Semi-Classical Spin Hydrodynamics: Current Status and the Road Ahead

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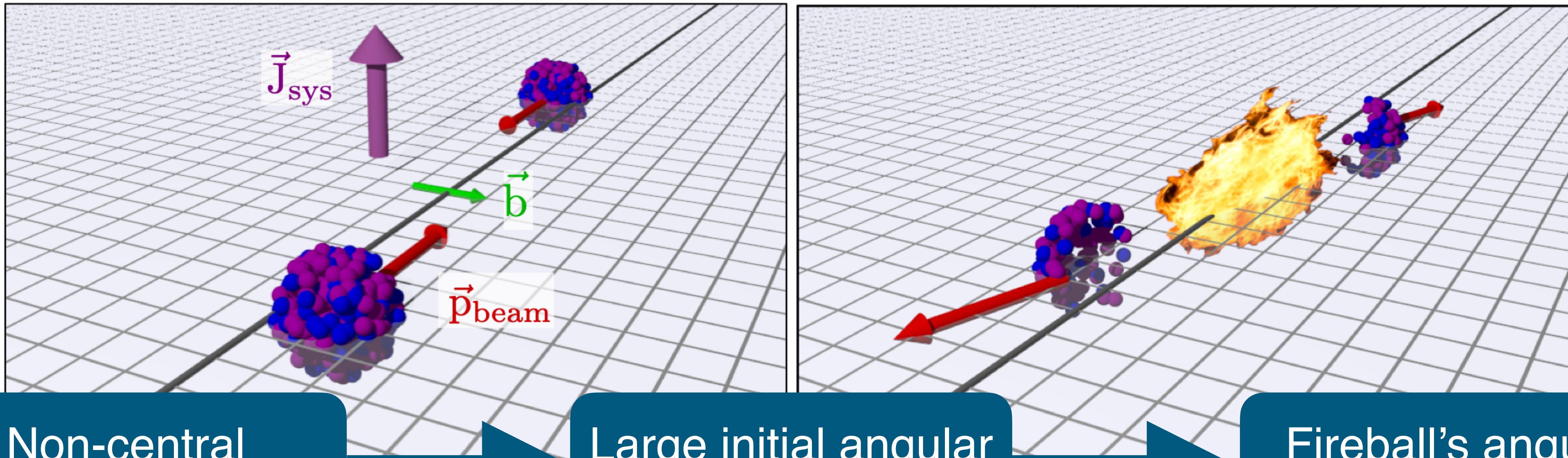
Semi-classical spin hydrodynamics

This formalism was developed in Frankfurt, mostly by **Nora Weickgenannt**, carried forward, brought to Florence and resummed by **David Wagner**, and put on a computer by **Sushant K. Singh**. Now, back in Frankfurt, it's being extended to include electromagnetic fields and curvature — pursued as the doctoral work by **Annamaria Chiarini.**

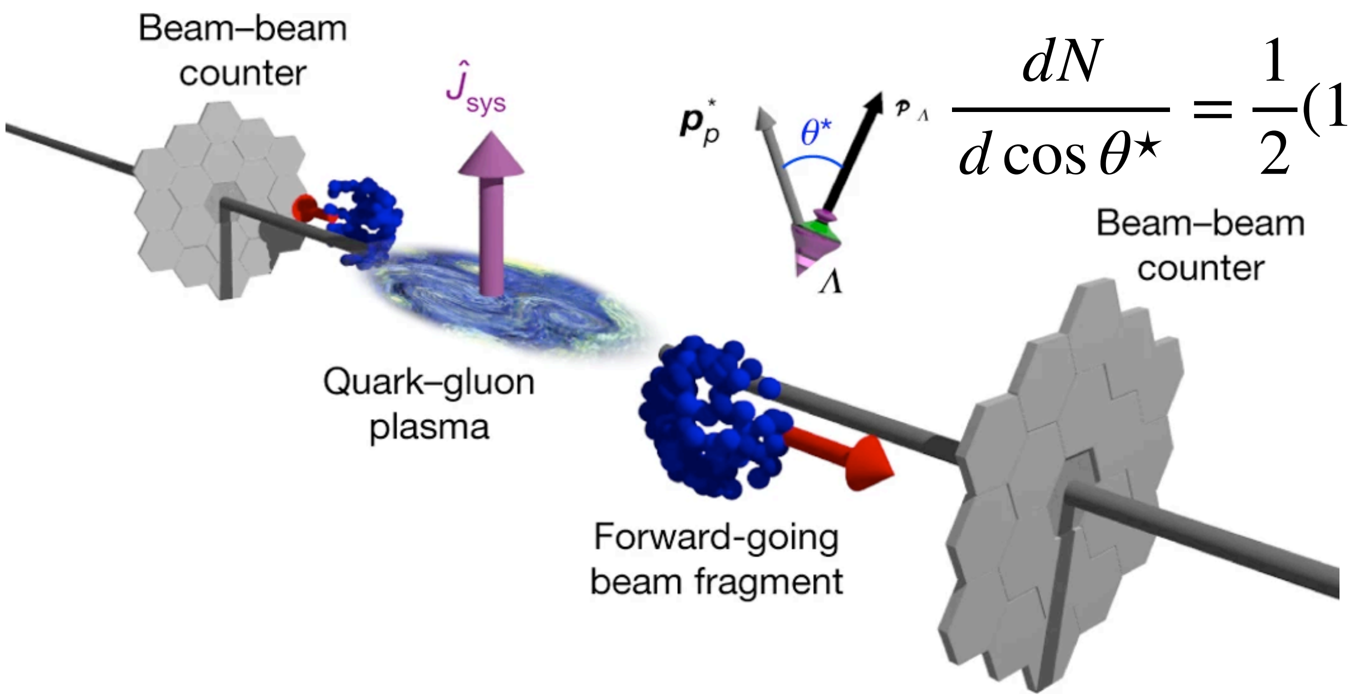
- **Nora Weickgenannt** et al. (2019) — **Kinetic theory** for massive spin-1/2 particles from the Wigner-function formalism
- Nora Weickgenannt et al. (2022) — Relativistic second-order dissipative spin hydrodynamics from the **method of moments**
- Nora Weickgenannt et al. (2022) — Relativistic dissipative spin hydrodynamics from kinetic theory with a **nonlocal collision term**
- **David Wagner** et al. (2024) — Damping of **spin waves**
- **David Wagner** (2024) — **Resummed** spin hydrodynamics from quantum kinetic theory
- **Annamaria Chiarini** et al. (2024) —Semi-Classical Spin Hydrodynamics in Flat and **Curved** Spacetime: **Covariance**, Linear Waves, and Bjorken Background
- Sapna, **Sushant K. Singh**, David Wagner (2025) — **Spin polarization of Lambda hyperons** from dissipative spin hydrodynamics

Angular momentum and polarization

[Becattini et al, 2018]



Non-central collisions → Large initial angular momentum → Fireball's angular momentum → Polarized final hadrons

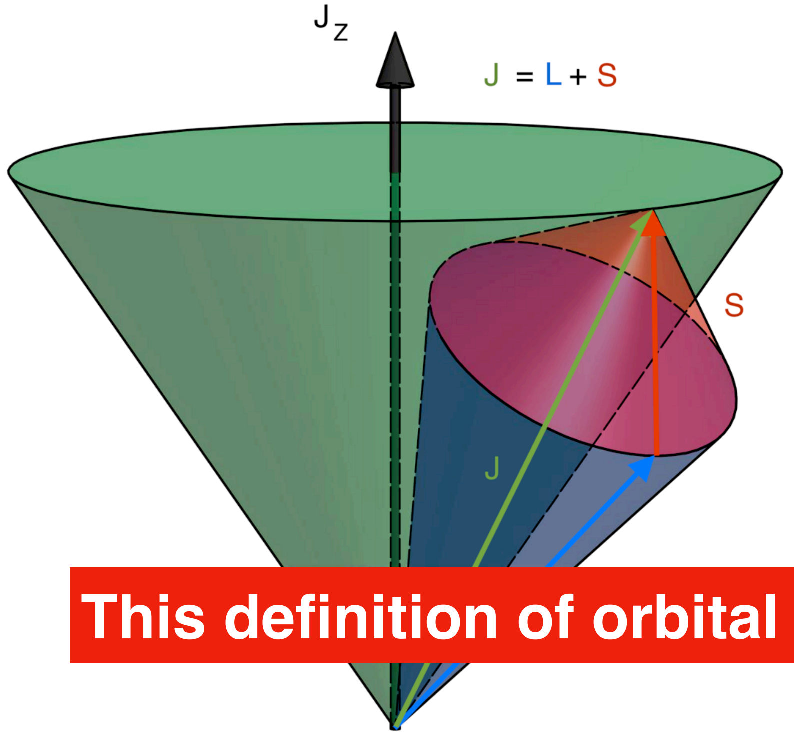


$$\frac{dN}{d\cos\theta^\star} = \frac{1}{2}(1 + \alpha_H |P_H| \cos\theta^\star)$$

[L. Adamczyk et al. (2017)]

Covariant spin hydrodynamics in flat and curved spacetime

Equations of spin hydrodynamics



Equations of Spin Hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0$$

Energy-momentum conservation

$$\partial_\lambda J^{\lambda\mu\nu} = 0$$

Angular momentum conservation

This definition of orbital angular momentum is specific to Cartesian coordinates

$$J^{\lambda\mu\nu} = L^{\lambda\mu\nu} + \mathcal{S}^{\lambda\mu\nu}$$

Decomposition of angular momentum

$$L^{\lambda\mu\nu} = 2T^{\lambda[\nu}x^{\mu]}$$

Orbital angular momentum

God of covariance

Notations and conventions

$$\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$

Mostly minus metric sign

$$A^{[\mu\nu]} \equiv \frac{1}{2}(A^{\mu\nu} - A^{\nu\mu})$$

Antisyemmtrization

$$A^{[\mu\nu]} \equiv \frac{1}{2}(A^{\mu\nu} - A^{\nu\mu})$$

Symmetrization

Recap: Killing vectors and conserved charges

Spacetime symmetries + a conserved symmetric energy-momentum tensor yield conserved charges

$$L^{\lambda r} \equiv -T^{\lambda\nu} K^r_\nu$$

$$D_\mu K^r_\nu + D_\nu K^r_\mu = 0$$

Killing vector field that generates rotations

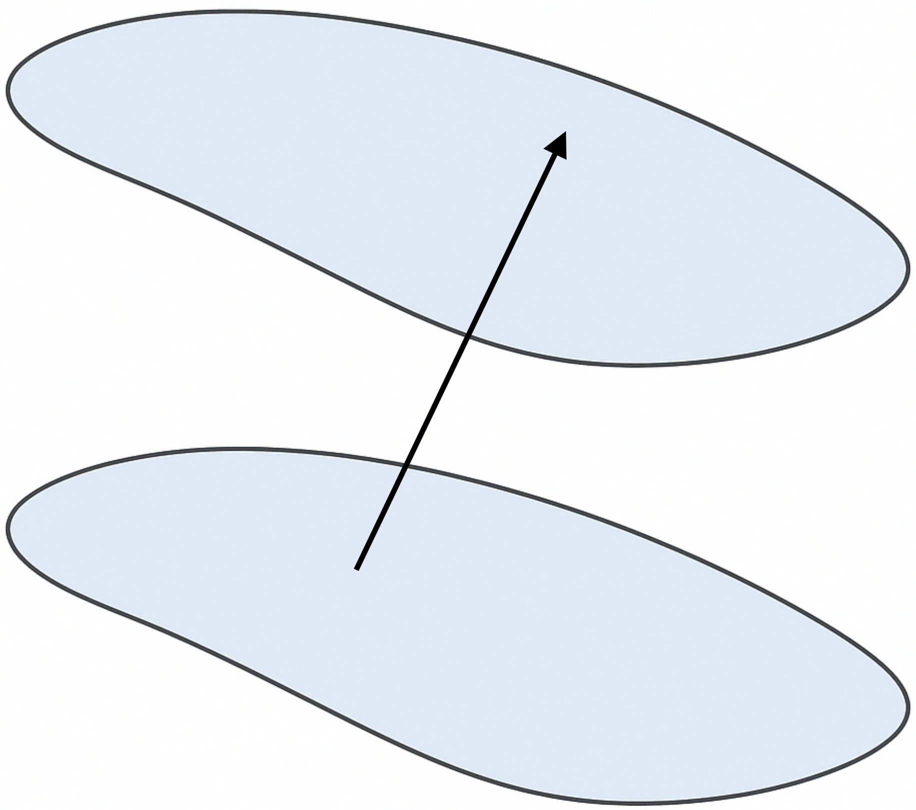
$$\nabla_\lambda L^{\lambda r} = -\frac{1}{2} T^{\lambda\nu} (\nabla_\lambda K^r_\nu + \nabla_\nu K^r_\lambda) = 0$$

Integrating over a Cauchy hyper-surface

$$L^r = \int_\Sigma \mathrm{d}\Sigma_\mu L^{\mu r}$$

The divergence theorem

$$\int_{\Sigma_2} \mathrm{d}\Sigma_\mu L^{\mu r} - \int_{\Sigma_1} \mathrm{d}\Sigma_\mu L^{\mu r} = \int \mathrm{d}V \nabla_\mu L^{\mu r}$$



The charge is conserved during the evolution

Non-symmetric energy-momentum tensor

What if the energy-momentum tensor is not symmetric?

$$T^{\mu\nu} = T^{(\mu\nu)} + T^{[\mu\nu]}$$

Nonsymmetric energy-momentum tensor

$$\nabla_\lambda L^{\lambda r} = -\frac{1}{2} T^{[\lambda\nu]}(\nabla_\lambda K^r_\nu - \nabla_\nu K^r_\lambda) \neq 0$$

$$T^{[\mu\nu]} = \frac{1}{2} D_\lambda \mathcal{S}^{\lambda\mu\nu}$$

Spin dynamics postulate

$$\nabla_\alpha \nabla_\beta K_\gamma = 0$$

In flat spacetime

The correct form of the conserved charge current is found

$$J^{\mu r} \equiv -T^{\mu\nu} K^r_\nu + \frac{1}{2} \mathcal{S}^{\mu\alpha\beta} K^r_{[\alpha;\beta]}$$

$$J^{\lambda r} = L^{\lambda r} + \mathcal{S}^{\lambda r}$$

Decomposition of angular momentum

$$\mathcal{S}^{\lambda r} = \frac{1}{2} \mathcal{S}^{\lambda\mu\nu} D_{[\nu} K^r_{\mu]}$$

Spin angular momentum

$$L^{\lambda r} \equiv -T^{\lambda\nu} K^r_\nu$$

Orbital angular momentum

Sanity check: 6x4 = 24

Or you can read our Appendix A

Extension to curved spacetime

Inspired by a discussion with A. Yarom in Trento, we thought about the curved space-time generalisation

The novelty is

$$K^r_{\alpha;\beta\gamma} = R^{\nu}{}_{\gamma\beta\alpha} K^r_{\nu}$$

$$\nabla_\mu J^{\mu r} = - K^r_\nu \left(\nabla_\mu T^{\mu\nu} + \frac{1}{2} R^{\nu}_{\alpha\beta\gamma} \mathcal{S}^{\alpha\beta\gamma} \right) - K^r_{[\alpha;\beta]} \left(T^{[\alpha\beta]} + \frac{1}{2} \nabla_\mu \mathcal{S}^{\mu\alpha\beta} \right)$$

Postulate that the divergence of the total angular momentum vanishes!

Resembles Joule-Heating term

$$D_\mu T^{\mu\nu} = - F^{\nu\lambda} J_\lambda$$

$$D_\mu T^{\mu\nu} = - \frac{1}{2} R^{\nu}_{\alpha\beta\gamma} \mathcal{S}^{\alpha\beta\gamma} \qquad T^{[\mu\nu]} = - \frac{1}{2} D_\lambda \mathcal{S}^{\lambda\mu\nu}$$

- Agrees with the variational approach [F.W. Hehl (1976) and A.D. Gallegos et al. (2021)]

Pseudo-gauge transformations

$$T^{\mu\nu\prime} = T^{\mu\nu} + \nabla_\lambda Z^{\lambda\mu\nu} \qquad \mathcal{S}^{\lambda\mu\nu\prime} = \mathcal{S}^{\lambda\mu\nu} - \Phi^{\lambda\mu\nu} \qquad Z^{\lambda\mu\nu} \equiv \frac{1}{2}\left(\Phi^{\lambda\mu\nu} - \Phi^{\mu\lambda\nu} - \Phi^{\nu\lambda\mu}\right)$$

Hydro currents are not unique

The change in angular momentum leads to a boundary term

$$\int d\Sigma_\mu \nabla_\mu A^{\mu\lambda r} = \int dS_{\mu\lambda} A^{\mu\lambda r}$$

$$J^{\mu r\prime} = J^{\mu r} - D_\lambda A^{\mu\lambda r} \qquad A^{\mu\lambda r} = Z^{\lambda\mu\nu} K^r_\nu$$

$$\nabla_\mu \nabla_\lambda Z^{\lambda\mu\nu} = -\frac{1}{2} R^\nu{}_{\lambda\alpha\beta} Z^{\alpha\beta\lambda}$$

Without these modifications the EOM do not transform properly!

$$\Downarrow$$

$$\nabla_\mu T^{\mu\nu\prime} = -\frac{1}{2} R^\nu_{\alpha\beta\gamma} \mathcal{S}^{\alpha\beta\gamma\prime} \qquad T^{[\mu\nu]\prime} = -\frac{1}{2}\nabla_\lambda \mathcal{S}^{\lambda\mu\nu\prime}$$

The EOM are covariant

On Equilibrium

Global thermodynamic equilibrium

Thermal Killing vector

$$\beta^\star = \lambda_I^\star K^I \qquad \lambda_I^\star = \frac{\partial S}{\partial Q^I}\bigg|_{\text{GTE}} \qquad \beta^\star \cdot \beta^\star > 0$$

It should not be confused with the beta vector!

The thermal Killing vector determines the fluid velocity and temperature

$$u_\mu = \frac{\beta^\star_\mu}{\sqrt{\beta^\star \cdot \beta^\star}} \qquad T = \frac{1}{\sqrt{\beta^\star \cdot \beta^\star}}$$

Although in GTE, it is equal to the beta vector.

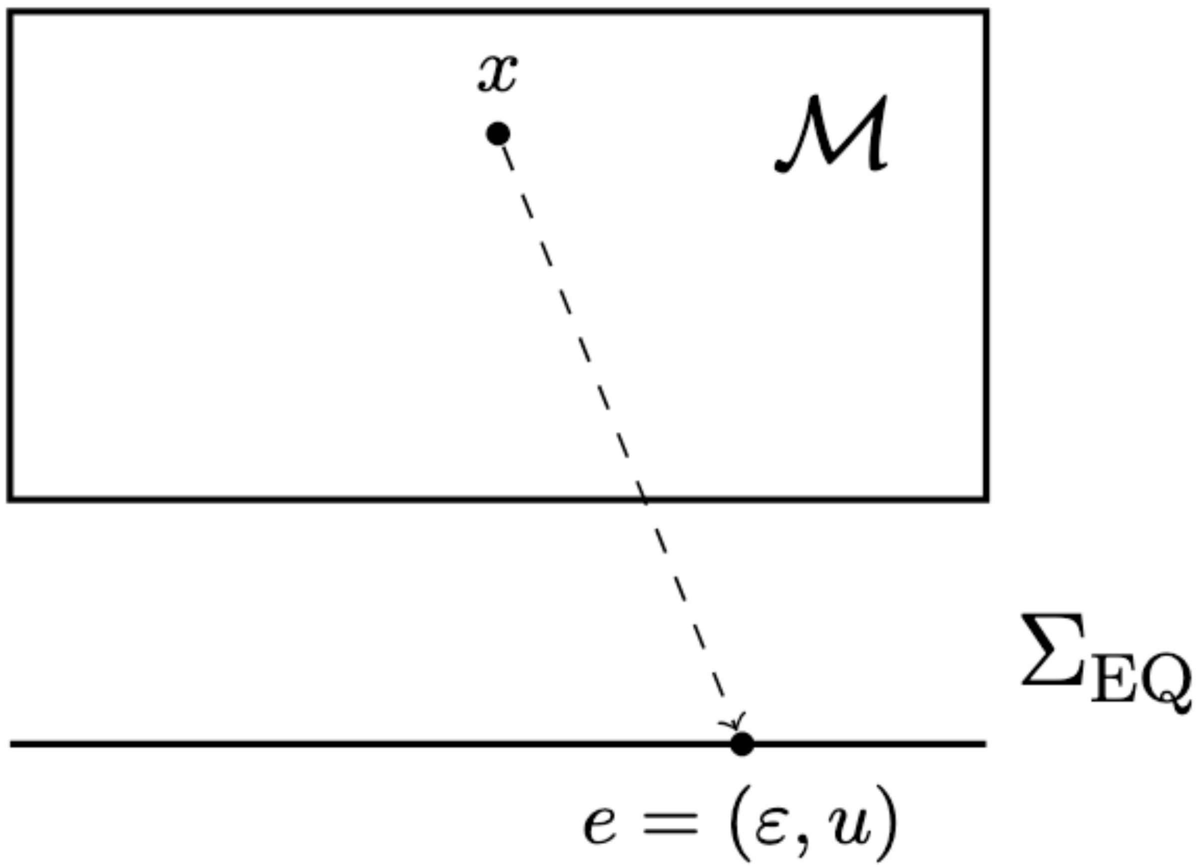
Thermal vorticity in equilibrium

$$\varpi^\star_{\mu\nu} \equiv -\nabla_{[\mu}\beta^\star_{\nu]}$$

GTE is dictated solely by the geometry of spacetime and the intensive parameters of the environment.

Local thermodynamic equilibrium as a map

Point-by-point mapping of each point to a point in the set of all possible (global) equilibrium states of the same fluid



Choose a u^μ

Matching condition
$T^{\mu\nu}(x)u_\mu(x)u_\nu(x) = \epsilon(e)$

Hydro currents are found from expanding around e

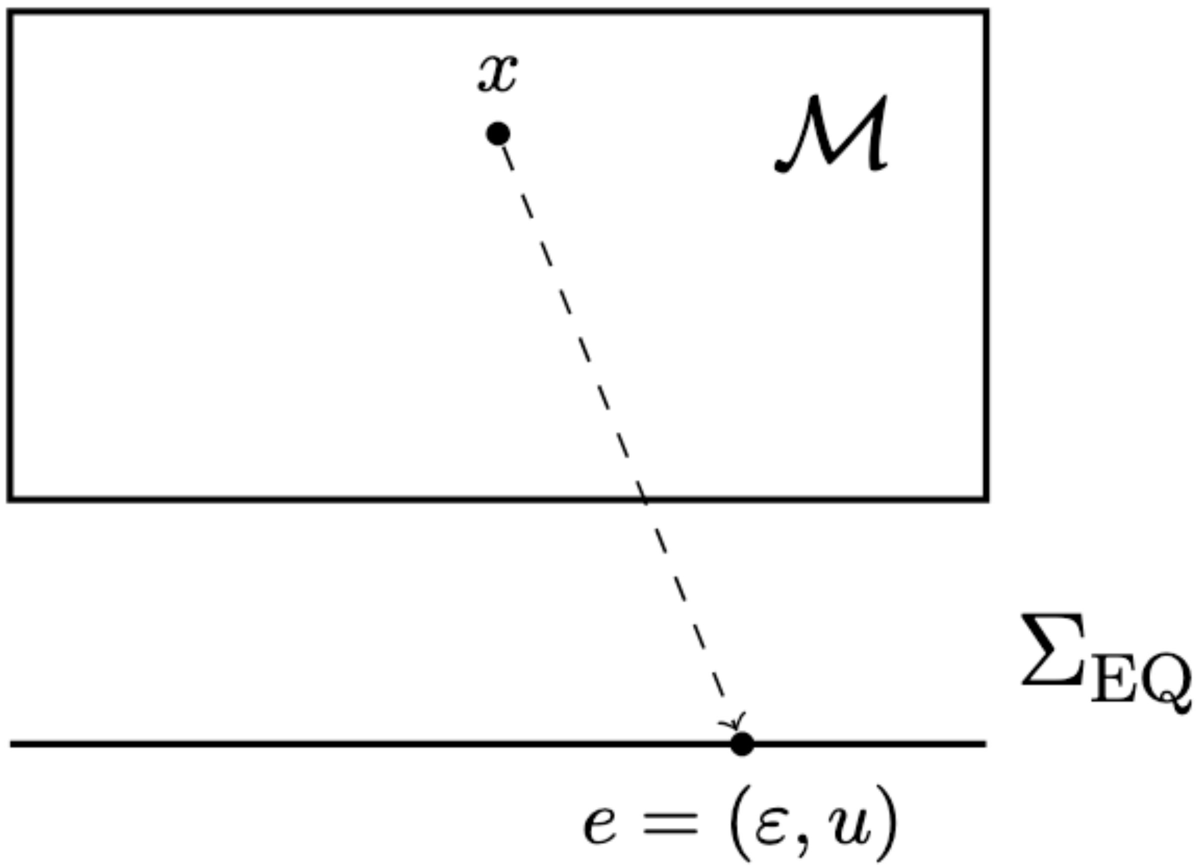
If the map is unique, then the fluid is in LTE

Local thermodynamic equilibrium as a map

In the work of Israel-Stewart the set were 4 (or 5)-dimensional

However the thermal vorticity provides us with 6 extra dimensions

With nonzero thermal vorticity the equilibrium state is anisotropic

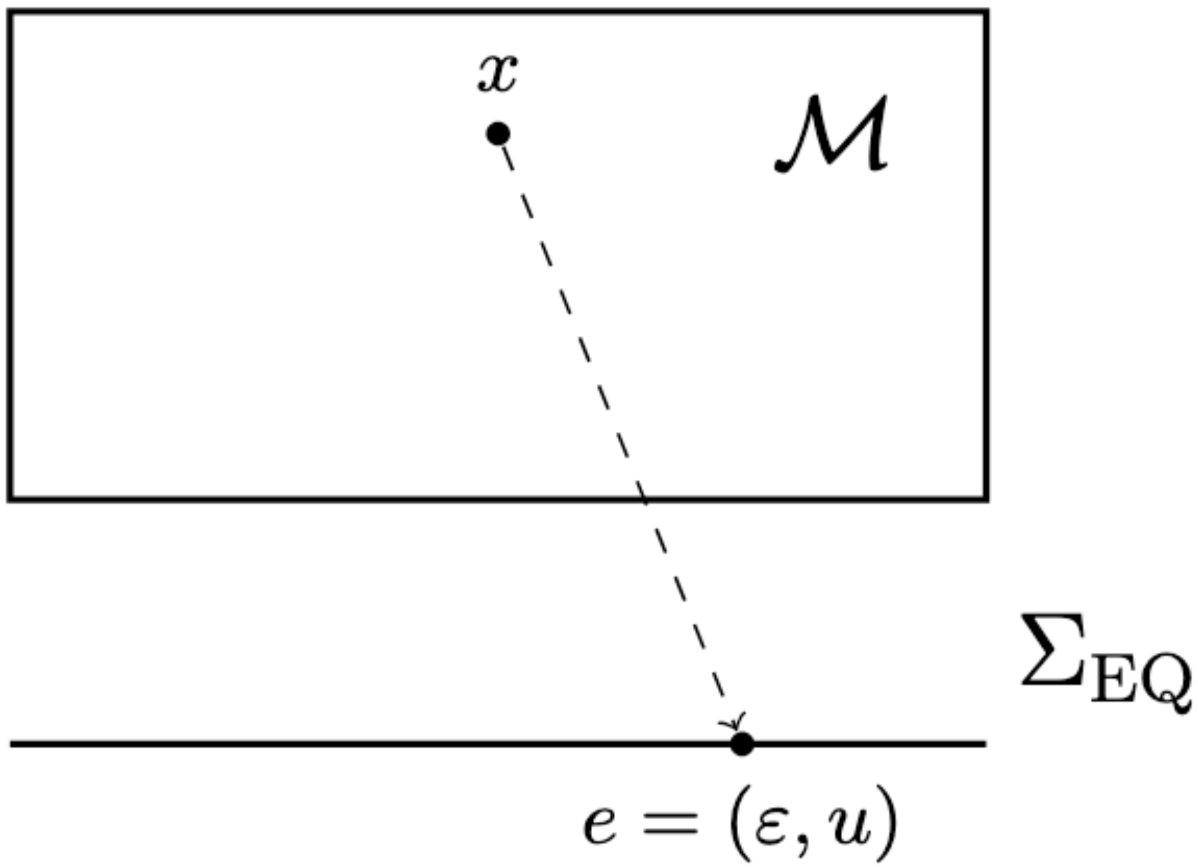


Power-counting confusion

$$(\varepsilon + P)\left(a_\mu - \frac{\nabla_\mu T}{T}\right) = -\Pi a_\mu + \nabla_\mu \Pi + \cdots$$

Local thermodynamic equilibrium as a map

In the work of Israel-Stewart the set were 4 (or 5)-dimensional



However the thermal vorticity provides us with 6 extra dimensions

With nonzero thermal vorticity the equilibrium state is anisotropic

We assume $\varpi \sim \mathcal{O}(\nabla)$

But will remind ourselves that thermal vorticity can exist in GTE

Enters the Spin Potential

- Thermal vorticity serves as the **intensive parameter** for angular momentum—akin to the chemical potential to temperature ratio.
- Out of Equilibrium: **spin potential** $\Omega_{\mu\nu}$

$$\Omega^{\mu\nu} = 2u^{[\mu}\kappa^{\nu]} + \epsilon^{\mu\nu\alpha\beta}u_\alpha\omega_\beta$$

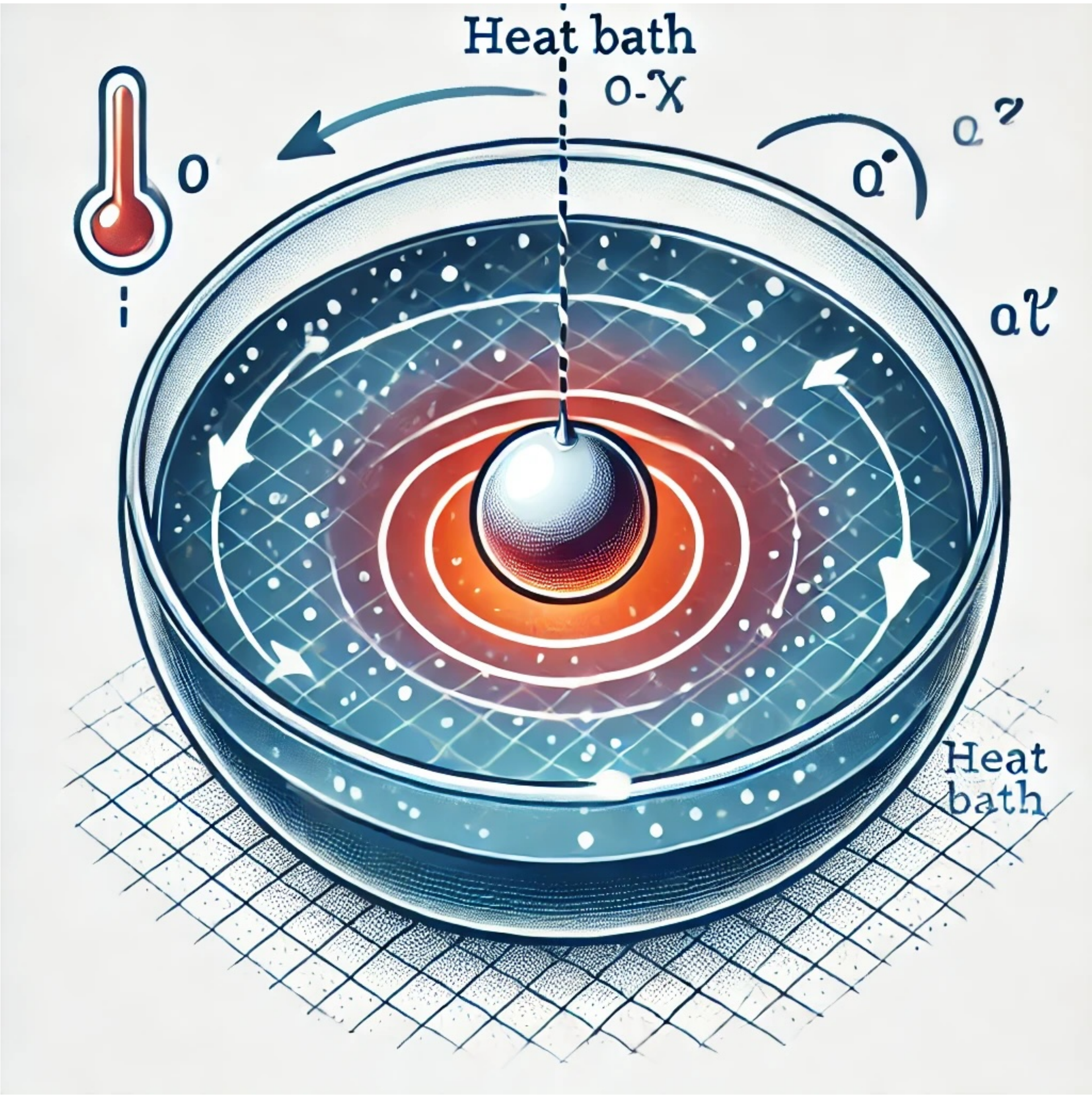
$$\kappa^\mu = -\Omega^{\mu\nu}u_\nu \qquad \omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu\Omega_{\alpha\beta}$$

Electric component **Magnetic component**

Global equilibrium

Acceleration $-\frac{1}{T}a^\mu$ **Fluid vorticity** $\frac{1}{T}\vartheta^\mu$

$$a^\nu \equiv u^\alpha\nabla_\alpha u^\nu \qquad \vartheta^\mu \equiv -\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu\nabla_\alpha u_\beta$$



Semi-classical spin hydrodynamics: the fun

Three assumptions of semiclassical spin hydro

Pre-Hydro

Spin Hydrodynamics

$D_\mu T^{\mu\nu} = 0$

$D_\lambda \mathcal{S}^{\lambda\mu\nu} = 2T^{[\mu\nu]}$

Polarization

Semi-classical expansion

$$\mathcal{S}^{\lambda\mu\nu} = (\hbar^0) + (\hbar) + (\hbar^2)$$

$$\downarrow$$

$$\mathcal{S}^{\lambda\mu\nu} = \hbar S^{\lambda\mu\nu}$$

Assumption I. spin tensor is small

Global equilibrium

$$T^{[\nu\mu]} = 0 \qquad D_\lambda S^{\lambda\mu\nu} = 0$$

Assumption II. In global equilibrium spin is independently conserved

$$T^{\mu\nu} = (\hbar^0) + (\hbar) + (\hbar^2)$$

$$\downarrow$$

Assumption III. No first-order contribution and symmetric at zeroth order

Pre-Hydro

No back-reaction

Spin Dynamics

$\hbar D_\lambda S^{\lambda\mu\nu} = 2T^{[\mu\nu]}$

Standard hydrodynamics

$D_\mu T^{(\mu\nu)} = \mathcal{O}(\hbar^2)$

Polarization

These assumptions are extracted from quantum-kinetic theory based formalism:

N. Weickgenannt et al, 2022

Ideal-spin hydrodynamics

24 unknowns - 6 equations

$$\hbar D_\lambda S^{\lambda\mu\nu} = 2T^{[\mu\nu]} \longrightarrow -\hbar^2\Gamma^{(\kappa)}u^{[\mu}\left(\kappa^{\nu]} + \varpi^{\nu]\alpha}u_\alpha\right) + \frac{1}{2}\hbar^2\Gamma^{(\omega)}\epsilon^{\mu\nu\rho\sigma}u_\rho\left(\omega_\sigma + \beta\Omega_\sigma\right) + \cdots$$

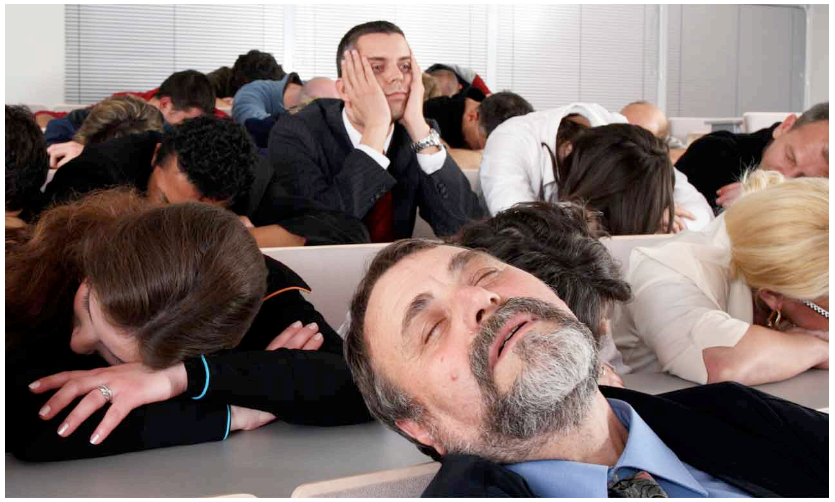
$$S^{\lambda\mu\nu} = S_0^{\lambda\mu\nu} + \delta S^{\lambda\mu\nu}$$

Extra equations of motion are needed

N. Weickgenannt et al, 2022

D. Wagner, 2024

The name is misleading, it is dissipative



Ideal-spin:
6 components of $\Omega^{\mu\nu}$

$$S_0^{\lambda\mu\nu} = Au^\lambda\Omega^{\mu\nu} + Bu^\lambda u_\alpha\Omega^{\alpha[\mu}u^{\nu]} + Cu^\lambda\Omega^{\alpha[\mu}\Delta^{\nu]}{}_\alpha + Du_\alpha\Omega^{\alpha[\mu}\Delta^{\nu]\lambda} + E\Delta^\lambda{}_\alpha\Omega^{\alpha[\mu}u^{\nu]}$$

A, B, C, D, E are functions of temperature

$$B - C - D + T\frac{\mathrm{d}E}{\mathrm{d}T} = 0 \qquad A = \frac{\hbar T^2}{4m^2}\frac{1}{\mathrm{d}T}(\varepsilon - 3P) \qquad B = \frac{\hbar T^2}{4m^2}\frac{\mathrm{d}\varepsilon}{\mathrm{d}T} \qquad C = D = E = -\frac{\hbar T^2}{4m^2}\frac{\mathrm{d}P}{\mathrm{d}T}$$

Constrained By assumption III

Quantum-kinetic theory values

Ideal-spin equations of motion

$\vec{\omega}$ $\vec{\kappa}$

Comoving time-derivative

Inv. temperature derivative

Expansion scalar

$$
\begin{aligned}
(A-B-C)\dot{\kappa}^{\langle\mu\rangle} \;=\; & -\left[\left(\frac{\partial A}{\partial\beta}-\frac{\partial B}{\partial\beta}-\frac{\partial C}{\partial\beta}\right)\dot{\beta}+\left(A-B-C+\frac{2}{3}D\right)\theta\right]\kappa^{\mu}+D\left(\vartheta^{\mu\lambda}+\sigma^{\mu\lambda}\right)\kappa_{\lambda}\\
& +\epsilon^{\mu\nu\rho\sigma}u_{\nu}\left[(E+2C-A)a_{\rho}\omega_{\sigma}+E\,\nabla_{\rho}\omega_{\sigma}+\left(\frac{\partial E}{\partial\beta}\nabla_{\rho}\beta\right)\omega_{\sigma}\right]\\
& +\hbar\Gamma^{(\kappa)}\left(\kappa^{\mu}+\varpi^{\mu\nu}u_{\nu}\right)-\hbar\Gamma^{(a)}\left(\nabla^{\mu}\beta+\beta a^{\mu}\right),
\end{aligned}
$$

$\vec{L} \leftrightarrow \overleftrightarrow{S}$

Shear tensor

Fluid vorticity

$$
\begin{aligned}
(A-2C)\dot{\omega}^{\langle\mu\rangle} \;=\; & -\left[\left(\frac{\partial A}{\partial\beta}-2\frac{\partial C}{\partial\beta}\right)\dot{\beta}+\left(A-2C-\frac{2}{3}E\right)\theta\right]\omega^{\mu}-E\left(\vartheta^{\mu\lambda}+\sigma^{\mu\lambda}\right)\omega_{\lambda}\\
& +\epsilon^{\mu\nu\rho\sigma}u_{\nu}\left[(B+C-A-D)a_{\rho}\kappa_{\sigma}+D\,\nabla_{\rho}\kappa_{\sigma}+\left(\frac{\partial D}{\partial\beta}\nabla_{\rho}\beta\right)\kappa_{\sigma}\right]\\
& -\hbar\Gamma^{(\omega)}\left(\omega^{\mu}+\beta\vartheta^{\mu}\right)
\end{aligned}
$$

Acceleration

Quite a mess but …

In a poor man’s toolbox

Hydrostatic

Linear hydro

Bjorken flow

Before turning to numerical solutions let’s look into the poor man’s toolbox

Standard hydrodynamics

We need a solution of hydrodynamics to feed into the spin equations

$$\tau_\kappa \dot{\kappa} + \cdots \kappa = \cdots \nabla \times \omega + \cdots \times \omega - \frac{\vec{a}}{T}$$

$$\tau_\omega \dot{\omega} + \cdots \omega = \cdots \nabla \times \kappa + \cdots \times \kappa + \frac{\overrightarrow{\Omega}}{T}$$

Relaxation-type equations:

Determine how the components of spin potential relax to the components of thermal vorticity

How long does it take for spin potential to relax to thermal vorticity?

Damping of spin waves

Hydrostatic

Fluid in equilibrium at rest

$$\tau_\kappa \dot{\vec{\kappa}} + \vec{\kappa} = \mu_\kappa \nabla \times \vec{\omega}$$

$$\tau_\omega \dot{\vec{\omega}} + \vec{\omega} = -\mu_\omega \nabla \times \vec{\kappa}$$

Fourier Transform

$$X(x) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} e^{ik\cdot x/\hbar} X(k)$$

Damped spin waves

Damping term

Wave frequency $\longrightarrow$ $\omega_0^2 - i\hbar a \omega_0 - v_{\mathfrak{s}}^2 \vec{k}^2 - \hbar^2 b = 0$ $\quad a = \frac{\tau_\kappa + \tau_\omega}{\tau_\kappa \tau_\omega} \quad b = \frac{1}{\tau_\kappa \tau_\omega} \quad v_{\mathfrak{s}}^2 = \frac{\Gamma^{(\kappa)} \tau_\kappa}{4\Gamma^{(\omega)} \tau_\omega}$

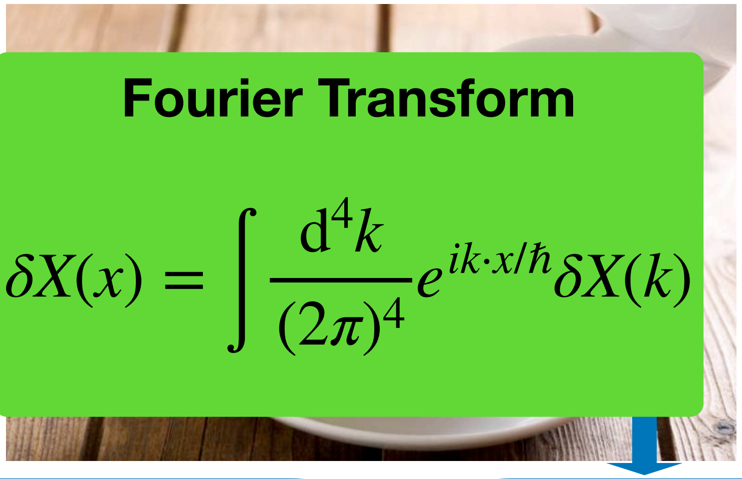
[D. Wagner, M.S, and D. H. Rischke Phys. Rev. Research 6, 043103]

Linear spin hydro

Linear hydro

Perturbing the equilibrium

$$\partial_\mu \delta T^{(\mu\nu)} = 0$$
$$\hbar \partial_\lambda \delta S^{\lambda\mu\nu} = \delta T^{[\nu\mu]}$$



Fourier Transform

$$\delta X(x) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} e^{ik\cdot x/\hbar} \delta X(k)$$

$$ik_\mu \delta T^{(\mu\nu)} = 0$$
$$ik_\lambda \delta S^{\lambda\mu\nu} = \delta T^{[\nu\mu]}$$

$$M_{\mathrm{f}}^{AB} \delta\varphi^B = \mathcal{O}(\hbar^2)$$
$$M_{\mathrm{s}}^{AB} \delta\psi^B + M_{\mathrm{fs}}^{AB} \delta\varphi^B = 0$$

Spin back-reaction **Fluid fields**

$$\begin{pmatrix} M_{\mathrm{f}} & \mathcal{O}(\hbar^2) \\ M_{\mathrm{fs}} & M_{\mathrm{s}} \end{pmatrix} \begin{pmatrix} \delta\varphi \\ \delta\psi \end{pmatrix} = \mathbf{0}$$

Fluid-induced source terms **Spin fields**

Det(M) = 0 $\longrightarrow$ characteristic equation

Spin and fluid modes decouple in the linear regime: Det(M) = Det(M_{f}) Det(M_{s})

Damped spin waves

$$\omega_0^2 - i\hbar a \omega_0 - v_{\mathfrak{s}}^2 \vec{k}^2 - \hbar^2 b = 0$$

Conformal Bjorken Flow

Conformal symmetry applied at classical order

Bjorken flow

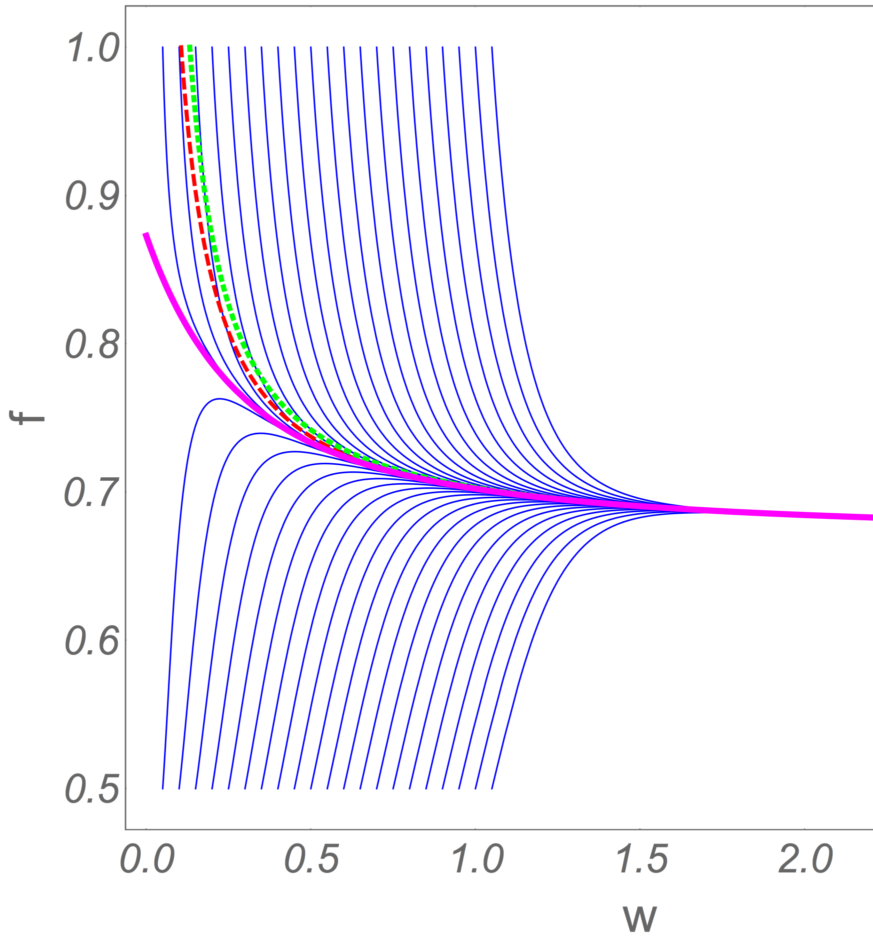
$3P, -P_\perp, -P_\perp, -P_\parallel\Big) + \mathcal{O}(\hbar^2)$

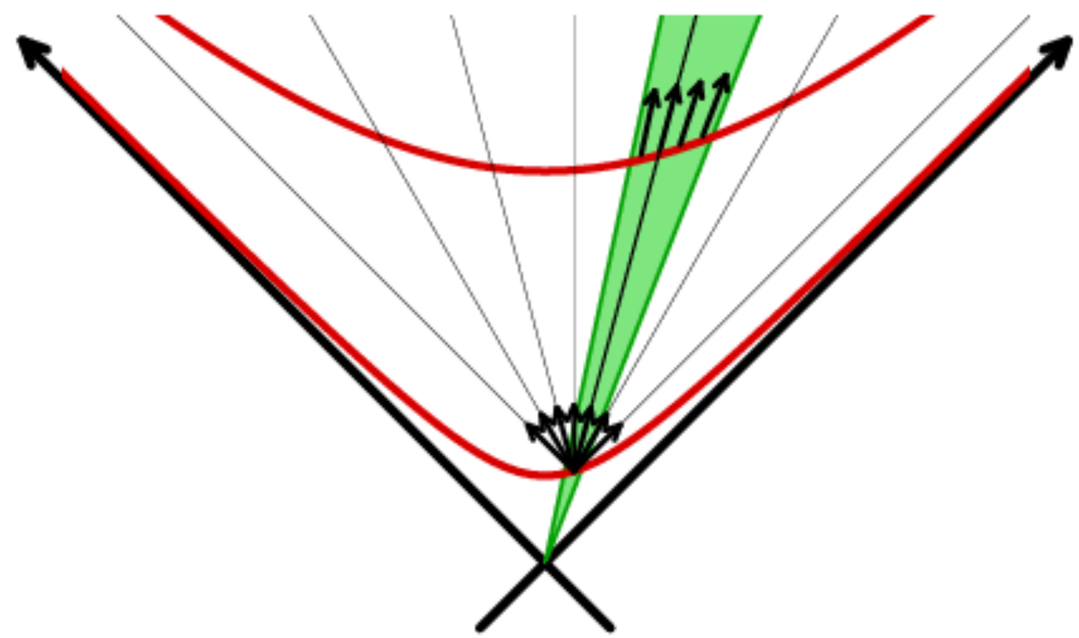
Redef. Heller-Spalinski

$f(w) = 1 + \tau T'(\tau) \quad w = T\tau$

Milne coordinates

ations of motion





roll approximation for f(w)

nformal symmetry roken at $\mathcal{O}(\hbar^2)$

Late time behaviour

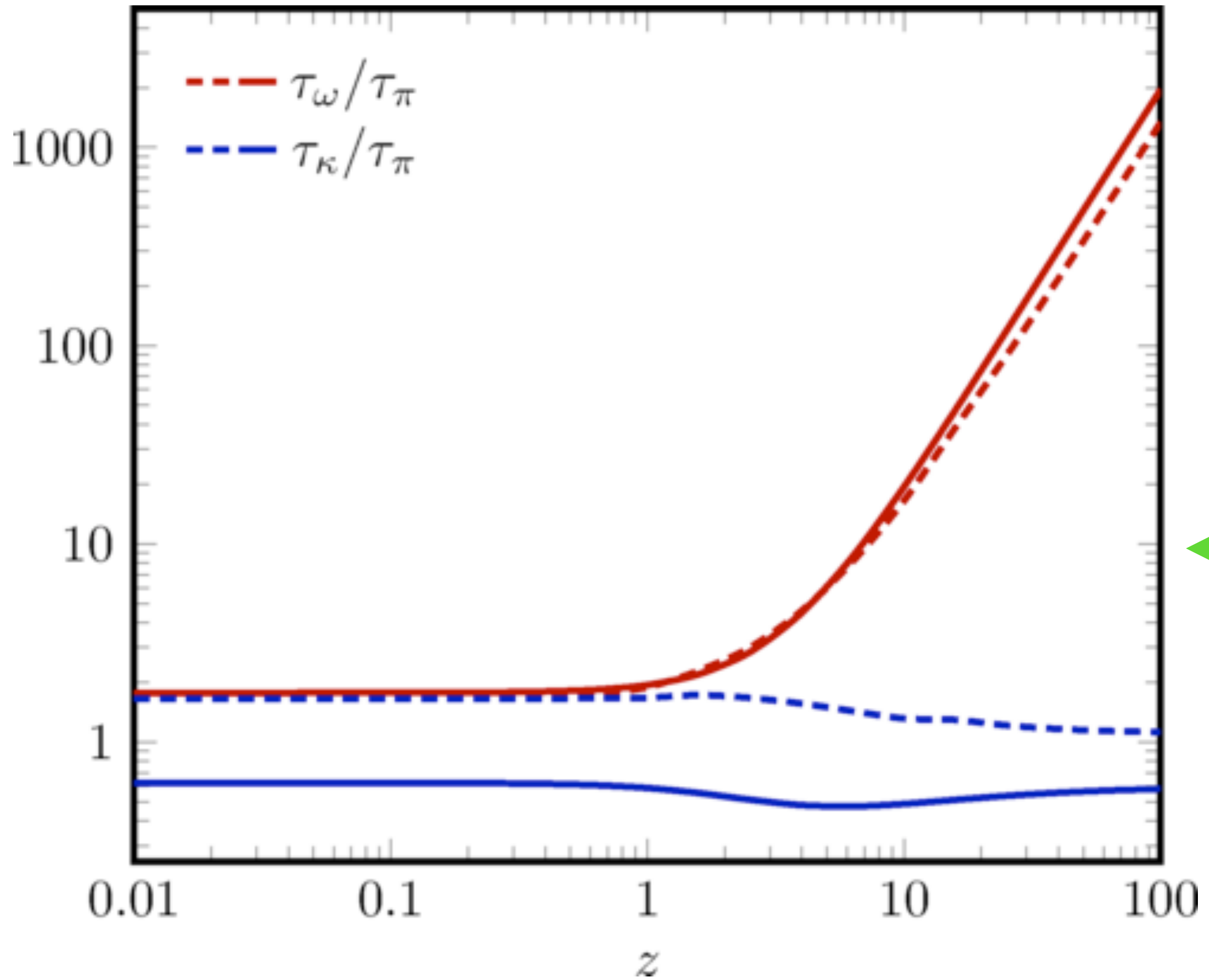
$$\mathbf{x} \sim \exp\left(-\frac{3\tau}{2\tau_{\mathbf{x}}}\right)$$

$$\tau_\omega \omega'(w) + G[f(w)]\omega = 0$$

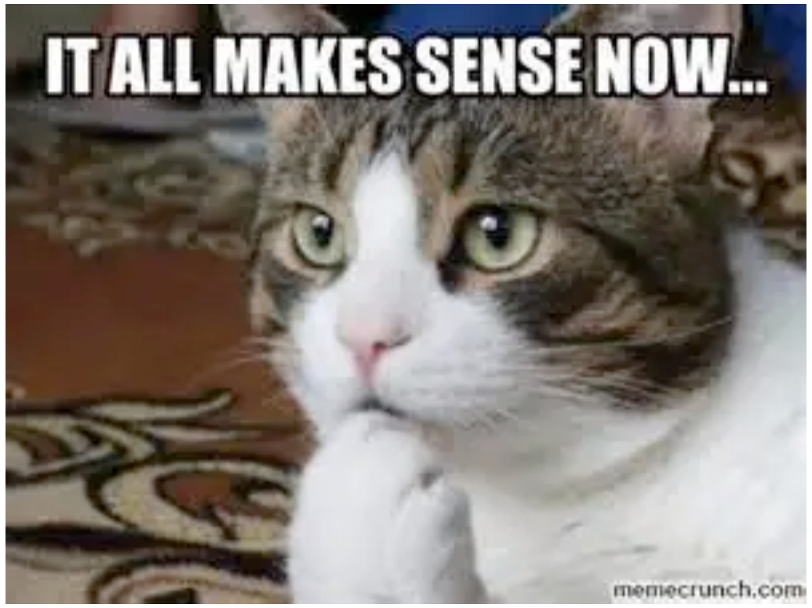
$\tau = \sqrt{t^2 - z^2} \quad \eta = \operatorname{arctanh}\frac{z}{t} \quad d^2s = d^2\tau - d^2x - d^2y - \tau^2 d^2\eta$

Timescales in spin hydrodynamics

The relaxation times τ_κ and τ_ω as functions of $z{=}m/T$ in units of the relaxation time of the shear-stress tensor τ_π. The solid lines denote the result for a scalar four-fermion interaction, while the dashed lines refer to (screened) one-gluon exchange.



High-energy collisions ← Low-energy collisions



Spin degrees of freedom relax quickly in **high-energy collisions**, while these timescales for low-energy collisions might be even larger than the lifetime of the fireball!

The triumph

Spin Polarization of Λ hyperons from Dissipative Spin Hydrodynamics

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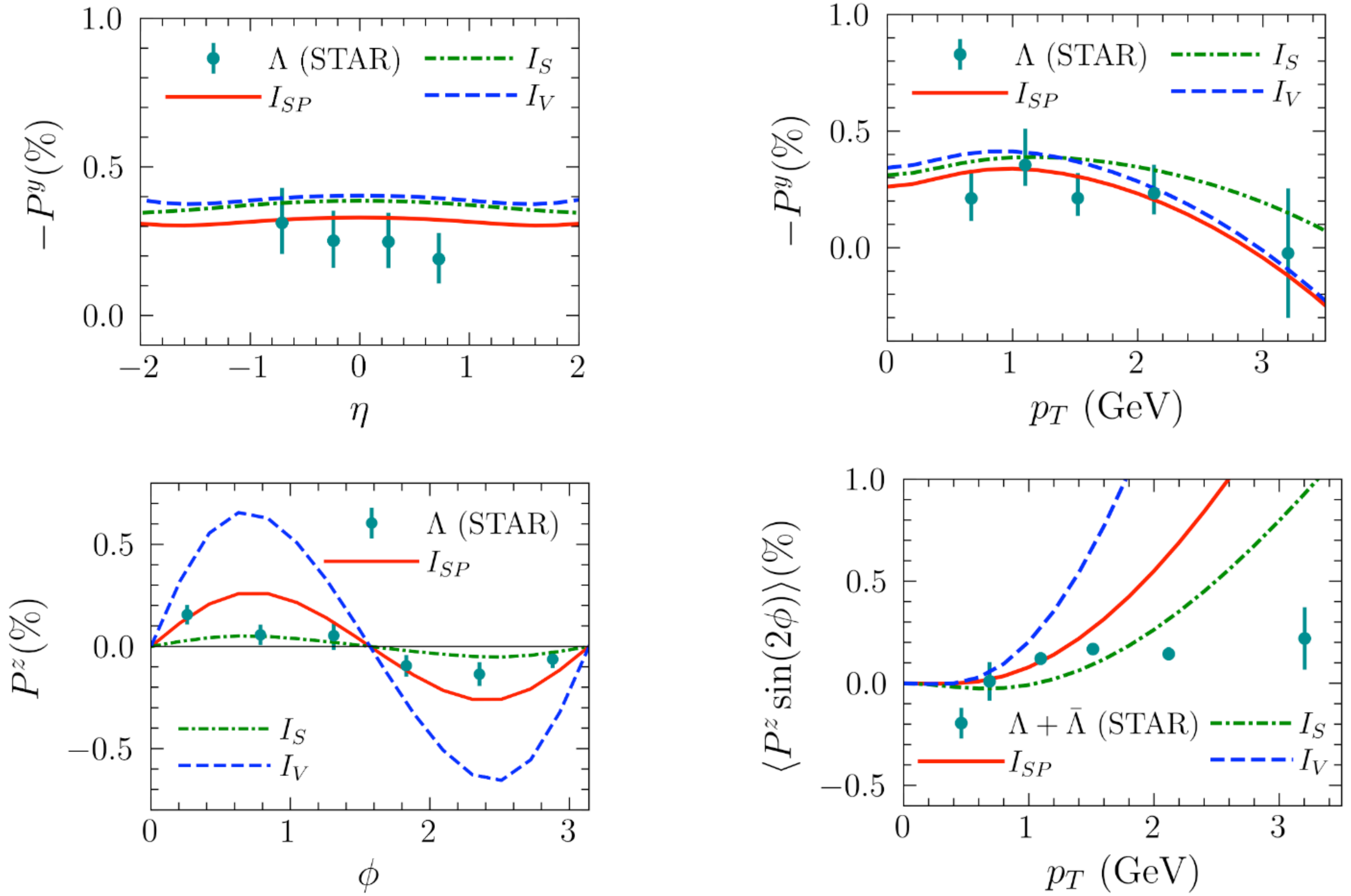
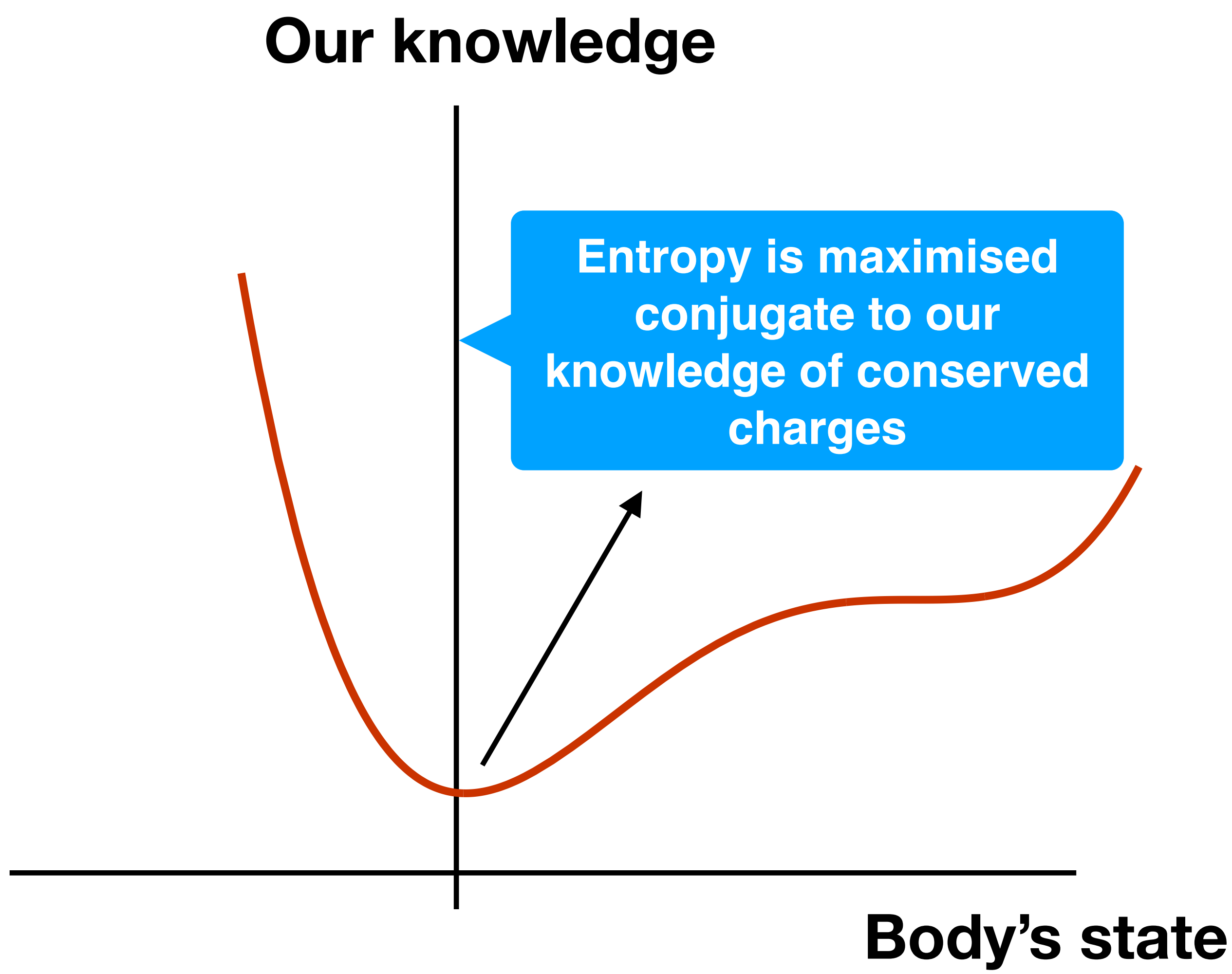
(Dated: March 31, 2025)

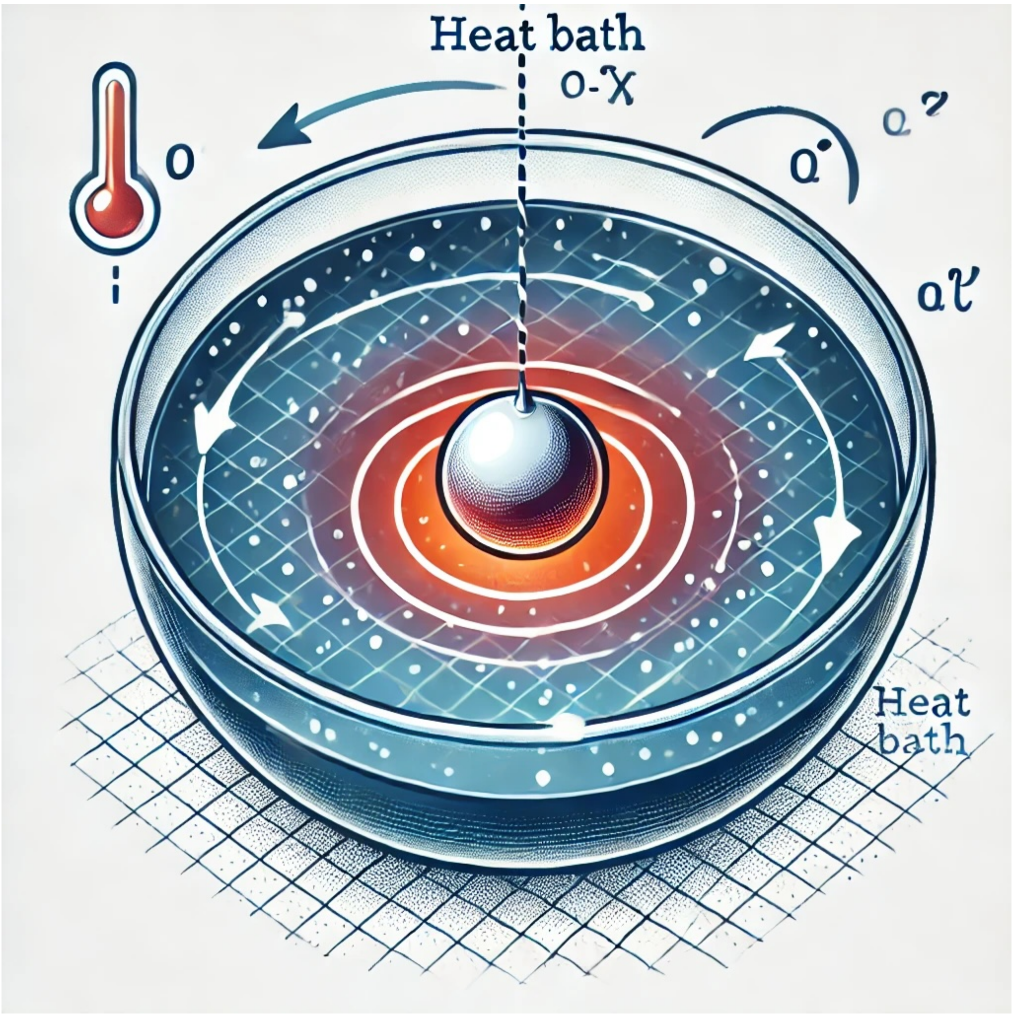


Semi-classical spin hydrodynamics: the price

A simple example of thermodynamic stability



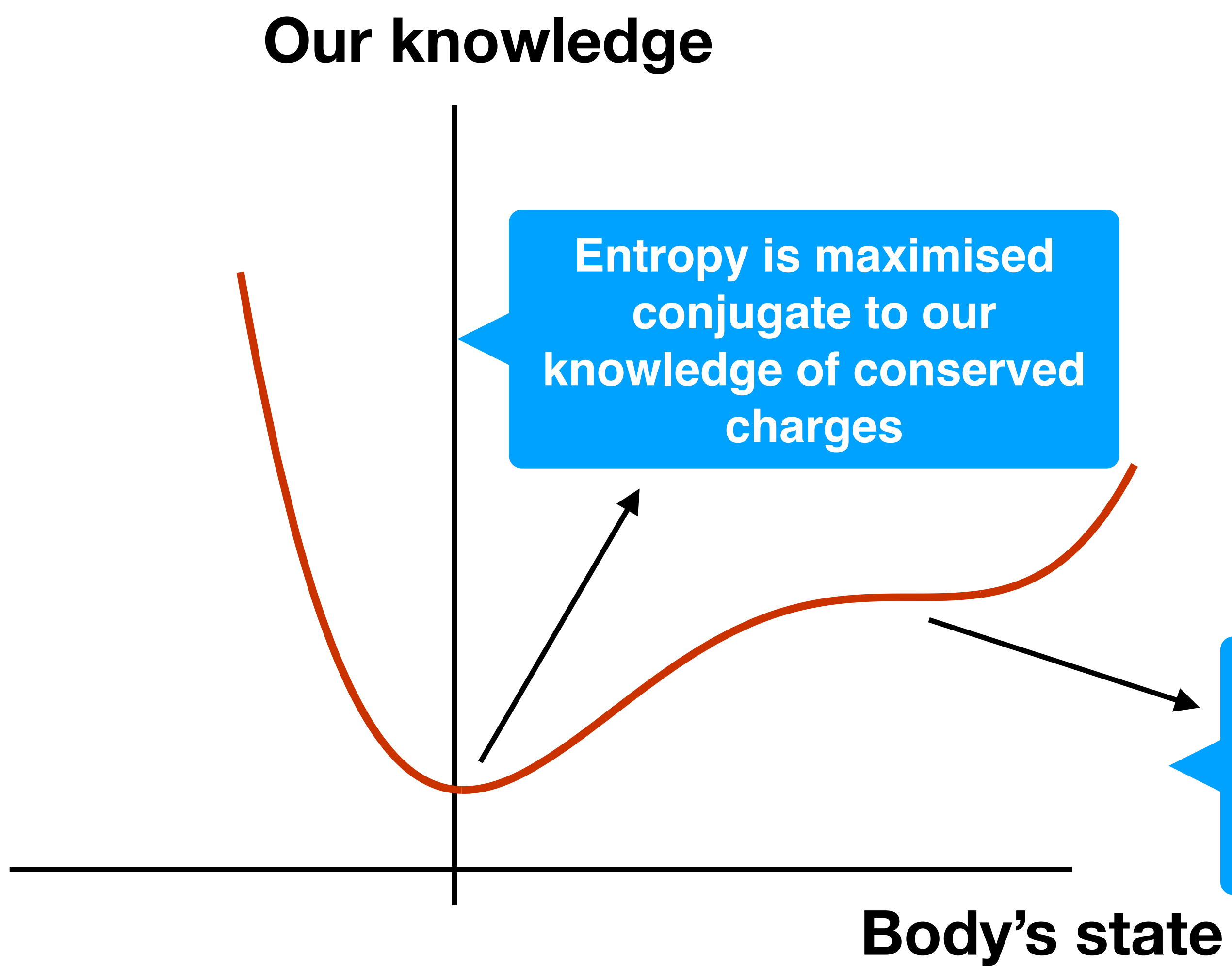
A small body in an environment



The environment performs work to shift the system slightly out of equilibrium

$$\delta W_{\min} = \delta E - T_E \delta S + P_E \delta V > 0$$

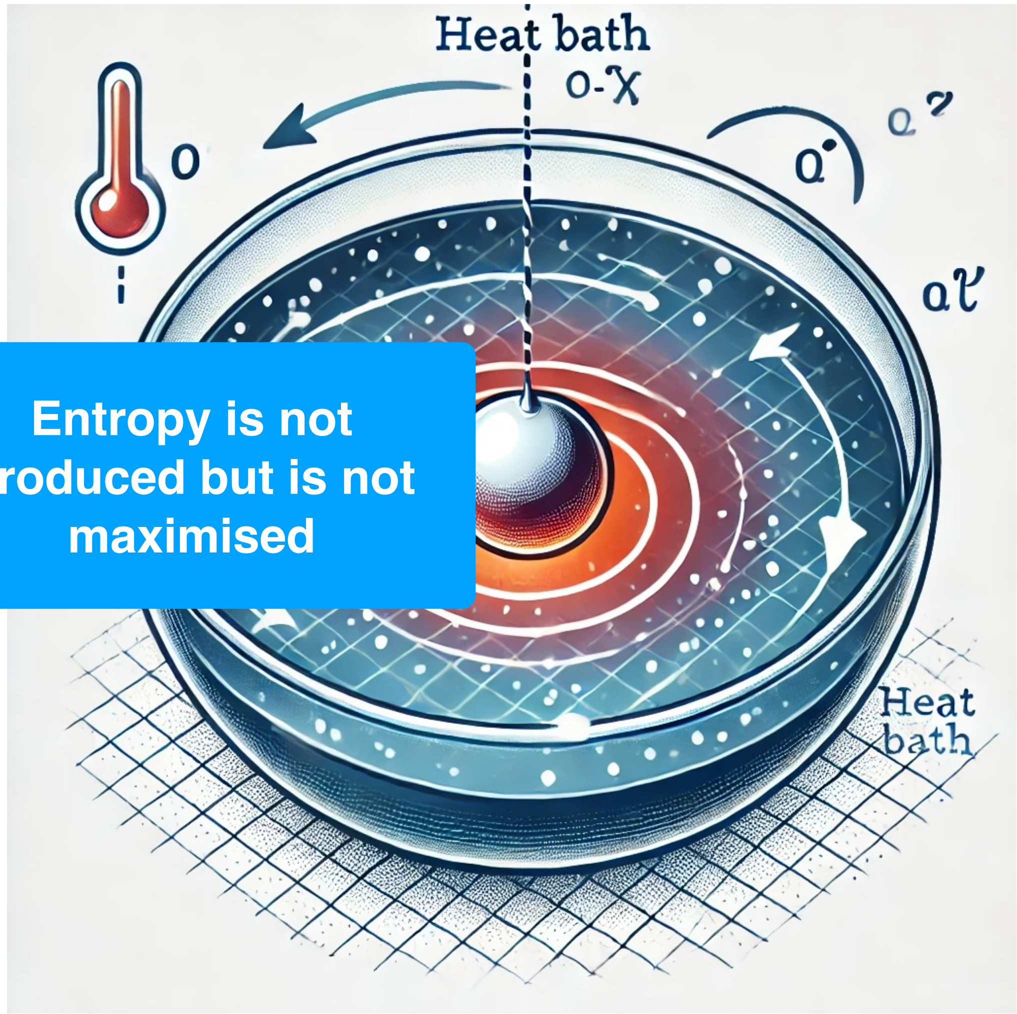
A simple example of thermodynamic stability



Entropy is maximised conjugate to our knowledge of conserved charges

Entropy is not produced but is not maximised

A small body in an environment



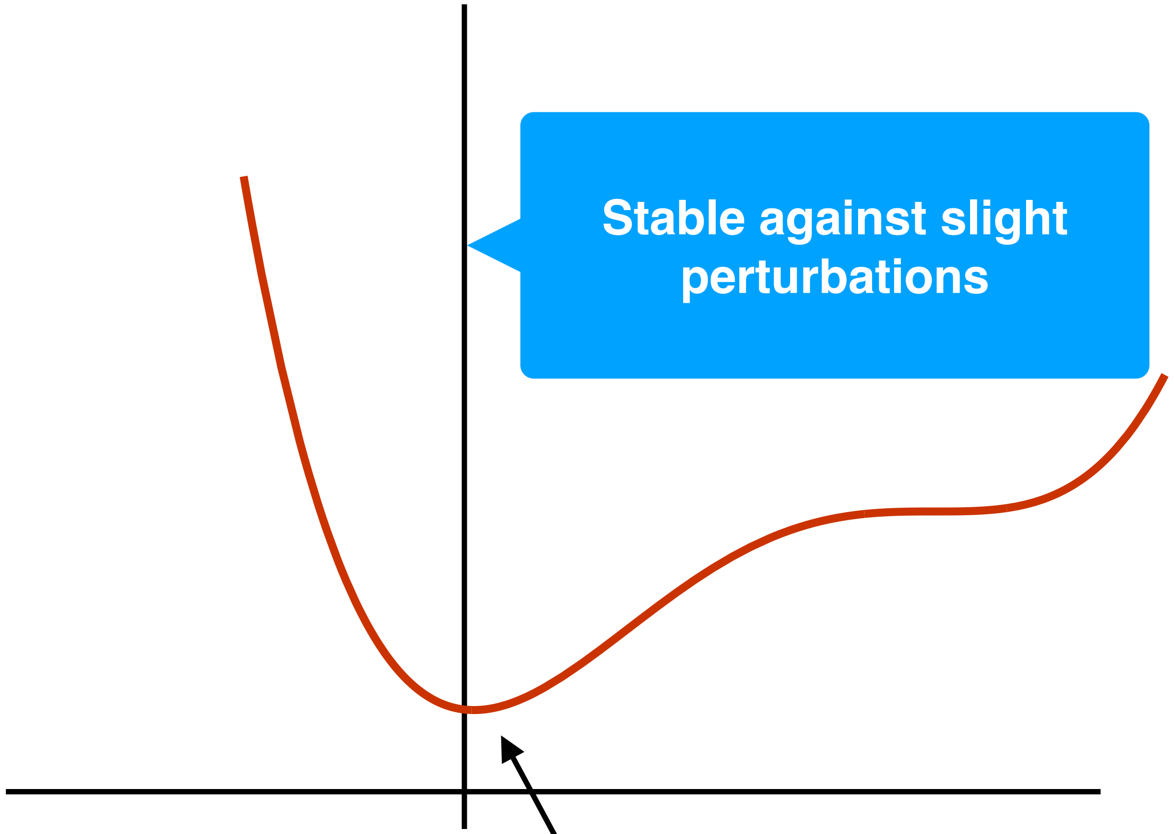
$$\delta W_{\min} = \delta E - T_E \delta S + P_E \delta V > 0$$

First-order derivative: the intensive parameters are equal to those of the bath

$$\left(\frac{1}{T_E} - \frac{\partial S}{\partial E}\right) \delta E + \left(\frac{P_E}{T_E} - \frac{\partial S}{\partial V}\right) \delta V$$

A simple example of thermodynamic stability

Our knowledge



Stable against slight perturbations

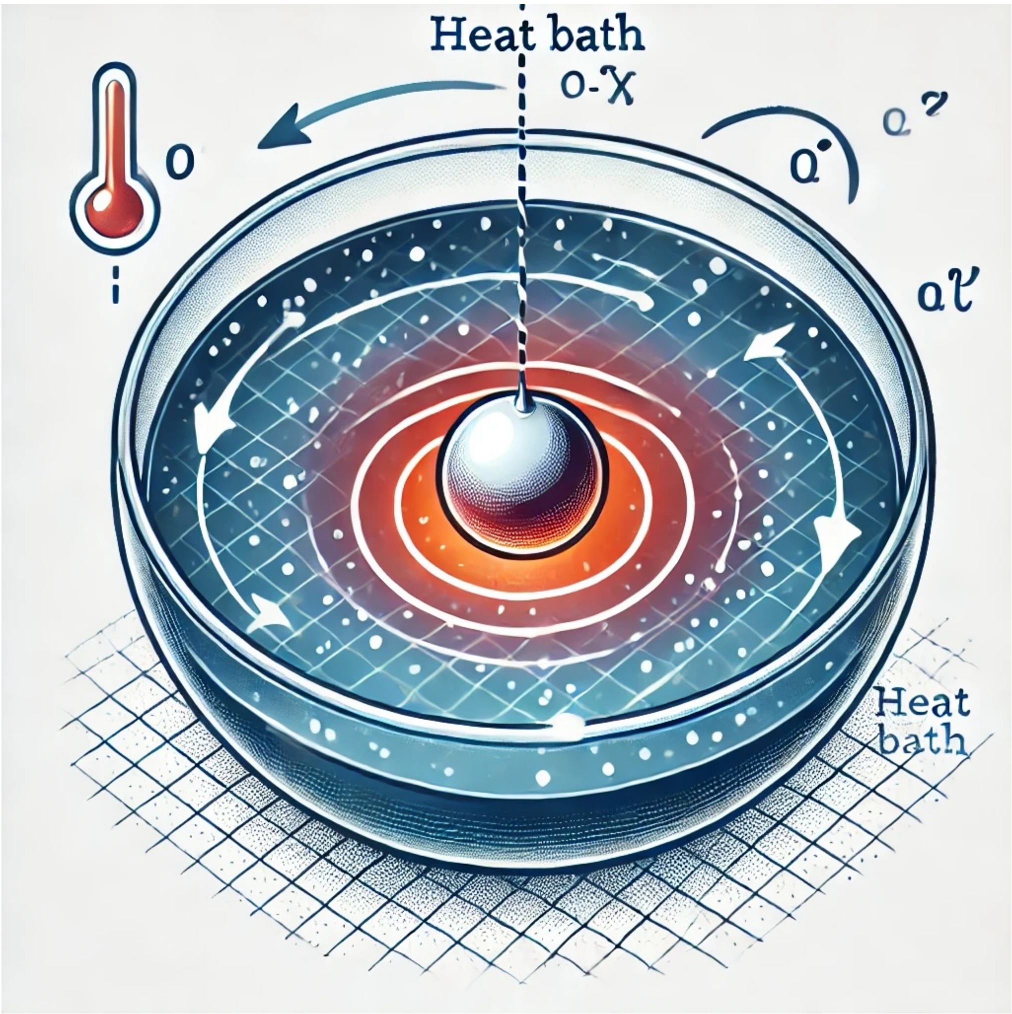
Body's state

Second-order derivative:thermodynamic inequalities

$$C_v > 0 \qquad \left.\frac{\partial P}{\partial V}\right|_T < 0 \qquad C_p > 0$$

$$\delta W_{\min} = \delta E - T_E \delta S + P_E \delta V > 0$$

A small body in an environment

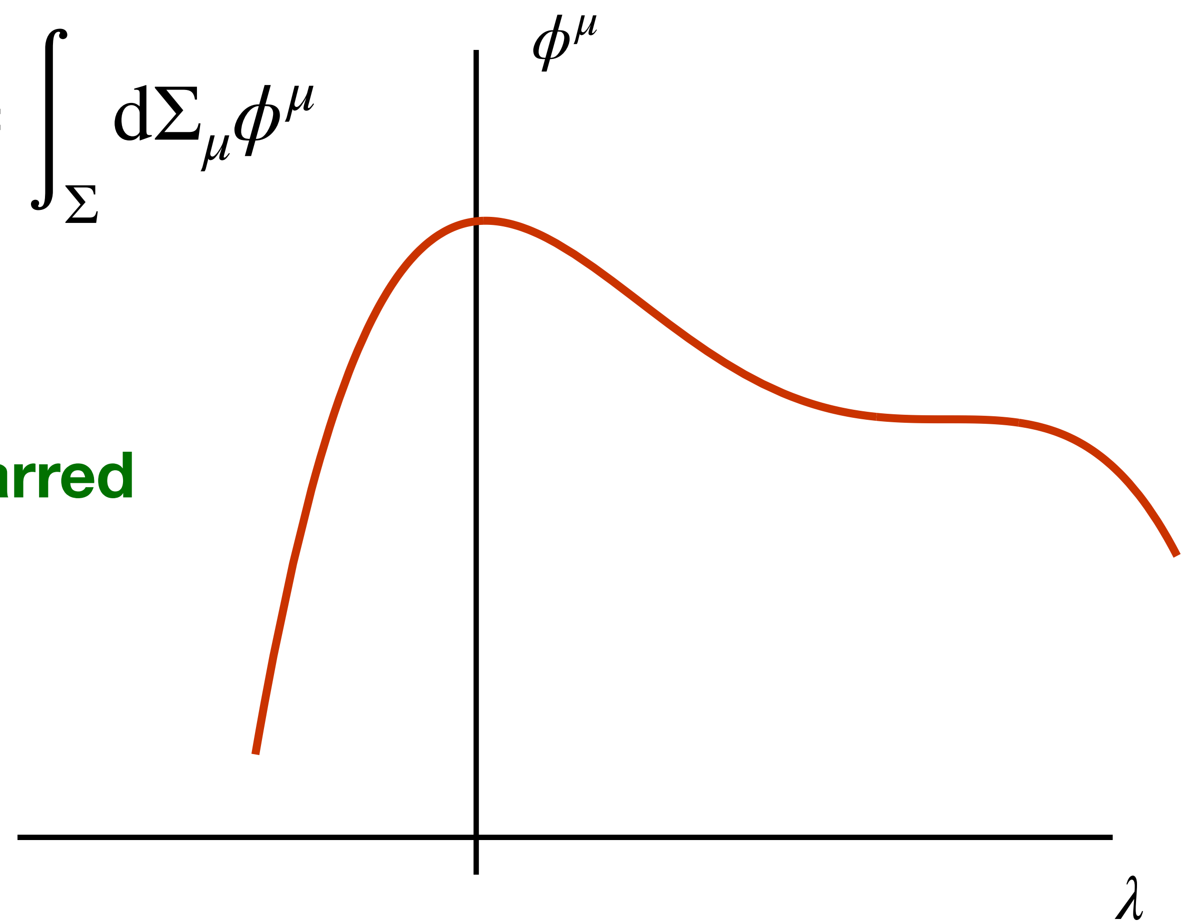


Thermodynamic stability

Gibbs stability criterion

$$\Phi \equiv S - \lambda_I^\star Q^I \leq \log Z_{GTE} \qquad \Phi = \int_\Sigma \mathrm{d}\Sigma_\mu \phi^\mu$$

$$\phi^\mu = S^\mu - T^{\mu\nu}\beta_\nu^\star + \frac{1}{2}\mathcal{S}^{\mu\alpha\beta}\varpi_{\alpha\beta}^\star$$



Slightly perturb the fields, keeping starred quantities constant

$$\psi \to \psi + \delta\psi \qquad \delta\psi = \frac{d\psi}{d\lambda}\delta\lambda \equiv \dot{\psi}\,\delta\lambda$$

At first-order, the stationary points are found

$$\dot{\phi}^\mu(0) = 0$$

At second-order, the information-current is found

$$E^\mu = -\frac{1}{2}\ddot{\phi}^\mu(0)$$ **future-directed and non-spacelike**

Stability conditions

The Maxwell-Israel-Stewart Miracle

The electromagnetic part of the information current is stable and causal by construction and, therefore, the stability criteria found for Israel-Stewart-type theories of hydrodynamics automatically extend to similar formulations of magnetohydrodynamics.

[L. Gavassino, MS, (2023)]

The price of truncating at first-order

We know things at second-order

$$\mathcal{O}(\hbar^2)$$

$$\phi^\mu = S^\mu - T^{\mu\nu}\beta^\star_\nu + \frac{1}{2}\mathcal{S}^{\mu\alpha\beta}\varpi^\star_{\alpha\beta}$$

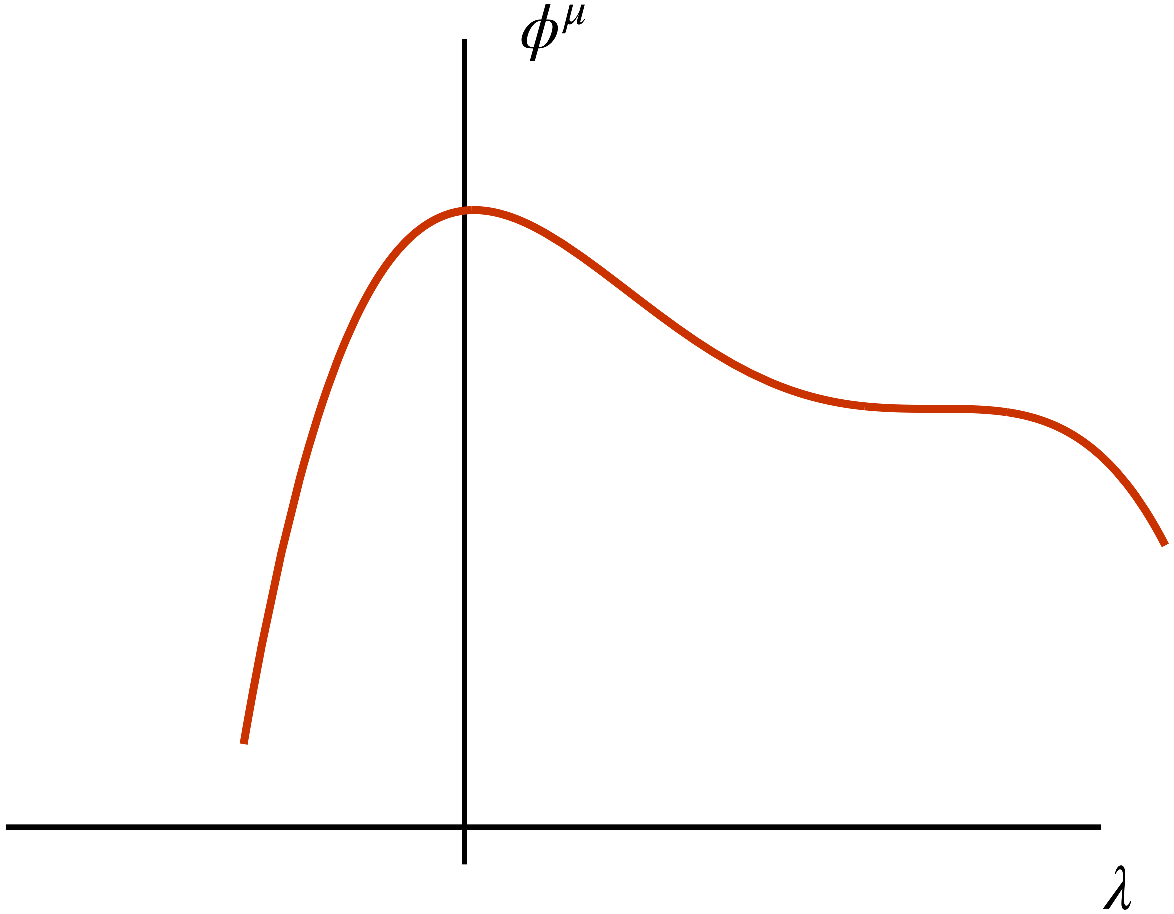
Missing information at $\mathcal{O}(\hbar^2)$

So we don't know everything!

To be consistent, we need to truncate second-order terms:

$$\phi^\mu = S^\mu_{\mathrm{f}} - T^{(\mu\nu)}_{\mathrm{f}}\beta^\star_\nu + \mathcal{O}(\hbar^2) \longrightarrow$$

Fluid stability criteria (already known)

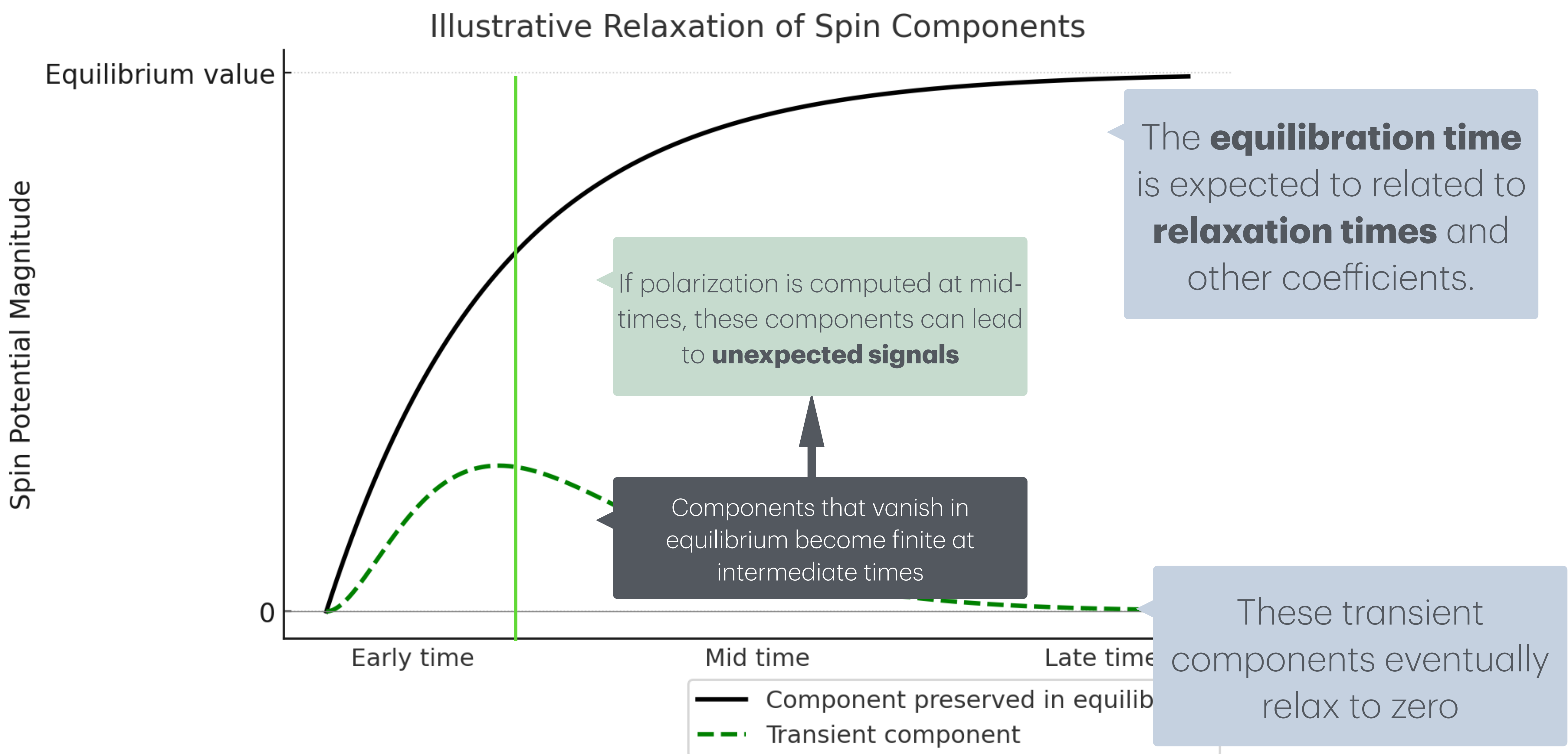


And truncate our thermodynamic identities

$$\mathrm{d}\epsilon = T\mathrm{d}s + \mathcal{O}(\hbar^2) \qquad \mathrm{d}P = s\mathrm{d}T + \mathcal{O}(\hbar^2) \qquad \epsilon + P = sT + \mathcal{O}(\hbar^2)$$

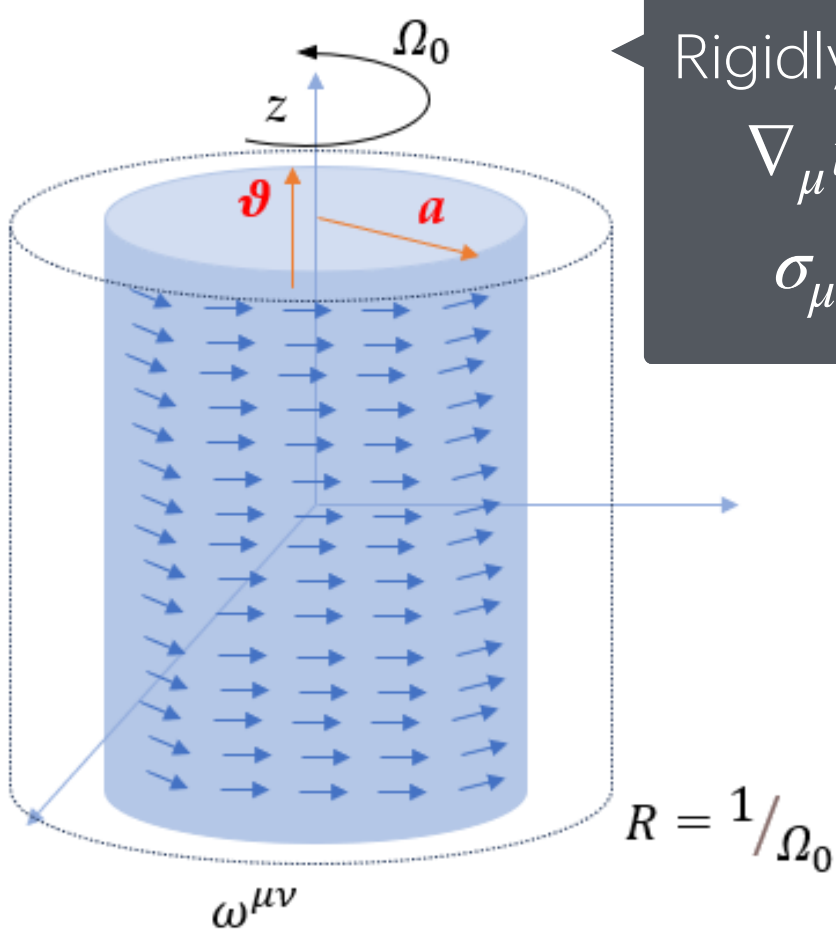
Outlook

How can we compare LTE and spin-hydro?



Adopted from QM 2025 poset of **Annamaria Chiarini**

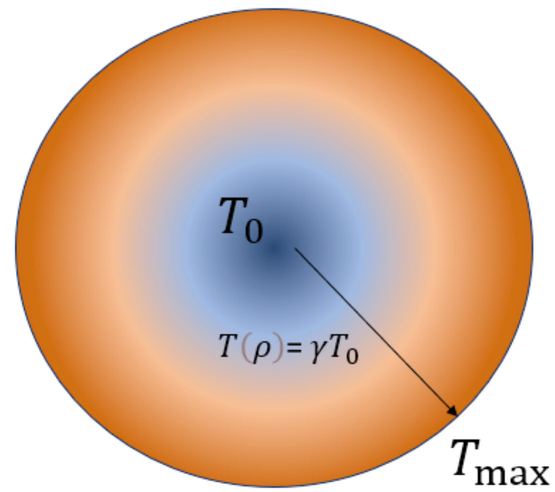
Rigidly rotating cylinder as a benchmark



Rigidly rotating

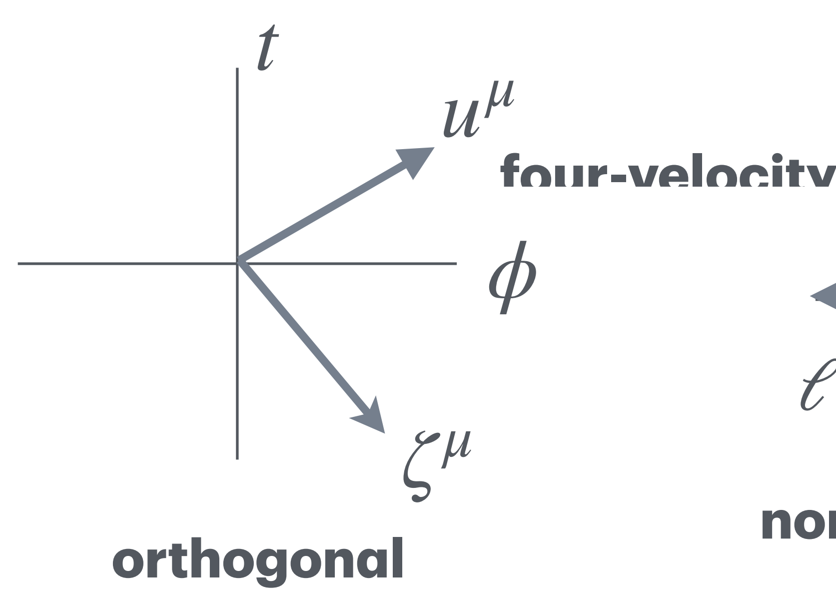
$$\nabla_\mu u^\mu = 0$$

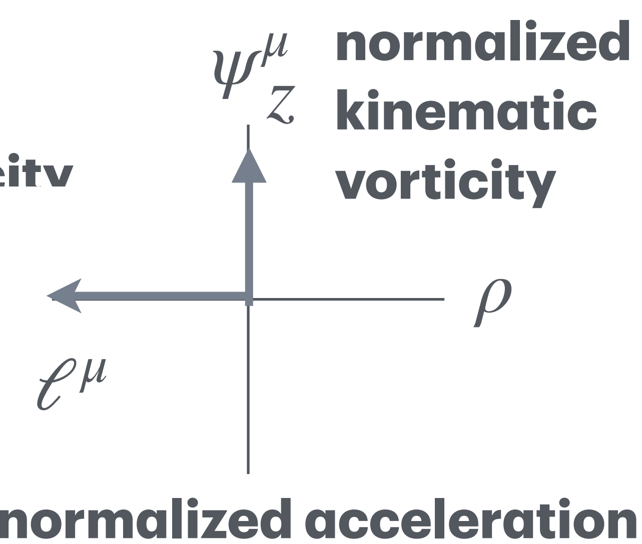
$$\sigma_{\mu\nu} = 0$$



Temperature increases with radius

$$\nabla_\mu T = T a_\mu$$





An orthonormal tetrad can be defined, built from the four-velocity, normalized acceleration, normalized kinematic vorticity, and a fourth vector orthogonal to all three.

Adopted from QM 2025 poset of **Annamaria Chiarini**

What is hiding at higher orders?

- Couplings between gauge fields and spin degrees of freedom, as well as anomalous transport effects
- Back-reaction from spin to the fluid
- Gravity-induced quantum effects
- Inherently anisotropic currents

Summary

- Covariant definitions for angular momentum currents
- Modifications of energy-momentum-conservation in curved spacetime
- Revised pseudo-gauge transformations in curved spacetime
- Spin and fluid modes decouple in the linear regime
- Information-current method: (1) at first order in $\hbar$ spin tensor does not modify fluid's stability conditions (2) the equilibrium currents are inherently anisotropic
- Spin potential damping in Bjorken flow is similar to damping of spin waves
- Spin potential relaxes quickly in high-energy collision and slowly in low-energy collisions