Spin polarization and alignment in heavy-ion collisions

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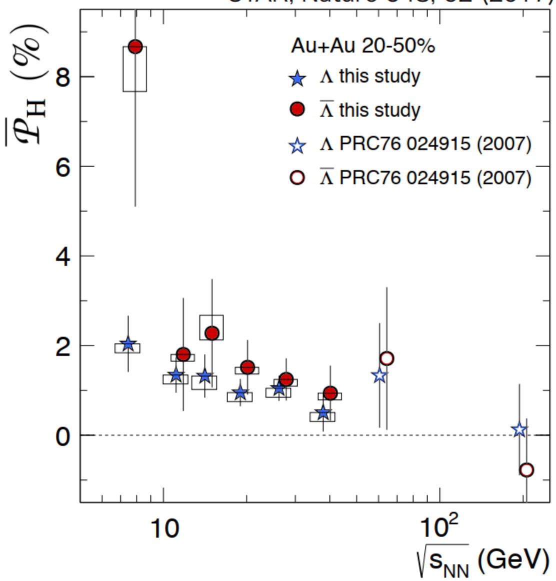
Outline

- **A microscopic model for spin-vorticity coupling that emerges from spin-orbit coupling in parton-parton scatterings**
- **Global and longitudinal spin polarization in heavy-ion collisions (solvable blast wave model)**
- **Spin Boltzmann (Kinetic) Equations for massive fermions and vector mesons**
- **Spin alignment of vector mesons**
- **Summary**

Rotation and Spin in HIC

STAR: global polarization of Λ hyperon

STAR, Nature 548, 62 (2017)

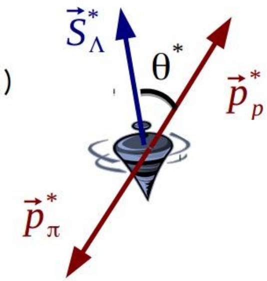


parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

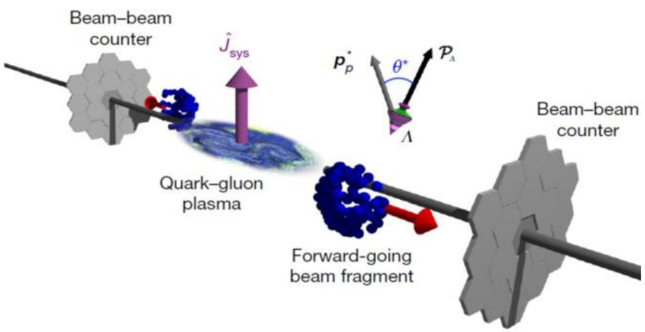
$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P_\Lambda} \cdot \mathbf{p_p^*})$$

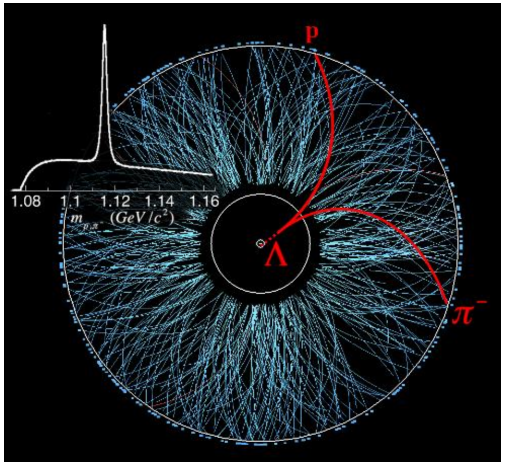
α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame



$\Lambda \rightarrow p + \pi^+$
(BR: 63.9%, c τ ~7.9 cm)

$\alpha_\Lambda = -\alpha_{\bar{\Lambda}} = 0.732 \pm 0.014$
Updated by BES III, PRL129, 131801 (2022)





ω = (9 ± 1)x10^21/s, the largest angular velocity that has ever been observed in any system

Some review articles on polarization in HIC

1. **Global and local spin polarization in heavy ion collisions: a brief overview, [phenomenology]**
   QW, Nucl. Phys. A 967, 225 (2017).

2. **Relativistic hydrodynamics for spin-polarized fluids, [theory]**
   Florkowski, Kumar, R. Ryblewski, Prog. Part. Nucl. Phys. 108, 103709 (2019).

3. **Polarization and Vorticity in the Quark–Gluon Plasma, [phenomenology]**
   Becattini, Lisa, Ann. Rev. Nucl. Part. Sci. 70, 395 (2020).

4. **Vorticity and Spin Polarization in Heavy Ion Collisions: Transport Models, [phenomenology]**
   Huang, Liao, QW, Xia, Lect. Notes Phys. 987, 281 (2021).

5. **Global polarization effect and spin-orbit coupling in strong interaction, [phenomenology]**
   Gao, Liang, QW, Wang, Lect. Notes Phys. 987, 195 (2021).

6. **Spin and polarization: a new direction in relativistic heavy ion physics, [theory+phenom.]**
   Becattini, Rept. Prog. Phys. 85, No.12, 122301 (2022)

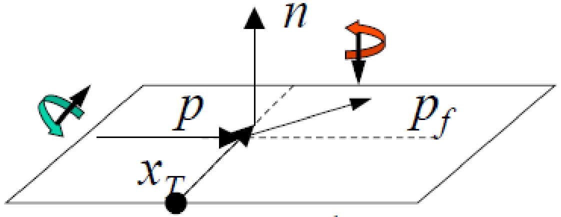
7. **Foundations and applications of quantum kinetic theory, [theory]**
   Hidaka, Pu, QW, Yang, Prog. Part. Nucl. Phys. 127, 103989 (2022).

8. **Spin polarization in relativistic heavy-ion collisions, [theory+experimet]**
   Becattini, Buzzegoli, Niida, Pu, Tang, QW, Int.J.Mod.Phys.E 33, 2430006 (2024).

Emergence of spin-vorticity coupling from spin-orbit coupling

Quark polarization in potential scatterings

- **Quark scatterings at small angle in static potential at impact parameter x_T**
- **Unpolarized and polarized cross sections**



$$A^0(q_T) = \frac{1}{q_T^2 + \mu^2}$$

screening mass

$$\boxed{\mu \sim T\sqrt{\alpha_S}}$$

$$\frac{d\sigma}{d^2\vec{x}_T} = \frac{d\sigma_+}{d^2\vec{x}_T} + \frac{d\sigma_-}{d^2\vec{x}_T} = 4C_T\alpha_s^2 K_0(\mu x_T)$$

$$\frac{d\Delta\sigma}{d^2\vec{x}_T} = \frac{d\sigma_+}{d^2\vec{x}_T} - \frac{d\sigma_-}{d^2\vec{x}_T} \propto \boxed{\vec{n}\cdot(\underline{\vec{x}_T \times \vec{p}})}$$

Spin quantization direction **OAM** **Spin-orbit coupling**

- **Polarization for small angle scattering and** $m_q \gg p, \mu$

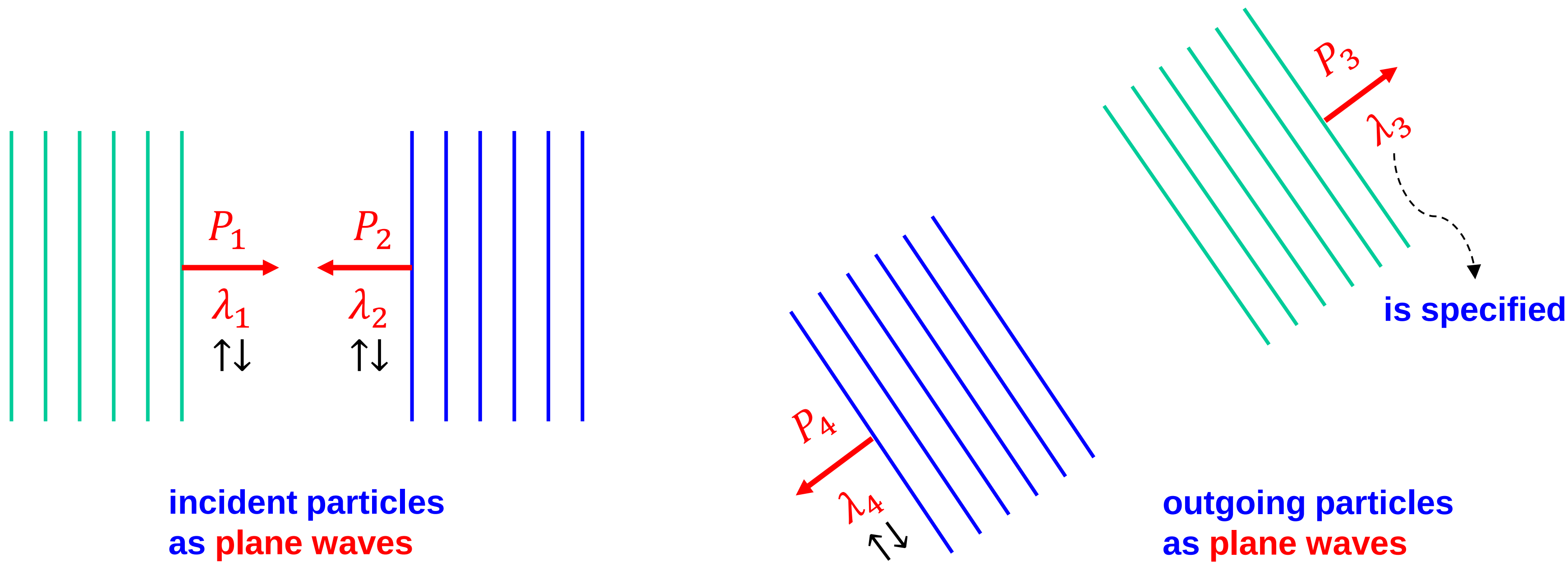
$$P_q \approx -\pi\frac{\mu p}{4m_q^2} \sim -\frac{\Delta E_{LS}}{E_0}$$

Liang, Wang, PRL 94, 102301(2005)

- **With initial polarization P_i, the final polarization P_f after one scattering is** $$P_f = P_i - \frac{(1-P_i^2)\pi\mu p}{2E(E+m) - P_i\pi\mu p}.$$

Huang, Huovinen, Wang, PRC84, 054910(2011)

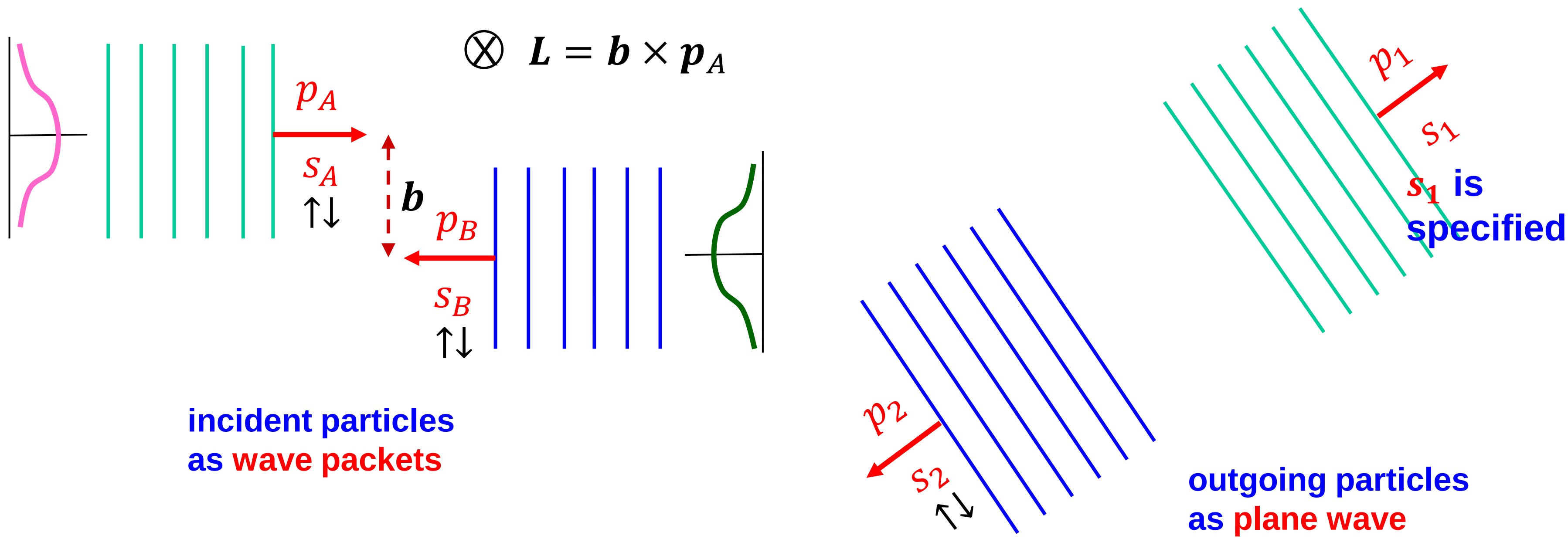
Collisions of particles as plane waves



Particle collisions as plane waves:
since there is no favored position for particles, so the OAM vanishing

$$\langle \hat{\boldsymbol{x}} \times \hat{\boldsymbol{p}} \rangle = \boldsymbol{0} \quad \Longrightarrow \quad \left(\frac{d\sigma}{d\Omega}\right)_{\lambda_3=\uparrow} = \left(\frac{d\sigma}{d\Omega}\right)_{\lambda_3=\downarrow}$$

Collisions of particles as wave packets



Particle collisions as wave packets: there is a transverse distance between two wave packets (impact parameter) giving non-vanishing OAM and then the polarization of one final particle

$$\boldsymbol{L} = \boldsymbol{b} \times \boldsymbol{p}_A \quad \Longrightarrow \quad \left(\frac{d\sigma}{d\Omega}\right)_{s_1=\uparrow} \neq \left(\frac{d\sigma}{d\Omega}\right)_{s_1=\downarrow}$$

Quark-quark scattering at fixed impact parameter

For the quark-quark scattering of spin-momentum states

$$q_1(P_1,\lambda_1)+q_2(P_2,\lambda_2)\rightarrow q_1(P_3,\lambda_3)+q_2(P_4,\lambda_4)$$

where $P_i=(E_i,\vec{p}_i)$ and λ_i denote spin states, the difference cross section (λ_3 is specified)

$c_{qq}=2/9$ (color factor)

$$d\sigma_{\lambda_3}=\frac{c_{qq}}{4F}\sum_{\lambda_1\lambda_2\lambda_4}\mathcal{M}(Q)\mathcal{M}^*(Q)(2\pi)^4\delta^{(4)}(P_1+P_2-P_3-P_4)\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}$$

fixed

sum over ↑↓

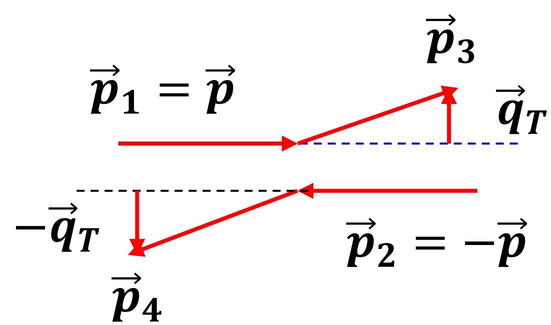
$Q=P_3-P_1=P_2-P_4$

(momentum transfer)

$F=4\sqrt{(P_1\cdot P_2)^2-m_1^2m_2^2}$ (flux factor)

Integrate $\vec{p}_4$ and $p_{3z}^{\pm}=\pm\sqrt{p^2-q_T^2}$

to remove $\delta^{(4)}(P_1+P_2-P_3-P_4)$



Quark-quark scattering at fixed impact parameter

We obtain $d\sigma_{\lambda_3}$ for scattered quark with spin state λ_3

$$d\sigma_{\lambda_3} = \frac{c_{qq}}{16F} \sum_{\lambda_1\lambda_2\lambda_4} \sum_{i=+,-} \frac{1}{(E_1+E_2)|p^i_{3z}|} \mathcal{M}(Q_i)\mathcal{M}^*(Q_i) \frac{d^2\vec{q}_T}{(2\pi)^2}$$

for small angle scattering, only $i=+$ is relevant

Jacobian

momentum transfer in small angle scattering

Then we can introduce impact parameter $\vec{x}_T = (x_T, \phi)$

$$d\sigma_{\lambda_3} = \frac{c_{qq}}{16F} \sum_{\lambda_1,\lambda_2,\lambda_4} \int d^2\vec{x}_T \int \frac{d^2\vec{q}_T}{(2\pi)^2} \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i(\vec{q}_T-\vec{k}_T)\cdot\vec{x}_T} \frac{\mathcal{M}(\vec{q}_T)\mathcal{M}^*(\vec{k}_T)}{\Lambda(\vec{q}_T)\Lambda(\vec{k}_T)}$$

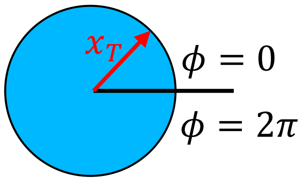
$\boxed{1}$

$$\Lambda(\vec{q}_T) = \sqrt{(E_1+E_2)|p^+_{3z}(q_T)|}$$

$\Rightarrow d^2\sigma_{\lambda_3}/d^2\vec{x}_T$

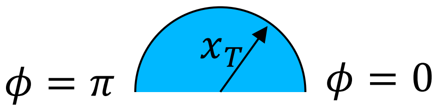
If we integrate over $\vec{x}_T$ in whole space we obtain

$$\sigma_{\lambda_3} = \int_0^\infty dx_T\, x_T \int_0^{2\pi} d\phi \frac{d^2\sigma_{\lambda_3}}{d^2\vec{x}_T} \Longrightarrow \sigma_\uparrow = \sigma_\downarrow$$



Quark-quark scattering at fixed impact parameter

If we integrate over $\vec{x}_T$ in half-space we obtain



$$\sigma_{\lambda_3} = \int_0^{\infty} dx_T\, x_T \int_0^{\pi} d\phi \frac{d^2\sigma_{\lambda_3}}{d^2\vec{x}_T} \Longrightarrow \sigma_{\uparrow} \neq \sigma_{\downarrow}$$

The differential cross section for spin-independent and spin-dependent part

$$\frac{d^2\sigma_{\lambda_3}}{d^2\vec{x}_T} = \frac{d^2\sigma}{d^2\vec{x}_T} + \lambda_3 \frac{d^2\Delta\sigma}{d^2\vec{x}_T}$$

$$\frac{d^2\sigma}{d^2\vec{x}_T} = \frac{1}{2}\left(\frac{d^2\sigma_{\uparrow}}{d^2\vec{x}_T} + \frac{d^2\sigma_{\downarrow}}{d^2\vec{x}_T}\right) = F(x_T)$$

$$\frac{d^2\Delta\sigma}{d^2\vec{x}_T} = \frac{1}{2}\left(\frac{d^2\sigma_{\uparrow}}{d^2\vec{x}_T} - \frac{d^2\sigma_{\downarrow}}{d^2\vec{x}_T}\right) = \underbrace{\vec{n}\cdot(\vec{x}_T \times \vec{p})}_{\text{spin-orbit coupling}}\Delta F(x_T)$$

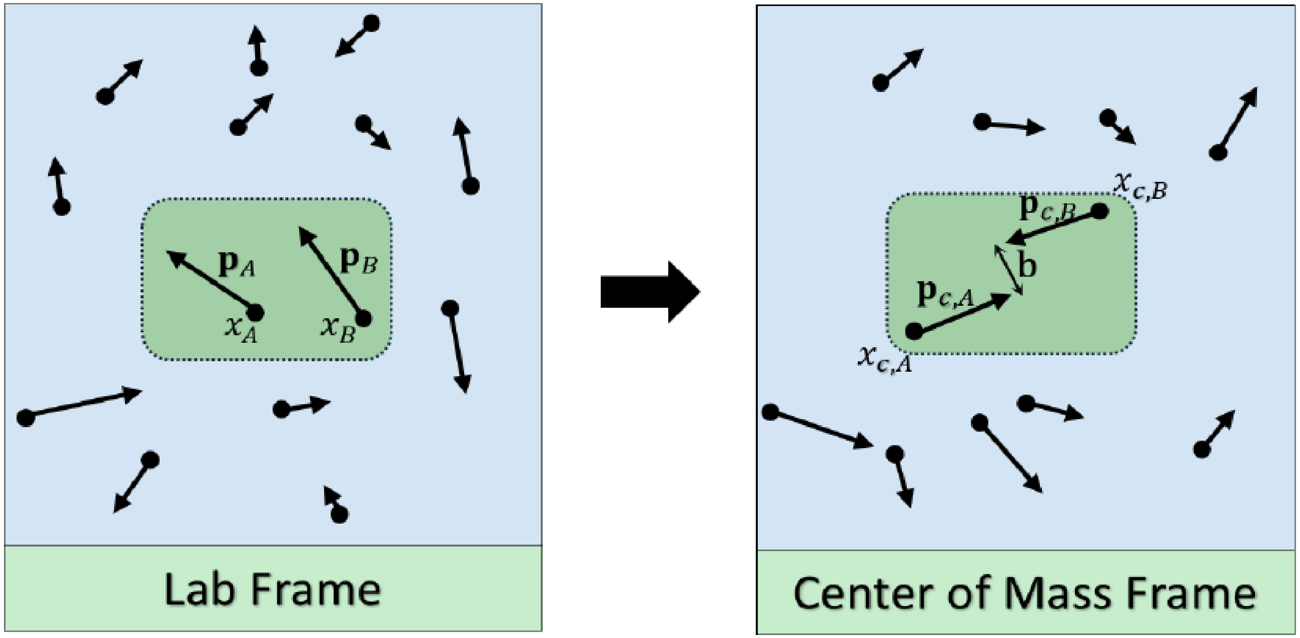
$$\boxed{\begin{aligned}\Delta\sigma &= \int_0^{\infty} dx_T x_T \int_0^{\pi} d\phi \frac{d^2\Delta\sigma}{d^2\vec{x}_T} \\ \sigma &= \int_0^{\infty} dx_T x_T \int_0^{2\pi} d\phi \frac{d^2\Delta\sigma}{d^2\vec{x}_T}\end{aligned}}$$

$$\Downarrow$$

$$\boxed{P_q = \frac{\Delta\sigma}{\sigma}}$$

Gao, Chen, Deng, et al., PRC 77, 044902 (2008)

Ensemble average in thermal QGP for global polarization through spin-orbit couplings in parton scatterings



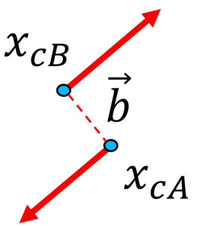
Zhang, Fang, QW, Wang, PRC 100, 064904 (2019)

Collisions of particles at different space-time points

- **Two incident particles at** $x_A = (t_A, \mathrm{x}_A)$ **and** $x_B = (t_B, \mathrm{x}_B)$ **in the lab frame**

$$t_A = t_B \xrightarrow{\boxed{\mathrm{x}_A \neq \mathrm{x}_B}} t_{c,A} \neq t_{c,B}$$

$$t_A \neq t_B \xrightarrow[\boxed{\mathrm{x}_{c,A} \neq \mathrm{x}_{c,B}}]{} t_{c,A} = t_{c,B}$$



CM frame

- **We impose the causality condition in CM frame for scattering of particles at two different space-time points (the time interval and longitudinal distance of two space-time points should be small enough for scattering to take place)**

$$\Delta t_c = t_{c,A} - t_{c,B} = 0$$
$$\Delta x_{c,L} = \hat{\mathrm{p}}_{c,A} \cdot (\mathrm{x}_{c,A} - \mathrm{x}_{c,B}) = 0$$

CM frame: collisions take place at the same time and longitudinal position but displaced by impact parameter

From spin-orbit coupling to spin-vorticity coupling: ensemble average

- **Quark polarization rate per unit volume: 10D + 6D integration**

$$
\begin{aligned}
\frac{d^4\mathbf{P}_{AB\to 12}(X)}{dX^4} &= \frac{\pi}{(2\pi)^4}\frac{\partial(\beta u_\rho)}{\partial X^\nu}\int \boxed{\frac{d^3p_A}{(2\pi)^3 2E_A}\frac{d^3p_B}{(2\pi)^3 2E_B}} \\
&\times |v_{c,A}-v_{c,B}| \boxed{[\Lambda^{-1}]^\nu_j} \mathbf{e}_{c,i}\epsilon_{ikh}\hat{\mathbf{p}}^h_{c,A} \\
&\times f_A(X,p_A)\, f_B(X,p_B)\,(p^\rho_A - p^\rho_B) \boxed{\Theta_{jk}(\mathbf{p}_{c,A})} \\
&\equiv \frac{\partial(\beta u_\rho)}{\partial X^\nu}\boxed{\mathbf{W}^{\rho\nu}}
\end{aligned}
$$

6D integral

Lorentz boost

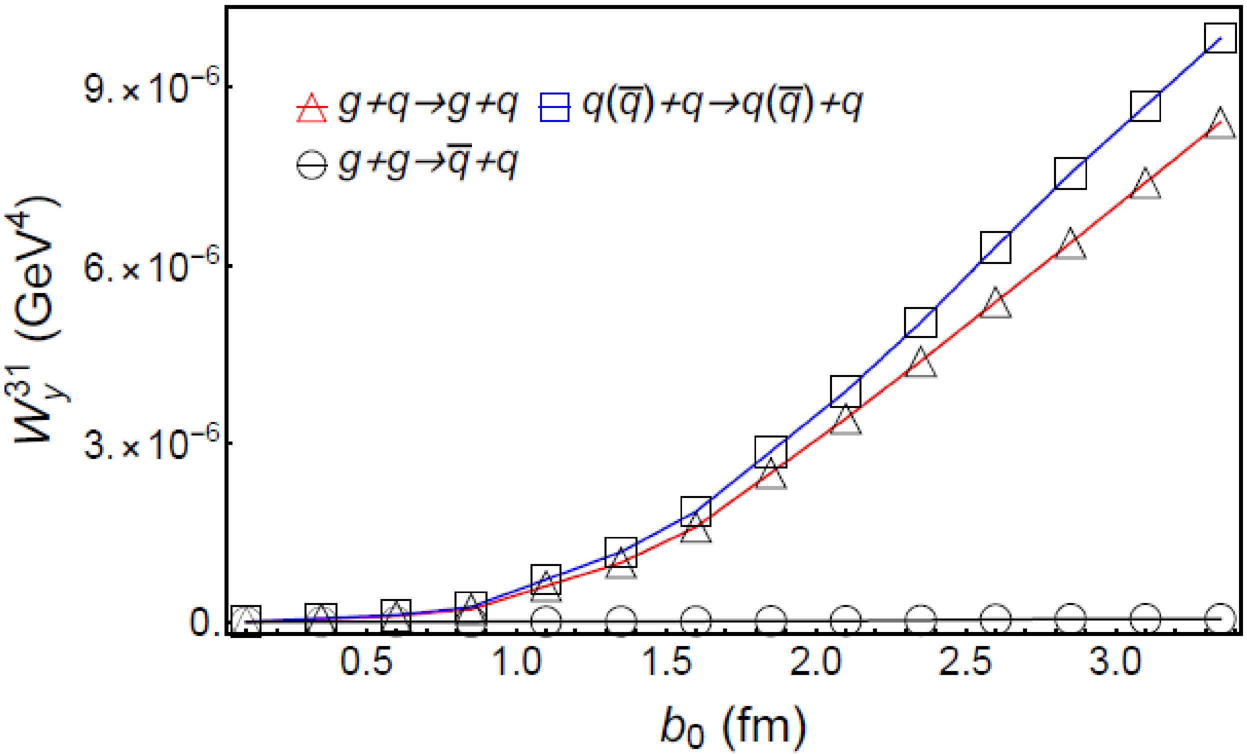
10D integral

16D integral !!

- **Numerical challenge !!! We have developed ZMCintegral-3.0, a Monte Carlo integration package that runs on multi-GPUs [Wu, Zhang, Pang, QW, Comp. Phys. Comm. (2020) (1902.07916)]**
- **Another challenge: there are more than 5000 terms in polarized amplitude squared for 2-to-2 parton scatterings**

$$
I_M^{q_a q_b \to q_a q_b}(s_2) = \sum_{s_A, s_B, s_1}\sum_{i,j,k,l} \mathcal{M}\left(\{s_A,k_A;s_B,k_B\}\to\{s_1,p_1;s_2,p_2\}\right)\mathcal{M}^*\left(\{s_A,k'_A;s_B,k'_B\}\to\{s_1,p_1;s_2,p_2\}\right)
$$

Numerical results for quark polarization



The cutoff b_0 is of the order of hydro length scale $1/\partial u(x)$ and larger than interaction scale $1/m_D$: $b_0 \sim \frac{1}{\partial u(x)} > \frac{1}{m_D}$

$$\frac{d^4 \mathrm{P}_{AB\rightarrow 12}(X)}{dX^4} = 2W\nabla_X \times (\beta \mathbf{u})$$

Zhang, Fang, QW, et al., PRC 100, 064904 (2019)

Local and global equilibrium for spin dof

Spin polarization vector on the freeze-out hypersurface from thermal vorticity

$$P_\omega^\mu = \frac{\int d\Sigma_\lambda p^\lambda f_{FD} \hat{P}_\omega^\mu}{\int d\Sigma_\lambda p^\lambda f_{FD}}$$

$$\hat{P}_\omega^\mu = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}(1-f_{FD})\omega_{\nu\sigma}p_\tau$$

$$\omega^{\mu\nu} = -\frac{1}{2}\left[\partial^\mu\left(\beta u^\nu\right) - \partial^\nu\left(\beta u^\mu\right)\right]$$

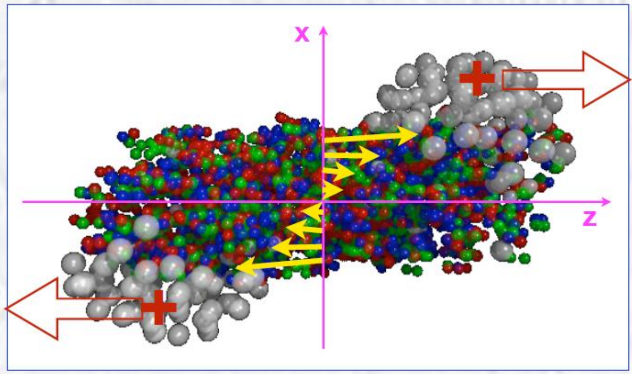
Becattini, Chandra, Del Zanna, Grossi, Ann. Phys. (2013); Fang, Pang, QW, Wang, Phys. Rev. C (2016)

Non-relativistic statistical mechanics

$$f \sim \exp\left[-\beta\left(E - \mu_i Q_i - \boldsymbol{\mu}_B \cdot \boldsymbol{B} - \underline{\boldsymbol{\omega}\cdot\boldsymbol{S}}\right)\right]$$

spin potential

Becattini, Karpenko, Lisa, Upsal, Voloshin, Phys. Rev. C (2017)



$$\boldsymbol{\omega} = \frac{1}{2}\nabla\times\mathbf{v} \quad \approx \frac{1}{2}\frac{\partial v_z}{\partial x}$$

Numerical results for P_y

AMPT transport model

-- Li, Pang, QW, Xia, PRC96, 054908(2017)

-- Wei, Deng, Huang, PRC99, 014905(2019)

UrQMD + vHLLE hydro

-- Karpenko, Becattini, EPJC 77, 213(2017)

PICR hydro

-- Xie, Wang, Csernai, PRC 95,031901(2017)

Chiral Kinetic Equation + Collisions

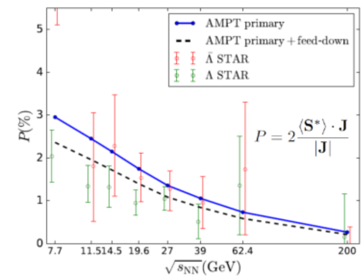
-- Sun, Ko, PRC96, 024906(2017)

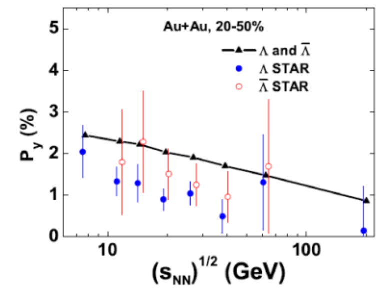
-- Liu, Sun, Ko, PRL125, 062301(2020)

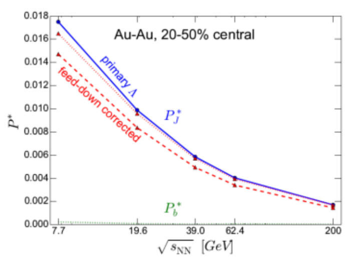
AVE+3FD

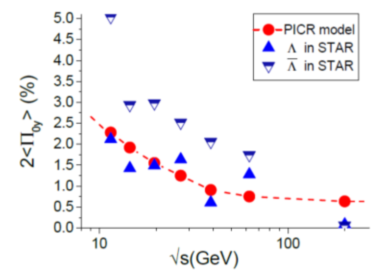
-- Ivanov, 2006.14328

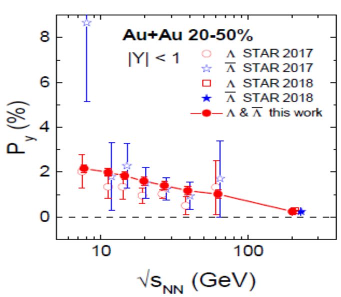
Other works …

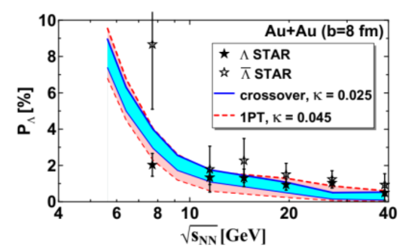






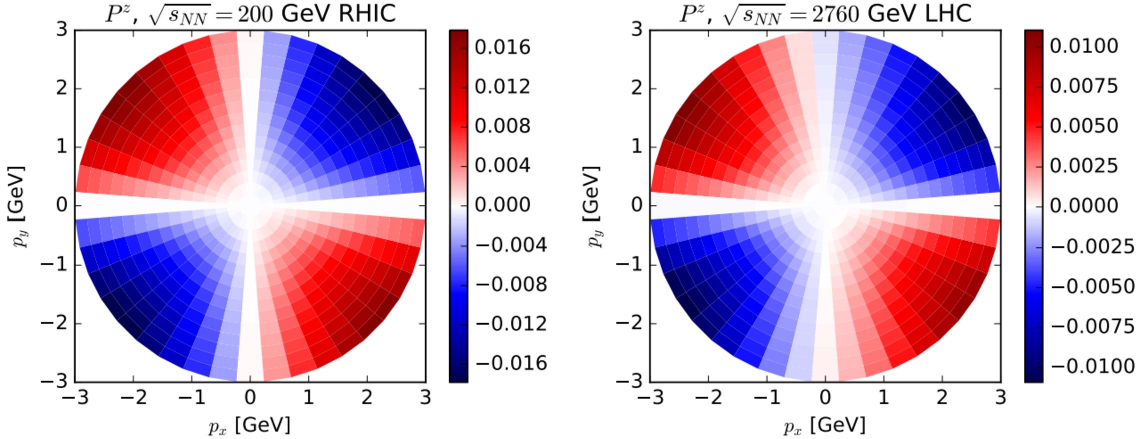




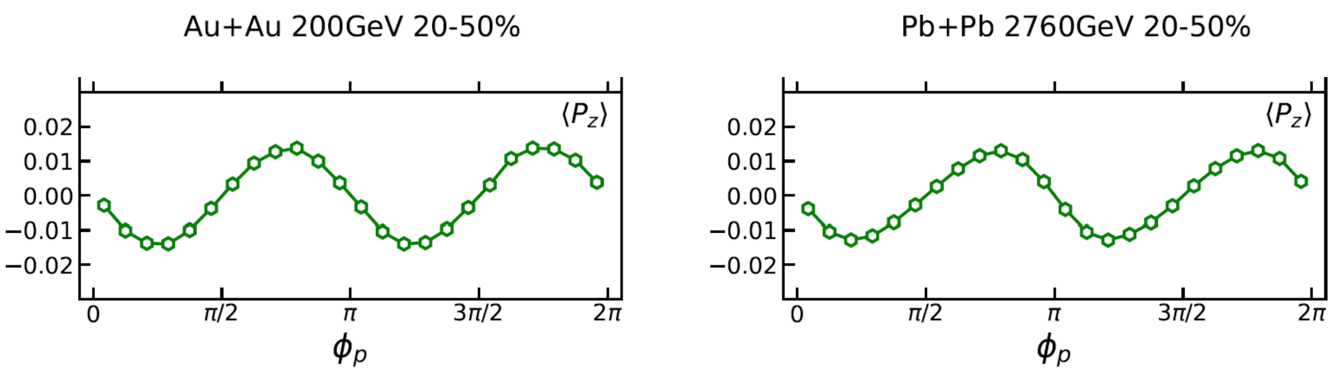


Longitudinal polarization P_z

Hydro predictions $P^z \sim -sin(2\phi)$ from thermal vorticity



Becattini, Karpenko PRL (2018)



Xai, Li, Tang, QW, PRC (2018)

Longitudinal polarization P_z

T-vorticity can explain the data

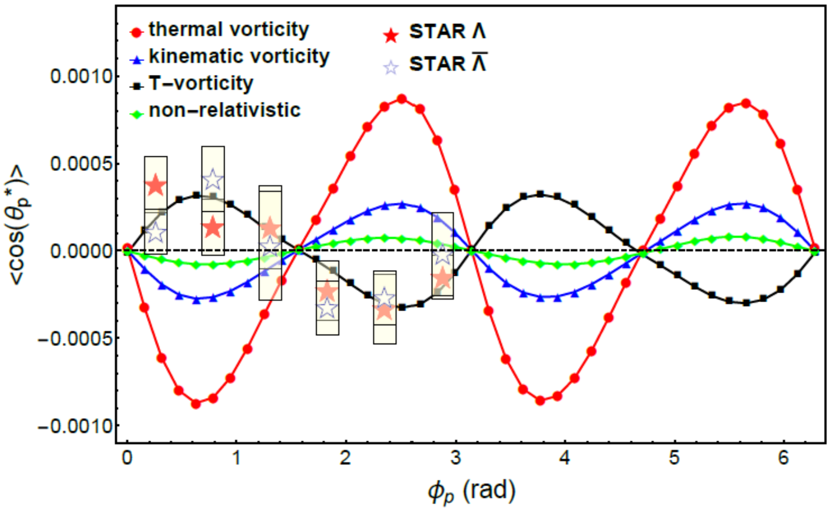
Wu, Pang, Huang, QW, PRR(2019); NPA (2021).

$$\omega_{\mu\nu}^{(NR)} = -\frac{1}{2}\left[\partial_\mu u_\nu - \partial_\nu u_\mu - u_\mu u^\alpha \partial_\alpha u_\nu + u_\nu u^\alpha \partial_\alpha u_\mu\right]$$

$$\omega_{\mu\nu}^{(K)} = -\frac{1}{2T}(\partial_\mu u_\nu - \partial_\nu u_\mu)$$

$$\omega_{\mu\nu}^{(T)} = -\frac{1}{2T^2}\left[\partial_\mu (T u_\nu) - \partial_\nu (T u_\mu)\right]$$

$$\omega_{\mu\nu}^{(th)} = -\frac{1}{2}\left[\partial_\mu (\beta u_\nu) - \partial_\nu (\beta u_\mu)\right]$$



Freeze-out formula

$$P_\omega^\mu = \frac{\int d\Sigma_\lambda p^\lambda f_{FD} \hat{P}_\omega^\mu}{\int d\Sigma_\lambda p^\lambda f_{FD}}$$

$$\hat{P}_\omega^\mu = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}(1-f_{FD})\Omega_{\nu\sigma}p_\tau$$

$$\Omega_{\mu\nu} = \frac{1}{T}\omega_{\mu\nu}^{(NR)},\ \frac{1}{T}\omega_{\mu\nu}^{(K)},\ \frac{1}{T^2}\omega_{\mu\nu}^{(T)},\ \omega_{\mu\nu}^{(th)}$$

Becattini, Chandra, Del Zanna, Grossi, Annals Phys. (2013);
Fang, Pang, QW, Wang, PRC(2016)

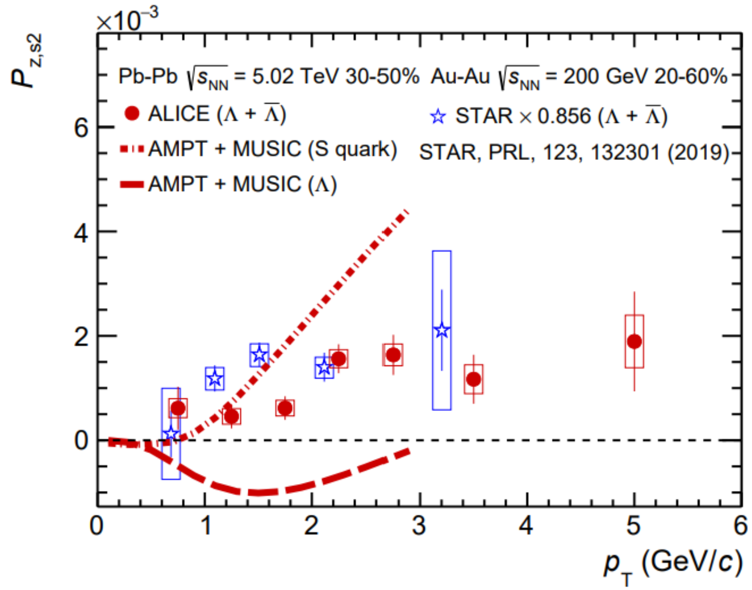
Longitudinal polarization P_z

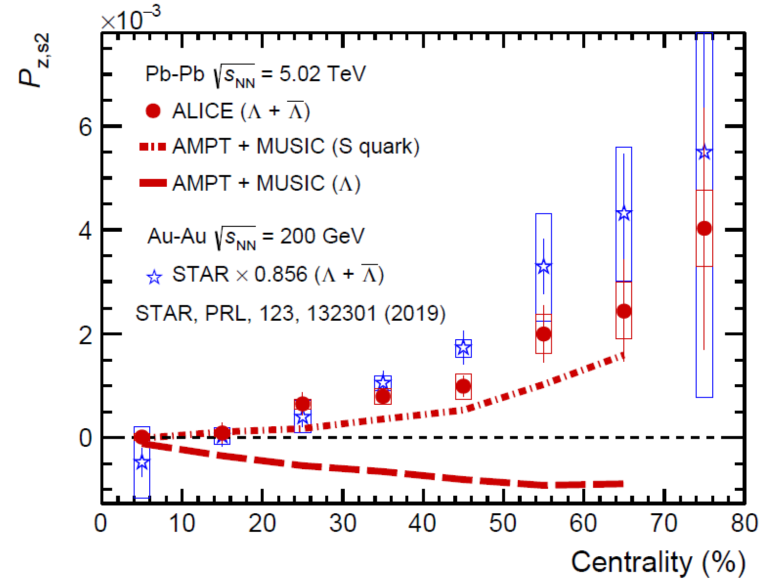
Polarization from shear stress tensor

$$\hat{P}^{\mu}_{\xi} = -\frac{1}{2m}\epsilon^{\mu\nu\sigma\tau}(1-f)\frac{p_\tau p^\rho}{E_p}\hat{t}_\nu \xi_{\rho\sigma}$$

$$\xi^{\mu\nu} = \frac{1}{2}\left[\partial^\mu\left(\beta u^\nu\right) + \partial^\nu\left(\beta u^\mu\right)\right]$$

Fu, Liu, Pang, et al. (2021);
Becattini, Buzzegoli, et al (2021);
Yi, Pu, Yang (2021)
Wu, Yi, Qin, Pu (2022);
Alzhrani, Ryu, Shen (2022);

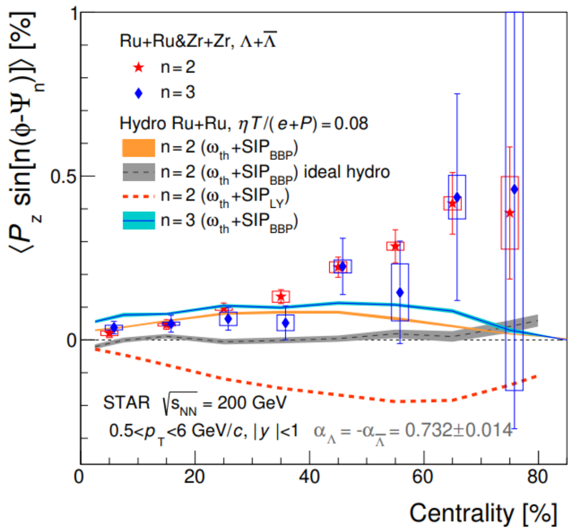


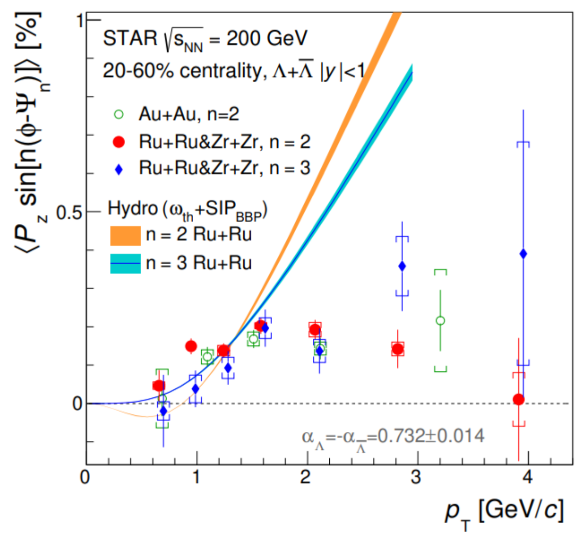


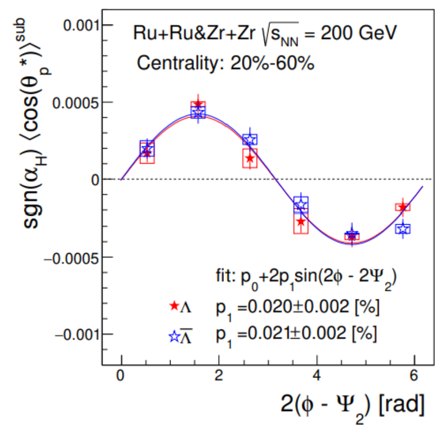
ALICE, PRL(2022); 2107.11183

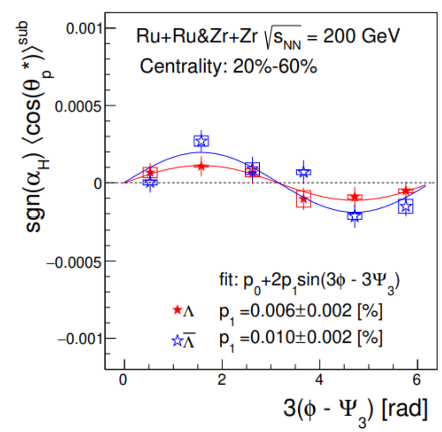
Longitudinal polarization P_z

Isobar collisions and third harmonics

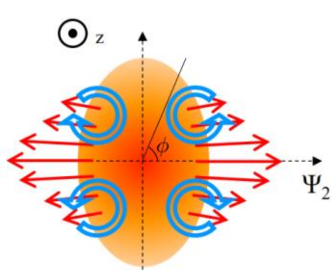


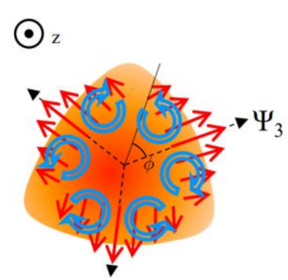






STAR, PRL(2023); 2303.09074
Alzhrani, Ryu, Shen (2022);



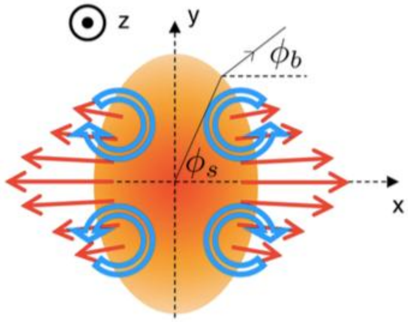


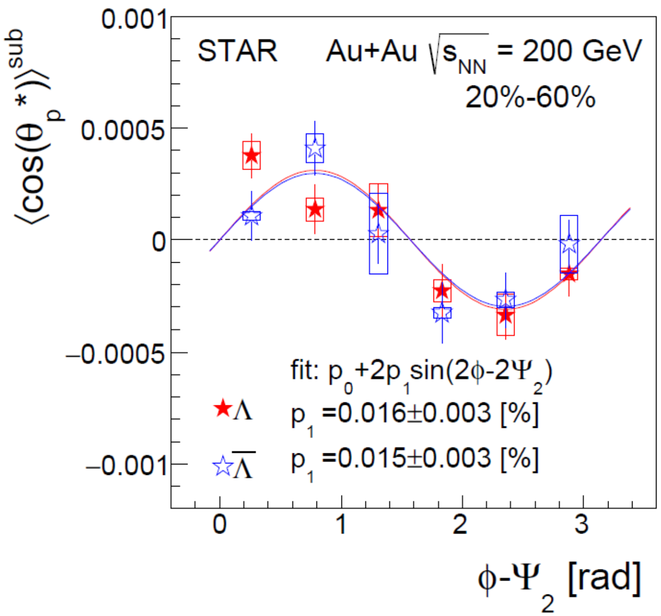
Longitudinal polarization P_z

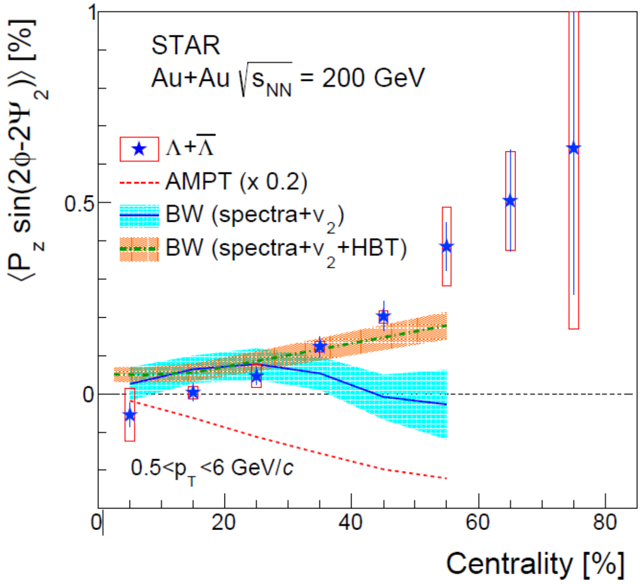
Blast wave picture (approximation)

$$\mathbf{v} \sim \mathbf{e}_r v_r \left[1 + v_2 \cos(2\phi)\right]$$

$$P^z \sim \partial_x v_y - \partial_y v_x \sim \frac{1}{r} v_2 v_r \sin(2\phi)$$







STAR, PRL13,132301 (2019); Voloshin, 1710.08934

A solvable blast-wave model

The particle's distribution function in phase space $f(x,p)$ is assumed to follow the Boltzmann distribution

$$f(x,p) \equiv f(p\cdot u) = \exp(-\beta p\cdot u)$$

flow-momentum correspondence: $\eta \sim Y$, $\phi_b \sim \phi_p$

$$= \exp\{-\beta\,[m_T \cosh\rho \cosh\underline{(\eta - Y)} - p_T \sinh\rho \cos\underline{(\phi_b - \phi_p)}]\}$$

where the flow four-velocity and the particle's four-momentum can be parameterized as

$$u^\mu(x) = (\cosh\eta\cosh\rho, \sinh\rho\cos\phi_b, \sinh\rho\sin\phi_b, \sinh\eta\cosh\rho)$$

$$p^\mu = (m_T\cosh Y, p_T\cos\phi_p, p_T\sin\phi_p, m_T\sinh Y)$$

and the transverse expansion of the fireball is described by the transverse rapidity

$$\rho(r,\phi_s,\eta) = \tilde{r}\,[\rho_0 + \underbrace{\rho_1(\eta)}_{\text{directed flow}}\cos(\phi_b) + \underbrace{\rho_2}_{\text{elliptic flow}}\cos(2\phi_b)]$$

$$\tilde{r} = \sqrt{\frac{(r\cos\phi_s)^2}{R_x^2} + \frac{(r\sin\phi_s)^2}{R_y^2}}$$

$$R = \frac{1}{2}(R_x + R_y), \quad \epsilon = \frac{1}{R}(R_x - R_y) \ll 1$$

$$\approx \frac{r}{R}\left[1 + \frac{1}{2}\epsilon\cos(2\phi_s)\right]$$

A solvable blast-wave model

The functional relation between ϕ_b and ϕ_s is

$$\tan\phi_b = \frac{R_x^2}{R_y^2}\tan\phi_s \approx (1-2\epsilon)\tan\phi_s$$

We use the ordering of parameters $\boxed{\alpha_1 \sim \rho_2 \sim \epsilon \ll \rho_0}$

We denote α_1, ρ_2 and ϵ are O(ϵ) quantities, while ρ_0 is an $O(1)$ quantity, so α_1, ρ_2 and ϵ can be treated as perturbations relative to ρ_0.

Physical observables can be computed on the freeze-out hyper-surface by

$$\langle O(p)\rangle = \frac{\int d^4x O(x,p) S(x,p)}{\int d^4x S(x,p)}$$

$O(x,p)$: Obserbavles
$S(x,p)$: emission function

$$S(x,p) = m_T \cosh(\eta - Y)\delta(\tau-\tau_f)\Theta(R-r)f(x,p)$$

Dong, Yin, Sheng, Yang, QW, 2311.18400

A solvable blast-wave model

The momentum integrated observables can be obtained by integration over all components of the on-shell momentum

$$\langle O\rangle = \frac{\int d^4x d^3\mathbf{p} E_p^{-1} O(x,p) S(x,p)}{\int d^4x d^3\mathbf{p} E_p^{-1} S(x,p)}$$

The integral elements of space-time and on-shell momentum are

$$d^4x = \tau r d\tau d\eta dr d\phi_s, \quad \frac{d^3\mathbf{p}}{E_p} = p_T dp_T dY d\phi_p$$

The partially integrated observables can also be obtained by integration over some components of the on-shell momentum

$$\langle O\rangle (p_T) = \frac{\int d^4x dY d\phi_p O(x,p) S(x,p)}{\int d^4x dY d\phi_p S(x,p)}$$

$$\langle O\rangle (\phi_p) = \frac{\int d^4x dp_T dY p_T O(x,p) S(x,p)}{\int d^4x dp_T dY p_T S(x,p)}$$

Dong, Yin, Sheng, Yang, QW, 2311.18400

A solvable blast-wave model

In the leading order of the flow-momentum correspondence with $\eta = 0$ and $\phi_b = \phi_p$, the analytical results for P_ω^i and P_ξ^i can be obtained to $O(\epsilon)$

Dong, Yin, Sheng, Yang, QW, 2311.18400

Directed flow

$$\hat{P}_\omega^y \approx \underline{\alpha_1} \frac{r}{4mTR\tau} (m_T \cosh\rho - p_T \sinh\rho) \underline{\cos^2\phi_p}$$

$$\hat{P}_\xi^y \approx \underline{\alpha_1} \frac{r}{4mTR\tau} \frac{p_T}{m_T} (p_T \cosh\rho - m_T \sinh\rho) \underline{\cos^2\phi_p}$$

$$\hat{P}_\omega^z \approx \frac{1}{4mT} \left[\underline{2\rho_2} \frac{1}{R} (m_T \cosh\rho - p_T \sinh\rho) - \underline{\epsilon} \frac{1}{r} m_T \sinh\rho \right] \underline{\sin(2\phi_p)}$$

$$\hat{P}_\xi^z \approx \frac{1}{4mT} \frac{p_T}{m_T} \left[\underline{2\rho_2} \frac{1}{R} (p_T \cosh\rho - m_T \sinh\rho) + \underline{\epsilon} \frac{1}{r} p_T \sinh\rho \right] \underline{\sin(2\phi_p)}$$

elliptic flow

ellipticity in emission area

A solvable blast-wave model

The average values P_y and P_z as functions of ϕ_p on the freeze-out hyper-surface

Dong, Yin, Sheng, Yang, QW, 2311.18400

$$
\begin{aligned}
P^y(\phi_p) &= \left\langle \hat{P}^y_\omega + \hat{P}^y_\xi \right\rangle (\phi_p) \\
&\approx \underline{\alpha_1} \frac{1}{4mT_f\tau_f R} \frac{1}{N_0} \left[N_1(2,1,2) + N_1(2,3,0) - 2N_2(2,2,1) \right] \underline{\cos^2 \phi_p} \\
P^z(\phi_p) &= \left\langle \hat{P}^z_\omega + \hat{P}^z_\xi \right\rangle (\phi_p) \\
&\approx \underline{\rho_2} \frac{1}{2mT_f R} \frac{1}{N_0} \left[N_1(1,1,2) + N_1(1,3,0) - 2N_2(1,2,1) \right] \underline{\sin(2\phi_p)} \\
&\quad - \underline{\epsilon} \frac{1}{4mT_f} \frac{1}{N_0} \left[N_2(0,1,2) - N_2(0,3,0) \right] \underline{\sin(2\phi_p)}
\end{aligned}
$$

where

$$
N_0 = \int_{p_T^{\min}}^{p_T^{\max}} dp_T \int_0^R dr\, r p_T m_T\, K_1(\beta m_T \cosh \underline{\bar{\rho}}) I_0(\beta p_T \sinh \underline{\bar{\rho}})
$$

$$
\boxed{\bar{\rho} \equiv (r/R)\rho_0}
$$

$$
N_1(n_1,n_2,n_3) = \int_{p_T^{\min}}^{p_T^{\max}} dp_T \int_0^R dr\, r^{n_1} p_T^{n_2} m_T^{n_3}\, \cosh \bar{\rho}\, K_1(\beta m_T \cosh \underline{\bar{\rho}}) I_0(\beta p_T \sinh \underline{\bar{\rho}})
$$

$$
N_2(n_1,n_2,n_3) = \int_{p_T^{\min}}^{p_T^{\max}} dp_T \int_0^R dr\, r^{n_1} p_T^{n_2} m_T^{n_3}\, \sinh \underline{\bar{\rho}}\, K_1(\beta m_T \cosh \underline{\bar{\rho}}) I_0(\beta p_T \sinh \underline{\bar{\rho}})
$$

A solvable blast-wave model

We can also obtain $P^y(p_T)$ and $P^z_{\sin(2\phi_p)}(p_T)$ by integration over ϕ_p instead of p_T

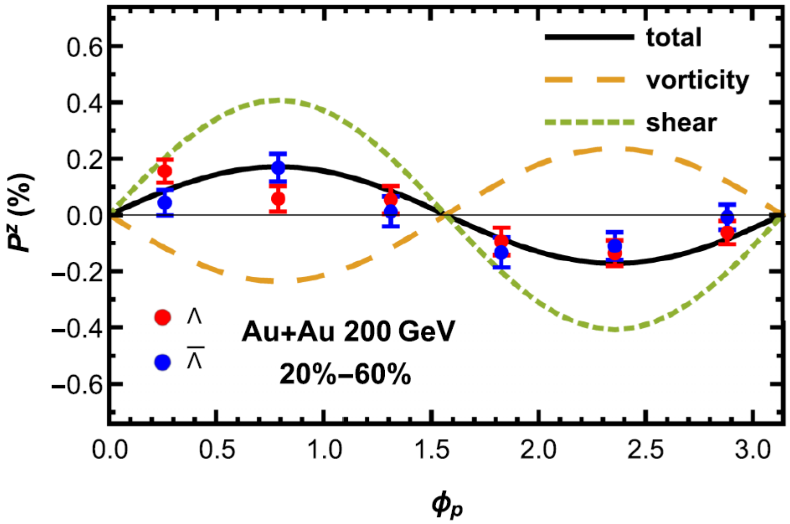
$$
\begin{aligned}
P^y(p_T) &= \left\langle \hat{P}^y_\omega + \hat{P}^y_\xi \right\rangle (p_T) \\
&\approx \alpha_1 \frac{1}{8mT_f R\tau_f} \frac{1}{N_0(p_T)} \left[N_{p1}(2,0,2) + N_{p1}(2,2,0) - 2N_{p2}(2,1,1)\right] \\
P^z_{\sin(2\phi)}(p_T) &\equiv \left\langle \left(\hat{P}^z_\omega + \hat{P}^z_\xi\right) \sin(2\phi_p) \right\rangle (p_T) \\
&\approx \rho_2 \frac{1}{4mT_f R} \frac{1}{N_0(p_T)} \left[N_{p1}(1,0,2) + N_{p1}(1,2,0) - 2N_{p2}(1,1,1)\right] \\
&\quad - \epsilon \frac{1}{8mT_f} \frac{1}{N_0(p_T)} \left[N_{p2}(0,0,2) - N_{p2}(0,2,0)\right]
\end{aligned}
$$

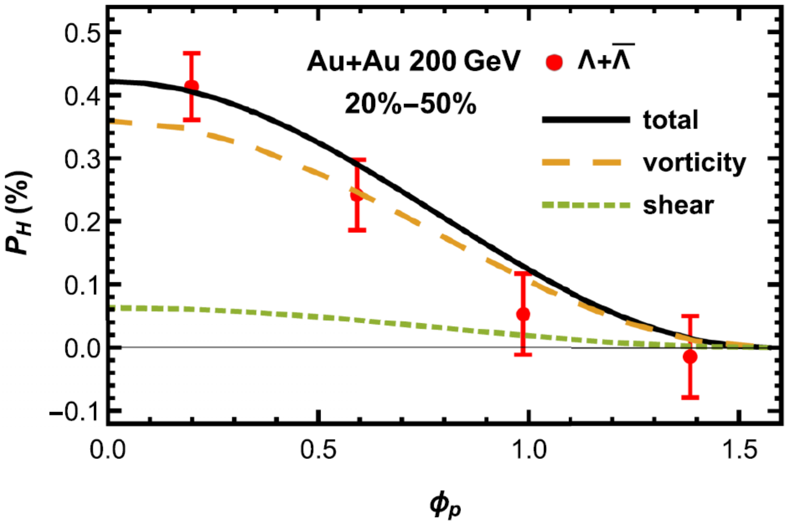
where $N_{p1,p2}(n_1, n_2, n_3)$ are integrals over r only (depending on p_T).

Dong, Yin, Sheng, Yang, QW, 2311.18400

Au+Au collisions at 200 GeV

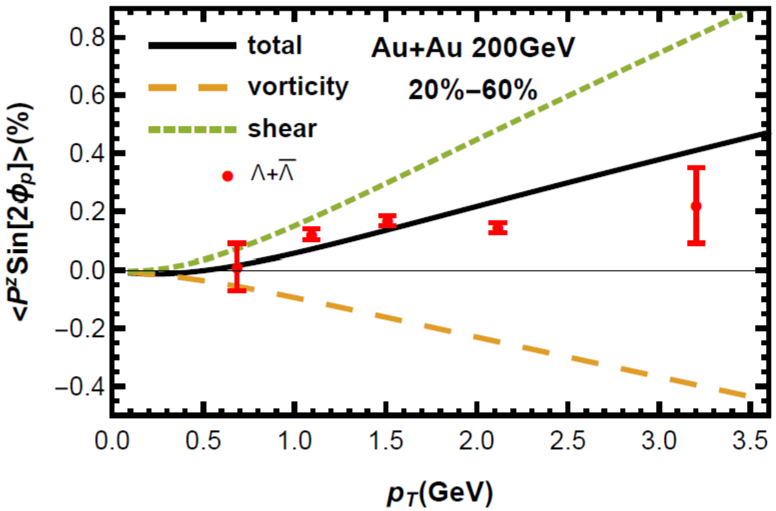
The results for P_z (left panel) and $P_H = -P_y$ (right panel) as functions of ϕ_p in Au+Au collisions at 200 GeV. We use data in 30-40% central collisions as an approximation. The P_T range is set to [0.5,6.0] GeV. The data of P_z and P_H are taken from Ref. [STAR:2019erd] and Ref. [STAR:2018gyt], respectively.

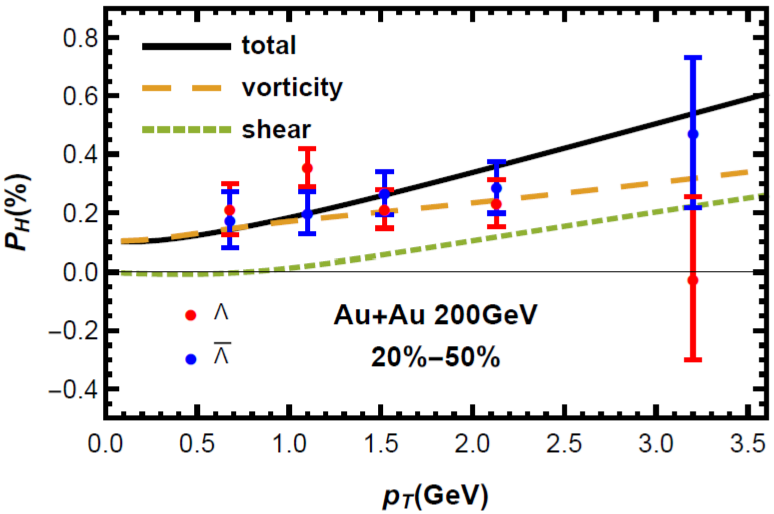




Au+Au collisions at 200 GeV

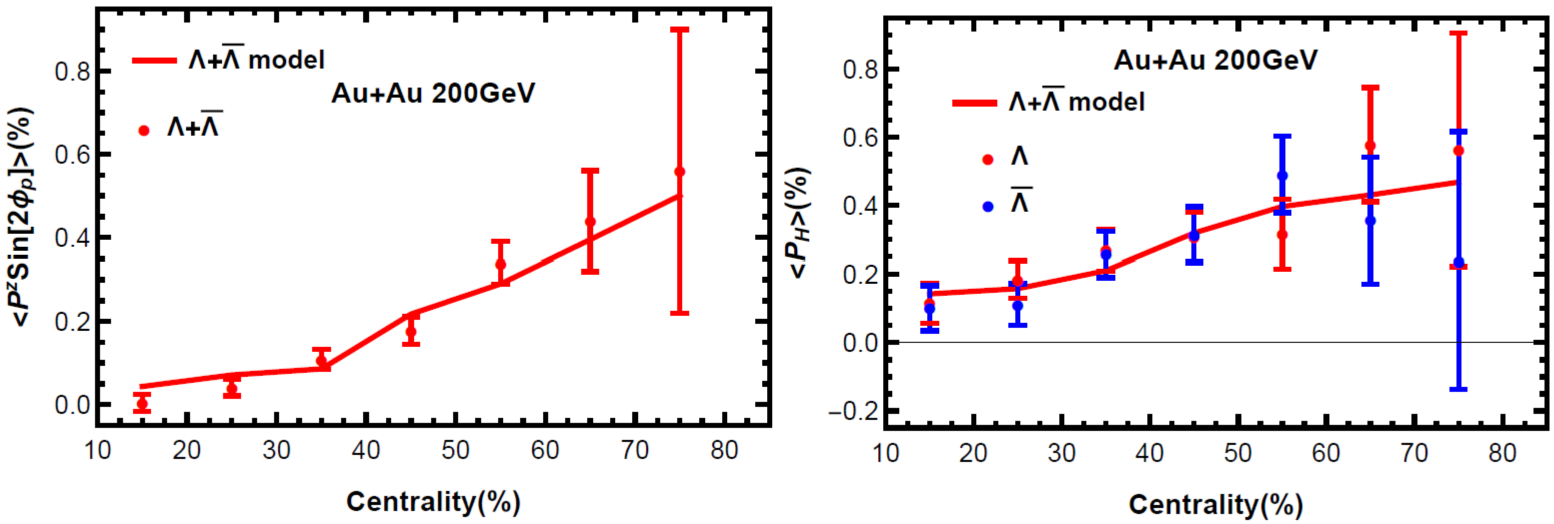
The results for $\langle P_z \sin(2\phi_p)\rangle$ and P_H as functions of p_T in Au+Au collisions at 200 GeV. We use the parameters of 30-40% central collisions as an approximation. The data of P_z and P_H are taken from Ref. [STAR:2019erd] and Ref. [STAR:2018gyt], respectively.





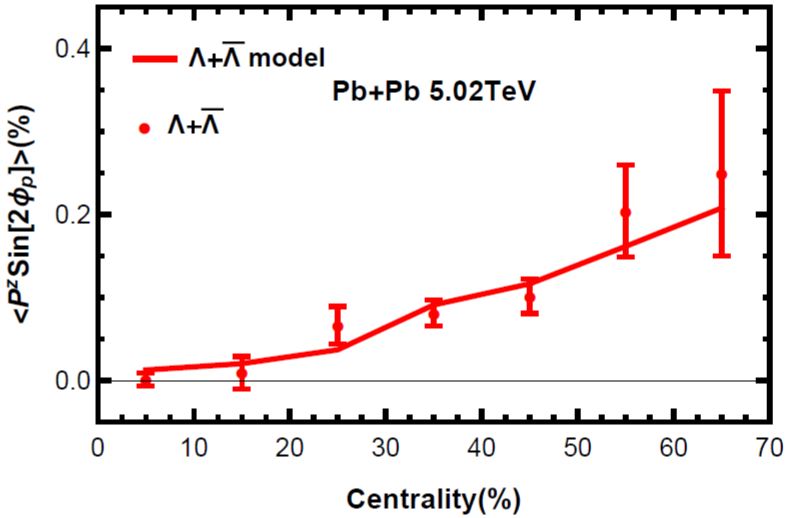
Au+Au collisions at 200 GeV

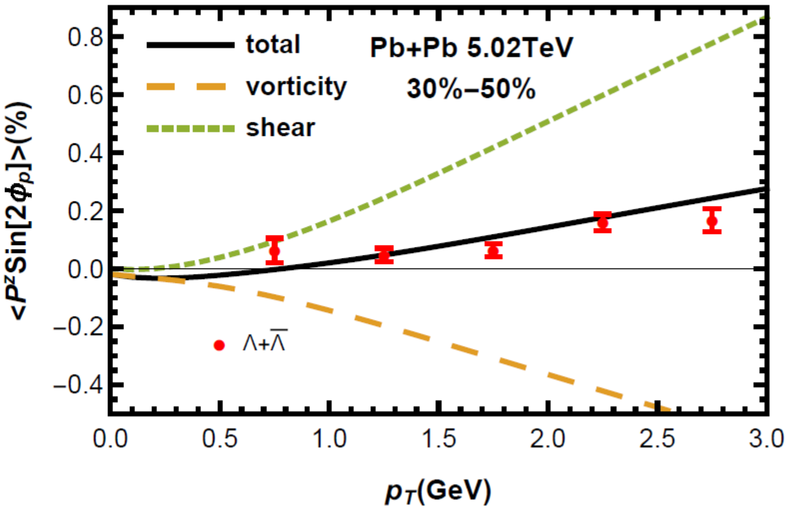
The result for the centrality dependence of $\langle P_z \sin(2\phi_p)\rangle$ and $\langle P_H\rangle$ in Au+Au collisions at 200 GeV. The p_T range is set to [0.5,6.0] GeV. The data of P_z and P_H are taken from Ref. [STAR:2019erd] and Ref. [STAR:2018gyt], respectively.



Pb+Pb collisions at 5.02 TeV

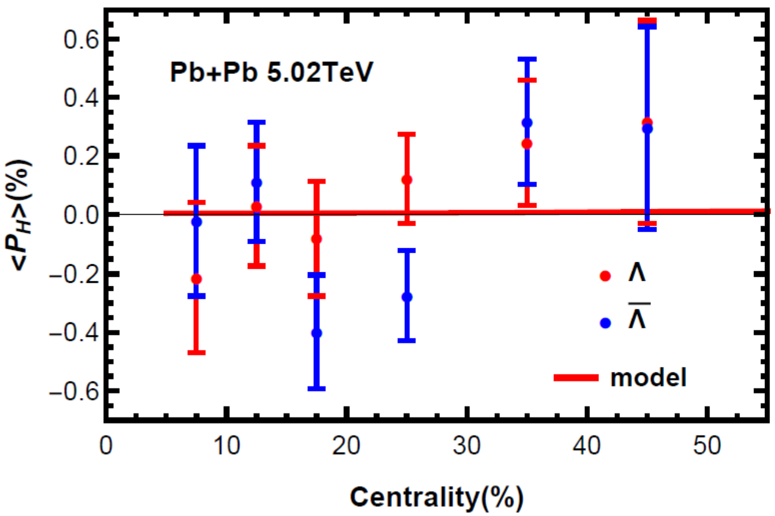
Spin polarization along the beam direction in Pb+Pb collisions at 5.02 TeV. The p_T range is set to [0.5,6.0] GeV in calculating the centrality dependence. The experimental data is from Ref. [ALICE:2021pzu].

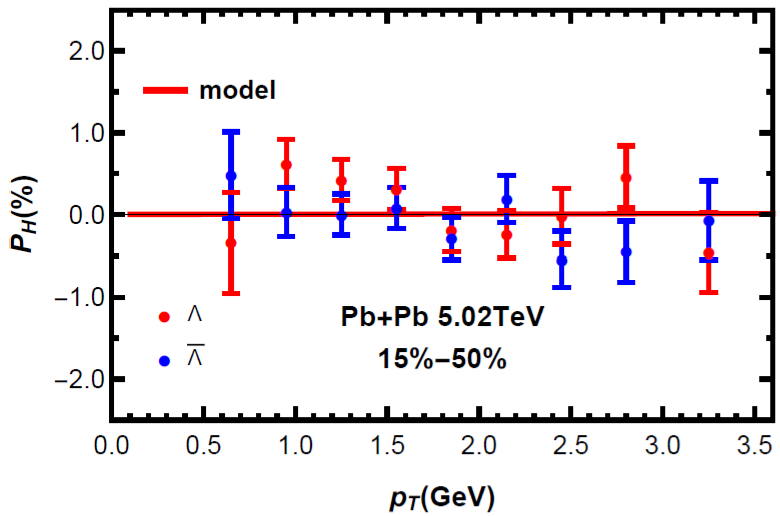




Pb+Pb collisions at 5.02 TeV

Spin polarization along the angular momentum direction in Pb+Pb collisions at 5.02 TeV. The p_T range is set to [0.5,6.0] GeV in calculating the centrality dependence. The experimental data is from Ref. [ALICE:2019onw].





Features: solvable blast-wave model

- This is an analytically solvable model for spin polarization based on the blast-wave picture of heavy-ion collisions with flow-momentum correspondence at the leading order.
- It not only gives the exact azimuthal angle dependences of spin polarization in the beam and angular momentum directions, but also gives their exact transverse momentum dependences.
- There are no contributions from temperature gradient.
- It can describe almost all available data for spin polarization with a few parameters constrained by transverse momentum spectra and collective flows of hadrons.
- It can be improved order by order through expansion in $\delta\phi = \phi_b - \phi_p$ and $\delta\eta = \eta - Y$.

Quantum kinetic equations with spin or Spin Boltzmann (kinetic) equations with Wigner functions

QKT for massive fermions in Wigner functions

- **Wigner function (4x4 matrix) for spin 1/2 massive fermions**

$$W_{\alpha\beta}(x,p) = \int d^4y \exp\left(\frac{i}{\hbar}p\cdot y\right)\left\langle \overline{\psi}_\beta\left(x-\frac{y}{2}\right)\psi_\alpha\left(x+\frac{y}{2}\right)\right\rangle$$

Heinz (1983); Vasak-Gyulassy-Elze (1987); Zhuang-Heinz (1996); Iancu-Blaizot (2001); QW-Redlich-Stoecker-Greiner (2002)

- **Wigner function decomposition in 16 generators of Clifford algebra**

spin 4-vector

$$W = \frac{1}{4}\left[\mathscr{F} + i\gamma^5\mathscr{P} + \gamma^\mu\mathscr{V}_\mu + \gamma^5\gamma^\mu\mathscr{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu}\right]$$

scalar p-scalar vector axial-vector tensor

$$j^\mu = \int d^4p\mathscr{V}^\mu, \qquad j_5^\mu = \int d^4p\mathscr{A}^\mu, \qquad T^{\mu\nu} = \int d^4p p^\mu\mathscr{V}^\nu$$

Recent reviews:
Hidaka-Pu-QW-Yang, PPNP (2022)
Gao-Liang-QW, IJMPA (2021)

Vasak-Gyulassy-Elze, Ann. Phys. 173, 462 (1987);
Elze-Gyulassy-Vasak, Nucl. Phys. B 276, 706 (1986);

Spin DOF: Matrix Valued Spin Distributions (MVSD)

Relativistic MVSD for fermion in QFT

$$p^\mu \equiv \frac{1}{2}(p_1^\mu + p_2^\mu) \qquad q^\mu \equiv p_1^\mu - p_2^\mu$$

$$f_{rs}(x,p) \equiv \int \frac{d^4 q}{2(2\pi)^3} \exp\left(-\frac{i}{\hbar} q \cdot x\right) \delta(\underline{p \cdot q}) \langle a^\dagger(\underline{s, \mathbf{p}_2}) a(\underline{r, \mathbf{p}_1}) \rangle$$

2 X 2 matrix

on-shell condition

$\boxed{r, s = \uparrow, \downarrow \text{ or } 1, 2}$

Relativistic MVSD can be parameterized in un-polarized and polarized parts

$\boxed{j = 1, 2, 3}$

$$f_{rs}^{(+)}(x, \mathbf{p}) = \frac{1}{2} \underline{f_q(x, \mathbf{p})} \left[\delta_{rs} - \underline{P_\mu^q(x, \mathbf{p})} \underline{n_j^{(+)\mu}(\mathbf{p})} \tau_{rs}^j \right],$$

$$f_{rs}^{(-)}(x, -\mathbf{p}) = \frac{1}{2} \underline{f_{\bar{q}}(x, -\mathbf{p})} \left[\delta_{rs} - \underline{P_\mu^{\bar{q}}(x, -\mathbf{p})} \underline{n_j^{(-)\mu}(\mathbf{p})} \tau_{rs}^j \right],$$

Pauli matrices in spin space (rs-space)

un-polarized dist.

spin polarization dist.

Four-vectors of three basis directions in rest frame of q and $\bar{q}$ (one is the spin quantization direction)

MVSD:
Becattini, Chandra, Del Zanna, Grossi (2013)
Sheng, Weickgenannt, et al. (2021)
Sheng, QW, Rischke (2022)

Spin Boltzmann equation for massive fermions

At leading order in $O(\hbar^0)$ spin Boltzmann equation (SBE) with local collision terms

$$\frac{1}{E_p} p\cdot\partial_x \mathrm{tr}\left[f^{(0)}(x,p)\right] = \mathscr{C}_{\mathrm{scalar}}\left[f^{(0)}\right]$$

$$\frac{1}{E_p} p\cdot\partial_x \mathrm{tr}\left[n_j^{(+)\mu}\tau_j f^{(0)}(x,p)\right] = \mathscr{C}^{\mu}_{\mathrm{pol}}\left[f^{(0)}\right]$$

$\Longrightarrow$ $f^{(0)}_{rs}(x,p)$

At next-to-leading order in $O(\hbar)$, SBE describes how $f^{(1)}(x,p)$ evolves for given $f^{(0)}(x,p)$ and $\partial_x f^{(0)}(x,p)$ [non-local terms]

$$\frac{1}{E_p} p\cdot\partial_x \mathrm{tr}\left[f^{(1)}(x,p)\right] = \mathscr{C}_{\mathrm{scalar}}\left[\underline{f^{(0)},\partial_x f^{(0)}}, f^{(1)}\right]$$

$$\frac{1}{E_p} p\cdot\partial_x \mathrm{tr}\left[n_j^{(+)\mu}\tau_j f^{(1)}(x,p)\right] = \mathscr{C}^{\mu}_{\mathrm{pol}}\left[\underline{f^{(0)},\partial_x f^{(0)}}, f^{(1)}\right]$$

determined by leading order SBE

$$\boxed{\partial_\mu u_\nu,\ \partial_\mu T,\ \partial_\mu \mu_B}$$

Convenient for simulation !

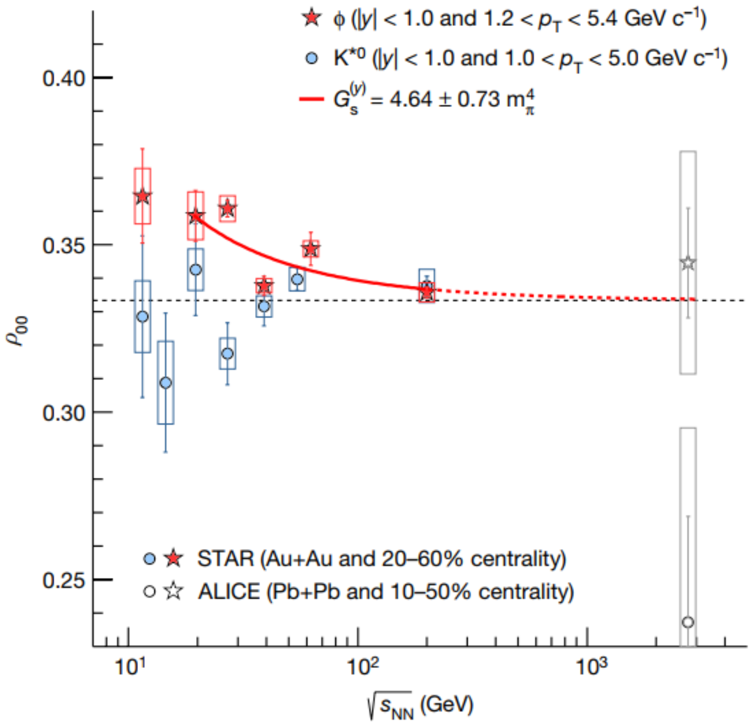
Sheng, Speranza, Rischke, QW, Weickgenannt (2021); Wagner, Weickgenannt, Rischke (2022);

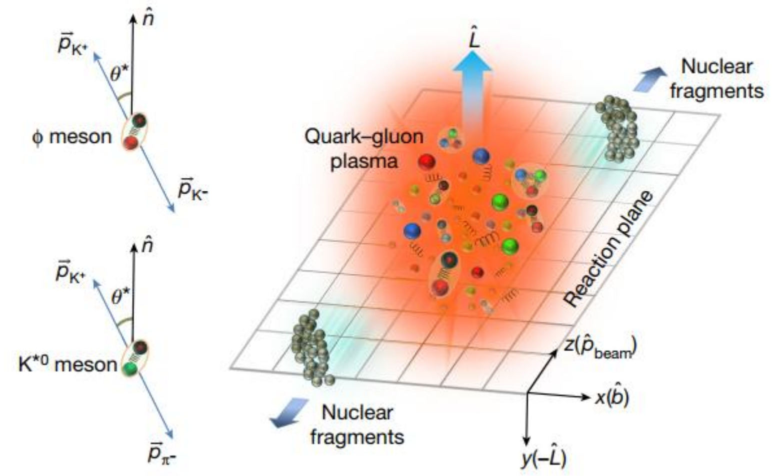
Spin transport for massive fermions from WF or KB equation was also studied in: Yang, Hattori, Hidaka (2020); Gao, Liang (2021); Wang, Zhuang (2021)

Spin alignment for vector mesons

STAR: global spin alignments of vector mesons

STAR, Nature 614, 244 (2023);





Theory prediction:
Sheng, Oliva, QW (2020);
Sheng, Oliva, et al., (2023, 2024).

$$P_\Lambda \sim \langle P_s \rangle, \quad P_{\bar{\Lambda}} \sim \langle P_{\bar{s}} \rangle$$

$$\rho_{00}^{\phi} - \frac{1}{3} \sim \langle P_s P_{\bar{s}} \rangle \neq \langle P_s \rangle \langle P_{\bar{s}} \rangle \sim P_\Lambda P_{\bar{\Lambda}}$$

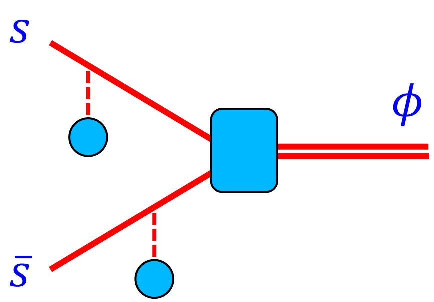
Implication of strong correlation or fluctuation between P_s and $P_{\bar{s}}$

Relativistic Spin Boltzmann (Kinetic) Equation for vector mesons in quark coalescence model

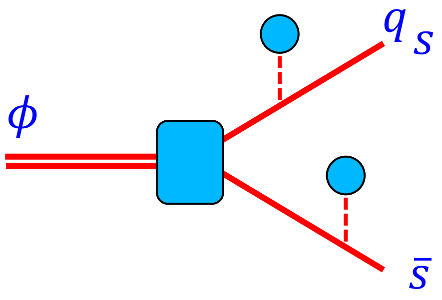
Sheng, Oliva, et al., 2206.05868, 2205.15689

Spin Boltzmann equations with collisons:
Sheng, Weickgennant, Speranza, Rischke, QW (2021);
Yang, Hattori, Hidaka (2020); Wagner, Weickgenannt, Speranza (2022); Wagner, Weickgenannt, Rischke (2022);

Review on QKE and SKE based on Wigner functions:
Hidaka, Pu, QW, Yang, Prog. Part. Nucl. Phys. 127 (2022) 103989



Quark coalescence to V-meson



V-meson dissociation to quarks

Quark coalescence model:
Greco, Ko, Levai (2003);
Fries, Mueller et al (2003);
Yang, Hwa (2003).

Wigner functions for vector mesons

The Wigner function can be defined from $G^{<}_{\mu\nu}(x_1, x_2)$ [or equivalently $G^{>}_{\mu\nu}(x_1, x_2)$] by taking a Fourier transform with respect to the relative position $y = x_1 - x_2$

$$G^{<}_{\mu\nu}(x,p) \equiv \int d^4y\, e^{ip\cdot y} G^{<}_{\mu\nu}(x_1,x_2) = \int d^4y\, e^{ip\cdot y} \left\langle A^{\dagger}_{\nu}(x_2) A_{\mu}(x_1) \right\rangle$$

Inserting the quantized field, we obtain the WF at the leading order $O(\hbar)$

$$\begin{aligned} G^{(0)<}_{\mu\nu}(x,p) &= 2\pi \sum_{\lambda_1,\lambda_2} \delta\left(p^2 - m_V^2\right) \Big\{ \theta(p^0) \underline{\epsilon_\mu(\lambda_1,\mathbf{p})\, \epsilon^*_\nu(\lambda_2,\mathbf{p})} \boxed{f^{(0)}_{\lambda_1\lambda_2}(x,\mathbf{p})} \\ &\quad + \theta(-p^0) \underline{\epsilon^*_\mu(\lambda_1,-\mathbf{p})\, \epsilon_\nu(\lambda_2,-\mathbf{p})} \left[\delta_{\lambda_2\lambda_1} + \boxed{f^{(0)}_{\lambda_2\lambda_1}(x,-\mathbf{p})} \right] \Big\} \end{aligned}$$

$$\begin{aligned} \epsilon_0 &= \mathbf{n}_y \\ \epsilon_{+1} &= -\frac{1}{\sqrt{2}}(\mathbf{n}_z + i\mathbf{n}_x) \\ \epsilon_{-1} &= \frac{1}{\sqrt{2}}(\mathbf{n}_z - i\mathbf{n}_x) \end{aligned}$$

$$\epsilon^\mu(\lambda,\mathbf{k}) = \left(\frac{\mathbf{k}\cdot\epsilon_\lambda}{m_V}, \epsilon_\lambda + \frac{\mathbf{k}\cdot\epsilon_\lambda}{m_V(E^V_{\mathbf{k}} + m_V)}\mathbf{k} \right)$$

where the MVSD for vector meson is defined as

$$f^{(0)}_{\lambda_1\lambda_2}(x,\mathbf{p}) \equiv \int \frac{d^4q}{2(2\pi\hbar)^3} \delta(p\cdot q) e^{-iq\cdot x/\hbar} \left\langle a^{\dagger}_V\left(\lambda_2, \mathbf{p} - \frac{\mathbf{q}}{2}\right) a_V\left(\lambda_1, \mathbf{p} + \frac{\mathbf{q}}{2}\right) \right\rangle$$

$\rho_{\lambda_1\lambda_2}$ **spin density matrix $\lambda_1, \lambda_2 = 1, 0, -1$**

Wigner functions for vector mesons

The decomposition of MVSD (spin density matrix)

$$f^{(0)}_{\lambda_1\lambda_2} = \mathrm{Tr}(f^{(0)})\left(\frac{1}{3}+\frac{1}{2}\underline{P_i\Sigma_i}+\underline{T_{ij}\Sigma_{ij}}\right)_{\lambda_1\lambda_2}$$

$\boxed{i,j=1,2,3 \text{ and } \lambda_1,\lambda_2=1,0,-1}$

Polarization part (cannot be measured in strong decay)

tensor part (can be measured in strong decay)

where Σ_i and Σ_{ij} are 3×3 traceless matrices and defined as

$$\Sigma_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \Sigma_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\Sigma_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad \Sigma_{ij} = \frac{1}{2}(\Sigma_i\Sigma_j+\Sigma_j\Sigma_i)-\frac{2}{3}\delta_{ij}$$

Wigner functions for vector mesons

Let us define an integrated or on-shell Wigner function

$$W^{\mu\nu}(x,p_{\text{on}}) = \frac{E_p}{\pi}\int_0^\infty dp_0 G_<^{\mu\nu}(x,p) = \sum_{\lambda_1,\lambda_2} \epsilon^\mu(\lambda_1,\mathbf{p})\,\epsilon^{\nu*}(\lambda_2,\mathbf{p})\,f^{(0)}_{\lambda_1\lambda_2}(x,\mathbf{p})$$

The on-shell Wigner function can be decomposed into the scalar ($\mathcal{S}$), polarization ($W^{[\mu\nu]}$) and tensor polarization ($\mathcal{T}^{\mu\nu}$) parts as

$$W^{\mu\nu}(x,p_{\text{on}}) = W^{[\mu\nu]} + W^{(\mu\nu)} = -\frac{1}{3}\Delta^{\mu\nu}_{p_{\text{on}}}\mathcal{S} + W^{[\mu\nu]} + \mathcal{T}^{\mu\nu}$$

$$\begin{aligned}
\mathcal{S} =& \mathrm{Tr}(f^{(0)}) = -\Delta^{\mu\nu}_{p_{\text{on}}} W_{\mu\nu} \\
W^{[\mu\nu]} =& \frac{1}{2}\mathrm{Tr}(f^{(0)}) \sum_{\lambda_1,\lambda_2} \epsilon^\mu(\lambda_1,\mathbf{p})\,\epsilon^{\nu*}(\lambda_2,\mathbf{p})\,\underline{P_i\Sigma^i_{\lambda_1\lambda_2}} \\
\mathcal{T}^{\mu\nu} =& \mathrm{Tr}(f^{(0)}) \sum_{\lambda_1,\lambda_2} \epsilon^\mu(\lambda_1,\mathbf{p})\,\epsilon^{\nu*}(\lambda_2,\mathbf{p})\,\underline{T_{ij}\Sigma^{ij}_{\lambda_1\lambda_2}}
\end{aligned}$$

We can extract $f_{00} \propto \rho_{00}$ by projecting $L_{\mu\nu}(p_{on})$ on $W^{\mu\nu}$

$$\begin{aligned}
L_{\mu\nu}(p_{\text{on}})W^{\mu\nu} =& \sum_{\lambda_1,\lambda_2} L_{\mu\nu}(p_{\text{on}})\epsilon^\mu(\lambda_1,\mathbf{p})\,\epsilon^{\nu*}(\lambda_2,\mathbf{p})\,f^{(0)}_{\lambda_1\lambda_2}(x,\mathbf{p}) \\
=& f^{(0)}_{00}(x,\mathbf{p}) + \frac{1}{3}\sum_{\lambda_1,\lambda_2} \epsilon^\mu(\lambda_1,\mathbf{p})\,\epsilon^*_\mu(\lambda_2,\mathbf{p})\,f^{(0)}_{\lambda_1\lambda_2}(x,\mathbf{p}) \\
=& f^{(0)}_{00}(x,\mathbf{p}) - \frac{1}{3}\mathrm{Tr}(f^{(0)})
\end{aligned}$$

$$L^{\mu\nu}(p_{\text{on}}) = \epsilon^{\mu,*}(0,p_{\text{on}})\,\epsilon^\nu(0,p_{\text{on}}) + \frac{1}{3}\Delta^{\mu\nu}(p_{\text{on}})$$

$$\frac{L_{\mu\nu}(p_{\text{on}})W^{\mu\nu}}{-\Delta^{\mu\nu}_{p_{\text{on}}}W_{\mu\nu}} = \frac{f^{(0)}_{00}(x,\mathbf{p})}{\mathrm{Tr}(f^{(0)})} - \frac{1}{3} = \rho_{00} - \frac{1}{3}$$

Wigner functions for vector mesons

From Kadanoff-Baym equation for Wigner functions, we obtain the spin Boltzmann (kinetic) equation in quasi-particle approximation

$$\frac{p}{E_p^V}\cdot\partial_x f_{\lambda_1\lambda_2}(x,\mathbf{p})\approx R^{\mathrm{coal}}_{\lambda_1\lambda_2}(\mathbf{p})-R^{\mathrm{diss}}(\mathbf{p})f_{\lambda_1\lambda_2}(x,\mathbf{p})$$

where $R^{coal}_{\lambda_1\lambda_2}$ and R^{diss} denote the coalescence and dissociation rates for the vector meson, i.e. the rates of $q\bar{q}\to M$ and $M\to q\bar{q}$ respectively. Schematically the formal solution reads

$$\begin{aligned} f_{\lambda_1\lambda_2}(x,\mathbf{p}) \quad &\sim \quad \frac{R^{\mathrm{coal}}_{\lambda_1\lambda_2}(\mathbf{p})}{R^{\mathrm{diss}}(\mathbf{p})}\left[1-\exp\left(-R^{\mathrm{diss}}(\mathbf{p})\Delta t\right)\right] \\ &\sim \quad \begin{cases} R^{\mathrm{coal}}_{\lambda_1\lambda_2}(\mathbf{p})\Delta t, & \text{for } \Delta t\ll 1/R^{\mathrm{diss}}(\mathbf{p}) \\ \frac{R^{\mathrm{coal}}_{\lambda_1\lambda_2}(\mathbf{p})}{R^{\mathrm{diss}}(\mathbf{p})}, & \text{for } \Delta t\gg 1/R^{\mathrm{diss}}(\mathbf{p}) \end{cases} \end{aligned}$$

$\rho_{\lambda_1\lambda_2}$ **spin density matrix $\lambda_1,\lambda_2=1,0,-1$**

Wigner functions for vector mesons

The spin density matrix element can be put into a compact form with an explicit dependence on the polarization vector of the quark and antiquark

$$\begin{aligned}
\rho^{V}_{\lambda_1\lambda_2}(x,\mathbf{p}) &= \frac{\Delta t}{32}\int \frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\frac{1}{E^{\overline{q}}_{p'}E^{q}_{\mathbf{p}-\mathbf{p}'}E^{V}_{p}} f_{\overline{q}}(x,\mathbf{p}')f_q(x,\mathbf{p}-\mathbf{p}') \\
&\times 2\pi\hbar\delta\left(E^{V}_{p}-E^{\overline{q}}_{p'}-E^{q}_{\mathbf{p}-\mathbf{p}'}\right)\epsilon^{*}_{\alpha}(\lambda_1,\mathbf{p})\epsilon_{\beta}(\lambda_2,\mathbf{p}) \\
&\times \mathrm{Tr}\left\{\Gamma^{\beta}\left(p'\cdot\gamma-m_{\overline{q}}\right)\left[1+\gamma_5\gamma\cdot \underline{P^{\overline{q}}}(x,\mathbf{p}')\right]\Gamma^{\alpha}\right. \\
&\times \left.\left[(p-p')\cdot\gamma+m_q\right]\left[1+\gamma_5\gamma\cdot \underline{P^{q}}(x,\mathbf{p}-\mathbf{p}')\right]\right\}
\end{aligned}$$

where the polarization for s and $\overline{s}$ are given by

$$\begin{aligned}
P^{\mu}_{s}(x,\mathbf{p}) &= \frac{g_\phi}{4m_s E^{s}_{p} T_{\mathrm{eff}}}\epsilon^{\mu\nu\rho\sigma}\underline{F^{\phi}_{\rho\sigma}}p_\nu\left[1-f_s(x,\mathbf{p})\right] \\
P^{\mu}_{\overline{s}}(x,\mathbf{p}) &= -\frac{g_\phi}{4m_s E^{\overline{s}}_{p} T_{\mathrm{eff}}}\epsilon^{\mu\nu\rho\sigma}\underline{F^{\phi}_{\rho\sigma}}p_\nu\left[1-f_{\overline{s}}(x,\mathbf{p})\right]
\end{aligned}$$

Effective ϕ field strength tensor

Spin density matrix element for vector mesons

The fusion (coalescence) collision kernel can be evaluated in the rest frame of ϕ meson, which gives ρ_{00}^{ϕ}

$$\rho_{00}(x,\underline{\mathbf{0}}) \approx \frac{1}{3} + C_1\left[\frac{1}{3}\boldsymbol{\omega}'\cdot\boldsymbol{\omega}' - (\epsilon_0\cdot\boldsymbol{\omega}')^2\right]$$
$$+ C_2\left[\frac{1}{3}\boldsymbol{\varepsilon}'\cdot\boldsymbol{\varepsilon}' - (\epsilon_0\cdot\boldsymbol{\varepsilon}')^2\right]$$
$$- \frac{4g_\phi^2}{m_\phi^2 T_{\mathrm{eff}}^2} C_1\left[\frac{1}{3}\mathbf{B}'_\phi\cdot\mathbf{B}'_\phi - (\underline{\epsilon_0}\cdot\mathbf{B}'_\phi)^2\right]$$
$$- \frac{4g_\phi^2}{m_\phi^2 T_{\mathrm{eff}}^2} C_2\left[\frac{1}{3}\mathbf{E}'_\phi\cdot\mathbf{E}'_\phi - (\underline{\epsilon_0}\cdot\mathbf{E}'_\phi)^2\right],$$

rest frame of ϕ meson

$$C_1 = \frac{8m_s^4 + 16m_s^2 m_\phi^2 + 3m_\phi^4}{120m_s^2(m_\phi^2 + 2m_s^2)},$$
$$C_2 = \frac{8m_s^4 - 14m_s^2 m_\phi^2 + 3m_\phi^4}{120m_s^2(m_\phi^2 + 2m_s^2)}.$$

All fields with prime are defined in the rest frame of ϕ meson

spin quantization direction

Features:

(1) Perfect factorization of x and p dependence;

(2) Perfect cancellation for mixing terms (protected by symmetry): all fields appear in squares, i.e. ρ_{00}^{ϕ} measures fluctuations of fields.

Surprising results!

Lorentz transformation for ϕ fields

We can express ρ_{00}^{ϕ} in terms of ϕ fields in the lab frame and obtain the dependence on momenta of ϕ mesons through Lorentz transformation

$$\mathbf{B}'_{\phi} = \gamma \mathbf{B}_{\phi} - \gamma \mathbf{v} \times \mathbf{E}_{\phi} + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{B}_{\phi}}{v^2}\mathbf{v},$$

$$\mathbf{E}'_{\phi} = \gamma \mathbf{E}_{\phi} + \gamma \mathbf{v} \times \mathbf{B}_{\phi} + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{E}_{\phi}}{v^2}\mathbf{v},$$

where $\gamma = E_{\mathbf{k}}^{\phi}/m_{\phi}$ **and** $\mathbf{v} = \mathbf{k}/E_{\mathbf{k}}^{\phi}$

Then we obtain factorization form of $\left\langle \rho_{00}^{\phi} \right\rangle$ in terms of lab-frame fields

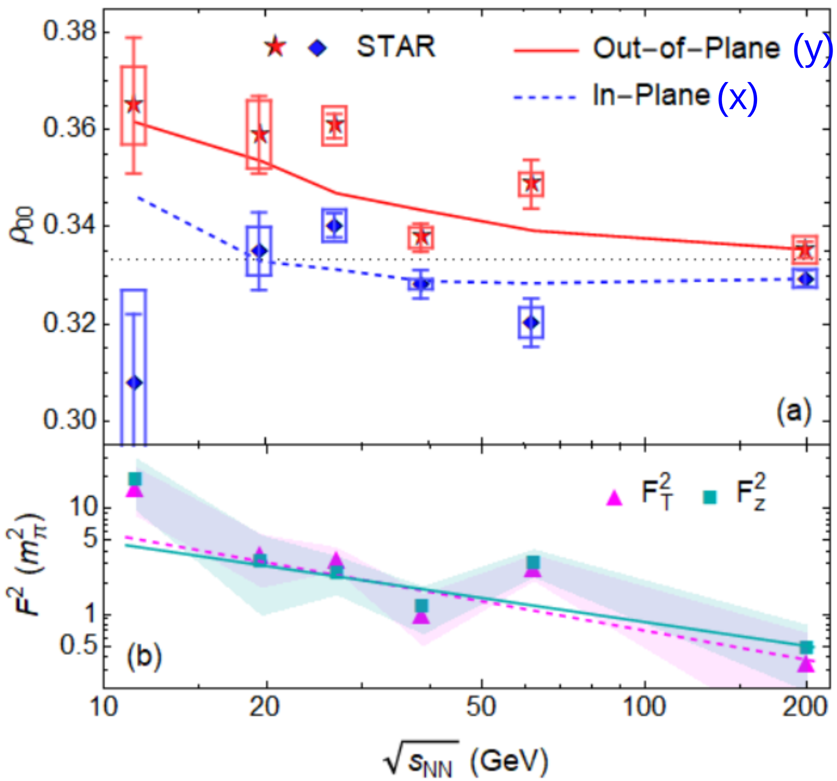
$$\left\langle \overline{\rho}_{00}^{\phi}(x,\mathbf{p}) \right\rangle_{x,\mathbf{p}} \approx \frac{1}{3} + \frac{1}{3}\sum_{i=1,2,3} \langle I_{B,i}(\mathbf{p})\rangle \frac{1}{m_{\phi}^2}\left[\left\langle \boldsymbol{\omega}_i^2 \right\rangle - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\text{eff}}^2}\left\langle (\mathbf{B}_i^{\phi})^2 \right\rangle\right] + \frac{1}{3}\sum_{i=1,2,3} \langle I_{E,i}(\mathbf{p})\rangle \frac{1}{m_{\phi}^2}\left[\left\langle \boldsymbol{\varepsilon}_i^2 \right\rangle - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\text{eff}}^2}\left\langle (\mathbf{E}_i^{\phi})^2 \right\rangle\right]$$

space-time average

momentum average

three basis directions in lab frame

STAR data on ρ_{00}^{y} and ρ_{00}^{x}

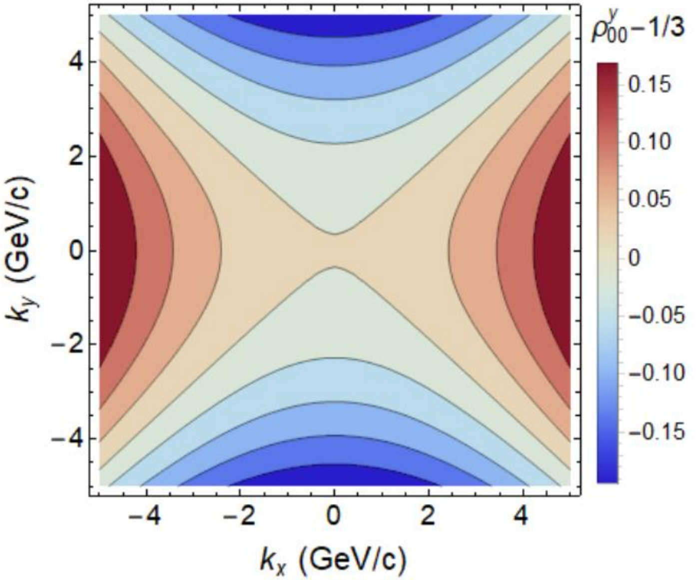


$$F_T^2 \equiv \langle E_{x,y}^2 \rangle = \langle B_{x,y}^2 \rangle, \quad F_z^2 \equiv \langle E_z^2 \rangle = \langle B_z^2 \rangle$$

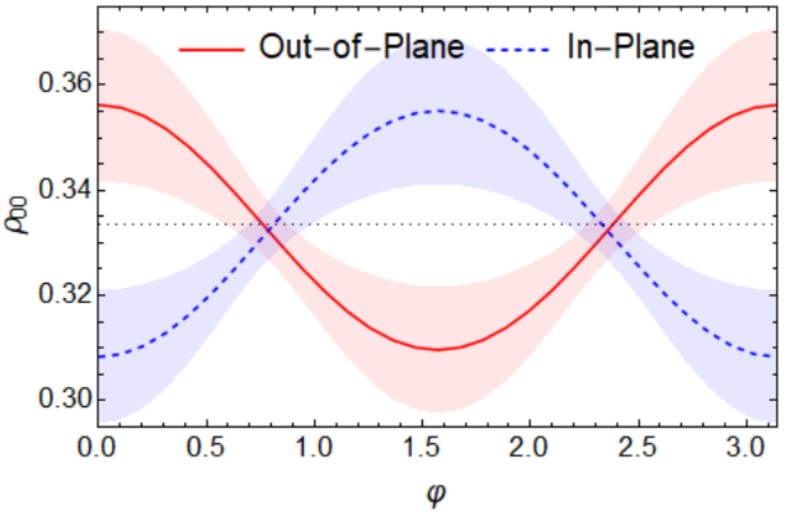
(a) The STAR's data on phi meson's ρ_{00}^{y} (out-of-plane, red stars) and ρ_{00}^{x} (in-plane, blue diamonds) in 0-80% Au+Au collisions as functions of collision energies. The red-solid line and blue-dashed line are calculated with values of F_T^2 and F_z^2 from fitted curves in (b).

(b) Values of F_T^2 (magenta triangles) and F_z^2 (cyan squares) with shaded error bands extracted from the STAR's data on the phi meson's ρ_{00}^{y} and ρ_{00}^{x} in (c). The magenta-dashed line (cyan-solid line) is a fit to the extracted F_T^2 (F_z^2) as a function of $\sqrt{s_{NN}}$ (see the text).

Prediction on ρ_{00}^{y} and ρ_{00}^{x}

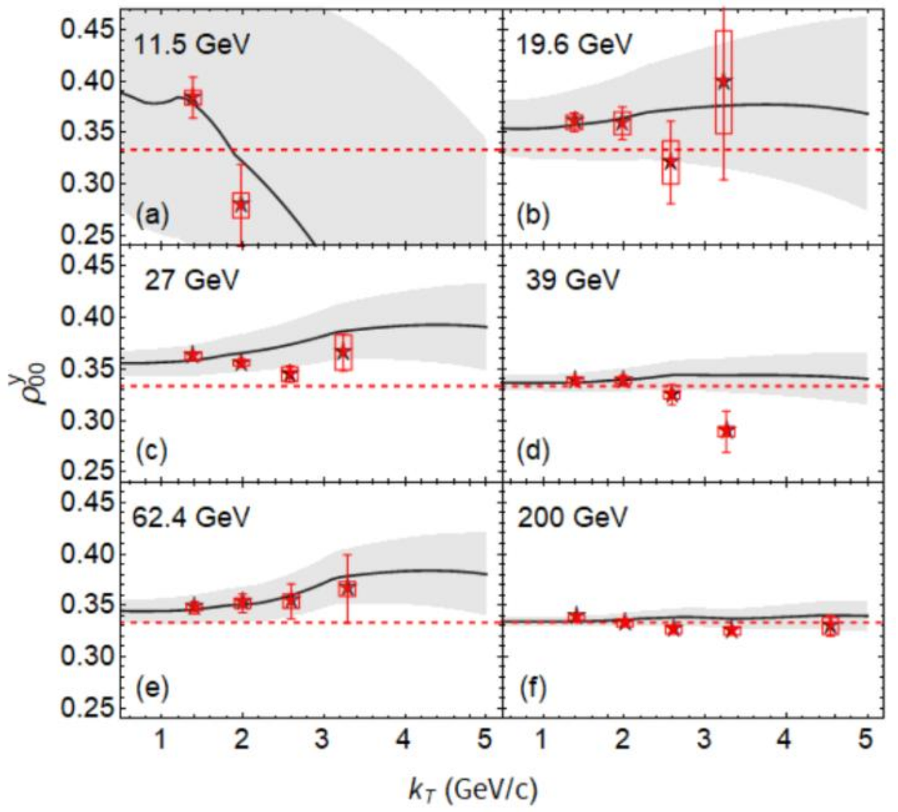


Contour plot of $\rho_{00}^{y}-1/3$ for ϕ mesons as a function of k_x and k_y in 0-80% Au+Au collisions at $\sqrt{s_{NN}}=200$ GeV.



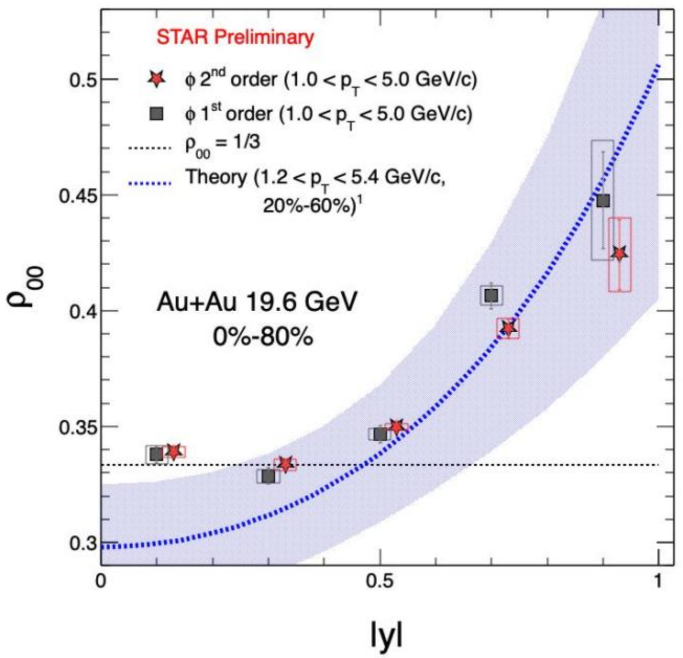
Calculated ρ_{00}^{y} (out-of-plane) and ρ_{00}^{x} (in plane) of ϕ mesons as functions of the azimuthal angle φ in 0-80% Au+Au collisions at $\sqrt{s_{NN}}=200$ GeV. Shaded error bands are from the extracted parameters F_T^2 and F_z^2.

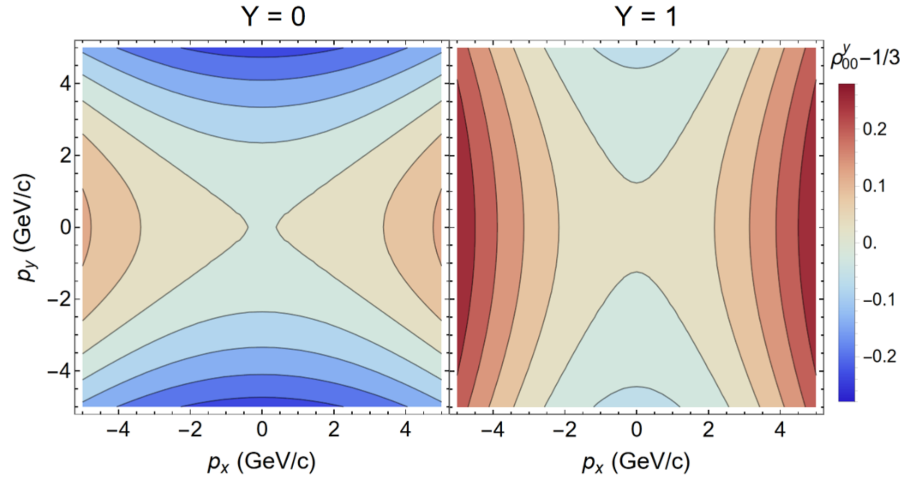
Transverse momentum spectra of ρ_{00}^{y}



Calculated ρ_{00}^{y} (solid line) of ϕ mesons as functions of transverse momenta in 0-80% Au+Au collisions at different colliding energies in comparison with STAR data. Shaded error bands are from the extracted parameters F_T^2 and F_z^2 .

STAR's new measurements and our prediction on rapidity dependence of ρ_{00}^{y}





If B^2 and E^2 is isotropic in all directions in lab frame, we have simple formula with clear physics

$$\langle \delta\rho_{00}^{y}\rangle(\mathbf{p}) = \frac{8}{3m_\phi^4}(C_1+C_2)F^2\left(\frac{p_x^2+p_z^2}{2} - p_y^2\right)$$

$$\propto \frac{1}{2}p_T^2\left[3\cos(2\varphi)-1\right] + \left(m_\phi^2 + p_T^2\right)\sinh^2 Y$$

Sheng, Pu, QW, PRC(2023); 2308.14038

Vector fields in Chiral quark model

Nuclear Physics B234 (1984) 189–212

Citations: 2272 (till September 22, 2023)

Fernandez, Valcarce, Straub, Faessler (1993)
Zhang, et al, (1997); Li, Ye, Lu (1997); Zhao, Li, Bennhold (1998)

CHIRAL QUARKS AND THE NON-RELATIVISTIC QUARK MODEL*

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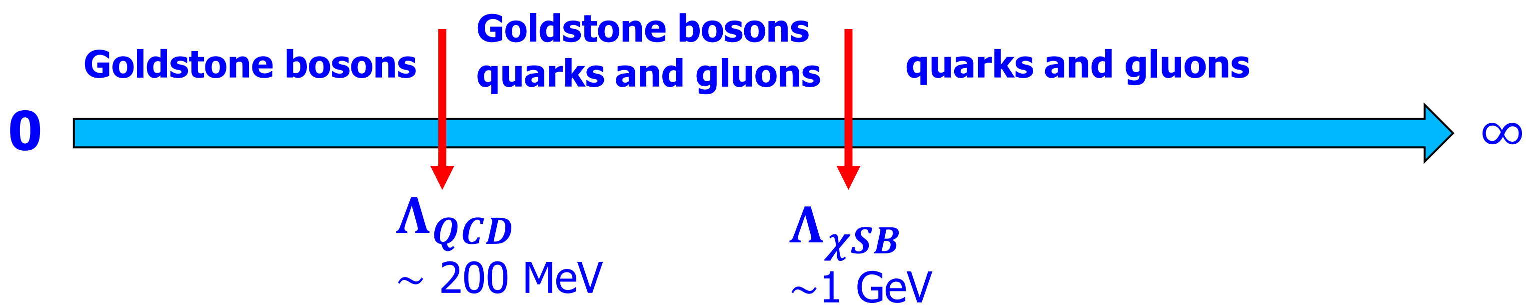
Received 18 July 1983

We study some of the consequences of an effective lagrangian for quarks, gluons and goldstone bosons in the region between the chiral symmetry breaking and confinement scales. This provides an understanding of many of the successes of the non-relativistic quark model. It also suggests a resolution to the puzzle of the hyperon non-leptonic decays.

Vector fields in Chiral quark model

- **Scale for strong interaction in dynamical process**



- **SU(3) Goldstone bosons by 3×3 matrix Σ and ξ,**

$$\begin{aligned}\Sigma &= \exp\left(i\frac{2\chi}{f}\right)\\ &= \exp\left(i\frac{\chi}{f}\right)\exp\left(i\frac{\chi}{f}\right)\\ &= \xi\xi\end{aligned}$$

f = **93 MeV**

$$\chi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

Vector field in Chiral quark model

- **Σ and ξ transform under $SU_L(3)\times SU_R(3)$ as**

$$\Sigma \rightarrow L\Sigma R^{\dagger}, \qquad \xi \rightarrow L\xi U^{\dagger} = U\xi R^{\dagger}$$

- **A set of color and flavor triplet quarks** $\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \qquad \psi = U\psi$

- **Lagrangian is invariant under $SU_L(3)\times SU_R(3)$ transformation**

$$\mathcal{L} = \overline{\psi}\left[i\gamma_\mu\left(\partial^\mu + igG_a^\mu T_c^a + \underline{ig_V V_a^\mu T_f^a}\right)\right]\psi - g_A\overline{\psi}\gamma_\mu\gamma_5\psi$$

$$+\frac{1}{4}f^2\mathrm{Tr}\left[\partial_\mu\Sigma^{\dagger}\partial^\mu\Sigma\right] - \frac{1}{2}\mathrm{Tr}_c(F_{\mu\nu}F^{\mu\nu}) - \frac{1}{2g_V^2}\mathrm{Tr}_f(V_{\mu\nu}V^{\mu\nu}) + \frac{m_V^2}{g_V^2}\mathrm{Tr}_f(V_\mu V^\mu)$$

T_c^a in color space
T_f^a in flavor space

3x3 matrix

$$\boxed{V^\mu = \frac{1}{2}\left(\xi^{\dagger}\partial^\mu\xi + \xi\partial^\mu\xi^{\dagger}\right) = g_V V_a^\mu T_f^a}$$

$$A^\mu = \frac{1}{2}i\left(\xi^{\dagger}\partial^\mu\xi - \xi\partial^\mu\xi^{\dagger}\right)$$

Effective vector fields induced by currents Goldstone boson fields

EOM for vector fields with quark currents

EOM for SU(3) vector fields produced by quark currents

$$\left(g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu\right) V_\nu^a + m_V^2 V_a^\mu = g_V \overline{\psi}\gamma^\mu T_f^a \psi$$

Flavor SU(3) vector meson multiplet

$$\omega_\mu = \frac{1}{\sqrt{2}}(u\overline{u} + d\overline{d}), \quad \phi_\mu = s\overline{s}$$

$$V_\mu^0 = \frac{1}{\sqrt{3}}(u\overline{u} + d\overline{d} + s\overline{s})$$

$$V_\mu^8 = \frac{1}{\sqrt{3}}(u\overline{u} + d\overline{d} + s\overline{s})$$

$$V_\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}(\rho_\mu^0 + \omega_\mu) & \rho_\mu^+ & K_\mu^{*+} \\ \rho_\mu^- & -\frac{1}{\sqrt{2}}(\rho_\mu^0 - \omega_\mu) & K_\mu^{*0} \\ K_\mu^{*-} & \overline{K}_\mu^{*0} & \phi_\mu \end{pmatrix}$$

$$\boxed{\sqrt{2}\mathrm{Tr}V_\mu = u\overline{u} + d\overline{d} + s\overline{s}}$$

$$K_\mu^0 = \frac{1}{\sqrt{2}}\left(V_\mu^6 - iV_\mu^7\right), \quad \omega_\mu = \sqrt{\frac{2}{3}}V_\mu^0 + \sqrt{\frac{1}{3}}V_\mu^8$$

$$\overline{K}_\mu^0 = \frac{1}{\sqrt{2}}\left(V_\mu^6 + iV_\mu^7\right), \quad \phi_\mu = \sqrt{\frac{1}{3}}V_\mu^0 - \sqrt{\frac{2}{3}}V_\mu^8$$

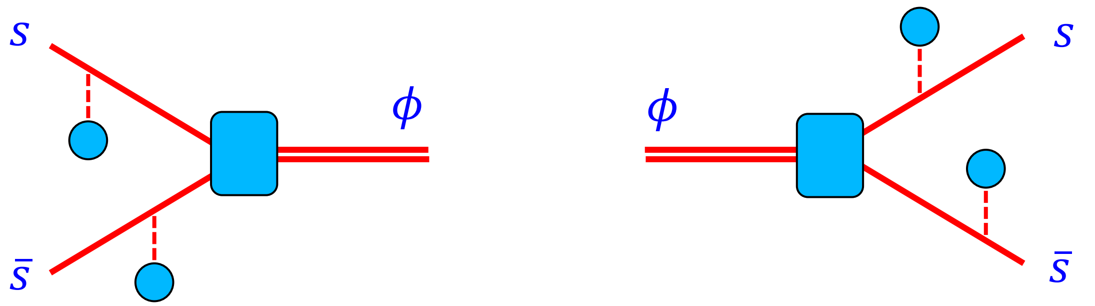
$$\rho_\mu^0 = V_\mu^3 = \frac{1}{\sqrt{2}}(u\overline{u} - d\overline{d})$$

$$\rho_\mu^+ = \frac{1}{\sqrt{2}}\left(V_\mu^1 - iV_\mu^2\right), \quad K_\mu^{*+} = \frac{1}{\sqrt{2}}\left(V_\mu^4 - iV_\mu^5\right)$$

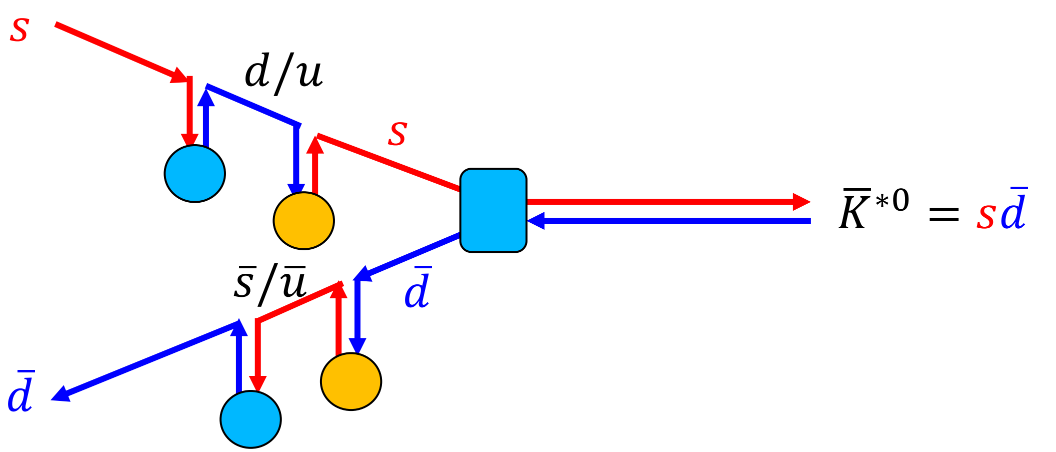
$$\rho_\mu^- = \frac{1}{\sqrt{2}}\left(V_\mu^1 + iV_\mu^2\right), \quad K_\mu^{*-} = \frac{1}{\sqrt{2}}\left(V_\mu^4 + iV_\mu^5\right)$$

Quarks polarized by vector fields in vector mesons

For quarkonium vector mesons (hidden flavor)



For open flavored vector mesons



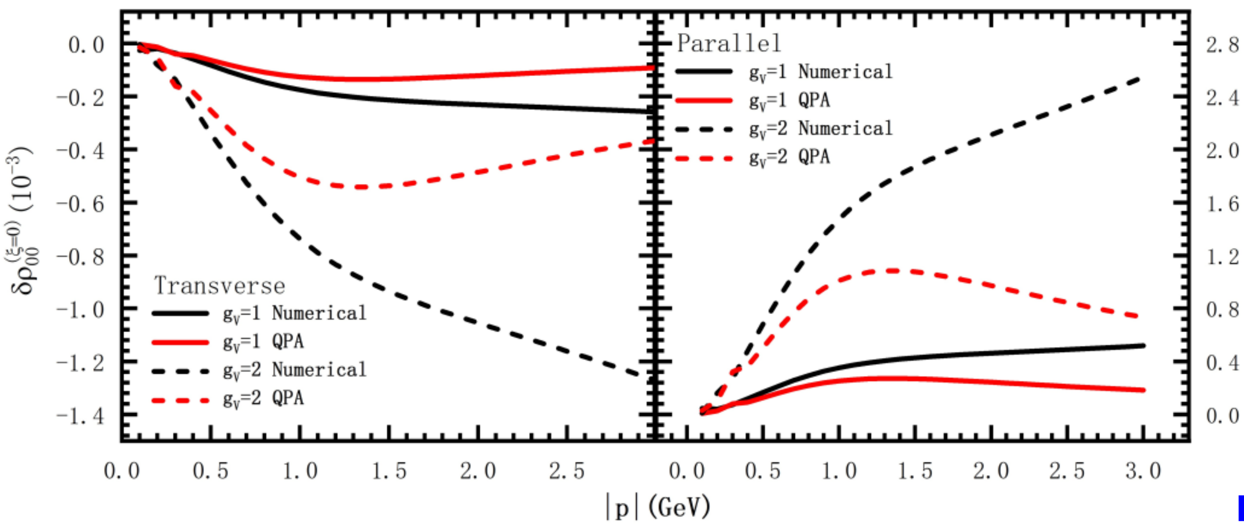
Spin alignment from self-energy and shear stress tensor in linear response theory

The correction to ρ_{00} from self-energy and shear stress tensor $\xi_{\mu\nu}$

$$\delta\rho_{00}(\mathbf{p}) = \delta\rho_{00}^{(\xi=0)}(\mathbf{p}) + \xi_{\mu\nu}C^{\mu\nu}(\mathbf{p})$$

$$\boxed{\delta\rho_{00}^{(\xi=0)} \sim (\Delta E_T - \Delta E_L)}$$

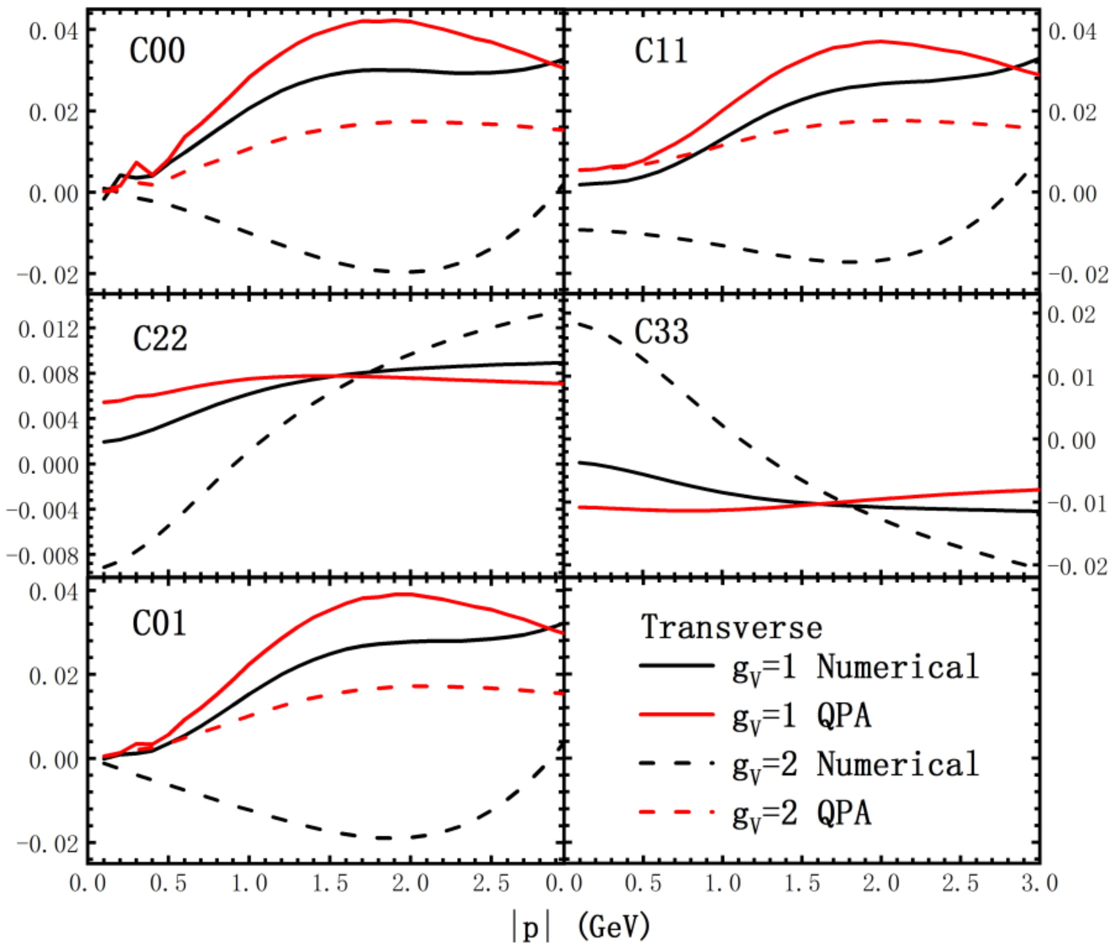
$$\boxed{\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu)}$$



The numerical results for $\delta\rho_{00}^{(\xi=0)}$ for the transverse (left) and parallel (right) configurations in which the momentum is transverse and parallel to the spin quantization direction z respectively. The results under the quasi-particle approximation (QPA) are shown for comparison.

Dong, Yin, Sheng, et al., 2311.18400, PRD (2024)

Spin alignment from self-energy and shear stress tensor in linear response theory



The numerical results for $C^{\mu\nu}$ for the transverse configuration in which the momentum is perpendicular to the spin quantization direction z. The results under the quasi-particle approximation (QPA) are shown for comparison.

Dong, Yin, Sheng, et al., 2311.18400, PRD (2024)

Take-home messages

- $\boldsymbol{P_\Lambda}$ **measures the fields (net mean field),** $\boldsymbol{\rho_{00}^{\phi}}$ **measures field squared (field correlation or fluctuation).**
- **The vector strong force field is induced by the current of pseudo-Goldstone boson during the hadronization**

Open questions for answers in the future

Global polarization:

- **We really need a comprehensive simulation solving the spin Boltzmann equation or spin hydro which includes non-equilibrium effects**

Spin alignment of vector mesons:

- **Any connection with QCD sum rules and QCD vacuum properties? Any connection with quark or gluon condensates (trace anomaly)?**
- **Implication for J/Psi polarization (gluon fields)?**
- **Any connection with effects from glasma fields? (Kuma, Mueller, Yang, 2023)**
- **Any connection with spin correlation of $\Lambda\bar{\Lambda}$? (Lv, Yu, et al., 2402.13721)**