Fluid dynamics of heavy quarks in the QGP

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GSI Darmstadt

GGI, Firenze, May 2025

Based on
FC, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, PRD 108 (2023) 11, 116011
Facen, FC, Grossi, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, in preparation

Outline

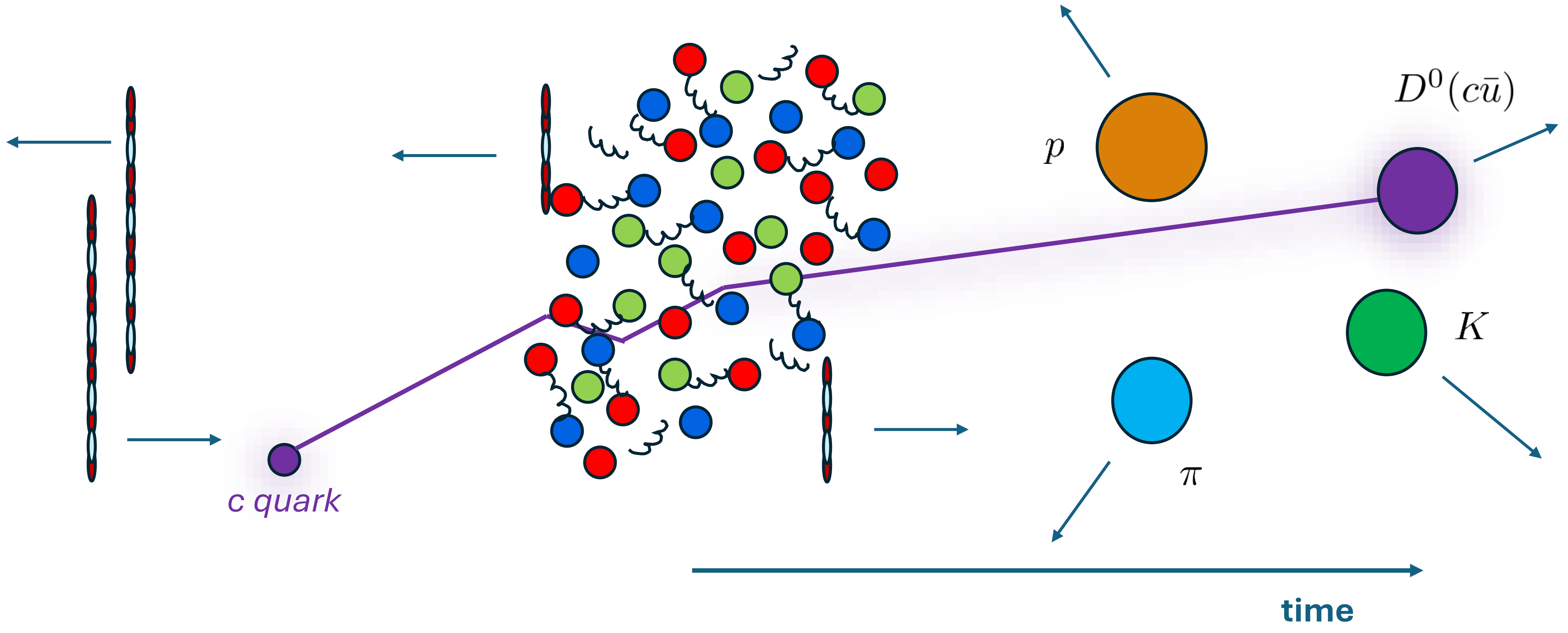
- Introduction and theoretical setup
- Numerical methods
- Phenomenology results
- Newest developments

Introduction
and
theoretical setup

Heavy quarks as probes of the QGP

Produced via hard scatterings at the beginning of the collision before the QGP is formed

$$\boxed{M_{c,b} \sim O(\mathrm{GeV}) \gg \mathrm{T_{QGP}}}$$



Heavy quarks as probes of the QGP

✗ Initially out of chemical and kinetic equilibrium

- In the low p_T region they provide a window to study **equilibration processes**

Heavy quarks as probes of the QGP

❌ Initially out of chemical and kinetic equilibrium

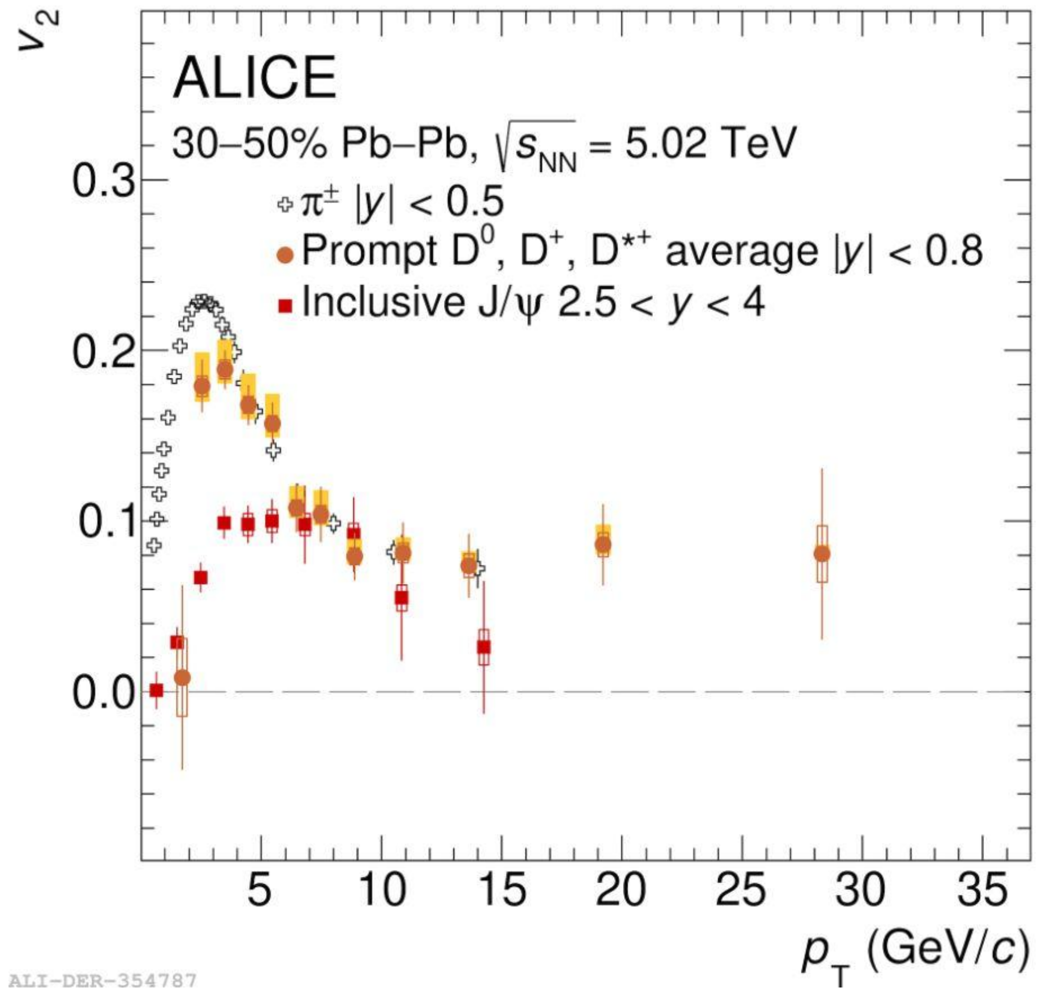
- In the low p_T region they provide a window to study **equilibration processes**

Significant measurements of J/ψ and D mesons of positive elliptic flow

- Do heavy quarks reach local kinetic equilibrium?
- Can they be considered as part of the medium itself?

This work: fluid-dynamic approach!

ALICE PLB 813 (2021) 136054



Fluid dynamics for heavy quarks

$$\left.\begin{array}{ll} \textbf{Energy-momentum conservation} & \nabla_\mu T^{\mu\nu} = 0 \\ \textbf{Equation of state} & P = P(T) \end{array}\right\} \text{Standard fluid dynamics}$$

Fluid dynamics for heavy quarks

Energy-momentum conservation $\nabla_\mu T^{\mu\nu} = 0$

Equation of state $P = P(T)$

$\Big\}$ Standard fluid dynamics

Conservation of $Q\bar{Q}$ **pairs** $\nabla_\mu N^\mu = 0$

$$N^\mu = n\, u^\mu + \nu^\mu$$

HQ density

HQ diffusion current

Fluid dynamics for heavy quarks

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HQ diffusion current

Equation of state $n(T,\alpha) = \frac{T}{2\pi^2} \sum_{i \in \mathrm{HRGc}} q_i M_i^2 e^{q_i \alpha} K_2(M_i/T)$

$$\boxed{\alpha = \mu_{Q\overline{Q}}/T}$$

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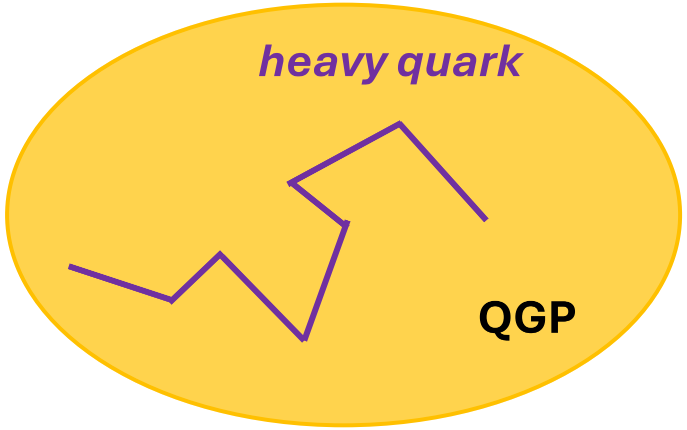
HQ density **HQ diffusion current**



Equation of state $n(T,\alpha) = \frac{T}{2\pi^2} \sum_{i \in \mathrm{HRGc}} q_i M_i^2 e^{q_i \alpha} K_2(M_i/T)$

Diffusion equation $\tau_n \partial_t \nu^i + \nu^i = n D_s \nabla^i (\mu/T)$

HQ diffusion coefficient D_s
= QCD property

HQ relaxation time τ_n
= timescale to approach fluid-dynamic regime

Fluid-dynamic transport coefficient

We computed the **relaxation time** and **diffusion coefficient** associated to charm and beauty quarks by integrating the first moment of the Fokker-Planck equation:

$$p^\mu \partial_\mu f(\mathbf{p},\mathbf{x},t) = \frac{\partial}{\partial p^i}\Big[\underbrace{A(\mathbf{p})}_{\text{Drag coefficient}}\, p^i f(\mathbf{p},\mathbf{x},t) - g^{ij}\frac{\partial}{\partial p^j} \underbrace{D(\mathbf{p})}_{\text{Momentum-diffusion coefficient}}\, f(\mathbf{p},\mathbf{x},t)\Big]$$

We obtain a diffusion equation that allows us the mapping:

$$\boxed{\tau_n = \frac{D_s I_{31}}{T P_0} \qquad \kappa_n = \frac{T^2}{D} n = D_s n}$$

Capellino *et al.* PRD 106 (2022) 034021

Where the **spatial diffusion coefficient** is defined as $D_s = \lim_{k\to 0} \frac{T}{M A(k)}$

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Drag coefficient

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Caveat: momentum dependence of A and D was neglected

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Where the **spatial diffusion coefficient** is defined as $D_s = \lim_{k \to 0} \frac{T}{M A(k)}$

Consistency with FP relaxation time

$$\tau_n = \frac{D_s I_{31}}{T P_0}$$

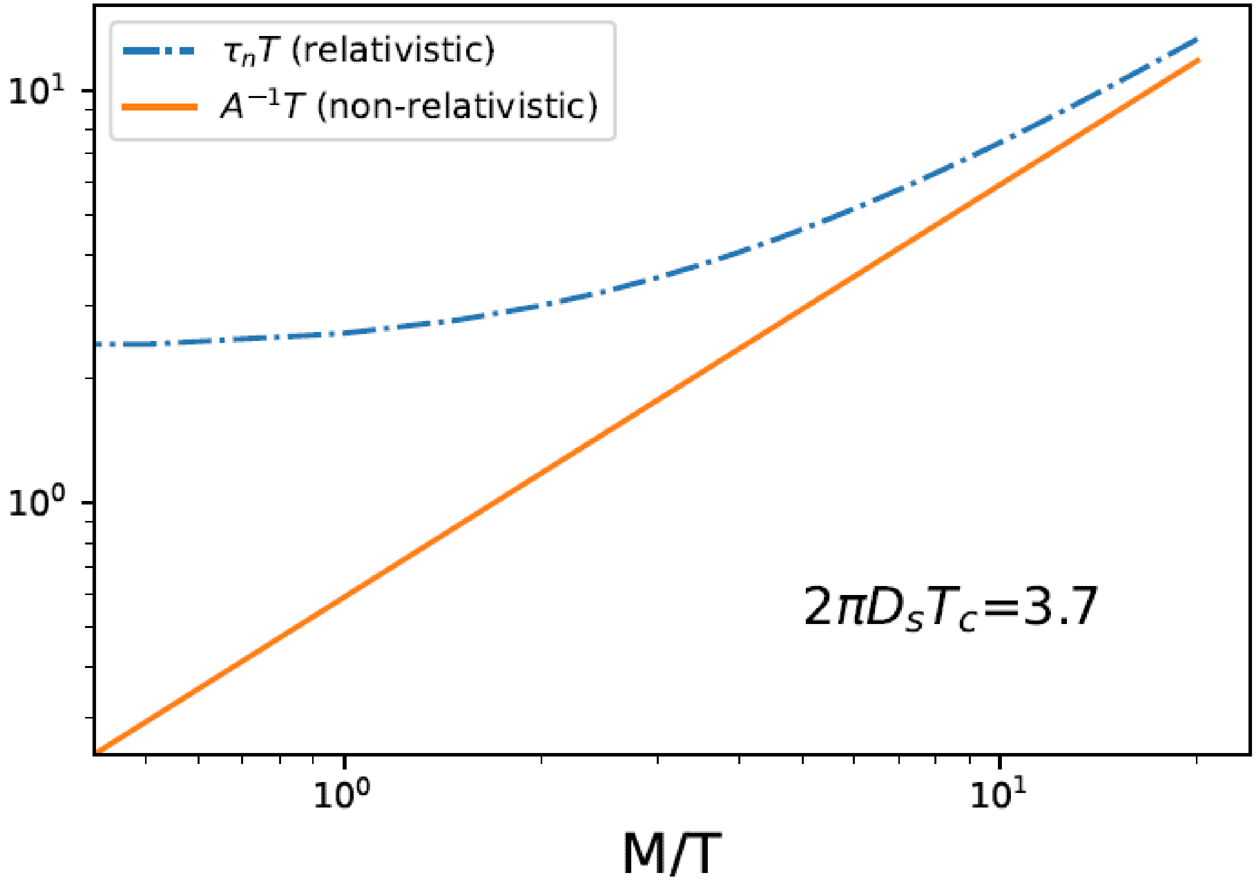
$$\kappa_n = \frac{T^2}{D} n = D_s n$$

$$I_{31} = \frac{1}{3} \int dP p^0 p^2 f_0(p)$$

$$p^0 \sim M \qquad I_{31} \sim M P_0$$

$$\tau_n \sim \frac{D_s M P_0}{T P_0} = D_s \frac{M}{T}$$

$$\tau_n = (2\pi D_s T) \frac{1}{2\pi} \frac{M}{T} = A^{-1} T$$



Consistency with FP relaxation time

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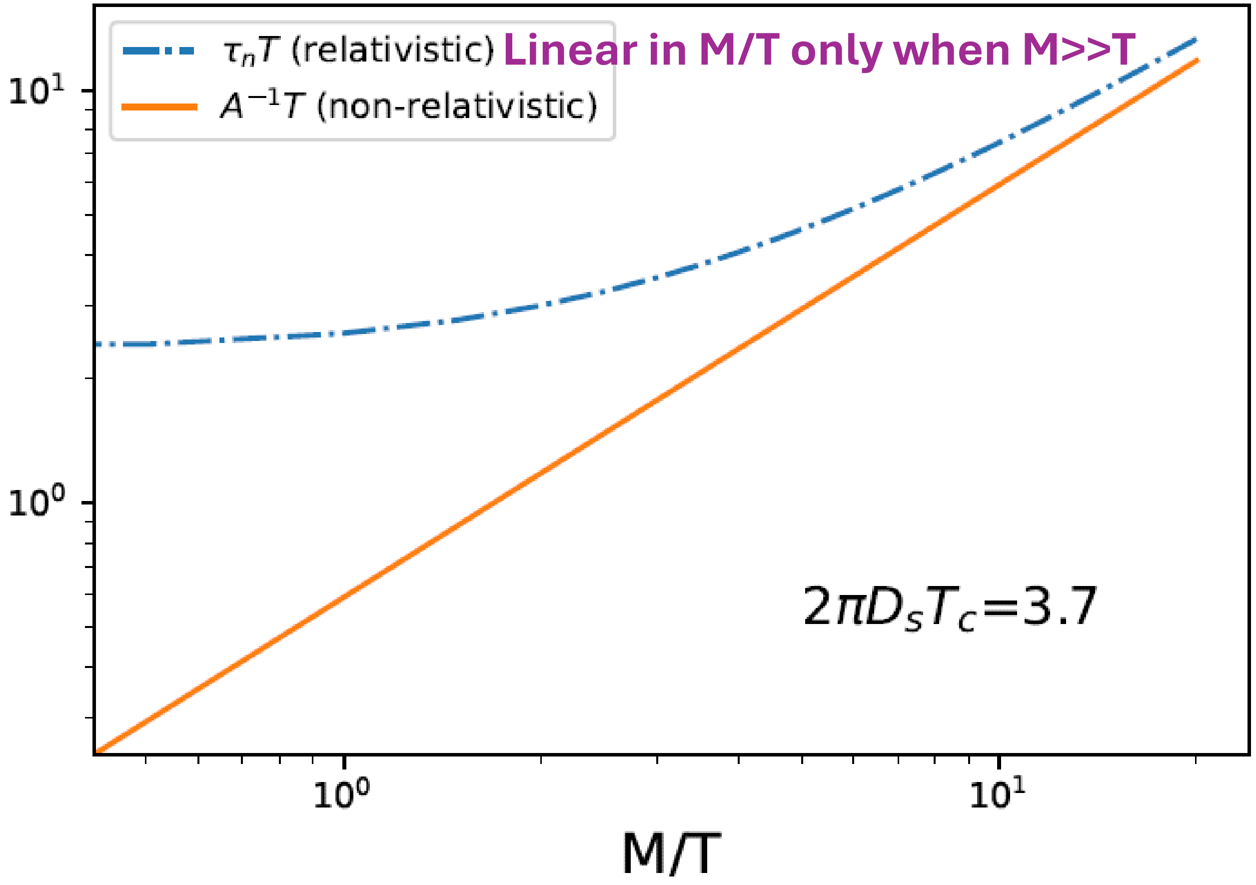
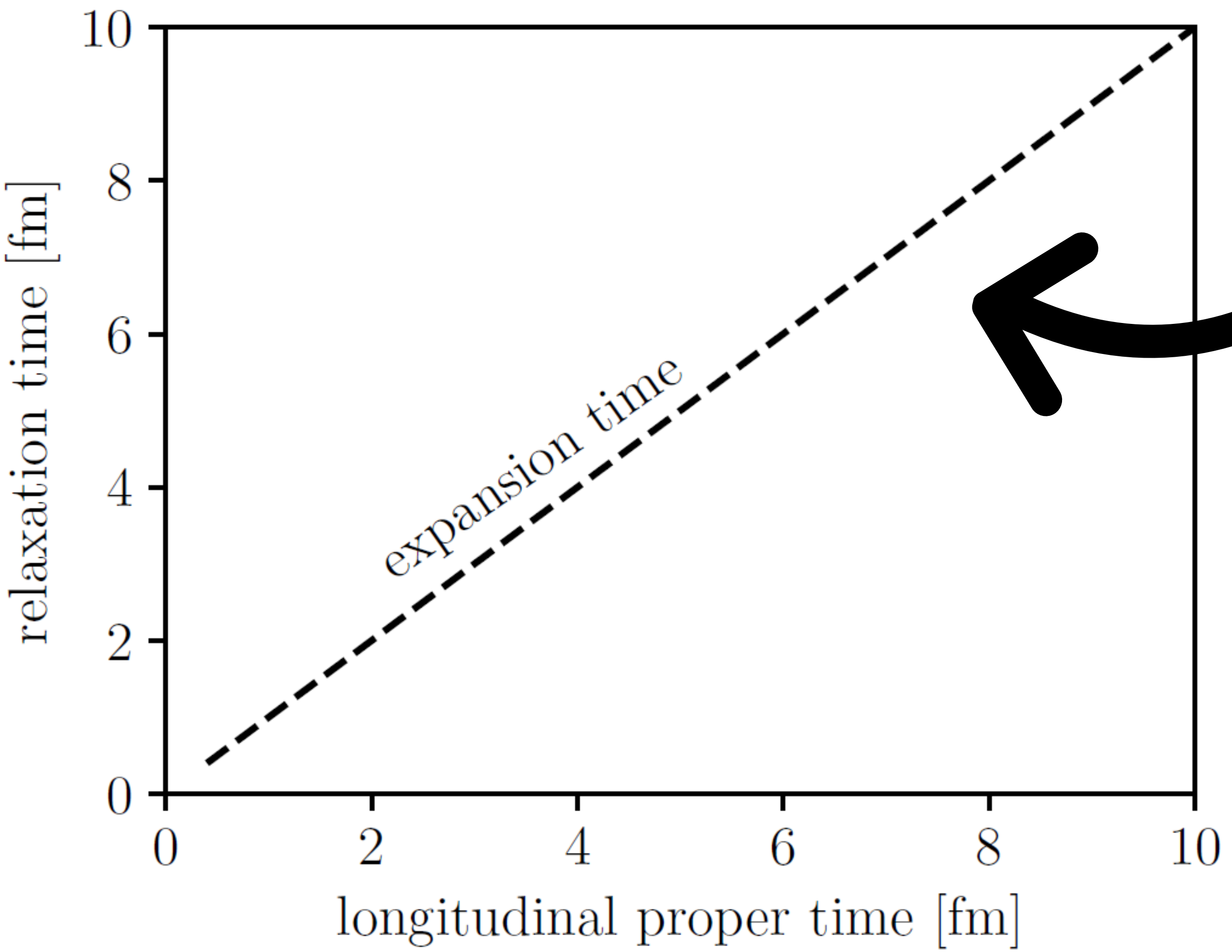


Charm quark relaxation time



QGP: Bjorken flow $v_x = v_y = 0 \quad v_z = z/t$

This plot: Capellino *et al.* PRD 106 (2022) 034021

lQCD 2023: Altenkort *et al.* PRL 130 (2023) 23, 231902
ALICE fits to data: ALICE JHEP 01 (2022) 174

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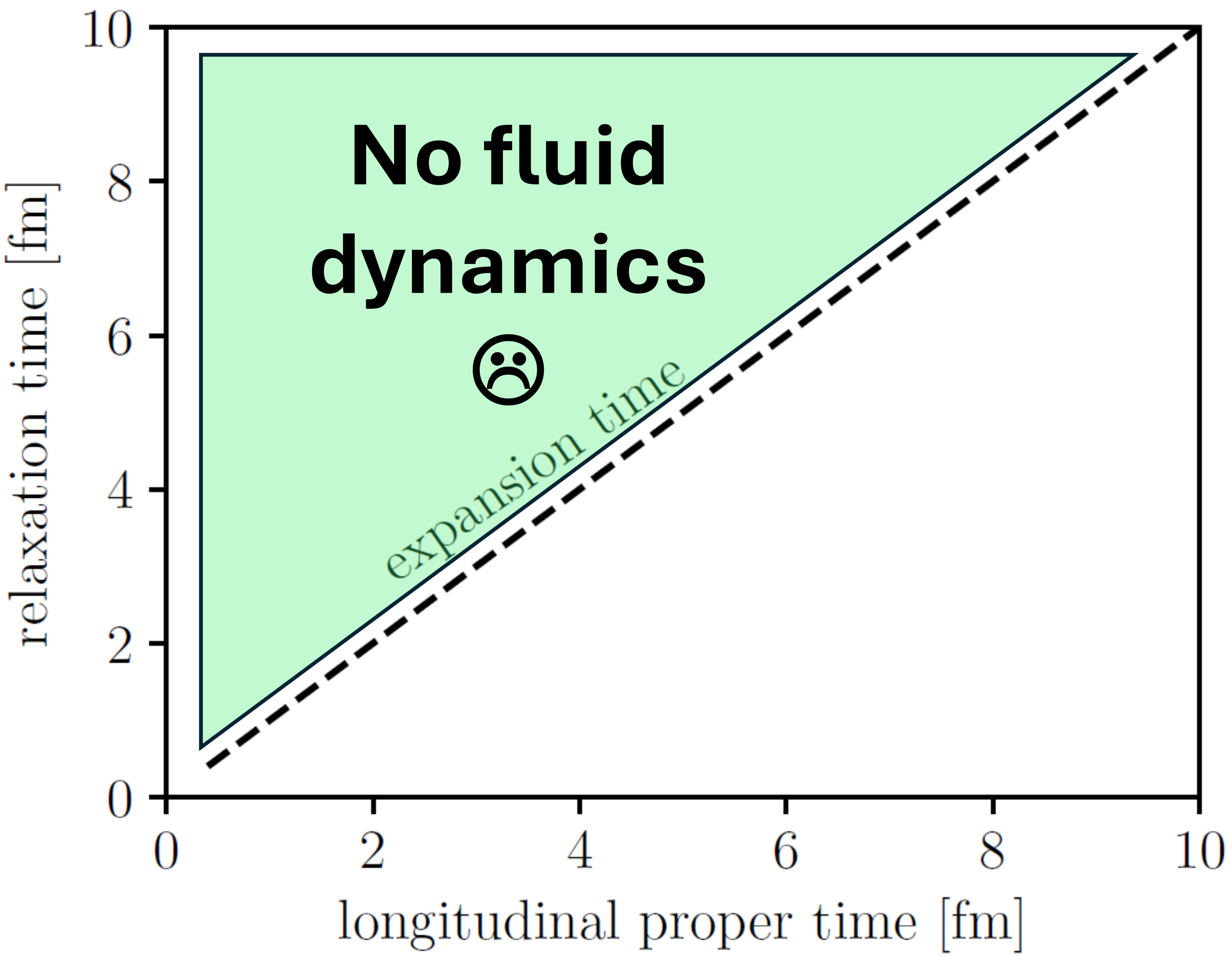

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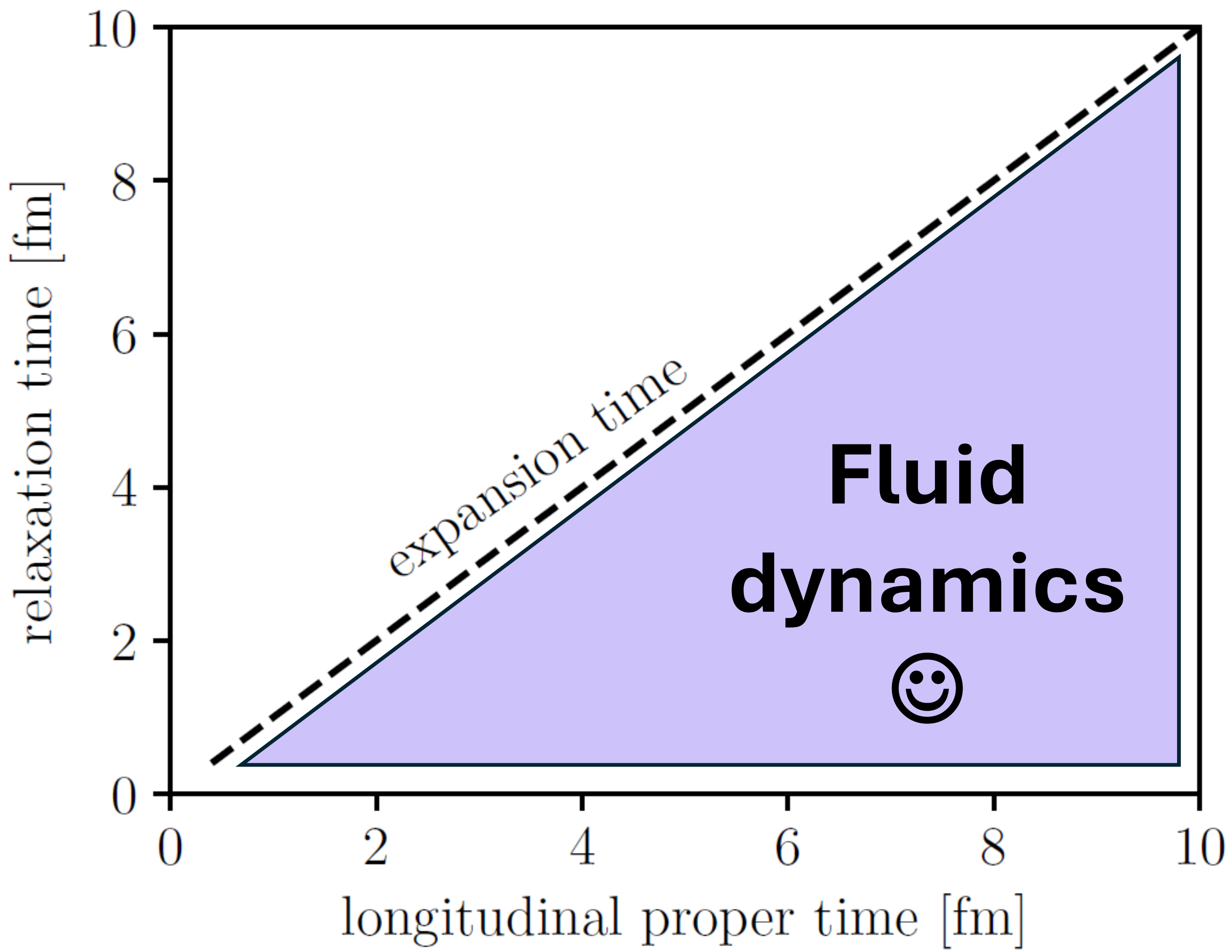

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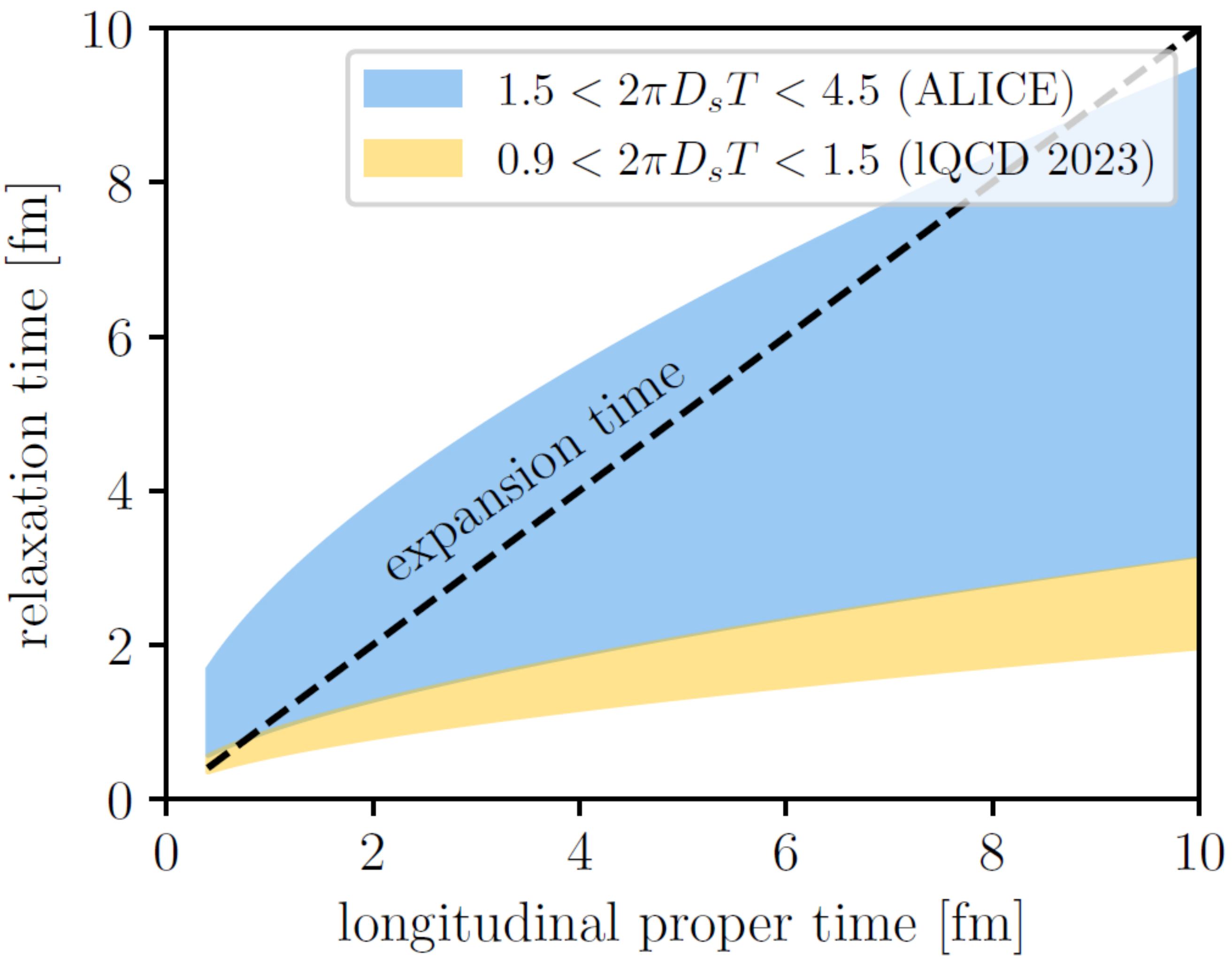

QGP: Bjorken flow $v_x = v_y = 0 \quad v_z = z/t$

$$\tau_n^{(\mathrm{charm})} \ll \text{expansion time}$$

Fluid dynamics is applicable!

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lQCD 2023: Altenkort *et al.* PRL 130 (2023) 23, 231902
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Initial conditions for fluid fields

- T is fixed by the light flavors with Trento
- $$n^{Q\overline{Q}}_{\text{hard}}(\tau_0, \vec{x}_\perp, y=0) = \frac{1}{\tau_0}\frac{d^3N^{Q\overline{Q}}}{d\vec{x}_\perp dy}\bigg|_{y=0}$$
- $$\frac{dN^{Q\overline{Q}}}{dy} = \langle N_{\text{coll}}\rangle \frac{1}{\sigma^{\text{in}}}\frac{d\sigma^{Q\overline{Q}}}{dy}$$

FONLL/pp data
momentum dependence is integrated out

- $$n^{Q\overline{Q}}_{\text{hard}}(\tau_0, \vec{x}_\perp, y=0) = \frac{1}{\tau_0} n_{\text{coll}}(\vec{x}_\perp)\frac{1}{\sigma^{\text{in}}}\frac{d\sigma^{Q\bar{Q}}}{dy}$$

Trento

- $$\boxed{n(T,\alpha) = n^{Q\overline{Q}}_{\text{hard}}}$$

Fixes the initial fugacity

Numerical methods

Discretization

The equations to solve for HQs are:

$$\nabla_\mu N^\mu = 0$$

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Viscous correction can be large: solve as a system of quasilinear PDEs (instead of ideal-viscous splitting)

$$\partial_t \phi + A(\phi)\partial_x \phi + S(\phi) = 0 \qquad \phi = (\alpha, \nu^r)$$

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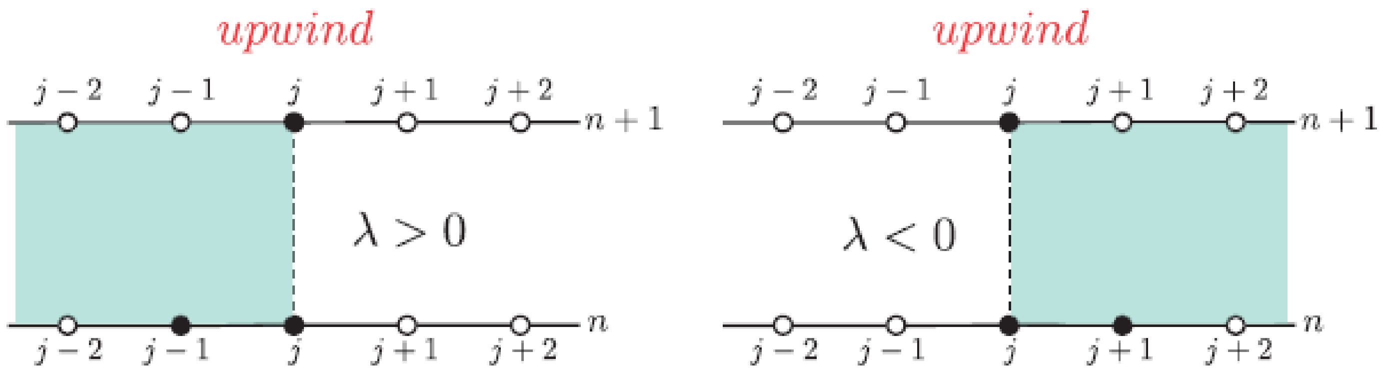
Naïve discretization: unstable!

$$\partial_t \phi_i + A(\phi_i)\partial_x \phi|_{x_i} + S(\phi_i) = 0 \qquad \partial_x \phi|_{x_i} \simeq \frac{1}{2\Delta x}(\phi_{i+1} - \phi_{i-1})$$

Discretization: upwinding



Rezzolla, Zanotti 2013

$$A^{+} = U \begin{bmatrix} \lambda_1^{+} & & \\ & \ddots & \\ & & 0 \end{bmatrix} U^{-1}, \quad A^{-} = U \begin{bmatrix} 0 & & \\ & \ddots & \\ & & \lambda_1^{-} \end{bmatrix} U^{-1}$$

$$\partial_t \phi_i + A^{+}(\phi_i)\partial_x \phi|_{x_i}^{-} + A^{-}(\phi_i)\partial_x \phi|_{x_i}^{+} + S(\phi_i) = 0$$

$$\partial_x \phi|_{x_i}^{-} = \frac{1}{\Delta x}(\phi_i - \phi_{i-1}), \; \partial_x \phi|_{x_i}^{+} = \frac{1}{\Delta x}(\phi_{i+1} - \phi_i)$$

Unambiguous discretization

Roe method

Express absolute value of A

$$|A| = A^+ - A^-$$

Such that

$$A^+ = \frac{1}{2}(A + |A|)\,, \quad A^- = \frac{1}{2}(A - |A|)$$

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The eom can be rewritten as

$$\partial_t \phi_i + \frac{1}{2} A(\partial_x \phi|^-_{x_i} + \partial_x \phi|^+_{x_i}) +$$
$$\frac{1}{2}|A|(\partial_x \phi|^-_{x_i} - \partial_x \phi|^+_{x_i}) + S(\phi_i) = 0.$$

Roe method

By rewriting the derivative operators

$$\frac{1}{2}(\partial_x\phi|^-_{x_i} + \partial_x\phi|^+_{x_i}) = \frac{1}{2\Delta x}(\phi_{i+1} - \phi_{i-1}) = \partial_x\phi|_{x_i}\ , \tag{B10}$$

$$\partial_x\phi|^-_{x_i} - \partial_x\phi|^+_{x_i} = \frac{1}{\Delta x}(\phi_{i+1} + \phi_{i-1} - 2\phi_i) = \Delta x\partial^2_x\phi|_{x_i}\ , \tag{B11}$$

We obtain the eom in the discretized form

$$\partial_t\phi_i + A\partial_x\phi|_{x_i} - \frac{1}{2}|A|\Delta x\partial^2_x\phi|_{x_i} + S(\phi_i) = 0$$

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Numerical viscosity
- **Stabilizes the equation**
- **Can be reduced by reducing lattice spacing**

Functional viscosity methods

Drawback: Roe method requires **full knowledge of all eigenvalues of A**

Solution: approximate the absolute value with a function (Castro et al.)

- Good approximation in the range *x* of the eigenvalues
- **Easy to calculate**
- $f(x) \geq 0$
- **Stability:** $f(x) \geq |x|$

Functional viscosity methods I

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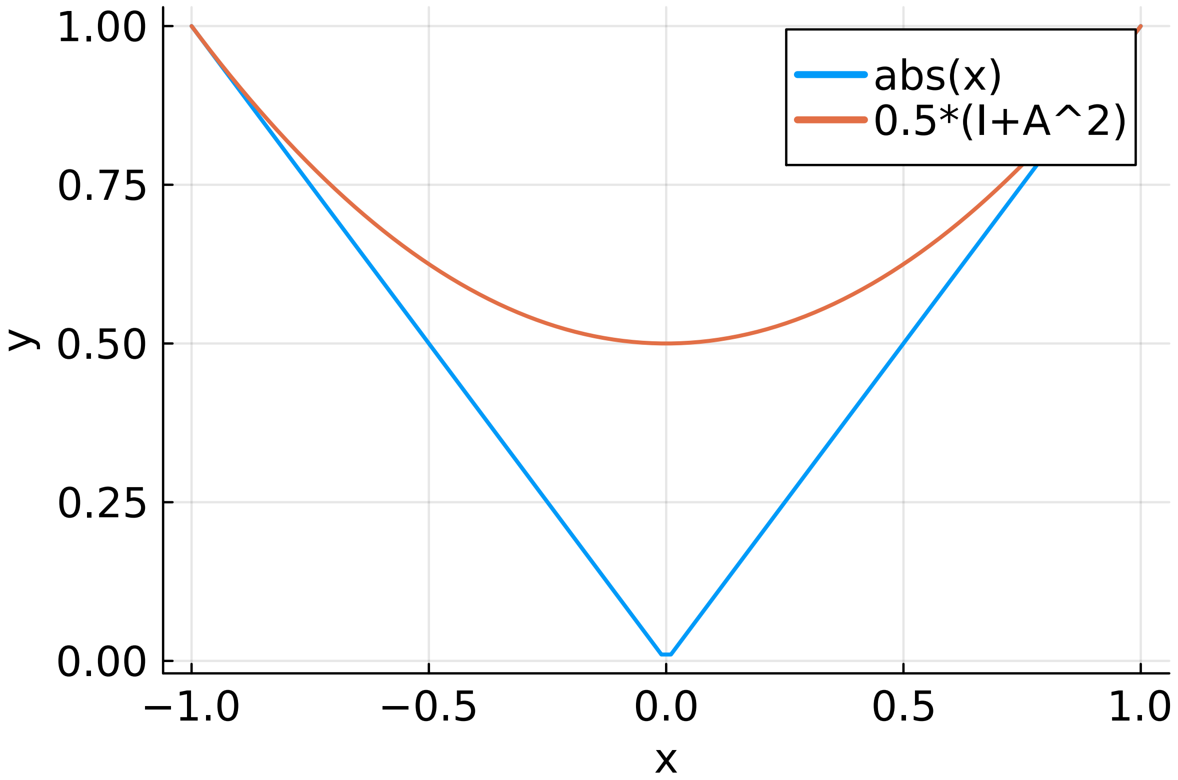
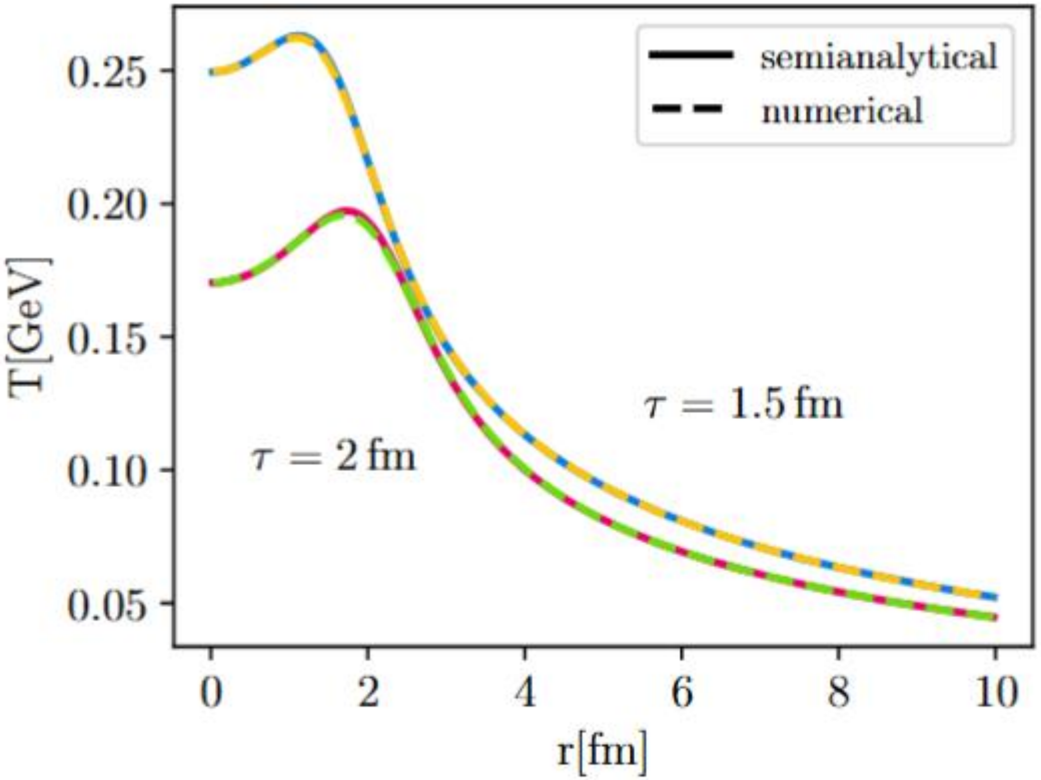
Simplest choice

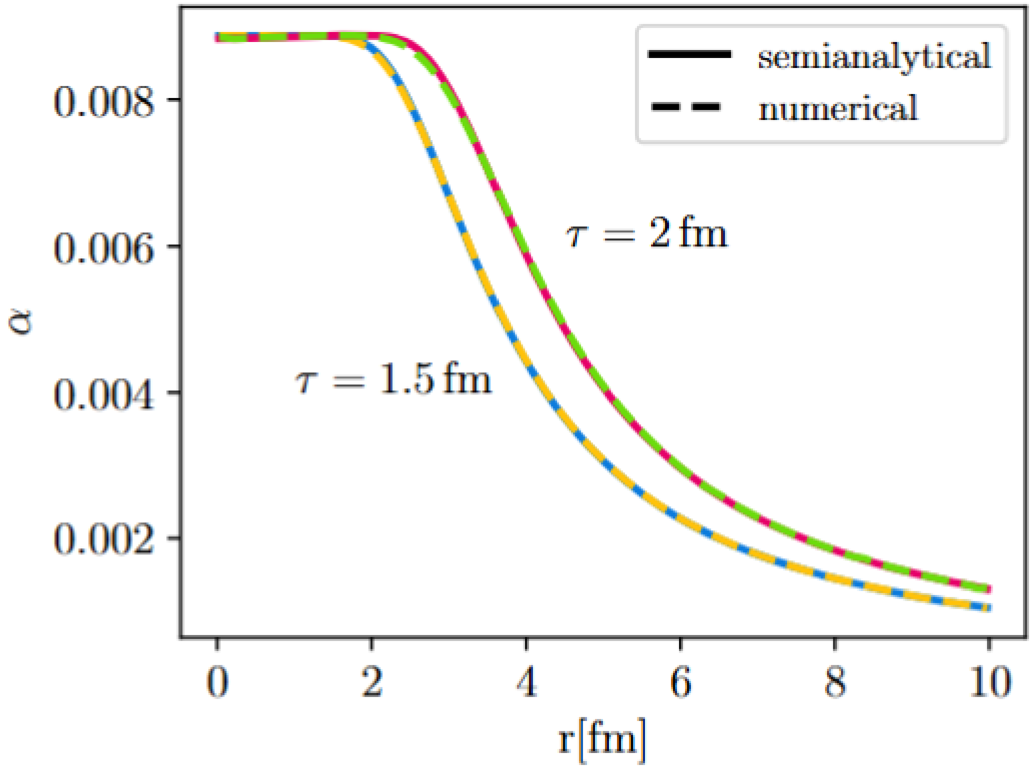
$$|A| \simeq \frac{1}{2}(I + A^2) + \mathcal{O}(A^4)$$



Code validation

Comparison against Gubser flow





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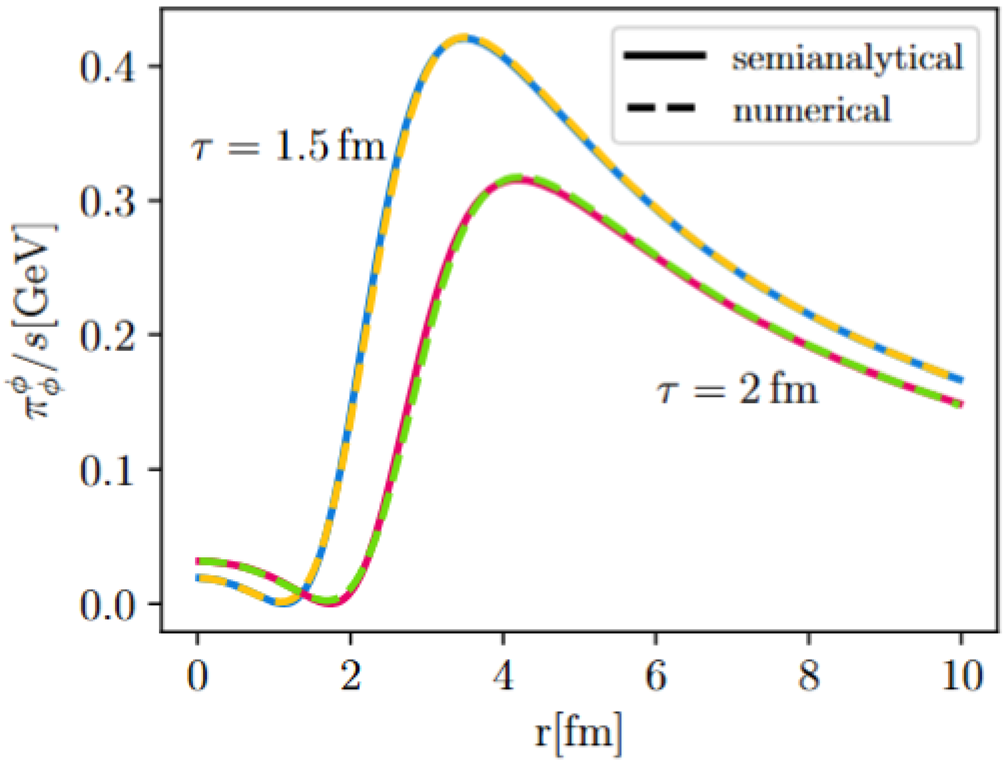
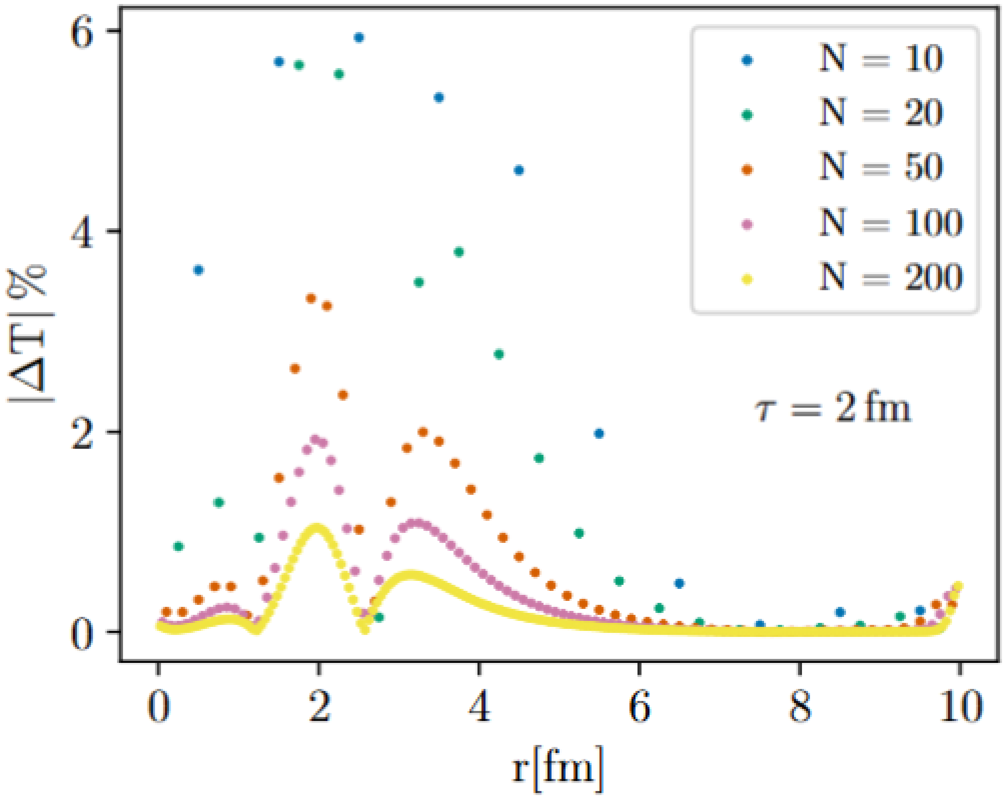


|Numerical/semianalytical -1|



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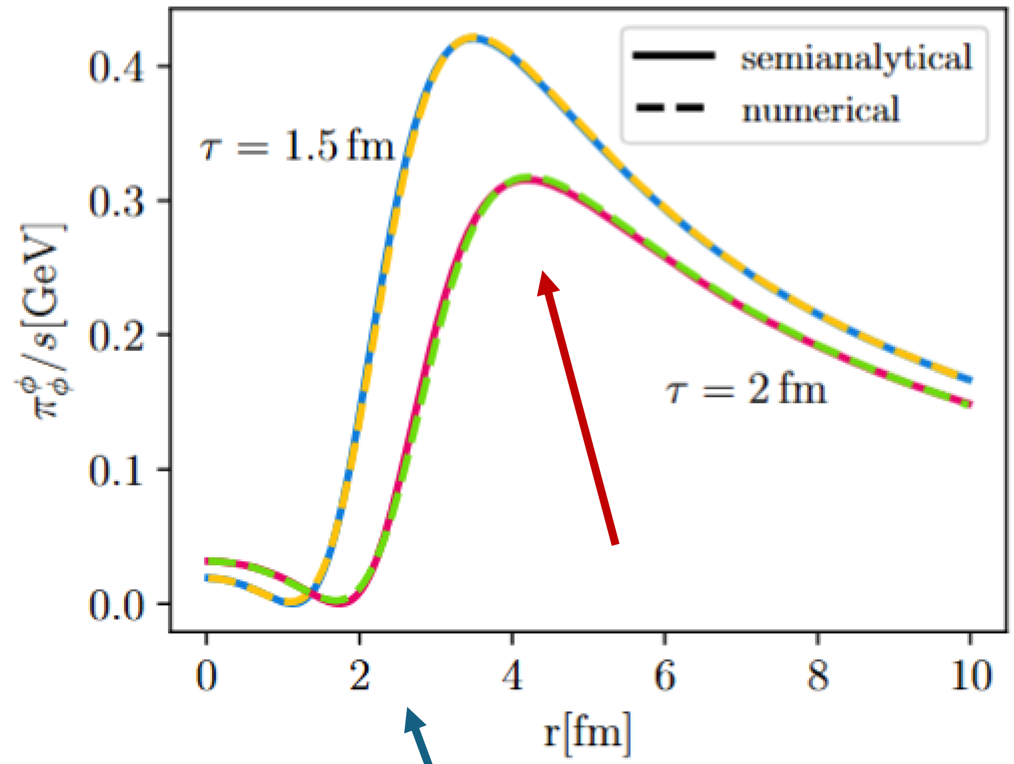

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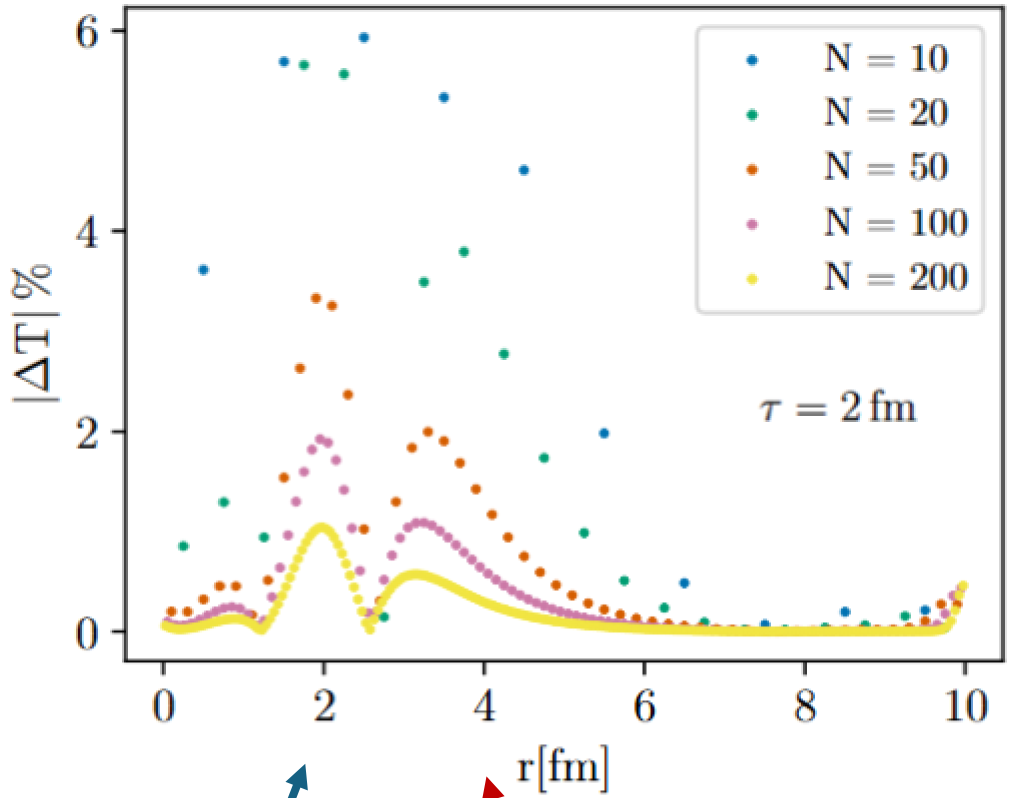
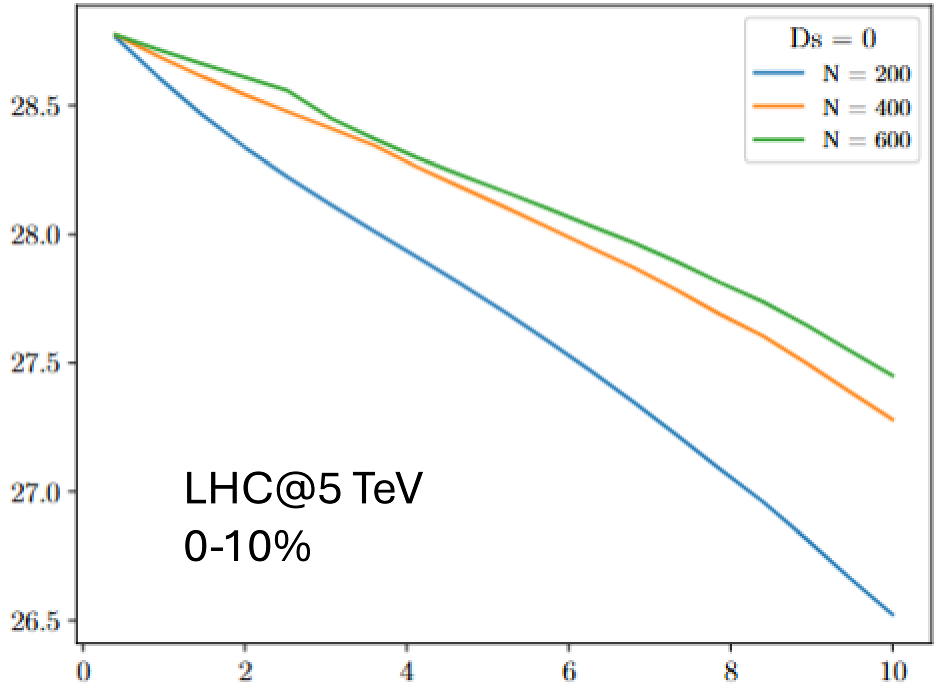


Dissipation is modified by numerical viscosity

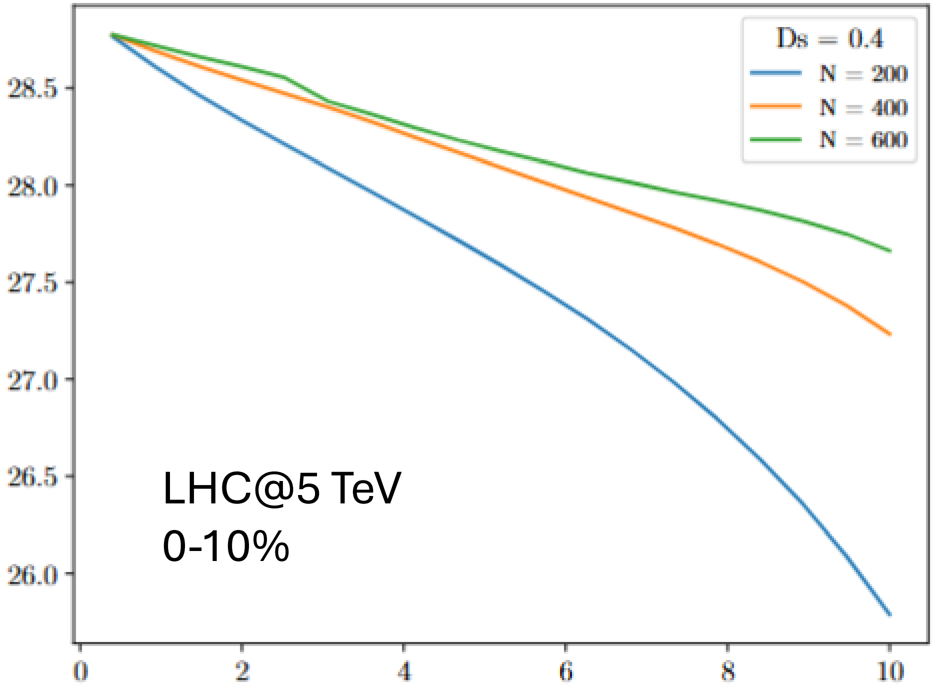
Accuracy on the conservation law

$$N_{Q\bar{Q}} = \int d^3x\sqrt{|g|}N^0(\vec{x}) \; = 2\pi\tau\int r(nu^\tau + \nu^\tau)dr$$

$N_{Q\bar{Q}}$



Time [fm/c]



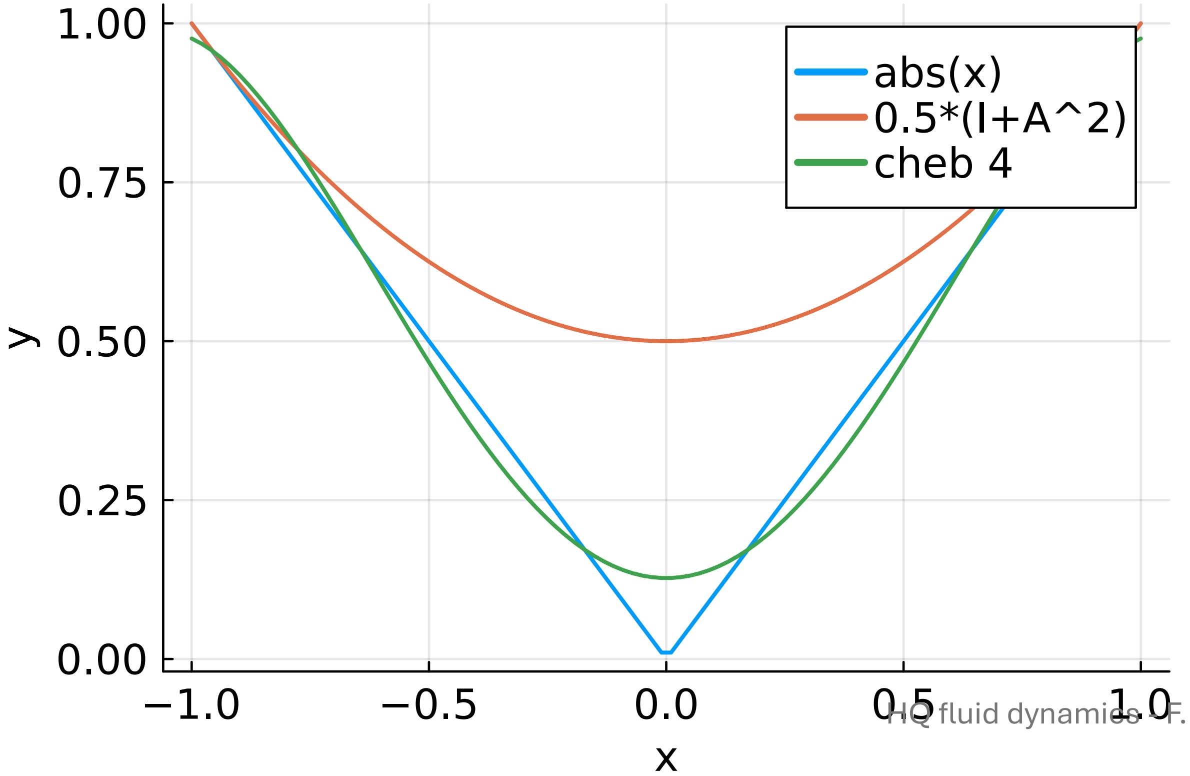
Time [fm/c]

Functional viscosity methods II

Chebyshev polynomials (Castro, Gallardo, Marquina, 2014)

$$\tau_{2p}(x) = \frac{2}{\pi} + \sum_{k=1}^{p} \frac{4}{\pi} \frac{(-1)^{k+1}}{(2k-1)(2k+1)} T_{2k}(x), \quad x \in [-1, 1],$$

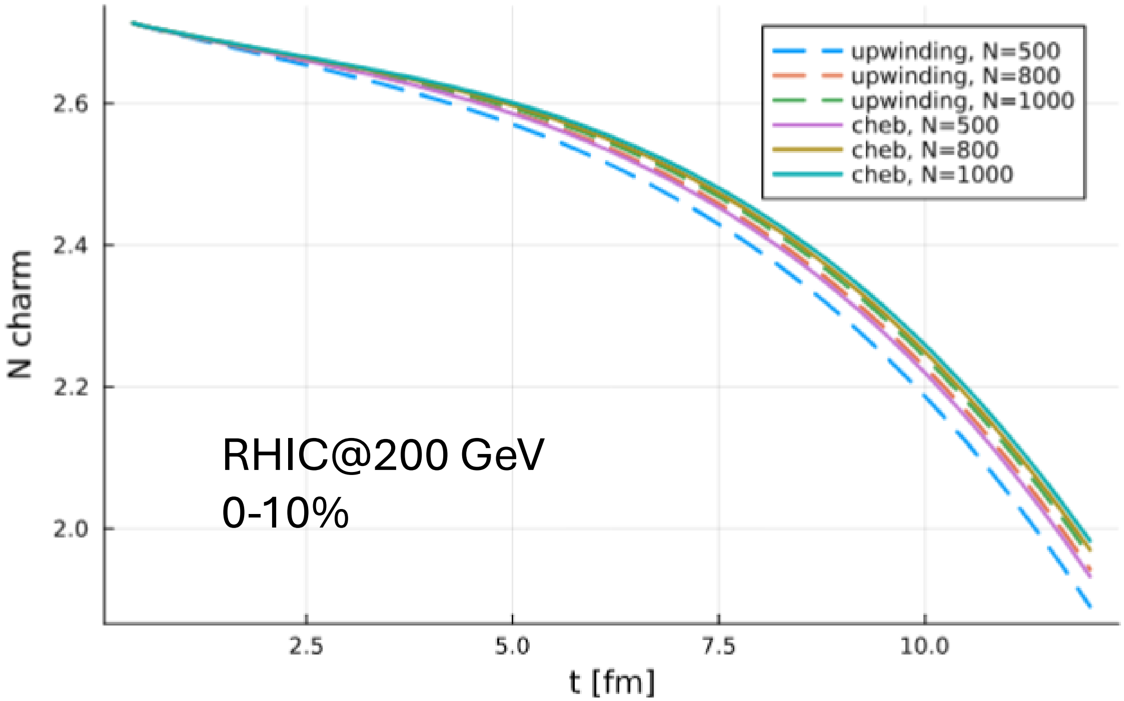
$$T_0(x) = 1, \quad T_2(x) = 2x^2 - 1, \quad T_{2k}(x) = 2T_2(x)T_{2k-2}(x) - T_{2k-4}(x).$$



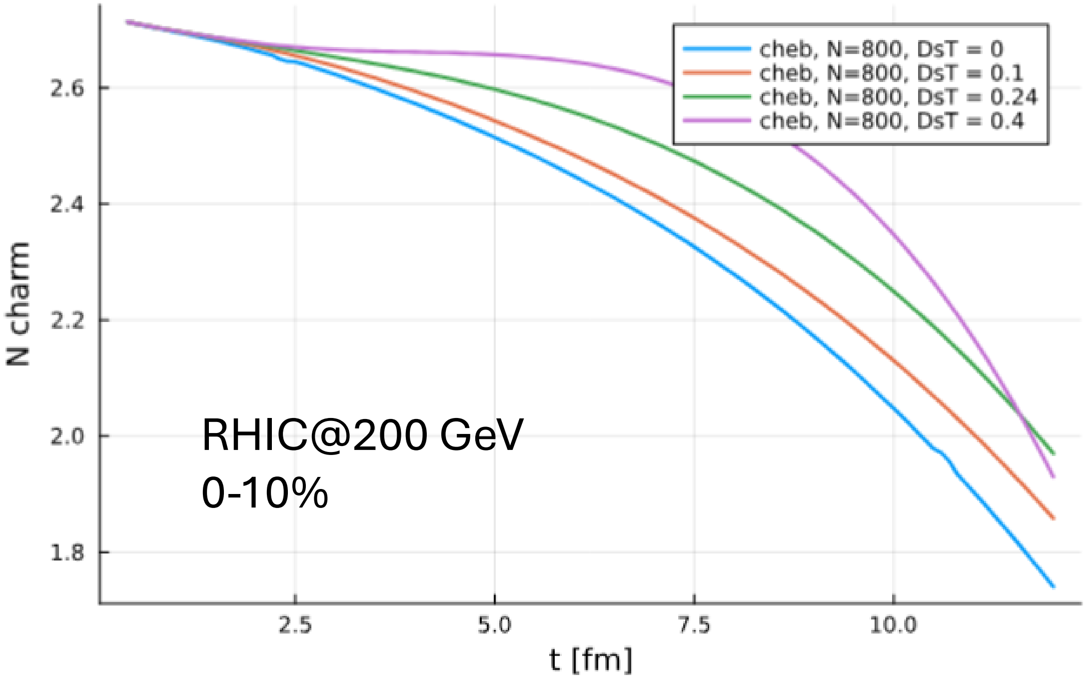
- $f(x) \geq |x|$ not verified but no difference is observed in computation (Castro et al.)
- Error decreases with 1/(2p+1)

Accuracy on the conservation law

Varying N points: comparison with previous method

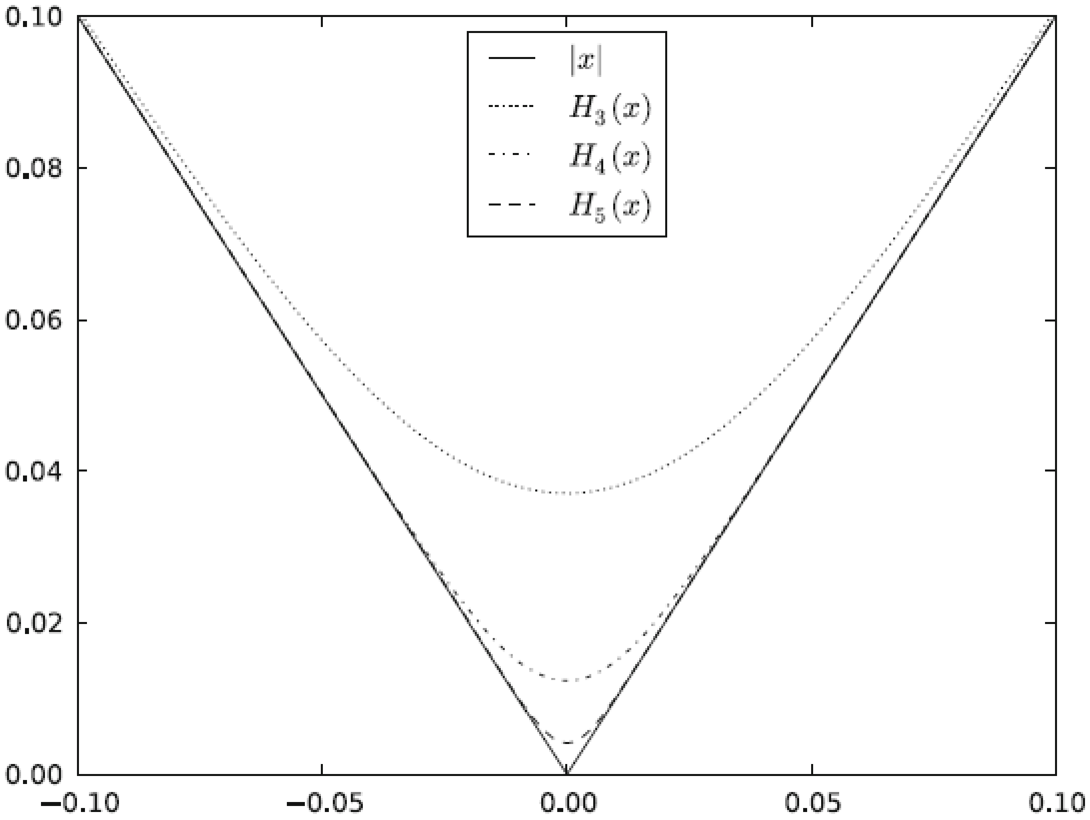


Varying heavy-quark diffusion



Can we do better?

Halley rational polynomials (Castro, Gallardo, Marquina, 2014)



$$H_{r+1}(x) = H_r(x)\,\frac{H_r(x)^2 + 3x^2}{3H_r(x)^2 + x^2}, \quad H_0(x) = 1.$$

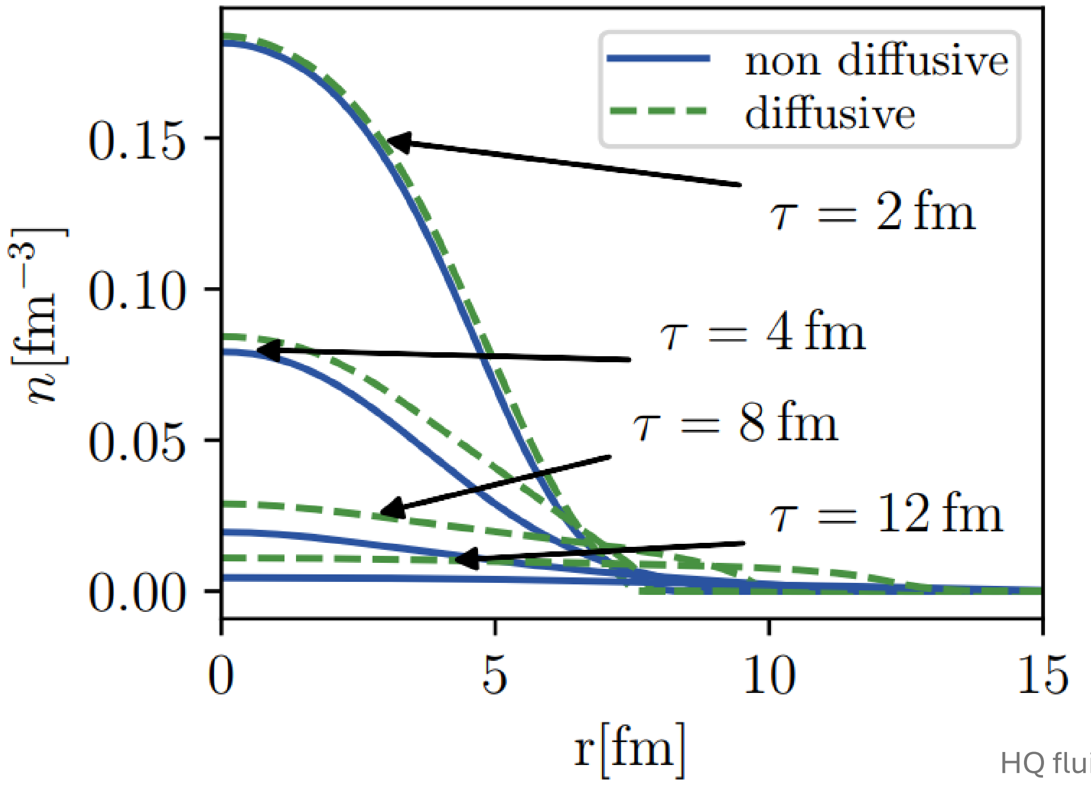
Phenomenology

Evolution of charm fields

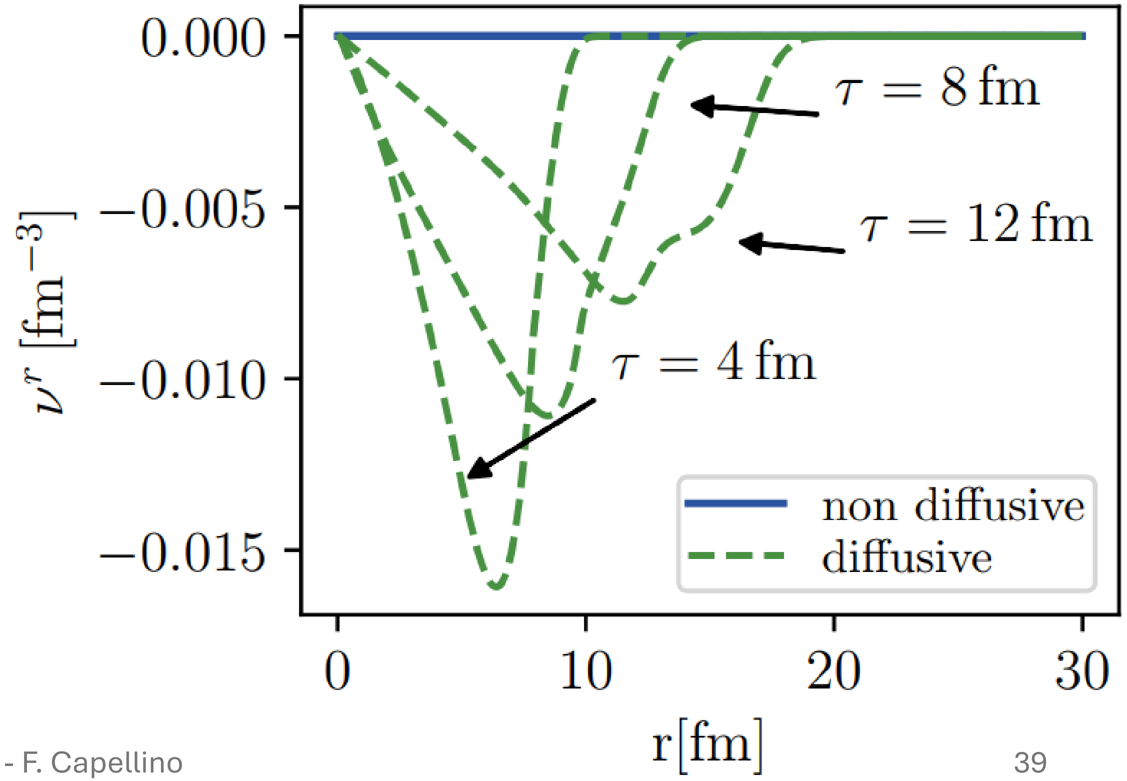
The evolution of the charm density and diffusion current depend on the **spatial diffusion coefficient** D_s .

The ratio $|\nu^r|/n$ is not always <<1 $\longrightarrow$ The larger D_s, the larger the out-of-equilibrium corrections coming from ν^r are throughout the QGP evolution.

charm density



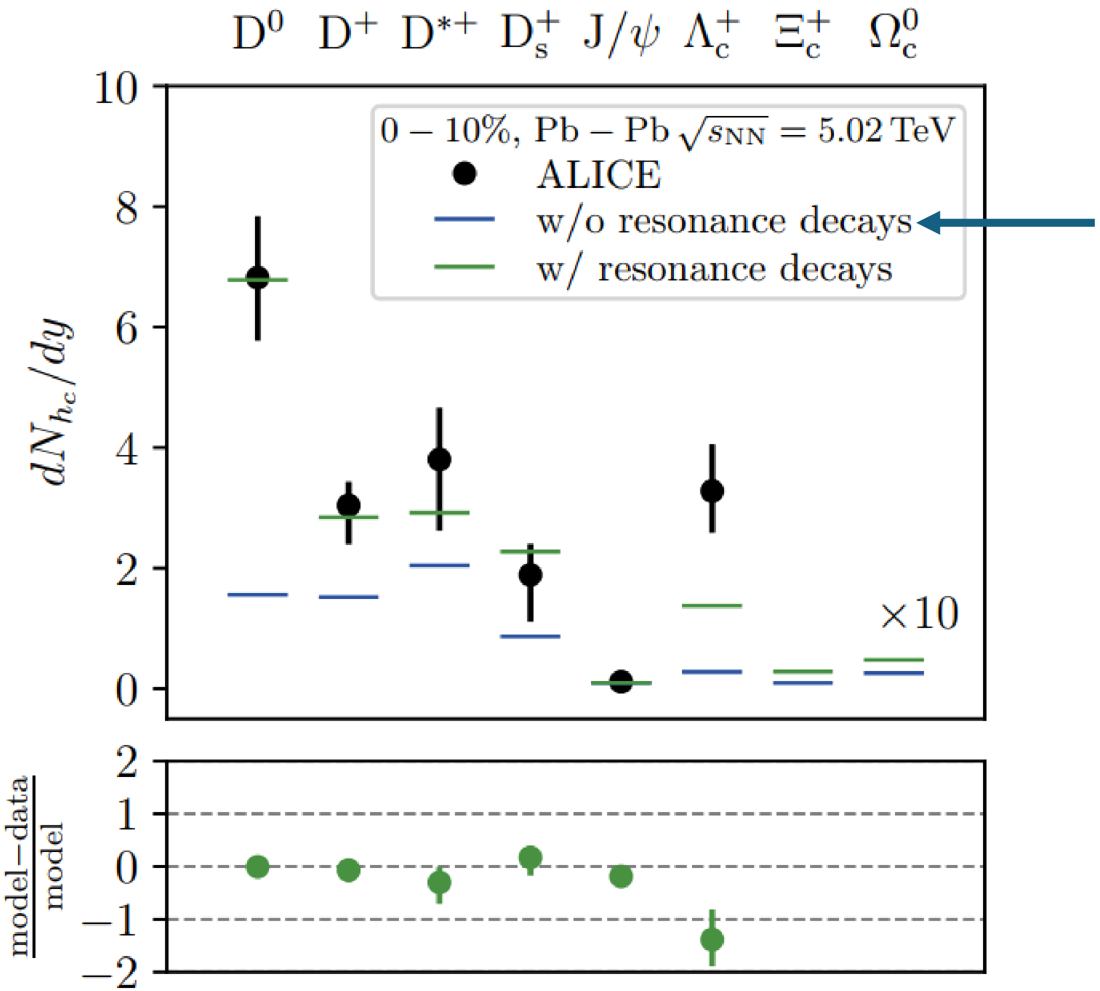
charm diffusion current



Charm hadron integrated yields *dN/dy*

- **Cooper-Frye at T_{fo} = 156.5 MeV**

$$\frac{dN_{hc}}{p_T\, dp_T\, dy} = \frac{g}{(2\pi)^3} \int_{\Sigma_{FO}} d\Sigma_\mu p^\mu f_{hc}(p^\mu, x^\mu)$$



Capellino *et al.* PRD 108 (2023) 11, 116011
ALICE JHEP 01 (2022) 174, ALICE PLB 849 (2024) 138451, ALICE PLB 839 137796 (2023), ALICE PLB 827 136986 (2022)

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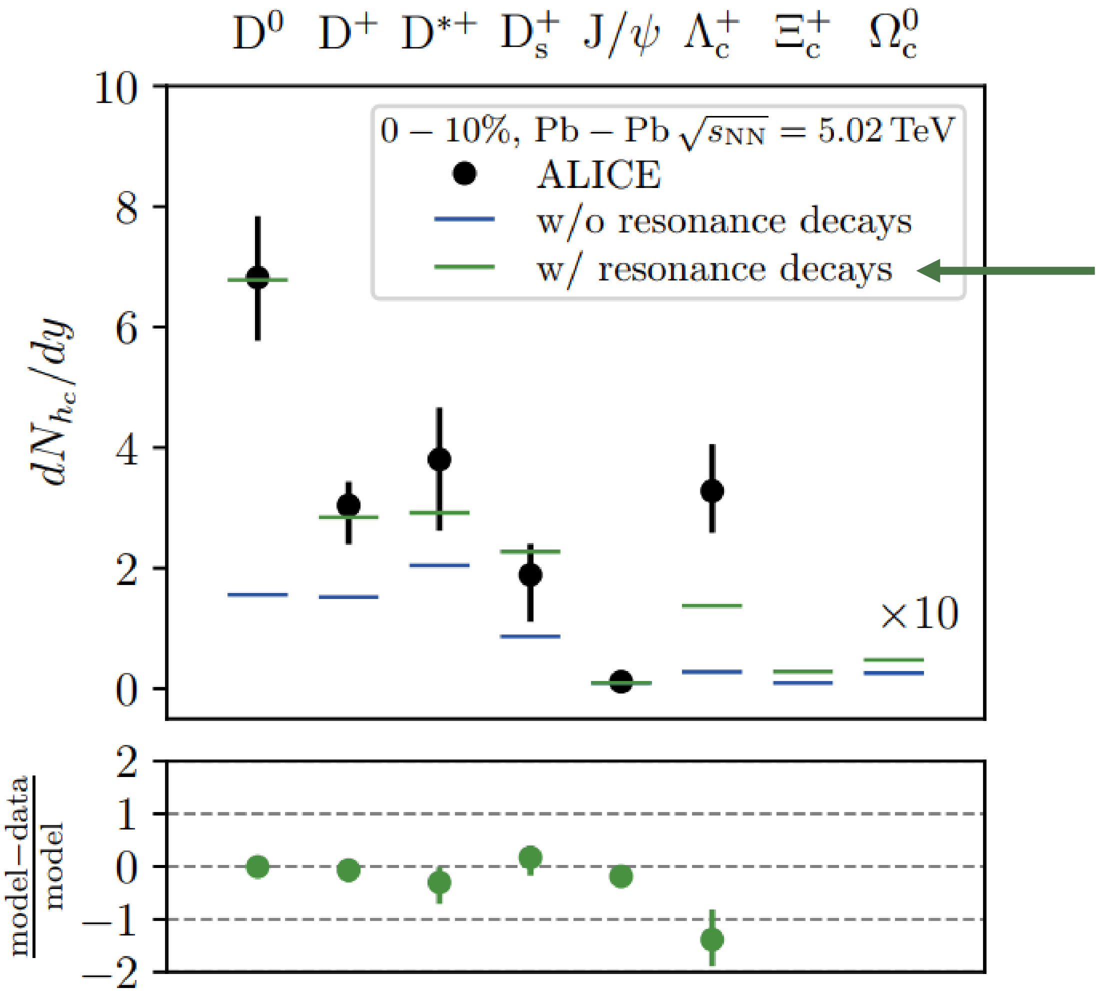
Mazeliauskas *et al.* Eur. Phys. J. C (2019) 79: 284



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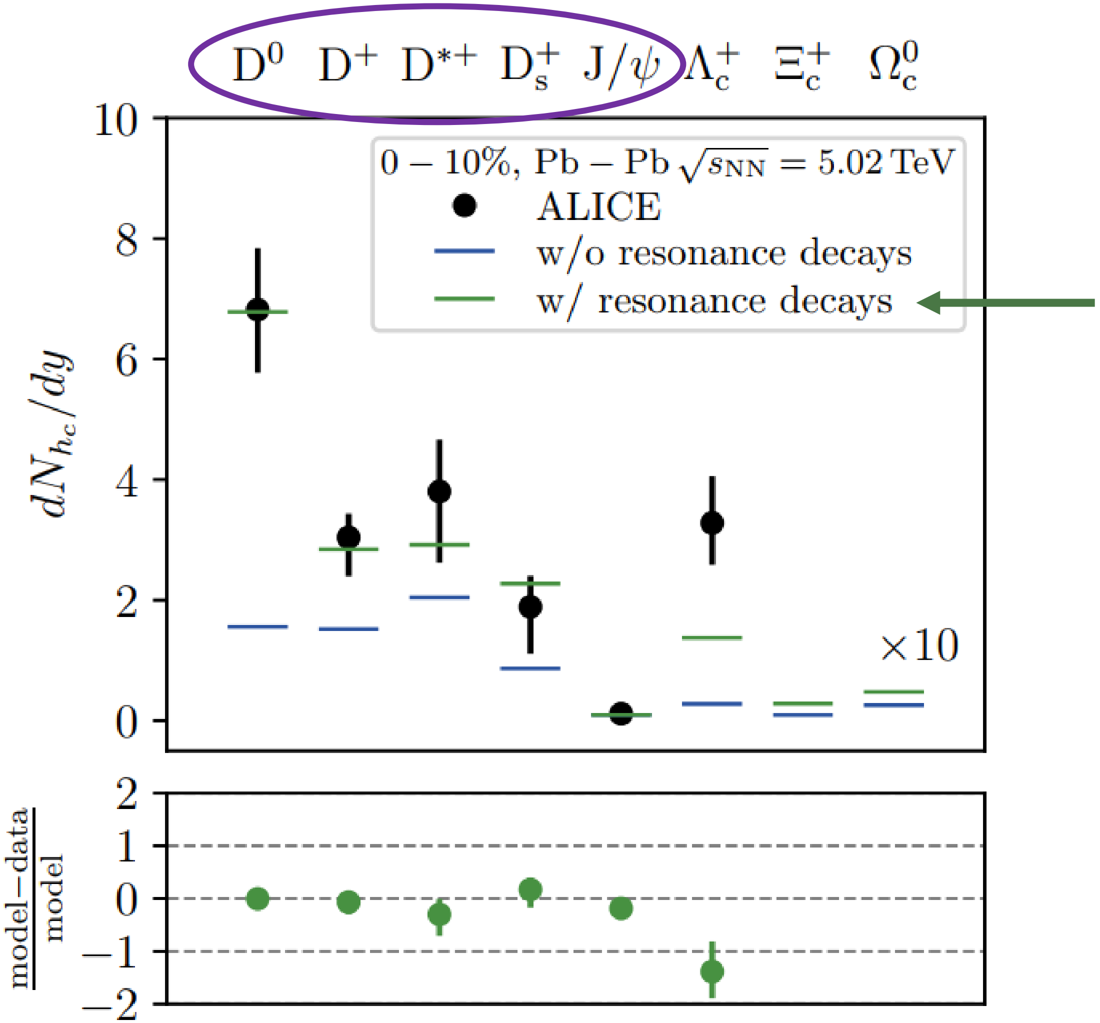
- **Mesons compatible with the experimental data**



Capellino *et al.* PRD 108 (2023) 11, 116011
ALICE JHEP 01 (2022) 174, ALICE PLB 849 (2024) 138451, ALICE PLB 839 137796 (2023), ALICE PLB 827 136986 (2022)

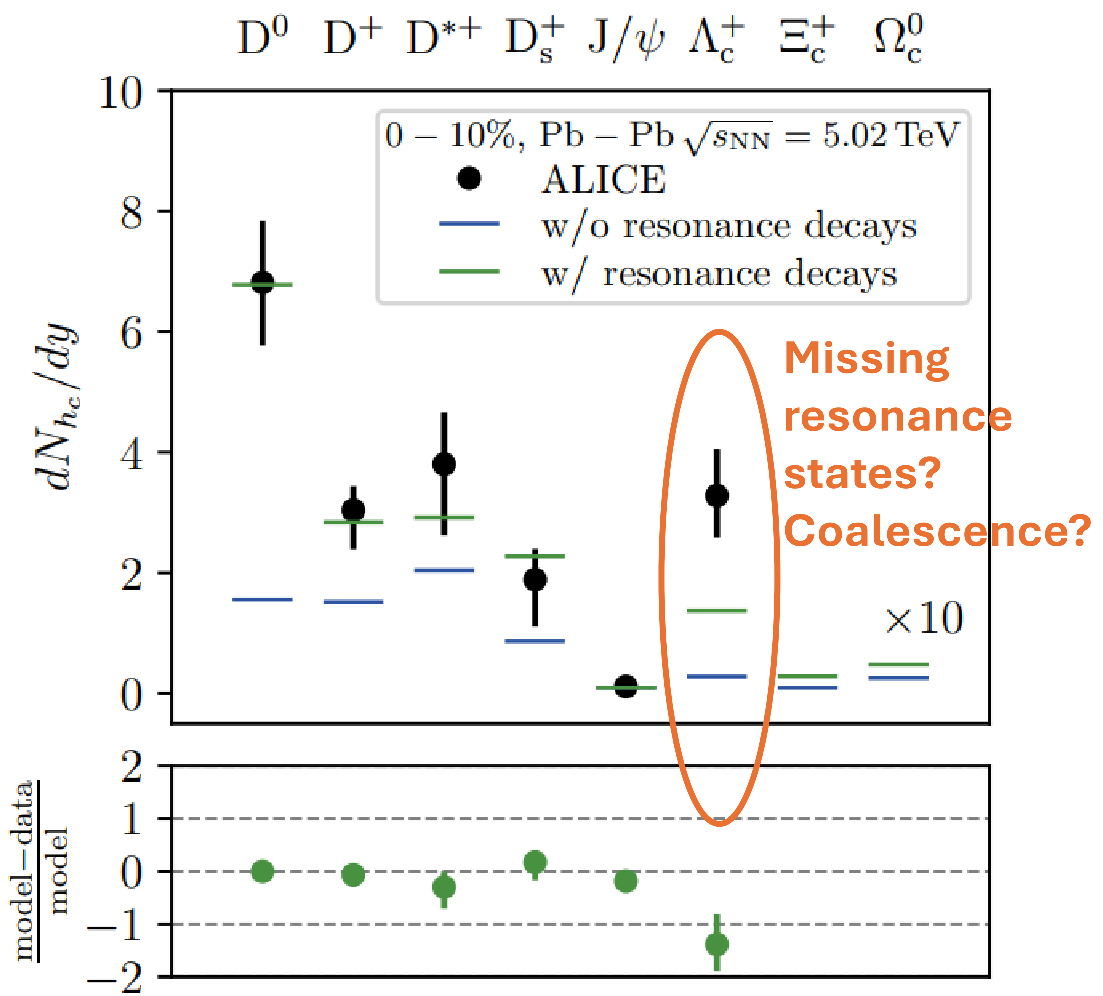
Charm hadron integrated yields *dN/dy*

- **Cooper-Frye at T_{fo} = 156.5 MeV + PDG resonances**

$$\frac{dN_{hc}}{p_T dp_T dy} = \frac{g}{(2\pi)^3} \int_{\Sigma_{FO}} d\Sigma_\mu p^\mu f_{hc}(p^\mu, x^\mu)$$

Mazeliauskas *et al.* Eur. Phys. J. C (2019) 79: 284

- **Mesons compatible with the experimental data**



Capellino *et al.* PRD 108 (2023) 11, 116011
ALICE JHEP 01 (2022) 174, ALICE PLB 849 (2024) 138451, ALICE PLB 839 137796 (2023), ALICE PLB 827 136986 (2022)

Charm hadron integrated yields *dN/dy*

Not yet measured!

- **Cooper-Frye at T_{fo} = 156.5 MeV + PDG resonances**

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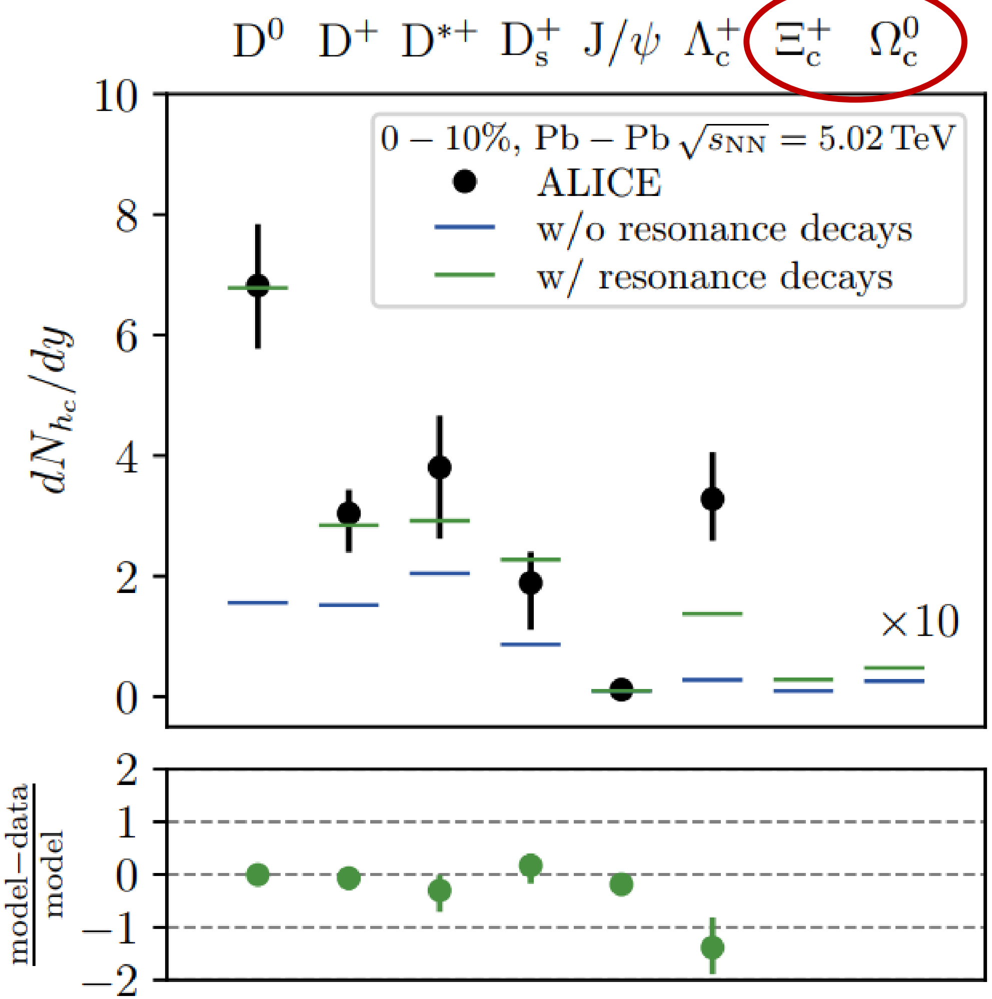

Capellino *et al.* PRD 108 (2023) 11, 116011
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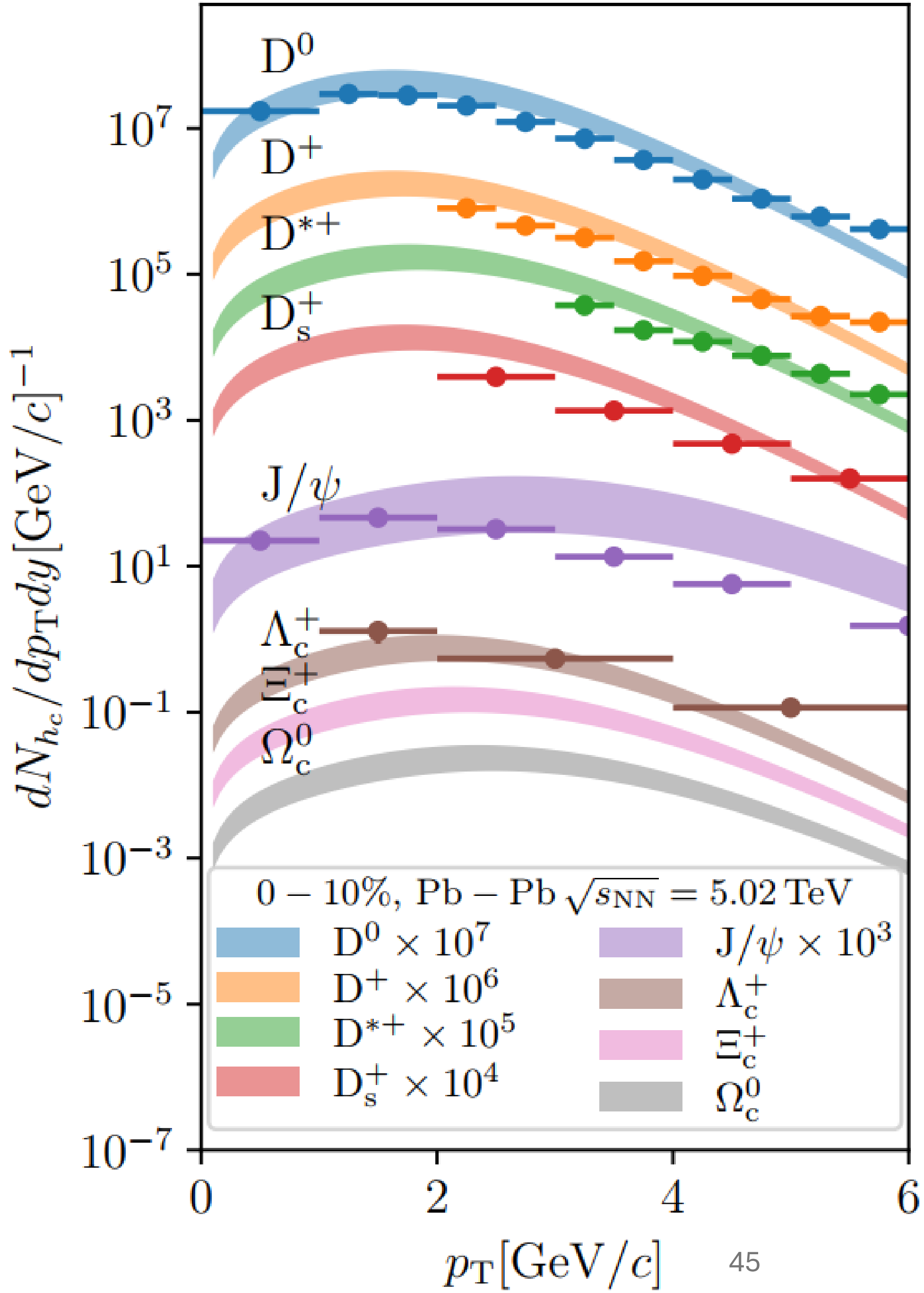
Momentum distributions

- Cooper-Frye at T_{fo} = 156.5 MeV + PDG resonances



Capellino *et al.* PRD 108 (2023) 11, 116011

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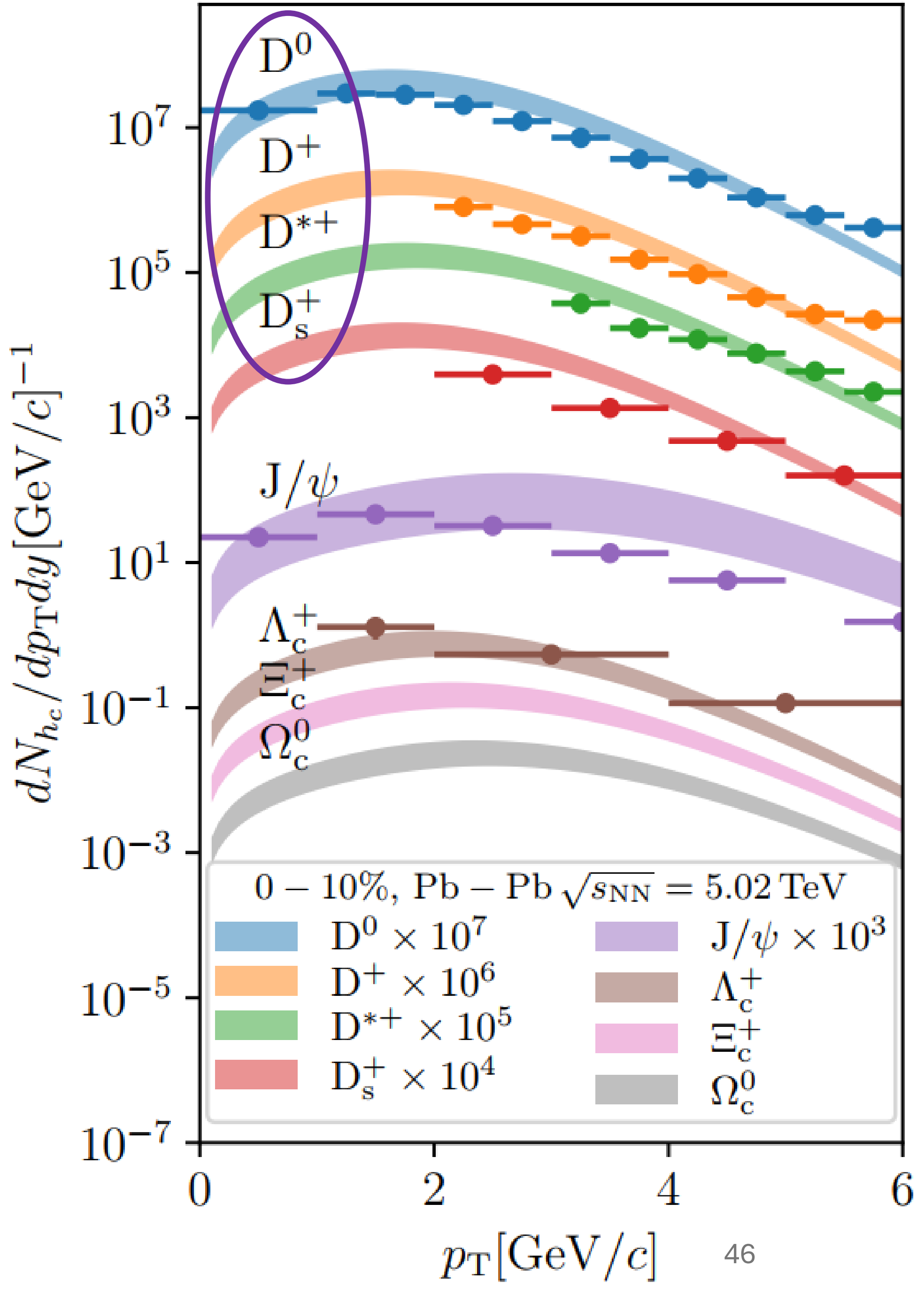
Momentum distributions

- Cooper-Frye at T_{fo} = 156.5 MeV + PDG resonances
- Fluid dynamics for D mesons up to 4-5 GeV



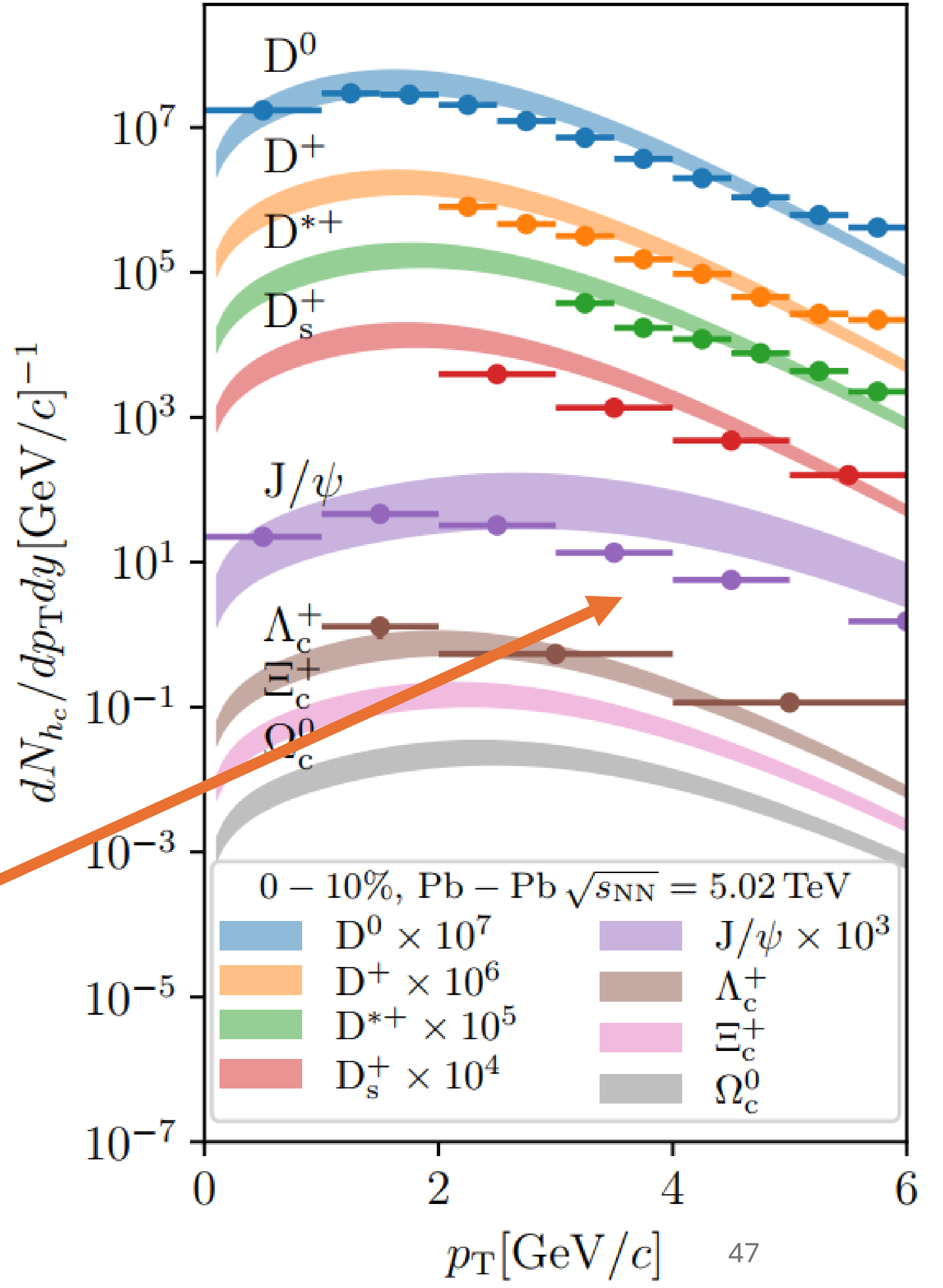
Capellino *et al.* PRD 108 (2023) 11, 116011

ALICE JHEP 01 (2022) 174
ALICE PLB 849 (2024) 138451
ALICE PLB 839, 137796 (2023)
ALICE PLB 827, 136986 (2022)

Momentum distributions

- Cooper-Frye at T_{fo} = 156.5 MeV + PDG resonances
- Fluid dynamics for D mesons up to 4-5 GeV

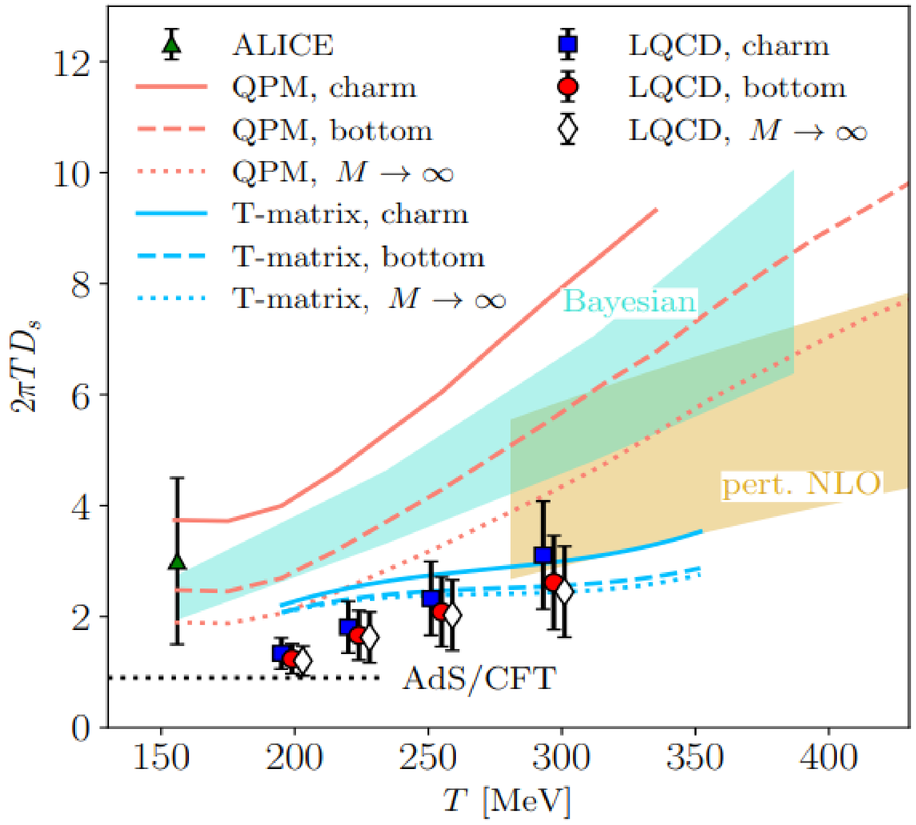
Primordial J/ψ ?



Capellino *et al.* PRD 108 (2023) 11, 116011

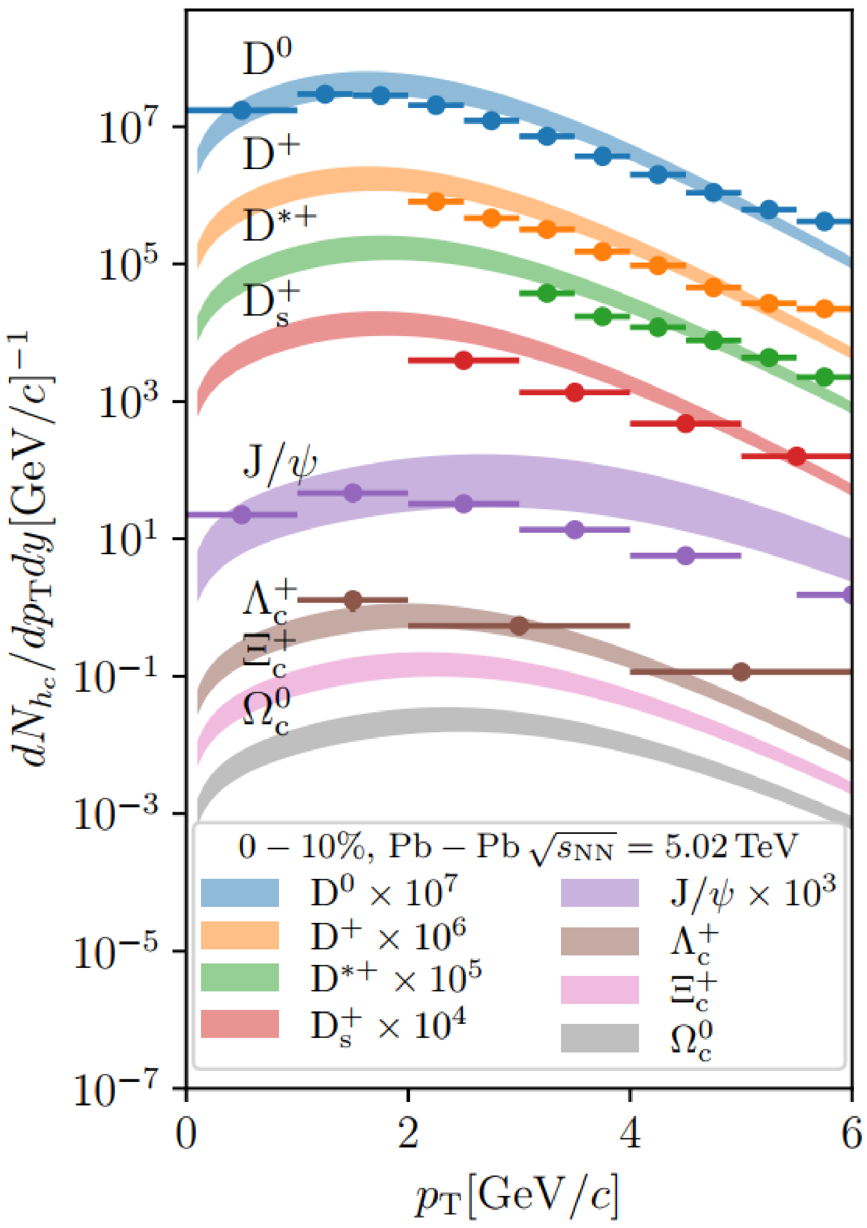
ALICE JHEP 01 (2022) 174
ALICE PLB 849 (2024) 138451
ALICE PLB 839, 137796 (2023)
ALICE PLB 827, 136986 (2022)

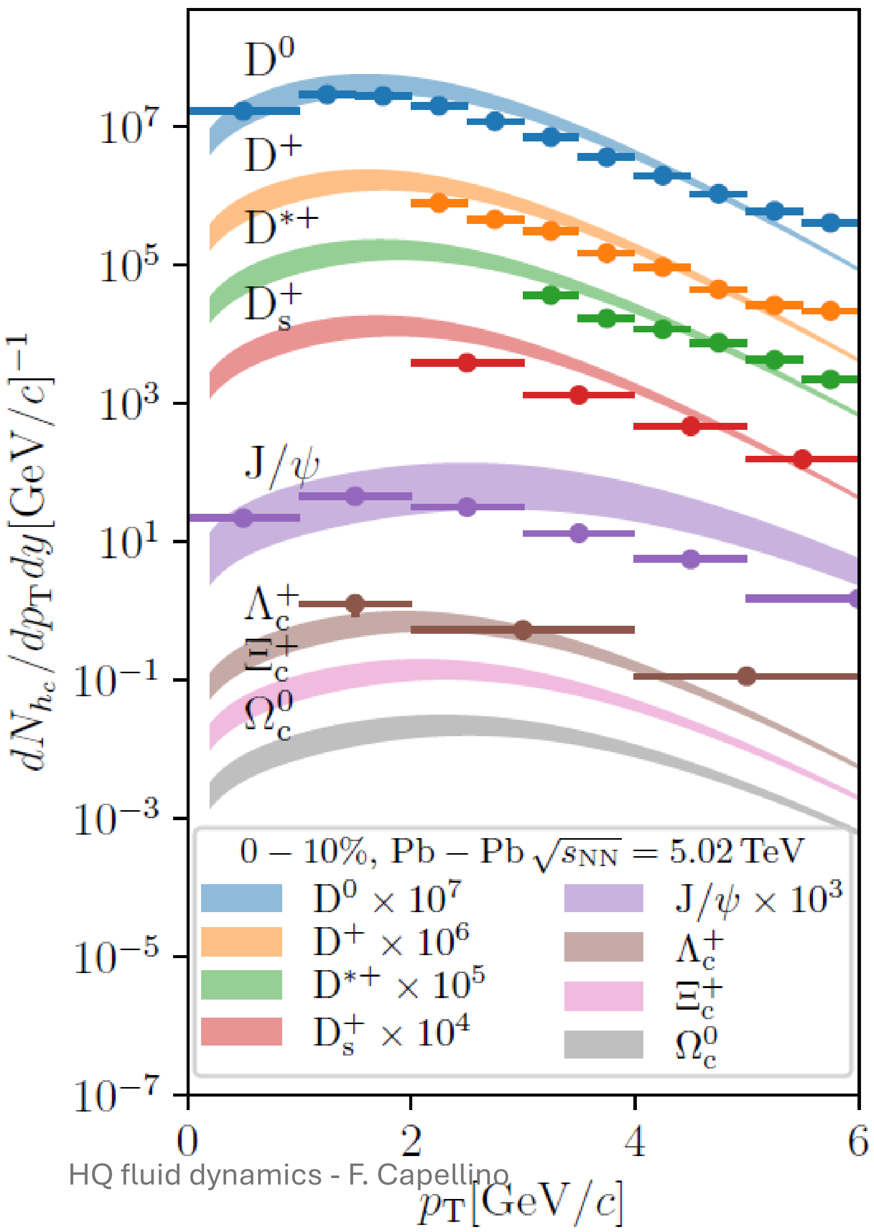
Temperature dependence of D_s



Linear fit to the central points of the LQCD infinite-mass diffusion coefficient to allow for temperature dependence of Ds

Temperature dependence of D_s





Left panel: **D_s at T=Tc**

Right panel: T**D_s linear in T**

- Spectra mostly sensitive on D_s at T=Tc
- The temperature dependence influences mostly the intermediate-high pT

New directions

Fluid dynamics of charm at the LHC

Fluid dynamics of beauty?

Smaller cross section

$M_b \approx 3M_c$
Larger relaxation time

RHIC top energies?

Smaller cross section

Lower temperatures:
Larger relaxation time

Finite μ_B

Smaller systems?

Smaller cross section

Lower temperatures:
Larger relaxation time

Deformations (v_n)

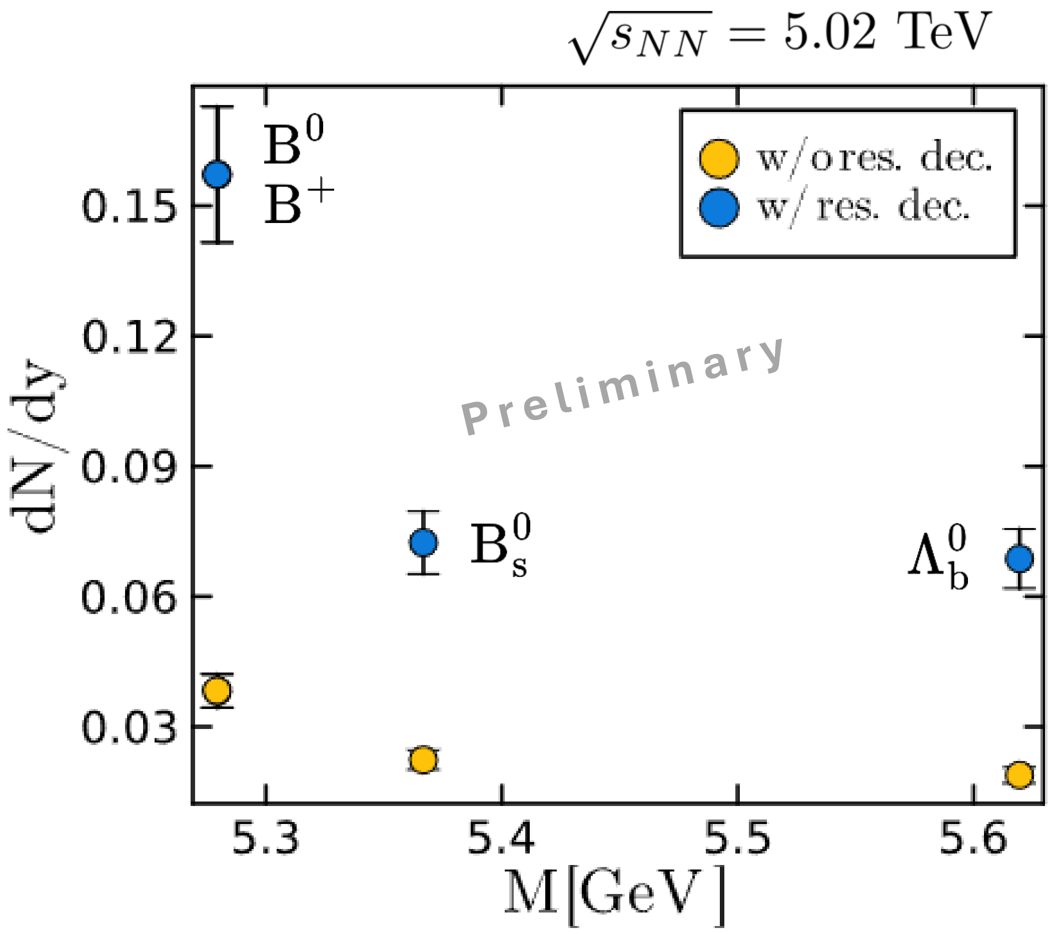
New directions

Fluid dynamics of charm at the LHC

Fluid dynamics of beauty?	RHIC top energies?	Smaller systems?
Smaller cross section	Smaller cross section	Smaller cross section
$M_b \approx 3M_c$ Larger relaxation time	Lower temperatures: Larger relaxation time	Lower temperatures: Larger relaxation time
	Finite μ_B	Deformations (v_n)

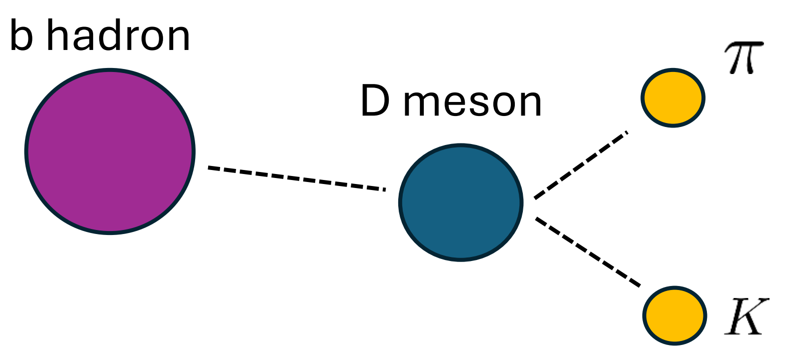
Beauty hadron integrated yields *dN/dy*

Cooper-Frye prescription at T_{fo} = 156.5 MeV + Resonance decays (FastReso)

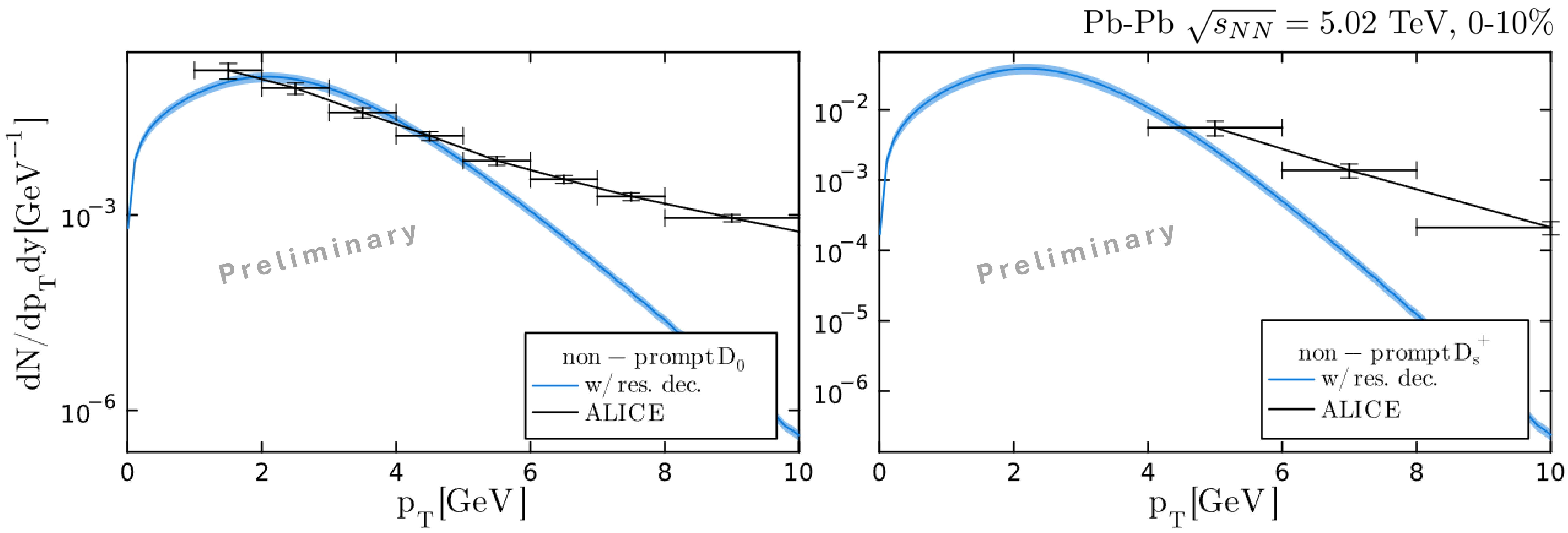


- Uncertainty from $b\bar{b}$ production cross section
  fugacity = $0.86 \cdot 10^9 - 1.05 \cdot 10^9$
- Presence of **currently unknown open bottom** states would lead to a change in hadrons' relative abundance

Non-prompt charm hadrons



- Pythia8 decay list arXiv:2203.11601 [hep-ph]
- Momentum distribution compatible within uncertainties up to 6 GeV
- Caveat: $D_s\!=\!0$

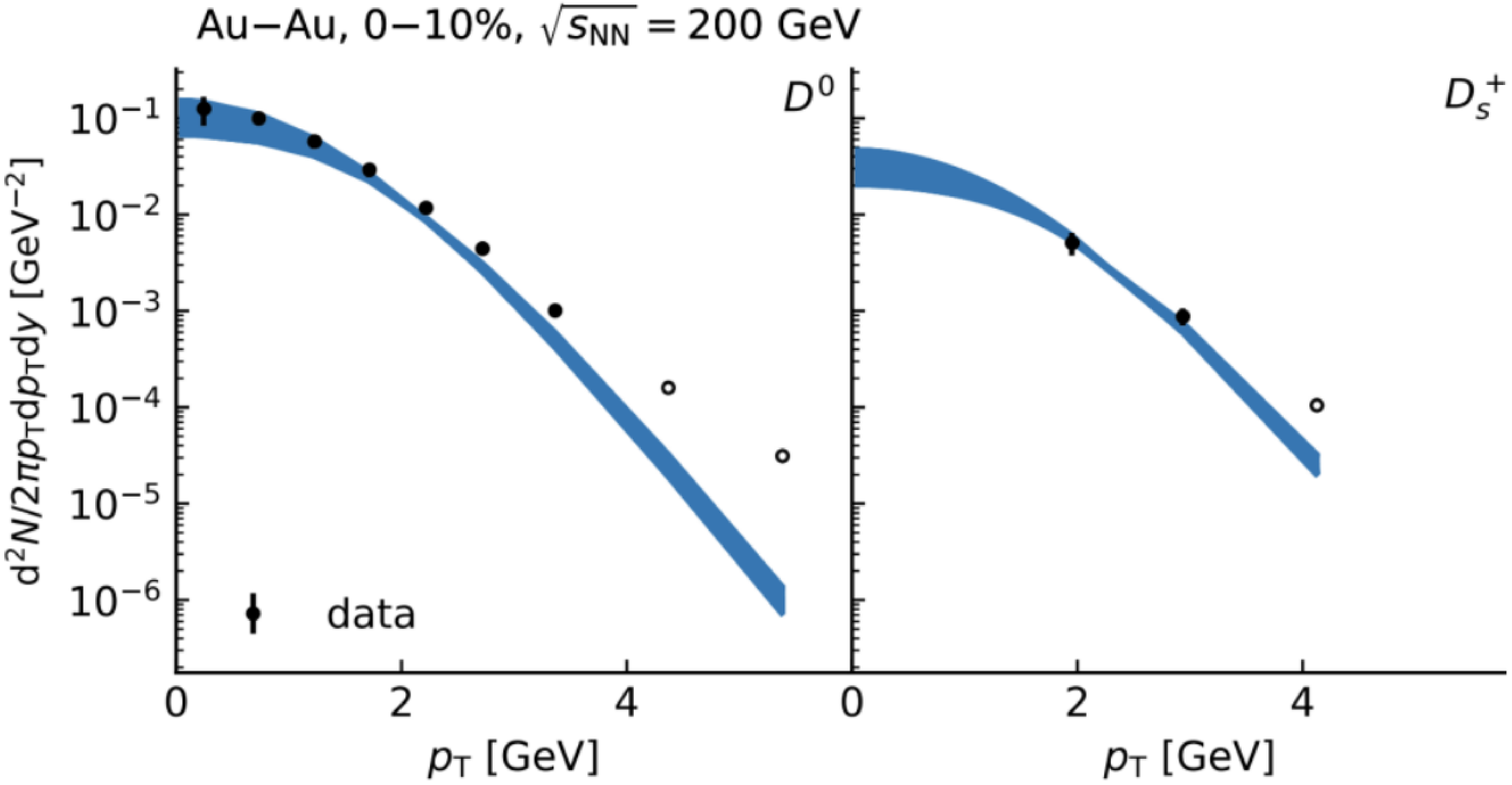


ALICE JHEP 12 (2022) 126, 2022
ALICE PLB 846 (2023) 137561, 2023

Top RHIC energies

2-3 ccbar per unit y produced!
Fluid dynamics still works!

Rossana Facen's Master Thesis

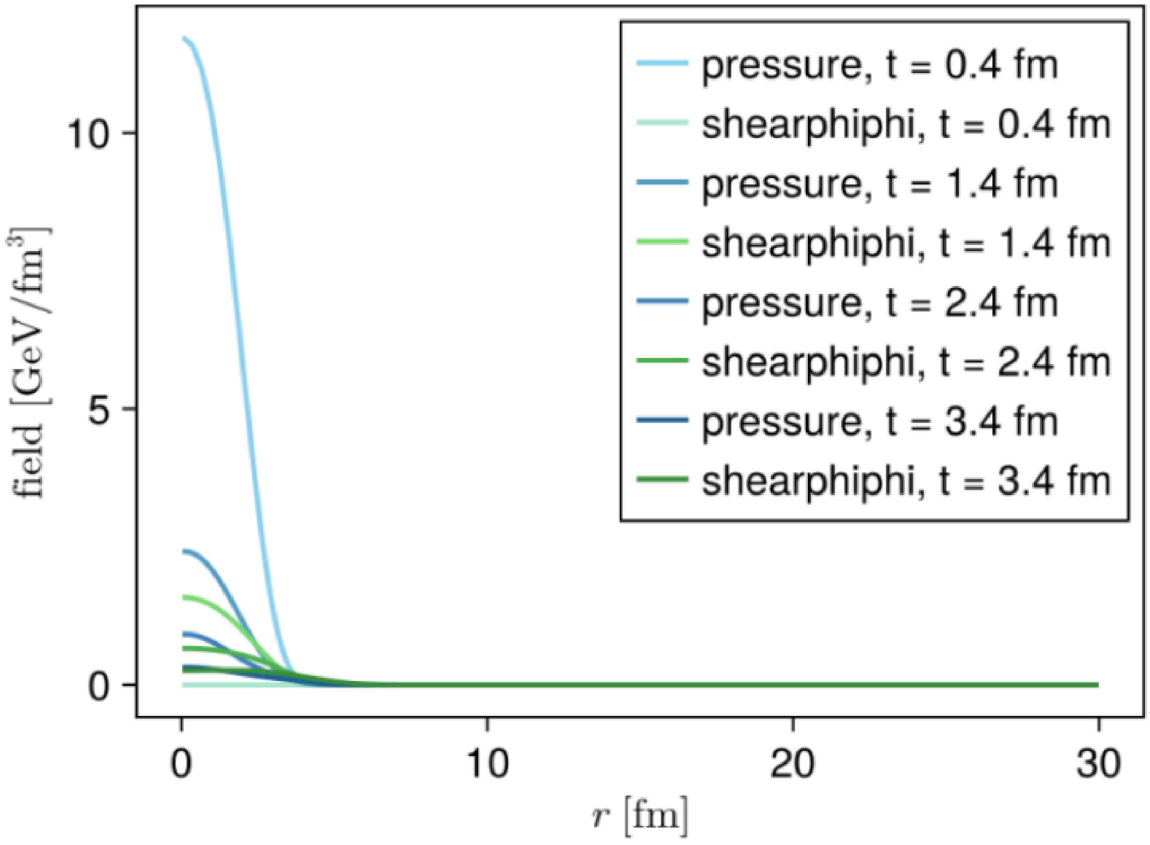


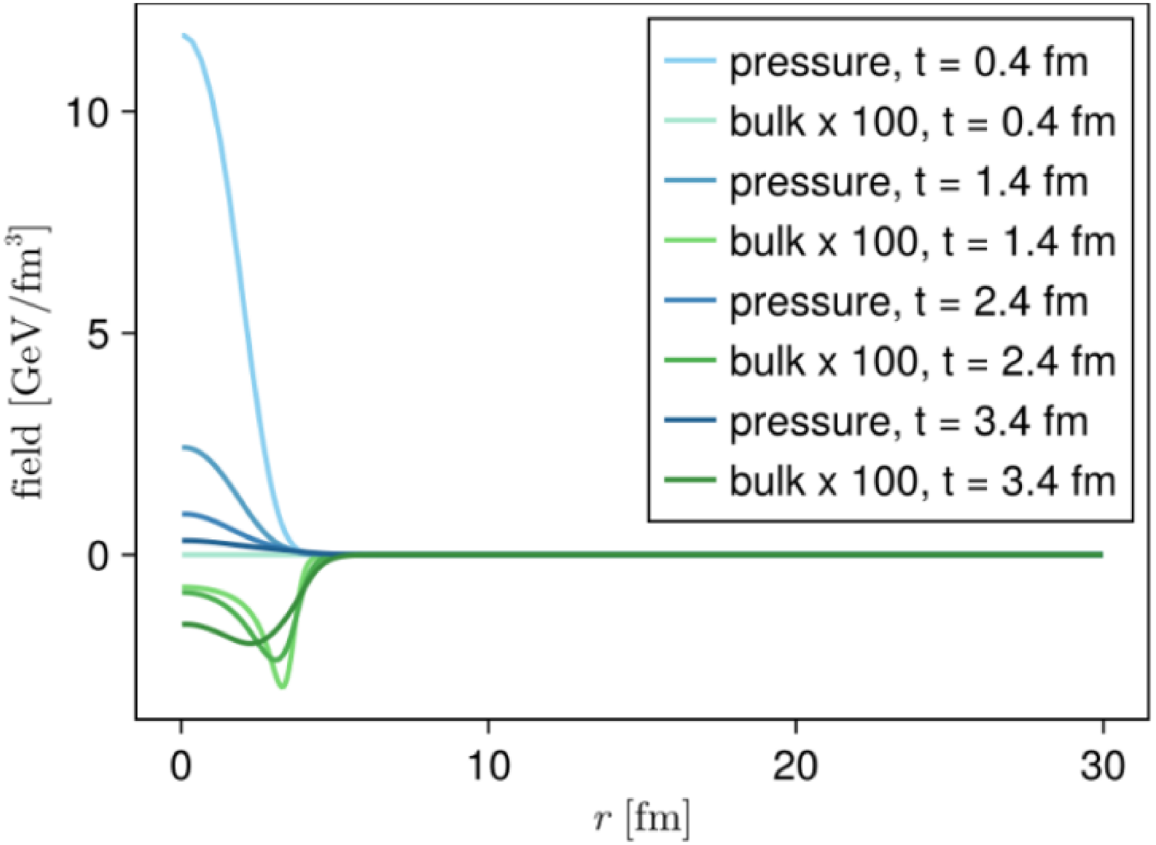
https://www.physi.uni-heidelberg.de/Publications/RossanaFacen_MasterThesis.pdf
PRC 99.3 (2019), PRL 127 (2021)

Fluid dynamics of QGP in OO collisions

Caveat: Assuming a thermalized QGP is produced, assuming fluid dynamics is applicable

Fluid dynamics is consistent for the bulk fields

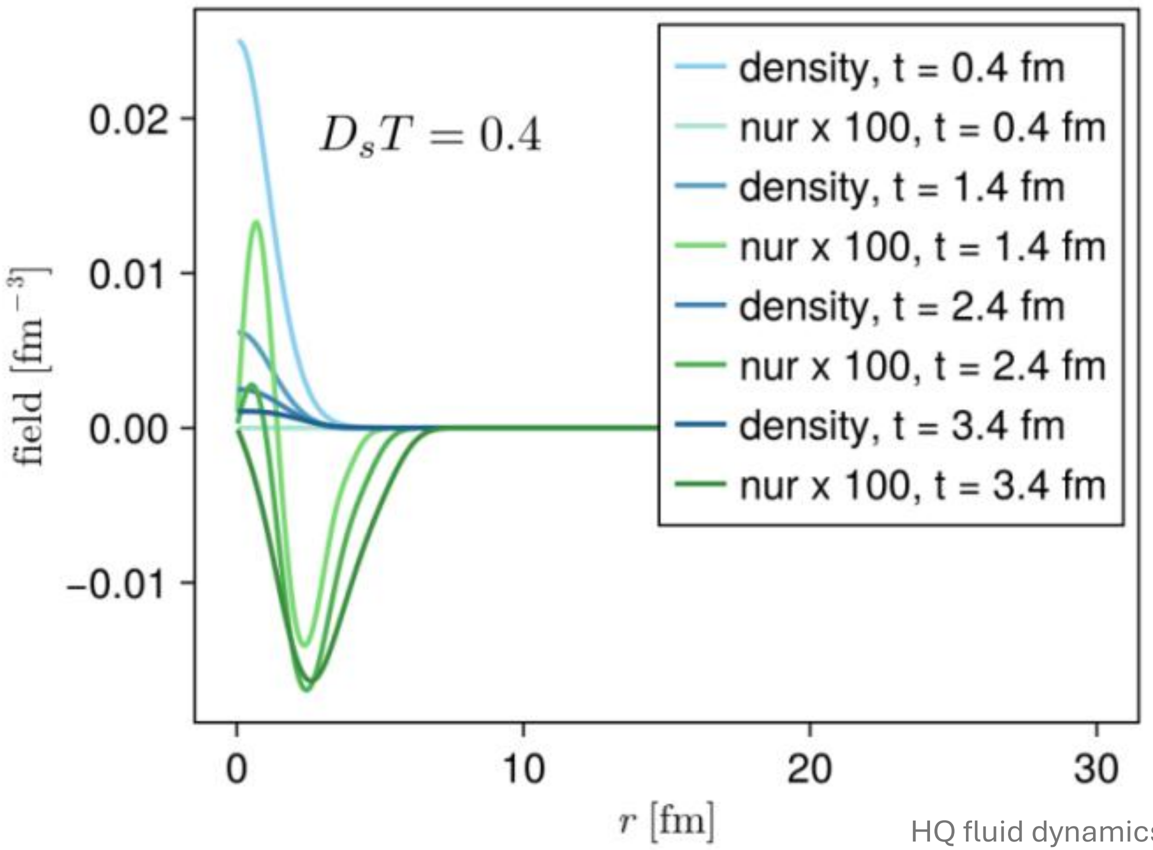


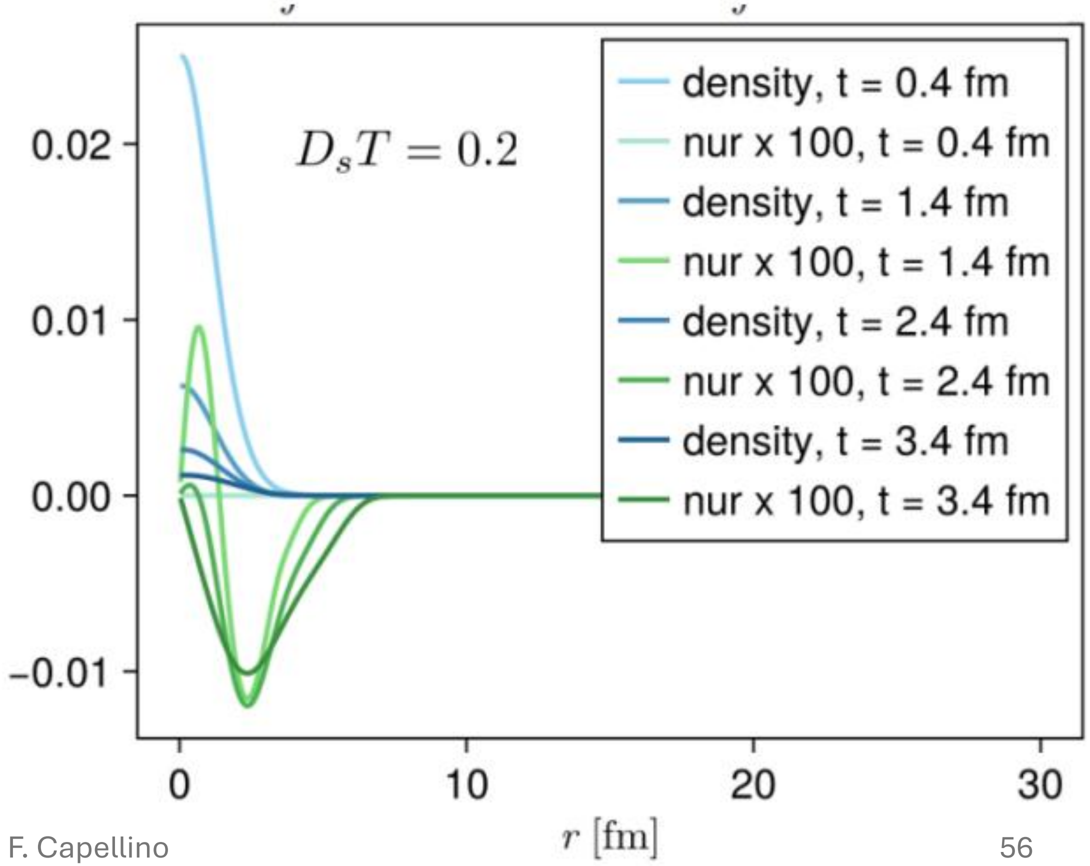


Fluid dynamics of QGP in OO collisions

Fluid dynamics is consistent for the heavy quark fields

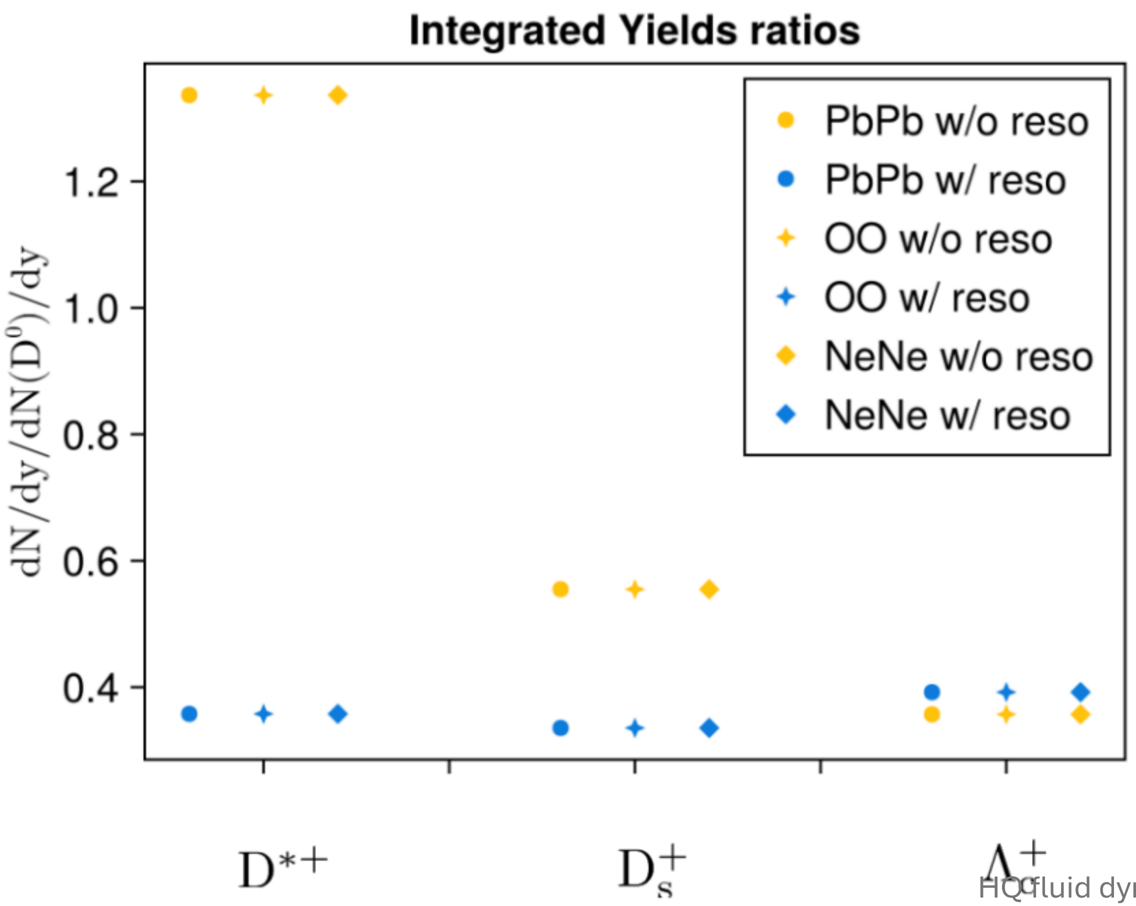
$$\boxed{\nu^r/n \ll 1}$$





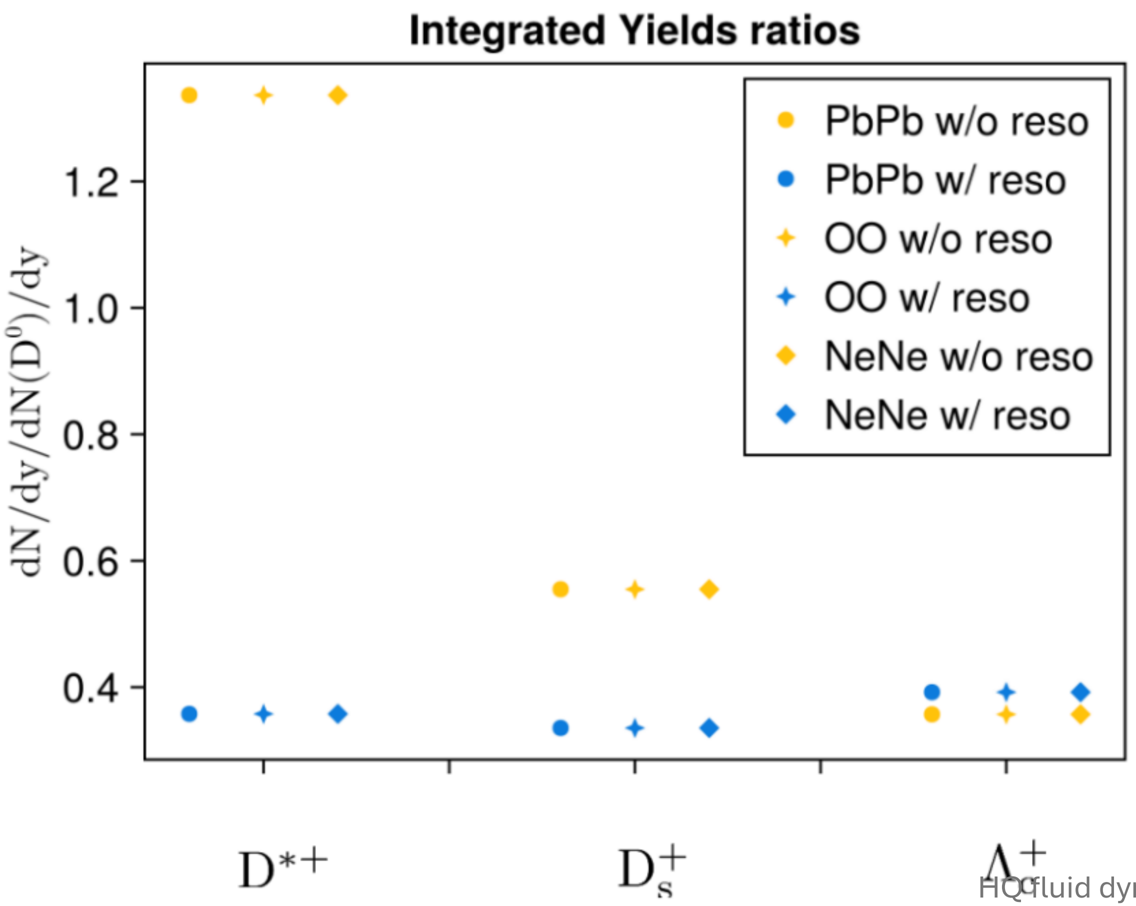
Take-home message from small systems

- Thermal production+resonance decays respects the ratio across different systems/energies
- Measuring these ratios immediately tells us if charm is produced thermally at freeze out or not
- **Looking forward to data!**

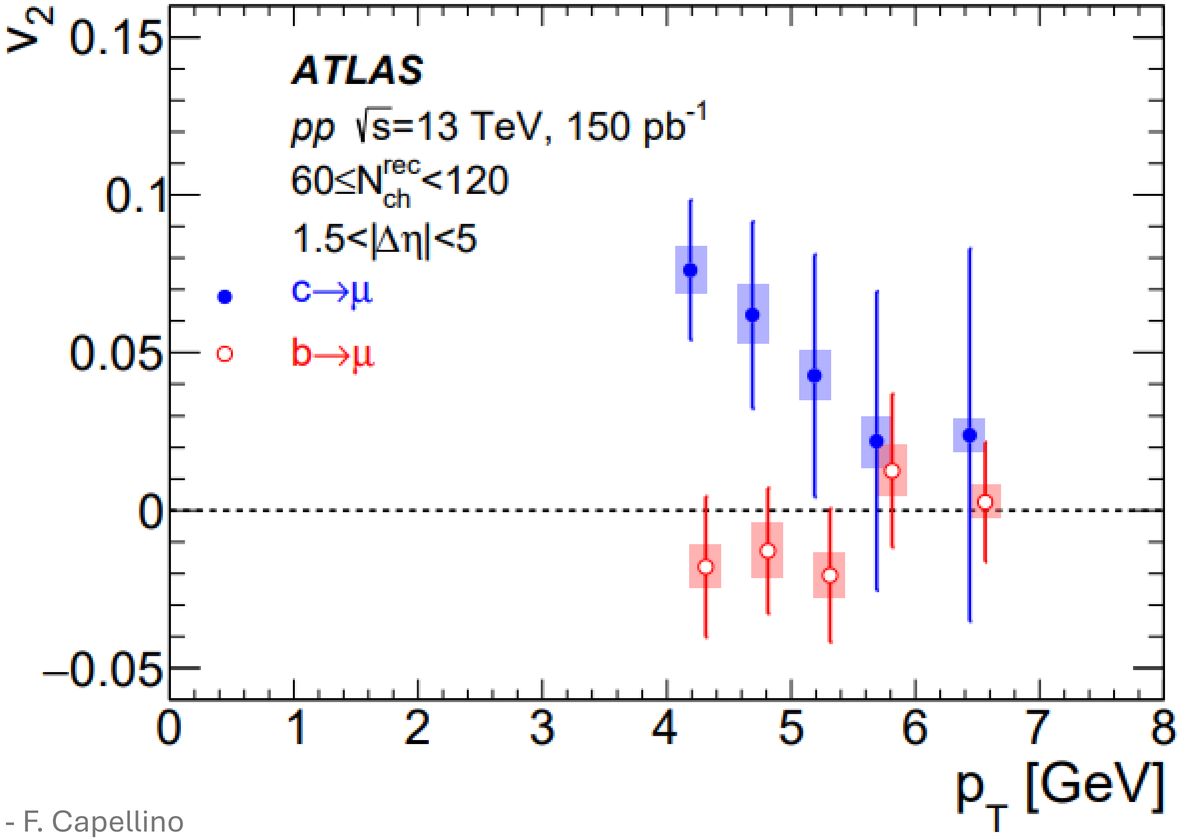


Take-home message from small systems

- Thermal production+resonance decays respects the ratio across different systems/energies
- Measuring these ratios immediately tells us if charm is produced thermally at freeze out or not
- **Looking forward to data!**
- Pushing the limits: **pp collisions?**



ATLAS coll., *PRL* 124 (2020) 8, 082301



Newest developments

Out-of-equilibrium corrections

Work in progress

$$f \sim \underbrace{e^{-E/T} e^{\mu/T}}_{n} \underbrace{+\delta f}_{\nu^\mu}$$

Current setup

$\nu^\mu = 0$ at τ_0

$\nu^\mu = 0$ at freeze out

Facen, Capellino, Grossi et al, in preparation

Out-of-equilibrium corrections

Work in progress

$$f \sim \underbrace{e^{-E/T} e^{\mu/T}}_{n} \underbrace{+\delta f}_{\nu^\mu}$$

Current setup

$\nu^\mu = 0$ at τ_0

$\boxed{\nu^\mu = 0 \quad \text{at freeze out}}$

Multi fluid setup

$$\tau_n \approx \sum_i \tau_n^{(i \in HRG_c)}$$

$$\delta f = \delta f(M_i, q_i)$$

Facen, Capellino, Grossi et al, in preparation

Out-of-equilibrium corrections

Work in progress

$$f \sim \underbrace{e^{-E/T} e^{\mu/T}}_{n} \underbrace{+\delta f}_{\nu^\mu}$$

Current setup

$$\boxed{\nu^\mu = 0 \quad \text{at} \quad \tau_0}$$

$\nu^\mu = 0$ at freeze out

$$\frac{d\sigma^{Q\bar{Q}}}{p_T dp_T d\varphi dy}$$ only depends on pT

$$\nu^\mu = \int \frac{d^3p}{E} p^{\langle\mu\rangle} \delta f(p) \quad = 0$$

Facen, Capellino, Grossi et al, in preparation

Out-of-equilibrium corrections

Work in progress

$$f \sim \underbrace{e^{-E/T} e^{\mu/T}}_{n} \underbrace{+\delta f}_{\nu^\mu}$$

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$\nu^\mu = 0$ at freeze out

$\dfrac{d\sigma^{Q\bar{Q}}}{p_T dp_T d\varphi dy}$ only depends on pT

$\nu^\mu = \int \dfrac{d^3p}{E} p^{\langle\mu\rangle} \delta f(p)$ = 0

BUT: HQ initial free-streaming would give nonzero radial component!

Facen, Capellino, Grossi et al, in preparation

Freeze out

Matching kinetic and hydro…

$$\tau_i = \frac{D_s}{T}\frac{I_{i,31}}{P_i}$$

$$\kappa_i = q_i n_i D_s$$

…we get an expression for δf_i …

$$\delta f_i = f_{i,\mathrm{eq}}\left(-\frac{1}{T}\right)\frac{q_i}{\sum_j q_j^2 n_j}\nu^\mu\, k_\mu$$

…which is weighted by each charge q_i

$$\delta f = \sum_i q_i \delta f_i$$

From Facen's slides DPG 2025

- Redefinition of total relaxation time for charm current
- Mass/charge dependent contribution out of eq. for each charm hadron
- $\delta f_i = \delta f_{i,\mathrm{shear}} + \delta f_{i,\mathrm{bulk}} + \delta f_{i,\mathrm{diff}}$

Facen, FC, Grossi, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, in preparation

Initial conditions: free streaming

$$f = f_{FS}(p_x, p_y, x - \frac{\tau p_x p^\tau}{p_T^2 + m^2}, y - \frac{\tau p_y p^\tau}{p_T^2 + m^2}, \eta + \ln\left[\frac{p^\tau}{p^\eta} + \frac{1}{\tau}\right])$$

Romatschke
Eur.Phys.J.C 75 (2015) 9, 429

Used to determine

$$n = \int dp_x dp_y dp_\eta f_{eq}(\tau_{FS})$$

$$\nu^x = \int dp_x dp_y dp_\eta p^x \delta f(\tau_{FS})$$

Facen, FC, Grossi, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, in preparation

Initial conditions: free streaming

Distribution function divided in eq. and out-of-eq component

$$f_{\text{eq}} = \frac{1}{\tau_0} n_{\text{coll}}(r) \frac{1}{\sigma^{\text{in}}} \left. \frac{d\sigma^{Q\bar{Q}}}{dp_x dp_y d\eta} \right|_{eq}$$

Boltzmann normalized to cross section

$$\delta f = \frac{1}{\tau_0} n_{\text{coll}}(r) \frac{1}{\sigma^{\text{in}}} \left. \frac{d\sigma^{Q\bar{Q}}}{dp_x dp_y d\eta} \right|_{\delta} = \frac{1}{\tau_0} n_{\text{coll}}(r) \frac{1}{\sigma^{\text{in}}} \left(\left. \frac{d\sigma^{Q\bar{Q}}}{dp_x dp_y d\eta} \right|_{FONLL} - \left. \frac{d\sigma^{Q\bar{Q}}}{dp_x dp_y d\eta} \right|_{eq} \right)$$

Facen, FC, Grossi, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, in preparation

Summary and Outlook

Summary

- Fluid dynamics of charm at the LHC and RHIC compatible with experimental data
- Fluid dynamics of beauty + smaller systems: looking forward to data

Outlook

- Flow coefficients: work in progress (Capellino, Floerchinger, Grossi, Kirchner)
- Out of equilibrium corrections (Facen, Capellino, Grossi et al)

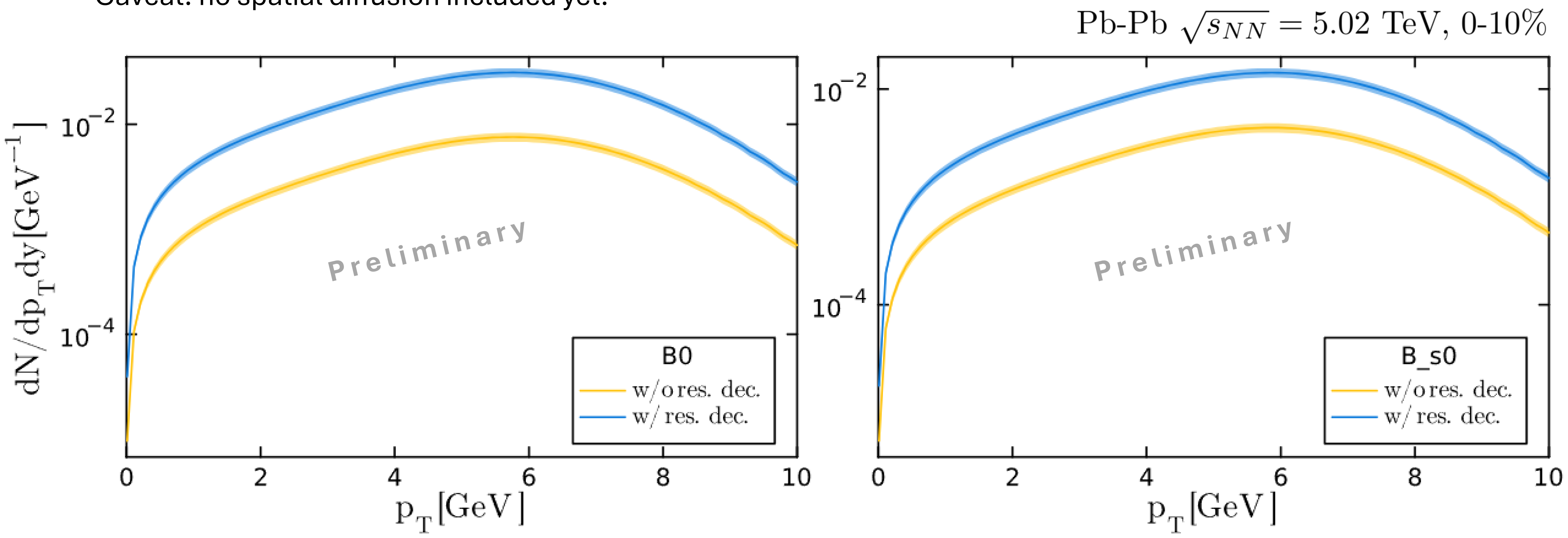
Thank you for your attention!

Back up

Open Beauty mesons

Cooper-Frye prescription at T_{fo} = 156.5 MeV (with Pythia8 decay list)

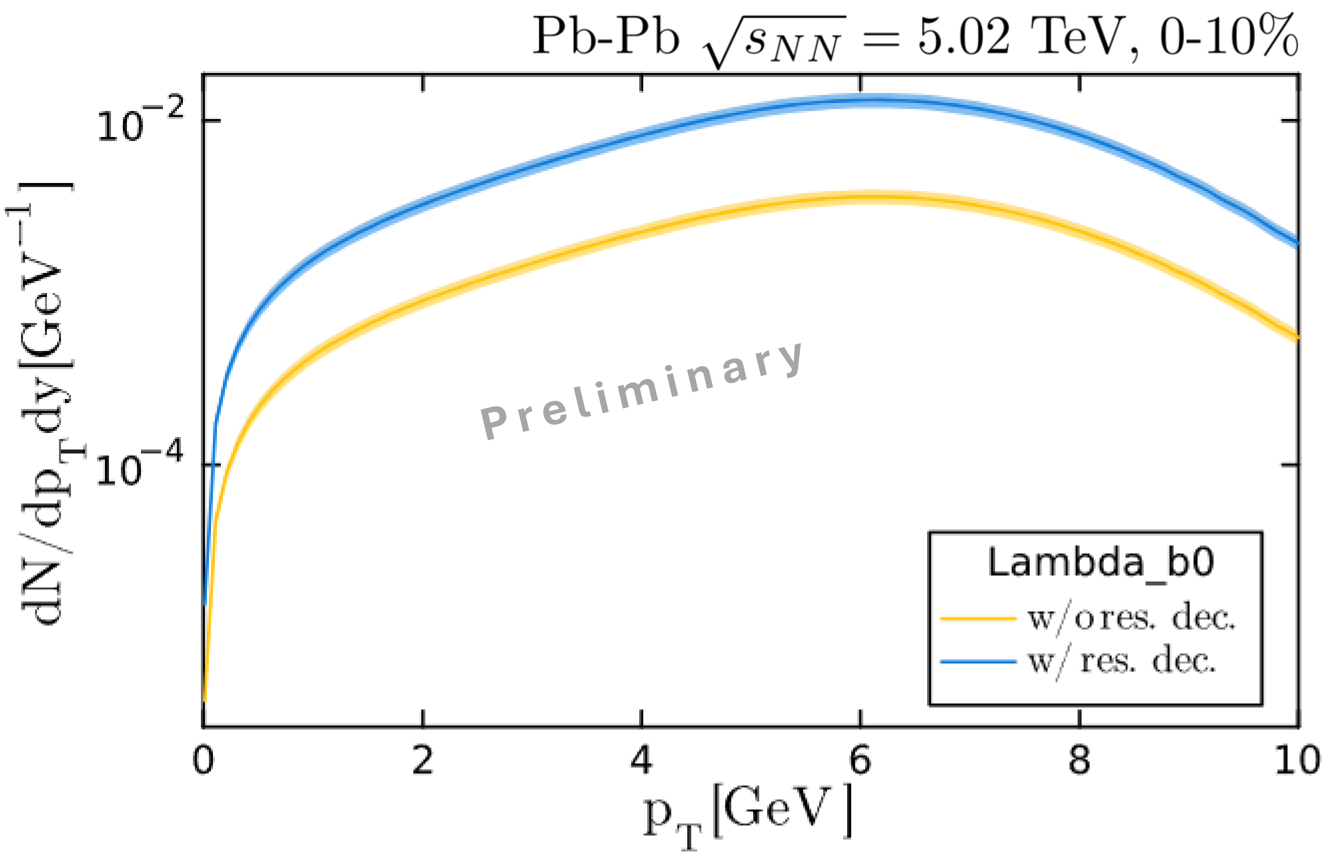
- Caveat: no spatial diffusion included yet!



Beauty baryons

Cooper-Frye prescription at T_{fo} = 156.5 MeV (with Pythia8 decay list)

- Caveat: no spatial diffusion included yet!

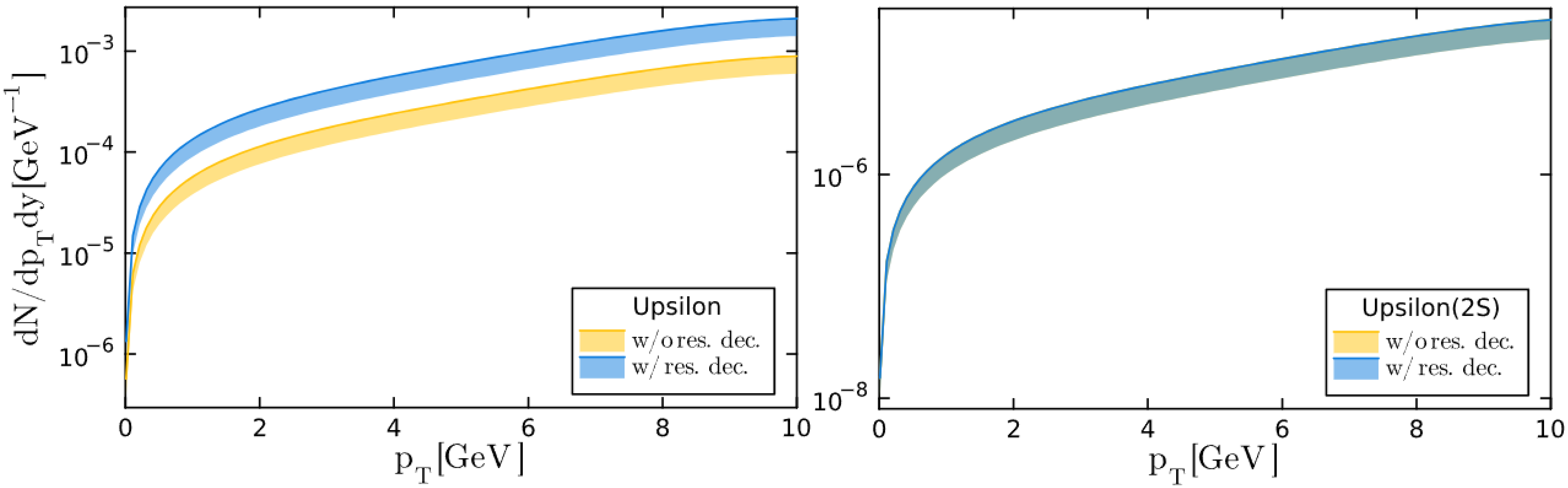


In the charm baryon sector:

❑ Deviation of 2.4σ for Λ_c^+:
- missing higher resonance states?
He, Rapp, PLB 795, 117 (2019)
Andronic *et al.* JHEP 07, 035 (2021)
- coalescence mechanisms?
Plumari *et al*. Eur. Phys. J. C 78, 348 (2018)
Beraudo *et al*., Eur. Phys. J. C 82, 607 (2022)

Deviation is expected also in beauty baryon sector!
Presence of yet **unknown resonances** would enhance the integrated yield.

Hidden Beauty mesons



Equation of motion for the HQ diffusion current

We derive hydrodynamic equations of motion from kinetic theory (Boltzmann) in a **Fokker-Planck approximation**

$$p^\mu \partial_\mu f(\mathbf{p},\mathbf{x},\mathrm{t}) = \frac{\partial}{\partial p^i}\left[A(p)p^i f(\mathbf{p},\mathbf{x},\mathrm{t}) - g^{ij}\frac{\partial}{\partial p^j} D(p) f(\mathbf{p},\mathbf{x},\mathrm{t})\right]$$

By integrating the first moment of the equation

$$\int dP p^\nu p^\mu \partial_\mu f(\mathbf{p},\mathbf{x},\mathrm{t}) = \int dP p^\nu \frac{\partial}{\partial p^i}\left[A(p)p^i f(\mathbf{p},\mathbf{x},\mathrm{t}) - g^{ij}\frac{\partial}{\partial p^j} D(p) f(\mathbf{p},\mathbf{x},\mathrm{t})\right]$$

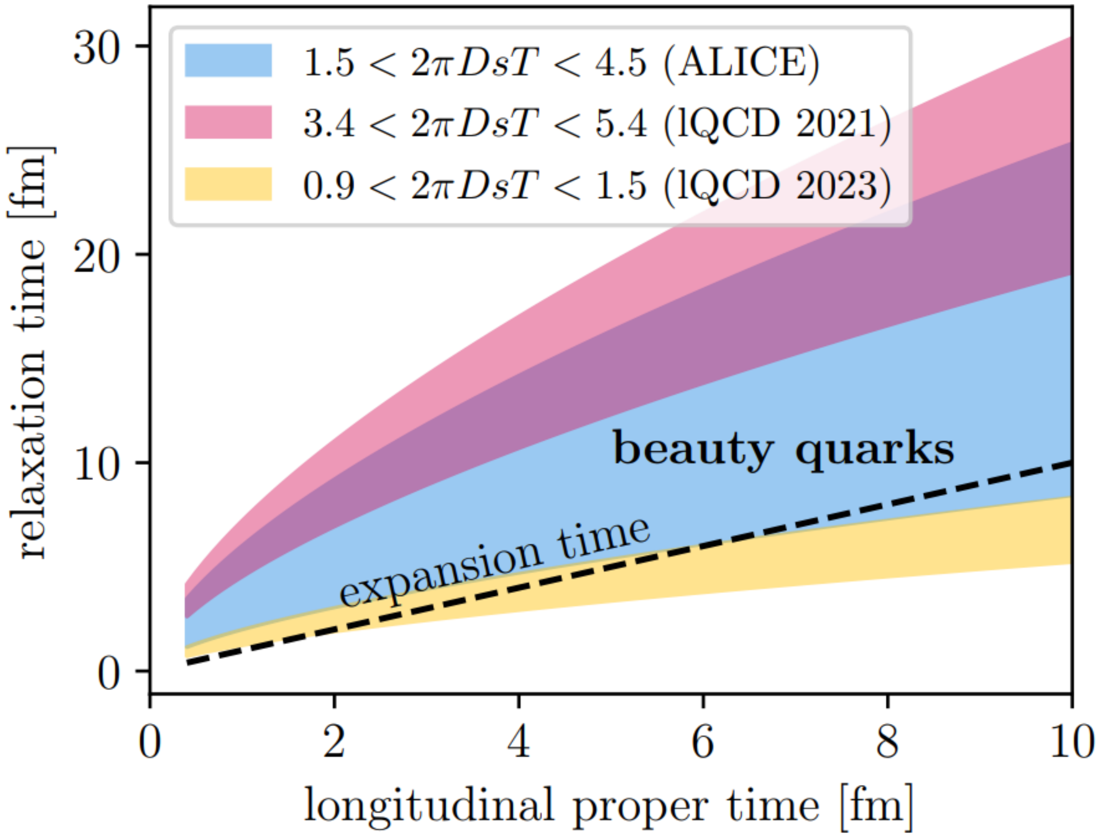
We obtain a relaxation-type equation for the diffusion current

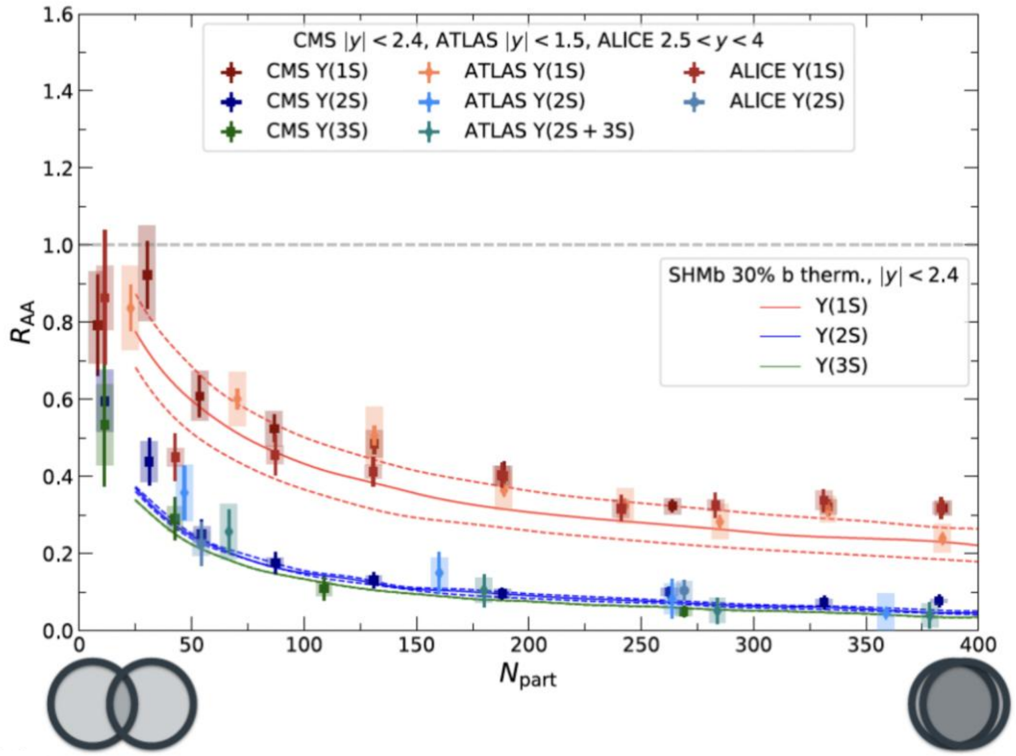
$$\tau_n \partial_t \nu^i + \nu^i = \kappa_n \nabla^i \left(\frac{\mu}{T}\right)$$

$$\boxed{\begin{aligned} \tau_n &= \frac{D_s I_{31}}{T P_o} \\ \kappa_n &= \frac{T^2}{D} n = D_s n \end{aligned}}$$

What about bottom quarks?

- Need of precise measurements for spectra and flow coefficients to study thermalization of bottom quarks in the QGP ⟶ Run3 at the LHC: new data for pp and Pb-Pb collisions
- Υ described by SHM if 30% of bottom quarks assumed to thermalize ⟶ **partial thermalization?**
- Presence of currently unknown open bottom states will lead to a reduction of the bottomonia yields





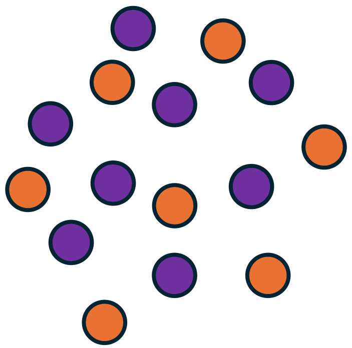
Capellino *et al.* PRD 106 (2022) 034021
Andronic *et al.* QM 2022

To thermalize or not to thermalize?

Chemical equilibrium
= unique local chemical potential

Kinetic equilibrium
= unique local temperature

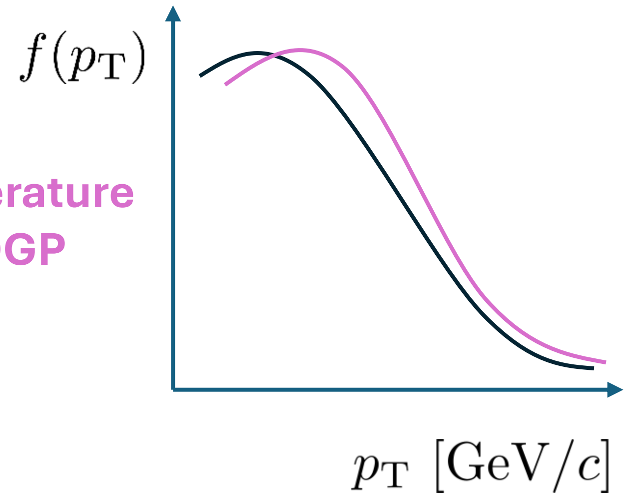
HQ: out of chemical equilibrium



$\mu \neq 0$

Fugacity factor

HQ: close to kinetic equilibrium



Same temperature T of the QGP

$$f \sim \underbrace{e^{-E/T}}_{\text{Thermal distribution}} \; e^{\mu/T} \; + \delta f$$

Momentum distributions:
Ratio plot

- ❑ **Cooper-Frye at T_{fo} = 156.5 MeV + resonances**
- ❑ **Fluid dynamics for D mesons up to 4-5 GeV** (up to 40% deviation)