Quasinormal modes and hydrodynamics of nonthermal fixed points

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Funded by the European Union

European Research Council
Established by the European Commission

2307.07545 with Mazeliauskas and Preis [PRL], **2502.01622** with De Lescluze and **2504.18754** with Berges, Denicol and Preis

Motivation

Motivation

Theory challenge: studying thermalization dynamics of closed quantum systems

2005.12299 with Berges, Mazeliauskas, Venugopalan

Highly relevant for → ultrarelativistic nuclear collisions at RHIC and LHC

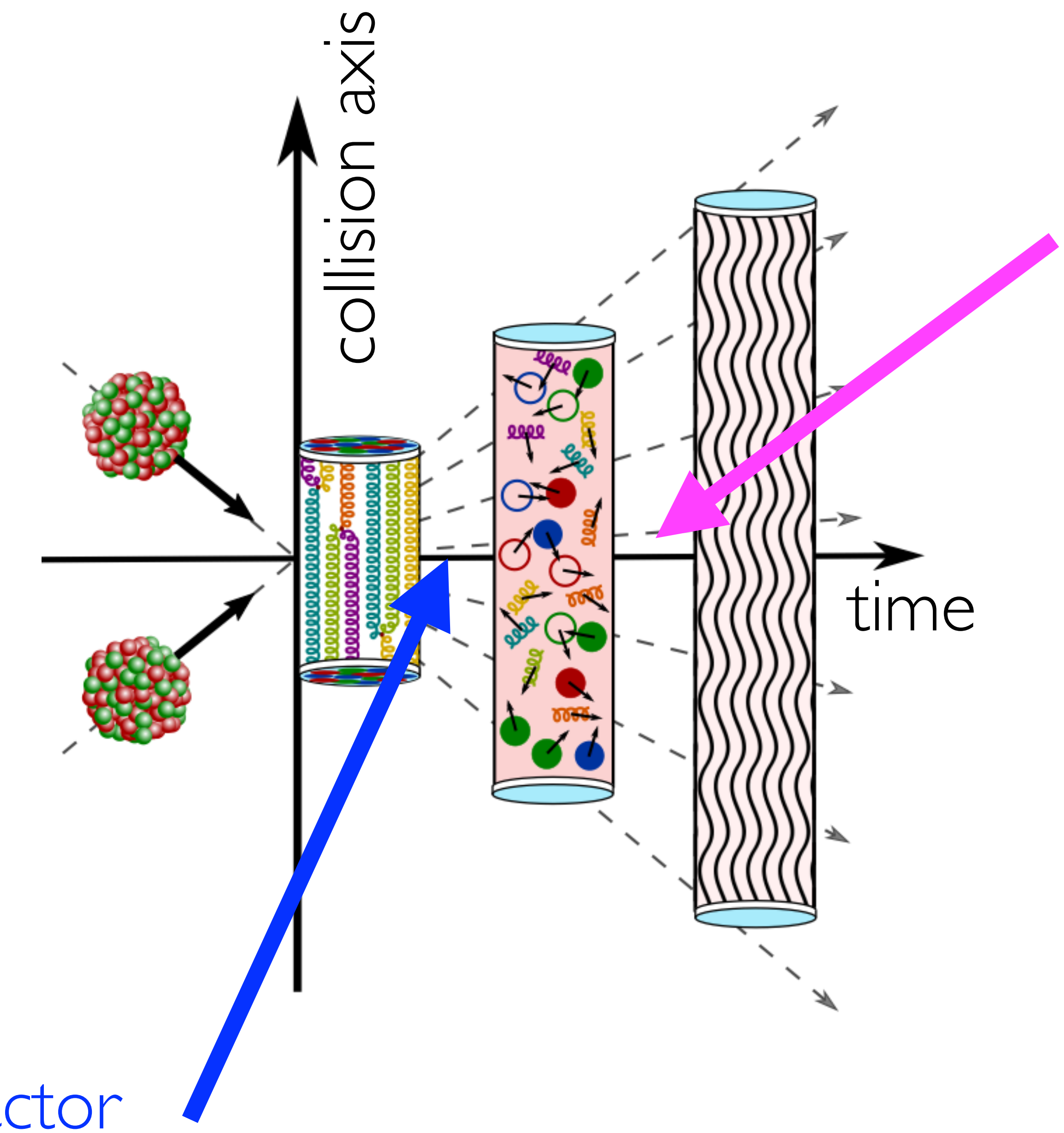
Highly relevant for → ultracold atoms experiments

The theory targets specific parametric regimes (strong or weak coupling)

Each regime has its own language → possibility of novel connections and perspectives

Attractors view on QCD thermalization

2005.12299 with Berges, Mazeliauskas and Venugopalan



hydrodynamic attractor

this week's talks by Blaizot, Spaliński

nonthermal attractor
(aka nonthermal fixed point)

FONDAZIONE BRUNO KESSLER

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Far from equilibrium ingredient: nonthermal fixed point(s)

Nonthermal attractor

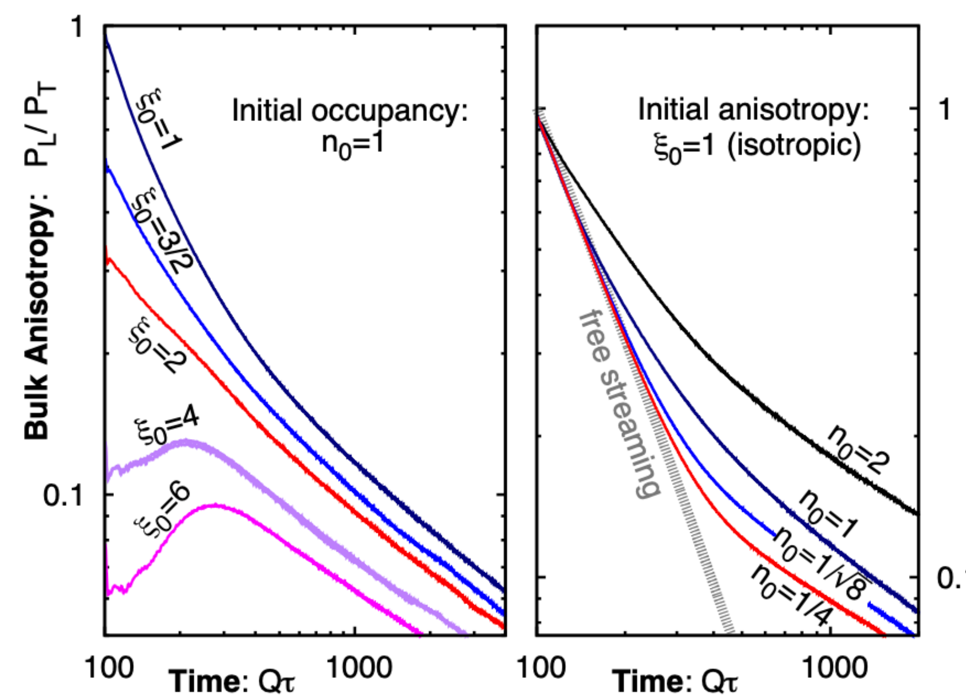
1303.5650 and **1311.3005** by Berges, Boguslavski, Schlichting and Venugopalan

After a collision at weak coupling: overoccupied gluons (strong classical fields):

$$f(p_T, p_z, \tau_0) = \frac{n_0}{8\pi\alpha_S}\Theta\left(Q - \sqrt{p_T^2 + (\xi_0 p_z)^2}\right) \quad \text{with} \quad \alpha_S = 10^{-5}$$

Ab initio simulations revealed significant insensitivity to plasma instabilities and free streaming, consistent with elastic scattering dominance among hard particles in the kinetic theory regime $Q\tau \sim log^2(1/\alpha_S) = O(10^2)$: stage I of the "bottom up" thermalization

hep-ph/0009237 by Baier, Mueller, Schiff and Son



Overoccupation is expected then to drive the system to a self similar regime

$$f(p_T, p_z, \tau) = (Q\tau)^\alpha f_S\left((Q\tau)^\beta p_T, (Q\tau)^\gamma p_z\right) \quad \text{with } \alpha = -\frac{2}{3}, \beta = 0, \gamma = \frac{1}{3}$$

Nonthermal attractor

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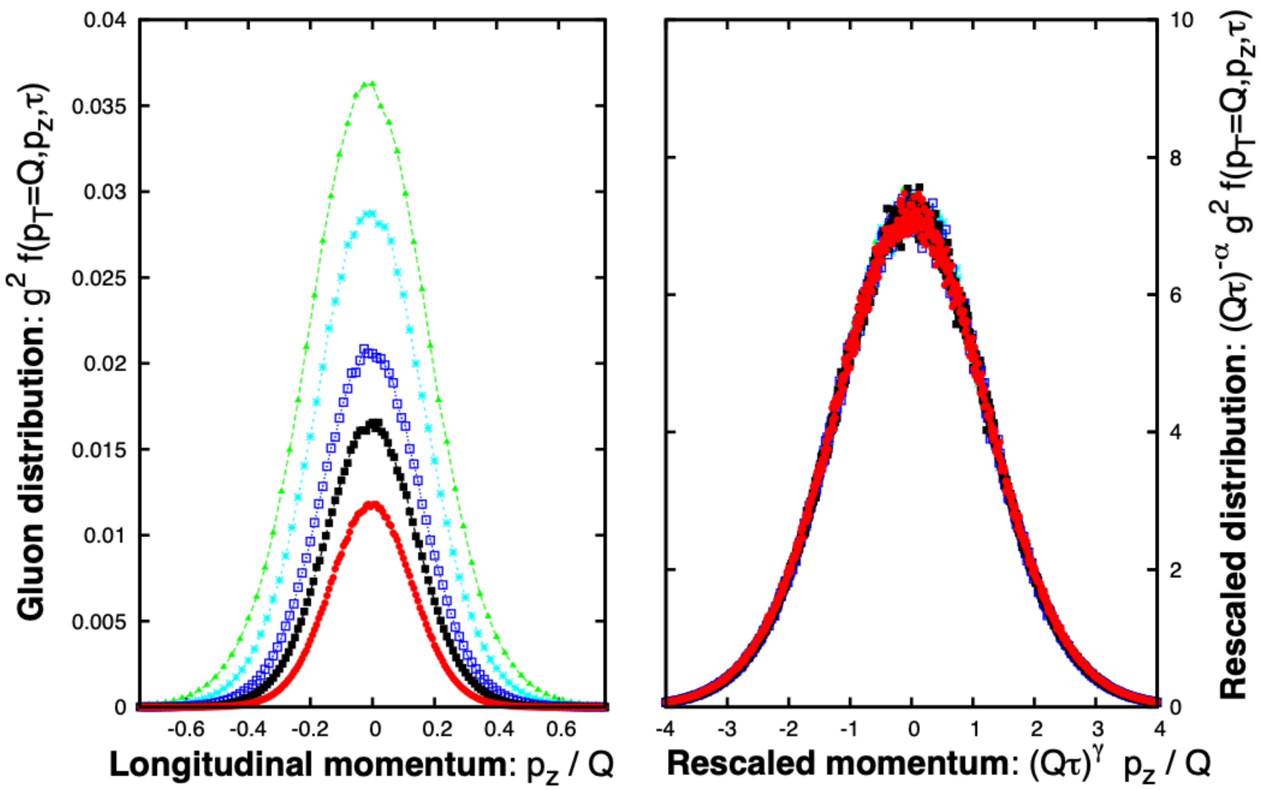
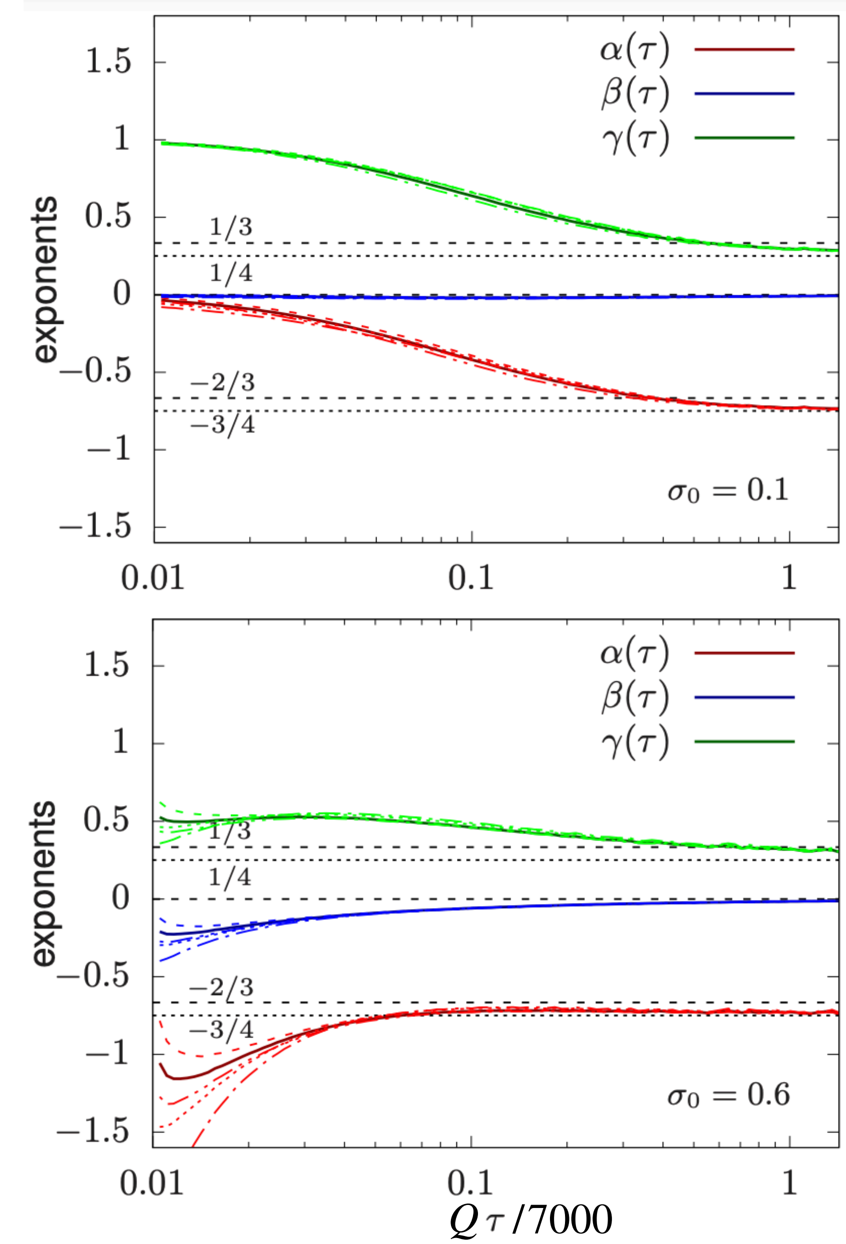


Classical statistical simulations of SU(2) gluons

1303.5650 and 1311.3005
by Berges, Boguslavski, Schlichting and Venugopalan



$g = 10^{-3}$ EKT with 2 massless quarks

1810.10554
by Berges and Mazeliauskas

Simplification in this talk: isotropy

No expansion: $ds^2 = -dt^2 + d\vec{x}^2$ and time dependence on t

Distribution function: $f(t, p \equiv |\vec{p}\,|)$

Nonthermal fixed point has only two scaling exponents:

$$f(t,p) \approx A(t) \times f_s\left(B(t)p\right) \quad \text{with} \quad A(t) = \left(t/t_{\mathrm{ref}}\right)^{\alpha} \quad \text{and} \quad B(t) = \left(t/t_{\mathrm{ref}}\right)^{\beta}$$

Isotropic nonthermal fixed points

e.g. **1810.08143** by Schmied, Mikheev and Gasenzer
or **2005.12299** with Berges, Mazeliauskas, Venugopalan

Ingredients:

weak coupling + overoccupied initial states $f(t=0,p) \gg f_{eq}(p)\Big|_{\text{same energy density}}$

Outcome:

simulations + experiments show prolonged self-similar evolution in time

$$f(t,p) \approx A(t) \times f_s\left(B(t)p\right) \quad \text{with} \quad A(t) = \left(t/t_{\text{ref}}\right)^{\alpha} \quad \text{and} \quad B(t) = \left(t/t_{\text{ref}}\right)^{\beta}$$

Observation: the choice of the origin of time

$$f(t,p) \approx A(t) \times f_s\left(B(t)p\right) \quad \text{with} \quad A(t) = \left(t/t_{\mathrm{ref}}\right)^{\alpha} \quad \text{and} \quad B(t) = \left(t/t_{\mathrm{ref}}\right)^{\beta}$$

Special points for scaling phenomena → $t = 0$

Special points for scaling phenomena → $t \to \infty$

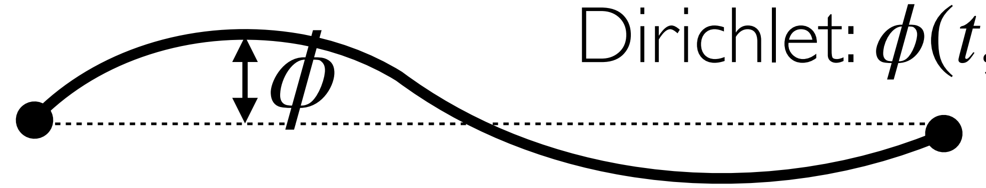
However, t used above has an origin chosen in a rather arbitrary way.

Near-equilibrium ingredient: quasinormal modes

Normal modes

Wave equation in a cavity $\quad \begin{aligned} \Box_{\text{cavity}}\, \phi &= 0 \\ \phi\Big|_{\text{bdries}} &= 0 \end{aligned} \quad \longrightarrow$ spectrum of normal modes e.g.

Dirichlet: $\phi(t, L) = 0$



Dirichlet: $\phi(t,0) = 0$

$$\Big\downarrow \; -\partial_t^2 \phi + \partial_x^2 \phi = 0$$

$$\phi_{\text{NM}} \sim e^{-i\omega_{\text{NM}} t} \sin\left(n\pi x / L\right) \quad \text{with} \quad \omega_{\text{NM}} = \pm\, n\pi/L$$

If excited, normal modes just oscillate (possibly interfering with each other)

Quasinormal modes e.g. **0905.2975** by Berti, Cardoso and Starinets

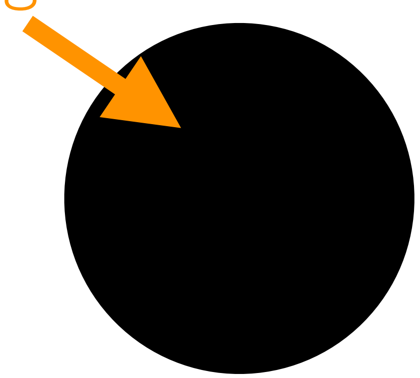
Option I: the equation breaks time symmetry explicitly, e.g.

$$\partial_t \phi - D\partial_x^2 \phi = 0 \quad \text{on a line} \quad \phi_{\mathrm{QNM}} \sim e^{-i\omega_{\mathrm{QNM}} t} e^{ikx} \quad \text{with} \quad \omega_{\mathrm{QNM}} = -\, iDk^2 \in \mathbb{C}$$

Option II: the boundary condition makes the problem non-Hermitian

ϕ purely ingoing at the horizon

black hole classically only absorbs : $\Box_{\text{black hole}}\, \phi = 0 \;+$



$\longrightarrow$ a tower of $\omega_{\mathrm{QNM}} \in \mathbb{C}$

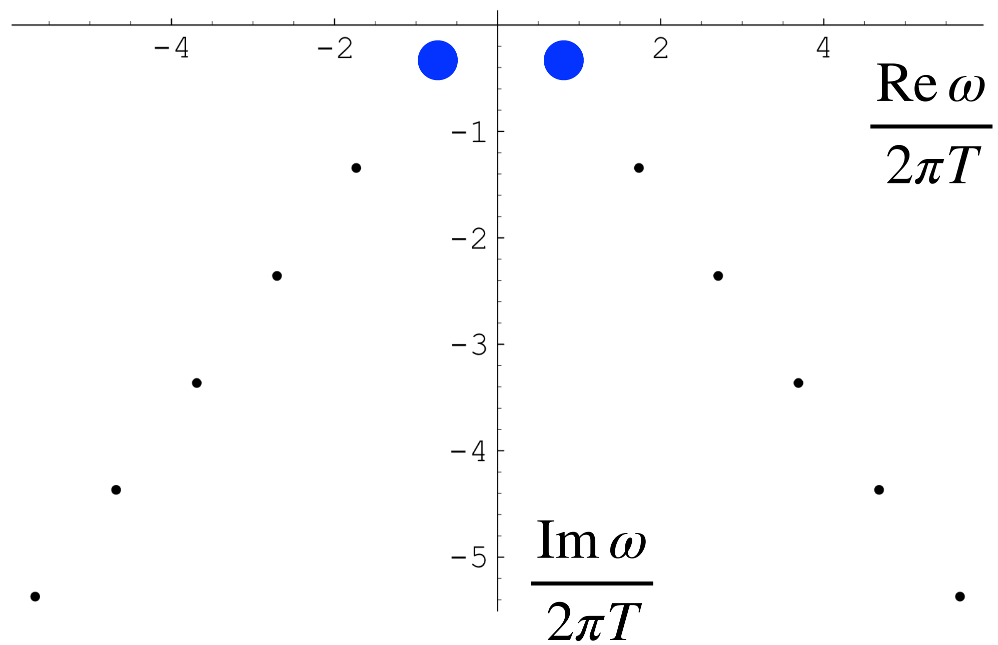
If excited, quasinormal modes decay in time (and often also oscillate)

Holographic quasinormal modes (QNMs)

Horowitz and Hubeny hep-th/9909056; Kovtun and Starinets hep-th/0506184

Strongly-coupled QFTs relax via dual QNMs: $\delta g_{ab} \sim \delta\langle T_{\mu\nu}\rangle \sim e^{-i\,\omega\,t+i\,\vec{q}\cdot\vec{x}}$

plot for $q = 2\pi T$



Consequences for thermalization → lots of short-lived excitations

→ a few long-lived hydrodynamic modes

Ingredient III: (relativistic) hydrodynamics

Relativistic hydrodynamics e.g. Florkowski, Heller & Spaliński 1707.02282

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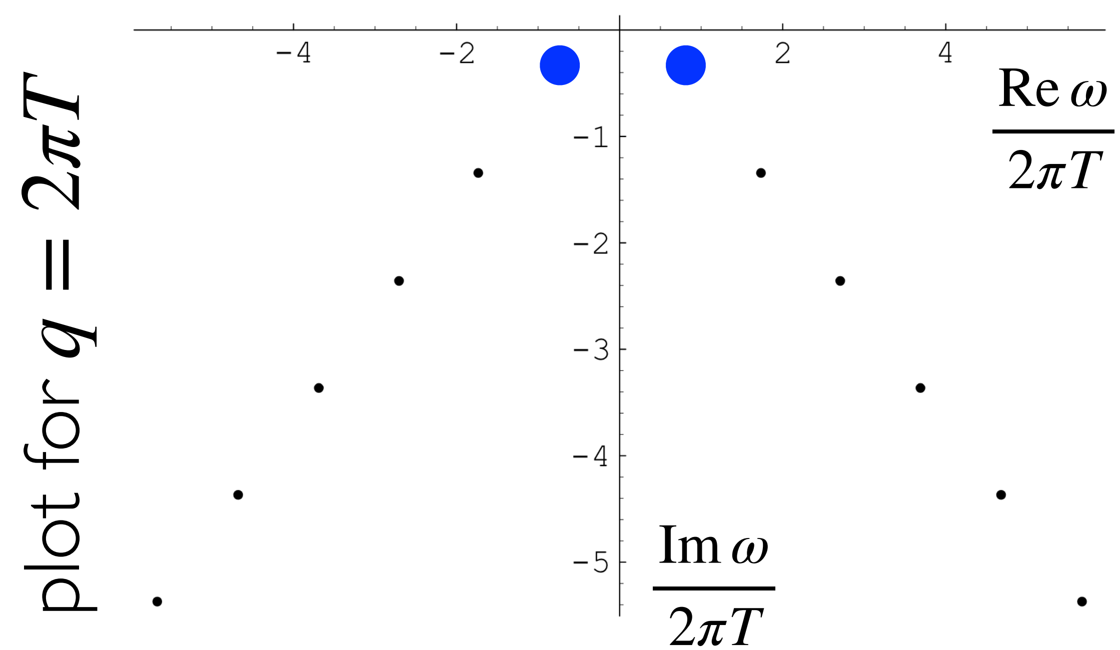


Relativistic Navier-Stokes equations dictate the properties of sound propagation:

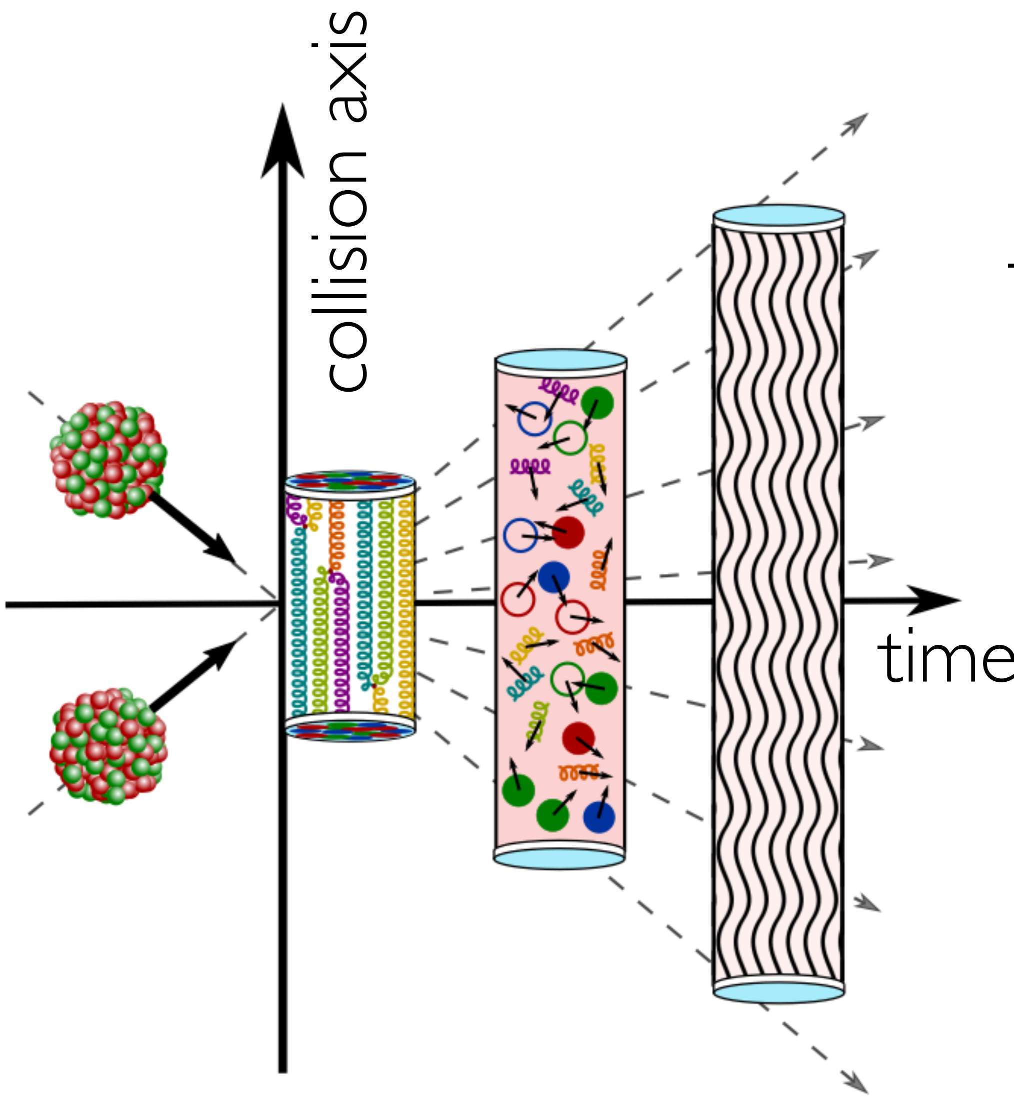
$$\omega = \pm\, c_s\, q - i\,\frac{4}{3T}\,\frac{\eta}{s}\,q^2 + \ldots$$

(shear) viscosity describes the dominant dissipative effect

Are there sound waves propagating on top of nonthermal fixed points?

Hydro and transient QNMs in action

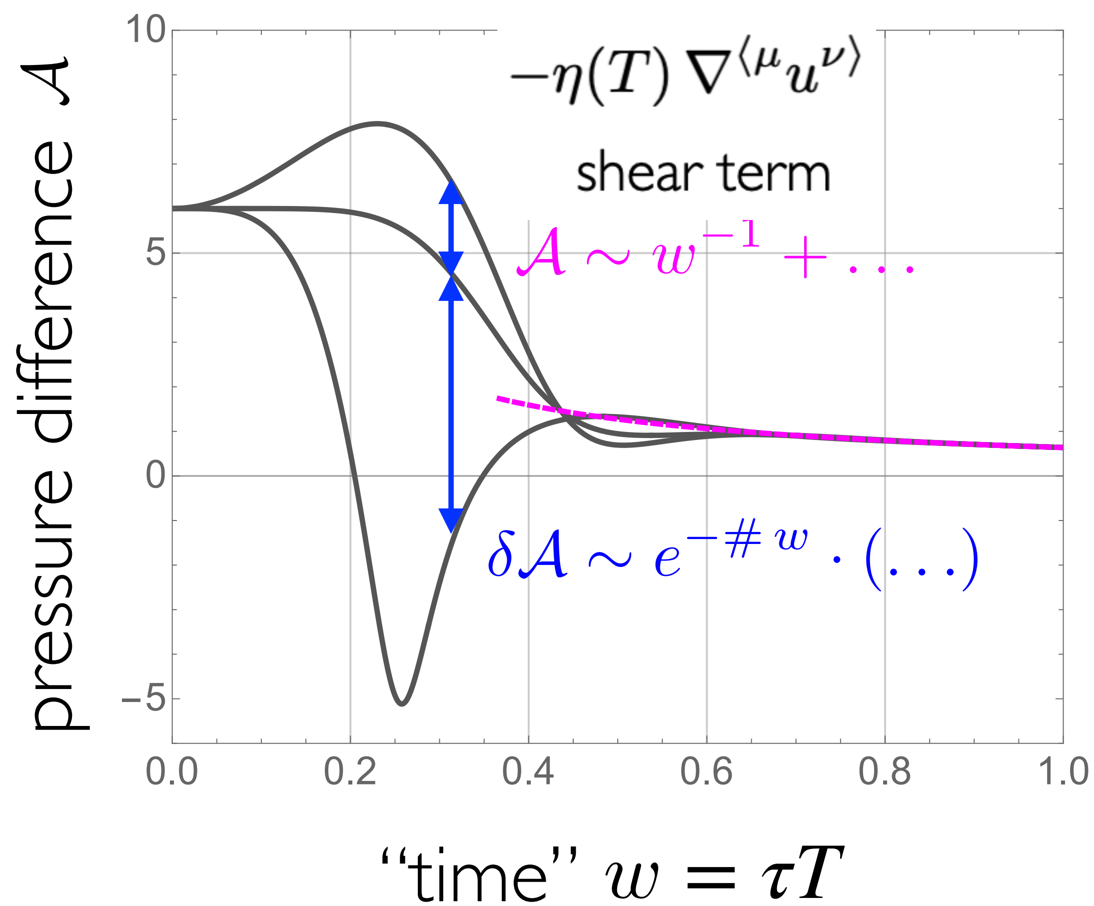
heavy-ion collisions
at RHIC and LHC



2005.12299
with Berges, Mazeliauskas & Venugopalan

behaviour in
of theoretical models
(here: holographic boost-invariant flow)

$$\frac{\pi^T{}_T - \pi^L{}_L}{\mathcal{P}(T)} = \mathcal{A}$$



1103.3452 with Janik & Witaszczyk

Setup

Kinetic theory setup

We will be using (relativistic) isotropic kinetic theory

$$\partial_t f(t,p) = C[f](t,p)$$

Example of $C[f]$ is the Fokker-Planck QCD collision kernel

see e.g. 1402.5049 by Blaizot, Wu & Yan

$$C_{FP}[f](t,p) \sim \mathcal{L}\left[I_a \frac{1}{p^2}\partial_p(p^2\partial_p f) + I_b \frac{1}{p^2}\partial_p(p^2 f(1+f))\right]$$

$$\cancel{\mathcal{L} = \int \frac{dq}{q} = \log \frac{\sqrt{\langle p^2 \rangle}}{m_D},} \qquad \mathscr{L} = \text{const},$$

where $$I_a = N_c \int \frac{d^3p}{(2\pi)^3} f(1+f),$$

$$I_b = 2N_c \int \frac{d^3p}{(2\pi)^3} \frac{f}{p}.$$

This collision kernel leads to a direct cascade in the UV

Result 1:
extending nonthermal fixed points to earlier times

Prescaling

1807.07514 by Schmied, Mikheev and Gasenzer
1810.10554 by Berges and Mazeliauskas

Scaling with slowly varying in time exponents

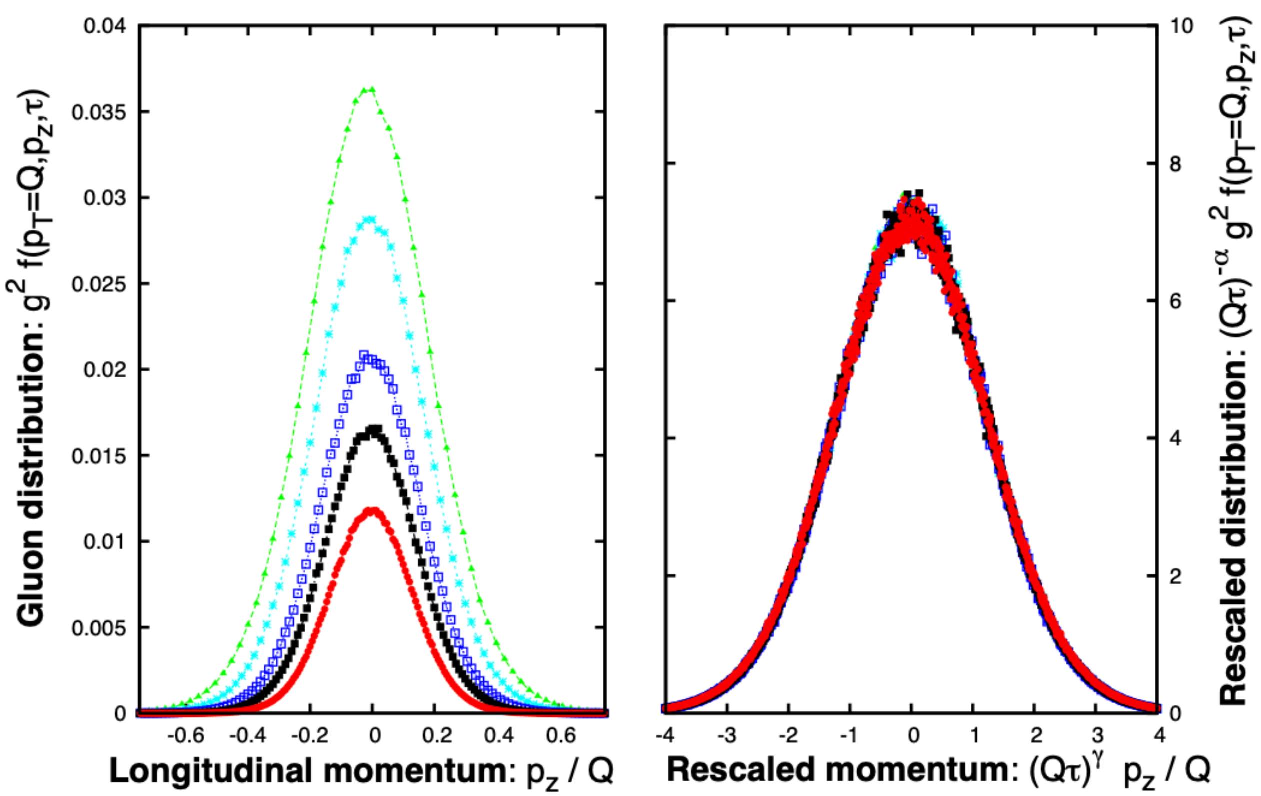
$$f(t,p) \approx A(t) \times f_s\left(B(t)p\right) \quad \text{with} \quad A(t) = \left(t/t_{\text{ref}}\right)^{\alpha(t)} \text{and} \quad B(t) = \left(t/t_{\text{ref}}\right)^{\beta(t)}$$

might precede the exact scaling with α, β constant

Important for theory and experiment, since makes scaling visible much earlier

Nonthermal attractor

$$f(p_T, p_z, \tau) = (Q\tau)^{\alpha} f_S\Big((Q\tau)^{\beta} p_T, (Q\tau)^{\gamma} p_z\Big)$$ with $\alpha = -\frac{2}{3}, \beta = 0, \gamma = \frac{1}{3}$



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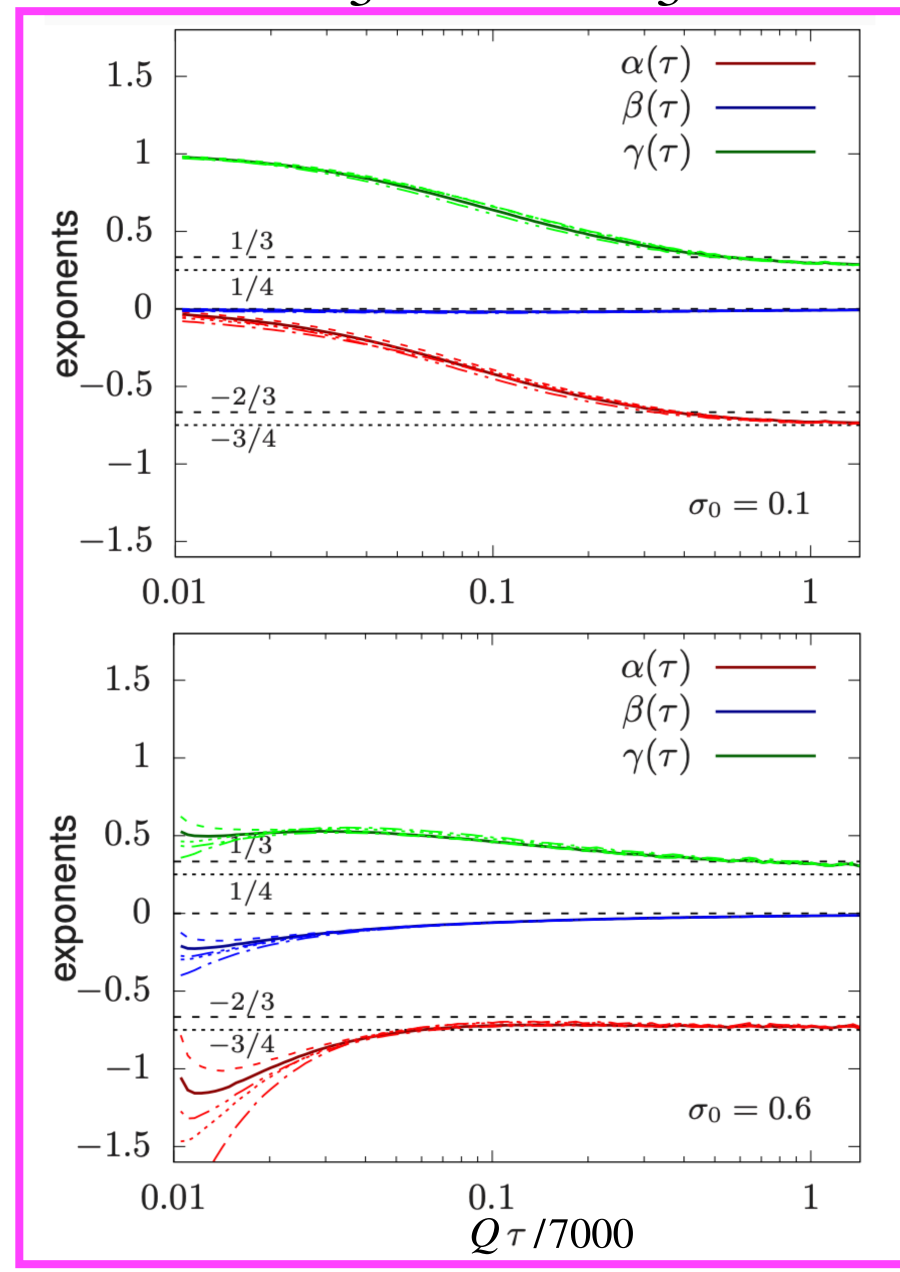

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The origin of (pre)scaling 2307.07545 with Mazeliauskas and Preis

Overoccupation simplifies $C[f(t,p)]$ as the highest power of f dominates

$$C[f(t,p)] \sim \int dP_1 \dots \, (\dots) f^{\kappa} \text{ with } \kappa_{2\leftrightarrow 2} = 3 \text{ and } \kappa_{1\leftrightarrow 2} = 2$$

This might make it possible to factor out t-dependence for $A(t) f_s\left(B(t)\, p\right)$:

$$C[f(t,p)] \equiv A(t)^{\mu_\alpha} B(t)^{\mu_\beta}\, C[f_s(\bar{p} \equiv B(t)p)]$$

Overoccupation leads to most of the energy (or particle number) in the relevant range of momenta, but the simplified kernel alters conservation properties

Imposing conservation of energy (or particle number) leads then to

$$A(t) \sim B(t)^{\sigma} \quad \text{with } \sigma = 4 \text{ (or } \sigma = 3)$$

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$\kappa = 3$

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Prescaling as dimensionality reduction

2307.07545 with Mazeliauskas and Preis

All in all, overoccupation + conservation **factorizes** then the Boltzmann equation

$$\frac{B(t)^{1-1/\beta_\infty}}{\partial_t B(t)} = \frac{1}{D_1} = \frac{[\sigma + \bar{\mathbf{p}} \cdot \partial_{\bar{\mathbf{p}}}]\, f_S(\bar{\mathbf{p}})}{\mathcal{C}[f_S](\bar{\mathbf{p}})} \quad \text{where} \quad \frac{1}{\beta_\infty} \equiv (1-\mu_\alpha)\sigma - \mu_\beta$$

time dependence (closed form eom for $B(t)$)

momentum dependence

Vast reduction of complexity in a sense of dimensionality reduction

Physics of prescaling 2307.07545 with Mazeliauskas and Preis

The solutions of

$$\boxed{\frac{B(t)^{1-1/\beta_\infty}}{\partial_t B(t)} = \frac{1}{D_1}} = \frac{[\sigma + \bar{\mathbf{p}}\cdot\partial_{\bar{\mathbf{p}}}]\, f_S(\bar{\mathbf{p}})}{\mathcal{C}[f_S](\bar{\mathbf{p}})} \quad \text{where} \quad \frac{1}{\beta_\infty} \equiv (1-\mu_\alpha)\sigma - \mu_\beta$$

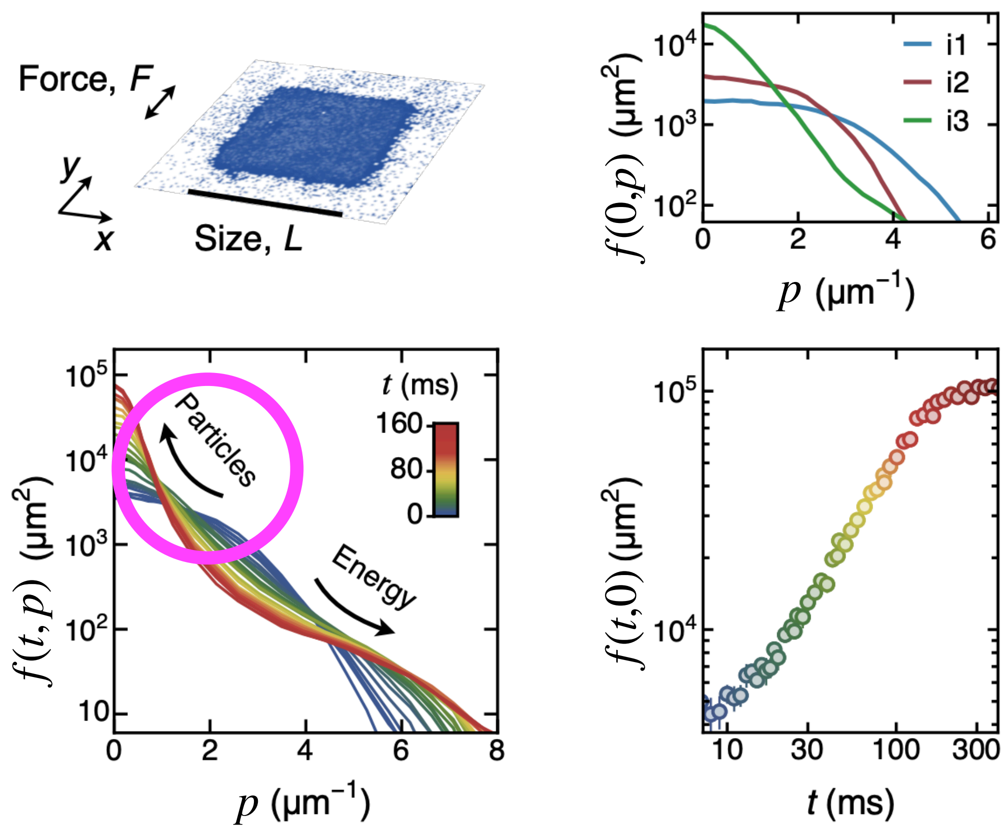
predict the prescaling as

$$B(t) = \left(\frac{t-t_*}{t_{\text{ref}}}\right)^{\beta_\infty} \approx \left(\frac{t}{t_{\text{ref}}}\right)^{\beta_\infty}\left(1 - \beta_\infty \frac{t_*}{t} + \ldots\right)$$

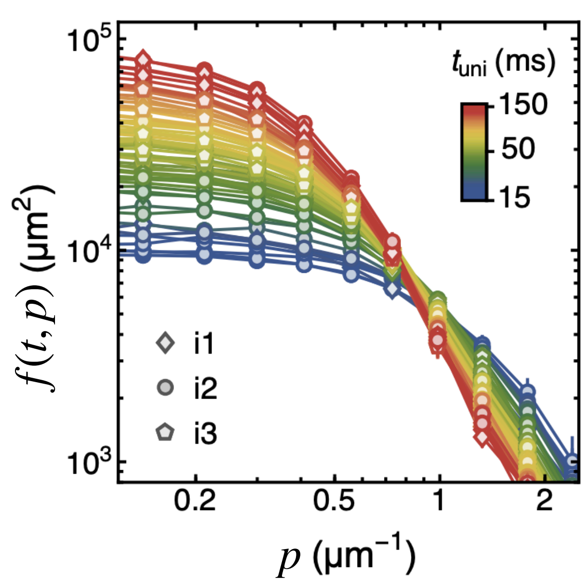
Prescaling is then just a scaling with an initial-condition dependent offset of time

Experimental confirmation

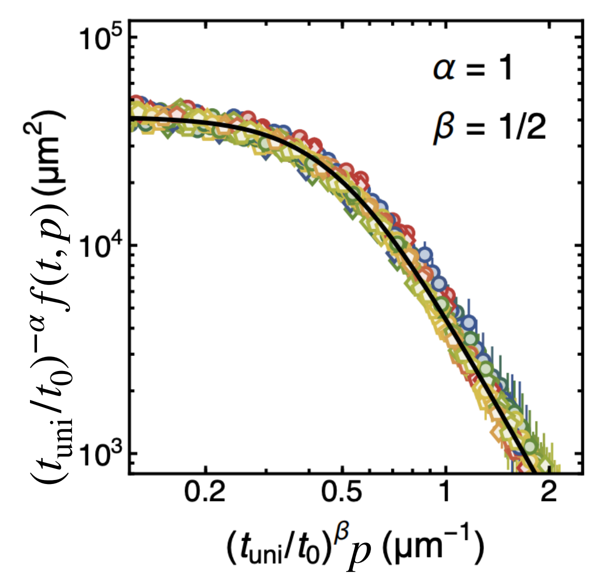
2312.09248 by Gazo, Karailiev, Satoor, Eigen, Gałka and Hadzibabic



We start with a quasi-pure interacting 2D condensate of 7×10^4 atoms of ^{39}K in the lowest hyperfine state, confined in a square box trap of size $L = 50\,\mu$m [40]. The interactions in the gas, characterized by the scattering length a, are tuneable via the magnetic Feshbach resonance at 402.7 G [41]. To prepare our far-from-equilibrium initial states, we temporarily turn off the interactions ($a \to 0$) and shake the gas with a spatially uniform oscillating force F (see Fig. 1A). This destroys the condensate and, as previously studied in 3D [42, 43], results in an isotropic highly nonthermal f distribution. After preparing one of the three different initial states i1–i3 shown in Fig. 1A, we stop the shaking, reinstate the interactions ($a \to 30\,a_0$, where a_0 is the Bohr radius), and let the gas relax. The states i1–i3 do not have a defined temperature, but $E = \int \varepsilon(k)\,\mathrm{d}k$, where $\varepsilon = 2\pi\hbar^2k^3n_k/(2m)$ and m is the atom mass, gives the total energy. We get $E/k_B = 4.1(3)$ mK, 2.2(3) mK, and 1.0(3) mK, for i1–i3 respectively; in all cases E is sufficiently low for a condensate to emerge during relaxation [44].



$$t_{\rm uni} \equiv t - t_*$$



Some perspectives from prescaling

Nonthermal fixed points = overoccupation + dimensionality reduction

(Pre)scaling is then a consequence of the equations of motion

Prescaling exhausts the dimensionally reduced ansatz $f(t,p) \approx A(t) \times f_s\left(B(t)p\right)$

Prescaling even at this level does not seem to have much to do with hydro

Result II:
transient QNMs of nonthermal fixed points

Key idea behind 2502.01622 with De Lescluze

There is sense in which nonthermal fixed points are static: $f_s(\bar{p})$

As a result, the notion of quasinormal mode approach makes sense

Quasinormal modes of nonthermal fixed points

2502.01622 with De Lescluze

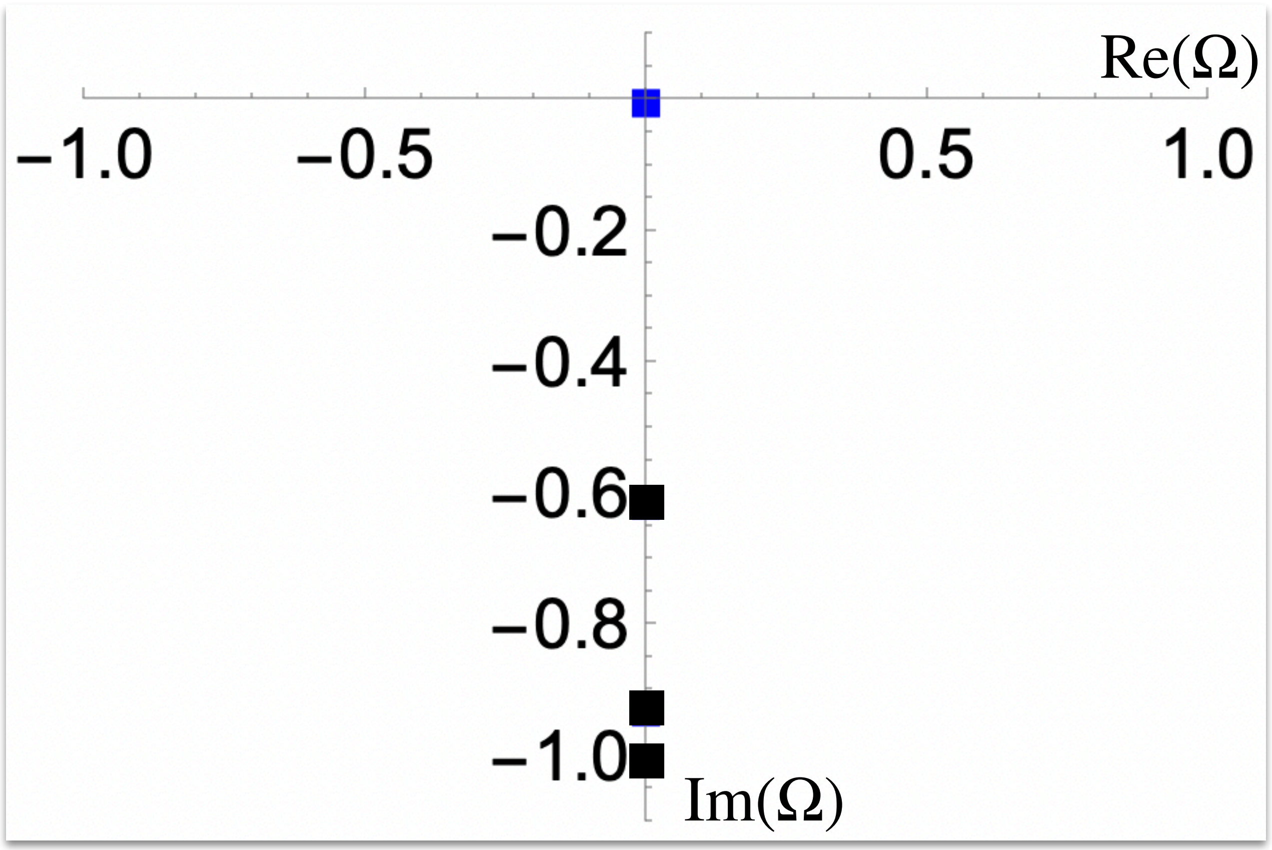
On a nonthermal fixed point:

$$f(t,p) \approx \left(t_{\mathrm{uni}}/t_0\right)^{\alpha} f_{\mathrm{scaling}}\left[\left(t_{\mathrm{uni}}/t_0\right)^{\beta} p\right]$$

with $t_{\mathrm{uni}} \equiv t - t_*$

directly before that:

$$\left(t_{\mathrm{uni}}/t_0\right)^{-\alpha} f(t,p) \approx f_{\mathrm{scaling}}\left[\left(t_{\mathrm{uni}}/t_0\right)^{\beta} p\right] + \sum_{\Omega} \left(t_{\mathrm{uni}}/t_0\right)^{-i\Omega} \delta f_{\Omega}\left[\left(t_{\mathrm{uni}}/t_0\right)^{\beta} p\right]$$



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Horowitz and Hubeny hep-th/9909056; Kovtun and Starinets hep-th/0506184

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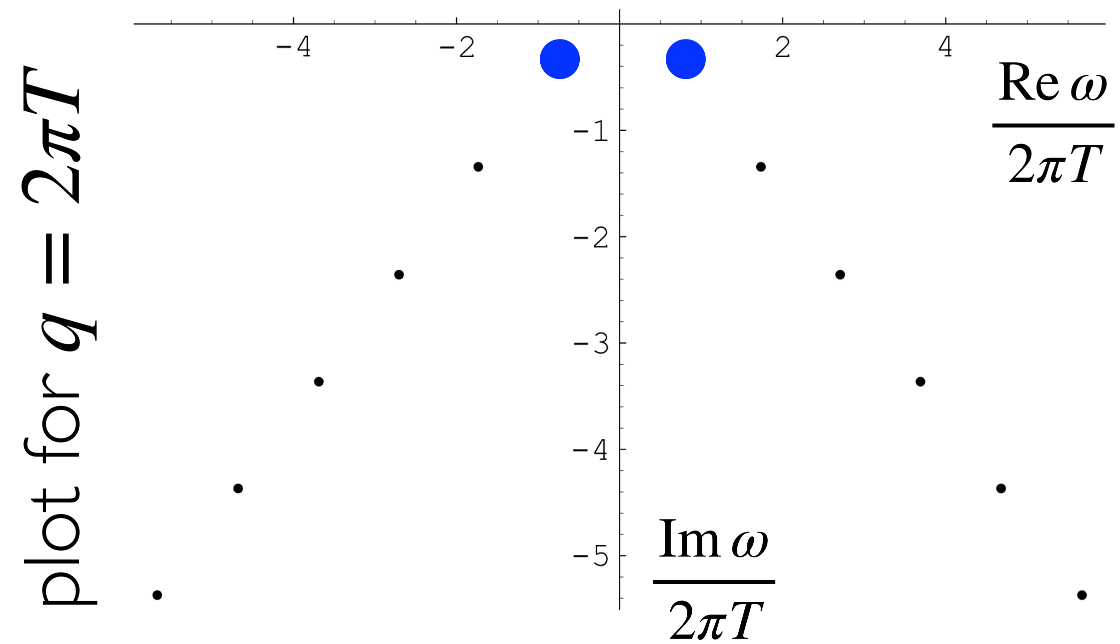

Consequences for thermalization → lots of short-lived excitations

→ a few long-lived hydrodynamic modes

How about nonthermal fixed points?

Quasinormal modes of nonthermal fixed points

2502.01622 with De Lescluze

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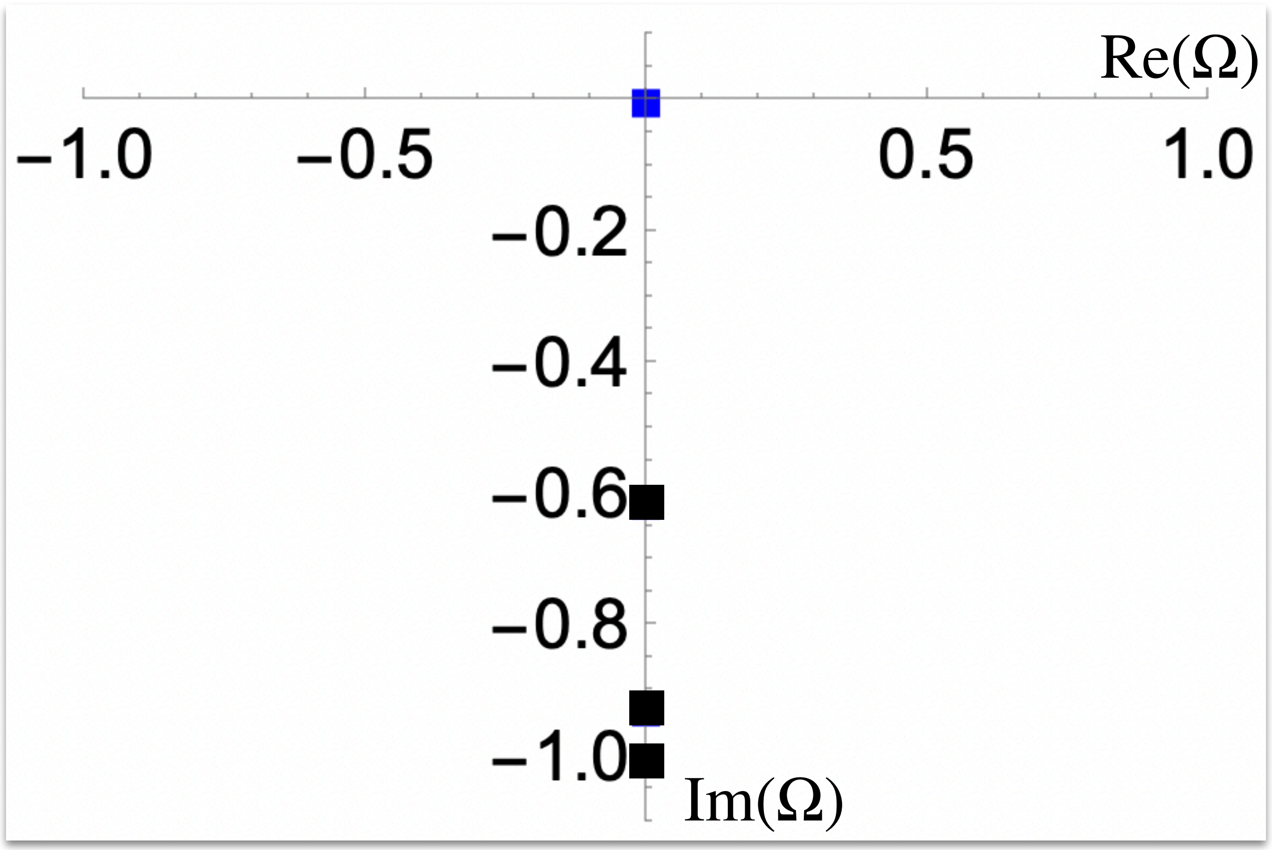

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with $t_{\rm uni} \equiv t - t_*$

directly before that:

$$\left(t_{\rm uni}/t_0\right)^{-\alpha} f(t,p) \approx f_{\rm scaling}\left[\left(t_{\rm uni}/t_0\right)^{\beta} p\right]$$

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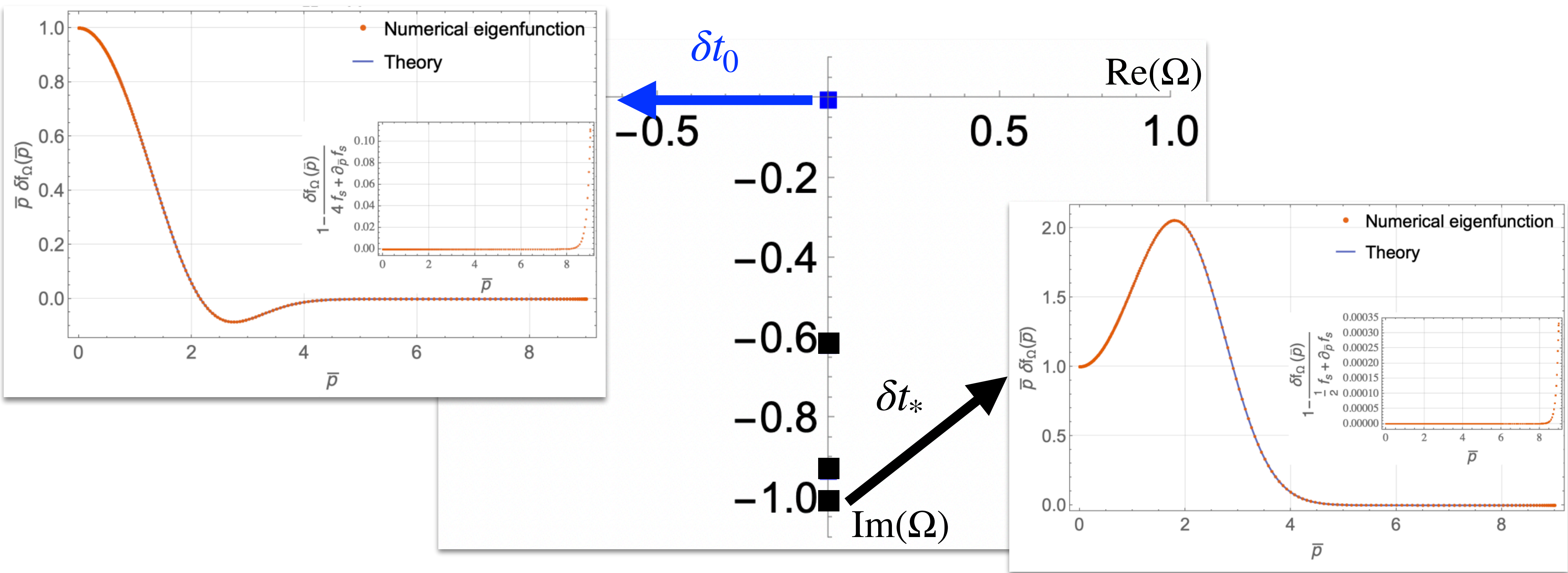

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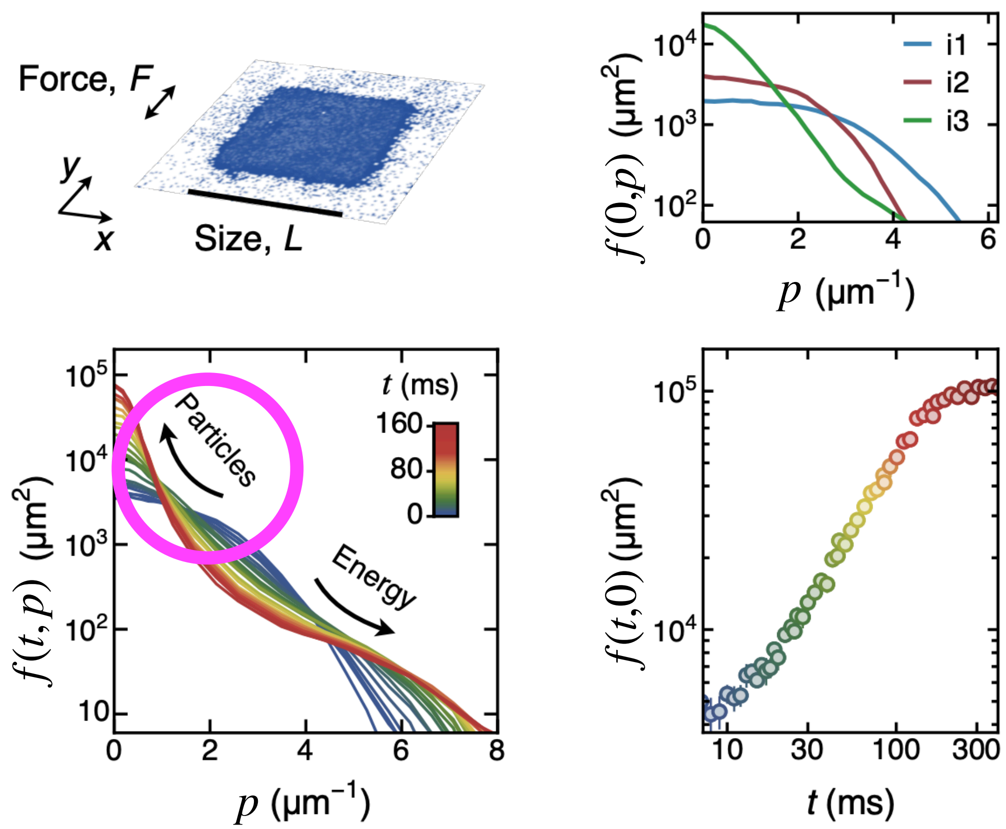
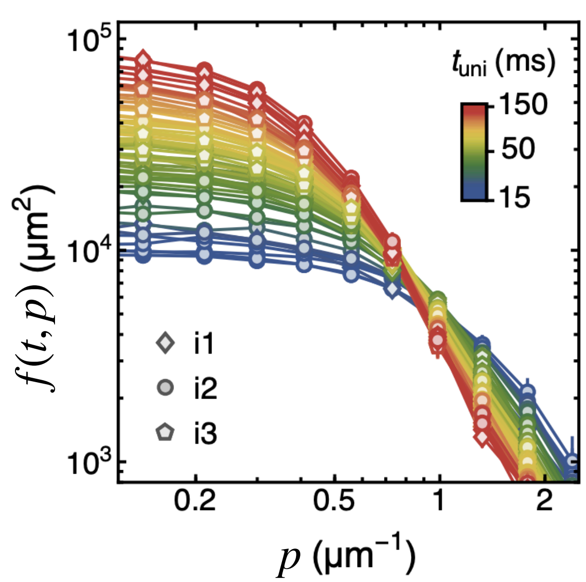
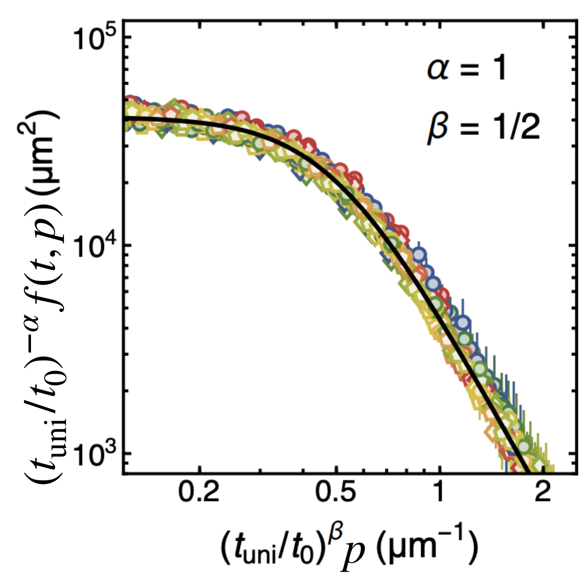
2312.09248 by Gazo, Karailiev, Satoor, Eigen, Gałka and Hadzibabic



We start with a quasi-pure interacting 2D condensate of 7×10^4 atoms of ^{39}K in the lowest hyperfine state, confined in a square box trap of size $L = 50\,\mu$m [40]. The interactions in the gas, characterized by the scattering length a, are tuneable via the magnetic Feshbach resonance at 402.7 G [41]. To prepare our far-from-equilibrium initial states, we temporarily turn off the interactions ($a \to 0$) and shake the gas with a spatially uniform oscillating force F (see Fig. 1A). This destroys the condensate and, as previously studied in 3D [42, 43], results in an isotropic highly nonthermal f distribution. After preparing one of the three different initial states i1–i3 shown in Fig. 1A, we stop the shaking, reinstate the interactions ($a \to 30\,a_0$, where a_0 is the Bohr radius), and let the gas relax. The states i1–i3 do not have a defined temperature, but $E = \int \varepsilon(k)\,\mathrm{d}k$, where $\varepsilon = 2\pi\hbar^2k^3n_k/(2m)$ and m is the atom mass, gives the total energy. We get $E/k_B = 4.1(3)$ mK, 2.2(3) mK, and 1.0(3) mK, for i1–i3 respectively; in all cases E is sufficiently low for a condensate to emerge during relaxation [44].



$$t_{\rm uni} \equiv t - t_*$$



Quasinormal modes of nonthermal fixed points

2502.01622 with De Lescluze

On a nonthermal fixed point:

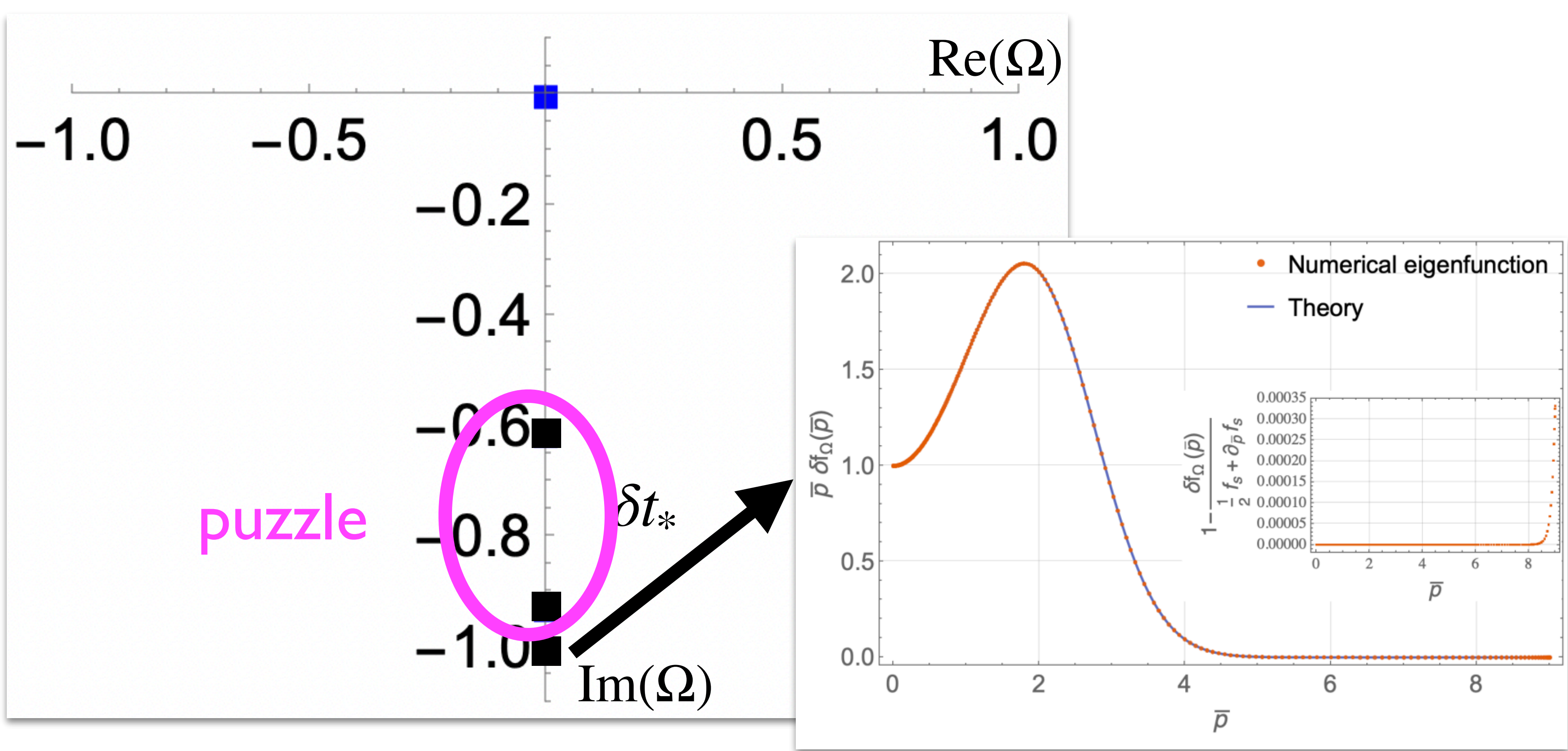
$$f(t,p) \approx \left(t_{\rm uni}/t_0\right)^{\alpha} f_{\rm scaling}\left[\left(t_{\rm uni}/t_0\right)^{\beta} p\right]$$

with $t_{\rm uni} \equiv t - t_*$

directly before that:

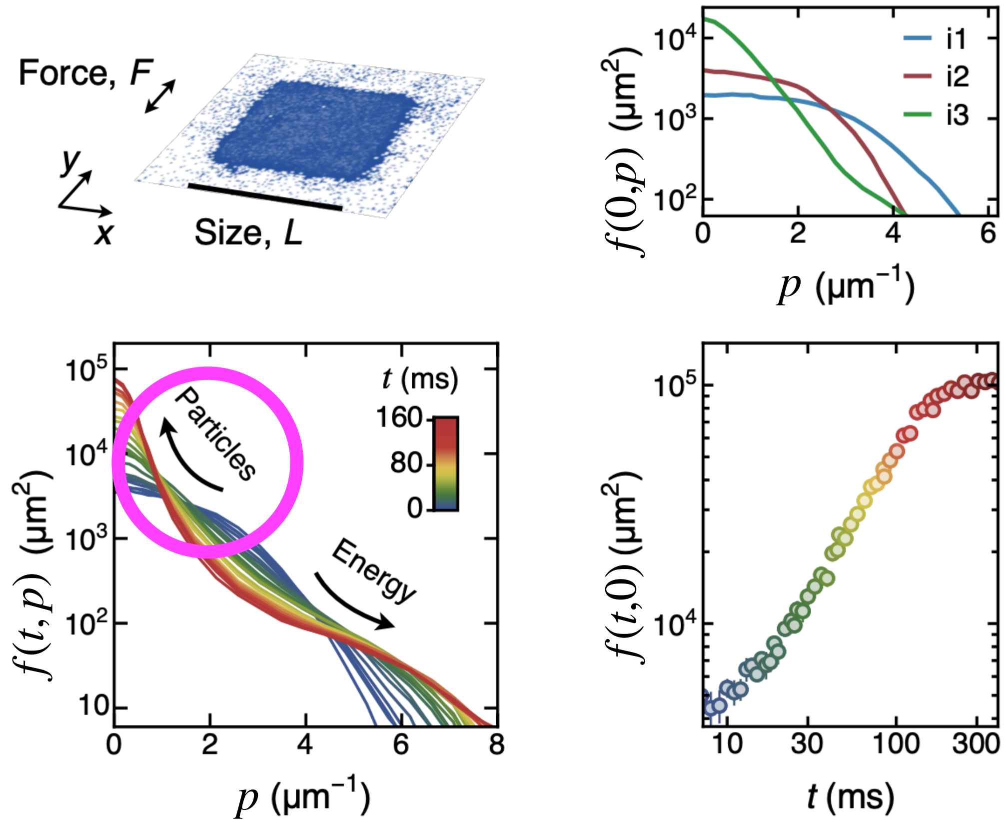
$$\left(t_{\rm uni}/t_0\right)^{-\alpha} f(t,p) \approx f_{\rm scaling}\left[\left(t_{\rm uni}/t_0\right)^{\beta} p\right]$$

$$+\sum_{\Omega} \left(t_{\rm uni}/t_0\right)^{-i\Omega} \delta f_{\Omega}\left[\left(t_{\rm uni}/t_0\right)^{\beta} p\right]$$



Experimental confirmation

2312.09248 by Gazo, Karailiev, Satoor, Eigen, Gałka and Hadzibabic



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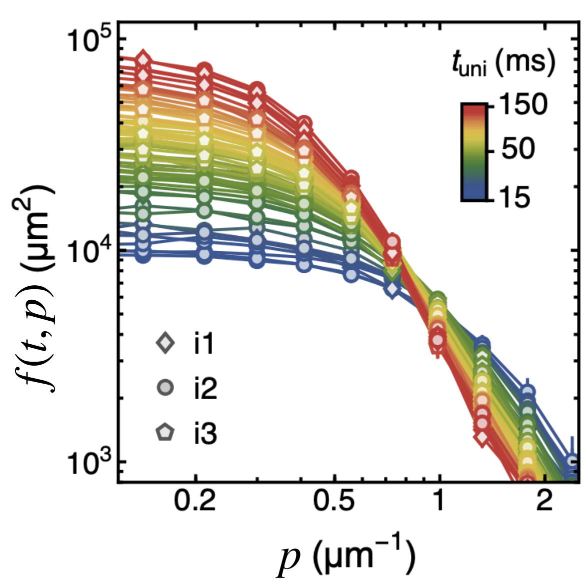
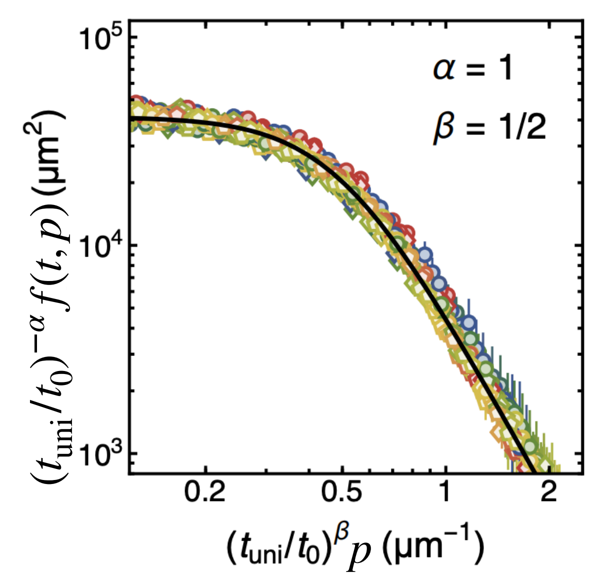


$$t_\mathrm{uni} \equiv t - t_*$$



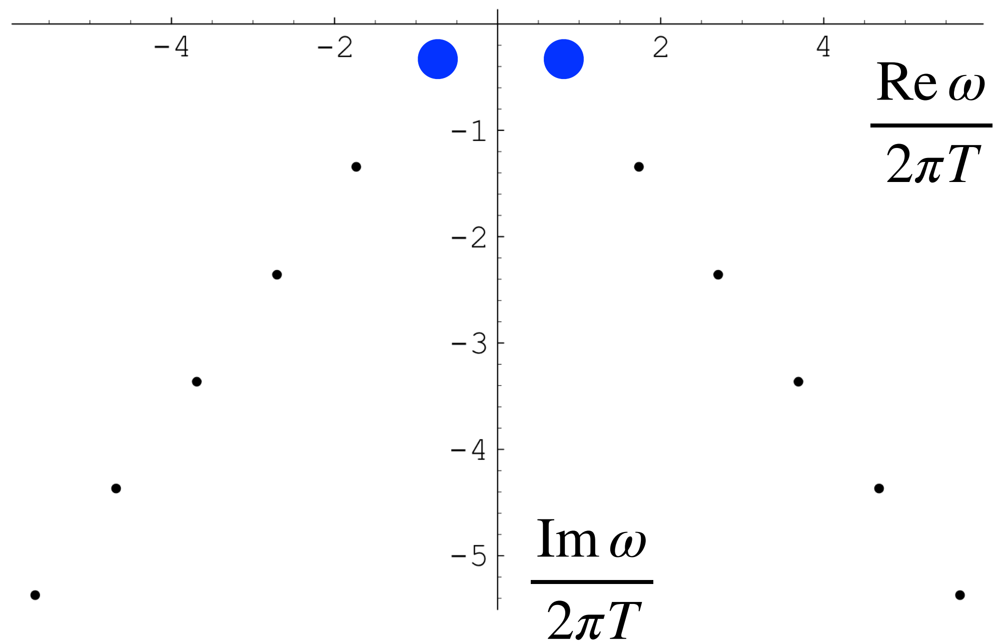
Result III:
hydrodynamics of nonthermal fixed points

Holographic quasinormal modes (QNMs)

Horowitz and Hubeny hep-th/9909056; Kovtun and Starinets hep-th/0506184

Strongly-coupled QFTs relax via dual QNMs: $\delta g_{ab} \sim \delta\langle T_{\mu\nu}\rangle \sim e^{-i\,\omega\,t + i\,\vec{q}\cdot\vec{x}}$

plot for $q = 2\pi T$



Consequences for thermalization → lots of short-lived excitations

→ a few long-lived hydrodynamic modes

How about nonthermal fixed points?

Hydrodynamics of nonthermal fixed points

2504.18754 with Berges, Denicol and Preis

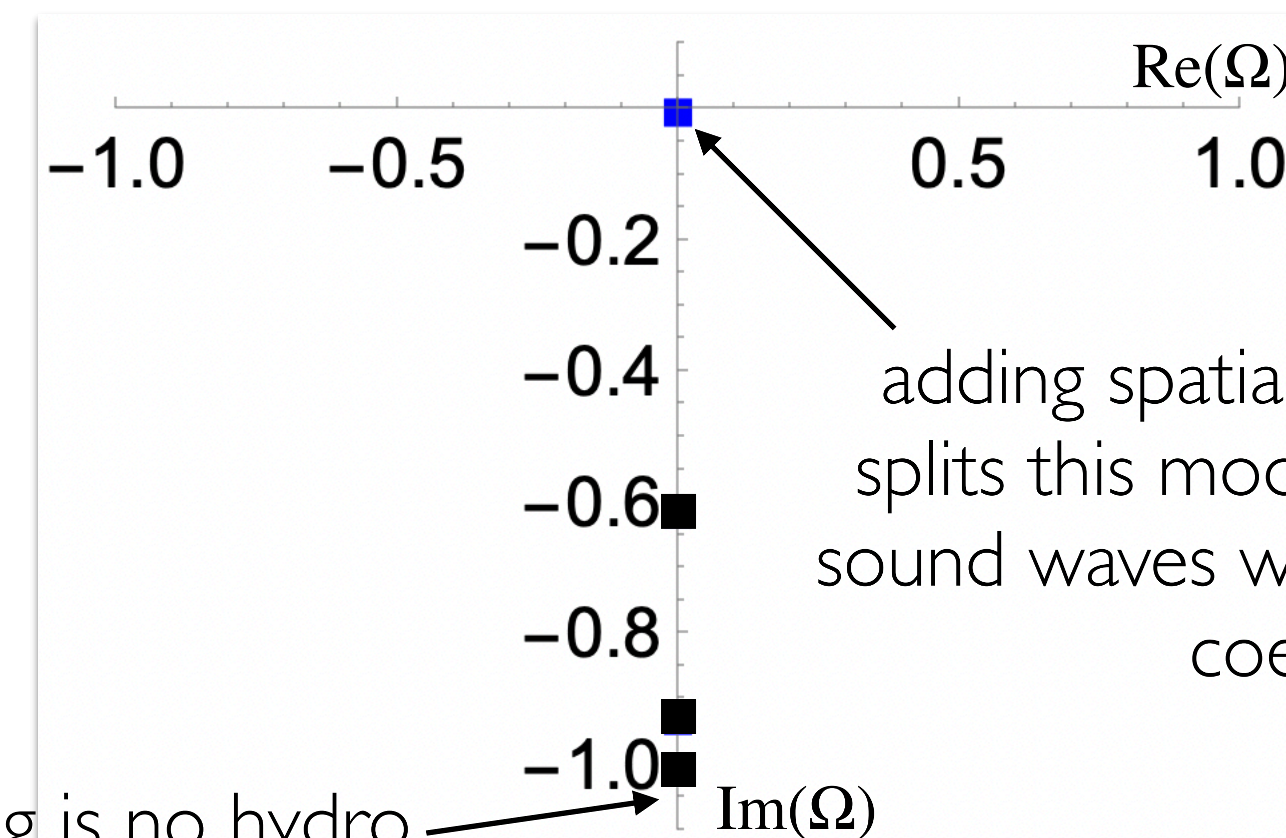
On a nonthermal fixed point:

$$f(t,p) \approx (t_{\rm uni}/t_0)^{\alpha} f_{\rm scaling}\left[(t_{\rm uni}/t_0)^{\beta} p\right]$$

with $t_{\rm uni} \equiv t - t_*$

directly before that:

$$(t_{\rm uni}/t_0)^{-\alpha} f(t,p) \approx f_{\rm scaling}\left[(t_{\rm uni}/t_0)^{\beta} p\right] + \sum_{\Omega} (t_{\rm uni}/t_0)^{-i\Omega} \delta f_{\Omega}\left[(t_{\rm uni}/t_0)^{\beta} p\right]$$



adding spatial momentum e^{iqx} splits this mode into propagating sound waves with scaling transport coefficients

indeed prescaling is no hydro

What we do in practice

2504.18754 with Berges, Denicol and Preis

Input:

$$f(t,p) = \left(t_{\rm uni}/t_0\right)^{\alpha} f_{\rm scaling}\left[\left(t_{\rm uni}/t_0\right)^{\beta} p\right] + \delta f(x^{\mu},p)$$

with $\delta f(t,p) \sim p_\mu p_\nu \pi^{\mu\nu}(x^\alpha)$

Method: take truncated low moments expansion of the Boltzmann equation that was used before to derive MIS/DNMR-type equations

Outcome for 2-2 scattering : $\tau_\pi(t) D\pi^{\mu\nu} = -\pi^{\mu\nu} + \eta(t)\sigma^{\mu\nu} + \ldots$ with

$$\eta(t) \sim (t_{\rm uni}/t_{\rm ref})^1$$

$$\frac{\eta(t)}{\tau_\pi(t)} = \frac{4}{15} \times \text{(energy density)}$$

Qualitative* crosscheck

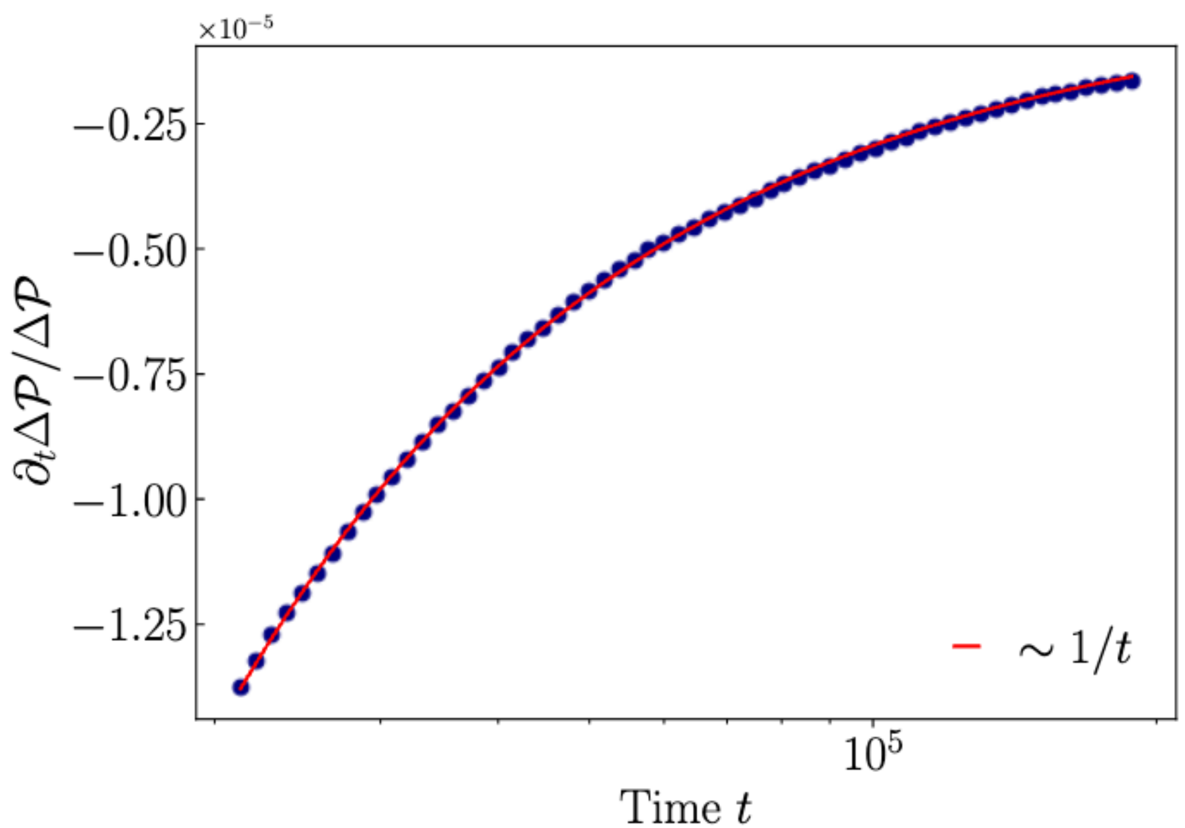
2504.18754 with Berges, Denicol and Preis

$$\tau_\pi(t) D\pi^{\mu\nu} = -\pi^{\mu\nu} + \eta(t)\sigma^{\mu\nu} + \ldots$$

↓

restrict to homogenous isotropization: $\pi^{\mu\nu} = \mathrm{diag}\left(0, -\frac{2}{3}\Delta P(t), \frac{1}{3}\Delta P(t), \frac{1}{3}\Delta P(t)\right)^{\mu\nu}$

↓



Quasinormal modes of nonthermal fixed points

2502.01622 with De Lescluze

On a nonthermal fixed point:

$$f(t,p) \approx (t_{\text{uni}}/t_0)^{\alpha} f_{\text{scaling}}\left[(t_{\text{uni}}/t_0)^{\beta} p\right]$$

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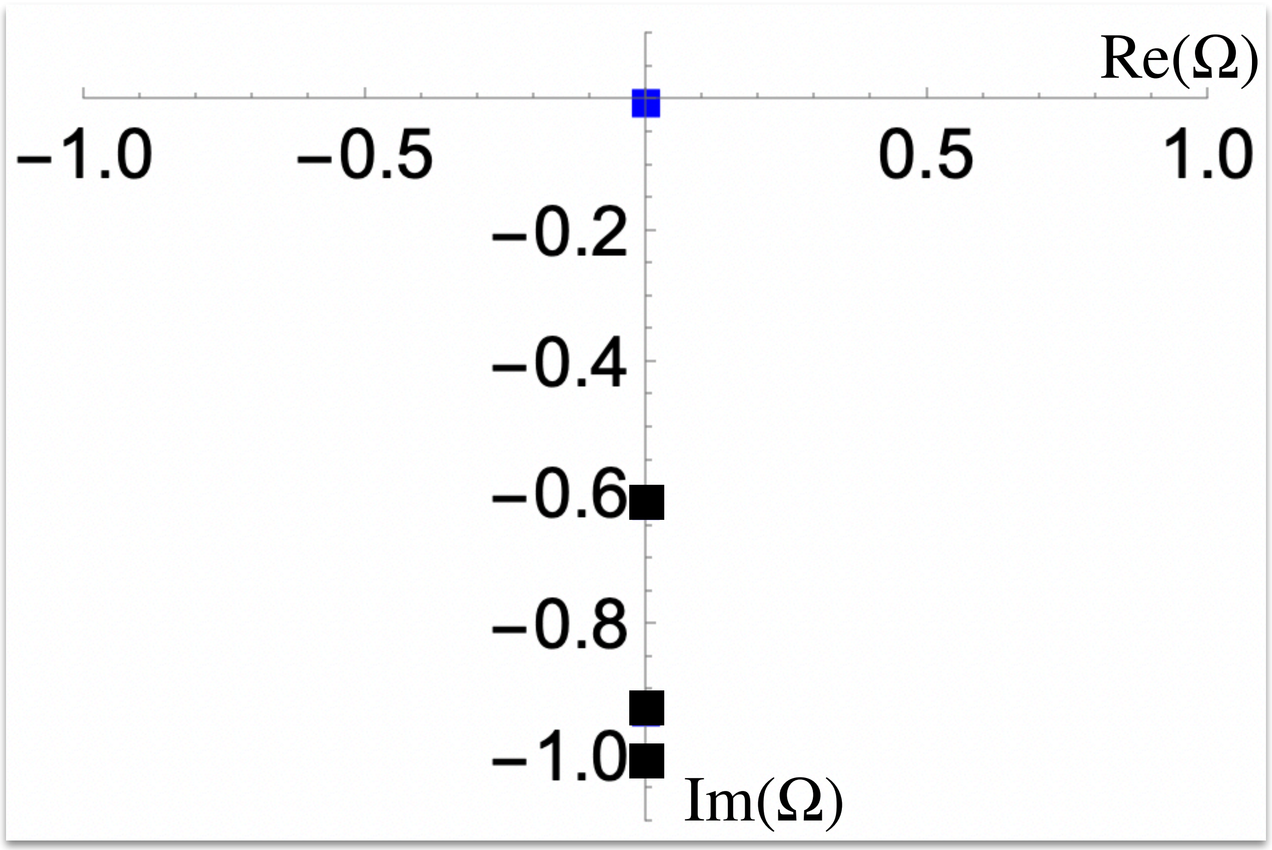


Qualitative* crosscheck

2504.18754 with Berges, Denicol and Preis

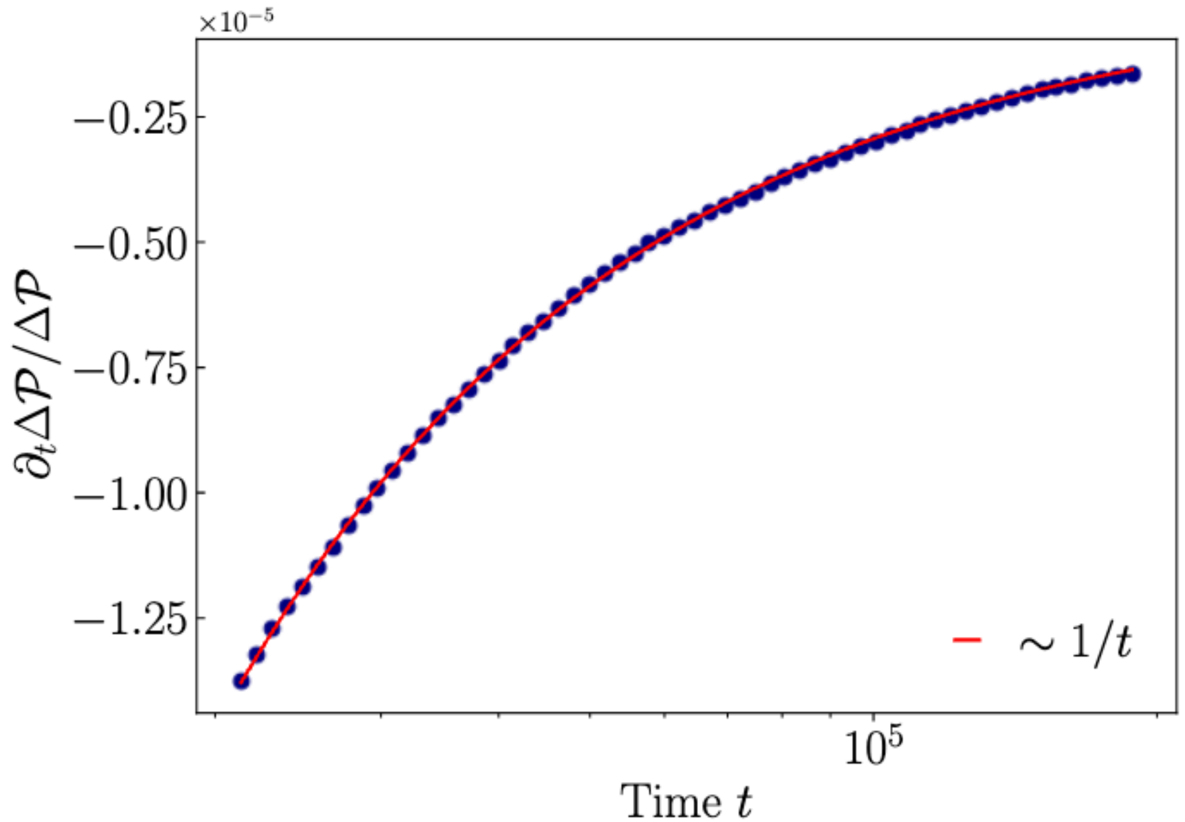
$$\tau_\pi(t) D\pi^{\mu\nu} = -\pi^{\mu\nu} + \eta(t)\sigma^{\mu\nu} + \dots$$

↓

restrict to homogenous isotropization: $\pi^{\mu\nu} = \mathrm{diag}\left(0, -\frac{2}{3}\Delta P(t), \frac{1}{3}\Delta P(t), \frac{1}{3}\Delta P(t)\right)^{\mu\nu}$

↓

$$\frac{\Delta \dot{P}}{\Delta P} = -\frac{1}{\tau_\pi(t)} = \frac{t_{\mathrm{ref}}}{\bar{\tau}_\pi(t - t_*)}$$



Implications 2504.18754 with Berges, Denicol and Preis

Comparing the nonthermal fixed point liquid with near-equilibrium liquids requires going beyond the η/s paradigm, as this ratio is now time dependent

One idea: consider the ratio $\dfrac{\eta}{\tau_\pi \times (\text{energy density})}$

→ $\dfrac{4}{15}$ both nonthermal fixed point and equilibrium

→ $\approx \dfrac{2}{3}$ in holography

Using $\tau_\pi(t) D\pi^{\mu\nu} = -\pi^{\mu\nu} + \eta(t)\sigma^{\mu\nu} + \ldots$ to model inhomogeneous nonthermal fixed point phenomena eying experimental confirmations in cold atomic gases

Summary

Summary

Prescaling is the scaling with correctly accounted for the origin of time
Consequence: nonthermal fixed points extend to earlier times

2307.07545 with Mazeliauskas and Preis

The emergence of nonthermal fixed points can be thought of as originating from the decay of transient quasinormal modes around them

2502.01622 with De Lescluze

Adding spatial momentum reveals hydrodynamic modes and opens a window on studying inhomogeneous nonthermal fixed points

2504.18754 with Berges, Denicol and Preis

Open problems (cold atoms / theory)

Prescaling is the scaling with correctly acc… …e
Consequence: nonthermal fixed … …es

…7.07545 with Mazeliauskas and Preis

experimentally confirmed in the Hadzibabic Lab

prescaling in the expanding geometry?

The emergence of nonthermal fix… …ght of as originating from the decay of transient qua… …around them

2502.01622 with De Lescluze

experiment?

less damped QNMs than the prescaling one?

Adding spatial momentum revea… …namic m… …s and opens a window on studying inhomog… …thermal f…

…xxx with Berges, Denicol and Preis

experiment?

aHydro for the expanding geometry nonthermal fixed point?

nonthermal fixed point with initial state fluctuations?