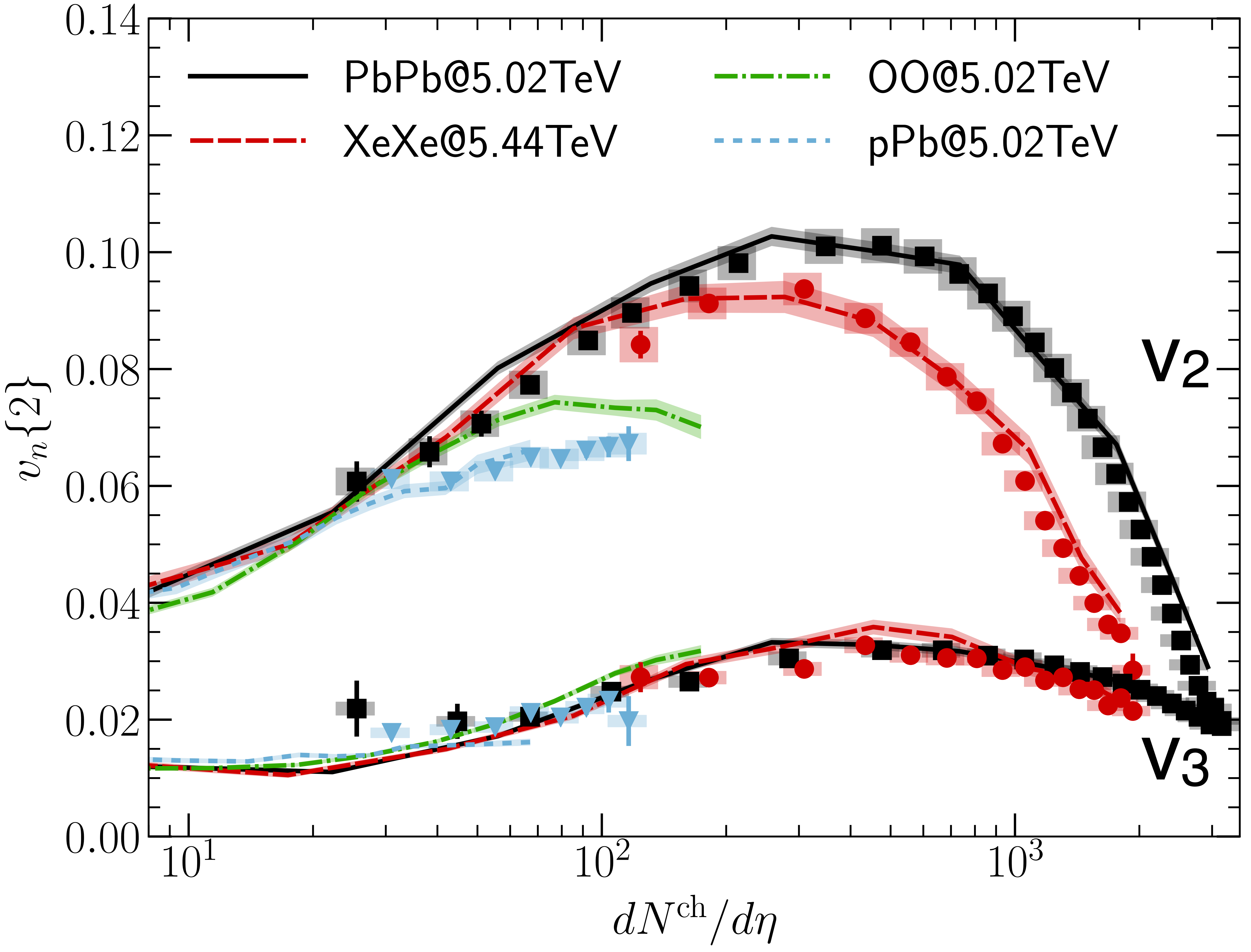


2 fm

0-5% p+Pb



CAUSALITY IN LINEARIZED PERTURBATIVE REGIONS

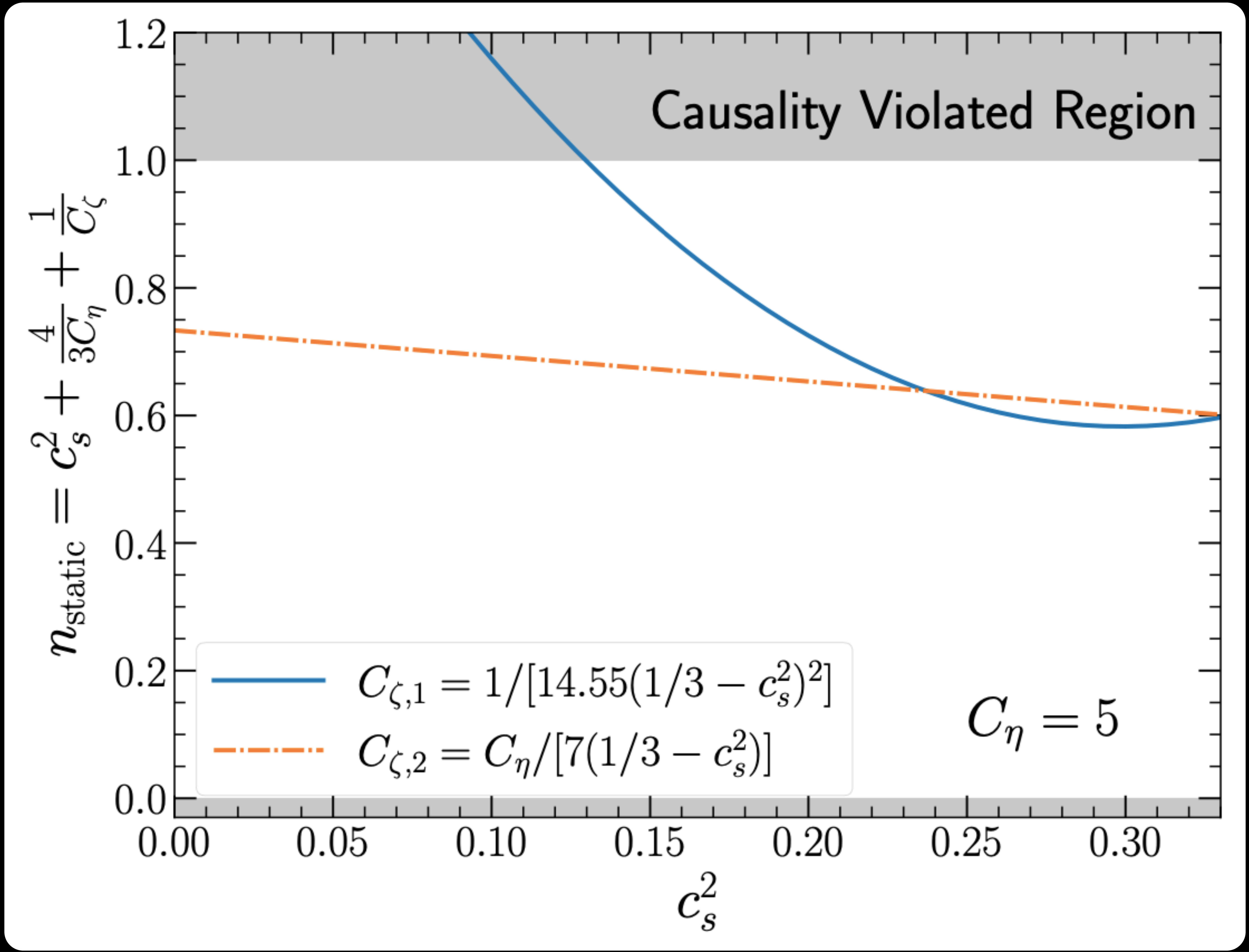
C. Chiu and C. Shen, Phys. Rev. C 103, 064901 (2021)

$$\tau_\pi = \frac{C_\eta}{T}\frac{\eta}{s} \qquad C_\eta \sim 2.6 - 7$$

$$\tau_\Pi = C_\zeta \frac{\zeta}{e+P}$$

$$0 \leq n_{\rm static} = c_s^2 + \frac{4}{3C_\eta} + \frac{1}{C_\zeta} \leq 1$$

- The $C_{\zeta,1}$ from transport theories requires $c_s^2 > 0.13$



CAUS… ZED PERTURBATIVE REGIONS

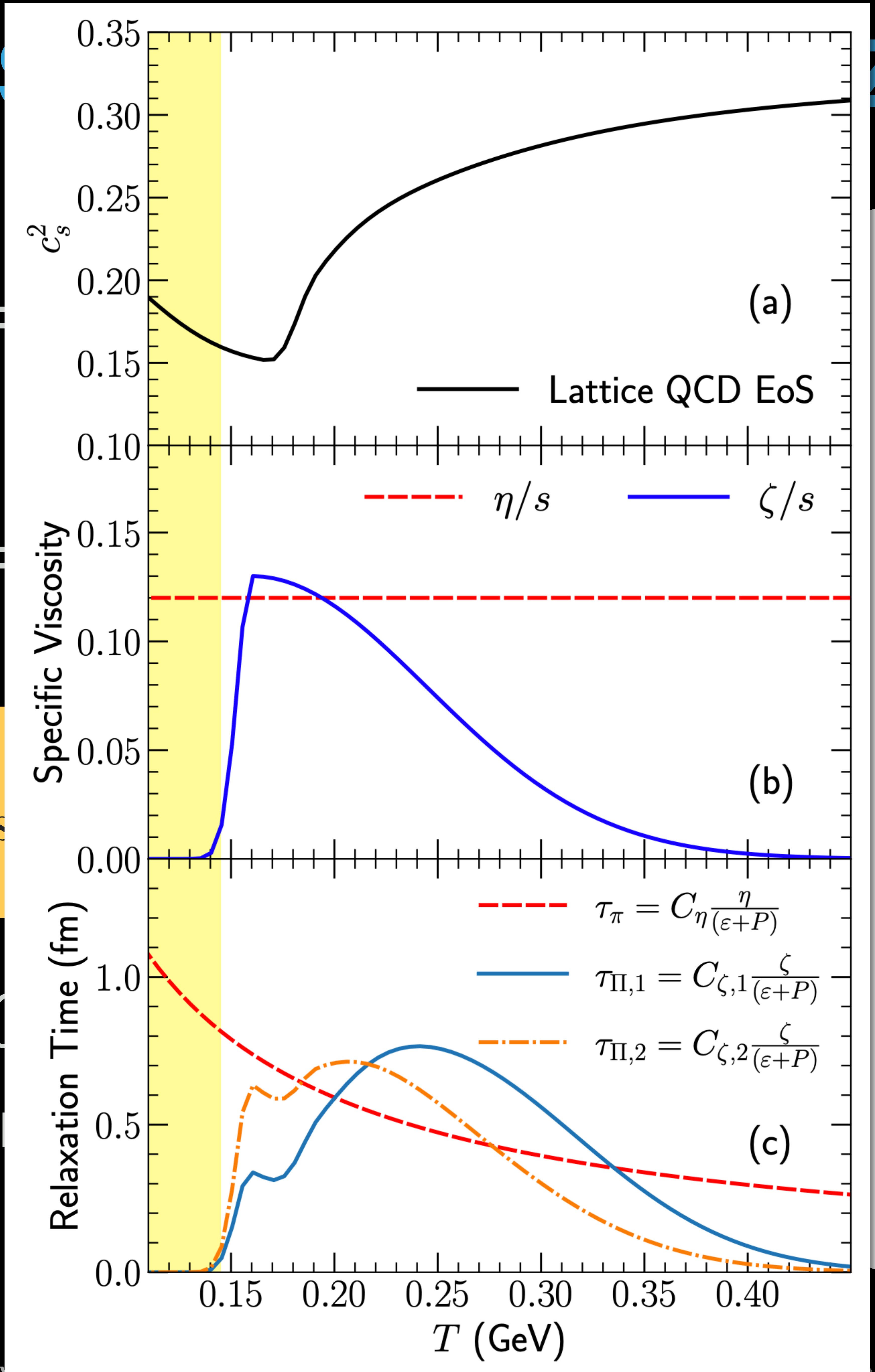
C. Chiu and C. Shen, Phys. Rev. C 103, 064901 (2021)

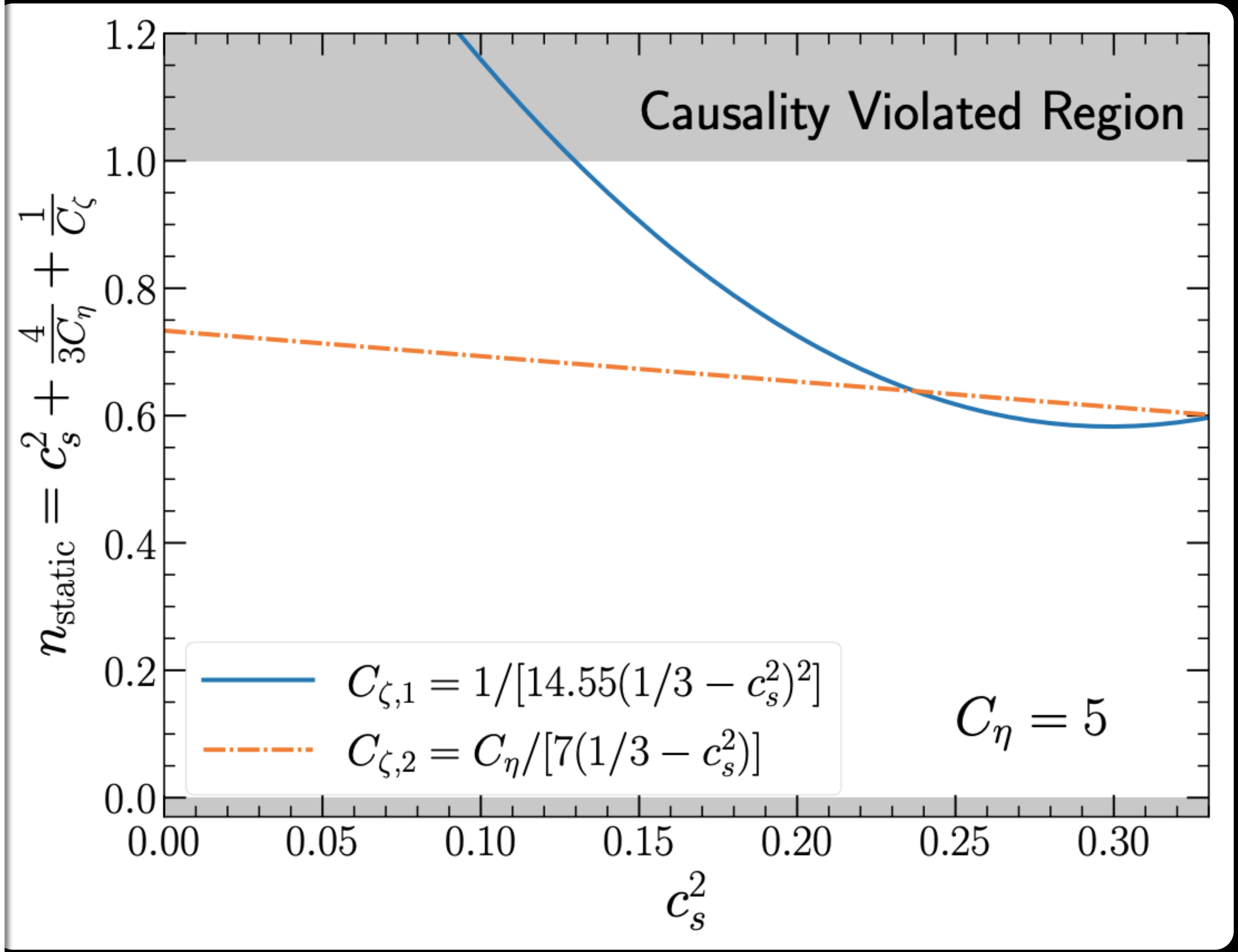
$\tau_\pi =$

$\tau_\Pi =$

$0 \leq n_s$

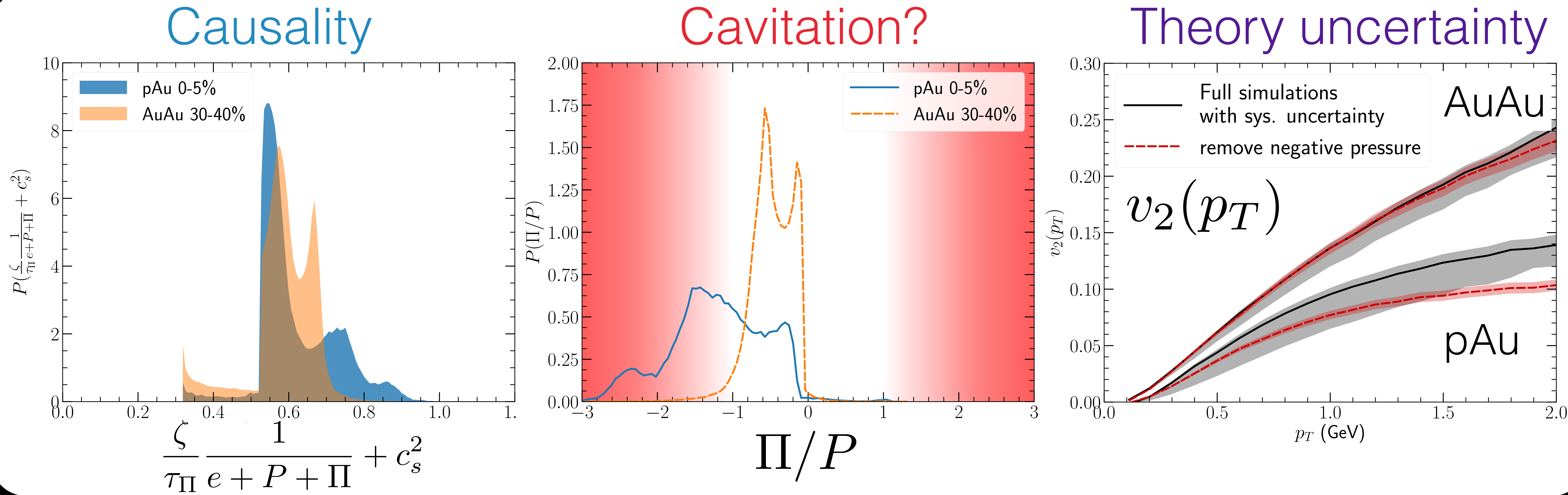
- The C … requi…





PUSHING HYDRODYNAMICS TO ITS LIMIT

F. S. Bemfica, M. M. Disconzi and J. Noronha, Phys. Rev. Lett. 122, 221602 (2019)
B. Schenke, C. Shen and P. Tribedy, arXiv:1908.06212 [nucl-th]



Theoretical uncertainty are estimated by turning on and off second-order transport coefficients and initial $\Pi^{\mu\nu}$

fast expansion $\Longrightarrow$ large negative bulk viscous pressure

FULL NON-LINEAR CAUSALITY CONDITIONS

F. S. Bemfica, M. M. Disconzi, V. Hoang, J. Noronha and M. Radosz, Phys. Rev. Lett. 126, 222301 (2021)

$$\partial_\mu T^{\mu\nu} = 0,$$
$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu},$$
$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu},$$

$\{\Lambda_i\}$ are eigenvalues of $\pi^{\mu\nu}$ with $\Lambda_3 > \Lambda_2 > \Lambda_1$ and $\Lambda_0 = 0$

$$n_1 \equiv \frac{1}{C_\eta} + \frac{\lambda_{\pi\Pi}}{2\tau_\pi}\frac{\Pi}{\varepsilon+P} - \frac{\tau_{\pi\pi}}{4\tau_\pi}\frac{|\Lambda_1|}{\varepsilon+P} \geq 0,$$

$$n_2 \equiv 1 - \frac{1}{C_\eta} + \left(1 - \frac{\lambda_{\pi\Pi}}{2\tau_\pi}\right)\frac{\Pi}{\varepsilon+P} - \frac{\tau_{\pi\pi}}{4\tau_\pi}\frac{\Lambda_3}{\varepsilon+P} \geq 0,$$

$$n_3 \equiv \frac{1}{C_\eta} + \frac{\lambda_{\pi\Pi}}{2\tau_\pi}\frac{\Pi}{\varepsilon+P} - \frac{\tau_{\pi\pi}}{4\tau_\pi}\frac{\Lambda_3}{\varepsilon+P} \geq 0,$$

$$n_4 \equiv 1 - \frac{1}{C_\eta} + \left(1 - \frac{\lambda_{\pi\Pi}}{2\tau_\pi}\right)\frac{\Pi}{\varepsilon+P} + \left(1 - \frac{\tau_{\pi\pi}}{4\tau_\pi}\right)\frac{\Lambda_a}{\varepsilon+P} - \frac{\tau_{\pi\pi}}{4\tau_\pi}\frac{\Lambda_d}{\varepsilon+P} \geq 0, (a \neq d)$$

$$n_5 \equiv c_s^2 + \frac{4}{3}\frac{1}{C_\eta} + \frac{1}{C_\zeta} + \left(\frac{2}{3}\frac{\lambda_{\pi\Pi}}{\tau_\pi} + \frac{\delta_{\Pi\Pi}}{\tau_\Pi} + c_s^2\right)\frac{\Pi}{\varepsilon+P} + \left(\frac{3\delta_{\pi\pi} + \tau_{\pi\pi}}{3\tau_\pi} + \frac{\lambda_{\Pi\pi}}{\tau_\Pi} + c_s^2\right)\frac{\Lambda_1}{\varepsilon+P} \geq 0,$$

$$n_6 \equiv 1 - \left(c_s^2 + \frac{4}{3}\frac{1}{C_\eta} + \frac{1}{C_\zeta}\right) + \left(1 - \frac{2}{3}\frac{\lambda_{\pi\Pi}}{\tau_\pi} - \frac{\delta_{\Pi\Pi}}{\tau_\Pi} - c_s^2\right)\frac{\Pi}{\varepsilon+P} + \left(1 - \frac{3\delta_{\pi\pi} + \tau_{\pi\pi}}{3\tau_\pi} - \frac{\lambda_{\Pi\pi}}{\tau_\Pi} - c_s^2\right)\frac{\Lambda_3}{\varepsilon+P} \geq 0.$$

FULL NON-LINEAR CAUSALITY CONDITIONS

F. S. Bemfica, M. M. Disconzi, V. Hoang, J. Noronha and M. Radosz, Phys. Rev. Lett. 126, 222301 (2021)

$$s_1 \equiv 1 - \frac{1}{C_\eta} - \frac{|\Lambda_1|}{\varepsilon + P} + \left(1 - \frac{\lambda_{\pi\Pi}}{2\tau_\pi}\right)\frac{\Pi}{\varepsilon + P} - \frac{\tau_{\pi\pi}}{2\tau_\pi}\frac{\Lambda_3}{\varepsilon + P} \geq 0,$$

$$s_2 \equiv \frac{1}{C_\eta} + \frac{\lambda_{\pi\Pi}}{2\tau_\pi}\frac{\Pi}{\varepsilon + P} - \frac{\tau_{\pi\pi}}{2\tau_\pi}\frac{|\Lambda_1|}{\varepsilon + P} \geq 0,$$

$$s_3 \equiv 6\frac{\delta_{\pi\pi}}{\tau_\pi} - \frac{\tau_{\pi\pi}}{\tau_\pi} \geq 0, \qquad s_4 \equiv \frac{\lambda_{\Pi\pi}}{\tau_\Pi} + c_s^2 - \frac{\tau_{\pi\pi}}{12\tau_\pi} \geq 0,$$

$$\begin{aligned} s_5 \equiv &\left(1 + \frac{\Pi}{\varepsilon + P}\right)(1 - c_s^2) \\ &- \left[\frac{4}{3}\frac{1}{C_\eta} + \frac{1}{C_\zeta} + \left(\frac{2}{3}\frac{\lambda_{\pi\Pi}}{\tau_\pi} + \frac{\delta_{\Pi\Pi}}{\tau_\Pi}\right)\frac{\Pi}{\varepsilon + P}\right. \\ &+ \left(\frac{\delta_{\pi\pi}}{\tau_\pi} + \frac{\tau_{\pi\pi}}{3\tau_\pi} + \frac{\lambda_{\Pi\pi}}{\tau_\Pi} + c_s^2\right)\frac{\Lambda_3}{\varepsilon + P} + \frac{|\Lambda_1|}{\varepsilon + P} \\ &\left. + \frac{\left(\frac{\delta_{\pi\pi}}{\tau_\pi} - \frac{\tau_{\pi\pi}}{12\tau_\pi}\right)\left(\frac{\lambda_{\Pi\pi}}{\tau_\Pi} + c_s^2 - \frac{\tau_{\pi\pi}}{12\tau_\pi}\right)\left(\frac{\Lambda_3}{\varepsilon + P} + \frac{|\Lambda_1|}{\varepsilon + P}\right)^2}{1 - \frac{1}{C_\eta} + \left(1 - \frac{\lambda_{\pi\Pi}}{2\tau_\pi}\right)\frac{\Pi}{\varepsilon + P} - \frac{|\Lambda_1|}{\varepsilon + P} - \frac{\tau_{\pi\pi}}{2\tau_\pi}\frac{\Lambda_3}{\varepsilon + P}}\right] \geq 0 \end{aligned}$$

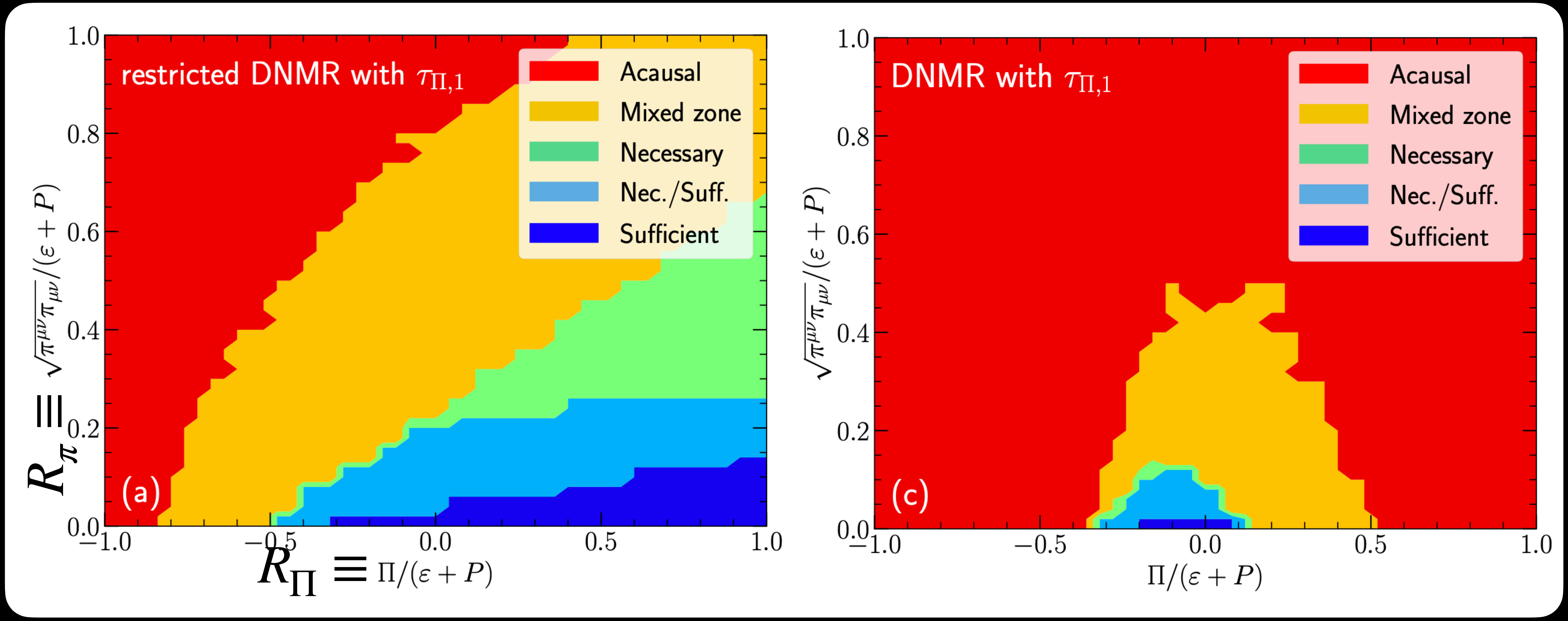
$$\begin{aligned} s_6 \equiv &\frac{1}{3C_\eta} + \frac{1}{C_\zeta} + c_s^2 + \left(\frac{\lambda_{\pi\Pi}}{6\tau_\pi} + \frac{\delta_{\Pi\Pi}}{\tau_\Pi} + c_s^2\right)\frac{\Pi}{\varepsilon + P} \\ &+ \left(\frac{\tau_{\pi\pi}}{6\tau_\pi} - \frac{\delta_{\pi\pi}}{\tau_\pi} + \frac{\lambda_{\Pi\pi}}{\tau_\Pi} - c_s^2\right)\frac{|\Lambda_1|}{\varepsilon + P} \geq 0, \end{aligned}$$

$$\begin{aligned} s_7 \equiv &\left[\frac{1}{C_\eta} + \frac{\lambda_{\pi\Pi}}{2\tau_\pi}\frac{\Pi}{\varepsilon + P} - \frac{\tau_{\pi\pi}}{2\tau_\pi}\frac{|\Lambda_1|}{\varepsilon + P}\right]^2 \\ &- \left(\frac{\delta_{\pi\pi}}{\tau_\pi} - \frac{\tau_{\pi\pi}}{12\tau_\pi}\right)\left(\frac{\lambda_{\Pi\pi}}{\tau_\Pi} + c_s^2 - \frac{\tau_{\pi\pi}}{12\tau_\pi}\right)\left(\frac{\Lambda_3}{\varepsilon + P} + \frac{|\Lambda_1|}{\varepsilon + P}\right)^2 \geq 0, \end{aligned}$$

$$\begin{aligned} s_8 \equiv &\frac{4}{3C_\eta} + \frac{1}{C_\zeta} + c_s^2 + \left(\frac{2}{3}\frac{\lambda_{\pi\Pi}}{\tau_\pi} + \frac{\delta_{\Pi\Pi}}{\tau_\Pi} + c_s^2\right)\frac{\Pi}{\varepsilon + P} \\ &- \left(\frac{\delta_{\pi\pi}}{\tau_\pi} + \frac{\tau_{\pi\pi}}{3\tau_\pi} - \frac{\lambda_{\Pi\pi}}{\tau_\Pi} + c_s^2\right)\frac{|\Lambda_1|}{\varepsilon + P} - \frac{\left(1 + \frac{\Pi}{\varepsilon + P} + \frac{\Lambda_2}{\varepsilon + P}\right)\left(1 + \frac{\Pi}{\varepsilon + P} + \frac{\Lambda_3}{\varepsilon + P}\right)}{3\left(1 + \frac{\Pi}{\varepsilon + P} - \frac{|\Lambda_1|}{\varepsilon + P}\right)^2} \\ &\times \left[1 + \frac{2}{C_\eta} + \left(1 + \frac{\lambda_{\pi\Pi}}{\tau_\pi}\right)\frac{\Pi}{\varepsilon + P} - \frac{|\Lambda_1|}{\varepsilon + P} + \frac{\tau_{\pi\pi}}{\tau_\pi}\frac{\Lambda_3}{\varepsilon + P}\right] \geq 0. \end{aligned}$$

VISUALIZE CAUSALITY CONDITIONS

C. Chiu and C. Shen, Phys. Rev. C 103, 064901 (2021)



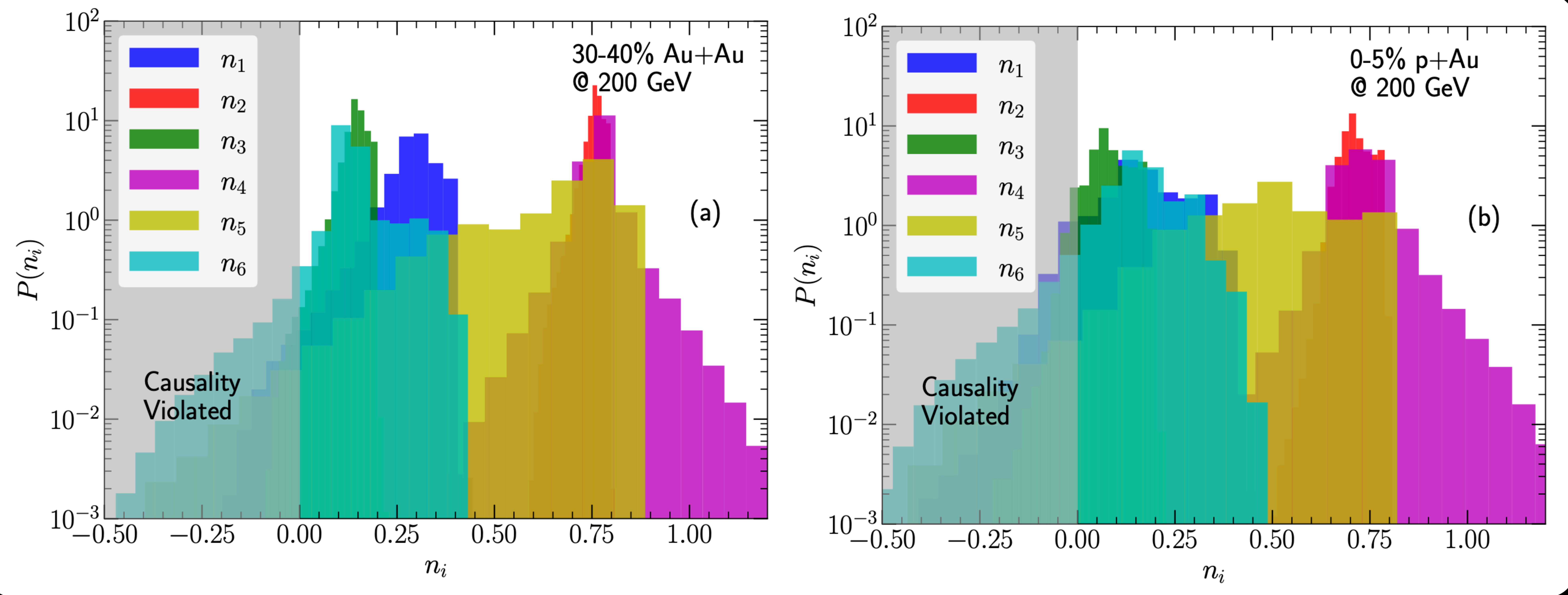
- Visualize causal regions in terms of inverse Reynolds numbers R_π and R_Π
- Non-zero second-order transport coefficients are important in determining the causal region

	restricted DNMR with $\tau_{\Pi,1}$	DNMR with $\tau_{\Pi,1}$
$\frac{\eta}{\tau_\pi(\varepsilon+P)} = \frac{1}{C_\eta}$	$\frac{1}{5}$	$\frac{1}{5}$
$\frac{\zeta}{\tau_\Pi(\varepsilon+P)} = \frac{1}{C_\zeta}$	$14.55(\frac{1}{3} - c_s^2)^2$	$14.55(\frac{1}{3} - c_s^2)^2$
$\frac{\delta_{\pi\pi}}{\tau_\pi}$	$\frac{4}{3}$	$\frac{4}{3}$
$\frac{\delta_{\Pi\Pi}}{\tau_\Pi}$	$\frac{2}{3}$	$\frac{2}{3}$
$\frac{\tau_{\pi\pi}}{\tau_\pi}$	0	$\frac{10}{7}$
$\frac{\lambda_{\pi\Pi}}{\tau_\pi}$	0	$\frac{6}{5}$
$\frac{\lambda_{\Pi\pi}}{\tau_\Pi}$	0	$\frac{8}{5}(\frac{1}{3} - c_s^2)$
φ_7	0	$\frac{9}{70}\frac{4}{\varepsilon+P}$

- For $R_\pi < 0.2$ and $-0.3 < R_\Pi < 0.1$, necessary causality conditions are fulfilled
- The sufficient conditions post stronger constraints on R_π and R_Π

EXAMINE HYDRODYNAMIC SIMULATIONS

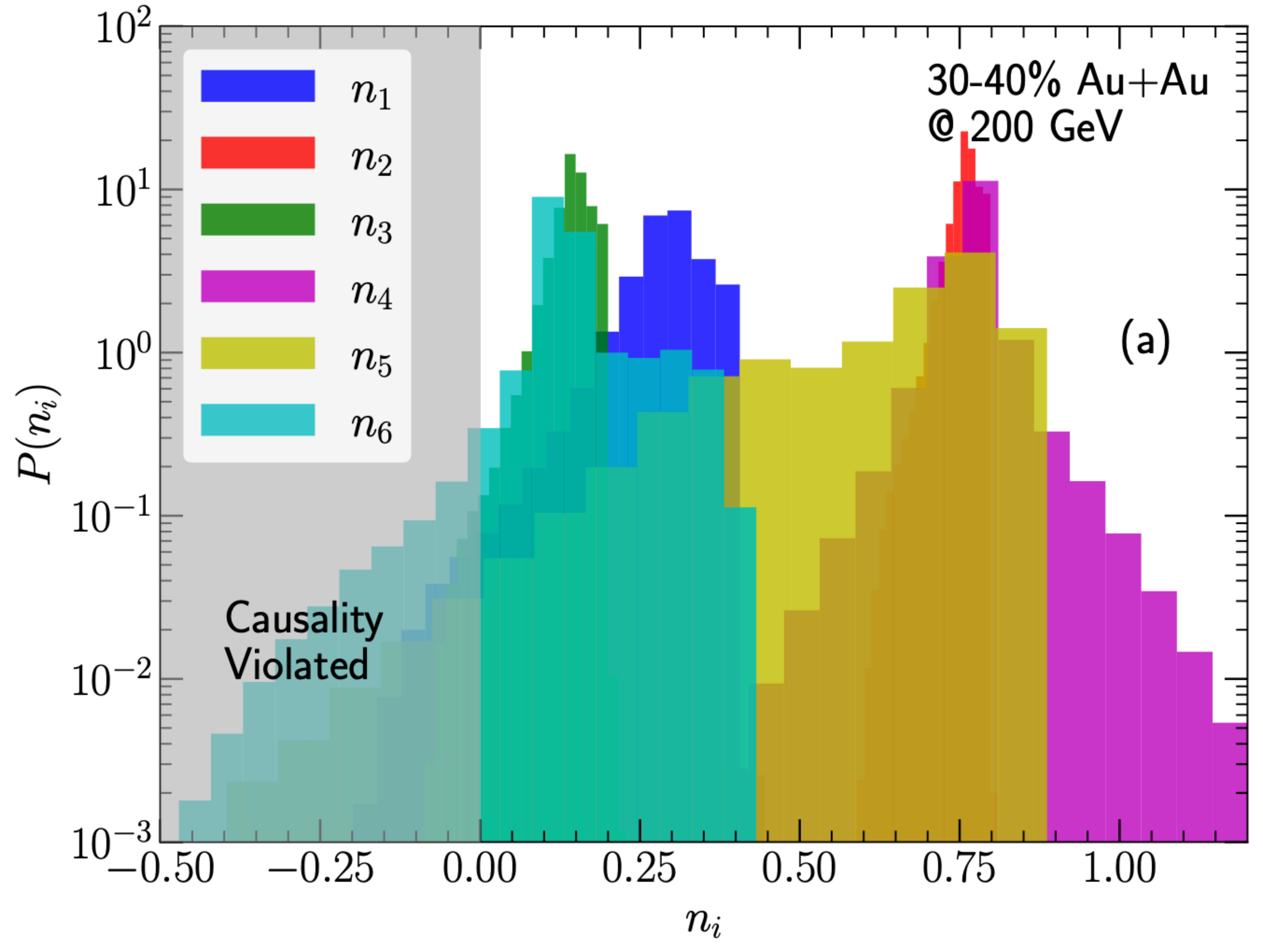
B. Schenke, C. Shen and P. Tribedy, Phys. Rev. C **102**, 044905 (2020)
C. Chiu and C. Shen, Phys. Rev. C 103, 064901 (2021)

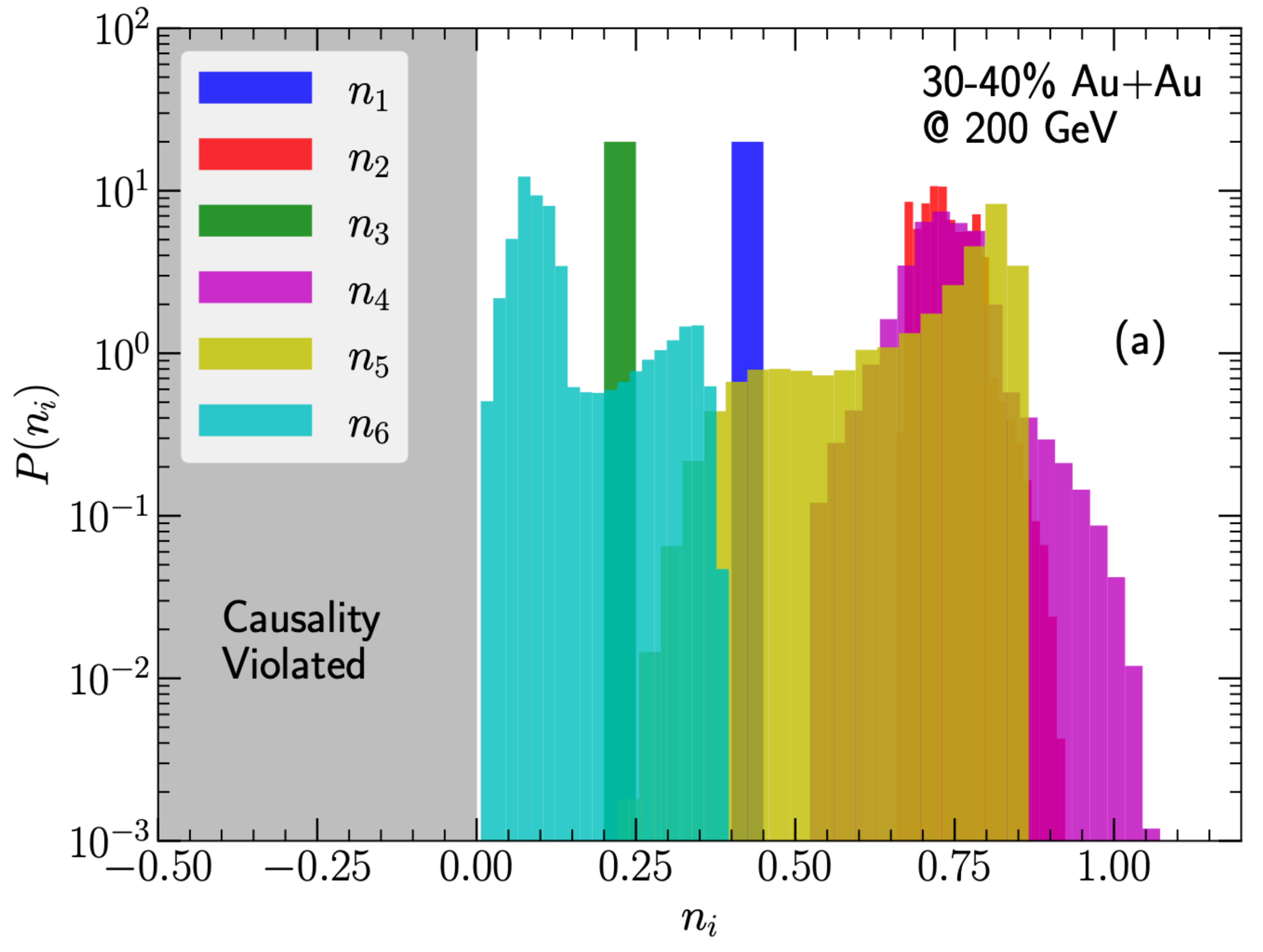


- The necessary conditions n_1, n_3, n_5, n_6 can be violated during event-by-event hydrodynamic simulation for heavy-ion collisions
- However, no numerical instability is observed because of regulations

IMPOSING CAUSALITY CONDITIONS AS REGULATION

C. Chiu and C. Shen, Phys. Rev. C 103, 064901 (2021)





- Imposing a global restriction on $R_\pi \leq \sqrt{2}P/(e+P)$ ($|\Lambda_{\max}| \leq P$) and $|R_\Pi| \leq P/(e+P)$ ($|\Pi| \leq P$) would regulate all the causality violations

IMPOSING CAUSALITY CONDITIONS AS REGULATION

C. Chiu and C. Shen, Phys. Rev. C 103, 064901 (2021)

- Using a different bulk viscous relaxation time $\tau_{\Pi,2} = \frac{5}{7(1/3 - c_s^2)} \frac{\zeta}{(e+P)}$, the restrictions on inverse Reynolds numbers can be relaxed to $R_\pi < 0.6$ and $|R_\Pi| < 0.6$

The choice of relaxation time is important!

A NUMERICAL REGULATION SCHEME

C. Chiu, G. Denicol, M. Luzum, and C. Shen, in preparation

$$n_1 \equiv \frac{1}{C_\eta} + \frac{\lambda_{\pi\Pi}}{2\tau_\pi}\frac{\Pi}{\varepsilon + P} - \frac{\tau_{\pi\pi}}{4\tau_\pi}\frac{|\Lambda_1|}{\varepsilon + P} \geq 0,$$

$$n_2 \equiv 1 - \frac{1}{C_\eta} + \left(1 - \frac{\lambda_{\pi\Pi}}{2\tau_\pi}\right)\frac{\Pi}{\varepsilon + P} - \frac{\tau_{\pi\pi}}{4\tau_\pi}\frac{\Lambda_3}{\varepsilon + P} \geq 0,$$

$$n_3 \equiv \frac{1}{C_\eta} + \frac{\lambda_{\pi\Pi}}{2\tau_\pi}\frac{\Pi}{\varepsilon + P} - \frac{\tau_{\pi\pi}}{4\tau_\pi}\frac{\Lambda_3}{\varepsilon + P} \geq 0,$$

$$n_4 \equiv 1 - \frac{1}{C_\eta} + \left(1 - \frac{\lambda_{\pi\Pi}}{2\tau_\pi}\right)\frac{\Pi}{\varepsilon + P} + \left(1 - \frac{\tau_{\pi\pi}}{4\tau_\pi}\right)\frac{\Lambda_a}{\varepsilon + P} - \frac{\tau_{\pi\pi}}{4\tau_\pi}\frac{\Lambda_d}{\varepsilon + P} \geq 0, (a \neq d)$$

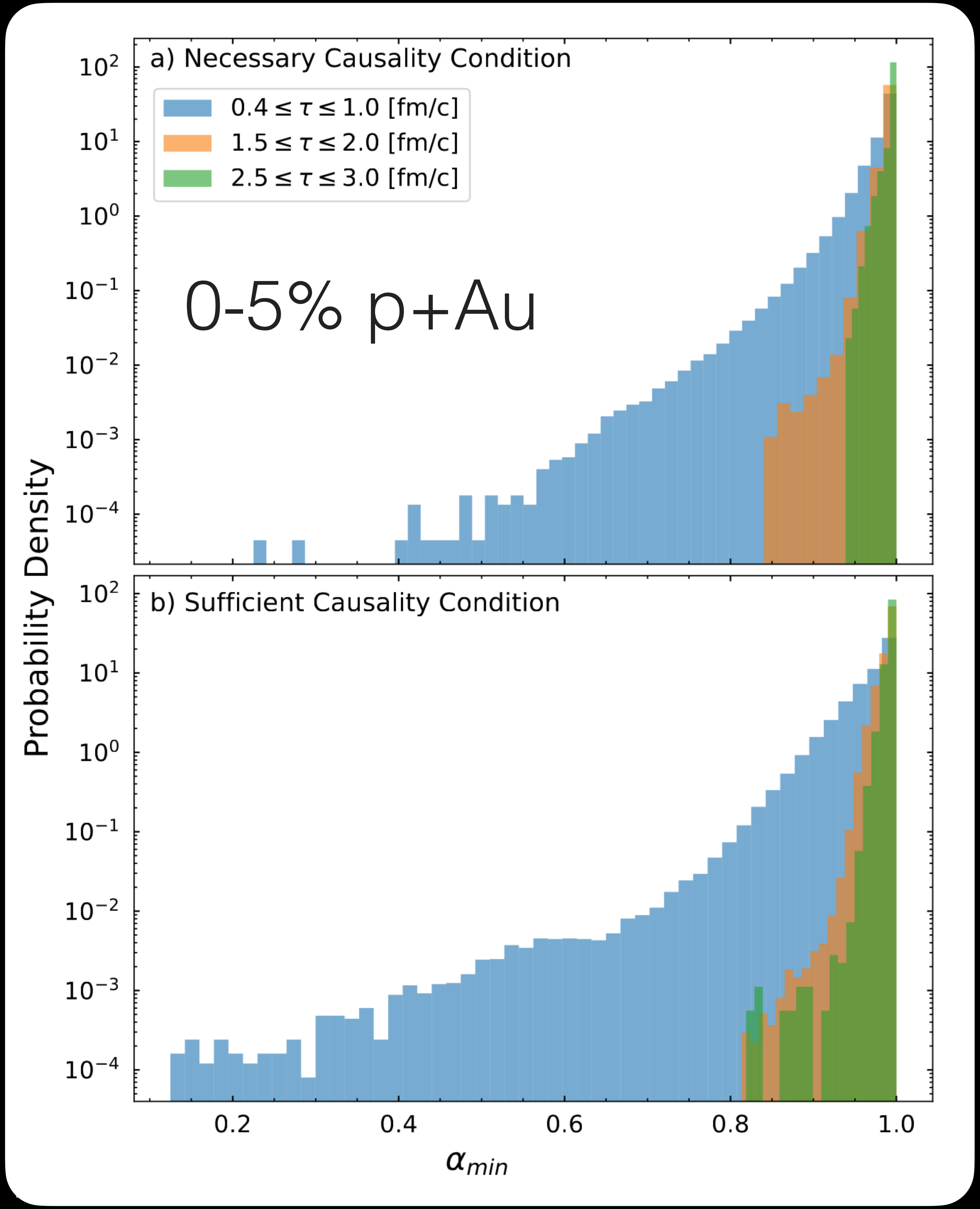
$$n_5 \equiv c_s^2 + \frac{4}{3}\frac{1}{C_\eta} + \frac{1}{C_\zeta} + \left(\frac{2}{3}\frac{\lambda_{\pi\Pi}}{\tau_\pi} + \frac{\delta_{\Pi\Pi}}{\tau_\Pi} + c_s^2\right)\frac{\Pi}{\varepsilon + P} + \left(\frac{3\delta_{\pi\pi} + \tau_{\pi\pi}}{3\tau_\pi} + \frac{\lambda_{\Pi\pi}}{\tau_\Pi} + c_s^2\right)\frac{\Lambda_1}{\varepsilon + P} \geq 0,$$

$$n_6 \equiv 1 - \left(c_s^2 + \frac{4}{3}\frac{1}{C_\eta} + \frac{1}{C_\zeta}\right) + \left(1 - \frac{2}{3}\frac{\lambda_{\pi\Pi}}{\tau_\pi} - \frac{\delta_{\Pi\Pi}}{\tau_\Pi} - c_s^2\right)\frac{\Pi}{\varepsilon + P} + \left(1 - \frac{3\delta_{\pi\pi} + \tau_{\pi\pi}}{3\tau_\pi} - \frac{\lambda_{\Pi\pi}}{\tau_\Pi} - c_s^2\right)\frac{\Lambda_3}{\varepsilon + P} \geq 0.$$

We solve $n_i(\alpha_i \pi^{\mu\nu}, \alpha_i \Pi) = 0$ for every fluid cell and define $\alpha_{\min} = \min\{\alpha_i\}$

Then we regulate viscous tensors as $\tilde{\pi}^{\mu\nu} = \alpha_{\min}\pi^{\mu\nu}$, $\tilde{\Pi} = \alpha_{\min}\Pi$

A NUMERICAL REGULATION SCHEME



C. Chiu, G. Denicol, M. Luzum, and C. Shen, in preparation

- Most of the numerical regulations are triggered during the first fm/c of the evolution, because the system is far out-of-equilibrium

C. Plumberg, D. Almaalol, T. Dore, J. Noronha and J. Noronha-Hostler, Phys. Rev. C105, L061901 (2022)

- Stronger regulations are needed to fulfill the sufficient causality conditions

A RESUMED NUMERICAL SCHEME

C. Chiu, G. Denicol, M. Luzum, and C. Shen, in preparation

We define shear and bulk inverse Reynolds numbers as

$$\tilde{R}_\pi \equiv \frac{2}{\sqrt{6}} \frac{\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}}{e+P} \qquad \tilde{R}_\Pi \equiv \frac{|\Pi|}{e+P}$$

The full necessary causality conditions can be relaxed to

$$1 - \tilde{R}_\Pi + \left(\frac{\tau_{\pi\pi}}{\tau_\pi} - 1\right)\tilde{R}_\pi \geq N_1 \geq 0 \qquad 1 - \tilde{R}_\Pi + \tilde{R}_\pi \geq N_2 \geq 0$$

$$N_1 \equiv \frac{1}{C_\eta} - \frac{\lambda_{\pi\Pi}}{2\tau_\pi}\tilde{R}_\Pi - \frac{\tau_{\pi\pi}}{4\tau_\pi}\tilde{R}_\pi \qquad N_2 \equiv c_s^2 + \frac{4}{3}\frac{1}{C_\eta} + \frac{1}{C_\zeta} - \left(\frac{2}{3}\frac{\lambda_{\pi\Pi}}{\tau_\pi} + \frac{\delta_{\Pi\Pi}}{\tau_\Pi} + c_s^2\right)\tilde{R}_\Pi + \left(\frac{3\delta_{\pi\pi} + \tau_{\pi\pi}}{3\tau_\pi} + \frac{\lambda_{\Pi\pi}}{\tau_\Pi} + c_s^2\right)\tilde{R}_\pi$$

We want to modify the equation of motion for the viscous stress tensor so that $\tilde{R}_\pi$ and $\tilde{R}_\Pi$ have bounds to fulfill the causality conditions

inspired by L. Gavassino and J. Noronha, Phys. Rev. D 109 (2024) 9, 096040

A RESUMED NUMERICAL SCHEME

C. Chiu, G. Denicol, M. Luzum, and C. Shen, in preparation

We modify the DNMR EoM when $\tilde{R}_\Pi$ or $\tilde{R}_\pi$ is large,

$$\dot{\Pi} = -\frac{1}{\tau_\Pi}(\Pi + \zeta\,\theta) - \frac{\delta_{\Pi\Pi}}{\tau_\Pi}\Pi\,\theta + \frac{\lambda_{\Pi\pi}}{\tau_\Pi}\pi^{\mu\nu}\sigma_{\mu\nu},$$

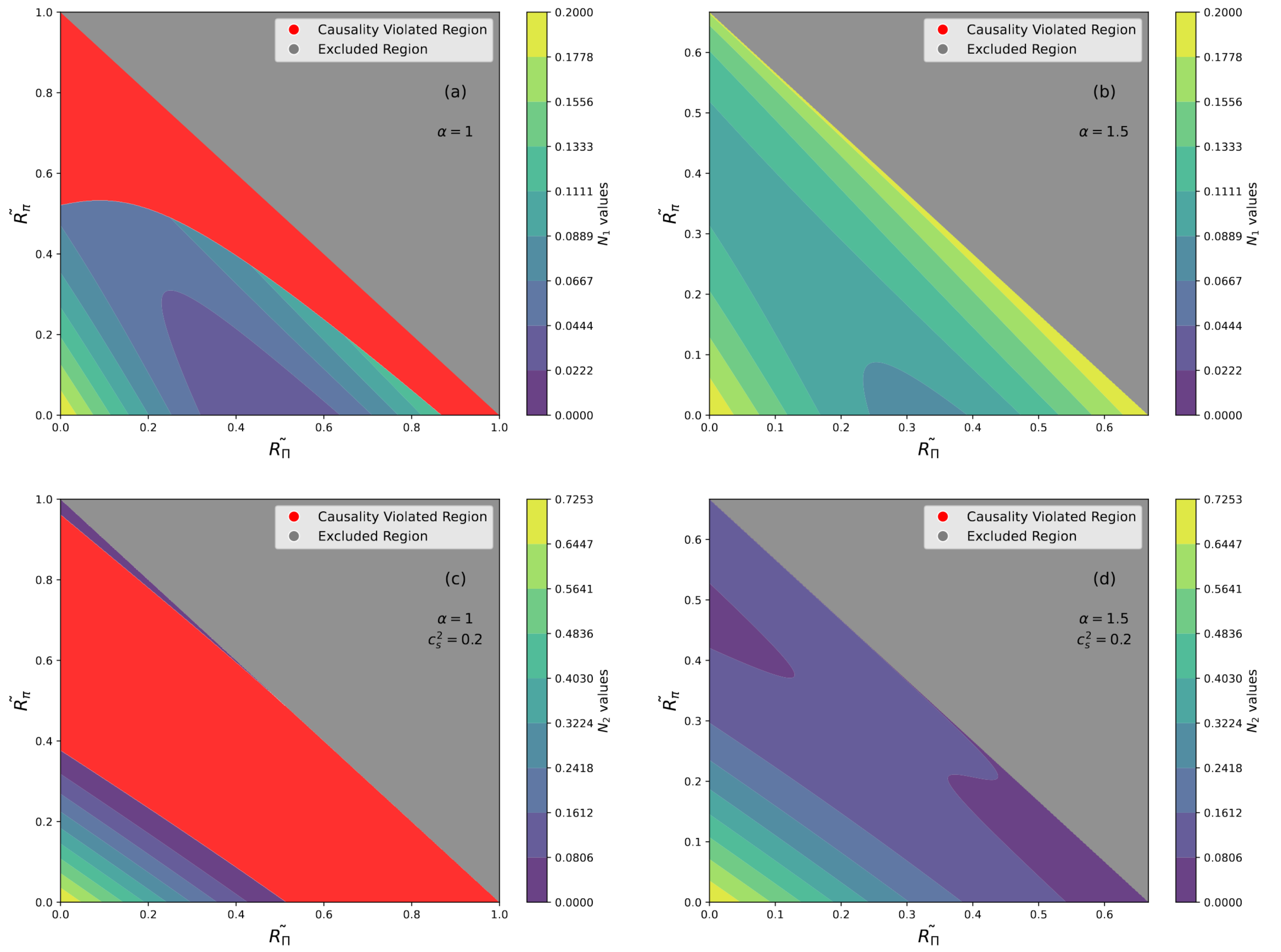
$$\dot{\pi}^{\langle\mu\nu\rangle} = -\frac{1}{\tau_\pi}(\pi^{\mu\nu} - 2\eta\,\sigma^{\mu\nu}) - \frac{\delta_{\pi\pi}}{\tau_\pi}\pi^{\mu\nu}\theta + \frac{\varphi_7}{\tau_\pi}\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \frac{\tau_{\pi\pi}}{\tau_\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \frac{\lambda_{\pi\Pi}}{\tau_\pi}\Pi\,\sigma^{\mu\nu},$$

$$\eta \to f\eta, \frac{\delta_{\pi\pi}}{\tau_\pi} \to f^2\frac{\delta_{\pi\pi}}{\tau_\pi}, \frac{\tau_{\pi\pi}}{\tau_\pi} \to f^2\frac{\tau_{\pi\pi}}{\tau_\pi}, \frac{\lambda_{\pi\Pi}}{\tau_\pi} \to f^2\frac{\lambda_{\pi\Pi}}{\tau_\pi}, \frac{\varphi_7}{\tau_\pi} \to f\frac{\varphi_7}{\tau_\pi}, \quad \zeta \to f\zeta, \frac{\delta_{\Pi\Pi}}{\tau_\Pi} \to f\frac{\delta_{\Pi\Pi}}{\tau_\Pi}, \frac{\lambda_{\Pi\pi}}{\tau_\Pi} \to f\frac{\lambda_{\Pi\pi}}{\tau_\Pi}$$

With the renomalization factor, $f = \dfrac{1}{1 + \mathrm{arctanh}^2(\alpha(\tilde{R}_\Pi + \tilde{R}_\pi))}$ $\quad 0 < \tilde{R}_\Pi + \tilde{R}_\pi < \dfrac{1}{\alpha}$

For small $\tilde{R}_\Pi$ and $\tilde{R}_\pi$, $f \approx 1 - \alpha^2(\tilde{R}_\Pi + \tilde{R}_\pi)^2$. The correction terms starts at the third order in gradients

IMPOSING NECESSARY CAUSALITY CONDITIONS

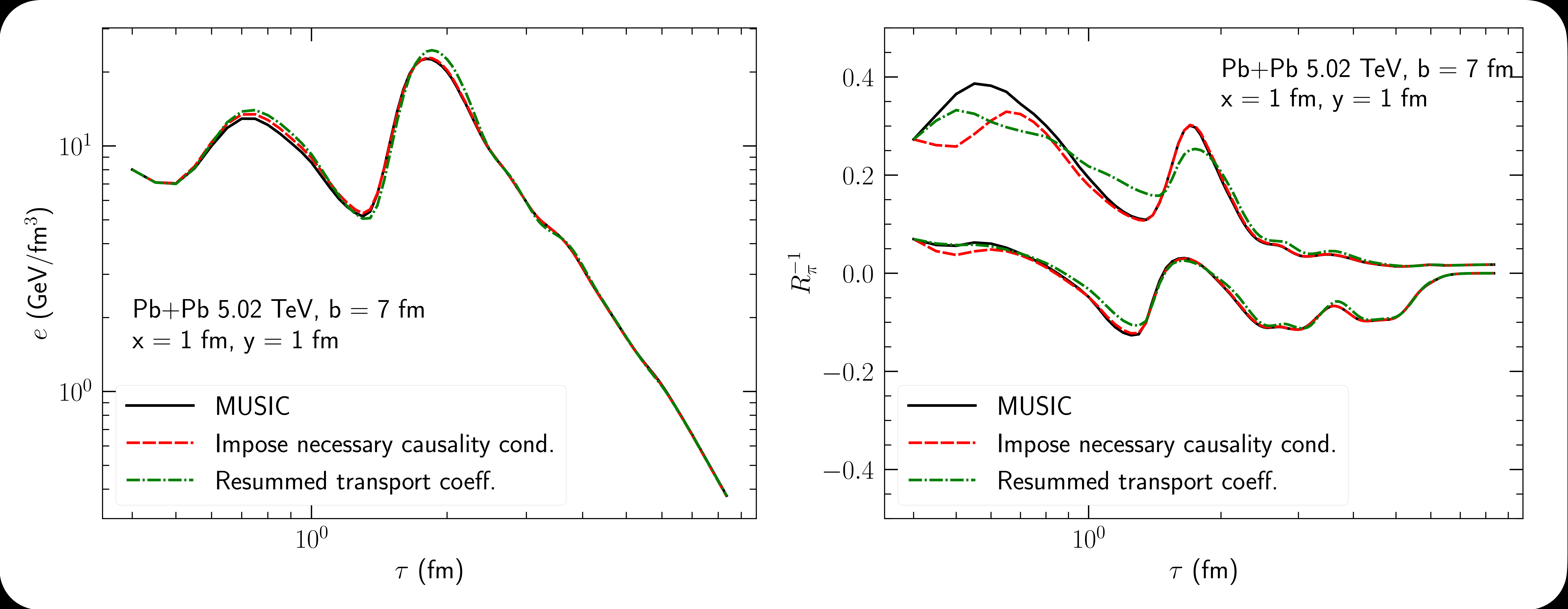


C. Chiu, G. Denicol, M. Luzum, and C. Shen, in preparation

- By adjusting the value for α in the renormalization factor, we can impose the maximum allowed values for inverse Reynolds numbers during the evolution
- With the standard DNMR choices of the transport coefficients and lattice QCD EoS, $\alpha = 1.5$ can ensure necessary causality conditions

COMPARISONS IN HYDRODYNAMIC EVOLUTION

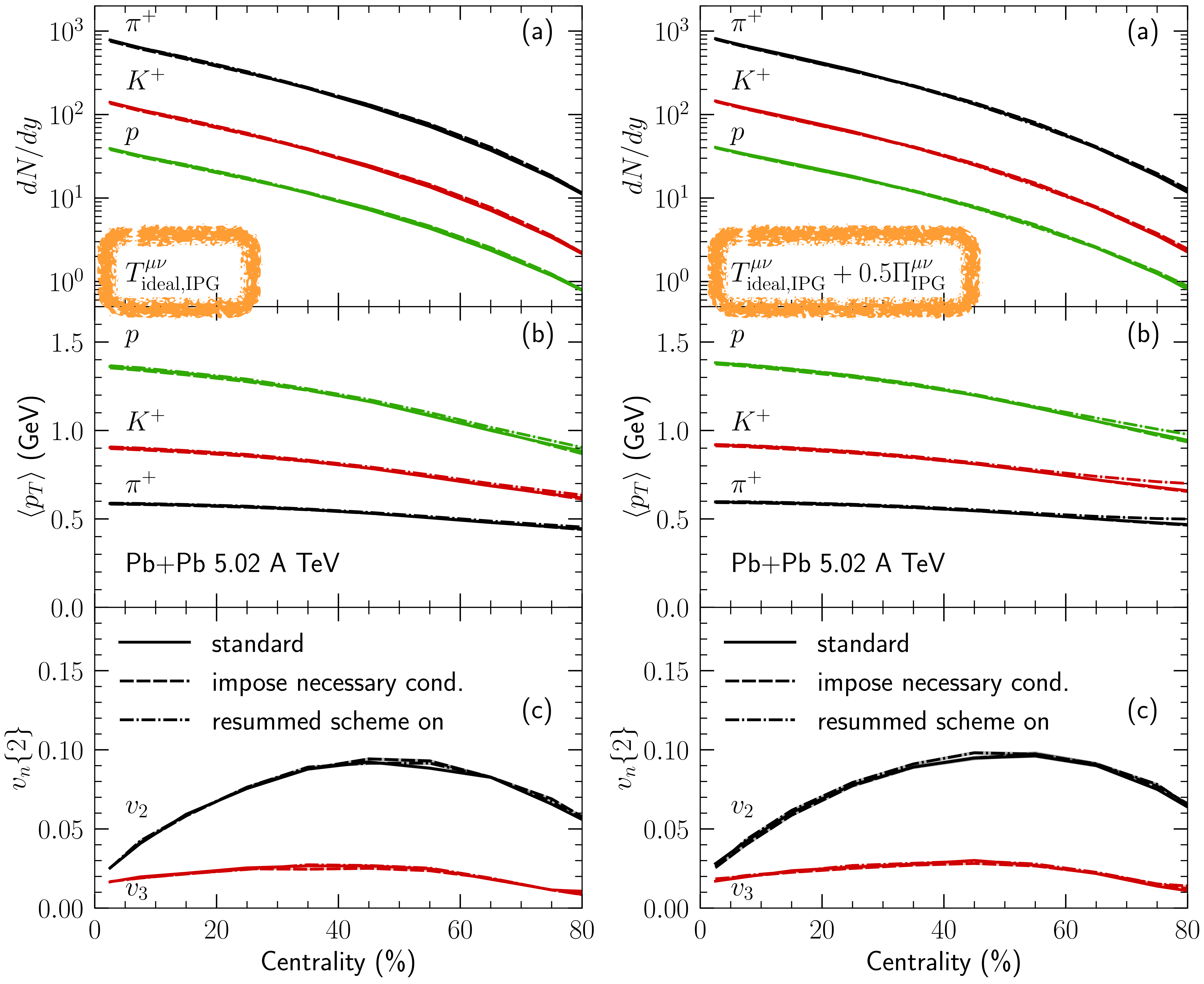
C. Chiu, G. Denicol, M. Luzum, and C. Shen, in preparation



- Difference are mostly present during the early-stage of the evolution

EFFECTS ON FINAL-STATE OBSERVABLES

C. Chiu, G. Denicol, M. Luzum, and C. Shen, in preparation



- Different numerical schemes show small differences in peripheral Pb+Pb collisions with IP-Glasma initial conditions, when the initial inverse Raynolds numbers are within allowed values

$$0 < \tilde{R}_\Pi + \tilde{R}_\pi < \frac{1}{\alpha} \text{ with } \alpha = 1.5$$

EFFECTS ON FINAL-STATE OBSERVABLES

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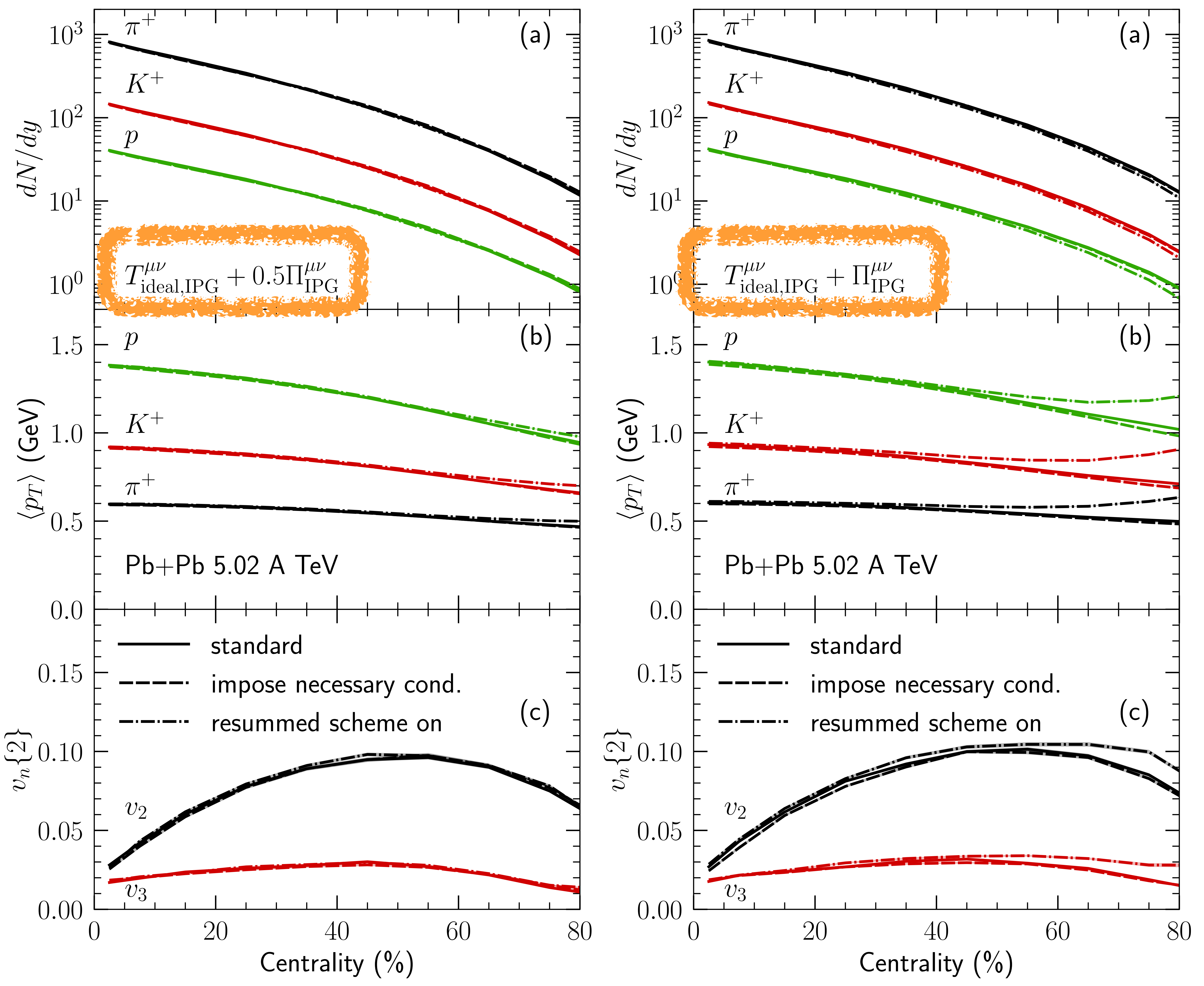

- Different numerical schemes show small differences in peripheral Pb+Pb collisions with IP-Glasma initial conditions, when the initial inverse Raynolds numbers are within allowed values

$$0 < \tilde{R}_\Pi + \tilde{R}_\pi < \frac{1}{\alpha} \text{ with } \alpha = 1.5$$

- The uncertainty from different numerical schemes increases when the systems are initially far from equilibrium

SUMMARY

- Causality conditions impose upper bounds for viscous stress tensors when one can match the pre-equilibrium phase $T^{\mu\nu}$ to second-order viscous hydrodynamics
- The choice of bulk and shear relaxation times can relax/tighten the causality constraints on the system's inverse Reynold's numbers
- We develop different numerical schemes to investigate the impact of causality constraints on final-state observables in heavy-ion collisions
- More robust formulation of hydrodynamics, such as maximum entropy hydrodynamics, could potentially reduce the theoretical uncertainty in describing the small system dynamics