VARIOUS ASPECTS OF PSEUDOGAUGE TRANSFORMATIONS

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Centro Nazionale di Studi Avanzati dell'Istituto Nazionale di Fisica Nucleare
Arcetri, Firenze

FOUNDATIONS AND APPLICATIONS OF RELATIVISTIC HYDRODYNAMICS
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SONATA BIS 8 Grant No. 2018/30/E/ST2/00432

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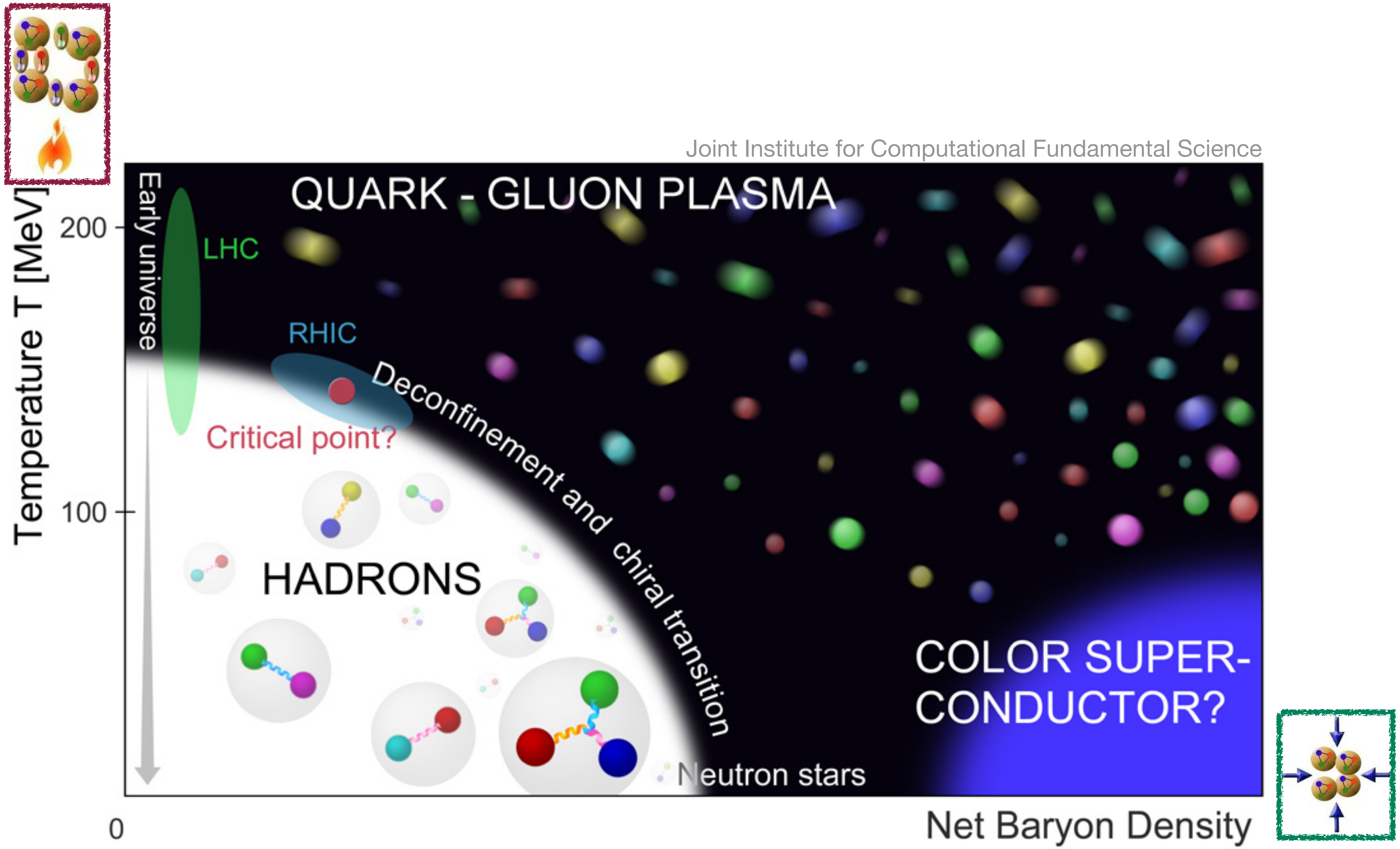
POLISH ACADEMY OF SCIENCES

PHASE DIAGRAM OF QCD IN EXTREME CONDITIONS



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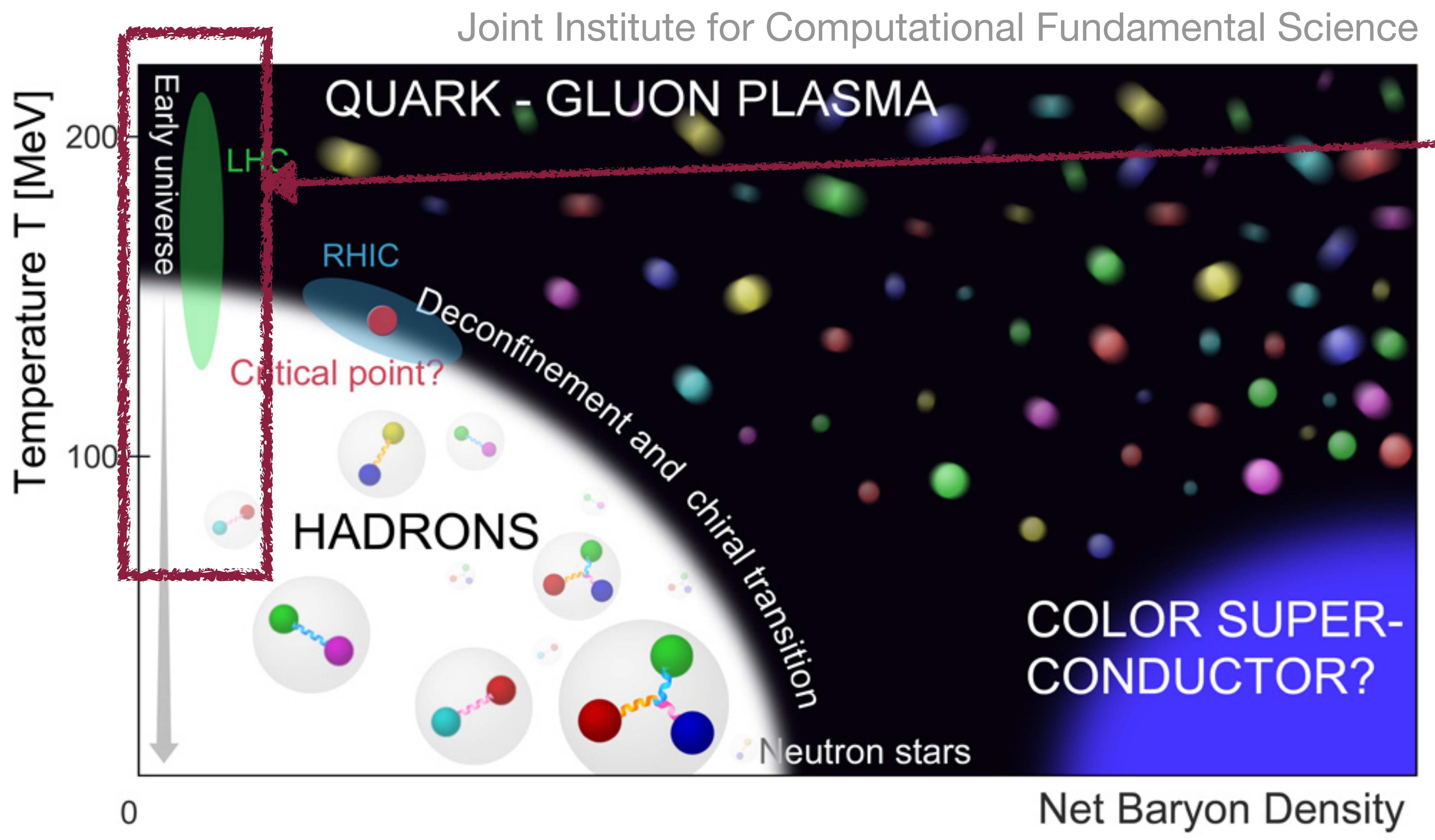
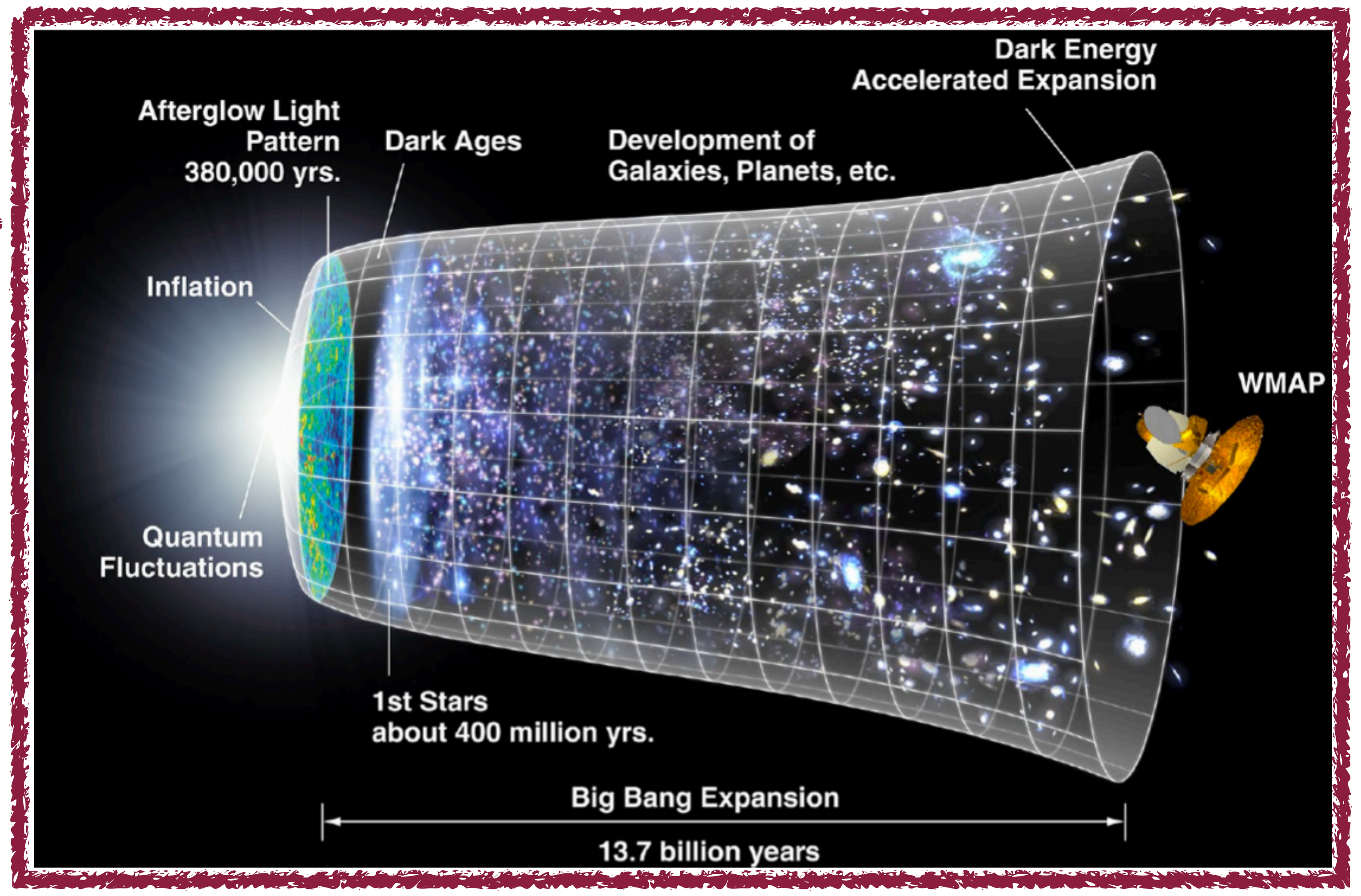


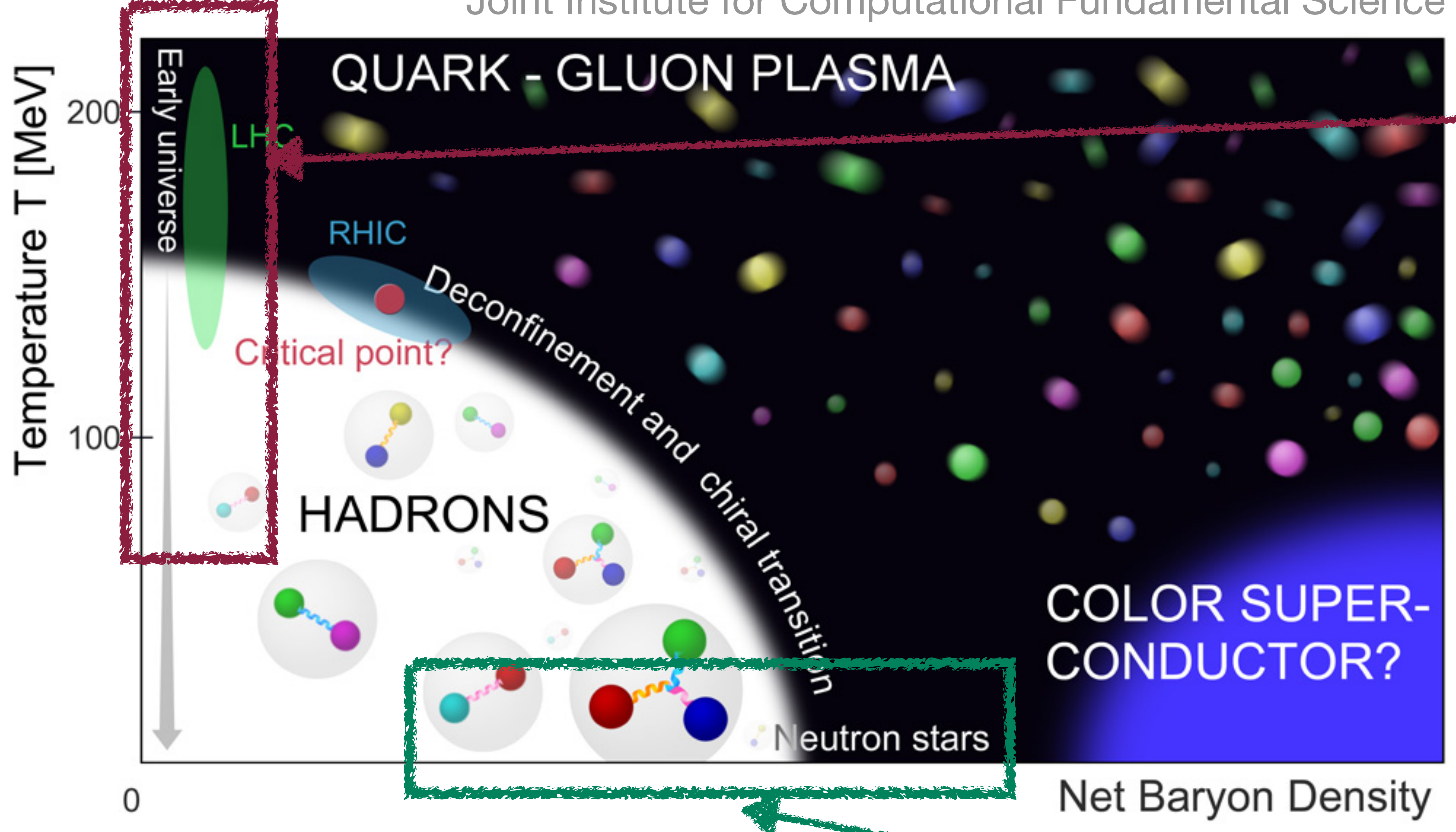
early Universe



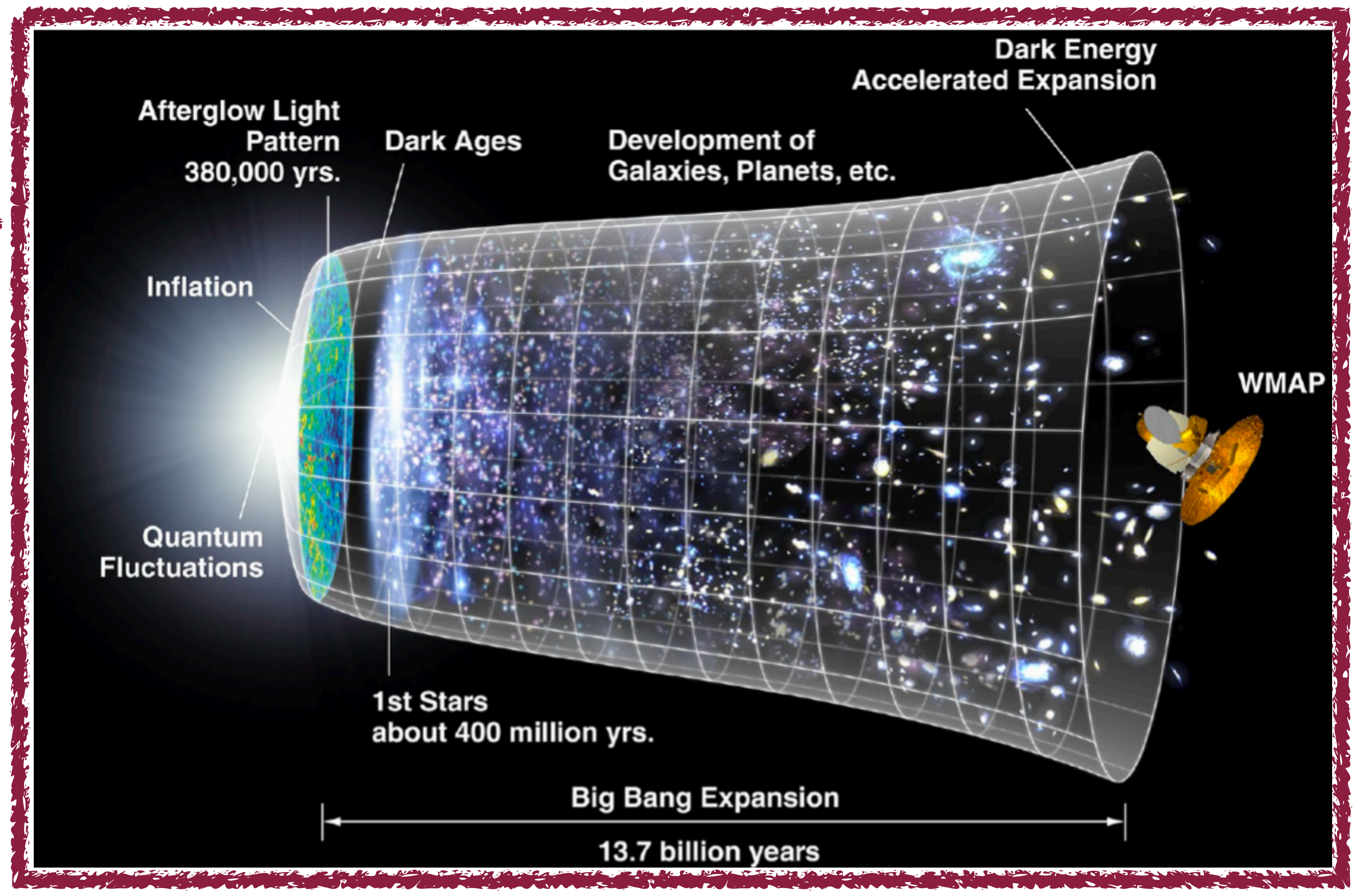
NASA

PHASE DIAGRAM OF QCD IN EXTREME CONDITIONS

Joint Institute for Computational Fundamental Science

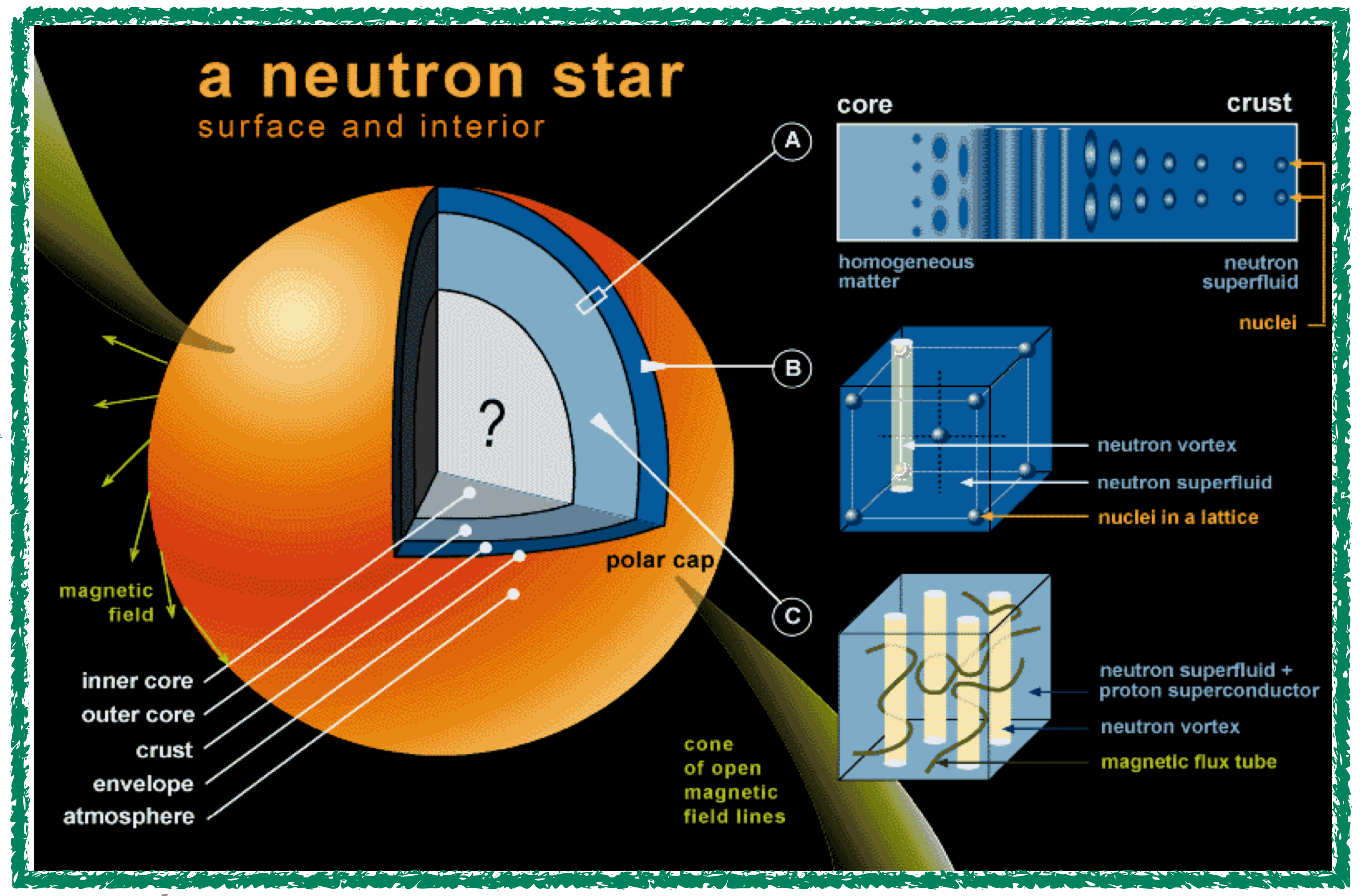


early Universe



NASA

neutron stars

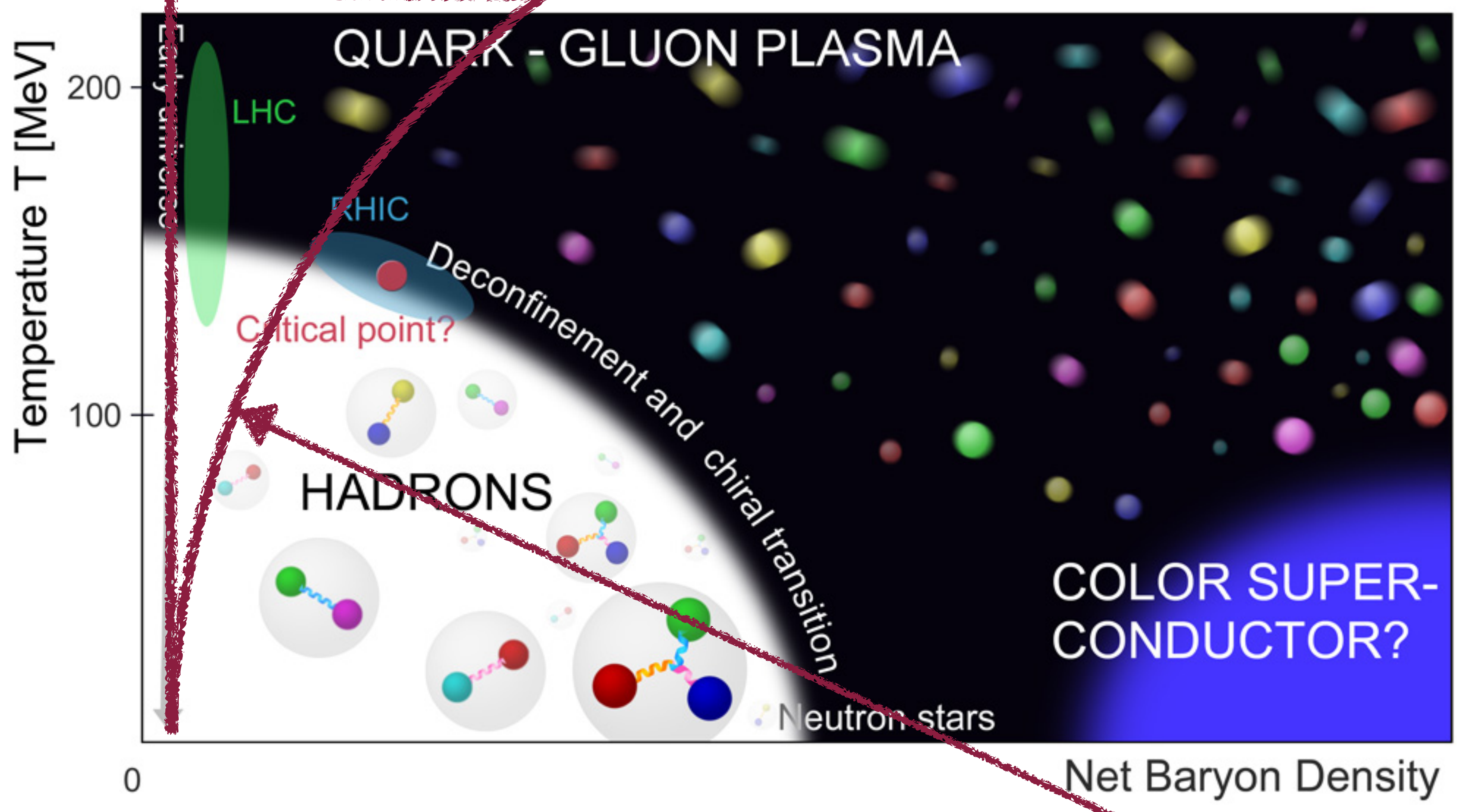


D.E. Á. Castillo, talk @RagTime 22

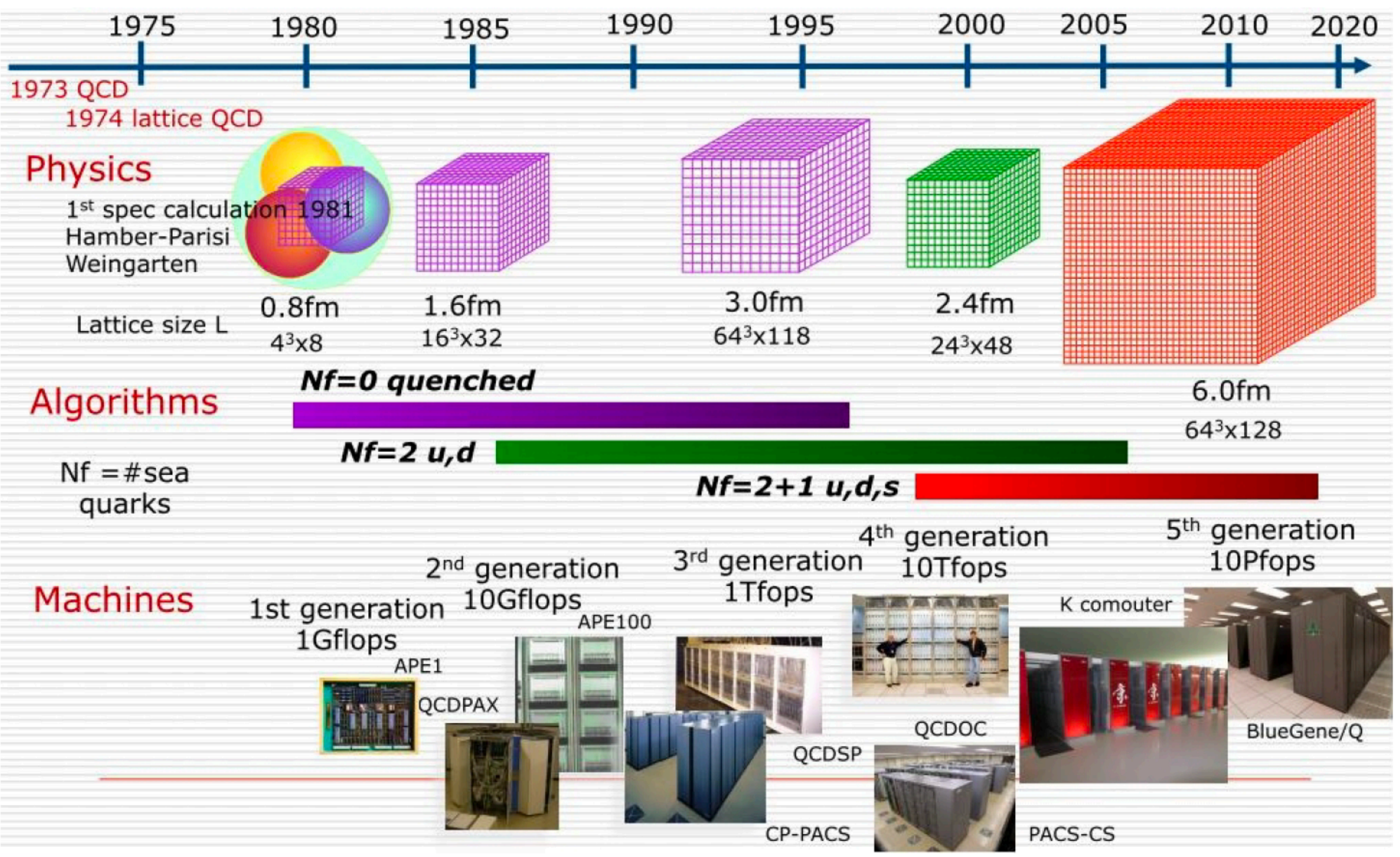
PHASE DIAGRAM OF QCD IN EXTREME CONDITIONS

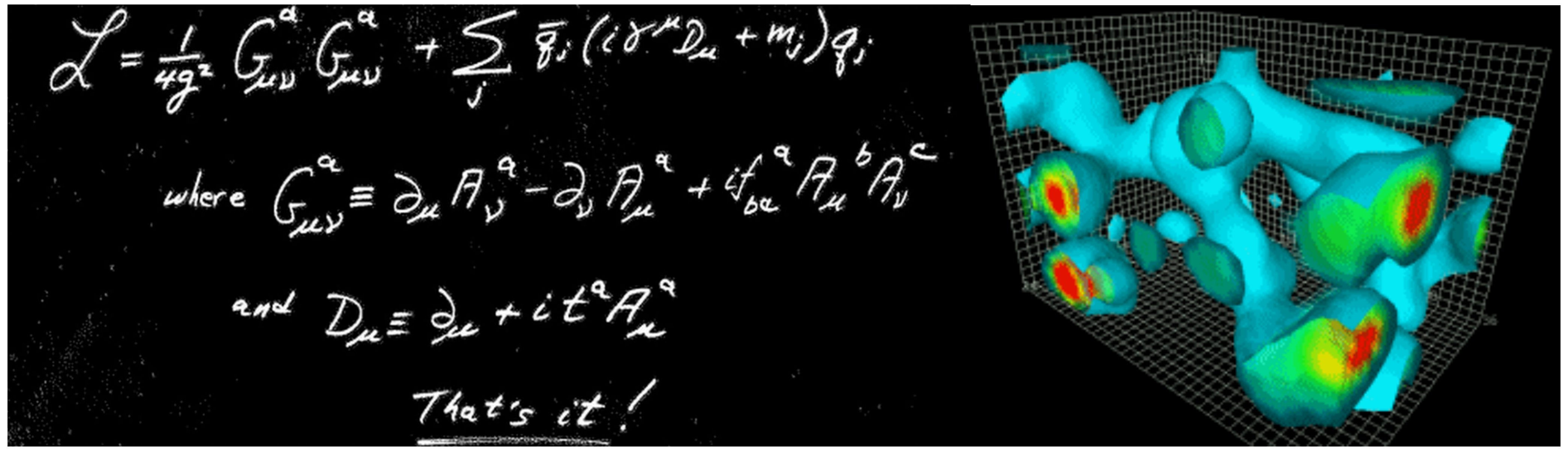
lattice QCD simulations

Joint Institute for Computational Fundamental Science

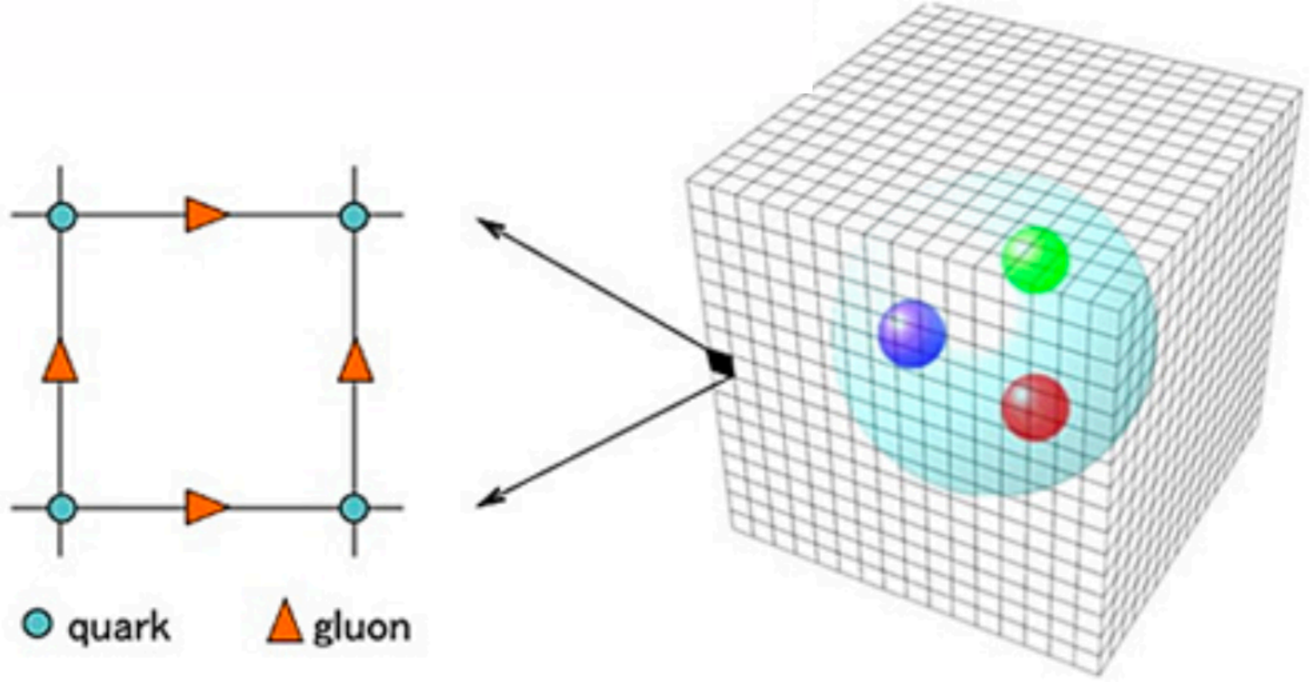


http://clrwww.in2p3.fr



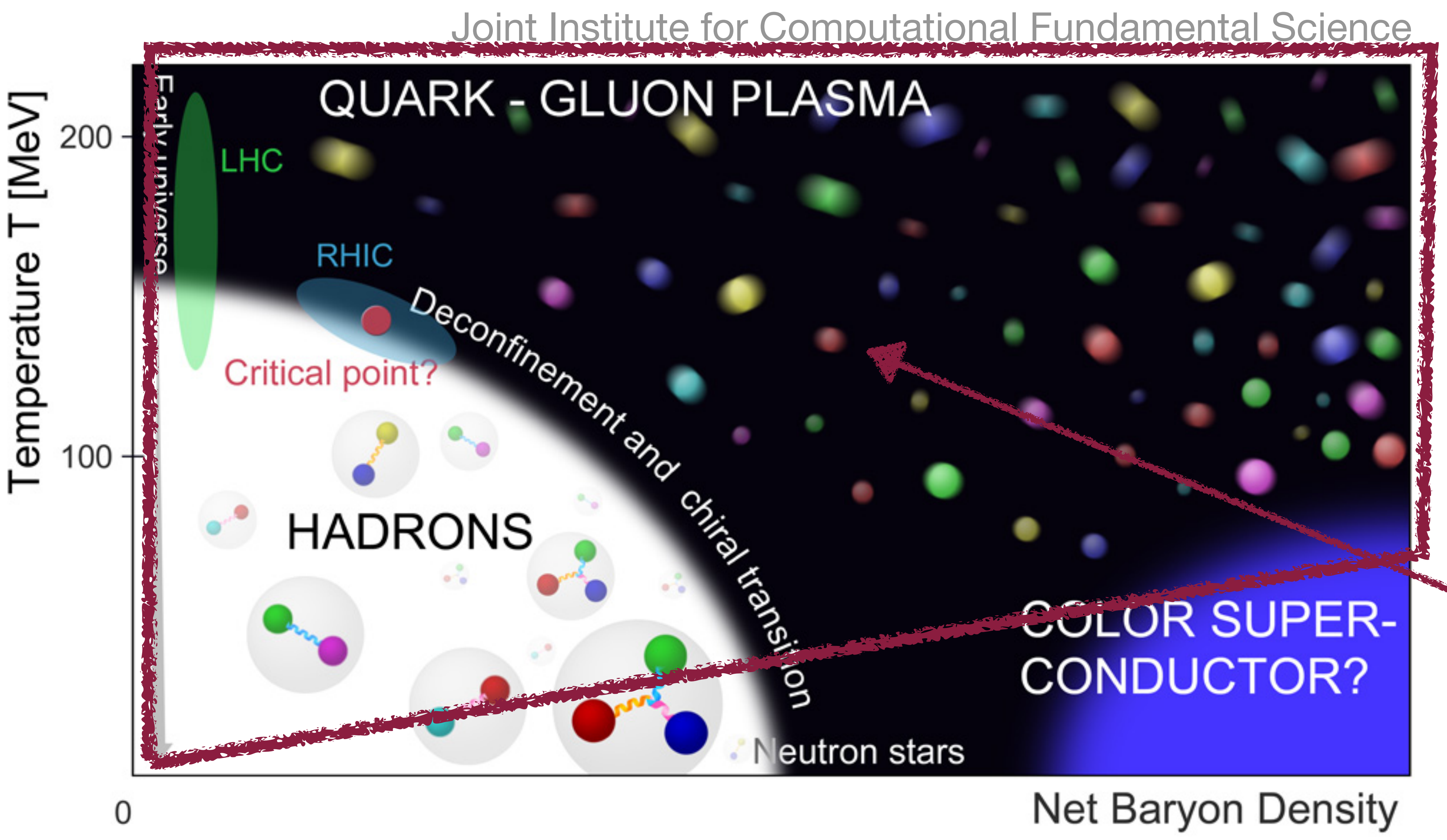


D. Leinweber (www.physics.adelaide.edu.au)



F. Karsch et al., Phys. Lett. B478, 447 (2000)

PHASE DIAGRAM OF QCD IN EXTREME CONDITIONS



heavy-ion collisions

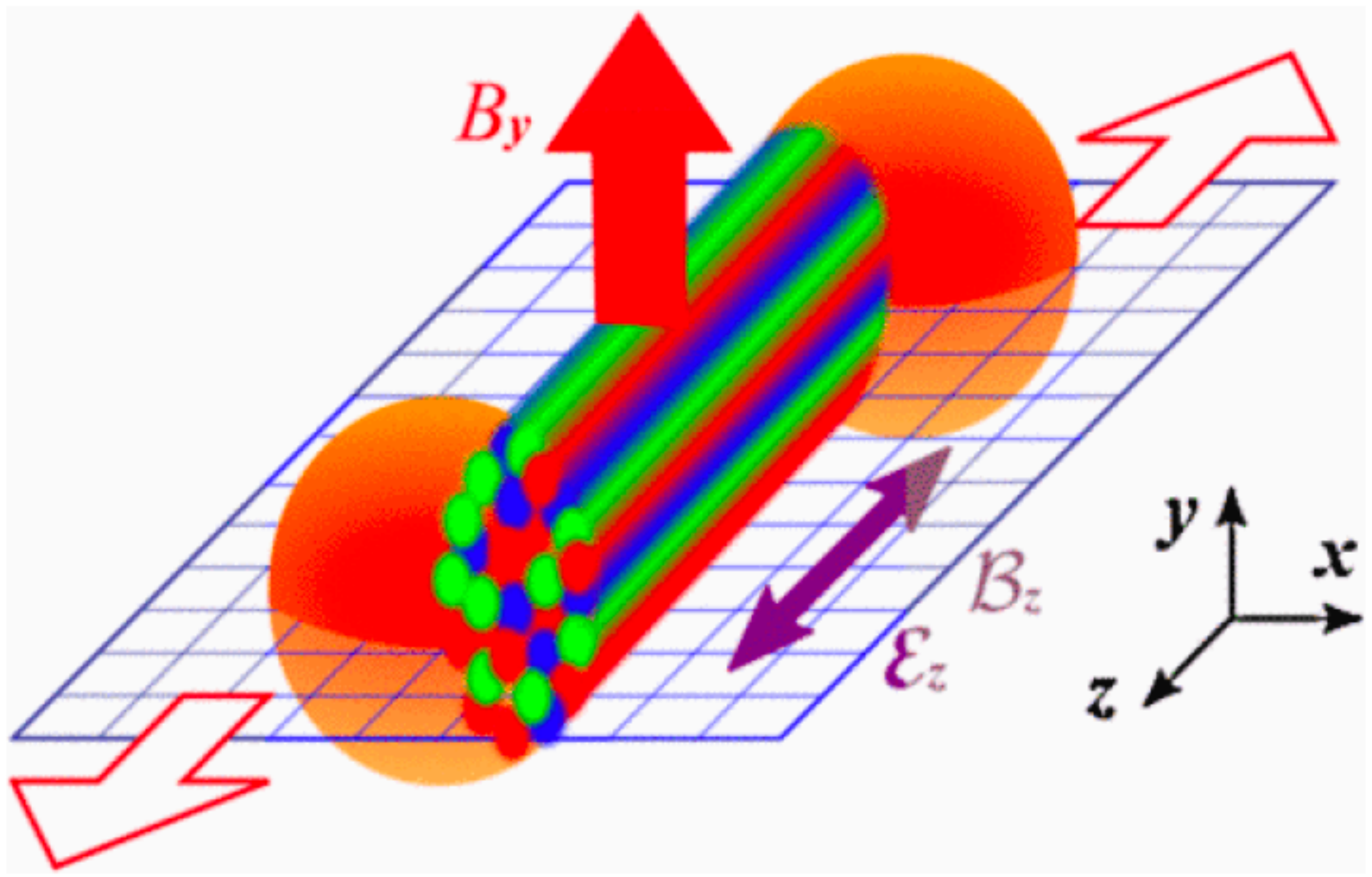
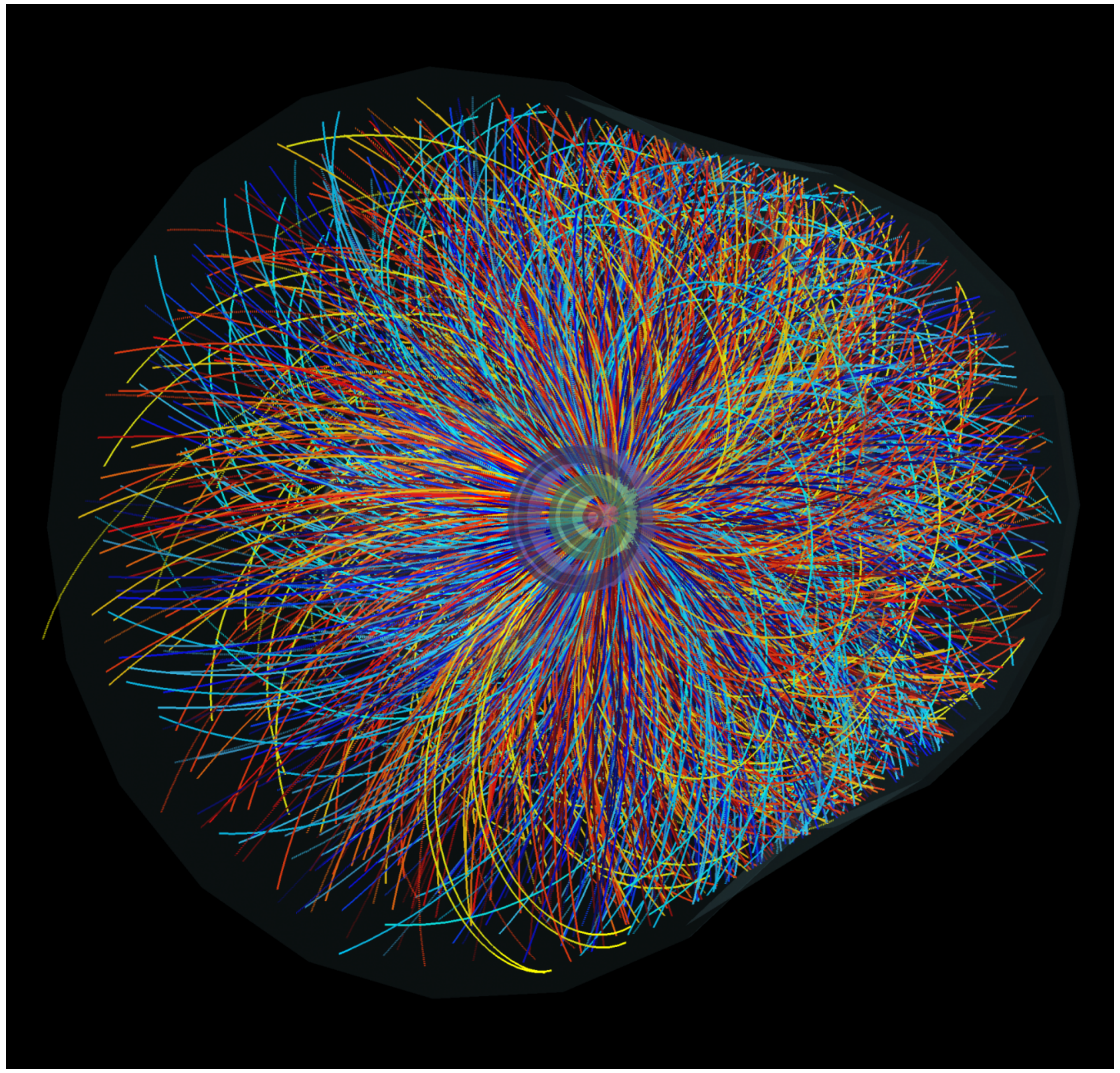


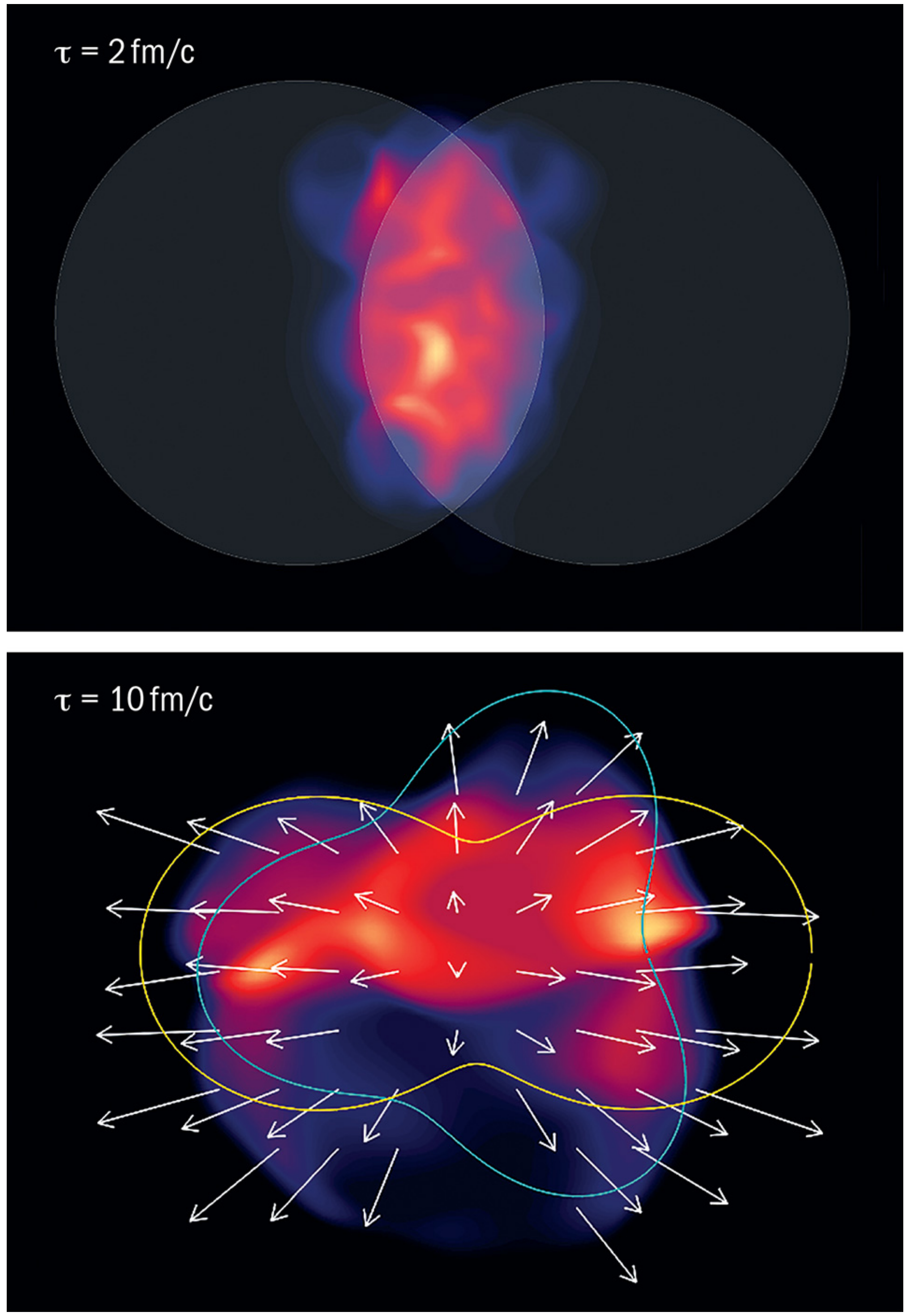
figure: K. Fukushima, D. E. Kharzeev, H. J. Warringa, PRL 104, 212001

MOMENTUM ANISOTROPY
PROBES THE MATTER PROPERTIES AND DYNAMICS



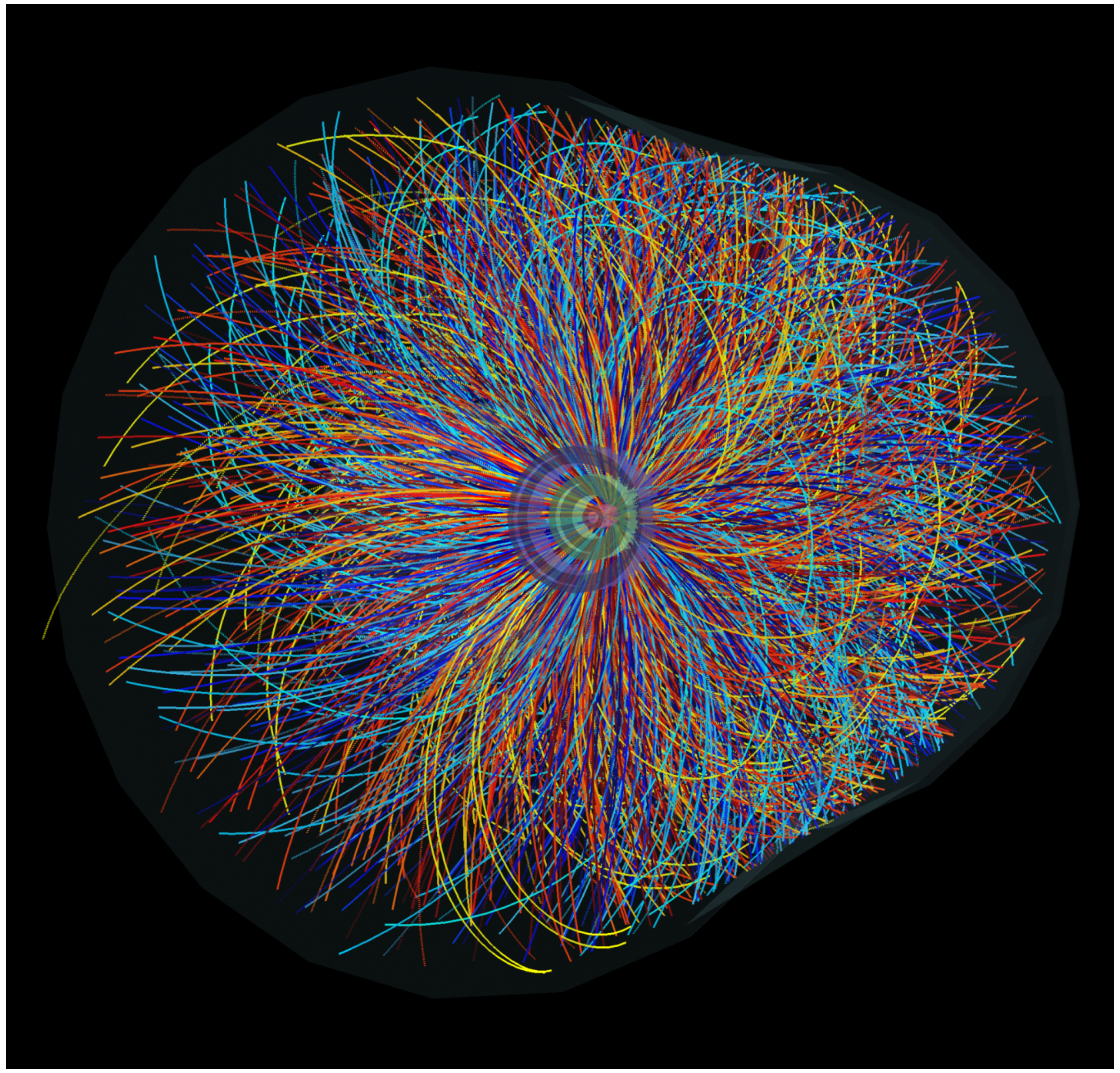
final state

MOMENTUM ANISOTROPY
PROBES THE MATTER PROPERTIES AND DYNAMICS



https://cerncourier.com/a/going-with-the-flow/

initial state



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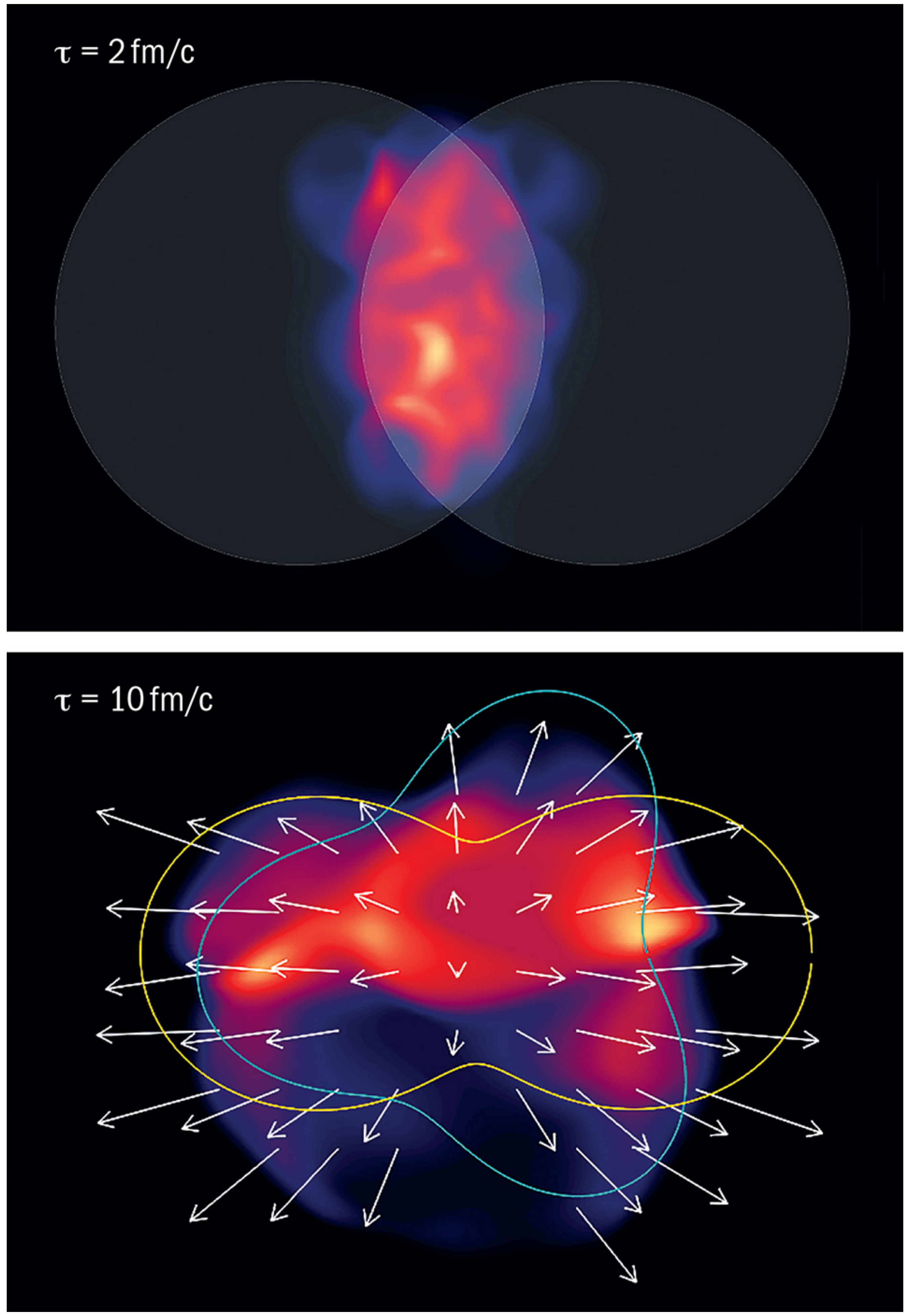


https://cerncourier.com/a/going-with-the-flow/

initial state

Hydrodynamics relies on the assumption that densities of various currents are known at the femtoscale

final state

MOMENTUM ANISOTROPY
PROBES THE MATTER PROPERTIES AND DYNAMICS

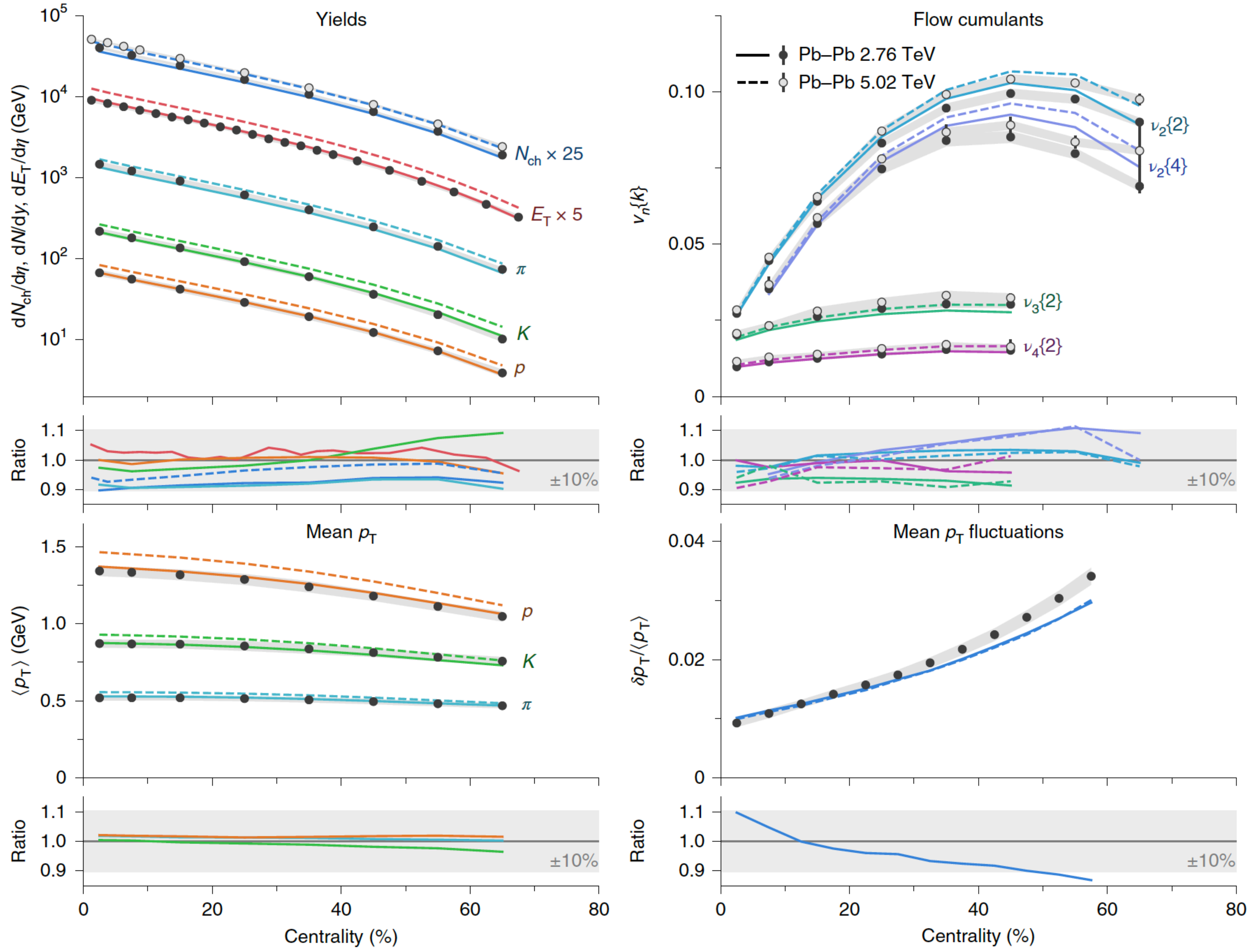
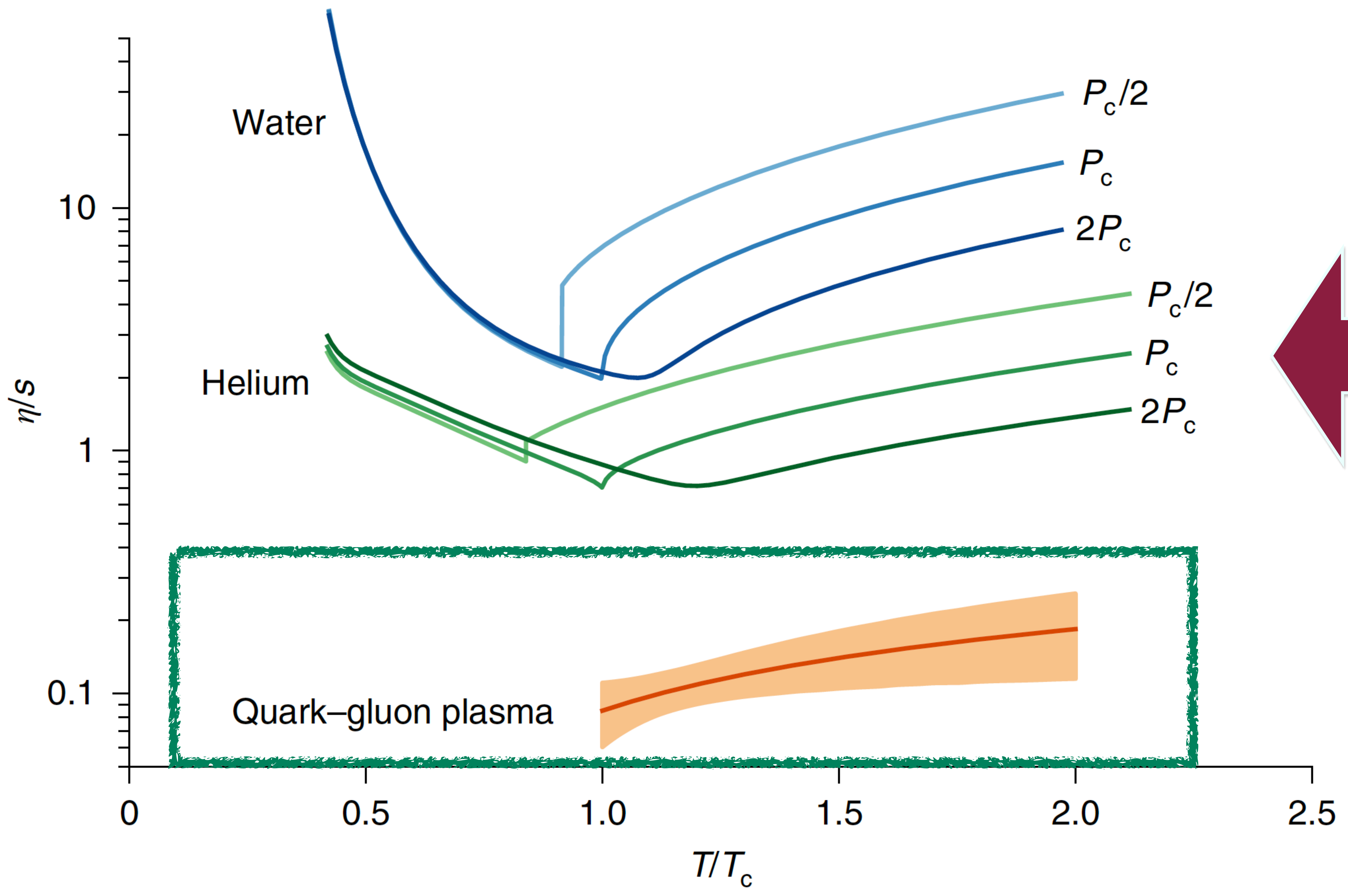
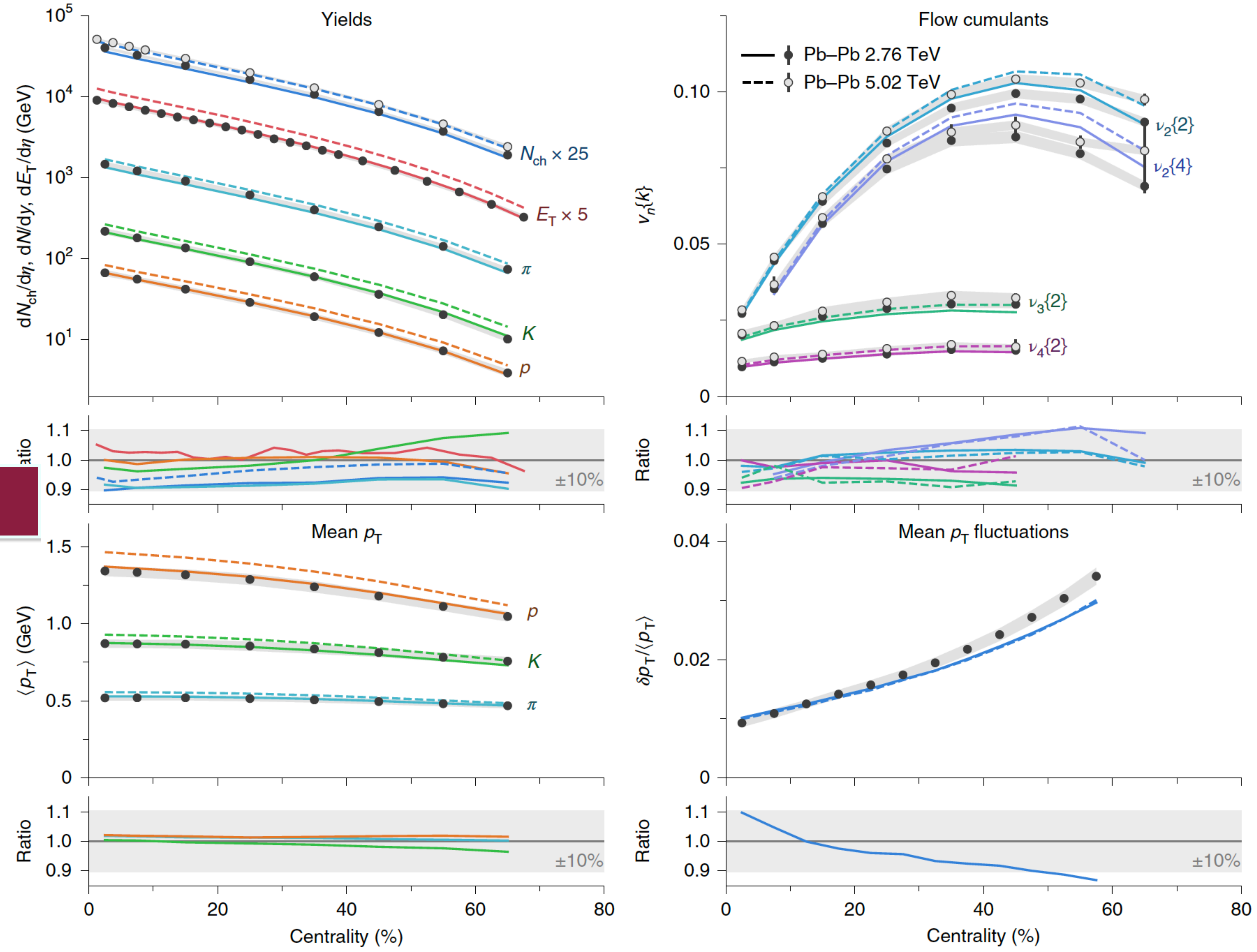


fig: Bernhard, Moreland, Bass, Nature Phys. 15, 1113–1117 (2019)

MOMENTUM ANISOTROPY
PROBES THE MATTER PROPERTIES AND DYNAMICS

The hot and dense QCD matter
behaves as an almost perfect fluid





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ANGULAR MOMENTUM AND MAGNETIC FIELD IN HIC

- **Large orbital angular momentum** (OAM)

  Becattini, Piccinini, Rizzo, PRC 77 (2008) 024906

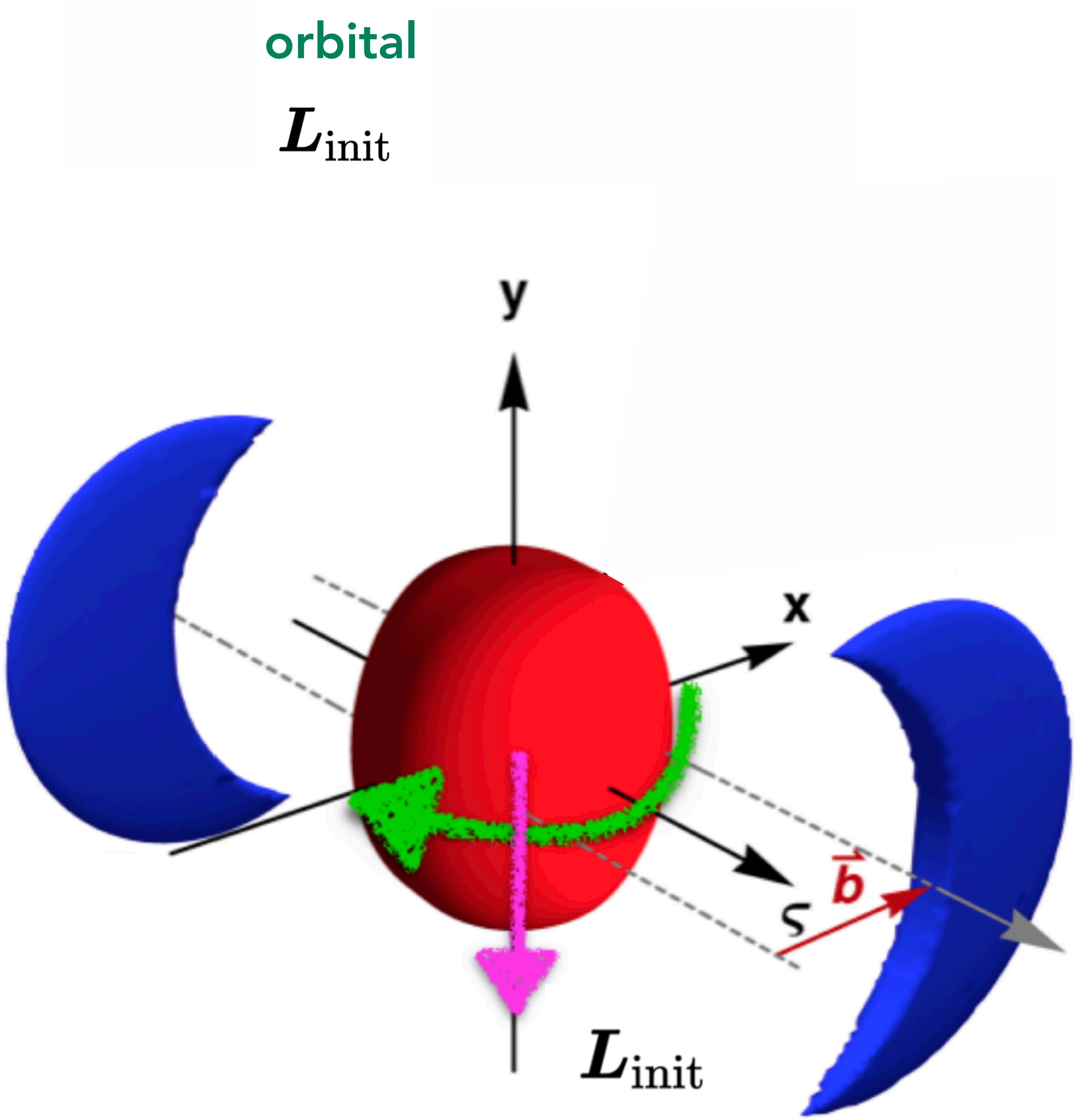

fig: R. R.

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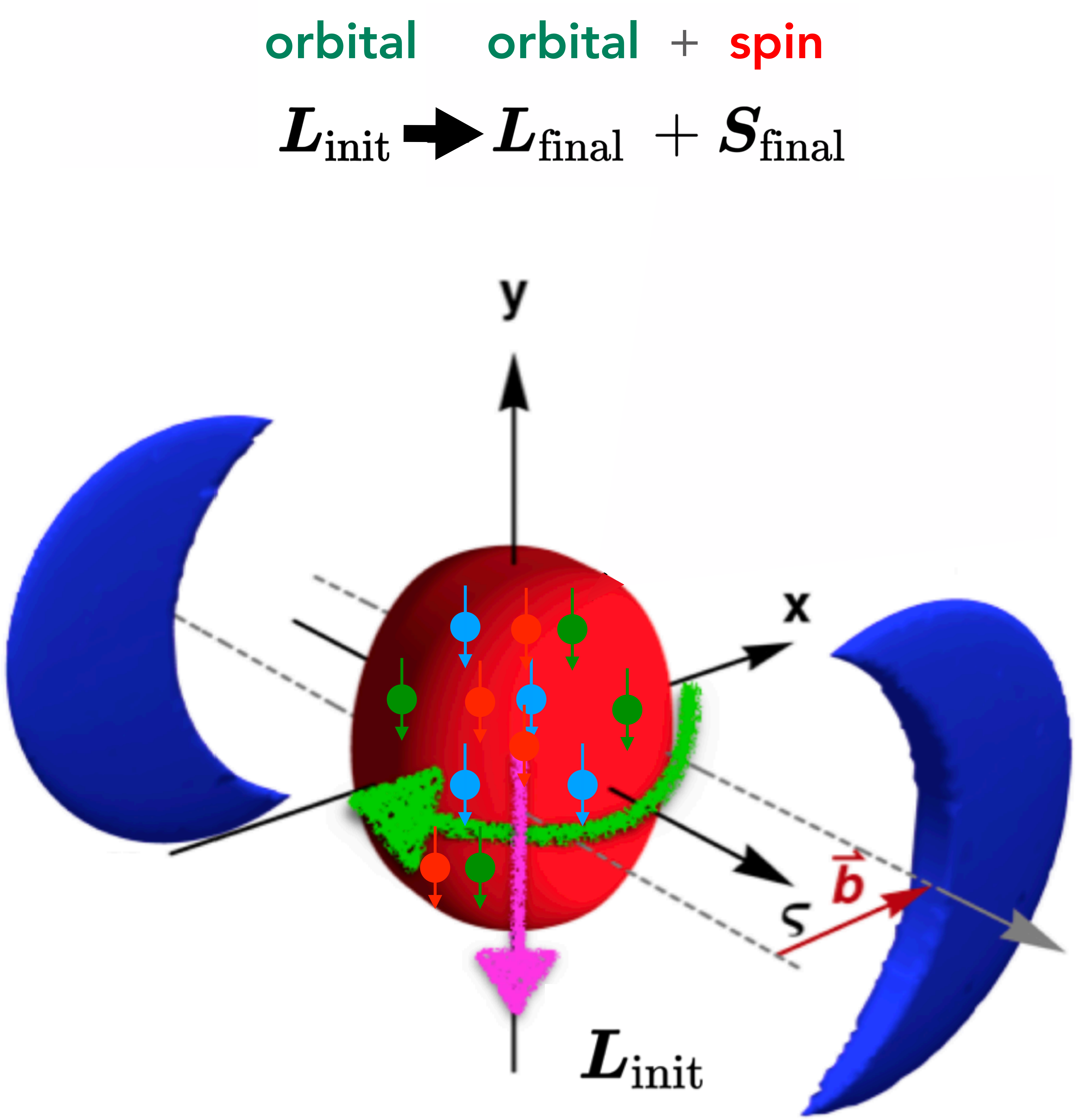

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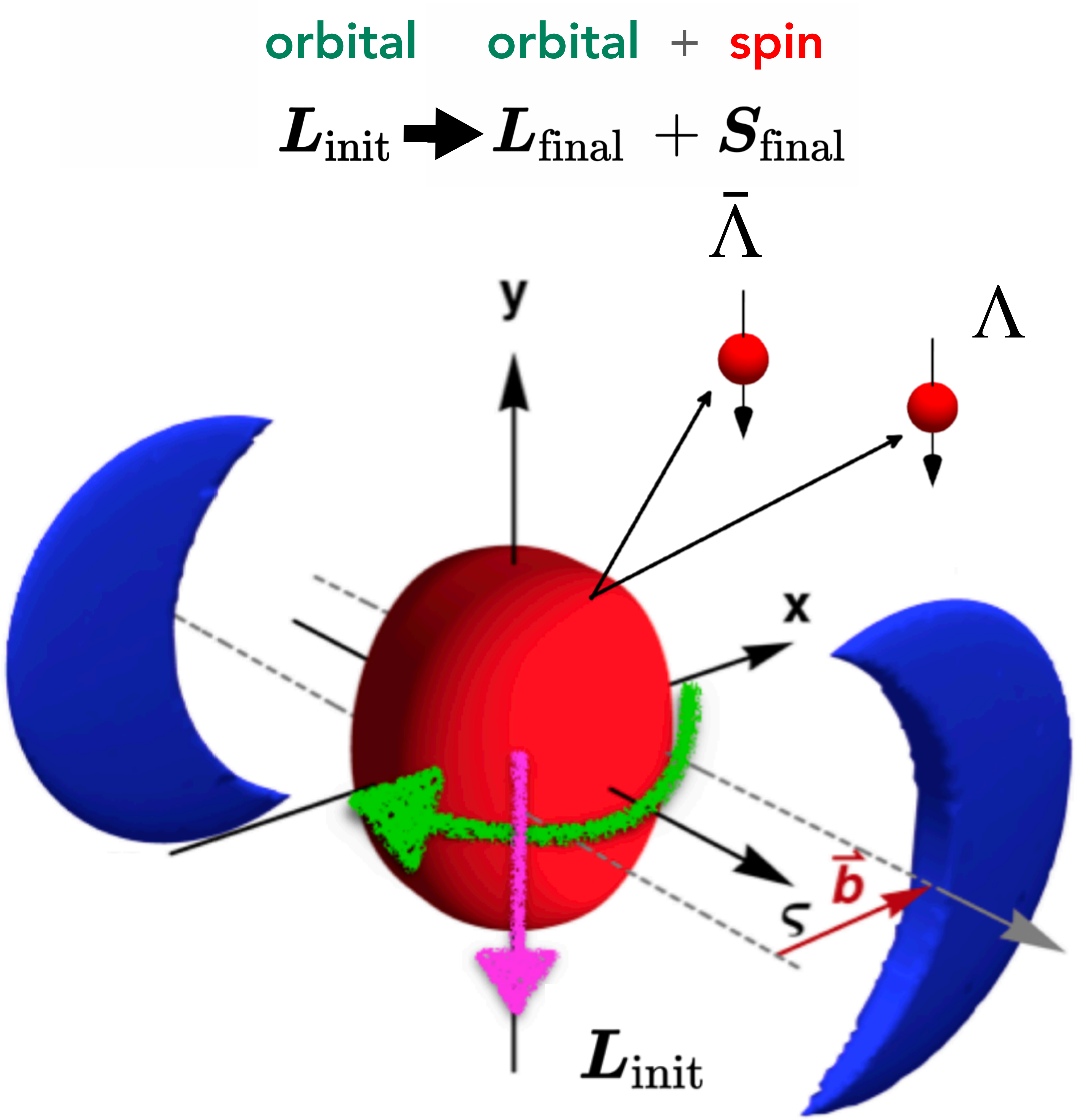
- OAM can be transferred to the spin of QGP constituents

- Emitted **particles (on average) are expected to be polarized** along the fireball's global angular momentum

  Liang, Wang PRL 94:102301 (2005)
  Betz, Gyulassy, Torrieri, PRC 76:044901 (2007)
  Gao, et al. PRC 77:044902 (2008)
  Becattini, Piccinini, et al. J. Phys. G 35:054001 (2008)



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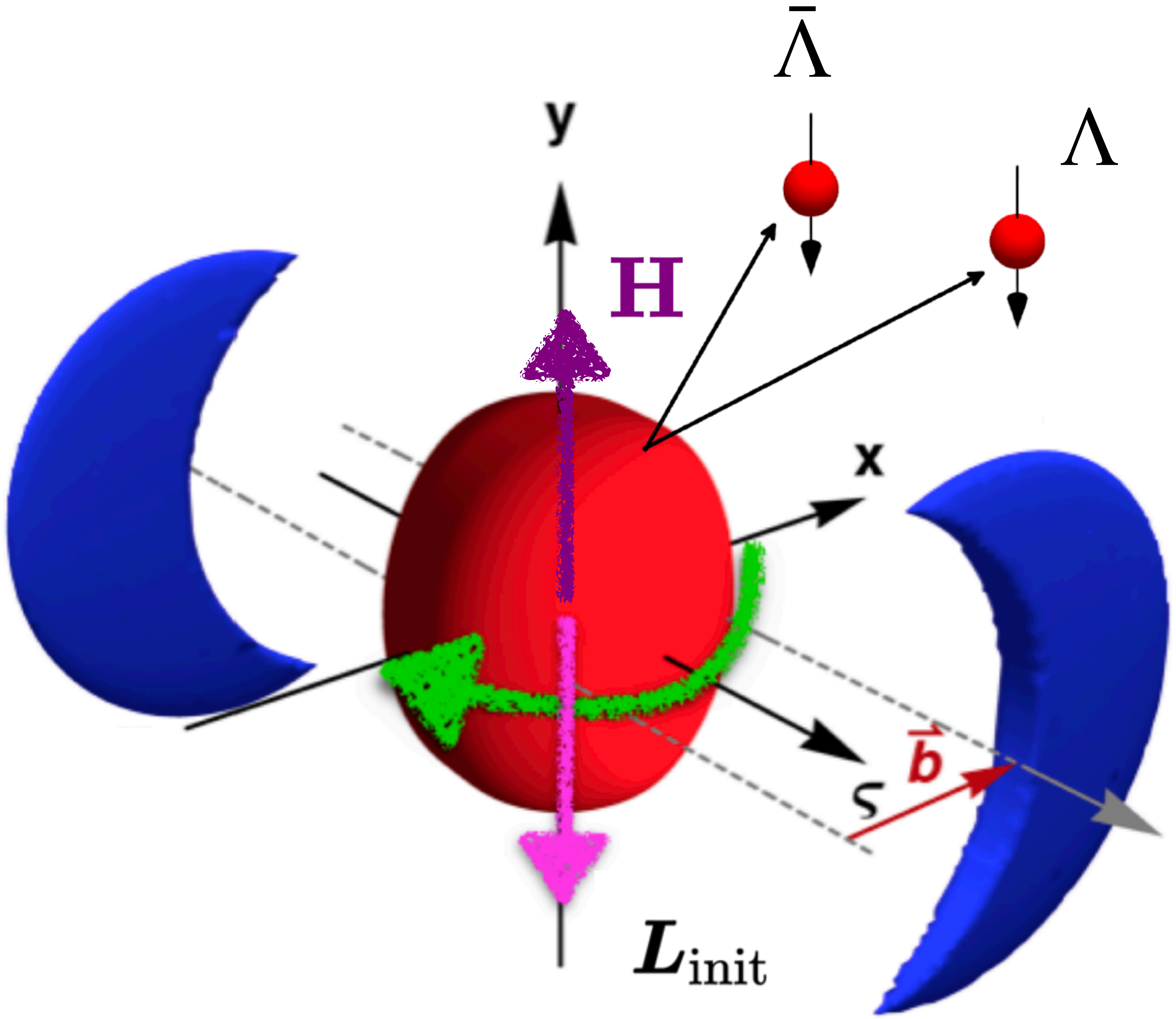
- **Large magnetic field** may be created initially
  Bzdak and Skokov, Phys. Lett. B 710 (2012) 171-174



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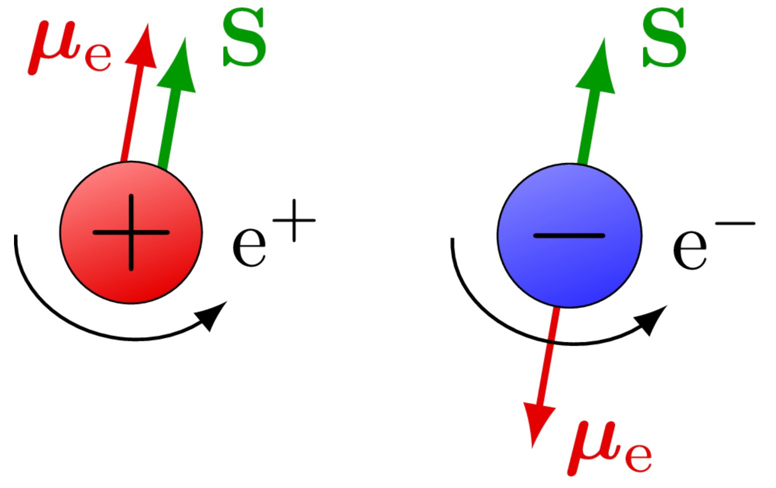
- **Large magnetic field** may be created initially

  Bzdak and Skokov, Phys. Lett. B 710 (2012) 171-174

- Alignment of magnetic moments is possible,
  which will split the signal for particles and antiparticles



$$\boldsymbol{\mu} = \frac{g_s q}{2m}\mathbf{S}$$

$$E = -\boldsymbol{\mu}\cdot\mathbf{B}$$

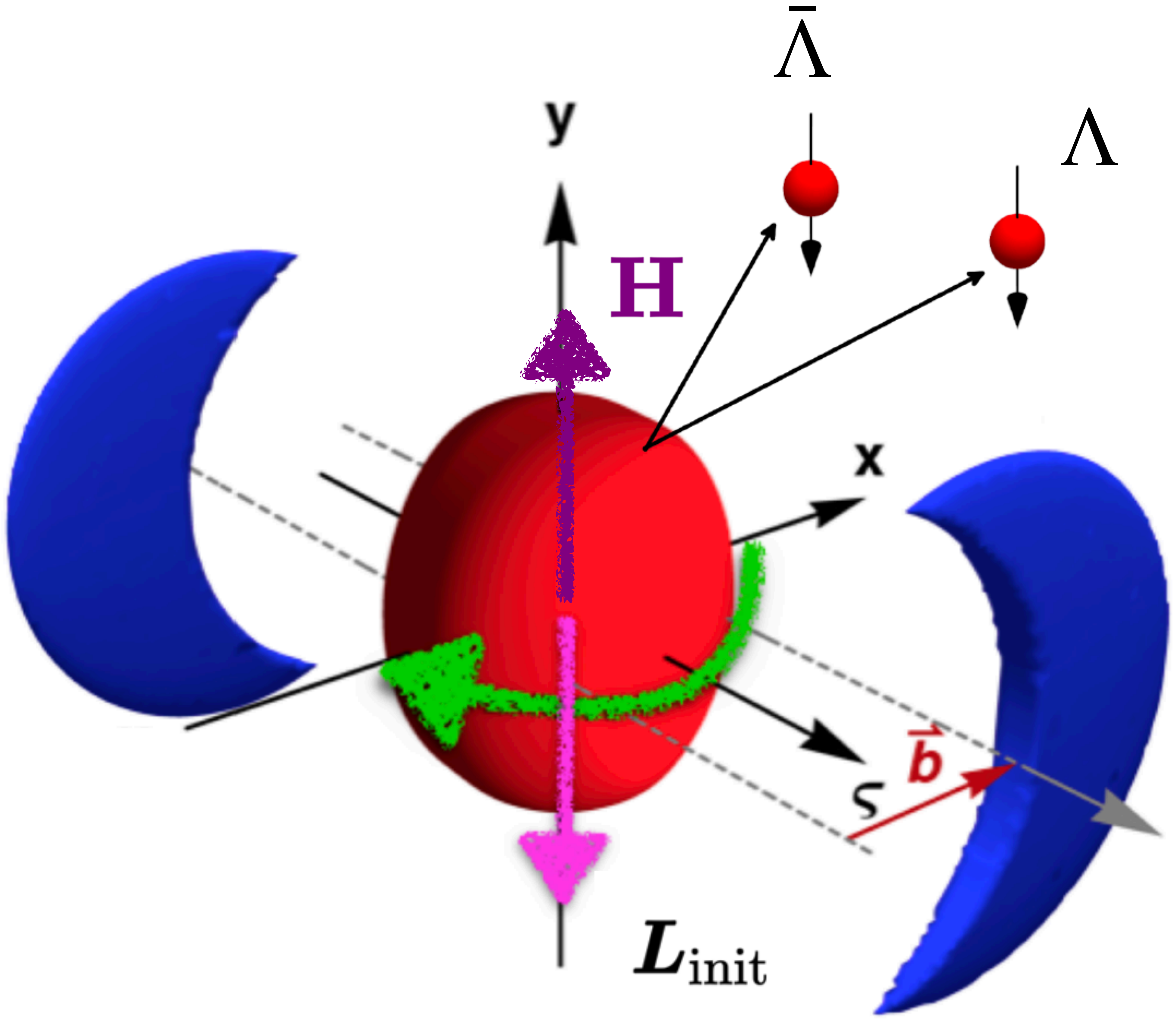


fig: R. R.

MEASUREMENT OF GLOBAL SPIN POLARIZATION

Self-analysing parity-violating weak decay allows to measure polarization of Λ hyperon

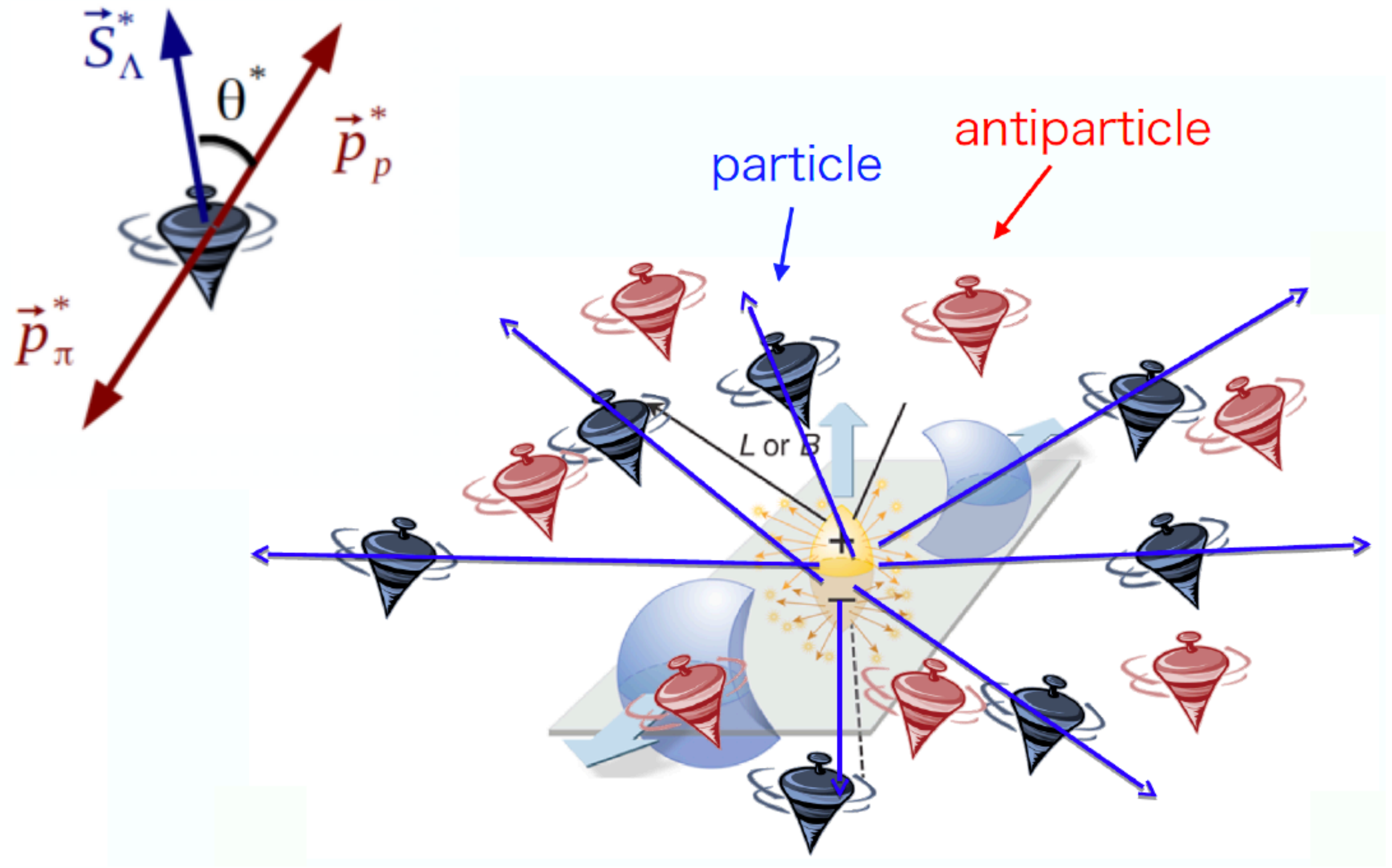


figure: T. Niida

$$\frac{dN}{d\cos\theta^*} = \frac{1}{2}\left(1 + \alpha_H |\vec{\mathcal{P}}_H| \cos\theta^*\right)$$

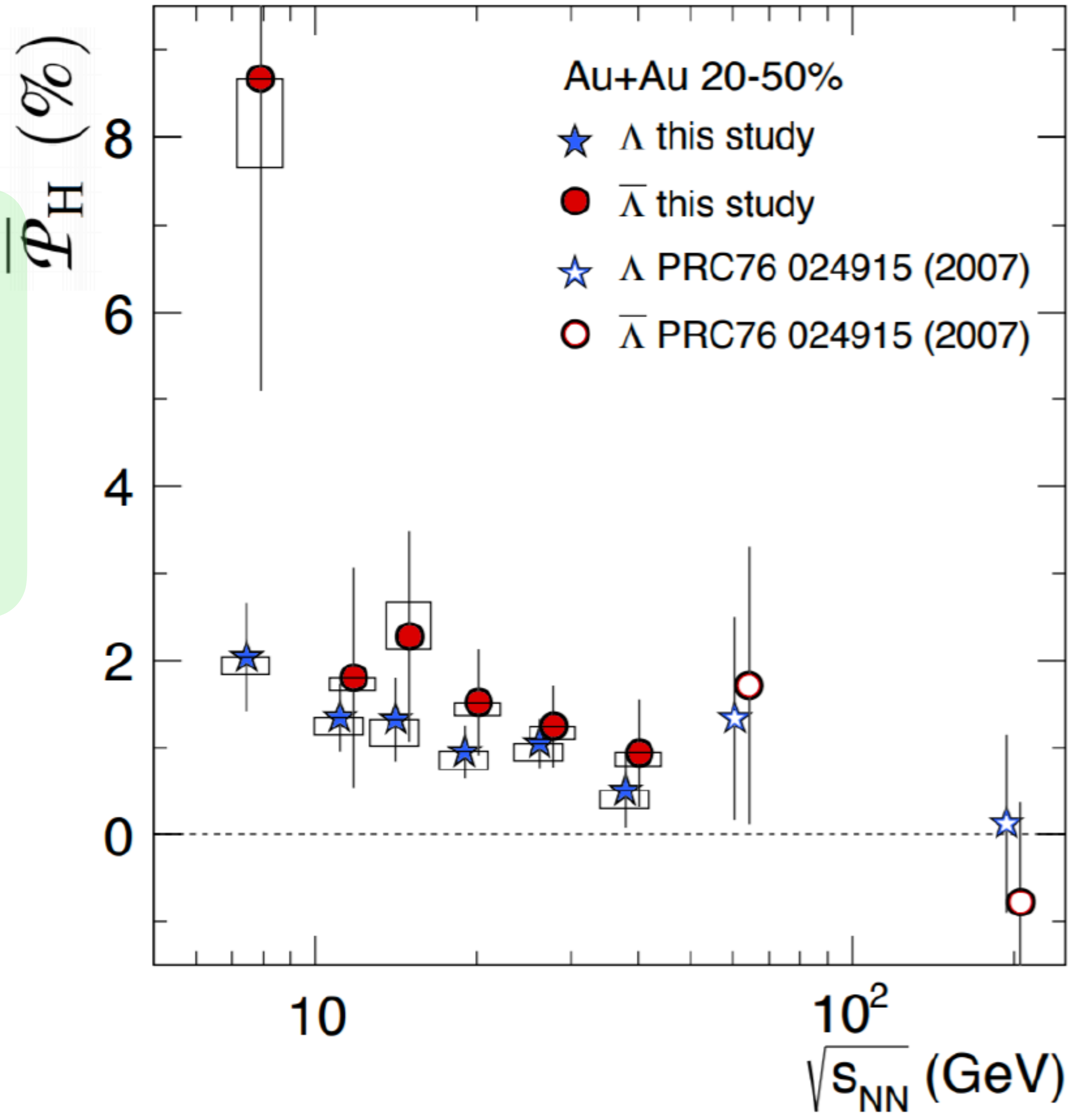
$(\alpha_\Lambda = 0.732)$

$$\overline{\mathcal{P}}_H \equiv \langle \vec{\mathcal{P}}_H \cdot \hat{J}_{sys} \rangle = \frac{8}{\pi\alpha_H} \frac{\left\langle \cos\left(\phi_p^* - \phi_{\hat{J}_{sys}}\right) \right\rangle}{R_{EP}^{(1)}}$$

MEASUREMENT OF GLOBAL SPIN POLARIZATION

global polarization ⟷ component along global angular momentum

unique measurement of mean spin orientation



Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65

INTERPRETARTION: SPIN-THERMAL APPROACH

Using quantum statistical field theory in **thermodynamic equilibrium** one can establish a link between **particle's spin** and **thermal vorticity**

F. Becattini, F. Piccinini, AP 323, 2452 (2008)
F. Becattini, F. Piccinini, J. Rizzo, PRC 77, 024906 (2008)
Becattini, Chandra, Del Zanna, Grossi, AP 338:32 (2013)
Fang, Pang, Wang, Wang, PRC 94:024904 (2016)
Becattini, Karpenko, Lisa, Upsal, and Voloshin PRC 95, 054902 (2017)

Spin vector of a single massive particle in RQM is given through by Pauli-Lubański vector

$$\widehat{S}^{\mu} = -\frac{1}{2m}\epsilon^{\mu\nu\rho\sigma}\widehat{J}_{\nu\rho}\widehat{P}_{\sigma};$$

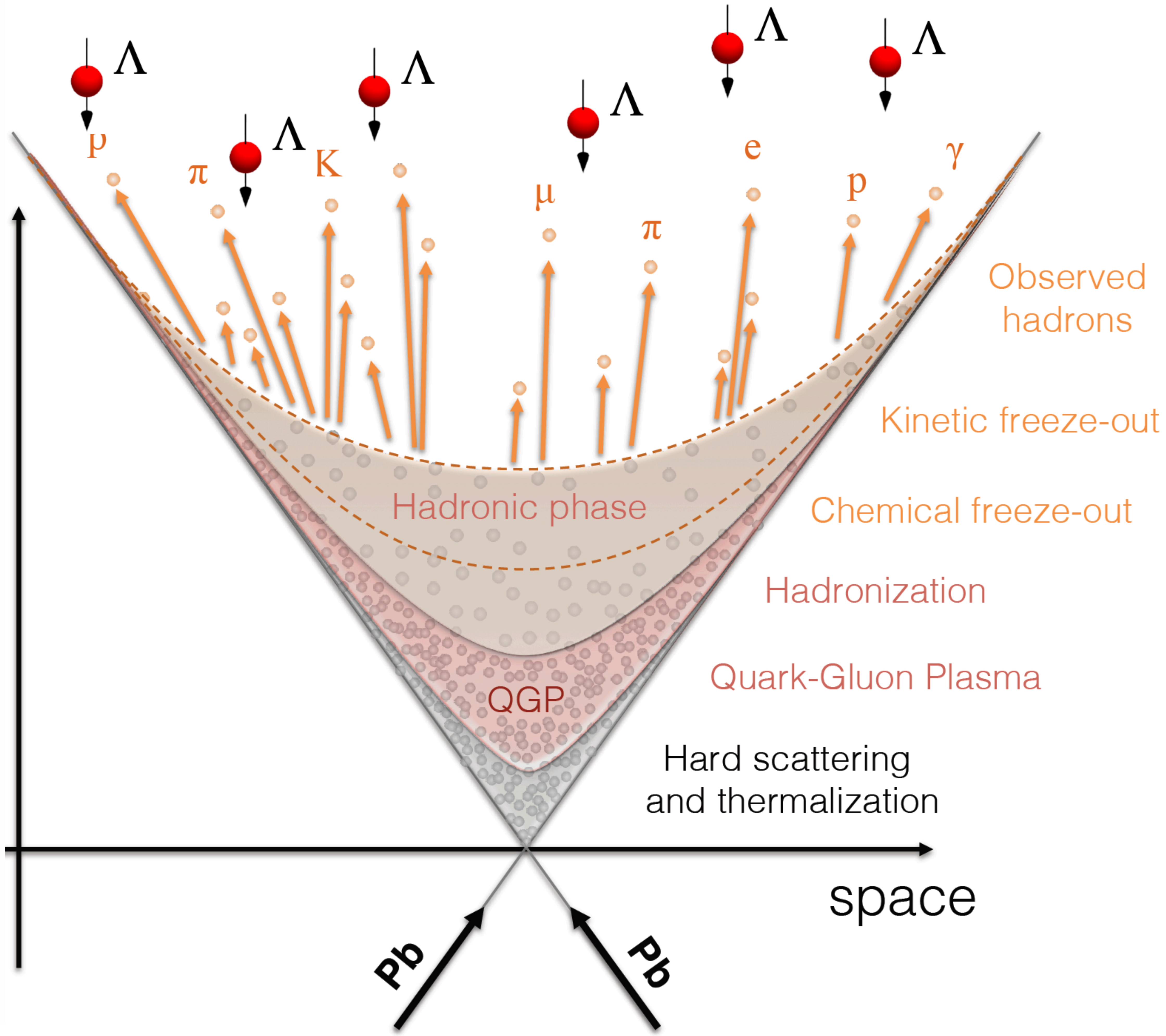


figure: D.D. Chinellato

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Becattini, Karpenko, Lisa, Upsal, and Voloshin PRC 95, 054902 (2017)

The **mean spin vector** of emitted particles is

$$S^{\mu}_{\varpi}(p) = -\frac{1}{8m_{\Lambda}}\epsilon^{\mu\nu\rho\sigma}p_{\sigma}\frac{\int d\Sigma\cdot p\; n_F(1-n_F)\varpi_{\nu\rho}}{\int d\Sigma\cdot p\; n_F}$$

Fluid's thermal vorticity is

$$\varpi_{\mu\nu} = \partial_{[\nu}\beta_{\mu]} = -\frac{1}{2}(\partial_{\mu}\beta_{\nu} - \partial_{\nu}\beta_{\mu})$$

$$\beta^{\mu} \equiv u^{\mu}/T$$

$$T, \mu_B\; u^{\rho}$$

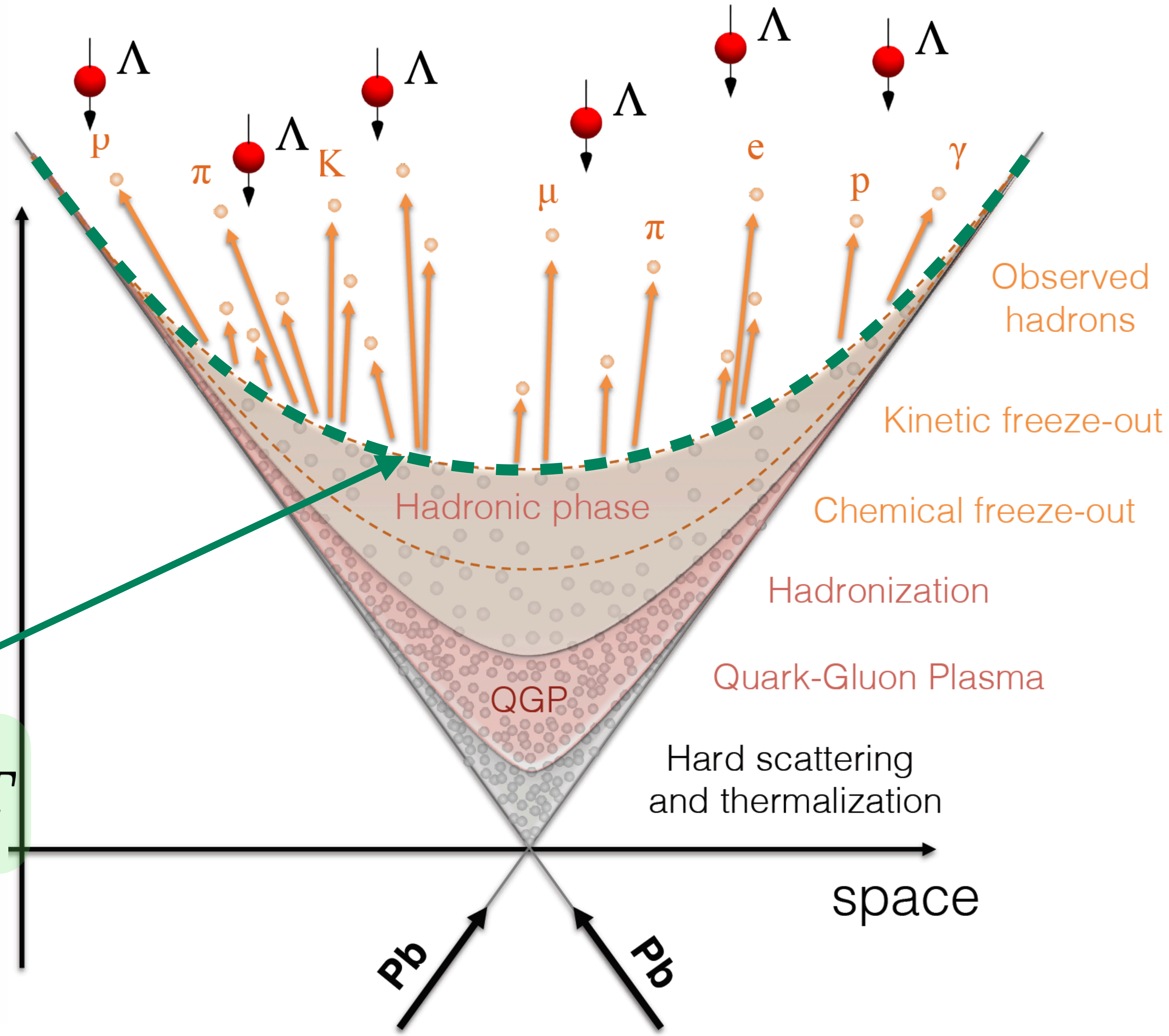


figure: D.D. Chinellato

GLOBAL POLARIZATION: MEASUREMENT VS SPIN-THERMAL APPROACH

Global polarization data agrees well with transport, hydrodynamics, and many other (effective) models which employ spin-thermal approach

Thermal vorticity determines global polarization !

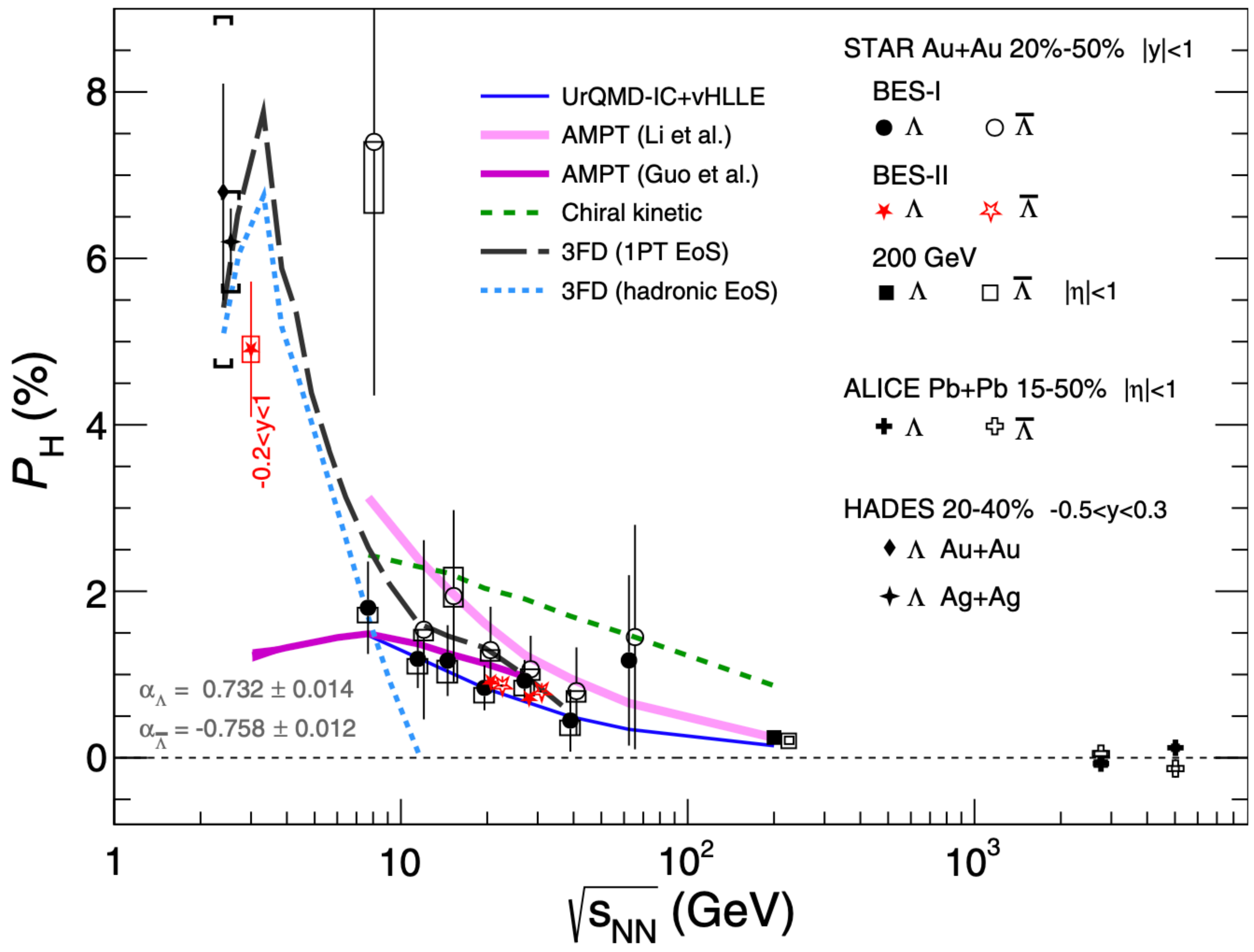
Spin DOFs seem to be in equilibrium

STAR:
L. Adamczyk et al., Nature 548 (2017) 62,
M. S. Abdallah et al., Phys. Rev. C 104 (2021) L061901
J. Adam et al., Phys. Rev. C 98 (2018) 014910,
M. I. Abdulhamid et al., Phys. Rev. C 108 (2023) 014910

ALICE:
S. Acharya et al., Phys. Rev. C 101 (2020) 044611,

HADES:
R. Abou Yassine et al., Phys. Lett. B 835 (2022) 137506

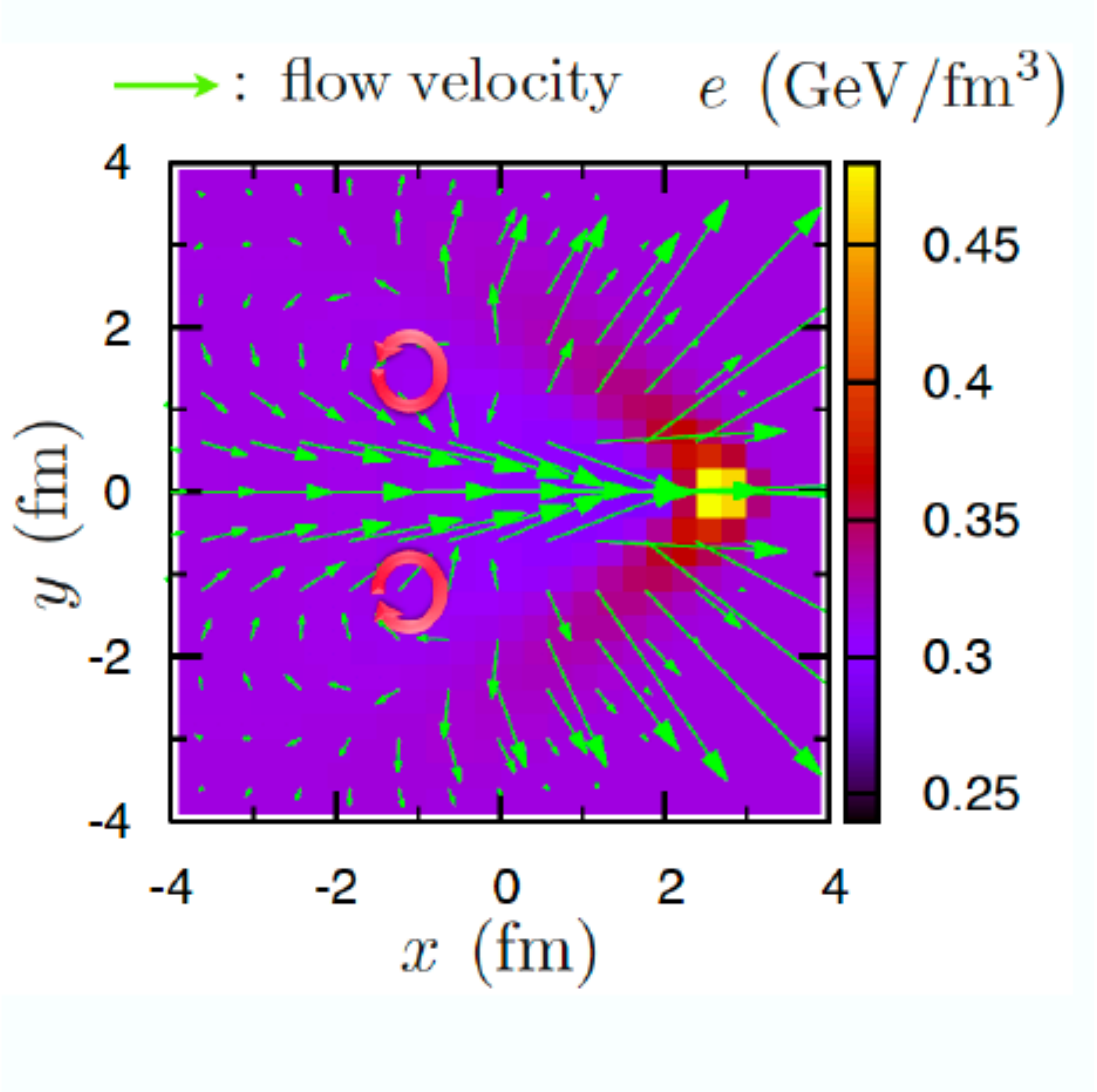
Becattini et al, IJMP EVol. 33, No. 6 (2024) 2430006

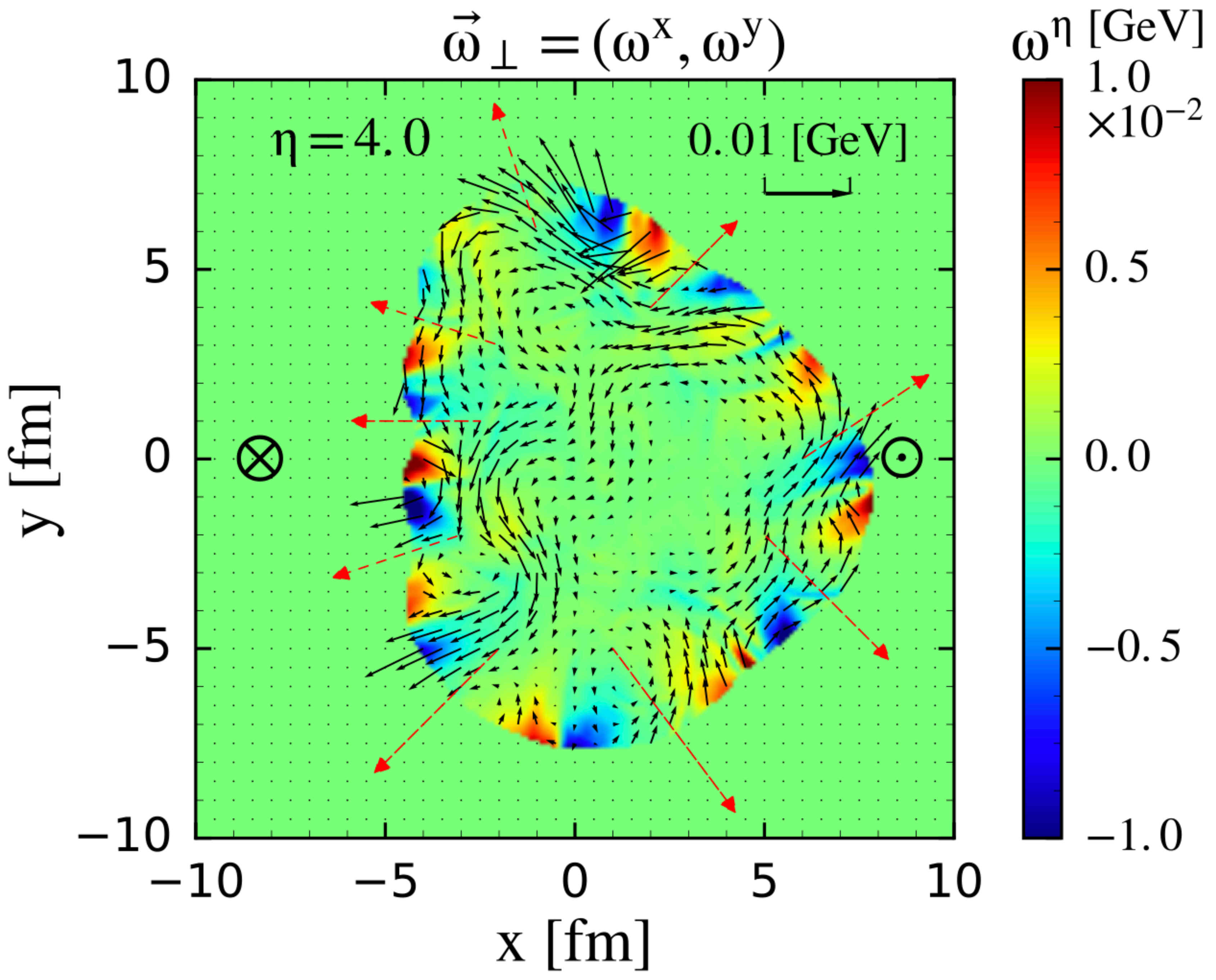


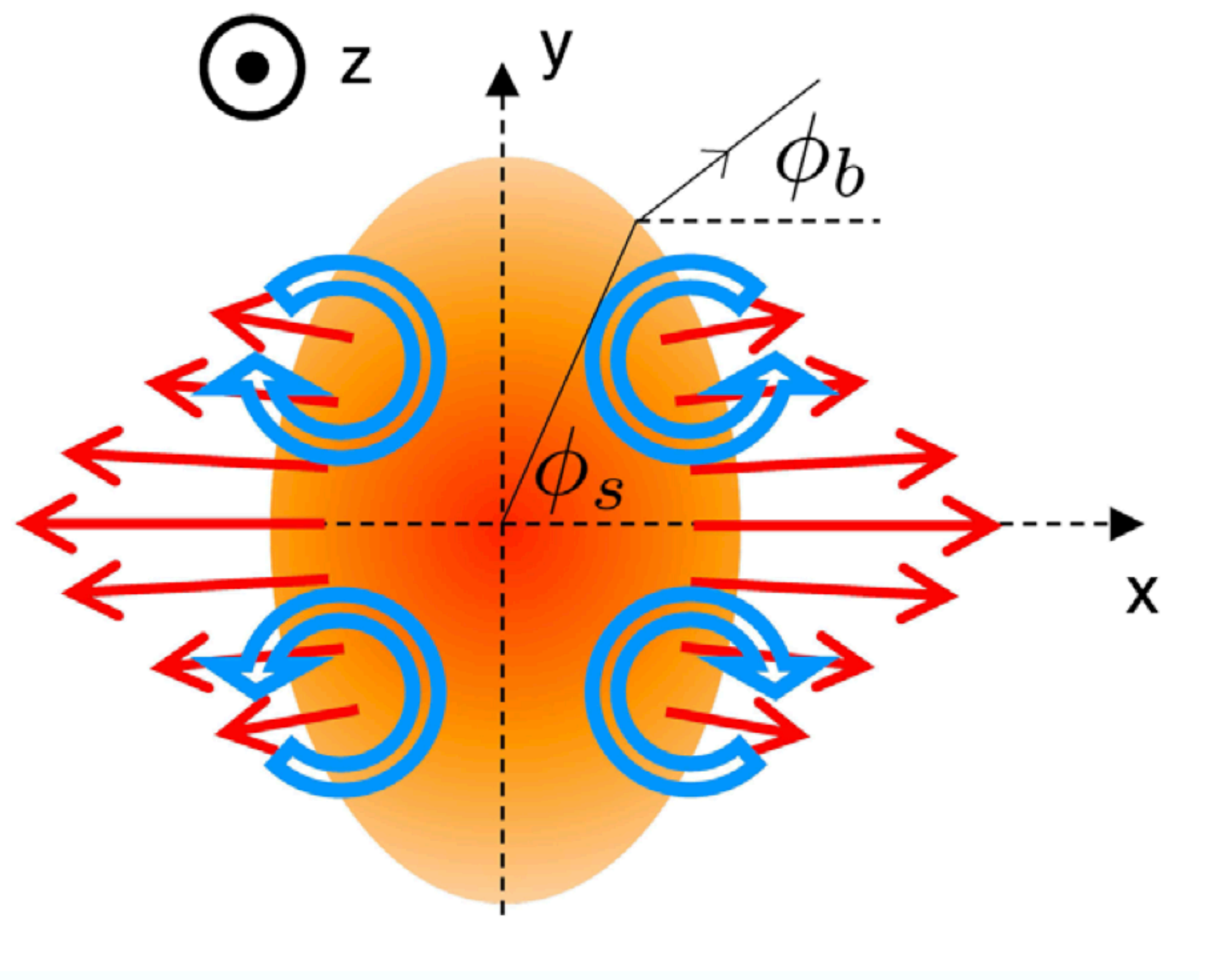
LONGITUDINAL POLARIZATION

Local **flow structures** in the plane transverse to the beam
lead to longitudinal (beam-direction) polarization

Pang, Petersen, Wang, Wang, Phys.Rev.Lett. 117 (2016) 19, 192301







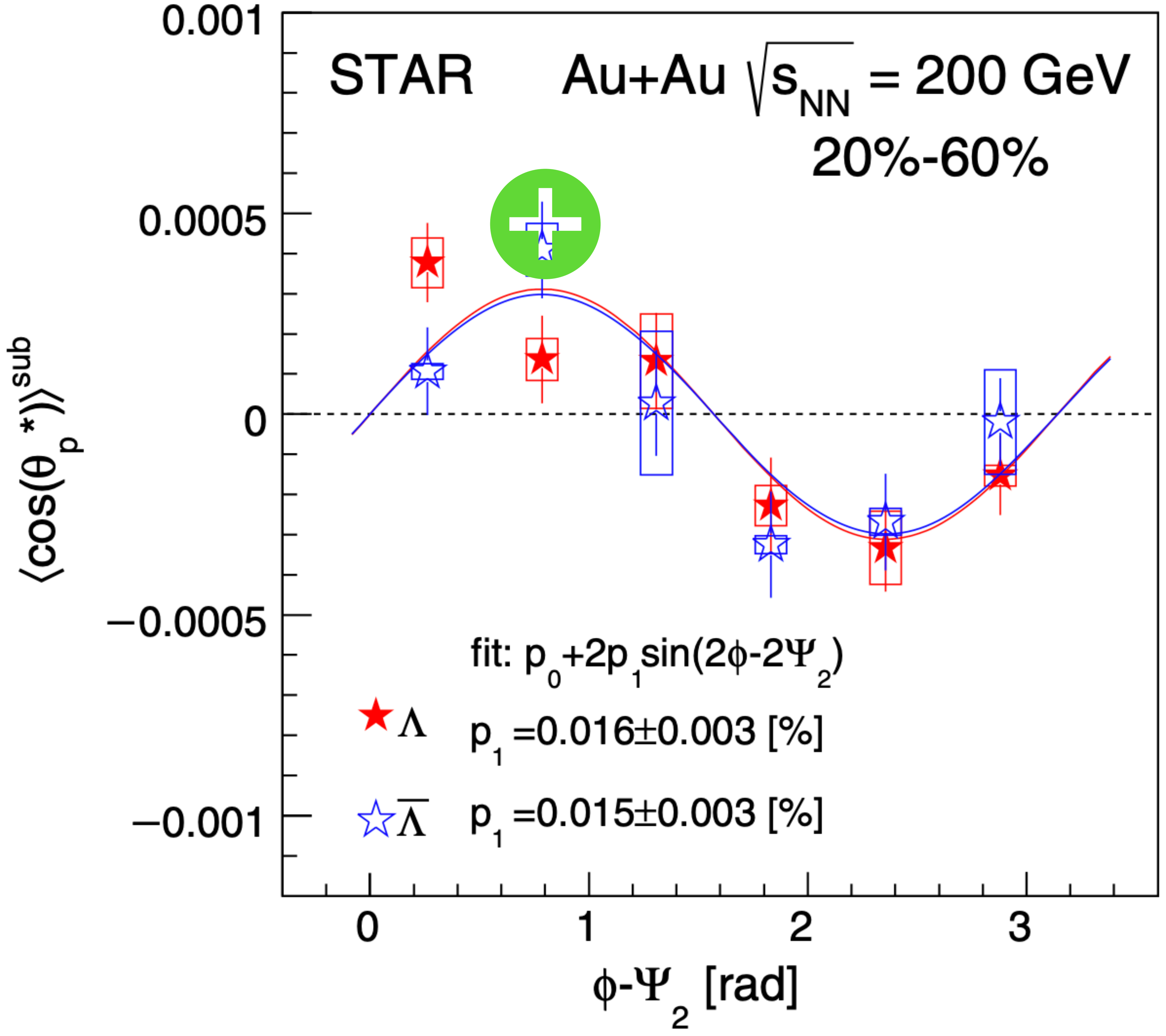
Tachibana, Hirano, Nucl.Phys.A 904-905 (2013) 1023c-1026c

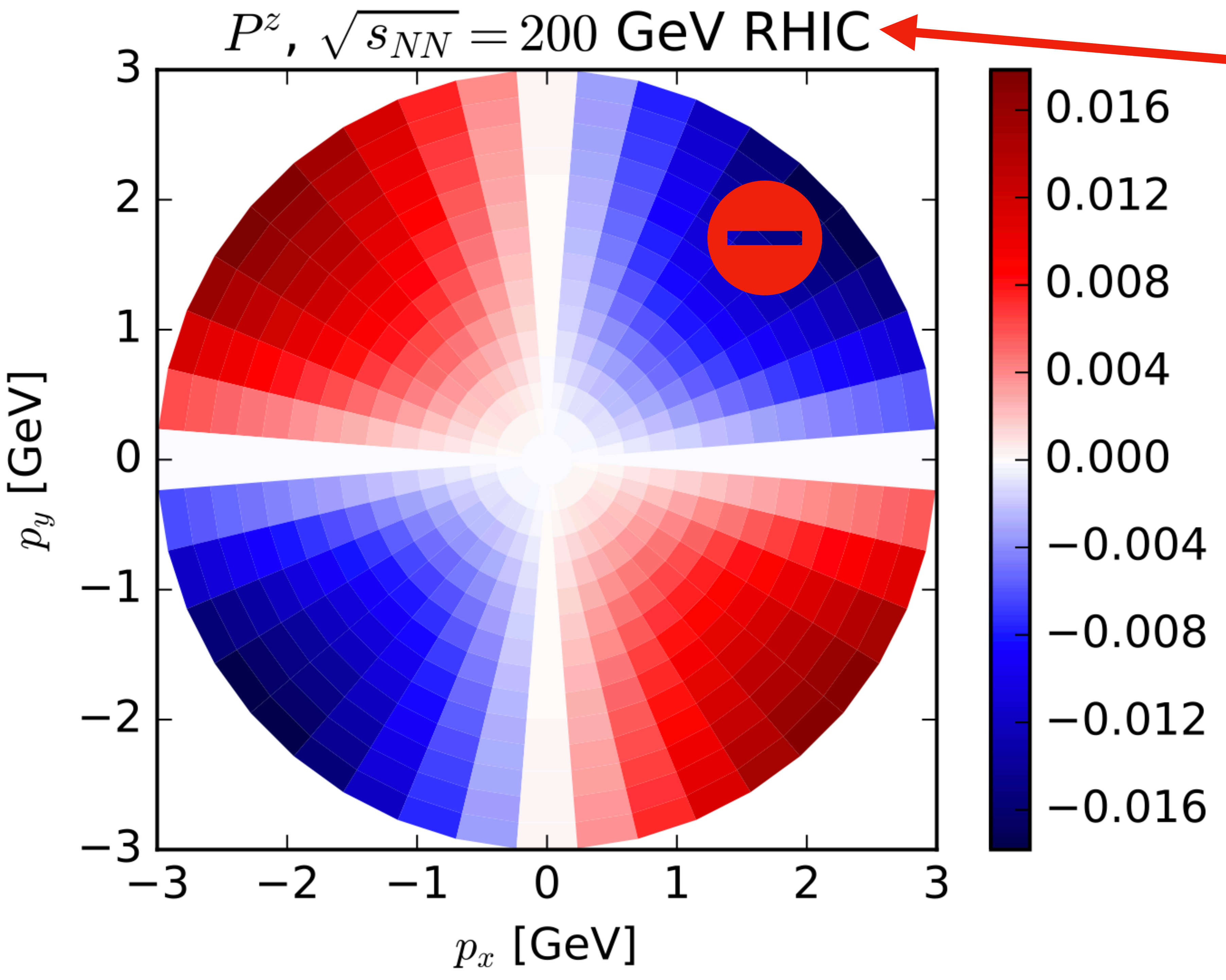
Adam et al (STAR Collaboration) Phys. Rev. Lett. 123, 132301

LONGITUDINAL POLARIZATION: MEASUREMENT VS SPIN-THERMAL APPROACH

Spin-thermal approach fails in reproducing local polarization observables using solely thermal vorticity

Adam et al (STAR Collaboration) Phys. Rev. Lett. 123, 132301

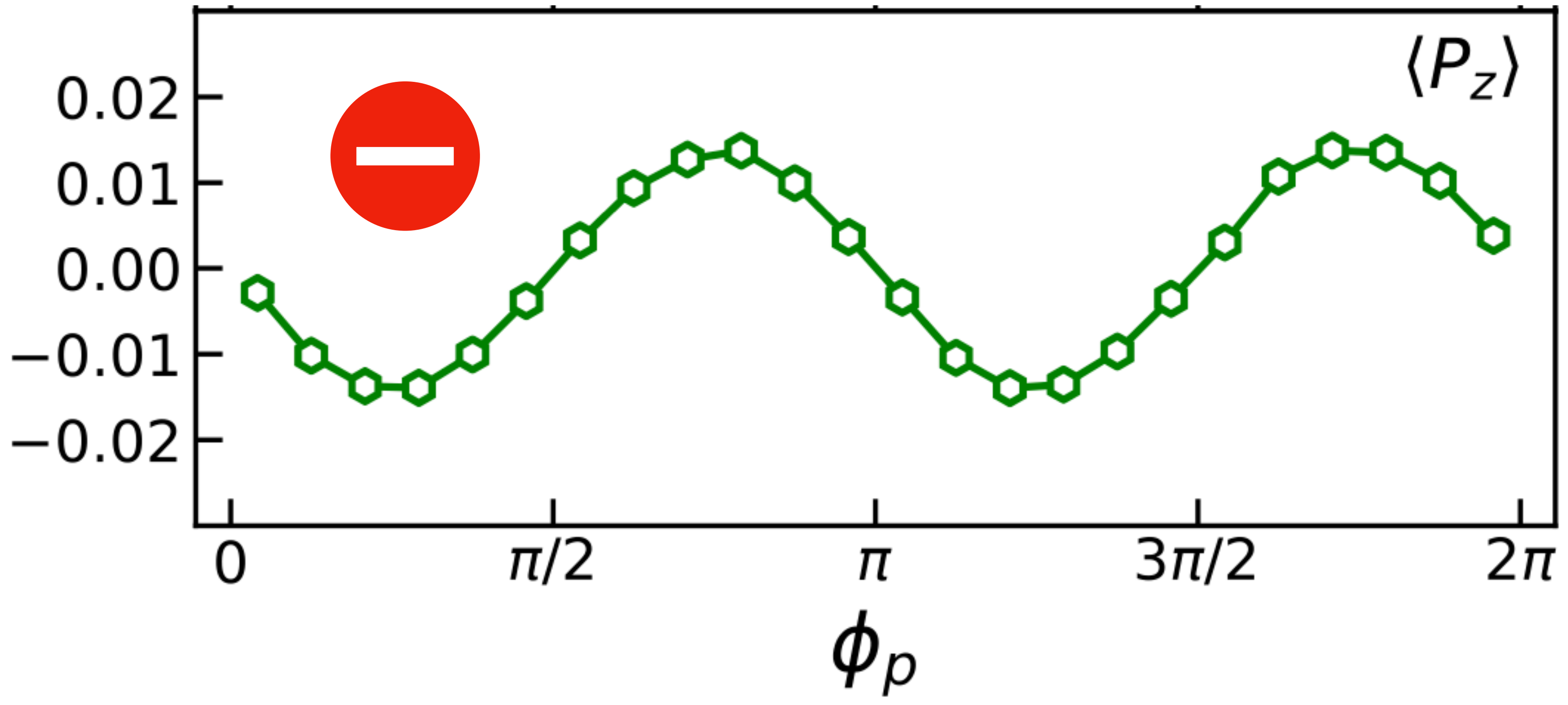




Becattini, Karpenko, Phys.Rev.Lett. 120 (2018) 1, 012302

$P^z = S^z_\varpi$

Hydrodynamics



Transport

Xia, Li, Tang, Wang, Phys.Rev.C 98 (2018) 024905

THERMAL SHEAR CONTRIBUTION

The spin polarization formula obtained at first order of thermodynamic gradients should be supplemented with the additional contribution from thermal shear

Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)
Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^\mu(p) = S^\mu_\varpi(p) + S^\mu_\xi(p)$$

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$$S^\mu(p) = \boxed{S^\mu_\varpi(p)} + S^\mu_\xi(p)$$

Where the first term is

Becattini, Inghirami, Rolando, Beraudo, DelZanna, DePace, Nardi, Pagliara, and Chandra, Eur. Phys. J. C 75, 406 (2015),

$$S^\mu_\varpi(p) = -\frac{1}{8m_\Lambda}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int d\Sigma\cdot p\, n_F(1-n_F)\varpi_{\nu\rho}}{\int d\Sigma\cdot p\, n_F}$$

$$\varpi_{\mu\nu} = \partial_{[\nu}\beta_{\mu]} = -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right)$$

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$$S^\mu(p) = S^\mu_\varpi(p) + \boxed{S^\mu_\xi(p)}$$

For the second term there are currently two prescriptions

First is **BBP**

Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)

$$S^\mu_{\xi,\mathrm{BBP}}(p) = -\frac{\epsilon^{\mu\nu\rho\sigma}}{4m_\Lambda}\frac{p_\sigma p^\lambda}{p\cdot\hat{t}}\frac{\int d\Sigma\cdot p\, n_F(1-n_F)\hat{t}_\nu\xi_{\lambda\rho}}{\int d\Sigma\cdot p\, n_F}$$

$$\xi_{\mu\nu} = \partial_{(\nu}\beta_{\mu)} = \frac{1}{2}\left(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu\right)$$

$$\hat{t} = (1,0,0,0)$$

THERMAL SHEAR CONTRIBUTION

The spin polarization formula obtained at first order of thermodynamic gradients should be supplemented with the additional contribution from thermal shear

Becattini, Buzzegoli, and Palermo, Phys. Lett. B 820, 136519 (2021)
Buzzegoli, Phys. Rev. C 105, 044907 (2022)

$$S^\mu(p) = S^\mu_\varpi(p) + \boxed{S^\mu_\xi(p)}$$

For the second term there are currently two prescriptions

Second is **LY**

Liu and Yin, JHEP 07, 188,

$$S^\mu_{\xi,\mathrm{LY}}(p) \;=\; -\frac{\epsilon^{\mu\nu\rho\sigma}}{4m_\Lambda}p_\sigma \frac{\int d\Sigma\cdot p\; n_F(1-n_F)\frac{p_\perp^\lambda u_\nu}{p\cdot u}\xi_{\rho\lambda}}{\int d\Sigma\cdot p\; n_F}$$

$$p^\perp_\mu \equiv \Delta_\mu^{\ \nu} p_\nu$$

Introducing the shear contribution allows to capture the local polarization data correctly.

DYNAMICAL MODELS: SPIN HYDRODYNAMICS

Problems with describing the longitudinal polarization triggered developments of hydrodynamics with spin degrees of freedom.

Perfect spin hydrodynamics was proposed

Florkowski, Friman, Jaiswal, Speranza, Phys. Rev. C97 (4) (2018) 041901
Florkowski, Friman, Jaiswal, RR, Speranza, Phys. Rev. D97 (2018) 116017

$$f^{\pm} = \exp[\pm\xi(x) - \beta_\mu(x)p^\mu]$$

$$X^{\pm} = \exp\left[\pm\xi(x) - \beta_\mu(x)p^\mu \pm \frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right]$$

$$f^{+}_{rs}(x,p) = \frac{1}{2m}\bar{u}_r(p)X^{+}u_s(p)$$

$$f^{-}_{rs}(x,p) = -\frac{1}{2m}\bar{v}_s(p)X^{-}v_r(p)$$

PHYSICAL REVIEW C **97**, 041901(R) (2018)

Rapid Communications

Relativistic fluid dynamics with spin

Wojciech Florkowski,[1,2,3] Bengt Friman,[4] Amaresh Jaiswal,[4,5] and Enrico Speranza[4,6]

DYNAMICAL MODELS: SPIN HYDRODYNAMICS

Since then spin hydrodynamics is being actively developed

Becattini and Tinti, Annals Phys. 325, 1566 (2010)
Montenegro and Torrieri, PRD 100, 056011 (2019)
Bhadury, Florkowski, Jaiswal, Kumar, and R. R, PRL 129, 192301 (2022)
Weickgenannt, Speranza, Sheng, Wang, and Rischke, PRL 127, 052301 (2021)
Li, Stephanov, and Yee, PRL 127, 082302 (2021)
Gallegos, Gursoy, and Yarom, JHEP 05, 139
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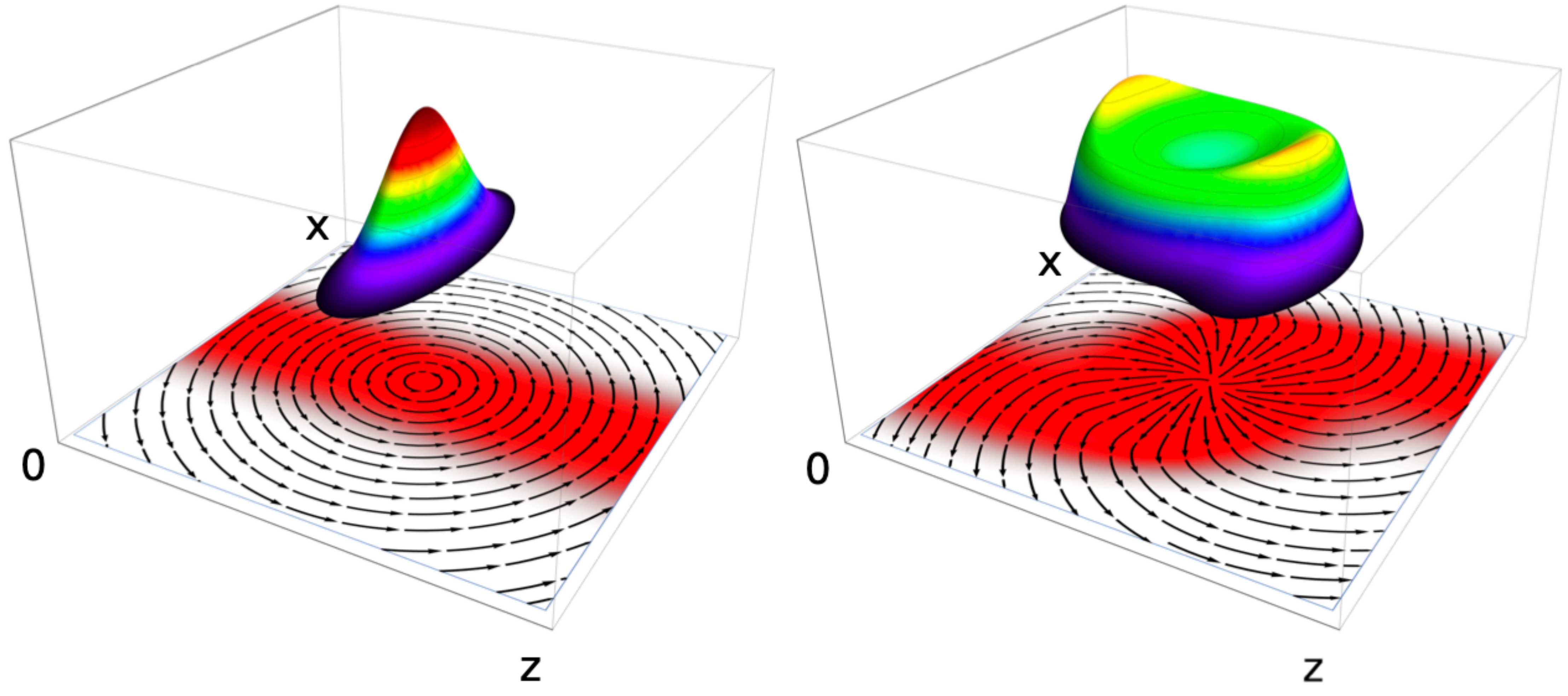
Becattini, Daher, Sheng Phys.Lett.B 850 (2024) 138533
Daher, Sheng, Wagner, Becattini, 2503.03713
…



Florkowski, Friman, Jaiswal, Ryblewski, Speranza, Nuclear Physics A 00 (2018) 1–4

CONSERVATION LAWS AND LAGRANGE MULTIPLIERS

conservation laws + (near) local equilibrium ➡ hydrodynamics

- conservation of charge (baryon number, electric charge, …)

$$\partial_\mu N^\mu(x) = 0 \quad \Rightarrow \quad \mu \equiv \xi T \qquad \text{(1 eq / charge)}$$

- conservation of energy and linear momentum

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad \Rightarrow \quad T, u^\nu \qquad \text{(4 eqs)}$$

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- conservation of energy and linear momentum

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad \Rightarrow \quad T, u^\nu \qquad \text{(4 eqs)}$$

- conservation of angular momentum

$$\partial_\lambda J^{\lambda\mu\nu}(x) = 0 \quad \Rightarrow \quad \Omega_{\mu\nu} \equiv T\omega_{\mu\nu} \qquad \text{(6 eqs)}$$

$\Omega_{\mu\nu}$: spin chemical potential

$\omega_{\mu\nu}$: spin polarization tensor

CONSERVATION LAWS FROM THE FIELD THEORY VIEWPOINT

A *relativistic fluid* can be regarded as the macroscopic, continuum limit of a quantum many-body system

The shared symmetries between the field-theoretic and relativistic hydrodynamic descriptions imply that both frameworks exhibit same conserved quantities.

In the context of a relativistic fluid, these **conserved quantities are defined as averages corresponding to the renormalized thermal expectation values of the respective conserved field operators**, computed using a density operator that depends on the state of the system

$$\mathcal{A} \equiv \langle \widehat{\mathcal{A}} \rangle = \mathrm{Tr}[\widehat{\rho}\widehat{\mathcal{A}}]_{\mathrm{ren}}$$

Thus, If the operator A is conserved, then the thermal expectation value is also conserved

$$\partial_\mu \mathcal{A}^\mu = \partial_\mu \mathrm{Tr}[\widehat{\rho}\widehat{\mathcal{A}}^\mu]_{\mathrm{ren}} = \mathrm{Tr}[\widehat{\rho}\,\partial_\mu \widehat{\mathcal{A}}^\mu]_{\mathrm{ren}} = 0.$$

CONSERVATION LAWS FROM THE FIELD THEORY VIEWPOINT

Noether's theorem states that the invariance of the action with respect to a group of global transformations corresponds to a **set of conserved currents**. The corresponding **conserved charge is the generator** of the starting transformation.

E. Noether. "Invariant Variation Problems". In: Gott. Nachr. 1918 (1918), pp.235–257
S. Weinberg, The Quantum Theory Of Fields Vol. 1,2 Cambridge University Press (1995)
Tinti, PhD Dissertation

$$\mathcal{A} = \int_\Omega \mathrm{d}^4 x \mathcal{L}(\phi^A, \partial\phi^A)$$

$$\begin{cases} x^\mu \longrightarrow \xi^\mu = x^\mu + \delta x^\mu = x^\mu + \varepsilon X^\mu \\ \phi^A(x) \longrightarrow \alpha^A(\xi) = \phi^A(x) + \delta\phi^A(x) = \phi^A(x) + \varepsilon \Phi^A \end{cases}$$

$$j^\mu = \left[\frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \phi^A\right)}(X\cdot\partial)\phi^A - \mathcal{L}X^\mu\right] - \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \phi^A\right)}\Phi^A$$

$$\partial \cdot j = 0$$

CANONICAL CURRENTS

For the **Dirac Lagrangian of noninteracting fields with spin 1/2**

Itzykson, Zuber, Quantum Field Theory, ISBN 9780486445687, 1980

$$\mathcal{L}_D(x) = \frac{i}{2}\bar{\psi}(x)\gamma^\mu \overleftrightarrow{\partial}_\mu \psi(x) - m\bar{\psi}(x)\psi(x)$$

the invariance of the action under continuous symmetries, via Noether's theorem, results in **canonical currents**

$$T_C^{\mu\nu} = \frac{i}{2}\bar{\psi}\gamma^\mu \overleftrightarrow{\partial}^\nu \psi - g^{\mu\nu}\mathcal{L}_D$$

$$J_C^{\lambda,\mu\nu} = x^\mu T_C^{\lambda\nu} - x^\nu T_C^{\lambda\mu} + S_C^{\lambda,\mu\nu} \qquad S_C^{\lambda,\mu\nu} = \frac{1}{4}\bar{\psi}\{\gamma^\lambda, \sigma^{\mu\nu}\}\psi = -\frac{1}{2}\epsilon^{\lambda\mu\nu\alpha}\bar{\psi}\gamma_\alpha\gamma_5\psi. \qquad \sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

which are conserved

$$\partial_\mu T_C^{\mu\nu} = 0 \qquad \partial_\lambda J_C^{\lambda,\mu\nu} = 0$$

The canonical EMT is not symmetric for free fields.
The spin tensor is not conserved.
The canonical global spin is not Lorentz covariant.

PSEUDO-GAUGE FREEDOM

The decomposition into spin and orbital parts is ambiguous. Having a pair of currents found, it is possible to generate a new pair using a **pseudo-gauge transformation (PGT)**

Hehl, Rep. Math. Phys. 9 (1) (1976) 55–82

$$T'^{\mu\nu} = T^{\mu\nu} + \frac{1}{2}\partial_\lambda G^{\lambda\mu\nu}$$

$$S'^{\lambda\mu\nu} = S^{\lambda\mu\nu} - \Phi^{\lambda\mu\nu} + \partial_\alpha Z^{\alpha\lambda\mu\nu}$$

$\leftarrow$

$$G^{\lambda\mu\nu} = \Phi^{\lambda\mu\nu} - \Phi^{\mu\lambda\nu} - \Phi^{\nu\lambda\mu}$$

$G^{\lambda\mu\nu}, \Phi^{\nu\lambda\mu}$ antisymmetric under $\lambda \leftrightarrow \mu$

$Z^{\alpha\lambda\mu\nu}$ antisymmetric under $\lambda \leftrightarrow \alpha$ and $\nu \leftrightarrow \mu$

Such that both pairs **fulfill the conservation equations**

$$\begin{cases} \partial_\mu T^{\mu\nu} = 0 \\ \partial_\lambda\left(S^{\lambda,\mu\nu} - x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}\right) = 0 \end{cases} \Rightarrow \begin{cases} \partial_\mu T^{\mu\nu} = 0 \\ \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu} \end{cases}$$

And **give the same generators of Poincaré algebra** through the spatial integrals of time components

$$\int_V \mathrm{d}^3\mathbf{x}\, T^{0\mu} = P^\mu \qquad \int_V \mathrm{d}^3\mathbf{x}\, J^{0\mu\nu} = \int_V \mathrm{d}^3\mathbf{x}\left(S^{0,\mu\nu} + x^\mu T^{0\nu} - x^\nu T^{0\mu}\right) = J^{\mu\nu}$$

Provided suitable boundary conditions are ensured on the superpotentials.

But give different global spins for different Phi's.

PSEUDO-GAUGE FREEDOM

The topic of PGTs was recently studied intensively in the context of nucleon spin …

E. Leader, C. Lorcé, Phys. Rep. 541 (3) (2014) 163–248,
M. Wakamatsu, Eur. Phys. J. A 55 (7) (2019) 123,

… and heavy-ion physics.

E. Speranza, N. Weickgenannt, Eur. Phys. J. A 57 (5) (2021) 155
N. Weickgenannt, D. Wagner, E. Speranza, Phys. Rev. D 105 (11) (2022) 116026,
S. Li, M.A. Stephanov, H.U. Yee, Phys. Rev. Lett. 127 (8) (2021) 082302, K. Fukushima, S. Pu, Phys. Lett. B 817 (2021) 136346,

Buzzegoli, Palermo, Phys.Rev.Lett. 133 (2024) 26, 262301
M. Buzzegoli, Phys. Rev. C 105 (4) (2022) 044907,

F. Becattini, L. Tinti, Phys. Rev. D 87 (2013) 2, 025029
F. Becattini, L. Tinti, Phys. Rev. D 84 (2011) 025013,
F. Becattini, W. Florkowski, E. Speranza, Phys. Lett. B 789 (2019) 419–425,

Daher, A. Das, W. Florkowski, R. Ryblewski, Phys.Rev.C 108 (2023) 2, 024902
Das, W. Florkowski, R. Ryblewski, R. Singh, Phys. Rev. D 103 (9) (2021) L091502,
Dey Bhadury, Florkowski, RR, Jaiswal Phys.Rev.D 110 (2024) 11, 116002
R. Singh, arXiv:2406.02127

Nevertheless, the consensus is not reached yet.

BELINFANTE-ROSENFELD PSEUDOGAUGE

In Einstein's general relativity, the Hilbert EMT is symmetric, as is obtained from the variation of the Lagrangian with respect to the metric tensor. Hence, it cannot coincide with the canonical EMT.

The **Belinfante-Rosenfeld (B) symmetrized EMT** is given by

F. Belinfante, Physica 6 (7) (1939) 887–898,
F. Belinfante,Physica 7 (5) (1940) 449–474,
L.J.H.C. Rosenfeld, Sur le tenseur d'impulsion-énergie, Palais des Académies,1940.

$$\Phi_B^{\lambda,\mu\nu} = S_C^{\lambda,\mu\nu},$$
$$Z_B^{\mu\nu\lambda\rho} = 0$$

Resulting currents are

$$T_B^{\mu\nu} = \frac{i}{4}\bar{\psi}\gamma^{(\mu}\overleftrightarrow{\partial}^{\nu)}\psi - g^{\mu\nu}\mathcal{L}_D,$$
$$S_B^{\lambda,\mu\nu} = 0.$$

This method gives a symmetric EMT and a vanishing spin tensor.

The spin contribution to the total angular momentum is hidden in the orbital part resulting from EMT.

No independent DOFs related to spin.

GLW PSEUDOGAUGE

One expects that the spin part of the angular momentum for non-interacting particles should be conserved. As a possible solution to this problem, **de Groot, van Leeuwen, and van Weert (GLW)** proposed to switch from the canonical forms to alternative forms, which may be achieved through the following PGT

S.R. De Groot, Relativistic Kinetic Theory. Principles and Applications, 1980.
Speranza, Weickgenannt, Eur. Phys. J. A 57 (5) (2021) 155

$$
\begin{aligned}
\Phi^{\lambda,\mu\nu}_{\rm GLW} &= \frac{i}{4m}\bar{\psi}\left(\sigma^{\lambda\mu}\overleftrightarrow{\partial}^{\nu} - \sigma^{\lambda\nu}\overleftrightarrow{\partial}^{\mu}\right)\psi, \\
Z^{\mu\nu,\lambda\rho}_{\rm GLW} &= 0.
\end{aligned}
$$

The resulting spin tensor is conserved.

It can be obtained within QKT approach.

HW PSEUDOGAUGE

An alternative version of the PGT that also leads to a conserved spin tensor was proposed by **Hilgevoord and Wouthuysen (HW)**. The idea is based on the fact that a Dirac spinor is also a solution of the Klein-Gordon equation. In this case, the superpotentials read

J. Hilgevoord, S. Wouthuysen, Nucl. Phys. 40 (1963) 1–12
J. Hilgevoord, E. De Kerf, Physica 31 (7) (1965) 1002–1016
Speranza, Weickgenannt, Eur. Phys. J. A 57 (5) (2021) 155

$$\Phi_{\mathrm{HW}}^{\lambda,\mu\nu} = \frac{i}{4m}\bar{\psi}\left(\sigma^{\lambda\mu}\overleftrightarrow{\partial}^{\nu} - \sigma^{\lambda\nu}\overleftrightarrow{\partial}^{\mu}\right)\psi - \frac{i}{4m}\bar{\psi}\left(g^{\lambda\mu}\sigma^{\nu\rho} - g^{\lambda\nu}\sigma^{\mu\rho}\right)\overleftrightarrow{\partial}_{\rho}\psi,$$

$$Z_{\mathrm{HW}}^{\mu\nu\lambda\rho} = -\frac{1}{8m}\bar{\psi}\left(\sigma^{\mu\nu}\sigma^{\lambda\rho} + \sigma^{\lambda\rho}\sigma^{\mu\nu}\right)\psi.$$

The resulting spin tensor is also conserved, and EMT is symmetric.
The global spin is the same as for **GLW** form.

The open question is whether there is a preferred PGT out of these?

SO(3) ALGEBRA OF SPIN OPERATORS

One can check if the spin operators can be considered as "good" angular momentum operators.

Dey, Florkowski, Jaiswal, RR, Physics Letters B 843 (2023) 137994

On general grounds, one expects that a set of angular momentum operators representing total, orbital, and spin parts satisfies the fundamental **SO(3) algebra for equal-time commutation relations (ETC)**:

$$
\begin{aligned}
\left[J^i(t), J^j(t)\right] &= i\epsilon^{ijk} J^k(t), \\
\left[L^i(t), L^j(t)\right] &= i\epsilon^{ijk} L^k(t), \\
\left[S^i(t), S^j(t)\right] &= i\epsilon^{ijk} S^k(t).
\end{aligned}
$$

A failure to correctly reproduce ETCs, may lead to fallacious or at least misleading conclusions.

E. Leader, C. Lorcé, [Phys. Rep. 541 (3) (2014) 163–248], Phys. Rep. 802 (2019) 23–24

It is known that SO(3) algebra of ETC for L and S holds in the case of the **canonical** spin tensor and **Belinfante** case. On the other hand, it is not obvious if it is fulfilled by the **GLW** and **HW** spin operators.

E. Leader, C. Lorcé, Phys. Rep. 541 (3) (2014) 163–248,

As the PGTs do not change the total "charges", ETC for J remain valid.

SO(3) ALGEBRA OF SPIN OPERATORS

We define the **total spin operator** as the integral

Dey, Florkowski, Jaiswal, RR, Physics Letters B 843 (2023) 137994

$$\frac{1}{2}S^k(t) = \frac{1}{2}\epsilon^{kij}\int d^3\mathbf{x}\, S^{0,ij}(t,\mathbf{x}) = \frac{1}{2}\epsilon^{kij}S^{ij}(t)$$

and verify the relation

$$\left[\frac{1}{2}S^i(t), \frac{1}{2}S^j(t)\right] = i\epsilon^{ijk}\frac{1}{2}S^k(t)$$

The angular momentum operators are functions of the field operators of the theory.
To check whether the commutation relation holds, one must know the fundamental commutation relations between fields and their conjugate momenta.

We consider a simple system of a **gas of relativistic fermions with spin 1/2 described by the Dirac equation.**
It is commonly considered a starting point for the construction of relativistic hydrodynamics of spin-polarized media.

Florkowski, Kumar, RR, Prog. Part. Nucl. Phys. 108 (2019) 103709

SO(3) ALGEBRA OF SPIN OPERATORS

Out of the PGTs considered, **only for the canonical one, the total spin operators fulfill the angular momentum SO(3) algebra**. This makes the canonical pseudogauge especially suitable for the treatment of the spin DOFs.

Dey, Florkowski, Jaiswal, RR, Physics Letters B 843 (2023) 137994

Clearly, a quantum mechanical treatment of spin observables within a PGT that breaks the SO(3) algebra is fundamentally inconsistent.

On the other hand, in any classical or semiclassical approaches to spin, where the operator character of the spin observables is neglected, other PGTs may be favored. Pseudogauges with the spin tensor conserved offer a possibility of making a direct link to thermodynamic and hydrodynamic approaches.

Florkowski, Kumar, R. R, Prog. Part. Nucl. Phys. 108 (2019) 103709

PAULI-LUBANSKI VECTOR

A natural question can be asked **if experimental procedures used to determine the spin polarization of particles can indicate themselves which PGT is the most appropriate to use.**

In our case, where we analyze a gas of relativistic particles, one considers a measurement of the spin polarization of particles with a given four-momentum.

For theoretical description of such processes one needs to consider the **phase space densities of the spin tensor**, which can be obtained, for instance, by a semiclassical expansion of the Wigner function

W. Florkowski, B. Friman, A. Jaiswal, R. Ryblewski, E. Speranza, Phys. Rev. D 97 (11) (2018) 116017

PAULI-LUBANSKI VECTOR

One can introduce the **phase-space density of the Pauli–Lubański (PL) four-vector** $\mathit{\Pi}_\mu = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}J^{\nu\alpha}p^\beta$

F.Becattini, V.Chandra, L.DelZanna, E.Grossi, Ann.Physics 338 (2013) 32–49
W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, Phys.Rev.C 97 (2018) 4, 041901
W. Florkowski, B. Friman, A. Jaiswal, R. R, E. Speranza, Phys. Rev. D 97 (11) (2018) 116017

$$E_p \frac{d\Delta\Pi_\mu(x,p)}{d^3p} = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\, \Delta\Sigma_\lambda(x)\, E_p \frac{dJ^{\lambda,\nu\alpha}(x,p)}{d^3p}\, \frac{p^\beta}{m}$$

The invariant angular momentum phase-space density of particles with given four-momentum may be replaced by the spin part since the two differ by terms proportional to four-momentum that do not contribute since they vanish if contracted with the Levi-Civita tensor.

For instance, using **GLW** PGT one finds

$$\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}E_p \frac{dS^{\lambda,\nu\alpha}(x,p)}{d^3p} = \frac{\hbar\cosh(\xi)}{(2\pi)^3}\,\frac{\sinh(\zeta)}{\zeta}\,e^{-p\cdot\beta}\,p^\lambda\tilde{\omega}_{\mu\beta}$$

The same result is obtained for the right-hand side if one uses the **canonical** spin tensor.

W. Florkowski, B. Friman, A. Jaiswal, R. Ryblewski, E. Speranza, Phys. Rev. D 97 (11) (2018) 116017

Hence, altough spin tensor depends on the PGT, the expression for the measured spin polarization is independent of it as the definition of the PL vector selects only one term that is common to all PGTs.

PSEUDO-GAUGE DEPENDENCE OF QUANTUM FLUCTUATIONS

Different PGTs may be interpreted as **different localizations of physical quantities**.

One can **study PGT dependence of quantum fluctuations of various components of the current densities**.

Our object of interest is **energy density operator** defined as the time-time (“00”) component of the EMT

We consider a subsystem Sa of the thermodynamic system SV composed of spin-1/2 particles with mass m described by the canonical ensemble characterized by the temperature T.
A. Das, W. Florkowski, R. R, and R. Singh, Acta Physica Polonica B Vol. 52 (2021),

The field operator has the standard form

$$\psi(t,\boldsymbol{x}) = \sum_r \int \frac{d^3k}{(2\pi)^3\sqrt{2\omega_{\boldsymbol{k}}}} \Big(U_r(\boldsymbol{k}) a_r(\boldsymbol{k}) e^{-ik\cdot x} + V_r(\boldsymbol{k}) b_r^\dagger(\boldsymbol{k}) e^{ik\cdot x} \Big)$$

Annihilation and creation operators for particles and antiparticles satisfy the canonical anti-commutation relations

PSEUDO-GAUGE DEPENDENCE OF QUANTUM FLUCTUATIONS

To perform thermal averaging, it is sufficient to know the **expectation values of the products of two and four creation and/or annihilation operators** (for both particles and antiparticles)

$$
\begin{aligned}
&\langle a_r^\dagger(\boldsymbol{k}) a_s(\boldsymbol{k}')\rangle = (2\pi)^3 \delta_{rs} \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}') f(\omega_{\boldsymbol{k}}),\\
&\langle a_r^\dagger(\boldsymbol{k}) a_s^\dagger(\boldsymbol{k}') a_{r'}(\boldsymbol{p}) a_{s'}(\boldsymbol{p}')\rangle\\
&= (2\pi)^6 \Big(\delta_{rs'}\delta_{r's} \delta^{(3)}(\boldsymbol{k}-\boldsymbol{p}')\ \delta^{(3)}(\boldsymbol{k}'-\boldsymbol{p})\\
&- \delta_{rr'}\delta_{ss'} \delta^{(3)}(\boldsymbol{k}-\boldsymbol{p})\ \delta^{(3)}(\boldsymbol{k}'-\boldsymbol{p}') \Big) f(\omega_{\boldsymbol{k}}) f(\omega_{\boldsymbol{k}'}).
\end{aligned}
$$

We define an **operator that represents the energy density** of a subsystem Sa placed at the origin

$$
\hat{T}_a^{00} = \frac{1}{(a\sqrt{\pi})^3} \int d^3\boldsymbol{x}\ \hat{T}^{00}(x)\ \exp\left(-\frac{\boldsymbol{x}^2}{a^2}\right)
$$

PSEUDO-GAUGE DEPENDENCE OF QUANTUM FLUCTUATIONS

To determine the **fluctuation of the energy density** of the subsystem Sa, we consider the variance

$$\sigma^2(a,m,T) = \langle :\hat{T}^{00}_a::\hat{T}^{00}_a:\rangle - \langle :\hat{T}^{00}_a:\rangle^2$$

and the normalized standard deviation

$$\sigma_n(a,m,T) = \frac{\left(\langle :\hat{T}^{00}_a::\hat{T}^{00}_a:\rangle - \langle :\hat{T}^{00}_a:\rangle^2\right)^{1/2}}{\langle :\hat{T}^{00}_a:\rangle}$$

Using the "tt" component of the canonical EMT we find the thermal expectation value of the operator for the subsystem Sa

$$\langle :\hat{T}^{00}_{\mathrm{Can},a}:\rangle = 4\int \frac{d^3k}{(2\pi)^3}\,\omega_{\boldsymbol{k}}\; f(\omega_{\boldsymbol{k}}) \equiv \varepsilon_{\mathrm{Can}}(T)$$

Agrees with the elementary kinetic theory considerations

Using the thermal expectation values of the products of two and four creation and/or annihilation operators we find the energy density fluctuation for the EMT in various PGTs ("tt" components agree for Bel and Can)

PSEUDO-GAUGE DEPENDENCE OF QUANTUM FLUCTUATIONS

Energy densities agree for all PGTs $\varepsilon_{\rm Can}(T) = \varepsilon_{\rm BR}(T) = \varepsilon_{\rm GLW}(T) = \varepsilon_{\rm HW}(T)$

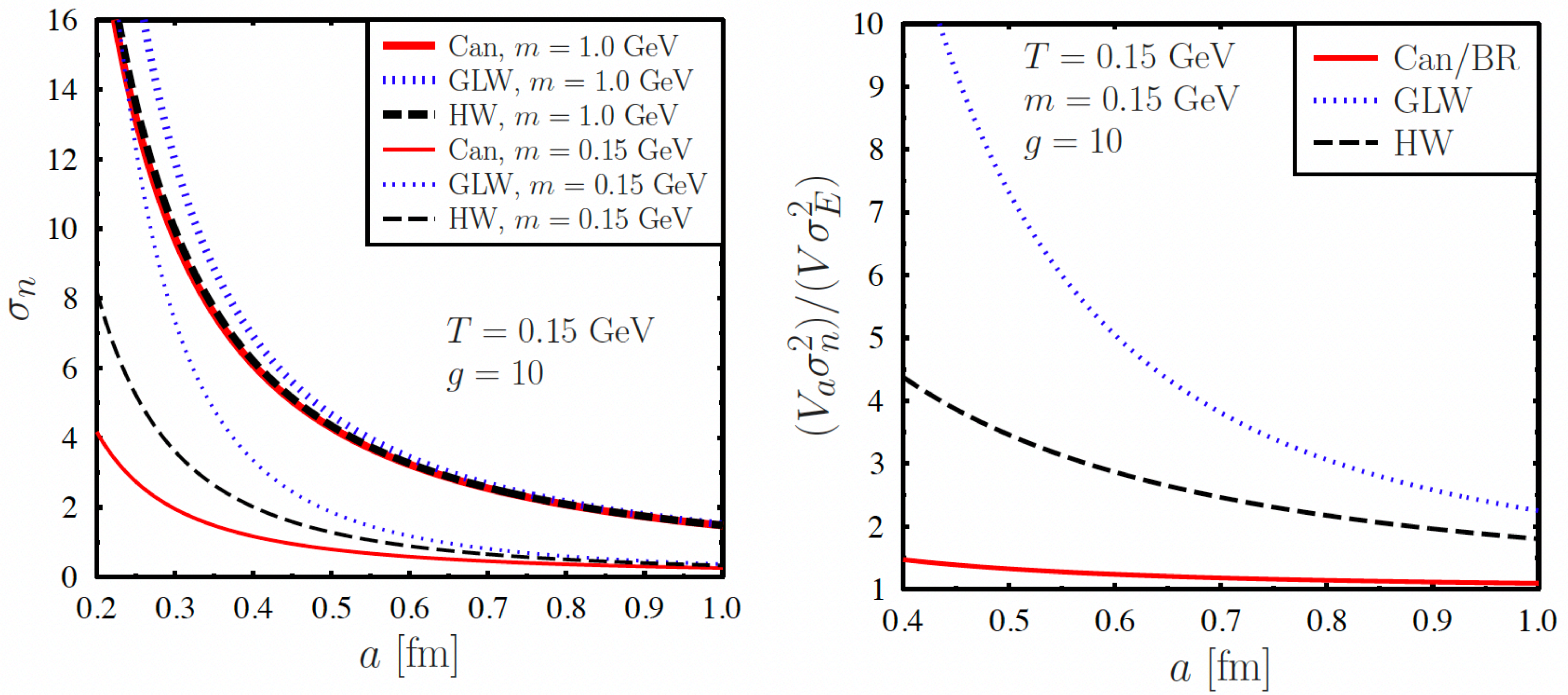
On the other hand, **the fluctuations are in general different for different PGTs**.

In the **thermodynamic limit**, i.e., the limiting case of very large system size quantum fluctuation as obtained here reduce to the known expression of **statistical fluctuation from classical statistical mechanics**

$$\begin{aligned}\sigma^2_{\rm Can} &= \frac{4\,g}{(2\pi)^{3/2}a^3}\int \frac{d^3k}{(2\pi)^3}\,\omega^2_{\boldsymbol{k}}\, f(\omega_{\boldsymbol{k}})(1-f(\omega_{\boldsymbol{k}}))\\ &= \sigma^2_{\rm GLW} = \sigma^2_{\rm HW}\end{aligned}$$

PSEUDO-GAUGE DEPENDENCE OF QUANTUM FLUCTUATIONS

Otherwise, fluctuations depend on the form of the EMT used in the calculations, although for sufficiently large subsystems, the results obtained in different PGTs converge and agree with the canonical-ensemble formula known from statistical physics.



On the practical side, the results of our calculations can be used to determine a scale of coarse graining for which the choice of the pseudo-gauge becomes irrelevant, which may be useful, in particular, in the context of hydrodynamic modeling of high-energy collisions.

CONSTRAINTS ON PSEUDO-GAUGE TRANSFORMATIONS

The superpotential can be decomposed wrt the flow four-vector

Biswas, Daher, Das, Florkowski, RR, Phys. Rev. D 108 (1) (2023) 014024.

$$\Phi^{\lambda\mu\nu} = u^\lambda \mathcal{S}^{\mu\nu} + \left(u^\nu \Delta^{\lambda\mu} - u^\mu \Delta^{\lambda\nu}\right) I + \left(u^\nu I_{(s)}^{\langle\lambda\mu\rangle} - u^\mu I_{(s)}^{\langle\lambda\nu\rangle}\right) + \left(u^\nu I_{(a)}^{\lambda\mu} - u^\mu I_{(a)}^{\lambda\nu}\right) + \Phi^{\langle\lambda\rangle\langle\mu\rangle\langle\nu\rangle},$$

where

$$\mathcal{S}^{\mu\nu} = u_\alpha \Phi^{\alpha\mu\nu}, \quad I = \frac{1}{3} I^\mu_{\ \mu}, \quad I^{\lambda\nu} = -u_\rho \Phi^{\langle\lambda\rangle\rho\nu}, \quad \Phi^{\langle\lambda\rangle\langle\mu\rangle\langle\nu\rangle} = \Delta^\lambda_{\ \alpha} \Delta^\mu_{\ \beta} \Delta^\nu_{\ \gamma} \Phi^{\alpha\beta\gamma}$$

$$H^{\alpha\beta\langle\gamma\rangle\delta\dots} \equiv \Delta^\gamma_{\ \rho} H^{\alpha\beta\rho\delta\dots} \qquad H^{\langle\alpha\beta\rangle} \equiv \Delta^{\mu\nu}_{\alpha\beta} H^{\alpha\beta}$$

$$\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu, \qquad \Delta^{\mu\nu}_{\alpha\beta} \equiv \frac{1}{2}\left(\Delta^\mu_{\ \alpha}\Delta^\nu_{\ \beta} + \Delta^\mu_{\ \beta}\Delta^\nu_{\ \alpha} - \frac{2}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}\right).$$

The number of independent DOFs on the two sides is the same,

$$24(\Phi) = 6(\mathcal{S}) + 1(I) + 5(I_{(s)}) + 3(I_{(a)}) + 9(\Phi)$$

CONSTRAINTS ON PSEUDO-GAUGE TRANSFORMATIONS

The EMT (before and after PGT) can be decomposed as follows

Biswas, Daher, Das, Florkowski, RR, Phys. Rev. D 108 (1) (2023) 014024.

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu - \mathcal{P}\Delta^{\mu\nu} + 2\mathcal{Q}^{(\mu}u^{\nu)} + \mathcal{T}^{\mu\nu} + 2\mathcal{H}^{[\mu}u^{\nu]} + \mathcal{F}^{\mu\nu}$$

where

$$\mathcal{E} = u_\mu u_\nu T^{\mu\nu} \qquad \mathcal{P} = -\frac{1}{3}\Delta_{\mu\nu}T^{\mu\nu}$$

$$\mathcal{Q}^\mu = \frac{1}{2}\left(\Delta^\mu_{\ \alpha}u_\beta + \Delta^\mu_{\ \beta}u_\alpha\right)T^{\alpha\beta} \qquad \mathcal{T}^{\mu\nu} = T^{\langle\mu\nu\rangle}$$

$$\mathcal{H}^\mu = \frac{1}{2}\left(\Delta^\mu_{\ \alpha}u_\beta - \Delta^\mu_{\ \beta}u_\alpha\right)T^{\alpha\beta} \qquad \mathcal{F}^{\mu\nu} = T^{\langle[\mu\nu]\rangle}$$

$$H^{\langle[\alpha\beta]\rangle} \equiv \frac{1}{2}\left(\Delta^\mu_{\ \alpha}\Delta^\nu_{\ \beta} - \Delta^\mu_{\ \beta}\Delta^\nu_{\ \alpha}\right)H^{\alpha\beta}$$

CONSTRAINTS ON PSEUDO-GAUGE TRANSFORMATIONS

The PGT induces changes of the coefficients in the decomposition of the EMT

Biswas, Daher, Das, Florkowski, RR, Phys. Rev. D 108 (1) (2023) 014024.

energy density
$$\mathcal{E}' = \mathcal{E} + I\theta - \nabla\cdot F - \theta_w + I_{(s)}^{\langle\alpha\beta\rangle}\sigma_{\alpha\beta} - I_{(a)}^{\alpha\beta}\Omega_{\alpha\beta},$$

isotropic pressure
$$\mathcal{P}' = \mathcal{P} - \frac{1}{3}\Big(2I\theta + 3DI + F\cdot a + \theta_w - I_{(s)}^{\langle\alpha\beta\rangle}\sigma_{\alpha\beta} + I_{(a)}^{\alpha\beta}\Omega_{\alpha\beta} + \partial_\alpha\Phi^{\langle\beta\rangle\ \ \alpha}_{\ \ \ \langle\beta\rangle}\Big),$$

heat flow
$$\begin{aligned}\mathcal{Q}'^{\mu} = {} & \mathcal{Q}^{\mu} + \frac{1}{2}[-Ia^{\mu} + DF^{\mu} + \theta F^{\mu} - 2D_F u^{\mu} - \nabla^{\mu}I + (F\cdot a)u^{\mu}] + u_\nu DI_{(s)}^{\langle\mu\nu\rangle} \\ & - \frac{1}{2}u^{\mu}\Big(I_{(s)}^{\langle\alpha\beta\rangle}\sigma_{\alpha\beta} - I_{(a)}^{\alpha\beta}\Omega_{\alpha\beta}\Big) \\ & - \frac{1}{2}\partial_\lambda\Big(I_{(s)}^{\langle\mu\lambda\rangle} + I_{(a)}^{\mu\lambda}\Big) - \frac{1}{2}\Big(\Phi^{\langle\mu\langle\nu\rangle\lambda\rangle} + \Phi^{\langle\nu\rangle\langle\mu\rangle\lambda}\Big)\partial_\lambda u_\nu + \frac{1}{2}\epsilon^{\langle\mu\rangle}_{\ \ \alpha\beta\gamma}\big(W^{\alpha\beta}\partial^{\gamma}u^{\nu} - u^{\alpha}\partial^{\beta}W^{\gamma}\big)\end{aligned}$$

shear-stress tensor
$$\begin{aligned}\mathcal{T}'^{\mu\nu} = {} & \mathcal{T}^{\mu\nu} - I\sigma^{\mu\nu} + F^{\langle\mu}a^{\nu\rangle} + DI_{(s)}^{\langle\mu\nu\rangle} + I_{(s)}^{\langle\mu\nu\rangle}\theta - I_{(s)}^{\langle\mu\lambda\rangle}\nabla_\lambda u^{\nu} \\ & + 2u^{(\mu}I_{(s)}^{\langle\nu)\lambda\rangle}a_\lambda + I_{(a)}^{\lambda\langle\mu}\nabla_\lambda u^{\nu\rangle} + \epsilon^{\alpha\beta\lambda\langle\mu}W_{\alpha\beta}u_\gamma\partial_\lambda u^{\nu\rangle} + \partial_\lambda\Phi^{\langle\mu\nu\rangle\lambda},\end{aligned}$$

SYMMETRIC EMT AND GRADIENT EXPANSION

A PGT **from symmetric to symmetric EMT** requires

Speranza, Weickgenannt, Eur. Phys. J. A 57 (5) (2021) 155

$$\partial_\lambda \Phi^{\lambda,\mu\nu} = T^{[\nu\mu]} \qquad T^{[\nu\mu]} = 0$$

An example of this PGT is the **de Groot, van Leeuwen, and van Weert** to **Belinfante** PGT

Florkowski, Kumar, RR, Prog. Part. Nucl. Phys. 108 (2019) 103709.

$$\left\{S^{\lambda,\mu\nu}_{\rm GLW}, T^{\mu\nu}_{\rm GLW}\right\} \Rightarrow \left\{S^{\lambda,\mu\nu}_{\rm B}, T^{\mu\nu}_{\rm B}\right\}$$

SYMMETRIC EMT AND GRADIENT EXPANSION

In the context of conventional relativistic hydrodynamics, consider **PGT constructed out of the hydrodynamic fields** such as temperature, chemical potential, and the flow vector

Restricting to the case where the **final EMT does not contain gradients of the order higher than one, the superpotential cannot be built from gradients of the hydrodynamic fields**

As a consequence of this strong assumption, we are left with the form

$$\Phi^{\lambda\mu\nu} = \left(u^\nu \Delta^{\lambda\mu} - u^\mu \Delta^{\lambda\nu}\right) I$$

SYMMETRIC EMT AND GRADIENT EXPANSION

The STS condition requires

$$\partial_\lambda \Phi^{\lambda\mu\nu} = (u^\nu \partial^\mu - u^\mu \partial^\nu) I + I(\partial^\mu u^\nu - \partial^\nu u^\mu) = 0$$

This represents six equations to be satified by one scalar function.

Consequently, such PGT does not exist in relativistic hydrodynamics unless the system (hydrodynamic expansion) possesses additional symmetries.

A possible solution to this issue involves introducing additional DOFs into the theory, as in spin hydrodynamics where the four-vector k and the pseudo-vector ω are used

Florkowski, Kumar, RR, Prog. Part. Nucl. Phys. 108 (2019) 103709.

STS condition is satisfied for the **one-dimensional Bjorken expansion**

Drogosz, Florkowski, Hontarenko, RR, Phys.Lett.B 861 (2025) 139244

$$u^\nu \nabla^\mu I - u^\mu \nabla^\nu I + I(u^\mu a^\nu - u^\nu a^\mu) + 2I\omega^{\mu\nu} = 0$$

RESIDUAL PGT FOR BJORKEN'S EXPANSION

Residual PGT becomes

$$\mathcal{E}' = \mathcal{E} + I\theta, \quad \mathcal{P}' = \mathcal{P} - DI - \frac{2}{3}I\theta, \quad \mathcal{Q}'^{\mu} = \mathcal{Q}^{\mu}, \quad \mathcal{T}'^{\mu\nu} = \mathcal{T}^{\mu\nu} - I\sigma^{\mu\nu}$$

One can easily check that the presence of the function $I(\tau)$ does not affect the EOMs

For simplicity, lets consider the case with **vanishing baryon chemical potential**

Change of the energy density, may interpreted as a change of the effective temperature (assuming EOS is given)

$$\mathcal{E}_{\mathrm{eq}}(T') = \mathcal{E}_{\mathrm{eq}}(T) + I(T)\,\theta \qquad \mathcal{P}_{\mathrm{eq}}(T') + \Pi'(T') = \mathcal{P}_{\mathrm{eq}}(T) + \Pi(T) - DI - \frac{2}{3}I\theta$$

$$\hookrightarrow T' \qquad \hookrightarrow \Pi'$$

$$\Pi(T) = -\zeta(T)\,\theta, \quad \Pi'(T') = -\zeta'(T')\,\theta, \quad \mathcal{T}^{\mu\nu} = 2\eta(T)\,\sigma^{\mu\nu}, \quad \mathcal{T}'^{\mu\nu} = 2\eta'(T')\,\sigma^{\mu\nu}.$$

RESIDUAL PGT FOR BJORKEN'S EXPANSION

For Bjorken expansion, the EOMs are reduced to a single equation of the form

$$\frac{d\,\mathcal{E}_{\rm eq}(T)}{d\tau}+\frac{\mathcal{E}_{\rm eq}(T)+\mathcal{P}_{\rm eq}(T)}{\tau}-\left[\frac{4}{3}\eta(T)+\zeta(T)\right]\theta^2=0$$

$$\frac{d\,\mathcal{E}_{\rm eq}(T')}{d\tau}+\frac{\mathcal{E}_{\rm eq}(T')+\mathcal{P}_{\rm eq}(T')}{\tau}-\left[\frac{4}{3}\eta'(T')+\zeta'(T')\right]\theta^2=0$$

$$\Rightarrow$$

$$\eta'(T')=\eta(T)-\frac{I(T)}{2}$$

$$\frac{4}{3}\left[\eta'(T)-\eta(T)\right]+\zeta'(T)-\zeta(T)=0$$

Both shear and bulk viscosities may change under PGT, however, their combination should remain unchanged

RESIDUAL PGT FOR BOOST-INVARIANT EXPANSION

For a conformal system trace of the EMT vanishes implying that both bulk viscosities vanish

All thermodynamic functions scale with temperature

$$\mathcal{E}_{\rm eq}(T) = aT^4, \quad \mathcal{P}_{\rm eq}(T) = \frac{a}{3}T^4, \quad \mathcal{S}_{\rm eq}(T) = \frac{4a}{3}T^3, \quad \eta(T) = \frac{4\bar{\eta}a}{3}T^3 \quad I(T) = bT^3$$

$$\eta'(T') = \eta(T) - \frac{I(T)}{2} \qquad \Rightarrow \qquad T' = T\left(1 - \frac{3b}{8a\bar{\eta}}\right)^{1/3}$$

$$\mathcal{E}_{\rm eq}(T') = \mathcal{E}_{\rm eq}(T) + I(T)\,\theta \qquad \Rightarrow \qquad T'^4 = T^4 + \frac{bT^3}{2\tau}$$

RESIDUAL PGT FOR BOOST-INVARIANT EXPANSION

For a nonconformal system one can derive differential equation for $I(\tau)$

$$\frac{dI}{d\tau} + \frac{I}{3\tau} + \mathcal{A}_{\rm eq}(T') - \mathcal{A}_{\rm eq}(T) = \frac{4}{3\tau}\left[\eta(T') - \eta(T)\right] + \frac{1}{\tau}\left[\zeta(T') - \zeta(T)\right]$$

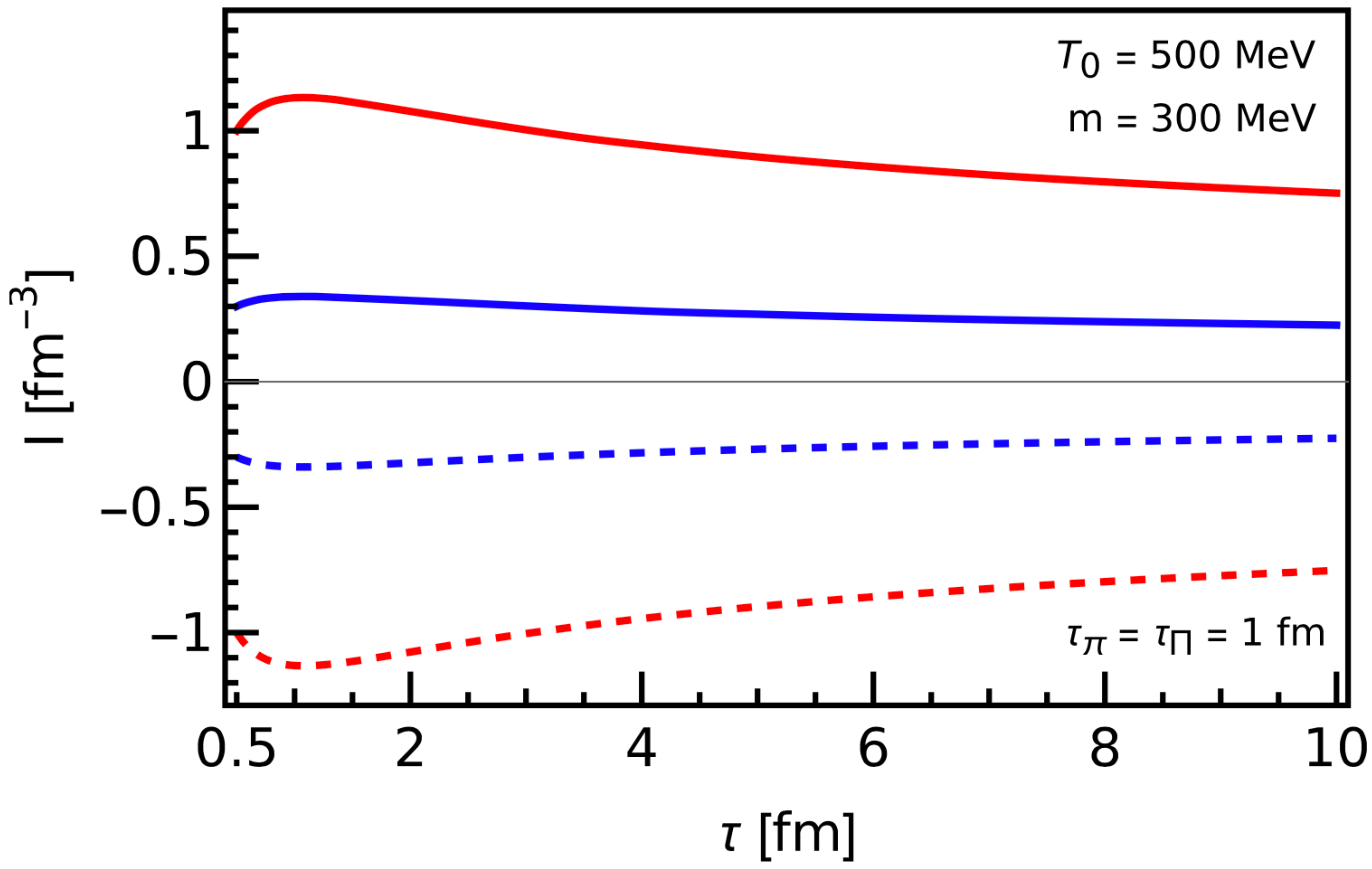
where the trace anomaly is introduced

$$\mathcal{A}_{\rm eq}(T) = \mathcal{P}_{\rm eq}(T) - \left(\frac{1}{3}\right)\mathcal{E}_{\rm eq}(T)$$

We solve these equations numerically using the EOS describing relativistic gas of classical massive particles,

Denicol, Florkowski, RR, Strickland, Phys. Rev. C 90 (4) (2014) 044905.

RESIDUAL PGT FOR BOOST-INVARIANT EXPANSION



SUMMARY

PGTs represent an uresolved puzzle of the spin studies in heavy-ion physics

PGTs are especially important when the densities of currents are assumed to have physical meaning, which is the case for conventional relativistic hydrodynamics and spin hydrodynamics

Various studies of implications of PGTs suggest both dependence and independence on their particular choice.

Studies of SO(3) algebra of ETCs as well as fluctuations of energy density show that the canonical choice is prefered

The residual PGT is possible in the simple case of Bjorken expansion, however, it is too restrictive for general cases.