# Unified Description of Hydrodynamic and Nambu–Goldstone Modes in Open and Closed Systems


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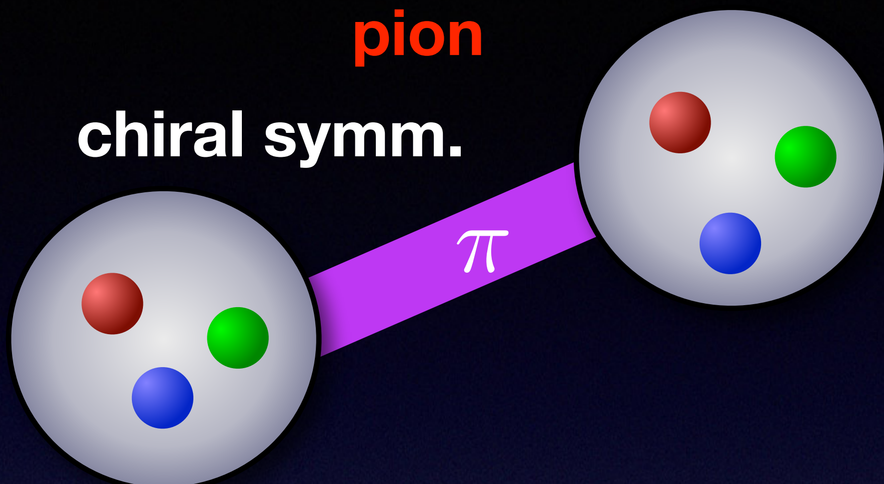
Y. Minami, YH, Phys.Rev. E97 (2018) 012130 , PTEP 2020 (2020) 3, 033A01
T. Hayata, YH, 1808.07636,
Y. Minami, H. Nakano, Y. Hidaka, Phys. Rev. Lett. 126 (2021) 14, 141601

# Gapless modes in nature

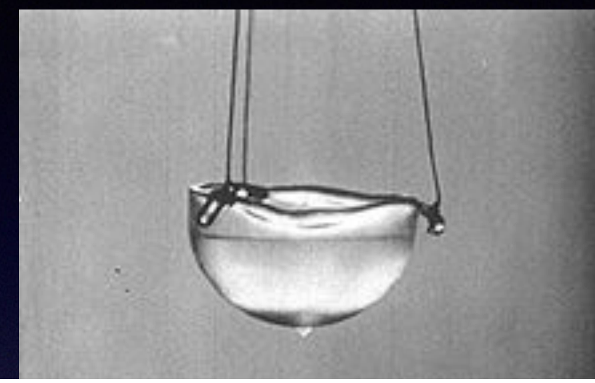
pion

chiral symm.



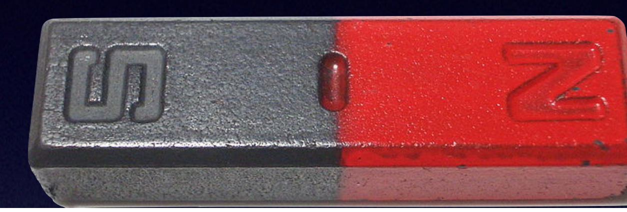

superfluid phonon

U(1) symm.



spin waves

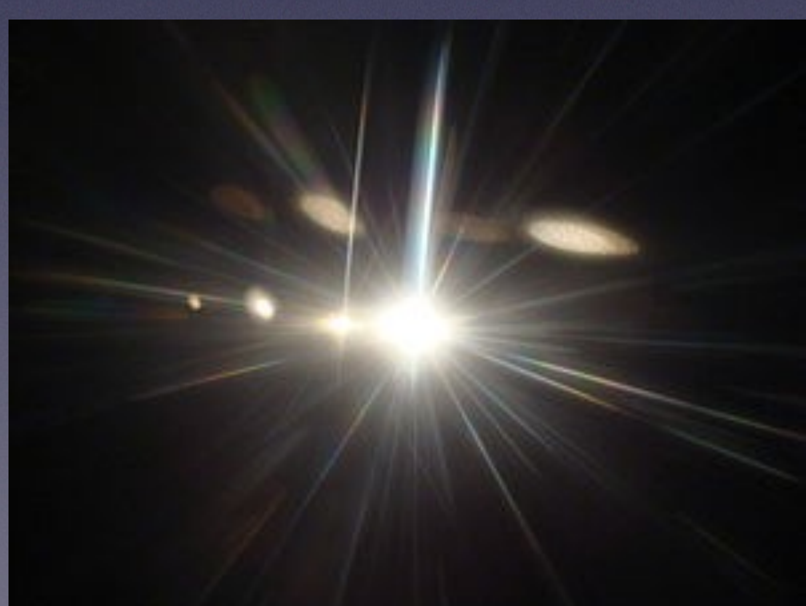
spin symm.




## Nambu-Goldstone modes

photon

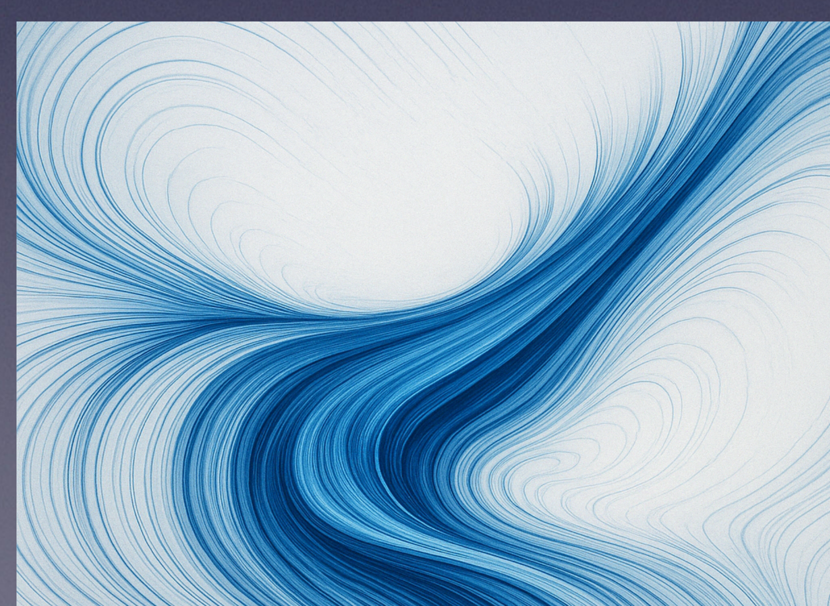
U(1) 1-form symm.





Hydrodynamic modes

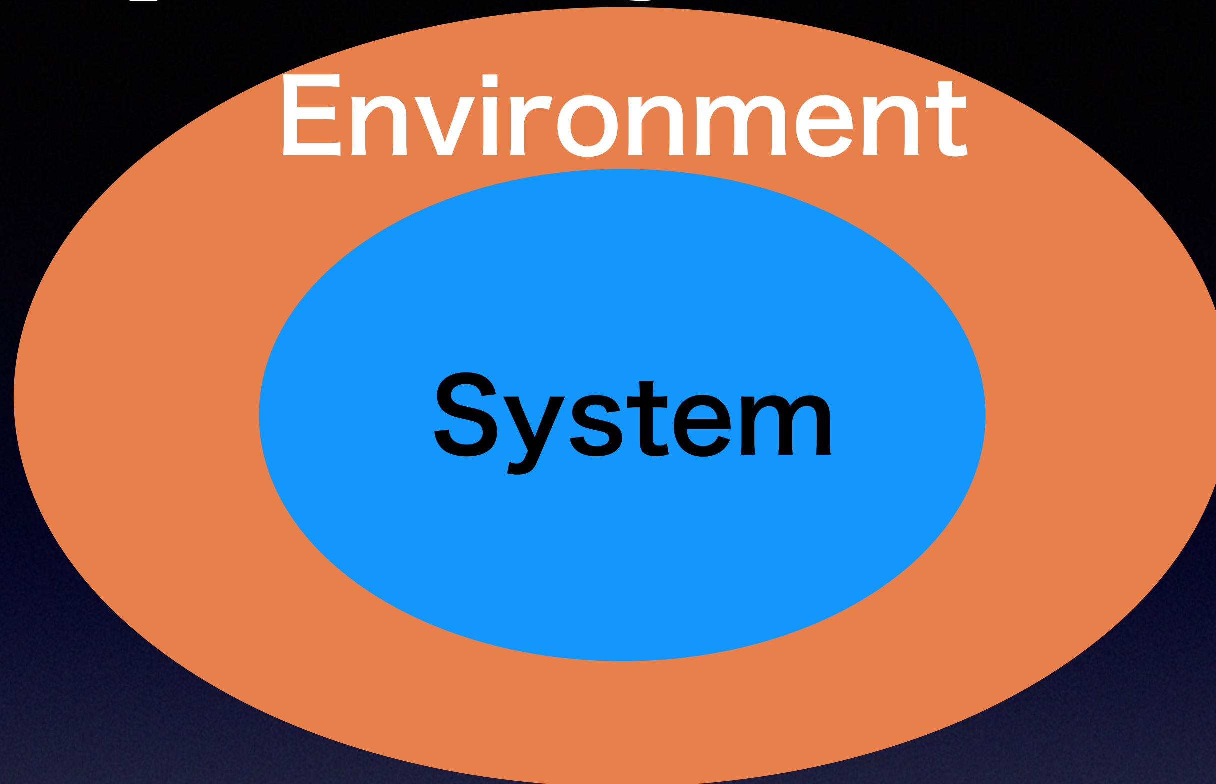
strong translation symm.



collective waves

rotational symm.

# Open systems





Environment: QGP
System: Heavy quarks



Environment: Air
System: Flock of birds

# Questions

## What is spontaneous symmetry breaking for hydrodynamics?

## What is symmetry of open systems?

Energy-momentum, particle number are not conserved.

## What is the dispersion relation for the Nambu-Goldstone (NG) modes?

propagating, diffusion, …

# Spontaneous symmetry breaking for hydrodynamics

# Strong and weak symmetry

Lessa, Ma, Zhang, Bi, Cheng, Wang ('24)  Sala, Gopalakrishnan, Oshikawa, You ('24)  Huang, Qi, Zhang, Lucas ('24)

## Let $\rho$ be a stationary state

**Consider unitary operator $U_g$ for global symmetry $G$**

**Ordinal global symmetry acts on $\rho$ as conjugation**

$$\rho \to U_g \rho U_g^\dagger$$

**(This symmetry is called weak symmetry)**

**For thermal equilibrium state,**

**if** $U_g \rho U_g^\dagger = \rho$ **symmetry is unbroken**

**Otherwise symmetry is spontaneous broken**

# Strong and weak symmetry

Consider unitary operator $U_g$ for Global symmetry $G$

One can consider

$$\rho \to U_g \rho \quad \text{or} \quad \rho U_g$$

If $U_g \rho = e^{i\theta} \rho$ Strong symmetry is unbroken

Otherwise strong symmetry is spontaneous broken

The vacuum $\rho = |\Omega\rangle\langle\Omega|$ is invariant under strong symmetry transformation

# Strong and weak symmetry

## Grand canonical distribution

$$\rho_{\mathrm{eq}} = \frac{e^{-\beta(H-\mu N)}}{\mathrm{tr}e^{-\beta(H-\mu N)}}$$ **breaks strong symmetry**

**but does not break weak symmetry**

**(Except Lorentz symmetry)**

$$U\rho_{\mathrm{eq}}U^{\dagger} = \rho_{\mathrm{eq}}$$

# Strong and weak symmetry

**Order parameter is susceptibility**

$$\chi = \frac{1}{V}\left( \int d^3x \int d^3y \langle j^0(x) j^0(y) \rangle - \left( \int d^3x \langle j^0(x) \rangle \right)^2 \right)$$

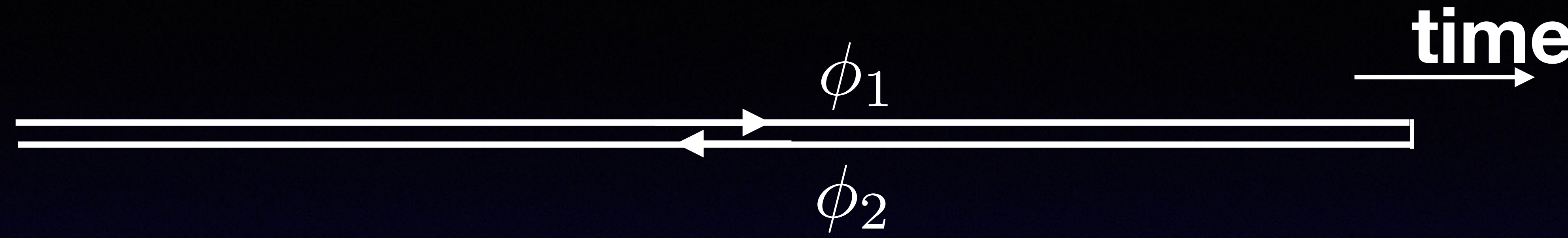
**For the eigenstate of** $\hat{Q} = \int d^3x j^0(x)$ $\quad \hat{Q} | \Psi \rangle = Q | \Psi \rangle$

$\chi$ **vanishes.**

**Micro canonical ensemble spontaneously breaks strong symmetry**

**(local observables give the same results as grand canonical)**

# Schwinger-Keldysh Path integral




$$Z = \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 \exp\left[iS[\phi_1] - iS[\phi_2]\right]$$

$S[\phi_1]$ **: forward evolution**

$S[\phi_2]$ **: backward evolution**

**Action is invariant under** $G_1 \times G_2$

**We consider effective theory associated with spontaneous breaking of** $G_1 \times G_2$**.**

# Construction of effective theory

## SSB of $G \to H$

- **Prepare** $\xi(x) = e^{i\pi(x)} \in G$

- **Gauging** $H$

  gauge transformation $\xi(x) \to \xi(x)h(x)$

  $h \in H$

**Write down gauge invariant action which is an effective theory for the coset $G/H$**

# Construction of effective theory

**Example) Chiral symmetry breaking in vacuum**

$$\mathrm{SU}(N)_R \times \mathrm{SU}(N)_L \to \mathrm{SU}(N)_V$$

$$\xi_R = e^{i\pi_R} \quad \xi_L = e^{i\pi_L}$$

**Imposing gauge invariance for $\mathrm{SU}(N)_V$**

$$\xi_R \to \xi_R h(x) \quad \xi_L \to \xi_L h(x)$$

**Gauge invariant combination is**

$$U = \xi_L \xi_R^\dagger$$

**Effective Lagrangian**

$$\mathscr{L} = F^2 \mathrm{tr} \partial^\mu U^\dagger \partial_\mu U + \cdots$$

# Hydrodynamics

**Strong translation symmetry is broken**

**Weak Lorentz symmetry is broken**

**The degrees of freedom are**

$X_1^\mu(\sigma^0, \boldsymbol{\sigma}) \quad X_2^\mu(\sigma^0, \boldsymbol{\sigma})$ $\qquad \sigma^A$ **: fluid space**

**Impose** $\sigma^i \to \sigma'^i(\sigma), \quad \sigma^0 \to \sigma^0$

$\sigma^0 \to f(\sigma^0, \sigma), \quad \sigma^i \to \sigma^i$ or $\sigma^0 \to f(\sigma), \quad \sigma^i \to \sigma^i$

cf. Crossley, Glorioso, Liu ('15) ('17)

**Note that imposing full diffeo recover the Lorentz invariance**

# Hydrodynamics

$$h_{AB} = \partial_A X^\mu \partial_B X^\nu g_{\mu\nu} \quad \partial_0 X^\mu = \beta^\mu = \beta u^\mu \quad h_{00} = \beta^2$$

$$S[X] = \int d^4\sigma \sqrt{-h}p(\beta)$$

**which is the effective action for perfect fluid**

**The effective action is**

$$S_{\rm SK} = S[X_1] - S[X_2] + S_{\rm diss}[X_1, X_2]$$

**dissipation and fluctuation**

# $\mathrm{U}(1)_1 \times \mathrm{U}(1)_2$ case

Huang, Qi, Zhang, Lucas ('24)

**NG field:** $e^{i\pi_1}, e^{i\pi_2}$

**Weak:** $\pi_1 \to \pi_1 + c \quad \pi_2 \to \pi_2 + c$

**Strong:** $\pi_1 \to \pi_1 + c \quad \pi_2 \to \pi_2 - c$

$R/A$ **basis:** $\pi_A = \pi_1 - \pi_2 \quad \pi_R = (\pi_1 + \pi_2)/2$

**Building block:** $\dot{\pi}_A \quad \partial_i \pi_A \quad \dot{\pi}_R$

**which are invariant under** $\pi_A \to \pi_A + c \quad \pi_R \to \pi_R + c(x)$

**If weak symmetry is spontaneously broken**

$\partial_i \pi_R$ **is also allowed**

# $\mathrm{U}(1)_1 \times \mathrm{U}(1)_2$ case

- **When weak symmetry is unbroken**

$$\mathscr{L} = \dot{\pi}_A \dot{\pi}_R - \Gamma(\partial_i \pi_A)(\partial_i \dot{\pi}_R) + \cdots$$

**E.o.m.** $\ddot{\pi}_R - \Gamma \partial_i^2 (\dot{\pi}_R) = 0$ ⇒ $\omega = -i\Gamma \boldsymbol{k}^2$

**diffusion mode**

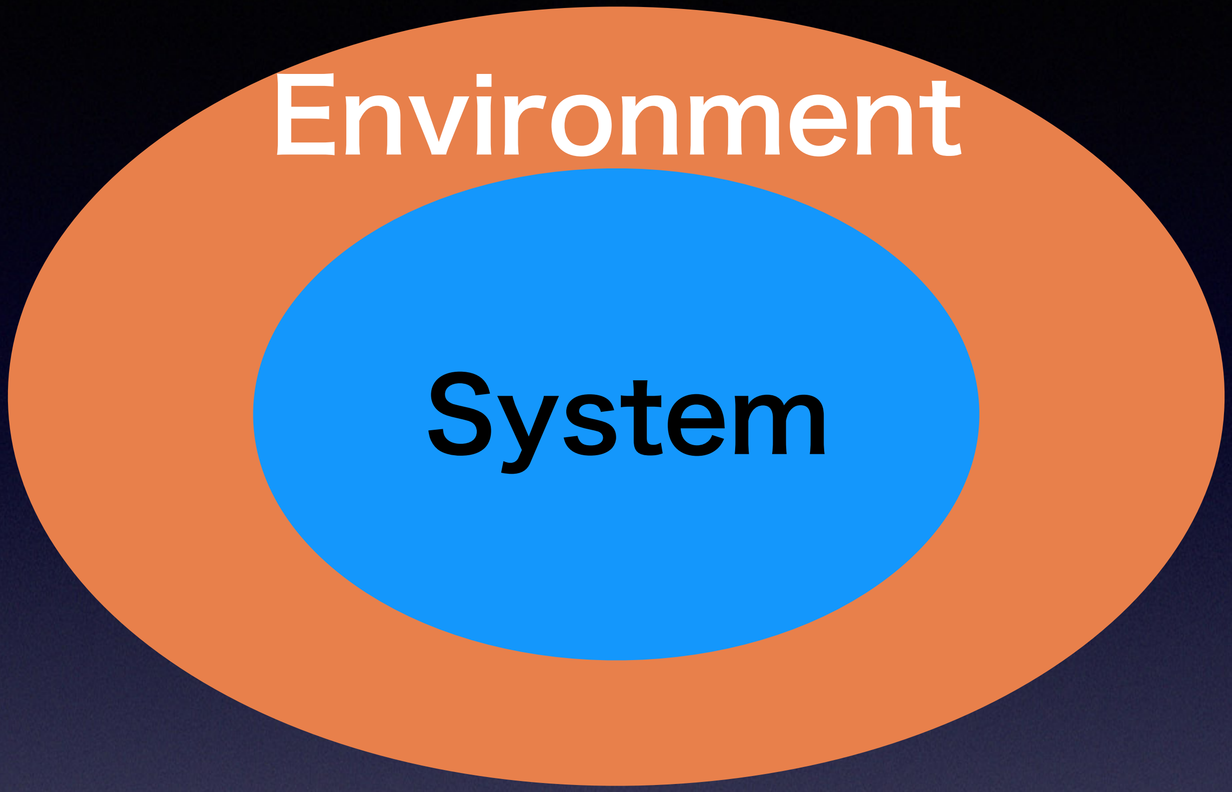
- **When Weak symmetry is spontaneously broken**

$$\mathscr{L} = \dot{\pi}_A \dot{\pi}_R - v_s^2 (\partial_i \pi_A)(\partial_i \pi_R) - \Gamma(\partial_i \pi_A)(\partial_i \dot{\pi}_R) + \cdots$$

**E.o.m.** $\ddot{\pi}_R - \Gamma \partial_i^2 \dot{\pi}_R - v_s^2 \partial_i^2 \pi_R = 0$ ⇒ $\omega = \pm v_s |\boldsymbol{k}| - i\frac{\Gamma}{2}\boldsymbol{k}^2$

**propagating mode**

# Open systems



# Questions

## Hamiltonian systems

**Continuum symmetry** $\Leftrightarrow$ $\partial_\mu J^\mu = 0$

## Open systems

$\partial_\mu J^\mu \neq 0$ because of friction

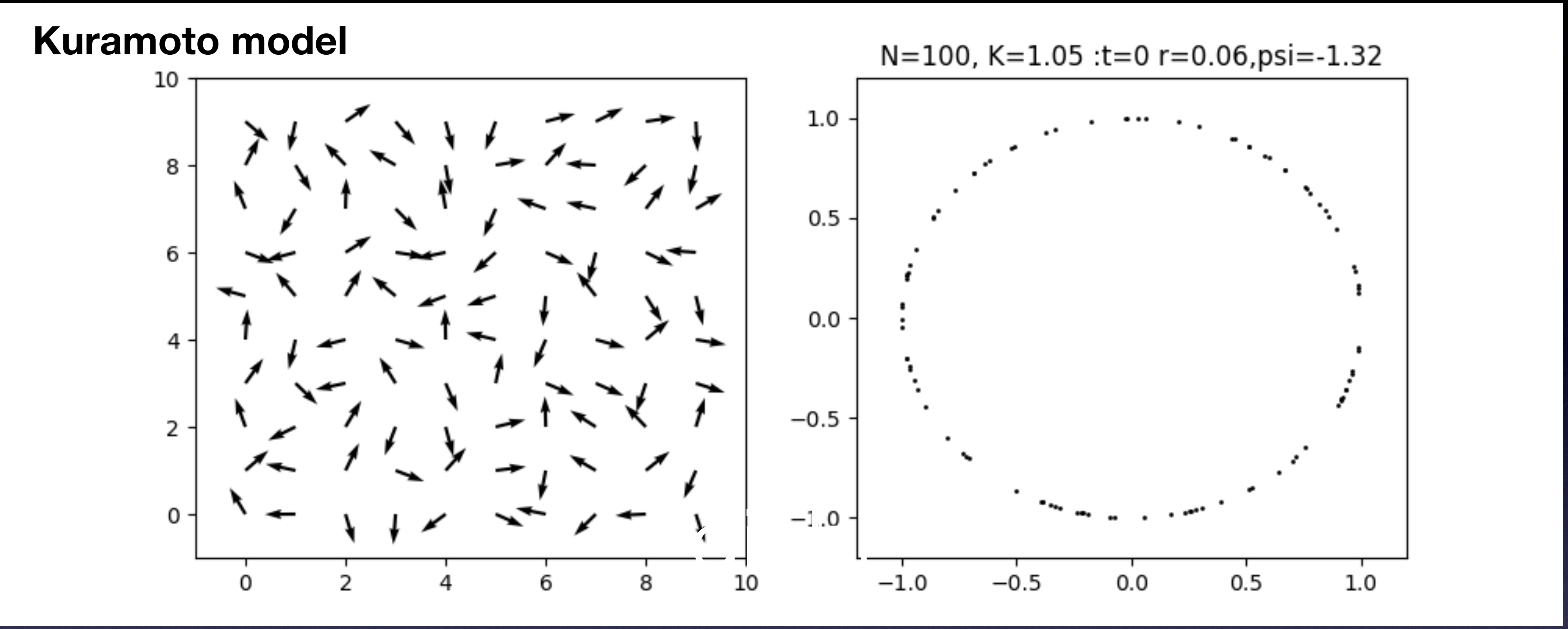
**What is the symmetry?**

**Is there any symmetry breaking?**

**Does a NG mode appear?**

# Symmetry breaking in open systems

## Synchronization




Metronome, fireflies, …

## Driven dissipative condensate

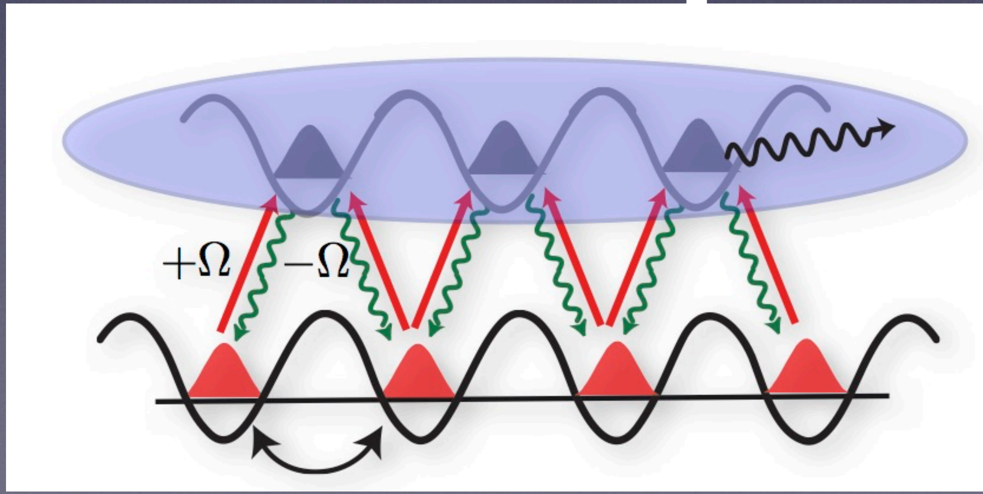


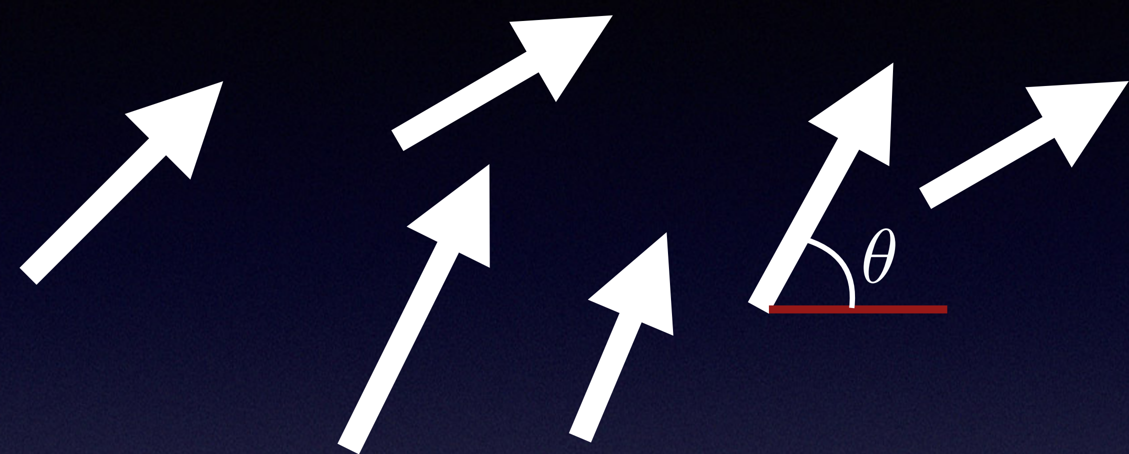

figure is taken from Diehl's website

**Driving force and dissipative causes a condensate.**

Diehl, Micheli, Kantian, Kraus, Büchler, Zoller, Nature Physics 4, 878 (2008);
Kraus, Diehl, Micheli, Kantian, Büchler, Zoller, Phys. Rev. A 78, 042307 (2008).

# Example) Vicsek model

T. Vicsek, et al., PRL (1995).



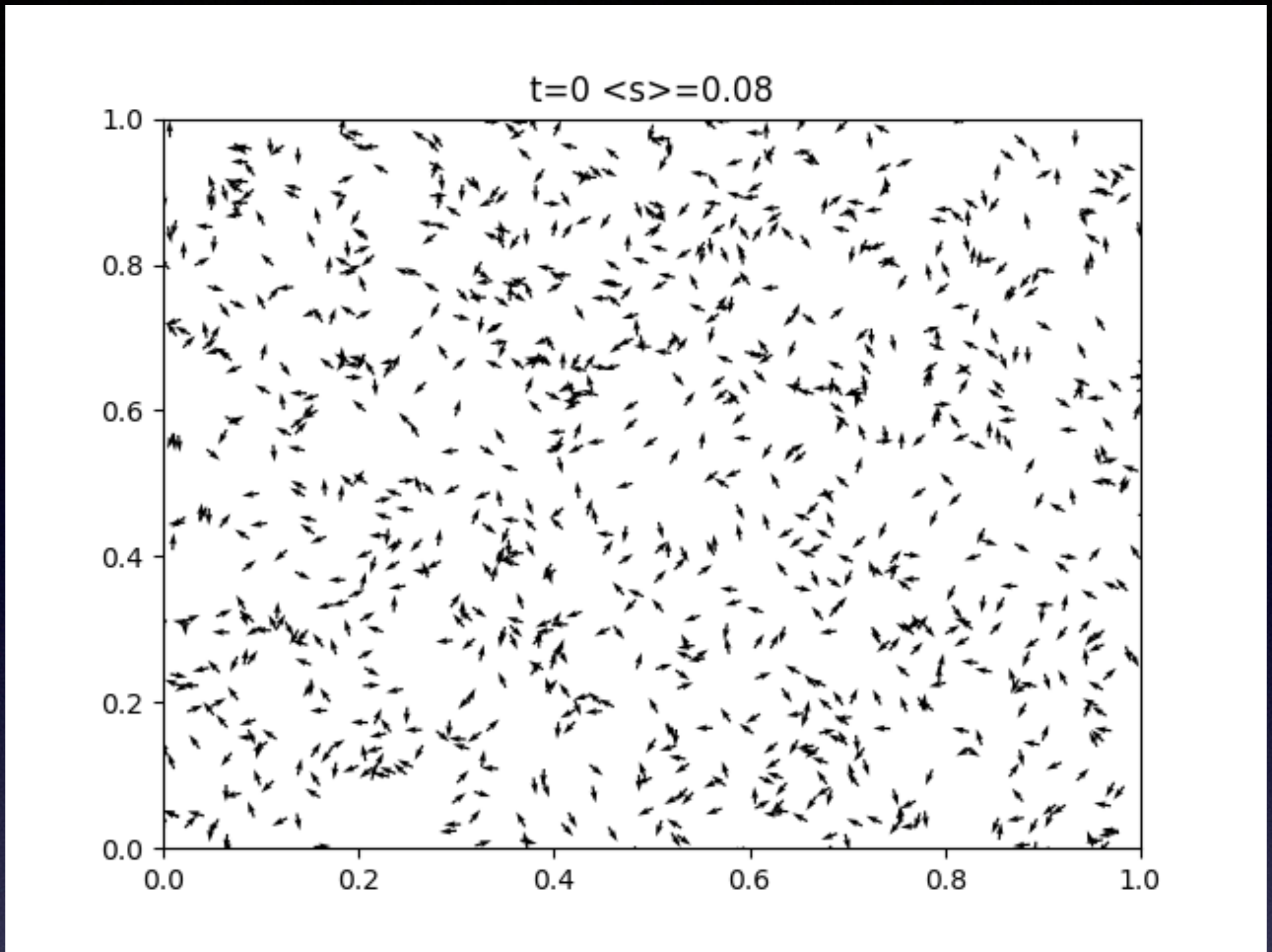

$$\boldsymbol{x}_i(t + \Delta t) = \boldsymbol{x}_i(t) + \boldsymbol{v}_i \Delta t$$

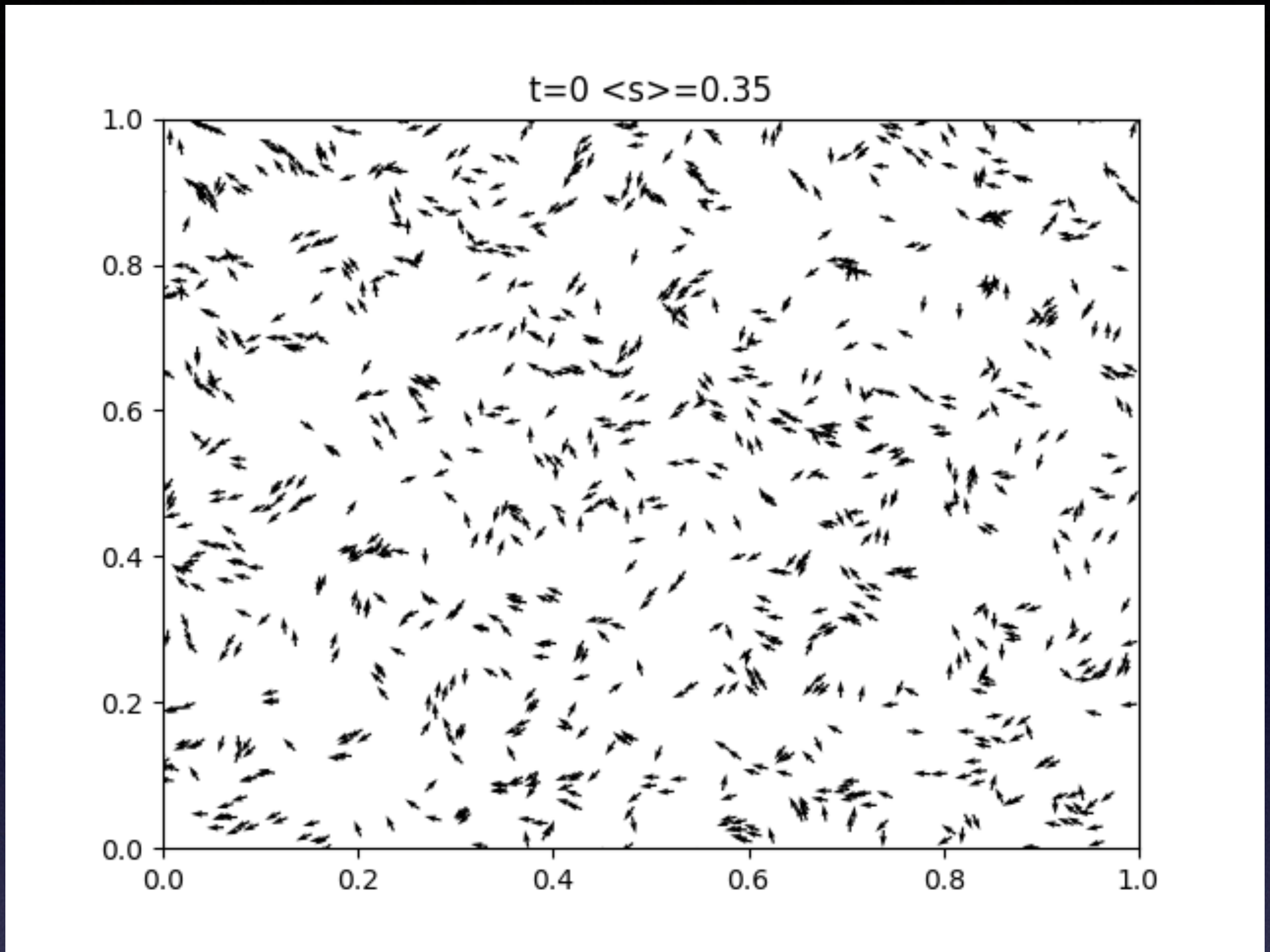
**velocity**

$$\boldsymbol{v}_i = v_0(\cos\theta_i, \sin\theta_i)$$

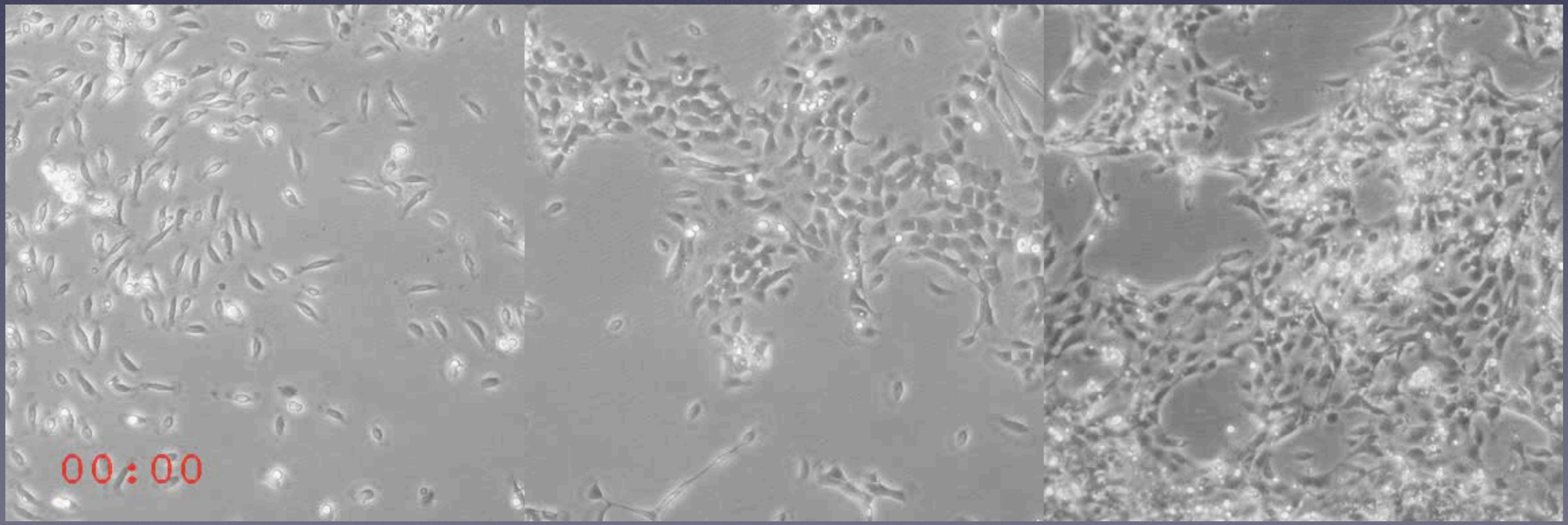
$$\theta_i(t + \Delta t) = \langle \theta_i(t) \rangle_r + \xi_i$$

**angle of velocity** **average angle** **noise**




**low density** **Some cells on fish skin** **high density**




B. Szabo, et al., Phys. Rev. E 74, 061908 (2006)

# Field theoretical model

## Ex.) NG mode in Active hydrodynamics

J. Toner, and Y. Tu, PRE (1998)

$$\partial\rho + \boldsymbol{\nabla}\cdot(\rho\boldsymbol{v}) = 0$$

$$\partial_t\boldsymbol{v} + (\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v} = \alpha\boldsymbol{v} - \beta\boldsymbol{v}^2\boldsymbol{v} - \boldsymbol{\nabla}P + D_L\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{v}) + D_l(\boldsymbol{v}\cdot\boldsymbol{\nabla})^2\boldsymbol{v} + \boldsymbol{f}$$

nonconserved term

**noise**

Steady state solution: $\boldsymbol{v}^2 = \alpha/\beta \equiv v_0^2$

Symmetry breaking: $O(3) \to O(2)$

Fluctuation: $\boldsymbol{v} = (v_0 + \delta v_x, \delta v_y, \delta v_z)$

⇒ $\omega = ck$ $\omega = -i\Gamma k^2$ **NG modes**

**propagating** **diffusive**

# Can we discuss symmetry breaking without ordinary conservation law?

# Conserved system

**Symmetry: Invariance of the action**

**Continuous symmetry ⇒conservation law**

**Ex: harmonic oscillator in 3d**

$$S = \int dt \left[ \frac{1}{2}\dot{x}^2 - \frac{1}{2}x^2 \right]$$

**EOM:** $\ddot{x} + x = 0$

**Rotation:** $x \to x + \epsilon \times x$

**Noether charge:** $L_R = x \times p \quad p = \dot{x}$

which is time independent:

$$\dot{L}_R = \dot{x} \times p + x \times \dot{p} = p \times p - x \times x = 0$$

# If there is a friction

$$\dot{x} = p \qquad \dot{p} = -\gamma p - x$$

**Equation is covariant, but**

**Angular momentum is not conserved**

$$\dot{L}_R = \dot{x} \times p + x \times \dot{p} = -\gamma x \times p$$

# We define the action

$$S = -\int dt \bar{x} \cdot (\ddot{x} + x + \gamma \dot{x})$$

auxiliary field

under rotation

$$x \to x + \epsilon \times x \quad \bar{x} \to \bar{x} + \epsilon \times \bar{x}$$

the action is invariant

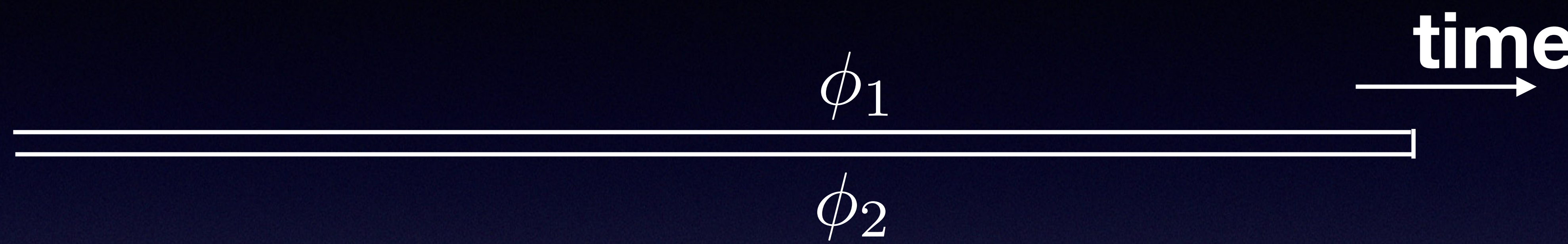
**Noether charge:** $L_A = x \times (2\dot{\bar{x}} - \gamma \bar{x})$

**Open system has a weak symmetry, but strong symmetry is explicitly broken.**

$$\frac{d}{dt} L_R \neq 0 \quad \frac{d}{dt} L_A = 0$$

# Symmetry of Open quantum system

## Schwinger-Keldysh Path integral




$$Z = \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 \exp\left[iS[\phi_1] - iS[\phi_2] + iS_{12}[\phi_1, \phi_2]\right]$$

complex

$Q_1, Q_2$ :Symmetry generators:

$S[\phi_1], S[\phi_2]$ are invariant.

$S_{12}[\phi_1, \phi_2]$ explicitly break the strong symmetry $Q_R = \frac{Q_1 + Q_2}{2}$

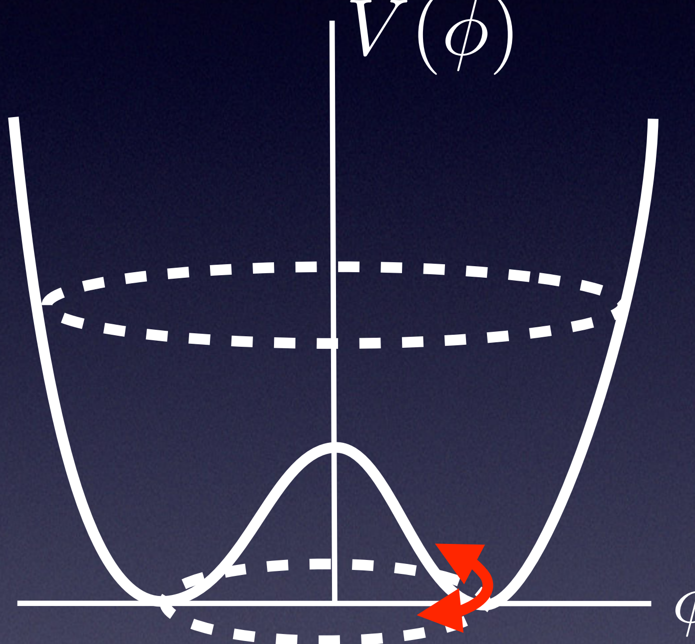
but keep the weak symmetry $Q_A = Q_1 - Q_2$

# Spontaneous symmetry breaking

Minami, YH ('18)

## Ex1) SU(2)xU(1) model

$$iS = \int d^4x \Big[ i\varphi_A^\dagger(-\partial_0^2 + \nabla^2 - \gamma\partial_0 - m^2 - 2\lambda|\varphi_R|^2)\varphi_R - A\varphi_A^\dagger\varphi_A \Big] + \cdots$$




$\varphi_{R/A}$ **two component complex field**

$$\varphi_R = (\pi_1 + i\pi_2, v + h + i\pi_3)$$

## Linear analysis

$$(\partial_0^2 + \gamma\partial_0 - \nabla^2)\pi_a = 0$$

$$\Rightarrow -\omega^2 - i\gamma\omega + k^2 = 0$$

$$\Rightarrow \omega = \frac{-i\gamma}{2} \pm \frac{i}{2}\sqrt{\gamma^2 - 4k^2} \sim -\frac{i}{\gamma}k^2, -i\gamma + \frac{i}{\gamma}k^2$$

**diffusion mode**

# Low-energy effective theory and dispersion relations

# $\mathrm{U}(1)_1 \times \mathrm{U}(1)_2$ model

**NG field:** $e^{i\pi_1}, e^{i\pi_2}$

**Weak:** $\pi_1 \to \pi_1 + c \quad \pi_2 \to \pi_2 + c$

**Strong:** $\pi_1 \to \pi_1 + c \quad \pi_2 \to \pi_2 - c$

$R/A$ **basis:** $\pi_A = \pi_1 - \pi_2 \quad \pi_R = (\pi_1 + \pi_2)/2$

**Building block:** $\dot{\pi}_A \quad \partial_i \pi_A \quad \dot{\pi}_R$

**which are invariant under** $\pi_A \to \pi_A + c \quad \pi_R \to \pi_R + c(\boldsymbol{x})$

**Strong symmetry is explicitly broken**

$\pi_A$ **is allowed**

**If weak symmetry is spontaneously broken**

$\partial_i \pi_R$ **is also allowed**

# $U(1)_1 \times U(1)_2$ model

- **When weak symmetry is unbroken**

$$\mathscr{L} = \gamma \pi_A \dot{\pi}_R + \dot{\pi}_A \dot{\pi}_R - \Gamma(\partial_i \pi_A)(\partial_i \dot{\pi}_R) + \cdots$$

**E.o.m.** $-\ddot{\pi}_R + \gamma \dot{\pi}_R - \Gamma \nabla^2 (\dot{\pi}_R) = 0$ ⇒ $\omega \simeq -i\gamma$

**damping mode**

- **When weak symmetry is spontaneously broken**

$$\mathscr{L} = \gamma \pi_A \dot{\pi}_R + \dot{\pi}_A \dot{\pi}_R - v_s^2(\partial_i \pi_A)(\partial_i \pi_R) - \Gamma(\partial_i \pi_A)(\partial_i \dot{\pi}_R) + \cdots$$

**E.o.m.** $\ddot{\pi}_R + \gamma \dot{\pi}_R - \Gamma \partial_i^2 \dot{\pi}_R - v_s^2 \partial_i^2 \pi_R = 0$ ⇒ $\omega \simeq -i\frac{v_s^2}{\gamma}\boldsymbol{k}^2$

**diffusion mode**

# Type-A and
# Type-B NG modes

# Nambu-Goldstone theorem

Nambu('60), Goldstone(61), Nambu Jona-Lasinio('61), Goldstone, Salam, Weinberg('62).

**For Lorentz invariant vacuum**
**Spontaneous breaking of global symmetry**

⇩

$$N_{NG} = N_{BS}$$

**# of NG modes** **# of broken symmetries**

**Dispersion relation:** $\omega = c|\boldsymbol{k}|$

# Exception of NG theorem

NG modes with $N_{\rm BS} \neq N_{NG}$ and $\omega \not\propto k$ exist

## NG modes in Kaon condensed CFL phase

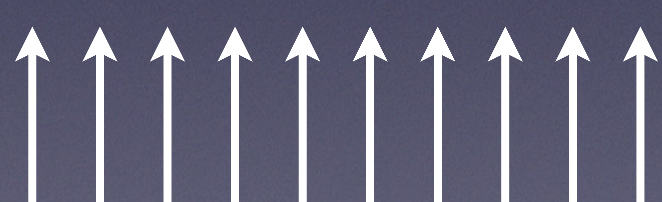
Miransky, Shovkovy ('01) Schafer, Son, Stephanov, Toublan, and Verbaarschot ('01)

$$SU(2)_I \times U(1)_Y \to U(1)_{\rm em}$$

$$N_{\rm BS} = 3, \quad N_{\rm NG} = 2$$

**Dispersion:** $\omega \propto k$ and $\omega \propto k^2$

## Magnon



spin rotation $SO(3) \to SO(2)$

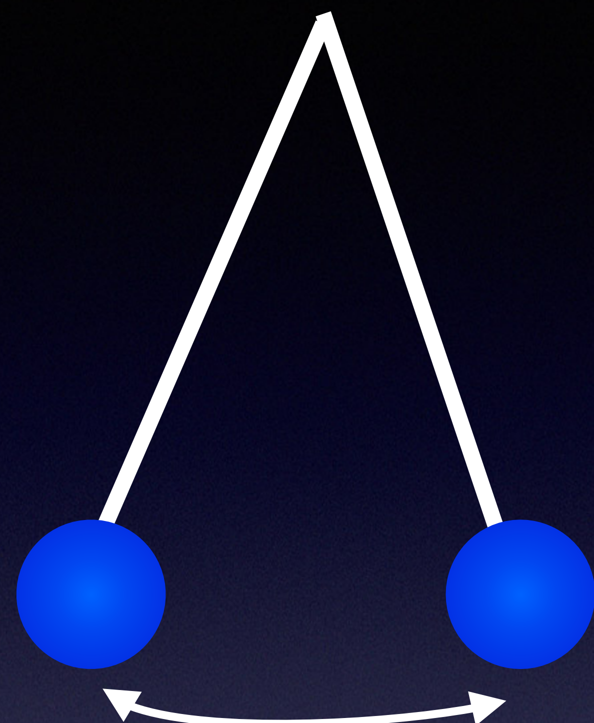
$$N_{\rm BS} = \dim(G/H) = 2 \quad N_{\rm NG} = 1$$

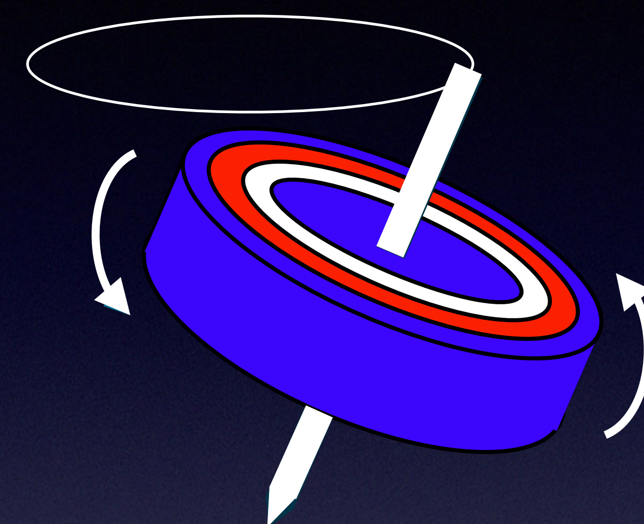
**Dispersion:** $\omega \propto k^2$

# Classification of NG modes

Watanabe, Murayama ('12), YH ('12)

cf. Takahashi, Nitta ('14), Beekman ('14)





**Type-A**

**Harmonic oscillation**

$$N_A = N_{\mathrm{BS}} - \mathrm{rank}\langle[iQ_a, Q_b]\rangle$$

**Ex. ) superfluid phonon**

**Type-B**

**Precession**

$$N_B = \frac{1}{2}\mathrm{rank}\langle[iQ_a, Q_b]\rangle$$

**Ex. ) magnon**

$$\Rightarrow \quad N_{\mathrm{NG}} = N_{\mathrm{BS}} - \frac{1}{2}\langle i[Q_a, Q_b]\rangle$$

# Effective Lagrangian approach

Leutwyler('94) Watanabe, Murayama ('12)

Write down all possible term

$$\mathcal{L} = \frac{1}{2}\rho_{ab}\pi^a\dot{\pi}^b + \frac{\bar{g}_{ab}}{2}\dot{\pi}^a\dot{\pi}^b - \frac{g_{ab}}{2}\partial_i\pi^a\partial_i\pi^b$$

+higher orders

No Lorentz symmetry:

The first derivative term may appear.

$$\Rightarrow \rho_{ab} \propto -i\langle[Q_a, j_b^0(x)]\rangle = \langle\delta_a j_b^0(x)\rangle$$

Watanabe, Murayama ('12)
YH ('12)

# Spontaneous symmetry breaking

Minami, YH ('18)

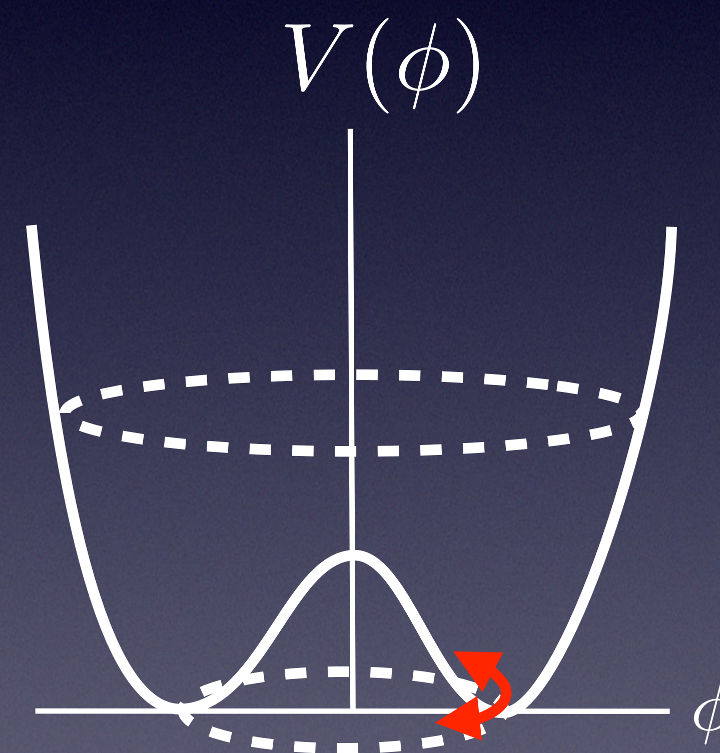
# Ex2) SU(2)xU(1)model

## with chemical potential μ

$$iS = \int d^4x \Big[ i\varphi_A^\dagger(-(\partial_0 + i\mu)^2 + \nabla^2 - \gamma\partial_0 - m^2 - 2\lambda|\varphi_R|^2)\varphi_R - A\varphi_A^\dagger\varphi_A \Big] + \cdots$$

$$\begin{pmatrix} -\partial_0^2 - \gamma\partial_0 + \nabla^2 & 2\mu\partial_0 \\ -2\mu\partial_0 & -\partial_0^2 - \gamma\partial_0 + \nabla^2 \end{pmatrix} \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} = 0,$$

$$\Rightarrow \quad \omega = \frac{k^2}{4\mu^2 + \gamma^2}(\pm 2\mu - i\gamma)$$



# Effective Lagrangian in open system

**Assumptions:**

- **strong spacetime translation are explicitly broken, and weak spacetime translation are unbroken.**

$$\mathscr{L} = \rho_{ab}\pi_A^a\dot{\pi}_R^b + \bar{g}_{ab}\dot{\pi}_A^a\dot{\pi}_R^b - g_{ab}(\partial_i\pi_A^a)(\partial_i\pi_R^b) - \Gamma_{ab}(\partial_i\pi_A^a)(\partial_i\dot{\pi}_R^b) + \cdots$$

**Symmetry matching implies** $\rho_{ab} = \langle \delta_{Ra} j^0_{Ab}(x) \rangle$

**E.O.M**

$$\rho_{ab}\dot{\pi}_R^b - \bar{g}_{ab}\ddot{\pi}_R^b + g_{ab}\partial_i^2\pi_R^b + \Gamma_{ab}\partial_i^2\dot{\pi}_R^b = 0$$

# Classification Degrees of Freedom

$$\rho_{ab}\dot{\pi}_R^b - \bar{g}_{ab}\ddot{\pi}_R^b + g_{ab}\partial_i^2\pi_R^b + \Gamma_{ab}\partial_i^2\dot{\pi}_R^b = 0$$

**DOF belonging to kernel of $\rho_{ab}$ = Type-A**

**DOF belonging to coimage of $\rho_{ab}$ = Type-B**

$$\omega = \begin{cases} \pm a_A|k| - ib_A|k|^2 & \text{Type-A propagation mode} \\ -i\gamma|k|^2 & \text{Type-A/B diffusion mode} \\ \pm a_B|k|^2 - ib_B|k|^2 - ic_B|k|^4 & \text{Type-B propagation mode} \\ -i\Gamma & \text{damping mode} \end{cases}$$

**Some relations (conjecture)**

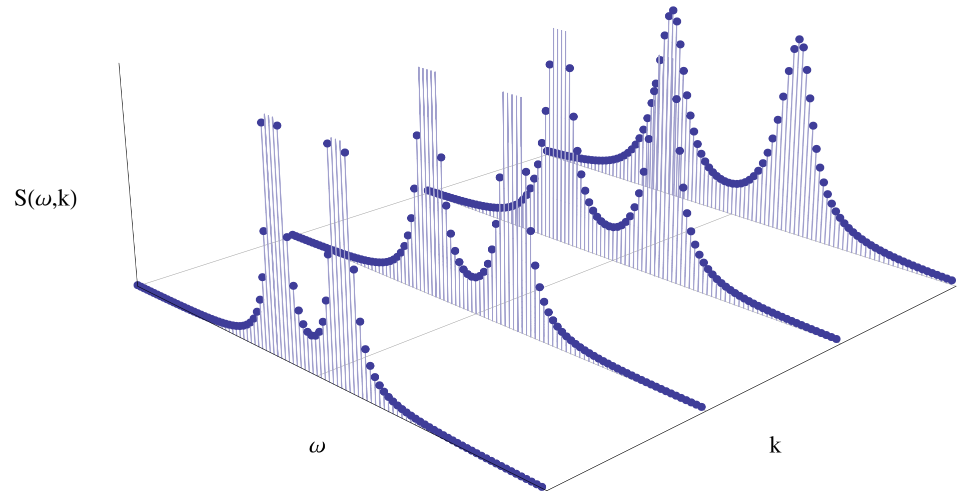
$$\dim(\ker\rho) = N_{\mathrm{A-prop}} + N_{\mathrm{A-diffusion}}$$

$$\dim\rho = 2N_{\mathrm{B-prop}} + N_{\mathrm{B-diffusion}} + N_{\mathrm{damp}}$$

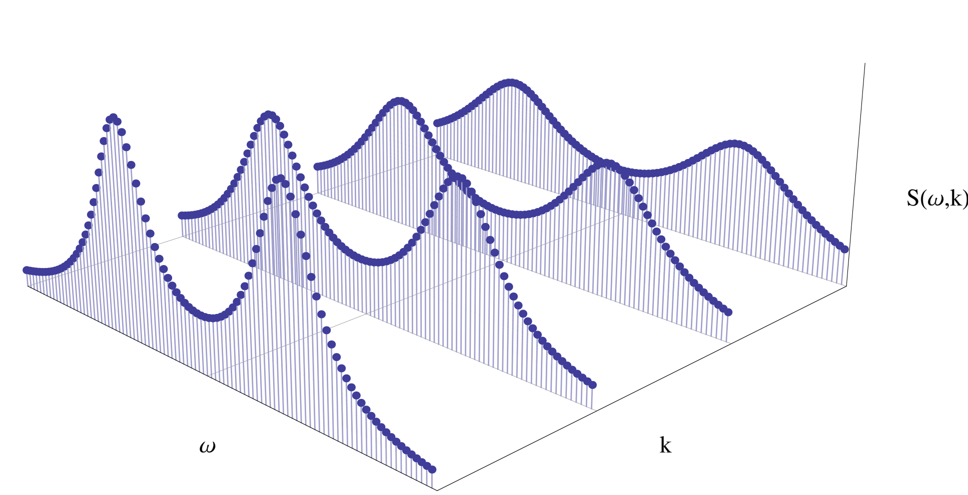
$$N_{\mathrm{BS}} = N_{\mathrm{A-prop}} + 2N_{\mathrm{B-prop}} + N_{\mathrm{B-diffusion}}$$

# Typical behaviors of spectra

isolated system

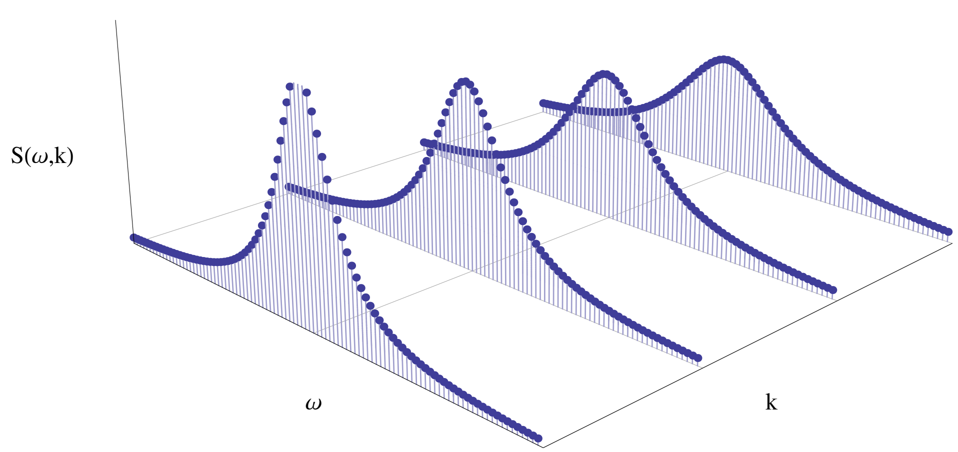



type-A

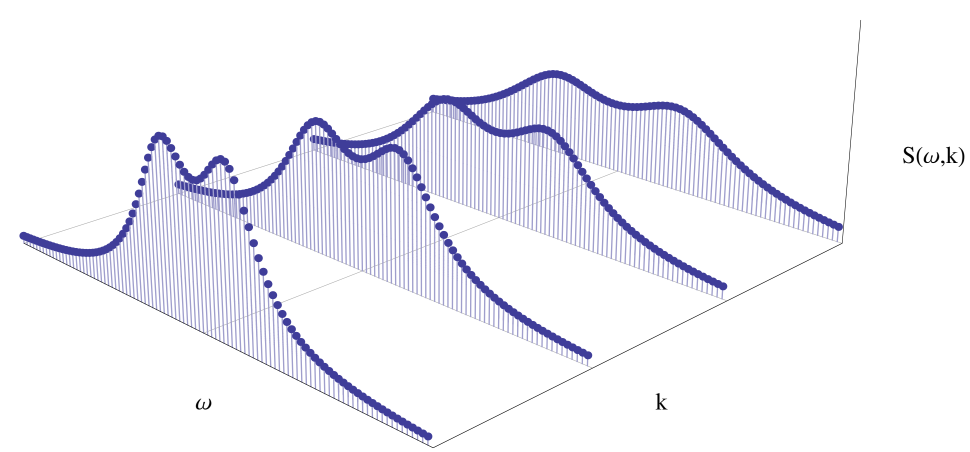



type-B

open system




type-$B_{\text{diffusion}}$




type-$B_{\text{prop}}$

# Summary

**Hydrodynamics: Strong to weak symmetry breaking**

**Ordinary symmetry breaking: weak symmetry breaking**

**Open system: explicit breaking of strong symmetry**

$$\omega = \begin{cases} \pm a_A|k| - ib_A|k|^2 & \text{Type-A propagation mode} \\ -i\gamma|k|^2 & \text{Type-A/B diffusion mode} \\ \pm a_B|k|^2 - ib_B|k|^2 - ic_B|k|^4 & \text{Type-B propagation mode} \\ -i\Gamma & \text{damping mode} \end{cases}$$

**Some relations (conjecture)** $\rho_{ab} = \langle \delta_{Rb} j^0_{Aa} \rangle$

$$\dim(\ker \rho) = N_{\mathrm{A-prop}} + N_{\mathrm{A-diffusion}}$$

$$\dim \rho = 2N_{\mathrm{B-prop}} + N_{\mathrm{B-diffusion}} + N_{\mathrm{damp}}$$

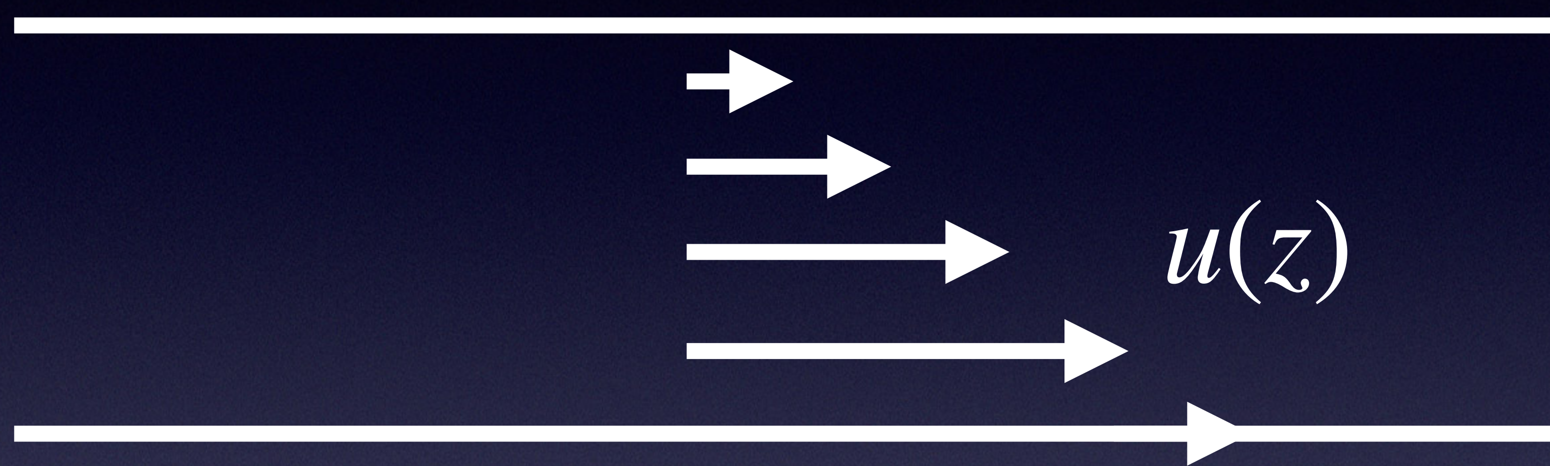
$$N_{\mathrm{BS}} = N_{\mathrm{A-prop}} + 2N_{\mathrm{B-prop}} + N_{\mathrm{B-diffusion}}$$

# Backup

## Exotic NG modes
## in nonequilibrium steady state

# Under shear flow

Y. Minami, H. Nakano, Y. Hidaka, 2009.10357




**moving wall with velocity $v$**

**What are the NG modes in non-equilibrium steady state?**

# Model: $O(N)$ scalar model

Y. Minami, H. Nakano, Y. Hidaka, 2009.10357

$$\partial_t \phi^a + (\boldsymbol{v} \cdot \nabla)\phi^a = -\Gamma \frac{\delta F}{\delta \phi^a} + \eta^a$$

$$\langle \eta^a(t, \boldsymbol{x}) \eta^b(t', \boldsymbol{y}) \rangle = 2T\Gamma \delta^{ab} \delta(t - t')\delta(\boldsymbol{x} - \boldsymbol{y})$$

$$\boldsymbol{v}(\boldsymbol{x}) = (\gamma x_2, 0, 0)$$

$$F = \int d\boldsymbol{x} \Big[ \frac{1}{2}(\nabla \phi^a)^2 - \frac{\mu^2}{2}(\phi^a)^2 + \frac{u^2}{4}((\phi^a)^2)^2 \Big]$$

**Steady state analysis**

⇒ **infinite number of modes with $k^{2/3}$**

$$\omega = \frac{\sqrt{3}}{2} \mathrm{sgn}(\gamma k_1) \Gamma^{1/3} t_n |\gamma k_1|^{2/3} - \frac{i}{2} \Gamma^{1/3} t_n |\gamma k_1|^{2/3}$$

# Quantum time crystal

with T. Hayata

**No quantum crystals in Hamiltonian system**

Oshikawa, Watanabe,Phys. Rev. Lett. 114, 251603 (2015)

**Time crystals can exist in open quantum system**

**What is the NG mode in time crystals?**
**Is it possible to appear a proagating mode?**

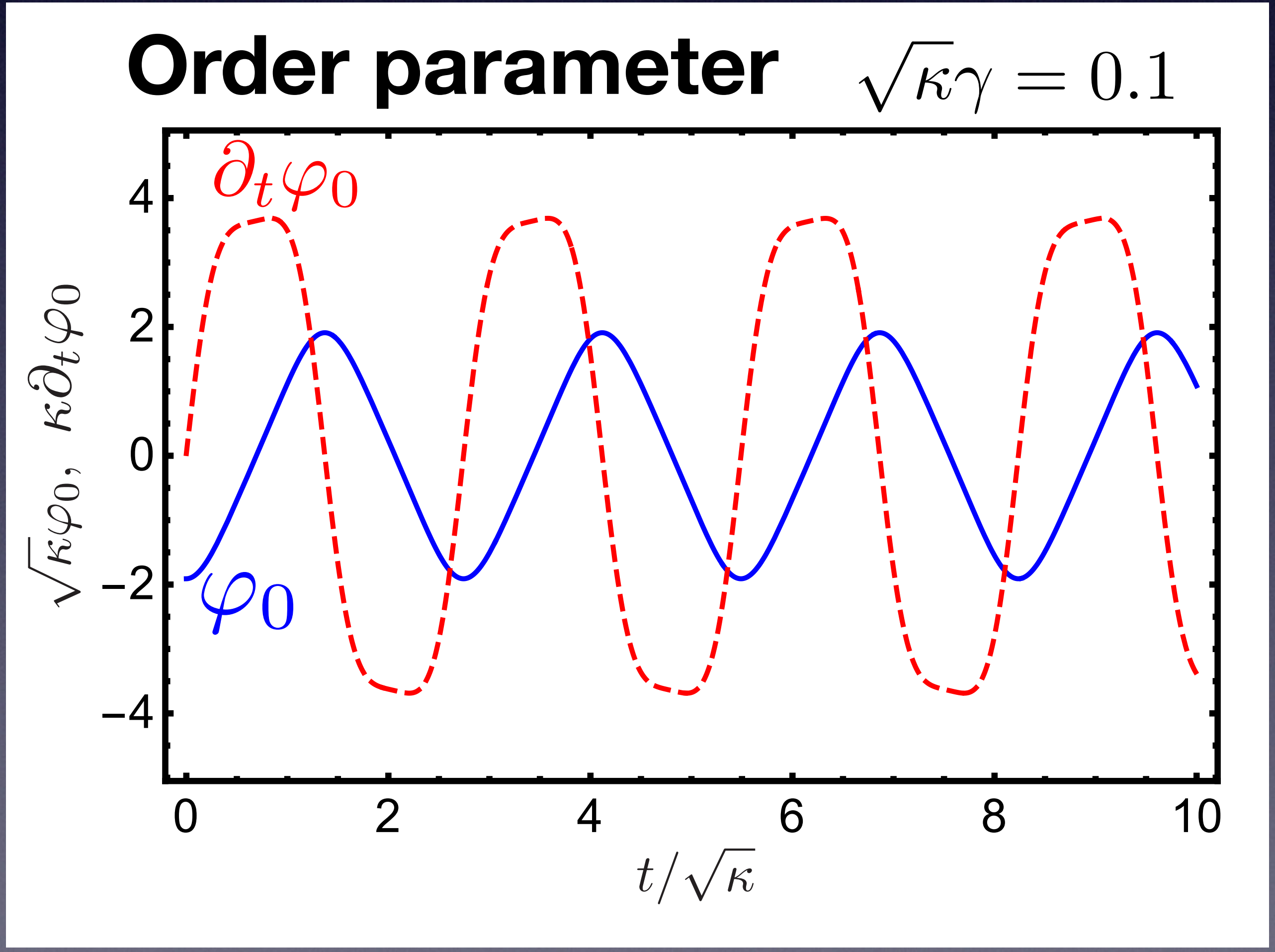
# Quantum time crystal

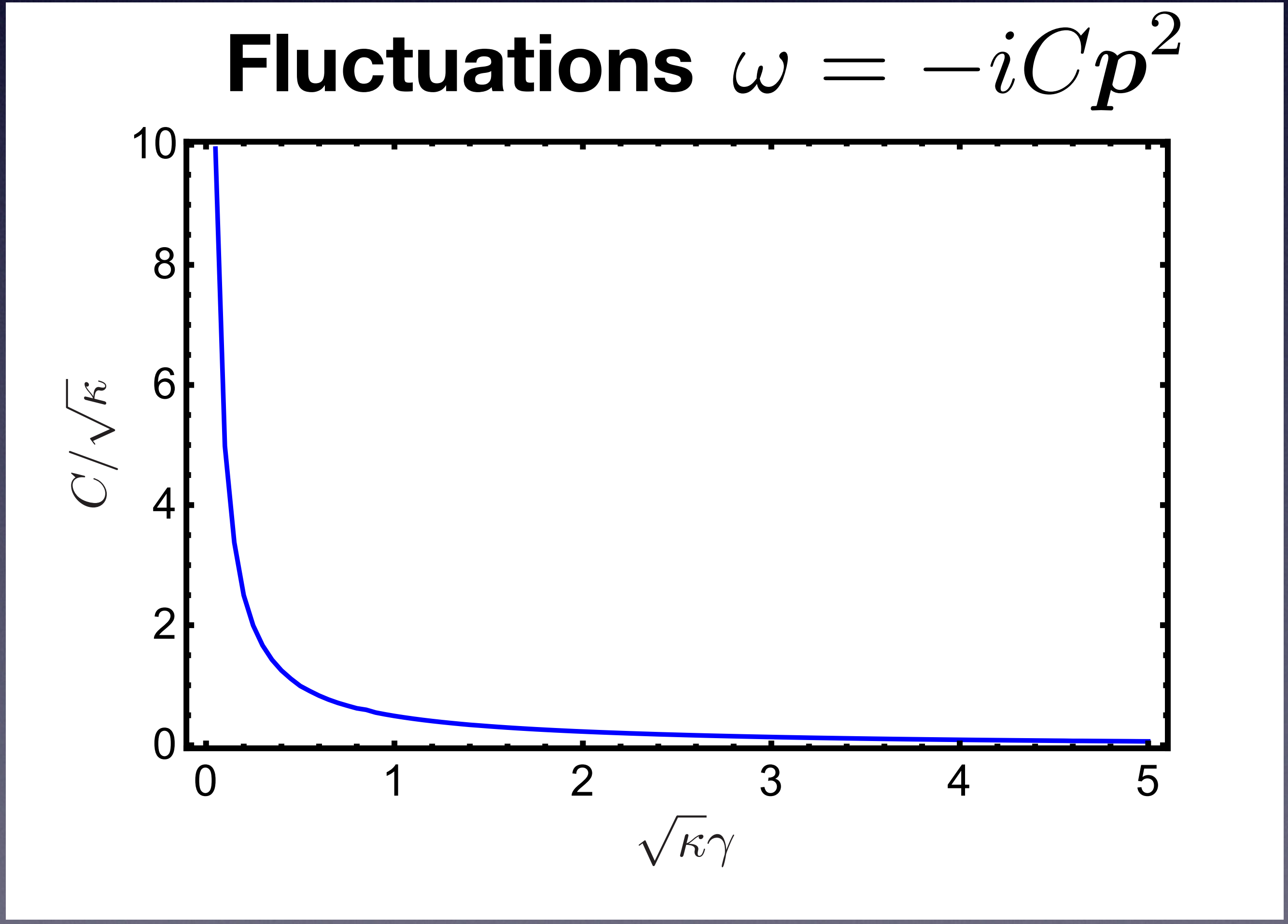
with T. Hayata

## Example 1) van del Pol type model

$$S = \int d^4x \phi_A \left(-\partial_t^2 + \nabla^2 + \gamma\left(1 - \kappa\phi_R^2\right)\partial_t - 2\lambda\phi_R^2\right)\phi_R \quad + iA(\phi_A)^2$$

Time translation symmetry is spontaneously broken.



# Quantum time crystal

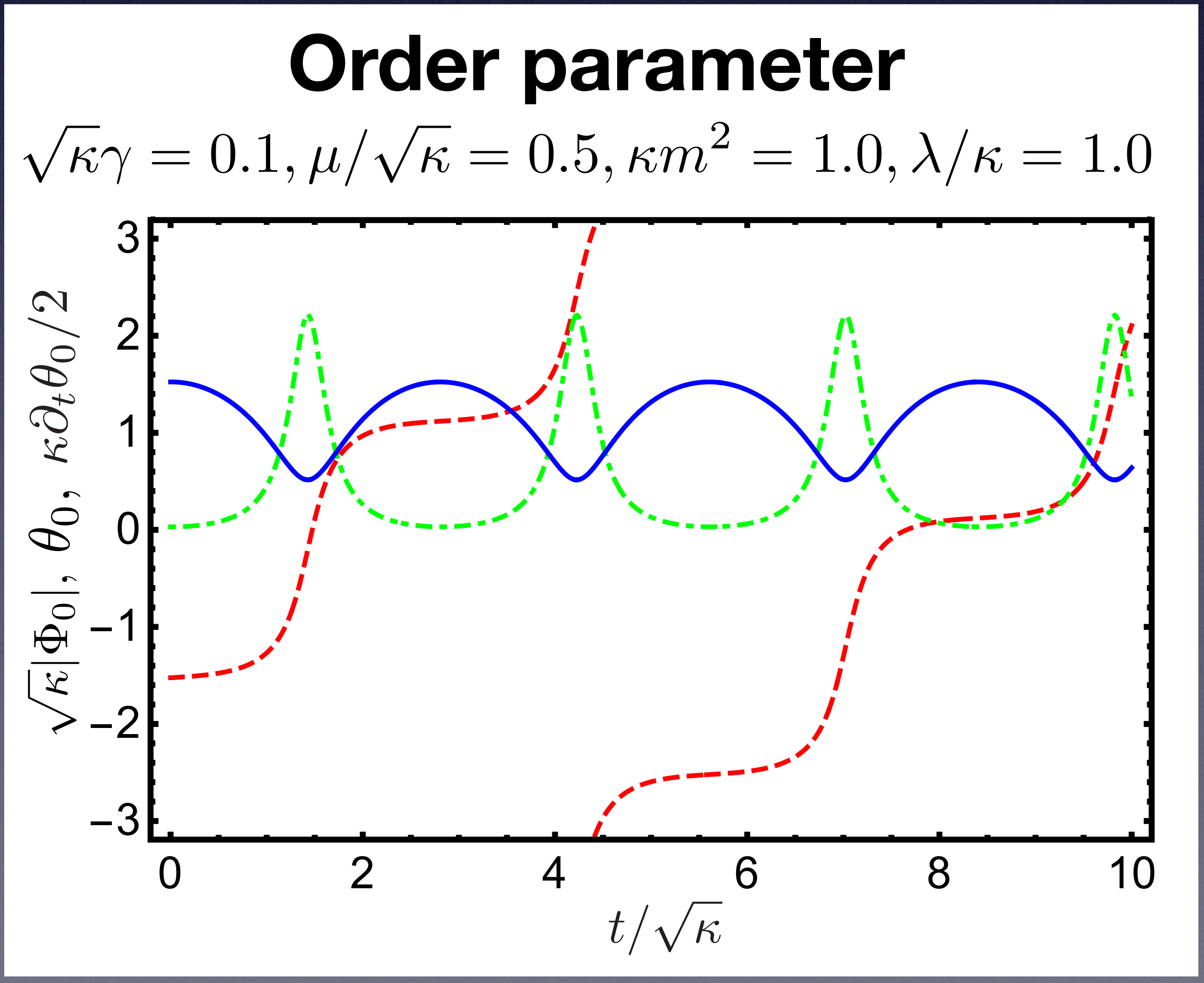
with T. Hayata

## Example 2) complex van del Pol type model

$$S = \int d^4x\, \Phi_A^* \Big( -(\partial_t + i\mu)^2 + \nabla^2 + \gamma\left(1 - \kappa|\Phi_R|^2\right)\partial_t - m^2 - 2\lambda|\partial_t\Phi_R|^2 \Big)\Phi_R$$

$$+ \Big(\text{Hermite conjugates}\Big) + iA|\Phi_A|^2$$

**Time translation and U(1) symmetries are spontaneously broken.**



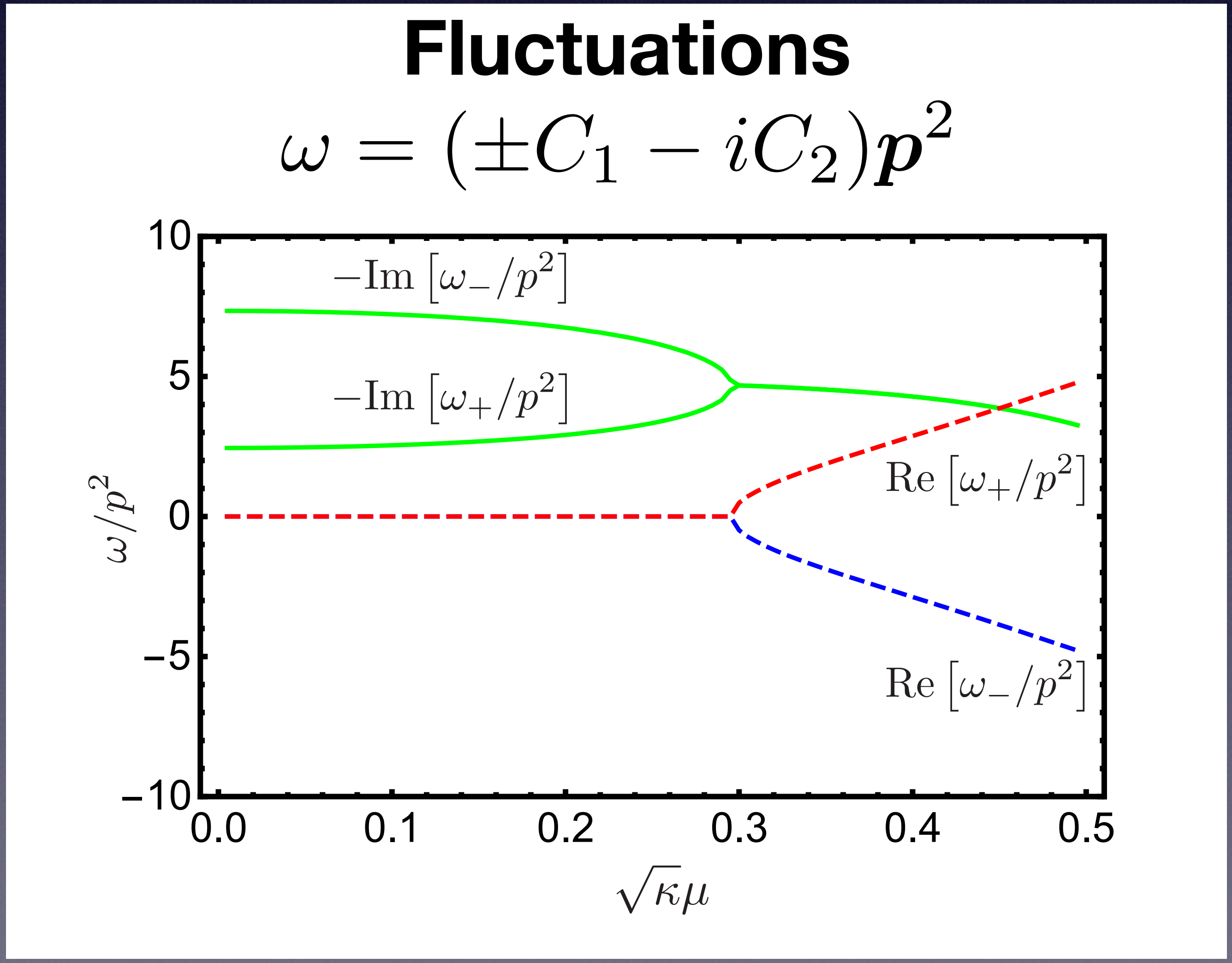
# Quantum time crystal

with T. Hayata

## Type-B modes

$$\langle \delta_R^T Q_A \rangle \neq 0$$

Time U(1)




**μ is large**

one propagating mode

**μ is small**

two diffusive modes
(over damping)

# Spontaneous symmetry breaking

Minami, YH ('18)

## SU(2)xU(1)model

## with complex potential

$$iS = \int d^4x \Big( i\varphi_A^\dagger((-\partial_0^2 + \nabla^2 - (\gamma + 2i\mu)\partial_0 - m_\mathrm{r}^2 - im_\mathrm{i}^2)\varphi_R$$

$$- 2(\lambda_\mathrm{r} + i\lambda_\mathrm{i})(\varphi_R^\dagger\varphi_R)\varphi_R) - A\varphi_A^\dagger\varphi_A \Big) + \cdots$$

**Assuming** $\varphi_R = (0, ve^{-i\omega_0 t})$

**Gap equation**

$$(\omega_0^2 - 2\mu\omega_0 - m_\mathrm{r}^2 - 2\lambda_\mathrm{r}v^2 + i(\gamma\omega_0 - m_\mathrm{i}^2 - 2\lambda_\mathrm{i}v^2))v = 0$$

**Symmetric phase** $v = 0$

**Broken phase** $v \neq 0, \quad \omega_0 \neq 0$

# Spontaneous symmetry breaking

Minami, YH ('18)

# SU(2)xU(1)model

## with complex potential

**Linear analysis**

$$\begin{pmatrix} -\partial_0^2 - \gamma\partial_0 + \nabla^2 & 2(\mu-\omega_0)\partial_0 \\ -2(\mu-\omega_0)\partial_0 & -\partial_0^2 - \gamma\partial_0 + \nabla^2 \end{pmatrix} \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} = 0,$$

$$\omega = \frac{k^2}{4(\mu-\omega_0)^2+\gamma^2}(\pm 2(\mu-\omega_0) - i\gamma)$$

**We still have quadratic dispersion**

**Similarly, we find**

$$\omega = -i\frac{\boldsymbol{k}^2}{\gamma + 2(\mu-\omega_0)\lambda_i/\lambda_r}$$

**Diffusive mode**